Project 2: Measuring Planetary Characteristics

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Section 1: Introduction and Motivation

Detecting exoplanets is an extremely daunting task. Not only is their light washed out by the blinding light of their stars, but their masses are also a small percentage of their systems' total mass. This requires astronomers to employ novel methods to reliably detect exoplanets. Two of these methods are the radial velocity and transit methods. In this paper we will explore these two methods in detail by looking at raw stellar data and employing various methods to extract exoplanet characteristics. By better understanding these methods, we get a better understanding of our detection limits and biases.

We obtain our raw data from the NASA Exoplanet Archive (NEA), which contains a catalog of confirmed exoplanets as well as their stellar data. For this project, we are specifically looking at the star system, GJ 436, which contains an exoplanet dubbed GJ 436 b. When the planet was discovered in 2004, GJ 436 was only the second M Dwarf star found to host a planet, but at the time GJ 436 b's mass and radius were fairly uncertain. We will specifically be looking at the photometric light curve of the system's star for the transit method, as well as the radial velocity curve for the radial velocity method, in order to constrain the mass and radius of the planet as astronomers did over time.

Section 2: Methods

We analyzed the radial velocity curve of GJ 436. The radial velocity of a star can be determined by the doppler shift of the star's light. If an object orbiting the star exerts strong enough gravitational attraction on the star, the star will have a measurable wobble as it orbits the system's center of mass. This wobble will result in the light of the star being blue shifted then redshifted in a periodic manner. This gives us an idea of the star's motion along our line of sight. If this is a simple gravitational two-body problem, we can use the laws of Keplerian motion to figure out the mass of the object causing the wobble. We end up with our measurement signal being given by,

$$K = \frac{M_p}{M_*}\sqrt{\frac{GM_*}{a}\sin(i)} \qquad (1)$$

where K is the semi amplitude of our signal in m/s, M_p and M_* are the mass of the planet and star respectively, a is the semi-major axis of the planet and i is the inclination angle of the planet's orbit. Since the inclination angle is often extremely difficult to figure out, masses using the radial velocity are minimum masses given by rearranging the equation above to get,

$$M_p sin(i) = KM_*\sqrt{\frac{a}{GM_*}} \qquad (2)$$

ut in our case, since the planets are also transiting, the inclination must be around 90°. We first extracted the periodicity of our planet by creating a Lomb-Scargle periodogram using the *scipy.signal* package. The periodogram will return a set of frequencies each with a corresponding power. If a planet is present, we will see a peak in the power at a frequency that corresponds to that planet's period. With the period, we can calculate the semi-major axis by using Kepler's Laws. We also found the semi amplitude of our signal by using the EXOFAST data fitting which also returned an uncertainty associated with *K*.

To obtain the radius of our planet, we needed to use the transit method. When observing a star, there's a possibility that a planet will pass in front of the star along our line of sight which will cause a dip in the flux received from the star. If the whole planet passes in front of the star, the planet will block out the star's flux by the planet's area. Then the depth of transit, *f*, will be given by the following,

$$f = \left(\frac{R_p}{R_*}\right)^2 \tag{3}$$

where R_p is the radius of the planet, and R_* is the radius of the star. So, given that a planet passes directly in front of the star and causes a dip large enough for a telescope to detect, we can effectively calculate the radius of the planet.
We analyzed the photometric light curve of GJ 436 using the *pylightcurve* package in python. We specifically make use of the package's integrated transit model to obtain an array of fluxes that best fit our photometric data. With this model of best fit, we can get an accurate value for *f*, the transit depth, thereby giving us a value for the planet's radius. We can also estimate the uncertainty in our radius measurement from the *pylightcurve* fit. Specifically, we take the array of data points when the transit is that full depth and find the standard deviation.

