Physics 224
The Interstellar Medium

Lecture #3

- Part I: Finish Collisional Processes
- Part II: Statistical Mechanics
- Part III: Quantum & Energy Levels
- Part IV (maybe): Radiative Transfer

Energy Levels of Atoms

$\boldsymbol{n}$ = principle quantum number

$\boldsymbol{l}$ = orbital angular momentum in units of ħ ($0 \leq \boldsymbol{l} < \boldsymbol{n}$)

$\boldsymbol{m_z}$ = proj. of angular mom. on z axis ($-\boldsymbol{l} \leq \boldsymbol{m_z} \leq \boldsymbol{l}$)

e- spin = -ħ/2 or +ħ/2

degenerate (same energy) w/o applied B-field

Energy Levels of Atoms

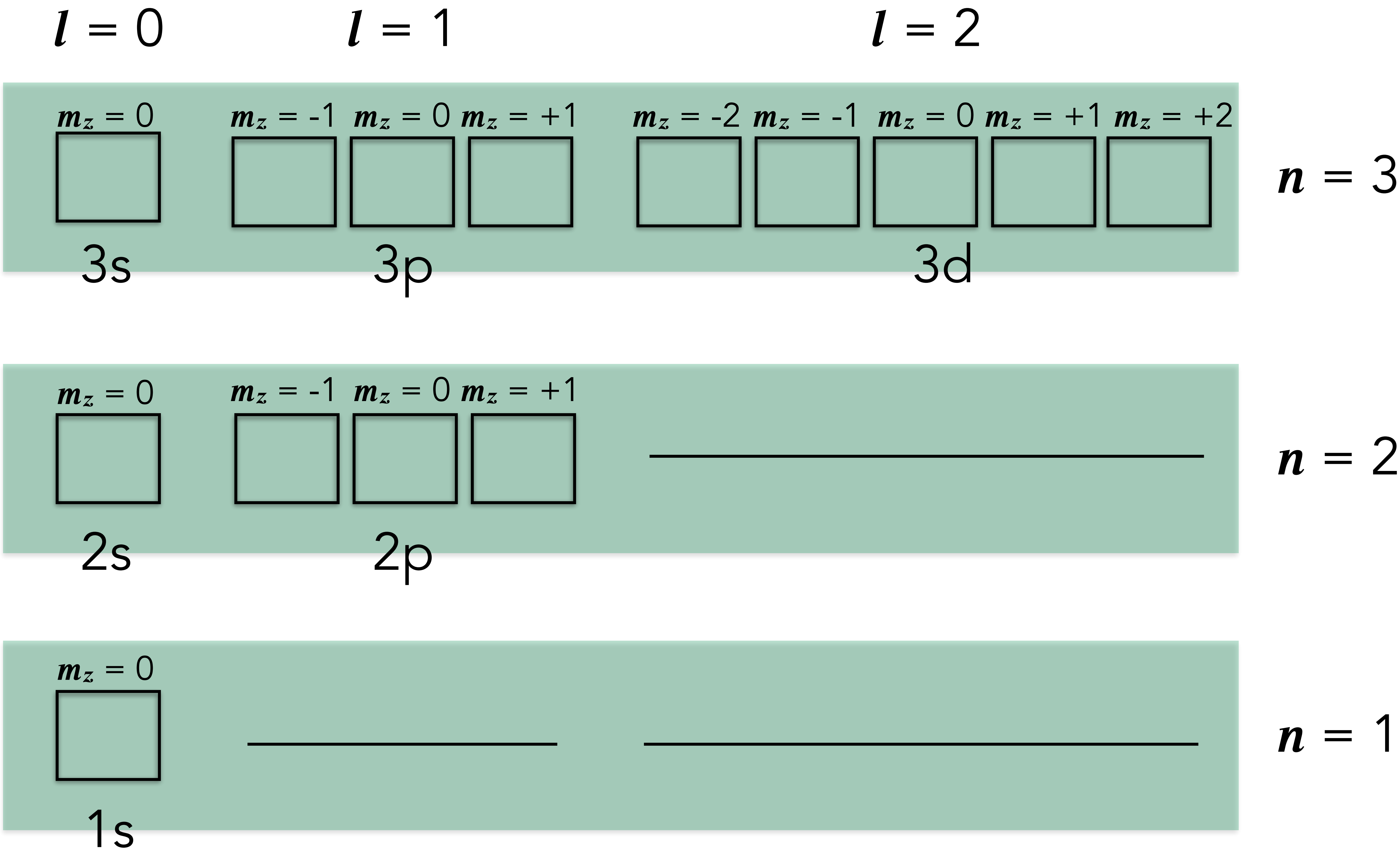
How do we arrange e- in a multi-electron atom?

Pauli exclusion principle says:

electrons can't share the same wavefunction ($\boldsymbol{n}$, $\boldsymbol{l}$, $\boldsymbol{m_z}$, spin)

For ground state configuration: fill up "subshells" from lowest energy up

subshell = combination of $\boldsymbol{nl}$ designated by number $\boldsymbol{n}$ and letter for $\boldsymbol{l}$ (0=s, 1=p, 2=d, 3=f, …)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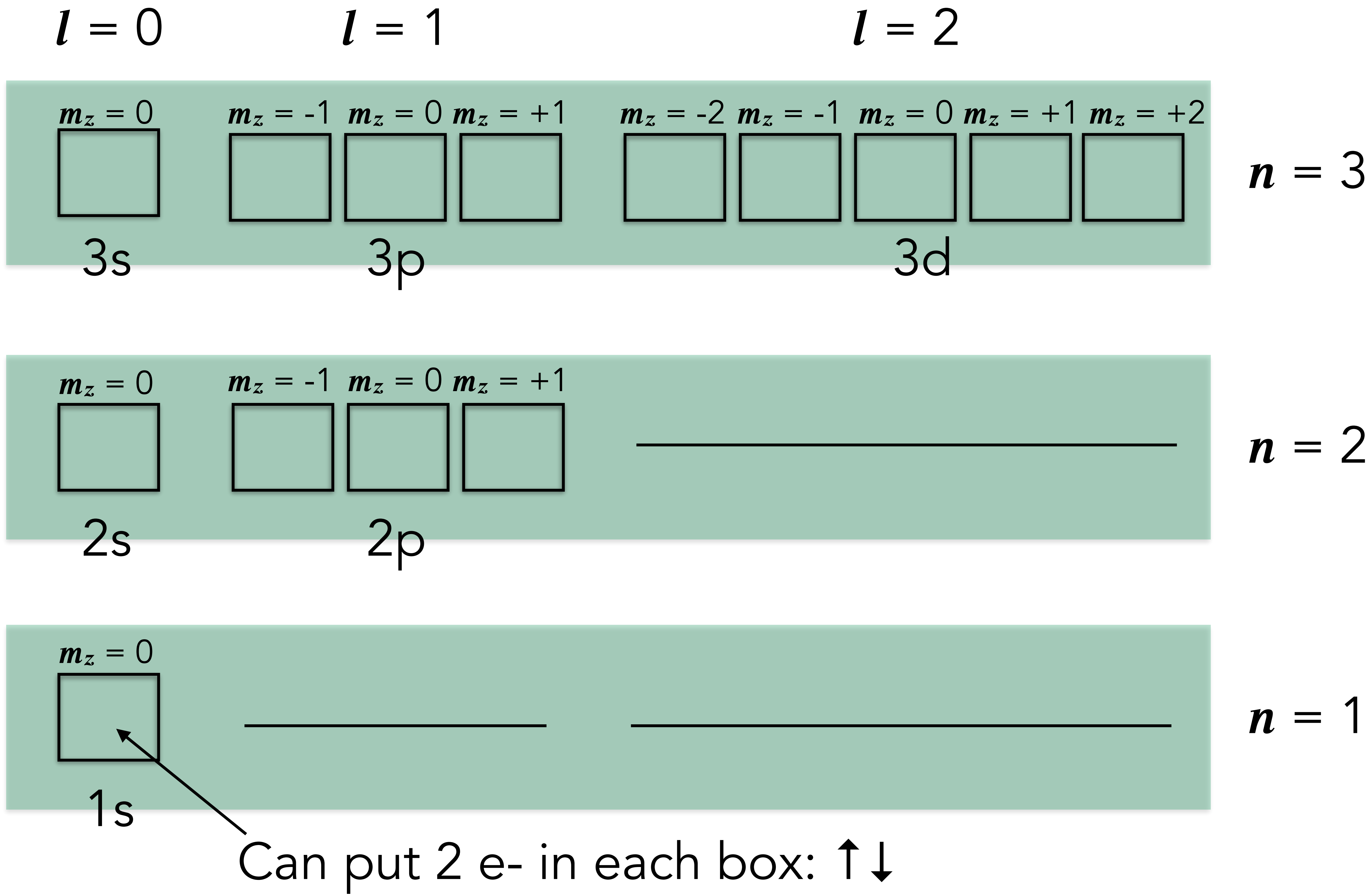
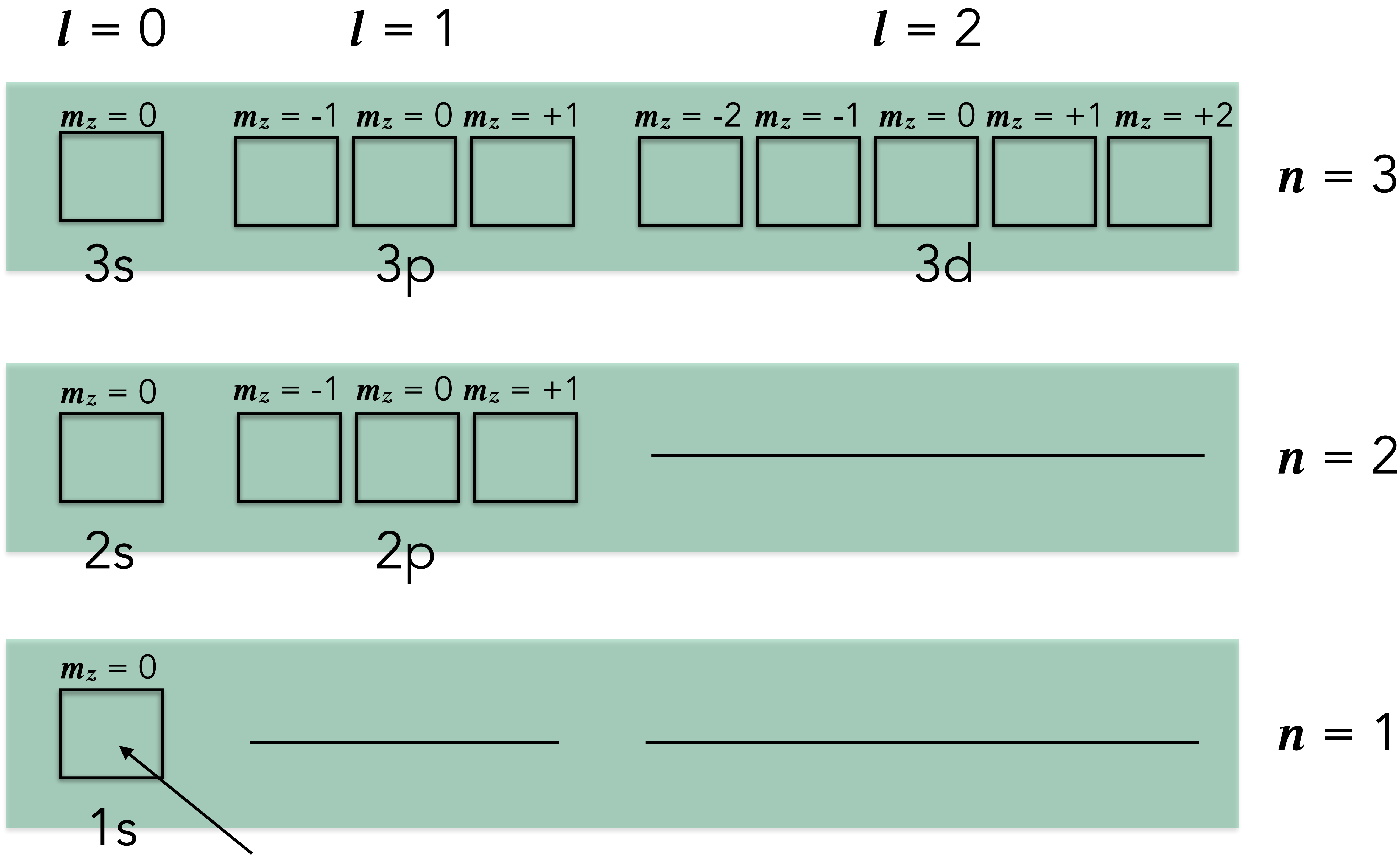
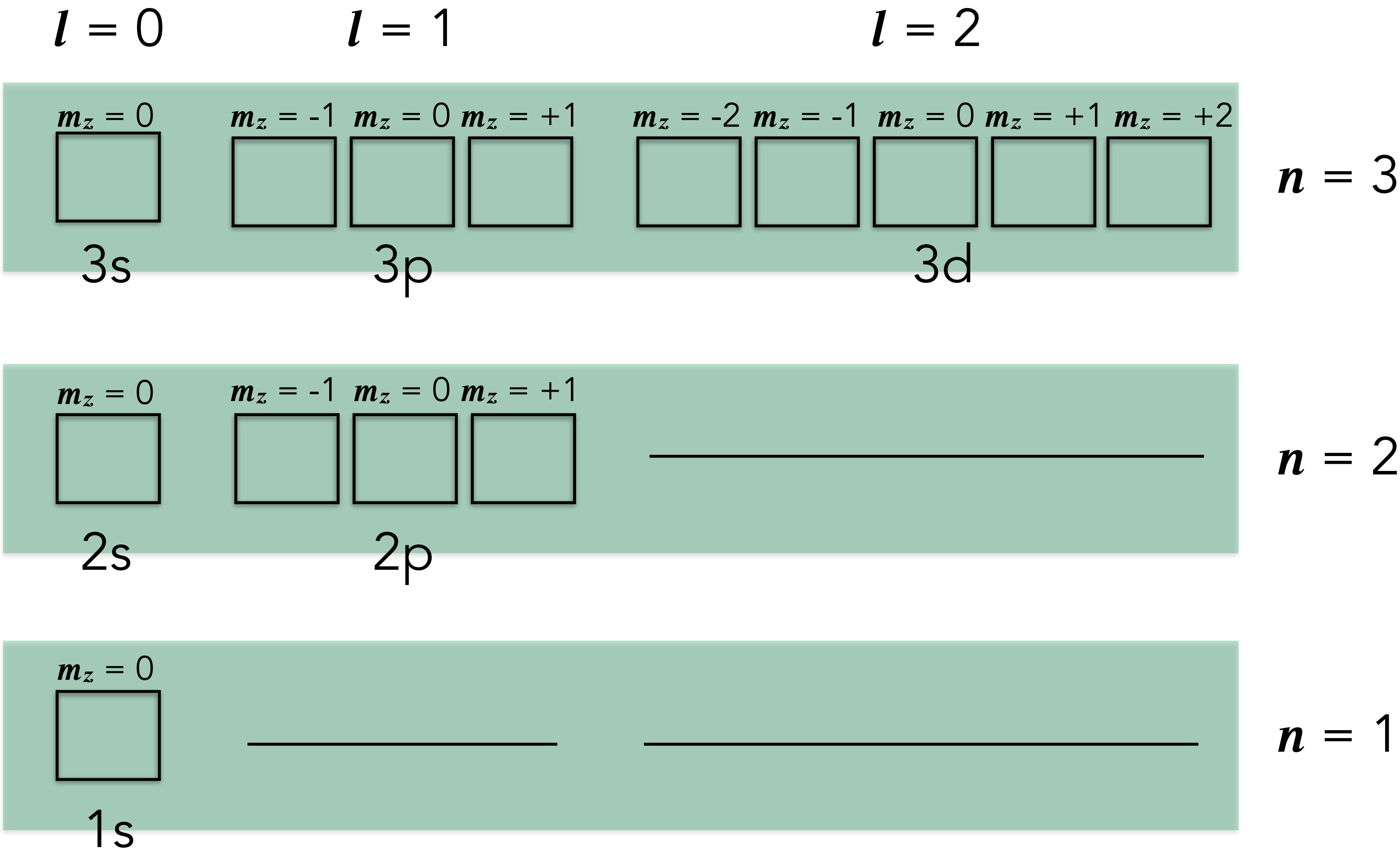
l = 1
l = 2
mz = 0
mz = -1
mz = 0
mz = +1
mz = -2
mz = -1
mz = 0
mz = +1
mz = +2
3s
3p
3d
n = 3
mz = 0
mz = -1
mz = 0
mz = +1
2s
2p
n = 2
mz = 0
1s
n = 1


