RNN implementation

Problem: (Optimization problems)

↓

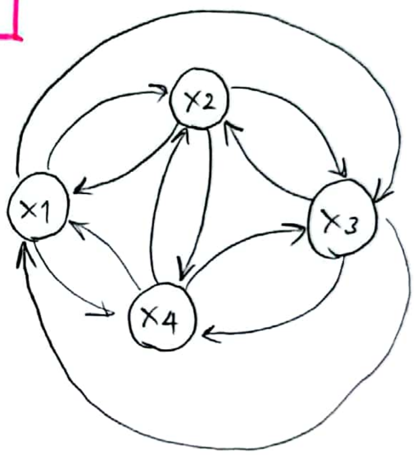
Create Energy func

↓

Compare created Energy func vs Std. Energy ⇒ Compute weights (ω_{ij}) and threshold (θ_i), C

↓

Create RNN



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Update the state of RNN

A). Using deterministic binary model

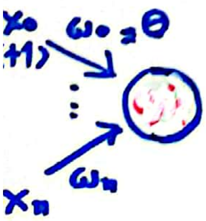
- Assign initial values to each neuron (x_i)
- Update one-by-one $s_i = \sum_{n=0}^{N} \omega_n x_n \Rightarrow \begin{cases} 0, & s_i < 0 \\ 1, & s_i \geq 0 \end{cases}$
- Repeat updating & check if you can find sol$^{\underline{n}}$.
- Change initial values and try again.

B). Probabilistic binary model

- Assign initial values to each neuron (x_i).
- Set α (gain) in sigmoid function (Try multiple values)
- Repeat updating neurons one-by-one

$$s_i = \sum_{n=0}^{N} \omega_n x_n \Rightarrow p_i = \text{sigmoid}(s_i) \begin{cases} \nearrow 0 \text{ with } 1-p_i \\ \searrow 1 \text{ with } p \end{cases}$$

(rand() < RAND_MAX * p)



- Count the number of occurances for each state. (The highest occur. of states = Sol$^{\underline{n}}$)

1). Gibb copies (Multiple RNN) 2). Ergodic theory (1 RNN)

2024. Jan. 10th

Application of RNN

So far we studied the operation (mechanism) of RNN by a very simple example of "the winner takes all"

$x_1 \rightleftarrows x_2 \rightleftarrows x_3$, $x_1 \leftrightarrow x_3$

Energy $E(x_1, x_2, x_3)$	x_1	x_2	x_3	
5	1	1	0	← Give initial values.
3	0	1	0	← Update x_1
3	0	0	0	← Update x_2
2	0	0	1	← Update x_3
1	1	0	1	← Update x_1
0	1	0	1	← Update x_2
⋮		⋮		

↓ Energy Decreases

Decrease

Update the state of the neuron one by one. (not at a time)

After updating states many times. it converges to the constant state

Deterministic model

In case of the Probabilistic Model, the energy sometimes increases.

In case of Probabilistic Model. The state of neurons with a lower energy appears more frequently than the state with a higer energy.

We can use the RNN to find the lowest energy state.

x_1	x_2	x_3	
0	1	1	← Initial values
1	1	1	← Update x_1
1	0	1	← x_2
1	0	0	← x_3
⋮	⋮	⋮	⋮

Repeat Updating 100 times.

100 states

The lowest energy state occurs most frequently

Find the maximum, and the state with the maximum occulance gives the minimum energy state.

Among 100 states

2×2×2 = 8 states

(x_1, x_2, x_3)			Number of Occurrance
0	0	0	21
0	0	1	17
0	1	0	⋮
0	1	1	⋮
⋮			

Total is 100

In case of the deterministic model, you can find a lower ~~en~~ energy state, by repeating updating many times and finding the converged state.

But, the converged state might be a local minimum.

The Deterministic Model might fail to find the global minimum, it might be caught by a local minimum.

RNN can be applied to optimization problems.

The optimization problems usually uses a ~~[illegible]~~ cost function and find the best combination of parameter values that minimize the cost function.

state of RNN

The energy function of RNN.

By using RNN, we can find the lowest energy state.

can be the solution of the optimization problem.

Application example : Finding a solution for a simultaneous equation. (A system of equations)

$$
\begin{cases}
x_1 - 2x_2 + x_3 - x_4 = 1 \\
-x_1 - x_2 + x_3 + 2x_4 = 2 \\
2x_1 + x_2 - x_3 + x_4 = 2 \\
-2x_1 + 3x_2 - 2x_3 - x_4 = -5
\end{cases}
$$

$$
\underbrace{x_1, x_2, x_3, x_4}_{\text{Binary variables}} = 0 \text{ or } 1
$$

Solution $x_1 = 1,\ x_2 = 0,\ x_3 = 1,\ x_4 = 1$

We use a RNN to find this solution.

