The Relative Contribution of Jumps to Total Price Variance *

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Abstract

We examine tests for jumps based on recent asymptotic results; we interpret the tests as Hausman-type tests. Monte Carlo evidence suggests that the daily ratio z-statistic has appropriate size, good power, and good jump detection capabilities revealed by the confusion matrix comprised of jump classification probabilities. We identify a pitfall in applying the asymptotic approximation over an entire sample. Theoretical and Monte Carlo analysis indicates that microstructure noise biases the tests against detecting jumps, and that a simple lagging strategy corrects the bias. Empirical work documents evidence for jumps that account for seven percent of stock market price variance.

Keywords: realized variance, quadratic variation, bipower variation, stochastic volatility.

JEL classification: G12, C51, C52.

Introduction

Observers of financial markets have long noted that financial movements exhibit unusual behavior relative to what would be expected from the Gaussian distribution. There are too many small changes (inliers) and too many large changes (outliers). Clark (1973) is perhaps the first to formally investigate this behavior using econometric methods. He provides an explanation based on an embryonic form of the now-familiar stochastic volatility model, as made formal in Taylor (1982, 1986), and studied extensively in the vast literature that follows (see Shephard (2005)). We now know that stochastic volatility can account for much of the dynamics of short-term financial price movements.

Modelling financial price changes in a way that implies the price series is the realization of a continuous time diffusive process plays a central role in modern financial economics. The assumption of local continuous Gaussianity, among other things, simplifies the hedging calculations that underly modern derivatives pricing. Furthermore, as is well known, the superposition of multiple diffusive stochastic volatility processes can potentially accommodate the unusual dynamics mentioned just above; some examples are Gallant *et al* (1999) and Alizadeh *et al* (2002), among others.

Although the diffusive models are of great analytical convenience, there remains the open issue of whether such models are empirically consistent with the extreme violent movements sometimes seen in financial price series. It is natural to ask whether jump diffusions, with discontinuous sample paths, provide a more appropriate empirical model for financial price

series. Jump diffusions have a long and rich history in financial economics dating back at least to Merton (1976).

Jump diffusion models present two practical problems, which some might view as nearly insurmountable while others might view as more minor nuisances. First, jump models are difficult to estimate, at least by simulation-based methods. The discontinuous sample paths create discontinuities in the econometric objective function, which have to be accommodated by rounding out the corners as in Andersen, Benzoni, and Lund (2002) and Chernov, Gallant, Ghysels, and Tauchen (2003). Still, the nonlinear optimization remains difficult. It could well be the case that approximate likelihood methods based on Duffie, Pan, and Singleton (2000) or Ait-Sahalia (2004) entail a better-behaved econometric objective function, but that empirical work remains, to our knowledge, undone. Second, jumps introduce additional parameters into the derivatives pricing problem such as the price of jump risk and the price of intensity risk if the intensity is state dependent. These risk parameters are hard to interpret, not estimable in the time series alone, and difficult to pin down in the cross section. A case in point is Andersen, Benzoni, and Lund (2002), who estimate jump models and explore the implications of many of the possible branches for candidate values of these risk parameters. Thus, it seems reasonable to attempt to preserve the simpler structure of purely diffusive models and thereby retain their convenience.

Chernov *et al* (2003) provide empirical evidence that there are alternative, mildly nonlinear, purely diffusive models that provide, at the daily level, dynamics comparable to those of jump diffusions. However, they are unable to reach any firm conclusion on the empirical validity of one class of models over the other, and it is self evident that higher frequency data are needed to provide more conclusive evidence on the empirical importance of jumps.

Barndorff-Nielsen and Shephard (2004a, 2006) develop a very powerful toolkit for detecting the presence of jumps in higher frequency financial time series. An appealing feature of their approach is that it does not require a fully observed state variable as in Ait-Sahalia (2002). Their basic idea is to compare two measures of variance, one of which includes the contribution of jumps, if any, to the total variance, while the second is robust to the jump contribution. A test of the statistical significance of the difference, suitably adjusted to improve asymptotic approximation, provides evidence on the presence of jumps. They implement the test on a high frequency data set of exchange rates, as do Andersen, Bollerslev, and Diebold (2004) on a broader set of assets; both papers adduce evidence that seemingly points to the presence of jumps on particular days of their data sets.

This paper evaluates the properties of these newly developed jump detection tests. For the Monte Carlo DGP we mainly use the single-factor log linear stochastic volatility model with jumps, which is the workhorse of applied econometrics on financial data. We supplement the

analysis with consideration of the two-factor model of Chernov *et al* (2003), a purely diffusive serious competitor to a jump diffusion. We examine size, power, and, in order to assess the tests' ability to identify correctly trading days on which a jump has occurred, we examine the confusion matrix, whose elements are the probabilities of correct and incorrect classification. We also consider tests designed to address the question of whether an entire data set is one generated from either a pure diffusion or jump diffusion model; to our knowledge, the full-sample type tests have not been previously considered or analyzed.

Jump detection tests are constructed from very high frequency financial price data, which are potentially seriously contaminated by market microstructure noise. We examine theoretically the robustness of a generic jump test to microstructure noise of the sort commonly considered in the literature, and we consider the appropriateness of a correction strategy due to Andersen, Bollerslev, and Diebold (2004). The theory delivers sharp predictions, which are assessed by further Monte Carlo analysis.

Our empirical work focuses on five minute returns on the S&P Index, cash 1997–2002, and futures 1982–2002, with the objective of identifying the empirical importance of jumps as a source of price variance.

The remainder of this paper is organized as follows. Section 1 sets up the notation and introduces the realized variance measures used for forming the jump test statistics. Section 2 reviews the joint asymptotic distribution of the realized measures, and section 3 summarizes the various jump detection tests. Section 4 reports on extensive Monte Carlo experiments that examine the behavior of the test statistics. Section 5 applies the tests to the S&P 500 Index cash and futures data. Section 6 introduces market microstructure noise, and it examines both analytically and by Monte Carlo the effects of the noise on the jump tests. This section also examines an adjustment for the noise and reports the outcome of applying the adjusted tests to the S&P 500 futures data. Finally, Section 7 contains the concluding remarks.

1 Setup

We consider a scalar log-price $p(t)$ evolving in continuous time as

$$dp(t) = \mu(t)dt + \sigma(t)dw(t) + d\mathcal{L}_J(t), \tag{1}$$

where $\mu(t)$ and $\sigma(t)$ are the drift and instantaneous volatility, $w(t)$ is standardized Brownian motion, $\mathcal{L}_J$ is a pure jump Lévy process with increments $\mathcal{L}_J(t) - \mathcal{L}_J(s) = \sum_{s \leq \tau \leq t} \kappa(\tau)$, and $\kappa(\tau)$ is the jump size. We adopt this notation from Basawa and Brockwell (1982). In this paper, we focus on a special class of the Lévy process called the Compound Poisson Process (CPP). It has constant jump intensity λ, and the jump size $\kappa(t)$ is independent identically distributed.

Throughout, time is measured in daily units, and for integer t we define the within-day geometric returns

$$r_{t,j} = p(t-1+j/M) - p(t-1+(j-1)/M), \quad j = 1, 2, \ldots, M,$$

where M is the sampling frequency.

Barndorff-Nielsen and Shephard (2004a) study general measures of realized within-day price variance, and two natural measures emerge from their work. The first is the now-familiar **Realized Variance**

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2,$$

and the other is the realized **Bipower Variation**

$$BV_t = \mu_1^{-2} (\frac{M}{M-1}) \sum_{j=2}^{M} |r_{t,j-1}||r_{t,j}| = \frac{\pi}{2} (\frac{M}{M-1}) \sum_{j=2}^{M} |r_{t,j-1}||r_{t,j}|,$$

where

$$\mu_a = \mathrm{E}(|Z|^a), \quad Z \sim N(0,1),\ a > 0.$$

We use a slightly different notation that absorbs μ_1^{-2} into the definition of the Bipower Variation and thereby makes it directly comparable to the Realized Variance.

As noted in Andersen, Bollerslev, and Diebold (2002), the Realized Variance satisfies

$$\lim_{M \to \infty} RV_t = \int_{t-1}^{t} \sigma^2(s) ds + \sum_{j=1}^{N_t} \kappa_{t,j}^2,$$

where N_t is the number of jumps within day t and $\kappa_{t,j}$ is the jump size. Thus the RV_t is a consistent estimator of the integrated variance $\int_{t-1}^{t} \sigma^2(s) ds$ plus the jump contribution. On the other hand, the results of Barndorff-Nielsen and Shephard (2004a), along with extensions in Barndorff-Nielsen, Graversen, Jacod, Podolskij, and Shephard (2005) and Barndorff-Nielsen, Graversen, Jacod, and Shephard (2005), imply that under reasonable assumptions about the dynamics (1)

$$\lim_{M \to \infty} BV_t = \int_{t-1}^{t} \sigma^2(s) ds.$$

Thus, BV_t provides a consistent estimator of the integrated variance unaffected by jumps. Evidently, the difference $RV_t - BV_t$ is a consistent estimator of the pure jump contribution and, as emphasized by Barndorff-Nielsen and Shephard (2004a, 2006), can form the basis of a test for jumps. Andersen, Bollerslev, Diebold (2004) use these results to generate evidence suggesting that there are too many large within-day movements in equity, fixed income, and foreign exchange prices to be consistent with the standard continuous time stochastic volatility model with Markov volatility dynamics.

We also consider the **Relative Jump** measure

$$RJ_t = \frac{RV_t - BV_t}{RV_t}, \tag{2}$$

which is an indicator of the contribution (if any) of jumps to the total within-day variance of the process. An equivalent statistic, $-RJ_t$, called the ratio statistic, is proposed and studied by Barndorff-Nielsen and Shephard (2006). We have a slight preference for the term Relative Jump since $100 \cdot RJ$ is a direct measure of the percentage contribution of jumps, if any, to total price variance.

Given a sample of T days, we denote the total realized variance as

$$RV_{1:T} = \sum_{t=1}^{T} RV_t,$$

and the total bipower variation as

$$BV_{1:T} = \sum_{t=1}^{T} BV_t.$$

The corresponding relative jump measure is

$$RJ_{1:T} = \frac{RV_{1:T} - BV_{1:T}}{RV_{1:T}}.$$

2 Asymptotic Distributions

Under the assumption of no jump and some other regularity conditions, Barndorff-Nielsen and Shephard (2006) first give the joint asymptotic distribution of RV_t and BV_t, conditional on the volatility path, as $M \to \infty$

$$M^{\frac{1}{2}} \left[\int_{t-1}^{t} \sigma^4(s) ds \right]^{-\frac{1}{2}} \begin{pmatrix} RV_t - \int_{t-1}^{t} \sigma^2(s) ds \\ BV_t - \int_{t-1}^{t} \sigma^2(s) ds \end{pmatrix} \xrightarrow{\mathcal{D}} N(0, \begin{bmatrix} v_{qq} & v_{qb} \\ v_{qb} & v_{bb} \end{bmatrix}),$$

where

$$\begin{bmatrix} v_{qq} & v_{qb} \\ v_{qb} & v_{bb} \end{bmatrix} = \begin{bmatrix} \mu_4 - \mu_2^2 & 2(\mu_3\mu_1^{-1} - \mu_2) \\ 2(\mu_3\mu_1^{-1} - \mu_2) & (\mu_1^{-4} - 1) + 2(\mu_1^{-2} - 1) \end{bmatrix},$$

and using $\mu_1 = \sqrt{\frac{2}{\pi}}$, $\mu_2 = 1$, $\mu_3 = 2\sqrt{\frac{2}{\pi}}$, $\mu_4 = 3$,

$$\begin{aligned} v_{qq} &= 2, \\ v_{qb} &= 2, \\ v_{bb} &= (\tfrac{\pi}{2})^2 + \pi - 3. \end{aligned}$$

The fact that asymptotically $v_{qb} = v_{qq}$ is no coincidence and reflects a situation exactly analogous to that of the Hausman (1978) test. Asymptotically, the situation is one with Gaussian errors, and RV_t is the most efficient estimate of the integrated variance $\int_{t-1}^{t} \sigma^2(s)ds$. The bipower variation is a less efficient estimator under the maintained assumption of no jumps, though it is also more robust. Thus, following the logic of the Hausman test:

PROPOSITION 1. Under the maintained assumptions of no jumps, then asymptotically $RV_t - BV_t$ is independent of RV_t conditional on the volatility path, and thus RJ_t in (2) is asymptotically the ratio of two conditionally independent random variables.

The proof is obvious by inspection. The above asymptotic distribution theory can be generalized considerably as in Barndorff-Nielsen, Graversen, Jacod, Podolskij, and Shephard (2005) and Barndorff-Nielsen, Graversen, Jacod, and Shephard (2005); see Barndorff-Nielsen and Shephard (2005b) for a survey.

The relative jump measure RJ_t has a natural notion of scale. If one is satisfied with this sense of scale, then there is no need to estimate the integrated quarticity $\int_{t-1}^{t} \sigma^4(s)ds$ as required for a standard deviation notion of scale.

To determine the scale of $RV_t - BV_t$ in units of conditional standard deviation, one needs to estimate the integrated quarticity $\int_{t-1}^{t} \sigma^4(s)ds$. Andersen, Bollerslev, and Diebold (2004) suggest using the jump-robust realized **Tri-Power Quarticity** statistic, which is a special case of the multipower variations studied in Barndorff-Nielsen and Shephard (2004a)

$$TP_t = M\mu_{4/3}^{-3}(\frac{M}{M-2})\sum_{j=3}^{M} |r_{t,j-2}|^{4/3}|r_{t,j-1}|^{4/3}|r_{t,j}|^{4/3}, \tag{3}$$

and they note that

$$TP_t \to \int_{t-1}^{t} \sigma^4(s)ds$$

even in the presence of jumps. There is a scale normalizing constant M in front of the summation because each absolute return is of order $\sqrt{\triangle t}$, so the product is of order $(\triangle t)^2$, and the summation $\triangle t$. M is $\frac{1}{\triangle t}$, which cancels out the summation order, and the whole expression approaches a well defined limit. As mentioned before, the value normalizing term is now $\mu_{4/3}^{-3}$ since each absolute return is raised to power 4/3 and there are three such terms in one product. Notice that the power of each absolute return should be strictly less than two for the statistics to be robust to jumps. If it is equal to two, the statistics will behave just like RV, picking up both the jump and the continuous-time parts, and, if it is greater than two, the whole expression will blow up to infinity, because of the interaction between the scale normalizing constant and the jump component. Another estimator, based on Barndorff-Nielsen and Shephard (2004a), is

the realized **Quad-Power Quarticity**

$$QP_t = M\mu_1^{-4}(\frac{M}{M-3})\sum_{j=4}^{M}|r_{t,j-3}||r_{t,j-2}||r_{t,j-1}||r_{t,j}|. \tag{4}$$

Given a sample of T days, the corresponding full-sample measures for TP and QP are:

$$TP_{1:T} = \sum_{t=1}^{T} TP_t,$$

$$QP_{1:T} = \sum_{t=1}^{T} QP_t.$$

3 Some Jump Test Statistics

3.1 Daily Statistics

One strategy is to use the above theoretical results to compute a measure of extreme movements on a day-by-day basis and then inspect for days where the price movements appear abnormally large, which would be indicative of at least one jump that day. Based on Barndorff-Nielsen and Shephard's (2006) theoretical results, Andersen, Bollerslev, and Diebold (2004) use the time series

$$z_{TP,t} = \frac{RV_t - BV_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}TP_t}} \tag{5}$$

to test for daily jumps. For each t, $z_{TP,t} \xrightarrow{\mathcal{D}} N(0,1)$ as $M \to \infty$, on the assumption of no jumps. Thus the sequence $\{z_{TP,t}\}_{t=1}^{T}$ provides evidence on the daily occurrence of jumps in the price process. Another closely related measure is

$$z_{QP,t} = \frac{RV_t - BV_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}QP_t}},$$

which uses the realized quad-power quarticity (4) in place of the realized tri-power quarticity (3) in the computation of the conditional scale of $RV_t - BV_t$.

Following Andersen, Bollerslev, Diebold and Labys (2001, 2003), Andersen, Bollerslev, Diebold and Ebens (2001), and Barndorff-Nielsen and Shephard (2005a), one might expect to be able to improve finite sample performance by basing the test statistics on the logarithm of the variation measures. In the case of (5), the statistic is

$$z_{TP,l,t} = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{TP_t}{BV_t^2}}}, \tag{6}$$

which is also used in Andersen, Bollerslev, and Diebold (2004). Another modification based on Barndorff-Nielsen and Shephard (2005a) entails the maximum adjustment

$$z_{TP,lm,t} = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\max(1, \frac{TP_t}{BV_t^2})}}. \tag{7}$$

Analogous to the logarithmic adjustment to $z_{TP,t}$, we also have the statistic

$$z_{QP,l,t} = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{QP_t}{BV_t^2}}},$$

and with the additional maximum adjustment, as used in Barndorff-Nielsen and Shephard (2004a), is

$$z_{QP,lm,t} = \frac{\log(RV_t) - \log(BV_t)}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\max(1, \frac{QP_t}{BV_t^2})}}.$$

We also recall the measure of the relative jump (equivalent to negative of the ratio statistic):

$$RJ_t = \frac{RV_t - BV_t}{RV_t},$$

and the statistics based on it are

$$z_{TP,r,t} = \frac{RJ_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{TP_t}{BV_t^2}}}, \tag{8}$$

$$z_{QP,r,t} = \frac{RJ_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{QP_t}{BV_t^2}}},$$

$$z_{TP,rm,t} = \frac{RJ_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\max(1, \frac{TP_t}{BV_t^2})}}, \tag{9}$$

$$z_{QP,rm,t} = \frac{RJ_t}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\max(1, \frac{QP_t}{BV_t^2})}}.$$

The QP-versions of these statistics are equivalent to the ratio jump statistics of Barndorff-Nielsen and Shephard (2006). By visual inspection, one can see that the denominators of the log tests and the ratio tests are the same. In other words, the numerators of the second and third pairs of the jump statistics have identical asymptotic distributions conditional on the volatility path. The intuition is that the first order Taylor expansions of the numerators of the log and the ratio test statistics around the asymptotic mean of RV_t and BV_t, i.e., the integrated variance $\int_{t-1}^{t} \sigma^2(s)ds$, are the same, thus the delta method generates the same asymptotic distribution.

