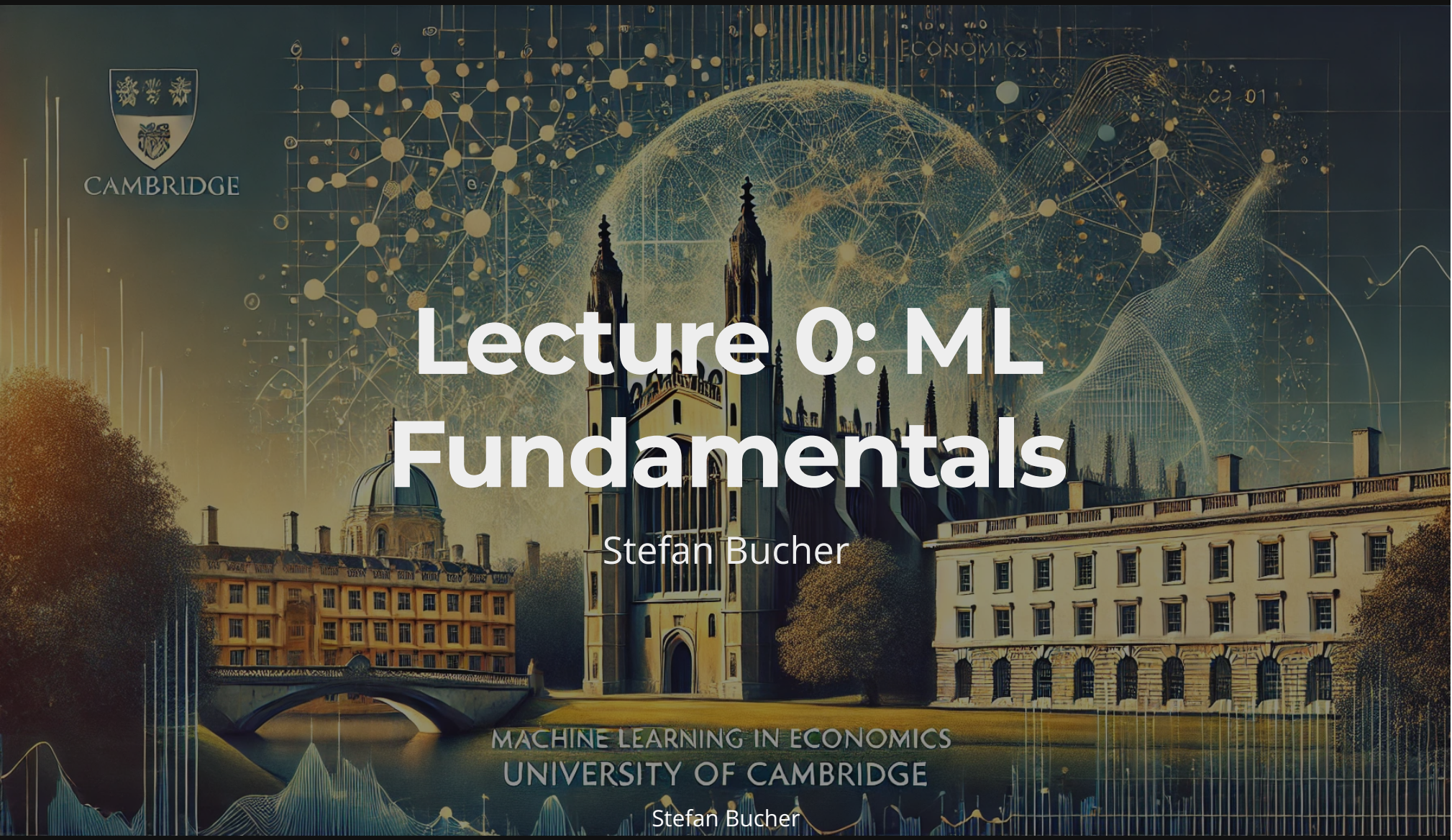

Lecture 0: ML Fundamentals

Stefan Bucher

Introduction to ML

Prince (2023, chap. 1 and Appendix C).[1]

