DIRICHLET'S CONDITIONS : (*)

→ $f(x)$ is defined and single valued except possibly at a finite number of points in the interval $(c, c+2l)$.

→ $f(x)$ is periodic in the interval $(c, c+2l)$

→ $f(x)$ and $f'(x)$ are piecewise continuous in the interval $(c, c+2l)$

→

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FOURIER SERIES

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

FOURIER COEFFICIENTS :

$$a_0 = \frac{1}{l}\int_{c}^{c+2l} f(x) \cdot dx$$

$$a_n = \frac{1}{l}\int_{c}^{c+2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{1}{l}\int_{c}^{c+2l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx.$$

$-a$ to $+a$ is also equal to $2l$

0 to $2l$ is also equal to $2l$.

$f(x) = x^2$

$f(-x) = x^2 = f(x)$

$f(-x) = f(x)$

Even fnc $\boxed{b_n = 0}$

$f(x) = x^3$

$f(-x) = -x^3$

Odd fnc $\boxed{a_0 = a_n = 0}$

Evn fnc : $f(x) = \cos x$ $f(-x) = f(x)$

Odd fnc : $f(x) = \sin x, \tan x$, ~~an~~ an zero.

$f(-x) = -f(x)$

Bernoulli's equation.

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 + \cdots$$

Integration by Parts.

$$\int u\,dv = uv - \int v\,du.$$

HALF RANGE [0 to l]

$$f(x) = \begin{cases} x & 0 < x < l \\ & l < x < 2l \end{cases}$$

$0, 2l$ → end points

l → discontinuous points.

$f(x)$ lies in the interval $0 < x < 2l$ such that

$0 < a < 2l$ then $x = a$ the continuous point

$\{f(x)\}$ at $x = a$ continuous point $= f(a)$.

$f(x)$ lies in the interval $a < x < b$ then for $x = a$ the

end point

$$\{f(x)\} \text{ at } x = a \text{ end point} = \frac{\{f(a) + f(b)\}}{2}$$

$$f(x) = \begin{cases} g(x)f \text{ or } a < x < b \\ h(x)f \text{ or } b < x < c \end{cases}$$

$$\{f(x)\} \text{ at } x = b \text{ discontinuous point} = \frac{\{g(b) + h(b)\}}{2}$$

Find the fourier series of period $2l$ for the function $f(x) = x(2l-x)$ in $(0,2l)$. Deduce the sum of

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

$$f(x) = x(2l-x) \qquad 0 \le x \le 2l$$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx$$

$$= \frac{1}{l}\int_0^{2l} x(2l-x)\,dx$$

$$= \frac{1}{l}\int_0^{2l} (2xl - x^2)\,dx$$

$$= \frac{1}{l}\left[2l\cdot\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{2l}$$

$$= \frac{1}{l}\left[l\cdot x^2 - \frac{x^3}{3}\right]_0^{2l}$$

$$= \frac{1}{l}\left[l\cdot 4l^2 - \frac{8l^3}{3} - 0\right]$$

$$= \frac{1}{l}\left[4l^3 - \frac{8l^3}{3}\right]$$

$$= \frac{1}{l}\left[\frac{12l^3 - 8l^3}{3}\right]$$

$$= \frac{4l^3}{3l}$$

$$a_0 = \frac{4l^2}{3}$$

$$u = x(2l-x)$$
$$u = 2xl - x^2$$
$$u' = 2l - 2x$$
$$u'' = -2$$
$$u''' = 0$$
$$v = \cos\left(\frac{n\pi x}{l}\right)$$
$$v_1 = \frac{\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}$$
$$v_2 = \frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}$$
$$v_3 = \frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}$$

$$a_n = \frac{1}{l}\int_0^{2l} x(2l-x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{1}{l}\left[\frac{(2xl - x^2)\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)} + \frac{(2l-2x)\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} + \frac{2\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right]_0^{2l}$$

$$= \frac{1}{l}\left[\frac{-2}{n^2\pi^2}\times l^3 - \frac{2l^3}{n^2\pi^2}\right]$$

$$= -\frac{4l^2}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_0^{2l} x(2l-x)\sin\left(\frac{n\pi x}{l}\right)dx.$$

$$= \frac{1}{l}\left[-(2lx-x^2)\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)} + \frac{(2l-2x)\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} - \frac{2\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right]_0^{2l}$$

$$= \frac{1}{l}\left[\frac{-2}{\left(\frac{n\pi}{l}\right)^3} + \frac{2}{\left(\frac{n\pi}{l}\right)^3}\right]$$

$$= 0.$$

$\sin 0 = 0$

$\sin \pi/2 = \cancel{4}\, +1$

$\sin \pi = 0$

$\sin(n\pi) = 0$

$\sin(2n\pi) = 0$

$\cos 0 = 1$

$\cos \pi/2 = 0$

$\cos \pi = -1$

$\cos(n\pi) = (-1)^n$

$\cos(2n\pi) = 1$

$u = 2lx - x^2$

$u' = 2l - 2x$

$u'' = -2$

$u''' = 0.$

$V = \sin\left(\frac{n\pi x}{l}\right)$

$V_1 = \dfrac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}$

$V_2 = \dfrac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}$

$V_3 = \dfrac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}$

Fourier series expansion

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$f(x) = \frac{\frac{4l^2}{3}}{2} + \sum_{n=1}^{\infty}\left(-\frac{4l^2}{n^2\pi^2}\right)\cos\left(\frac{n\pi x}{l}\right) + 0.$$

$$f(x) = \frac{2l^2}{3} - \frac{4l^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\pi x}{l}\right)$$

$$= \frac{2l^2}{3} - \frac{4l^2}{\pi^2}\left[\frac{\cos\left(\frac{\pi x}{l}\right)}{1^2} + \frac{\cos\left(\frac{2\pi x}{l}\right)}{2^2} + \frac{\cos\left(\frac{3\pi x}{l}\right)}{3^2} + \cdots\right]$$

