Arithmetic Operations

1- Addition

base 2

Binary

	Base 5	Hexa
1 0 1 1 0	3 4 2	D A 1
+	+	+
1 0 0 1 1	1 2 4	6 F
101001	1021	E10

9
4
3

6

10
11

2- Subtraction

1 0 1 1 0
-
1 0 0 1 1

⇒ 10110
01100 → 10110
01101 +
100011

3- Multiplication

```
    1 0 1 1 0
  *     1 0 1
-------------
    1 0 1 1 0
  0 0 0 0 0
1 0 1 1 0
-------------
1 1 0 1 1 1 0
```

4- Division

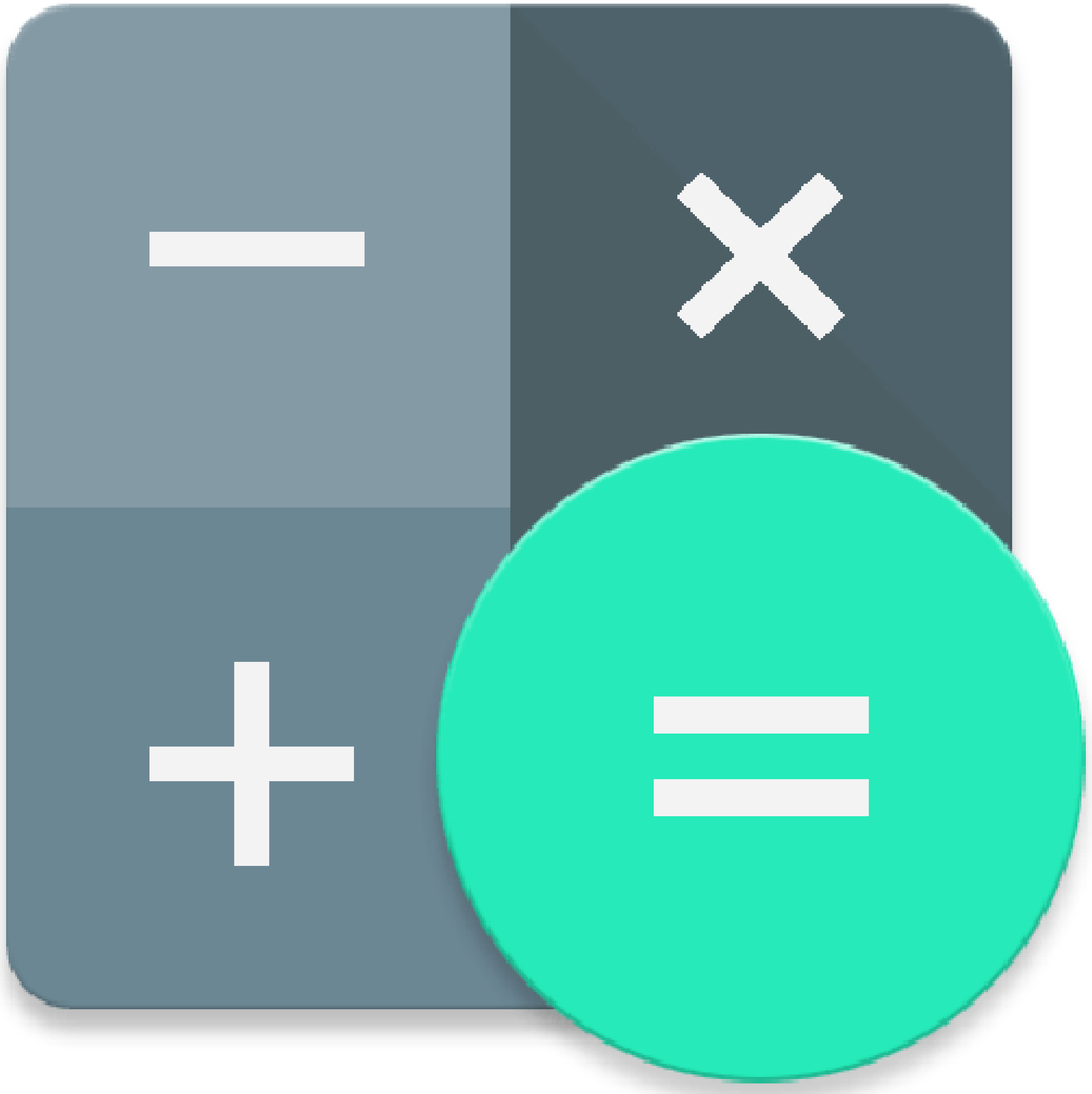
1100010 / 101

```
          0 0 1 0 0 1 1
        ---------------
5  1 0 1 | 1 1 0 0 0 1 0
           1 0 1
         -------
           0 0 1 0
               0 0
         -------
             1 0 0
             0 0 0
         -------
             1 0 0 1
               1 0 1
         -------
               1 0 0 0
               1 0 1
```

011

exercise 10000111 / 101

result is 11011 and reminder is 0



Logic Gates

Why Logic Gates ?

- We send information through computers using wires that represent 1s and 0s.

- Computers need a way to manipulate those 1s and 0s.

- Computers use logic gates to transform the 1s and 0s from input wires.

