Delta-to-Y Equivalent Circuit (simplifying techniques)

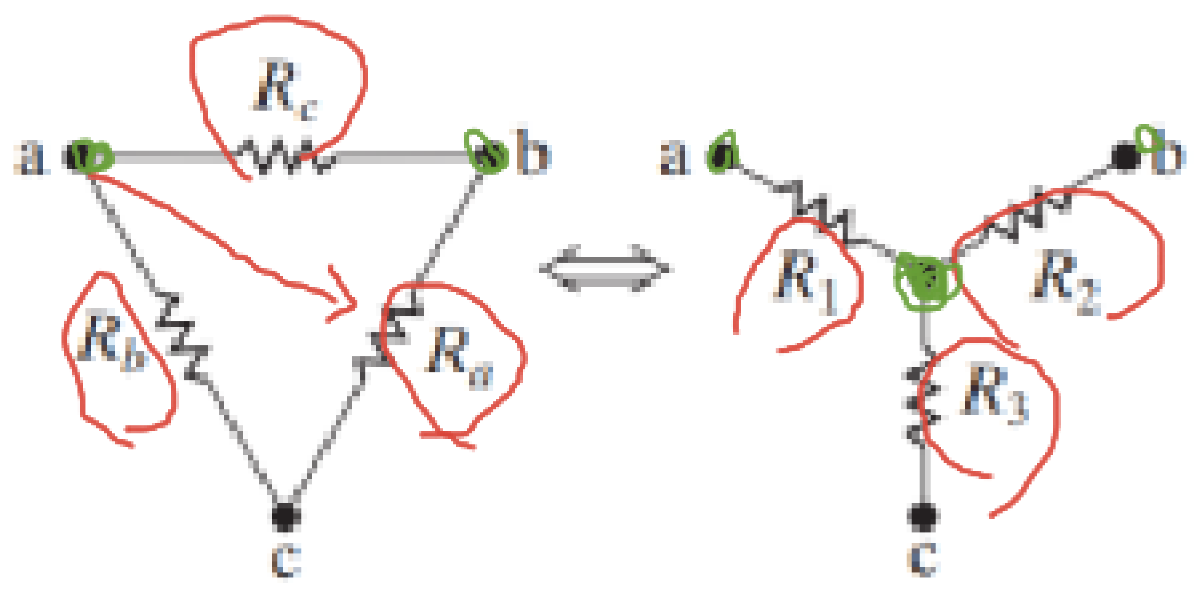
proof:

Let's assume;

$$R_{ab} = (R_a + R_b) \| R_c = R_1 + R_2 \quad ①$$

$$R_{bc} = (R_b + R_c) \| R_a = R_2 + R_3 \quad ②$$

$$R_{ca} = (R_c + R_a) \| R_b = R_1 + R_3 \quad ③$$



$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c},$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c},$$

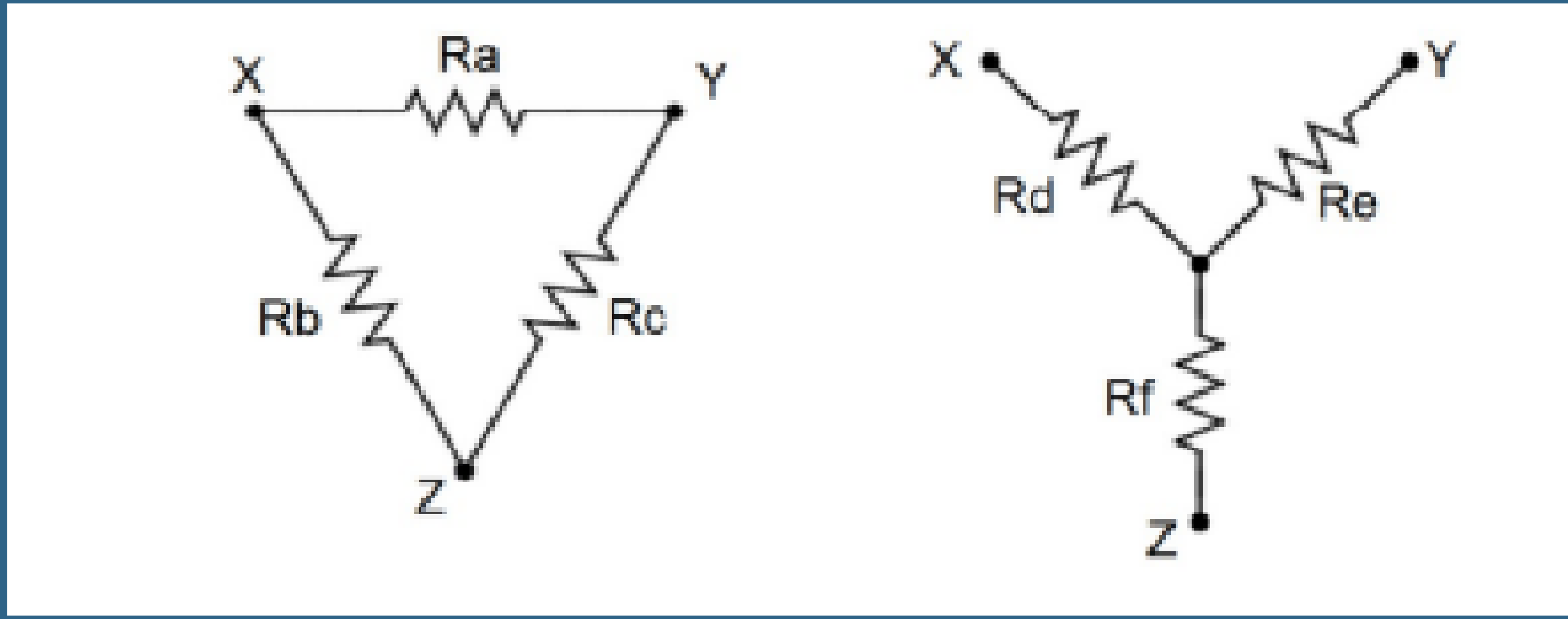
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}.$$

