# TMA4300Ex2

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```
# Loading libraries
library(ggplot2)

# Set seed so that the task can be reproduced
set.seed(42)
```

## Exercise 1: Derivations

From the definition of conditional probability, we know that

$$\begin{aligned} \mathrm{p}(\boldsymbol{\eta}, \boldsymbol{u}, \kappa_u, \kappa_v | \boldsymbol{y}) &\propto \mathrm{p}(\boldsymbol{y} | \boldsymbol{\eta}, \boldsymbol{u}, \kappa_u, \kappa_v) \mathrm{p}(\boldsymbol{\eta} | \boldsymbol{u}, \kappa_u, \kappa_v) \mathrm{p}(\boldsymbol{u} | \kappa_u, \kappa_v) \mathrm{p}(\kappa_u | \kappa_v) \mathrm{p}(\kappa_v) \\ &\propto \mathrm{p}(\boldsymbol{\eta}, \boldsymbol{u}, \kappa_u, \kappa_v | \boldsymbol{y}). \end{aligned}$$

By inserting the corresponding probabilities, this becomes

$$\begin{aligned} \mathrm{p} &\propto \left( \prod_{i=1}^{n} (E_i e^{\eta_i})^{y_i} \, e^{E_i e^{\eta_i}} \right) |\kappa_v \mathrm{I}|^{\frac{1}{2}} \, e^{-\frac{\kappa_v}{2}(\boldsymbol{\eta}-\boldsymbol{u})^T(\boldsymbol{\eta}-\boldsymbol{u})} \kappa_u^{(n-1)/2} e^{-\frac{\kappa_u}{2}\boldsymbol{u}^T \mathbf{R} \boldsymbol{u}} \kappa_u^{\alpha_u - 1} e^{-\beta_u \kappa_u} \kappa_v^{\alpha_v - 1} e^{-\beta_v \kappa_v} \\ &\propto \kappa_u^{\frac{n-1}{2} + \alpha_u - 1} \kappa_v^{\frac{n}{2} + \alpha_v - 1} \exp\left\{ -\beta_u \kappa_u - \beta_v \kappa_v - \frac{\kappa_v}{2} (\boldsymbol{\eta} - \boldsymbol{u})^T (\boldsymbol{\eta} - \boldsymbol{u}) - \frac{\kappa_u}{2} \boldsymbol{u}^T \mathbf{R} \boldsymbol{u} + \sum_i (y_i \eta_i - E_i e^{\eta_i}) \right\}. \end{aligned}$$