

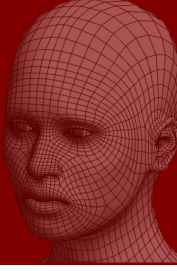
# Advanced Image Processing

## Lesson 4: Mesh processing

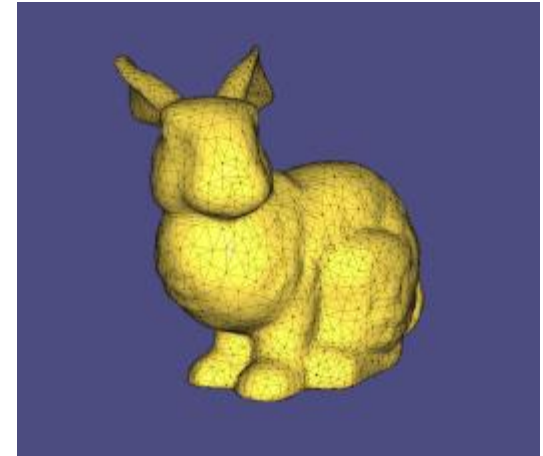
Speaker: Alice OTHMANI, PhD  
Associate professor at UPEC

Email: [alice.othmani@u-pec.fr](mailto:alice.othmani@u-pec.fr)

## Geometry processing



- Geometry processing involves working with a shape.
- Shape is a basic property of most objects.
- The shape can live in a space of arbitrary dimensions.

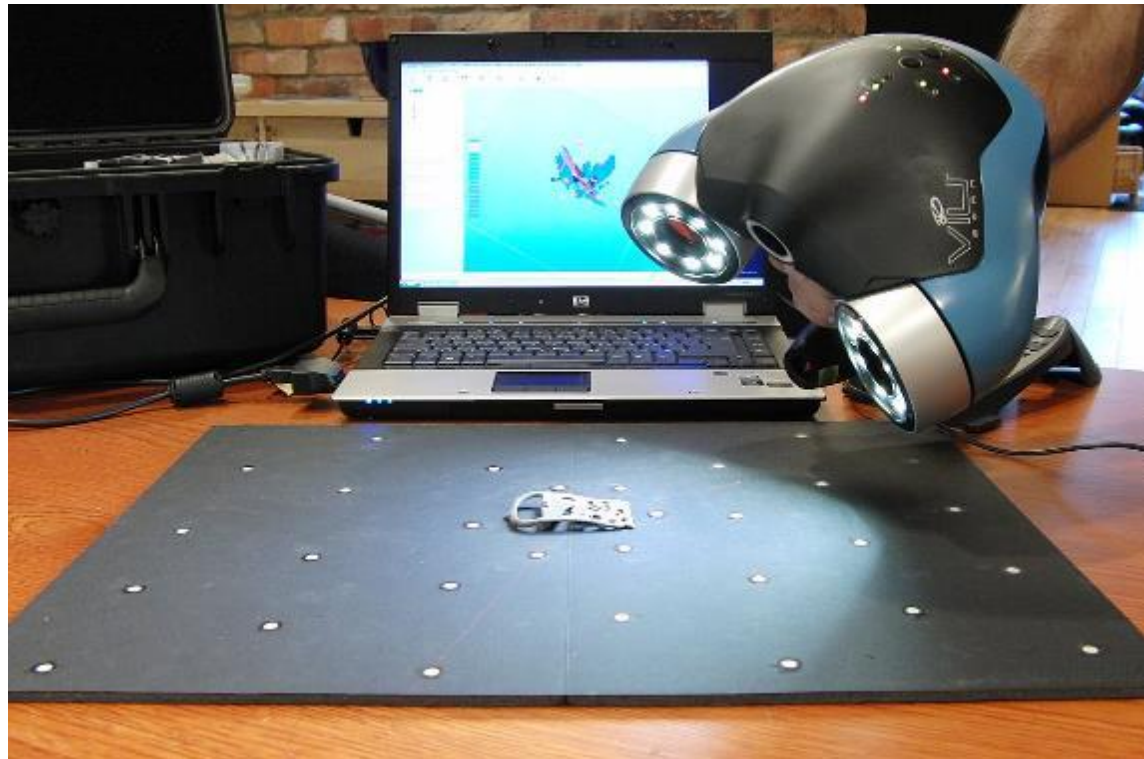


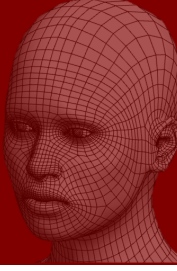
A mesh of the famous Stanford bunny. Shapes are usually represented as a mesh, a collection of polygons that delineate the contours of the shape.

## Acquiring 3D geometry

- 3D scanning is the process of analyzing a real-world object or environment to collect data on its **shape** and possibly its **appearance** (e.g. **colour**).

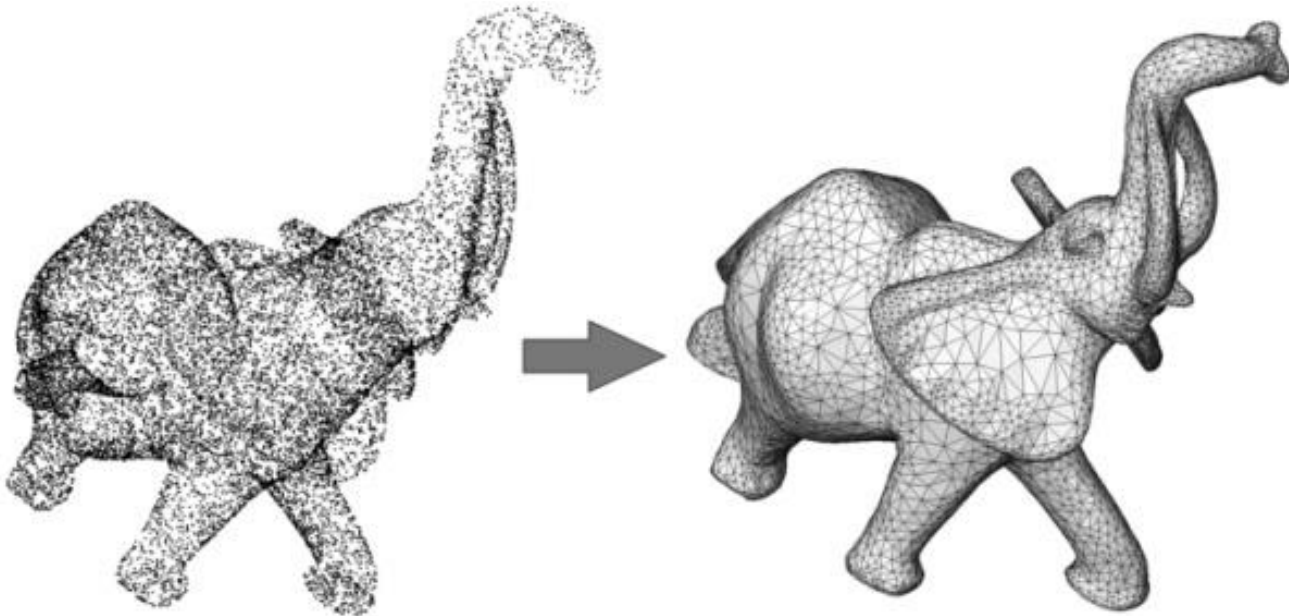
The collected data can then be used to construct digital 3D models.





## Acquiring 3D geometry

- The purpose of a 3D scanner is usually to create a **3D model**.
- This 3D model consists of a **point cloud** of geometric samples on the surface of the subject.
- These points can then be used to extrapolate the shape of the subject (a process called **reconstruction**).

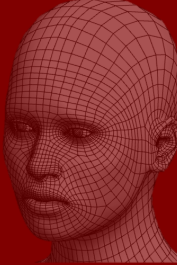


# Acquiring 3D geometry



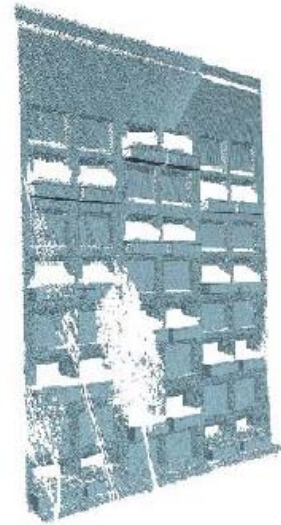
## Range Scanners





## Acquiring 3D geometry

### Range Scanners

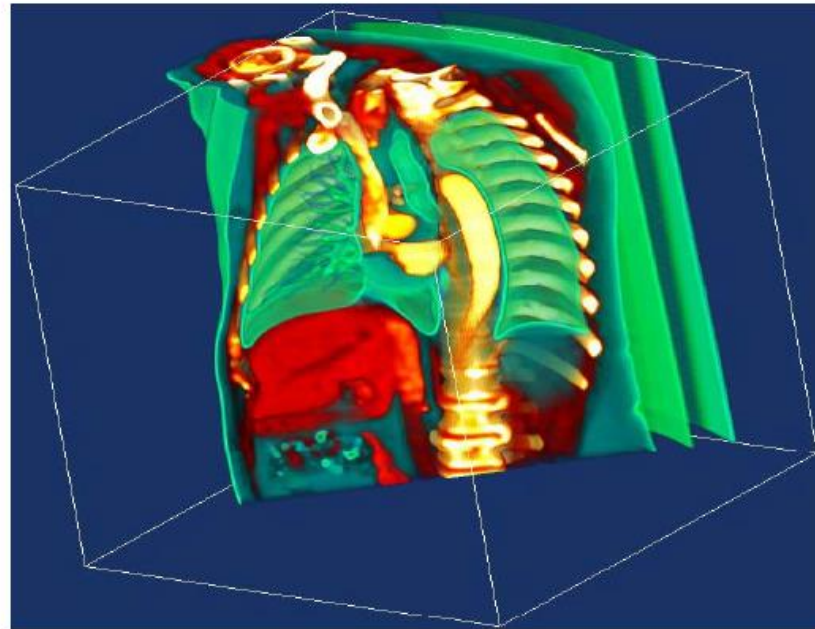
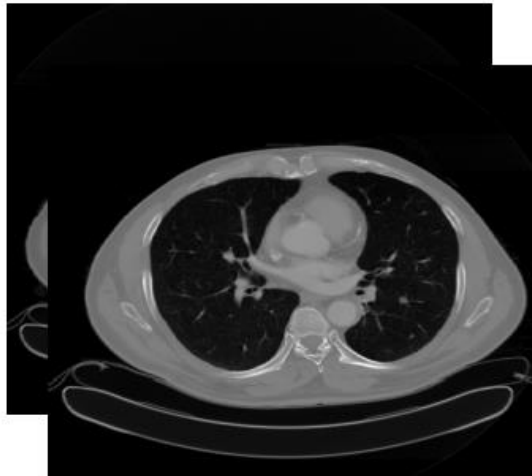