$x = l$ is a continuous point

$$f(x) = f(a)$$

$$f(l) = l(2l - l)$$

$$= \frac{2l^2}{3} - \frac{4l^2}{\pi^2}\left[-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right]$$

$$2l^2 - l^2 = \frac{2l^2}{3} - \frac{4l^2}{\pi^2}\left[-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right]$$

$$l^2 - \frac{2l^2}{3} = -\frac{4l^2}{\pi^2}\left[-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right]$$

$$\frac{l^2}{3} = -\frac{4l^2}{\pi^2}\left[-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right]$$

$$\frac{\cancel{l^2}}{3} = \frac{4\cancel{l^2}}{\pi^2}\left[\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$\frac{\pi^2}{12} = \left[\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots\right]$$

20/02

Find the fourier series expansion of $f(x)$ with period $2l$ defined by $f(x) = (l-x)^2$ in $(0, 2l)$. Hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$

$$f(x) = (l-x)^2 \qquad 0 \le x \le 2l$$

$$a_0 = \frac{1}{l}\int_0^{2l} (l-x)^2 \, dx$$

$$= \frac{1}{l}\left[\frac{(l-x)^3}{-3}\right]_0^{2l}$$

$$= \frac{1}{l}\left[\frac{(l-2l)^3}{-3} - \left(\frac{l}{-3}\right)\right]$$

$$= \frac{1}{l}\left[\frac{-l^3}{-3} + \frac{l^3}{3}\right]$$

$$= \frac{1}{l}\left[\frac{2l^3}{3}\right].$$

$$a_0 = \frac{2l^2}{3}$$

$$a_n = \frac{1}{l}\int_0^{2l} (l-x)^2 \cos\left(\frac{n\pi x}{l}\right) dx$$

$u = (l-x)^2$

$u' = -2(l-x) = -2l + 2x$

$u'' = 2$

$u''' = 0$.

$v = \cos\left(\frac{n\pi x}{l}\right)$

$v_1 = \frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}$

$v_2 = \frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}$

$v_3 = \frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}$

$$= \frac{1}{l}\left[(l-x)^2 \frac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)} + (-2l+2x)\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} + 2\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right)\right]_0^{2l}$$

$$= \frac{1}{l}\left[\frac{(-2l+4l)\,l^2}{n^2\pi^2} - \frac{(-2l)\,l^2}{n^2\pi^2}\right]$$

$$= \frac{1}{l}\left[\frac{2l^3}{n^2\pi^2} + \frac{2l^3}{n^2\pi^2}\right] = \frac{4l^3}{l(n^2\pi^2)} = \frac{4l^2}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_0^{2l} (l-x)^2 \sin\left(\frac{n\pi x}{l}\right) dx$$

$u = (l-x)^2$

$u' = -2(l-x) = -2l + 2x$

$u'' = 2$

$u''' = 0.$

$v = \sin\left(\frac{n\pi x}{l}\right)$

$v_1 = \frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}$

$v_2 = \frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}$

$v_3 = \frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}$

$$= \frac{1}{l}\left[(l-x)^2\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}\right) + \frac{(-2l+2x)\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} + 2\left(\frac{\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right)\right]_0^{2l}$$

$$= \frac{1}{l}\left[\left(\frac{-l^2}{\left(\frac{n\pi}{l}\right)} + \frac{2}{\left(\frac{n\pi}{l}\right)^3}\right) - \left(\frac{-l^2}{\left(\frac{n\pi}{l}\right)} + \frac{2}{\left(\frac{n\pi}{l}\right)^3}\right)\right]$$

$$= \frac{1}{l}\left[\frac{-l^2}{\frac{n\pi}{l}} + \frac{2l^3}{n^3\pi^3} + \frac{l^2}{\frac{n\pi}{l}} - \frac{2l^3}{n^3\pi^3}\right]$$

$$= 0.$$

Fourier series expansion

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{\frac{2l^2}{3}}{2} + \sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2}\cos\left(\frac{n\pi x}{l}\right) + 0$$

$$= \frac{l^2}{3} + \sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2}\cos\left(\frac{n\pi x}{l}\right)$$

$$= \frac{l^2}{3} + \frac{4l^2}{\pi^2}\left[\frac{\cos\frac{\pi x}{l}}{1^2} + \frac{\cos\frac{2\pi x}{l}}{2^2} + \frac{\cos\frac{3\pi x}{l}}{3^2} + \dots\right]$$

When $x = 0$ 0 is end point.

$f(x)_{x=0} = f(-x)$ ...

$f(x)_{x=0} = l$ ...

$l^2 = \frac{l^2}{3} + \frac{4l^2}{\pi^2}$ ...

22|2 Find the f...

and hence dedu...

$(c, c+2l) = (0,$ ...

$c = 0$

$l = 1$ as

$c + 2l = 2$

$f(x) = \begin{cases} \pi x \dots \\ \pi(2 \dots \end{cases}$

$a_0 = \frac{1}{l}$ ...

put $l = 1$

$a_0 = \pi$ ...

