

In [14]: *# dummy variables*

```
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
import statsmodels.api as sm
import statsmodels.formula.api as smf
```

In [15]: *#Example: automobile data*

*#https://www.kaggle.com/toramky/automobile-dataset*

```
carsdata=pd.read_csv('Automobile_data.csv')
carsdata.sample(5)
```

Out[15]:

	symboling	normalized- losses	make	fuel- type	aspiration	num- of- doors	body- style	drive- wheels	en loc
141	0	102	subaru	gas	std	four	sedan	fwd	
37	0	106	honda	gas	std	two	hatchback	fwd	
97	1	103	nissan	gas	std	four	wagon	fwd	
39	0	85	honda	gas	std	four	sedan	fwd	
203	-1	95	volvo	diesel	turbo	four	sedan	rwd	

5 rows x 26 columns

In [16]: *# categorical variables: fuel-type*

```
carsdata['city-mpg'].replace('?', np.nan, inplace= True)
carsdata['fuel-type'].replace('?', np.nan, inplace= True)
cars=carsdata.dropna()

#check the summary of 'fuel-type': two levels: gas and diesel
print(cars['fuel-type'].value_counts())
```

```
gas      185
diesel   20
Name: fuel-type, dtype: int64
```

In [19]: *# regression city-mpg~fuel-type, names wouldn't be recognized in smf.ols*

```
cars['citympg']=cars['city-mpg']
cars['fueltype']=cars['fuel-type']
reg = smf.ols('citympg ~ fueltype', data=cars).fit()
reg.summary()

# in the summary it only showed 1 level: gas
```

Out [19]:

## OLS Regression Results

<b>Dep. Variable:</b>	citympg	<b>R-squared:</b>	0.066
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.061
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	14.23
<b>Date:</b>	Mon, 20 Sep 2021	<b>Prob (F-statistic):</b>	0.000212
<b>Time:</b>	14:11:04	<b>Log-Likelihood:</b>	-668.48
<b>No. Observations:</b>	205	<b>AIC:</b>	1341.
<b>Df Residuals:</b>	203	<b>BIC:</b>	1348.
<b>Df Model:</b>	1		
<b>Covariance Type:</b>	nonrobust		
	<b>coef</b>	<b>std err</b>	<b>t</b> <b>P&gt; t </b> <b>[0.025</b> <b>0.975]</b>
<b>Intercept</b>	30.3000	1.418	21.374 0.000 27.505 33.095
<b>fueltype[T.gas]</b>	-5.6297	1.492	-3.773 0.000 -8.572 -2.687
<b>Omnibus:</b>	17.025	<b>Durbin-Watson:</b>	0.846
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	18.937
<b>Skew:</b>	0.668	<b>Prob(JB):</b>	7.73e-05
<b>Kurtosis:</b>	3.658	<b>Cond. No.</b>	6.25

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [17]: #similarly, drivewheels has 3 levels
cars['drivewheels']=cars['drive-wheels']
print(cars['drivewheels'].value_counts())
```

```
→ fwd    120
→ rwd    76
→ 4wd     9
```

Name: drivewheels, dtype: int64

```
In [20]: reg1 = smf.ols('citympg ~ drivewheels', data=cars).fit()
reg1.summary()

#two levels: fwd, rwd showed up, 4wd as the baseline level doesn't show
```

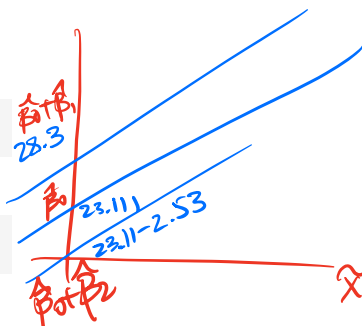
Out [20]:

## OLS Regression Results

<b>Dep. Variable:</b>	citympg	<b>R-squared:</b>	0.324
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.317
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	48.38
<b>Date:</b>	Mon, 20 Sep 2021	<b>Prob (F-statistic):</b>	6.81e-18
<b>Time:</b>	14:11:09	<b>Log-Likelihood:</b>	-635.31
<b>No. Observations:</b>	205	<b>AIC:</b>	1277.
<b>Df Residuals:</b>	202	<b>BIC:</b>	1287.
<b>Df Model:</b>	2		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
<u>Intercept</u>	23.1111	1.802	12.825	0.000	19.558	26.664
<u>drivewheels[T.fwd]</u>	5.2056	1.868	2.786	0.006	1.522	8.890
<u>drivewheels[T.rwd]</u>	-2.5322	1.906	-1.329	0.185	-6.290	1.225

<b>Omnibus:</b>	15.581	<b>Durbin-Watson:</b>	1.175
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	19.666
<b>Skew:</b>	0.537	<b>Prob(JB):</b>	5.36e-05
<b>Kurtosis:</b>	4.072	<b>Cond. No.</b>	10.2



Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [22]: sm.stats.anova_lm(reg1, typ=2) #in anova, it's analysis as a whole
# reduce: null model
# full: y~drivewheels_fwd+ drivewheels_rwd
# notice: df of drivewheels is 2, since there are two parameters to estimate
```

