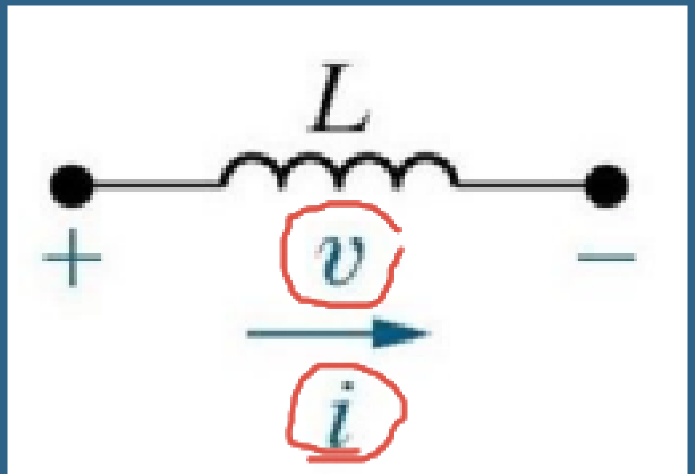
Inductance

$v \propto \frac{di}{dt} \rightarrow v = L\frac{di}{dt}$

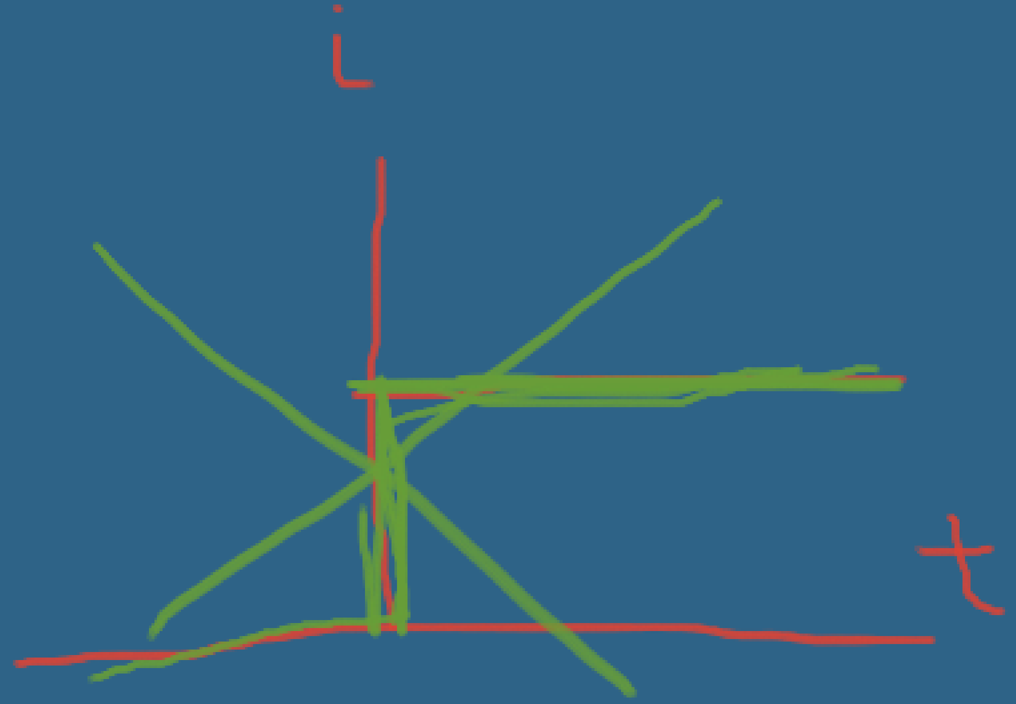


• The voltage across the terminals of an inductor is proportional to the time rate of change of the current in the inductor.

• First, if the current is constant, the voltage across the ideal inductor is zero. Thus, the inductor behaves as a short circuit in the presence of a constant, or dc current.

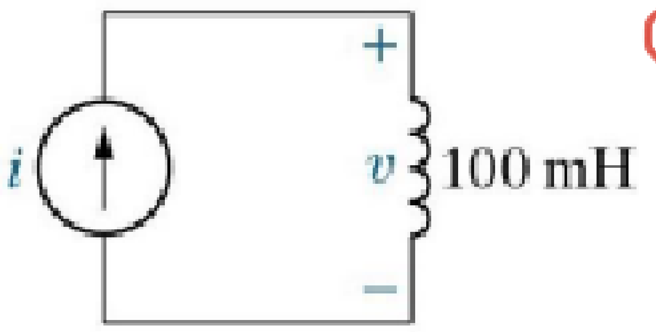
• Second, current cannot change instantaneously in an inductor; that is, the current cannot change by a finite amount in zero time.