$= \pi$ ...

$$f(x)_{x=0} = \frac{f(-x)+f(x)}{2}$$

$$f(x)_{x=0} = \frac{l^2+l^2}{2} = \frac{2l^2}{2} = l^2$$

$$l^2 = \frac{l^2}{3} + \frac{4l^2}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$l^2 - \frac{l^2}{3} = \frac{4l^2}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$\frac{2\cancel{l^2}}{3}\times\frac{\pi^2}{4\cancel{l^2}} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$$

$$\frac{\pi^2}{6} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$$

12) Find the fourier series expansion of $f(x) = \begin{cases} \pi x & 0\le x\le 1 \\ \pi(2-x) & 1\le x\le 2 \end{cases}$ and hence deduce the value of $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$ (or) $\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}$

$(c, c+2l) = (0,2)$

$c = 0.$

$l = 1$ as

$c+2l = 2.$

$$f(x) = \begin{cases} \pi x & 0\le x\le 1 \\ \pi(2-x) & 1\le x\le 2 \end{cases}$$

$$a_0 = \frac{1}{l}\int_0^1 f(x)\,dx + \frac{1}{l}\int_1^2 f(x)\,dx$$

Put $l = 1$

$$a_0 = \pi\int_0^1 x\,.\,dx + \pi\int_1^2 2-x\,.\,dx$$

$$= \pi\left[\frac{x^2}{2}\right]_0^1 + \pi\left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{\pi}{2}[1-0] + \pi\left[4 - \frac{4}{2} - 2 + \frac{1}{2}\right]$$

$$= \frac{\pi}{2} + \pi\left[\frac{\cancel{8}-\cancel{4}-\cancel{4}+1}{2}\right]$$

$$= \frac{\pi}{2} + \pi\left[\frac{1}{2}\right] = \frac{2\pi}{2} = \pi$$

$$a_0 = \pi$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{l}\int_0^2 f(x)\cos(n\pi x)\,.\,dx$$

$$= \int_0^1 \pi x \cos(n\pi x)\,dx + \int_1^2 \pi(2-x)\cos(n\pi x)\,dx$$

$$= \pi \int_0^1 \underset{u}{x}\,\underset{v}{\cos(n\pi x)}\,dx + \pi \int_1^2 \underset{u}{(2-x)}\,\underset{v}{\cos(n\pi x)}\,dx$$

$u = x$ $v = \cos(n\pi x)$

$u' = 1$ $v_1 = \frac{\sin(n\pi x)}{n\pi}$

$u'' = 0$ $v_2 = -\frac{\cos(n\pi x)}{(n\pi)^2}$

$u = 2-x$ $v = \cos(n\pi x)$

$u' = -1$ $v_1 = \frac{\sin(n\pi x)}{n\pi}$

$u'' = 0.$ $v_2 = \frac{-\cos(n\pi x)}{(n\pi)^2}$

$$= \pi\left[\frac{x}{n\pi}\sin(n\pi x) + \frac{1}{(n\pi)^2}\cos(n\pi x)\right]_0^1 +$$

$$\pi\left[\frac{(2-x)}{n\pi}\sin(n\pi x) - \frac{\cos n\pi x}{n^2\pi^2}\right]_1^2$$

$$= \pi\left[\frac{1}{n^2\pi^2}(-1)^n - \frac{1}{n^2\pi^2}\right] +$$

$$\pi\left[-\frac{1}{n^2\pi^2} + \frac{1}{n^2\pi^2}(-1)^n\right]$$

$$= \frac{1}{n^2\pi}\left[(-1)^n - 1\right] + \frac{1}{n^2\pi}\left[(-1)^n - 1\right] \quad \text{if } n \neq 0.$$

$$a_n = \begin{cases} 0 & \text{if } n \text{ is even} \neq 0. \\ \frac{-4}{n^2\pi} & \text{if } n \text{ is odd.} \end{cases}$$

$$b_n = \frac{1}{l}\int_0^{c+2l} f(x)\sin n\pi x\,dx$$

$$= 1\int_0^1 \pi x.\sin n\pi x\,.dx + \int_1^2 \pi(2-x).\sin n\pi x\,.dx$$

$u = x \quad v = \sin n\pi x \qquad u = 2-x \quad v = \sin n\pi x$

$u' = 1 \quad v_1 = \frac{-\cos n\pi x}{n\pi} \qquad u' = -1 \quad v_1 = \frac{-\cos n\pi x}{n\pi}$

$v_2 = -\frac{\sin n\pi x}{(n\pi)^2} \qquad v_2 = -\frac{\sin n\pi x}{(n\pi)^2}$

$$b_n = \pi\left[x\left(\frac{-\cos n\pi x}{n\pi}\right) + \frac{\sin n\pi x}{(n\pi)^2}\right]_0^1 +$$

$$\pi\left[(2-x)\left(\frac{-\cos n\pi x}{n\pi}\right) - \frac{\sin n\pi x}{(n\pi)^2}\right]_1^2$$

$s(n\pi x)$

$\frac{\sin(n\pi x)}{n\pi}$

$\frac{-\cos(n\pi x)}{(n\pi)^2}$

$$= \pi\left[-\frac{1}{n\pi}(-1)^n\right] + \pi\left[\frac{+(-1)^n}{n\pi}\right]$$

$$= -\frac{1}{n}(-1)^n + \frac{1}{n}(-1)^n$$

$b_n = 0$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1,3,\dots\infty}^{\infty} \frac{1}{n^2}\cos n\pi x$$

