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Image analysis of changes in surface color of chocolate

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Abstract

Blooming or the migration of fat to the surface of chocolate results in color changes and development of non-uniform color patterns. These phenomena were assessed during storage of milk chocolate tablets (cycling temp. between 16 and 28 °C for 52 days) by a computer vision system and image analysis. Eight features were extracted from images (L^*, a^* and b^* values, whiteness index, chroma, hue, % bloom and energy of Fourier). Major changes occurred after day 36 of storage, coincidental with visual perception. Initially, white specks emerged on the brown background but were superseded by the development of a whitish color extending over most of the surface. L^*, whiteness index, a^* and chroma correlated well with values taken with a commercial colorimeter ($R^2 > 0.70$). Changes in image texture (energy of Fourier) followed a similar trend as color changes. The sequential forward selection strategy allowed correct classification of 97.8% of samples into four classes with only five features. The computer vision system has the capability to quantify overall changes as well as particular features over the whole chocolate surface thus enabling customization and standardization for quality assessment.

Keywords: Chocolate; Bloom; Color; Image analysis; Computer vision; Classification

1. Introduction

Chocolate blooming induced by exposure to high ambient temperatures involves a gradual change in color and loss of gloss giving a grayish appearance to the surface of chocolate. Blooming is a major quality problem for the chocolate industry (ca. $70 billion annual sales) after products leave the factory. The exact causes of blooming are still ignored but temperature fluctuations and improper tempering conditions promote fat migration through the particle matrix of chocolate and subsequently recrystallization on the surface. The surface-dulling appearance of bloom is due to the scattering of light by clusters of fat crystals (5 μm or greater) that protrude from the surface of the chocolate (Aguilera, Mayor, & Michel, 2004; Lohman & Hartel, 1994).

Bloom has been assessed by color techniques using a spot colorimeter (Bricknell & Hartel, 1998) or simply by visual inspection (Ali, Selamat, Che Man, & Suria, 2001). However, the decolorization of the surface of chocolate tablets is not homogeneous but emerges in random color patterns. Commercial colorimeters function over small areas chosen by an operator (normally 2–5 cm^2) present obvious limitations when non-homogeneous color patterns arise. Sometimes this shortcoming is circumvented by taking the average of several measurements over the entire surface. Alternatively, samples are homogenized using a blender to achieve a uniform color, but this method destroys usable information (e.g., color patterns) and render the sample unusable for longitudinal time studies. Previous approaches may be satisfactory for

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quality control of samples having more or less homogeneous colors but not for engineering analysis of patterned colored surfaces.

Computer vision is an alternative technique for color evaluation and quantification (Yam & Papakadis, 2004). Basically, a computer vision system generally consists of five basic components: the illuminant, a digital camera, an image capture board (frame grabber or digitizer), computer hardware and a software to process the images. A standardized lighting system is crucial to reduce reflection, shadows and some noise, thus enhancing the image quality and assuring repeatability. For high quality, a high-resolution charge-coupled-device (CCD) camera is normally used in conjunction with a frame grabber to capture a color image of the food sample. Once an image is obtained, feature extraction and quantitative analysis may be perfomed (Sun, 2000).

The CIELAB or $L^*a^*b^*$ space was used to describe color. This color space is device-independent, creating consistent colors regardless of the device used to acquire the image. L^* is the luminance or lightness component, which ranges from 0 to 100, while a^* (green to red) and b^* (blue to yellow) are two chromatic components, with values varying from −120 to +120 (Papadakis, Abdul-Malek, Kamden, & Yam, 2000). The cylindrical version of the $L^*a^*b^*$ system resembles the Munsell color order system corresponding to the perceptual attributes of lightness (L^*), hue ($h°$) and chroma (C^*) (McLaren, 1976).

Blooming induces a non-uniform pattern of colors over the surface of a chocolate bar. Image texture is a term used to describe the spatial variability of the gray levels of pixel over the whole image and thus provides useful information about color patterns. Analysis of the Fourier spectrum of an image permits extraction of a single descriptor (number) that represents the image texture, based on the contents of the image (e.g., pixel values and spatial pixel distribution) (Augusteijn, Clemens, & Shaw, 1995). Color parameters as well as Fourier descriptors constitute features derived from an image. Organizing a feature space into regions corresponding to several classes is called classification, and it is supposed to represent the high-end task for computer-based image analysis (Russ, 2002).