In Section 4 below we examine, among other things, the quality of the asymptotic normal approximation to each of these statistics under the null hypothesis of no jumps.

3.2 Full Sample Statistics

We also consider the finite sample properties of the test statistics for jumps in a given sample. In this case, the test statistics are computed over the entire sample instead of on a day-by-day basis.

A natural asymptotically normal test statistic, following Andersen, Bollerslev, and Diebold (2004), is

$$z_{TP,1:T} = \frac{RV_{1:T} - BV_{1:T}}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}TP_{1:T}}}. \tag{10}$$

Another, based on the results of Barndorff-Nielsen and Shephard (2004a), is

$$z_{QP,1:T} = \frac{RV_{1:T} - BV_{1:T}}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}QP_{1:T}}}. \tag{11}$$

The log versions of these statistics are

$$z_{TP,l,1:T} = \frac{\log(RV_{1:T}) - \log(BV_{1:T})}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{TP_{1:T}}{BV_{1:T}^2}}}, \tag{12}$$

$$z_{QP,l,1:T} = \frac{\log(RV_{1:T}) - \log(BV_{1:T})}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{QP_{1:T}}{BV_{1:T}^2}}}. \tag{13}$$

With the additional maximum adjustment, the statistics become

$$z_{TP,lm,1:T} = \frac{\log(RV_{1:T}) - \log(BV_{1:T})}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\max(\frac{1}{T}, \frac{TP_{1:T}}{BV_{1:T}^2})}}, \tag{14}$$

$$z_{QP,lm,1:T} = \frac{\log(RV_{1:T}) - \log(BV_{1:T})}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\max(\frac{1}{T}, \frac{QP_{1:T}}{BV_{1:T}^2})}}. \tag{15}$$

There are similar full-sample statistics based on $RJ_{1:T}$ using $TP_{1:T}$ and $QP_{1:T}$ as well:

$$z_{TP,r,1:T} = \frac{RJ_{1:T}}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{TP_{1:T}}{BV_{1:T}^2}}}, \tag{16}$$

$$z_{QP,r,1:T} = \frac{RJ_{1:T}}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\frac{QP_{1:T}}{BV_{1:T}^2}}}, \tag{17}$$

$$z_{TP,rm,1:T} = \frac{RJ_{1:T}}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\max(\frac{1}{T}, \frac{TP_{1:T}}{BV_{1:T}^2})}}, \tag{18}$$

$$z_{QP,rm,1:T} = \frac{RJ_{1:T}}{\sqrt{(v_{bb} - v_{qq})\frac{1}{M}\max(\frac{1}{T}, \frac{QP_{1:T}}{BV_{1:T}^2})}}. \tag{19}$$

A simple t-test on the **Relative Jump** measure is

$$t_{RJ,classic} = \frac{\frac{1}{T}\sum_{t=1}^{T} RJ_t}{\sqrt{v_{classic}}}, \tag{20}$$

where $v_{classic}$ is the classical estimate of the variance of the mean computed under the assumption of no serial dependence. One can also form

$$t_{RJ,GMM} = \frac{\frac{1}{T}\sum_{t=1}^{T} RJ_t}{\sqrt{v_{HAC}}}, \tag{21}$$

where v_{HAC} is a HAC estimator of the variance of the mean. A bootstrap version is

$$t_{RJ,boot} = \frac{\frac{1}{T}\sum_{t=1}^{T} RJ_t}{\sqrt{v_{boot}}}, \tag{22}$$

where v_{boot} is a bootstrap estimate of the variance of the mean. Finally, by bootstrapping $t_{RJ,classic}$ one can get a bootstrap confidence interval (t_{low}, t_{up}) for the t-statistic and form a test that way. We have not computed these more complicated t statistics because the evidence and theory suggest that the RJ_t are essentially serially uncorrelated, if the jump part follows the CPP process. Interested readers are referred to Gonçalves and Meddahi (2005) for an in-depth study of the test statistics based on the bootstrap variance of the realized variance measures.

4 Monte Carlo Analysis

4.1 Setup

We analyze the behavior of the various tests above under two classes of models for the log price process p_t. The first is a stochastic volatility jump diffusion model of the form

SV1FJ

$$\begin{aligned} dp(t) &= \mu\, dt + \exp\left[\beta_0 + \beta_1 v(t)\right] dw_p(t) + d\mathcal{L}_J(t) \\ dv(t) &= \alpha_v v(t)\, dt + dw_v(t) \end{aligned} \tag{23}$$

where the w's are standard Brownian motions, $\mathrm{Corr}(dw_p, dw_v) = \rho$ is the leverage correlation, $v(t)$ is a stochastic volatility factor, $\mathcal{L}_J(t)$ is a Compound Poisson Process with constant jump intensity λ and random jump size distributed as $N(0, \sigma^2_{jmp})$. The model **SV1FJ** has one stochastic volatility factor and a jump; it has been widely studied. Barndorff-Nielsen and Shephard (2004a) term this a stochastic volatility model with rare jumps. A special case without a jump term is

SV1F

$$\begin{aligned} dp(t) &= \mu\, dt + \exp\left[\beta_0 + \beta_1 v(t)\right] dw_p(t) \\ dv(t) &= \alpha_v v(t)\, dt + dw_v(t) \end{aligned}$$

We also follow Chernov *et al* (2003) and consider a two factor stochastic volatility model

SV2F

$$\begin{aligned} dp(t) &= \mu\, dt + \text{s-exp}\left[\beta_0 + \beta_1 v_1(t) + \beta_2 v_2(t)\right] dw_p(t) \\ dv_1(t) &= \alpha_{v1} v_1(t)\, dt + dw_{v1}(t) \\ dv_2(t) &= \alpha_{v2} v_2(t)\, dt + [1 + \beta_{v2} v_2(t)] dw_{v2}(t) \end{aligned} \tag{24}$$

where $v_1(t)$ and $v_2(t)$ are stochastic volatility factors. The process $v_1(t)$ is a standard Gaussian process while $v_2(t)$ exhibits a feedback term in the diffusion function. The feedback is found to be important in Chernov *et al* (2003). The function s-exp means the usual exponential function with a polynomial function splined in at very high values of its argument to ensure that the system (24) with $\beta_{v2} \neq 0$ satisfies the growth conditions for a solution to exist and for the Euler scheme to work. The s-exp function is considered more fully below. The leverage correlations are $\text{Corr}(dw_p, dw_{v1}) = \rho_1$ and $\text{Corr}(dw_p, dw_{v2}) = \rho_2$.

The **SV2F** model has continuous sample paths but it can generate quite rugged appearance price series via the volatility feedback and the exponential function, which is splined only at relatively high values of its argument. One of our objectives is to examine whether the various jump test statistics will falsely signal jumps under a process with continuous sample paths.

Table 1 shows the parameter settings used in the simulations for the **SV1FJ** model, with, of course, **SV1F** representing the null hypothesis. The parameter values are based on the empirical results reported in Andersen, Benzoni, and Lund (2002), Andersen, Bollerslev, Diebold (2004), and Chernov *et al* (2003). The three values of the volatility mean reversion parameter α_v represent very slow mean reversion, with a half life of two years (2×252 trading days), medium mean reversion, with a half life just over one week, and very strong mean reversion with a half life of 0.50 days. The very slow and very strong values for the mean reversion parameter are based on Chernov *et al* (2003) while the medium value is based on Andersen, Benzoni, and Lund (2002). There are four values for the jump intensity (λ). The smallest jump intensity is estimated by Andersen, Benzoni, and Lund (2002) for the daily S&P 500 Cash Index, while the other three come from Andersen, Bollerslev, and Diebold (2004) for the high-frequency data on the DM/$ spot, S&P 500 Index futures, and U.S. T-Bond futures markets. The parameter σ_{jmp} varies over a range that includes Andersen, Benzoni, and Lund's (2002) estimate of about 1.50 percent. The value of the leverage parameter is from Andersen, Benzoni, and Lund (2002), though our experiments suggest that the findings are not very sensitive to this parameter. The scaling parameters, i.e., the β's, are selected on the basis of the studies and some experiments guided by plots of simulated data. For computational reasons, the above-mentioned empirical studies use different normalizations, different timing conventions, and they simulated arithmetic, not geometric returns, so it is difficult to match up the values exactly to those previously estimated.

The simulation details are as follows: the basic unit of time is one day throughout. Simulations of the diffusion parts of the **SV1FJ** and **SV2F** models are generated using the basic Euler scheme with an increment of 1 second per tick on the Euler clock. We first simulate the log price level, then compute the 1-minute, 3-minute, 5-minute and 30-minute geometric returns by taking the difference of the corresponding log price levels, with the objective to see how sampling frequency affects the properties of the test statistics. In order to make results comparable across sampling intervals, it is important to simulate a **single** Brownian motion at a very fine time interval, use the Euler scheme to solve for a simulation from the nonlinear model, and then sample that series at coarser intervals. The jump component is simulated by drawing the jump times from the exponential distribution and the jump size from $\mathrm{N}(0, \sigma^2_{jmp})$.

The simulation of the **SV2F** model requires some special attention to satisfy the growth conditions (Kloeden and Platen, 1992, p. 128) of Ito's theorem. Following Chernov *et al* (2003) we spline smoothly to the far right-hand side of the exponential function the growth function itself, so the model satisfies the regularity conditions by construction. The knot point for the spline is a value of the argument of the exponential that implies a 100 percent annualized volatility, which is unlikely to occur in the normal U.S. financial markets. Inspection of very long simulations revealed the spline has no essential effect, except for attenuating the influences of a very few really large values of the argument.

Figure 1 and Figure 2 show the simulated realizations of length 10,000 days (about 40 years) at the daily frequency from the **SV1F** and **SV1FJ** models, with medium mean reversion, $\sigma_{jmp} = 1.50$, and $\lambda = 0.014$ in the latter case. The daily returns seen in the second panel appear reasonable for a financial time series. Likewise, Figure 3 shows a simulation at the daily frequency from the **SV2F** model which looks very similar to the plots of daily returns found in the financial econometrics literature.

4.2 Monte Carlo Findings

4.2.1 Daily Statistics

We first consider the characteristics of the daily statistics computed over a long simulated realizations, $\{z_{TP,t}\}_{t=1}^{N}$ and $\{z_{QP,t}\}_{t=1}^{N}$, of length N=45,000.

4.2.1.1. Size

Figure 4 shows QQ plots of the raw, log-max, and ratio-max adjusted versions of $z_{TP,t}$ defined in (5), (7), (9) above. Since the log and ratio adjusted versions are similar, they are not shown here. The data generation process is the null case in Table 1, $\sigma_{jmp} = 0.00$, with medium mean reversion ($\alpha_v = -0.100$) and 5-minute returns. Since large values of the z statistics discredit the null hypothesis of no jumps, we are only interested in the right-hand tail. As is clear from

the figures, the raw statistic has a size distortion towards over rejecting in the range 2.00 to 3.00, about 0.99 to 0.999 significance level, which is the usual range considered for these daily z test statistics. However, the log transformation and the statistic based on RJ correct the size distortions, except perhaps in the extreme right-hand tail. Although the boundary in the maximum adjustment is hit a lot, the QQ plots do not appear to change much when we add the maximum adjustment. A similar situation shows up when we study the full sample statistics. Apparently, the deviations from the boundary are not serious, thus they do not affect the value of the z statistics significantly.

As seen from Figures 5, the raw statistic $z_{QP,t}$ has the same size problem and the effects of the log, ratio and max adjustments are the same as with $z_{TP,t}$. It appears that the choice between TP_t and QP_t does not matter in any important way for the estimation of $\int_{t-1}^{t} \sigma^4(s)ds$. Interestingly, Figures 6 indicates that the relative jump statistic, RJ_t, is very well approximated by a Gaussian distribution.

With the exception of the $z_{TP,rm,t}$ statistic, the sampling frequency has a significant impact on the size. As the sampling frequency decreases, i.e., the sampling interval increases, the actual sizes of all statistics except $z_{TP,rm,t}$ increase above the Monte Carlo confidence band, which can be seen in Figure 7 for the medium mean reversion case. The behavior of the size for the slow mean reversion and the fast cases are the same, so they are not shown here.

Figure 8 shows a simulation of length 1400 days of the five versions of $z_{TP,t}$ statistics under the null of no jumps and $\alpha_v = -0.100$. We choose 1400 because that is very close to the sample size for the Cash Index in the empirical application below. The size problem is apparent in the top panel as is the correction due to the log adjustment and ratio adjustment. Note that $z_{TP,rm,t}$ in the bottom panel appears to have the best size property among the five statistics.

4.2.1.2. Jump Detection

For evidence on the ability of the tests to detect jumps under the SV1FJ setting, Figure 9 shows a simulation of the versions of $z_{TP,t}$ under the same conditions as in Figure 8 except with $\sigma_{jmp} = 1.50$ and $\lambda = 0.014$. As is clear, the statistics do a very good job of picking out the jumps.

To see how the parameter settings affect the detection ability of the test statistics, we report the confusion matrix under different parameter settings. The matrix consists of four cells: the upper left cell is the proportion of the statistic smaller than the 99% standard Gaussian critical value among days without jumps, the upper right cell is the proportion greater than the 99% critical value among the no-jump days; the lower left cell is the proportion smaller than the 99% critical value among the days that jumps occur on, and the lower right one is the proportion greater than the 99% critical value among the jump days. The diagonal elements of this matrix

represent the ability of the test statistic to tell correctly whether or not there is a jump(s) in a particular day, and the off-diagonal represent the proportion of days when the statistic signals the wrong answer. The row sums of this matrix equal unity. Under the asymptotic theory presented in the previous sections, we expect the (1,1) element of this matrix to be close to 99%, as the sampling interval goes to zero; however, the (2,2) element of this matrix needs not approach unity, as explained in Subsection 4.2.1.3. below.

Table 3 contains the confusion matrices for the three test statistics, $z_{TP,t}$, $z_{TP,lm,t}$ and $z_{TP,rm,t}$ over different jump intensities and sampling frequencies under medium mean reversion ($\alpha_v = -0.100$) and jump size 1.50. The mean reversion does not significantly impact the daily statistics, and the jump size positively affects the rejection frequency in the expected manner, so we do not report variations in these two parameters here. By comparing the first rows of the matrices for the three test statistics, we can see that $z_{TP,t}$ tends to signal more false jumps when there is no jump in a particular day, and the problem becomes more severe with longer sampling intervals. On the other hand, $z_{TP,rm,t}$ is more robust with a false rejection rate close to the nominal size of 1 percent. By examining the second rows of the matrices, we see that as the sampling interval increases from 1 minute to 30 minutes, the test statistics signal fewer instances of jumps when jumps occur because of the time averaging effect. An increase in jump intensity, λ, has a positive effect on the detection rate. The reason is that the statistics detect jumps through the proportion of the total price variation attributable to jumps. When λ increases, the expected number of jumps per day increases. This increases the expected accumulated jump contribution per day, making it more likely that the statistics detect the jumps.

4.2.1.3. Power

The above jump detection results are most likely to be important for daily application purposes, but from the statistical view point, we are also interested in the power of the tests. Table 4 reports the power of the three test statistics, $z_{TP,t}$, $z_{TP,lm,t}$ and $z_{TP,rm,t}$, over different jump intensities and jump sizes under medium mean reversion using 5-minute simulated returns. Jump intensity and jump size have positive effects on power. These results are intuitive. Table 5 shows the power property of $z_{TP,rm,t}$ over different jump intensities, jump sizes and sampling frequencies. Just like the effect on the jump detection rate, the sampling frequency impacts power positively. So combining the results in size, jump detection rate and power, we can see that, in the absence of market microstructure noise, for lower sampling frequency the statistics not only neglect true jumps when there are jumps (lower jump detection rate and lower power), but also signal more false jumps when there is no jump (larger size). So high-frequency data is necessary for jump detection when we use this types of statistics.

In addition to the effects of different parameters on the test statistics, an important phe-

nomenon is apparent in these two tables. The test is inconsistent; that is, its power will not approach one as the sample size goes to infinity. The reason is that, for any finite jump intensity, the underlying jump-diffusion process does not have jump(s) every day. The time interval between two jumps is exponentially distributed, and the probability of having no jump for a day is $e^{-\lambda}$, which is not zero for any $\lambda < \infty$. Even when λ is as high as 2 (unlikely to occur in the empirical data), such a probability is still 0.135. Since the test statistics signal jumps for only a very small portion of the no-jump days, their power, defined to be the proportion of signalled jump days over the whole sample, will not approach one.

4.2.1.4. The SV2F Model

The above results show that the test statistics have excellent size property under the SV1F(J) model. By way of contrast, however, Figure 10 shows a simulation of the $z_{TP,t}$ statistics under the **SV2F** model described above. The underlying model does not contain jumps, though the simulation indicates detection of spurious jumps. The figure suggests the test statistics have incorrect size. Table 6 for 5 minute returns further illustrates this point. The 95% confidence band for the nominal size of 5% over 45,000 simulation days is [0.04799, 0.05201], that for the size of 1% is [0.00908, 0.01092], that for the size of 0.5% is [0.00435, 0.00565], and that for the size of 0.1% is [0.00071, 0.00129]. However, all the empirical sizes are outside these confidence bands, although the size of the $z_{TP,rm,t}$ is least affected. The finding of over rejection is in contrast to that of Barndorff-Nielsen and Shephard (2006), who utilize the superposition of two square-root (CIR) volatility processes. It suggests that the curvature of the volatility functions influences the properties of the test statistics somewhat, although the size distortions in Table 6 are not very large from a practical point of view.

4.2.2 Monte Carlo Assessment of Full Sample Statistics

We now consider the test of the null hypothesis that a given data set has been generated from a DGP without jumps versus the alternative of one with jumps. This question is different from the one just considered, where the issue was whether or not a jump occurred on a particular day. Tests of this null hypothesis are based on the full sample, rather than being computed day-by-day. The candidate test statistics are displayed in Equations (10)–(22) above.

