TRANSFORM OF PERIODIC FUNCTIONS

$$L[f(t)] = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t) \cdot dt$$

$T = 2\pi$ $\quad f(x+T) = f(x)$ is true then

Find the laplace transform of the Half wave rectifier function. $f(t) = \begin{cases} E\sin\omega t, & 0<t<\pi/\omega \\ 0 & , \pi/\omega < t < 2\pi/\omega \end{cases}$

$T = \frac{2\pi}{\omega}$

$$L\{f(t)\} = \frac{1}{1-e^{-\frac{2\pi s}{\omega}}} \int_0^{\frac{2\pi}{\omega}} e^{-st} f(t) dt$$

$$= \frac{1}{1-e^{-\frac{2\pi s}{\omega}}} \left[\int_0^{\pi/\omega} e^{-st} E\sin\omega t \, dt + 0 \right]$$

$$= \frac{E}{1-e^{-\frac{2\pi s}{\omega}}} \int_0^{\pi/\omega} e^{-st} \sin\omega t \, dt$$

$$= \frac{E}{1-e^{-\frac{2\pi s}{\omega}}} \left[\frac{e^{-st}}{s^2+\omega^2} \left(-s\sin\omega t - \omega\cos\omega t \right) \right]_0^{\pi/\omega}$$

$\int e^{at} \sin bt \, dt$

$$= \frac{E}{1-e^{-2\pi s/\omega}} \left[\frac{e^{-\frac{s\pi}{\omega}} \omega}{s^2+\omega^2} + \frac{\omega}{s^2+\omega^2} \right]$$

$$= \frac{E}{1-e^{-\frac{2\pi s}{\omega}}} \left[\frac{e^{-s\pi/\omega} \omega + \omega}{s^2+\omega^2} \right]$$

$$= \frac{E\omega\left(1+e^{-\frac{\pi s}{\omega}}\right)}{\left(1-e^{-\frac{\pi s}{\omega}}\right)\left(1+e^{-\frac{\pi s}{\omega}}\right)(s^2+\omega^2)}$$

$$= \frac{E\omega}{\left(1-e^{-\pi/\omega}\right)(s^2+\omega^2)}$$

Find the Laplace transform of

$$f(x) = \begin{cases} t & 0<t<a \\ 2a-t & a<t<2a \end{cases} \quad \text{with}$$

$$\boxed{\frac{1-e^{-x}}{1+e^{-x}} = \tanh\left(\frac{x}{2}\right)}$$

$f(t+2a) = f(t)$.

$$L[f(t)] = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t)\, dt$$

$T = 2a$

$$L[f(t)] = \frac{1}{1-e^{-2as}} \int_0^{2a} e^{-st} f(t)\, dt$$

$$= \frac{1}{1-e^{-2as}} \left[\int_0^a e^{-st} . t . dt + \int_a^{2a} e^{-st} (2a-t)\, dt \right]$$

$$= \frac{1}{1-e^{-2as}} \left\{ \left[t\frac{e^{-st}}{-s} - 1\left(\frac{e^{-st}}{s^2}\right) \right]_0^a + \left[(2a-t)\left(\frac{e^{-st}}{-s}\right) - (-1)\left(\frac{e^{-st}}{s^2}\right) \right]_a^{2a} \right\}$$

$u = t \quad v = e^{-st}$

$u' = 1 \quad v_1 = \frac{e^{-st}}{-s}$

$v_2 = \frac{e^{-st}}{s^2}$

$u = 2a-t \quad v = e^{-st}$

$u' = -1 \quad v_1 = \frac{e^{-st}}{-s}$

$v_2 = \frac{e^{-st}}{s^2}$

$$= \frac{1}{1-e^{-2as}}\left\{\left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^a + \left[-(2a-t)\left(\frac{e^{-st}}{s}\right) + \left(\frac{e^{-st}}{s^2}\right)\right]_a^{2a}\right\}$$