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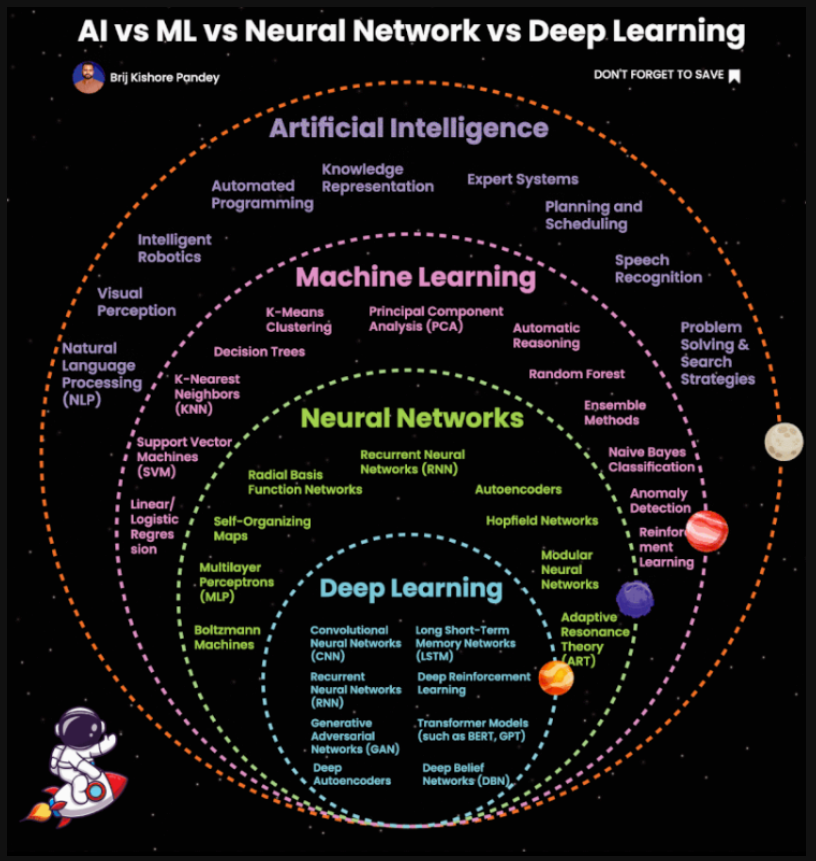
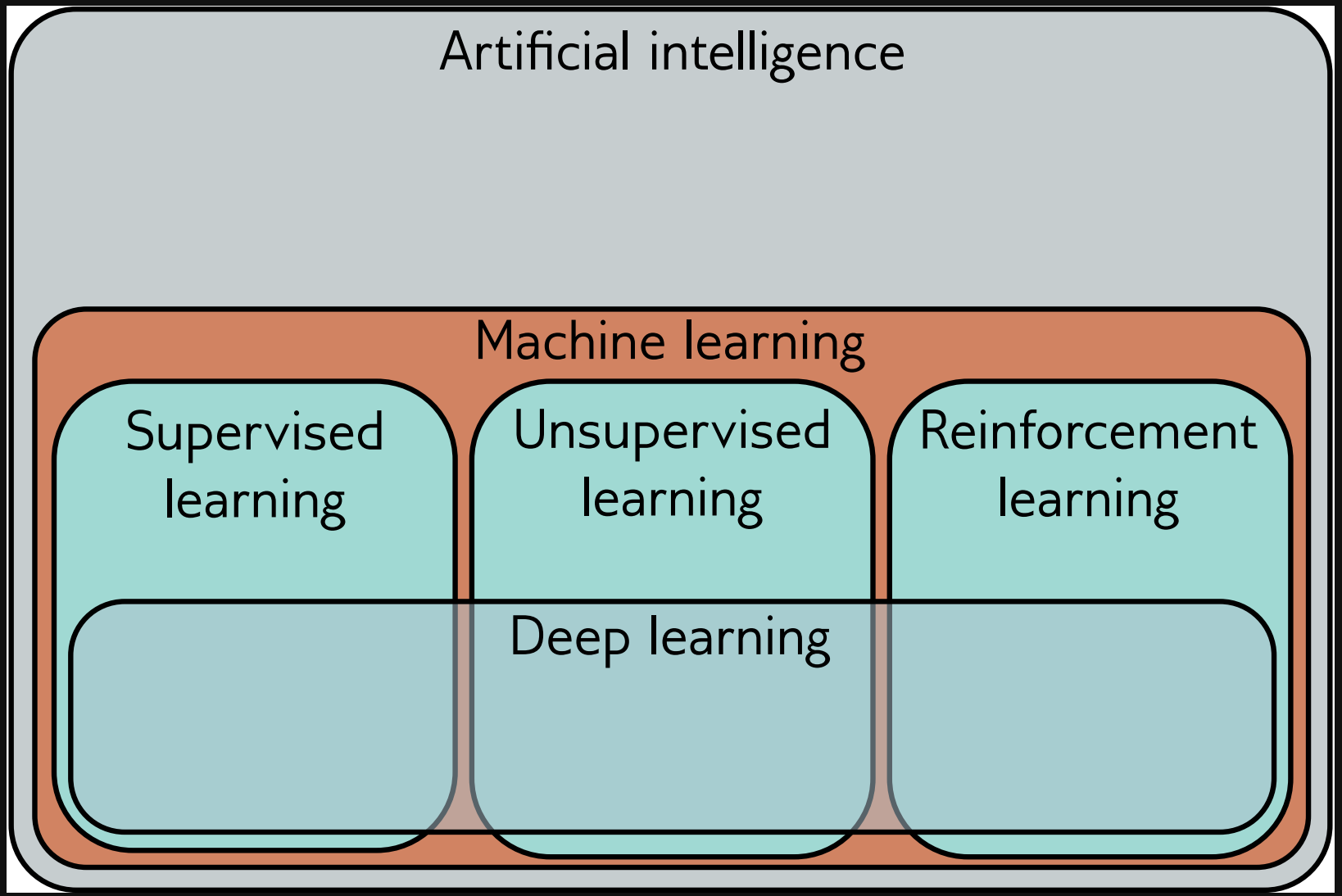
What is Machine Learning (ML)?



