Response of RL and RC circuits

magnetic electrical

- we established that inductors and capacitors can store energy.

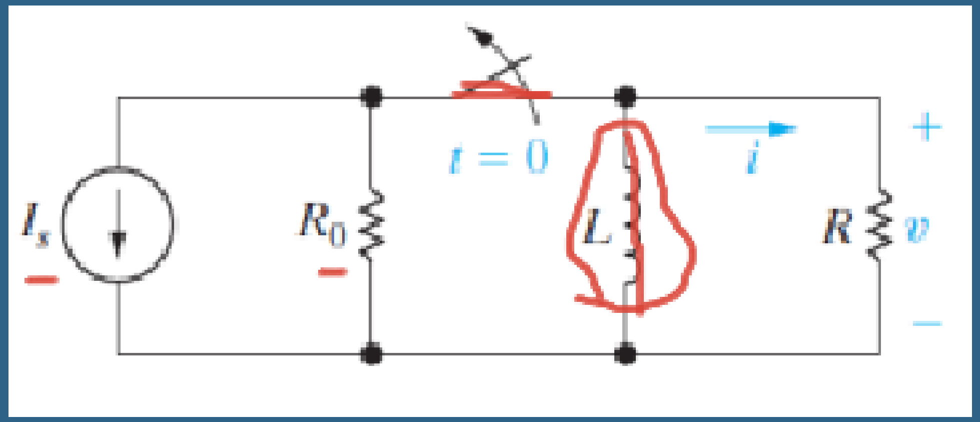
- We analyze RL and RC circuits for I & V ==> when energy is either released or acquired by an inductor or capacitor in response to an abrupt change in a dc voltage or current source.

- Natural Response: When an inductor or capacitor is abruptly disconnected from its dc source, we find I & V that arise when stored energy in an inductor or capacitor is suddenly released to a resistive network.

- Step Response: We find the I & V that arise when energy is being acquired by an inductor or capacitor when a dc voltage or current source is suddenly applied

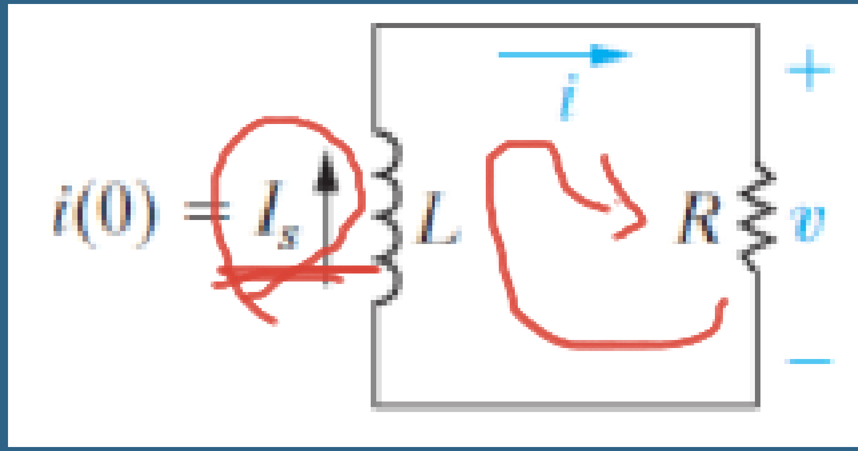
Natural Response of RL and RC circuits

- We assume that the independent current source generates a constant current Is and that the switch has been in a closed position for a long time.



