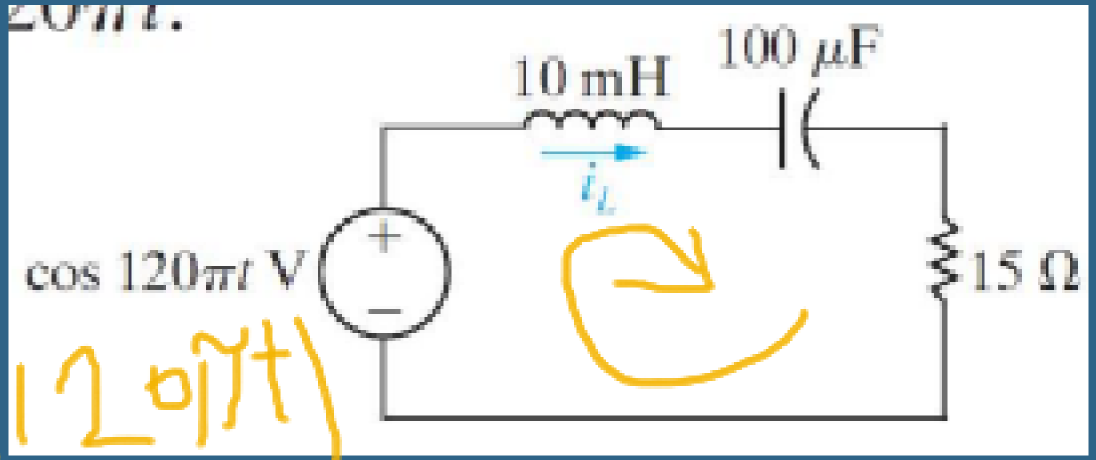
Transient Effect of AC source

Calculate iL



$i_L(t)$

$$15 i_L + 0.01 \frac{di_L}{dt} + \frac{1}{100*10^{-6}} \int_0^t i_L(x)\,dx = \cos(120\pi t)$$

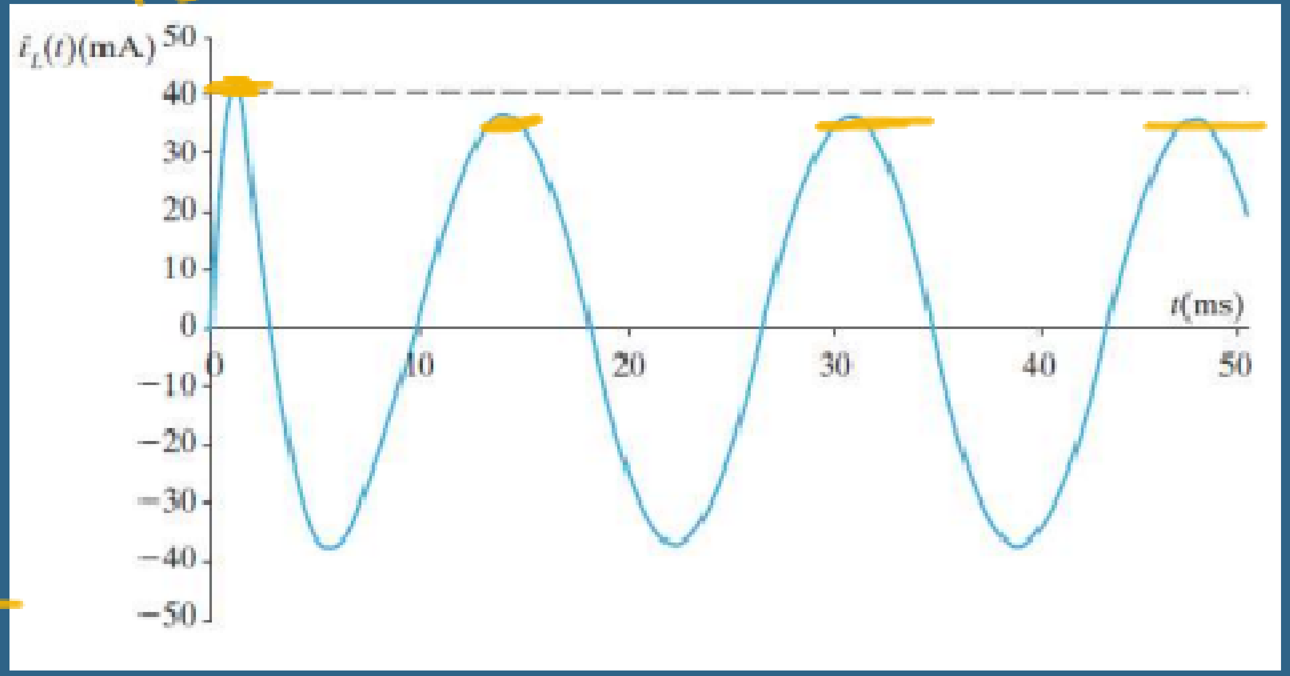
$$15 I_L + 0.01 s I_L + 10^4 \frac{I_L}{s} = \frac{s}{s^2+(120\pi)^2}$$

