Modeling Problems

- Non-Normal Resids

- False Linearity b/w Y & X

NonNormal Resids can be detected through:

1. Histogram of resids

2. QQ plot

3. Tests for Normality: Anderson Darling, Shapiro Wilks, Reyan Joiner, Kolmogorov Smirnov

Reminder: problems associated w/ non Normal resids

- t-test results unreliable

- CI/PIs are wrong

But if $n >>> 0$ then asymptotically the dist of $\hat{\beta}$ converges to a Normal dist.

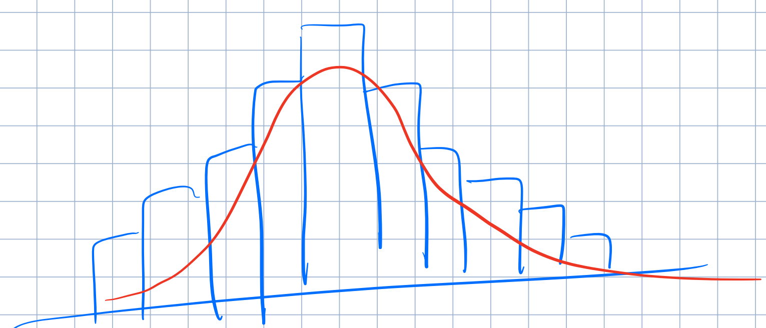
$$\underline{\sqrt{n}(\hat{\beta}-\beta)} \xrightarrow{\mathcal{D}} N(0, Var(\hat{\beta}))$$

$$\Updownarrow$$

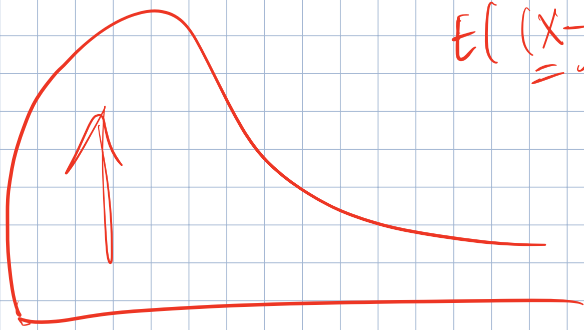
$$\hat{\beta} \overset{a}{\sim} N(\beta, Var(\hat{\beta}))$$

CLT argument

Ex



Ways to Deviate from Normality



$E((\underline{X}-\mu)^k)$

Skewness $\leftarrow$ measures asymmetry

pop qty: $E\left(\left(\frac{e_i}{\sigma}\right)^3\right)$

sample skew: $\frac{1}{n}\sum_{i=1}^{n}\left(\frac{e_i}{\hat{\sigma}}\right)^3$

<u>Kurtosis</u> $\leftarrow$ measures "peakedness" of dist

pop qty: $E\left(\left(\frac{e_i}{\sigma}\right)^4\right)$

sample kurtosis: $\frac{1}{n}\sum_{i=1}^{n}\left(\frac{e_i}{\hat{\sigma}}\right)^4$