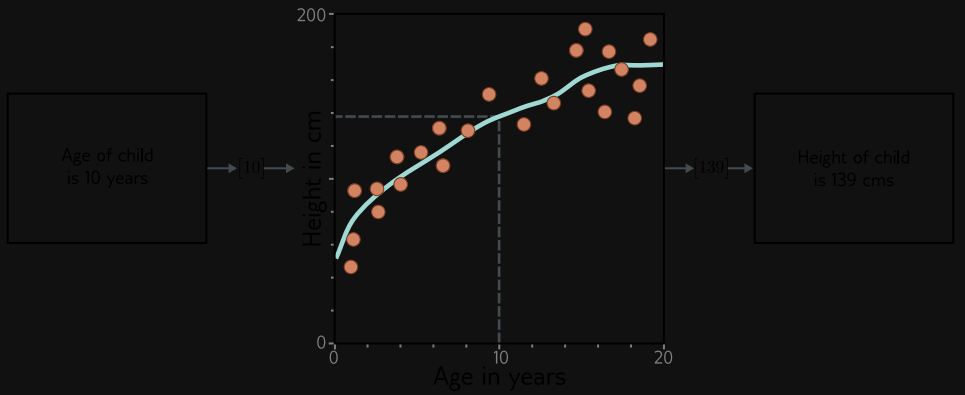
> "Machine learning is a field of study that gives computers the ability to learn [from data] without being explicitly programmed." — Arthur Samuel (1959)