## Acquiring 3D geometry

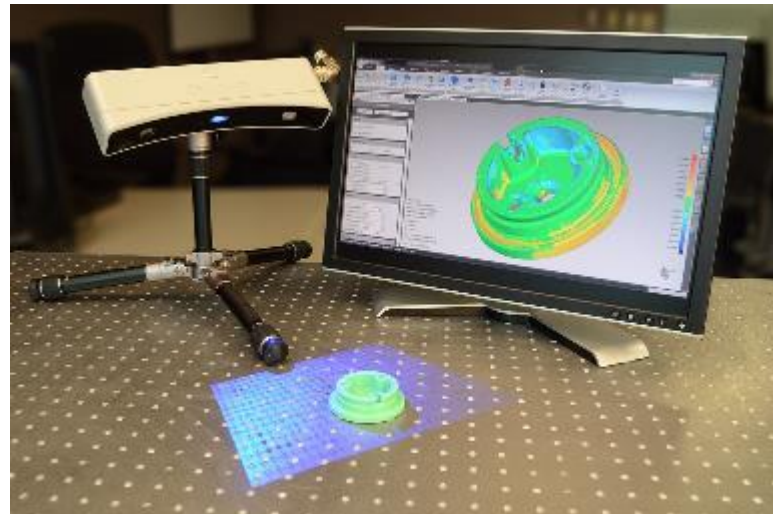
### Tomography





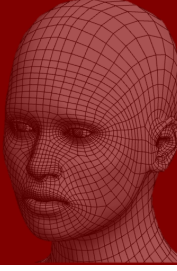
## Geometry processing

- A shape can be instantiated through one of three methods: a **model**, a **mathematical representation**, or a **scan**.
- After a shape is born, it can be analyzed and edited repeatedly in a cycle.
- Editing may involve **denoising**, **deforming**, or performing **rigid transformations**.





## Geometry Capture and representation



- Geometry processing is about the Creation & manipulation of 3D geometry



“SFMedu: A Structure from Motion System for Education”, Jianxiong Xiao  
<http://3dvision.princeton.edu/courses/SFMedu/>



# Geometry Capture and representation

ICCV 2017

## BodyFusion

Real-time Capture of Human Motion and Surface Geometry  
Using a Single Depth Camera

Tao Yu<sup>1,2</sup>, Kaiwen Guo<sup>2</sup>, Feng Xu<sup>2</sup>, Yuan Dong<sup>2</sup>, Zhaoqi Su<sup>2</sup>,  
Jianhui Zhao<sup>1</sup>, Jianguo Li<sup>3</sup>, Qionghai Dai<sup>2</sup>, Yebin Liu<sup>2</sup>

Beihang University, Beijing, China<sup>1</sup>

Tsinghua University, Beijing, China<sup>2</sup>

Intel Labs China, Beijing, China<sup>3</sup>

***A novel real-time motion tracking and fusion method called BodyFusion that reconstructs non-rigid surface motions of human performers using a single consumer-level depth camera.***

## Geometry representations: Meshes

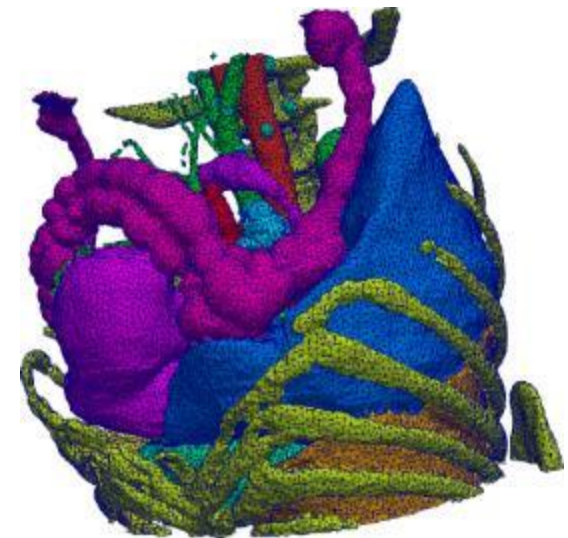
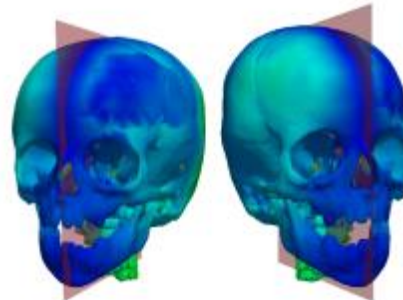
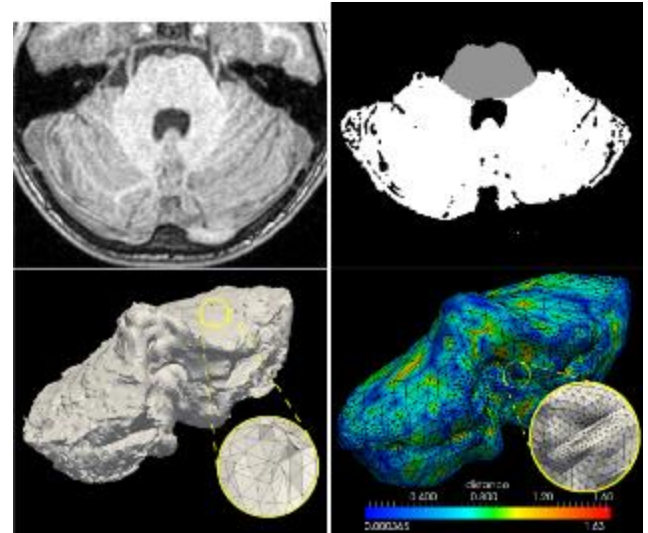


- **Focus on discrete (polygonal mesh) models**

Typically triangular

- **Why?**

- Simplicity – ease of description & transfer
- Base data for rendering software/hardware
- Input to most simulation/analysis tools
- Output of most acquisition tools (CT, MRI, laser, etc..)



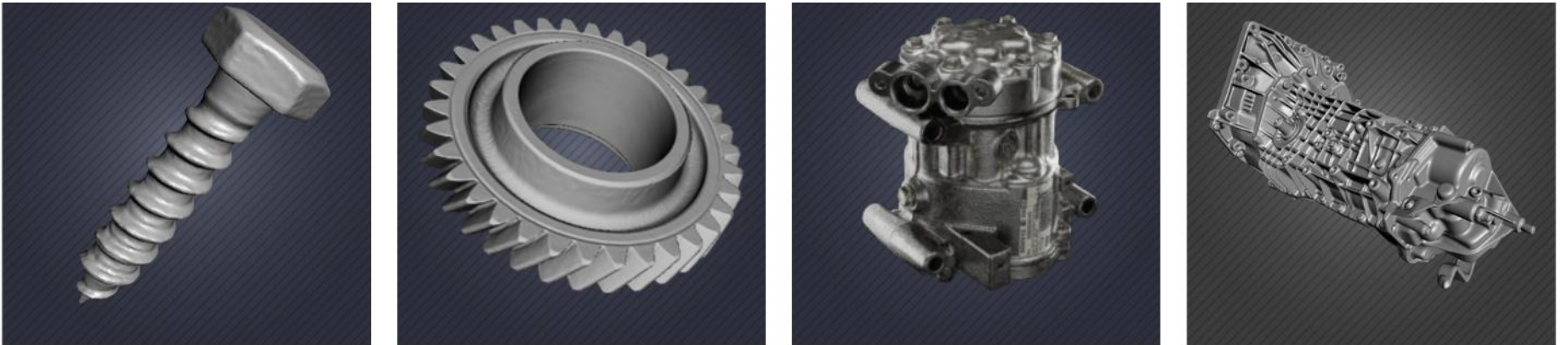
# Applications



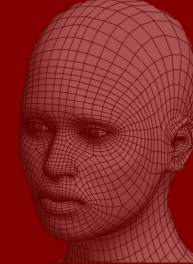
## 3D Shape Capture for Heritage Preservation



## Industrial design and manufacturing





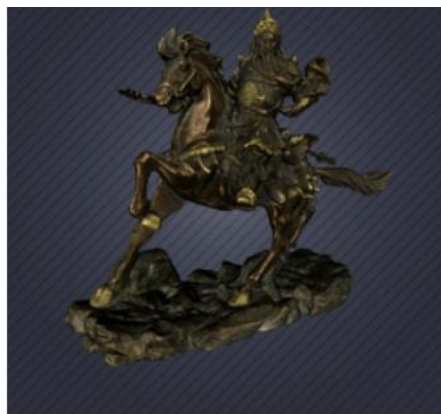
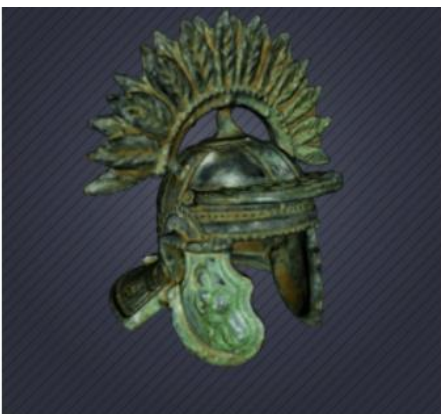
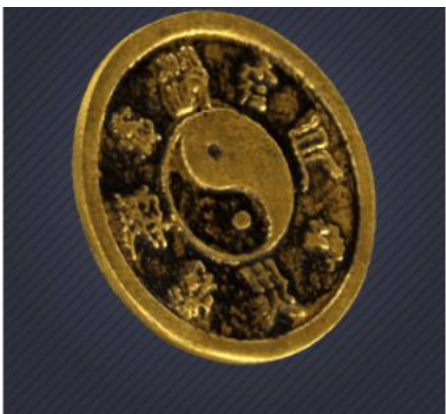