$$= \frac{1}{1-e^{-2as}}\left\{\left(-a\frac{e^{-sa}}{s} - \frac{e^{-as}}{s^2}\right) + \left(\frac{1}{s^2}\right) + \left(\frac{e^{-2as}}{s^2}\right) - \left(-a\frac{e^{-as}}{s} + \frac{e^{-as}}{s^2}\right)\right\}$$

$$= \frac{1}{1-e^{-2as}}\left\{\cancel{-\frac{ae^{-sa}}{s}} - \frac{e^{-as}}{s^2} + \frac{1}{s^2} + \frac{e^{-2as}}{s^2} + \cancel{\frac{ae^{-as}}{s}} - \frac{e^{-as}}{s^2}\right]$$

$$= \frac{1}{1-e^{-2as}}\left[\frac{1-2e^{-as}+e^{-2as}}{s^2}\right]$$

$$= \frac{1}{1-e^{-2as}}\left[\frac{(1-e^{-as})^2}{s^2}\right]$$

$$= \frac{(1-e^{-as})^2}{1^2-(e^{-as})^2 \cdot s^2}$$

$$= \frac{\cancel{(1-e^{-as})}(1-e^{-as})}{\cancel{(1-e^{-as})}(1+e^{-as})\, s^2}$$

$$= \frac{1}{s^2}\left(\frac{1-e^{-as}}{1+e^{-as}}\right)$$

$$= \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$$

Find the Laplace transform of the periodic function
$$f(t) = \begin{cases} t & 0 < t < 2 \\ 4-t & 2 \le t \le 4 \end{cases}$$

$T = 4$

$$L[f(t)] = \frac{1}{1-e^{4s}} \int_0^4 e^{-st} f(t).dt$$

$$= \frac{1}{1-e^{4s}} \left[\int_0^2 e^{-st} t.dt + \int_2^4 e^{-st} (4-t)\, dt \right]$$

$u = t$
$u' = 1$
$v = e^{-st}$
$v_1 = \frac{e^{-st}}{-s}$
$v_2 = \frac{e^{-st}}{s^2}$

$$= \frac{1}{1-e^{4s}} \left\{ \left[\frac{t.e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right]_0^2 + \left[\frac{(4-t)e^{-st}}{-s} + \frac{e^{-st}}{s^2} \right]_2^4 \right\}$$

$u = 4-t$
$u' = -1$
$v = e^{-st}$
$v_1 = \frac{e^{-st}}{-s}$
$v_2 = \frac{e^{-st}}{s^2}$

$$= \frac{1}{1-e^{4s}} \left\{ \left[-\frac{t e^{-st}}{s} - \frac{e^{-st}}{s^2} \right]_0^2 + \left[-\frac{(4-t)e^{-st}}{s} + \frac{e^{-st}}{s^2} \right]_2^4 \right\}$$

$$= \frac{1}{1-e^{4s}} \left\{ -\frac{\cancel{2e^{-2t}}}{\cancel{s}} - \frac{e^{-2t}}{s^2} + \frac{1}{s^2} + \frac{e^{-4t}}{s^2} + \frac{\cancel{2e^{-2t}}}{\cancel{s}} - \frac{e^{-2t}}{s^2} \right\}$$

$$= \frac{1}{1-e^{-4s}} \left\{ \frac{1 - 2e^{-2t} + e^{-4t}}{s^2} \right\}$$

$$= \frac{1}{(1-e^{-4s})} \frac{(1-e^{-2t})^2}{s^2}$$

$$= \frac{(1-e^{-2t})(1-e^{-2t})}{1^2-(e^{-2s})^2 . s^2}$$

$$= \frac{(1-e^{-2t})\cancel{(1-e^{-2t})}}{(1+e^{-2t})\cancel{(1-e^{-2t})} . s^2}$$