$$= \frac{100 s^2}{[s^2+1500s+10^6][s^2+(120\pi)^2]} = \frac{K}{(s+750-661.44j)}$$

$$\frac{K_1^*}{(s+750+661.44j)}$$

$K_1 = 0.07357\angle -97.89^\circ$

$K_2 = 0.018345 \angle 56.6^\circ$



$$147.14\, e^{-750t} \cos(661.44t - 97.89^{\circ}) + 76.69 \cos(1204t + 56.61) \text{ mA}$$

Poles and Zeros

Poles ==> The roots of the denominator polynomial [F(s) becomes infinitely large].

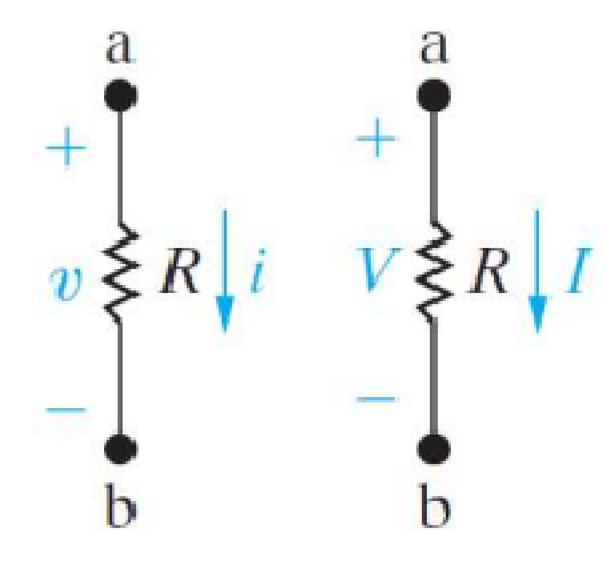
Zeros ==> The roots of the numerator polynomial [F(s) becomes zero].

$$F(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_n)}{(s+p_1)(s+p_2)\cdots(s+p_m)}$$