The objectives of this study were: (i) to characterize the color of the surface of chocolate tablets using a digital camera and a commercial spot colorimeter; (ii) to follow color changes on the surface of chocolate under accelerated storage conditions that induced bloom; (iii) to identify and separate features of interest such as white specks and the gray-background area, and determine textural features by Fourier spectra analysis, and; (iv) to utilize the features extracted from images to discriminate between different stages (or classes) during blooming.

2. Materials and methods

2.1. Materials

Rectangular (5.4 cm × 13.7 cm) tablets of commercial milk chocolate ("Trencito", Nestle-Chile) from the same production lot were purchased in a local supermarket in Santiago, Chile. Five chocolate tablets were transferred to the storage chamber and exposed to 12 h cooling/heating cycles between 16 and 28 °C for 52 days. The chamber consisted of an insulated wooden box with an air conditioning system (for flow of hot and cold air) where the fluctuating temperature was controlled and registered using a computer program that activated the incoming air current into the chamber. Relative humidity never exceeded 50%.

2.2. Image acquisition and capture

A color digital camera model PowerShot A70 (Canon, USA) connected to a computer USB interface IFC-300PCU (Canon, USA) was mounted on a stand inside a large box impervious to light and having internal black surfaces. Samples were transferred from the storage chamber to the dark box and images acquired every 2 days during storage. The iris was operated in manual mode, with the lens aperture at $f = 8$ and speed 1/20 (no zoom, no flash) to achieve high uniformity and repeatability. The camera was gray-balanced before each imaging session. Uniform diffuse lighting was used to illuminate the samples. The lighting system consisted of four CIE source D_{65} lamps (60 cm length and 18 W; Model TLD/965, Philips, Singapore) placed above the sample at a 45° angle to maximize diffuse reflection responsible for color. The angle between the camera lens axis and the sample was around 90° to reduce gloss. A Kodak gray card with 18% reflectance was used as a white reference to standardize the illumination level. The gray-level image (1600 × 1200 pixels) of this card was divided into 192 blocks, each one of 100 × 100 pixels. The mean of the histogram of gray values for each block was calculated and located within the gray-value interval [98, 152] to assure the 18% reflectance ($L^* = 50$) (www.luminous-landscape.com/tutorials/understan-ding-series/understanding-histograms; www.brucelindbloom.com). After calibration, samples were placed in the field of view of the camera and an image of 1600 × 1200 pixels (approximately covering the whole area of the tablet) was acquired and stored in JPEG (Joint Photographic Experts Group, a standard for compressing digital photographic images) format of high resolution and superfine quality.

2.3. Segmentation of original images

Background segmentation was performed by the technique proposed by Belkasim, Ghazal, and Basir (2003) and combined with a Gaussian lowpass filter which permits pre-smoothing of noisy images. This technique is based on maximizing the correlation between the phase of the gray-level image in the Fourier power spectrum and the phase of the thresholded version of the image, resulting in an optimal threshold value. The segmentation algorithm, written in Matlab 6.5 (The MathWorks, Inc., Natick, MA, USA) is explained in Exhibit 1.

2.4. Color measurement

Color was measured by a computer vision system and image analysis (CVSIA) as well as with a hand-held colorimeter. In CVSIA, color images of chocolate tablets were digitized into pixels (24 bits/pixel) containing levels of the three primary colors: red, green and blue (RGB) and converted into *XYZ* tristimulus values, a widely used, device-independent color standard developed by CIE based on color-matching experiments with human observers. In the following step, CIEXYZ data were converted to CIELAB or $L^*a^*b^*$ values (Gonzalez & Woods, 1992; www.brucelindbloom.com) using an algorithm written in Matlab 6.5 and ran in a Pentium III computer (256 MB, 30 GB hard disk). At each sampling time during storage the five chocolate tablets were analyzed and the average and standard deviation calculated.

