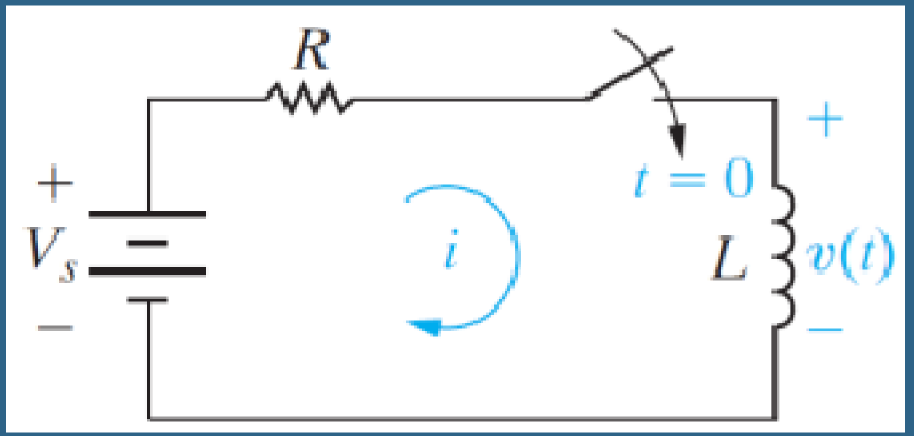
The Step Response (Transient) of an RL Circuit

The response of a circuit to the ~~sudden application~~ of a constant voltage or ~~current source.~~



$$V_s = Ri + L\frac{di}{dt},$$

$$V_s - Ri = L\frac{di}{dt}$$

$$\left(\frac{V_s}{L} - \frac{Ri}{L}\right)dt = di$$

$$\left(i - \frac{V_s}{R}\right)\left(\frac{-R}{L}\right)dt = di$$

$$\frac{di}{i - \frac{V_s}{R}} = \frac{-R}{L}dt$$

Let $z = i - \frac{V_s}{R}$ $\quad dz = di$

$$\frac{dz}{z} = -\frac{R}{L}dt$$

$$\ln(z) = \frac{-R}{L}dt$$

$$\ln\left(i - \frac{V_s}{R}\right)\Big|_{I_0}^{i} = \frac{-R}{L}t\Big|_0^t$$

$$\ln\left(i - \frac{V_s}{R}\right) - \ln\left(I_0 - \frac{V_s}{R}\right) = \frac{-R}{L}t$$

