MLRF Lecture 01

J. Chazalon, LRDE/EPITA, 2019

Introduction to Twin it!

Lecture 01 part 03

Twin it! overview

A poster game

- X bubbles, all different but
- Y bubbles, which have 1 (and only 1) twin

Your goals

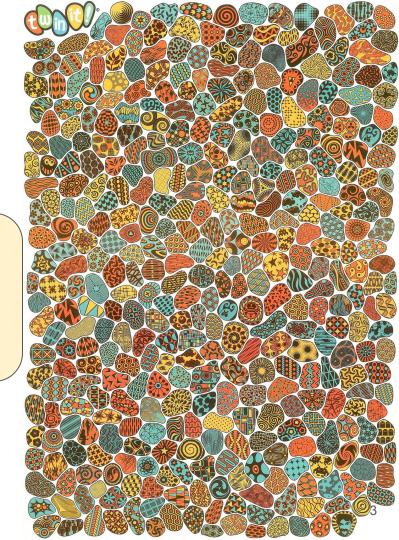
- ..
- Find the pairs
- ...

Already done

- Scan the poster
- Stitch the tiles
- Normalize the contrast

Discussion (3 minutes):

- 1. How can we <u>decompose</u> the problem?
- 2. How can we make <u>sure</u> our solution works?
- 3. What should we focus on?



Twin it! overview

A poster game

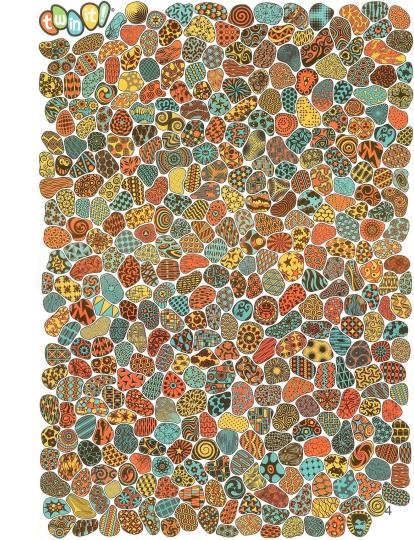
- X bubbles, all different but
- Y bubbles, which have 1 (and only 1) twin

Your goals

- Isolate each bubble
- Find the pairs
- Check it works

Already done

- Scan the poster
- Stitch the tiles
- Normalize the contrast



Twin it! underlying problems

1. Isolate each bubble ⇒ **Segmentation**We provide pre-computed results for this step.

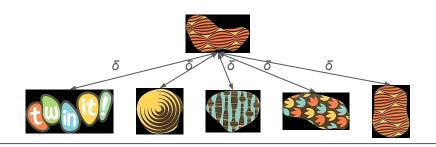






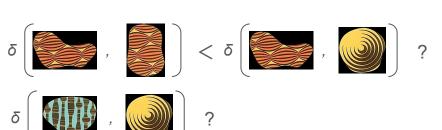
2. Find the pair ⇒ Matching

We will focus on this one.
We will use **Template Matching**.



3. Check it works ⇒ Evaluation

We will understand the challenges of this one.



Template Matching

Lecture 01 part 04

Why template matching?

A simple method which will be useful to understand

- Evaluation challenges
- The ideas behind keypoint detection (next lecture)

It can work in the Twin it! case

- Twice the same texture (in two bubbles of different shape)
- Textures at the same scale,
 without rotation
 nor intensity change
- Only need to cope with translation (and some small noise)