The color of the surface of the five chocolate tablets was also measured using a HunterLab Miniscan™ XE colorimeter model 45/0 LAV (Hunter Associates Inc., Reston, VA) after calibration with white and black glass standards. Three equidistant spots were examined on the major axis of each chocolate bar. Since the spot diameter of the instrument was 12.5 mm, the total area of the tablet from which information was taken was 3.68 cm^2. The colorimeter yielded L^*, a^* and b^* values for each spot, which were converted to whiteness index (WI) values according to the expression: $WI = 100 - [(100 - L^*)^2 + (a^{*2}) + (b^{*2})]^{0.5}$. Besides, the hue angle ($h°$) was calculated from $h° = \arctan(b^*/a^*)$ and chroma (C^*) from $C^* = [(a^*)^2 + (b^*)^2]^{0.5}$. These same formulas were used to convert color data from the CVSIA.

2.5. White specks and whitish background analysis

Original background (brown), white specks and whitish background (bloomed) were determined from CVSIA data as percentage of the total area of the tablet. Each image was equalized and binarized after dividing the gray-scale histogram into three zones: pixels over a gray-value of 250 were assigned to white specks; those in the interval between 30 and 249 were defined as belonging to the whitish background, and; pixels between 0 and 29 were ascribed to the original background (brown). The segmentation algorithm was written in Matlab 6.5.

- Read the original image (RGB).
- Apply Gaussian filter to original image.
- Transform filtered original image to a gray-level image and obtain blue component image (B) of the original image.
- Obtain the discrete Fourier transform (DFT) of the gray-level image.
- From the DFT obtain the phase; $\phi_{\text{gray-level}}(u,v)$.
- Obtain the minimun (min) and maximun (max) gray level value of the B image:

  For threshold = min to max do

  binary image = For B obtain the binary image based on each threshold

  $\phi_{\text{threshold}}(u,v)$ = Obtain the phase (DFT) of the image binary

  $C(\phi_{\text{gray level}}, \phi_{\text{threshold}})$ = obtain the maximum correlation between $\phi_{\text{gray level}}(u,v)$ and $\phi_{\text{threshold}}(u,v)$

  End
- Find the optimun threshold value (OTV) corresponding to the maximum value of C.
- Apply (OTV) to B and obtain the binary image.
- Detect boundary in the binary image of B.
- Obtain the segmented original image.

Exhibit 1. The segmentation algorithm.

2.6. Image texture analysis

The Fourier power spectra were constructed from the gray-level images and calculated according to Augusteijn et al. (1995). Fourier spectra calculated to determine the prominent peaks in one principal direction (45°) defined texture patterns and the location of the peaks in the frequency plane gave the fundamental spatial period of the patterns. Energy of Fourier (E) was calculated from the Fourier power spectrum of each image and values saved in the feature vector of the corresponding image. If $F(j,k)$ is the matrix containing the amplitudes of the spectrum, E is defined as (Augusteijn et al., 1995),

$$E = \left[\sum_{j,k} | F(j,k)|^2 \right]^{1/2}. \tag{1}$$

An average E was calculated for each day of measurement (i.e., from the five tablets). A normalized $E^* = [(E - E_{\min})/(E_{\max} - E_{\min})]$ was defined to compare values through the whole storage period, where $E_{\min}$ and $E_{\max}$ are the minimum and maximum E values found during the whole storage period. Thus, E^* ranges between 0 and 1.

2.7. Selection and classification of features

One of the most important steps in pattern recognition is the selection of an appropriate set of features with good discriminative power. In order to reduce the computational time required the features for classification have to be non-correlated while providing information about the extent of bloom. Defining the blooming stages of chocolate (classes) required first an "expert" visual inspection (supervised method) of all images. Since blooming is a progressive phenomenon four classes of bloomed chocolate were arbitrarily defined according to the time of storage: class 1 (0–33 days), class 2 (36–39 days), class 3 (42–45 days) and class 4 (48–52 days).

A total of eight features per class (L^*, a^* and b^* values, WI, C^*, $h°$, % bloom and energy of Fourier) were computed for all samples. Percent bloom was defined as the sum of % whitish background and % white specks. Two methods of feature selection were compared to define the reduced feature set. The first method used all eight selected features. A second method used, sequential forward selection (SFS) (Aha & Bankert, 1996; Dash & Liu, 1997), is a common search method in image classification. SFS proceeds by adding one feature at the time to a current suboptimal feature set. At each stage, the feature to be included is selected from among the remaining available features. This cycle is repeated until improvement by adding the new feature yields a maximum value of the criterion function used. Since features in the reduced feature set were selected according to their discrimination ability, discriminant analysis was used as selection criterion.

