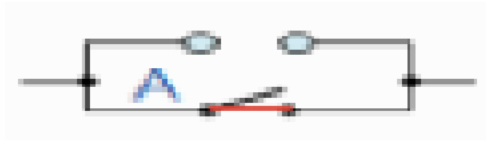
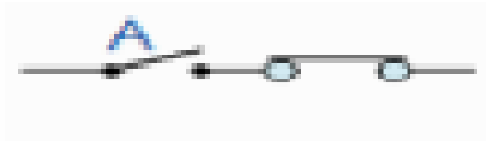
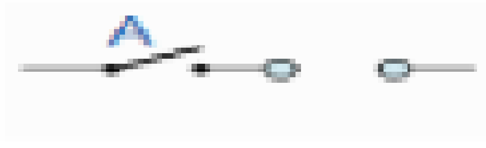
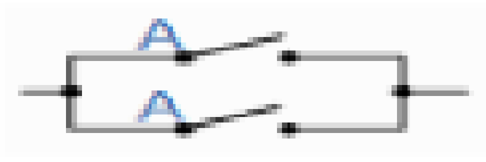
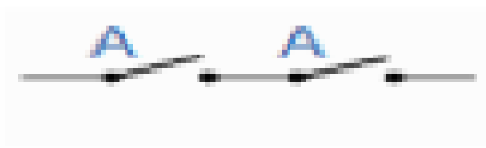
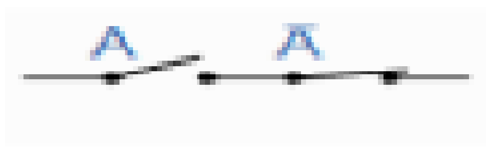
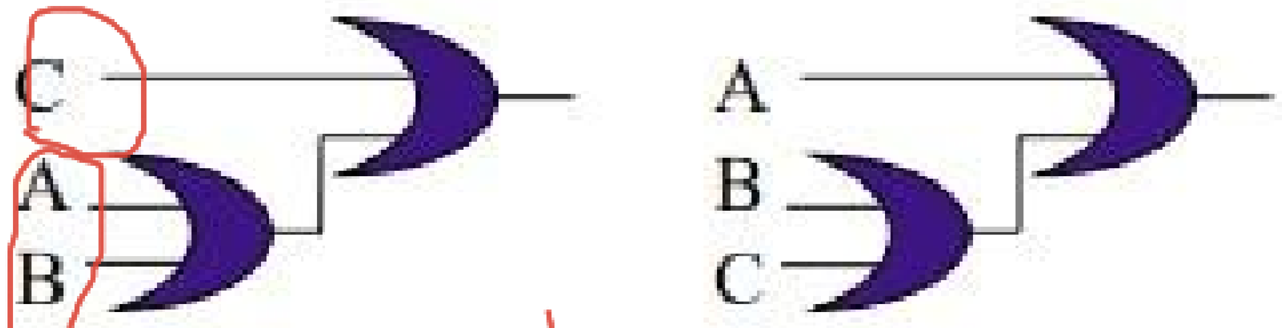
Boolean Algebra Laws

Associative laws	(a · b) · c = a · (b · c)	(a +b)+c = a+(b+c)
Commutative laws	a·b = b·a	a+b = b+a
Distributive laws	a·(b+c) = a·b + a·c	a+(b·c)=(a+b)·(a+c)
Identity laws	a·1 = a	a+0 = a
Compliment laws	a·a' = 0	a + a' = 1
Double Negation Law	A`` = A	

Boolean Expression	Equivalent Switching Circuit
$A + 1 = 1$	
$A + 0 = A$	
$A . 1 = A$	
$A . 0 = 0$	
$A + A = A$	
$A . A = A$	
$A + \overline{A} = 1$	
$A . \overline{A} = 0$	
$A+B = B+A$	
$A.B = B.A$	