I_f $\quad I_f$ $\quad \tau = \frac{L}{R}$

$$\ln\left(\frac{i - I_f}{I_0 - I_f}\right) = \text{''}$$

$$\frac{i - I_f}{I_0 - I_f} = e^{(-R/L)t}$$

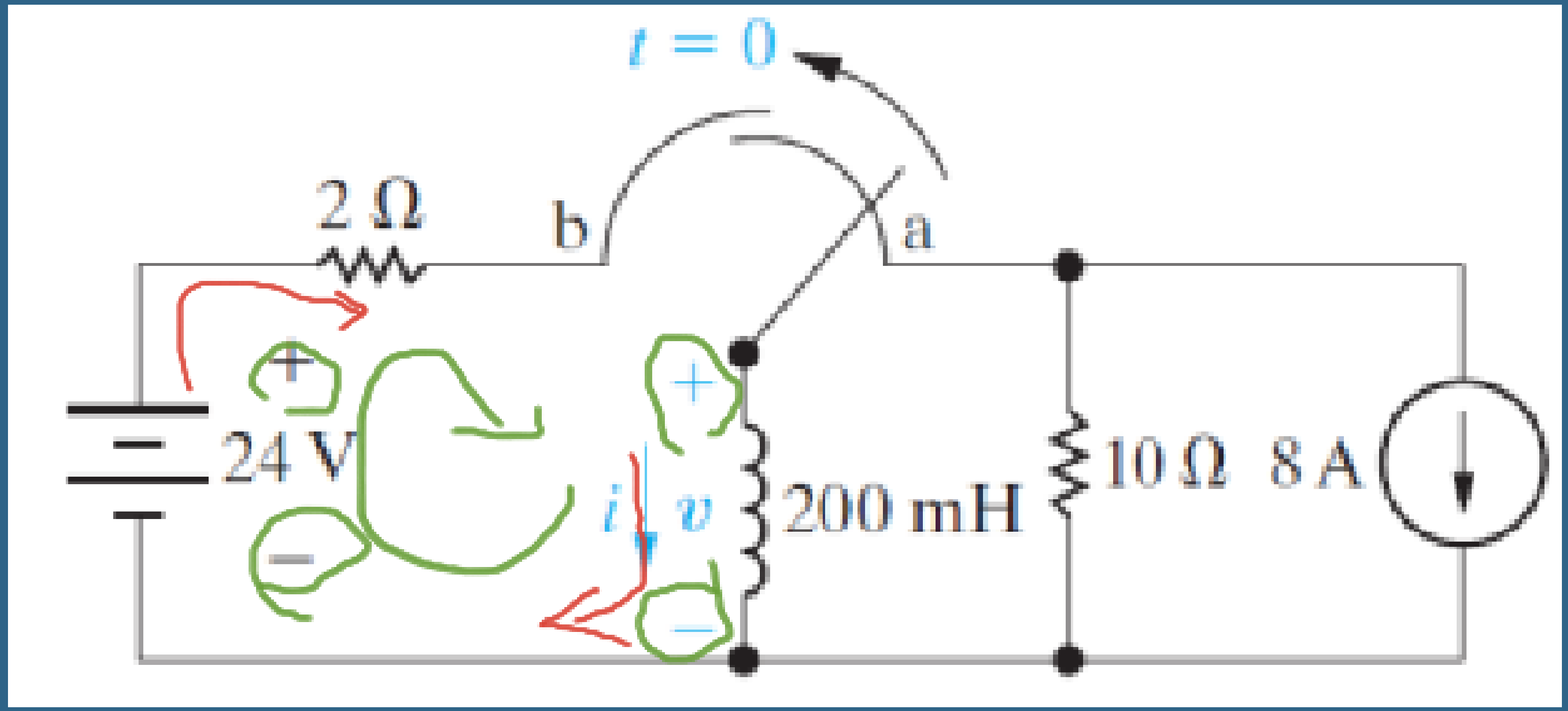
$$i = I_f + (I_0 - I_f)e^{\left(-\frac{R}{L}\right)t}$$

$$i(t) = I_f + (I_0 - I_f)e^{-t/\tau}.$$

The Step Response (Transient) of an RL Circuit

Steps:

1- Determine Io (usually @ t < 0)
2- Calculate tau ==> Req
3- Calculate I_final (t -> infinity)
4- Write the equation of Inductor current
5- Calculate any required quantities



→ $I_0 = -8A$

→ $\tau = \frac{L}{R} = 0.1 sec$

→ $I_f = \frac{24}{2} = 12A$

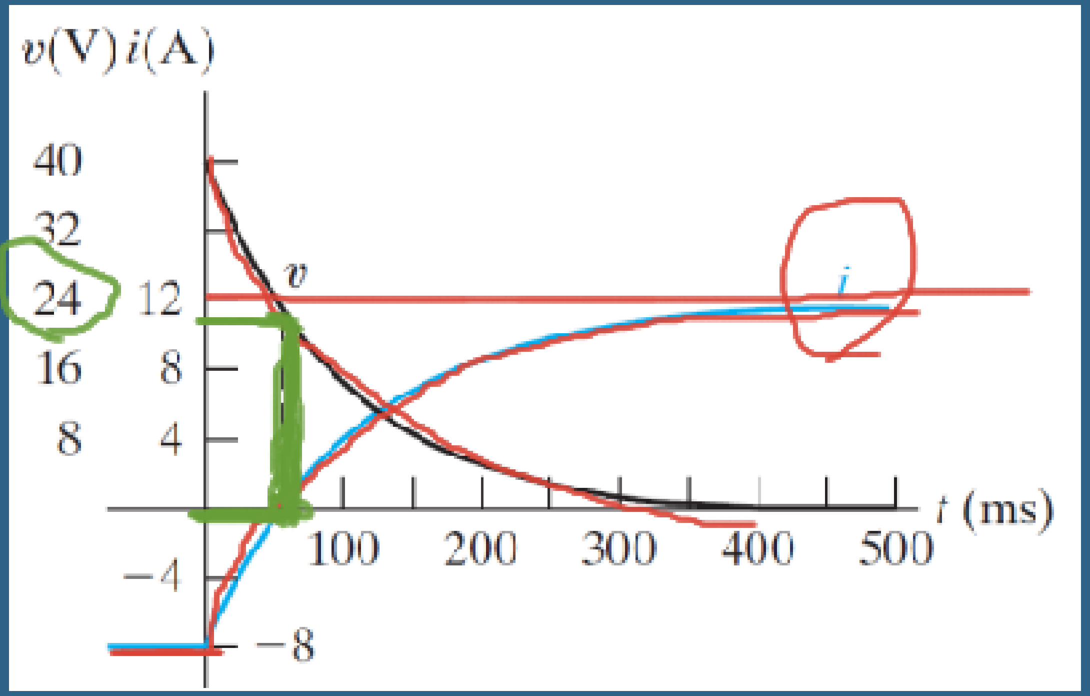
→ $i = I_f + (I_0 - I_f)\, e^{-t/\tau} = 12 - 20\, e^{-10t}$