ex: Calculator

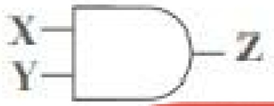
+ * / −

*

AND

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

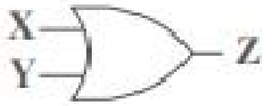
$Z = X \cdot Y$



OR

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

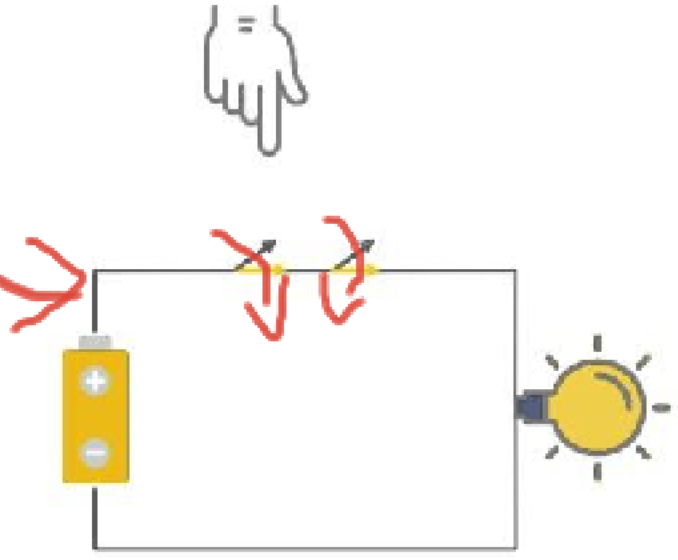
$Z = X + Y$



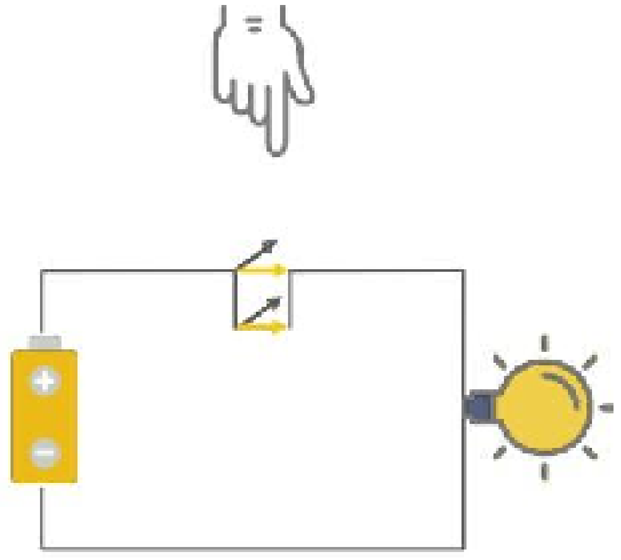
NOT

X	Z
1	0
0	1

$\overline{X} = Z$

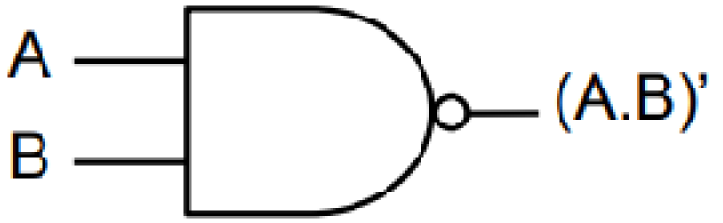


AND



OR

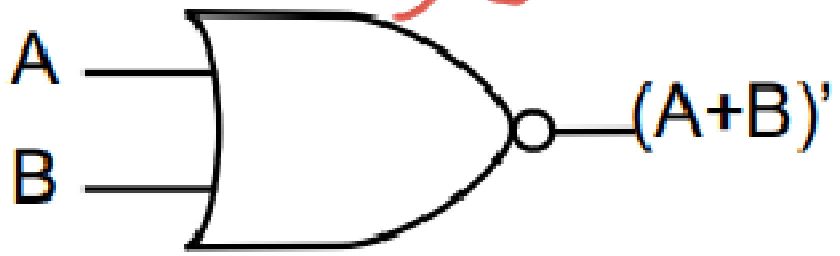
NAND



Truth Table

A	B	(A.B)'
0	0	1
0	1	1
1	0	1
1	1	0

