

## Homework 6 (Part 1)

● Graded

### Student

Yugam Surana

### Total Points

67 / 74 pts

### Question 1

Probability, Part I

14 / 14 pts

1.1 (no title)

7 / 7 pts

✓ + 7 pts Correct

+ 0 pts Incorrect

1.2 (no title)

7 / 7 pts

✓ + 7 pts Correct

+ 0 pts Incorrect

### Question 2

Probability, Part II

7 / 14 pts

2.1 (no title)

7 / 7 pts

✓ + 7 pts Correct

+ 0 pts Incorrect

2.2 (no title)

0 / 7 pts

+ 7 pts Correct

✓ + 0 pts Incorrect

### Question 3

Probability, Part III

14 / 14 pts

✓ + 14 pts Correct

+ 0 pts Incorrect

### Question 4

Chain Rule

16 / 16 pts

✓ + 16 pts Correct

+ 0 pts Incorrect

### Question 5

#### Bayes' Nets and Probability

16 / 16 pts

5.1 (no title)

11 / 11 pts

✓ + 11 pts Correct

+ 0 pts Incorrect

5.2 (no title)

5 / 5 pts

✓ + 5 pts Correct

+ 0 pts Incorrect

### Question 6

#### Bayes' Nets Independence (Optional)

0 / 0 pts

✓ + 0 pts Correct

+ 0 pts Incorrect

### Question 7

#### D-Separation (Optional)

0 / 0 pts

✓ + 0 pts Correct

+ 0 pts Incorrect

### Question 8

#### Feedback

0 / 0 pts

✓ + 0 pts Correct

### Q1 Probability, Part I

14 Points

Below is a table listing the probabilities of three binary random variables. Fill in the correct values for each marginal or conditional probability below.

$X_0$	$X_1$	$X_2$	$P(X_0, X_1, X_2)$
0	0	0	0.160
1	0	0	0.100
0	1	0	0.120
1	1	0	0.040
0	0	1	0.180
1	0	1	0.200
0	1	1	0.120
1	1	1	0.080

#### Q1.1

7 Points

$$P(X_0 = 1, X_1 = 0, X_2 = 1)$$

0.2

$$P(X_0 = 0, X_1 = 1)$$

0.24

$$P(X_2 = 0)$$

0.42

**Q1.2**

**7 Points**

$$P(X_1 = 0 \mid X_0 = 1)$$

0.714

$$P(X_0 = 1, X_1 = 0 \mid X_2 = 1)$$

0.345

$$P(X_0 = 1 \mid X_1 = 0, X_2 = 1)$$

0.526

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## Q2 Probability, Part II

14 Points

You are given the prior distribution  $P(X)$ , and two conditional distributions  $P(Y | X)$  and  $P(Z | Y)$  as below (you are also given the fact that  $Z$  is independent from  $X$  given  $Y$ ).

All variables are binary variables.

Compute the following joint distributions based on the chain rule.

$X$	$P(X)$	$Y$	$X$	$P(Y X)$	$Z$	$Y$	$P(Z Y)$
0	0.500	0	0	0.600	0	0	0.100
1	0.500	1	0	0.400	1	0	0.900
		0	1	0.900	0	1	0.700
		1	1	0.100	1	1	0.300

### Q2.1

7 Points

$$P(X = 0, Y = 0)$$

0.3

$$P(X = 1, Y = 0)$$

0.45

$$P(X = 0, Y = 1)$$

0.2

$$P(X = 1, Y = 1)$$

0.05

Q2.2

7 Points

$$P(X = 0, Y = 0, Z = 0)$$

$$P(X = 1, Y = 1, Z = 0)$$

$$P(X = 1, Y = 0, Z = 1)$$

$$P(X = 1, Y = 1, Z = 1)$$

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### Q3 Probability, Part III

14 Points

For each of the following four subparts, you are given three joint probability distribution tables. For each distribution, please identify if the given independence / conditional independence assumption is true or false.

For your convenience, we have also provided some marginal and conditional probability distribution tables that could assist you in solving this problem.

$X$	$Y$	$P(X, Y)$
0	0	0.240
1	0	0.160
0	1	0.360
1	1	0.240

$X$	$P(X)$
0	0.600
1	0.400

$Y$	$P(Y)$
0	0.400
1	0.600

$X$  is independent from  $Y$ .