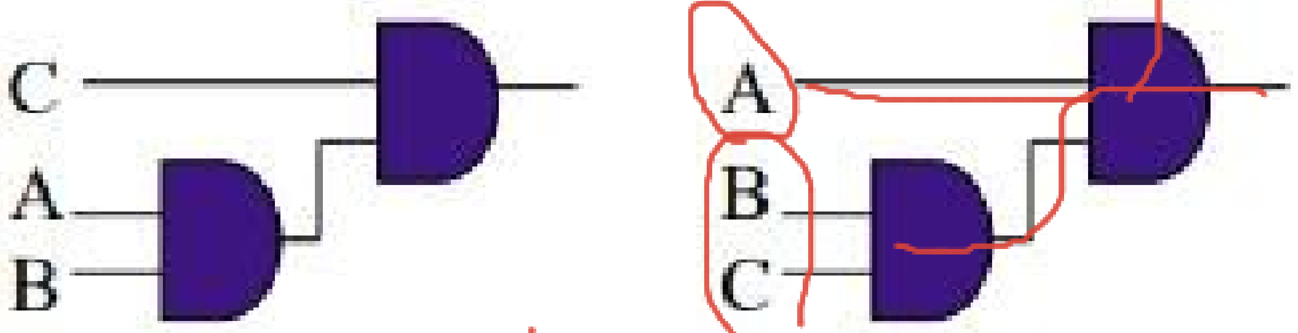
Why ?

Timing!



$(A+B)+C$ | $(B+C)+A$

delay



$(A \cdot B) \cdot C$ | $(B \cdot C) \cdot A$

Why Simplifying Boolean Algebra?

- Gate-level minimization

- Absorptive Law: This law enables a reduction in a complicated expression to a simpler one by absorbing like terms.

$A(1+B) = A$

1- A + (A.B) $\rightarrow A \cdot 1 + (A \cdot B) \Rightarrow A \cdot (1+B) + A \cdot B$

2- A(A + B) $\rightarrow A + AB = A$

$\hookrightarrow A + AB + \cancel{AB} = A + AB$

$A(\cancel{1+B}) = A$

3- A + A`B

$A \cdot 1 + \bar{A}B = A(1+B) + \bar{A}B$

$A + AB + \bar{A}B = A + (\cancel{A + \bar{A}})B$

$= \boxed{A+B}$

Prove that: $A + A`B = A + B$

- Simplify the following Boolean expression:
- F(A, B, C, D) = A'BCD + AB'CD' + AB'CD + ABC'D + ABCD' + ABCD

$$\bar{A}BCD + ABCD = (\bar{A}+A)BCD = BCD$$

$$ABD$$

$$A\bar{B}C\bar{D} + A\bar{B}CD + ABC\bar{D} + ABCD$$

$$AC\bar{D} \qquad ACD$$

$$AC$$

$$AC + BCD + ABD = AC + BD(C+A)$$

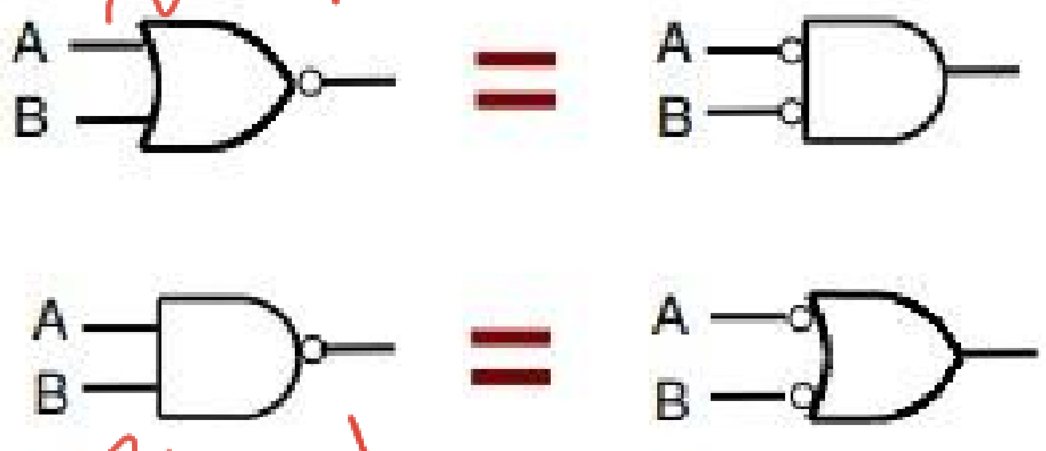
DeMorgan's Theorem

- The complement of the product is the sum of the complements.
- The complement of the sum is the product of the complements.

