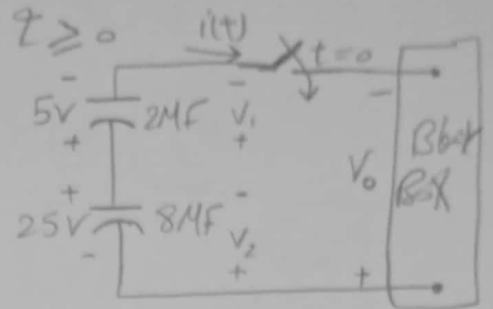


Handwritten Assignment #2

1

1) resulting current is $800 e^{-25t}$ mA @ $t \geq 0$

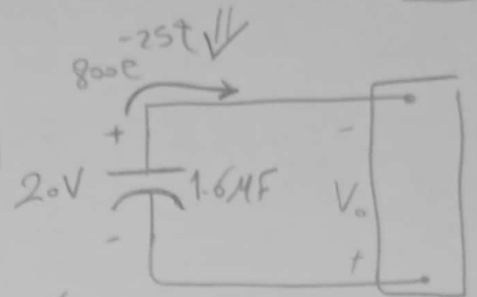


@ equivalent Capacitor:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2 \times 8) \times 10^{-12}}{(2+8) \times 10^{-6}} = 1.6 \mu F$$

$$i = C \frac{dV}{dt} \rightarrow i dt = C dV \rightarrow \int dV = \frac{1}{C} \int i dt$$

$$\therefore V_o = \frac{1}{C} \int i(t) dt + V_o(t_0) \quad V_o(t_0) = -20$$



$$V_o = \frac{1}{1.6 \times 10^{-6}} \int_0^t (800 e^{-25\tau}) d\tau - 20$$

$$= \frac{800}{1.6(-25)} e^{-25\tau} \Big|_0^t - 20 = (-20 e^{-25t} - (-20)) - 20$$

$$\therefore V_o(t) = -20 e^{-25t} \text{ V} @ t \geq 0 \quad \# (1)$$

b) $V_1(t) = ?$

$$V_1(t) = \frac{1}{C} \int i(\tau) d\tau + V_o(t_0) \quad V_o(t_0) = 5V$$

$$= \frac{1}{2 \times 10^{-6}} \int_0^t (800 e^{-25\tau}) d\tau + 5 = \frac{800}{2 \times -25} e^{-25\tau} \Big|_0^t + 5 = (-16 e^{-25t} - (-16))$$

$$= 21 - 16 e^{-25t} \text{ V} @ t \geq 0 \quad \# (2)$$

c) $V_2(t) = ?$

Similar as b) $\rightarrow V_2(t) = \frac{800}{8 \times -25} e^{-25\tau} \Big|_0^t - 25 = (-4 e^{-25t} - (-4)) - 25$

$$= -4 e^{-25t} - 21 \text{ V} @ t \geq 0 \quad \# (3)$$

d) energy to black box = ? $0 \leq t \leq \infty$

$$P = V_i = -(-20 e^{-25t})(800 \times 10^{-6} e^{-25t}) = 16 \times 10^{-3} e^{-50t} \text{ watt}$$

$$W = \int_0^{\infty} P dt = \int_0^{\infty} 16 \times 10^{-3} e^{-50t} dt = \frac{16 \times 10^{-3}}{-50} e^{-50t} \Big|_0^{\infty} = -3.2 \times 10^{-4} (0 - 1)$$

$$= 320 \mu J \quad \# (4)$$

© Energy initially Stored in Series Capacitors = ?

2

$$W_{\text{initial}} = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \left[(2 \times 10^{-6}) (5)^2 + (8 \times 10^{-6}) (25)^2 \right] = 2525 \mu\text{J}$$

$$W_{\text{initial}} = 2525 \mu\text{J} \quad \# \text{ (5)}$$

$$\textcircled{f} W_{\text{trapped}} = (-W_{\text{delivered}}) + W_{\text{initial}} = 2525 - 320 = 2205 \mu\text{J}$$

$$W_{\text{trapped}} = 2205 \mu\text{J} \quad \# \text{ (6)}$$

$$\textcircled{g} W_{\text{trapped}} = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \left[(2 \times 10^{-6}) (21)^2 + (8 \times 10^{-6}) (-21)^2 \right] = 2205 \mu\text{J}$$

$$W_{\text{trapped}} = 2205 \mu\text{J} \quad \# \text{ (7)}$$

→ which agree with answers obtained in (f)

2 Switch is @ a for a long time & @ $t=0$ it moves to b.

@ $i(t)=?$ @ $t \geq 0$

$$\therefore i_L(0^-) = \frac{150}{180} (12) = 10A$$

(using current divider)

$$\therefore i_L(0^-) = 10A$$

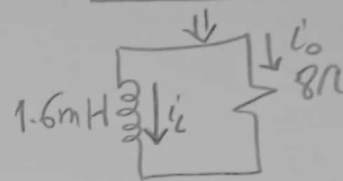
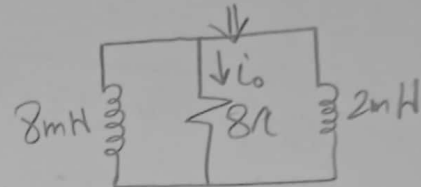
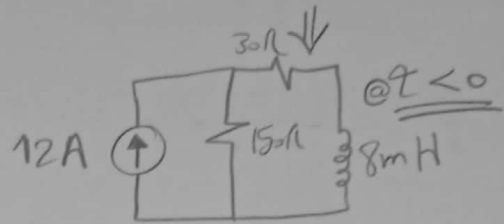
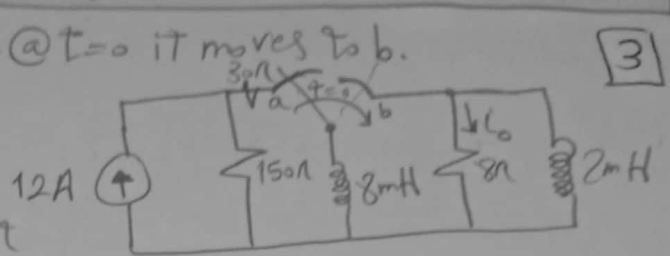
⇒ After switch moves to b

