

# Formula Sheet

<b>p</b>	<b>q</b>	<b>not p</b>	<b>p and q</b>	<b>p or q</b>	<b>p xor q</b>	<b>p nand q</b>	<b>p nor q</b>	<b>p xnor q</b>	$(p \rightarrow q)$	$(p \leftrightarrow q)$
F	F	T	F	F	F	T	T	T	T	T
F	T	T	F	T	T	T	F	F	T	F
T	F	F	F	T	T	T	F	F	F	F
T	T	F	T	T	F	F	F	T	T	T

  

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## Logical Equivalence ( $\equiv$ ) Laws

Identity (I)	$(p \wedge T) \equiv p$	$(p \vee F) \equiv p$
	$(T \wedge p) \equiv p$	$(F \vee p) \equiv p$
Universal Bound (UB)	$(p \wedge F) \equiv F$	$(p \vee T) \equiv T$
	$(F \wedge p) \equiv F$	$(T \vee p) \equiv T$
Idempotent (ID)	$(p \wedge p) \equiv p$	$(p \vee p) \equiv p$
Commutative (COM)	$(p \wedge q) \equiv (q \wedge p)$	$(p \vee q) \equiv (q \vee p)$
Associative (ASS)	$(p \wedge (q \wedge r)) \equiv ((p \wedge q) \wedge r)$	$(p \vee (q \vee r)) \equiv ((p \vee q) \vee r)$
Distributive (DIST)	$(p \vee (q \wedge r)) \equiv ((p \vee q) \wedge (p \vee r))$ $((r \wedge q) \vee p) \equiv ((r \vee p) \wedge (q \vee p))$	$(p \wedge (q \vee r)) \equiv ((p \wedge q) \vee (p \wedge r))$ $((r \vee q) \wedge p) \equiv ((r \wedge p) \vee (q \wedge p))$
Absorption (ABS)	$(p \vee (p \wedge q)) \equiv p$ $((q \wedge p) \vee p) \equiv p$	$(p \wedge (p \vee q)) \equiv p$ $((q \vee p) \wedge p) \equiv p$
Negation (NEG)	$(p \wedge \sim p) \equiv F$ $(\sim p \wedge p) \equiv F$	$(p \vee \sim p) \equiv T$ $(\sim p \vee p) \equiv T$
Double Negation (DNEG)	$\sim \sim p \equiv p$	
Complement (CP)	$\sim T \equiv F$	$\sim F \equiv T$
DeMorgan's (DM)	$\sim(p \wedge q) \equiv (\sim p \vee \sim q)$	$\sim(p \vee q) \equiv (\sim p \wedge \sim q)$
Definition of Exclusive OR (XOR)	$(p \oplus q) \equiv ((p \vee q) \wedge \sim(p \wedge q))$	$(p \oplus q) \equiv ((p \wedge \sim q) \vee (\sim p \wedge q))$
Definition of Implication (IMP)	$(p \rightarrow q) \equiv (\sim p \vee q)$	
Contrapositive (CONTP)	$(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$	
Definition of Biconditional (BIC)	$(p \leftrightarrow q) \equiv ((p \rightarrow q) \wedge (q \rightarrow p))$	$(p \leftrightarrow q) \equiv \sim(p \oplus q)$

## Rules of Inference

Modus Ponens: [M.PON]	$(p \rightarrow q)$ $p$ $\therefore q$	Modus Tollens: [M.TOL]	$(p \rightarrow q)$ $\sim q$ $\therefore \sim p$
Generalization: [GEN]	$p$ $\therefore (p \vee q)$	Specialization: [SPEC]	$(p \wedge q)$ $\therefore p$
Conjunction: [CONJ]	$p$ $q$ $\therefore (p \wedge q)$	Elimination: [ELIM]	$(p \vee q)$ $\sim q$ $\therefore p$
Transitivity: [TRANS]	$(p \rightarrow q)$ $(q \rightarrow r)$ $\therefore (p \rightarrow r)$	Proof by cases: [CASE]	$(p \rightarrow r)$ $(q \rightarrow r)$ $\therefore ((p \vee q) \rightarrow r)$
Resolution: [RES]	$(p \vee q)$ $(\sim p \vee r)$ $\therefore (q \vee r)$	Contradiction: [CONTD]	$(p \rightarrow F)$ $\therefore \sim p$