$\overline{A+B} \equiv \bar{A} \cdot \bar{B} \rightarrow$ Truth Table

$\overline{A \cdot B} \equiv \bar{A} + \bar{B} \rightarrow$ " "

NOR



Nand

$$Y = \overline{(A+\overline{BD})\overline{C}} = \overline{(A + \overline{BD})} + \overline{\overline{C}}$$

$$= (\bar{A} \cdot \overline{\overline{BD}}) + C$$

$$= \boxed{(\bar{A} \cdot B \cdot D) + C}$$

$$Y = \overline{(\overline{ACE}+\overline{D}) + B}$$

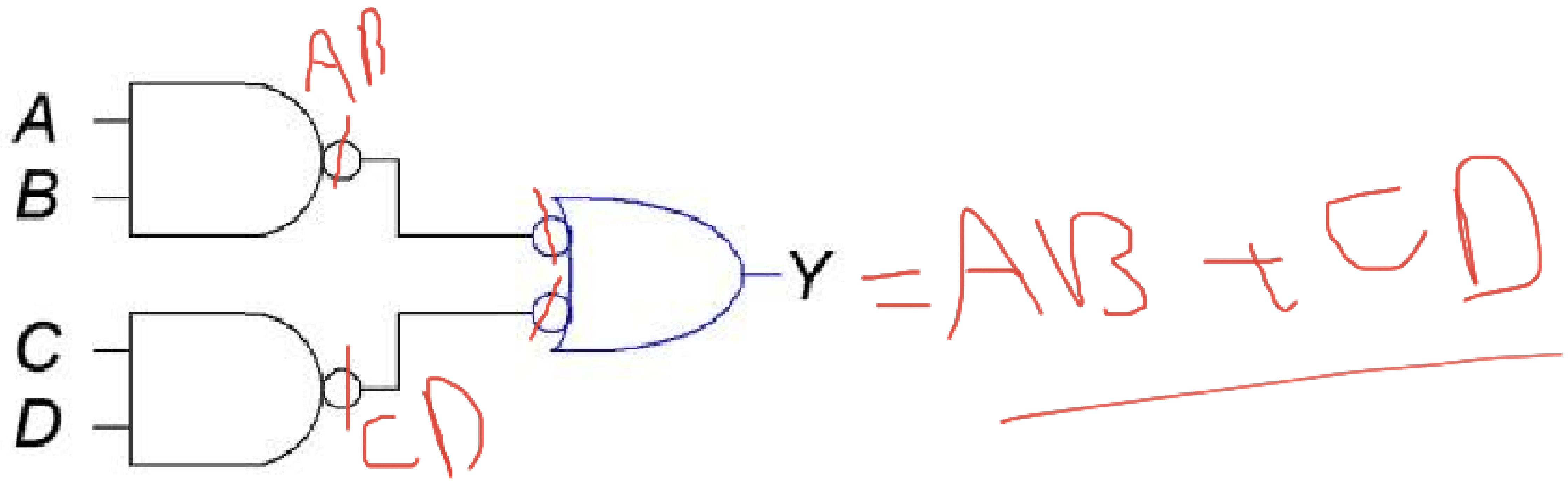
$$= \overline{(\overline{A}CE+\overline{D})}\cdot\overline{B}$$

$$= (\overline{\overline{A}CE}\cdot\overline{\overline{D}})\cdot\overline{B}$$

$$= ((\overline{\overline{A}}+\overline{C}+\overline{E})\cdot D)\cdot\overline{B}$$

$$\boxed{(A+\overline{C}+\overline{E})\cdot D\cdot\overline{B}}$$

- What is the Boolean expression for this circuit?



Consensus Theorem

A B C

AB + A`C + BC = AB + A`C ==>ANDing form

Verify Using Truth Table
==> They must be the same

Proof:

$(A+\bar{A})$

$AB + \bar{A}C + BC \cdot 1$

$AB + \bar{A}C + ABC + \bar{A}BC$

$\bar{A}C(1+B)$

$AB + \bar{A}C$

(A + B)(A` + C)(B + C) = (A + B)(A` + C) ==> ORing form

$(AB + AC + B + BC)(\bar{A} + C)$

$(B + AB + AC)(\bar{A} + C)$

$(B + AC)(\bar{A} + C)$

$B(\bar{A} + C) + AC(\bar{A} + C)$

$A\bar{A} + AC$

$AC + A\bar{A}$

$B(\bar{A} + C) + A(\bar{A} + C)$

$(\bar{A} + C)(A + B)$

K-map:
a graphical representation used in digital design and logic optimization to simplify Boolean algebra expressions and reduce the number of logic gates required.
K-maps are particularly useful for minimizing logic functions.