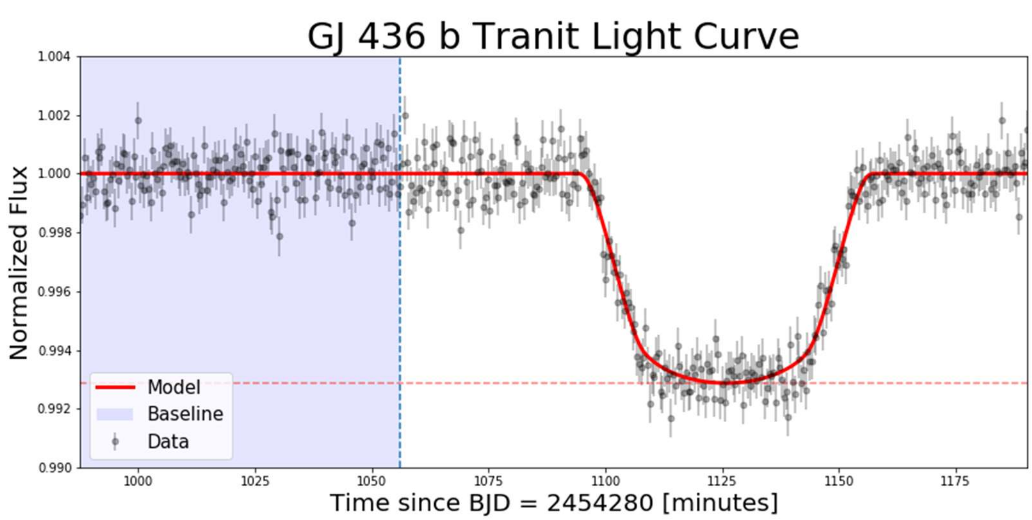
Now that we have both the planet's mass and radius along with their uncertainties, we can effectively calculate its density. The density is going to be given by,

$$\rho = \frac{3M_p}{4\pi {R_p}^3} \tag{4}$$

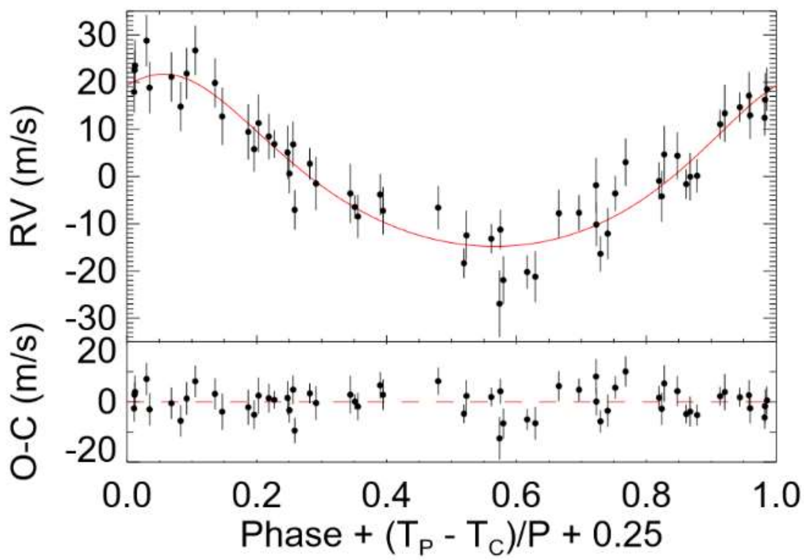
The uncertainty in the density can be computed by propagating the errors from the radius and mass. Since these two variables are divided by one another, and since radius is cubed, the fractional error is calculated as:

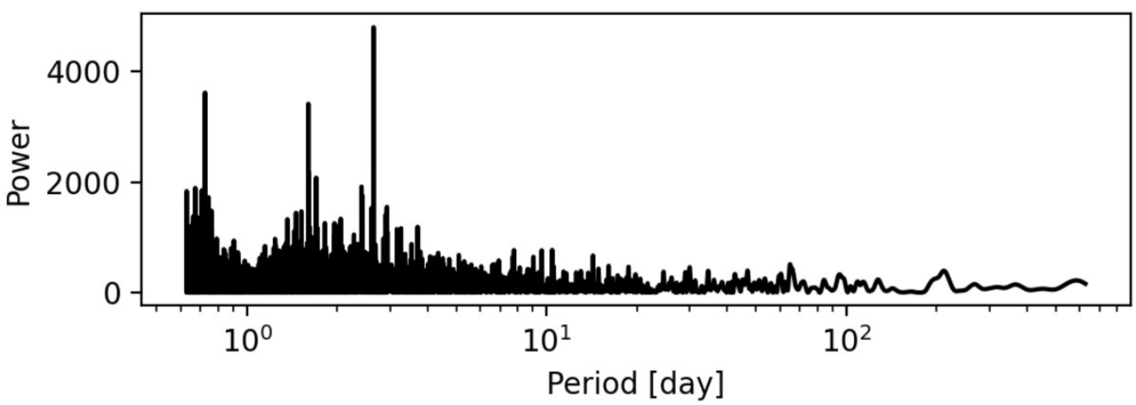
$$\left(\frac{\delta\rho}{\rho}\right) = \left(\frac{\delta M}{M}\right) + 3\left(\frac{\delta R}{R}\right) \tag{5}$$

This value is then multiplied by the calculated density to derive the true error.
<u>Section 3: Results</u>



After analyzing the transit data, the final radius of GJ 436 b was determined to be 24979.308 km ± 70.542 km. The uncertainty was determined using the baseline data outside of the transit, shown in blue. The minimum flux detected occurs at 99.28% of the baseline, depicted by the horizontal red line.