l = 0
l = 1
l = 2
m_z = 0
m_z = -1 m_z = 0 m_z = +1
m_z = -2 m_z = -1 m_z = 0 m_z = +1 m_z = +2
n = 3
3s
3p
3d
m_z = 0
m_z = -1 m_z = 0 m_z = +1
n = 2
2s
2p
m_z = 0
n = 1
1s
Can put 2 e- in each box: ↑↓

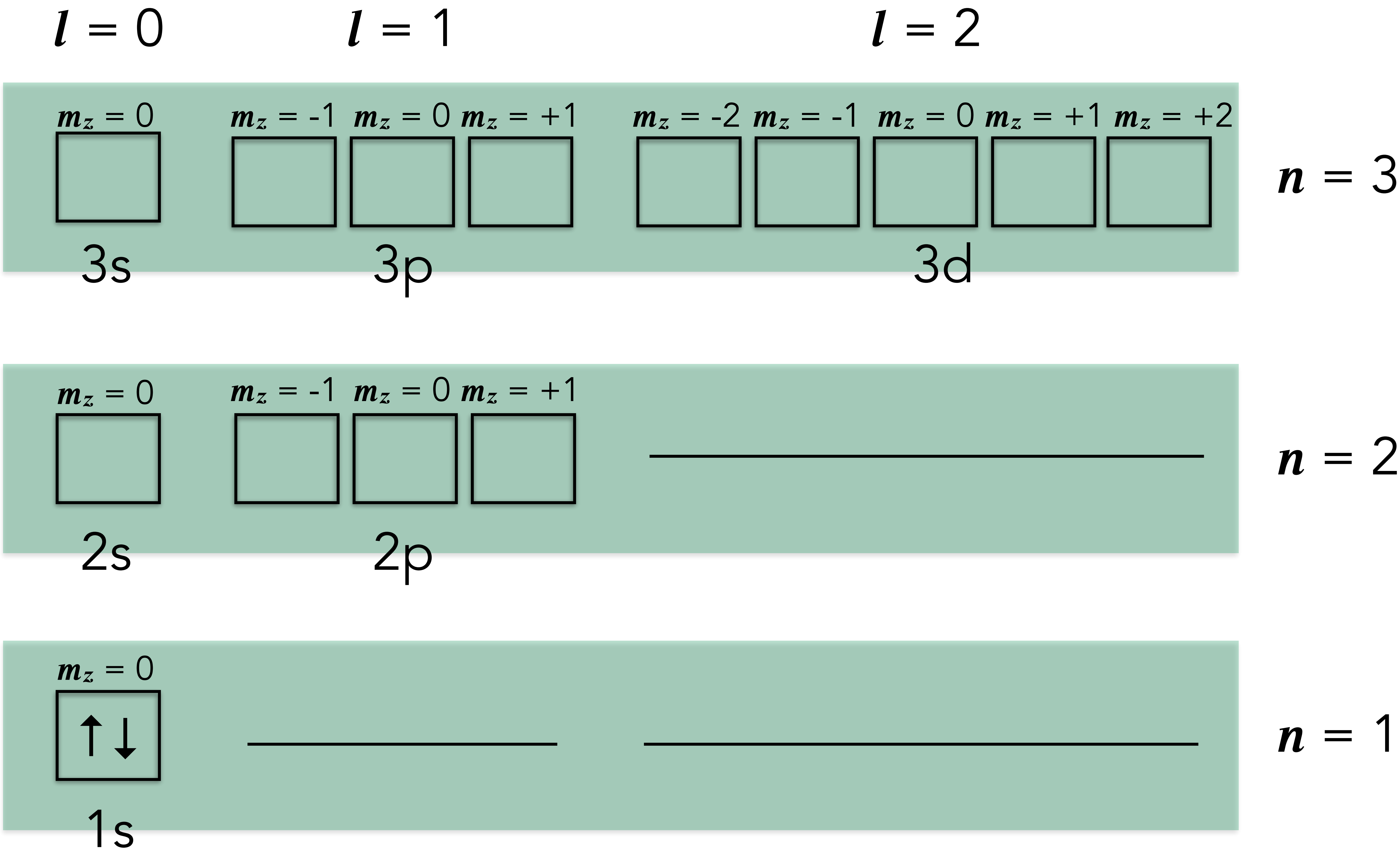


Can put 2 e- in each box: ↑↓

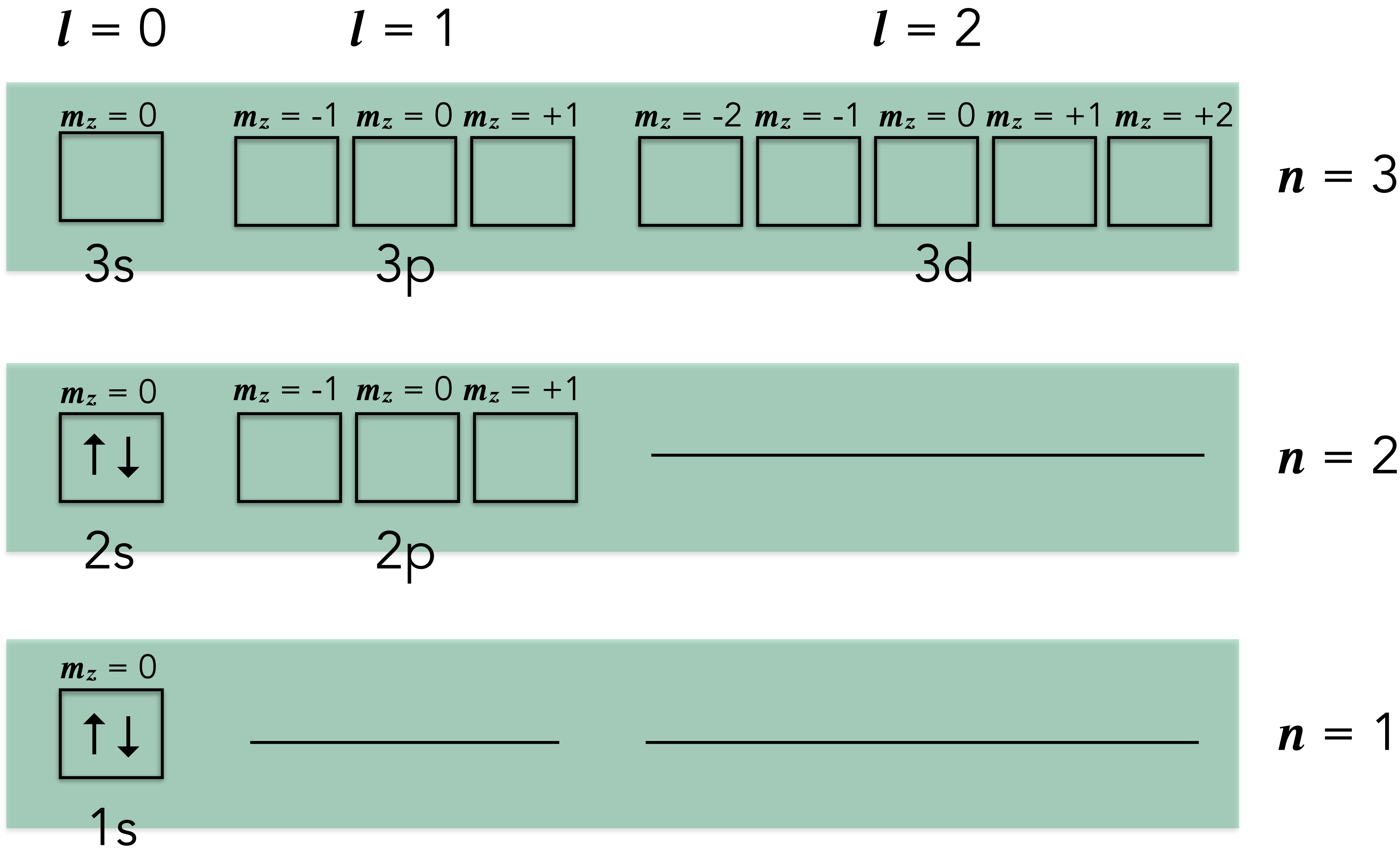
$\therefore$ degeneracy of subshell = $2(l+1)$