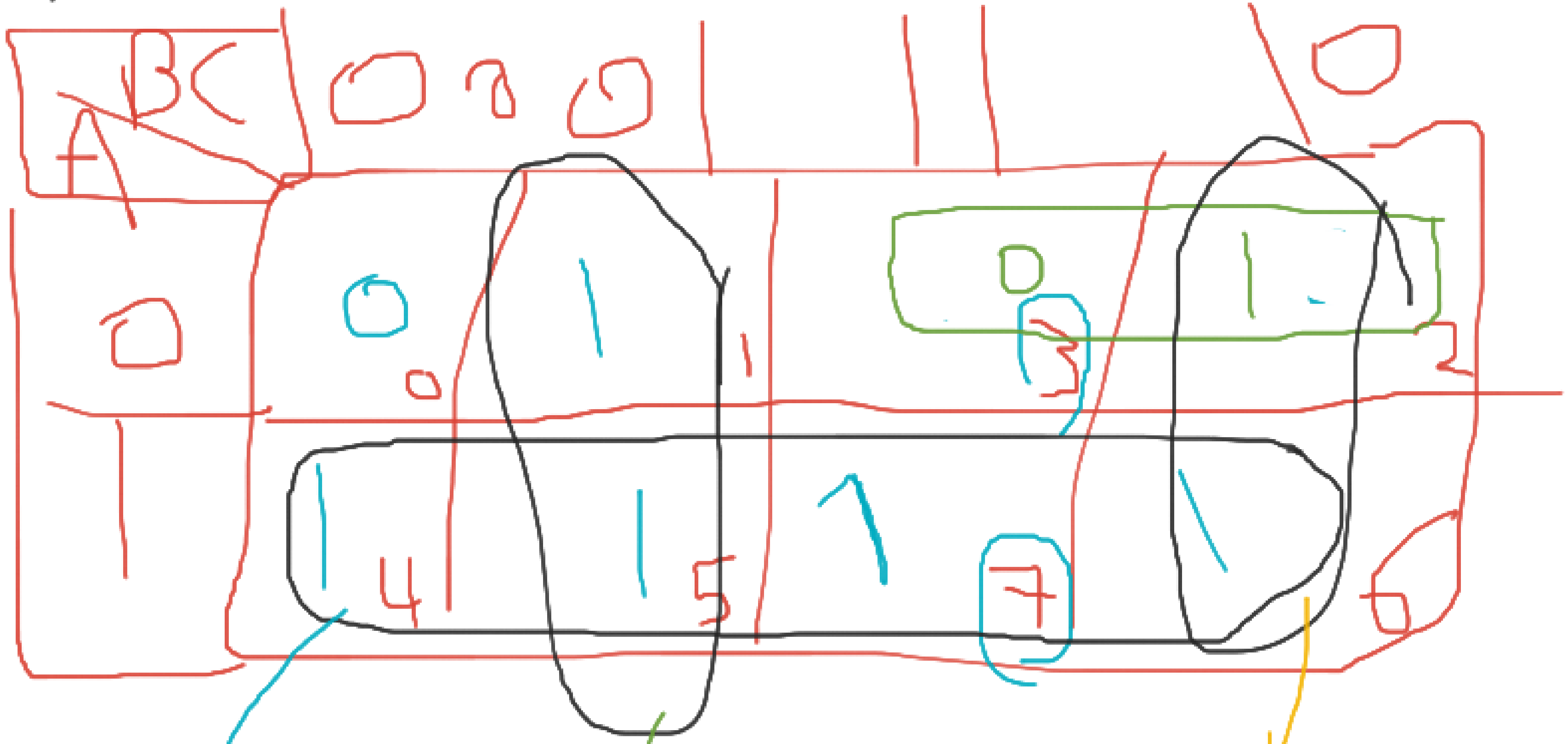
Gray code ensures that only one variable/bit changes between each pair of adjacent cells.

