Online Coverage of a Mobile Robot

2 Online Algorithms Competitively Analysed

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Teaser

Figure: Not very efficient, but cute.

Outline

Introduction

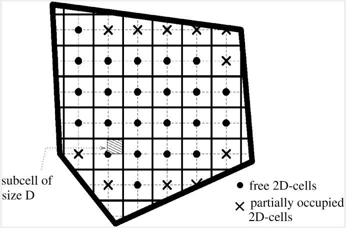


Figure: The work-area grid

- When online: robot has no apriori knowledge about environment
- Let D be the tool size
- Subdivide the work-area into a grid of cells of size D
- Let n be the number of free cells
- Obstacles are allowed

Definitions

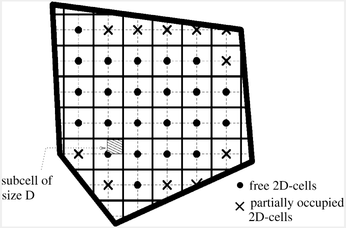


Figure: The work-area grid

- Boundary cells share at least one point with grid boundary
- Let m be the number boundary cells
- Let $l_{\mathcal{A}}$ be the length of the path, generated by Algorithm $\mathcal{A}$
- $l_{\mathcal{A}}$ defines the cost for $\mathcal{A}$

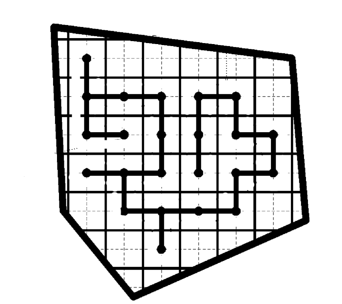


Figure: A trivial solution: DFS

- l_{DFS} is always $2nD$
- STC Algorithms are upper bounded by $(n+m)D$
- Both have comp. ratio of $2-\epsilon$
- In practice $m \ll n$

Related Work

- 1994 Paper by Kalyanasundaram and Pruhs: Constructing competitive tours from local information
  - 16-competitive algorithm: ShortCut
- 2002 On the Competitive Complexity of Navigation Tasks
  - 2-competitive algorithm: URC

Spiral STC

Idea

- Constructing minimal spanning trees online
- Grid of coarser 2D cells
- Traversal of spanning tree edges
- Internal covering of 2D cells
- Spiral-like patterns

Recursive Function $STC1(w, x)$:

1. Mark the current cell x as an old cell.
2. While x has a new obstacle-free neighboring cell:
   2.1 Scan for first **new neighbor of** x **in ccw order**, start with parent cell w. Call this y.
   2.2 Construct spanning-tree edge from x to y.
   2.3 Move to subcell of y **following right-side of spanning tree edges.**
   2.4 Execute STC1(x, y).
3. If $x \neq S$, move back from x to a subcell of w **along right-side of spanning tree edges.**

