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Using Laplace transform evaluate $\int_0^\infty e^{-3t} t^2 \cdot dt$

$$L[f(t)] = \int_0^\infty e^{-st} f(t) \cdot dt$$

$$L[t^n] = \frac{n!}{s^{n+1}}$$

$$L[t^2]_{s \to s+3} = \left[\frac{2!}{s^{2+1}}\right]_{s \to s+3}$$

$$= \left[\frac{2}{s^3}\right]_{s \to s+3}$$

$$= \frac{2}{(3)^3}$$

$$= \frac{2}{27}$$

Using Laplace transform evaluate $\int_0^\infty t e^{-2t} \sin 3t \, dt$

$$L[f(t)] = \int_0^\infty t e^{-2t} \sin 3t \, . dt$$

$$L[f(t)] = -\frac{d}{ds} L[\sin 3t]_{s \to 2}$$

$$= -\frac{d}{ds}\left[\frac{3}{s^2+3^2}\right]_{s\to 2}$$

$$= -\left[\frac{-3}{(s^2+9)^2}(2s)\right]_{s\to 2}$$

$$= \frac{6s}{(s^2+9)^2}$$

$$= \frac{6(2)}{(4+9)^2}$$

$$= \frac{6\times 2}{13\times 13}$$

$$= \frac{12}{169}$$

$$L[y(t)] = L(y) \quad \Big| \quad L\left[\int_0^t f(t)\, dt\right] = \frac{1}{s} L[f(t)]$$

$$L[y'(t)] = s\, L[y(t)] - y(0)$$

$$L[y''(t)] = s^2 L[y(t)] - s\, y(0) - y'(0)$$

If $L[\sin\sqrt{t}] = \frac{\sqrt{\pi} e^{-\frac{1}{4s}}}{2s^{3/2}}$ then Find the L.T of

$$\frac{\cos\sqrt{t}}{\sqrt{t}}$$

$$f(t) = \sin\sqrt{t}$$

$$f'(t) = \frac{1}{2}\frac{\cos\sqrt{t}}{\sqrt{t}} \qquad f(0) = 0$$

$L[f'(t)] = sF(s) - f(0)$

$L\left[\frac{\cos\sqrt{t}}{2\sqrt{t}}\right] = s\,L[\sin\sqrt{t}] - f(0)$

$= s.\ \frac{\sqrt{\pi}}{2s^{3/2}} e^{-1/4s} - 0$

$L\left[\frac{\cos\sqrt{t}}{2\sqrt{t}}\right] = \frac{\sqrt{\pi}}{2s^{1/2}} e^{-1/4s}$

$L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right] = \sqrt{\frac{\pi}{s}}\, e^{-1/4s}$

Find Laplace Transform of $\int_0^t \frac{e^{-t}\sin t}{t} .dt$

$L\left[\int_0^t f(t).dt\right] = \frac{1}{s} L[f(t)]$

$L\left[\frac{f(t)}{t}\right] = \int_s^\infty L[f(t)]\, ds$

$L\left[\int_0^t f(t).\,dt\right] = \frac{1}{2}\int_s^\infty e^{-t}\sin t\, ds$

$= \frac{1}{2}\left[\int_s^\infty L[\sin t]_{s\to s+1}\right] ds$

$= \frac{1}{2}\int_s^\infty \left[\frac{1}{s^2+1}\right]_{s\to s+1}$

$= \frac{1}{2}\left[\int_s^\infty \left[\frac{1}{(s+1)^2+1}\right] ds\right.$

$= \frac{1}{2}\left[\tan^{-1}(s+1)\right]_s^\infty$

$$\boxed{\int \frac{1}{x^2+a^2}\, dx = \tan^{-1}\left(\frac{x}{a}\right)}$$

$= \frac{1}{s}\left[\frac{\pi}{2} - \tan^{-1}(s+1)\right]$

$= \frac{1}{s} \cot^{-1}(s+1)$

$L\left[e^{-t}\int_0^t \frac{\sin t}{t}\, dt\right]$

$L\left[\int_0^t \frac{\sin t}{t} . dt\right]$

$L\left[t \int_0^t e^{-4t} \sin 3t . dt\right]$

DSP paper

Unit step function / Heavyside's unit step function.