Circuit Elements in the s Domain

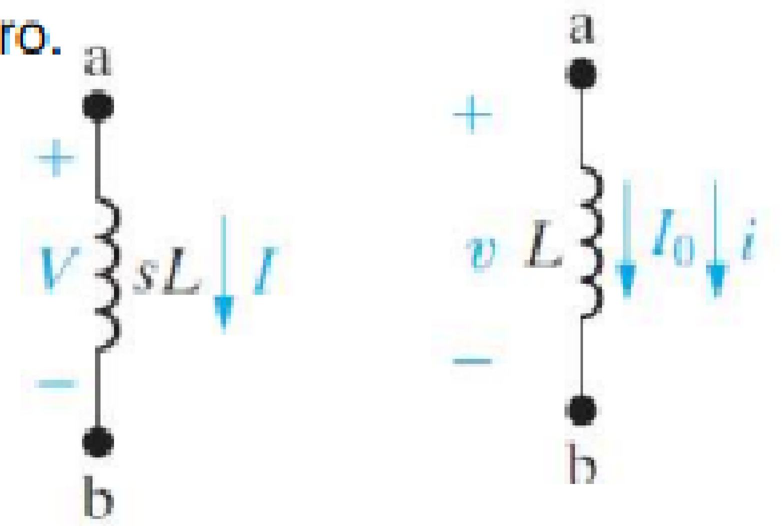
Resistor

$v = iR \longrightarrow V = IR$



Inductor

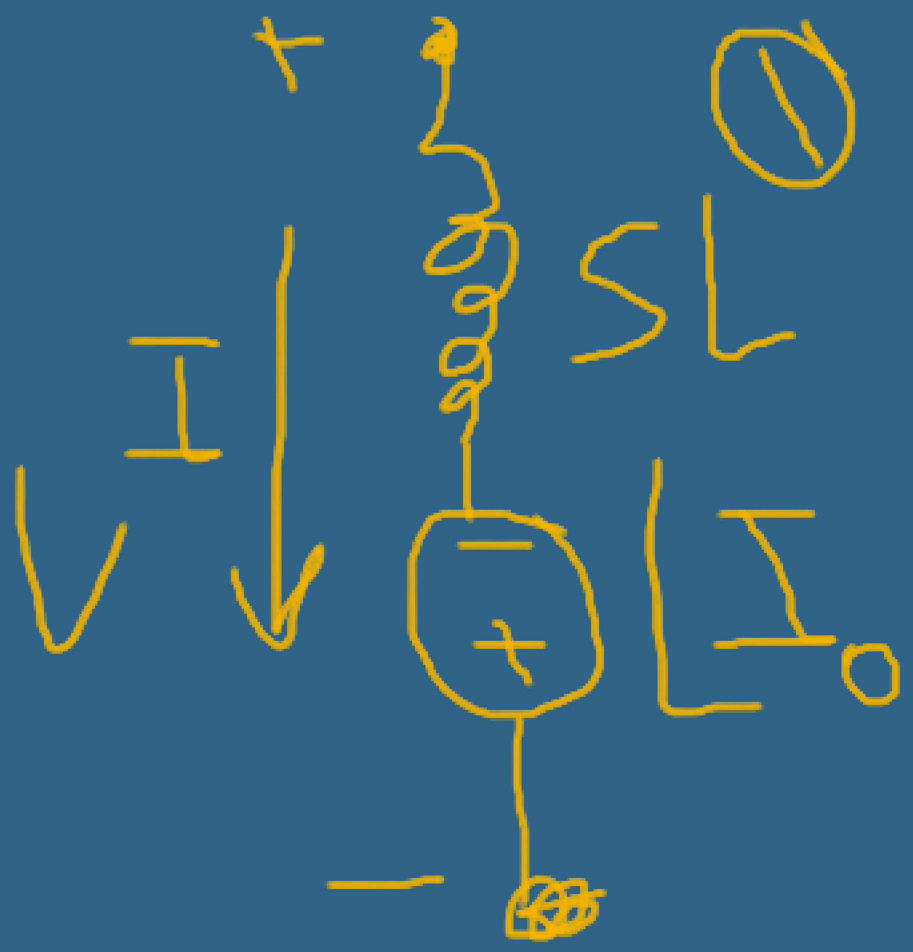
the initial current is zero.