Proof of Delta to Y

$$R_{ab} = (R_a+R_b)\,\|\,R_c = R_1+R_2 \quad ①$$

$$R_{bc} = \boxed{(R_b+R_c)\,\|\,R_a} = R_2+R_3 \quad ②$$

$$R_{ca} = (R_c+R_a)\,\|\,R_b = R_1+R_3 \quad ③$$



$R_{XY} = R_d + R_e = R_a \,\|\, (R_b+R_c)$ eq. 1
$R_{XZ} = R_d + R_f = R_b \,\|\, (R_a+R_c)$ eq. 2
$R_{ZY} = R_e + R_f = R_c \,\|\, (R_b+R_a)$ eq. 3

$R_d = R_aR_b/(R_a+R_b+R_c)$ eq. 4

Similarly,

$R_e = R_aR_c/(R_a+R_b+R_c)$ eq. 5
$R_f = R_bR_c/(R_a+R_b+R_c)$ eq. 6

equ ① − equ ③ $\quad \cancel{R_1}+R_2-\cancel{R_1}-R_3$

$$R_2-R_3 = \underbrace{(R_a+R_b)\,\|\,R_c - (R_c+R_a)\,\|\,R_b}_{} \quad ④$$

④ + ②: $2R_2 = \quad + (R_b+R_c)\,\|\,R_a$

R_{1+5}

$$(R_a + R_b) \parallel R_c \rightarrow \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} + (1)$$

$$(R_c + R_a) \parallel R_b \rightarrow \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} - (2)$$

$$\frac{2R_aR_c}{R_a + R_b + R_c} = 2R_2$$

$$(R_b + R_c) \parallel R_a \rightarrow \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} + (3)$$

Proof of Y to Delta

4, 5

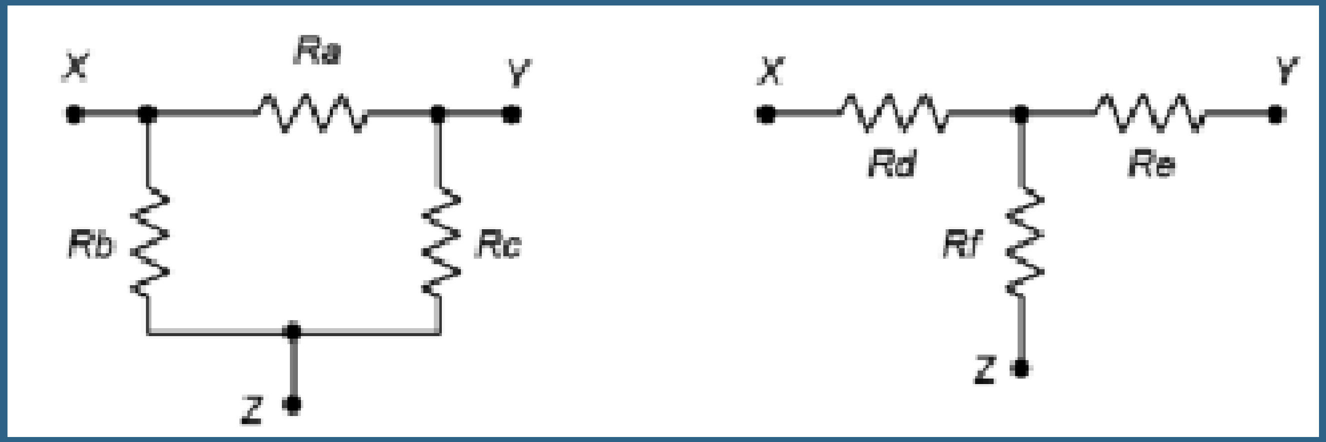
$R_d / R_e = (R_aR_b/(R_a+R_b+R_c)) / (R_aR_c/(R_a+R_b+R_c)) = R_aR_b / R_aR_c = R_b / R_c$