☒ True

☐ False

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$X$	$Y$	$P(X, Y)$
0	0	0.540
1	0	0.360
0	1	0.060
1	1	0.040

$X$	$P(X)$
0	0.600
1	0.400

$X$	$Y$	$P(X Y)$
0	0	0.600
1	0	0.400
0	1	0.600
1	1	0.400

$X$  is independent from  $Y$ .

☒ True

☐ False

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$X$	$Y$	$Z$	$P(X, Y, Z)$
0	0	0	0.280
1	0	0	0.070
0	1	0	0.210
1	1	0	0.140
0	0	1	0.060
1	0	1	0.060
0	1	1	0.030
1	1	1	0.150

$X$	$Z$	$P(X Z)$
0	0	0.700
1	0	0.300
0	1	0.300
1	1	0.700

$Y$	$Z$	$P(Y Z)$
0	0	0.500
1	0	0.500
0	1	0.400
1	1	0.600

$X$	$Y$	$Z$	$P(X, Y Z)$
0	0	0	0.400
1	0	0	0.100
0	1	0	0.300
1	1	0	0.200
0	0	1	0.200
1	0	1	0.200
0	1	1	0.100
1	1	1	0.500

$X$  is independent from  $Y$  given  $Z$ .

☐ True

☒ False

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$X$	$Y$	$Z$	$P(X, Y, Z)$
0	0	0	0.140
1	0	0	0.140
0	1	0	0.060
1	1	0	0.060
0	0	1	0.048
1	0	1	0.192
0	1	1	0.072
1	1	1	0.288

$X$	$Z$	$P(X Z)$
0	0	0.500
1	0	0.500
0	1	0.200
1	1	0.800

$Y$	$Z$	$P(Y Z)$
0	0	0.700
1	0	0.300
0	1	0.400
1	1	0.600

$X$	$Y$	$Z$	$P(X, Y Z)$
0	0	0	0.350
1	0	0	0.350
0	1	0	0.150
1	1	0	0.150
0	0	1	0.080
1	0	1	0.320
0	1	1	0.120
1	1	1	0.480

$X$  is independent from  $Y$  given  $Z$ .

☒ True

☐ False

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#### Q4 Chain Rule

16 Points

Select all expressions that are equivalent to the specified probability using the given independence assumptions.