$x = 1$ is a continuous point of $f(x)$

$$f(x)_{x=1} = f(1)$$

$$\pi = \frac{\pi}{2} - \frac{4}{\pi}\left[-\frac{1}{1^2} - \frac{1}{3^2} - \frac{1}{5^2} - \dots\infty\right]$$

$$\pi = \frac{\pi}{2} + \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$$

$$\pi - \frac{\pi}{2} = \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$$

$$\frac{\pi}{2} = \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$$

$$\frac{\pi^2}{8} = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$$

24/2/25

Obtain the fourier series of the function $f(x) = x\cos x$, $0 \le x \le 2\pi$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x \cos x \, dx$$

$u = x$ $\quad v = \cos x \, dx$

$u' = 1$ $\quad v_1 = \sin x$

$u'' = 0$ $\quad v_2 = -\cos x$.

$$= \frac{1}{2\pi}\left[x\sin x + \cos x\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[0 + 1 - 0 - 1\right]$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cdot \cos\left(\frac{n\pi x}{l}\right) dx \quad (l = \pi)$$

$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$$= \frac{1}{\pi}\int_0^{2\pi} x \cos x \cos nx \, dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\left[\cos(n+1)x + \cos(n-1)x\right] dx.$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x (\cos(n+1)x \, dx + \frac{1}{2\pi}\int_0^{2\pi} x \cdot \cos(n-1)x \, dx.$$

$u = x$ $\quad v = \cos(n+1)x$

$u' = 1$ $\quad v_1 = \frac{\sin(n+1)x}{n+1}$

$u'' = 0$ $\quad v_2 = \frac{-\cos(n+1)x}{(n+1)^2}$

$$= \frac{1}{2\pi}\left[x\frac{\sin(n+1)x}{n+1} + \frac{\cos(n+1)x}{(n+1)^2}\right]_0^{2\pi} +$$

$$\frac{1}{2\pi}\left[x\frac{\sin(n-1)x}{n-1} + \frac{\cos(n-1)x}{(n-1)^2}\right]_0^{2\pi}$$

$u = x$ $\quad v = \cos(n-1)x$

$u' = 1$ $\quad v_1 = \frac{\sin(n-1)x}{(n-1)}$

$u'' = 0$ $\quad v_2 = \frac{-\cos(n-1)x}{(n-1)^2}$

$$= \frac{1}{2\pi}\left[\frac{1}{(n+1)^2} - \frac{1}{(n+1)^2}\right] + \frac{1}{2\pi}\left[\frac{1}{(n-1)^2} - \frac{1}{(n-1)^2}\right]$$

$a_n = 0$ for $n \neq 1$. if 1 $a_n = \infty$.

If $f(x) = \sin(kx)$ or $\cos(kx)$ we calculate a_0, a_n, b_n & additionally a_k and b_k.

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx.$$

$l = \pi$.

$$a_1 = \frac{1}{l}\int_c^{c+2l} f(x) \cos\left(\frac{\pi x}{l}\right) dx.$$

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} x \cos x . \cos x . dx$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$u =$

$$= \frac{1}{2\pi}\int_0^{2\pi} x (1+\cos 2x). dx.$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} x . dx + \int_0^{2\pi} x . \cos 2x . dx\right]$$

$u = x$ $v = \cos 2x$
$u' = 1$ $v_1 = \frac{\sin 2x}{2}$
$u'' = 0$ $v_2 = \frac{-\cos 2x}{4}$

$$= \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} + \frac{1}{2\pi}\left[x . \frac{\sin 2x}{2} + \frac{\cos 2x}{4}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{4\pi^2}{2}\right] + \frac{1}{2\pi}\left[\frac{1}{4} - \frac{1}{4}\right]$$

$$= \frac{4\pi^2}{4\pi} + 0$$

$$a_1 = \pi$$

$= x \cos x,$

$v = \cos x \, dx$
$v_1 = \sin x$
$v_2 = -\cos x$

$\cos(A+B) +$
$\cos(A-B)]$

x $v = \cos(n+1)x$
1 $v_1 = \frac{\sin(n+1)x}{n+1}$
0 $v_2 = \frac{-\cos(n+1)x}{(n+1)^2}$

$v = \cos(n-1)x$
$v_1 = \frac{\sin(n-1)x}{(n-1)}$
$v_2 = \frac{-\cos(n-1)x}{(n-1)^2}$

$$bn = \frac{1}{\pi}\int_0^{2\pi} x.\cos x \sin nx \; dx$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\,[\sin(n+1)x + \sin(n-1)x]\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x \sin(n+1)x\,dx + \frac{1}{2\pi}\int_0^{2\pi} x.\sin(n-1)x.\,dx$$

$u = x \quad v = \sin(n+1)$

$u' = 1 \quad v_1 = \frac{-\cos(n+1)}{n+1}$

$u'' = 0 \quad v_2 = \frac{-\sin(n+1)}{(n+1)^2}$

$$= \frac{1}{2\pi}\left[-x\frac{\cos(n+1)x}{n+1} + \frac{\sin(n+1)x}{(n+1)^2}\right]_0^{2\pi} +$$

$$\frac{1}{2\pi}\left[-x\frac{\cos(n-1)}{n-1}x + \frac{\sin(n-1)x}{(n-1)^2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{-2\pi}{n+1} - 0\right] + \frac{1}{2\pi}\left[\frac{-2\pi}{n-1} - 0\right]$$

$$= \frac{1}{2\pi}\left[\frac{-2\pi}{n+1} - \frac{2\pi}{n-1}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-2\pi(n-1) - 2\pi(n+1)}{n^2-1^2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-2\pi n + \cancel{2\pi} - 2n\pi - \cancel{2\pi}}{n^2-1^2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-4\pi n}{n^2-1}\right]$$

$$= \frac{-2n}{n^2-1}$$

$\frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$u = x \quad v = \sin(n+1)$