Out [22]:

	sum_sq	df	F	PR(>F)
<b>drivewheels</b>	2827.740080	2.0	48.379345	6.809467e-18
<b>Residual</b>	5903.381871	202.0	NaN	NaN

```
In [23]: # same for MLR with categorical variables
cars['engine_size']=cars['engine-size']
reg2 = smf.ols('citympg ~ engine_size+ drivewheels + fueltype', data=cars).fit()
reg2.summary()
```

Out [23]:

## OLS Regression Results

<b>Dep. Variable:</b>	citympg	<b>R-squared:</b>	0.599
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.591
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	74.59
<b>Date:</b>	Mon, 20 Sep 2021	<b>Prob (F-statistic):</b>	1.36e-38
<b>Time:</b>	14:16:28	<b>Log-Likelihood:</b>	-581.84
<b>No. Observations:</b>	205	<b>AIC:</b>	1174.
<b>Df Residuals:</b>	200	<b>BIC:</b>	1190.
<b>Df Model:</b>	4		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
<b>Intercept</b>	38.9139	1.954	19.911	0.000	35.060	42.768
<b>drivewheels[T.fwd]</b>	4.5714	1.449	3.156	0.002	1.715	7.428
<b>drivewheels[T.rwd]</b>	0.2171	1.537	0.141	0.888	-2.813	3.247
<b>fueltype[T.gas]</b>	-7.0471	0.994	-7.090	0.000	-9.007	-5.087
<b>enginesize</b>	-0.0795	0.009	-9.319	0.000	-0.096	-0.063

<b>Omnibus:</b>	33.046	<b>Durbin-Watson:</b>	1.095
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	84.717
<b>Skew:</b>	0.689	<b>Prob(JB):</b>	4.02e-19
<b>Kurtosis:</b>	5.832	<b>Cond. No.</b>	1.19e+03

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 1.19e+03. This might indicate that there are strong multicollinearity or other numerical problems.

In [24]: `sm.stats.anova_lm(reg2, typ=2)`

Out [24]:

	sum_sq	df	F	PR(>F)
<b>drivewheels</b>	699.332669	2.0	19.958355	1.250114e-08
<b>fueltype</b>	880.634747	1.0	50.265123	2.260697e-11
<b>enginesize</b>	1521.368276	1.0	86.837095	2.213074e-17
<b>Residual</b>	3503.959410	200.0	NaN	NaN

```
In [10]: sm.stats.anova_lm(reg2, typ=2)
```

```
Out[10]:
```

	sum_sq	df	F	PR(>F)
<b>drivewheels</b>	699.332669	2.0	19.958355	1.250114e-08
<b>fueltype</b>	880.634747	1.0	50.265123	2.260697e-11
<b>enginesize</b>	1521.368276	1.0	86.837095	2.213074e-17
<b>Residual</b>	3503.959410	200.0	NaN	NaN

```
In [13]: # when the variable is categorical, but the levels are incidated as numbers
# or if you want to analyze an ordinal variable as categorical
# we can "force" it to be categorical using "C()" in smf.ols

credit = pd.read_csv("Credit.csv")
credit.sample(5)
```

```
Out[13]:
```