Perhaps the most interesting tests are those based on $z_{TP,1:T}$ and $z_{QP,1:T}$ and their transforms as defined in equations (10)–(19). Table 7 shows the rejection frequencies under the one factor **SV1FJ** and two-factor **SV2F** models, for 1000 repetitions of a time series with 1400 days. The tests appear to have excellent size and power properties under the **SV1FJ** model with the strong and medium mean reversions, while, as might be expected, the power of each test under the slow mean reversion is somewhat lower, because the diffusive part of the model accounts for

a relatively larger share of the variance. However, the tests seem to have incorrect size under the **SV2F** model for this sample size.

The finding that, under the **SV2F** model, the full-sample statistics incorrectly reject the null of no jump more often than the daily statistics is notable, but needs to be interpreted properly. The realized Bipower Variation and Realized Variance, though consistent, are not unbiased for the integrated variance. That is, for any finite sampling frequency M, it can be the case that, if the expectations exist,

$$\mathrm{E}\left(RV_t - BV_t\right) \doteq \mathcal{B}(M) \neq 0, \tag{25}$$

although one expects $\mathcal{B}(M)$ to be rather small. However, relative to the daily statistics, the full sample statistics increase the time span without shrinking the sampling interval, so in effect the small bias gets inflated to $T \cdot \mathcal{B}(M)$ and the z statistics can become very large. Therefore, the larger the sample size, the more apparent the difference between $RV_{1:T}$ and $BV_{1:T}$, and hence the greater the rejection frequency of the full sample z statistics.

There are other strategies for forming full sample test statistics. For example, one can add the sum of squared daily z statistics and treat it as a Chi-squared random variable. Another is to compute over the simulated sample of length T the proportion of days on which the individual z-statistics are statistically significant at some given level α, and then form the usual pivotal statistic based on the Gaussian approximation to the Binomial. We experimented with these strategies, and always found the same conclusions as just described above. When replicated over many samples of length T, the tests can incorrectly over-reject the null of no jumps, because a tiny daily bias gets inflated by the factor T.

Full sample tests can potentially become consistent tests for jumps with proper size, if a way can be found to eliminate the bias. There are possible strategies to knock it out based on the behavior of $T \cdot \mathcal{B}(M)$ for different M, but they all appear to us to entail additional assumptions and models that would be inconsistent with the nonparametric character of the jump detection strategies. So, for now, the daily statistics are perhaps more reliable than the full sample statistics, at least in terms of their size, though the full sample statistics offer some tantalizing longer term possibilities for development of consistent tests. For the related asymptotic results as both T and M go to infinity, see Corradi and Distaso (2004) in the framework of testing the specification of stochastic volatility models.

5 Empirical Application

The data set consists of five minute observations on the S&P Index; the cash data is from April 21, 1997 to October 22, 2002 and the futures data is from April 21, 1982 (the beginning of the S&P futures contract) to December 9, 2002. We eliminated a few days where trading

was thin and the market was open for a shortened session. Figure 11 shows plots of the daily price level and the daily geometric returns of the Cash Index and the Index futures. For the within-day computations, we used five minute data after applying a standard adjustment for the deterministic pattern of volatility over the trading day.

To investigate jumps, we first consider Figures 12 and 13, which shows in each panel a time series plot of the five versions (5), (6), (7), (8), and (9) of the $z_{TP,t}$ statistic computed over this data set, along with the upper 0.99 percent and 0.999 percent critical values of the standard Gaussian distribution. From the Monte Carlo evidence generated in Section 4, the second to the last panels provide the more reliable evidence on days on which large jump(s) occurred conditional on the **SV1FJ** model (23). Evidently, the statistics indicate far more jumps than would be expected under a purely diffusive model satisfying the rather mild regularity conditions of Barndorff-Nielsen and Shephard (2006). Similarly, there appear to be many more jumps than could possibly be generated as false jumps under the continuous **SV2F** model, which casts doubt on the validity of that model in describing the very high frequency character of stock prices.

We now consider the relative contribution of jumps to total price variance. Table 8 shows that the proportion of days that the daily z statistics identify as having jump(s) is larger for the Cash Index than those for the Index futures in the corresponding periods or in the full sample. Moreover, Table 9 shows that about 4.4% to 4.6% of the total realized variance comes from the jump component in the Index futures, and, somewhat surprisingly, smaller than the value (7.328%) in the Cash Index. A similar relationship holds for the full sample statistics ($RJ_{1:T}$) as well. On the other hand, $RV_{1:T}$ and $BV_{1:T}$ for the Index futures are larger than those for the Cash Index, for both the shorter (1997–2002) and the longer samples (1982–2002).

Taken together, the results reported in Figures 12 and 13 along with those in Tables 8 and 9 indicate that jumps are a statistically important component of aggregate stock price movements. This evidence is generated using statistical techniques validated in the Monte Carlo work of Section 4, which makes the case for jumps all that more compelling. The jumps appear to contribute about 4.4 to 7.3 percent to the total variance of daily stock price movements.

6 Market Microstructure Noise

Our empirical evidence on the jumps in the financial price process is consistent with the findings in Andersen, Bollerslev, and Diebold (2004). Eraker, Johannes and Polson (2003) also find similar proportions of return variance due to jumps in S&P 500 and Nasdaq 100 index returns using their jumps in volatility and returns models. These sets of findings suggest more instances of jumps and higher jump contribution to total return variance than most of the existing literature in this aspect does. However, since we are using high-frequency data, the observed return

series can be contaminated by market microstructure noise. That raises the issue of whether the jumps detected by the above statistics are true jumps or spurious ones induced by the market microstructure noise. To address this question, we conduct the following analysis of the effects of the market microstructure noise on jump detection.

6.1 The Noise Structure

We now assume the observed log price at time t_j is

$$p(t_j) = p^*(t_j) + u_{t,j} \tag{26}$$

where $p^*(t)$ is the true continuous-time log price assumed to satisfy the conditions given in Section 1 above, and $u_{t,j}$ is the microstructure noise. We follow Aït-Sahalia, Mykland and Zhang (2005), Zhang, Mykland and Aït-Sahalia (2004), and Bandi and Russell (2005a), among others, and assume that the $u_{t,j}$'s are i.i.d.; for tractability, we additionally assume the $u_{t,j}$'s are $\mathrm{N}(0, \sigma^2_{mn})$. We discuss further below the (important) deviations from this setup. For detailed studies on the realized volatility measures under non-i.i.d. market microstructure noise, see the above literature as well as Bandi and Russell (2005b), and Hansen and Lunde (2004a,b); a comprehensive survey is Barndorff-Nielsen and Shephard (2005b).

Under these assumptions, the observed return is

$$r_{t,j} = r^*_{t,j} + \epsilon_{t,j} \tag{27}$$

where $r^*_{t,j}$ is the true geometric return based on $p^*(t_j)$ and $\epsilon_{t,j}$ is an MA(1) process with $\mathrm{Var}(\epsilon_{t,j}) = 2\sigma^2_{mn}$, and $\mathrm{Cov}(\epsilon_{t,j}, \epsilon_{t,j-1}) = -\sigma^2_{mn}$. As above, we assume equally spaced sampling, where $\delta = t_j - t_{j-1}$ denotes the sampling interval and is equal to $\frac{1}{M}$. Let σ^2_d denote the unconditional daily variance of the true returns, so, as noted by Aït-Sahalia *et al* (2005),

$$\pi_{mn}(\delta) = \frac{2\sigma^2_{mn}}{\sigma^2_d \delta + 2\sigma^2_{mn}} \tag{28}$$

is the proportion of the variance of the return over the interval $[t_j - \delta, t_j]$ attributable to the microstructure noise.

6.2 Staggered Returns

Recall from Section 1 the definition of the **Bipower Variation**

$$BV_t = \mu_1^{-2} (\frac{M}{M-1}) \sum_{j=2}^{M} |r_{t,j-1}||r_{t,j}|.$$

The effect of the microstructure noise introduced in Subsection 6.1 above is to induce correlation in the two adjacent returns, $r_{t,j-1}$ and $r_{t,j}$, appearing in this summation. Andersen,

Bollerslev, and Diebold (2004) suggest breaking the correlation by using staggered returns as in $|r_{t,j-2}||r_{t,j}|$, or more generally as in $|r_{t,j-(1+i)}||r_{t,j}|$, where the nonnegative integer i denotes the offset. The asymptotics of various bipower measures using staggered returns are studied in depth by Barndorff-Nielsen and Shephard (2004b). Here we consider the generalized bipower measure based on staggered returns

$$BV_{i,t} = \mu_1^{-2} \left(\frac{M}{M-1-i} \right) \sum_{j=2+i}^{M} |r_{t,j-(1+i)}||r_{t,j}|, \quad i \geq 0. \tag{29}$$

The above reduces to the **Bipower Variation** defined in Section 1 if $i = 0$. Likewise, we consider the generalized tri-power quarticity measure

$$TP_{i,t} = M\mu_{4/3}^{-3} \left(\frac{M}{M-2(1+i)} \right) \sum_{j=1+2(1+i)}^{M} |r_{t,j-2(1+i)}|^{4/3} |r_{t,j-(1+i)}|^{4/3} |r_{t,j}|^{4/3}, \quad i \geq 0, \tag{30}$$

and the quad-power quarticity measure

$$QP_{i,t} = M\mu_1^{-4} \left(\frac{M}{M-3(1+i)} \right) \sum_{j=1+3(1+i)}^{M} |r_{t,j-3(1+i)}||r_{t,j-2(1+i)}||r_{t,j-(1+i)}||r_{t,j}|, \quad i \geq 0. \tag{31}$$

With $BV_{i,t}$, $TP_{i,t}$, $QP_{i,t}$ used in place of BV_t, TP_t, QP_t, the asymptotic theory of Section 2 for the various z-test statistics of Section 3 is exactly the same for each fixed i as $M \to \infty$.

6.3 Some Analytics

To analyze the relationship between the Realized Variance and the Bipower Variation in the presence of microstructure noise, we retain the simple standard Gaussian i.i.d. structure of Subsection 6.1 above; later we discuss the implications of deviations from this setup. With these assumptions for the noise, assuming no leverage, working conditional on the volatility process, and with the t subscript suppressed, we can write the j^{th} within-day return as

$$r_j \stackrel{\mathcal{D}}{=} (\sigma_j^2 \delta + \sigma_{mn}^2)^{\frac{1}{2}} Z_j \tag{32}$$

where the Z_j's are standard Gaussian random variables and σ_j^2 is the integrated variance averaged over the sampling interval of width δ. The correlations of the Z_j's are

$$\rho_{j,k} \equiv \text{Corr}(Z_j, Z_{j-k}) = \begin{cases} \frac{-\sigma_{mn}^2}{\sqrt{(\sigma_j^2\delta+\sigma_{mn}^2)(\sigma_{j-k}^2\delta+\sigma_{mn}^2)}} & |k| = 1, \\ 0 & |k| \geq 1 \end{cases} \tag{33}$$

All of the jump-test z-statistics displayed in Section 3 have the common structure of a studentized measure of discrepancy

$$z = \frac{\mathcal{D}(RV, BV)}{\sqrt{\text{Avar}\left[\mathcal{D}(RV, BV)\right]}} \tag{34}$$

where the numerator of the z statistic, $\mathcal{D}(\cdot,\cdot)$, is a measure of the discrepancy between the realized variance RV and the bipower variation BV, and the denominator is the square root of the asymptotic variance of the discrepancy under the Barndorff-Nielsen and Shephard theory. The asymptotic variance of $\mathcal{D}(\cdot,\cdot)$ depends upon the integrated quarticity which is estimated by either the realized tri-power quarticity or the realized quad-power quarticity.

To gain insight into the behavior of the z-statistic (34), we consider the conditional expectations, given the volatility process, of the numerator and the realized tri-power quarticity term for the denominator in (34) for the basic case where the discrepancy $\mathcal{D}(\cdot,\cdot)$ is simply the difference, as in (5) above. For simplicity, we ignore the small sample adjustment $M/(M-1)$ along with end effects due to lagging, and we presume the same number of terms $M = 1/\delta$ in the summations.

The relevant conditional expectation for the numerator is thus

$$\mathrm{E}_\sigma\left(\sum_1^M r_j^2\right) - \mathrm{E}_\sigma\left(\frac{\pi}{2}\sum_1^M |r_{j-(1+i)}||r_j|\right) \tag{35}$$

where, as before, $i \geq 0$ is the offset, and E_σ denotes the expectation computed conditional on the trajectory of volatility. From (32) we have that (35) is

$$\begin{aligned}&\textstyle\sum_1^M(\sigma_j^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}\left[(\sigma_j^2\delta+\sigma_{mn}^2)^{\frac{1}{2}} - (\sigma_{j-(1+i)}^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}\right] \quad + \\ &\textstyle\sum_1^M(\sigma_j^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}(\sigma_{j-(1+i)}^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}\left[\frac{2}{\pi} - g(\rho_{j,1+i})\right]\frac{\pi}{2},\end{aligned} \tag{36}$$

where

$$g(\rho) = \mathrm{E}\left(|(1-\rho^2)^{\frac{1}{2}}Z_a + \rho Z_b||Z_b|\right), \quad Z_a, Z_b \text{ independent } \mathrm{N}(0,1), \tag{37}$$

is the expected value of the product of the absolute values of two correlated standard Guassian random variables. The function $g(\rho)$ is symmetric about zero with $g(0) = 2/\pi$, $g(1) = 1$, and it is easily numerically evaluated. For an explicit expression for $g(\rho)$ and its properties, see Barndorff-Nielsen and Shephard (2004b) Theorem 3 and Remark 5. Application of the mean value theorem to the square-root function $(\cdot)^{\frac{1}{2}}$ appearing in the first square bracket in (36) gives

$$\begin{aligned}&\mathrm{E}_\sigma\left(\textstyle\sum_1^M r_j^2\right) - \mathrm{E}_\sigma\left(\frac{\pi}{2}\sum_1^M |r_{j-(1+i)}||r_j|\right) = \\ &\textstyle\sum_1^M \frac{1}{2}\left[\frac{\sigma_j^2\delta+\sigma_{mn}^2}{\sigma_j^2\delta+\sigma_{mn}^2+\phi_j(\sigma_j^2-\sigma_{j-(1+i)}^2)\delta}\right]^{\frac{1}{2}}\left(\sigma_j^2 - \sigma_{j-(1+i)}^2\right)\delta + \\ &\textstyle\sum_1^M(\sigma_j^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}(\sigma_{j-(1+i)}^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}\left[\frac{2}{\pi} - g(\rho_{j,1+i})\right]\frac{\pi}{2}\end{aligned}$$

where $|\phi_j| < 1$. Hence the conditional expectation of the numerator of the z-statistic (34) is the sum of two terms

$$\mathrm{E}_\sigma\left(\sum_1^M r_j^2\right) - \mathrm{E}_\sigma\left(\frac{\pi}{2}\sum_1^M |r_{j-(1+i)}||r_j|\right) = A_1 + A_2 \tag{38}$$

where

$$A_1 = \frac{1}{M}\sum_1^M \frac{1}{2}\left[\frac{\sigma_j^2\delta+\sigma_{mn}^2}{\sigma_j^2\delta+\sigma_{mn}^2+\phi_j(\sigma_j^2-\sigma_{j-(1+i)}^2)\delta}\right]^{\frac{1}{2}}\left(\sigma_j^2-\sigma_{j-(1+i)}^2\right), \tag{39}$$

$$A_2 = \sum_1^M(\sigma_j^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}(\sigma_{j-(1+i)}^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}\left[\frac{2}{\pi}-g(\rho_{j,1+i})\right]\frac{\pi}{2}, \tag{40}$$

and recall $\delta = \frac{1}{M}$. For any fixed $i \geq 0$, the first term A_1 converges to zero as δ goes to 0, so it is robust with respect to the microstructure noise. As for the second term, if returns are not staggered so $i = 0$, it does not converge to 0, and the numerator is biased in the negative direction against finding jumps. In contrast, if returns are staggered, $i > 0$, then because of the i.i.d. assumption $\rho_{j,1+i} = 0$ and the term A_2 vanishes, leaving only the term A_1. It is important to keep in mind that the Gaussian assumption determines the characteristics of the function $g(\rho)$ in (37), and so that assumption plays a key role in our results regarding the behavior of the term A_2.

We now consider the denominator of (34), and we examine the case where the realized tri-power quarticity is used to estimate the asymptotic variance in (34). The relevant conditional expectation is

$$\mu_{4/3}^{-3}\ \mathrm{E}_\sigma\left(\sum_{j=1}^M |r_{j-2(1+i)}|^{4/3}|r_{j-(1+i)}|^{4/3}|r_j|^{4/3}\right). \tag{41}$$

Using the representation (32), the above becomes

$$\sum_{j=1}^M(\sigma_{j-2(1+i)}^2\delta+\sigma_{mn}^2)^{2/3}(\sigma_{j-(1+i)}^2\delta+\sigma_{mn}^2)^{2/3}(\sigma_j^2\delta+\sigma_{mn}^2)^{2/3}\mu_{4/3}^{-3}h(\rho_{j-(1+i),1+i},\rho_{j,1+i}) \tag{42}$$

where $\rho_{j,k}$ is defined in (33) above, and the function h is given by

$$h(\rho_a,\rho_b) = \mathrm{E}\left(|(1-\frac{\rho_a^2}{1-\rho_b^2})^{\frac{1}{2}}\,Z_a+\frac{\rho_a}{\sqrt{1-\rho_b^2}}\,Z_b|^{4/3}\cdot|(1-\rho_b^2)\frac{1}{2}\,Z_b+\rho_b Z_c|^{4/3}\cdot|Z_c|^{4/3}\right)$$

The function h is equivalently written as

$$h(\rho_a,\rho_b) = \mathrm{E}(|Y_a|^{4/3}|Y_b|^{4/3}|Y_c|^{4/3})$$

where Y_a, Y_b, Y_c are three standard Gaussian random variables with pairwise correlations $\mathrm{Corr}(Y_a,Y_b) = \rho_a$, $\mathrm{Corr}(Y_b,Y_c) = \rho_b$, $\mathrm{Corr}(Y_a,Y_c) = 0$. The function is easily numerically evaluated.