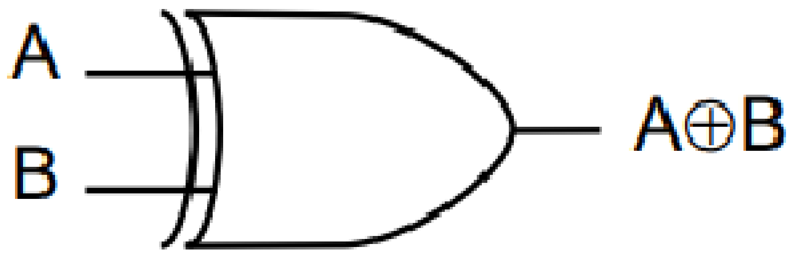
NOR



Truth Table

A	B	(A+B)'
0	0	1
0	1	0
1	0	0
1	1	0

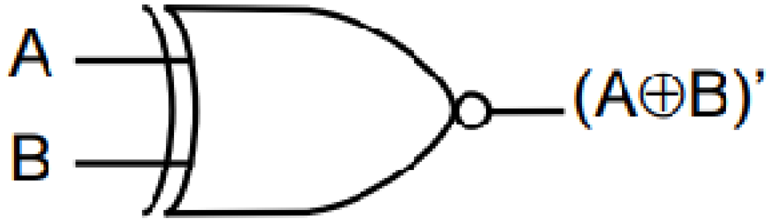
XOR



Truth Table

A	B	(A⊕B)
0	0	0
0	1	1
1	0	1
1	1	0

XNOR



Truth Table

A	B	(A⊕B)'
0	0	1
0	1	0
1	0	0
1	1	1

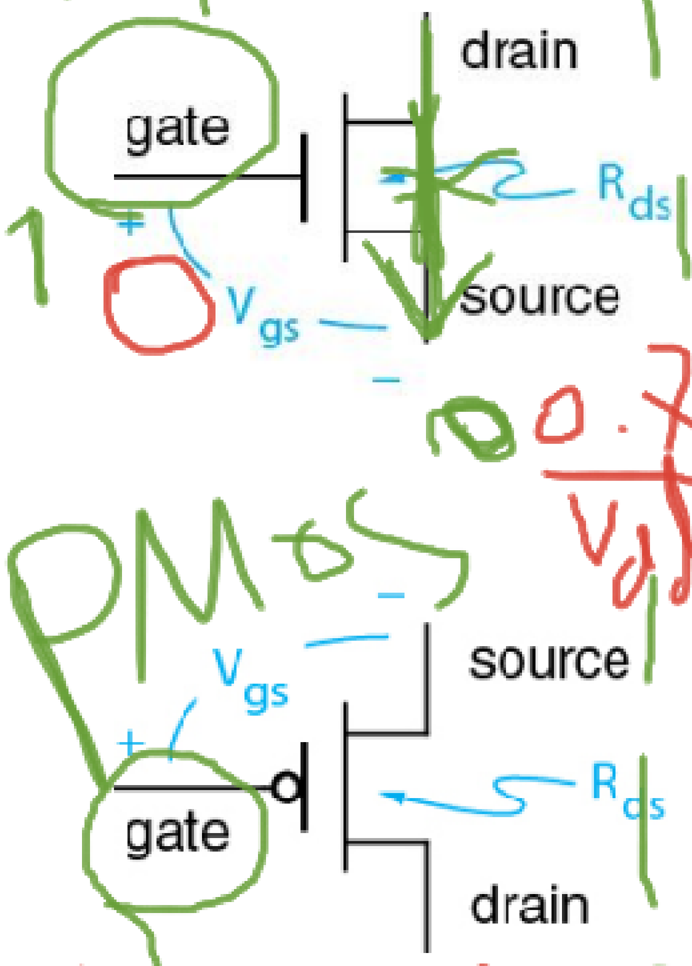
Subtraction

$$A - B = \boxed{A + \bar{B} + 1}$$

Transistor Level

NMos

pull down



PMos

pull up

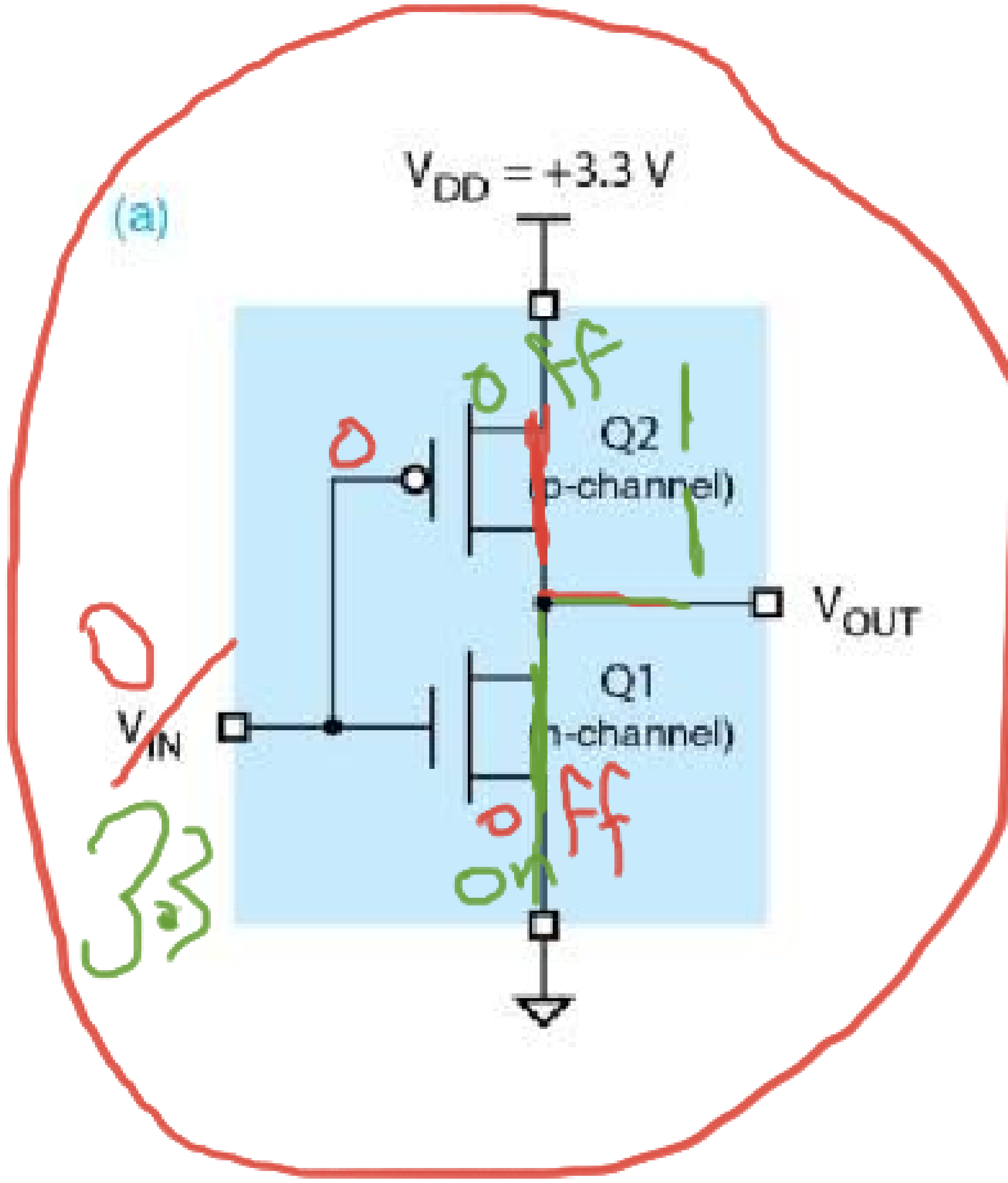
0.7

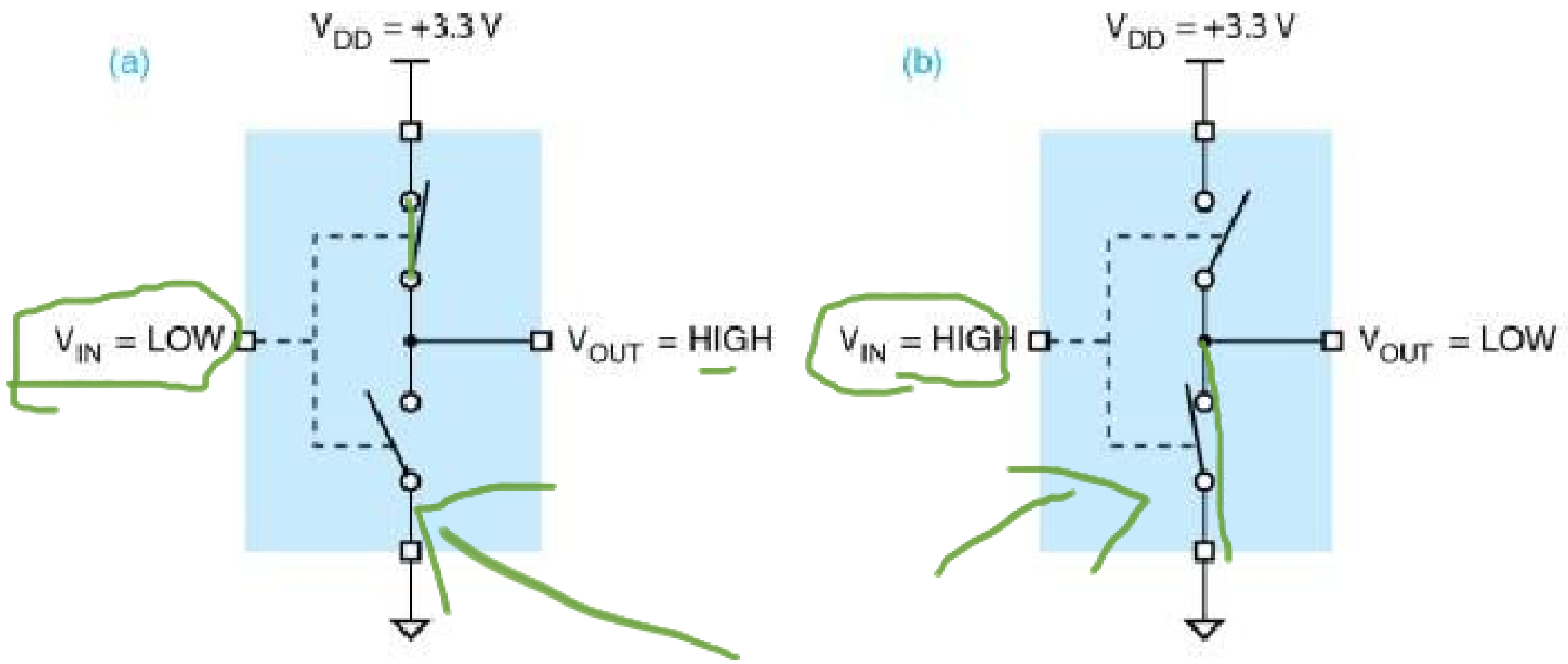
$V_{dd} - V_{th}$

0.2

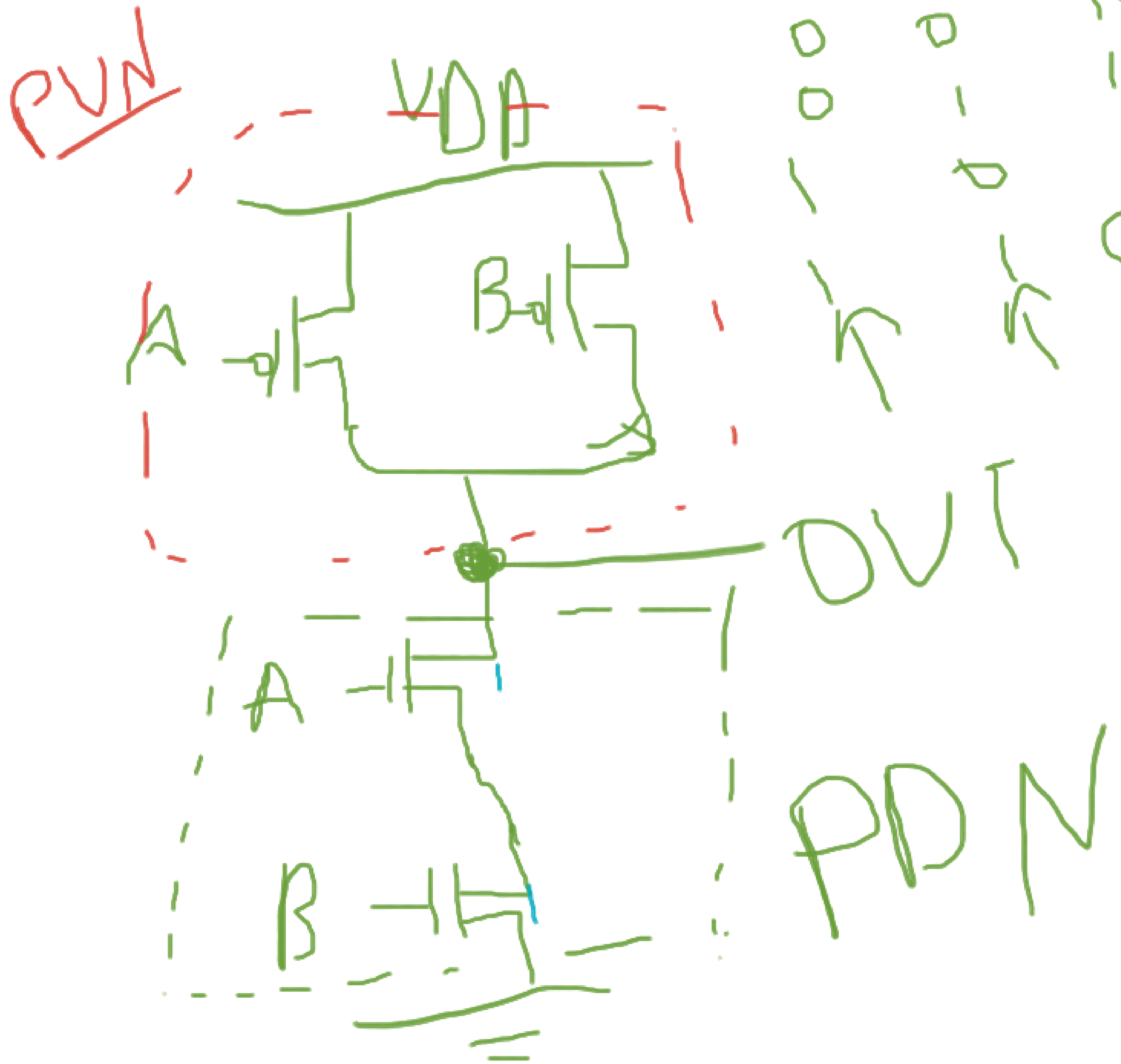