## Applications

### Science and education



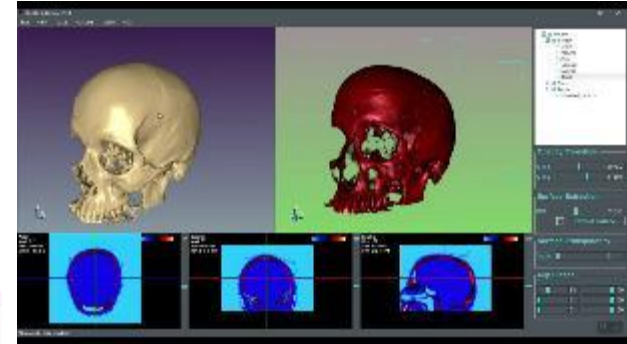
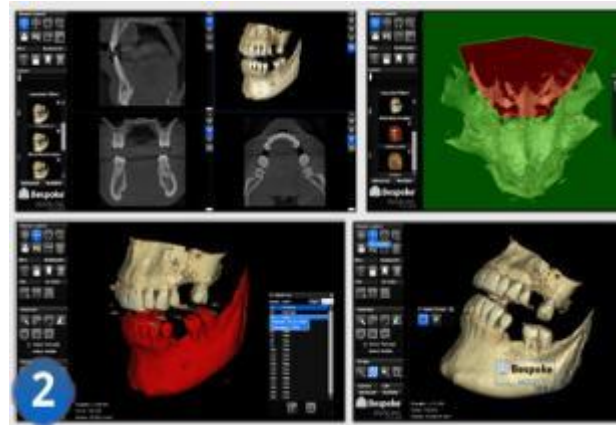
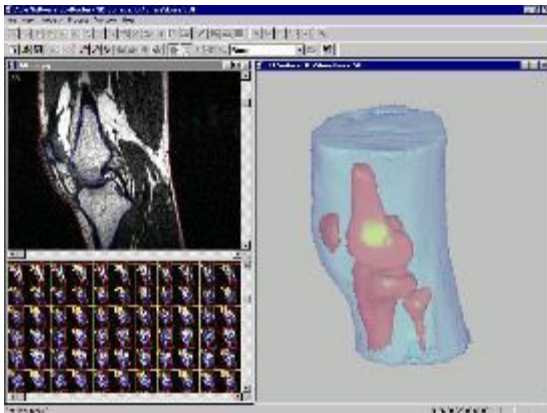
### Art and design

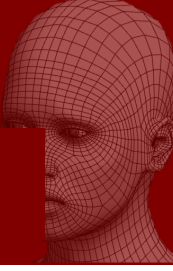




# Applications

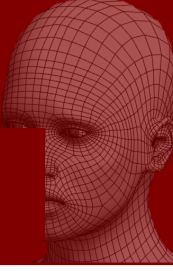
## Medical Imaging





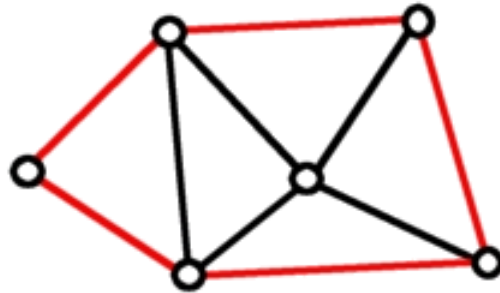
## What is a Mesh?

- A Mesh is a pair  $(P, K)$ , where  $P$  is a set of point positions  $P = \{p_i \in R^3 \mid 1 \leq i \leq n\}$  and  $K$  is an **abstract simplicial complex** which contains all topological information.
- $K$  is a set of subsets of  $\{1, \dots, N\}$ :
  - **Vertices**  $v = \{i\} \in V$
  - **Edges**  $e = \{i, j\} \in E$
  - **Faces**  $f = \{i_1, i_2, \dots, i_{n_f}\} \in F$
- $K = V \cup E \cup F$

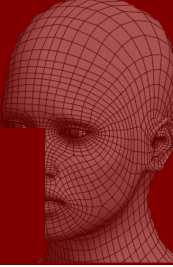


## What is a Mesh?

- Each edge must belong to at least one face, i.e.  
$$v = \{j\} \in V \text{ iff } \exists e = \{i, j\} \in E$$
- Each vertex must belong to at least one edge, i.e.  
$$e = \{j, k\} \in E \text{ iff } \exists f = \{i_1, \dots, j, k, \dots, i_{n_f}\} \in F$$
- An edge is a **boundary edge** if it only belongs to one face

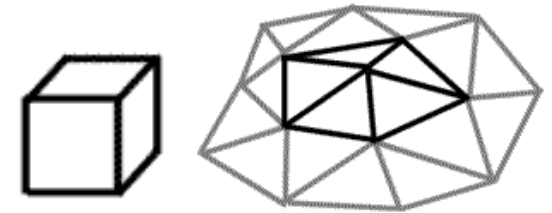


## What is a Mesh?

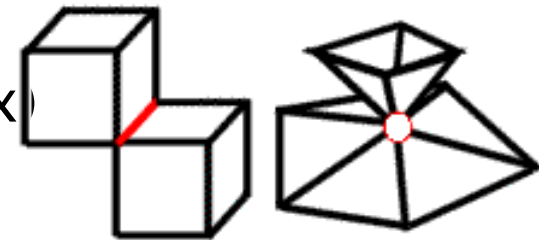


- A mesh is a **manifold** if

- Every edge is adjacent to one (boundary) or two faces
- For every vertex, its adjacent polygons form a disk (internal vertex) or a half-disk (boundary vertex)



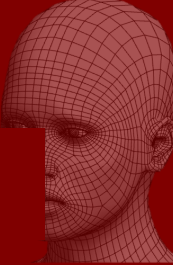
Manifold



Non-manifold

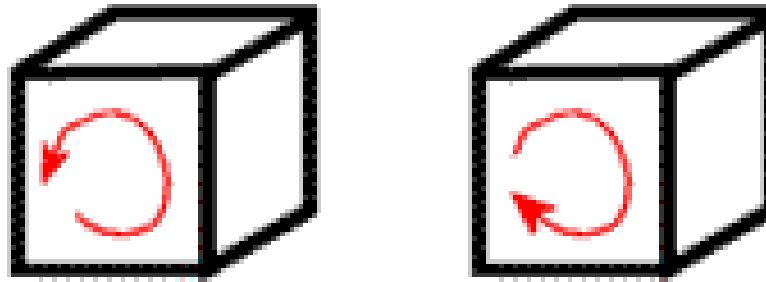
- A mesh is a **polyhedron** if

- It is a manifold mesh and it is closed (no boundary)
- Every vertex belongs to a cyclically ordered set of faces (local shape is a disk)



## Orientation of Faces

- Each face can be assigned an orientation by defining the ordering of its vertices
- Orientation can be **clockwise** or **counter-clockwise**.



- The orientation determines the normal direction of face. Usually **counterclockwise** order is the “front” side.



## Euler Formula

- The relation between the number of vertices, edges, and faces.

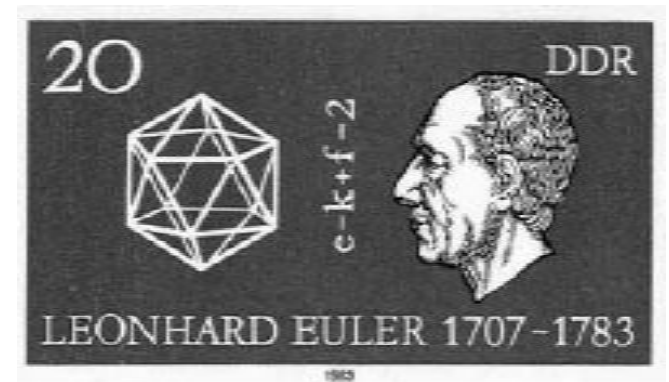
$$V - E + F = 2$$

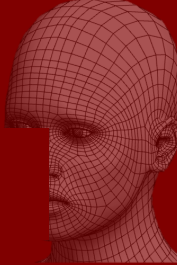
- where

V : number of vertices

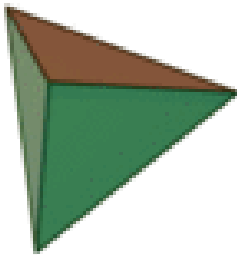
E : number of edges

F : number of faces



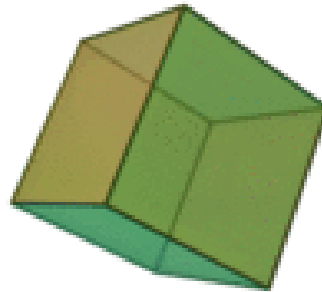


## Euler Formula



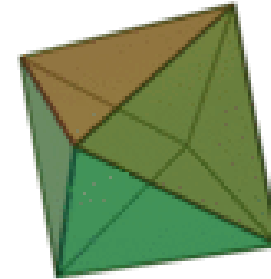
### ■ Tetrahedron

- $V = 4$
- $E = 6$
- $F = 4$
- $4 - 6 + 4 = 2$



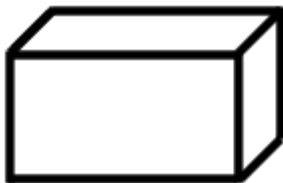
### ■ Cube

- $V = 8$
- $E = 12$
- $F = 6$
- $8 - 12 + 6 = 2$



### ■ Octahedron

- $V = 6$
- $E = 12$
- $F = 8$
- $6 - 12 + 8 = 2$



$$\begin{aligned} V &= 8 \\ E &= 12 \\ F &= 6 \\ 8 - 12 + 6 &= 2 \end{aligned}$$



$$\begin{aligned} V &= 8 \\ E &= 12 + 1 = 13 \\ F &= 6 + 1 = 7 \\ 8 - 13 + 7 &= 2 \end{aligned}$$