Lets build the ground state of Na: 11 electrons



Lets build the ground state of Na: 11 electrons



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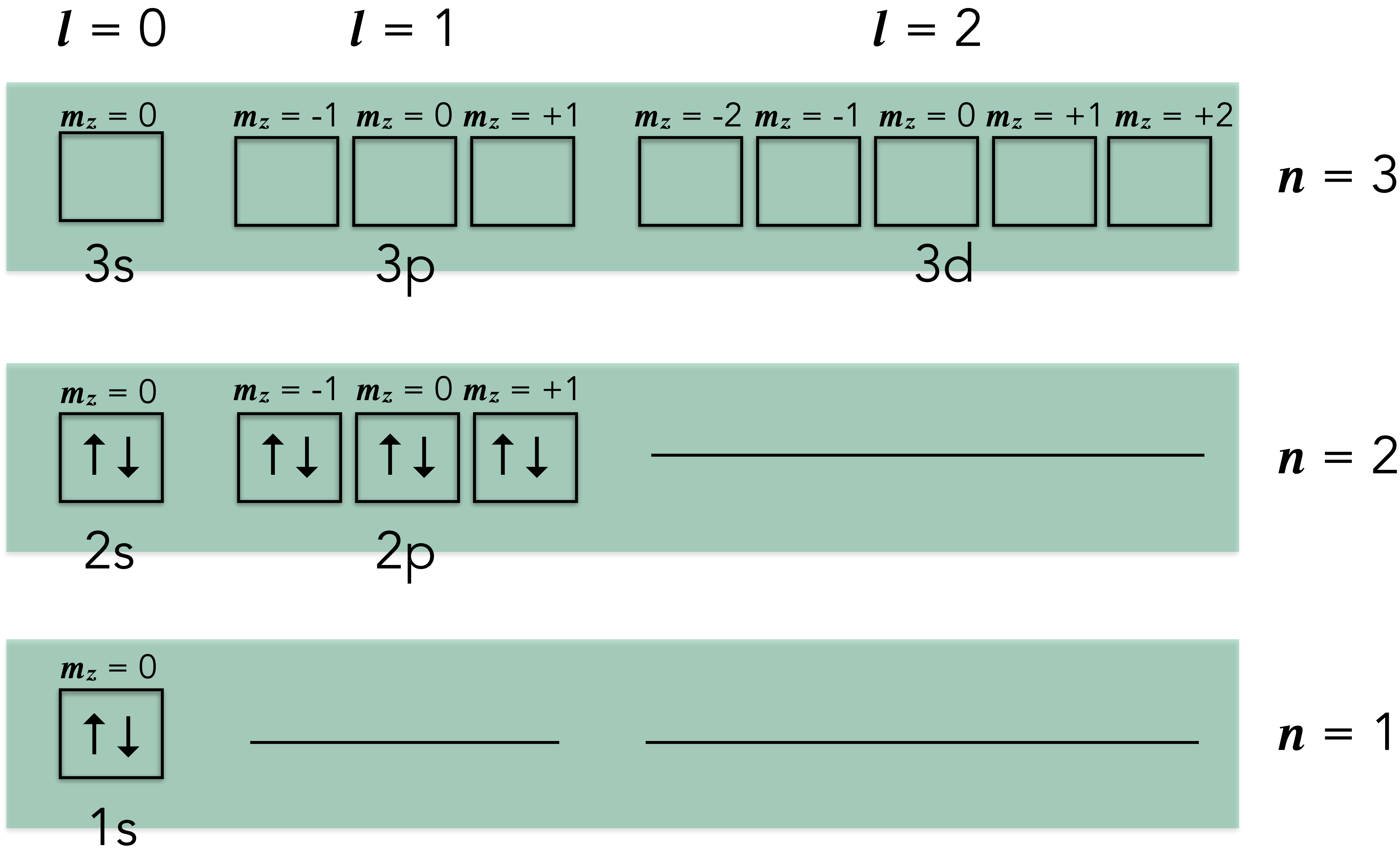
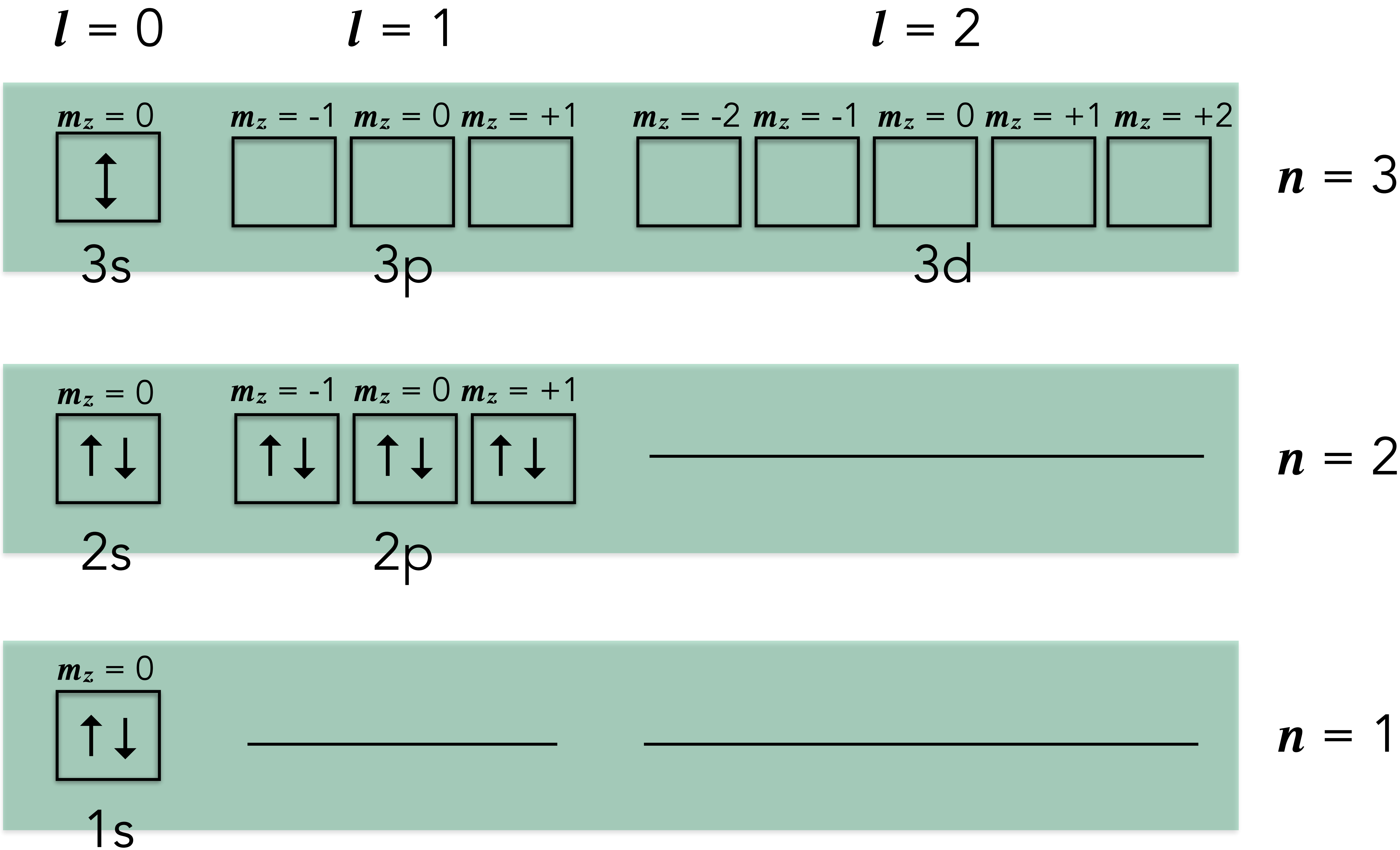
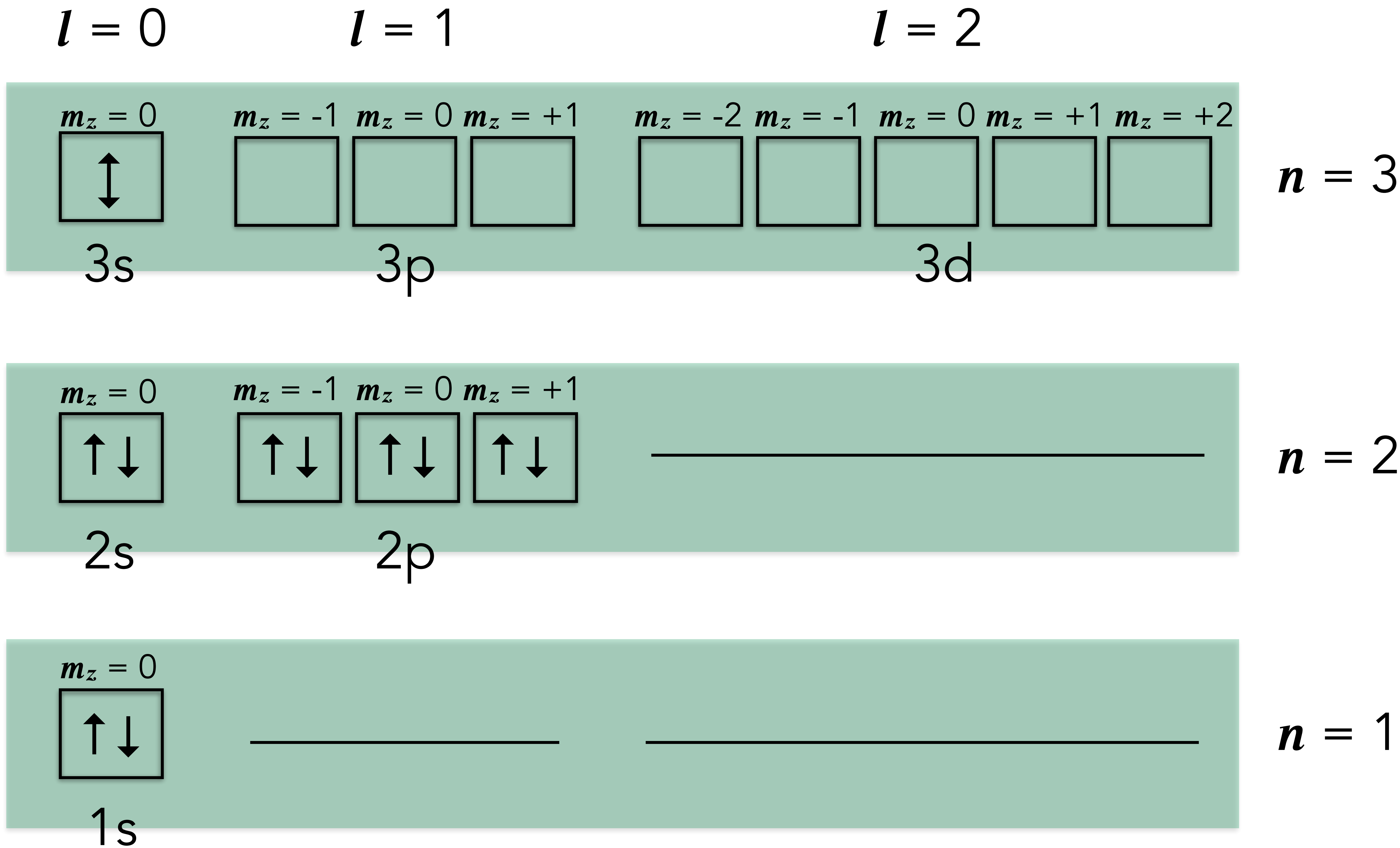