Therefore, before the stored energy is released,

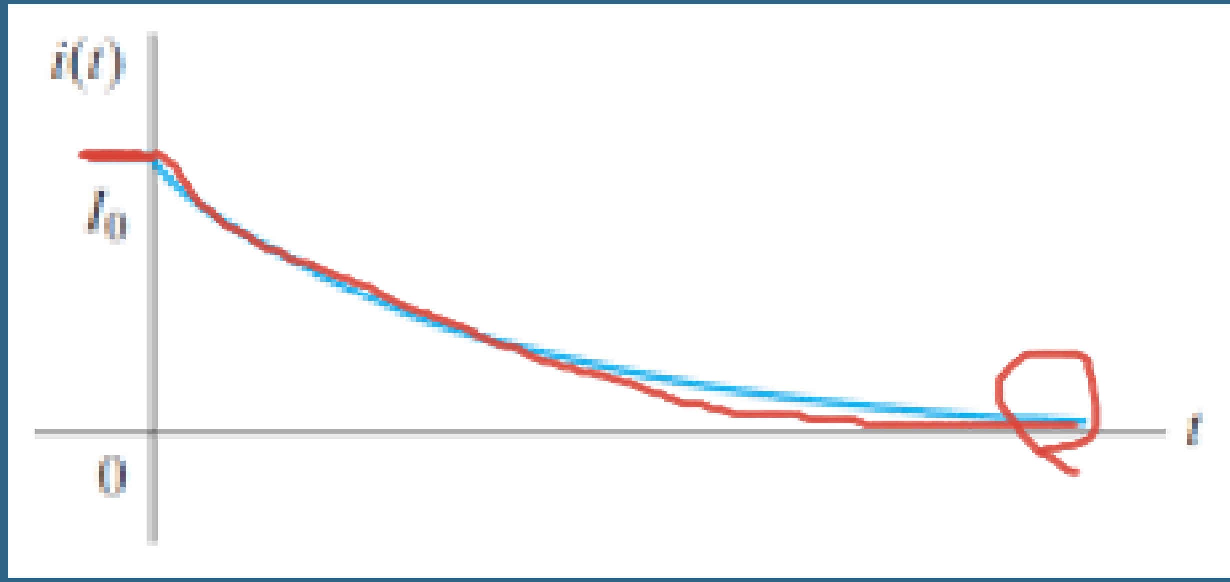
$$L\frac{di}{dt} + Ri = 0,$$



$$i(t) = I_0 e^{-(R/L)t}, \quad t \geq 0,$$

- The inductor behaves like a short circuit.
- The whole current is in inductive branch and No current in R or Ro.

$$\tau = \frac{L}{R}$$



- The ~~time constant~~ is an important parameter for fir~~st order~~ circuits. You can express the time elapsed after switching as an integer multiple of t.
- one-time constant after the inductor begins releasing its stored energy to the resistor, the current has been reduced to approximately 0.37 of its initial value.

t	$e^{-t/\tau}$	t	$e^{-t/\tau}$
τ	3.6788×10^{-1}	6τ	2.4788×10^{-3}
2τ	1.3534×10^{-1}	7τ	9.1188×10^{-4}
3τ	4.9787×10^{-2}	8τ	3.3546×10^{-4}
4τ	1.8316×10^{-2}	9τ	1.2341×10^{-4}
5τ	6.7379×10^{-3}	10τ	4.5400×10^{-5}

$$L\frac{di}{dt} + Ri = 0$$

$$L\frac{di}{dt} = -Ri$$

$$L\frac{di}{\cancel{dt}}\cancel{dt} = -Ri\,dt$$

$$\int_{i_0}^{i} \frac{1}{i}\,di = \int_0^t -\frac{R}{L}\,dt$$

$$\ln(|i|)\Big|_{i_0}^{i} = -\frac{R}{L}t$$

$$\ln(i(t)) - \ln(i_0) = -\frac{R}{L}t$$

$$\ln\left[\frac{i(t)}{i(0)}\right] = -\frac{R}{L}t$$

log

$$\frac{i(t)}{i(0)} = e^{(-R/L)t}$$

$$i(t) = I_0\, e^{-\frac{R}{L}t}$$

Natural Response of RL and RC circuits

Steps:

1- Determine Io (usually @ t < 0)

2- Calculate time constant (tau) ==> don't forget Req

3- Write equation of Inductor

4- Calculate any required quantity.