# Mesh processing pipeline



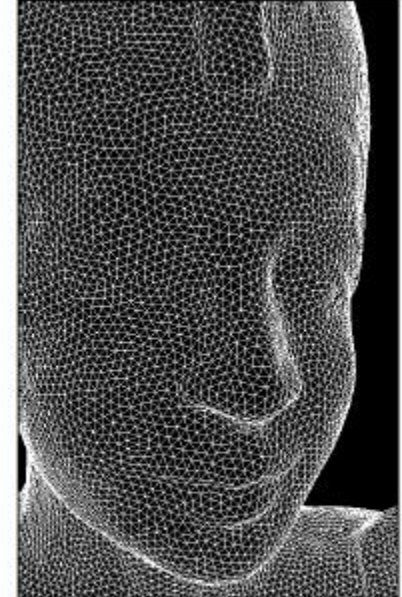
Scan



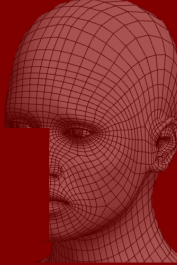
Reconstruct



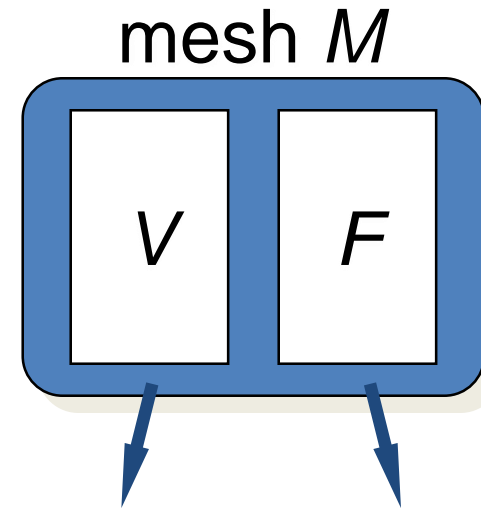
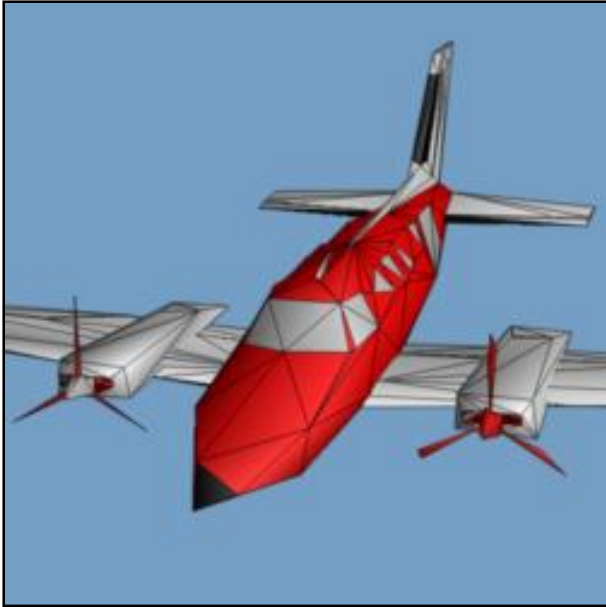
Clean



Remesh



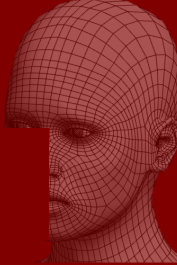
## Traditional Mesh Representation



Vertex 1	$x_1$	$y_1$	$z_1$	Face	1	2	3
Vertex 2	$x_2$	$y_2$	$z_2$	Face	3	2	4
...				Face	4	2	7
				...			

A blue arrow points from the  $z_2$  coordinate of Vertex 2 to the first face (Face 1) in the list.

(appearance attributes:  
normals, colors, textures, ...)



## Traditional Mesh Representation

```
v -6.4796930e-002 1.5210615e-001 -3.6185520e-002
v -6.4400320e-002 1.5834400e-001 -5.4256370e-002
v -6.6178120e-002 1.4218350e-001 -9.3766300e-003
v -6.7751430e-002 1.4605207e-001 -2.3333300e-002
v -6.4731580e-002 1.5410067e-001 -4.0464820e-002
v -2.4265590e-002 1.5687690e-001 -7.8509300e-003
v -1.5723180e-002 1.6312344e-001 -1.6396570e-002
v -7.0887660e-002 1.4404618e-001 -1.4908480e-002
v -4.4341830e-002 1.5113809e-001 -5.6859800e-003
v -6.2896810e-002 1.4694778e-001 -1.3098620e-002
v -6.3755400e-002 1.4428875e-001 -1.1395730e-002
v -6.8214560e-002 1.4390932e-001 -1.4984170e-002
v -5.0271440e-002 1.4336563e-001 1.5153000e-003
v -2.8535590e-002 1.6208479e-001 -1.4786030e-002
v -6.5810700e-002 1.4359119e-001 -1.2585380e-002
v -5.6179200e-002 1.3774406e-001 -4.0674300e-003
v -6.8866880e-002 1.4723338e-001 -2.8739870e-002
v -6.0965420e-002 1.7002113e-001 -6.0839390e-002
v -1.3895490e-002 1.6787168e-001 -2.1897230e-002
v -6.9413000e-002 1.5121847e-001 -4.4538540e-002
v -5.5039800e-002 5.7309700e-002 1.6990900e-002
f 1069 1647 1578
f 1058 909 939
f 421 1176 238
f 1055 1101 1042
f 238 1059 1126
f 1254 30 1261
f 1065 1071 1
f 1037 1130 1120
f 1570 2381 1585
f 2434 2502 2473
f 1632 1654 1646
f 1144 1166 669
f 1202 1440 305
```

Exemple de mesh file with  
the format OBJ





## Progressive Mesh

- New representation of triangular meshes.
- Simplify meshes through sequence of **edge collapse** transformations.
- Record the sequence of inverse transformations (**vertex splits**).
- It is necessary to undertake as many simplifications as needed to achieve the minimal model.
- hierarchical structure which helps to create a model in the chosen level of detail.

Hoppe, Progressive mesh, Siggraph 96

Hoppe, View-dependent Refinement of Progressive Meshes, Siggraph 97



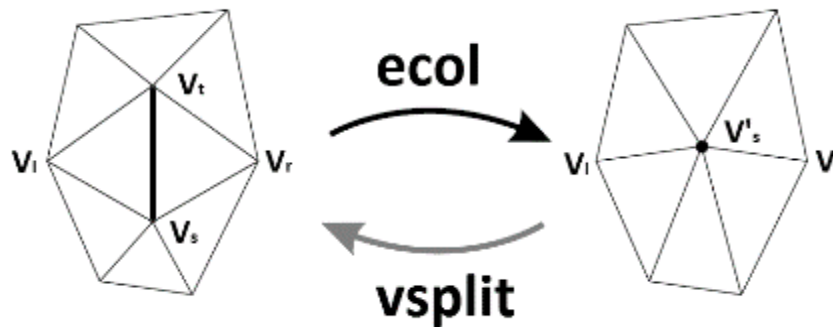
## Progressive Mesh

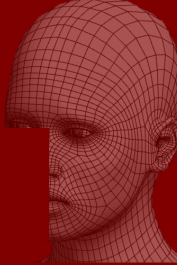
### ▪ Edge collapse

This simplistic operation - ecol takes two connected vertices and replaces them with a single vertex. Two triangles  $\{v_s, v_t, v_l\}$  and  $\{v_t, v_s, v_r\}$  which were connected by the edge are also removed during this operation.

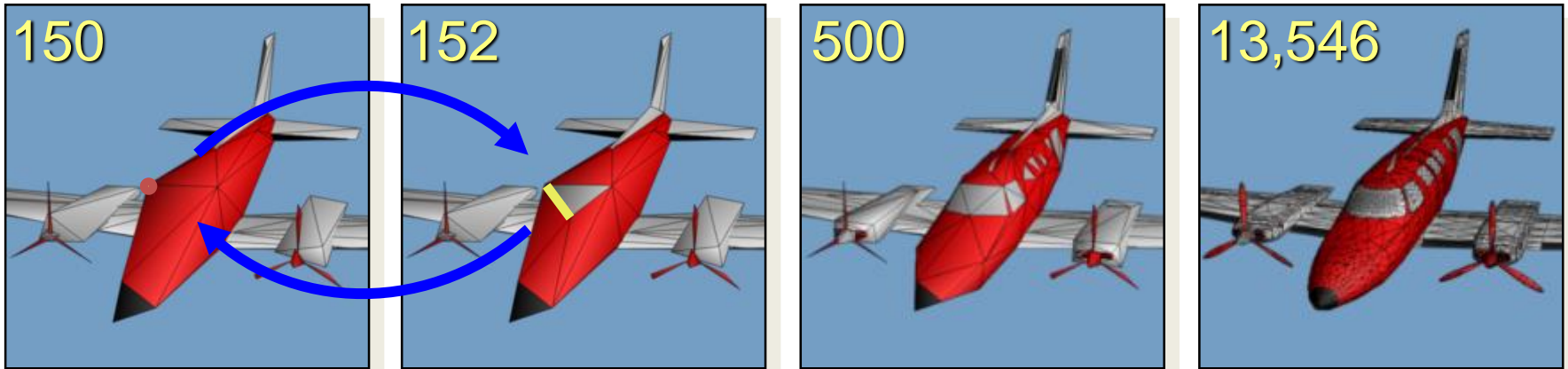
### ▪ Vertex split

Vertex split (vsplit) is the inverse operation to the edge collapse that divides the vertex into two new vertexes. Therefore, a new edge  $\{v_t, v_s\}$  and two new triangles  $\{v_s, v_t, v_l\}$  and  $\{v_t, v_s, v_r\}$  arise.