$i = \text{constant} \rightarrow \boxed{v = 0}$



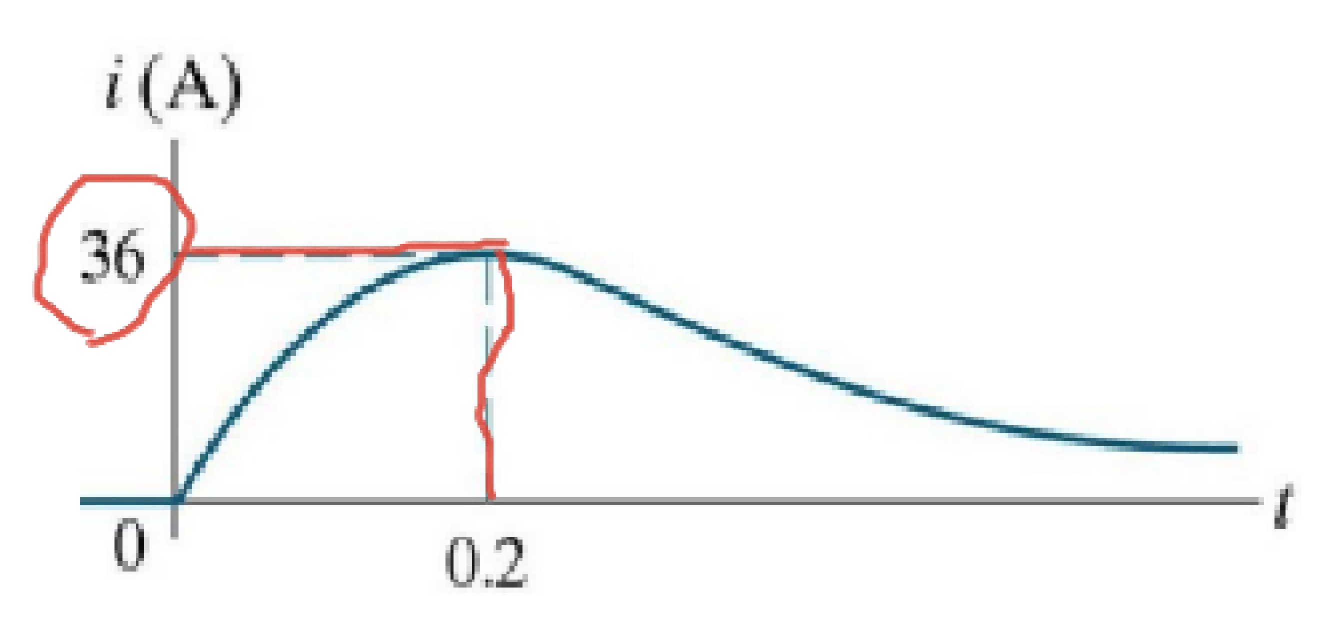
Voltage Across an Inductor

a) Sketch the current waveform.



$i = 0, \quad t < 0$

$i = 10te^{-5t}\text{ A}, \quad t > 0$



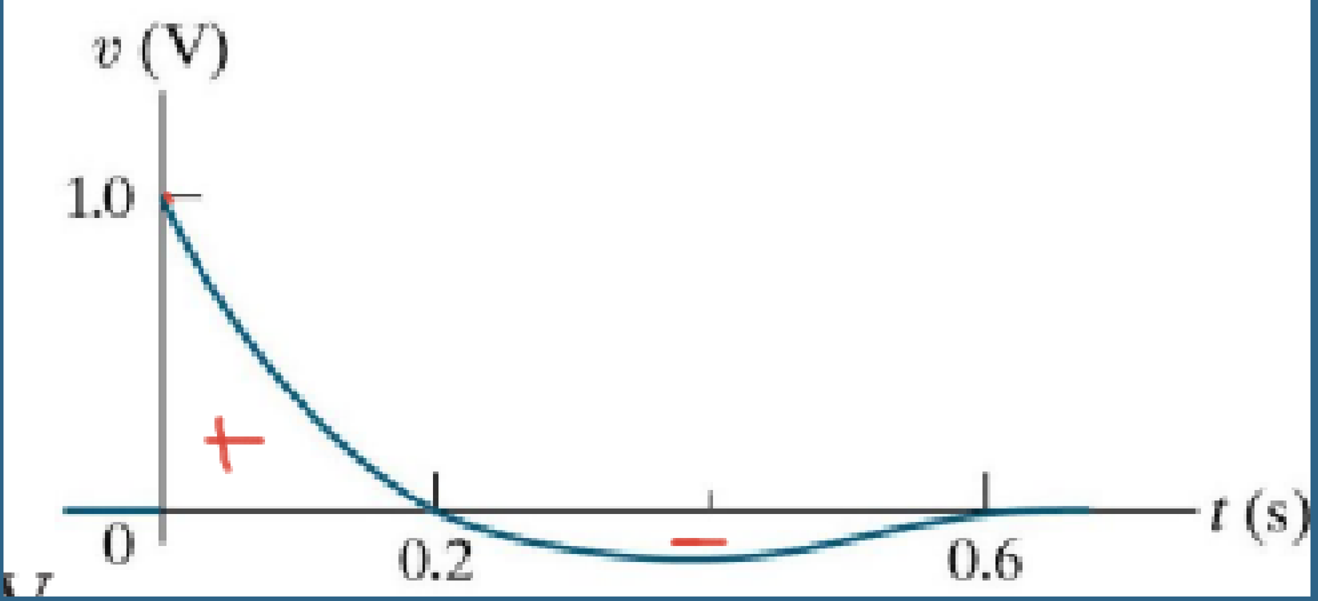
a) At what instant of time is the current maximum?

$$\frac{di}{dt} = 0 \rightarrow (10t)(-5)e^{-5t} + e^{-5t}(10) = 0 \rightarrow t = 0.2\ \text{sec}$$

- Express the voltage across the terminals of the 100 mH inductor as a function of time.