Solution

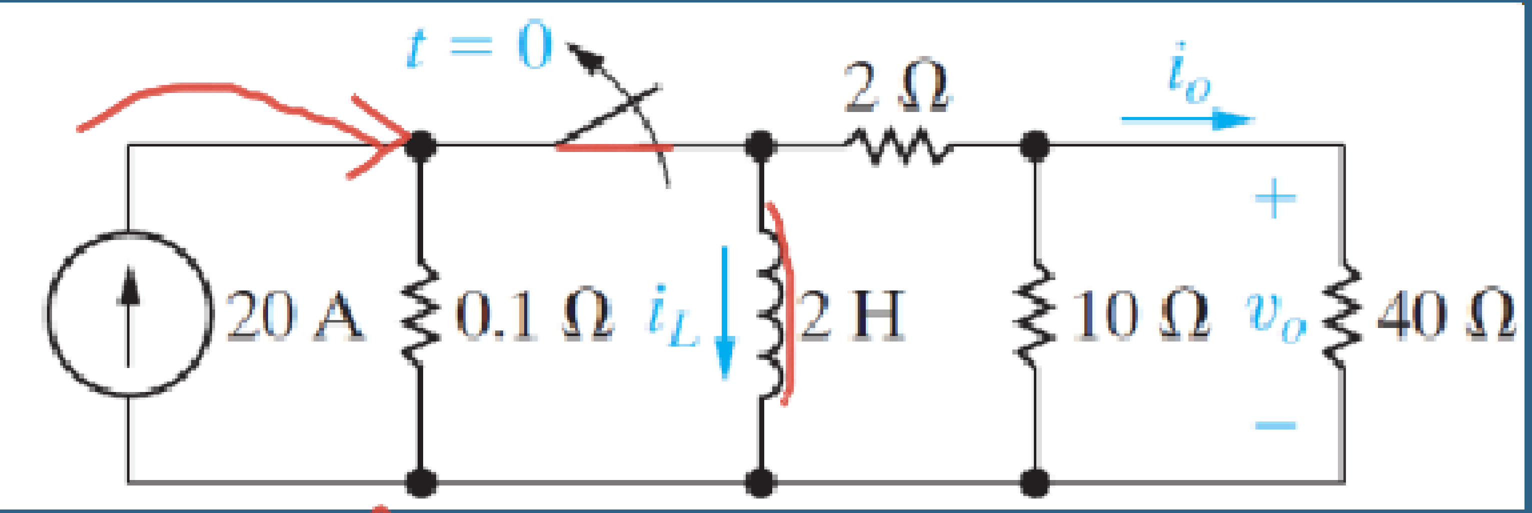
The switch in the circuit shown in Fig. 7.6 has been closed for a long time before it is opened at $t = 0$. Find

a) $i_L(t)$ for $t \geq 0$,

b) $i_o(t)$ for $t \geq 0^+$,

c) $v_o(t)$ for $t \geq 0^+$,

d) the percentage of the total energy stored in the 2 H inductor that is dissipated in the 10 Ω resistor.