	Unnamed: 0	Income	Limit	Rating	Cards	Age	Education	Gender	Student	M
<b>358</b>	359	30.111	4336	339	1	81	18	Male	No	
<b>165</b>	166	25.383	4527	367	4	46	11	Male	No	
<b>283</b>	284	49.927	6396	485	3	75	17	Female	No	
<b>92</b>	93	30.733	2832	249	4	51	13	Male	No	
<b>184</b>	185	158.889	11589	805	1	62	17	Female	No	

```
In [12]: # example: number of cards
# without changing anything, it will be analyzed as numeric

reg3= smf.ols('Balance~Cards', data=credit).fit()
reg3.summary()

#not a perfect example since there are too many levels, but you get the idea
# then we see compared to having just one card, if there's significant change
```

Out [12]:

## OLS Regression Results

<b>Dep. Variable:</b>	Balance	<b>R-squared:</b>	0.007
<b>Model:</b>	OLS	<b>Adj. R-squared:</b>	0.005
<b>Method:</b>	Least Squares	<b>F-statistic:</b>	2.997
<b>Date:</b>	Mon, 26 Oct 2020	<b>Prob (F-statistic):</b>	0.0842
<b>Time:</b>	16:22:31	<b>Log-Likelihood:</b>	-3017.9
<b>No. Observations:</b>	400	<b>AIC:</b>	6040.
<b>Df Residuals:</b>	398	<b>BIC:</b>	6048.
<b>Df Model:</b>	1		
<b>Covariance Type:</b>	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
<b>Intercept</b>	434.2861	54.569	7.958	0.000	327.006	541.566
<b>Cards</b>	28.9869	16.743	1.731	0.084	-3.929	61.903

<b>Omnibus:</b>	28.964	<b>Durbin-Watson:</b>	1.957
<b>Prob(Omnibus):</b>	0.000	<b>Jarque-Bera (JB):</b>	26.603
<b>Skew:</b>	0.566	<b>Prob(JB):</b>	1.67e-06
<b>Kurtosis:</b>	2.437	<b>Cond. No.</b>	8.37


Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [13]: #change it to categorical
reg3= smf.ols('Balance~C(Cards)', data=credit).fit()
reg3.summary()
```

Out [13]:

## OLS Regression Results

Dep. Variable:		Balance		R-squared:		0.023	
Model:		OLS		Adj. R-squared:		0.003	
Method:		Least Squares		F-statistic:		1.144	
Date:		Mon, 26 Oct 2020		Prob (F-statistic):		0.332	
Time:		16:22:31		Log-Likelihood:		-3014.7	
No. Observations:		400		AIC:		6047.	
Df Residuals:		391		BIC:		6083.	
Df Model:		8					
Covariance Type:		nonrobust					
		coef	std err	t	P> t	[0.025	0.975]
	Intercept	531.1373	64.286	8.262	0.000	404.748	657.527
	C(Cards)[T.2]	-58.1720	77.236	-0.753	0.452	-210.023	93.679
	C(Cards)[T.3]	-39.0742	77.663	-0.503	0.615	-191.763	113.615
	C(Cards)[T.4]	45.2794	84.024	0.539	0.590	-119.916	210.475
	C(Cards)[T.5]	-8.1373	101.645	-0.080	0.936	-207.977	191.702
	C(Cards)[T.6]	149.6809	152.622	0.981	0.327	-150.381	449.743
	C(Cards)[T.7]	497.6127	238.379	2.087	0.037	28.947	966.278
	C(Cards)[T.8]	106.8627	463.574	0.231	0.818	-804.547	1018.272
	C(Cards)[T.9]	-149.1373	463.574	-0.322	0.748	-1060.547	762.272
Omnibus:		28.038	Durbin-Watson:		1.928		
Prob(Omnibus):		0.000	Jarque-Bera (JB):		26.387		
Skew:		0.568	Prob(JB):		1.86e-06		
Kurtosis:		2.459	Cond. No.		22.5		

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [ ]:

Ex:  $y$  = cal in coffee

$X$  = level of sugar: (low, med, high)

Dummy

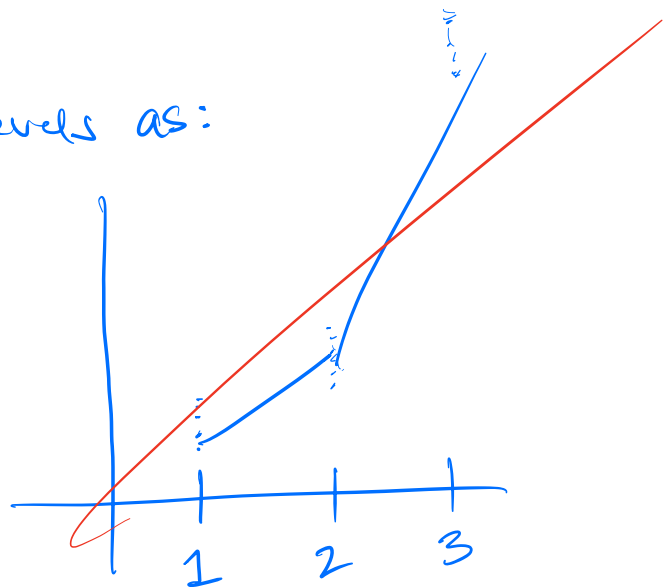
$X_{\text{med}}, X_{\text{high}}$   
 $\Rightarrow X \rightarrow \quad 0 \quad 1$

low is baseline

Alternatively,

what if I encode the levels as:

$X = \begin{cases} 1 & \text{if low} \\ 2 & \text{if med} \\ 3 & \text{if high} \end{cases}$

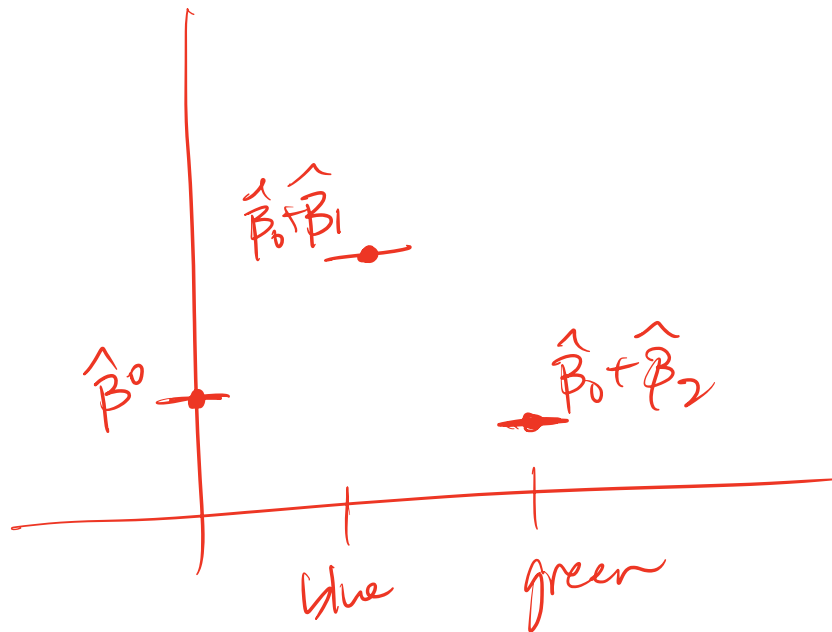


$$y = \beta_0 + \beta_1 X + \varepsilon_i$$

→ this type of encoding is ok  
if you are ok with the assumption  
that there is a true linear effect  
for category levels &  $y$ .



Ex: eye color



2 Cat:

eye & Gender  
 $X_1$   $X_2$

$X_1$  - blue

$X_2$  - M

$X_1$  - green

$X_2$  - NB

$$Y_i = \beta_0 + \beta_1 X_{1, \text{blue}} + \beta_2 X_{1, \text{green}}$$

$$+ \beta_3 X_{2, \text{M}} + \beta_4 X_{2, \text{NB}} + \epsilon_i$$

only main effects  $\Rightarrow$  no interactions

What is an interaction?

$Y$  = stylishness

$X_1$  = wearing casual pink shirt

$X_2$  = wearing business pants

There is potential for interaction  $\rightarrow$   
we need an interaction:

$$Y_i = \beta_0 + \beta_1 X_{1-\text{shirt}} + \beta_2 X_{2-\text{pants}} + \beta_3 X_{1-\text{shirt}} X_{2-\text{pants}} + \epsilon_i$$

$$\hat{\beta}_1 = 2$$

$$\hat{\beta}_2 = 1$$

$$\hat{\beta}_3 = -6$$



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...

Finding new ways for explaining interaction is a passion of mine.



7:08 AM · Apr 29, 2020