Two arrays of intensities

128	128	10
126	126	9
126	126	9

135	130	12
127	128	8
126	128	9

Two arrays of intensities

Take the difference

128	128	10
126	126	9
126	126	9

135	130	12
127	128	8
126	128	9

I₁

I,

-7	-2	-2
-1	-2	1
0	-2	0

$$R(x,y) = I_1(x,y) - I_2(x,y)$$
 R

Two arrays of intensities

Take the **absolute** difference

128	128	10
126	126	9
126	126	9

135	130	12
127	128	8
126	128	9

I₁

I,

7	2	2
1	2	1
0	2	0

$$R(x,y)=|I_1(x,y)-I_2(x,y)|$$
 R

Two arrays of intensities

Take the **squared** difference

128	128	10
126	126	9
126	126	9

135	130	12
127	128	8
126	128	9

I₁

I,

49	4	4
1	4	1
0	4	0

$$R(x,y) = (I_1(x,y) - I_2(x,y))^2$$
 R

Two arrays of intensities

Take the **squared** difference

Sum the differences

128	128	10
126	126	9
126	126	9

135	130	12
127	128	8
126	128	9

I₁

I,

49	4	4
1	4	1
0	4	0

$$S=\sum\limits_{x,y}(I_1(x,y)-I_2(x,y))^2$$

R

Two arrays of intensities

Take the **squared** difference

Sum the differences

(Opt.) Normalize so the results belongs to [0, 1].

0: closest / match

1: farthest / no match

	L
"Sum of squared	/
differences" or "SSD"	
	-

128	128	10
126	126	9
126	126	9

135	130	12
127	128	8
126	128	9

49	4	4
1	4	1
0	4	0



$$\sum\limits_{x,y}(I_1(x,y){-}I_2(x,y))^2$$

\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	$2 \sim \tau$
-1 (x, y)	$\{\cdot\} \setminus I_2(x,y)^2$
$\lambda / \angle I (w, g)$	$\angle 12(\omega,9)$
1/ru	r n
$V^{\omega,g}$	ω, g

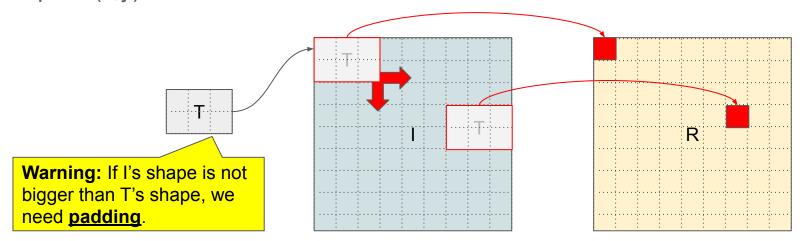
K

Template Matching: Sliding comparison

I₁ is a small template T to match against I₂ (just I after).

We rewrite the preceding formula to compute a map R of the shape of I.

Each pixel of R will have the value of the SSD when the top-left pixel of T in on the pixel (x,y) of I.



Several approaches ⇒ Practice session

Sum of squared differences

$$R(r,c) = \sum_{r',c'} (T(r',c') - I(r+r',c+c')^2$$

Cross correlation

$$R(r,c) = \sum\limits_{r',c'} (T(r',c') \cdot I(r+r',c+c'))$$

Correlation coefficient

$$R(r,c) = \sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))$$

where:

$$egin{aligned} T'(r',c') &= T(r',c') - 1/(w\cdot h) \cdot \sum_{r'',c''} T(r'',c'') \ I'(r+r',c+c') &= I(r+r',c+c') - 1/(w\cdot h) \cdot \sum_{r'',c''} I(r+r'',c+c'') \end{aligned}$$

Simply divide by the mean of pixel values

Normed SSD

$$R(r,c) = rac{\sum_{r',c'} (T(r',c') - I(r + r',c + c')^2}{\sqrt{\sum_{r',c'} (T(r',c'))^2 \cdot \sum_{r',c'} I(r + r',c + c')^2}}$$

Normed CCORR

$$R(r,c) = rac{\sum_{r',c'} (T(r',c') \cdot I(r+r',c+c'))}{\sqrt{\sum_{r',c'} T(r',c')^2 \cdot \sum_{r',c'} I(r+r',c+c')^2}}$$

