Project 3 - Measuring Stellar Elemental Abundance

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Section 1: Introduction and Motivation

Looking at the stars in our universe is interesting for a variety of reasons. One reason why analyzing a star's spectrum deserves special attention is that it gives us important information about its composition, information which not only tells us about the star itself but also the planets around it. Stars make up the vast majority of the mass of their system and the composition of a star could allow us to draw conclusions about the composition of its planets. With this in mind, in this project, we look to determine the solar abundance of sodium and magnesium based on the solar spectrum.

Section 2: Methods

We use the curve of growth method to determine the stellar abundance of sodium and magnesium. We begin by examining the respective spectral lines, looking at the light wavelengths generated as electrons within the atom transition from the higher to lower energy subshell. Based on the area inside the curve of the spectral line, we can draw a rectangle of equivalent width that has the same area and the height of the continuum emission.

With a curve of growth plot (Figure 1), we find the number of atoms in the absorbing state using the following equations:

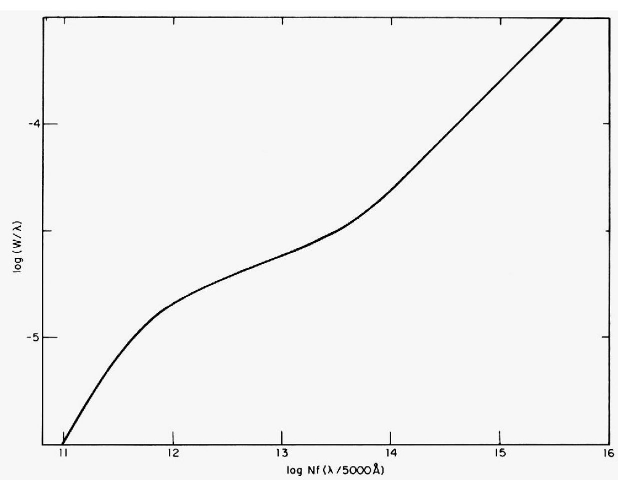

Figure 9.22 A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)

Figure 1

$$log(N f(\lambda/5000\mathring{A})) \sim 14.8 \qquad N = \frac{10^{14.8}}{f \exp\left(\frac{\lambda}{5000\mathring{A}}\right)} \qquad (1)$$

In the equation, f refers to the oscillator strength and λ is the wavelength in Ångström.

Next, we use the Boltzmann equation to determine the ratio of atoms in the ground state to those in excited states.

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\exp\left(-\frac{E_2 - E_1}{kT}\right) \quad (2)$$

Here, N_2 refers to the atoms in excited states and N_1 to those in ground states, g_1, and g_2 are the number of individual states which are degenerate in energy for *the 3s and 3p* states. *A neutral atom of sodium holds only one electron in its third orbital; in an excited state, that one electron would jump from the s subshell to the p subshell, occupying one of the available energy states there. If we define the s subshell as having 2 available energy states, then the p subshell is defined as having 6.*[1] E_1 and E_2 are the energies for those states in Joules, k is the Boltzmann constant (1.380649×10^{-23} $kg{*}m^2/s^2{*}K$) and T is the temperature of the gas in Kelvin, which in this instance we can take to be the effective temperature of the star.

Next, we use the Saha equation to determine the ratio of neutral to ionized atoms.

$$\frac{N_{II}}{N_I} = \frac{2kT}{P_e}\frac{Z_{II}}{Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(-\frac{\chi}{kT}\right) \quad (3)$$

N_{II} symbolizes the ionized atoms while N_I refers to the neutral ones. N_I is also equal to the sum of N_1 and N_2 from the previous equation. We're given an electron pressure (P_e) of 1.0 N/m^2 and need the ionization energy (χ), and partition functions (Z_I and Z_{II}) for the respective element.

After this, we use the three previously determined values of number density, the ratio of excited atoms, and the ratio of ionized atoms to calculate the total column density.

$$N_1 \times \left(1 + \frac{N_2}{N_1}\right) \times \left(1 + \frac{N_{II}}{N_I}\right) \qquad (4)$$

Lastly, we compute the abundance relative to hydrogen and convert the value into the three different categories used by physicists, stellar astronomers, and galactic astronomers, using the equation formats in Figure 2 where oxygen is used as an example.

Physicist	Astronomers working galaxies	Astronomers working on stars
Mole ratio between O and H	Log of mole ratio between O and H and add an offset of 12	$log\left(\frac{N_O/N_H}{\left(N_O/N_H\right)_\odot}\right)$
N_O/N_H	12 + log(O/H)	[O/H]

Figure 2

Section 3: Results

3.1 Sodium

We use the 5896 line of the sodium doublet lines for the calculations of the different number densities. The red lines in the plot are used as a reference and to limit the area for the rectangle of equivalent width.

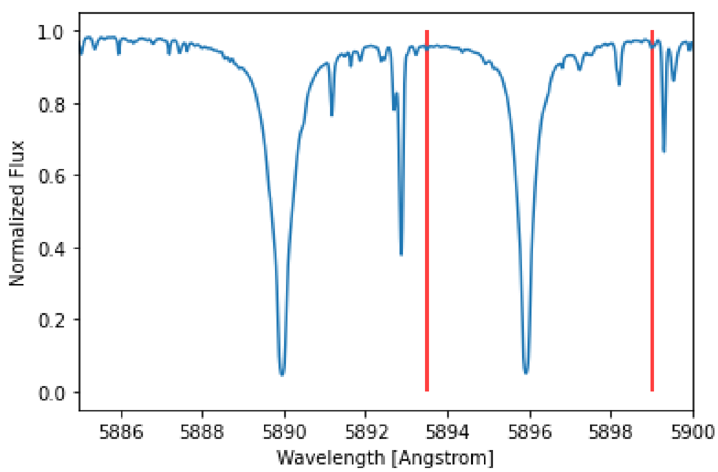

Figure 3

We calculate the equivalent width of a rectangle with the height of the continuum spectrum to be 0.7123 Ångström so that its area is equal to that of the spectral line, using the method described in section 2.