$u' = 1 \quad v_1 = \frac{-\cos(n+1)}{n+1}$

$u'' = 0 \quad v_2 = \frac{-\sin(n+1)}{(n+1)^2}$

$$b_1 = \frac{1}{l}\int_c^{c+2l} f(x)\sin\left(\frac{\pi x}{l}\right).\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x \cos x \,.\, \sin x \,.\, dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\left[\sin(1+1)x - \sin(1-1)x\right].\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x \,.\, \sin 2x \,dx - \frac{1}{2\pi}\int_0^{2\pi} x \,.\, \sin 0x$$

$u = x \quad v = \sin 2x$

$u' = 1 \quad v_1 = \frac{-\cos 2x}{2}$

$u'' = 0 \quad v_2 = \frac{-\sin 2x}{4}$

$$= \frac{1}{2\pi}\left[-x\frac{\cos 2x}{2} + \frac{\sin 2x}{4}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{-2\pi}{2} + 0\right]$$

$$= -\frac{1}{2}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{a_0}{2} + a_1\cos\left(\frac{\pi x}{\pi}\right) + \sum_{n=2}^{\infty} a_n \cos(nx) + b_1 \sin\left(\frac{\pi x}{\pi}\right) + \sum_{n=2}^{\infty} b_n \sin(nx)$$

$$= \pi\cos x - \frac{1}{2}\sin x + \sum_{n=2}^{\infty}\left(\frac{-2n}{n^2-1}\right)\sin nx$$

$$= \pi\cos x - \frac{1}{2}\sin x - 2\sum_{n=2}^{\infty}\frac{n}{n^2-1}\sin nx$$

1) $f(x) = \begin{cases} x & \text{in } 0 < x < 2 \\ 0 & \text{in } 2 < x < 4 \end{cases}$ Find the fourier series.

2) $f(x) = \begin{cases} 1 & \text{in } 0 < x < 1 \\ 2 & \text{in } 1 < x < 3 \end{cases}$

1 $f(x) = \begin{cases} x & \text{in } 0 < x < 2 \\ 0 & \text{in } 2 < x < 4 \end{cases}$

$2l = 4$
$l = 2$
$c = 0$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

$$a_0 = \frac{1}{2}\int_0^2 x \, dx + \frac{1}{2}\int_2^4 0 \, dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{1}{4}[4]$$

$$a_0 = 1$$

$$a_n = \frac{1}{2}\left[\int_0^2 f(x) \cos\left(\frac{n\pi x}{l}\right) dx + \int_2^4 f(x) \cos\left(\frac{n\pi x}{l}\right) dx\right]$$

$$= \frac{1}{2}\left[\int_0^2 x \cos\frac{n\pi x}{2} \, dx + 0.\right]$$

$u = x$ $\quad v = \cos\frac{n\pi x}{2}$
$u' = 1$ $\quad v_1 = \dfrac{\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)}$
$u'' = 0$ $\quad v_2 = \dfrac{-\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^2}$

$$= \frac{1}{2}\left[x \cdot \frac{\sin\left(\frac{n\pi x}{2}\right)}{\frac{n\pi}{2}} + \frac{\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^2}\right]_0^2$$

$$= \frac{1}{2}\left[\frac{4}{n^2\pi^2}(-1)^n - \frac{4}{n^2\pi^2}\right]$$

$$= \frac{4}{2n^2\pi^2}\left[(-1)^n - 1\right] \qquad = \begin{cases} \dfrac{-4}{n^2\pi^2} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases} \quad n \neq 0$$

$$= \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{2}\left[\int_0^2 f(x)\sin\frac{n\pi x}{l}\,dx + \int_2^4 f(x)\sin\frac{n\pi x}{l}\,dx\right]$$

$$= \frac{1}{2}\left[\int_0^2 x \sin\frac{n\pi x}{2}\,dx + 0\right]$$

$u = x \qquad v = \sin\frac{n\pi x}{2}$

$u' = 1 \qquad v_1 = \frac{-\cos\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)}$

$u'' = 0 \qquad v_2 = \frac{-\sin\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)^2}$

$$= \frac{1}{2}\left[-x\cdot\frac{\cos\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)} + \frac{\sin\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)^2}\right]_0^2$$

$$= \frac{1}{2}\left[-2\,\frac{(-1)^n}{\left(\frac{n\pi}{2}\right)} + 0\right]$$

$$= \frac{-2}{2\left(\frac{n\pi}{2}\right)}(-1)^n$$

$$= \frac{-2\times\cancel{2}}{\cancel{2}n\pi}(-1)^n$$

$$b_n = \frac{-2}{n\pi}(-1)^n$$

$$f(x) = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=1,3,5..}^{\infty}\frac{1}{n^2}\cos\frac{n\pi x}{2} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2}$$

2. $f(x) = \begin{cases} 1 & \text{in } 0 < x < 1 \\ 2 & \text{in } 1 < x < 3 \end{cases}$

$2l = 3$

$l = \frac{3}{2}$

$c = 0$

$$a_0 = \frac{2}{3}\left[\int_0^1 f(x)\,dx + \int_1^3 f(x).dx\right]$$

$$= \frac{2}{3}\left[\int_0^1 1.dx + \int_1^3 2.\,dx\right]$$

$$= \frac{2}{3}\left\{\left[x\right]_0^1 + \left[2x\right]_1^3\right\}$$

$$= \frac{2}{3}\left(\left[1-0\right] + \left[6-2\right]\right)$$

$$= \frac{2}{3}\left[1+4\right]$$

$$a_0 = \frac{10}{3}.$$

$$a_n = \frac{2}{3}\left[\int_0^1 f(x)\cos\left(\frac{n\pi x}{l}\right)dx + \int_1^3 f(x).\cos\left(\frac{n\pi x}{l}\right)dx\right]$$

$$= \frac{2}{3}\left[\int_0^1 1\ \cos\left(\frac{n\pi x}{\frac{3}{2}}\right).dx + \int_1^3 2.\ \cos\left(\frac{n\pi x}{\frac{3}{2}}\right).dx\right]$$

