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Section: 2 B.N: 12

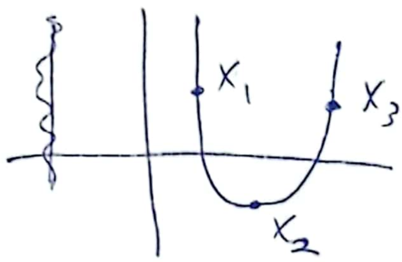
(Assignment #2)

problem $\boxed{2.16}$: $H = \{ h_c(x) = \text{sign}\left( \sum_{i=0}^{D} c_i x^i \right) \}$

(a) To solve $\sum_{i=0}^{D} c_i x^i$ we need $(D+1)$ points.
The resulting polynomial will pass through all of these points
So choosing $(D+1)$ points & Assigning y value with the
~~Sign we can need (+1/-1)~~
Sign we need $(+1/-1)$.
After solving → we get coeff. that produce the signs
on $(D+1)$ points. So H shatters $(D+1)$ points

ex: for $D=2$

points $\left.\begin{matrix}(x_1, +1)\\(x_2, -1)\\(x_3, +1)\end{matrix}\right\}$ solve to get $c_0, c_1, c_3$




→ We choose any +ve number other than one or any -ve number other than -ve one #

ⓑ Same approach as in part ⓐ.

We can only control $(D+1)$ points.

When check $(D+2) \Longrightarrow$ extra point which its sign cannot be controlled

$\Longrightarrow$ This case cause the 2 neighbouring points will have same sign so not generating all dichotomies

ex: if $D=1$


Can not be solved by a straight line

( no other orientation we let it be solved )

so $H$ don't shatter $(D+2)$ points #

From ⓐ & ⓑ : Vc dimension of $H$ is exactly $(D+1)$

$\cancel{2\alpha X_i^2 - 2X_i}$

$\boxed{X_i^2 - \alpha X_i^2 - 2abX_i - b_i^2}$

**2.24**

ⓐ $E_{in}(g) = \sum_{i=1}^{2} (f(X_i) - h(X_i))^2 = \sum_{i=1}^{2} (X_i^2 - (aX_i + b))^2$

We need $\frac{\partial E_{in}(g)}{\partial a} = \frac{\partial E_{in}(g)}{\partial b} = 0$

$\frac{\partial E_{in}(g)}{\partial a} = -2 \sum_{i=1}^{2} X_i (X_i^2 - aX_i - b) = 0 \longrightarrow \text{I}$

$\frac{\partial E_{in}(g)}{\partial b} = -2 \sum_{i=1}^{2} X_i^2 - (aX_i + b) = 0 = -2\sum_{i=1}^{2} (X_i^2 - aX_i - b) \rightarrow \text{II}$

$\text{I} * X_1 - \text{II} * X_2$

$\therefore \left.\begin{array}{l} X_1^2 - aX_1 - b = 0 \\ X_2^2 - aX_2 - b = 0 \end{array}\right\} \Rightarrow \begin{array}{l} a = X_1 + X_2 \\ b = -X_1 X_2 \end{array}$

$\overset{P}{g}(X) = (\overbrace{X_1 + X_2}^{a}) X - \overbrace{X_1 X_2}^{b}$

$\bar{g}(X) = E_D[g^D(X)] = E_D[(X_1 + X_2) X - X_1 X_2]$

$= E_D[X_1 X] + E_D[X_2 X] - E_D[X_1 X_2]$

$= \boxed{E_D[X_1] X + E_D[X_2] X - E_D[X_1] E_D[X_2]}$ # due to independence of $X_1$ & $X_2$

$\Rightarrow$ Data follow uniform distribution $[-1, 1]$

$E_D(X_1) = 0$ & $E_D(X_2) = 0$   $E_D(X) = 0$

$\bar{g}(X) = 0 * X + 0 * X - 0 * 0 = \boxed{0}$ #

d

Variance $= E_X[E_D[(g^D(x)-\bar{g}(x))]]$

$= E_X[E_D((X_1+X_2)X - X_1X_2 - 0)^2] = E_X[E_D((X_1+X_2)X - X_1X_2)^2]$

$= E_X[E_D((X_1+X_2)^2X^2 + X_1^2X_2^2 - 2X_1X_2(X_1+X_2)X)]$

$= E_X[X^2 E_D(X_1^2+X_2^2+2X_1X_2) + E_D(X_1^2X_2^2) - 2X E_D(X_1^2X_2+X_1X_2^2$

$\Rightarrow$ Data is uniformally distributed $[-1,1]$

$\therefore E(X)=0$ & $E(X^2)=\frac{1}{3}$ & $E(X^3)=0$ $E(X^4)=\frac{1}{5}$

Variance $= E_X[X^2(\frac{1}{3}+\frac{1}{3}) + (\frac{1}{3}\cdot\frac{1}{3})]$ other terms with $X_1$ & $X_2$ yields zeros

$= E_X(\frac{2}{3}X^2+\frac{1}{9}) = \frac{2}{3}*\frac{1}{3}+\frac{1}{9} = \boxed{\frac{1}{3}}$

Bias $= E_X[(\bar{g}(x)-f(x))^2] = E_X[(0-X^2)^2] = E_X[X^4] = \boxed{\frac{1}{5}}$

$E_{out}$ = Variance + Bias $= \frac{1}{3}+\frac{1}{5} = \boxed{\frac{8}{15}}$ #

$E_X[X] = \frac{1}{2}\int_{-1}^{1} X\,dX = \frac{1}{2}\cdot\frac{X^2}{2}\Big|_{-1}^{1} = \frac{1}{2}[\frac{1}{2}-\frac{1}{2}] = \boxed{\text{Zero}}$

$E_X[X^2] = \frac{1}{2}\int_{-1}^{1} X^2\,dX = \frac{1}{2}\frac{X^3}{3}\Big|_{-1}^{1} = \frac{1}{2}[\frac{1}{3}+\frac{1}{3}] = \boxed{\frac{1}{3}}$

$E[X^3] = \boxed{\text{Zero}}$

$E[X^4] = \frac{1}{2}\int_{-1}^{1} X^4\,dX = \frac{1}{2}\frac{X^5}{5}\Big|_{-1}^{1} = \frac{1}{2}[\frac{1}{5}+\frac{1}{5}] = \boxed{\frac{1}{5}}$ #