Therefore,

$R_b / R_c = R_d / R_e$

$R_b = R_cR_d / R_e$

$R_a \Rightarrow R_c, R_d, R_e$



$R_d = R_aR_b/(R_a+R_b+R_c)$ eq. 4

Similarly,

$R_e = R_aR_c/(R_a+R_b+R_c)$ eq. 5

$R_f = R_bR_c/(R_a+R_b+R_c)$ eq. 6

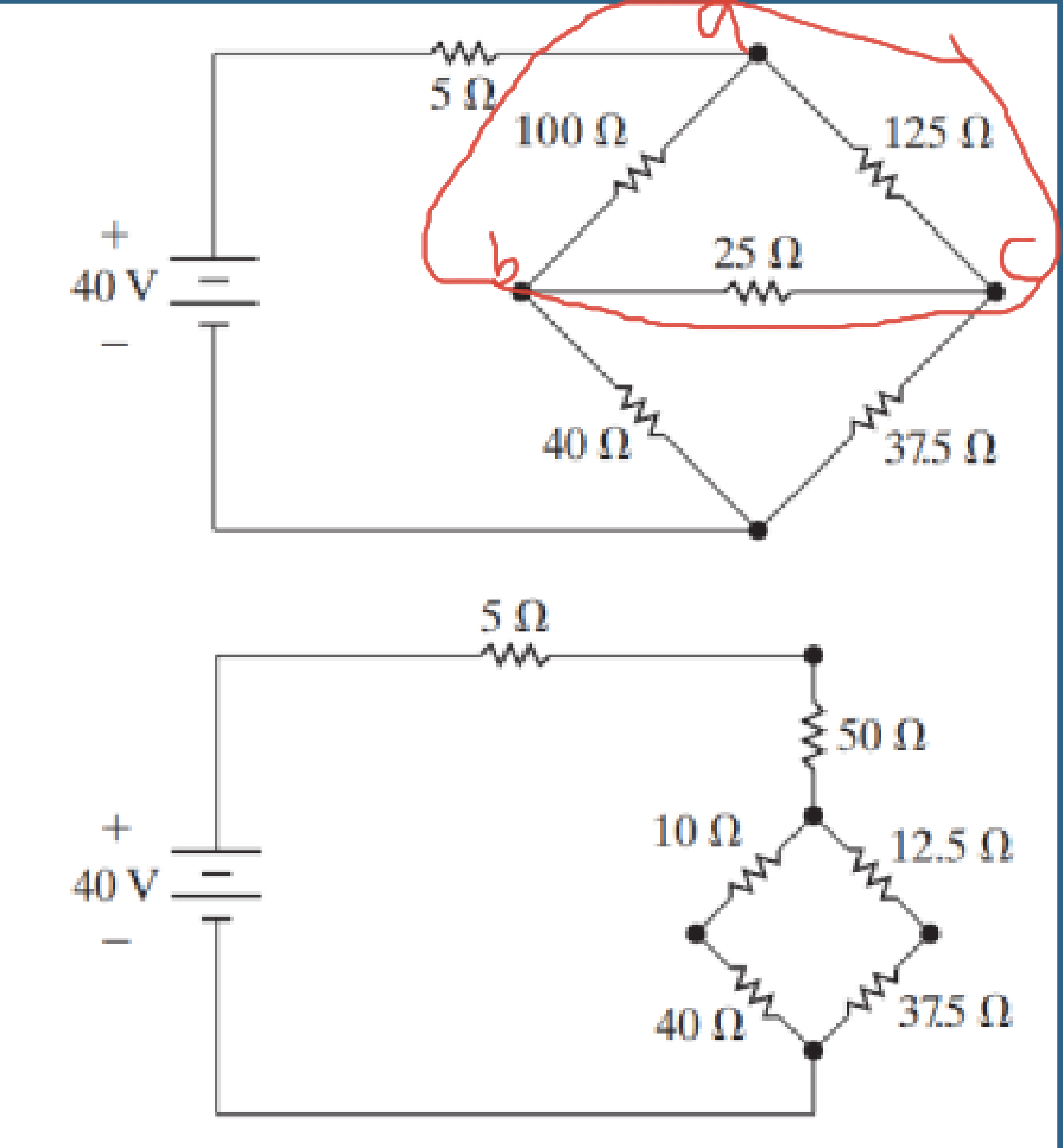
This process can be repeated for eq. 4 and 6 to obtain an expression for R_a. The two expressions for R_a and R_b can then be substituted into eq. 4 to obtain an expression for R_c that utilizes only R_d, R_e and R_f. A similar process is followed for R_a and R_b resulting in:

$$R_a = (R_dR_e + R_eR_f + R_dR_f)/R_f$$
$$R_b = (R_dR_e + R_eR_f + R_dR_f)/R_e$$
$$R_c = (R_dR_e + R_eR_f + R_dR_f)/R_d$$

$$\frac{100 \times 125}{100+25+125}$$

12500

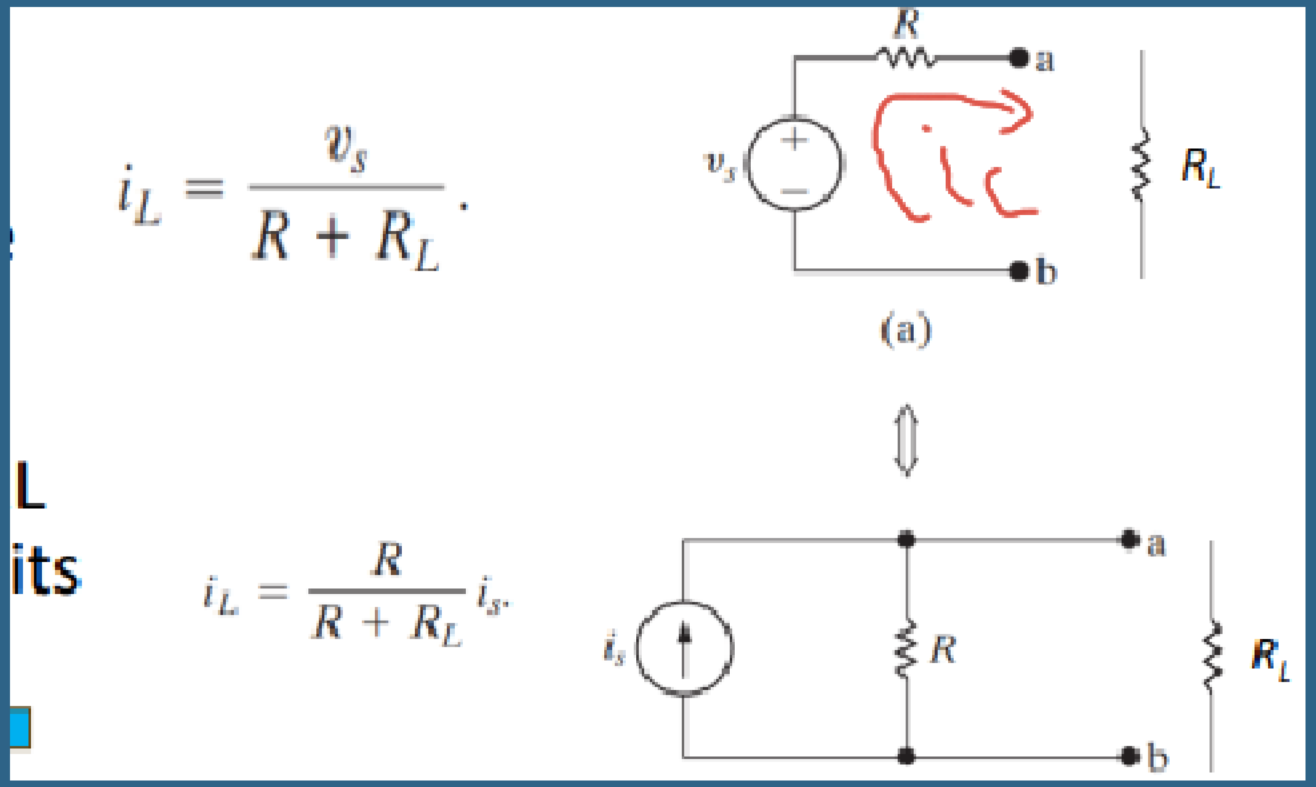
250



Source Transformations (simplifying techniques)

- A source transformation allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor or vice versa.
- We assume that both circuits are loaded with the same load resistance RL.

In order to be equivalent the current thought RL should be the same in both circuits