The expression (42) indicates that the effect of the microstructure noise is to inflate the realized tri-power quarticity for two reasons, although the inflation can be substantially mitigated by staggering returns. First, the noise variance σ_{mn}^2 inflates terms like $(\sigma_j^2\delta+\sigma_{mn}^2)^{\frac{1}{2}}$. Second, if

there is no staggering, then adjacent returns are negatively correlated, so their magnitudes are positively correlated in (41), and therefore

$$\mu_{4/3}^{-3} h(\rho_{j-1,1}, \rho_{j,1}) > 1. \tag{43}$$

Table 10 shows the value of the factor $\mu_{4/3}^{-3} h$ for different values of the correlations, and it suggests the inflation of the realized tri-power quarticity can be substantial, as high as 25 to 39 percent. On the other hand, if the returns are staggered via $i \geq 1$, then the inflation due to the correlation of adjacent returns is absent.

The upshot of our analytical analysis is that without staggering, our results suggest the jump test statistics to be biased downwards, i.e., in favor of finding fewer jumps in the presence of market microstructure noise. The reason is that the numerator of the test statistic is negatively biased. At the same time, the noise inflates the estimate of the integrated quarticity used to form the estimate of scale in the denominator of (34). The two effects together bias the z_{TP} statistic against rejection. We expect the same logic to work for the other forms of the z statistics.

If returns are staggered, $i \geq 1$, in forming the requisite power measures (29)–(31) above, then the term A_2 is knocked out of the numerator, and $\mu_{4/3}^{-3} h = 1$. In this case, the asymptotic approximation of the Barndorff-Nielsen and Shephard theory is expected to be more accurate. Nonetheless, one must keep in mind the presumption of i.i.d. Gaussian microstructure noise. If the microstructure noise is non-Gaussian, then the decomposition (39)–(40) is unavailable, although the results are still suggestive; we defer non-Gaussian noise to future work. If the microstructure noise is Gaussian, but serially correlated such as an MA(k) process, then one can reasonably expect an appropriately long lag i for the staggering to work, but again we defer formal verification to later work.

Clearly, our separate analysis of the conditional expectations of the numerator and denominator of the z jump statistic can only provide some guidance of what to expect for the z-statistic in the presence of microstructure noise. A formal theoretical analysis of the z-statistic itself would entail complicated higher order approximations, so we turn to a Monte Carlo assessment instead.

6.4 Monte Carlo Assessment of the Effects of Microstructure Noise

We need to run the simulations over a plausibly wide range of values for the standard deviation σ_{mn} of the microstructure noise that appears in equation (26). To calibrate σ_{mn}, we use equation (28) with a presumed return volatility of one percent per day, so that

$$\pi_{mn}(5/390) = \frac{2\sigma_{mn}^2}{1 \cdot (5/390) + 2\sigma_{mn}^2} \tag{44}$$

is the proportion of the variance of the five-minute return attributable to microstructure noise for a market open 6.50 hours per day. We consider values of $\pi_{mn}(5/390)$ ranging from 0.00 to 0.50 in the increment of 0.10, which thereby determines the grid of values for σ_{mn} and also values of $\pi_{mn}(\delta)$ for all other sampling frequencies as well; see Table 11. The upper value of $\pi_{mn}(5/390) = 0.50$ implies that 50 percent of the variance of the five minute return is accounted for by the noise, which appears to be more than adequately high enough for an index cash or an index future like those usually found in most empirical work, such as Madhavan, Richardson and Roomans (1997), Hasbrouck (1993), Kaul and Nimalendran (1990), and Conrad, Kaul and Nimalendran (1991), among others.

The following subsections summarize the Monte Carlo findings with the market microstructure noise.

6.4.1 Size

Table 12 shows the size of the three z_{tp} statistics under different noise sizes (σ_{mn}) and sampling intervals, with i =0, 1, and 2. It is clear from this table that without staggering, the z statistics' rejection frequency is seriously downward biased at the 1-minute level, but not at the 30-minute level, since the market microstructure noise impact decreases as the sampling interval increases. Moreover, the larger the noise size, the more serious the under-rejection is, as explained in the above Section 6.3.

Staggering in BV_t and TP_t, one the other hand, restores the nominal size, and makes the rejection rate robust to the noise proportion. Extra lagging ($i = 2$) retains the robustness feature, but leads to a little over-rejection. Thus, to achieve the correct empirical size of the statistics, we can only stagger the returns up to the level that breaks the serial dependence of the observed returns induced by the market microstructure noise. Any extra lagging might introduce the finite-sample bias due to the longer interval covered in each return product term, making the asymptotic approximation worse off.

6.4.2 Power and Jump Detection

Tables 13 summarizes the power of the three z_{TP} statistics, and Table 14 shows the jump detection rate of the $z_{TP,rm,t}$ statistic, under different noise sizes and sampling intervals, with i =0, 1, and 2. Without staggering, noise size reduces the power and the jump detection rate of the statistics, due to the noise contribution of the total price variance, which has different characteristics from those of the jump part. Staggering helps to improve the jump detection rate at the high frequency levels (1-minute and 3-minute returns).

We did the same experiment for the constant volatility setup as the benchmark case. It turns out that the above features hold true under the constant volatility setup, showing the

robustness of the results.

6.5 Empirical Data Revisited

The Monte Carlo evidence points toward the conclusion that in Section 5 above the jumps detected in the S&P data are bona fide jumps, instead of the market microstructure noise. We re-conduct the same calculation, using staggered returns from $i = 0$ up to $i = 2$. Here we only use the S&P 500 Index futures, because the Cash Index is made up of 500 underlying stocks without a sensible notion of the market microstructure noise. For comparability, we use the same subperiods as those in Section 5 above.

The empirical evidence, especially over the entire sample period April 21, 1982 – December 9, 2002, is generally consistent with the above analytical and Monte Carlo findings. Staggering suggests a slightly more important role for jumps both in terms of the number of jump days and the jump contribution in the total price variance, as exhibited in Tables 15 and 16.

7 Conclusion

The Monte Carlo evidence suggests that, under the arguably realistic scenarios considered here, the recently developed z-tests for jumps perform impressively and are not easily fooled. Computed on a daily basis in ratio form, with a maximum adjustment in the estimate of scale, the size and power properties are excellent, and the test statistics appear to do an outstanding job of identifying the days on which jump(s) occur. Under modeling assumptions that include Gaussian microstructure noise, the effect of market microstructure noise is to bias the tests against finding jumps. That bias can be corrected by using Andersen, Bollerslev, and Diebold's (2004) technique of staggering returns sufficiently to diminish the effects of the local serial correlation induced by microstructure noise. Although any Monte Carlo study is necessarily limited, and the theoretical work is constrained by the assumptions, it nonetheless seems reasonable to make valid nonparametric claims for the reliability of the tests. That claim could be further buttressed (or possibly discredited) by additional research using more complicated data generating processes such as those with regime shifts or jumps in volatility. A good starting point for the latter would be the pure jump nonnegative OU processes introduced in Barndorff-Nielsen and Shephard (2001), with generalizations reviewed and simulation schemes presented in Todorov and Tauchen (2004). Another avenue to strengthen the nonparametric claim would be to check whether the jump tests can uncover possible serial correlation in jumps as suggested by evidence in Andersen, Bollerslev, and Huang (2005).

The Monte Carlo work also suggests that the new test statistics, while powerful, are not consistent tests. We believe that only by computing the statistics over a full sample, instead

of day-by-day, can a consistent test be developed. However, our Monte Carlo work identifies a serious pitfall in this effort. Successful development requires a nonparametric way of knocking out a small sample bias that otherwise gets greatly magnified when summed over many days. We defer this task to future work.

The empirical work indicates strong evidence for jumps, where jumps account for about 4.5 to 7.0 percent of the total daily variance of the S&P Index, cash or futures. Interestingly, that evidence is obtained with a test likely biased away from finding jumps, and redoing the tests with the adjustment for microstructure noise raises the test statistics somewhat. The case for jumps thus appears compelling. The presence of jumps perforce precludes the prevalent assumption of local continuity and suggests that financial econometricians will need to enrich considerably the class of parametric models used to model very high frequency price series. The recent book by Cont and Tankov (2004) provides a powerful toolkit for creating more elaborate models. Although parametric modelling of very high frequency time series with jumps presents an intriguing statistical challenge, there is the serious issue of how economically important is the relatively small average contribution of 4.5 to 7.0 percent of total daily price variance. That question can only be addressed by examining the portfolio optimization behavior of an economic agent facing price series generated by the enhanced statistical models just discussed.

8 Figure Legends

Figure 1: Simulated realization from the SV1F model, daily frequency, for 10,000 days, using 5-minute returns under medium mean reversion ($\alpha_v = -0.100$).

Figure 2: Simulated realization from the SV1FJ model, daily frequency, for 10,000 days, using 5-minute returns under medium mean reversion ($\alpha_v = -0.100$), $\sigma_{jmp} = 1.50$, and $\lambda = 0.014$.

Figure 3: Simulated realization from the SV2F model, daily frequency, for 10,000 days, using 5-minute returns.

Figure 4: QQ Plots, z_{TP} daily statistics, for 45,000 days, using 5-minute returns under medium mean reversion ($\alpha_v = -0.100$).

Figure 5: QQ Plots, z_{QP} daily statistics, for 45,000 days, using 5-minute returns under medium mean reversion ($\alpha_v = -0.100$).

Figure 6: QQ Plot and Kernel Density Plot, RJ_t daily statistic for 45,000 days, using 5-minute returns under medium mean reversion ($\alpha_v = -0.100$).

Figure 7: The size of the jump statistics over different sampling intervals under the medium mean reversion (α_v=-0.100). The nominal size is 0.05 for the upper left subplot, 0.005 for the lower left subplot, 0.01 for the upper right subplot, and 0.001 for the lower right subplot. The two horizontal lines are the 95% Monte Carlo confidence bands corresponding to the nominal size. The sample size is 45,000 days and the return horizon is 5 minutes.

Figure 8: Simulated time series of $z_{TP,t}$'s under the SV1F model. The five panels show simulations of the basic statistic, the log version, the max-log version, the ratio version and the max-ratio version for $\sigma_{jmp} = 0$ under medium mean reversion. The horizontal lines are the upper 0.99 and 0.999 critical values of the standard Gaussian distribution. The sample size is 1400 days and the return horizon is 5 minutes.

Figure 9: Simulated time series of $z_{TP,t}$'s under the SV1FJ model. The five panels show simulations of the basic statistic, the log version, the max-log version, the ratio version and the max-ratio version for $\sigma_{jmp} = 0$ and $\lambda = 0.014$ under medium mean reversion. The horizontal lines are the upper 0.99 and 0.999 critical values of the standard Gaussian distribution. The bottom panel shows the jumps in the simulated realization. The sample size is 1400 days and

the return horizon is 5 minutes.

Figure 10: Simulated time series of $z_{TP,t}$'s under the **SV2F** model. The five panels show simulations of the basic statistic, the log version, the max-log version, the ratio version and the max-ratio version. The horizontal lines are the upper 0.99 and 0.999 critical values of the standard Gaussian distribution. The sample size is 1400 days and the return horizon is 5 minutes.

Figure 11: The top two panels show the daily closing price and the daily geometric return of the S&P Cash Index, April 21, 1997–October 22, 2002. The bottom two panels show the daily closing price and the daily geometric return of the S&P Index futures, April 21, 1982–December 9, 2002.

Figure 12: The five panels show observed values of the five daily jump statistics $z_{TP,t}$, $z_{TP,l,t}$, $z_{TP,lm,t}$, $z_{TP,r,t}$ and $z_{TP,rm,t}$, computed using the five-minute returns on the S&P Cash Index, April 21, 1997–October 22, 2002. The horizontal lines are the upper 0.99 and 0.999 critical values of the standard Gaussian distribution.

Figure 13: The five panels are the time series plots of the observed values of the daily statistics $z_{TP,t}$, $z_{TP,l,t}$, $z_{TP,lm,t}$, $z_{TP,r,t}$ and $z_{TP,rm,t}$, computed using the five-minute returns on the S&P Index futures, April 21, 1982–December 9, 2002. The horizontal lines are the upper 0.99 and 0.999 critical values of the standard Gaussian distribution.

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Table 1: Experimental design for SV1FJ models

μ	0.030
β_0	0.000
β_1	0.125
α_v	{ -0.137e-2, -0.100, -1.386 }
ρ	-0.620
λ	{0.014, 0.058 , 0.082 , 0.118 }
	{0.50, 1.00 , 1.50 , 2.00 }
σ_{jmp}	{0.00 ... 2.50 by 0.50 }
$ntick$	60
$nstep$	390

Table 2: Experimental design for SV2F model

μ	0.030
β_0	-1.200
β_1	0.040
β_2	1.500
α_{v1}	-0.137e-2
α_{v2}	-1.386
β_{v2}	0.250
$\rho_{p,v1}$	-0.300
$\rho_{p,v2}$	-0.300

Table 3: Confusion Matrices

Interval			$\lambda = 0.014$		$\lambda = 0.118$		$\lambda = 1.000$		$\lambda = 2.000$	
			(NJ)	(J)	(NJ)	(J)	(NJ)	(J)	(NJ)	(J)
1-minute	$z_{TP,t}$	(NJ)	0.980	0.020	0.981	0.019	0.989	0.011	0.995	0.005
		(J)	0.205	0.795	0.178	0.822	0.132	0.868	0.080	0.920
	$z_{TP,lm,t}$	(NJ)	0.986	0.014	0.986	0.014	0.993	0.007	0.996	0.004
		(J)	0.208	0.792	0.184	0.816	0.136	0.864	0.084	0.916
	$z_{TP,rm,t}$	(NJ)	0.988	0.012	0.989	0.011	0.994	0.006	0.997	0.003
		(J)	0.214	0.786	0.187	0.813	0.139	0.861	0.086	0.914
3-minute	$z_{TP,t}$	(NJ)	0.968	0.032	0.969	0.031	0.982	0.018	0.992	0.008
		(J)	0.273	0.727	0.261	0.739	0.198	0.802	0.125	0.875
	$z_{TP,lm,t}$	(NJ)	0.981	0.019	0.982	0.018	0.989	0.011	0.996	0.004
		(J)	0.286	0.714	0.274	0.726	0.208	0.792	0.132	0.868
	$z_{TP,rm,t}$	(NJ)	0.988	0.012	0.988	0.012	0.993	0.007	0.997	0.003
		(J)	0.292	0.708	0.285	0.715	0.219	0.781	0.141	0.859
5-minute	$z_{TP,t}$	(NJ)	0.960	0.040	0.963	0.037	0.980	0.020	0.991	0.009
		(J)	0.302	0.698	0.314	0.686	0.239	0.761	0.160	0.840
	$z_{TP,lm,t}$	(NJ)	0.977	0.023	0.978	0.022	0.988	0.012	0.994	0.006
		(J)	0.347	0.653	0.337	0.663	0.257	0.743	0.175	0.825
	$z_{TP,rm,t}$	(NJ)	0.986	0.014	0.987	0.013	0.993	0.007	0.997	0.003
		(J)	0.360	0.640	0.358	0.642	0.274	0.726	0.191	0.809
30-minute	$z_{TP,t}$	(NJ)	0.894	0.106	0.899	0.101	0.943	0.057	0.974	0.026
		(J)	0.558	0.442	0.543	0.457	0.489	0.511	0.432	0.568
	$z_{TP,lm,t}$	(NJ)	0.954	0.047	0.957	0.043	0.975	0.025	0.988	0.012
		(J)	0.620	0.380	0.641	0.359	0.590	0.410	0.540	0.460
	$z_{TP,rm,t}$	(NJ)	0.986	0.014	0.987	0.013	0.992	0.008	0.996	0.004
		(J)	0.744	0.257	0.749	0.251	0.706	0.294	0.671	0.329

Fixed parameters: level of significance = 0.01, $\sigma_{jmp} = 1.50$, $\alpha_v = -0.10$. The columns represent the jump days signaled by the statistics and the rows are the actual days on which jumps occur in the simulation.

Table 4: Power for different jump statistics

	λ	σ_{jmp}: 0.5	1.0	1.5	2.0	2.5
$z_{TP,t}$	0.500	0.14718	0.25651	0.30640	0.33291	0.34789
	1.000	0.24060	0.42038	0.49578	0.53458	0.55727
	1.500	0.31969	0.54704	0.63264	0.67322	0.69687
	2.000	0.39227	0.64382	0.73042	0.77042	0.79278
$z_{TP,lm,t}$	0.500	0.12260	0.23538	0.28713	0.31613	0.33236
	1.000	0.21082	0.39747	0.47767	0.51931	0.54389
	1.500	0.28569	0.52367	0.61540	0.65993	0.68567
	2.000	0.35569	0.62182	0.71567	0.75967	0.78309
$z_{TP,rm,t}$	0.500	0.10493	0.21949	0.27309	0.30480	0.32216
	1.000	0.18669	0.37782	0.46264	0.50660	0.53282
	1.500	0.25767	0.50193	0.59967	0.64758	0.67436
	2.000	0.32433	0.59936	0.70044	0.74709	0.77291

Fixed parameters: level of significance=0.010, $\alpha_v = -0.100$, and 5-minute returns.