$u = 1$ $v = \cos\left(\frac{n\pi x}{\frac{3}{2}}\right)$

$u' = 0$ $v_1 = \frac{\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}$

$$= \frac{2}{3}\left\{\left[\frac{\sin\frac{2n\pi x}{3}}{\left(\frac{2n\pi}{3}\right)}\right]_0^1 + \left[2.\ \frac{\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}\right]_1^3\right\}$$

$u = 2$ $v = \cos\left(\frac{n\pi x}{\frac{3}{2}}\right)$

$u' = 0$ $v_1 = \frac{\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}$

$$= \frac{2}{3}\left[\left\{\frac{3}{2n\pi}\sin\frac{2n\pi}{3}\right\} + 2.\left\{-\frac{3}{2n\pi}\sin\frac{2n\pi}{3}\right\}\right]$$

$$= \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}n\pi}\sin\frac{2n\pi}{3} + 2 \times \frac{\cancel{2}}{\cancel{3}}\left(-\frac{\cancel{3}}{\cancel{2}n\pi}\sin\frac{2n\pi}{3}\right)$$

$$= \frac{1}{n\pi}\sin\frac{2n\pi}{3} - \frac{2}{n\pi}\sin\frac{2n\pi}{3} = -\frac{1}{n\pi}\sin\frac{2n\pi}{3}$$

$$b_n = \frac{2}{3}\left[\int_0^1 f(x)\sin\frac{n\pi x}{l}\,dx + \int_1^3 f(x)\sin\frac{n\pi x}{l}\cdot dx\right]$$

$$= \frac{2}{3}\left[\int_0^1 1\cdot\sin\frac{n\pi x}{\left(\frac{3}{2}\right)}\cdot dx + \int_1^3 2\cdot\sin\left(\frac{n\pi x}{\frac{3}{2}}\right)dx\right]$$

$$= \frac{2}{3}\left\{\left[-\frac{\cos\left(\frac{2n\pi x}{3}\right)}{\frac{2n\pi}{3}}\right]_0^1 + \left[-2\frac{\cos\left(\frac{2n\pi x}{3}\right)}{\frac{2n\pi}{3}}\right]_1^3\right\}$$

$u = 1$ $\quad v = \sin\frac{2n\pi x}{3}$

$u' = 0$ $\quad v_1 = \frac{-\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}$

$$= \frac{2}{3}\left[\left\{\frac{3}{2n\pi}\left(-\cos\frac{2n\pi}{3}\right) + \frac{3}{2n\pi}\right\}\right.$$
$$\left.\left[-2\left\{\frac{3}{2n\pi} - \frac{3}{2n\pi}\cos\frac{2n\pi}{3}\right\}\right]\right]$$

$u = 2$ $\quad v = \sin 2\cdot\frac{n\pi x}{3}$

$u' = 0$ $\quad v_1 = \frac{-\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}$

$$= \frac{2}{3}\left\{\frac{3}{2n\pi}\left[1 - \cos\frac{2n\pi}{3}\right] - \frac{2\times 3}{2n\pi}\left[1 - \cos\frac{2n\pi}{3}\right]\right\}$$

$$= \frac{1}{n\pi}\left[1 - \cos\frac{2n\pi}{3}\right] - \frac{2}{n\pi}\left[1 - \cos\frac{2n\pi}{3}\right]$$

$$= -\frac{1}{n\pi}\left[1 - \cos\frac{2n\pi}{3}\right]$$

$$f(x) = \frac{5}{3} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\sin\frac{2n\pi}{3}\right)\cos\frac{2n\pi x}{3}$$
$$- \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(1 - \cos\frac{2n\pi}{3}\right)\sin\frac{2n\pi x}{3}$$

25/2/25

Expand $f(x) = x^2$ when, $-\pi < x < \pi$ in a fourier series of periodicity of 2π Hence deduce that

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ to $\infty = \frac{\pi^2}{6}$.

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots$ to $\infty = \frac{\pi^2}{12}$

(iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ to $\infty = \frac{\pi^2}{8}$

$c = -\pi$

$c+2l = \pi$

$-\pi + 2l = \pi$

$2l = \pi + \pi$

$l = \pi$

$f(x) = x^2$

$f(-x) = (-x)^2$

$f(-x) = x^2$

$f(-x) = f(x)$

$f(x)$ is even fnc.

$b_n = 0$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \, dx$$

$$= \frac{1}{\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{3\pi}\left[\pi^3 - (-\pi^3)\right]$$

(or) $\int_{-a}^{a} = 2\int_{0}^{a}$

$$= \frac{1}{3\pi}\left[2\pi^3\right]$$

$$a_0 = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos\left(\frac{n\pi x}{\pi}\right) . dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x^2 \cos nx . dx$$