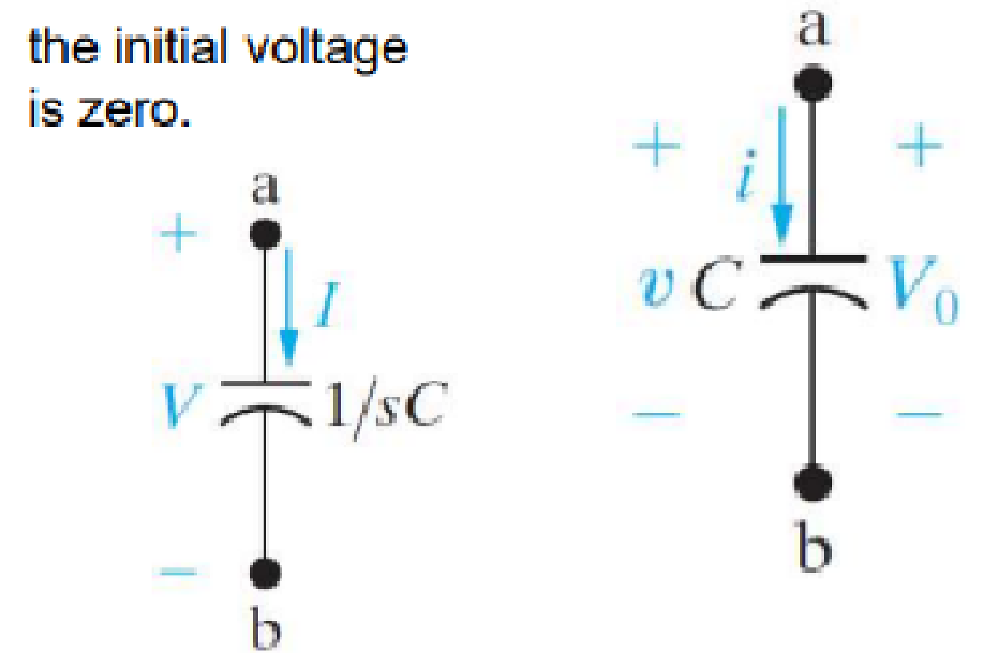
$$v = L\frac{di}{dt} \rightarrow V = L[sI - \cancel{I_0}]$$