The Three Pillars of ML

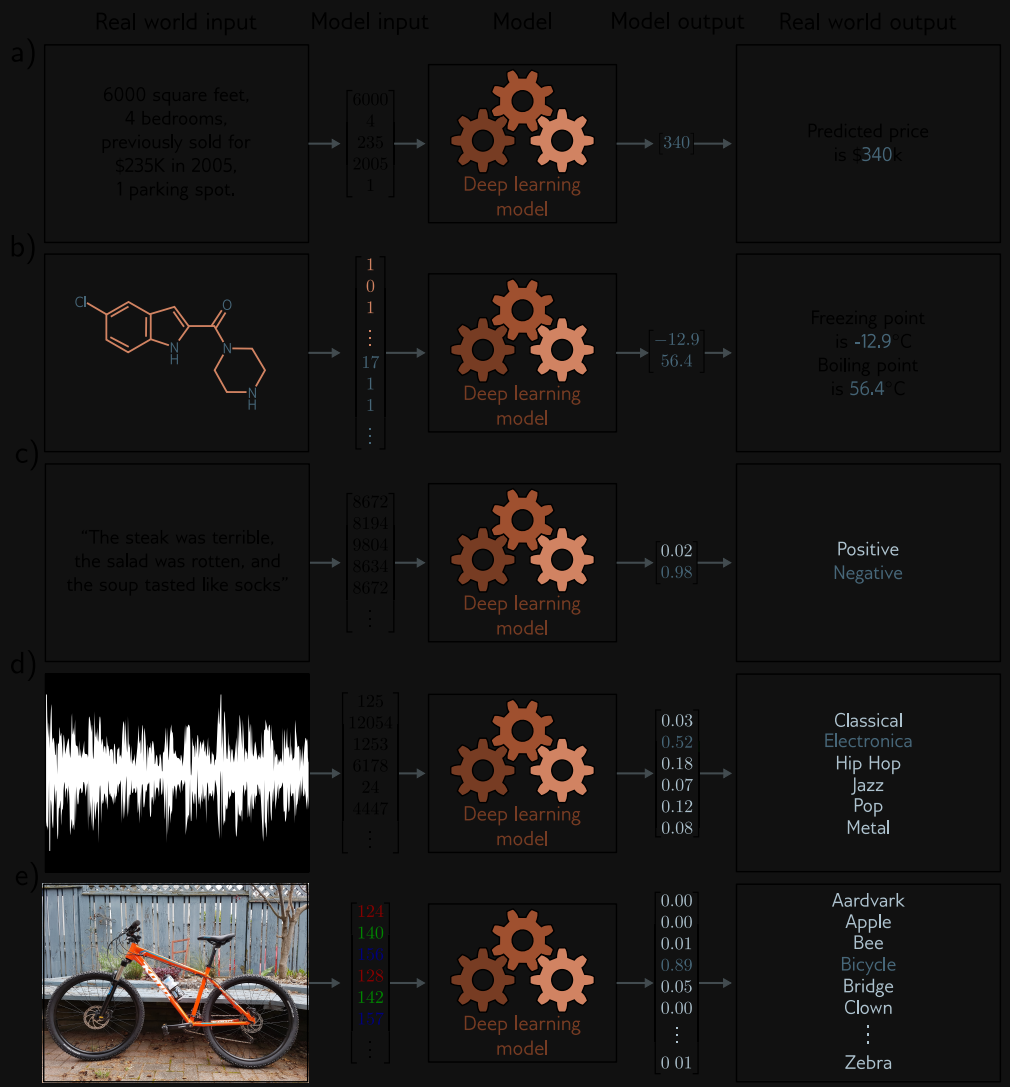


Supervised Learning

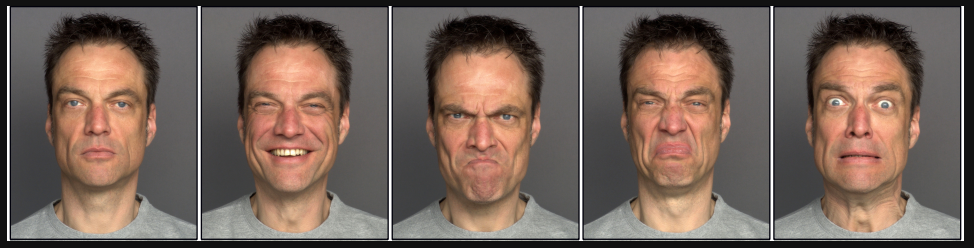
- Economists use regressions to predict outcomes
- ML does the same
- Key: Learning input-output mapping