$$V = L\left[e^{-5t}(10 - 50t)\right]$$

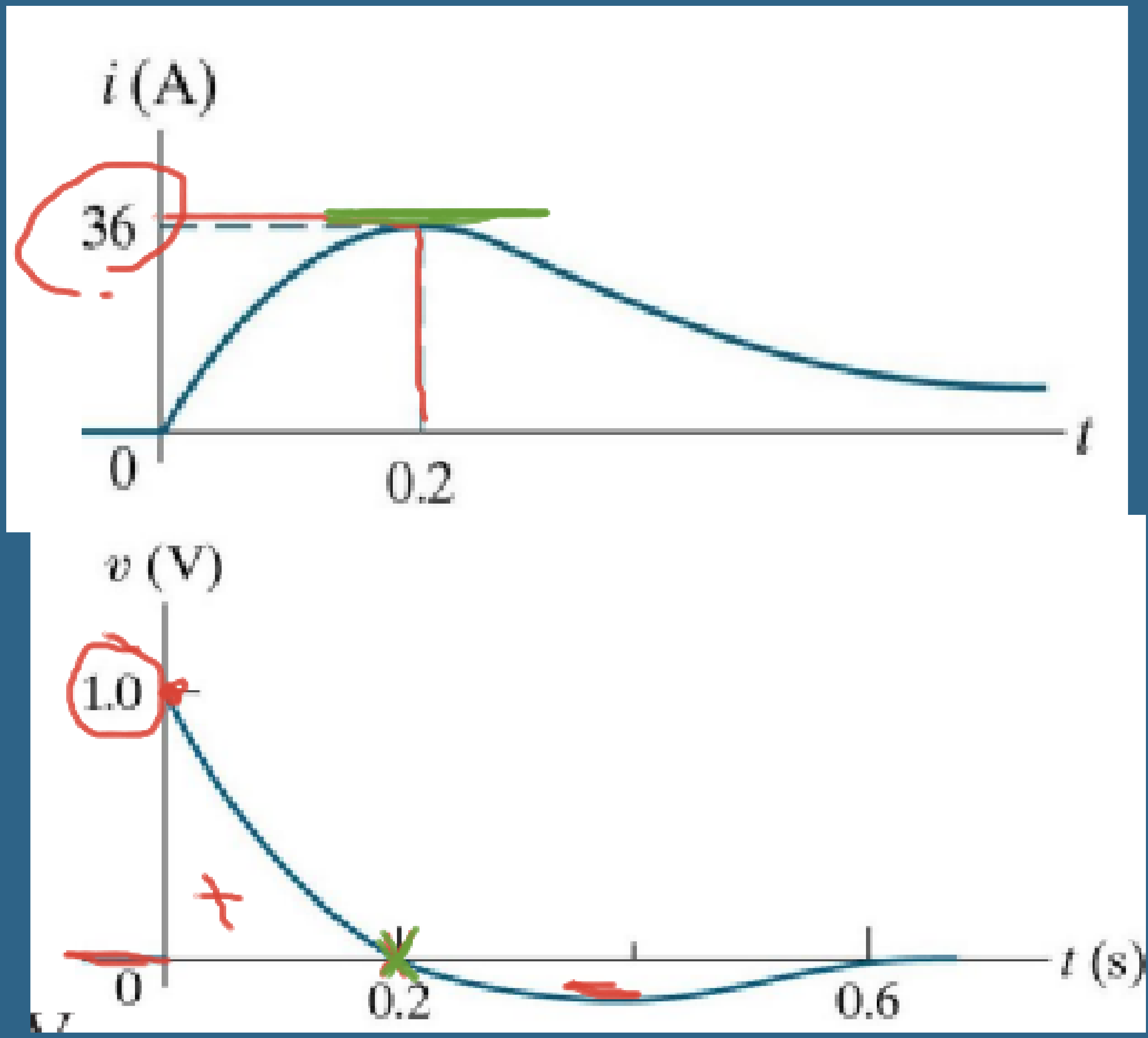
$$= e^{-5t}(1 - 5t)$$



e) Are the voltage and the current at a maximum at the same time?

$V \propto \frac{di}{dt} \Rightarrow$ V is not $\propto$ i

No



f) At what instant of time does the voltage change polarity?

@ 0.2 sec

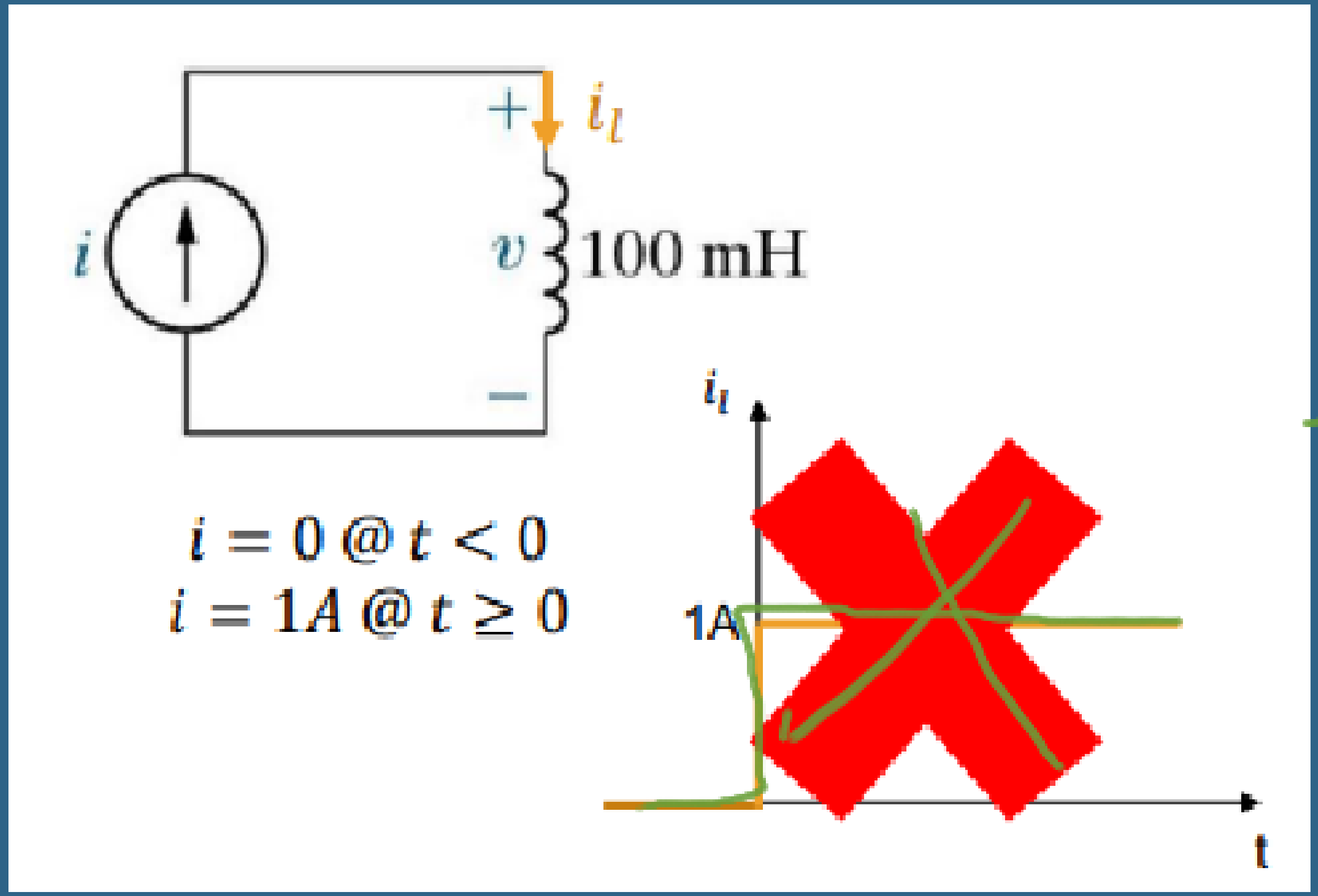
g) Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