Visualization

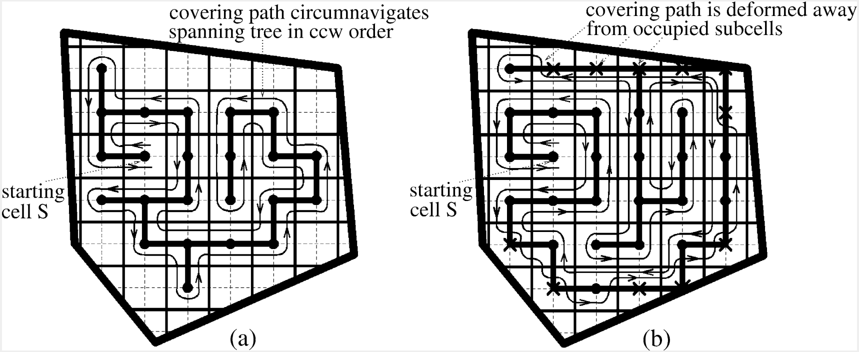


Figure: Visualization of the Spiral STC Algorithms

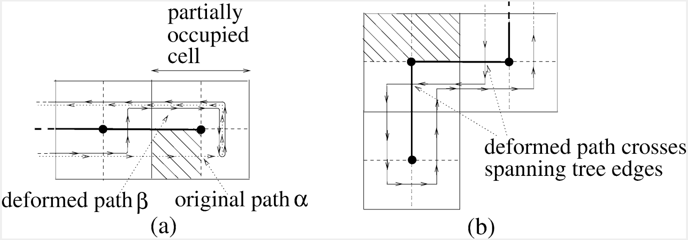


Figure: Deforming the path to circumvent boundarys

- Keep distance of $\frac{D}{2}$ to boundary

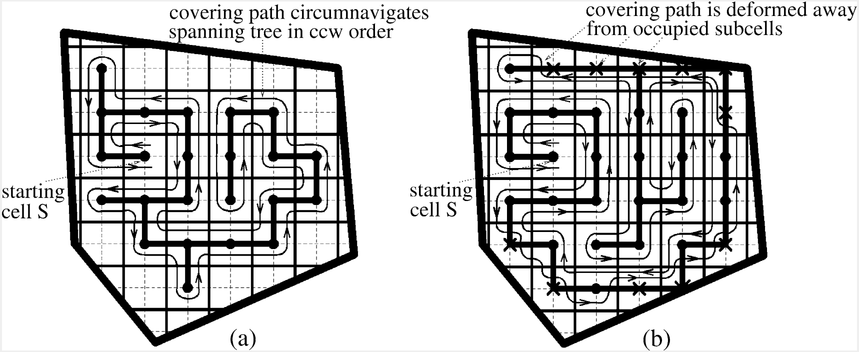


Figure: Visualization of the Spiral STC Algorithms

Scan STC

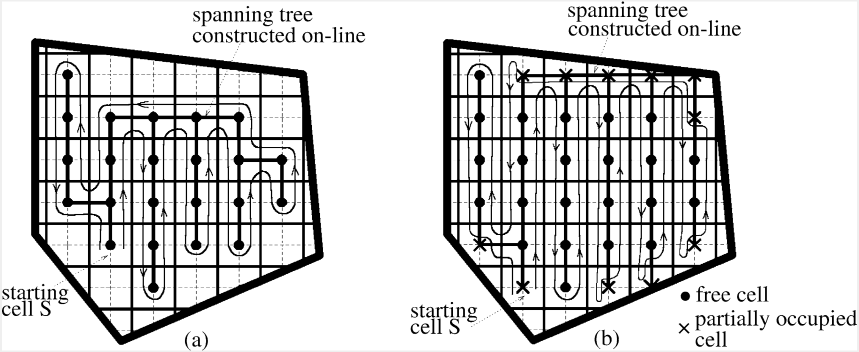


Figure: Visualization of the Scan STC Algorithms

Differences to Spiral STC

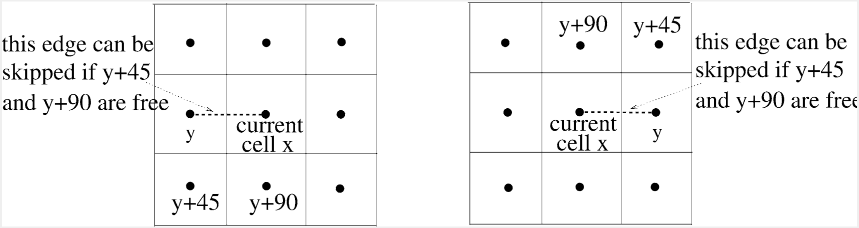


Figure: How Scan STC achieves scan like patterns

Analysis of the Algorithms

Bounds

- $l_{2D-STC} \leq nD$ but ignores partially occupied 2D cells
- $l_{Full-STC} \leq (n+m)D$
- STC Algorithms run in $O(n)$ time and have $O(n)$ space complexity