$$= \frac{1}{s^2} \left(\frac{1-e^{-2t}}{1+e^{-2t}} \right)$$

$$= \frac{1}{s^2} \tan h\left(\frac{2t}{2}\right)$$

$$= \frac{1}{s^2} \tan ht.$$

INVERSE LAPLACE TRANSFORM

Find $L^{-1}\left[\frac{1}{s^2-25}\right]$

$$L^{-1}\left[\frac{1}{s^2-25}\right] = L^{-1}\left[\frac{1}{5}\cdot\frac{5}{s^2-25}\right]$$

$$= \frac{1}{5}L^{-1}\left[\frac{5}{s^2-25}\right] \;(h)$$

$$= \frac{1}{5}\sin h\, 5t$$

Find $L^{-1}\left[\frac{1}{(s-2)^2+1}\right]$

$$L^{-1}\left[\frac{1}{(s-2)^2+1}\right] = e^{2t}L^{-1}\left[\frac{1}{s^2+1}\right]$$

$$= e^{2t}\sin t$$

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Find $L^{-1}\left[\frac{s-3}{s^2+4s+13}\right]$

$$L^{-1}\left[\frac{s-3}{s^2+4s+13}\right] = L^{-1}\left[\frac{s-3}{(s+2)^2+9}\right]$$

$$= L^{-1}\left[\frac{s+2-5}{(s+2)^2+3^2}\right]$$

$$= L^{-1}\left[\frac{s+2}{(s+2)^2+3^2}\right] - 5L^{-1}\left[\frac{1}{(s+2)^2+3^2}\right]$$

$$= e^{-2t}L^{-1}\left[\frac{s}{s^2+3^2}\right] - \frac{5}{3}e^{-2t}L^{-1}\left[\frac{3}{(s+2)^2+3^2}\right]$$

$$= e^{-2t}\cos 3t - \frac{5}{3}e^{-2t}\sin 3t$$

$$L^{-1}\left[\frac{s+a}{(s+a)^2+b^2}\right] = e^{-at} L^{-1}\left[\frac{s}{s^2+b^2}\right]$$

$$L^{-1}\left[\frac{a}{(s+b)^2+a^2}\right] = e^{-bt} L^{-1}\left[\frac{a}{s^2+a^2}\right]$$

$$\boxed{L^{-1}[s\,F(s)] = \frac{d}{dt} L^{-1}[F(s)] = \frac{d}{dt} f(t)}$$

$$L^{-1}\left[\frac{2s-5}{9s^2-25}\right]$$

$$L^{-1}\left[\frac{1}{2(s-1)^2+3^2}\right]$$

Find $L^{-1}\left[\frac{s}{(s+3)^2}\right]$

$$L^{-1}\left[\frac{s}{(s+3)^2}\right] = L^{-1}\left[s\,\frac{1}{(s+3)^2}\right]$$

$$= \frac{d}{dt} L^{-1}\left[\frac{1}{(s+3)^2}\right]$$

$$= \frac{d}{dt}\left(e^{-3t} L^{-1}\left[\frac{1}{s^2}\right]\right) \qquad L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$$

$$= \frac{d}{dt} e^{-3t}\, t$$

$$= e^{-3t}(1) + t(-3)e^{-3t} \qquad udv = uv' - vu'$$

$$= e^{-3t}(1-3t)$$

$$\boxed{L^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t L^{-1}[f(t)]\,dt = \int_0^t f(t)\cdot dt}$$