Vin	Q1	Q2	Vout
0	off	on	3.3V
Vdd 3.3	on	off	0

NOT

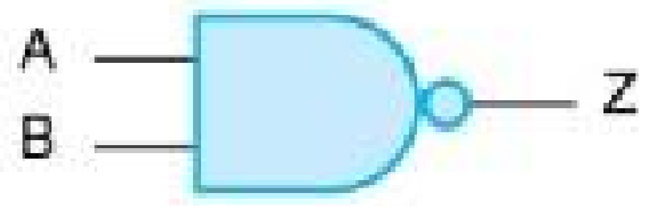




Draw Transistor Level



A	B	Q1	Q2	Q3	Q4	Z
LOW	LOW	off	on	off	on	HIGH
LOW	HIGH	off	on	on	off	HIGH
HIGH	LOW	on	off	off	on	HIGH
HIGH	HIGH	on	off	on	off	LOW



Draw Transistor Level

(b)

A	B	Q1	Q2	Q3	Q4	Z
LOW	LOW	off	on	off	on	HIGH
LOW	HIGH	off	on	on	off	LOW
HIGH	LOW	on	off	off	on	LOW
HIGH	HIGH	on	off	on	off	LOW

(c)

Boolean Algebra

the mathematics associated with binary numbers

- 3 different ways to represent logic functions:

1. Equation: a mathematical representation of a logic function

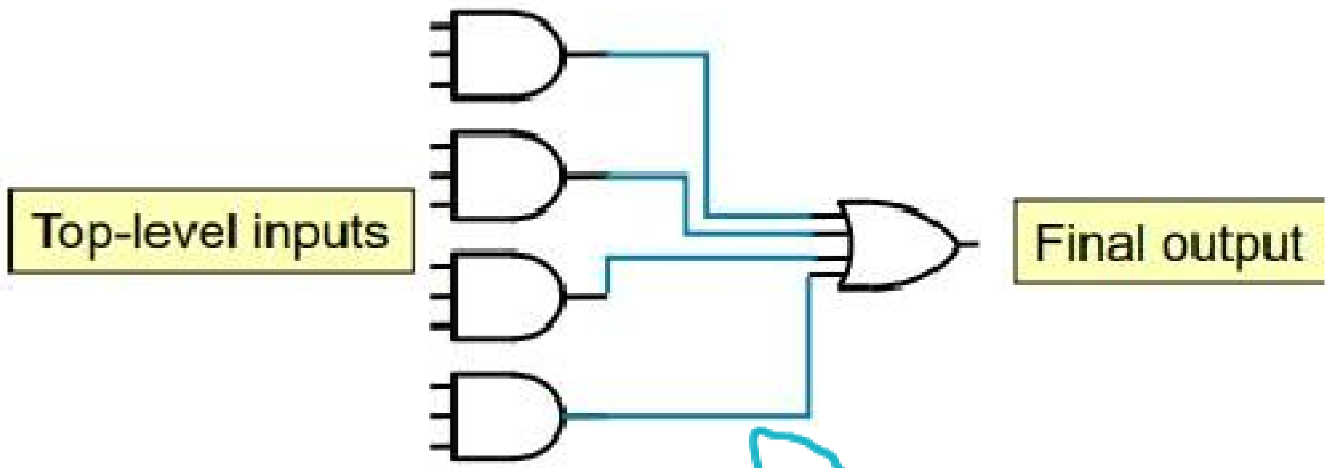
$$out = \bar{s}a\bar{b} + \bar{s}ab + s\bar{a}b + sab$$

Final logic output

Each letter variable represents a top-level input to the logic function

Mathematical operations (i.e. addition and multiplication) are boolean algebra operations

2. Gates: a visual block representation of the function



3. Truth Table: indicates what the output will be for every possible input combination

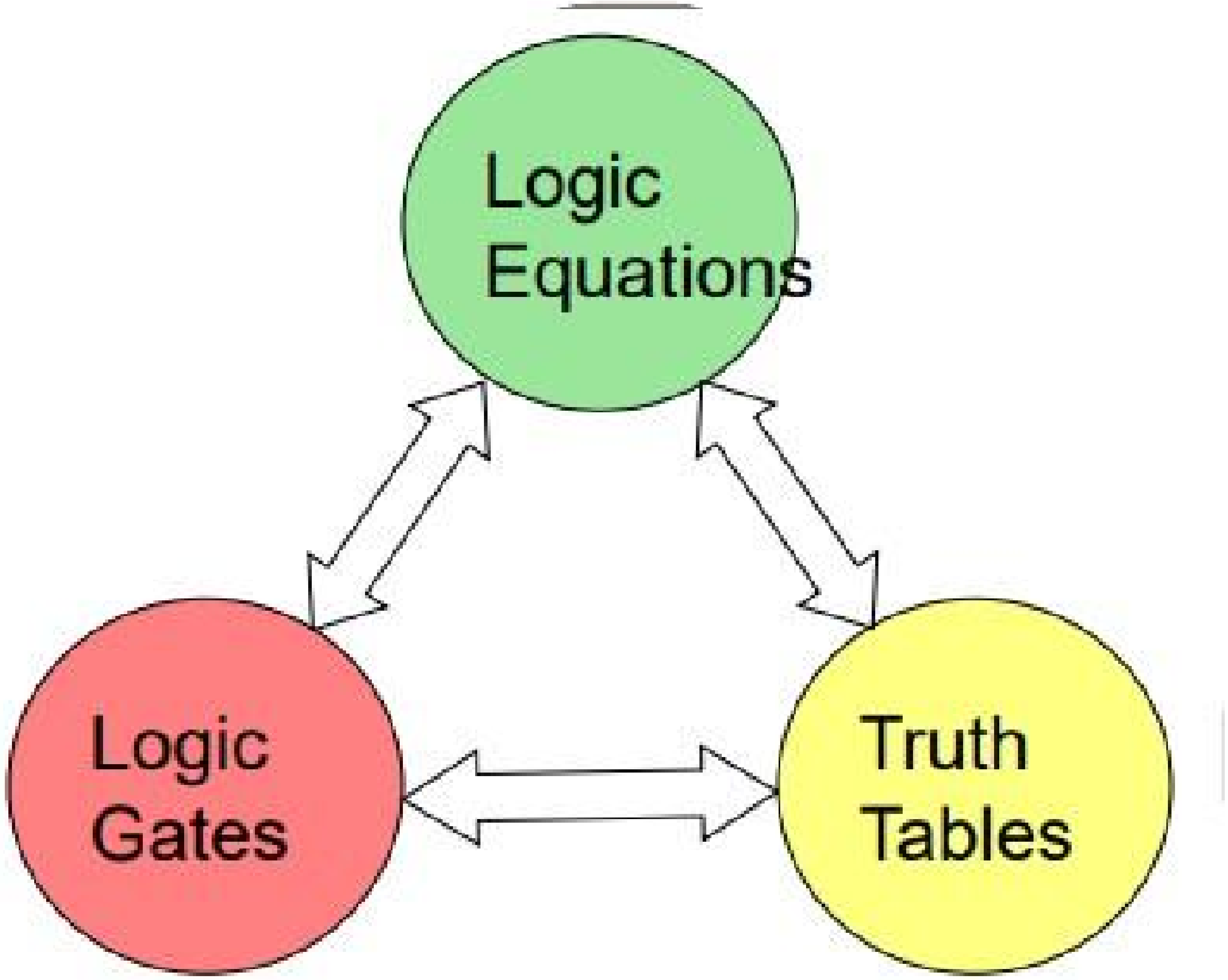
2^n

If there are n inputs (left-hand columns) there will be 2^n entries (rows) in the table
EX: 3 inputs require $2^3 = 8$ rows

	A	B	C	Z
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
	1	0	0	0
	1	0	1	1
	1	1	0	1
7	1	1	1	1