Theorem 1

Theorem 1

Spiral-STC and Scan-STC cover the work-area grid using a path of total length $l \leq (n + m)D$

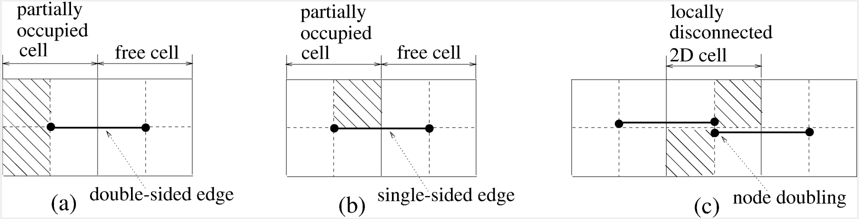
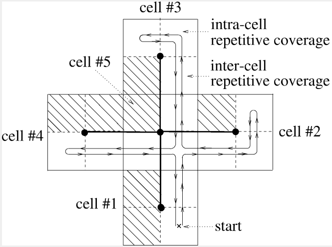


Figure: Single sided edges versus double sided edges.

- Entry Edges, Exit Edges
- Single Sided Edge, Double Sided Edge

- Repetitive coverages
  - When a subcell is covered, then uncovered and later covered again
  - Inter-cell repetitive coverage: Between two 2D cells
  - Intra-cell repetitive coverage: Within a 2D cell



cell	inter-cell repetitive coverage	intra-cell repetitive coverage	total repetitive coverage	number of boundary subcells
1	0	1	1	2
2	1	2	3	3
3	1	2	3	3
4	1	1	2	2
5	1	2	3	3
total			12	13

Figure: Repetitive coverage counting of a 2D cell.

Theorem 1 - Preliminaries

- Single sided edge $\implies$ inter-cell repetitive coverage in cell **from which it exits**
- Counting convention: Such rep. coverage is counted for cell **into which it enters**
- No change in total amount of repetitive coverages

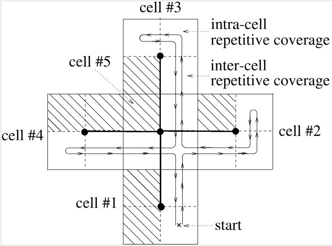


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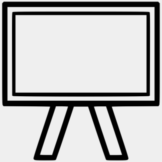
Figure: Repetitive coverage counting of a 2D cell.

Claim 1: Total repetitive coverages bounded by m

- Suffices to show for each 2D cell:
  Total amount repetitive coverages $\leq$ Total amount of boundary cells
- Consider 4 cases
  - 1 free subcell
    - Double sided entry edge
    - Single sided entry edge
  - 2 free subcells
    - Double sided entry edge
    - Single sided entry edge

  ...

Claim 1 - Case 1



- 1 free subcell
- only single sided entry edge
- 1 inter-cell repetitive coverage
- 1 boundary cell

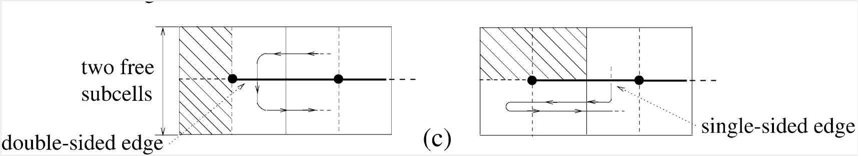


Figure: 2 free subcells, double sided entry edge (left) and single sided entry edge (right).

color boundary subcells

- 0 repetitive coverages
- 2 boundary cells

- 2 repetitive coverages
- 2 boundary cells

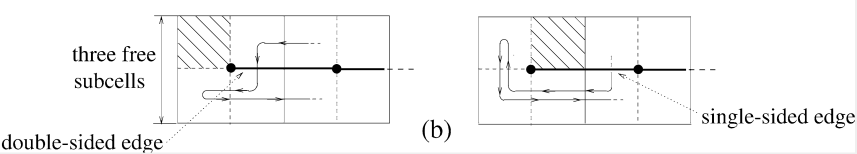


Figure: 3 free subcells, double sided entry edge (left) and single sided entry edge (right).

