The Leverage Cycle

With Exogenous Limits on Leverage and Belief Updating

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For Macroeconomic Instability and Financial Markets

1 Introduction

In 2010, John Geanakoplos published a paper entitled "The Leverage Cycle;" in which, he argues that in times of crises classical economic thought which points to the interest rate as the mechanism by which asset prices fluctuate is incomplete (Geanakoplos, 2010). Rather he points to margins, or leverage, as the workhorse driving asset price booms and busts. Geanakoplos introduces the example of the prospective home buyer who looks to use the home as collateral for a loan and is concerned with two things: the interest rate on the loan, and how much money the loan amount is as a fraction of the home price. If the home price is $1 million dollars and the loan is for the amount of $700,000, then the margin amount is 30% (e.g. the portion of the home price that has to be financed with cash). Margin fluctuations have direct effects on asset demand, price, and volatility.

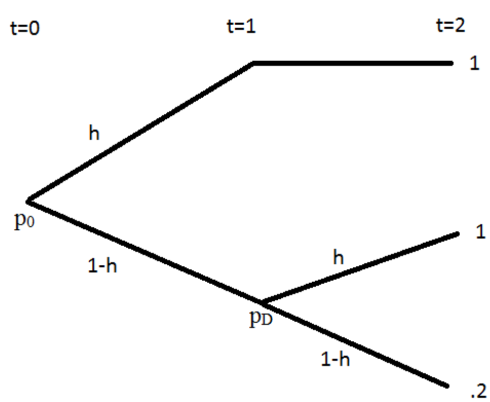
In order to prove his theory, he introduces a sort of economic thought experiment with a three period model (t=0, 1, and 2), endogenous margins, and investors with a range of static beliefs regarding an assets future payoff. At t=0, agents are endowed with one unit of the asset and one unit of cash; those who choose to buy the asset are optimistic about its future payoffs and those who choose to sell are pessimistic. In one scenario all agents have no access to leverage, and in another scenario all agents have access to one-period risk-free debt (i.e. the loan amount at t=0 is equal to the minimum collateral price at t=1). In the leveraged scenario, optimistic investors take out as much leverage as possible at t=0 in order to maximize their expected returns, but if the economy gets bad news at t=1 then they have to liquidate their investments to repay the loan and ultimately end up bankrupt. Furthermore, if the economy gets bad news at t=1 and the asset price tumbles (a function of margin calls, margin raises, and worsening fundamentals), agents beliefs about the future probabilities of an assets payoff remain unchanged; they have static beliefs.

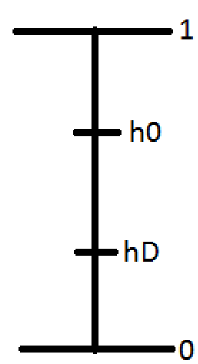
In a 1997 paper, Andrei Shleifer and Robert Vishny introduced a model of performance-based arbitrage (PBA) which sought to illustrate the risks arbitrageurs have to face when attempting to correct asset price discrepancies across markets (Shleifer and Vishny, 1997). Their model introduces three classes of agents: noise traders, arbitrageurs (e.g. hedge fund managers), and investors (e.g. those who deposit their money into hedge funds). Shleifer and Vishny setup a three period model; in which, arbitrageurs know an assets fundamental value, v, at t=0, 1, and 2 but noise traders and investors only discover the assets fundamental value at t=2. If noise trader sentiment deepens between t=0 and t=1, driving the asset price further below v at t=1 than it had been at t=0, then investors are inclined to assume that their money manager is not very good and that they should pull their money out of the fund. In a sense, investors have dynamic beliefs that update with price movements.

John Geanakoplos argues that financial regulators should look to control margins as well as interest rates during times of crisis in order to minimize economic volatility. The question of what the relationship is between leverage availability and asset price volatility is yet to be flushed out in sufficient detail to make sound policy recommendations. Likewise, in times of crises, agents are constantly updating their beliefs regarding an assets future payoff. The fact that agents have static beliefs in Geanakoplos current model is a limitation. An investor who seeks to optimize his/her timing and maximize profit would do well to take into account belief updating when looking at the leverage cycle.