$u = x^2$ $\quad v = \cos nx$

$u' = 2x$ $\quad v_1 = \frac{\sin nx}{n}$

$u'' = 2$ $\quad v_2 = -\frac{\cos nx}{n^2}$

$v_3 = -\frac{\sin nx}{n^3}$

$$= \frac{2}{\pi}\left[x^2 \frac{\sin nx}{n} + 2x\frac{\cos nx}{n^2} - 2\frac{\sin nx}{n^3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{2\pi}{n^2}(-1)^n - 0\right].$$

$$= \frac{2}{\pi}\left[\frac{2\pi}{n^2}(-1)^n\right]$$

$$= \frac{4}{n^2}(-1)^n.$$

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$$

When $x = \pi$ (end point)

$$(f(x))_{x=\pi} = \frac{f(-\pi) + f(\pi)}{2}$$

$$= \frac{(-\pi)^2 + (\pi)^2}{2}$$

$$= \pi^2$$

$$\pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}(-1)^n$$

$$\pi^2 - \frac{\pi^2}{3} = 4\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$\frac{2\pi^2}{3} = 4\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \rightarrow (1)$$

(ii) $x = 0$ (mid point)

$[f(x)]_{x=0}$ is constant function

$$0 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos(0)}{n^2}$$

$$-\frac{\pi^2}{3} = 4\left[-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots\right]$$

$$\frac{\pi^2}{12} = \left[\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots\right]. \rightarrow (2)$$

(iii) (1) + (2)

$$\frac{\pi^2}{6} + \frac{\pi^2}{12} = 2\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

$$\frac{2\pi^2 + \pi^2}{12 \times 2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

$$\frac{\cancel{3}\pi^2}{\cancel{12}^{4} \times 2} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

$$\frac{\pi^2}{8} = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right] \rightarrow (3).$$

$f(x) = \sin(kx)$ in $(-l, l)$

Doubt (24)

$f(x) = |\cos x|$ in $(-\pi, \pi)$

$c = -\pi$

$c + 2l = \pi$

$-\pi + 2l = \pi$

$l = \pi$

$f(x) = |\cos x|$

$f(-x) = |\cos(-x)| = |\cos x| = f(x)$

$b_n = 0$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} |\cos x| \, dx.$$

S | A
T | C

as cos is -ve in 2nd quadrant

$$= \frac{2}{\pi}\int_0^{\pi/2} \cos x \, dx \ominus \int_{\pi/2}^{\pi} \cos x \, dx$$

$$= \frac{2}{\pi}\left[\sin x\right]_0^{\pi/2} - \left[\sin x\right]_{\pi/2}^{\pi}$$

$$= \frac{2}{\pi}\Big[[1-0] - [0-1]\Big]$$

$$= \frac{2}{\pi}[1+1]$$

$$a_0 = \frac{4}{\pi}$$

$$\boxed{\sin(n\pm1)\frac{\pi}{2} = \sin\frac{n\pi}{2}\cos\frac{\pi}{2} \pm \cos\frac{n\pi}{2}\cdot\sin\frac{\pi}{2} = \pm\cos\frac{n\pi}{2}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\left(\frac{n\pi x}{l}\right) dx$$

$$\cos A\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} |\cos x| \cos\left(\frac{n\pi x}{\pi}\right) dx$$

$$= \frac{2}{\pi}\int_0^{\pi} |\cos x| \cos(n\cancel{x}) \cdot dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x \cdot \cos n\cancel{x} \, dx - \int_{\pi/2}^{\pi} \cos x \cdot \cos n\pi \, dx\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{2}\int_0^{\pi/2} [\cos(n+1)x + \cos(n-1)x] \, dx - \frac{1}{2}\int_{\pi/2}^{\pi} [\cos(n+1)x + \cos(n-1)x] \, dx\right]$$

$$= \frac{1}{\pi}\left\{\frac{\sin(n+1)x}{n+1} + \frac{\sin(n-1)x}{n-1}\right\}_0^{\pi/2} - \frac{1}{\pi}\left\{\frac{\sin(n+1)x}{n+1} + \frac{\sin(n-1)x}{n-1}\right\}_{\pi/2}^{\pi}$$

$$= \frac{1}{\pi}\left\{\frac{\sin(n+1)\pi/2}{n+1} + \frac{\sin(n-1)\pi/2}{n-1}\right\} + \frac{1}{\pi}\left\{\frac{\sin(n+1)\pi/2}{n+1} + \frac{\sin(n-1)\pi/2}{n-1}\right\} \quad [\uparrow]$$

$$= \frac{2}{\pi}\left(\frac{1}{n+1} + \frac{1}{n-1}\right)\left\{\sin(n+1)\frac{\pi}{2} + \sin(n-1)\frac{\pi}{2}\right\}$$

26/2

Find the fourier series of $f(x) = x$ in the interval $(-4, 4)$

$f(x) = x$

$c = -4$

$c + 2l = 4$

$-4 + 2l = 4$

$2l = 8$

$l = 4$

$f(x) = x$

$f(-x) = -x$

$f(x)$ is odd fnc

a_0 and $a_n = 0$

$$b_n = \frac{1}{4}\int_{-4}^{4} x \cdot \sin\frac{n\pi x}{4}\,dx$$

$u = x \quad v = \sin\frac{n\pi x}{4}$

$u' = 1 \quad v_1 = \frac{-\cos\left(\frac{n\pi x}{4}\right)}{\frac{n\pi}{4}}$

$v_2 = \frac{-\sin\left(\frac{n\pi x}{4}\right)}{\left(\frac{n\pi}{4}\right)^2}$

$$= \frac{1}{4}\left[-x \cdot \frac{\cos\left(\frac{n\pi x}{4}\right)}{\frac{n\pi}{4}} - \frac{\sin\left(\frac{n\pi x}{4}\right)}{\left(\frac{n\pi}{4}\right)^2}\right]_{-4}^{4}$$

$$= \frac{1}{4}\left[-4\,\frac{(-1)^n}{\frac{n\pi}{4}} - 4\,\frac{(-1)^n}{\frac{n\pi}{4}}\right]$$