## Progressive Mesh Representation



$M^0$   
base mesh



$M^1$

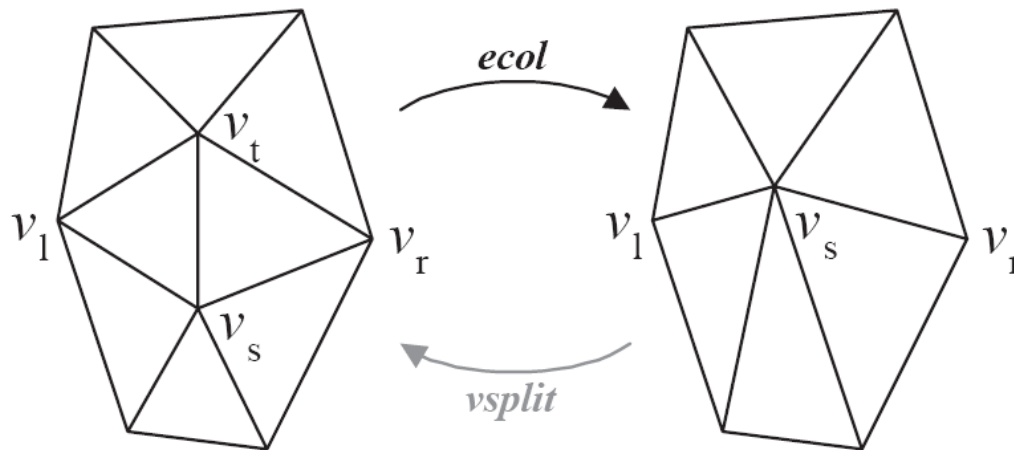


$M^{175}$



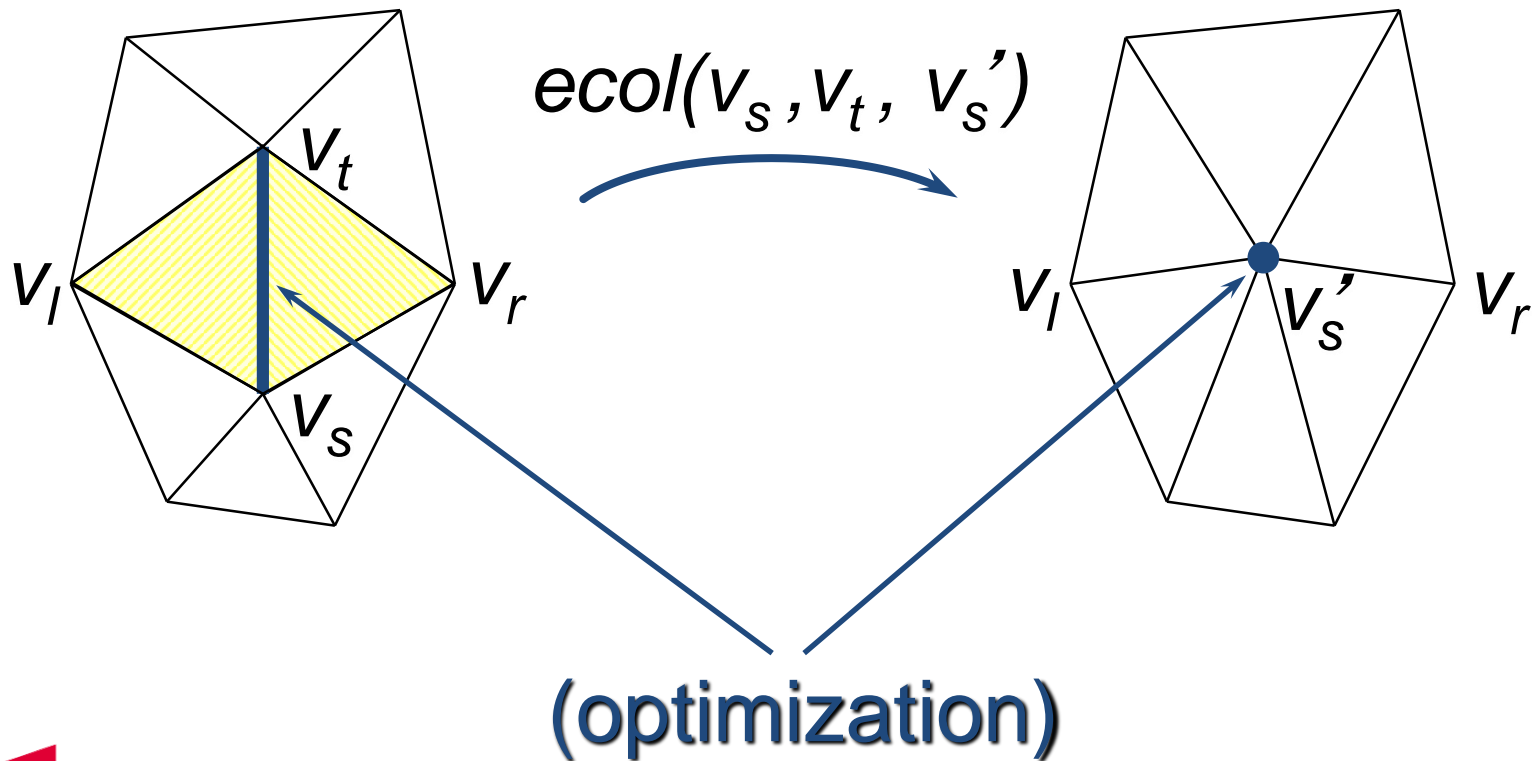
$M^n$

Original mesh

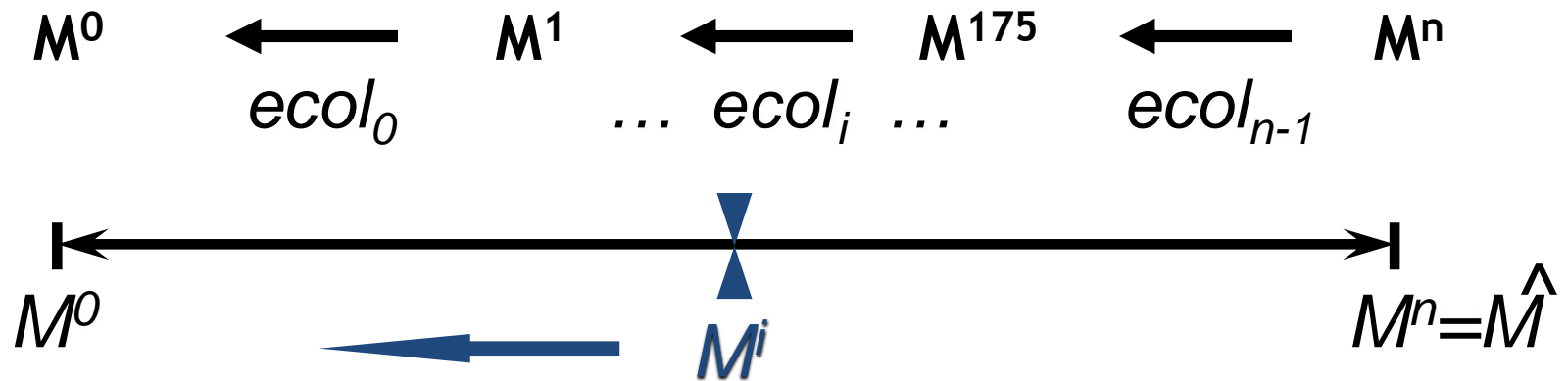
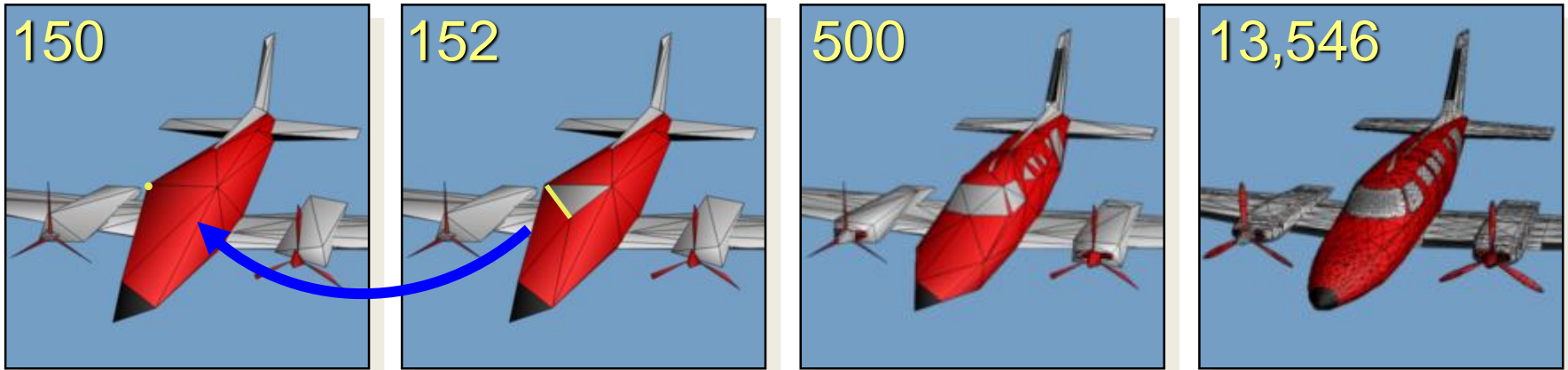