Omnibus K^2 test for Normality

$$K^2_{stat} = \left[Z_1(g_1)\right]^2 + \left[Z_2(g_2)\right]^2$$

skew ... kurtosis

Z_1, Z_2 special transformation

Under H_0: $K^2_{stat} \sim \chi^2_2$

H_0: dist is Normal

H_1: significant deviation from Normality exists

Jarque Bera Test

similar to Omnibus test

tries to combine skew & kurtosis

H_0: dist is Normal

H_1: significant deviation from Normality exists

$$JB_{stat} = \frac{n-p}{6}\left((g_1)^2 + \frac{1}{4}(g_2)^2\right)$$

False Assumption of Linearity

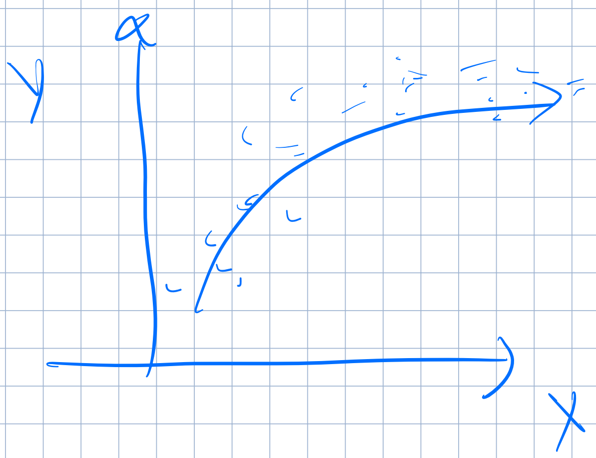
When Y & X are not linearly related we want to transform them until the relationship is linear

Ways to Do This:

Watch out for interpretation

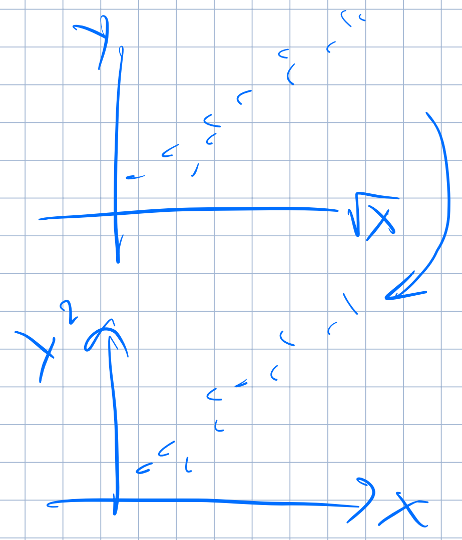
1. Transform Y → may affect hetero/homo skedasticity
2. Transform X → nice when the only prob is nonlinearity
3. Transform both