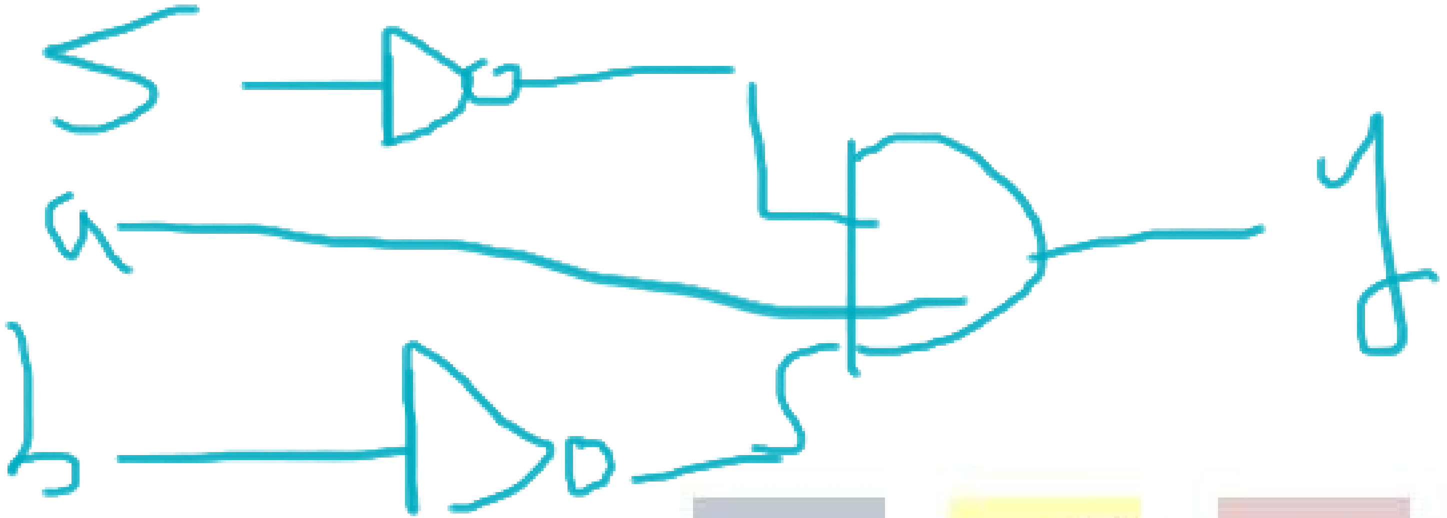
There will always be at least one output (right-hand columns)

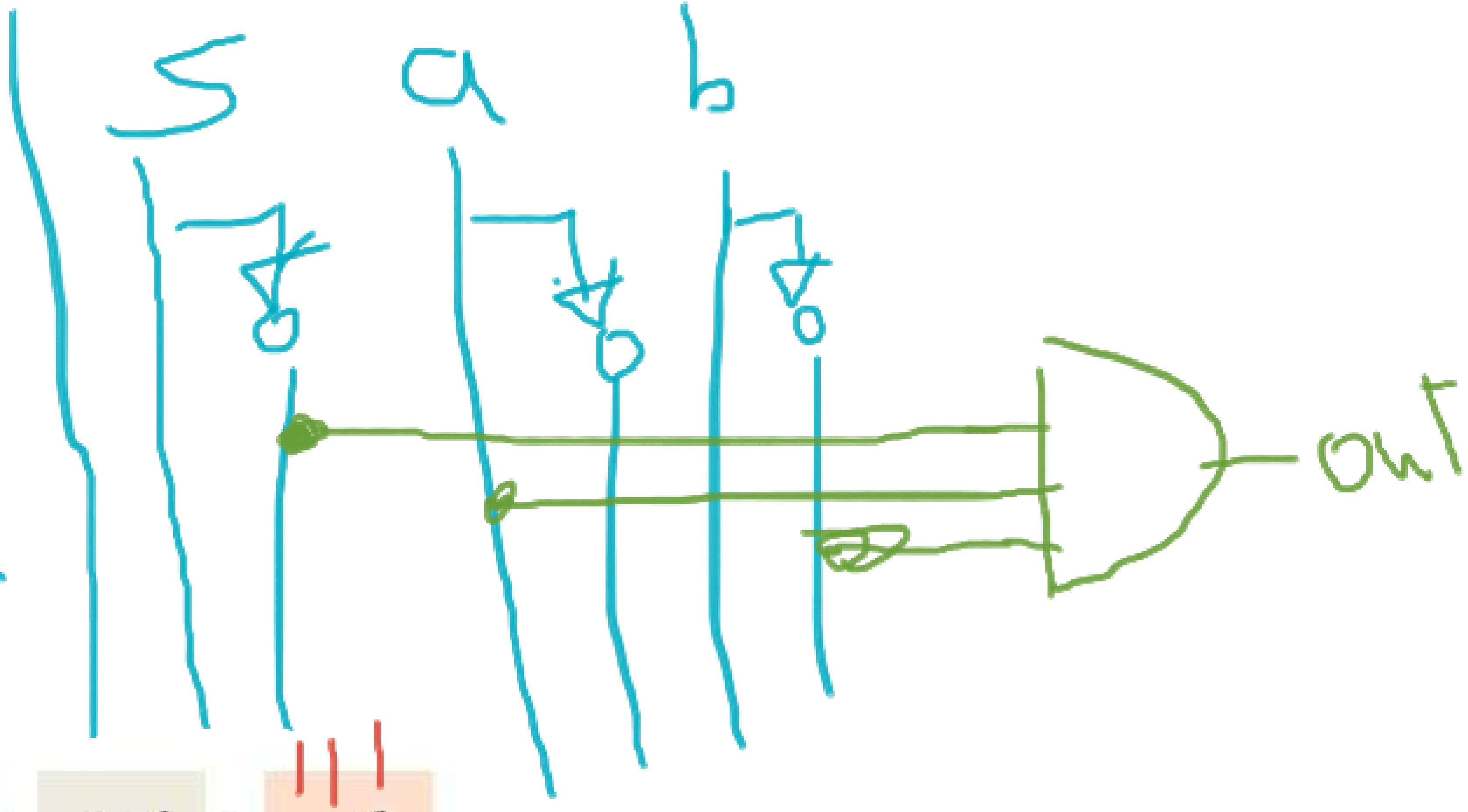
For each input combination (row) outputs will be either 0 or 1


Logic
Equations
Logic
Gates
Truth
Tables

1- Equations to Gates

$$\boxed{y=\bar{s}\cdot a\cdot\bar{b}}$$





$$out=\underset{010}{\bar{s}a\bar{b}}+\underset{011}{\bar{s}ab}+\underset{101}{s\bar{a}b}+\underset{111}{sab}$$