2.8. Statistical analysis

Statistical analysis was done using Statgraphics for Windows software, Version 5.0 (Manugistic Inc., Rockville, MD, USA). Methods applied were simple regression, analysis of variance and discriminant analysis with 95% confidence level.

3. Results and discussion

3.1. Images of chocolate tablets during storage (using CVSIA)

Fig. 1 shows a gallery of images of the same chocolate tablet as a function of storage time. Major color changes in the surface did not occur until after 33 days of storage. The presence of surface bloom was manifested in two distinctive ways. First, (around day 33) a few round white specks appeared against the original brown background, probably due to rapid migration of liquid fat through flaws or pores in the surface. (Adenier, Chaveron, & Ollivon, 1993) reported that at the onset of blooming fat accumulated at the edge of holes or along cracks in the surface of chocolate. Later (around day 36), a whitish homogeneous background gradually set in, slowly replacing the original brown background of the chocolate tablets and superimposed on the white specks. Visually, at the end of storage most of the chocolate surface was whitish except for a few brown spots and lines, and brown narrow zones around the edges of the tablet. Some of the original white specks turned less noticeable as the whitish background became more intense. The evolution of changes in the pattern of color distribution defined by white and brown backgrounds and white specks is quantitatively depicted in Fig. 2 as the % of total area. White specks reached a peak of 2.59% of the total area at day 39 and decreased to a final value of 1.6% at the end of storage, due to masking by the whitish background. The whitish background increased rapidly from day 36 to occupy 88.9% of the area of the tablet at day 48 and remained almost constant thereafter. Even at the end of the experiment around 9.6% of the area of the tablet was "brown" (as specks, lines or along the edges). At this point it can only be surmised that brown color zones occur due to absence of fat migration to the surface. This phenomenon may originate during molding of the chocolate mass or by sealing of pores by the aluminum wrap. Thus, it is not unexpected that different chocolate samples developed dissimilar color patterns during blooming (data not shown).

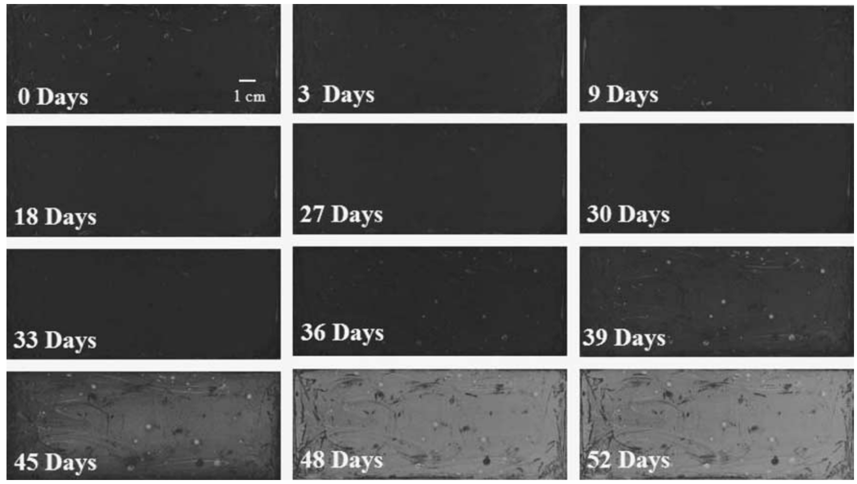

Fig. 1. Gallery of digital images showing changes in color and color patterns in the surface of one chocolate tablet during storage.

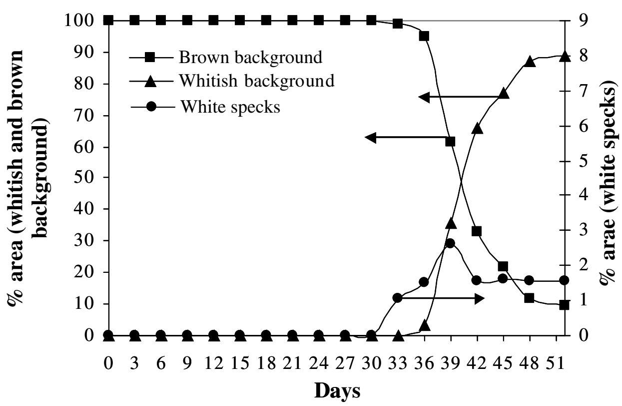

Fig. 2. Changes in the % area (brown background, whitish background and white specks) of chocolate surface during storage (average values for all tablets).