→ $v = L\frac{di}{dt} = 40\, e^{-10t}$

The Step Response (Transient) of an RL Circuit

- Yes. In the instant, after the switch has been moved to position b, the inductor current is 8 A counterclockwise around the newly formed closed path.

- This current causes a 16 V drop across the 2 resistors. This voltage drop adds to the 24 V drop across the source, producing a 40 V drop across the inductor.



$$24 = 40e^{-10t}$$

$$t = 51.08\text{ ms}$$

$$24 = 40e^{-10t}$$

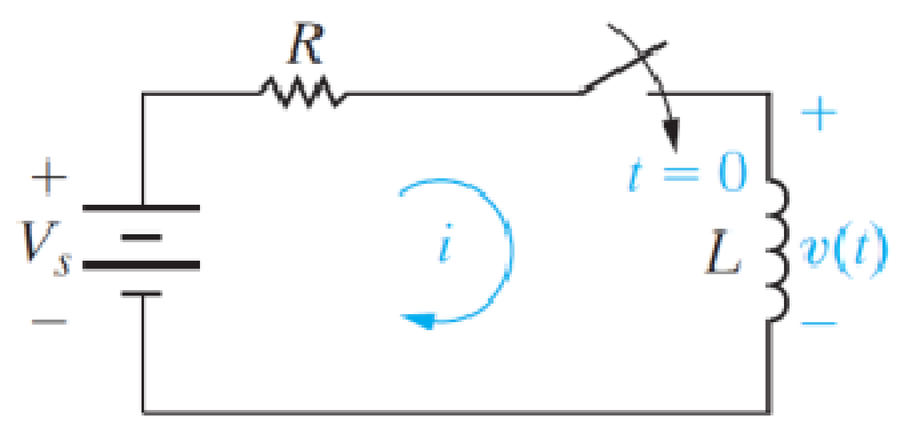
$$t = \frac{1}{10} \ln \frac{40}{24}$$

