Task2theory

Before we can implement the Metropolis-Hastings part of the sampler, we need to simplfy the expression for the acceptance propability α. If we first consider the ratio between true the full conditionals of $\boldsymbol{\eta}^*$ and $\boldsymbol{\eta}$ we can simply insert into the expression found in 1c)

$$\frac{\mathrm{p}(\boldsymbol{\eta}^*|\boldsymbol{y},\boldsymbol{u},\kappa_u,\kappa_v)}{\mathrm{p}(\boldsymbol{\eta}|\boldsymbol{y},\boldsymbol{u},\kappa_u,\kappa_v)} = \exp\left\{-\frac{\kappa_v}{2}\boldsymbol{\eta}^{*T}\boldsymbol{\eta}^* + \boldsymbol{\eta}^{*T}(\kappa_v\boldsymbol{u}+\boldsymbol{y}) - \exp(\boldsymbol{\eta}^*)^T\mathbf{E} + \frac{\kappa_v}{2}\boldsymbol{\eta}^T\boldsymbol{\eta} - \boldsymbol{\eta}^T(\kappa_v\boldsymbol{u}+\boldsymbol{y}) + \exp(\boldsymbol{\eta})^T\mathbf{E}\right\}.$$

Here $\boldsymbol{\eta}^*$ is the proposed m'th step, $\boldsymbol{\eta}$ the value of the $(m-1)$'th step while $\boldsymbol{u}$, κ_u and κ_v are the m'th step values. The ratio between the proposal distributions can also be found by insertion

$$\begin{aligned}\frac{\mathrm{q}(\boldsymbol{\eta}|\boldsymbol{\eta}^*,\boldsymbol{y},\boldsymbol{u},\kappa_u,\kappa_v)}{\mathrm{q}(\boldsymbol{\eta}^*|\boldsymbol{\eta},\boldsymbol{y},\boldsymbol{u},\kappa_u,\kappa_v)} = &\frac{\Big|\kappa_v\mathrm{I}+\mathrm{diag}(c(\boldsymbol{\eta}^*))\Big|^{\frac{1}{2}}}{\Big|\kappa_v\mathrm{I}+\mathrm{diag}(c(\boldsymbol{\eta}))\Big|^{\frac{1}{2}}}\cdot\\ &\exp\Big\{-\frac{1}{2}\boldsymbol{\eta}^T\Big(\kappa_v\mathrm{I}+\mathrm{diag}(c(\boldsymbol{\eta}^*))\Big)\boldsymbol{\eta}+\boldsymbol{\eta}^T(\kappa_u\boldsymbol{u}+b(\boldsymbol{\eta}^*))+\\ &\frac{1}{2}\boldsymbol{\eta}^{*T}\Big(\kappa_v\mathrm{I}+\mathrm{diag}(c(\boldsymbol{\eta}))\Big)\boldsymbol{\eta}^*-\boldsymbol{\eta}^{*T}(\kappa_u\boldsymbol{u}+b(\boldsymbol{\eta}))\Big\}.\end{aligned}$$

(dette er stygt, men jeg vet ikke hvordan det kan bli penere). Multiplying these two ratios and using the fact that $b(\boldsymbol{z}) = \boldsymbol{y} + \mathrm{diag}(c(\boldsymbol{z}))\boldsymbol{z} - c(\boldsymbol{z})$ and that $\Sigma c(\boldsymbol{z}) = \exp(\boldsymbol{z})^T\mathbf{E}$, we get

$$\alpha = \min\left\{1, \frac{\prod_i(\kappa_v + c(\eta_i^*))}{\prod_i(\kappa_v + c(\eta_i))}\exp\left[c(\boldsymbol{\eta})^T\Big(\mathrm{diag}(\boldsymbol{\eta}^*)(\frac{1}{2}\boldsymbol{\eta}^*-\boldsymbol{\eta})+\vec{1}\Big) - c(\boldsymbol{\eta}^*)^T\Big(\mathrm{diag}(\boldsymbol{\eta})(\frac{1}{2}\boldsymbol{\eta}-\boldsymbol{\eta}^*)+\vec{1}\Big)\right]\right\}$$