# A framework for assessing morphological diversity of modular robots

## 1 Possible values for morphological descriptors

For assessing a body morphology we designed and utilized eight morphological descriptors which describe interesting aspects of it. Each $i$ morphological descriptor was normalized to range between 0 or 1, and is explained hereafter. To enable comparison of descriptors in the experimental setup, they are calculated referent to a parameter $m$ of maximum components allowed in a body. Given an $m$, each descriptor can assume a discrete number of values, and the calculation for this number of values is described hereafter.

**Branching** (Fig.1) refers to clustered attachments of components in the body, i.e., attachment very close to each other: $b$ is the number $b_1$ of components which have all its four faces attached, divided by a practical limit $b_2$, given a total $t$ of components. The limit is the maximum amount of four-sided-attachment components that the body could have, if containing the same amount of components arranged in a different way. (1).

$$b = \begin{cases} b_1/b_2, & \text{if } b_1 >= b_2 \text{ and } t >= 5 \\ 0 & \text{otherwise} \end{cases}$$

$$b_2 = \lfloor (t-2)/3 \rfloor$$

(1)

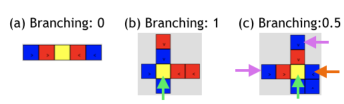


**Fig. 1.** (a) has no four-sided-attachment component, (b) has the maximum it could have, and (c) has one but could have two, if using the components pointed by pink arrows to be attached to the one pointed by the orange arrow.

The maximum amount of values is $n_1$ (Alg.1).

**Result:** $n_1$ = count of distinct values from vector $V$
$i = 0$
**for** *size from 5 to m* **do**
  $limit = \lfloor (size - 2)/3 \rfloor$
  **for** *item from 0 to limit* **do**
    $V[i++] = item/limit$
  **end**
**end**

**Limbs** (Fig.2) describes the abundance of extremities which a body has: $l$ is the number of components $l_1$ which have only one face attached to another component (except for the core-component, i.e., head), divided by a practical limit $t_1$. The limit is the maximum amount of one-sided-attachment components that the body could have, if containing the same amount of components arranged in a different way. (2)

$$l = \begin{cases} l_1/t_1, & \text{if } l_1 >= t_1 \text{and } t_1 > 0 \\ 0 & \text{otherwise} \end{cases}$$
$$t_1 = \begin{cases} 2 * \lfloor (t-6)/3 \rfloor + (t-6) \pmod 3 + 4, & \text{if } t >= 6 \\ t-1 & \text{otherwise} \end{cases} \tag{2}$$

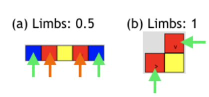


**Fig. 2.** (a) has four components that could be extremities, but only the two pointed by green arrows are; (b) has the maximum number of extremities it could have.

The maximum amount of values is $n_2$ (Alg.2).

**Result:** $n_2$ = count of distinct values from vector $V$
$i = 0$
**for** *size from 1 to m* **do**
  **if** *size <=5* **then**
    $limit = size - 1$
  **else**
    $limit = 2 * \lfloor (size - 6)/3 \rfloor + ((size - 6) \pmod 3) + 4$
  **end**
  **if** *limit <0* **then**
    **for** *item from 1 to limit* **do**
      $V[i++] = item/limit$
    **end**
  **else**
    $V[i++] = 0$
  **end**
**end**

**Length of limbs** (Fig.3) describes how extensive the limbs of the body are: $e$ is the number of components $e_1$ which have two of its faces attached to other components (except for the core-component, i.e., head), divided by a practical limit $e_2$, given a total $t$ of components. The limit is the maximum amount of two-sided-attachment components[1] that the body could have, if containing the same amount of components arranged in a different way. (3)

$$e = \begin{cases} e_1/e_2, & \text{if } e_1 >= e_2 \text{ and } t >= 3 \\ 0 & \text{otherwise} \end{cases}$$

$$e_2 = t - 2$$

(3)

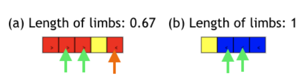


**Fig. 3.** while in (b) the maximum possible quantity of components was used as the extension of a limb, in (a), the component pointed by an orange arrow was used as an extra limb.

The maximum amount of values is $n_3$ (Alg.3).

[1] The types of components would have to be necessarily the same.

**Result:** $n_3$ = count of distinct values from vector $V$
$i = 0$
**for** *size from 3 to m* **do**
  $limit = size - 2$
  **for** *item from 0 to limit* **do**
    $right = size - 1 - left$
    $V[i++] = item/limit$
  **end**
**end**

**Coverage** (Fig.4) describes how covered the area of the body is. The greater this number, the less empty space there is between neighbor components regarding the angles created them in the 2D area of the body: $c$ is the total number $t$ of components of a body divided by the $t_1$ supported number of components in the area of the body, where $t_1$ is calculated as the $t_2$ components that would fit in a column as long as the length of the body, times the $t_3$ components that would fit in a row as long as the width of the body. (4)

$$\begin{aligned} c &= t/t_1 \\ t_1 &= t_2 * t_3 \end{aligned} \tag{4}$$

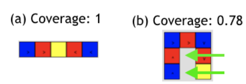


**Fig. 4.** while in (a) all the area created by the body contains components, in (b), there is space for two more components.

