(Assignment #1)

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$\boxed{1.6}$ 10 marbles, red $\to \mu$, $\mu = 0.05$, $\mu = 0.5$, $\mu = 0.8$, $(\nu = 0)$

(a) $\mu = 0.05$

$p(\text{not red}) = (1-0.05)^{10} = \boxed{0.59874}$

$\mu = 0.5 \to p(\text{not red}) = (1-0.5)^{10} = \boxed{9.77 * 10^{-4}}$

$\mu = 0.8 \longrightarrow p(\text{not red}) = (1-0.8)^{10} = \boxed{1.02 * 10^{-7}}$

(b) We want to Calculate the probability of having at least one Sample with no red marbles

let $X \to$ the event of @ least having one Sample with Zero red marbels

$p(X) = 1 - p(\bar{X})$ where $\bar{X} \to$ here represent having no Sample that is free from red marbles

→ To get a certain number of Success out of n trials, we will use the binomial distribution

$p(K) = \binom{n}{k} p^{k} (1-p)^{n-k}$ $k \to$ # of Success, $p \to$ probability of a Success

$p(X) = 1 - p(\bar{X})$,

$= 1 - \cancel{\binom{n}{0}} (\cancel{p(\text{no red})})^{0} . (1 - p(\text{no red}))^{n}$

$= 1 - (1 - p(\text{no red}))^{n}$

$\mu = 0.05 \longrightarrow p(X) = 1 - (1 - 0.59874)^{1000} \approx \boxed{1}$

$\mu = 0.5 \longrightarrow p(X) = 1 - (1 - 9.77 * 10^{-4})^{1000} \approx \boxed{0.624}$

$\mu = 0.8 \longrightarrow p(X) = 1 - (1 - 1.02 * 10^{-7})^{1000} \approx \boxed{1.02 * 10^{-4}}$.

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ⓒ Assume that X is the event of at least having one sample with zero red marbles

$p(X) = 1 - p(\bar{X})$

$= 1 - \binom{n}{0} (p(\text{no red}))^0 \, p\,(1 - p(\text{no red}))^n$

$= 1 - (1 - p(\text{no red}))^n$

$\mu = 0.05 \rightarrow \therefore p(X) = 1 - (1 - 0.59874)^{1{,}000{,}000} \approx \boxed{1}$

$\mu = 0.5 \rightarrow p(X) = 1 - (1 - 9.77 * 10^{-4})^{1{,}000{,}000} \approx \boxed{1}$

$\mu = 0.8 \rightarrow p(X) = 1 - (1 - 1.02 * 10^{-7})^{1{,}000{,}000} \approx \boxed{0.0973}$

$\infty \infty \infty \infty \infty \infty \infty \infty \infty$

$\boxed{2\text{-}5}$ prove by induction: $\sum_{i=0}^{D} \binom{N}{i} \le N^D + 1$, hence $m_H(N) \le N^{d_{vc}} + 1$

① for $N = 1$ & $D = 0$ $\therefore \sum_{i=0}^{D} \binom{N}{i} = \binom{1}{0} = 1 \le N^D + 1 \le 1^0 + 1 \le 2$ ✓

for $N = 1$ & $D = 1$ $\therefore \sum_{i=0}^{D} \binom{N}{i} = \binom{1}{0} + \binom{1}{1} = 2 \le N^D + 1 \le 1^1 + 1 \le 2$ ✓

$\therefore$ The inequality holds for the base case # Ⓘ

② Assume $\sum_{i=0}^{D} \binom{N}{i} \le N^D + 1$ $\qquad \sum_{i=0}^{D} \binom{N+1}{i} \le (N+1)^D + 1$?

using: $\binom{N}{i-1} + \binom{N}{i} = \binom{N+1}{i}$

$$\rightarrow \sum_{i=0}^{D} \binom{N+1}{i} = \sum_{i=0}^{D} \left[\binom{N}{i} + \binom{N}{i-1} \right] = \sum_{i=0}^{D} \binom{N}{i} + \sum_{j=0}^{D-1} \binom{N}{j}$$

$$\le N^D + 1 + N^{D-1} + 1$$

$$\le (N^D + N^{D-1} + 1) + 1$$

$$\le \boxed{(N+1)^D} + 1$$

Still valid as we removed a term & replaced it with a bigger term
$(N+1)^D = N^D + D N^{D-1} + \text{---}$

$\therefore \sum_{i=0}^{D} \binom{N}{j} \le N^D + 1$

$\because m_H(N) \le \sum_{i=0}^{d_{vc}} \binom{N}{i}$ & $\sum_{i=0}^{d_{vc}} \binom{N}{i} \le N^{d_{vc}} + 1$

$$\boxed{\therefore m_H(N) \le N^{d_{vc}} + 1} \;\#$$