Lets build the ground state of Na: 11 electrons

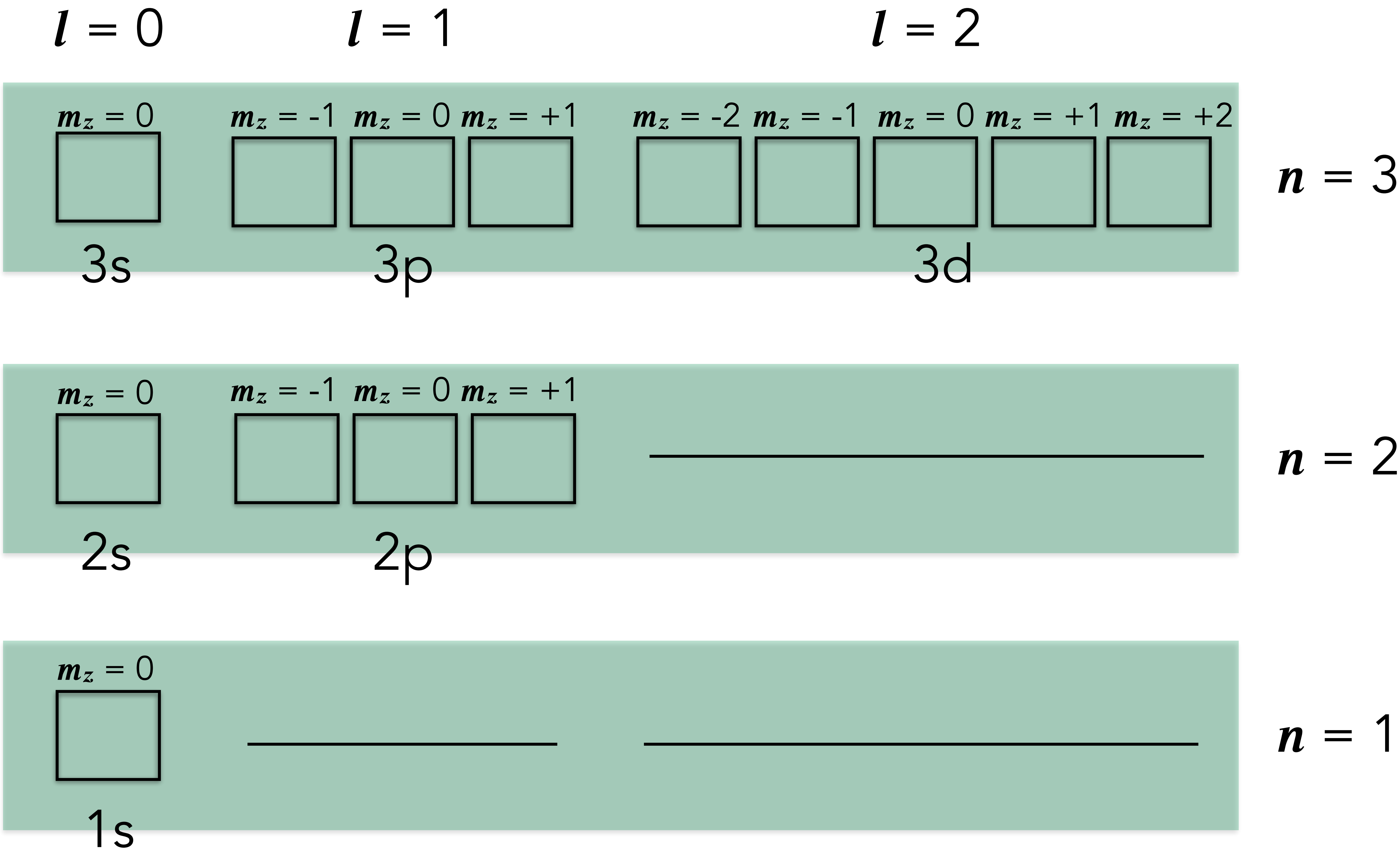


Lets build the ground state of Na: 11 electrons

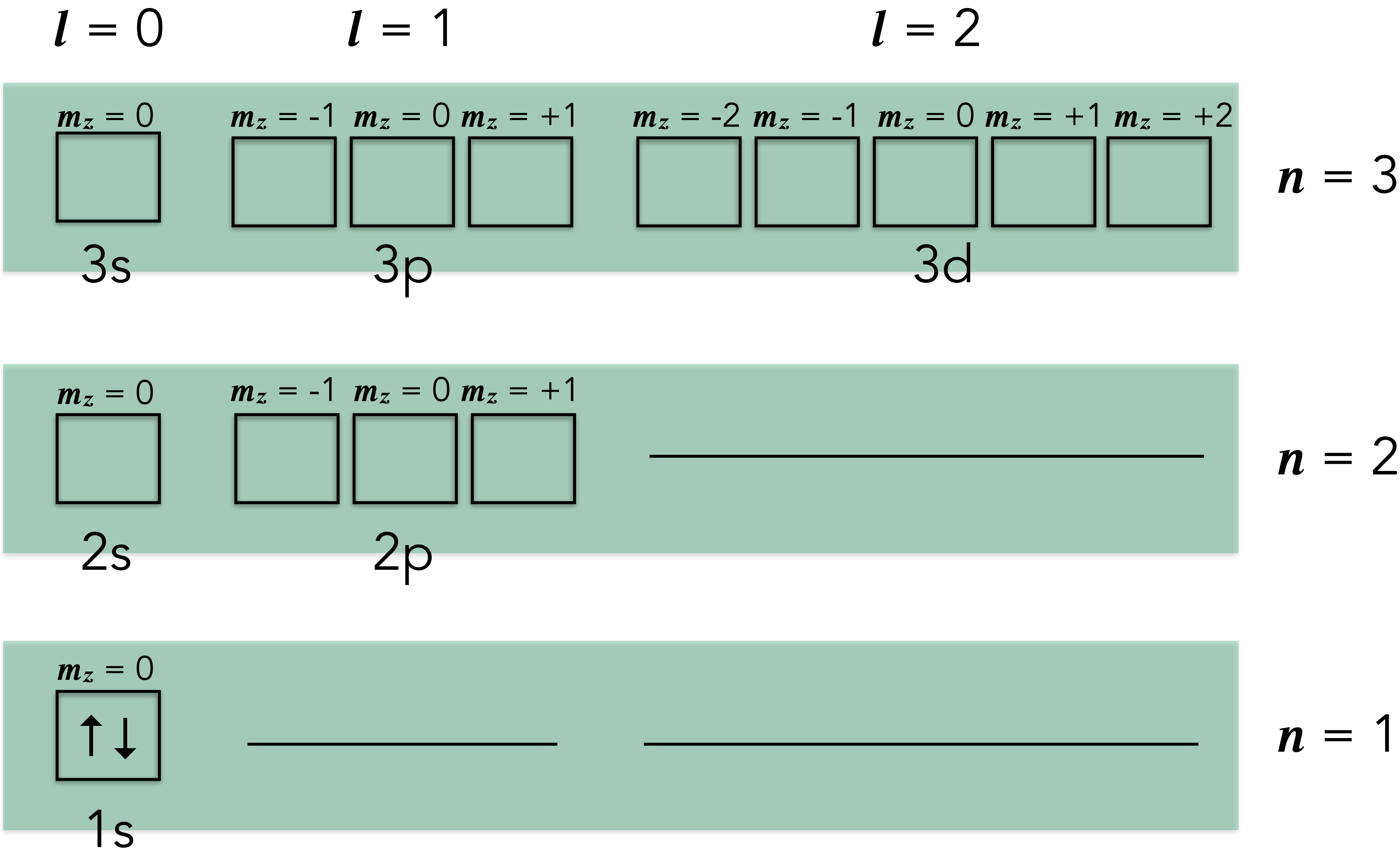


Lets build the ground state of Na: 11 electrons

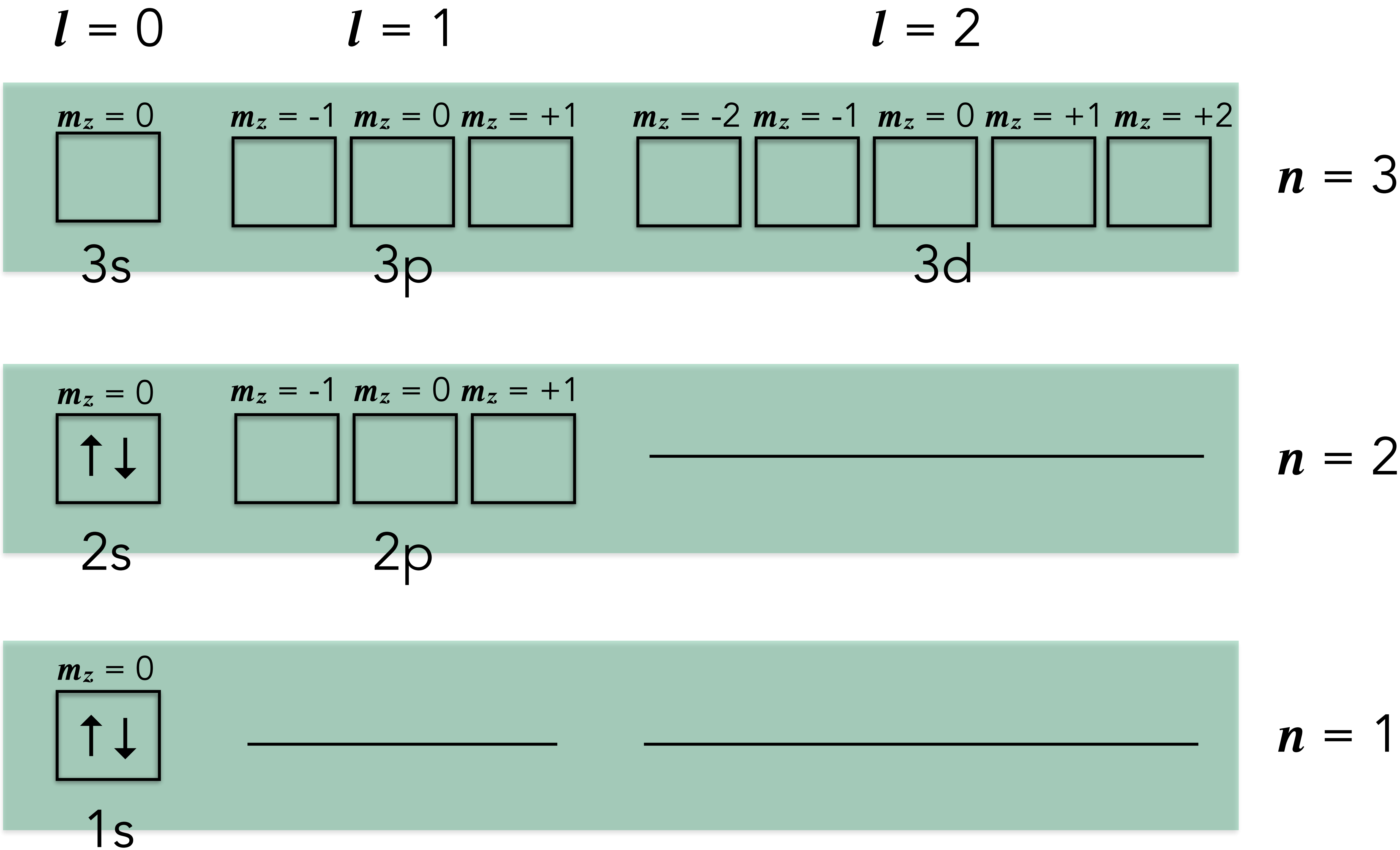
$\rightarrow 1s^2\ 2s^2\ 2p^6\ 3s$



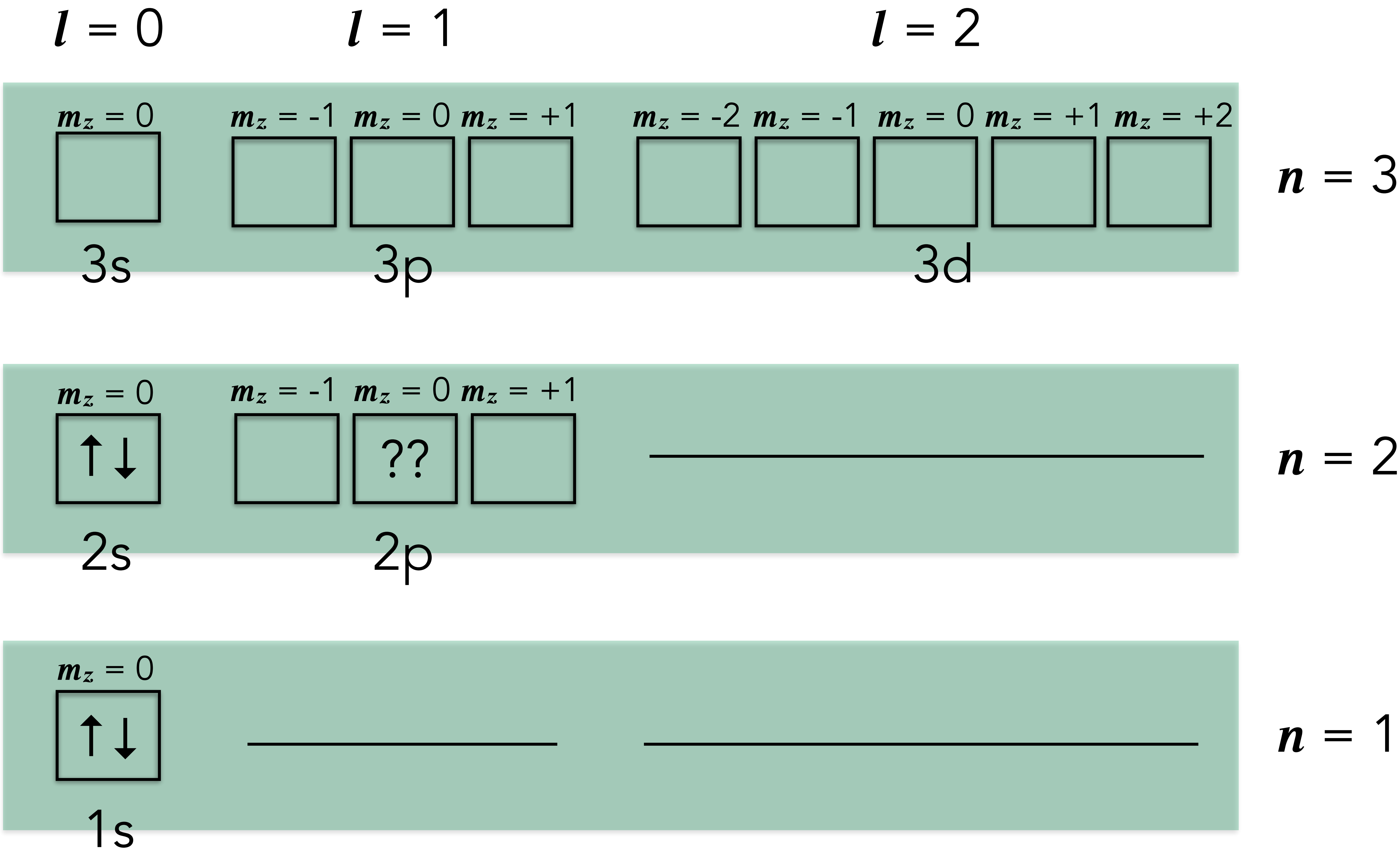
Lets build the ground state of C: 6 electrons



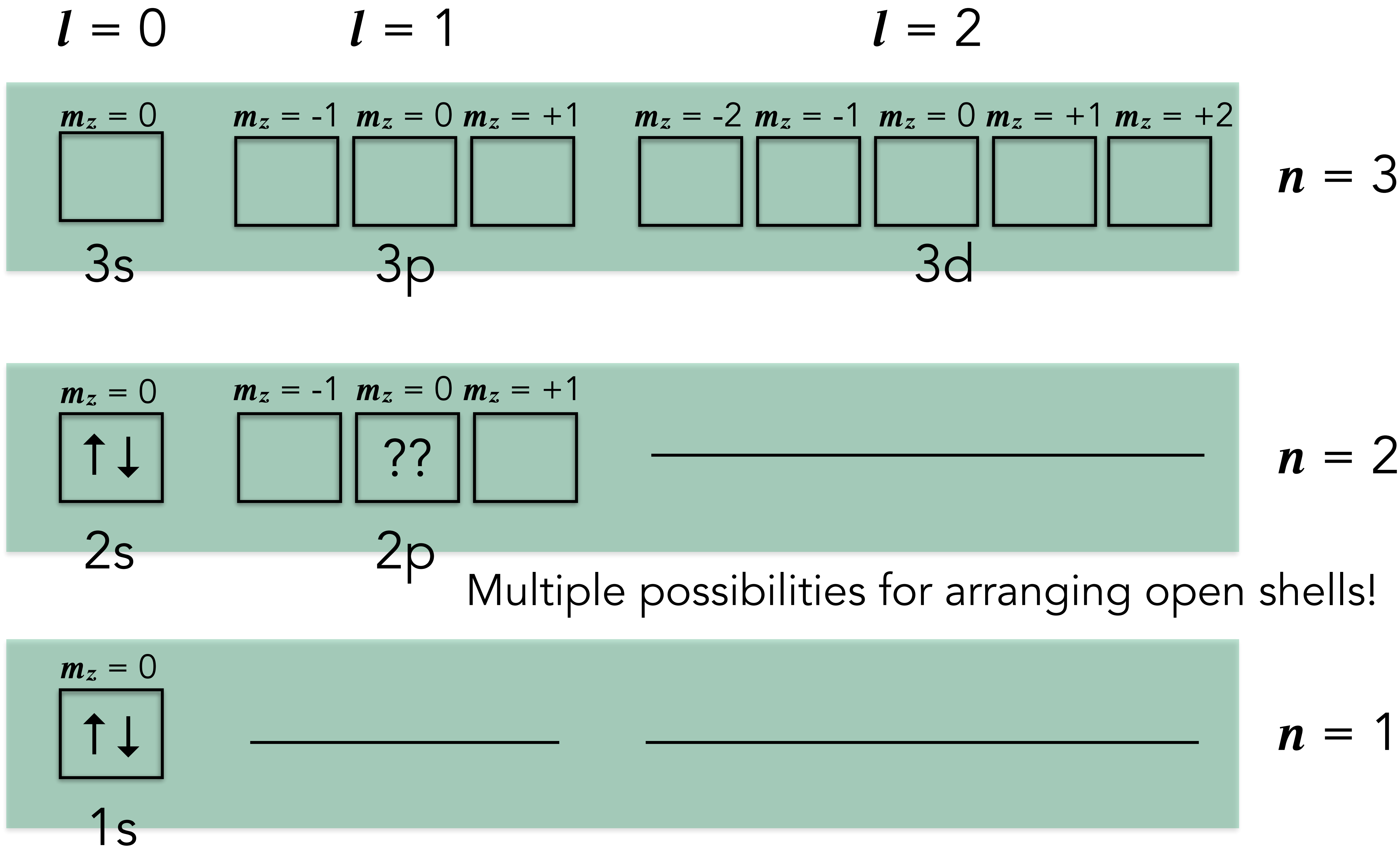
Lets build the ground state of C: 6 electrons



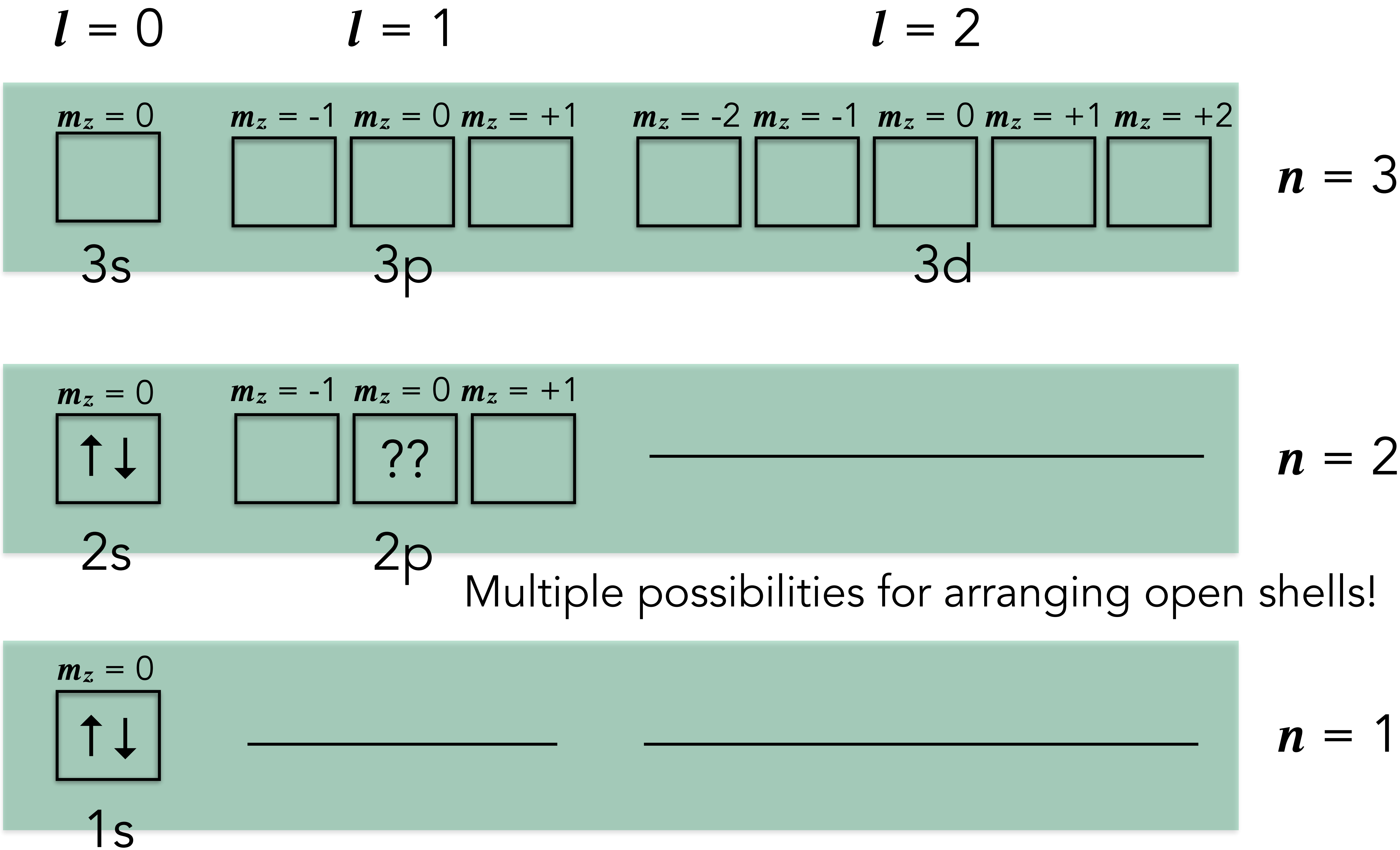
Lets build the ground state of C: 6 electrons



Lets build the ground state of C: 6 electrons



Lets build the ground state of C: 6 electrons



Multiple possibilities for arranging open shells!

Lets build the ground state of C: 6 electrons

$\rightarrow 1s^2\ 2s^2\ 2p^2$

Multiple possibilities for distributing e- in unfilled subshell, lead to different overall spin & angular momentum

$\mathbf{L}$ = vector sum of angular momentum

$\mathbf{S}$ = vector sum of spin angular momentum

Different combinations of $\mathbf{L}$ and $\mathbf{S}$ have different energies.

- *z* component of the total angular momentum can have values between -$\mathbf{L}$ and $\mathbf{L}$, i.e. 2($\mathbf{L}$+1) degenerate levels
- *z* component of the total spin can have values between -$\mathbf{S}$ and $\mathbf{S}$, i.e. 2($\mathbf{S}$+1) degenerate levels

Each $\mathbf{L}$ and $\mathbf{S}$ has (2$\mathbf{L}$+1)(2$\mathbf{S}$+1) possible m_z & spin combinations.

Spectroscopic Notation

The "Spectroscopic Term"

Spin $\mathbf{S}$

$$^{2\mathbf{S}+1}\mathcal{L}^{p}_{\mathbf{J}}$$

"parity" = $\Sigma \boldsymbol{l}_i$

blank if even, 1 if odd