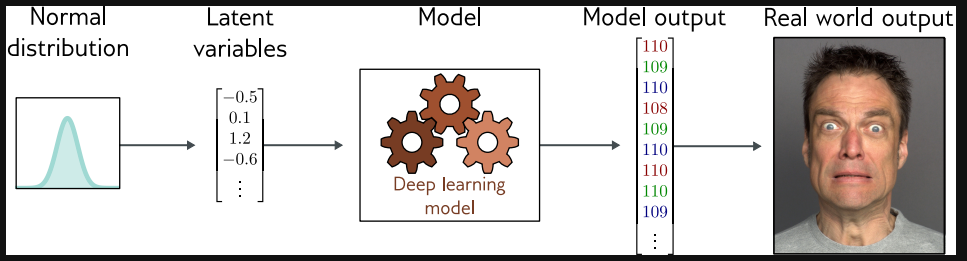
Regression & Classification



Unsupervised Learning

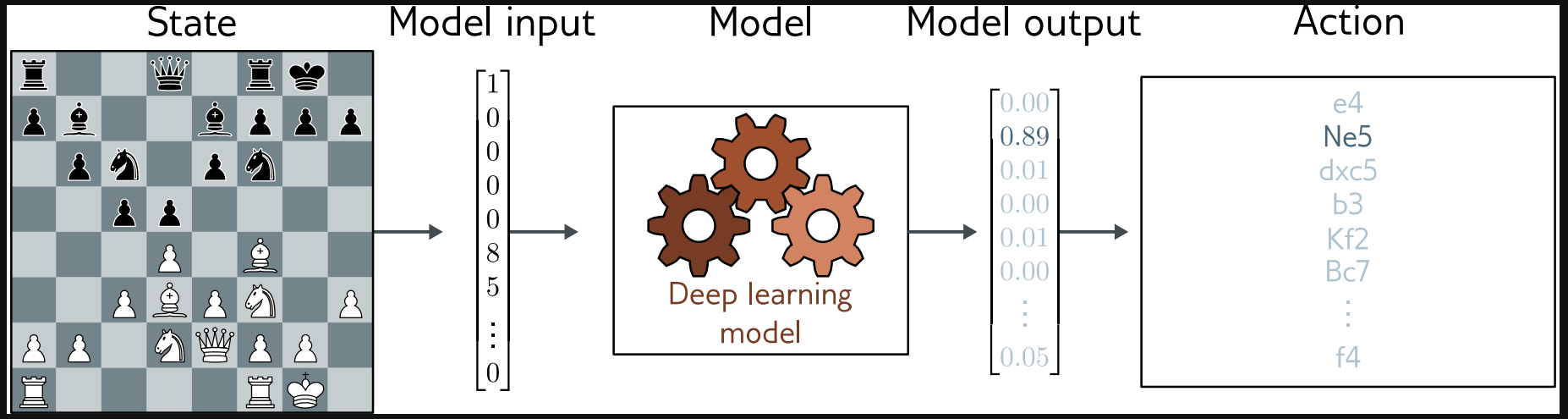


Key: Learning a distribution



Focus on generative unsupervised models