$$V = LIs - LI_0$$

$$I = \frac{V + LI_0}{sL} = \frac{V}{sL} + \frac{I_0}{s}$$

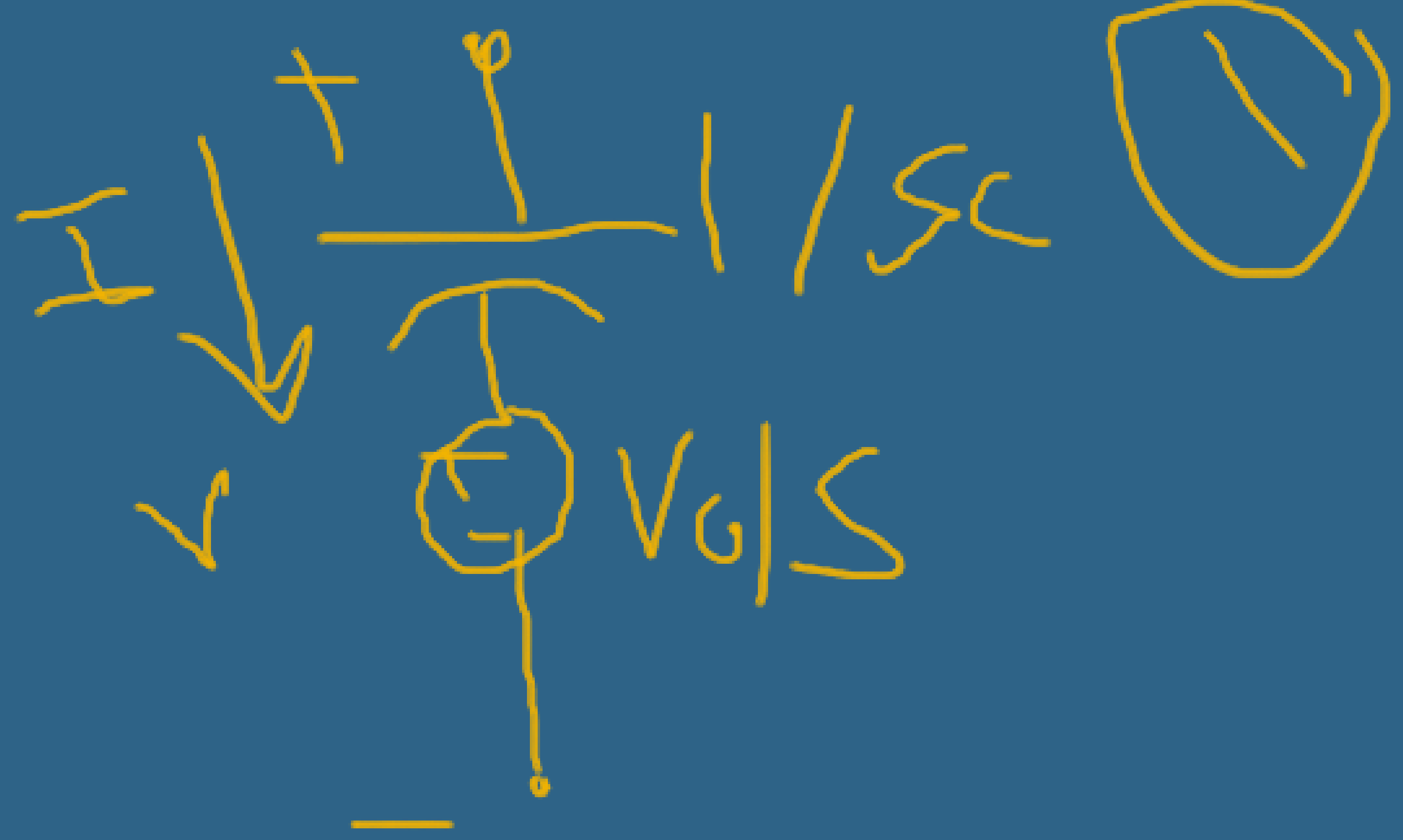


Capacitor



$$i = C\frac{dv}{dt} \rightarrow I = CsV - CV_0$$

$$V = \frac{I + CV_0}{Cs} = \frac{I}{Cs} + \frac{V_0}{s}$$



LAPLACE TRANSFORM METHOD

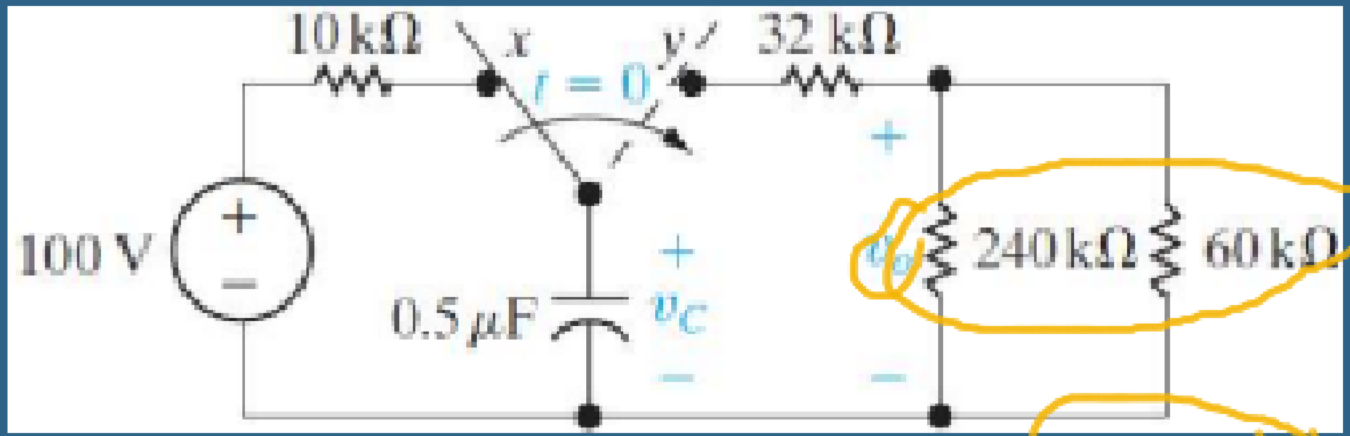
Steps:

1- Initial conditions for ind & cap.