Find $L^{-1}\left[\frac{3s}{2s+9}\right]$

$$L^{-1}\left[\frac{3s}{2s+9}\right] = \frac{3}{2}L^{-1}\left[\frac{s}{s+\frac{9}{2}}\right]$$

$$= \frac{3}{2}\frac{d}{dt}L^{-1}\left[\frac{1}{(s+\frac{9}{2})}\right]$$

$$= \frac{3}{2}\frac{d}{dt}\left(e^{-9/2\,t}\right)$$

$$= -\frac{27}{4}e^{-\frac{9}{2}t}$$

$L^{-1}[s\,F(s)] = \frac{d}{dt}L^{-1}[F(s)]$

$\frac{d}{dt}e^{at} = a\cdot e^{at}$

$L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$

$L^{-1}\left[\frac{1}{s(s^2-2s+5)}\right]$

$$L^{-1}\left[\frac{1}{s(s^2-2s+5)}\right] = \int_0^t L^{-1}\left[\frac{1}{s^2-2s+5}\right]dt$$

$$= \int_0^t L^{-1}\left[\frac{1}{s^2-2s+1+4}\right]dt$$

$$= \int_0^t L^{-1}\left[\frac{1}{(s-1)^2+2^2}\right]dt$$

$$= \int_0^t e^t L^{-1}\left[\frac{1}{s^2+2^2}\right]dt$$

$$= \frac{1}{2}\int_0^t e^t \sin 2t\,.\,dt$$

$$= \frac{1}{2}\left[\frac{e^t}{1+2^2}(\sin 2t - 2\cos 2t)\right]_0^t$$

$$= \frac{1}{2}\left[\frac{e^t}{1+2^2}(\sin 2t - 2\cos 2t) - \frac{e^0}{1+2^2}(0-2)\right]$$

$\int e^{at}\sin bt\,.\,dt = \frac{e^{at}}{a^2+b^2}(a\sin bt - b\cos bt)$

$L^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t L^{-1}[f(t)]dt$

$= \int_0^t f(t)\,dt$

$$= \frac{1}{10}\left[e^{t}(\sin 2t - 2\cos 2t) + 2\right]$$

$$\boxed{\begin{aligned} L^{-1}[F'(s)] &= -t L^{-1}[F(s)] = -t f(t) \\ L^{-1}[F(s)] &= -\frac{1}{t} L^{-1}[F'(s)] = -\frac{1}{t}\frac{d}{ds} L^{-1}[F(s)] \end{aligned}}$$

Find $L^{-1}\left[\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right]$

$$L^{-1}\left[\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right] = -\frac{1}{t} L^{-1}\left[\frac{d}{ds}\left[\log(s^2+a^2) - \log(s^2+b^2)\right]\right]$$

$$= -\frac{1}{t} L^{-1}\left\{\frac{2s}{s^2+a^2} - \frac{2s}{s^2+b^2}\right\}$$

$$= -\frac{2}{t} L^{-1}\left[\frac{s}{s^2+a^2}\right] + \frac{2}{t} L^{-1}\left[\frac{s}{s^2+b^2}\right]$$

$$= -\frac{2}{t}\cos at + \frac{2}{t}\cos bt$$

$$= \frac{2}{t}(\cos bt - \cos at)$$

Find $L^{-1}\left[\frac{s}{(s+2)^2}\right]$

$$L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$$

$$L^{-1}\left[s \cdot \frac{1}{(s+2)^2}\right] = \frac{d}{dt} L^{-1}\left[\frac{1}{(s+2)^2}\right]$$

$$= \frac{d}{dt}\left(e^{-2t} L^{-1}\left[\frac{1}{s^2}\right]\right)$$

$$= \frac{d}{dt}\, e^{-2t} \cdot t$$

$$= e^{-2t}(1) + t(e^{-2t})(-2)$$

$$= e^{-2t} - 2te^{-2t} = e^{-2t}[1-2t]$$

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INVERSE LAPLACE TRANSFORM USING PARTIAL FRACTION METHOD

FACTORS ARE LINEAR AND DISTINCT

$$F(s) = \frac{p(s)}{(s+a)(s+b)} \Rightarrow F(s) = \frac{A}{s+a} + \frac{B}{s+b}$$

FACTORS ARE LINEAR AND REPEATED

$$F(s) = \frac{p(s)}{(s+a)(s+b)^2} \Rightarrow F(s) = \frac{A}{s+a} + \frac{B}{s+b} + \frac{C}{(s+b)^2}$$