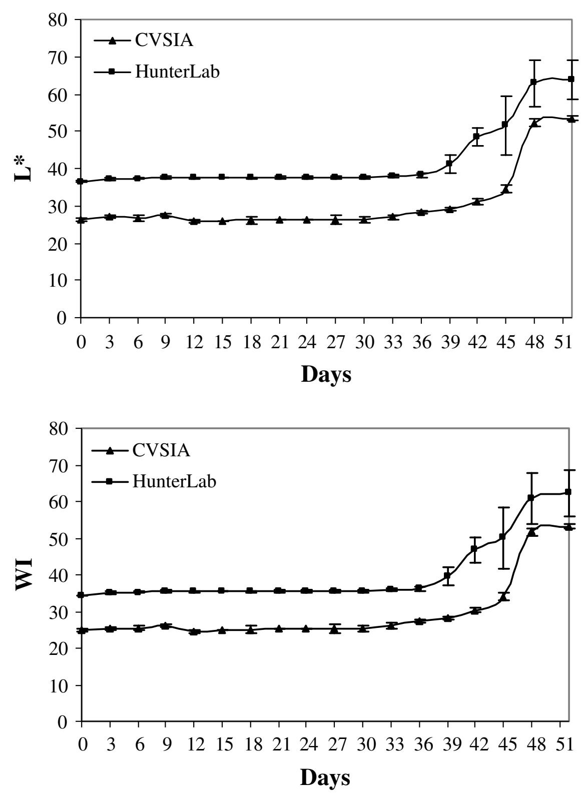

Fig. 3. L^* and WI values of chocolate bars during storage as measured by CVSIA and HunterLab colorimeter (average values and std. deviation, five tablets at each sampling time).

3.2. Changes in measured surface color during storage

Differences in color data acquired by CVSIA and with the colorimeter may be explained as follows. First, the light → specimen → detector interaction was different for both color measuring systems (e.g., in the HunterLab the angle of light incidence is 45°). Second, the area of the specimen from which color data was acquired was larger for the CVSIA (whole tablet = 74 cm^2) than for the colorimeter (area of three spots = 3.7 cm^2). Lastly, the colorimeter was hand-held by the operator and touched the sample while in CVSIA the camera remained fixed and removed from the sample.

Fig. 3 shows changes in L^* and WI from chocolate samples during storage measured with CVSIA and the HunterLab. Both parameters remained almost constant for the first 36 days in accordance with the visual impression of surfaces, as shown in Fig. 1 (tablet 1, others tablets not shown). L^* and WI from bloomed samples increased rapidly afterwards and seemed to reach an asymptotic value at the end of the storage period. A lag period before the onset of bloom has been observed and measured by several researchers and by different means: whiteness index (Bricknell & Hartel, 1998), magnetic resonance imaging (Miquel & Hall, 2002) and laser microscopy (Quevedo, 2003). A major drawback of the WI, a widely used color parameter in chocolate storage studies, is that it is an average value taken from a few locations over the surface. Although it shows the general drift from the original brownish color (low WI) to a grey-whitish color of bloomed chocolate (high WI), data exhibit rather large standard deviations that seem to increase as blooming proceeds (Tietz & Hartel, 2000). This may be due to the non-homogeneous white color development mentioned before. Furthermore, WI fails to reveal the presence of white specks or color patterns These limitations are understandable in the case of the hand-held instrument since the operator tends to avoid such discontinuities while performing the measurements. Since blooming depends on formulation, microstructure imparted to chocolate during fabrication and storage conditions, among others, color changes are likely to be specific for each chocolate item (Aguilera et al., 2004).