2- Laplace-transform (Tables) ==> I, V, and other components.

3- Analyze the s-domain circuit using resistive circuit analysis techniques.

4- Inverse-Laplace-transform the s-domain voltages and currents using partial fraction expansion.



48k

The Natural Response of an RC Circuit

using first-order circuit analysis techniques.

Use the Laplace Transform to find v(t) for t > 0+.

① $v_0 = \boxed{100V}$

② s-domain $Z_C = \frac{1}{sC} = \frac{2*10^6}{s}\Omega$

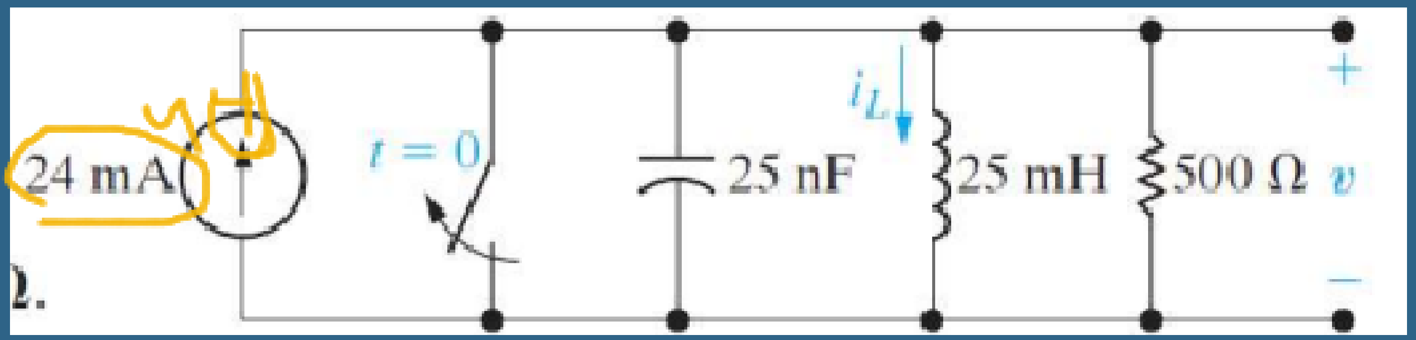
③ $V_0 = \frac{48}{48+32+\frac{2*10^6}{s}}\left(\frac{100}{s}\right) \to \frac{60}{s+25} \to \boxed{60e^{-25}u(t)}$

The Step Response of an RLC Circuit

==> the initial current in the inductor is 29 mA

==> the initial voltage across the capacitor is 50 V.

Use the Laplace transform method to find v(t) for t > 0.



$\frac{24}{s}$

$$Z_c = \frac{1}{sC} = \frac{40\times10^{6}}{s}\,\Omega$$

$$Z_L = sL = 0.025s$$

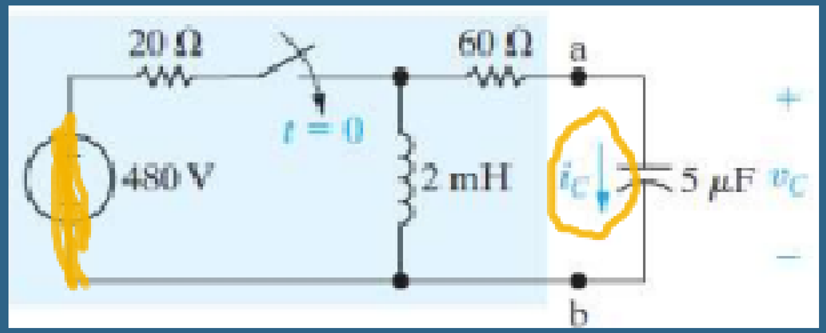
$$V = Z_{eq}\left(\frac{0.024}{s} + 1.25\times10^{-6} - \frac{0.029}{s}\right) = \frac{50s - 2.0\times10^{4}}{s^2 + 80000s + 16\times10^{8}}$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_R} + \frac{1}{Z_C} + \frac{1}{Z_L} \qquad Z_{eq} = V$$