FACTORS ARE QUADRATIC AND DISTINCT

$$F(s) = \frac{p(s)}{(s^2+as+b)(s^2+cs+db)}$$

$$F(s) = \frac{As+B}{(s^2+as+b)} + \frac{Cs+D}{(s^2+cs+d)}$$

Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)}$

$$\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$1 = A(s+2) + B(s+1)$$

Sub $s=-2$ $\quad B = -1$

Sub $s=-1$ $\quad A = 1$

$$L^{-1}\left[\frac{1}{(s+1)(s+2)}\right] = L^{-1}\left[\frac{1}{s+1}\right] - L^{-1}\left[\frac{1}{s+2}\right]$$

$$= e^{-t} + e^{-2t}$$

Find the inverse Laplace transform of $\frac{s+2}{s(s+1)(s+3)}$

$$F(s) = \frac{s+2}{s(s+1)(s+3)}$$

$$F(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$s+2 = A(s+1)(s+3) + B(s)(s+3) + C(s)(s+1)$$

When $s=-1$

$$1 = B(-1)(2)$$

$$-\frac{1}{2} = B$$

When $s=-3$

$$-1 = C(-3)(-2)$$

$$\frac{-1}{6} = C$$

When $s=0$

$$2 = A(3)$$

$$A = \frac{2}{3}$$

$$\boxed{\begin{aligned} A &= \frac{2}{3} \\ B &= -\frac{1}{2} \\ C &= -\frac{1}{6} \end{aligned}}$$

$$F(s) = \frac{2}{3s} - \frac{1}{2(s+1)} - \frac{1}{6(s+3)}$$

$$L^{-1}[F(s)] = \frac{2}{3}L^{-1}\left[\frac{1}{s}\right] - \frac{1}{2}L^{-1}\left[\frac{1}{s+1}\right] - \frac{1}{6}L^{-1}\left[\frac{1}{s+3}\right]$$

$$L^{-1}[F(s)] = \frac{2}{3} - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}$$

Find the inverse laplace transform of $\frac{2s}{s^4+4}$

$$F(s) = \frac{2s}{s^4+4}$$

$(a+b)^2 = a^2+b^2+2ab$

$(a+b)^2 - 2ab = a^2+b^2$

$$F(s) = \frac{2s}{(s^2)^2+2^2}$$

$$F(s) = \frac{2s}{(s^2+2)^2 - (4s^2)}$$

$$= \frac{2s}{(s^2+2)^2 - (2s)^2}$$

$$= \frac{2s}{(s^2+2+2s)(s^2+2-2s)}$$

By partial fraction

$$F(s) = \frac{As+B}{s^2+2+2s} + \frac{Cs+D}{s^2+2-2s}$$

$$2s = As+B\,(s^2+2-2s) + (Cs+D)(s^2+2+2s)$$

$s=0$

$0 = 2B + 2D$

$B+D = 0$

$B = -D$.