10 → 01

10 — 00
10 — 11

Rules:
- Every 1 must be circled at least once
- Each circle must span a power of 2 (i.e. 1, 2, 4) squares in each direction
- Each circle must be as large as possible
- A circle may wrap around the edges
- A “don't care” (X) is circled only if it helps minimize the equation

1,2,4,~~7~~

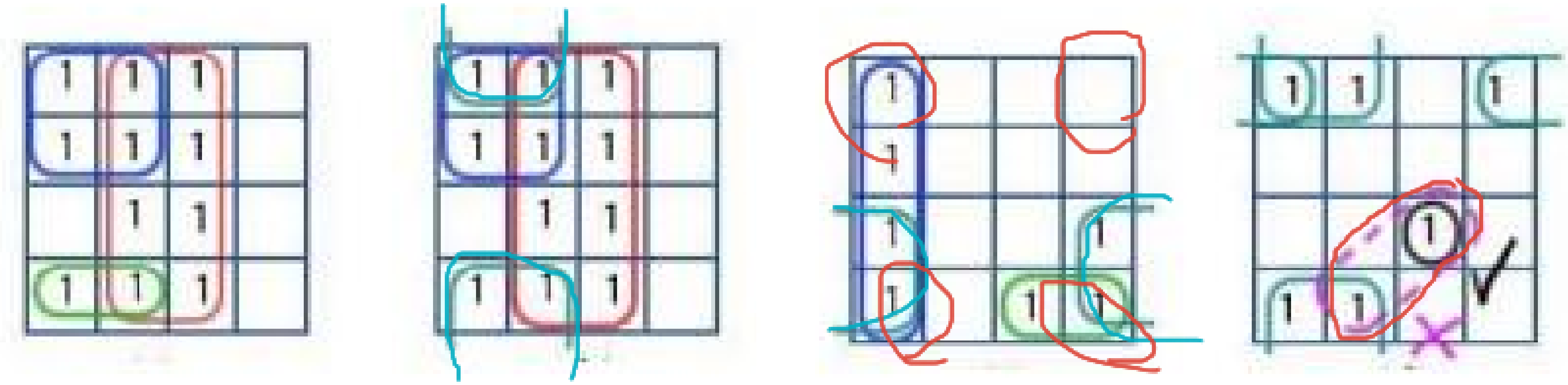


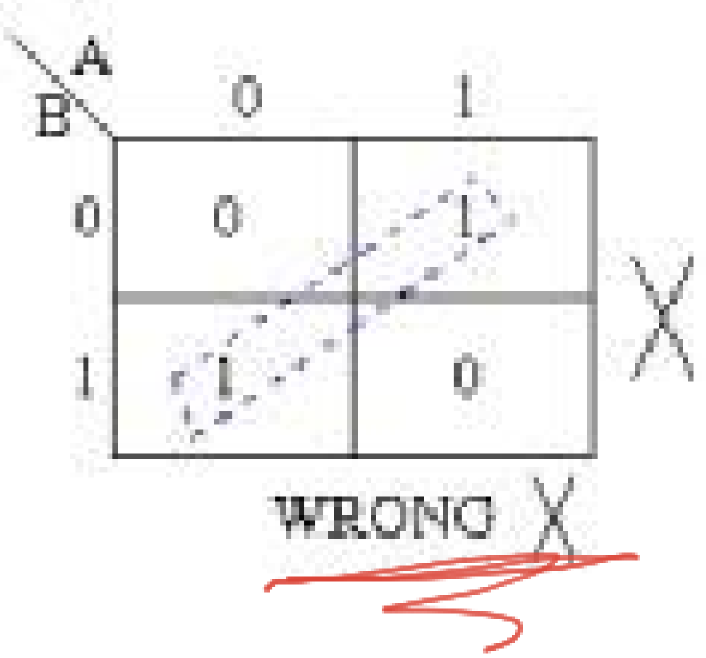
	A	B	C	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$A + \bar{B}C + B\bar{C}$