2- Equations to Truth Tables

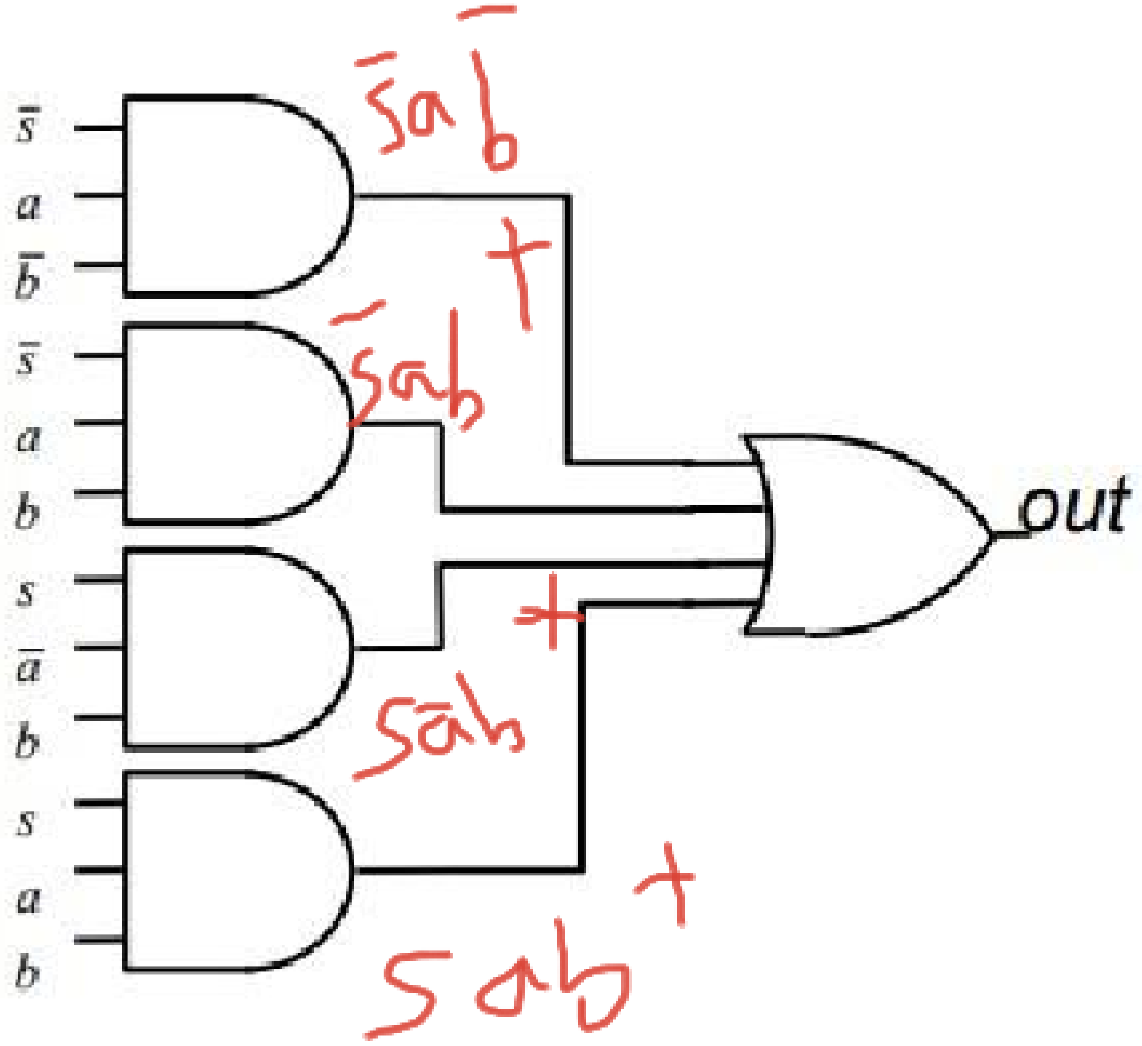
8

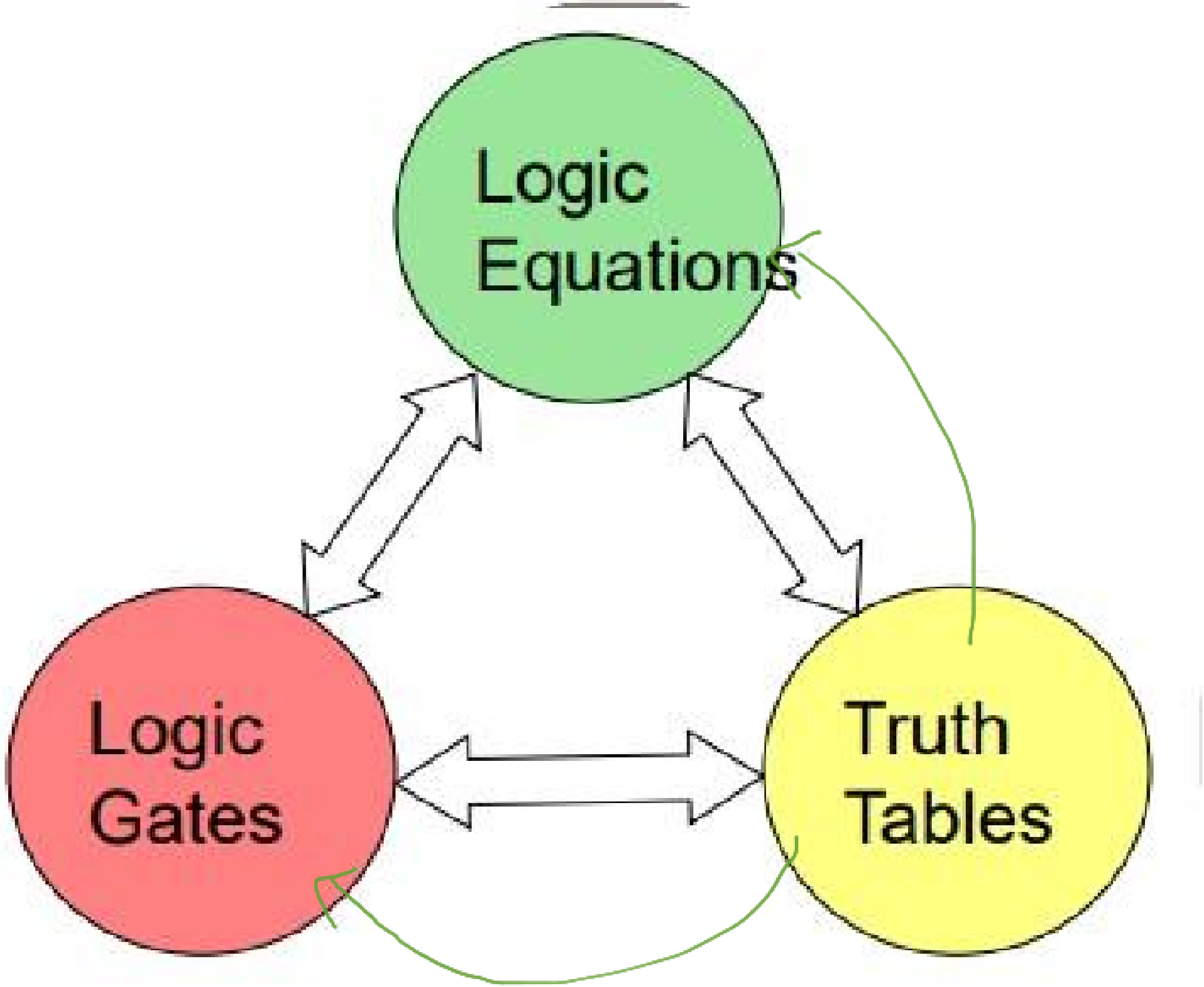
s	a	b	out
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