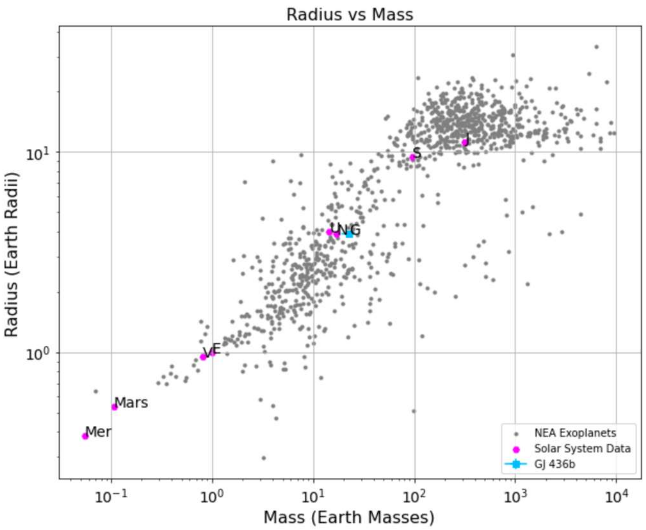


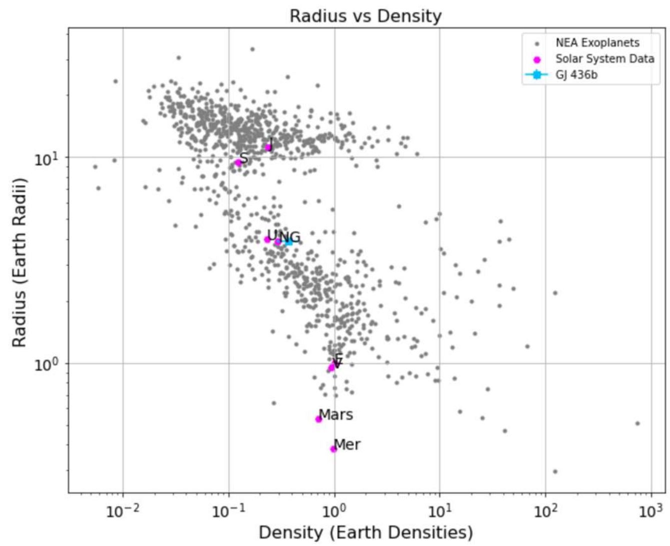


After analyzing the radial velocity data, we obtain a semi-amplitude of, *K = 18.24 ± 2.7 m/s* and other planetary parameters. EXOFAST gives a warning that it is not accurate when using star's with a mass less than 60% of the Sun which could possibly account for the relatively large uncertainty. We also obtained the periodicity of *2.64 days* when looking at the Lomb-Scargle periodogram. Using the planetary parameters from EXOFAST we calculated the mass of GJ 436b to be, $M_p = 22.84 \pm 3.38\ M_\oplus$ using the equation for Msin(i) from above. Additionally, we used the python package *radvel* which gave us a mass of $M_p = 22.44 \pm 3.33\ M_\oplus$, a value quite close to our EXOFAST result.

Density is calculated by using equation 4 above, and the uncertainty calculated with equation 5. Using a simple Python function to do this calculation, we have a density of 0.374 Earth densities. Using the equation, the uncertainty is 15.6% of the density value, or .058 earth densities.

However, planetology is a comparative science, and to understand whether our values really make sense, we need to compare them to a broader population.