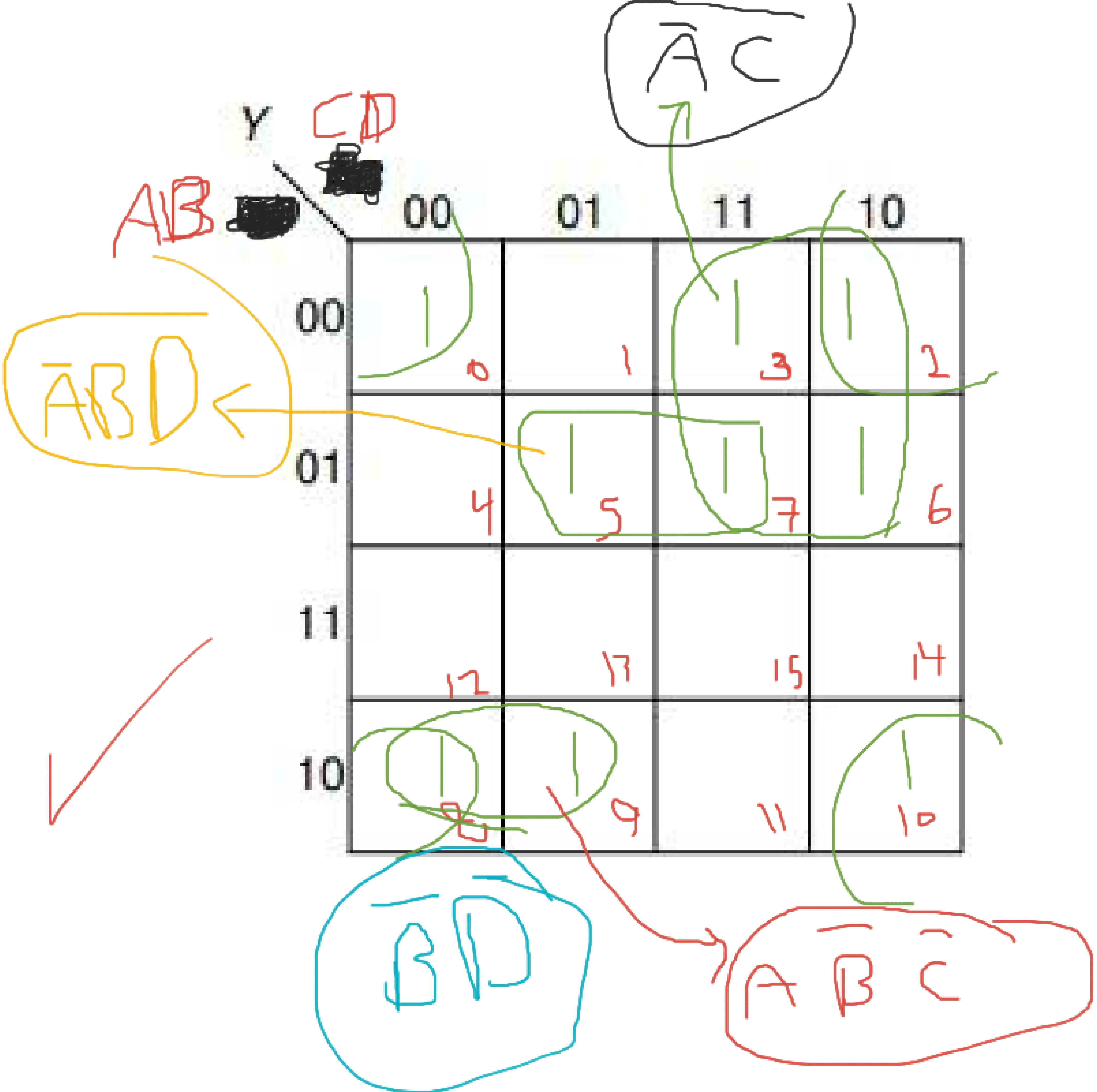
$A + (B \oplus C)$

Notes





	A	B	C	D	Y
0	0	0	0	0	1
	0	0	0	1	0
	0	0	1	0	1
	0	0	1	1	1
	0	1	0	0	0
	0	1	0	1	1
	0	1	1	0	1
	0	1	1	1	1
	1	0	0	0	1
	1	0	0	1	1
	1	0	1	0	1
	1	0	1	1	0
	1	1	0	0	0
	1	1	0	1	0
	1	1	1	0	0
15	1	1	1	1	0



- don't cares can be treated as1s or 0s
- depending on which is more advantageous

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

$A + C + \bar{B}\bar{D}$

C

Y (AB \ CD)	00	01	11	10
00	1		1	1
01		X	1	1
11	X	X	X	X
10	1	1	X	X

A

$\bar{B}\bar{D}$