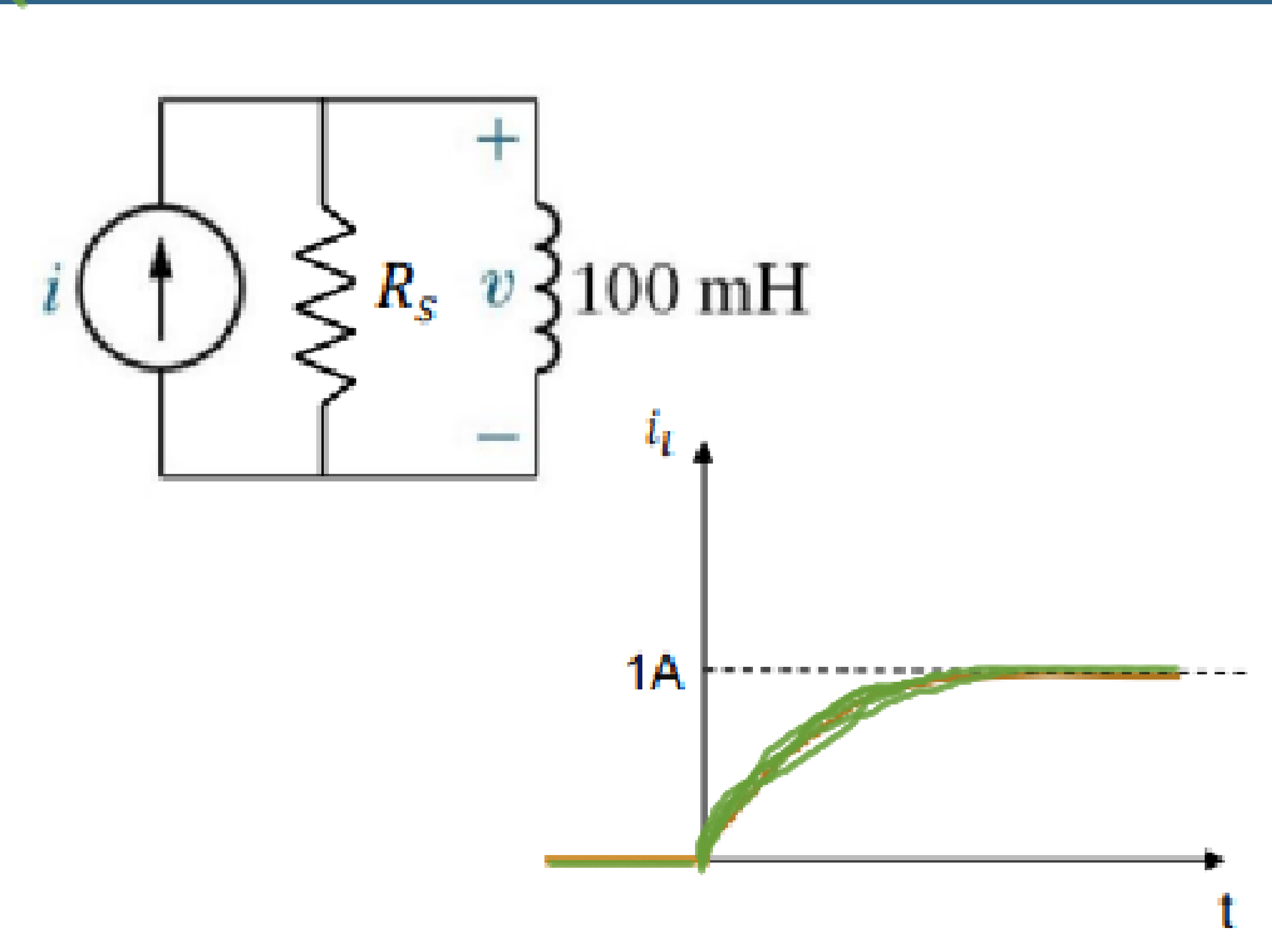
yes @ t = 0 but I is not

Current across an inductor

Assume an ideal dc current source connected to an inductor at t=0, current cannot change instantaneously across an inductor, where does the current go ?

practial





Current in an inductance

To find i as a function of v:

$$v = L\,di/dt$$

$$v = L\frac{di}{dt} \quad dt$$

$$\int v\,dt = \int L\,di$$

$$\int_{i_0}^{i_f} di = \frac{1}{L}\int_{t_0}^{t_f} v\,dt$$

$$i_f - i_0 = \frac{1}{L}\int_{t_0}^{t_f} v\,dt$$

$$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$$

$$i = \frac{1}{L}\int_{0}^{t} v\,dt$$

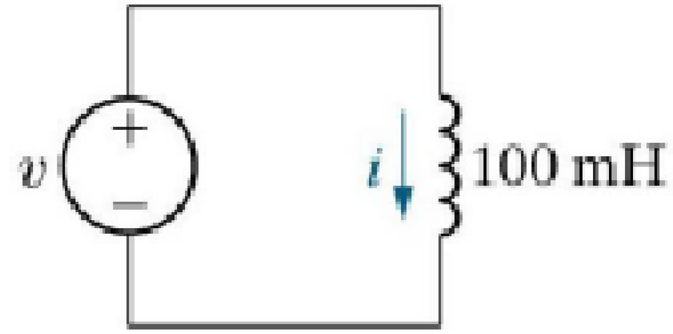
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0),$$