- 1 repetitive coverages
- 3 boundary cells

- 3 repetitive coverages
- 3 boundary cells

Claim 1 - Case 4

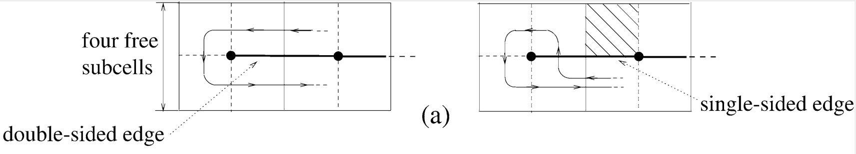


Figure: 4 free subcells, double sided entry edge (left) and single sided entry edge (right).

- No repetitive coverages
- No boundary cells

- 2 repetitive coverages
- 2 boundary cells

Proof.

Claim 1: Total repetitive coverages bounded by m

- Suffices to show for each 2D cell:
  Total amount repetitive coverages $\leq$ Total amount of boundary cells
- Consider 4 cases
  - ...
  - 3 free subcells
    - Double sided entry edge
    - Single sided entry edge
  - 4 free subcells
    - Double sided entry edge
    - Single sided entry edge

$\square$

Theorem 1

Proof.

- Every cell covered at least once: n
- (Claim 1) Number of repetitive coverages is at most m
- $\implies l_{STC} \leq (n+m)D$

$\square$

Optimality and Competitive Ratio

- Area of grid: $A := n \cdot D^2$
- Area of boundary cells: $\delta A := m \cdot D^2$
- $l_{opt} \geq \frac{A}{D} = \frac{nD^2}{D} = n \cdot D$
- $l_{stc} \leq (n + m) \cdot D$

$$\frac{l_{stc}}{l_{opt}} \leq \frac{(n+m) \cdot D}{n \cdot D} = \ldots = 1 + \frac{mD^2}{nD^2} = 1 + \frac{\delta A}{A} = 2 - \epsilon$$

with $\epsilon = (1 - \frac{\delta A}{A})$ and $0 \leq \epsilon \leq 1$ as $A \geq \delta A$ and $A, \delta A \geq 0$

Universal Lower Bound

Universal Lower Bound

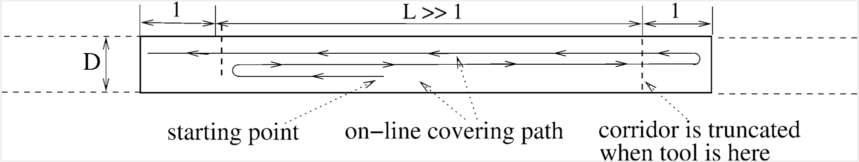


Figure: Trivial corridor example.

- Simple Corridor with width D
- Corridor truncated when L portion is covered
- $l \geq 2L + 3$
- $l_{opt} = L + 2$
- Does not show bounds for tour req.
- Different starting positions

Theorem 2

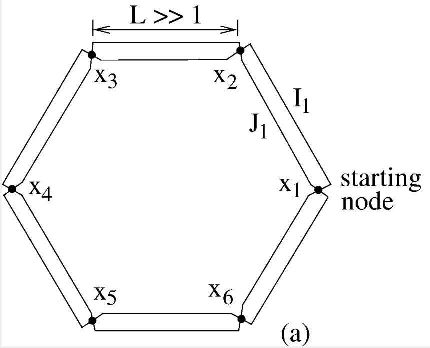


Figure: Double ring environment.