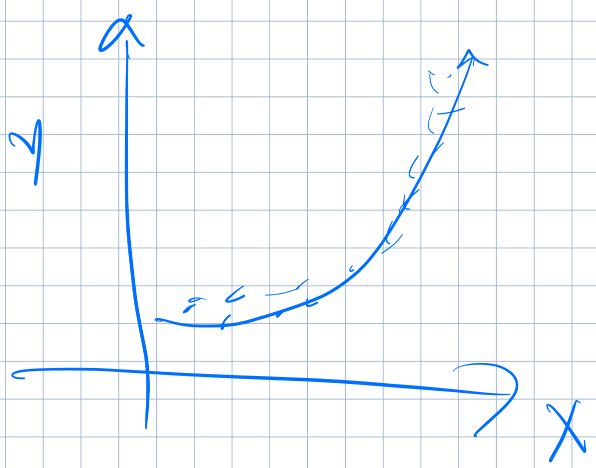
Heuristics for Choosing Transformations



$f(x) = \log x$

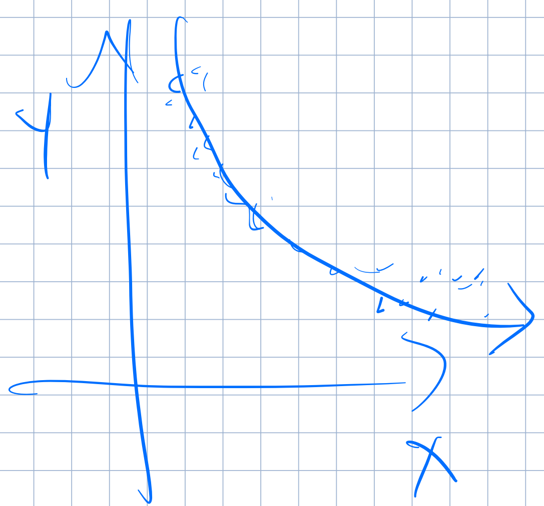
$f(x) = \sqrt{x}$





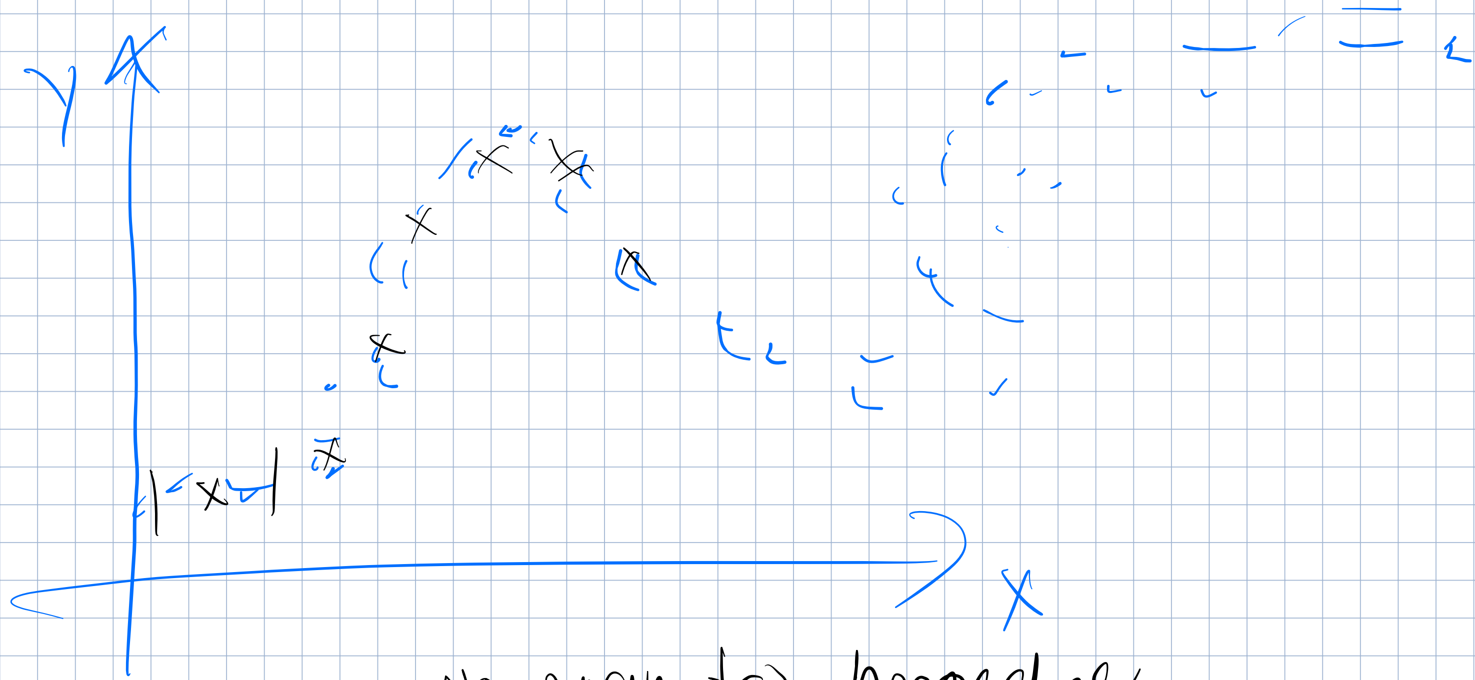
$f(x) = x^2$

$f(x) = e^x$



$f(x) = 1/x$

$f(x) = e^{-x} = \frac{1}{e^x}$



Nonparametric Approaches

Ex:

$$\log y = \beta_0 + \beta_1 \log X$$

$$e^{\log y} = e^{\beta_0 + \beta_1 \log X}$$

$$Y = e^{\beta_0} e^{\beta_1 \log X}$$

$$Y = e^{\beta_0} e^{\log X^{\beta_1}}$$

$$Y = e^{\beta_0} X^{\beta_1}$$

$$Y = a X^{\beta_1}$$

can interpret original scale w/ this if you want

$$a(X+1)^{\beta_1} - a X^{\beta_1}$$

SVD & Changing Basis of X

$\hat{\beta} = (X^T X)^{-1} X^T Y$

$X = U D V^T$

$$D = \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_p \end{pmatrix}, \quad d_p \approx 0$$