The maximum amount of values is $n_4$ (Alg.4).

**Result:** $n_4$ = count of distinct values from vector $V$
$i = 0$
**for** *size from 1 to m* **do**
  **for** *length from 1 to ceiling(size/2)* **do**
    $width = size - length + 1$
    $area = length * width$
    **for** *item from (length+width-1) to area* **do**
      $V[i++] = item/area$
    **end**
  **end**
**end**

**Joints** (Fig.5) describes how movable the body is: $j$ is the number $j_1$ of effective joints, i.e., joints which have both of its opposite faces attached to a core-component or a brick, divided by a practical limit $j_2$, given a total $t$ of components. The limit represents the maximum amount of components with two opposite faces attached that the body could have, if containing the same amount of components arranged in a different way. (5)

$$j = \begin{cases} j_1/j_2, & \text{if } j_1 >= j_2 \text{ and } t >= 3 \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$
$$j_2 = \lfloor (t-1)/2 \rfloor$$

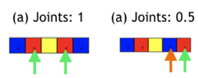


**Fig. 5.** although both cases have two joints, in (b) the second joint is not effective, and would be only if the component pointed by the green arrow was switched with the one pointed by the orange arrow.

The maximum amount of values is $n_5$ (Alg.5).

**Result:** $n_5$ = count of distinct values from vector $V$
$i = 0$ **for** *size from 3 to m* **do**
  $limit = truncate((size - 1)/2)$
  **for** *item from 0 to limit* **do**
    $V[i++] = item/limit$
  **end**
**end**

**Proportion** (Fig.6) describes the 2D ratio of the body: $p$ is the shortest side $p_1$ divided by the longest side $p_2$, after measuring both dimensions of length and width of the body. (6)

$$p = p_1/p_2 \qquad (6)$$


**Fig. 6.** (a) is disproportional and (b) is proportional.

The maximum amount of values is $n_6$ (Alg.6).

**Result:** $n_6$ = count of distinct values from vector $V$
$i = 0$
**for** *size from 1 to m* **do**
  **for** *left from 1 to ceiling(size/2)* **do**
    $width = size - left + 1$
    $V[i++] = length/width$
  **end**
**end**

**Symmetry** (Fig.7) describes the reflexive symmetry of the body: $z$ is the maximum value of horizontal symmetry $z_h$ and vertical symmetry $z_v$. For calculating each of these symmetry values, a referential center for the body is defined as the core-component. For both horizontal $h$ and vertical $v$ axes, a spine is determined as a line dividing the body into two parts according to the center and this axis. Each value is the number $o$ of components that have a mirrored component on the other side of the spine (each match of components accounts for two), divided by the total number $q$ of compared components. The spine is not accounted in the comparison. (7)

$$z = \max_{z_v z_h} \qquad z_v = o_v/q_v \qquad z_h = o_h/q_h \tag{7}$$

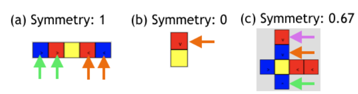


**Fig. 7.** (a) has the components pointed by green arrows horizontally reflected by the components pointed by orange arrows; (b) has no components reflected; (c) has the component pointed by the orange arrow vertically reflected by the components pointed by the green arrow, but no reflection for the component pointed by the pink arrow.

The maximum amount of values is $n_7$ (Alg.7).

**Result:** $n_7$ = count of distinct values from vector $V$
$i = 0$
**for** *size from 3 to m* **do**
  **for** *left from ceiling((size-1)/2) to (size-1)* **do**
    $right = size - 1 - left$
    $V[i++] = (right * 2)/(left + right)$
  **end**
**end**

**Size** (Fig.8) describe the extent of the body in terms of number of components: $s$ is the total number $t$ of components of a body divided by the maximum number

$m$ of components permitted in any body. For these experiments $m$ was set to 20. (8)

$$s = t/m \tag{8}$$

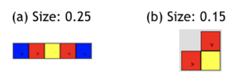


**Fig. 8.** (a) is bigger than (b).

The maximum amount of values is $n_8$ (Alg.8).

**Result:** $n_8$ = count of distinct values from vector $V$
$i = 0$
**for** *size from 5 to m* **do**
  $limit = truncate((size - 2)/3)$
  **for** *item from 0 to limit* **do**
    $V[i++] = item/limit$
  **end**
**end**