for $1s^2\ 2s^2\ 2p^2$
parity = 0+0+0+0+1+1
= 2

Orbital Angular Momentum

S, P, D, F (for $\mathbf{L}$=0,1,2,3)

Total angular momentum

$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

Possible Terms for 2 electrons in p

Term (deg.)	L=0	L=1	L=2
S=0	^1S (1)	^1P (3)	^1D (5)
S=1	^3S (3)	^3P (9)	^3D (15)

$$^{2\mathbf{S}+1}\mathcal{L}$$

$$(2\mathbf{L}+1)(2\mathbf{S}+1)$$

= 36 total

Not all of these work - lets see why…

"Non-Equivalent" electrons (i.e. 2p3p, different $\boldsymbol{n}$)
36 combinations allowed:

(m_z, m_s)	(+1, +1/2)	(0, +1/2)	(-1, +1/2)	(+1, -1/2)	(0, -1/2)	(-1, -1/2)
(+1, +1/2)	+2, +1	+1, +1	0, +1	+2, 0	+1, 0	0, 0
(0, +1/2)	+1, +1	0, +1	-1, +1	+1, +1	0, 0	-1, 0
(-1, +1/2)	0, +1	-1, +1	-2, +1	0, 0	-1, 0	-2, 0
(+1, -1/2)	+2, 0	+1, 0	0, 0	+2, -1	+1, -1	0, -1
(0, -1/2)	+1, 0	0, 0	-1, 0	+1, -1	0, -1	-1, -1
(-1, -1/2)	0, 0	-1, 0	-2, 0	0, -1	-1, -1	-2, -1

"Equivalent" electrons (i.e. $2p^2$, same $\boldsymbol{n}$)

only 15 combinations allowed (b.c. Pauli & some are identical)

(m_z, m_s)	(+1, +1/2)	(0, +1/2)	(-1, +1/2)	(+1, -1/2)	(0, -1/2)	(-1, -1/2)
(+1, +1/2)	+2, +1	+1, +1	0, +1	+2, 0	+1, 0	0, 0
(0, +1/2)	+1, +1	0, +1	-1, +1	+1, +1	0, 0	-1, 0
(-1, +1/2)	0, +1	-1, +1	-2, +1	0, 0	-1, 0	-2, 0
(+1, -1/2)	+2, 0	+1, 0	0, 0	+2, -1	+1, -1	0, -1
(0, -1/2)	+1, 0	0, 0	-1, 0	+1, -1	0, -1	-1, -1
(-1, -1/2)	0, 0	-1, 0	-2, 0	0, -1	-1, -1	-2, -1

Possible Terms for 2 electrons in p

Term (deg.)	L=0	L=1	L=2
S=0	^1S (1)	^1P (3)	^1D (5)
S=1	^3S (3)	^3P (9)	^3D (15)

$$^{2\mathbf{S}+1}\mathcal{L}$$

$(2\mathbf{L}+1)(2\mathbf{S}+1)$

$= 36$ total

Only 15 states allowed, so some terms don't work when electrons are equivalent - which ones?