Accordingly, this paper aims to answer two questions. First, how do exogenous limits on leverage affect asset pricing? Second, how does adding belief updating to variable leverage availability affect Geanakoplos' model? This paper attempts to answer those questions by synthesizing the Leverage Cycle with the PBA model in order to create a more realistic environment in which the leverage cycle may be studied. We will first modify the leverage cycle by introducing agents who are exogenously preempted from using leverage. This will allow us to see the relationship between leverage availability and price volatility more clearly, while acting as a first step towards incorporating PBA into the leverage cycle. With unleveraged agents in the model, not all investors who bought the asset at t=0 get wiped out at t=1. It is with these investors who are allowed to continue holding the asset that PBA will come into effect.

2 The Leverage Cycle





The Leverage Cycle is a three period model with one asset, subjective beliefs, h, about the assets future payoff, and one-period risk-free debt contracts using the asset as collateral. Agents beliefs are between 0, most pessimistic, and 1, most optimistic. At t=0, agents make their first trades. At t=1, the economy either gets good news or bad news. If the economy gets good news (occurs with probability h), there is no more future uncertainty; however, if the economy gets bad news (with probability 1-h), there is remaining uncertainty about the assets payoff. At t=2, the asset either pays 1 or .2. The price of the asset at t=0 is p_0 and the price of the asset at t=1 when the economy gets bad news is p_D (notice this is the maximum loan an agent can take at t=0 for debt to be risk-free). There are two marginal buyers along the belief spectrum:

1. Agent h_0 who is indifferent between buying the asset at t=0 or holding cash and potentially buying the asset at t=1
2. Agent h_D who is indifferent between buying the asset in the down state at t=1 and holding cash

There are four unknowns (h_0, h_D, p_0, p_D) and we can setup a system of four equations: (1) market clearing at t=0, (2) marginal buyer, h_0, valuation at t=0, (3) market clearing in the down state at t=1, and (4) marginal buyer, h_D, valuation at t=1.

$$(1-h_0)\frac{1+p_0}{p_0-p_D}=1 \tag{1}$$

$$h_0\frac{1-p_D}{p_0-p_D}=h_0+h_0(1-h_0)\frac{1-.2}{p_D-.2} \tag{2}$$

$$(h0-hD)\frac{1+p_0}{p_D-.2}=1 \tag{3}$$

$$h_D+(1-h_D)(.2)=p_D \tag{4}$$

3 Exogenous Limits on Leverage

Geanakoplos uses the leverage cycle to illustrate two types of equilibria. One is a no-borrowing equilibrium, in which, no agent has access to leverage. The other is a leverage equilibrium wherein every agent has access to one-period risk-free debt. He discovers that the price crash, $\frac{p_0-p_D}{p_0}*100$, is amplified from 23.7% in the no-borrowing equilibrium to 26.7% in the leverage equilibrium. This is because agents with beliefs above h_0 take out as much leverage against the asset as possible and are forced to liquidate if the economy gets bad news at t=1 in order to repay their debt. It is easy to extrapolate from these two equilibria and argue that as leverage increases in the economy, economic volatility will increase in lockstep. However, the relationship between credit availability and price volatility is not so straightforward.

In order to explore the relationship between access to credit and economic volatility, we can modify the leverage cycle by introducing agents with heterogeneous access to credit markets. In this modified model α agents have access to credit markets and $1-\alpha$ agents do not have access to credit markets. We proceed to modify our system of four equations to reflect the change:

$$\alpha(1-h_0)\frac{1+p_0}{p_0-p_D}+(1-\alpha)(1-h_0)\frac{1+p_0}{p_0}=1 \tag{5}$$