3.3. Comparison between measuring systems

Fig. 4 shows the correlation between L^*, a^*, b^*, WI, C^* and $h°$ values obtained by the CVSIA and Hunterlab colorimeter for all data acquired during storage. Although L^* and WI showed higher correlation coefficients ($R^2 = 0.912$ and 0.910, respectively) than a^* and C^* ($R^2 = 0.768$ and 0.709, respectively), there were significant differences ($p < 0.01$) between data obtained by both techniques. Correlation coefficients for b^* and $h°$ values were even smaller ($R^2 = 0.505$ and 0.444, respectively). Colorimeters are used primarily for quality control applications and measure the color (typically using the CIE system) of luminous (or externally illuminated) objects at somewhat large angular subtense. These devices do not involve large spatial coverage, have one sensor per color channel and give a single average color value over their aperture. In a computer visual inspec-

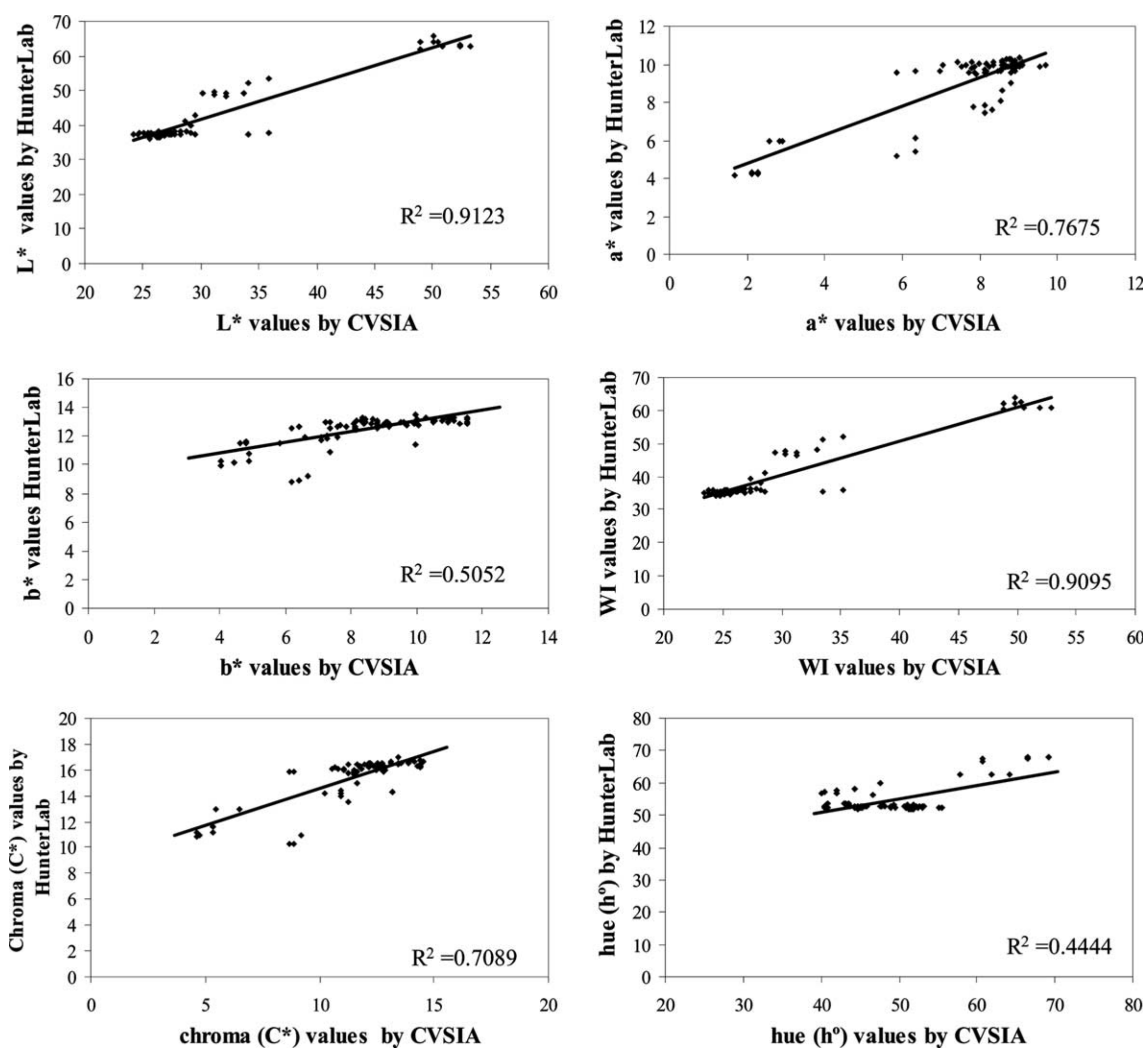

Fig. 4. Relationship between color parameters of the surface of chocolate samples as determined by the CVSIA system and HunterLab colorimeter.

tion system, a camera acquires an image of the whole sample under controlled and reproducible conditions and a computer with specialized software is used to conduct pre-defined visual tasks. Another major advantage of computer vision systems is that the image on which measurements are performed can be saved and becomes a permanent record for later examination, comparison or further analysis. It is anticipated that computer vision inspection of food products will be consistent, efficient and cost effective.