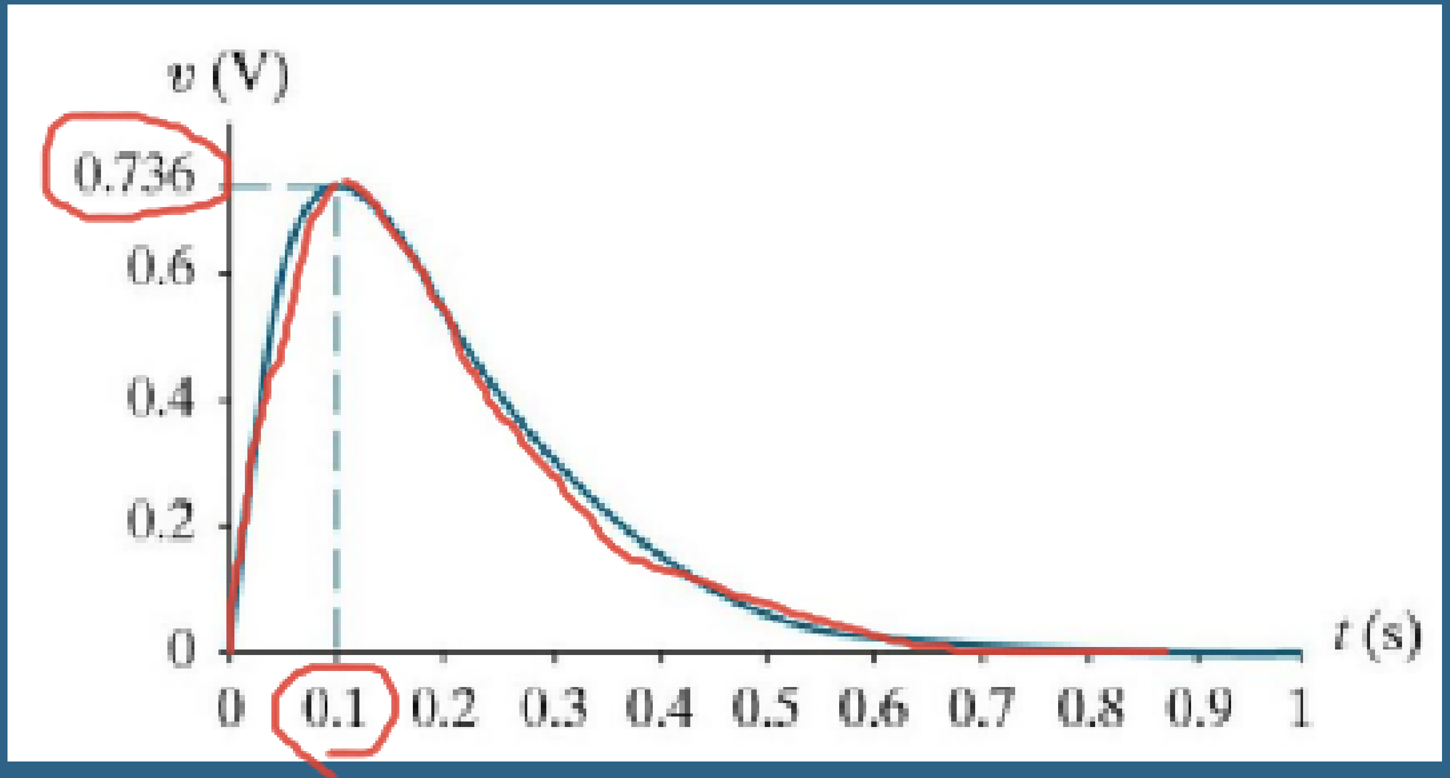
Most of the time $t_0 = 0$

Current in an Inductor

$$v(t) = 20te^{-10t}\ \text{V}$$

for $t > 0$. Also assume $i = 0$ for $t \le 0$.





Get the i equation,then draw it:

$$i = \frac{1}{L}\int_0^t v + K_0$$

100mH

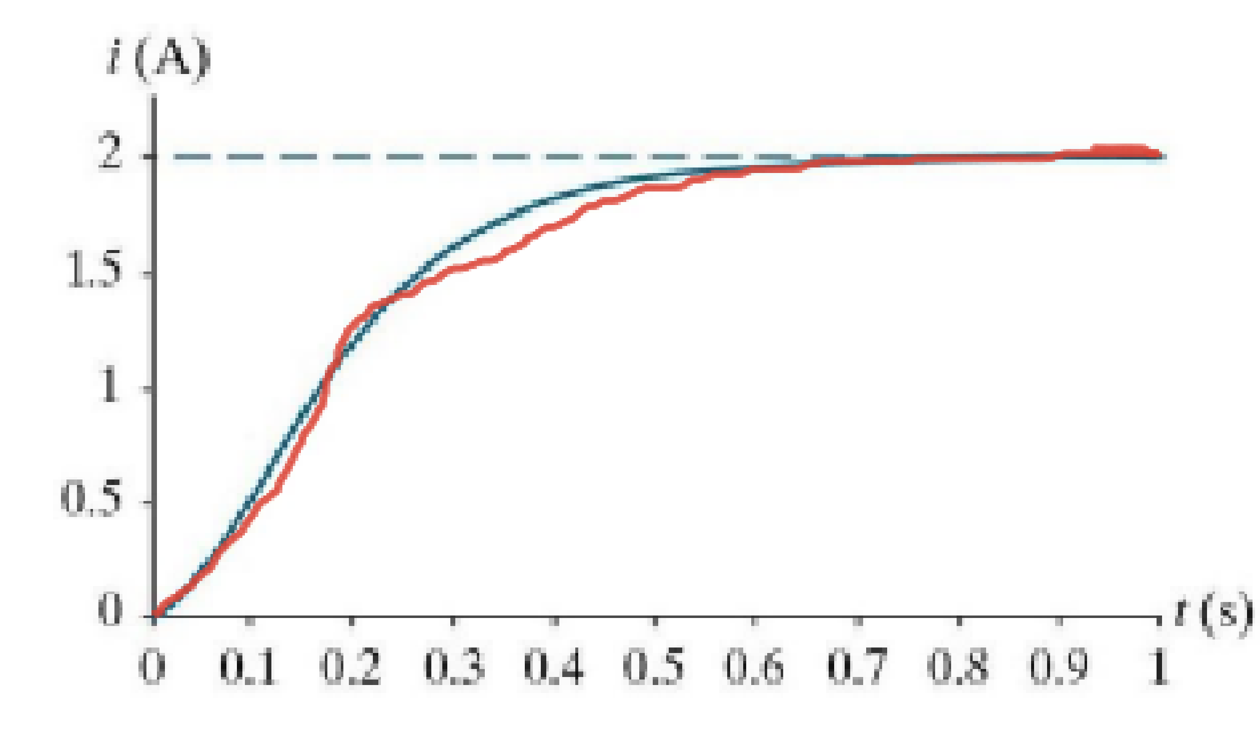
$$20te^{-10t}$$

$$20t \quad e^{-10t}$$

$$20 \quad e^{-10t} \quad -10$$

$$2(1 - 10te^{-10t} - e^{-10t})\ \text{A}$$

$t \ge 0$



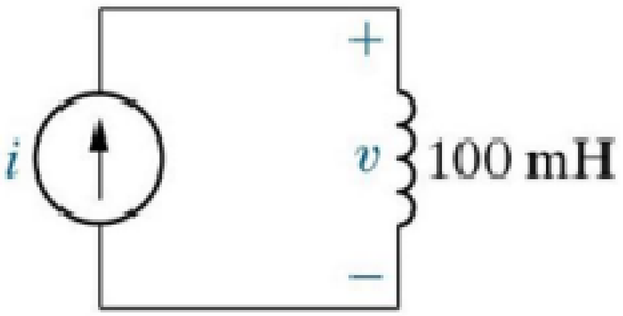
Power and Energy in the Inductor

$w = pt$

Power

$p = vi = Li\frac{di}{dt} = v\left[\frac{1}{L}\int_0^t v\,d\tau + i_0\right] = \frac{w}{t}$