$$= \frac{1}{4}\left[-16\,\frac{(-1)^n}{n\pi} - 16\,\frac{(-1)^n}{n\pi}\right]$$

$$= \frac{1}{\cancel{4}}\left[\frac{-\cancel{16}^{\,4}}{n\pi}\right]\left[(-1)^n + (-1)^n\right]$$

$$= \frac{-4}{n\pi}\left[(-1)^{n-1} + (-1)^n\right] = \frac{-8}{n\pi}\left[(-1)^n\right]$$

$$f(x) = \frac{-8}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(-1)^n \sin\frac{n\pi x}{4} = \boxed{\frac{-8}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(-1)^n \sin\frac{n\pi x}{4}}$$

interval

$\frac{\sin\frac{n\pi x}{4}}{}$

$= \frac{-\cos\left(\frac{n\pi x}{4}\right)}{\frac{n\pi}{4}}$

$\frac{-\sin\left(\frac{n\pi x}{4}\right)}{\left(\frac{n\pi}{4}\right)^2}$

$f(x) = x^2 + x$ in $(-2, 2)$ find $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

$$f(-x) = (-x)^2 + (-x)$$
$$= x^2 - x$$
$$\neq f(x)$$
$$\neq -f(x)$$

is neither odd nor even

$c = -2$

$c + 2l = 2$

$-2 + 2l = 2$

$2l = 4$

$l = 2$

$$a_0 = \frac{1}{2}\int_{-2}^{2} x^2 + x \, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-2}^{2}$$

$$= \frac{1}{2}\left[\frac{8}{3} + \frac{4}{2} - \left(\frac{-8}{3} + \frac{4}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{8}{3} + \cancel{\frac{4}{2}} + \frac{8}{3} - \cancel{\frac{4}{2}}\right]$$

$$= \frac{1}{\cancel{2}}\left[\cancel{2}\left(\frac{8}{3}\right)\right]$$

$$\boxed{a_0 = \frac{8}{3}}$$

$$a_n = \frac{1}{\cancel{2}}\int_{-2}^{\cancel{2}} f(x) \cdot \cos\frac{n\pi x}{l} \cdot dx$$

$$= \frac{1}{2}\int_{-2}^{2}(x^2 + x) \cos\frac{n\pi x}{2} \cdot dx$$

$$= \frac{1}{2}\left[\int_{-2}^{2} \underbrace{x^2 \cdot \cos\frac{n\pi x}{2}}_{\text{even}} \cdot dx + \int_{-2}^{2} \underbrace{x \cdot \cos\frac{n\pi x}{2}}_{\text{odd.}} \cdot dx\right]$$

$u = x^2$

$u' = 2x$

$u'' = 2$

$$= \frac{1}{2}\left[2\int_{0}^{2} x^2 \cdot \cos\frac{n\pi x}{2} \cdot dx\right]$$

$V = \cos\frac{n\pi x}{2}$

$V_1 = \frac{\sin\frac{n\pi x}{2}}{\frac{n\pi}{2}}$

$V_2 = \frac{-\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^2}$

$V_3 = \frac{-\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^3}$

$$= \left[x^2 \frac{\sin\frac{n\pi x}{2}}{\frac{n\pi}{2}} + 2x \frac{\cos\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)^2} - 2\frac{\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^3}\right]_0^2$$

$$= \left[\frac{4(-)^n}{\frac{n^2\pi^2}{4}} - 0\right] = \frac{16(-1)^{n-1}}{n^2\pi^2} \text{ if } n \neq 0$$

$\frac{\pi x}{4)}$

$$\boxed{a_n = \frac{16(-1)^n}{n^2\pi^2}} \text{ for } n\neq 0.$$

$$b_n = \frac{1}{2}\int_{-2}^{2}(x^2+x)\sin\frac{n\pi x}{2}\,dx.$$

$u = x \quad v = \sin\frac{n\pi x}{2}$

$u' = 1 \quad v_1 = \frac{-\cos\frac{n\pi x}{2}}{\frac{n\pi}{2}}$

$v_2 = \frac{-\sin\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)^2}$

$$= \frac{1}{2}\int_{-2}^{2}x^2\sin\frac{n\pi x}{2}\,dx + \frac{1}{2}\int_{-2}^{2}x.\sin\frac{n\pi x}{2}$$

$$= \int_{0}^{2} x.\sin\frac{n\pi x}{2}\,dx$$

$x^2\sin\frac{n\pi x}{2}$ is odd fnc.

$x.\sin\frac{n\pi x}{2}$ is even fnc.

$$= \left[-x.\frac{\cos\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)} + \frac{\sin\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)^2}\right]_0^2$$

$$= \left[-\frac{4}{n\pi}(-1)^n\right] = -\frac{4}{n\pi}(-1)^n$$

$$f(x) = \frac{\frac{8}{3}}{2} + \frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi x}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2}$$

$$f(x) = \frac{4}{3} + \frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\frac{n\pi x}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2}$$

when $\cancel{x}$ 2 end point

$$f(x)_{x=2} = \frac{f(-2)+f(2)}{2}$$

$$= \frac{(-2)^2+(-2) + (2^2+2)}{2}$$

$$f(x) = \frac{4 \cancel{-2} + 4 \cancel{+2}}{2} = 4$$

$$\frac{\pi^2}{6} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\infty$$

$$4 = \frac{4}{3} + \frac{16}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}(-1)^n$$

$$4 - \frac{4}{3} = \frac{16}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$\frac{8}{3}\times\frac{\pi^2}{16} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$$