Given no independence assumptions,  $P(A, B \mid C) =$

☐  $\frac{P(C|A)P(A|B)P(B)}{P(C)}$

☐  $\frac{P(B,C|A)P(A)}{P(B,C)}$

☒  $P(A \mid B, C)P(B \mid C)$

☐  $\frac{P(A|C)P(B,C)}{P(C)}$

Given that A is independent of B given C,  $P(A, B \mid C) =$

☐  $\frac{P(C|A)P(A|B)P(B)}{P(C)}$

☐  $\frac{P(B,C|A)P(A)}{P(B,C)}$

☒  $P(A \mid B, C)P(B \mid C)$

☒  $\frac{P(A|C)P(B,C)}{P(C)}$

Given no independence assumptions,  $P(A \mid B, C) =$

☐  $\frac{P(C|A)P(A|B)P(B)}{P(C)}$

☒  $\frac{P(B,C|A)P(A)}{P(B,C)}$

☐  $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$

☒  $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

Given that A is independent of B given C,  $P(A \mid B, C) =$



$$\frac{P(C|A)P(A|B)P(B)}{P(C)}$$



$$\frac{P(B,C|A)P(A)}{P(B,C)}$$



$$\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$$

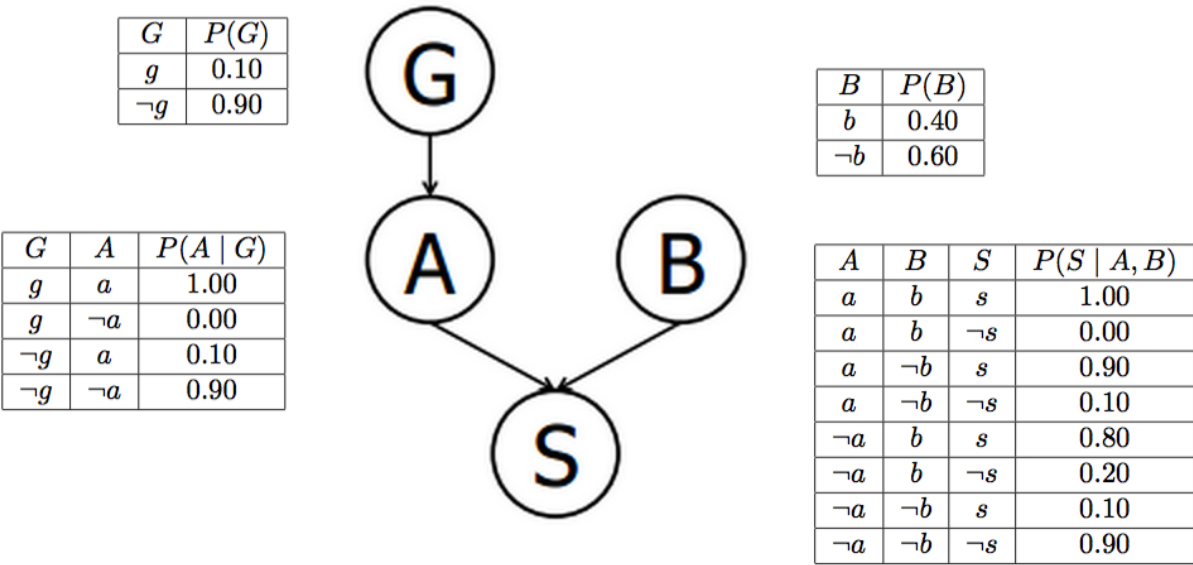


$$\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$$

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Q5 Bayes' Nets and Probability  
16 Points

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding probability tables for this situation are shown below.



**Q5.1****11 Points**

Compute  $P(g, a, b, s)$ .

0.04

What is the probability that a patient has disease A?

0.19

What is the probability that a patient has disease A given that they have disease B?

0.19

What is the probability that a patient has disease A given that they have symptom S and disease B?

0.227

**Q5.2****5 Points**

What is the probability that a patient has the disease carrying gene variation G given that they have disease A?

0.526

What is the probability that a patient has the disease carrying gene variation G given that they have disease B?

0.1

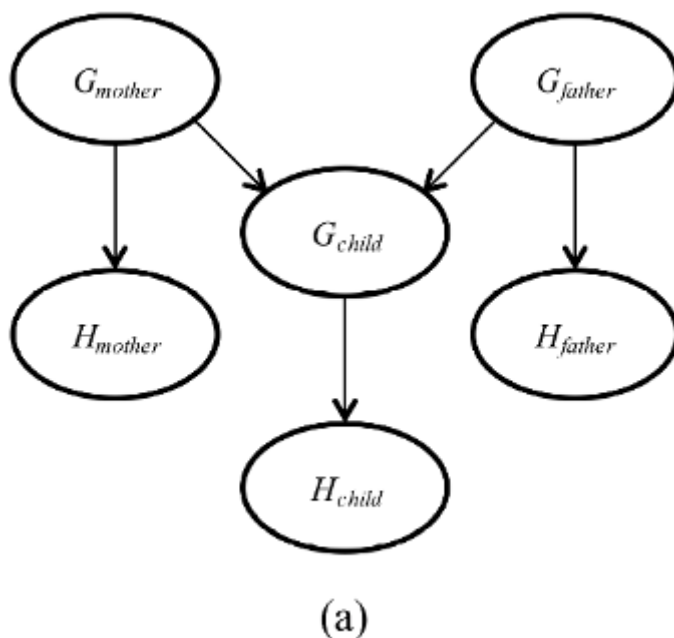
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### Q6 Bayes' Nets Independence (Optional)