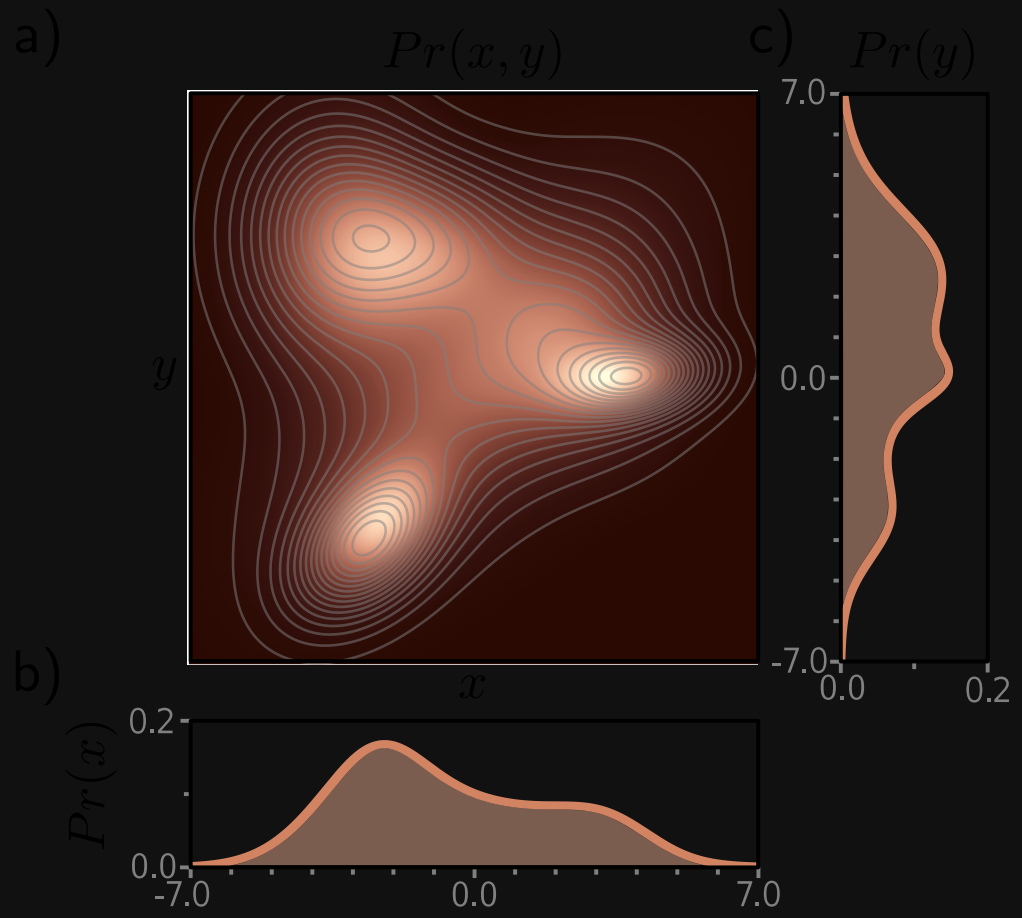
Reinforcement Learning



Key: Learning an action policy

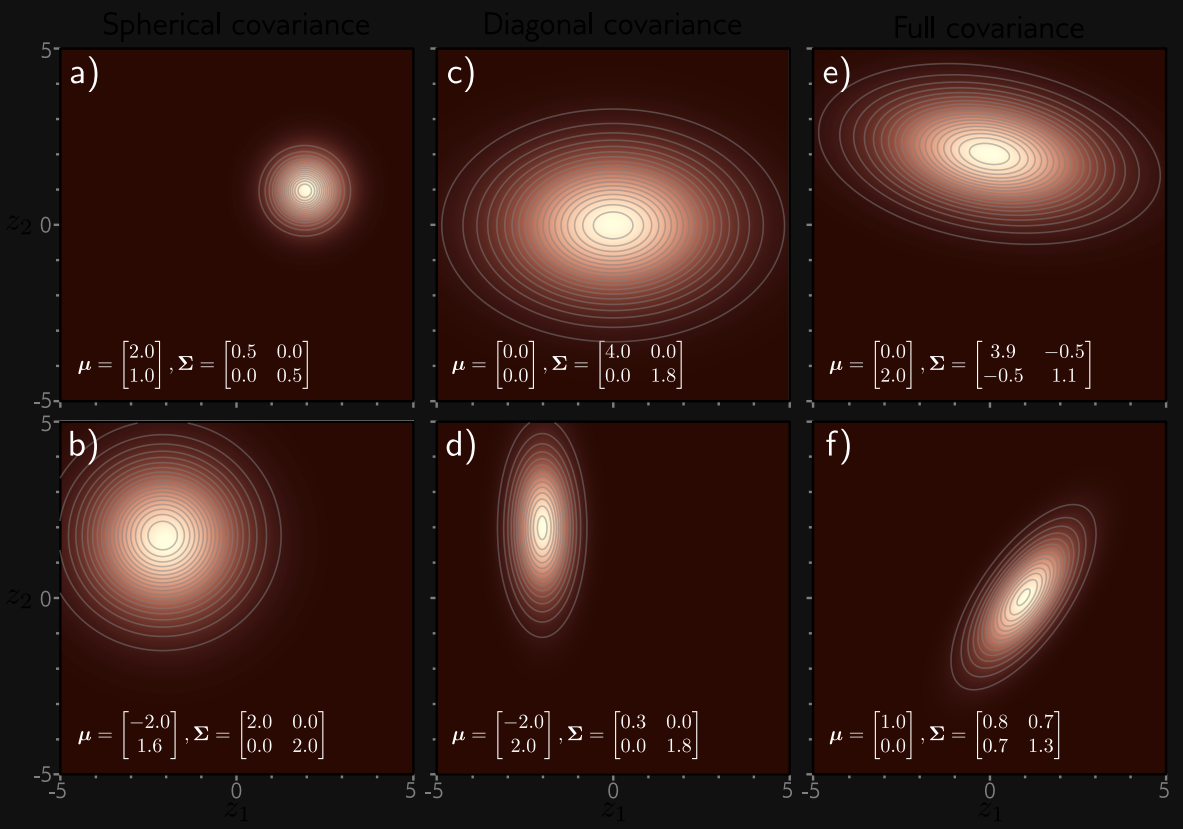
Probability and Information Fundamentals