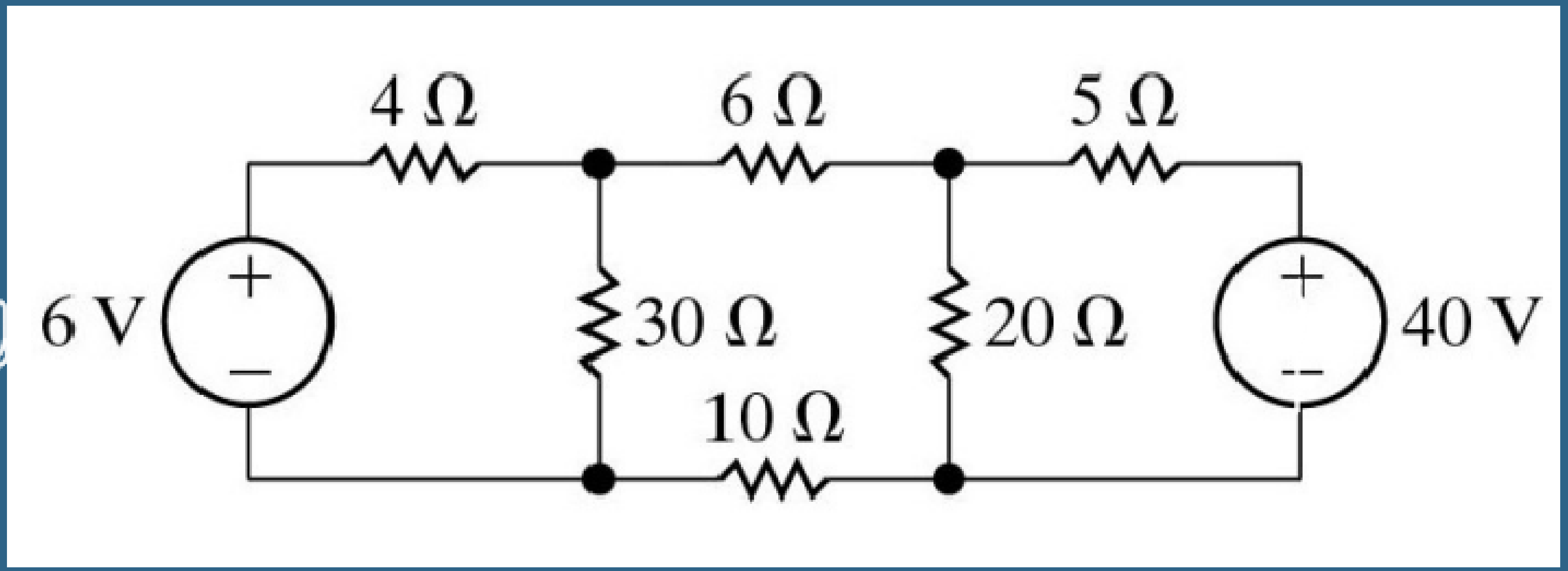
$$i_L = \frac{V_s}{R+R_L}$$

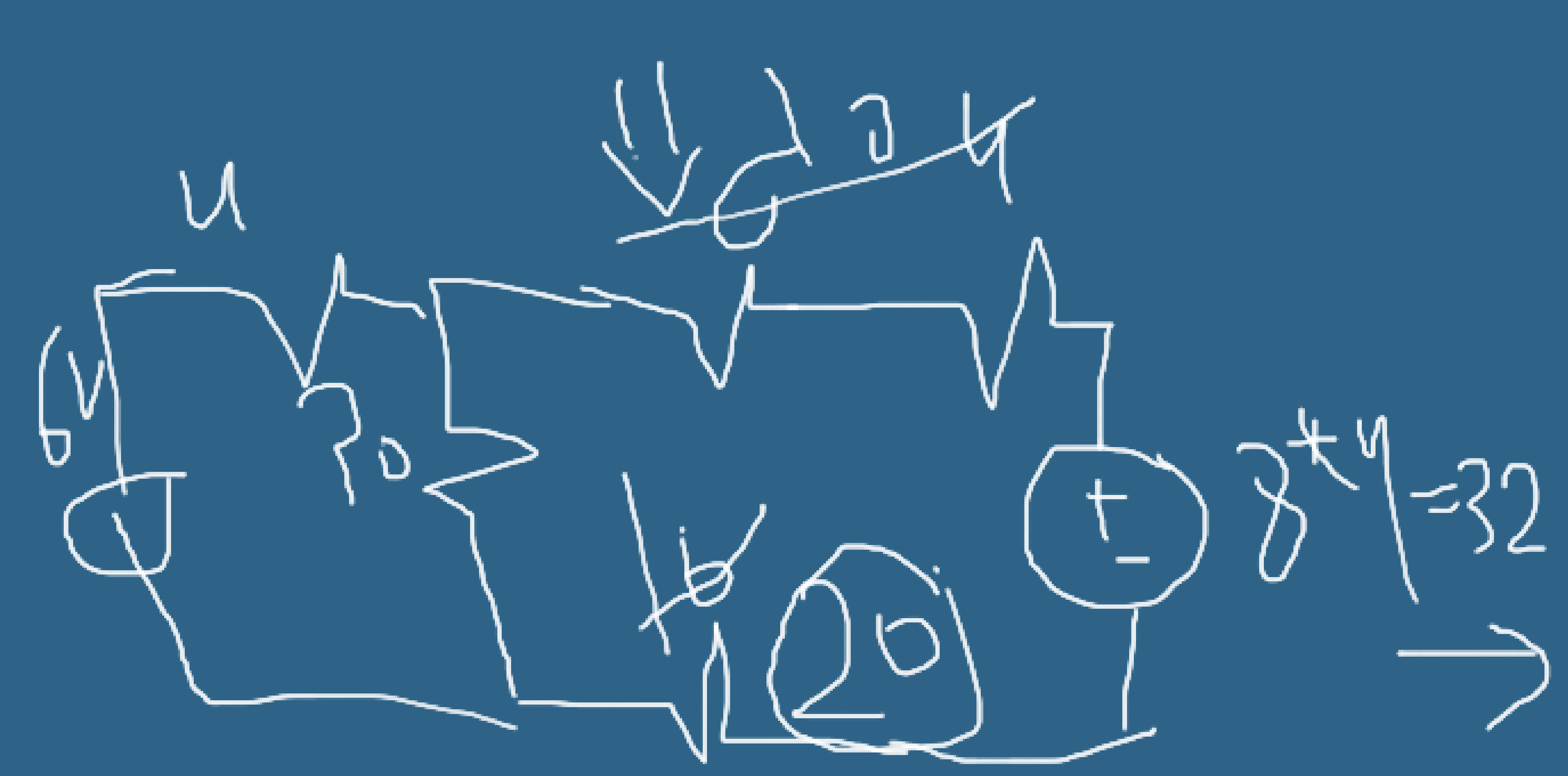