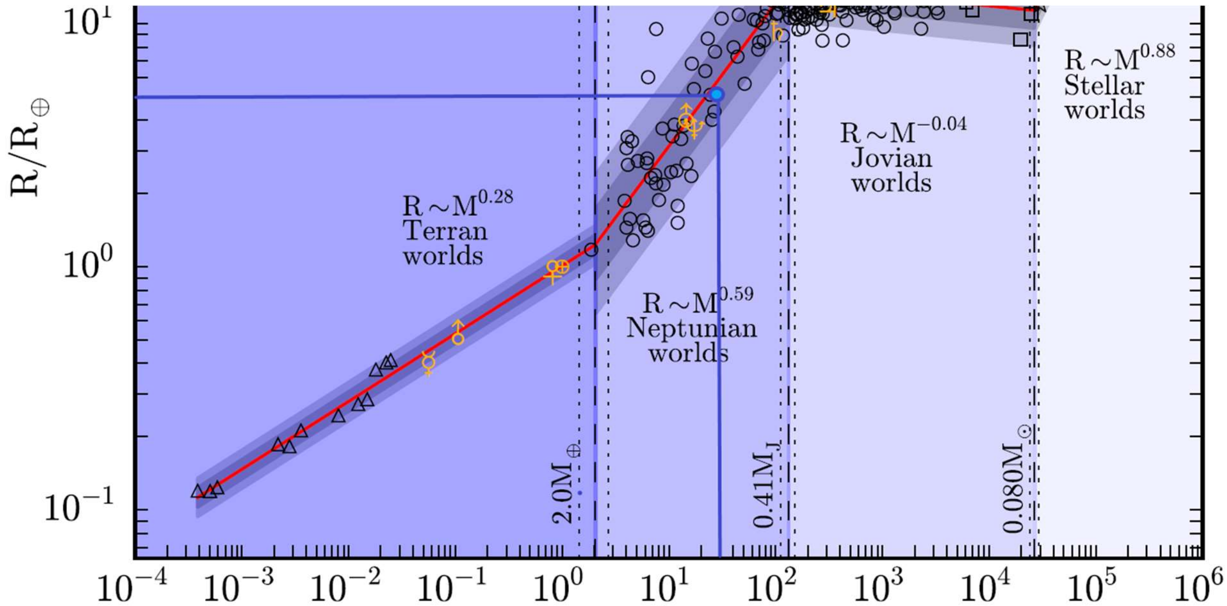




The radius vs. mass plot shows the planets with known mass and radius in the NEA data set of confirmed exoplanets. For comparison, it also shows the planets in our solar system. As we can see, the mass and radius do indeed place this planet around Uranus and Neptune.

For those planets whose mass and radius are known, we can also calculate their density. The radius vs. density plot shows the known densities of the planets in the NEA sample, along with the planets in our solar system, calculated using formula #4. We have plotted density vs radius, and in this we can see how in general the smaller planets are more dense and rocky, while the larger gas giant planets are more fluffy and less compact. We can see that GJ 436 b lies in between the extremes, smaller than Jupiter and less dense than Earth. This again is what we expect for a Neptunian planet.

Having made comparison to the other planets in the NEA sample, we now require some way to check that our mass measurement and radius measurement, which were both derived from separate methods of observation, fit together and make sense with one another. For this, we turn to a mass/radius relationship outlined in Chen & Kipping (2016). These authors analyzed a sample of 316 well-considered objects, and by comparing the characteristics of these worlds, they developed a broken power law which describes the mass/radius relationship we might expect to see for different types of exoplanets. They developed this power law into a Python package called “Forecaster,” which predicts the mass (or radius) of an object given its radius (or mass). A section of Figure 3 from this paper is shown below, with our determined mass and radius plotted against the M-R line:



Our data point falls just below the line within a reasonable margin of error. For a Neptunian world, they have observed that the Radius should be equal to mass to the 0.59^{th} power. Our mass of 22.52 is equivalent to 1.313 Neptune masses and taking this to the 0.59^{th} power gives 1.174 Neptune radii, or 4.56 Earth radii, which is reasonably close to our expected radius of 3.916. This is evidence that the mass derived from our RV curve and the radius derived from our transit curve match one another fairly well and make sense physically.

Section 4: Conclusion and Discussion

Using various techniques, we were able to constrain GJ 436b's mass and radius. We were then able to compare our measurements with other similar exoplanets and found that our planet was more dense than gas giants but less dense than rocky planets, suggesting that GJ 436b was a Neptune-like world. This was further supported by finding the mass to radius relationship of our planet. Exoplanets that can be detected via both the transit and radial velocity methods are incredibly important as they allow us to compute the density of the planet, thereby giving us a better idea of the planet type, as well as the distribution of different types of planets.

References

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