$t > a \quad 1$

$t < a \quad 0$

Unit Impulse function

INITIAL VALUE THEOREM:

$L[f(t)] = F(s)$ then $Lt_{t \to 0} f(t) = Lt_{s \to \infty} sF(s)$

Verify initial value theorem for the function $1+e^{-2t}$

$f(t) = 1+e^{-2t}$

$L[f(t)] = F(s) = L[1+e^{-2t}] = L[1] + L[e^{-2t}]$

$F(s) = \frac{1}{s} + \frac{1}{s+2}$

By initial value theorem

L.H.S $\lim_{t \to 0} [f(t)] = \lim_{t \to 0} (1+e^{-2t}) = 1+1 = 2$

R.H.S

$\lim_{s \to \infty} [s F(s)] = \lim_{s \to \infty}\left[s.\left(\frac{1}{s} + \frac{1}{s+2}\right)\right]$

$= \lim_{s \to \infty}\left[\frac{s}{s}\left(\frac{1}{1} + \frac{1}{1+\frac{2}{s}}\right)\right]$

$= [1+1] = 2$

$\boxed{LHS = RHS}$

FINAL VALUE THEOREM :

$L[f(t)] = F(s)$ then $Lt_{t\to\infty} f(t) = Lt_{s\to 0} sF(s)$

Verify ~~initial~~ ~~and~~ final value theorem for the function $1+e^{-t}(\sin t+\cos t)$

LHS $\Rightarrow$ $Lt_{t\to\infty} f(t) = \lim_{t\to\infty} (1+e^{-t}(\sin t+\cos t))$

$= 1+0 = 1$

$L(f(t)) = L[1+e^{-t}(\sin t+\cos t)]$

$= L(1) + L(e^{-t}\sin t) + L(e^{-t}\cos t)$

$= \frac{1}{s} + \frac{1}{(s+1)^2+1} + \frac{s+1}{(s+1)^2+1}$

RHS $= \lim_{s\to 0}\left[\frac{1}{s} + \frac{1}{(s+1)^2+1} + \frac{s+1}{(s+1)^2+1}\right]$

$=$

TRANSFORM OF PERIODIC FUNCTIONS

$$L[f(t)] = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t) \cdot dt$$

$T = 2\pi$ $\quad f(x+T) = f(x)$ is true then

Find the laplace transform of the Half wave rectifier function.

$$T = \frac{2\pi}{\omega}$$

$$L\{f(t)\} = \frac{1}{1-e^{-\frac{2\pi s}{\omega}}} \int_0^{\frac{2\pi}{\omega}} e^{-st} f(t).dt$$

$$= \frac{1}{1-e^{-\frac{2\pi s}{\omega}}} \left[\int_0^{\pi/\omega} e^{-st} E \sin\omega t\, dt + 0 \right]$$

$$= \frac{E}{1-e^{-\frac{2\pi s}{\omega}}} \int_0^{\pi/\omega} e^{-st} \sin\omega t\, dt$$

$\int e^{at} \sin bt\, dt$

$$= \frac{E}{1-e^{-\frac{2\pi s}{\omega}}} \left[\frac{e^{-st}}{s^2+\omega^2} (-s\sin\omega t - \omega\cos\omega t) \right]_0^{\pi/\omega}$$

$$= \frac{E}{1-e^{-2\pi s/\omega}} \left[\frac{e^{-\frac{s\pi}{\omega}}\omega}{s^2+\omega^2} + \frac{\omega}{s^2+\omega^2} \right]$$

$$= \frac{E}{1-e^{-\frac{2\pi s}{\omega}}} \left[\frac{e^{-s\pi/\omega}\omega + \omega}{s^2+\omega^2} \right]$$

$$= \frac{E\omega\left(1+e^{-\frac{\pi s}{\omega}}\right)}{\left(1-e^{-\frac{\pi s}{\omega}}\right)\left(1+e^{-\frac{\pi s}{\omega}}\right)(s^2+\omega^2)}$$

$$\boxed{\frac{1-e^{-x}}{1+e^{-x}} = \tanh\left(\frac{x}{2}\right)}$$

$$f(t) = \begin{cases} t & 0 < t < 2 \\ 4-t & 2 \le t \le 4. \end{cases}$$