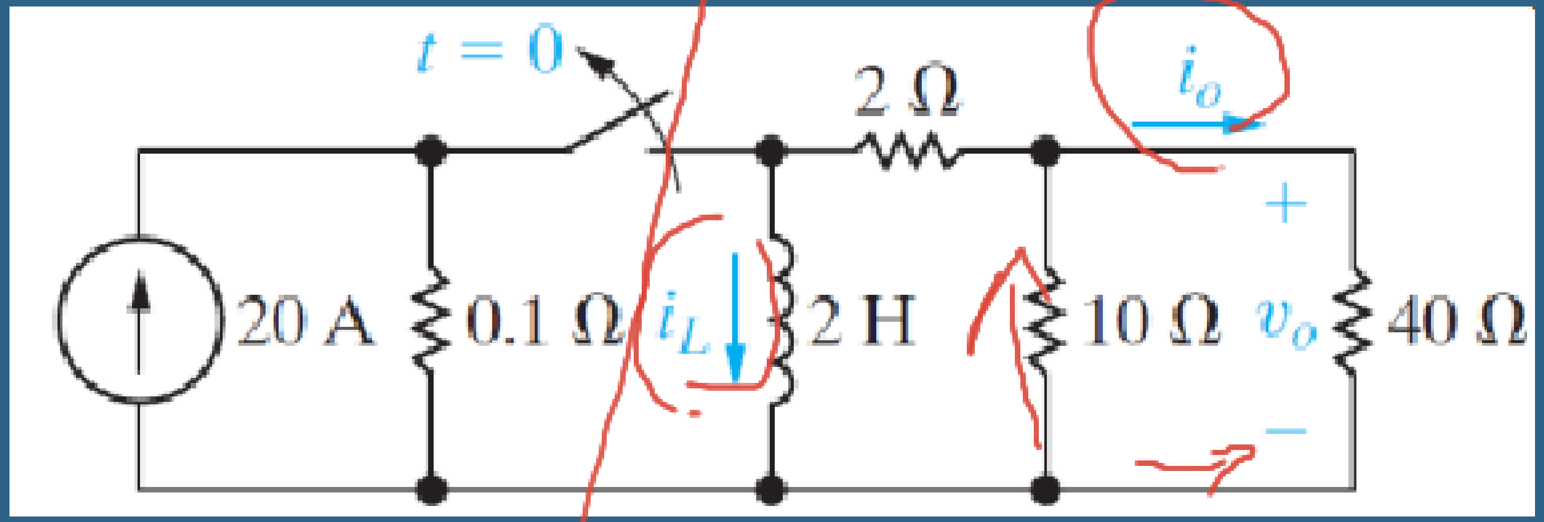


$\rightarrow i_L = 20A$

$\rightarrow \tau = ? \Rightarrow \frac{L}{R} = \frac{2}{10} = 0.2$ sec

$R_{eq} = (10 \| 40) + 2 = 10\,\Omega$

$\rightarrow I_L(t) = I_0 e^{-t/\tau} = 20 e^{-t/0.1} = 20 e^{-5t}$



$$i_o = \frac{10}{10+40}\left(i_L^{+} - 1\right)$$

$$= -4e^{-5t}$$

$$v_o(t) = i_o(t)R = -160e^{-5t}\,V$$

$$w = \int p\,dt$$

$$p = \frac{dw}{dt}$$

$$P_{10\Omega} = \frac{v_o^2}{10} = 256e^{-10t}\ \text{watt}$$

$$W_{10\Omega} = \int_0^{\infty} {}'' \, dt = 256\,J$$

$$w(0) = \frac{1}{2}L\,i(t_0)^2$$

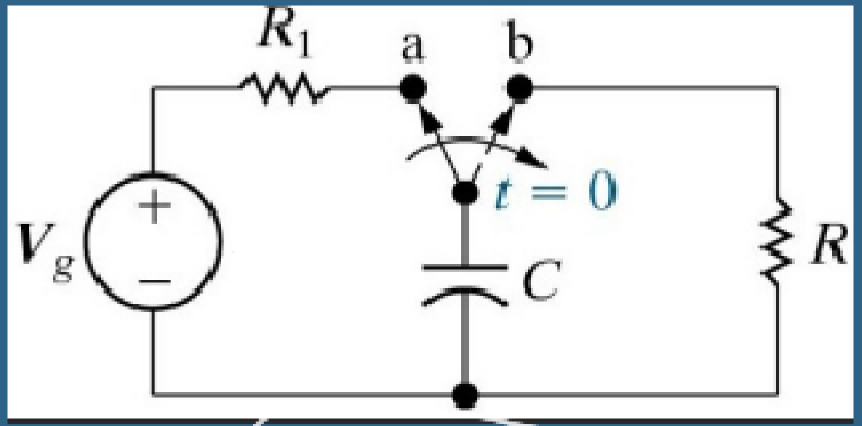
$$= \frac{1}{2}(2)(20)^2$$

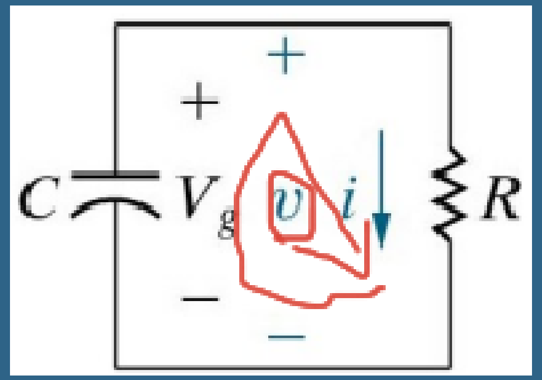