Table 5: Power of $z_{TP,rm,t}$ for different sampling frequencies

	λ	σ_{jmp}: 0.5	1.0	1.5	2.0	2.5
1-minute	0.500	0.20420	0.29744	0.33033	0.34693	0.35651
	1.000	0.36213	0.50140	0.54920	0.57158	0.58462
	1.500	0.48840	0.64236	0.69351	0.71711	0.72993
	2.000	0.58822	0.74833	0.79222	0.81340	0.82502
3-minute	0.500	0.13633	0.24873	0.29624	0.32167	0.33607
	1.000	0.24687	0.42813	0.49971	0.53400	0.55520
	1.500	0.33969	0.56329	0.64049	0.67773	0.69902
	2.000	0.42362	0.66578	0.74324	0.77909	0.79756
5-minute	0.500	0.10493	0.21949	0.27309	0.30480	0.32216
	1.000	0.18669	0.37782	0.46264	0.50660	0.53282
	1.500	0.25767	0.50193	0.59967	0.64758	0.67436
	2.000	0.32433	0.59936	0.70044	0.74709	0.77291
30-minute	0.500	0.02500	0.06642	0.11384	0.15609	0.18787
	1.000	0.03451	0.10747	0.19082	0.25720	0.30787
	1.500	0.04102	0.13958	0.24260	0.32618	0.38818
	2.000	0.04893	0.16556	0.28342	0.37276	0.43624

Fixed parameters: level of significance = 0.010, $\alpha_v = -0.100$.

Table 6: SV2F model results, daily

	size(5%)	size(1%)	size(0.5%)	size(0.1%)
$z_{TP,t}$	0.13589	0.07616	0.06209	0.04056
$z_{TP,l,t}$	0.12024	0.05931	0.04544	0.02664
$z_{TP,lm,t}$	0.11709	0.05642	0.04284	0.02436
$z_{TP,r,t}$	0.10171	0.04167	0.03016	0.01471
$z_{TP,rm,t}$	0.09829	0.03867	0.02747	0.01276

Fixed parameters: 5-minute returns; see Table 2 for the others.

Table 7: Rejection frequencies, full sample

	z_{TP}		$z_{TP,lm}$		$z_{TP,rm}$		z_{QP}		$z_{QP,lm}$		$z_{QP,rm}$	
σ_{jmp}	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05
SV1FJ: $\alpha_v = -1.3860$, Fast Mean Reversion												
0.00	0.015	0.056	0.015	0.055	0.015	0.055	0.015	0.057	0.015	0.057	0.015	0.056
0.50	0.106	0.306	0.106	0.306	0.105	0.303	0.106	0.307	0.106	0.307	0.105	0.303
1.00	0.885	0.948	0.882	0.947	0.882	0.947	0.885	0.948	0.883	0.947	0.883	0.947
1.50	0.994	0.995	0.994	0.995	0.994	0.995	0.994	0.995	0.994	0.995	0.994	0.995
2.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
SV1FJ: $\alpha_v = -0.100$, Medium Mean Reversion												
0.00	0.009	0.059	0.008	0.059	0.008	0.058	0.009	0.059	0.008	0.059	0.008	0.058
0.50	0.067	0.209	0.064	0.209	0.064	0.206	0.068	0.212	0.066	0.210	0.064	0.206
1.00	0.743	0.878	0.740	0.878	0.739	0.878	0.743	0.878	0.742	0.878	0.739	0.878
1.50	0.983	0.990	0.983	0.990	0.983	0.990	0.983	0.990	0.983	0.990	0.983	0.990
2.00	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
2.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
SV1FJ: $\alpha_v = -0.137e-02$, Slow Mean Reversion												
0.00	0.004	0.032	0.003	0.031	0.003	0.031	0.003	0.033	0.003	0.032	0.003	0.032
0.50	0.081	0.137	0.079	0.136	0.079	0.134	0.080	0.139	0.080	0.138	0.079	0.134
1.00	0.229	0.288	0.228	0.288	0.228	0.286	0.228	0.288	0.228	0.288	0.228	0.287
1.50	0.295	0.355	0.293	0.354	0.292	0.352	0.295	0.353	0.294	0.352	0.291	0.351
2.00	0.361	0.426	0.361	0.425	0.357	0.424	0.361	0.427	0.357	0.425	0.357	0.424
2.50	0.430	0.478	0.429	0.478	0.427	0.477	0.433	0.481	0.430	0.479	0.426	0.478

SV2F: Two Factor Continuous Model

	z_{TP}		$z_{TP,lm}$		$z_{TP,rm}$		z_{QP}		$z_{QP,lm}$		$z_{QP,rm}$	
Interval	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05
30-min	0.495	0.670	0.475	0.658	0.452	0.639	0.548	0.716	0.534	0.696	0.519	0.681
5-min	0.084	0.200	0.079	0.195	0.074	0.188	0.089	0.206	0.084	0.204	0.079	0.198
3-min	0.036	0.129	0.035	0.127	0.035	0.124	0.039	0.132	0.038	0.128	0.038	0.127
1-min	0.045	0.124	0.044	0.123	0.044	0.119	0.045	0.125	0.044	0.124	0.043	0.122

Fixed parameters: 5-minute returns and $\lambda = 0.014$ for **SV1FJ** model.

Table 8: Proportion of days identified as jumps by the daily statistics

level of significance	0.500	0.950	0.990	0.995	0.999
	S&P 500 Cash Index				
	(04/21/1997 – 10/22/2002)				
$z_{TP,t}$	0.74215	0.28488	0.18335	0.14609	0.10299
$z_{TP,l,t}$	0.74215	0.26150	0.14536	0.12199	0.07159
$z_{TP,lm,t}$	0.74215	0.24543	0.12929	0.09715	0.05698
$z_{TP,r,t}$	0.74215	0.23886	0.11907	0.08839	0.04383
$z_{TP,rm,t}$	0.74215	0.22060	0.09204	0.06866	0.03214
	S&P 500 Index Futures				
	(04/21/1997 – 10/22/2002)				
$z_{TP,t}$	0.66108	0.20918	0.11152	0.09402	0.05175
$z_{TP,l,t}$	0.66108	0.18440	0.09329	0.06851	0.03717
$z_{TP,lm,t}$	0.66108	0.17566	0.07726	0.05831	0.03061
$z_{TP,r,t}$	0.66108	0.16618	0.06778	0.04227	0.02041
$z_{TP,rm,t}$	0.66108	0.14942	0.05758	0.03499	0.01603
	S&P 500 Index Futures				
	(04/21/1982 – 12/09/2002)				
$z_{TP,t}$	0.64886	0.20960	0.11705	0.09583	0.05997
$z_{TP,l,t}$	0.64886	0.18801	0.09526	0.07231	0.04319
$z_{TP,lm,t}$	0.64886	0.17875	0.08427	0.06479	0.03779
$z_{TP,r,t}$	0.64886	0.16815	0.07038	0.05110	0.02680
$z_{TP,rm,t}$	0.64886	0.15523	0.06190	0.04223	0.02140

Table 9: Summary of RV, BV and RJ.

Statistics	RV	BV	$RV - BV$	RJ
	S&P 500 Cash Index			
	(04/21/1997 – 10/22/2002)			
Sample Mean	1.26702	1.18084	0.08618	0.07328
Std of Mean	0.04086	0.03746	0.00340	0.00294
Full-Sample Statistics	1734.55618	1616.57673	117.97945	0.06802
	S&P 500 Index Futures			
	(04/21/1997 – 10/22/2002)			
Sample Mean	2.10718	2.01193	0.09525	0.04445
Std. of Mean	0.06690	0.06266	0.00424	0.00279
Full-Sample Statistics	2891.05330	2760.37224	130.68106	0.04520
	S&P 500 Index Futures			
	(04/21/1982 – 12/09/2002)			
Sample Mean	1.63393	1.46074	0.17319	0.04562
Std. of Mean	0.21083	0.12375	0.08708	0.00152
Full-Sample Statistics	8473.55466	7575.38272	898.17194	0.10600

Table 10: Values of $\mu_{4/3}^{-3} h(\rho_a, \rho_b)$

	ρ_b	-0.50	-0.40	-0.30	-0.20	-0.10	0.00
ρ_a							
-0.50		1.39	1.32	1.27	1.23	1.21	1.21
-0.40		1.32	1.25	1.19	1.15	1.13	1.13
-0.30		1.27	1.19	1.13	1.08	1.07	1.06
-0.20		1.23	1.15	1.08	1.04	1.02	1.02
-0.10		1.21	1.13	1.07	1.02	1.01	1.00
0.00		1.21	1.13	1.06	1.02	1.00	1.00

This table is computed by numerical integration using Gauss-Hermite Quadrature.

Table 11: Proportions of noise contribution under different sampling intervals in Monte Carlo

Interval	π_{mn}					
1 minute	0.000	0.357	0.556	0.682	0.769	0.833
3 minute	0.000	0.156	0.294	0.417	0.526	0.625
5 minute	0.000	0.100	0.200	0.300	0.400	0.500
30 minute	0.000	0.018	0.040	0.067	0.100	0.143
σ_{mn}:	0.000	0.027	0.040	0.052	0.065	0.080

Table 12: Size of different jump statistics under market microstructure noise

Interval	σ_{mn}:	0.000	0.027	0.040	0.052	0.065	0.080
			(i = 0)				
1-minute	$z_{TP,t}$	0.020	0.007	0.002	0.001	0.000	0.000
	$z_{TP,lm,t}$	0.014	0.005	0.002	0.000	0.000	0.000
	$z_{TP,rm,t}$	0.012	0.004	0.001	0.000	0.000	0.000
3-minute	$z_{TP,t}$	0.032	0.027	0.020	0.014	0.009	0.005
	$z_{TP,lm,t}$	0.019	0.017	0.013	0.008	0.005	0.003
	$z_{TP,rm,t}$	0.012	0.011	0.008	0.005	0.003	0.002
5-minute	$z_{TP,t}$	0.041	0.038	0.034	0.029	0.022	0.017
	$z_{TP,lm,t}$	0.023	0.022	0.019	0.015	0.012	0.009
	$z_{TP,rm,t}$	0.014	0.014	0.011	0.009	0.007	0.005
30-minute	$z_{TP,t}$	0.107	0.107	0.107	0.107	0.107	0.107
	$z_{TP,lm,t}$	0.047	0.047	0.047	0.046	0.046	0.046
	$z_{TP,rm,t}$	0.014	0.015	0.015	0.015	0.014	0.014
			(i = 1)				
1-minute	$z_{TP,t}$	0.021	0.020	0.021	0.021	0.022	0.022
	$z_{TP,lm,t}$	0.015	0.015	0.015	0.015	0.015	0.015
	$z_{TP,rm,t}$	0.012	0.012	0.011	0.012	0.012	0.012
3-minute	$z_{TP,t}$	0.031	0.034	0.033	0.032	0.032	0.032
	$z_{TP,lm,t}$	0.020	0.020	0.021	0.020	0.019	0.019
	$z_{TP,rm,t}$	0.014	0.014	0.014	0.013	0.013	0.013
5-minute	$z_{TP,t}$	0.042	0.041	0.041	0.042	0.041	0.041
	$z_{TP,lm,t}$	0.024	0.023	0.024	0.024	0.024	0.022
	$z_{TP,rm,t}$	0.014	0.014	0.015	0.014	0.014	0.014
30-minute	$z_{TP,t}$	0.129	0.128	0.128	0.127	0.127	0.126
	$z_{TP,lm,t}$	0.058	0.057	0.058	0.059	0.058	0.057
	$z_{TP,rm,t}$	0.021	0.020	0.020	0.020	0.020	0.020
			(i = 2)				
1-minute	$z_{TP,t}$	0.022	0.021	0.021	0.022	0.023	0.023
	$z_{TP,lm,t}$	0.016	0.015	0.015	0.016	0.016	0.017
	$z_{TP,rm,t}$	0.013	0.011	0.011	0.012	0.013	0.013
3-minute	$z_{TP,t}$	0.034	0.035	0.034	0.035	0.035	0.035
	$z_{TP,lm,t}$	0.022	0.021	0.021	0.022	0.021	0.022
	$z_{TP,rm,t}$	0.015	0.014	0.014	0.015	0.015	0.015
5-minute	$z_{TP,t}$	0.044	0.042	0.043	0.043	0.043	0.042
	$z_{TP,lm,t}$	0.025	0.024	0.024	0.024	0.024	0.024
	$z_{TP,rm,t}$	0.016	0.014	0.014	0.015	0.015	0.015
30-minute	$z_{TP,t}$	0.144	0.145	0.145	0.146	0.146	0.145
	$z_{TP,lm,t}$	0.070	0.069	0.069	0.068	0.068	0.068
	$z_{TP,rm,t}$	0.025	0.026	0.026	0.025	0.025	0.026

Fixed parameters: level of significance = 0.010, $\alpha_v = -0.100$.

Table 13: Rejection frequencies of different jump statistics under market microstructure noise

Interval	σ_{mn}:	0.000	0.027	0.040	0.052	0.065	0.080
			(i = 0)				
1-minute	$z_{TP,t}$	0.032	0.018	0.012	0.010	0.008	0.007
	$z_{TP,lm,t}$	0.026	0.016	0.011	0.009	0.008	0.007
	$z_{TP,rm,t}$	0.023	0.015	0.011	0.009	0.008	0.006
3-minute	$z_{TP,t}$	0.042	0.036	0.030	0.023	0.017	0.013
	$z_{TP,lm,t}$	0.029	0.027	0.022	0.017	0.013	0.010
	$z_{TP,rm,t}$	0.022	0.021	0.018	0.014	0.011	0.009
5-minute	$z_{TP,t}$	0.050	0.048	0.043	0.037	0.030	0.025
	$z_{TP,lm,t}$	0.033	0.032	0.028	0.024	0.020	0.017
	$z_{TP,rm,t}$	0.023	0.023	0.020	0.018	0.015	0.013
30-minute	$z_{TP,t}$	0.112	0.113	0.113	0.112	0.112	0.111
	$z_{TP,lm,t}$	0.052	0.052	0.052	0.051	0.050	0.050
	$z_{TP,rm,t}$	0.018	0.019	0.019	0.018	0.018	0.018
			(i = 1)				
1-minute	$z_{TP,t}$	0.032	0.031	0.031	0.031	0.031	0.030
	$z_{TP,lm,t}$	0.026	0.026	0.025	0.025	0.025	0.024
	$z_{TP,rm,t}$	0.023	0.022	0.022	0.021	0.021	0.020
3-minute	$z_{TP,t}$	0.042	0.044	0.043	0.042	0.041	0.040
	$z_{TP,lm,t}$	0.030	0.031	0.030	0.029	0.028	0.027
	$z_{TP,rm,t}$	0.024	0.024	0.024	0.023	0.022	0.021
5-minute	$z_{TP,t}$	0.052	0.051	0.051	0.051	0.049	0.049
	$z_{TP,lm,t}$	0.033	0.033	0.033	0.033	0.032	0.030
	$z_{TP,rm,t}$	0.024	0.023	0.024	0.022	0.022	0.021
30-minute	$z_{TP,t}$	0.134	0.133	0.133	0.132	0.132	0.131
	$z_{TP,lm,t}$	0.063	0.062	0.063	0.063	0.062	0.061
	$z_{TP,rm,t}$	0.024	0.024	0.024	0.024	0.023	0.023
			(i = 2)				
1-minute	$z_{TP,t}$	0.034	0.032	0.031	0.031	0.032	0.032
	$z_{TP,lm,t}$	0.027	0.026	0.025	0.025	0.025	0.025
	$z_{TP,rm,t}$	0.024	0.022	0.021	0.021	0.022	0.022
3-minute	$z_{TP,t}$	0.044	0.045	0.043	0.044	0.044	0.044
	$z_{TP,lm,t}$	0.032	0.031	0.031	0.031	0.030	0.030
	$z_{TP,rm,t}$	0.025	0.024	0.024	0.024	0.024	0.023
5-minute	$z_{TP,t}$	0.053	0.051	0.052	0.051	0.051	0.050
	$z_{TP,lm,t}$	0.035	0.033	0.033	0.032	0.032	0.032
	$z_{TP,rm,t}$	0.025	0.023	0.023	0.023	0.023	0.023
30-minute	$z_{TP,t}$	0.150	0.151	0.150	0.151	0.151	0.150
	$z_{TP,lm,t}$	0.074	0.074	0.073	0.073	0.072	0.073
	$z_{TP,rm,t}$	0.029	0.029	0.029	0.029	0.028	0.029

Fixed parameters: level of significance = 0.010, $\alpha_v = -0.100$, $\sigma_{jmp} = 1.50$, $\lambda = 0.014$.

Table 14: Confusion Matrices, large rare jumps and market microstructure noise ($z_{TP,rm,t}$)

Interval		$\sigma_{mn} = 0.000$		$\sigma_{mn} = 0.027$		$\sigma_{mn} = 0.052$		$\sigma_{mn} = 0.080$	
				(i = 0)					
		(NJ)	(J)	(NJ)	(J)	(NJ)	(J)	(NJ)	(J)
1-minute	(NJ)	0.988	0.012	0.996	0.004	1.000	0.000	1.000	0.000
	(J)	0.214	0.786	0.260	0.740	0.393	0.607	0.558	0.442
3-minute	(NJ)	0.988	0.012	0.989	0.011	0.995	0.005	0.998	0.002
	(J)	0.292	0.708	0.308	0.692	0.377	0.623	0.461	0.539
5-minute	(NJ)	0.986	0.014	0.986	0.014	0.991	0.009	0.995	0.005
	(J)	0.360	0.640	0.354	0.646	0.396	0.604	0.503	0.497
30-minute	(NJ)	0.986	0.014	0.985	0.015	0.985	0.015	0.986	0.014
	(J)	0.744	0.257	0.724	0.276	0.734	0.266	0.740	0.260
				(i = 1)					
1-minute	(NJ)	0.988	0.012	0.989	0.011	0.988	0.012	0.988	0.012
	(J)	0.224	0.776	0.253	0.747	0.321	0.679	0.432	0.568
3-minute	(NJ)	0.986	0.014	0.986	0.014	0.987	0.013	0.988	0.012
	(J)	0.286	0.714	0.305	0.695	0.341	0.659	0.416	0.584
5-minute	(NJ)	0.986	0.014	0.986	0.014	0.986	0.014	0.986	0.014
	(J)	0.344	0.656	0.370	0.630	0.422	0.578	0.481	0.519
30-minute	(NJ)	0.980	0.020	0.980	0.020	0.980	0.020	0.981	0.019
	(J)	0.708	0.292	0.721	0.279	0.718	0.282	0.750	0.250
				(i = 2)					
1-minute	(NJ)	0.988	0.012	0.989	0.011	0.988	0.012	0.987	0.013
	(J)	0.221	0.779	0.266	0.734	0.331	0.669	0.409	0.591
3-minute	(NJ)	0.985	0.015	0.986	0.014	0.985	0.015	0.985	0.015
	(J)	0.286	0.714	0.325	0.675	0.383	0.617	0.442	0.558
5-minute	(NJ)	0.984	0.016	0.986	0.014	0.985	0.015	0.985	0.015
	(J)	0.360	0.640	0.390	0.610	0.435	0.565	0.490	0.510
30-minute	(NJ)	0.975	0.025	0.974	0.026	0.975	0.025	0.974	0.026
	(J)	0.737	0.263	0.730	0.270	0.737	0.263	0.750	0.250

Fixed parameters: level of significance = 0.010, $\alpha_v = -0.100$, $\sigma_{jmp} = 1.50$, $\lambda = 0.014$.