$w = \frac{1}{2}Li^2$



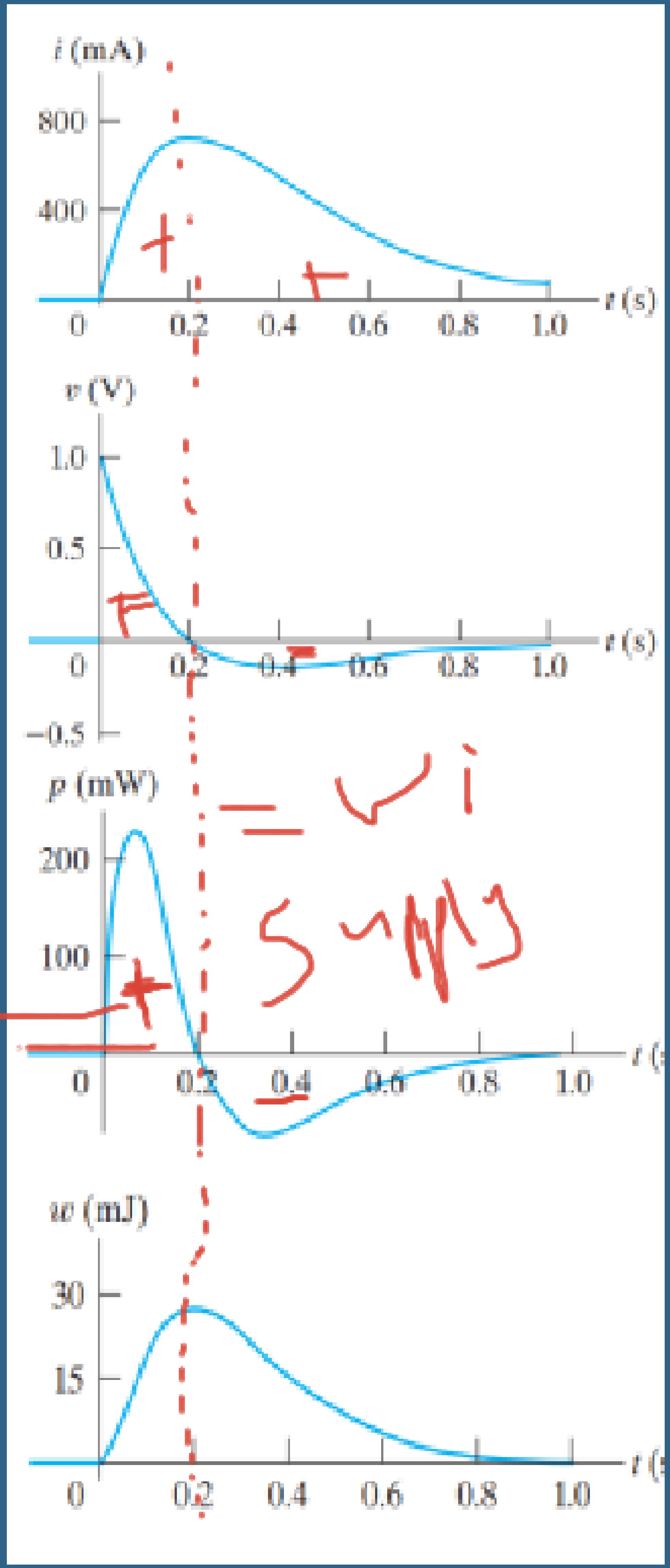
$i = 0, \quad t < 0$

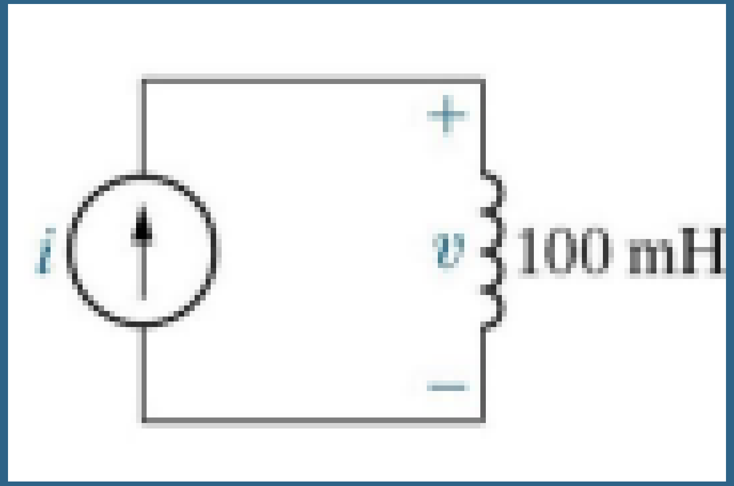
$i = 10te^{-5t}\,\text{A}, \quad t > 0$

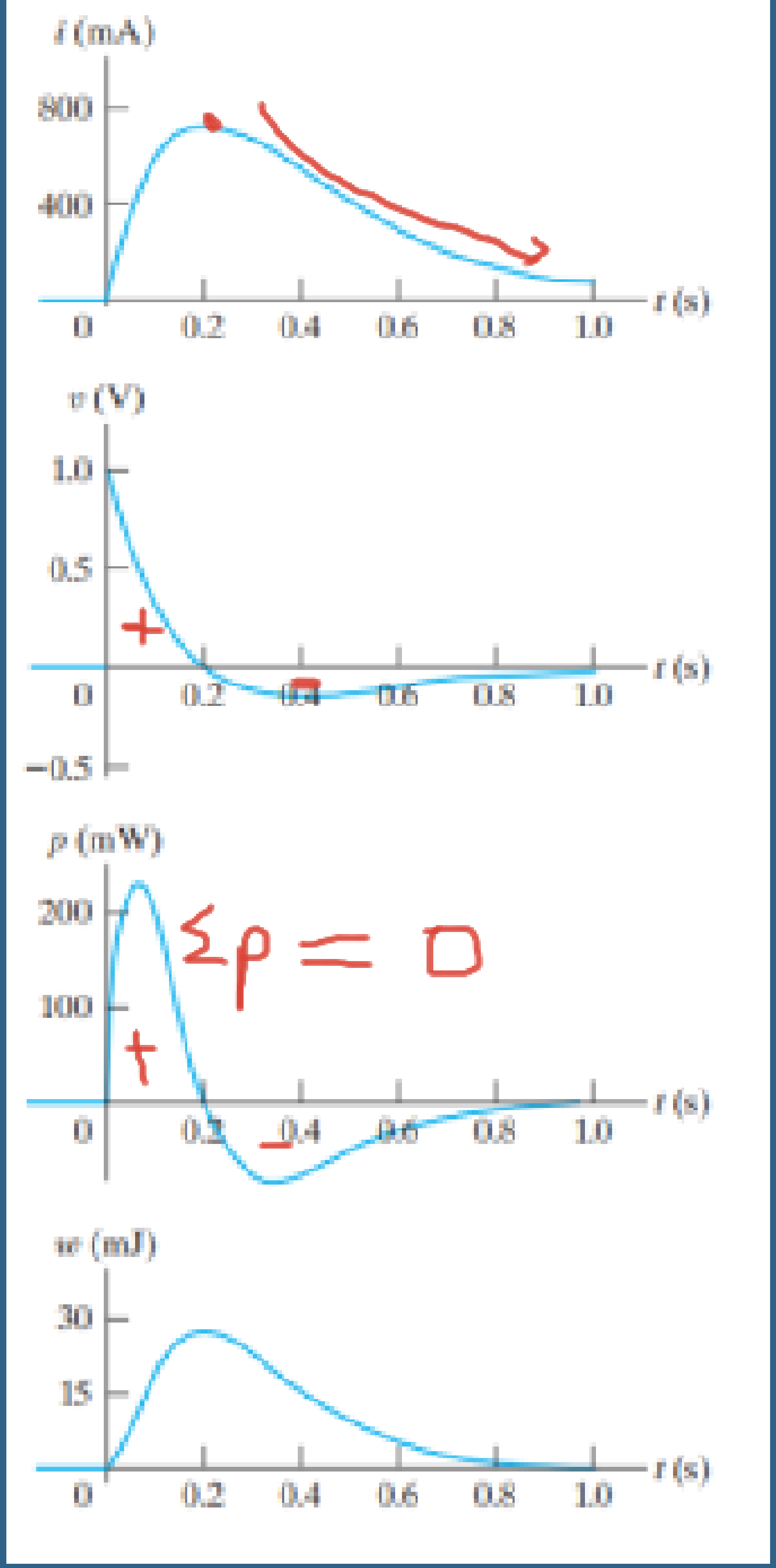
$2e^{-5t}(1-5t)$

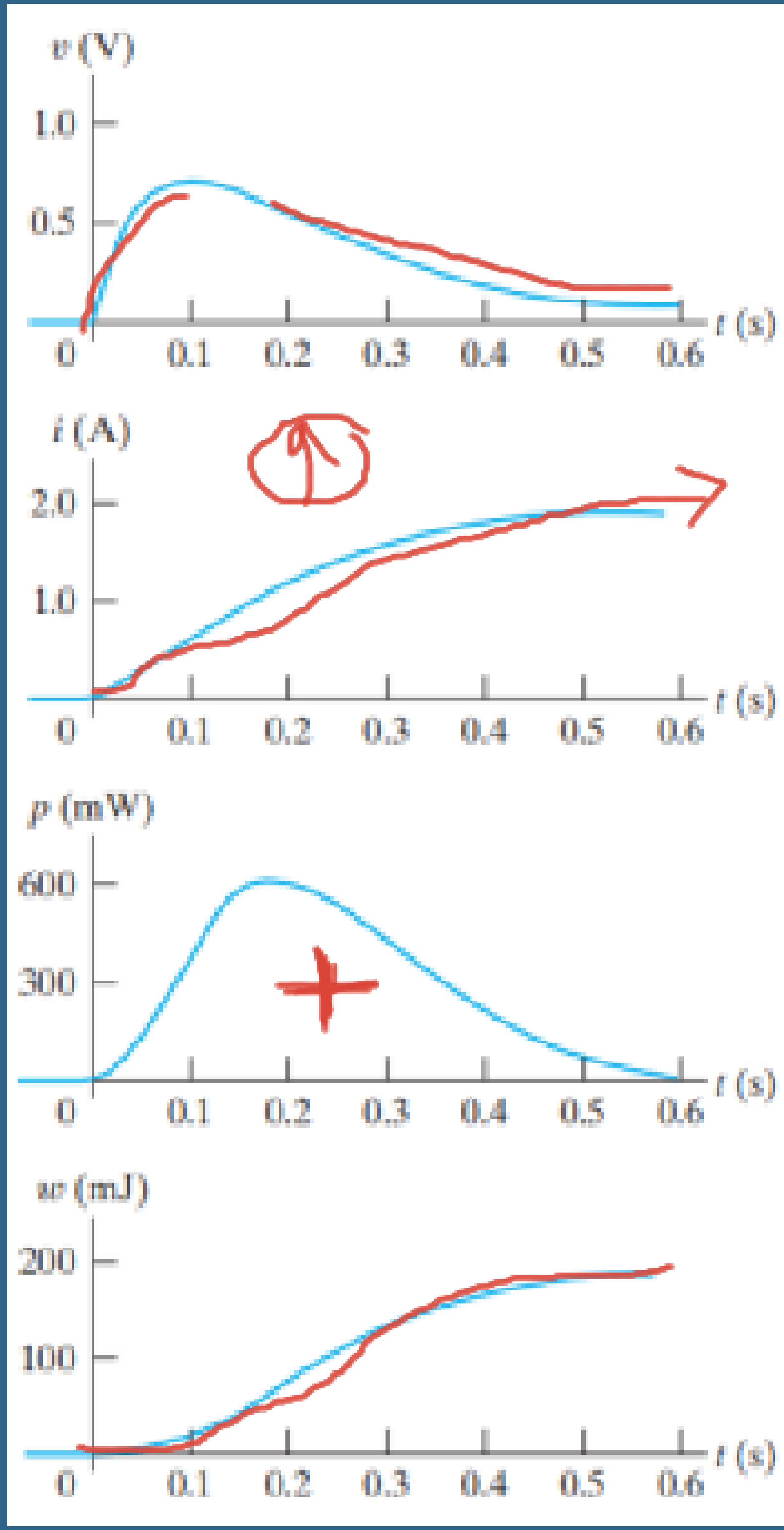
$p = vi$