Joint and Marginal Distributions



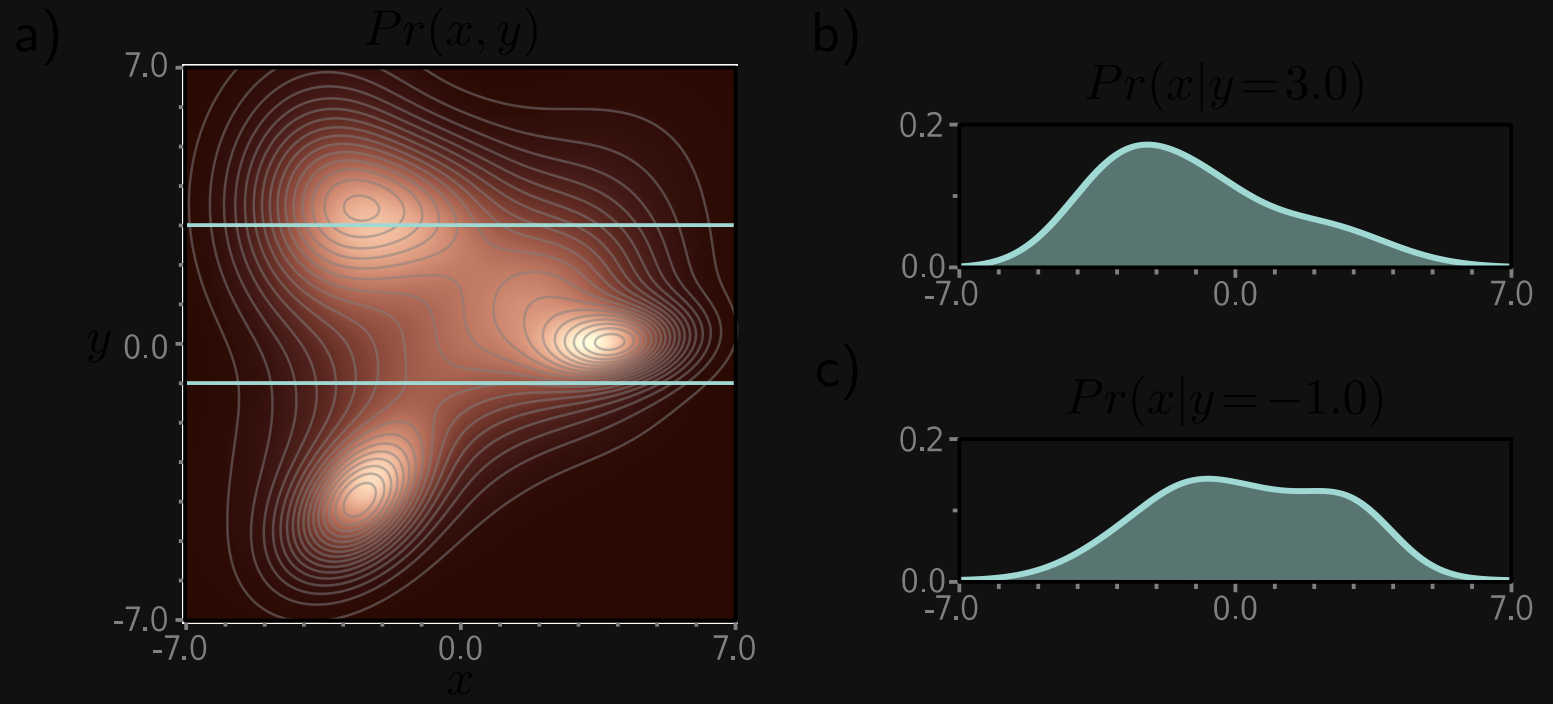
$$P(x) = \int P(x, y)dy$$

Bivariate Gaussian



$$P(\mathbf{x}; \mu, \mathbf{\Sigma}) \propto \exp\left[-\frac{1}{2}(\mathbf{x} - \mu)^T \mathbf{\Sigma}^{-1}(\mathbf{x} - \mu)\right]$$

Conditional Dist. & Bayes' Rule



$$\frac{P(x, y)}{P(y)} \equiv P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

Kullback-Leibler Divergence

KL divergence from (model) Q to (true) P

$$D_{KL}[P||Q] = \int_{-\infty}^{\infty} \log\left[\frac{p(x)}{q(x)}\right] p(x)dx$$

or relative entropy of P with respect to Q captures “distance” (*not* a metric!).

In Bayesian inference for instance, it can measure information gain from prior Q to posterior P.

Shannon Entropy

Entropy is a measure of uncertainty

$$H(X) = -\sum_{x} p(x) \log p(x) = \log(N) - D_{KL}(p(x)||p_{uni\,f}(x))$$

In continous case differential entropy (Shannon)

$$h(X) = -\int_{-\infty}^{\infty} p(x) \log p(x) dx$$

or better limiting density of discrete points (Jaynes)

$$H(X) = \log(N) - D_{KL}(p(x)||m(x))$$

with $m(x)$ the limiting density of discrete points.