$$L_{eq} = \frac{8 \times 2 \times 10^{-6}}{(8+2) \times 10^{-3}} = 1.6mH$$

$$T = \frac{L}{R} = \frac{1.6 \times 10^{-3}}{8} = 200 \times 10^{-6}$$

$$i_L(t) = I_0 e^{-t/T} = 10 e^{-\frac{t}{200 \times 10^{-6}}} = 10 e^{-5000t} A$$

$$\therefore i_L(t) = 10 e^{-5000t} A \quad \# ①$$



b) energy = $\frac{1}{2} L i^2 = \frac{1}{2} (1.6 \times 10^{-3}) (10)^2 = 80mJ \quad \# ②$

c) $\frac{95}{100} \times \text{energy} = \frac{95}{100} \times 80 \times 10^{-3} = \frac{19}{250}$, $w = \int_0^{t_0} i_L^2(t) \cdot R dt$

$$\frac{19}{250} = \int_0^{t_0} 8 (10 e^{-5000t})^2 dt$$

$$R.H.S = 800 \int_0^{t_0} e^{-10000t} dt = \frac{800}{-10000} e^{-10000t} \Big|_0^{t_0} = -8 \times 10^{-2} (e^{-10000t_0} - 1)$$

$$\therefore \frac{19}{250} = 76 \times 10^{-3} = -8 \times 10^{-2} (-1 + e^{-10000t_0})$$

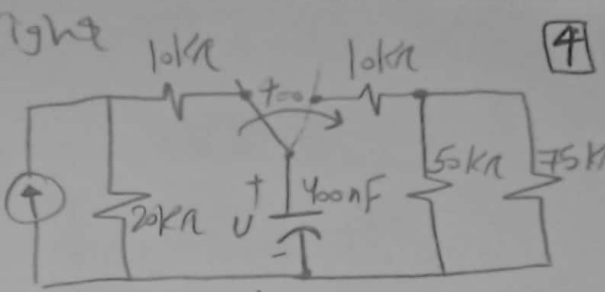
$$\therefore e^{-10000t_0} = 0.05 \rightarrow t_0 = 299.57 \mu s$$

$$\frac{t_0}{T} = \frac{299.57 \times 10^{-6}}{200 \times 10^{-6}} = 1.498 \quad \text{So } t_0 \approx 1.498T \quad \# ③$$

③ Switch on left for $t < 0$ & $t = 0$ switch to right

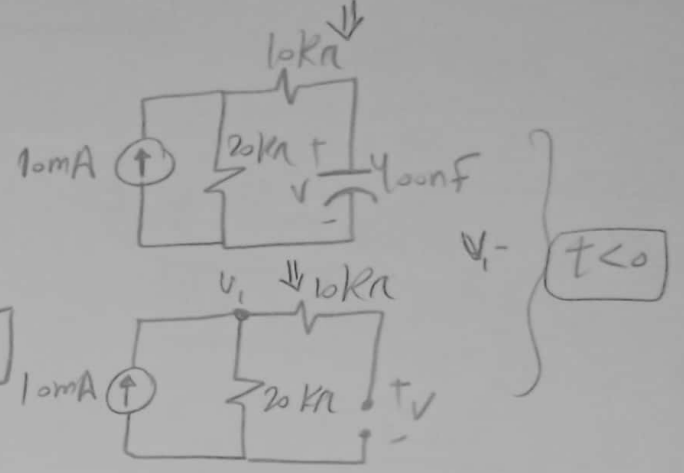
① $V_{\text{initial}} = ?$

$V(0^-) = (10 \times 10^{-3}) (20 \times 10^3) = 200 \text{ V}$ #① 10 mA



② $W_{\text{initial}} = \frac{1}{2} C V^2 = \frac{1}{2} (400 \times 10^{-9}) (200)^2$

$W_{\text{initial}} = 8 \text{ mJ}$ #②

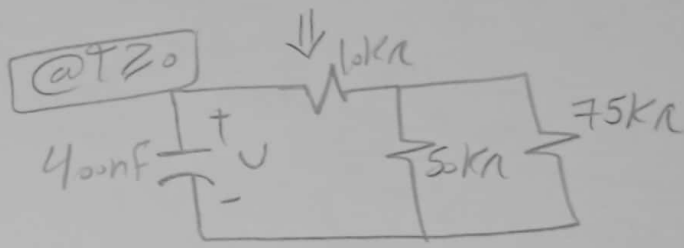


③ $\tau = CR = (400 \times 10^{-9}) (400 \times 10^3) = 0.16$

$R_{5,75} = \frac{(50 \times 75) 10^6}{(50 + 75) 10^3} = 30 \text{ k}\Omega$

$R_{\text{eq}} = 30 + 10 = 40 \text{ k}\Omega$

$\therefore \tau = 16 \text{ ms}$ #③



④ $V(t) = ?$ for $t \geq 0$
 $V = V_0 e^{-t/\tau} = 200 e^{-\frac{t}{0.16}} = 200 e^{-6.25t} \text{ V}$ for $t \geq 0$ #④

☺
 Kerim Mahmood