Possible Terms for 2 electrons in p

Term (deg.)	L=0	L=1	L=2
S=0	^1S (1)	^1P (3)	^1D (5)
S=1	^3S (3)	^3P (9)	^3D (15)

$$^{2S+1}\mathcal{L}$$

Possible Terms for 2 electrons in p

Term (deg.)	L=0	L=1	L=2
S=0	^1S (1)	^1P (3)	^1D (5)
S=1	^3S (3)	^3P (9)	~~^3D (15)~~

$$^{2S+1}\mathcal{L}$$

^3D has L = 2 (m_z = 2, 1, 0, -1, -2) and S=1 (m_s = 1, 0, -1)

$m_z = \pm 2$, $m_s = \pm 1$ not in the table so ^3D cannot be a valid term.

"Equivalent" electrons (i.e. $2p^2$, same $\boldsymbol{n}$)
only 15 combinations allowed (b.c. Pauli & some are identical)

(m_z, m_s)	(+1, +1/2)	(0, +1/2)	(-1, +1/2)	(+1, -1/2)	(0, -1/2)	(-1, -1/2)
(+1, +1/2)	+2, +1	+1, +1	0, +1	+2, 0	+1, 0	0, 0
(0, +1/2)	+1, +1	0, +1	-1, +1	+1, +1	0, 0	-1, 0
(-1, +1/2)	0, +1	-1, +1	-2, +1	0, 0	-1, 0	-2, 0
(+1, -1/2)	+2, 0	+1, 0	0, 0	+2, -1	+1, -1	0, -1
(0, -1/2)	+1, 0	0, 0	-1, 0	+1, -1	0, -1	-1, -1
(-1, -1/2)	0, 0	-1, 0	-2, 0	0, -1	-1, -1	-2, -1

Possible Terms for 2 electrons in p

Term (deg.)	L=0	L=1	L=2
S=0	^1S (1)	^1P (3)	^1D (5)
S=1	^3S (3)	^3P (9)	~~^3D (15)~~

$^{2S+1}\mathcal{L}$

= 21 remaining
still too many

^1S includes: $\boldsymbol{m_z} = 0$ and $\boldsymbol{m}_s = 0$

^3S includes: $\boldsymbol{m_z} = 0$ and $\boldsymbol{m}_s = +1, 0, -1$

^1P includes: $\boldsymbol{m_z} = +1, 0, -1$ and $\boldsymbol{m}_s = 0$

^3P includes: $\boldsymbol{m_z} = +1, 0, -1$ and $\boldsymbol{m}_s = +1, 0, -1$

^1D includes: $\boldsymbol{m_z} = +2, +1, 0, -1, -2$ and $\boldsymbol{m}_s = 0$

"Equivalent" electrons (i.e. $2p^2$, same $\boldsymbol{n}$)
only 15 combinations allowed (b.c. Pauli & symmetry)

(m_z, m_s)	**(+1, +1/2)**	**(0, +1/2)**	**(-1, +1/2)**	**(+1, -1/2)**	**(0, -1/2)**	**(-1, -1/2)**
(+1, +1/2)	+2, +1	+1, +1	0, +1	+2, 0	+1, 0	0, 0
(0, +1/2)	+1, +1	0, +1	-1, +1	+1, +1	0, 0	-1, 0
(-1, +1/2)	0, +1	-1, +1	-2, +1	0, 0	-1, 0	-2, 0
(+1, -1/2)	+2, 0	+1, 0	0, 0	+2, -1	+1, -1	0, -1
(0, -1/2)	+1, 0	0, 0	-1, 0	+1, -1	0, -1	-1, -1
(-1, -1/2)	0, 0	-1, 0	-2, 0	0, -1	-1, -1	-2, -1

Possible Terms for 2 electrons in p

Term (deg.)	L=0	L=1	L=2
S=0	^1S (1)	^1P (3)	^1D (5)
S=1	^3S (3)	^3P (9)	~~^3D (15)~~

$$^{2S+1}\mathcal{L}$$

= 21 remaining

^1S includes: $m_z = 0$ and $m_s = 0$

^3S includes: $m_z = 0$ and $m_s = +1, 0, -1$

^1P includes: $m_z = +1, 0, -1$ and $m_s = 0$

^3P includes: $m_z = +1, 0, -1$ and $m_s = +1, 0, -1$

$\boxed{^1D \text{ includes: } m_z = +2, +1, 0, -1, -2 \text{ and } m_s = 0}$

must have ^1D

"Equivalent" electrons (i.e. $2p^2$, same $\boldsymbol{n}$)
only 15 combinations allowed (b.c. Pauli & symmetry)

(m_z, m_s)	(+1, +1/2)	(0, +1/2)	(-1, +1/2)	(+1, -1/2)	(0, -1/2)	(-1, -1/2)
(+1, +1/2)	+2, +1	+1, +1	0, +1	+2, 0	+1, 0	0, 0
(0, +1/2)	+1, +1	0, +1	-1, +1	+1, +1	0, 0	-1, 0
(-1, +1/2)	0, +1	-1, +1	-2, +1	0, 0	-1, 0	-2, 0
(+1, -1/2)	+2, 0	+1, 0	0, 0	+2, -1	+1, -1	0, -1
(0, -1/2)	+1, 0	0, 0	-1, 0	+1, -1	0, -1	-1, -1
(-1, -1/2)	0, 0	-1, 0	-2, 0	0, -1	-1, -1	-2, -1

Possible Terms for 2 electrons in p

Term (deg.)	L=0	L=1	L=2
S=0	^1S (1)	^1P (3)	^1D (5)
S=1	^3S (3)	^3P (9)	$\cancel{^3D\ (15)}$

$^{2S+1}\mathcal{L}$

= 5 accounted for
need 10 more,
16 remaining

^1S includes: $m_z = 0$ and $m_s = 0$

^3S includes: $m_z = 0$ and $m_s = +1, 0, -1$

^1P includes: $m_z = +1, 0, -1$ and $m_s = 0$

^3P includes: $m_z = +1, 0, -1$ and $m_s = +1, 0, -1$

Possible Terms for 2 electrons in p

Term (deg.)	L=0	L=1	L=2
S=0	^1S (1)	^1P (3)	^1D (5)
S=1	^3S (3)	^3P (9)	$\cancel{^3D\ (15)}$

$^{2S+1}\mathcal{L}$

= 5 accounted for
need 10 more,
16 remaining

$\boxed{^1S \text{ includes: } m_z = 0 \text{ and } m_s = 0}$

^3S includes: $m_z = 0$ and $m_s = +1, 0, -1$

^1P includes: $m_z = +1, 0, -1$ and $m_s = 0$

$\boxed{^3P \text{ includes: } m_z = +1, 0, -1 \text{ and } m_s = +1, 0, -1}$

Only one way
to get 9 states

“Equivalent” electrons (i.e. $2p^2$, same $\boldsymbol{n}$)
only 15 combinations allowed (b.c. Pauli & symmetry)

(m_z, m_s)	(+1, +1/2)	(0, +1/2)	(-1, +1/2)	(+1, -1/2)	(0, -1/2)	(-1, -1/2)
(+1, +1/2)	+2, +1	+1, +1	0, +1	+2, 0	+1, 0	0, 0
(0, +1/2)	+1, +1	0, +1	-1, +1	+1, +1	0, 0	-1, 0
(-1, +1/2)	0, +1	-1, +1	-2, +1	0, 0	-1, 0	-2, 0
(+1, -1/2)	+2, 0	+1, 0	0, 0	+2, -1	+1, -1	0, -1
(0, -1/2)	+1, 0	0, 0	-1, 0	+1, -1	0, -1	-1, -1
(-1, -1/2)	0, 0	-1, 0	-2, 0	0, -1	-1, -1	-2, -1

It gets complicated & tedious to do this for more electrons or for excited states. Just look it up!

Table 7.2 Terms arising from some configurations of non-equivalent and equivalent electrons

Non-equivalent electrons		Equivalent electrons	
Configuration	Terms	Configuration	Terms[a]
s^1s^1	$^{1,3}S$	p^2	$^1S, ^3P, ^1D$
s^1p^1	$^{1,3}P$	p^3	$^4S, ^2P, ^2D$
s^1d^1	$^{1,3}D$	d^2	$^1S, ^3P, ^1D, ^3F, ^1G$
s^1f^1	$^{1,3}F$	d^3	$^2P, ^4P, ^2D(2), ^2F,$
p^1p^1	$^{1,3}S, ^{1,3}P, ^{1,3}D$		$^4F, ^2G, ^2H$
p^1d^1	$^{1,3}P, ^{1,3}D, ^{1,3}F$	d^4	$^1S(2), ^3P(2), ^1D(2),$
p^1f^1	$^{1,3}D, ^{1,3}F, ^{1,3}G$		$^3D, ^5D, ^1F, ^3F(2),$
d^1d^1	$^{1,3}S, ^{1,3}P, ^{1,3}D, ^{1,3}F, ^{1,3}G$		$^1G(2), ^3G, ^3H, ^1I$
d^1f^1	$^{1,3}P, ^{1,3}D, ^{1,3}F, ^{1,3}G, ^{1,3}H$	d^5	$^2S, ^6S, ^2P, ^4P, ^2D(3),$
f^1f^1	$^{1,3}S, ^{1,3}P, ^{1,3}D, ^{1,3}F, ^{1,3}G,$		$^4D, ^2F(2), ^4F, ^2G(2),$
	$^{1,3}H, ^{1,3}I$		$^4G, ^2H, ^2I$