### Binary Representation

$x_3$	$x_2$	$x_1$	$x_0$	HEX	unsigned	signed	$x_3$	$x_2$	$x_1$	$x_0$	HEX	unsigned	signed
0	0	0	0	0	0	0	1	0	0	0	8	8	-8
0	0	0	1	1	1	1	1	0	0	1	9	9	-7
0	0	1	0	2	2	2	1	0	1	0	A	10	-6
0	0	1	1	3	3	3	1	0	1	1	B	11	-5
0	1	0	0	4	4	4	1	1	0	0	C	12	-4
0	1	0	1	5	5	5	1	1	0	1	D	13	-3
0	1	1	0	6	6	6	1	1	1	0	E	14	-2
0	1	1	1	7	7	7	1	1	1	1	F	15	-1

### Domains

$\mathbb{Z}$	Integers	$\{\dots, -2, -1, 0, 1, 2, \dots\}$	$\mathbb{Q}$	Rational Numbers	$\{\frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z}^*\}$
$\mathbb{Z}^+$	Positive Integers	$\{x \in \mathbb{Z}, x > 0\}$	$\bar{\mathbb{Q}}$	Irrational Numbers	$\{x \in \mathbb{R}, x \notin \mathbb{Q}\}$
$\mathbb{Z}^*$	Non-zero Integers	$\{x \in \mathbb{Z}, x \neq 0\}$	$\mathbb{N}_0$	Natural Numbers	$\{0, 1, 2, 3, \dots\}$
$\mathbb{R}$	Real Numbers	$\{\dots, -\frac{20}{6}, 0, 1, \sqrt{2}, \pi, \dots\}$	$\mathbb{N}_1$		$\{1, 2, 3, \dots\}$

### Set Operations

S contains a	$a \in S$
S does not contain a	$\sim(a \in S) \equiv a \notin S$
A is a subset of B	$A \subseteq B \equiv \forall x \in U, x \in A \rightarrow x \in B$
A is not a subset of B	$A \not\subseteq B \equiv \exists x \in U, x \in A \wedge x \notin B$
A is a proper subset of B	$A \subset B \equiv \forall x \in U, (x \in A \rightarrow x \in B) \wedge \exists y \in U, y \in B \wedge y \notin A$
A is equal to B	$A = B \equiv \forall x \in U, x \in A \leftrightarrow x \in B$
A is different to B	$A \neq B \equiv \exists x \in U, x \in A \oplus x \in B$
A union B	$C = A \cup B = \{x \in U \mid x \in A \vee x \in B\}$
A intersection B	$C = A \cap B = \{x \in U \mid x \in A \wedge x \in B\}$
B minus A	$C = B - A = \{x \in U \mid x \in B \wedge x \notin A\}$
Complement of A	$C = A^c = \{x \in U \mid x \notin A\}$

### Big O()

$f \in O(g) \Leftrightarrow \exists c \in \mathbb{R}^+, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n_0 \rightarrow f(n) \leq c \cdot g(n)$
$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n) < O(n!)$

### Regular Expressions

.	Matches any character
[xy]	Matches one character from those listed
[x - z]	Matches one character from the range of characters listed
	Matches one element from those separated by pipes
*	Matches the previous element 0 or more times
+	Matches the previous element 1 or more times
?	Matches the previous element 0 or 1 time
{m, n}	Matches the preceding element from m to n times
{n}	Matches the preceding element exactly n times
\s	Matches a whitespace character
\d	Matches a digit, same as [0-9]
\w	Matches an alphanumeric character and underscore, same as [A-Za-z0-9_]