3- Gates to Equations

4- Gates to Truth-Table

s	a	b	out
0	0	0	
1	1	1	




Logic
Equations
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Truth
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5- ~~Truth Table to Equations~~ ==> ~~SOP Vs POS~~

S	A	B	OUT
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

SOP : 0 → False
1 → True

$$(\bar{S}.A.\bar{B}) + (\bar{S}AB) + (S\bar{A}B) + (S.A.B)$$

POS : 1 → False
0 → True

$$(S+\bar{A}+B).(S+\bar{B}+\bar{A}).(\bar{S}+A+\bar{B}).(\bar{S}+\bar{A}+\bar{B})$$

Give a truth table and a standard sum of products expression that describes:

- F = A xor B xor C

SoP

0 → F

1 → T

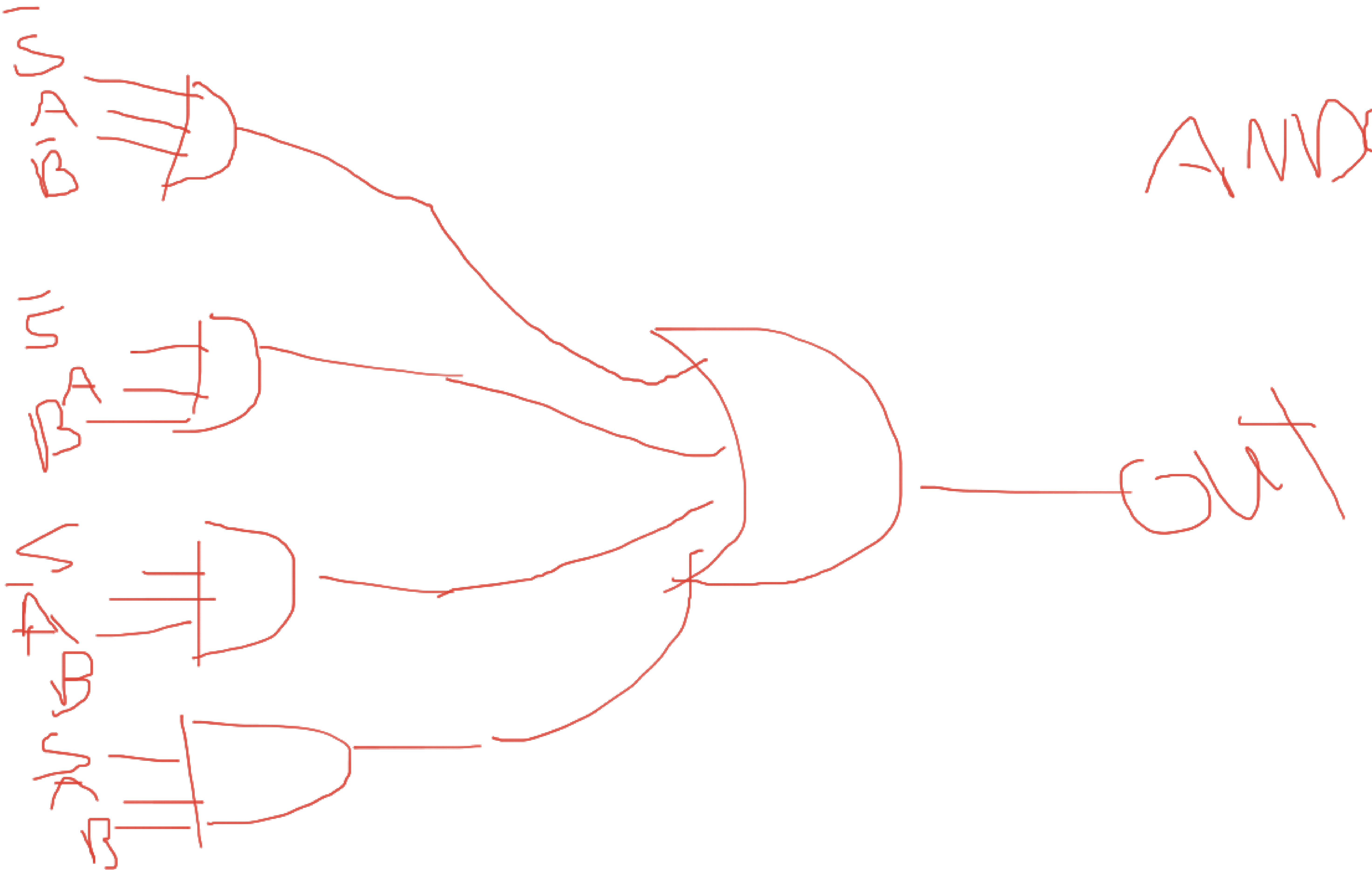
1's odd → out = $\boxed{1}$

// even → // = $\boxed{0}$

$(\bar{A}\bar{B}C) + (\bar{A}B\bar{C}) + (A\bar{B}\bar{C}) + (ABC)$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

6- Truth Tables to Gates



S	A	B	OUT
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

or

Equation to Gates: A`B`C`+A B`C`+ ABC