$$v(t) = \mathscr{L}^{-1}\left\{\frac{-2.2\times10^{6}}{(s+40{,}000)^2} + \frac{50}{(s+40{,}000)}\right\}$$

$$= (-2.2\times10^{6}\,te^{-40{,}000t} + 50e^{-40{,}000t})u(t)\ \text{V}.$$

Creating a Thévenin Equivalent in the s Domain



① $Z_L = sL = \frac{s}{500}\ \Omega$

② $Z_C = \frac{1}{sC} = \frac{2\times10^5}{s}\ \Omega$

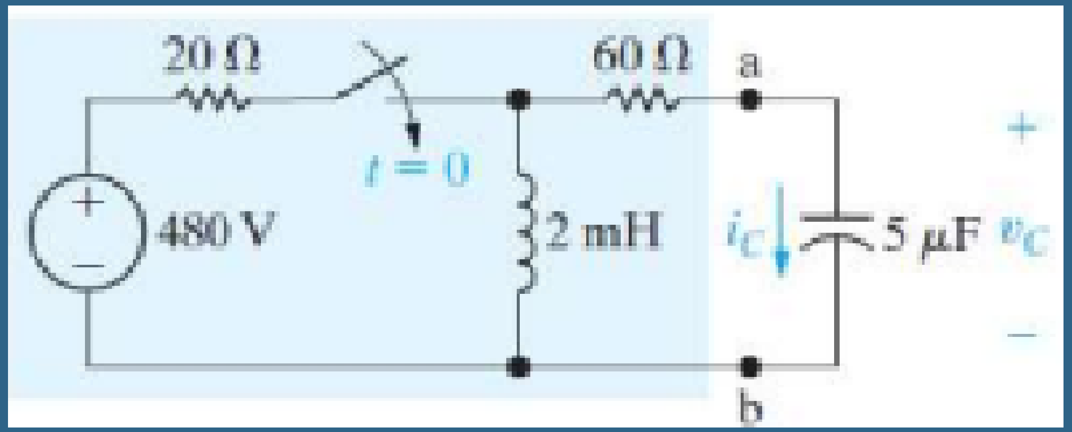
$V_{Th} = \frac{(480/s)(0.002s)}{20 + 0.002s} = \frac{480}{s + 10^4}$

$\frac{48}{5}$

$Z_{Th} = (20 \,\|\, 2\text{mH}) + 60 = \frac{80(s + 7500)}{s + 10^4}$

$i_C = (-30{,}000te^{-5000t} + 6e^{-5000t})u(t)$ A.

$v_C = 12 \times 10^5 te^{-5000t}u(t)$ V.



V_{th}

$$i_c = \frac{\dfrac{480}{s+10^4}}{\dfrac{80(s+7500)}{s+10^4} + \dfrac{2\times10^5}{s}}$$

$$= \frac{6s}{(s+5000)^2} \ \#$$

$$\lim_{s\to\infty} sI(s) \equiv \lim_{t\to 0} i(t) \rightarrow \text{initial current}$$

$$\lim_{s\to\infty} sI(s) = \lim_{s\to\infty} \frac{6s^2 / s^2}{\left(\frac{s}{s} + \frac{5000}{s}\right)^2} = \lim_{\frac{1}{s}\to 0} \frac{6}{\left(1 + \frac{5000}{s}\right)^2}$$

$$= \boxed{6}$$

$$\boxed{0}$$

$$\lim_{s\to 0} sI(s) \equiv \lim_{t\to\infty} i(t) \Rightarrow \text{final current}$$