$$i_L = \frac{R}{R+R_L} i_s$$

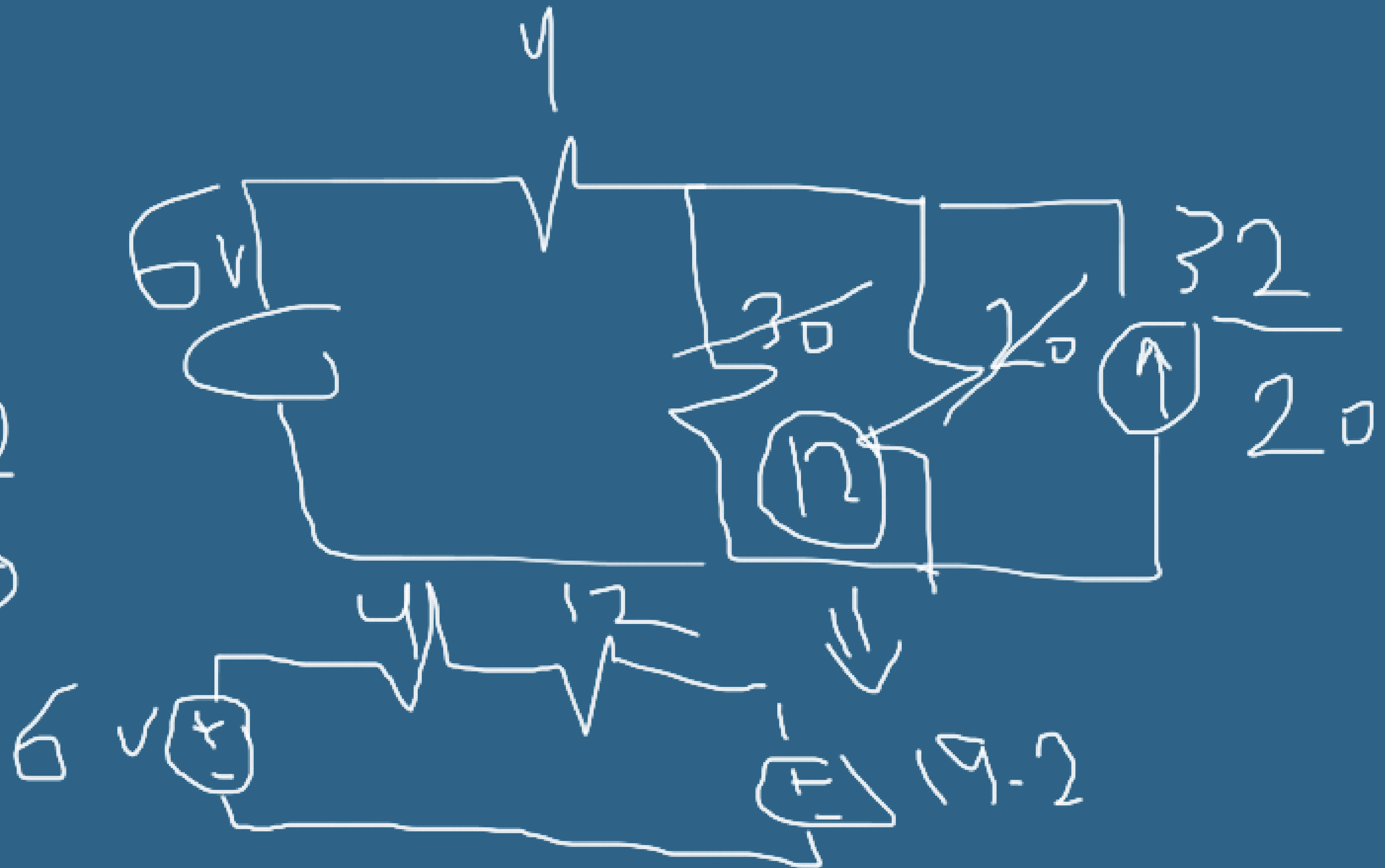
$$\frac{V_s}{\cancel{R+R_L}} = \frac{R}{\cancel{R+R_L}} i_s$$