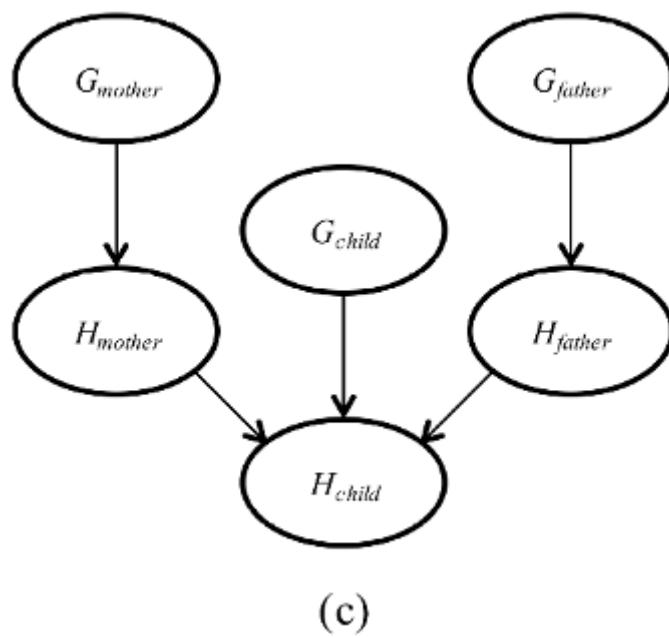
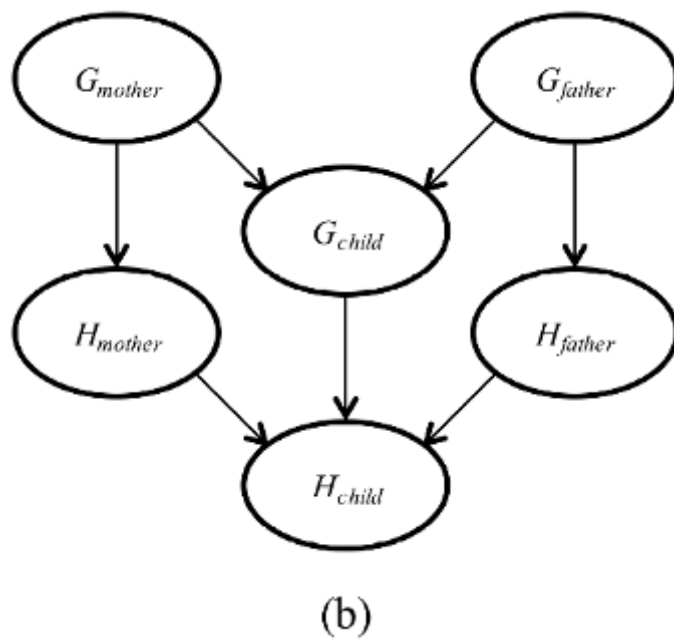
0 Points

Let  $H_x$  be a random variable denoting the handedness of an individual  $x$ , with possible values  $l$  or  $r$ . A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene  $G_x$ , also with values  $l$  or  $r$ , and perhaps actual handedness turns out mostly the same (with some probability  $s$ ) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability  $m$  of a random mutation flipping the handedness.

The following three images are possible models involving the genes  $G$  and handednesses  $H$ .







Which of the three networks above claim that  $P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$ ?

☐ (a)

☐ (b)

☐ (c)

Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

☐ (a)

☐ (b)

☐ (c)

Which of the three networks is the best description of the hypothesis?

☐ (a)

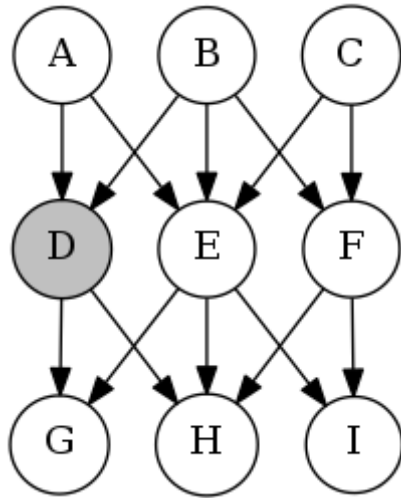
☐ (b)

☐ (c)

### Q7 D-Separation (Optional)

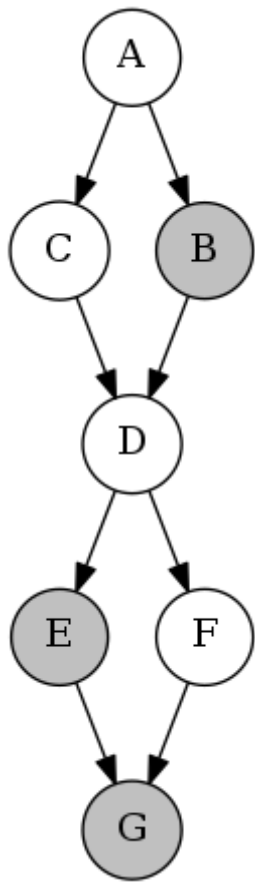
0 Points

You are given several graphical models below, and each graphical model is associated with an independence (or conditional independence) assertion. Please specify if the assertion is true or false.