## Simplification: Edge Collapse

- **Idea:** apply a sequence of edge collapses:

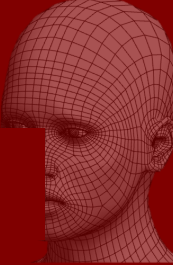


## Simplification: Edge Collapse

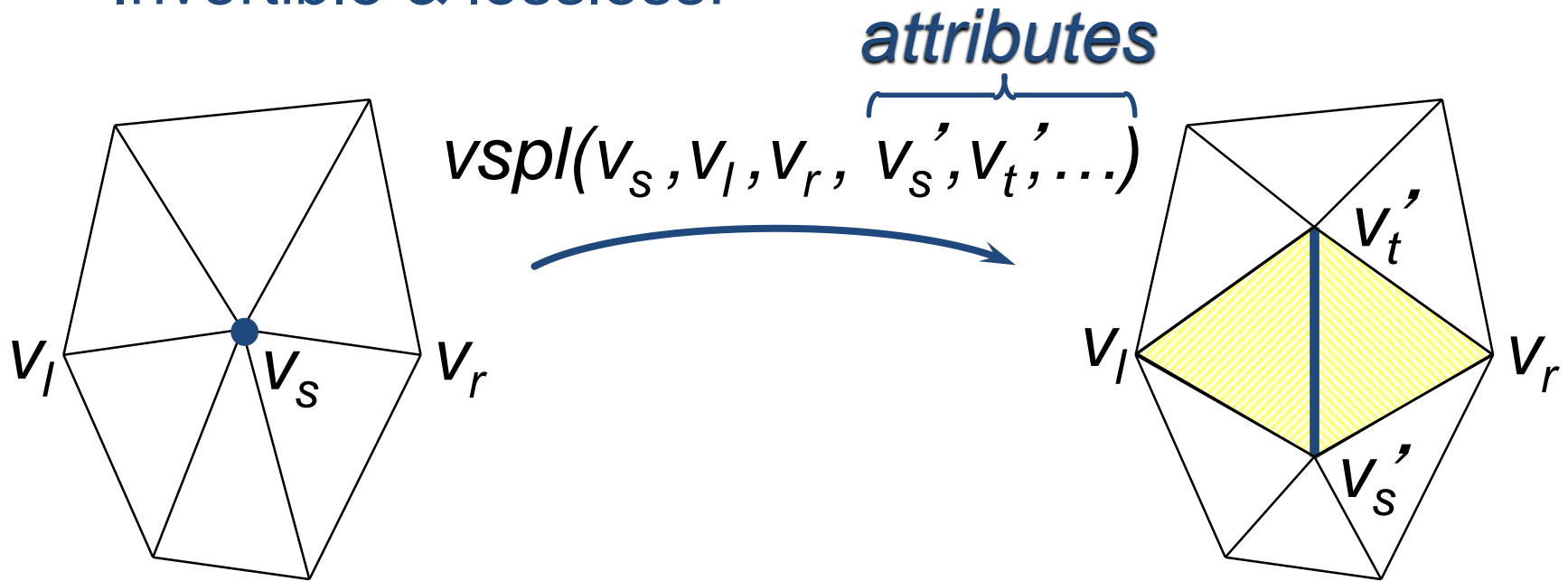


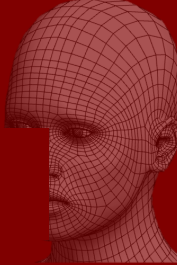


## Reconstruction: Vertex Split

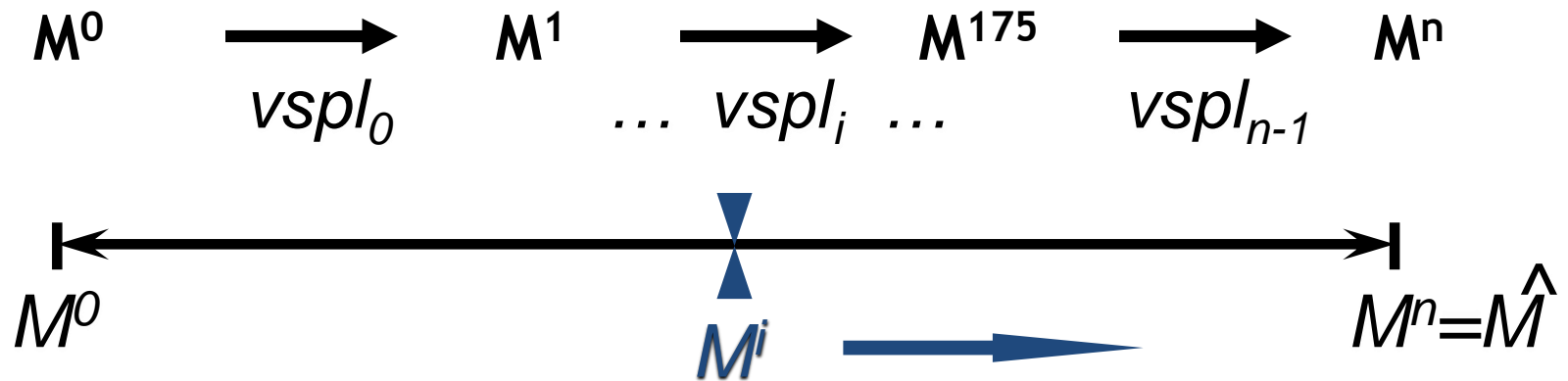
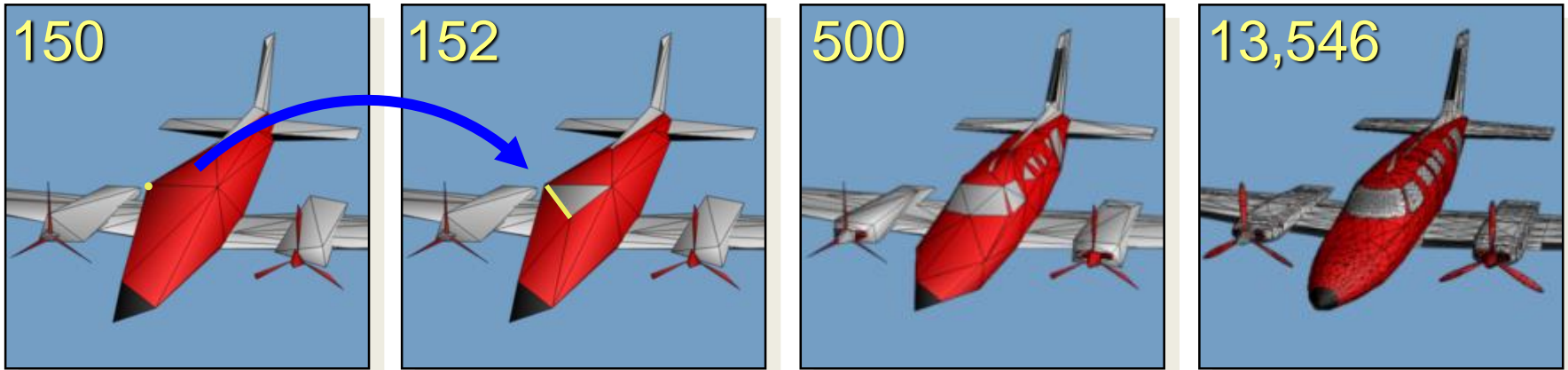


### ■ Invertible & lossless!





## Reconstruction: Vertex Split



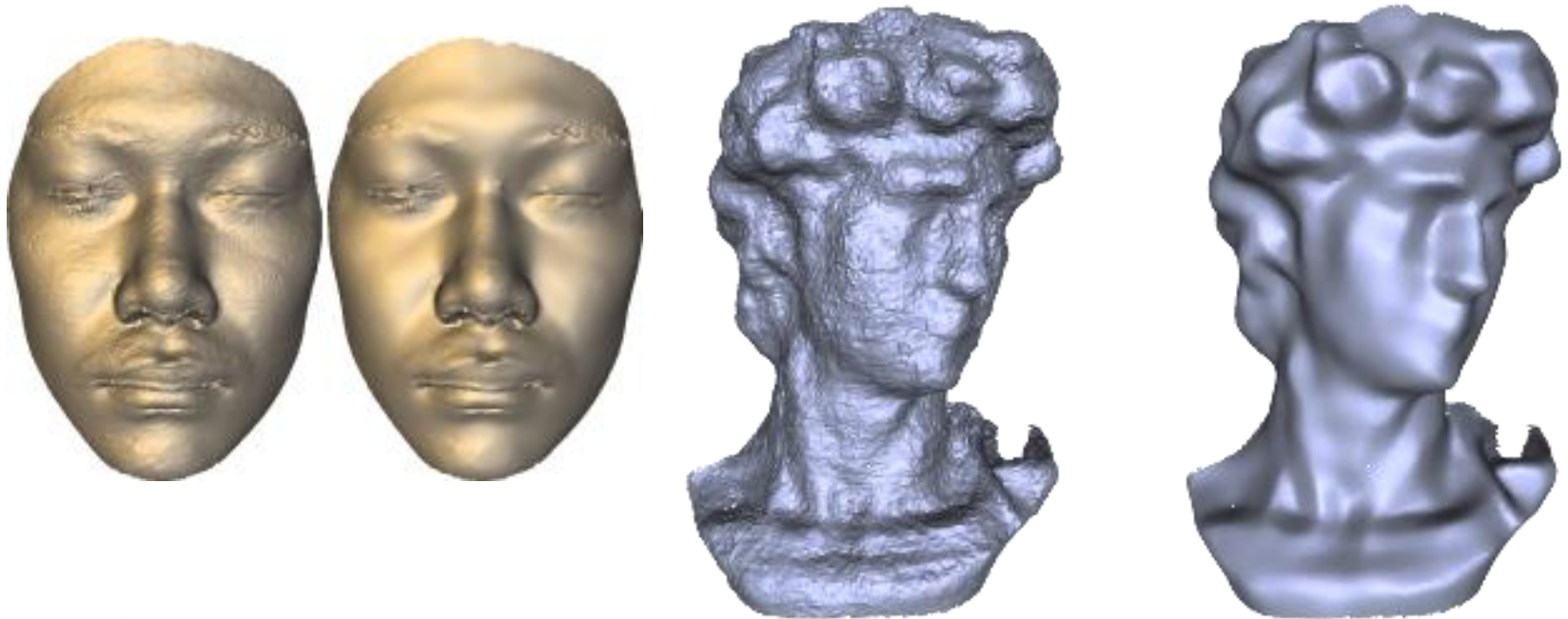


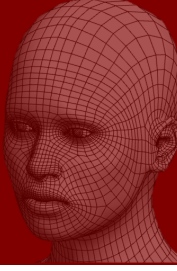
## Mesh smoothing (Fairing, Filtering, denoising)