Table 15: Proportion of days identified as jumps by the daily statistics

level of significance	0.500	0.950	0.990	0.995	0.999
		S&P 500 Index Futures			
		(04/21/1982 – 04/18/1997)			
		(i = 0)			
$z_{TP,t}$	0.64384	0.20941	0.11925	0.09651	0.06293
$z_{TP,lm,t}$	0.64384	0.17980	0.08699	0.06716	0.04019
$z_{TP,rm,t}$	0.64384	0.15732	0.06346	0.04469	0.02327
		(i = 1)			
$z_{TP,t}$	0.68112	0.25754	0.15415	0.12850	0.08726
$z_{TP,lm,t}$	0.68112	0.22263	0.11687	0.09228	0.05500
$z_{TP,rm,t}$	0.68112	0.19751	0.08858	0.06319	0.03411
		(i = 2)			
$z_{TP,t}$	0.71549	0.29799	0.18879	0.16288	0.11528
$z_{TP,lm,t}$	0.71549	0.25939	0.14728	0.11819	0.07377
$z_{TP,rm,t}$	0.71549	0.23559	0.11396	0.08726	0.04707
		(04/21/1997 – 10/22/2002)			
		(i = 0)			
$z_{TP,t}$	0.66108	0.20918	0.11152	0.09402	0.05175
$z_{TP,lm,t}$	0.66108	0.17566	0.07726	0.05831	0.03061
$z_{TP,rm,t}$	0.66108	0.14942	0.05758	0.03499	0.01603
		(i = 1)			
$z_{TP,t}$	0.62026	0.20408	0.11953	0.09184	0.06195
$z_{TP,lm,t}$	0.62026	0.17055	0.08163	0.06341	0.03134
$z_{TP,rm,t}$	0.62026	0.15452	0.05977	0.03790	0.02332
		(i = 2)			
$z_{TP,t}$	0.60714	0.19096	0.11006	0.09111	0.06706
$z_{TP,lm,t}$	0.60714	0.16327	0.08309	0.06924	0.04519
$z_{TP,rm,t}$	0.60714	0.14213	0.06560	0.05248	0.02551
		(04/21/1982 – 12/09/2002)			
		(i = 0)			
$z_{TP,t}$	0.64886	0.20960	0.11705	0.09583	0.05997
$z_{TP,lm,t}$	0.64886	0.17875	0.08427	0.06479	0.03779
$z_{TP,rm,t}$	0.64886	0.15523	0.06190	0.04223	0.02140
		(i = 1)			
$z_{TP,t}$	0.66506	0.24277	0.14462	0.11859	0.08060
$z_{TP,lm,t}$	0.66506	0.20845	0.10740	0.08465	0.04859
$z_{TP,rm,t}$	0.66506	0.18569	0.08099	0.05650	0.03105
		(i = 2)			
$z_{TP,t}$	0.68608	0.26938	0.16776	0.14366	0.10220
$z_{TP,lm,t}$	0.68608	0.23351	0.13016	0.10490	0.06595
$z_{TP,rm,t}$	0.68608	0.21037	0.10085	0.07790	0.04126

Table 16: Summary of RV, BV and RJ.

Statistics	RV	BV	$RV - BV$	RJ
	S&P 500 Index Futures			
	(04/21/1982 – 04/18/1997)			
	(i = 0)			
Sample Mean	1.45438	1.25291	0.20146	0.04601
Std of Mean	0.28803	0.16804	0.12000	0.00182
Full-Sample Statistics	5500.46066	4738.52390	761.93676	0.13852
	(i = 1)			
Sample Mean	1.45438	1.26816	0.18622	0.06017
Std of Mean	0.28803	0.20411	0.08393	0.00189
Full-Sample Statistics	5500.46066	4796.18426	704.27640	0.12804
	(i = 2)			
Sample Mean	1.45438	1.24790	0.20647	0.07269
Std of Mean	0.28803	0.19699	0.09105	0.00198
Full-Sample Statistics	5500.46066	4719.57255	780.88811	0.14197
	(04/21/1997 – 10/22/2002)			
	(i = 0)			
Sample Mean	2.10718	2.01193	0.09525	0.04445
Std of Mean	0.06690	0.06266	0.00424	0.00279
Full-Sample Statistics	2891.05330	2760.37224	130.68106	0.04520
	(i = 1)			
Sample Mean	2.10718	2.01341	0.09378	0.04095
Std of Mean	0.06690	0.06133	0.00557	0.00302
Full-Sample Statistics	2891.05330	2762.39293	128.66038	0.04450
	(i = 2)			
Sample Mean	2.10718	2.02129	0.08589	0.03556
Std of Mean	0.06690	0.06211	0.00479	0.00311
Full-Sample Statistics	2891.05330	2773.21121	117.84209	0.04076
	(04/21/1982 – 12/09/2002)			
	(i = 0)			
Sample Mean	1.63393	1.46074	0.17319	0.04562
Std of Mean	0.21083	0.12375	0.08708	0.00152
Full-Sample Statistics	8473.55466	7575.38272	898.17194	0.10600
	(i = 1)			
Sample Mean	1.63393	1.47225	0.16168	0.05500
Std of Mean	0.21083	0.14980	0.06103	0.00160
Full-Sample Statistics	8473.55466	7635.08331	838.47135	0.09895
	(i = 2)			
Sample Mean	1.63393	1.45973	0.17420	0.06266
Std of Mean	0.21083	0.14467	0.06616	0.00168
Full-Sample Statistics	8473.55466	7570.17627	903.37839	0.10661

Figure 1:

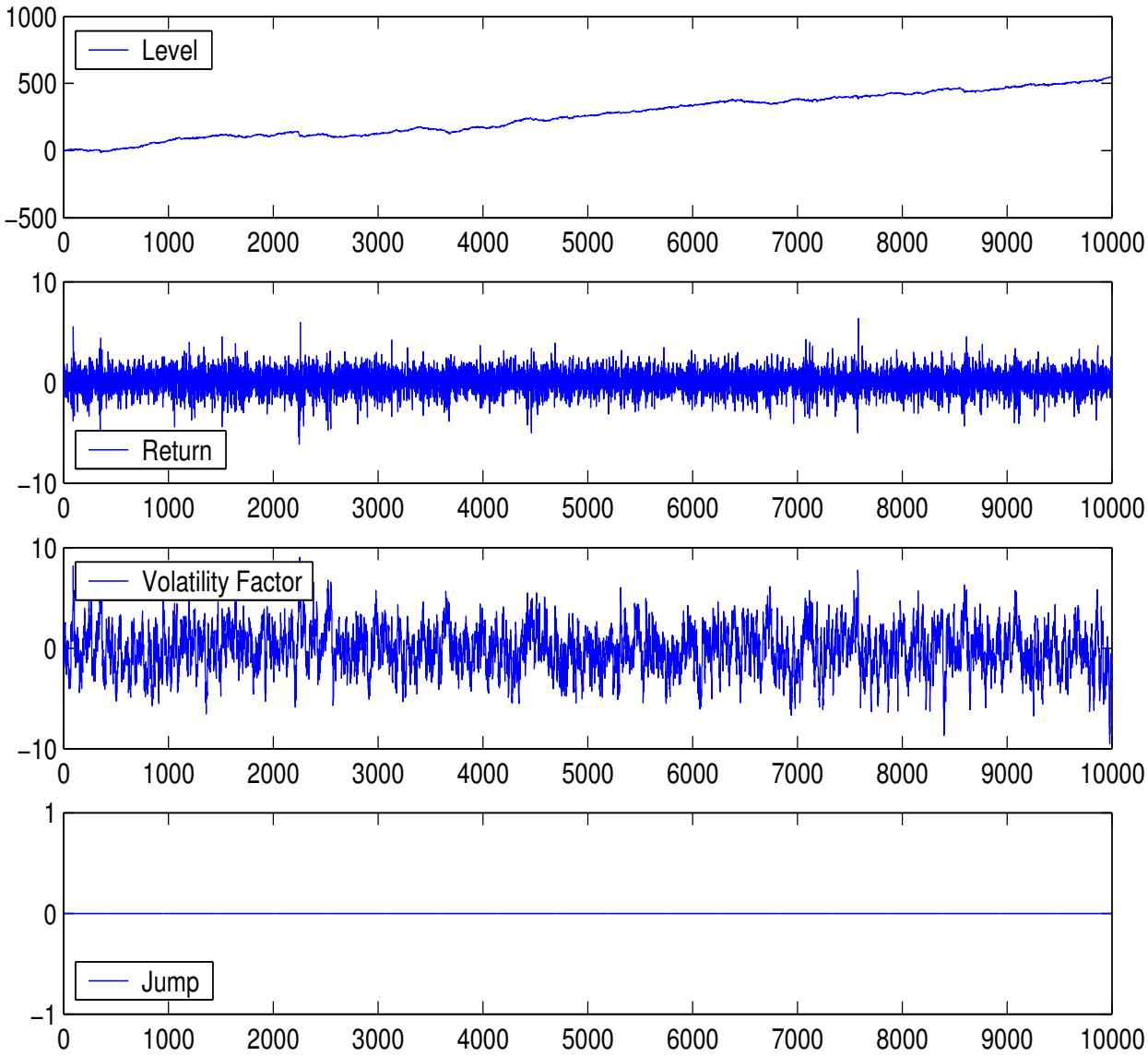

Level
Return
Volatility Factor
Jump
1000
500
0
−500
10
−10
1
−1
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

Figure 2:

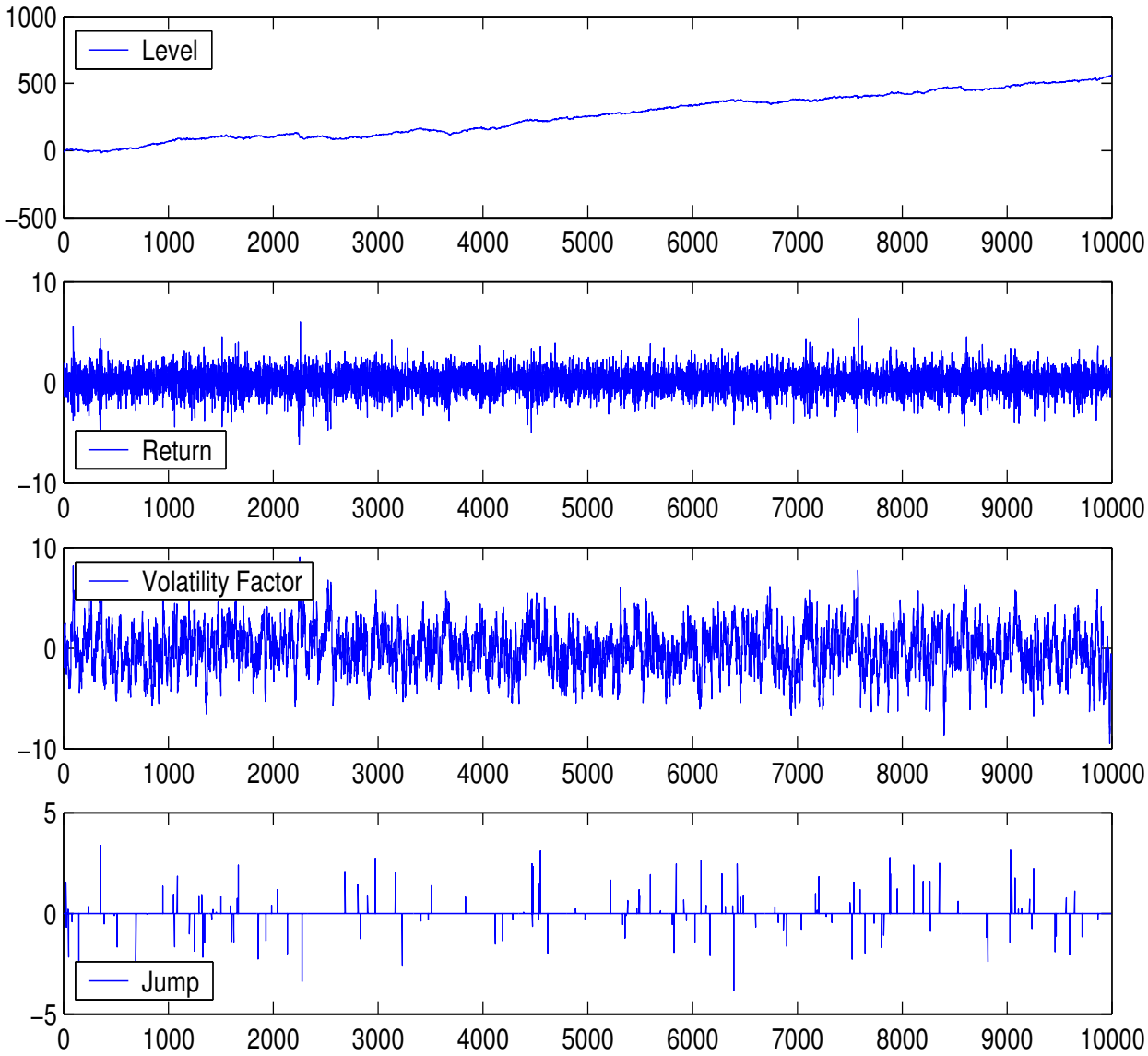

Level
1000
500
0
−500
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Return
10
0
−10
Volatility Factor
Jump
5
−5

Figure 3:

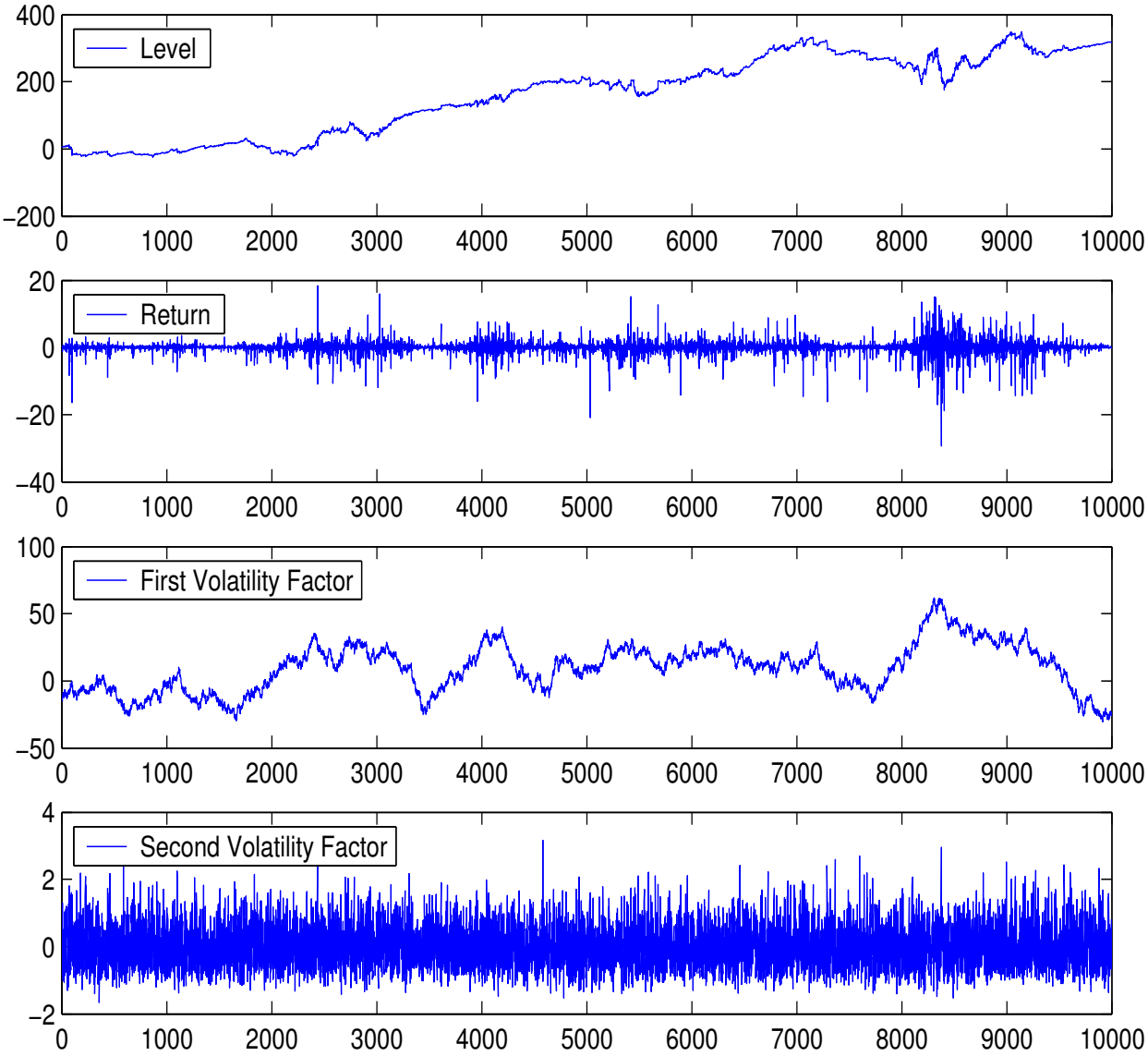

Level
400
200
0
−200
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Return
20
0
−20
−40
First Volatility Factor
100
50
0
−50
Second Volatility Factor
4
2
0
−2