- Lower bound for any online algorithm
- W.l.o.g point robot on graph
- Detection range $1-\delta$
- Let $x_1, ..., x_k$ nodes of ring (ccw. order)
- I_i, J_i paralell edges connecting x_i and x_{i+1}
- L length of I_i and J_i

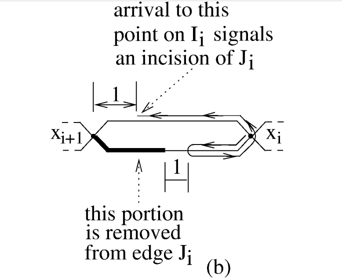


Figure: Incision procedure

- At instant when robot is 1 distance away from x_{i+1} on I_i
- Truncate J_i at distance 1 of already visited part
- $H_i :=$ length of J_i that was covered before incision
- $H := \sum_{i=1}^{k} H_i$
- Incision special case at starting position

- Consider best case scenario (say ccw. order)
- First Loop: $I_1, \ldots I_k$ + $2 \cdot H_1, \ldots 2 \cdot H_k$
- Remaining part of $J_1, \ldots J_k$ covered in second loop
- Cost on $I_1, \ldots I_k$ is: $2kL$
- Cost on $J_1, \ldots J_k$ is: $2H$ in first round
- And $2(H + k)$ in second round

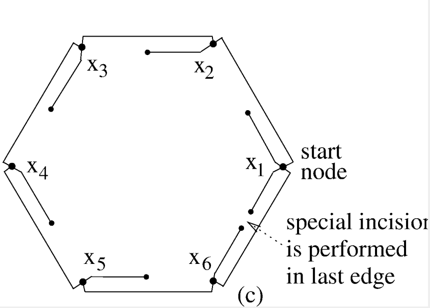


Figure: Incisions in double ring environment.

Theorem 2

Proof.

- Online Lower Bound: $l_{\mathcal{A}} \geq 2kL + 4H + 2k$
- Offline Upper Bound: $l_{opt} \leq kL + 2(H + k)$
- $\implies l_{\mathcal{A}} \geq (2 - \epsilon)l_{opt}$ where $\epsilon = \frac{2k}{kL+2(H+k)} \ll 1$ as $L \gg 1$

$\square$

Conclusion

Conclusion

- Spiral STC and Scan STC are $2 - \epsilon$ competitive online algorithms for the coverage problem
- In practice, where $m \ll n$ the algorithms are close to optimal (comp. ratio $2 - \epsilon$ and ϵ approaches 1)
- Online Algorithm can not get better than $2 - \epsilon$ competitive
- Spiral STC and Scan STC are optimal online algorithms

Thanks!

References

Yoav Gabriely and Elon Rimon.
Competitive on-line coverage of grid environments by a mobile robot.
Computational Geometry, 24(3):197–224, 2003.

Christian Icking, Thomas Kamphans, Rolf Klein, and Elmar Langetepe.
On the competitive complexity of navigation tasks.
In Gregory D. Hager, Henrik Iskov Christensen, Horst Bunke, and Rolf Klein, editors, *Sensor Based Intelligent Robots*, pages 245–258, Berlin, Heidelberg, 2002. Springer Berlin Heidelberg.

Bala Kalyanasundaram and Kirk R. Pruhs.
Constructing competitive tours from local information.
Theoretical Computer Science, 130(1):125–138, 1994.

Optimality and Competitive Ratio - Details

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- $l_{opt} \geq \frac{A}{D} = \frac{nD^2}{D} = n \cdot D$
- $l_{stc} \leq (n + m) \cdot D$

$$\frac{l_{stc}}{l_{opt}} \leq \frac{(n+m) \cdot D}{n \cdot D} = \frac{(n+m)D^2}{nD^2} = \frac{nD^2 + mD^2}{nD^2} = \frac{nD^2 \cdot (1 + \frac{mD^2}{nD^2})}{nD^2}$$

$$= 1 + \frac{mD^2}{nD^2} = 1 + \frac{\delta A}{A} = 2 - \epsilon$$

with $\epsilon = (1 - \frac{\delta A}{A})$ and $0 \leq \epsilon \leq 1$ as $A \geq \delta A$ and $A, \delta A \geq 0$