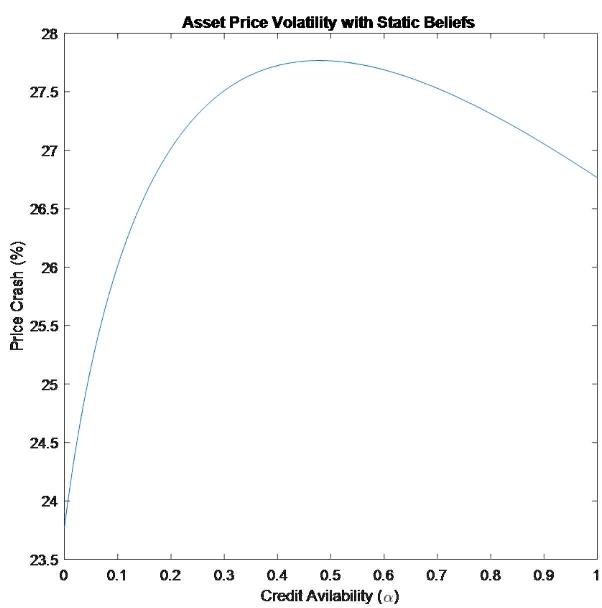
$$(1-\alpha)[h_0\frac{1}{p_0}+h_0(1-h_0)\frac{1}{p_0}+(1-h_0)^2\frac{.2}{p_0}]+(\alpha)[h_0\frac{1-p_D}{p_0-p_D}]=$$
$$(1-\alpha)[h_0+h_0(1-h_0)\frac{1}{p_D}+(1-h_0)^2\frac{.2}{p_D}]+(\alpha)[h_0+h_0(1-h_0)\frac{1-.2}{p_D-.2}] \tag{6}$$

$$\alpha[(h0-hD)\frac{1+p_0}{p_D-.2}]+(1-\alpha)[(h_0-h_D)\frac{1+p_0}{p_D}]+(1-\alpha)[(1-h0)\frac{1+p_0}{p_0}]=1 \tag{7}$$

$$h_D+(1-h_D)(.2)=p_D \tag{8}$$

This is to say that at t=0, some agents α will be allowed to use leverage to buy the asset, and some agents, $1-\alpha$, will not be allowed to use leverage. Market clearing at t=0 is modified to reflect marginal buyer h_0's indifference between purchasing the asset now (with probability α that he/she gets leverage)and holding cash to purchase the asset in the down state at t=1 (again with probability α that he/she gets leverage). Market clearing at t=1 is modified using the same rationale.

We can now plot the relationship between credit availability, α, and the price crash.



As we can see, the relationship between credit availability and the price crash between t=0 and t=1 is non-linear. In a sense there are two competing forces:

1. Forced selling of the asset by leveraged buyers with beliefs above h_0 when the economy gets bad news

2. Asset purchases in the down state by agents with beliefs $h_D < h < h_0$ pushing the down-state price, p_D, up

As the number of leveraged buyers of the asset at t=0 increases, the price crash increases as a result of more forced liquidations at t=1. Likewise, as the number of leveraged buyers at t=1 increases, the price crash decreases; this is because access to credit for those agents with beliefs $h_D < h < h_0$ has a big effect on the down-state price, p_D. Credit effectively allows them to buy more of the asset and therefore increase its price, p_D. This directly translates into a smaller price crash.

As the above figure indicates, the first force dominates when credit availability, α, is between 0 and .5. But for credit availability greater than .5, increases in α have a calming effect on the price crash as the second force comes to dominate. Indeed the least economic volatility exists when there is no access to credit. However if the economy has an $\alpha = .7$, pursuing a policy of decreasing access to credit may actually cause more harm than good. Hence, reducing access to credit for investors during a time of crises has to be studied carefully before implementation.