Mutual Information

KL divergence of joint from product of marginals

$$I(X;Y) = \iint P_{(X,Y)}(x,y) \log\left[\frac{P_{(X,Y)}(x,y)}{P_X(x)P_Y(y)}\right] dxdy$$
$$= H(Y) - H(Y|X)$$

is the expected reduction in entropy (information gain).

- Captures statistical dependence (zero iff independent)
  - also nonlinear depend. (unlike linear correlation)

Evidence Lower Bound (ELBO)

$$L(\phi, \theta; x) = \int \log \frac{p_\theta(x, z)}{q_\phi(z|x)} q_\phi(z|x) dz$$
$$= \log p_\theta(x) - D_{KL}(q_\phi(z|x)||p_\theta(z|x)) \leq \log p_\theta(x)$$

is a lower bound on the *evidence* $\log p_\theta(x)$ for data x.

In variational Bayesian inference, loss minimization

$$\min_{\theta,\phi} -L(\phi, \theta; x)$$

simultaneously maximizes evidence so that the easy generative model $p_\theta(x|z)p(z)$ is good and minimizes KL divergence so that discriminative model $q_\phi(z|x)$ approximates posterior $p_\theta(z|x)$ well, yielding

$$p(x) \approxeq \frac{p_\theta(x|z)p(z)}{q_\phi(z|x)}$$

References

Prince, Simon J. D. 2023. *Understanding Deep Learning*. Cambridge, Massachusetts: The MIT Press.