**Input:** Noisy mesh (scanned or other)

**Output:** Smooth mesh

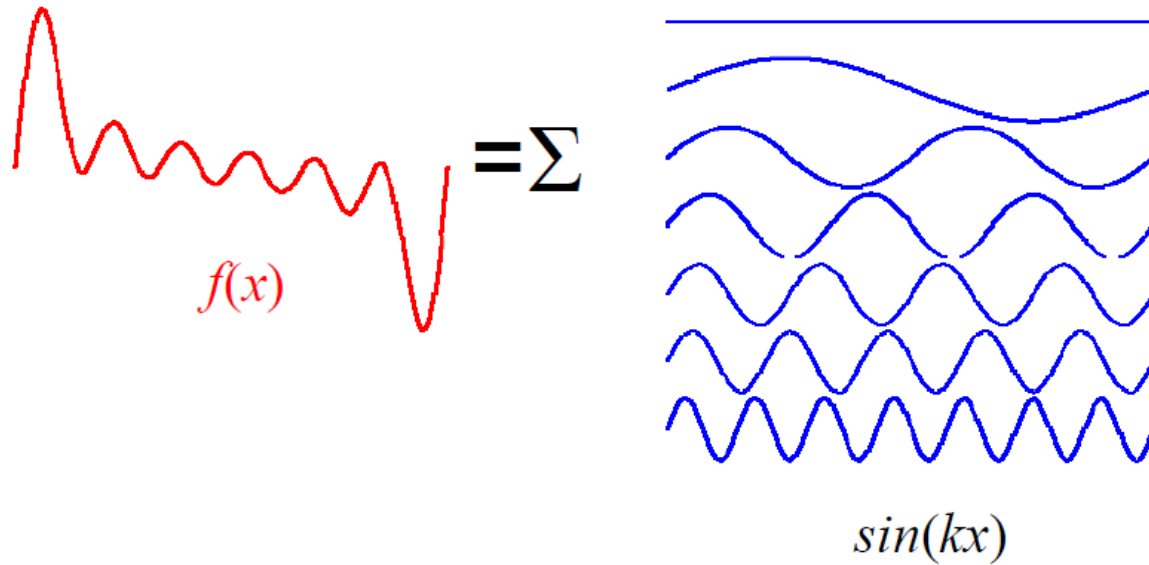
**How:** Filter out high frequency noise