Figure 4:

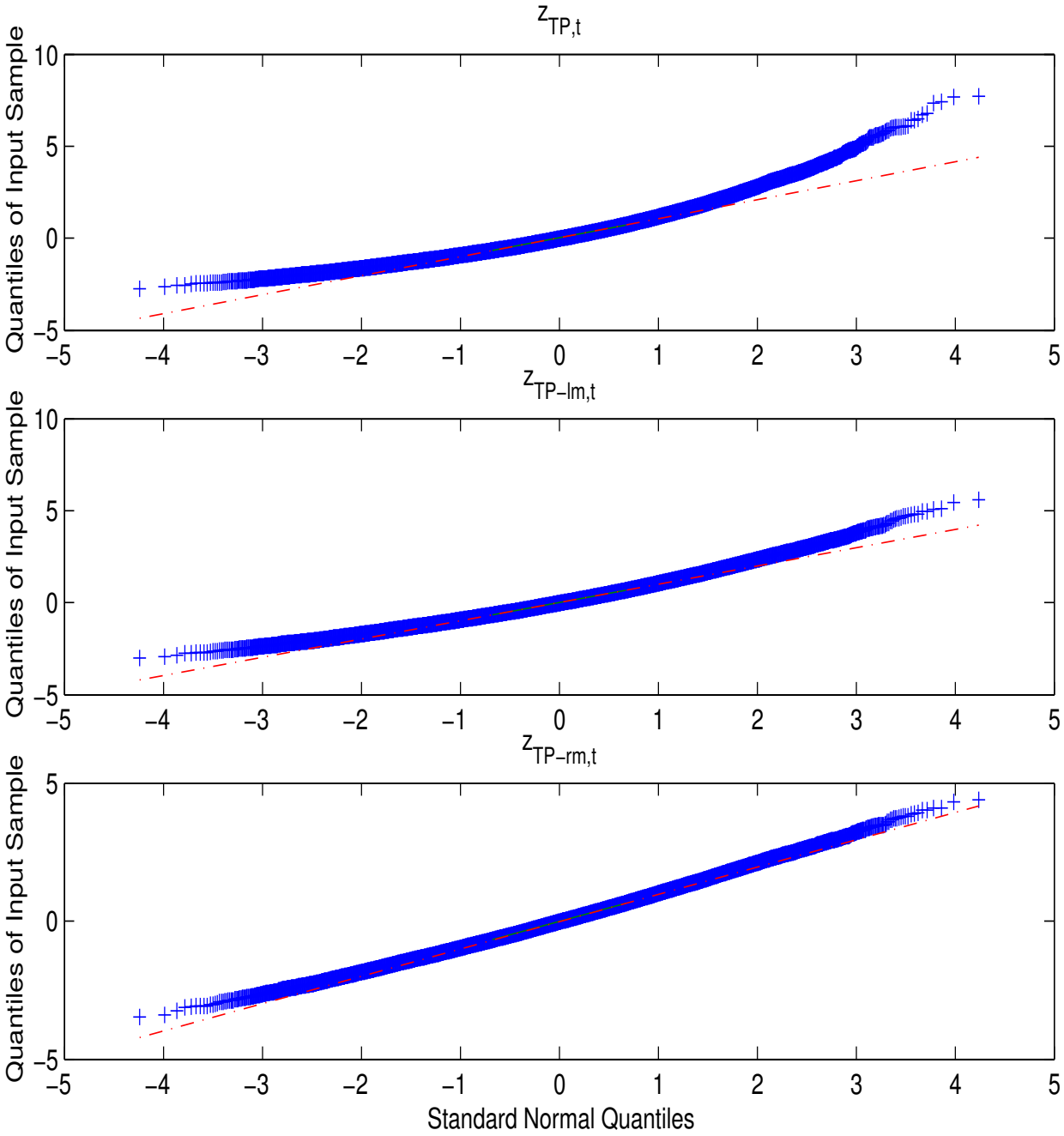

$z_{TP,t}$
$z_{TP-lm,t}$
$z_{TP-rm,t}$
Quantiles of Input Sample
Standard Normal Quantiles
10
5
0
−5
−4
−3
−2
−1
0
1
2
3
4
5

Figure 5:

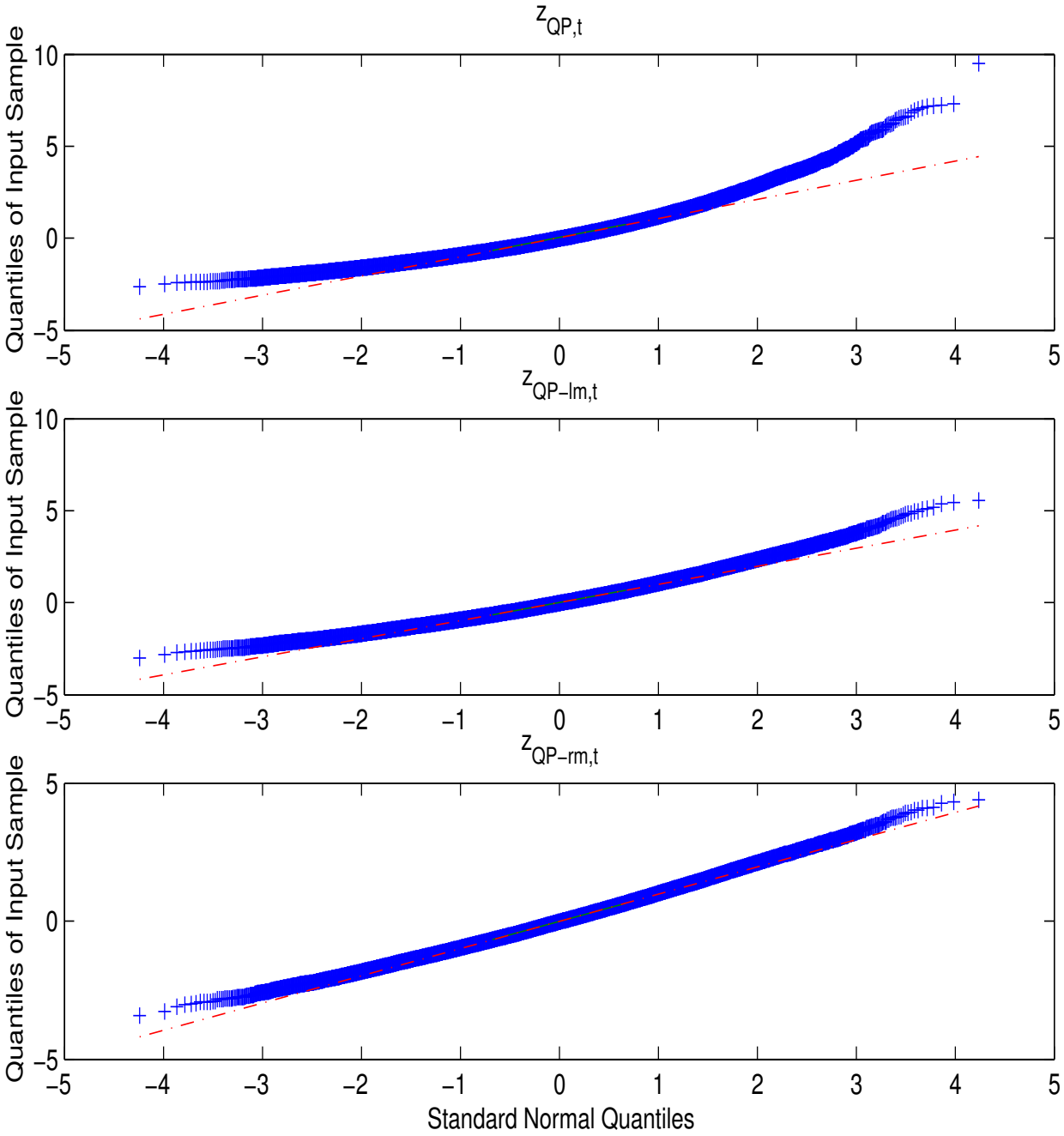

$z_{QP,t}$
$z_{QP-lm,t}$
$z_{QP-rm,t}$
Quantiles of Input Sample
Standard Normal Quantiles
10
5
0
−5
−4
−3
−2
−1
1
2
3
4

Figure 6:

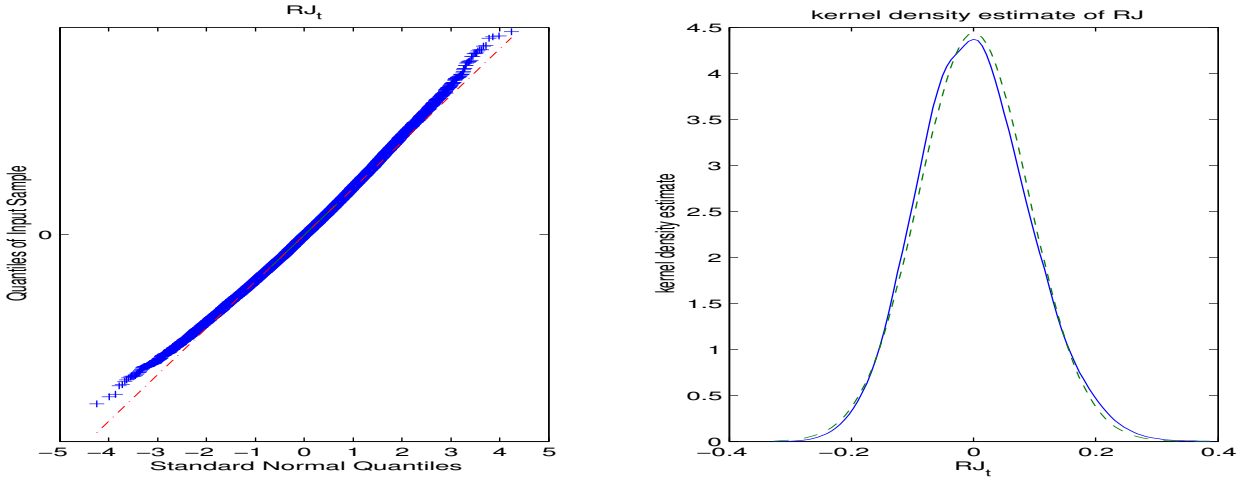


Figure 7:

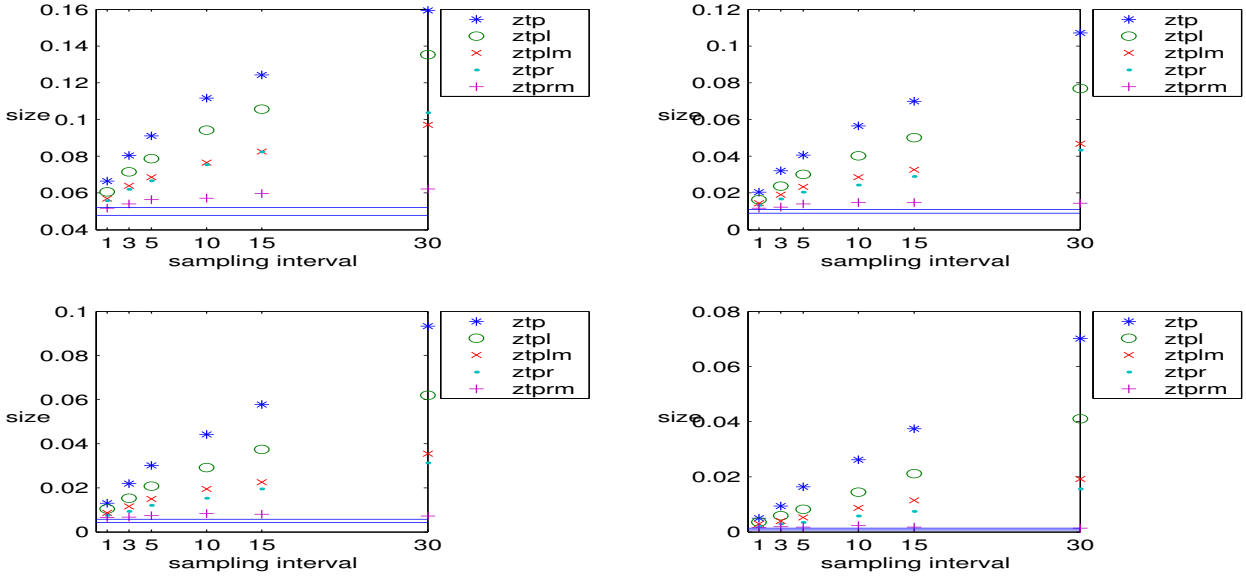


Figure 8:

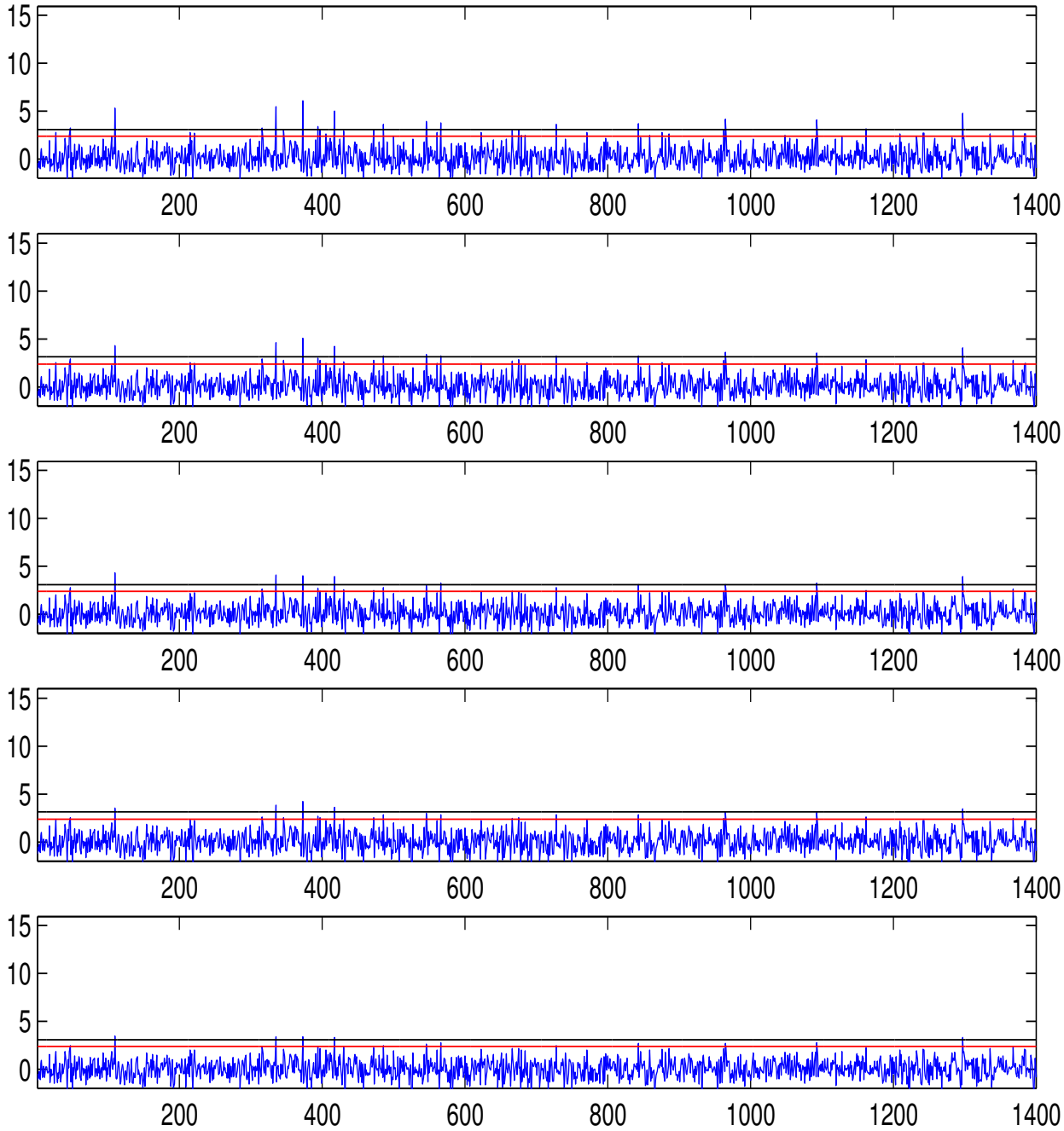


Figure 9:

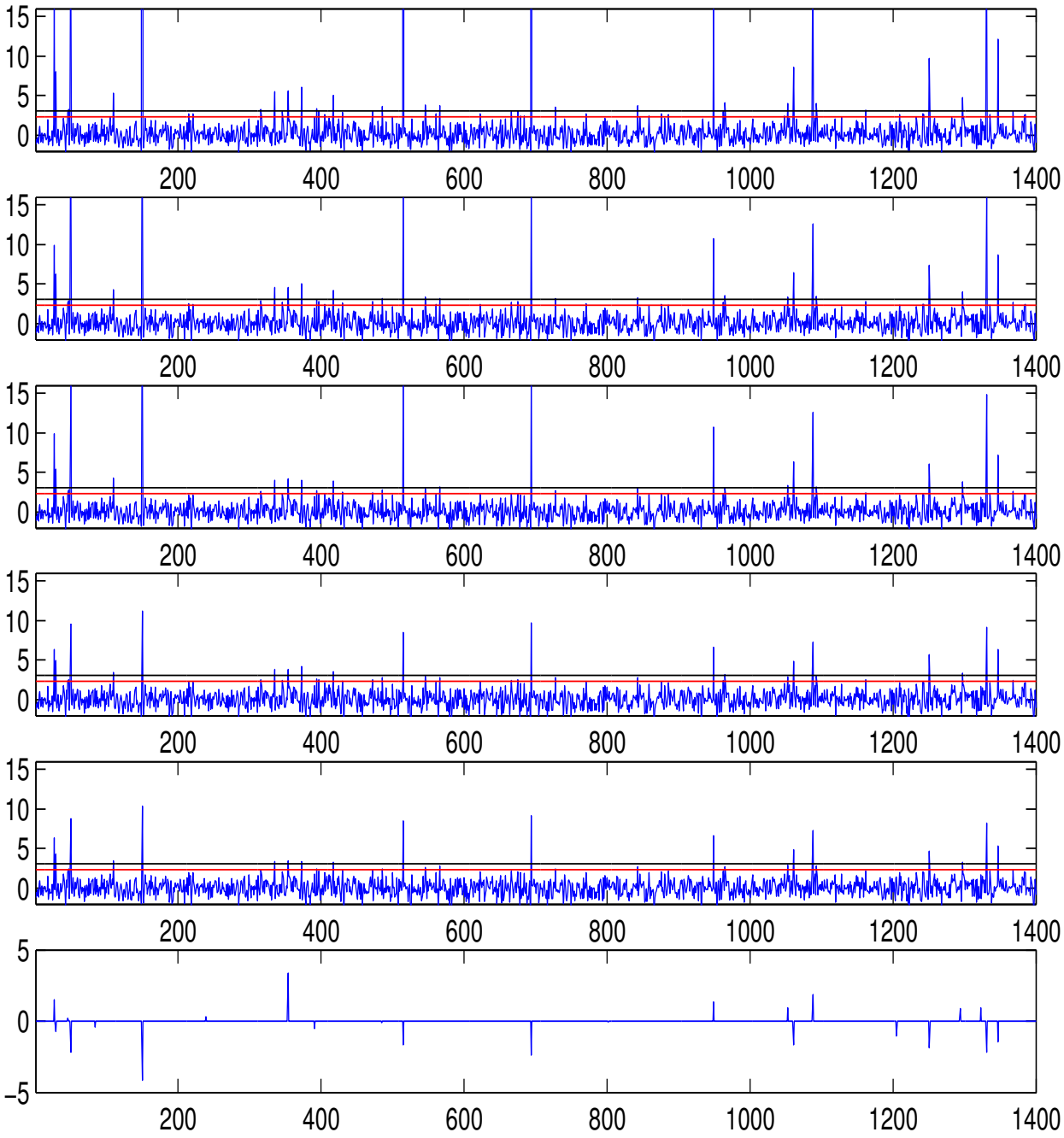

15
10
5
0
200
400
600
800
1000
1200
1400
15
10
5
0
200
400
600
800
1000
1200
1400
15
10
5
0
200
400
600
800
1000
1200
1400
15
10
5
0
200
400
600
800
1000
1200
1400
15
10
5
0
200
400
600
800
1000
1200
1400
5
0
−5
200
400
600
800
1000
1200
1400

Figure 10:

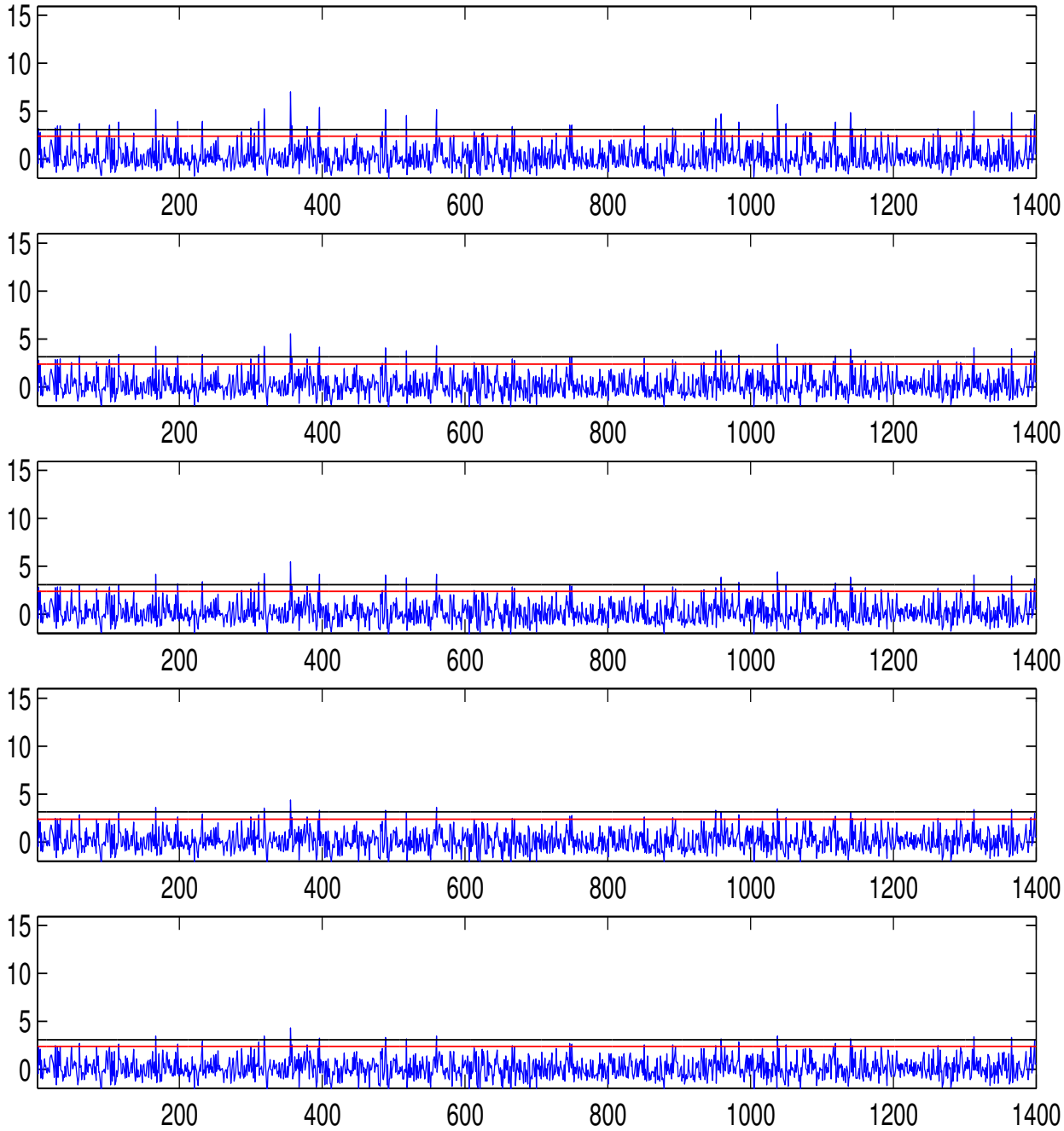


Figure 11:

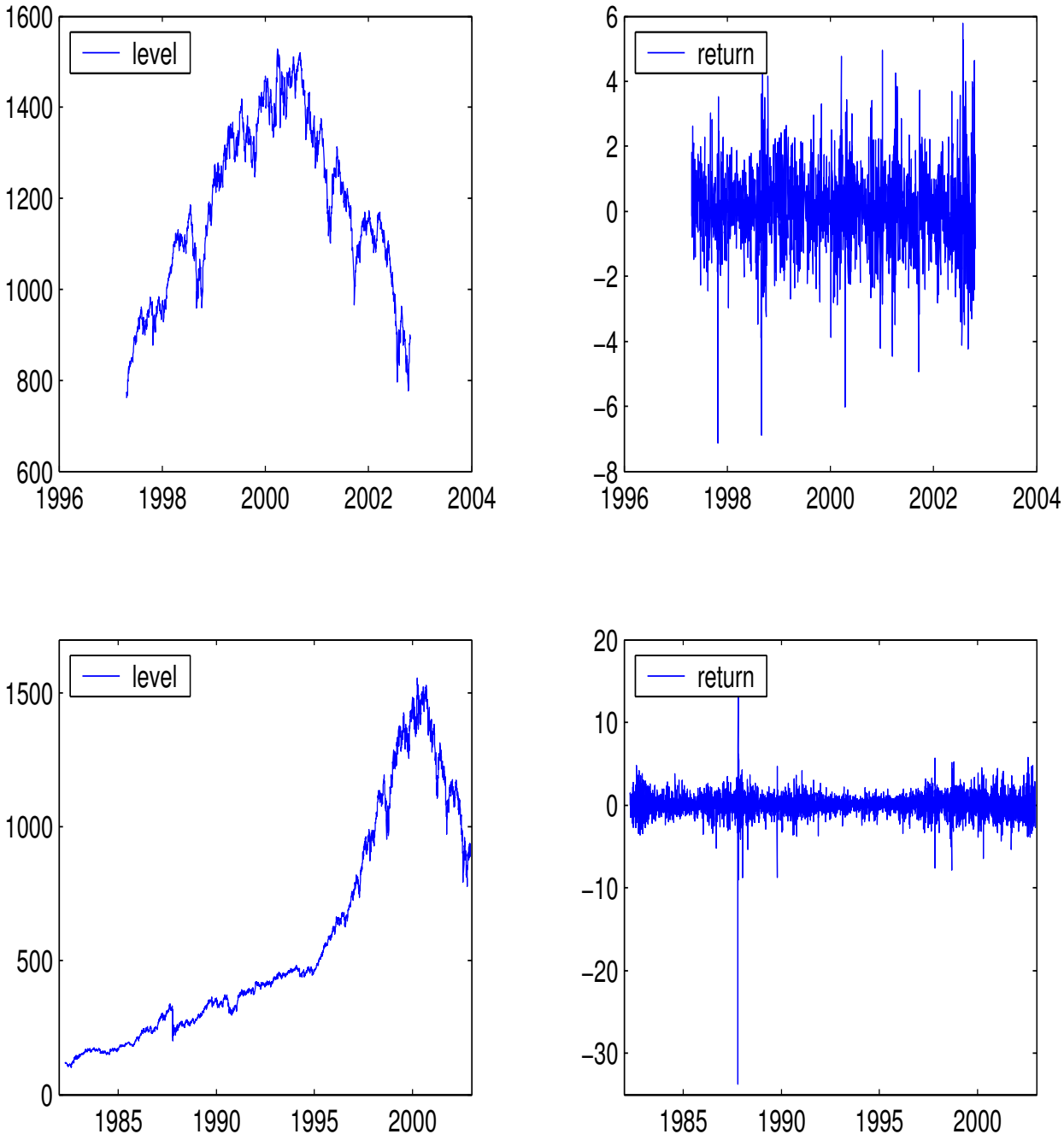

level
1600
1400
1200
1000
800
600
1996
1998
2000
2002
2004
return
6
4
2
0
−2
−4
−6
−8
1996
1998
2000
2002
2004
level
1500
1000
500
0
1985
1990
1995
2000
return
20
10
0
−10
−20
−30
1985
1990
1995
2000

Figure 12:

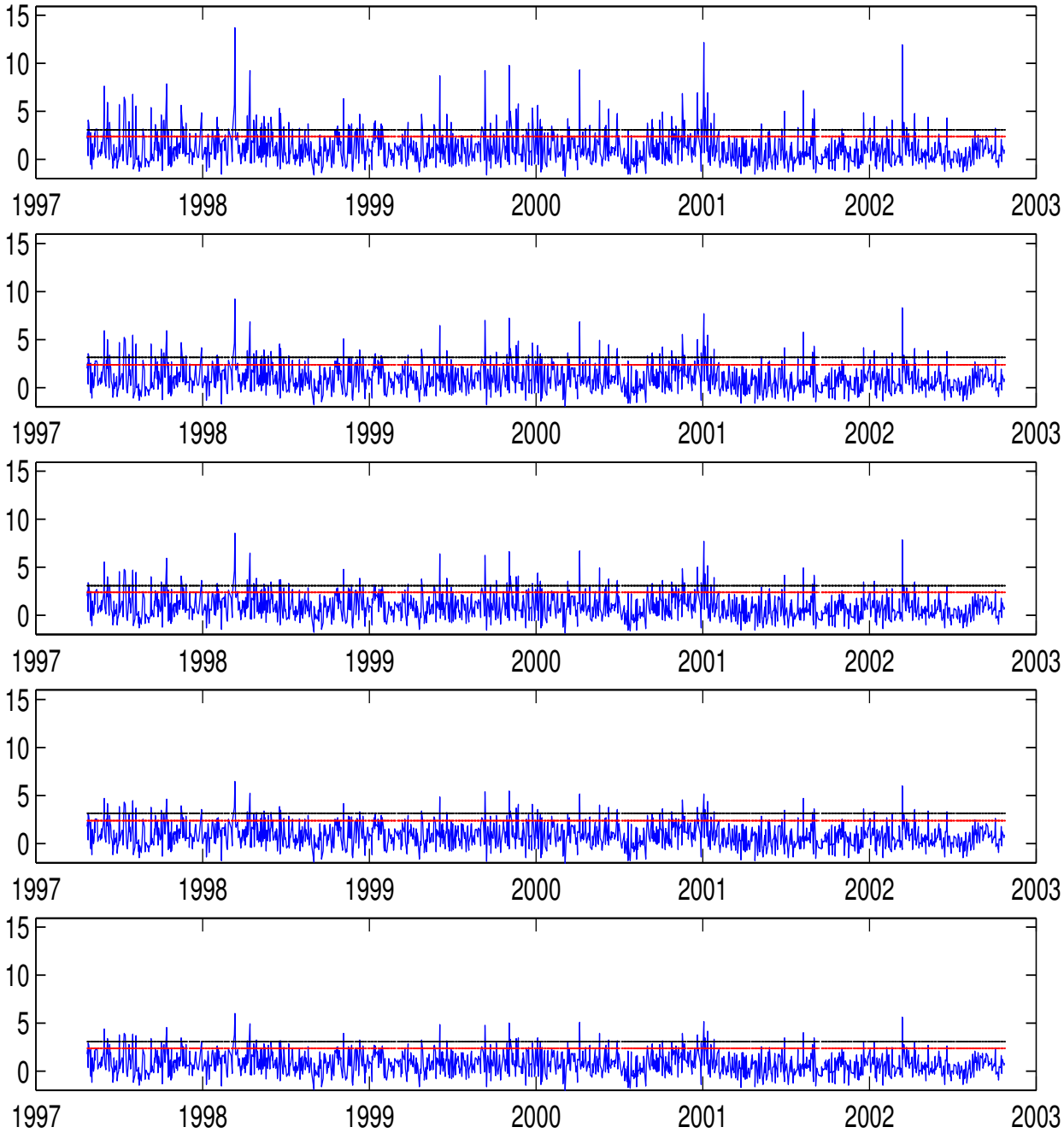
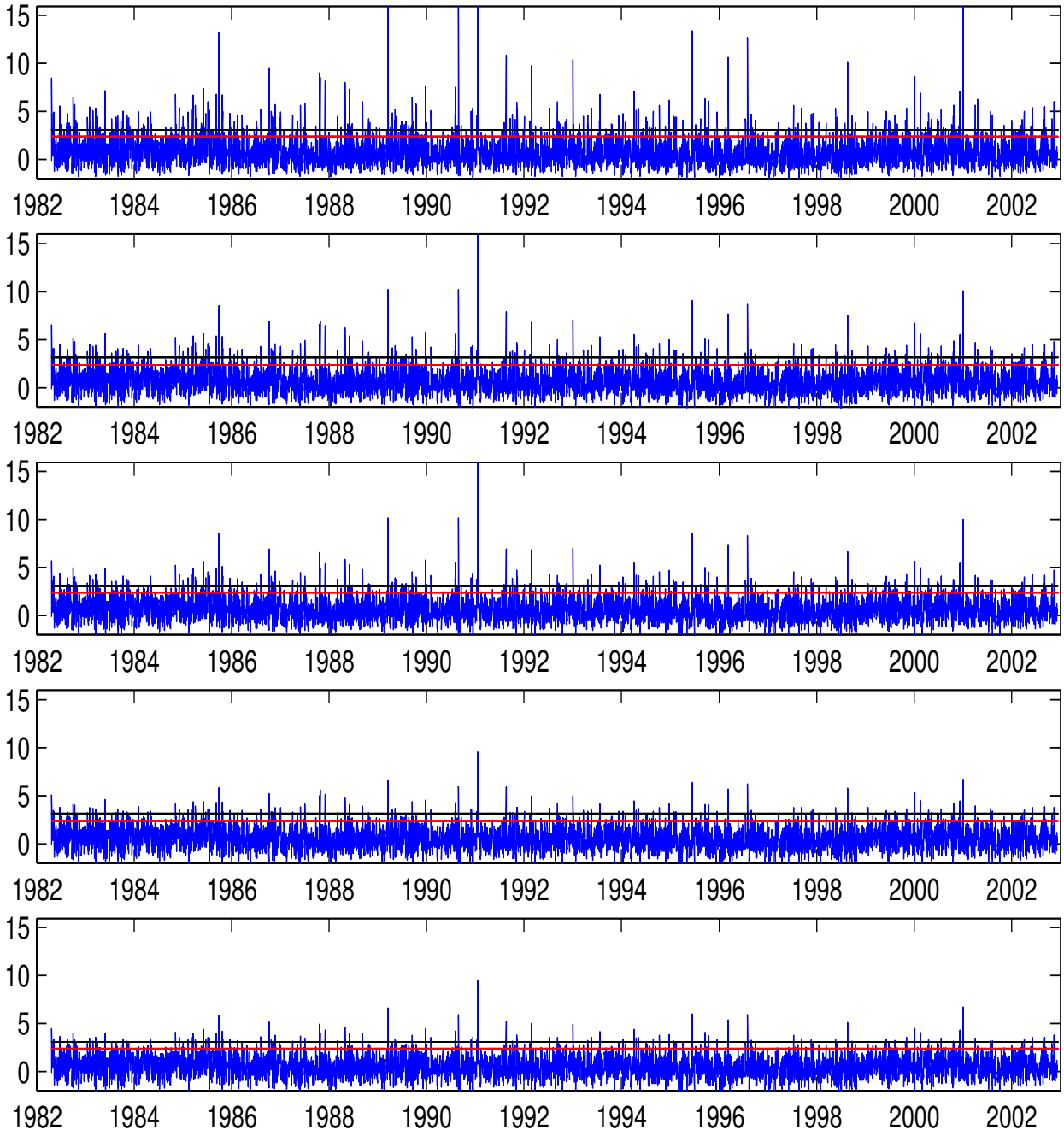


Figure 13:


15
10
5
0
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002