$s=1$

$2 = A+B + (C+D)(5)$

$2 = A+B+5C+5D$

Comparing co.eff of s^2

$0 = -2A + B + 2C + D$

Comparing co.eff of s^3

$0 = A+C$

Comparing co.eff of s

$2 = 2A - 2B + 2C + 2D$

$2 = 2(A-B+C+D)$

$1 = A-B+C+D$

$1 = A+C - B+D$

$1 = -B+D$

$1 = D+D$

$1 = 2D$

$\boxed{D = \frac{1}{2}}$

$-B+D=1$

$-B+\frac{1}{2}=1$

$-B=1-\frac{1}{2}$

$\boxed{B=-\frac{1}{2}}$

$A+C=0$

$A=-C$

$A+B+5C+5D=2$

$-C+B+5C+5D=2$

$B+4C+5D=2$

$-\frac{1}{2}+4C+\frac{5}{2}=2$

$\cancel{\frac{4}{2}}\,2+4C=2$

$4C=2-2$

$\boxed{C=0}$

$A=-C$

$-A=C$

$\boxed{A=0}$

$$F(s)=\frac{-\frac{1}{2}}{s^2+2+2s}+\frac{\frac{1}{2}}{s^2+2-2s}$$

$$F(s)=-\frac{1}{2(s^2+2+2s)}+\frac{1}{2(s^2+2-2s)}$$

$$F(s)=-\frac{1}{2}L^{-1}\left[\frac{1}{s^2+2+2s}\right]+\frac{1}{2}L^{-1}\left[\frac{1}{s^2+2-2s}\right]$$

$$=-\frac{1}{2}L^{-1}\left[\frac{1}{(s+1)^2+1}\right]+\frac{1}{2}L^{-1}\left[\frac{1}{(s-1)^2+1}\right]$$

$$=-\frac{1}{2}e^{-t}L^{-1}\left[\frac{1}{s^2+1}\right]+\frac{1}{2}e^{+t}L^{-1}\left[\frac{1}{s^2+1}\right]$$

$$=-\frac{1}{2}e^{-t}\sin t+\frac{1}{2}e^{t}\sin t$$

Find the value ...

$f(t)=e^t$

$f(u)=e^u$

$f(t)*g(t)$

$e^t * \sin t$

$=\frac{e^u}{1+1}[1 \sin$ ...

$=\left[\frac{e^t}{2}(1 \sin($ ...

$-\frac{1}{2}(\sin$ ...

$=\frac{1}{2}[e^t -$ ...

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Laplace

CONVOLUTION THEOREM

convolution product

$$L[f(t) * g(t)] = F(s)\,G(s)$$

$$L^{-1}[F(s)\,G(s)] = f(t) * g(t)$$

$$f(t) * g(t) = \int_0^t f(u)\,g(t-u)\,du$$

Find the value of $e^t * \sin t$.

$f(t) = e^t$ $\quad g(t) = \sin t$

$f(u) = e^u$ $\quad g(t-u) = \sin(t-u)$

$$f(t) * g(t) = \int_0^t f(u)\, g(t-u)\, du$$

$$e^t * \sin t = \int_0^t e^u \sin(t-u)\, du$$

$$= \frac{e^u}{1+1}\Big[1 \sin(-u+t) - (-1)\cos(-u+t)\Big]_0^t$$

$$\int e^{ax} \sin(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}\Big[a\sin(bx+c) - b\cos(bx+c)\Big]$$

$$= \Big[\frac{e^t}{2}\big(1 \cdot \sin(-t+t) + \cos(-t+t)\big)$$

$$- \frac{1}{2}\big(\sin(0+t) + \cos(0+t)\big)\Big]$$

$$= \frac{1}{2}\Big[e^t - (\sin t + \cos t)\Big]$$

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$$F(s) = \frac{s}{(s^2+a^2)} \qquad G(s) = \frac{s}{s^2+b^2}$$

$$g(t) = L^{-1}[G(s)]$$

$$f(t) = L^{-1} F(s) = L^{-1}\left(\frac{s}{s^2+a^2}\right)$$

$$= L^{-1}\left[\frac{s}{s^2+b^2}\right]$$

$$= \cos at$$

$$= \cos bt$$

$$f(u) = \cos au \qquad g(u) = \cos bu$$

$$f(t-u) = \cos a(t-u)$$

SOLUTION OF LINEAR ORDINARY DIFFERENTIAL EQUATION OF SECOND ORDER WITH CONSTANT COEFFICIENT USING LAPLACE TRANSFORM TECHNIQUES :

$$L[y''(t)] = s^2 L[y(t)] - s\,y(0) - y'(0)$$

$$L[y'(t)] = s L[y(t)] - y(0)$$

$$L[y(t)] = L(y)$$

Solve the equation $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 2e^{-3t}$, $y(0)=1$ and $y'(0) = -2$ by Laplace transform.