3.4. Image texture analysis of the chocolate surfaces

As shown in Fig. 1, blooming changes the visual appearance of the surface of chocolate bars from a more or less homogeneous brown color to a final intricate pattern of white background and specks, and a few browns areas. Changes in normalized energy of Fourier are expected to delimitate periods where major changes in the pattern or texture of the image occur. The descriptor E^* stays close to zero for the first 36 days of storage meaning that image texture remained unchanged (Fig. 5). Major changes occur between days 39 and 45 (with the steepest slope of the curve around day 42) followed by a slower progress until the end of the storage period. The shape of the curve seems to indicate that texture image (as described here) trails development of the whitish background by a few days (see Fig. 2).

3.5. Classification of bloomed samples

Table 1 shows the selection performance for the two classification methods and the use of discriminant analysis as the classification criterion. Both methods permitted the correct classification of 90 images (18 sampling days and five tablets each time) in four different classes of bloomed chocolate with an accuracy of 97.78%. However, the SFS method gave the same performance using only five features: WI (white index), C^* (chroma), $h°$ (hue), E^* (energy of Fourier) and % bloom, and, consequently, less computational expense. Fig. 6 shows, results of the discriminant analysis to classify bloomed chocolate samples into classes using the SFS method. The proposed discriminant functions permitted the correct prediction of the class of bloomed chocolate at the 95% confidence level. However, no perfect distinction was possible between classes 1 and 2, corresponding to the lag period of storage when feature differences between samples are small. Untrained panelists asked to discriminate visually between chocolate samples coincided in that these stages were the most difficult to discriminate among.

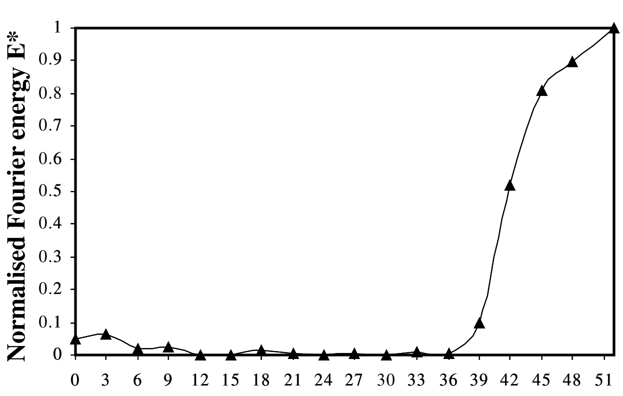

Fig. 5. Changes in image texture during blooming of chocolate expressed as normalized Fourier energy E^* (data from the five samples at each sampling time).

Table 1
Selection of features using two methods of classification

Method	Feature	Performance (%)
Sequential forward selection	WI, C^*, $h°$, E^*, % bloom	97.78
All features	L^*, a^*, b^*, WI, C^*, $h°$, E^*, % bloom	97.78

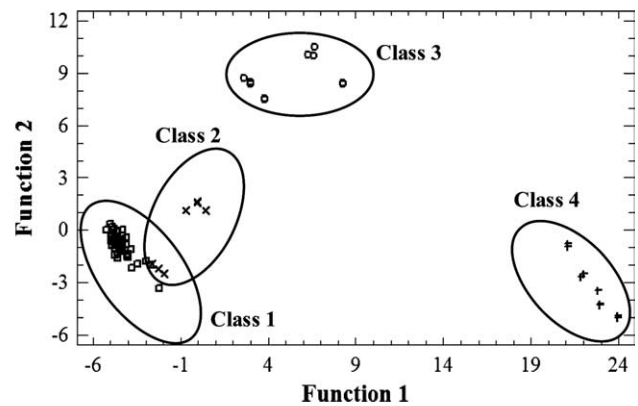

Fig. 6. Classification of different stages during surface bloom of chocolate tablets, using linear discriminant functions. Function 1 = 0.7865WI − 0.3854C^* + 0.2666$h°$ + 0.0209E^* + 0.3186% bloom; Function 2 = −0.6217WI + 0.4397C^* − 0.6965$h°$ + 1.0380E^* + 0.3103% bloom. Legend: (□) class 1, (×) class 2, (○) class 3 and (+) class 4.

