RAISE - Resource Allocation Intelligent Support Engine
Model Description

Problem Statement

When facing a pandemic disease like currently COVID-19 a major challenge is to fairly allocate scarce resources (Emanuel et al., 2020) within regions (e.g., hospitals, cities, provinces or nations). The objective of the RAISE project is to provide decision support in such cases based on an optimization model.

Input Data

Data needed for the optimization model are:

- Regions i to be covered in the problem
- Resources r to be allocated
- Demand Q_i^r of resource r in region i
- Available supply A_i^r of resource r in region i

Most challenging is the estimation of Q_i^r which can be based on different sources (e.g., expert knowledge, prediction model, population data). A method to achieve the latter is described for example in Koyuncu and Erol (2010). Input data used is:

- Total population P_i of region i
- Risk groups g in which the population can be divided
- Population P_{ig} in region i of risk group g
- Infection rate α_{ig} for region i and risk group g
- Demand d_g^r of resource r for each patient of risk group g

Then Q_i^r can be calculated for each i and r as:

$$Q_i^r = \sum_g P_{ig} * \alpha_{ig} * d_g^r \tag{1}$$

Fairness and Preprocessing

Cost in any form (e.g., monetary cost, time, distance) plays a crucial role in allocating goods because of limited available budget. This is covered in the RAISE model by using a cost matrix with entries c_{ij}^r giving the cost for allocating one unit of resource r from region i to region j. In a our model we focus on time as cost factor.

Nevertheless, another important factor in allocating scarce resources in a pandemic situation is fairness. Different concepts of fair allocation can be used as for example described by Katoh et al. (2013), who focus on covering fairness completely in the opjective function of the optimization model. In the RAISE project we build our idea of covering fairness on suggestions by Arora et al. (2010) and Mondal et al. (2019) and use a preprocessing step to fairly distribute the shortage of scarce resources between regions and risk groups.

To achieve this we introduce two auxiliary variables:

- Corrected demand $\tilde{Q}_i^r$ of resource r in region i
- Corrected supply $\tilde{A}_i^r$ of resource r in region i (only required in case of surplus of resource r)

The total shortage S^r can be easily calculated as $S^r = \sum_i Q_i^r - A_i^r$. To enable the inclusion of ethical considerations, factors δ_{ig}^r for each region r, risk group g and resource r are introduced with $\sum_i \sum_g \delta_{ig}^r = 1 \; \forall \; r$. Then the shortage S_i^r for each resource r and region i is calculated as:

$$S_i^r = \sum_g S^r * \delta_{ig}^r \tag{2}$$

with $\sum_r \sum_i S_i^r = S^r$ and $S_i^r \in \mathbb{Z}$.

Finally, the corrected demand $\tilde{Q}_i^r = Q_i^r - S_i^r$ is caclculated for each i and r. The corrected supply is simply set to $\tilde{A}_i^r = A_i^r$ for each i and r. This holds for $S^r > 0$.
If $S^r < 0$ the corrected supply $\tilde{A}_i^r$ is calculated accordingly and the corrected demand $\tilde{Q}_i^r$ is set just to the given one.

Mathematical Model

Based on the preprocessing step we have to solve a transportation problem with decision variable x_{ij}^r, which reflects the units of resource r allocated from region i to region j.

The objective function is given by:

$$minimize \sum_r \sum_i \sum_j x_{ij}^r * c_{ij}^r \tag{3}$$

subject to

$$\sum_j x_{ij}^r = \tilde{A}_i^r \; \forall \; i, \; r \tag{4}$$

$$\sum_i x_{ij}^r = \tilde{Q}_j^r \; \forall \; j, \; r \tag{5}$$

$$x_{ij}^r \geq 0 \tag{6}$$

$$x_{ij}^r \in \mathbb{Z} \tag{7}$$

Summary

An overview of the steps required to get a fair allocation of scarce resources is displayed in Figure 1.

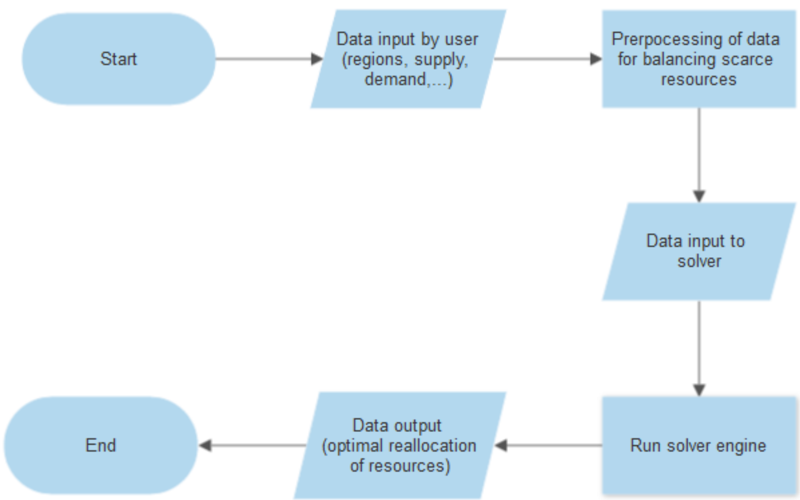

Figure 1: Overview of process steps

References

Arora, H., Raghu, T., and Vinze, A. (2010). Resource allocation for demand surge mitigation during disaster response. *Decision Support Systems*, 50(1):304 – 315.

Emanuel, E. J., Persad, G., Upshur, R., Thome, B., Parker, M., Glickman, A., Zhang, C., Boyle, C., Smith, M., and Phillips, J. P. (2020). Fair allocation of scarce medical resources in the time of covid-19. *New England Journal of Medicine*, pages 1–7.

Katoh, N., Shioura, A., and Ibaraki, T. (2013). *Resource Allocation Problems*, pages 2897–2988. Springer New York, New York, NY.

Koyuncu, M. and Erol, R. (2010). Optimal resource allocation model to mitigate the impact of pandemic influenza: A case study for turkey. *Journal of medical systems*, 34:61–70.

Mondal, T., Boral, N., Bhattacharya, I., Das, J., and Pramanik, P. (2019). Distribution of deficient resources in disaster response situation using particle swarm optimization. *International Journal of Disaster Risk Reduction*, 41:101308.