4 Belief Updating

The original leverage cycle, as laid out by John Geanakoplos does not allow for agents to buy the asset at t=0 and then sell it at t=1. This is a structural limitation resulting from two features in the model:

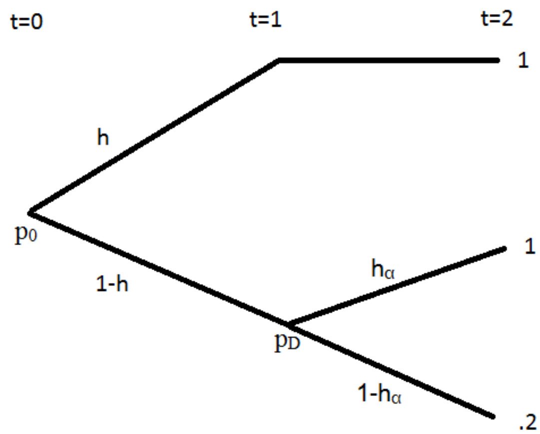
1. All agents have static beliefs - the probability of the up-state is the same at t=0 and t=1

2. Since agents who buy the asset at t=0 are forced to sell in the down-state to repay their debts, there is no room for updated beliefs among agents with beliefs $h > h_0$

It is important to correct this limitation in order to take into account the real-life practice of performance based money allocation. In this paper, we will synthesize the PBA model with the Leverage Cycle by introducing two classes of agents in three-period model. One class of agents are depositors and the other class are investors. Depositors deposit their money with the best performing investors as defined by the price movement of the investor's asset holdings over the previous two time periods. This three-period model has α investors with access to credit and $1-\alpha$ investors without access to credit markets.Their beliefs are heterogeneous and are distributed between 0, most pessimistic, and 1, most optimistic.

With the exogenous limits on leverage placed in the model in the previous section, it is now possible to have some fraction, $1-\alpha$, of investors with beliefs $h > h_0$ continue to hold the asset at t=1. We can further incorporate a belief updating feature for these investors by modifying their beliefs about the asset's time-2 payoff at t=1. In effect, their beliefs, h, are transformed at t=1 into h_α.

This is modeled as follows:



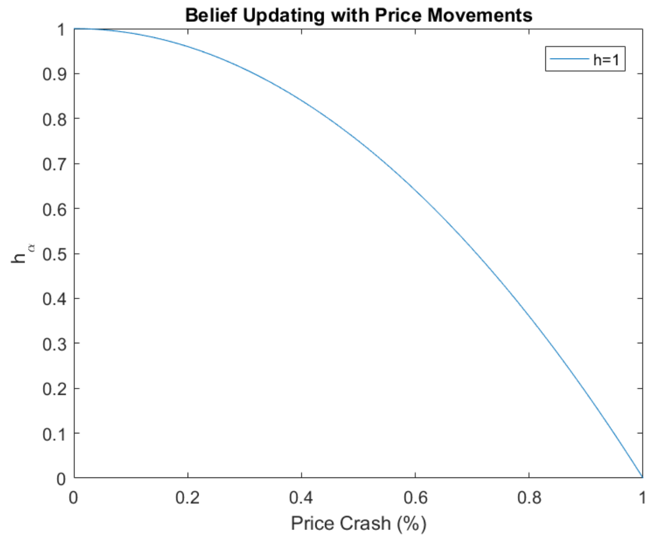
In this paper, we will model belief updating as a function of the price crash. This is to mimic the effect of depositors pulling money out of investor funds when the asset price falls between t=0 and t=1 in Shleifer and Vishny's PBA model. Belief updating follows the below functions, where k and g are some constants that represents the impact power of the price crash on agents' beliefs:

$$h_\alpha = h_0(1 - k(\frac{p_0 - p_D}{p_0})^2) \tag{9}$$

$$h_\gamma = h_0(1 - g(\frac{p_0 - p_D}{p_0})^2) \tag{10}$$

$$h_\delta = h_D(1 - g(\frac{p_0 - p_D}{p_0}))^2 \tag{11}$$

This function was chosen because of its parabolic nature. As the price crash increases, the rate at which depositors become more pessimistic about an investor's talent increases - therefore pulling more and more money out. Also notice that the impact power of the price crash is different between h_α, h_γ, and h_δ even though the first two are not different marginal buyers. Presumably those depositors whose money is deposited with an investor who holds the asset at t=0 have a stronger belief updating reaction at t=1 than those who don't, hence $k > g$.