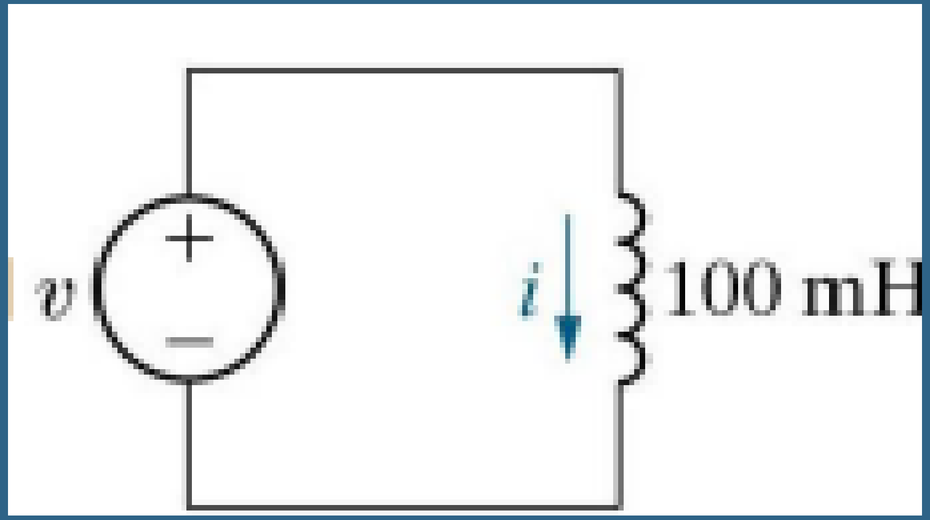
Energy




i
+
v
−
100 mH


i (mA)
800
400
t (s)
v (V)
1.0
0.5
−0.5
+
p (mW)
200
100
Σp = 0
w (mJ)
30
15
0
0.2
0.4
0.6
0.8
1.0


v (V)
1.0
0.5
i (A)
2.0
1.0
p (mW)
600
300
+
w (mJ)
200
100
t (s)
0
0.1
0.2
0.3
0.4
0.5
0.6


+
v
−
i
100 mH