What should we do at first?

We have to make a cost function at first, so that the minimum of the cost function can give the solution of the simultaneous equation.

$$E(x_1, x_2, x_3, x_3) = \boxed{(x_1 - 2x_2 + x_3 - x_4 - 1)^2}$$
$$+ (-x_1 - x_2 + x_3 + 2x_4 - 2)^2$$
$$+ (2x_1 + x_2 - x_3 + x_4 - 2)^2$$
$$+ (-2x_1 + 3x_2 - 2x_3 - x_4 + 5)^2$$

This part takes 0 as its minimum. It becomes 0 when the first equation is satisfied.

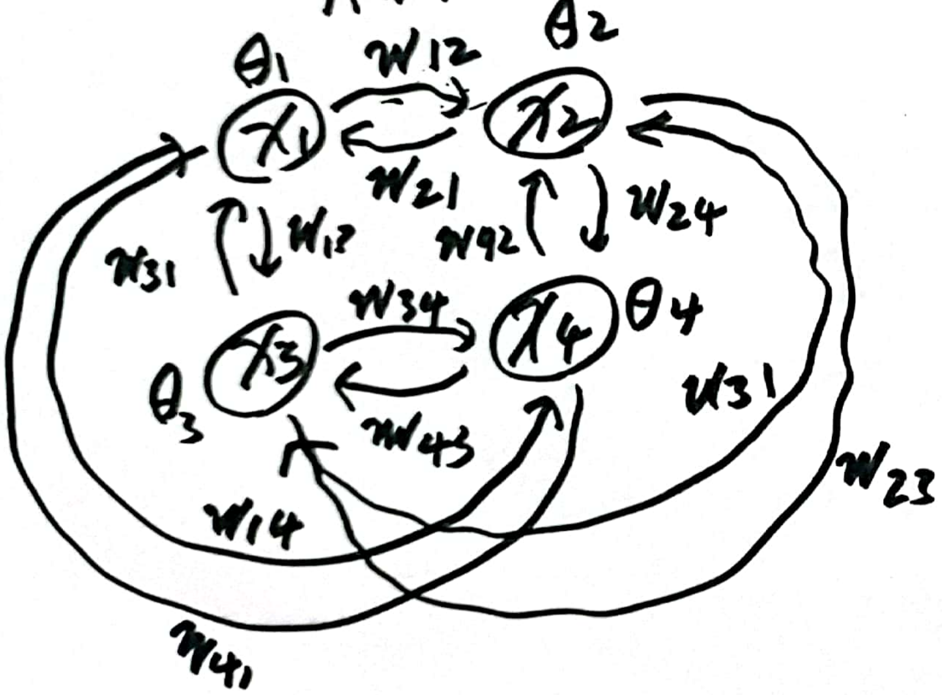
This cost function is also the energy function of RNN.

By reducing this cost (energy) function and finding its minimum (0), we can solve the ~~#~~ simultaneous equation.

The second thing we should do is to transform this cost (energy) function to the standard form of the energy function.

The standard form of the energy function.

RNN



The standard form of the energy function

$$E(x_1, x_2, x_3, x_4) = -\frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} w_{ij} x_i x_j + \sum_{i=1}^{4} \theta_i x_i$$

$w_{ij} = w_{ji}$ Symmetric condition

$w_{ii} = 0$ No self connection.

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Transform into the standard form

$$E(x_1, x_2, x_3, x_4) = \underbrace{x_1^2}_{=x_1} + 4\overbrace{x_2^2}^{=x_2} + \overbrace{x_3^2}^{=x_3} + \overbrace{x_4^2}^{=x_4} + 1$$

$$-4x_1x_2 + 2x_1x_3 - 2x_1x_4 - 2x_1$$

$$-4x_2x_3 + 4x_2x_4 + 4x_2$$

$$-2x_3x_4 - 2x_3$$

$$+ 2x_4$$

x_i : binary

$$x_i = \begin{cases} 0 & x_i^2 = 0 \\ 1 & x_i^2 = 1 \end{cases}$$

$$x_i^2 = x_i$$

$$=$$

$$+ \underbrace{x_1^2}_{=x_1} + \underbrace{x_2^2}_{=x_2} + \overbrace{x_3^2}^{=x_3} + 4\overbrace{x_4^2}^{=x_4} + 4$$

$$+2x_1x_2 - 2x_1x_3 - 4x_1x_4 + 4x_1$$

$$-2x_2x_3 - 4x_2x_4 + 4x_2$$

$$+ 4x_3x_4 - 4x_3$$

$$-8x_4$$

$+ 4\overset{=x_1}{x_1^2} + \overset{=x_2}{x_2^2} + \overset{=x_3}{x_3^2} + \overset{=x_4}{x_4^2} + 4$