$$L[y''(t)] + 6L[y'(t)] + 9L(y) = 2L(e^{-3t})$$

$$s^2 L[y(t)] - s\,y(0) - y'(0) + 6[s L(y(t)) - y(0)] + 9L[y(t)] = \frac{2}{s+3}$$

$$s^2 L(y(t)) - s \cdot 1 + 2 + 6s\,L[y(t)] - 6 + 9\,L[y(t)] = \frac{2}{s+3}$$

$$L[y(t)]\,(s^2+6s+9) = \frac{2}{s+3} + 4 + s$$

$$y(t) = L^{-1}\left[\frac{2}{(s+3)^3}\right] + L^{-1}\left[\frac{(4+s)}{(s+3)^2}\right]$$

$$y(t) = 2L^{-1}\left[\frac{1}{(s+3)^3}\right] + L^{-1}\left[\frac{s+3+1}{(s+3)^2}\right]$$

$= 2e^{-3t} L^{-1}\left[\frac{1}{s^3}\right] + L^{-1}\left[\frac{s+3}{(s+3)^2}\right] + L^{-1}\left[\frac{1}{(s+3)^2}\right]$

$= 2e^{-3t}\frac{t^2}{2} + L^{-1}\left[\frac{1}{s+3}\right] + e^{-3t} L^{-1}\left[\frac{1}{s^2}\right]$

$= t^2 e^{-3t} + e^{-3t} + e^{-3t} t.$

$= e^{-3t}\left[1+t+t^2\right]$

Solve the equation $y'' + y' - 2y = 3\cos 3t - 11\sin 3t$

$y(0) = 0 \quad y'(0) = 6$ by Laplace transform.

$L[y''(t)] + L[y'(t)] - 2L(y) = 3L(\cos 3t) - 11L(\sin t)$

$s^2 L[y(t)] - s\,y(0) - y'(0) + [s\,L(y(t)) - y(0)] - 2L[y(t)]$

$= \frac{3s}{s^2+9} - \frac{33}{s^2+9}$

$s^2 L(y(t)) - s(0) - 6 + s\,L[y(t)] - 0 - 2L[y(t)]$

$= \frac{3s}{s^2+9} - \frac{33}{s^2+9}.$

$L[y(t)]\,(s^2+s-2) = \frac{3s-33}{s^2+9} + 6$

$L[y(t)] = \frac{3s-33+6(s^2+9)}{(s^2+9)(s^2+s-2)}$

$L[y(t)] = \frac{6s^2+3s+21}{(s^2+9)(s^2+s-2)}$

$$y(t) = L^{-1}\left[\frac{6s^2+3s+21}{(s^2+9)(s^2+s-2)}\right]$$

$$\frac{6s^2+3s+2}{(s^2+9)(s^2+s-2)} = \frac{6s^2+3s+21}{(s^2+9)(s+2)(s-1)}$$

$$\frac{6s^2+3s+2}{(s^2+9)(s+2)(s-1)} = \frac{As+B}{s^2+9} + \frac{C}{s+2} + \frac{D}{s-1}$$

$A = 0 \quad B = 3 \quad C = -1 \quad D = 1$

$$y(t) = L^{-1}\left[\frac{3}{s^2+9} - \frac{1}{s+2} + \frac{1}{s-1}\right] = \sin 3t - e^{-2t} + e^t$$

28/03

Z - TRANSFORM

$$\boxed{Z\{f(n)\} = f(z) = \sum_{n=1}^{\infty} f(n)\, z^{-n}}$$

[one-sided]

1. Find $z[a^n]$

$$Z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$z\{a^n\} = \sum_{n=0}^{\infty} a^n z^{-n} = \sum_{n=0}^{\infty} \frac{a^n}{z^n} = \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

2. Find Z

Z

Fin