$$= 400\,J$$

The natural response of an RC Circuit

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

KCL





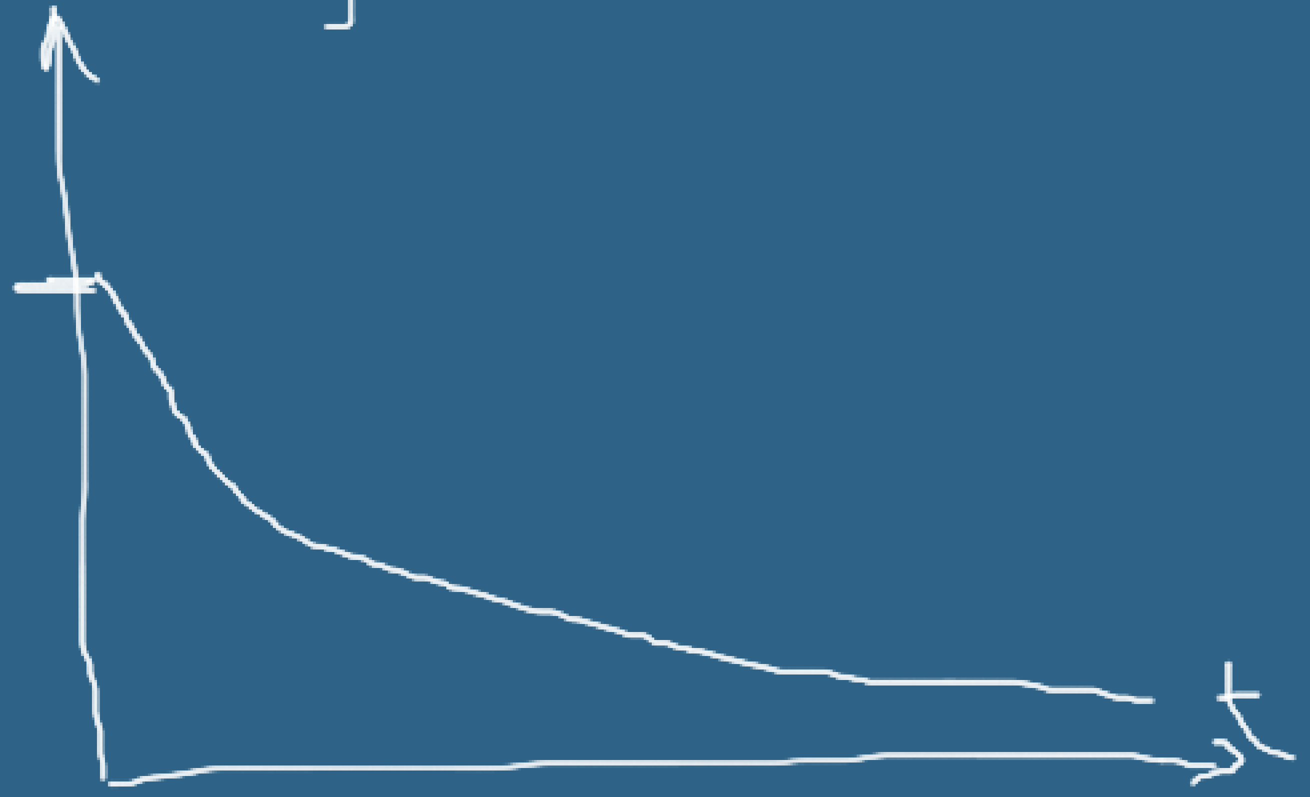
$$\tau = RC$$

$$v(t) = V_0e^{-t/\tau}$$

$$C\frac{dv}{\cancel{dt}}\,\cancel{dt} = \frac{-v}{R}\,dt$$

$$\frac{1}{v}dv = \frac{-1}{RC}dt$$

$$v(t) = V_0e^{-t/\tau}$$



The natural response of an RC Circuit

Steps:

1- Determine ~~Io~~ Vo (usually @ t < 0)

2- Calculate time constant (tau) ==> don't forget Req

3- Write equation of ~~Inductor~~

4- Calculate any required quantity.

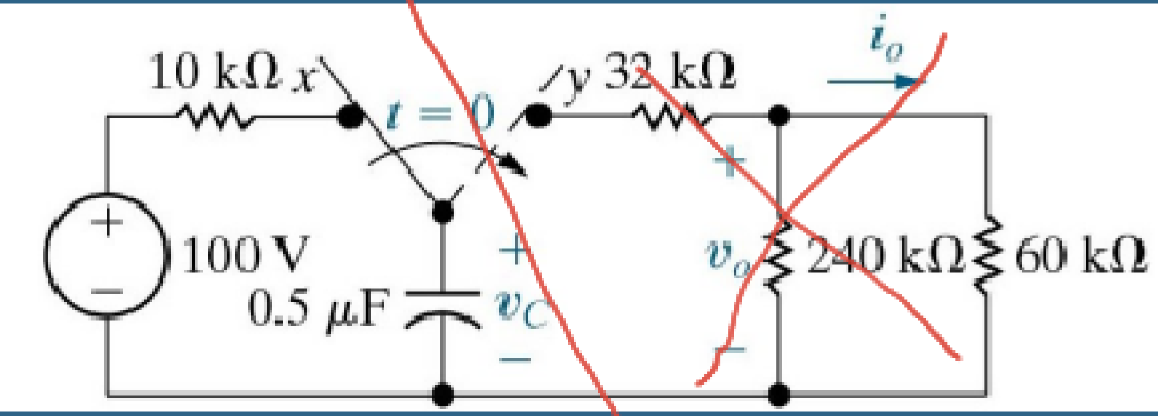
The switch in the circuit shown in Fig. 7.15 has been in position x for a long time. At $t = 0$, the switch moves instantaneously to position y. Find

a) $v_C(t)$ for $t \geq 0$,

b) $v_o(t)$ for $t \geq 0^+$,

c) $i_o(t)$ for $t \geq 0^+$, and

d) the total energy dissipated in the 60 kΩ resistor.



$\rightarrow V_0 = \boxed{100V}$

$\rightarrow \tau = ? \rightarrow R_{eq} = (60 \| 240) + 32 = \boxed{80 k\Omega}$

$RC = 40ms$

$\rightarrow v_c(t) = 100 e^{-25t}$

$v_0(t) = \frac{240 \| 60}{(240 \| 60) + 32} v_c(t)$

$$i_o(t) = \frac{v_o(t)}{60k\Omega} = \boxed{e^{-25} \, mA}$$

$$P_{60k\Omega} = i_o^2(t) R = 60 e^{-50t} \text{ watt}$$

$$W_{60k\Omega} = \int_0^{\infty} \quad dt = \boxed{1.2 \, mJ} \quad \#$$