$+ 4x_1x_2 - 4x_1x_3 + 4x_1x_4 - 8x_1$

$- 2x_2x_3 + 2x_2x_4 - 4x_2$

$- 2x_3x_4 + 4x_3$

$- 4x_4$

$+ 4\overset{=x_1}{x_1^2} + 9\overset{=x_2}{x_2^2} + 4\overset{=x_3}{x_3^2} + \overset{=x_4}{x_4^2} + 25$

$- 12x_1x_2 + 8x_1x_3 + 4x_1x_4 - 20x_1$

$- 12x_2x_3 - 6x_2x_4 + 30x_2$

$- \cancel{6x_3x_4 + 30x_2}$

$- 4x_3x_4 - 20x_3$

$- 10x_4$

Standard Form

$$E(x_1, x_2, x_3, x_4) = -\frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} w_{ij} x_i x_j + \sum_{i=1}^{4}\theta_i x_i \underline{\underline{+\ C}}$$

$w_{12} x_1 x_2 + w_{21} x_2 x_1$ symmetric $= 2 w_{12} x_1 x_2$

constant number It does not affect the minima as it is constant

$$= -\left(w_{12} x_1 x_2 + w_{13} x_1 x_3 + w_{14} x_1 x_4 + w_{23} x_2 x_3 + w_{24} x_2 x_4 + w_{34} x_3 x_4\right)$$