4. Conclusions

Blooming is a complex color phenomenon that encompasses all surfaces of a chocolate tablet and has to be analyzed accordingly. Computer vision and image analysis are appropriate techniques to capture spatial changes and to measure and analyze color evolution during blooming development. Moreover, these techniques are relatively simple, versatile and can be implemented at low cost. The method proposed has the capability to quantify overall changes as well as particular features on the chocolate surface over time thus enabling customization and standardization for quality assessment.

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References

Adenier, H., Chaveron, H., & Ollivon, M. (1993). Mechanism of fat bloom development on chocolate. In G. Charalambous (Ed.), *Shelf life studies of foods and beverages* (pp. 353–389). London: Elsevier Science.

Aguilera, J. M., Michel, M., & Mayor, G. (2004). Fat migration in chocolate: Diffusion or capillary flow in a particulate solid? – A hypothesis paper. *Journal of Food Science, 69*(7), R167–R174.

Aha, D. W., & Bankert, R. L. A (1996). Comparative evaluation of sequential feature selection algorithms. In D. Fisher & J. H. Lenz (Eds.), *Artificial intelligence and statistics* (pp. 199–206). New York: Springer.

Ali, A., Selamat, J., Che Man, Y. B., & Suria, A. M. (2001). Effect of storage temperature on texture, polymorphic structure, bloom formation and sensory attributes of filled dark chocolate. *Food Chemistry, 72*(4), 491–497.

Augusteijn, M. F., Clemens, L. E., & Shaw, K. A. (1995). Performance evaluation of texture measures for ground cover identification in satellite images by means of a neural network classifier. *IEEE Transactions in Geoscience and Remote Sensing, 33*(3), 616–626.

Belkasim, S., Ghazal, A., & Basir, O. A. (2003). Phase-based optimal image thresholding. *Digital Signal Processing, 13*(4), 636–655.

Bricknell, J., & Hartel, R. W. (1998). Relation of fat bloom in chocolate to polymorphic transition of cocoa butter. *Journal American Oil Chemist's Society, 75*(11), 1609–1614.

Dash, M., & Liu, H. (1997). Feature selection methods for classification. *Intelligent Data Analysis, An International Journal, 1*(3).

Gonzalez, R. C., & Woods, R. E. (1992). *Digital image processing*. Boston, MA, USA: Addison-Wesley Longman Publishing Co., Inc.

Lohman, M., & Hartel, R. W. (1994). Effect of milk fat fractions on fat in dark chocolate. *Journal American Oil Chemistry Society, 71*(3), 267–275.

McLaren, K. (1976). The development of CIE (L^*,a^*,b^*) uniform color space. *Journal Society Dyers Colour*, 338–341.

Miquel, M. E., & Hall, L. D. (2002). Measurement by MRI of storage changes in commercial chocolate confectionery products. *Food Research International, 35*, 993–998.

Papadakis, S., Abdul-Malek, S., Kamden, R. E., & Yam, K. L. (2000). Versatile and inexpensive technique for measuring color of foods. *Food Technology, 54*(12), 48–51.

Quevedo R. (2003). *Characterization of food surfaces* (96 pp.). PhD dissertation, Universidad Católica de Chile, Santiago, Chile.

Russ, J. C. (2002). *The image processing handbook* (4th ed.). Boca Raton: CRC Press.

Sun, D. W. (2000). Inspecting pizza topping percentage and distribution by a computer vision method. *Journal of Food Engineering, 44*(4), 245–249.

Tietz, R. A., & Hartel, R. W. (2000). Effects of minor lipids on crystallization of milk–cocoa butter blends and bloom formation in chocolate. *Journal of the American Oil Chemists Society, 77*, 763–771.

Yam, K. L., & Papakadis, S. E. (2004). A simple digital imaging method for measuring and analysing color of food surfaces. *Journal of Food Engineering, 61*(1), 137–142.

www.luminous-landscape.com/tutorials/understanding-series/understanding-histograms.

www.brucelindbloom.com.