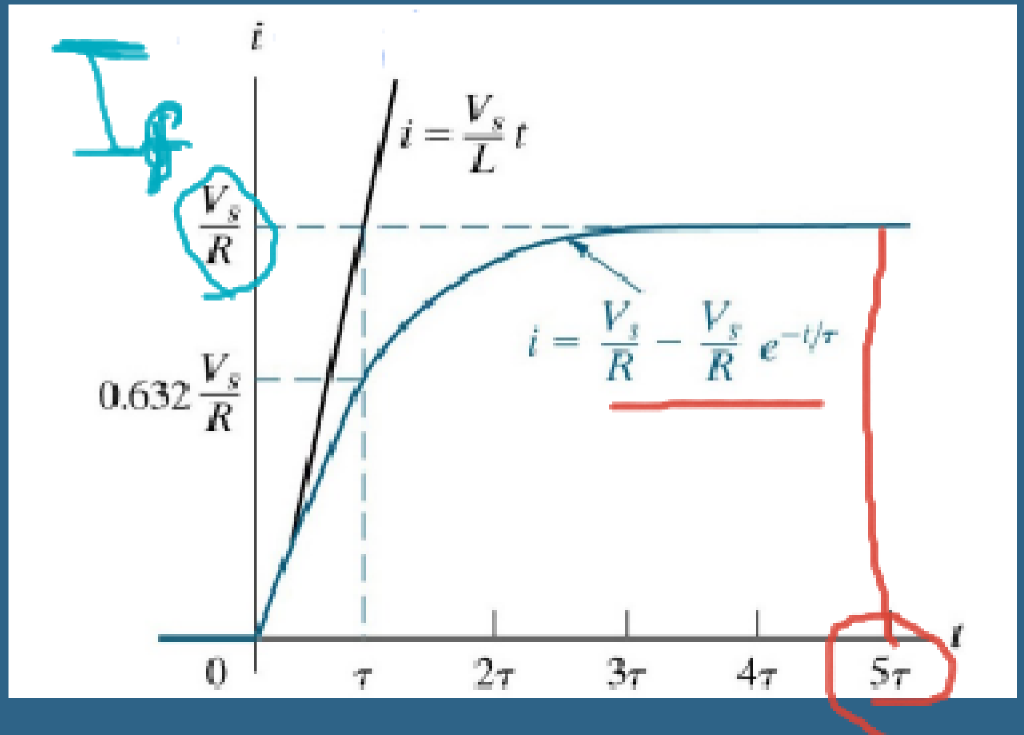
$$= 51.08\text{ ms.}$$

Get V equation:

$$i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-(R/L)t}.$$

$I_0 = 0$





Start with $v = L\frac{di}{dt}$

$$v = \cancel{L}\left(+\frac{V_s}{\cancel{R}}\right)\left(+\frac{\cancel{R}}{\cancel{L}}\right) e^{-R/L\, t}$$

$$= V_s e^{(-R/L)t}$$

$$v = V_s e^{-(R/L)t}$$

- When the ~~initial inductor~~ current is zero

The Step Response of an RC Circuit

Steps:

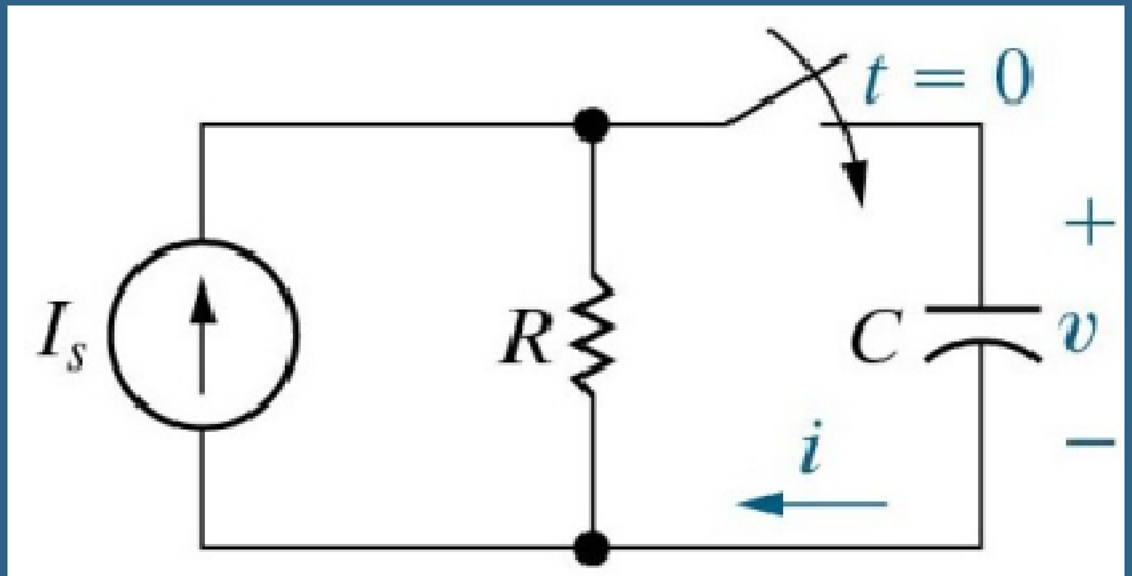
1- Determine Vo (usually @ t < 0)

2- Calculate tau ==> Req

3- Calculate V_final (t -> infinity)