Normed CCOEFF

$$R(r,c) = rac{\sum_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum_{r',c'} T'(r',c')^2 \cdot \sum_{r',c'} I'(r+x',c+c')^2}}$$

Always the same normalization

Several approaches ⇒ Practice session

Sum of squared differences

$$R(r,c) = \sum\limits_{r'.c'} (T(r',c') - I(r+r',c)$$

Normed SSD

$$R(r,c) = \sum\limits_{r',c'} (T(r',c') - I(r+r',c] ext{ Both very similar: just a local normalization}, c) = rac{\sum_{r',c'} (T(r',c') - I(r+r',c+c')^2}{\sqrt{\sum_{r',c'} (T(r',c'))^2 \cdot \sum_{r',c'} I(r+r',c+c')^2}}$$

Cross correlation

$$R(r,c) = \sum\limits_{r',c'} (T(r',c') \cdot I(r+r',c+c'))$$

Normed CCORR

$$R(r,c) = rac{\sum_{r',c'} (T(r',c') \cdot I(r+r',c+c'))}{\sqrt{\sum_{r',c'} T(r',c')^2 \cdot \sum_{r',c'} I(r+r',c+c')^2}}$$

Correlation coefficient

$$R(r,c) = \sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))$$

Normed CCOEFF

$$R(r,c) = rac{\sum_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum_{r',c'} T'(r',c')^2 \cdot \sum_{r',c'} I'(r+x',c+c')^2}}$$

where:

$$egin{aligned} T'(r',c') &= T(r',c') - 1/(w\cdot h) \cdot \sum_{r'',c''} T(r'',c'') \ I'(r+r',c+c') &= I(r+r',c+c') - 1/(w\cdot h) \cdot \sum_{r'',c''} I(r+r'',c+c'') \end{aligned}$$

Simply divide by the mean of pixel values Always the same normalization

Several approaches ⇒ Practice session

$$\begin{array}{lll} \textbf{Sum of squared differences} & \textbf{The smaller (close to 0),} \\ R(r,c) = \sum\limits_{r',c'} (T(r',c') - I(r+r',c+c')^2 & R(r,c) = \frac{\sum\limits_{r',c'} (T(r',c') - I(r+r',c+c')^2}{\sqrt{\sum\limits_{r',c'} (T(r',c'))^2 \cdot \sum\limits_{r',c'} I(r+r',c+c')^2}} \\ \textbf{Cross correlation} & \textbf{The larger,} \\ R(r,c) = \sum\limits_{r',c'} (T(r',c') \cdot I(r+r',c+c')) & R(r,c) = \frac{\sum\limits_{r',c'} (T(r',c') \cdot I(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T(r',c')^2 \cdot \sum\limits_{r',c'} I(r+r',c+c')^2}} \\ \textbf{Correlation coefficient} & \textbf{The larger,} \\ R(r,c) = \sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c')) & R(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c')}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c'))}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c')}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c')}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2 \cdot \sum\limits_{r',c'} I'(r+r',c+c')^2}} \\ \textbf{R}(r,c) = \frac{\sum\limits_{r',c'} (T'(r',c') \cdot I'(r+r',c+c')}{\sqrt{\sum\limits_{r',c'} T'(r',c')^2$$

where:

$$T'(r',c') = T(r',c') - 1/(w \cdot h) \cdot \sum_{r'',c''} T(r'',c'')$$
 $I'(r+r',c+c') = I(r+r',c+c') - 1/(w \cdot h) \cdot \sum_{r'',c''} I(r+r'',c+c'')$

Simply divide by the mean of pixel values

Always the same normalization

Cross correlation: 2 things to know

$$R(r,c) = \sum\limits_{r',c'} (T(r',c') \cdot I(r+r',c+c'))$$

More robust to intensity shifts (as long as gradients "agree") than SSD

SSD: X+offset - X = offset CCORR: (X+offset) \cdot X \cong X²

Base version requires to normalize T by its mean

Otherwise large image values always produce better matches Not necessary for CCOEFF