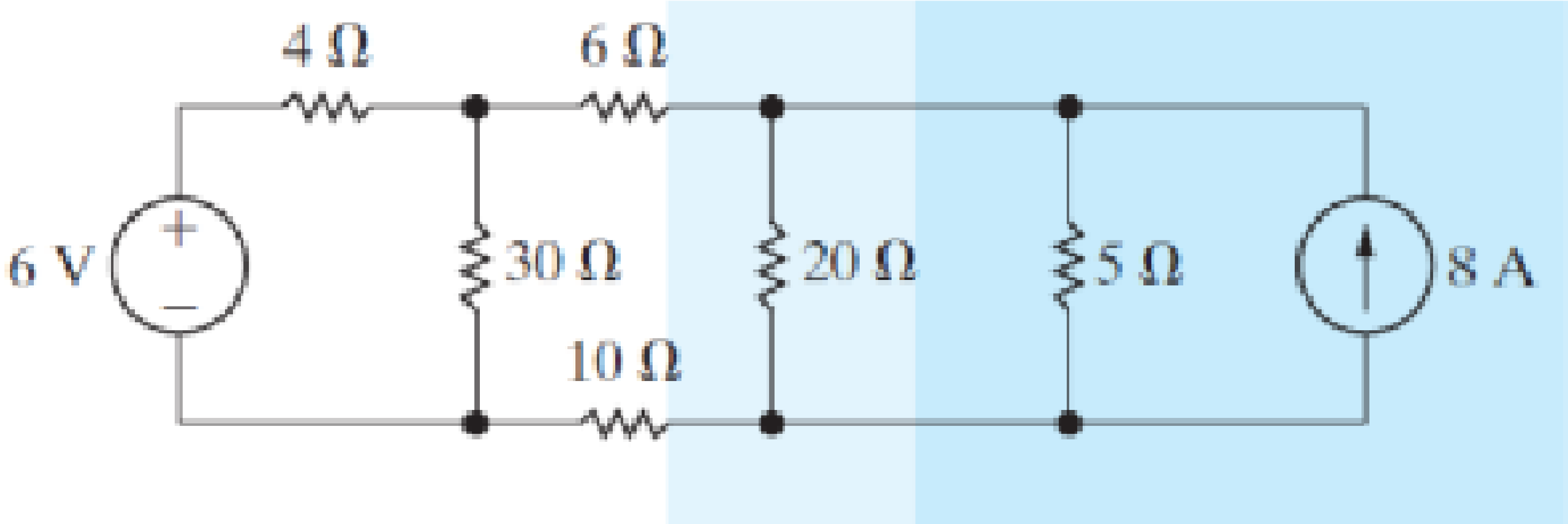
$$i_s = \frac{V_s}{R}$$

Source Transformations

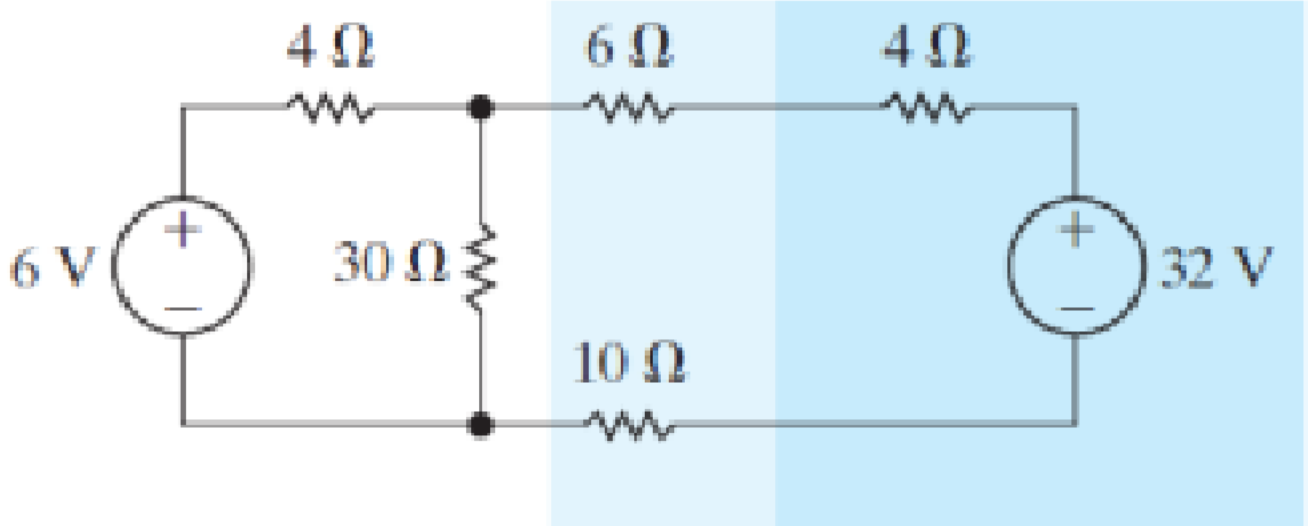