We can now begin to think about reworking our system of four equations to reflect agents with price-based belief updating. Our market clearing conditions at t=0 and t=1 remain unchanged. However, the marginal buyer, h_0, has to now be indifferent between buying the asset at t=0 and potentially updating their beliefs if the economy gets bad news, and holding cash at t=0 and purchasing the asset at t=1. The marginal buyer also knows that if he/she chooses to hold cash at t=0 and the economy gets bad news, he/she will update less than those who held the asset throughout (because $k > g$).

The marginal buyer, h_D, updates his/her beliefs less than those who purchased the asset at t=0, and therefore has a golden opportunity to buy the asset at a deep discount in the down state. The reason these agents update at all is because there may be some residual bad press lingering in the minds of their depositors regarding the asset in question (making them more likely to pull their money out), and there may be some fear among the investors that what happened before will happen again. This is something Seth Klarman warns his readers about in his book *Margin of Safety*; a natural human tendency is to confuse probabilistic independence for dependence (Klarman, 1991).

The equations are now as follows:

$$\alpha(1-h_0)\frac{1+p_0}{p_0-p_D}+(1-\alpha)(1-h_0)\frac{1+p_0}{p_0}=1 \tag{12}$$

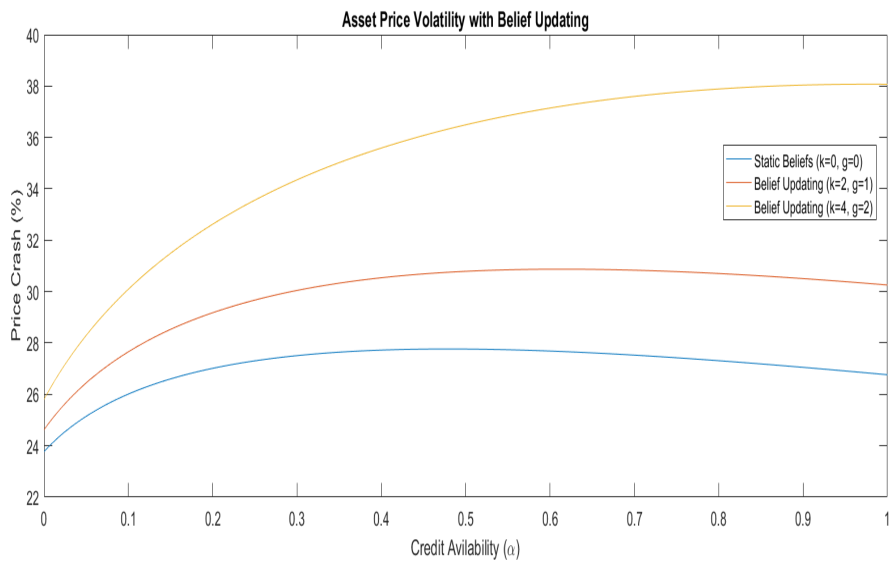
$$(1-\alpha)[h_0\frac{1}{p_0}+h_\alpha(1-h_0)\frac{1}{p_0}+(1-h_0)(1-h_\alpha)\frac{.2}{p_0}]+(\alpha)[h_0\frac{1-p_D}{p_0-p_D}]=$$

$$(1-\alpha)[h_0+h_\gamma(1-h_0)\frac{1}{p_D}+(1-h_0)(1-h_\gamma)\frac{.2}{p_D}]+(\alpha)[h_0+h_\gamma(1-h_0)\frac{1-.2}{p_D-.2}] \tag{13}$$

$$\alpha[(h0-hD)\frac{1+p_0}{p_D-.2}]+(1-\alpha)[(h_0-h_D)\frac{1+p_0}{p_D}]+(1-\alpha)[(1-h0)\frac{1+p_0}{p_0}]=1 \tag{14}$$

$$h_\delta + (1 - h_\delta)(.2) = p_D \tag{15}$$

We can plot the results for different levels of belief updating to see the effect of updating and credit availability on the price crash. With $g = \frac{1}{2}k$, we produce the following results:



This figure reveals four key findings from the price-based belief updating extension:

1. As the impact power of price movement on beliefs, k and g, increases, so too does the price crash

2. Even with credit availability, α, set to zero, belief updating increases the price crash. This means that even in a no-borrowing equilibrium there may be price movements that are not resultant from fundamental changes

3. Belief updating has differential effects at low and high levels of leverage availability. At low α, updated beliefs only worsen the price crash marginally, but at high α the crash is amplified on orders of magnitude

4. The curvature of the price crash to credit availability curve changes as belief updating is amplified. The curve becomes more linear as belief updating is amplified. Whereas with static beliefs, increases in credit availability after $\alpha = .5$ dampened the price crash, with amplified belief updating ($k = 4, g = 2$), increases in credit availability after $\alpha = .5$ made things worse for the economy

While the first three findings are fairly intuitive, the fourth finding is unexpected. One explanation for the phenomenon goes back to the competing forces of the agents who are forced to liquidate at t=1 and those who can buy the asset with leverage at t=1. For static beliefs and moderate belief updating ($k = 2, g = 1$), as leverage availability increases above $\alpha = .5$, the effect of those agents who purchase the asset in the down state at t=1 overwhelms that of the agents who are forced to liquidate. The price crash is dampened because these agents can buy more of the asset and therefore drive its down-state price, p_D, up. With amplified belief updating ($k = 4, g = 2$), those investors who would've bought the asset at t=1 are no longer so optimistic about its future payoffs. Either they are worried that depositors will overreact to a purchase decision, or they themselves think the asset payoff probabilities are not independent. Even with $g = \frac{1}{2}k$, the belief updating effect for those agents who didn't buy the asset at t=0 overwhelms the leveraged-returns that they could get.

5 Conclusion

This paper has sought to answer two key questions: (1) how do exogenous limits on leverage affect asset pricing, and (2) how does adding belief updating to agents' optimizing decisions affect the Leverage Cycle?

By incorporating exogenous limits on leverage to Geanakoplos' model, we've illustrated that the relationship between access to credit and economic volatility is not linear under assumptions of static agent beliefs. Increasing access to credit after a threshold value may actually work to stabilize asset price fluctuations by allowing some agents to pickup the slack when other agents are forced to sell.

Additionally incorporating belief updating into the leverage cycle produced a number of insights. Perhaps most interesting of which is that the curvature on the price-crash to credit-availability curve changed as belief updating was amplified. As agents became more sensitive to downward price movements and built their future expectations on the past, the effect of increased access to credit ceased to have a dampening effect on the price crash. After some threshold value of belief sensitivity, increased leverage simply meant a larger price crash.

Aside from a policy perspective, these results are also meaningful for an investor who seeks to optimize returns. Namely it is in an agent's best interest to not update their beliefs in the down state at t=1 if the probabilities of an asset's future payoff are truly independent of the past. The agents who fight the urge to link the past with the future and ignore depositor fear will earn the highest returns. Under Shleifer and Vishny's performance based arbitrage model, these investors would also have a large influx of money at t=2, when the expected return to their investment is realized and their performance is transmitted to depositors.

6 Bibliography

1. Geanakoplos, John. "The Leverage Cycle." University of Chicago Press, 2010.

2. Klarman, Seth. *Margin of Safety: Risk-Averse Value Investing Strategies for the Thoughtful Investor.* Harper Business, 1991.

3. Shleifer, Andrei and Summers, Lawrence H. "The Noise Trader Approach to Finance." Journal of Economic Perspectives Vol. 4 (2), 1990.

4. Shleifer, Andrei and Vishny, Robert W. "The Limits of Arbitrage." The Journal of Finance Vol. 52 (1), 1997.