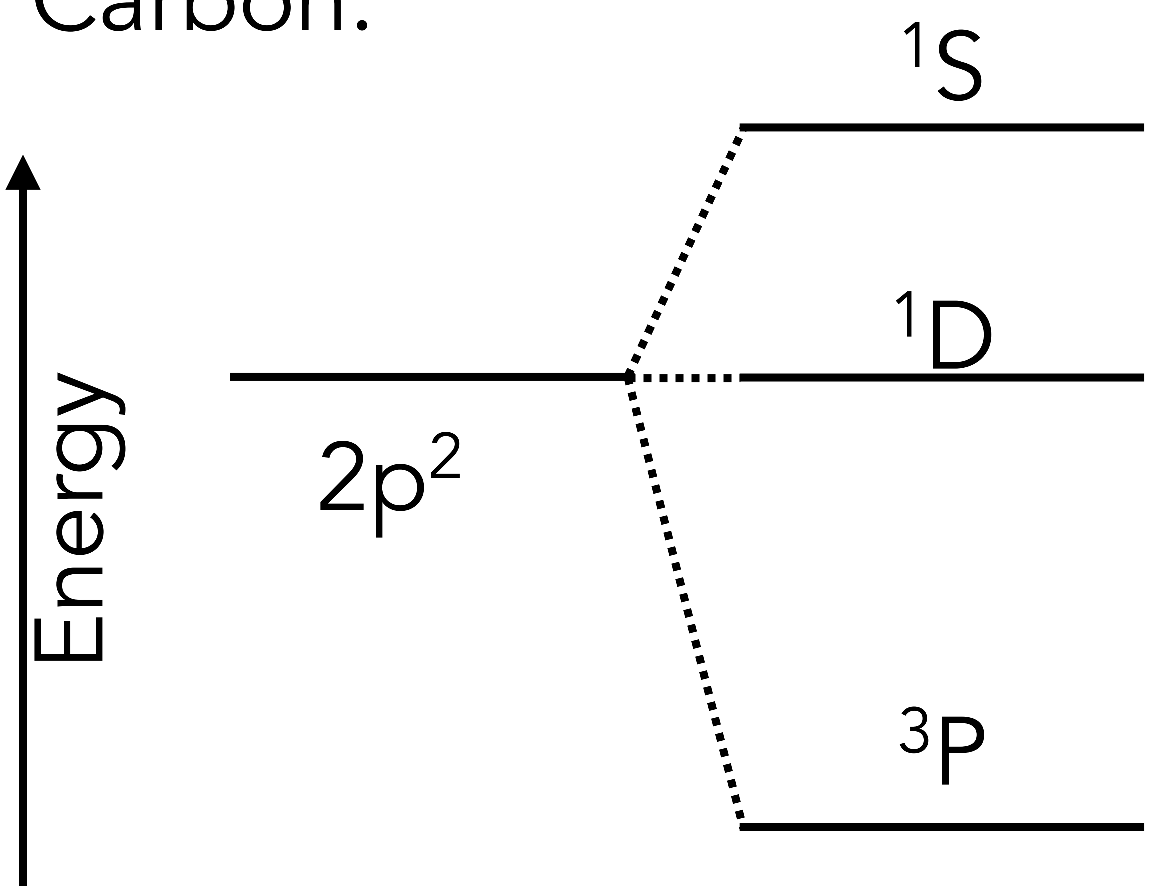
[a] The numbers in brackets indicate that a particular term occurs more than once.

from *Modern Spectroscopy* by Hollas

Energy Levels $\longleftrightarrow$ Terms

$$^{2\mathbf{S}+1}\mathcal{L}^{\,p}_{\mathbf{J}}$$

Carbon:



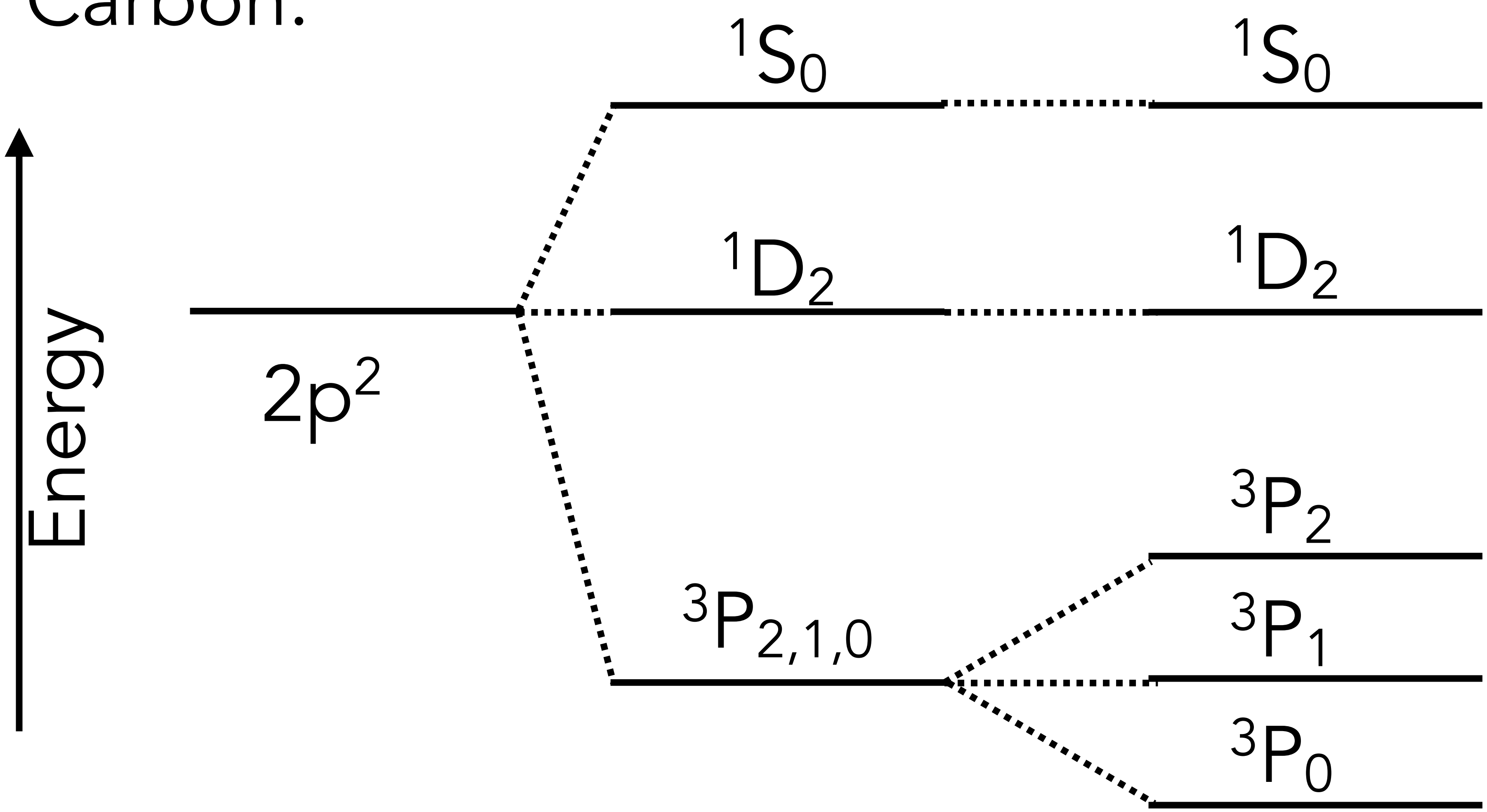
"Hund's Rules"

1) Terms w/larger spin generally have lower energy.
2) For terms with given configuration and spin, larger L has lower energy.
3) Higher J = higher energy if shell is less than half full (opposite otherwise).

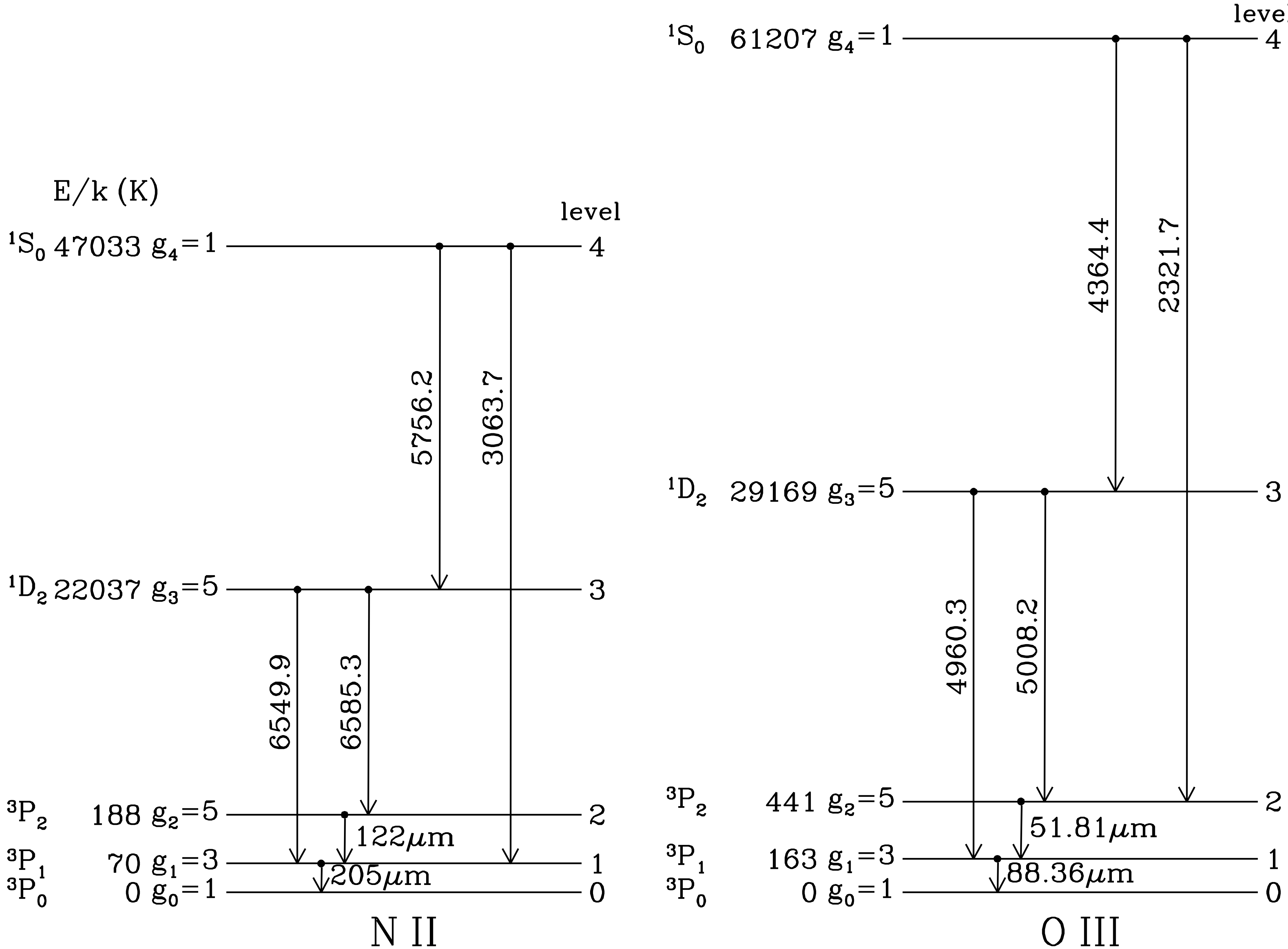
Energy Levels $\longleftrightarrow$ Terms

$$^{2\mathbf{S}+1}\mathcal{L}^{p}_{\mathbf{J}}$$

Carbon:



Other examples of np^2 ground state configurations



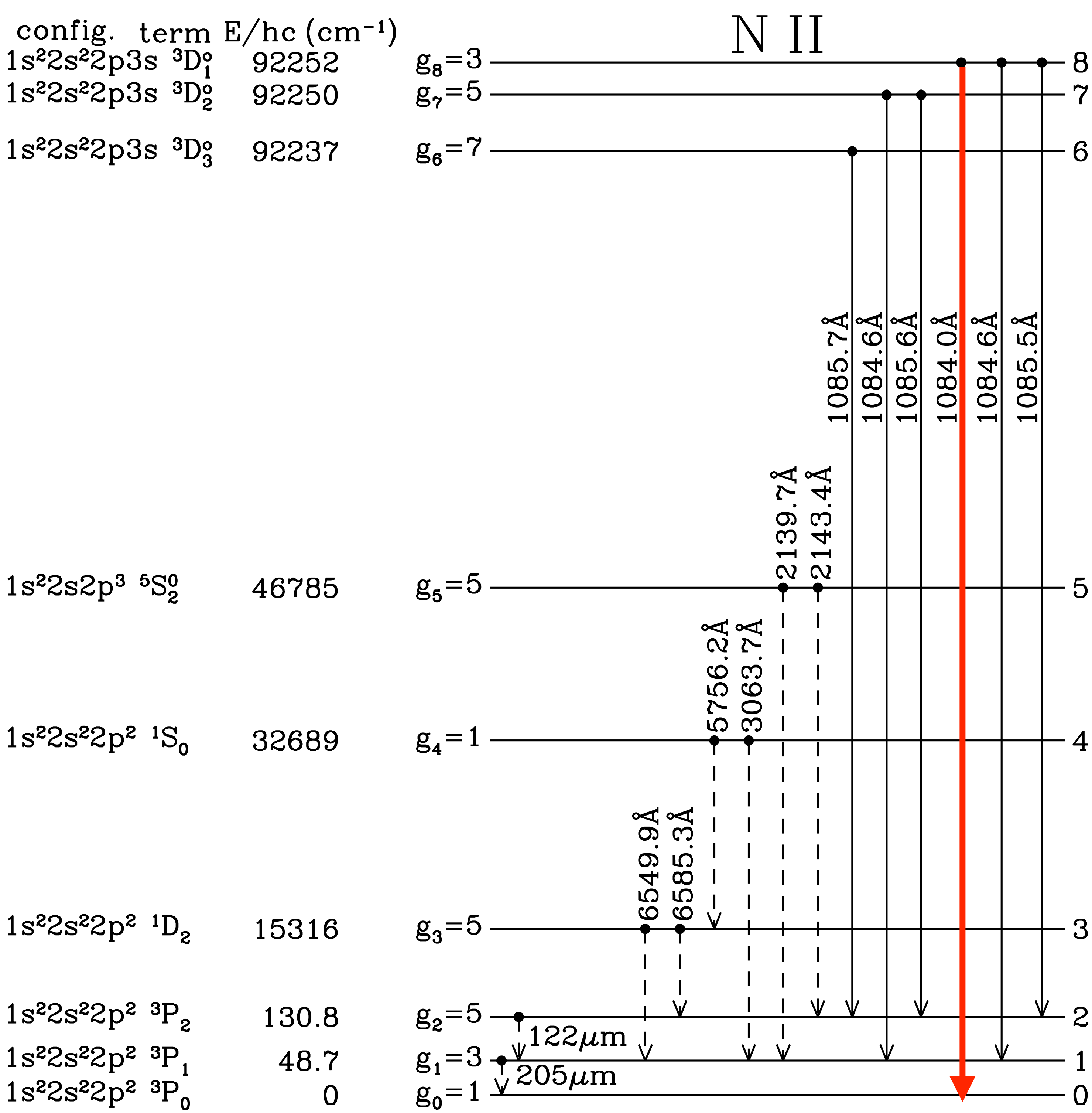
Selection Rules for Transitions

We can now figure out the energy levels, what about the transitions between them?