$$+ \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4 + C$$

$\times \frac{1}{2}$

$10 \begin{cases} 4 \\ -2 \\ -4 \\ 12 \end{cases}$

$\underline{\underline{w_{12} = 10}}$

$4 \begin{cases} 2 \\ 4 \\ 2 \\ 4 \end{cases}$

$w_{34} = 4$

$-4 \begin{cases} -2 \\ 2 \\ 4 \\ -8 \end{cases}$

$\underline{\underline{w_{13} = -4}}$

$-2 \begin{cases} 2 \\ 4 \\ -4 \\ -4 \end{cases}$

$\underline{\underline{w_{14} = -2}}$

$4 \begin{cases} 4 \\ 2 \\ 2 \\ 12 \end{cases}$

$w_{23} = 20$

$4 \begin{cases} -4 \\ 4 \\ -2 \\ 6 \end{cases}$

$w_{24} = 4$

$34 \begin{cases} -2 \\ 4 \\ -8\ 4 \\ -20\ 4 \end{cases}$

$\theta_1 = -34$

$\begin{cases} 44 \\ 4 \\ -4 \\ 30\ 20 \end{cases}$ 49

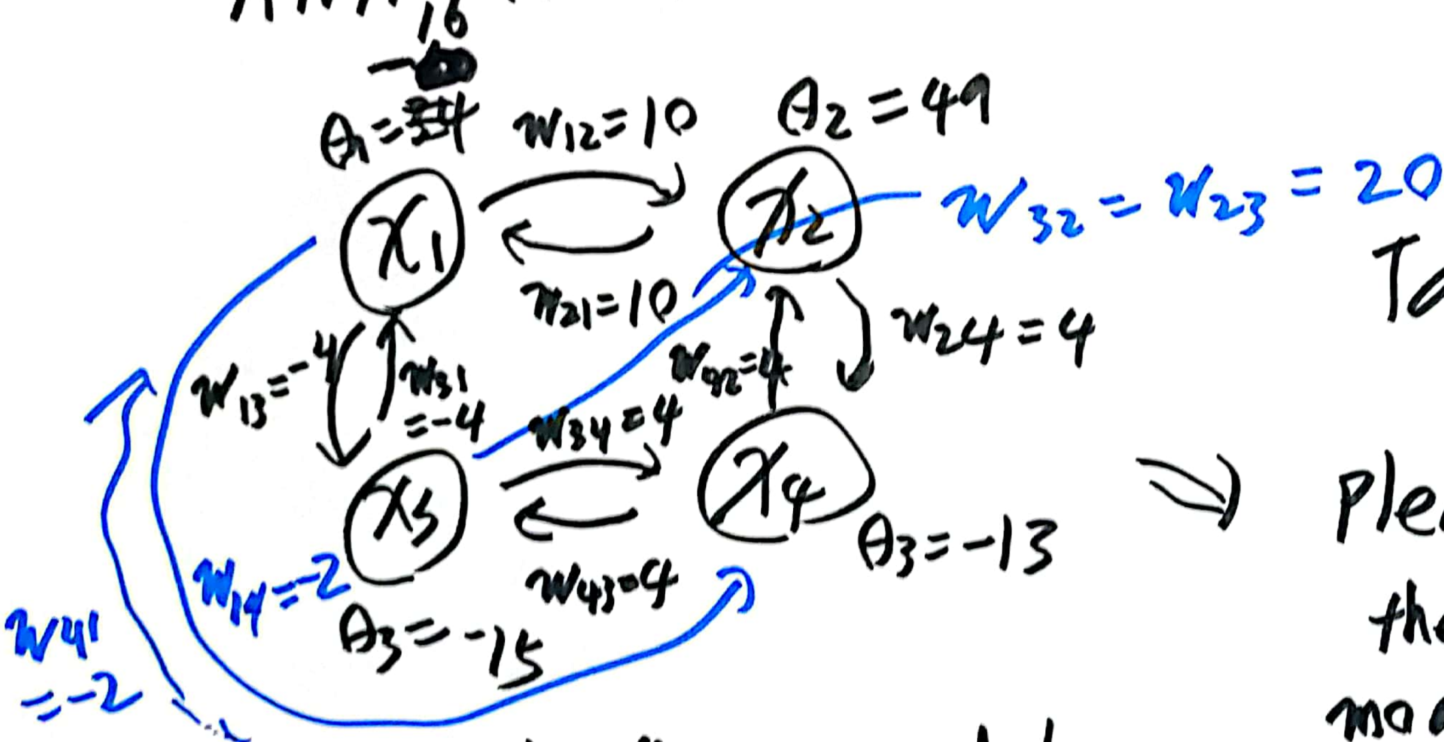
$\theta_2 = 49$

θ_3 $-15 \begin{cases} -2 \\ -4 \\ 4 \\ -20\ 4 \end{cases}$

$\begin{cases} 2 \\ -8\ 4 \\ -4 \\ -10 \end{cases}$ -13

$\theta_4 = -13$

RNN to solve the simultaneous equation.



Task 3

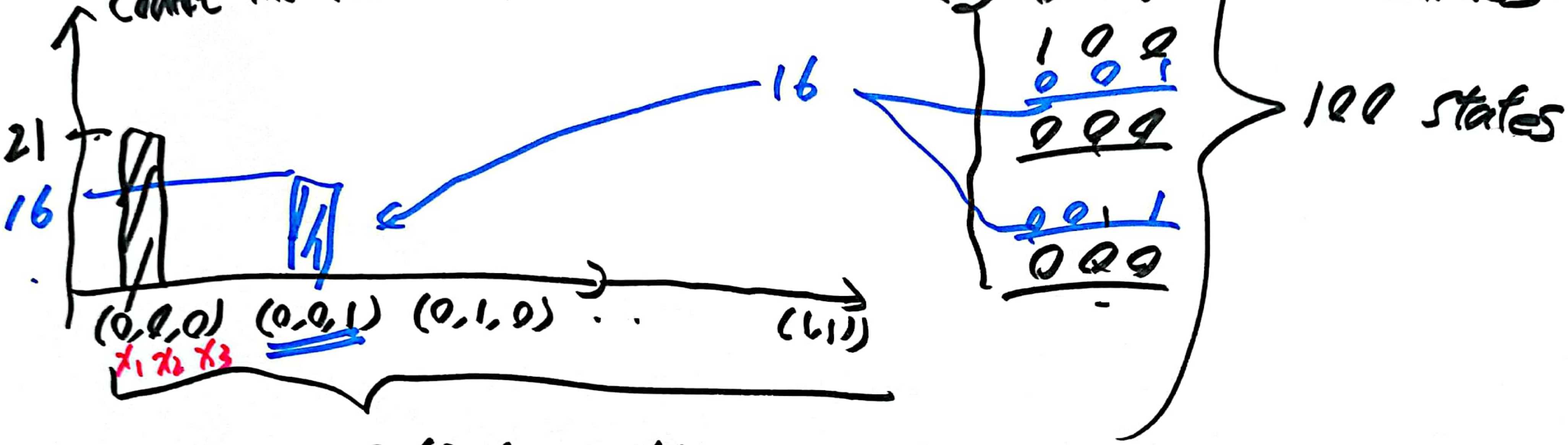
$\Rightarrow$ Please use the deterministic model, confirm the decrease of energy. if Check the convergent state is a solution or it might be a local minimum.

apply probablistic model, and check if the most frequent state is the solution.

You find

Histogram for the probablistic

Count the occurrance



$2 \times 2 \times 2 = \cancel{\#} 8$