Turning to the Boltzmann equation (2), for a sodium atom, the difference in energy between the 3s subshell (-5.14 eV) and the 3p subshell (-3.04) is 2.1 eV, or 3.365 x 10^{-19} Joules.[2] We use this value in the exponent of the equation. We define the effective temperature of the sun as 5780 Kelvin,[3] which is useful as an approximation of the temperature of the sun's photosphere. Entering these values into the Boltzmann equation, we arrive at a ratio of excited sodium atoms to ground state sodium atoms of 0.044 (4.4%).

Next, we solve for the Saha equation (3). Using ionization energy of 5.1 eV and values of 2.4 and 1.0 for the partition functions Z_I and Z_{II}, we find a ratio of 2522 ionized sodium atoms in the sun's photosphere for every 1 neutral atom. This makes sense, as most heavy atoms such as sodium would be ionized in such a high-temperature environment, and sodium is prone to ionization given the single electron in its outer shell.

Putting these values together with the number of sodium atoms in an absorbing state, which we calculated earlier as 6.54×10^{14} atoms per square centimeter, we compute the column density of sodium atoms in the photosphere (4) as 1.72×10^{18} atoms per cm^2. Comparing this value to hydrogen, we find a mole ratio of 2.61×10^{-6}, which corresponds to an abundance of 6.42. This is quite close to the literature value of 6.3.[4]

3.2 Magnesium

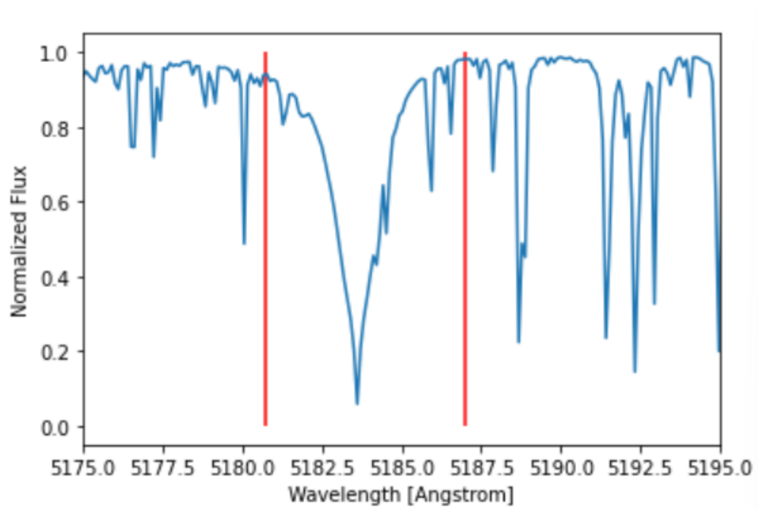

Figure 4

For our magnesium abundance calculation, we used the magnesium triplet lines. Specifically, we used the b1 line which occurs at a wavelength of 5183.6 Å. We calculated the equivalent width of the absorption to be 1.73 Å which corresponds to a number density of 1.87×10^{15} ground-state magnesium atoms per cm^2. Next, we use the Boltzmann equation (2) with the only difference from the sodium case being the difference in the ground state and excited state energy which can be found from the wavelength of the transition. We calculated the ratio of atoms in the excited state to the ground state to be 0.025.

We then attempted to compute the ratio of ionized magnesium to neutral magnesium by using the Saha equation (3). The electron pressure is the same as the sodium case but we now have an ionization energy of 7.65 eV. We also expect the partition function of magnesium to be different from sodium, but repeated attempts to calculate the partition function resulted in incorrect values, so we used the sodium values for an approximation. This gave us a very low value of 15.1 for our ionization ratio. Given the temperature of the sun, we expect this value to be much higher. Due to this low ionization ratio, our column density was only 3.1×10^{16} atoms per cm^2. Comparing this value to hydrogen, we find a mole ratio of 4.66×10^{-8} which corresponds to an abundance of approximately 6.1. As expected, this is much lower than the actual solar value which is about 7.54. Our value could be ameliorated by using the proper values in the Saha equation.

Section 4: Discussion and Conclusion

In this project, we worked to calculate chemical abundance values for two elements within the sun's photosphere, sodium (Na) and magnesium (Mg). For sodium, we were able to calculate a value that was a reasonable approximation to the value in the literature. What we can see from our calculations is that most of the sodium in the sun is not in an excited state, but heavily ionized, and that even though the column density seems impressive it is still almost diminishingly small compared to the column density of hydrogen. For magnesium, we unfortunately did not find the correct partition function and were therefore unable to calculate the correct value of its abundance. What we do see in our comparison with the literature, however, is that magnesium is more abundant in the sun than sodium, despite being a heavier element. We also see that once again, very little of it is in an excited state, indicating once again that most of it is either neutral or ionized. As mentioned in the beginning, we would expect that the planets in our solar system would have comparable chemical abundances as the sun.

Contribution Statement:

Andy Miller worked out the Boltzmann and Saha equation, and the column density for sodium, and assisted in calculating values for magnesium.

Karish Seebaluck calculated the equivalent width and number of atoms in the absorbing state for sodium and magnesium and worked out the Boltzmann and Saha equation(kinda), and the column density for magnesium.

Leandra Hogrefe wrote the paper.

Jacob Borinson created and presented the presentation.

References

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3. Reinhold et al, “The Sun is less active than other solar-like stars,” Science, Volume 368, Issue 6490, pp. 518-521 (2020)
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