$\vdots$

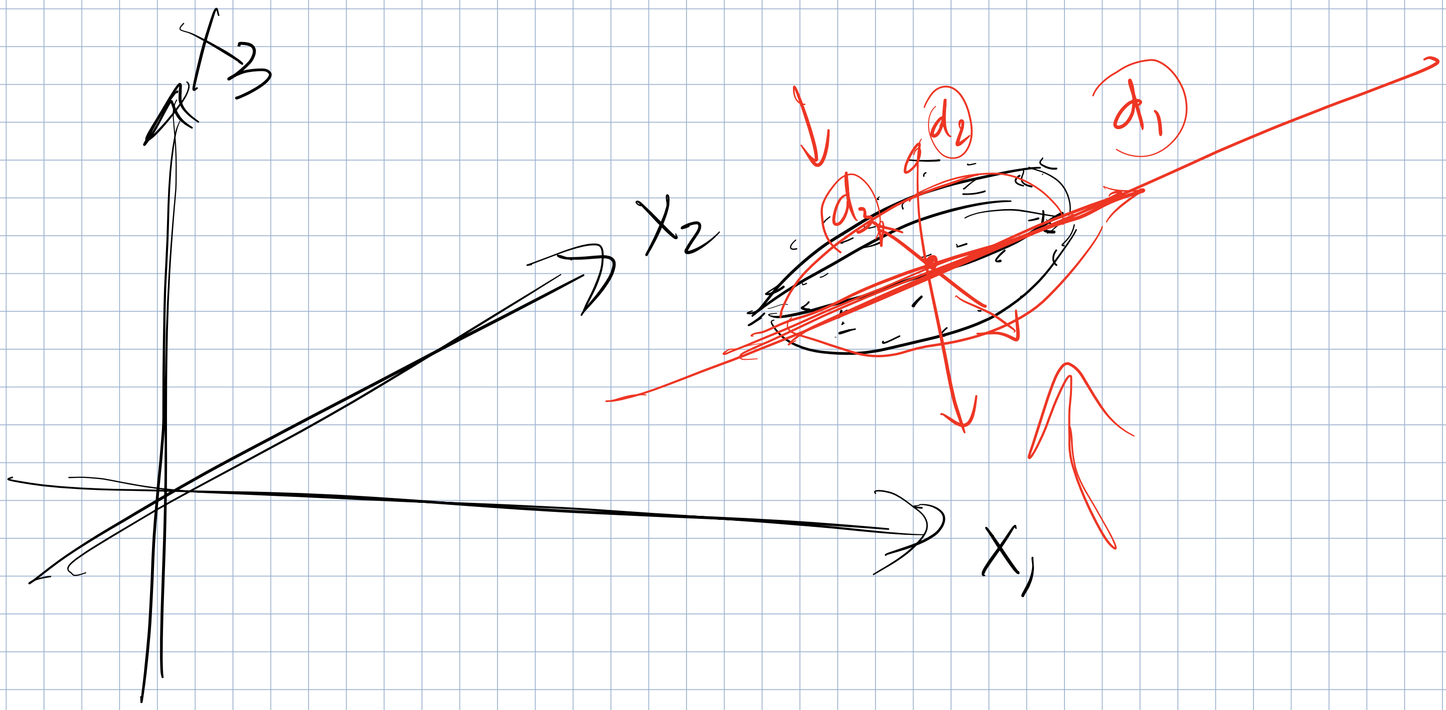
.000001

.000002

$\hat{\beta} = (\ldots) D^{-1} (\ldots)$

$$\begin{pmatrix} 1/d_1 & & \\ & \ddots & \\ & & 1/d_p \end{pmatrix}$$

$\frac{1}{.000001}$ vs $\frac{1}{.000002}$



x3
x2
x1
d1
d2
d3