It is guaranteed that  $G$  is independent of  $H$  given  $D$

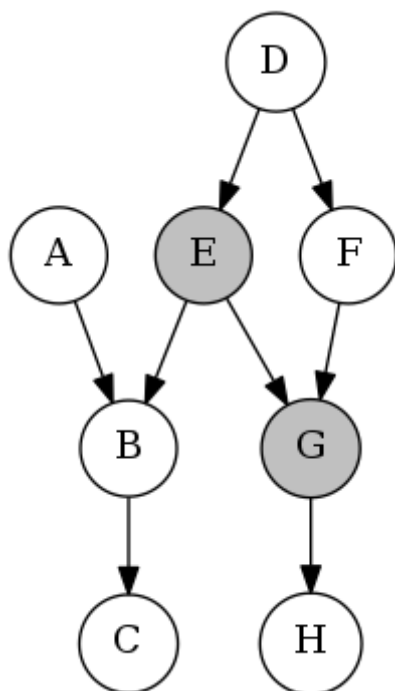
- ☐ True
- ☐ False
-



It is guaranteed that  $A$  is independent of  $D$  given  $E, B, G$

☐ True

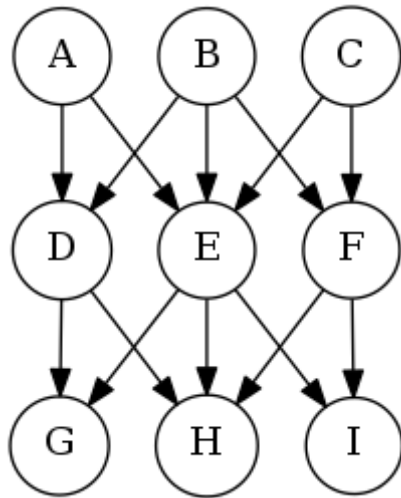
☐ False



It is guaranteed that  $H$  is independent of  $B$  given  $G, E$

☐ True

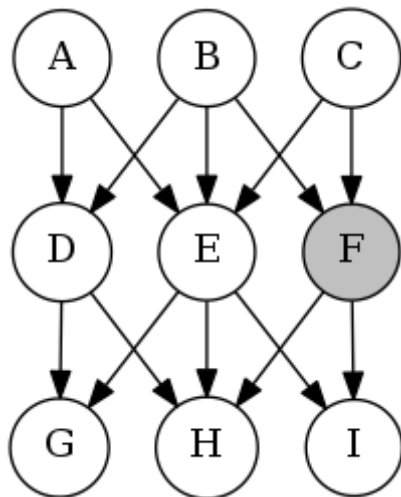
☐ False



It is guaranteed that  $A$  is independent of  $C$

☐ True

☐ False

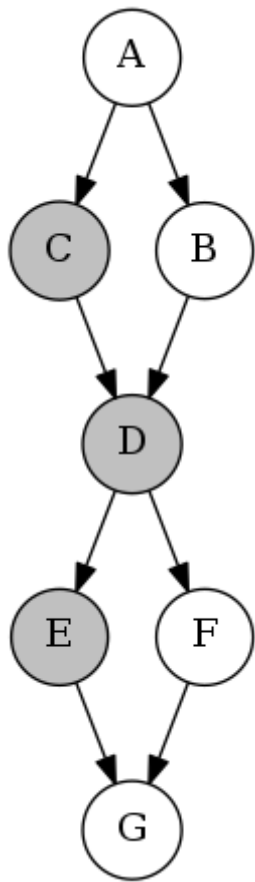


It is guaranteed that  $D$  is independent of  $C$  given  $F$

☐ True

☐ False

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It is guaranteed that  $G$  is independent of  $B$  given  $C, E, D$

☐ True

☐ False

### Q8 Feedback

0 Points

Optionally, you can provide any feedback for the course that you have here.  
Anything you write here will not affect your grade and will stay anonymous.

Good homework