4- Write the equation of Capacitor Voltage

5- Calculate any required quantities



$$C\frac{dv}{dt} + \frac{v}{R} = I_s.$$

$$v = I_s R + (V_0 - I_s R)e^{-t/RC}, \quad t \geq 0.$$

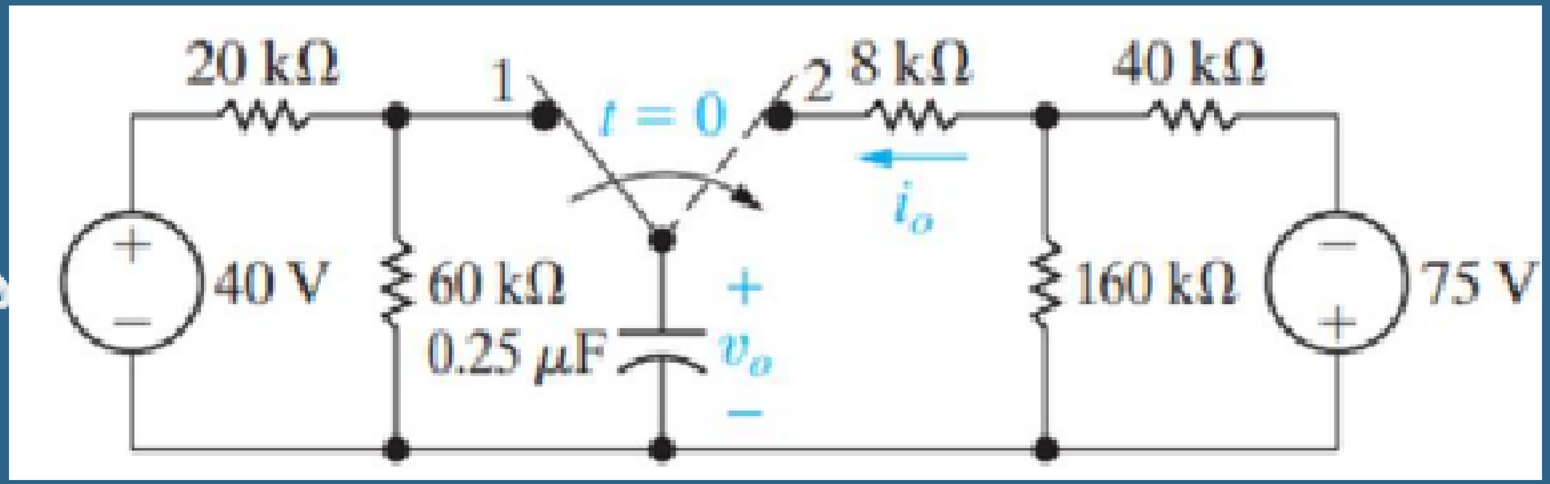
$$v(t) = V_f + (V_0 - V_f)e^{-t/\tau}$$

Solution

initial

$\rightarrow V_0 = \frac{60}{60+20}(40)$

$= 30V$



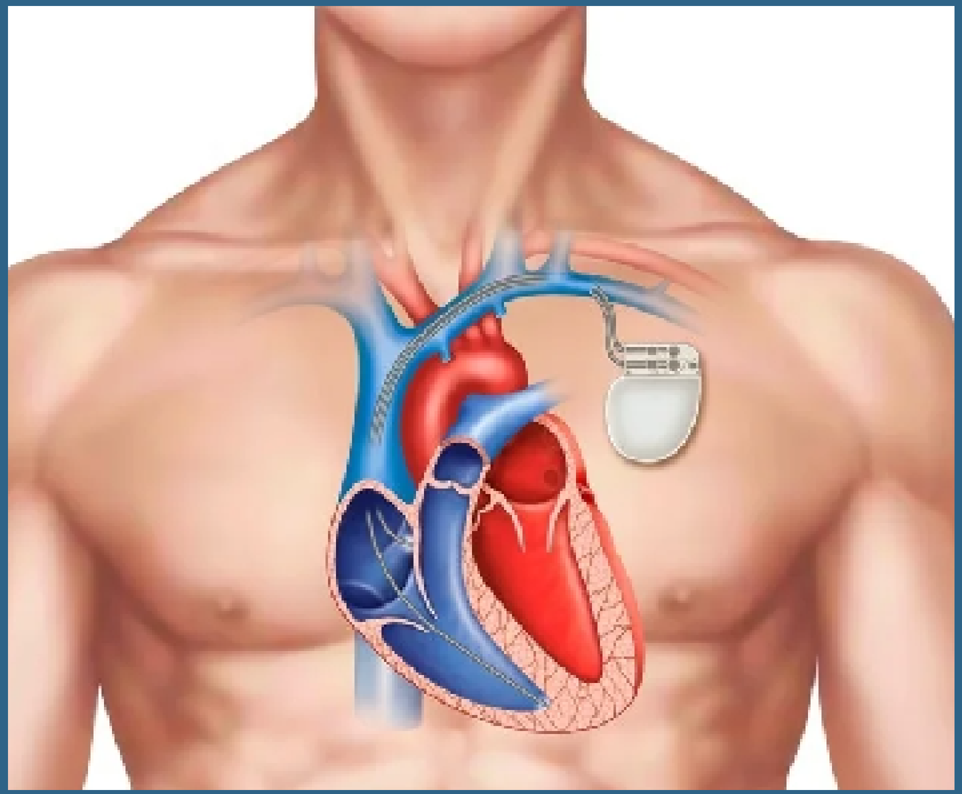
$\rightarrow \tau = RC = 10ms$

$\rightarrow V_f = \frac{160}{160+40}(75) = (-60V)$

$v(t) = V_f + (V_i - V_f)e^{-t/\tau} = -60 + 90e^{-100t}$

$i = C\frac{dv}{dt} = \checkmark$

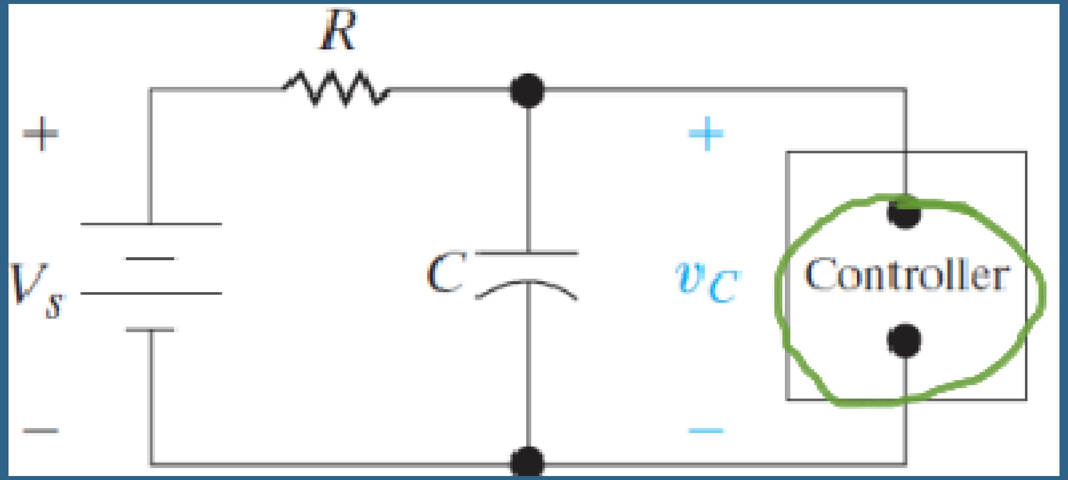
Real App ( ) Artificial pacemaker



- The RC circuit can be used in an artificial pacemaker to establish a normal heart rhythm by generating periodic electrical impulses.

- The "controller" behaves as an o.c until the voltage drop across the capacitor reaches a preset limit.

- Once that limit is reached, the capacitor discharges its stored energy in the form of an electrical impulse to the heart, starts to recharge, and then the process repeats.



$V_0 = v_C(0) = 0;$

$V_f = v_C(\infty) = V_s;$ and $\tau = RC.$

$$v_o = V_f + (V_0 - V_f)e^{-t/\tau}$$

$$v_C(t) = V_s(1 - e^{-t/RC}).$$

To find H = (1/tc)* 60 , we should estimate tc

0.75 Vs = $V_s(1 - e^{-t/RC}).$

$$H = \frac{60}{-RC \ln 0.25} \quad \text{[beats per minute]}$$