1) Parity must change
2) $\Delta J = 0, \pm 1$, but $J=0 \rightarrow 0$ is forbidden
3) $\Delta S = 0$
4) $\Delta L = 0, \pm 1$, but $L=0 \rightarrow 0$ is forbidden
5) if one e- then $\Delta \boldsymbol{l} = 0$

<u>All rules satisfied: "allowed" electric dipole transition</u>

NII 1084.0 Å $^3P_0$ - $^3D_1^\circ$



✓1) Parity must change

✓2) $\Delta J = 0, \pm 1$, but $J=0 \rightarrow 0$ is forbidden

✓3) $\Delta S = 0$

✓4) $\Delta L = 0, \pm 1$, but $L=0 \rightarrow 0$ is forbidden

✓5) if one e- then $\Delta \boldsymbol{l} = 0$

$$A_{ul} = 2.18 \times 10^8 \ s^{-1}$$

$$1/A_{ul} = 4.6 \text{ ns}$$

Selection Rules for Transitions

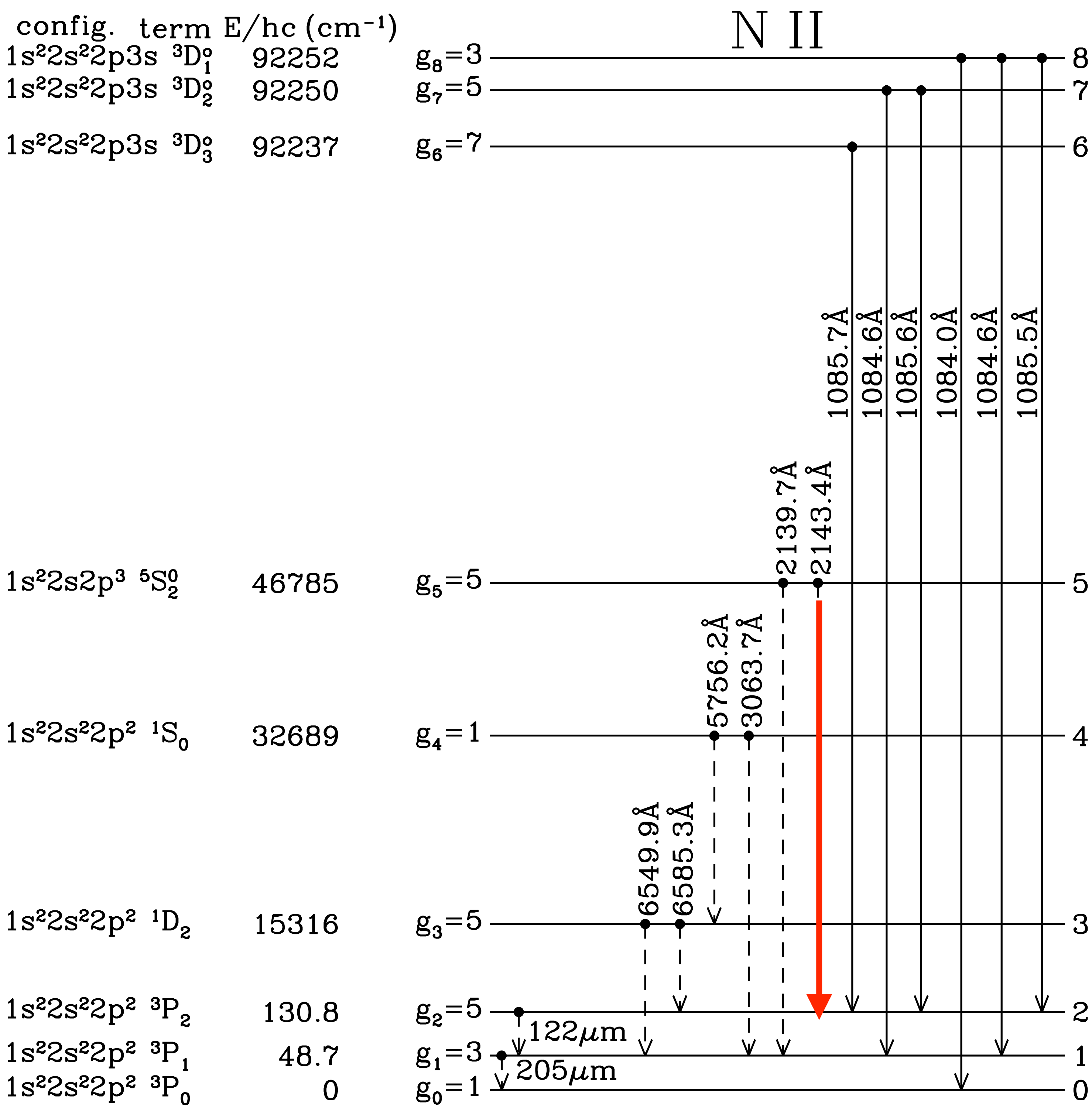
We can now figure out the energy levels, what about the transitions between them?

1) Parity must change
2) $\Delta J = 0, \pm 1$, but $J=0 \rightarrow 0$ is forbidden
3) $\Delta S = 0$
4) $\Delta L = 0, \pm 1$, but $L=0 \rightarrow 0$ is forbidden
5) if one e- then $\Delta \boldsymbol{l} = 0$

<u>All rules except $\Delta S = 0$: "semi-forbidden" or "intercombination" or "intersystem" electric dipole transition</u>

N II] 2143.4 Å $^5S_2^o$ - $^3P_2$

single bracket for "semi-forbidden"



1) Parity must change

2) $\Delta J = 0, \pm 1$, but $J=0 \rightarrow 0$ is forbidden

3) $\Delta S = 0$

4) $\Delta L = 0, \pm 1$, but $L=0 \rightarrow 0$ is forbidden

5) if one e- then $\Delta \boldsymbol{l} = 0$

$$A_{ul} = 1.27 \times 10^2 \ s^{-1}$$

$$1/A_{ul} = 7.9 \ ms$$

Selection Rules for Transitions

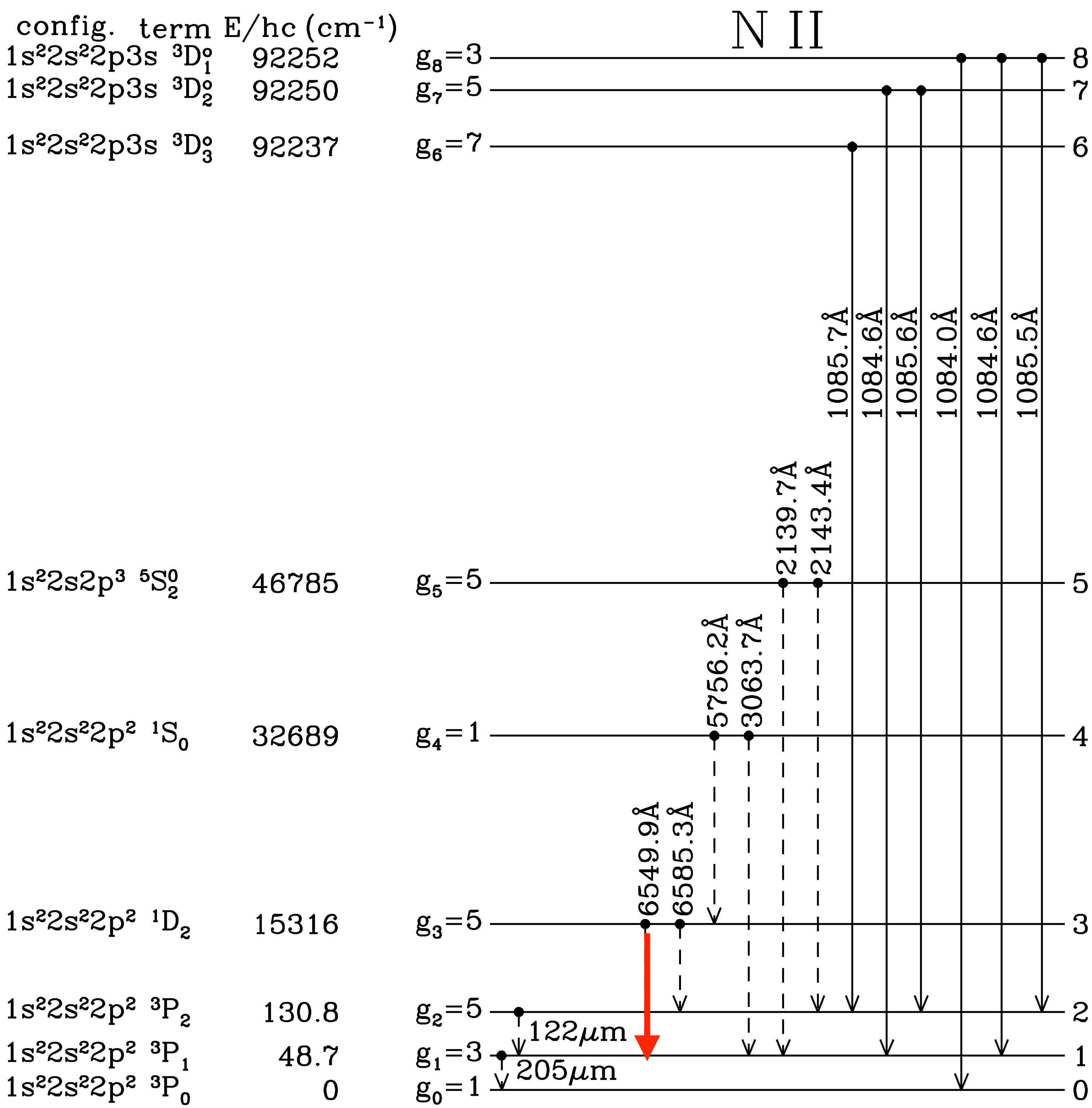
We can now figure out the energy levels, what about the transitions between them?

1) Parity must change
2) $\Delta J = 0, \pm 1$, but $J=0 \rightarrow 0$ is forbidden
3) $\Delta S = 0$
4) $\Delta L = 0, \pm 1$, but $L=0 \rightarrow 0$ is forbidden
5) if one e- then $\Delta \boldsymbol{l} = 0$

<u>More rules broken: "forbidden" transition, either magnetic dipole or electric quadrupole usually</u>

[N II] 6549.9 Å $^1D_2$ - $^3P_1$

double bracket for "forbidden"



1) Parity must change

2) $\Delta J = 0, \pm 1$, but $J=0 \rightarrow 0$ is forbidden

3) $\Delta S = 0$

4) $\Delta L = 0, \pm 1$, but $L=0 \rightarrow 0$ is forbidden

5) if one e- then $\Delta l = 0$

$$A_{ul} = 9.2 \times 10^{-4}\ s^{-1}$$

$$1/A_{ul} \sim 20 \text{ min}$$

<u>Forbidden transitions are very important in astronomy!</u>

Collisions populate the levels of the ground state

There is a low probability for transitions
so the line is generally optically thin

When there is a radiative transition, that energy
escapes! Very important for cooling!