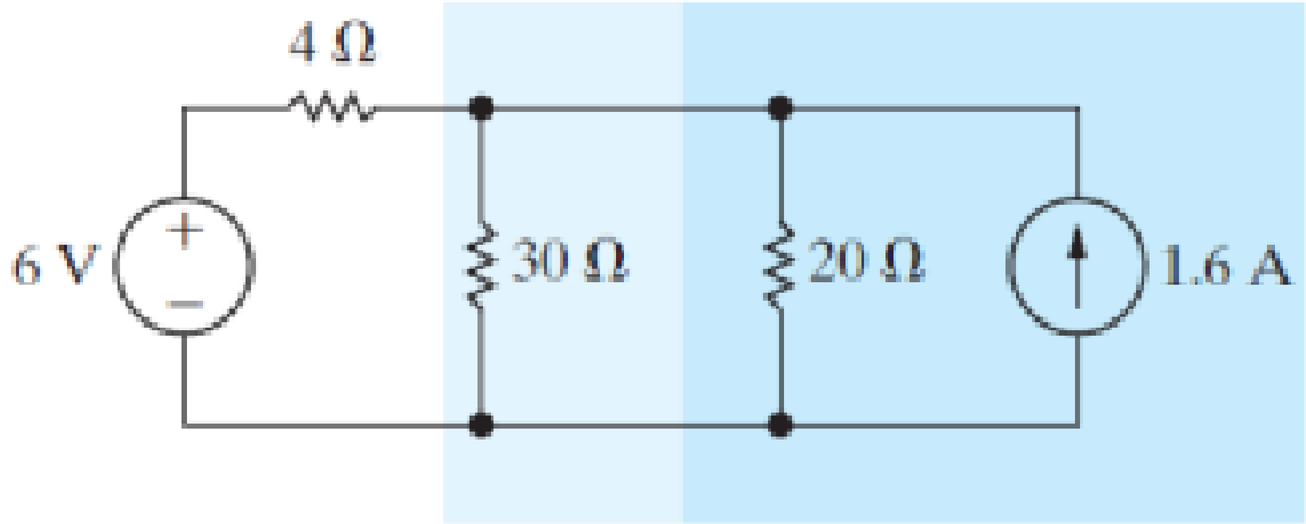




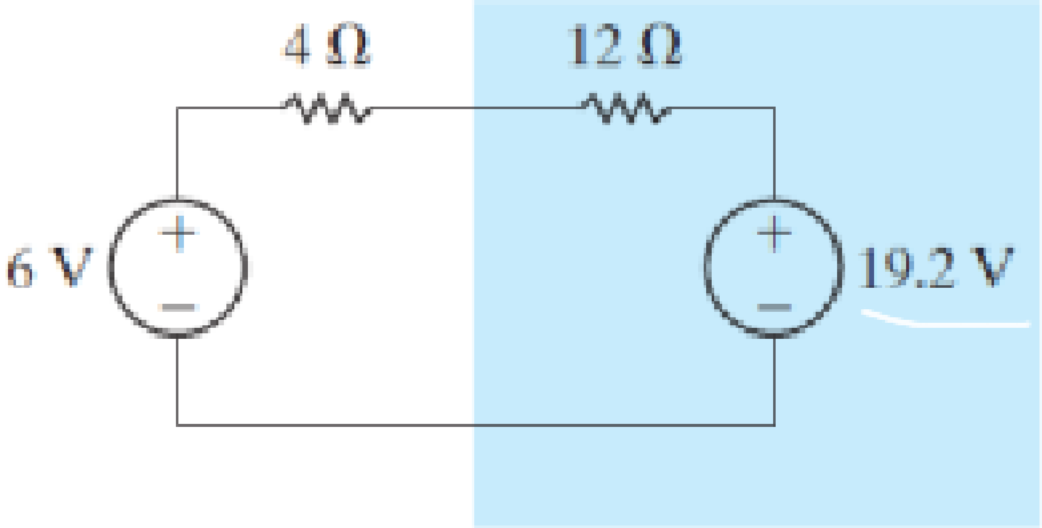
(a) First step



(b) Second step



(c) Third step



(d) Fourth step