## Smoothing by Filtering

# Fourier Transform

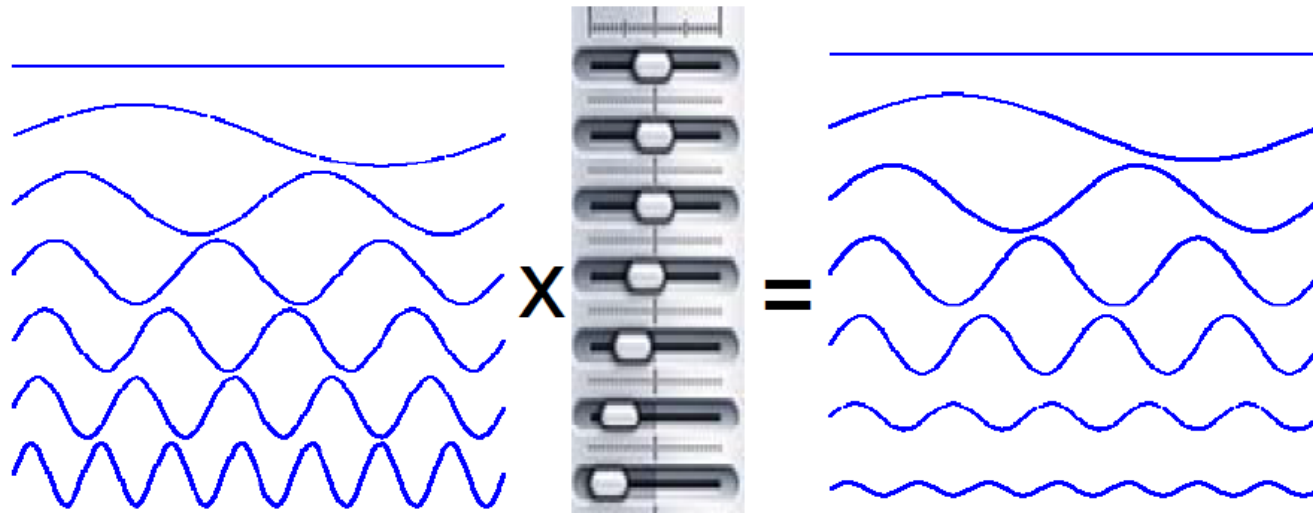


Slides by Levy et al., SigAsia Course 2009

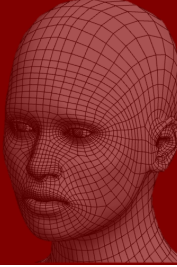


## Smoothing by Filtering

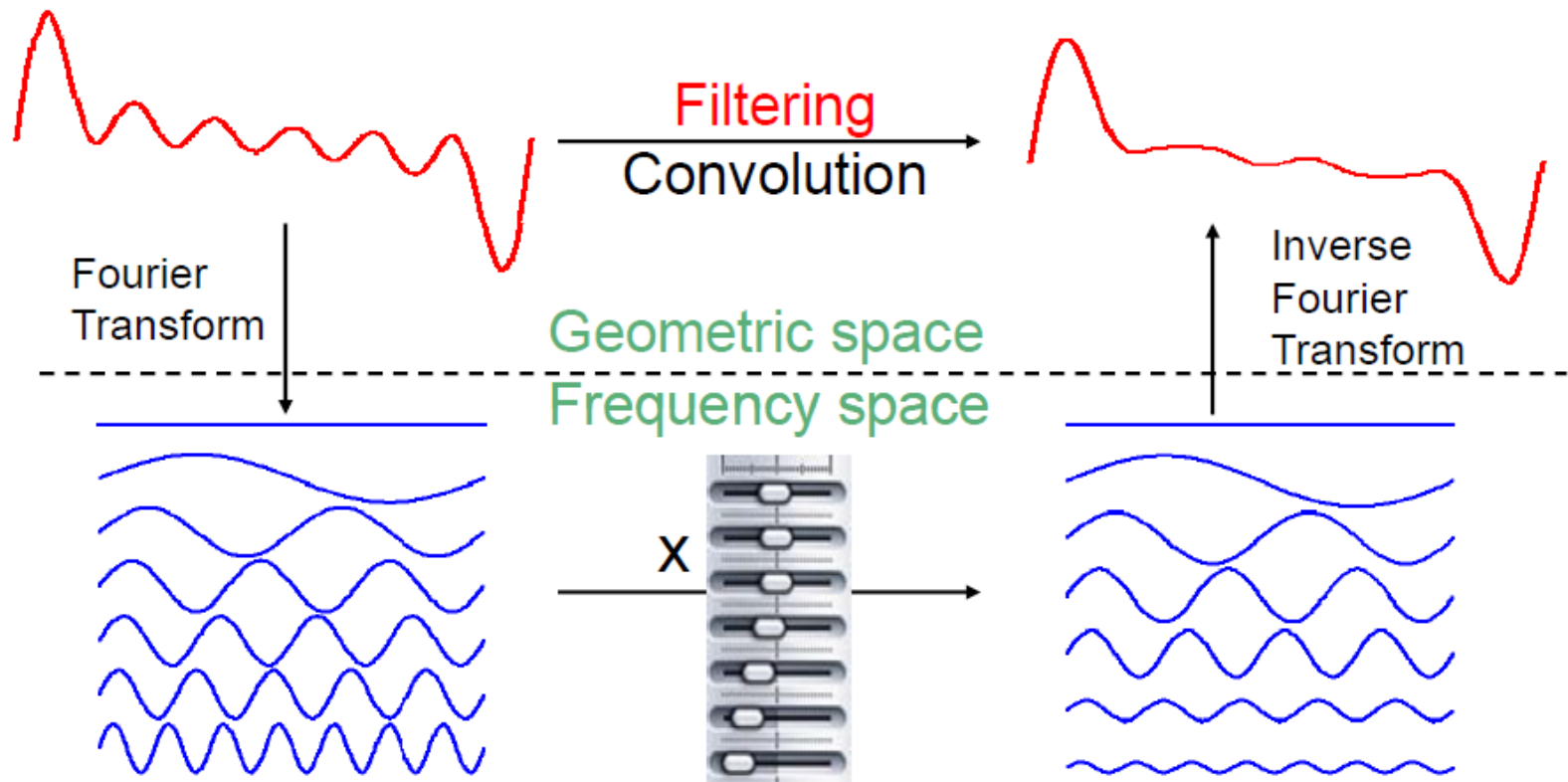
# Fourier Transform



Slides by Levy et al., SigAsia Course 2009



## Smoothing by Filtering



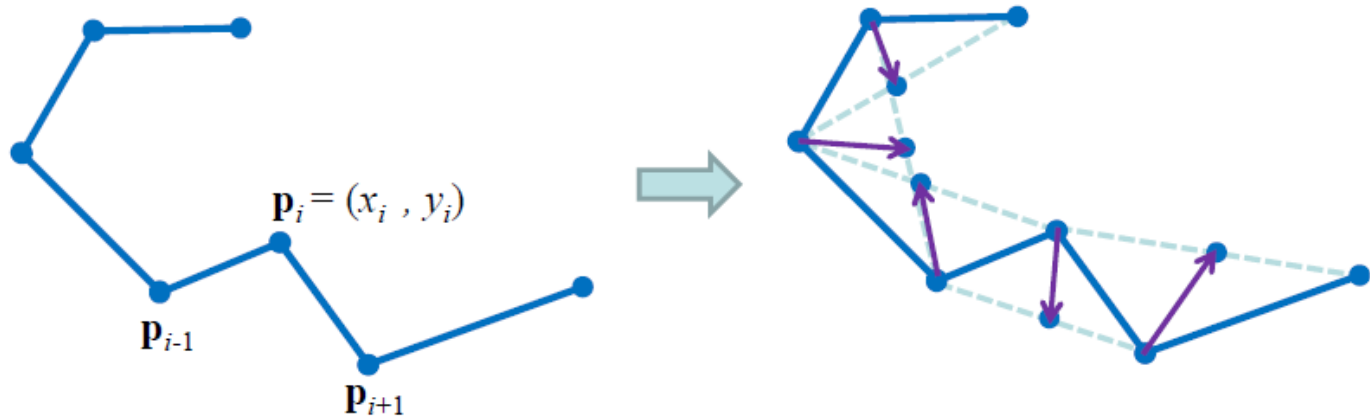
Slides by Levy et al., SigAsia Course 2009





## Laplacian Smoothing on Meshes

An easier problem: How to smooth a curve?

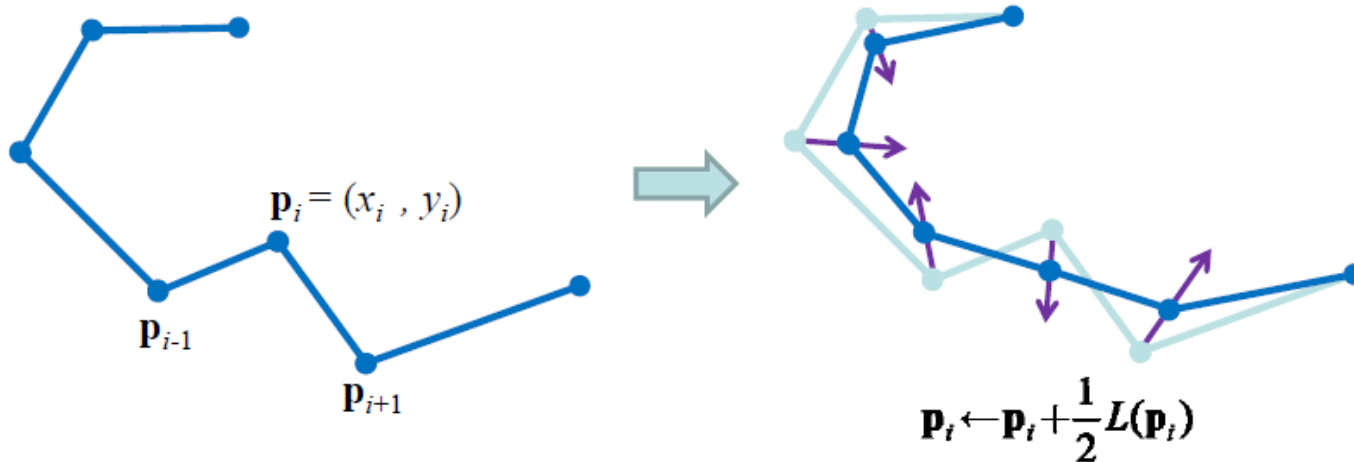


$$L(\mathbf{p}_i) = \frac{(\mathbf{p}_{i-1} + \mathbf{p}_{i+1})}{2} - \mathbf{p}_i$$
$$L(\mathbf{p}_i) = \frac{1}{2}(\mathbf{p}_{i+1} - \mathbf{p}_i) + \frac{1}{2}(\mathbf{p}_{i-1} - \mathbf{p}_i)$$



## Laplacian Smoothing on Meshes

An easier problem: How to smooth a curve?



Finite difference  
discretization of second  
derivative  
= Laplace operator in  
one dimension

$$L(p_i) = \frac{1}{2}(p_{i+1} - p_i) + \frac{1}{2}(p_{i-1} - p_i)$$



## Laplacian Smoothing on Meshes

### Algorithm:

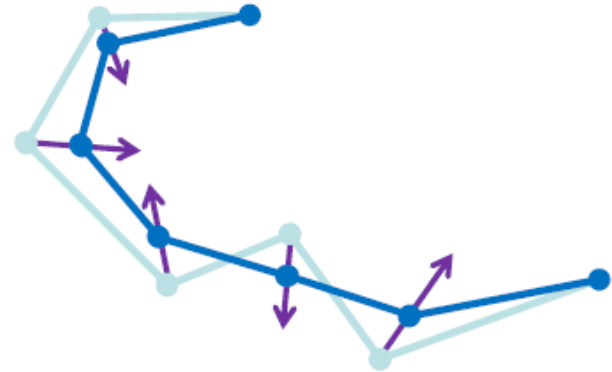
Repeat for  $m$  iterations (for non boundary points):

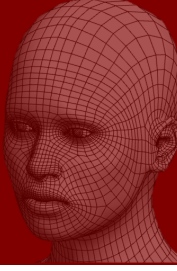
$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda L(\mathbf{p}_i)$$

For which  $\lambda$ ?

$$0 < \lambda < 1$$

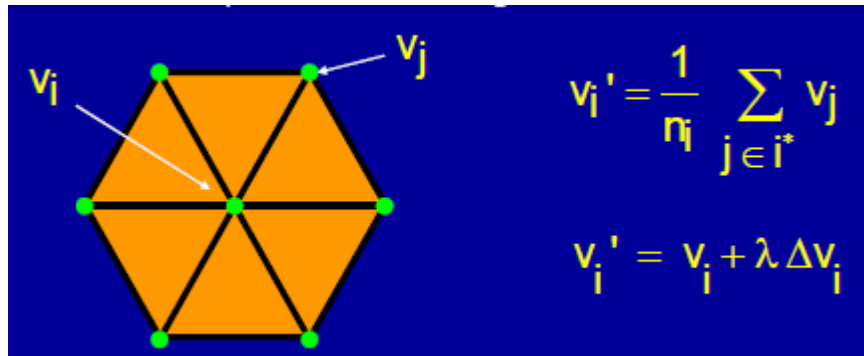
Closed curve converges to?  
Single point





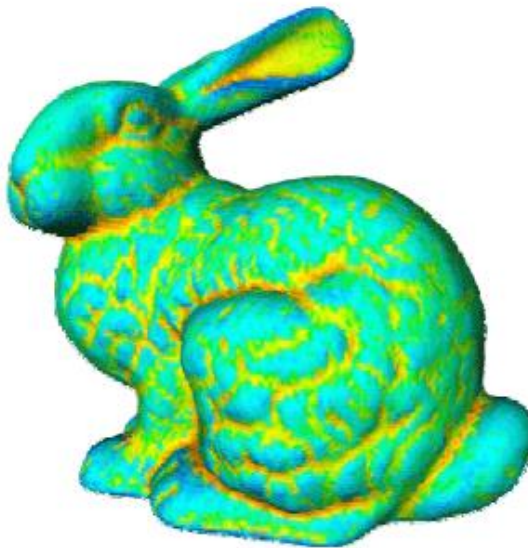
## Laplacian Smoothing on Meshes

- Keep boundary vertices fixed
- Move each internal vertex to the barycenter of its neighbors

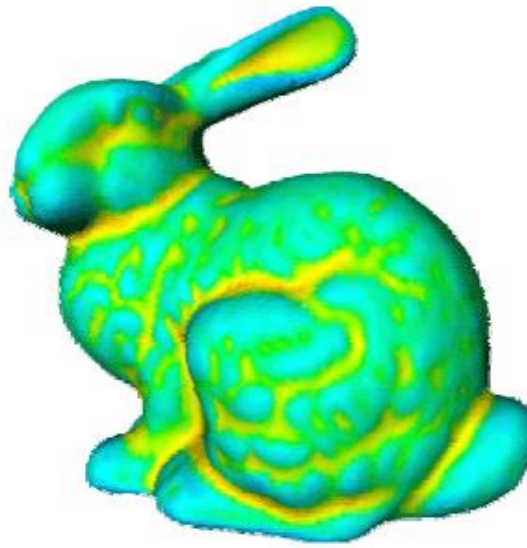




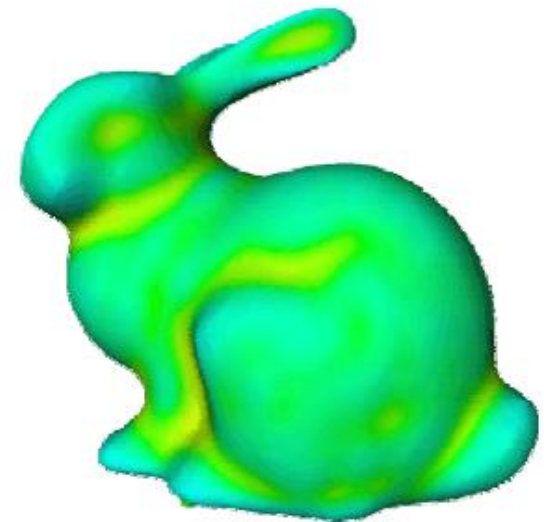
## Laplacian Smoothing on Meshes



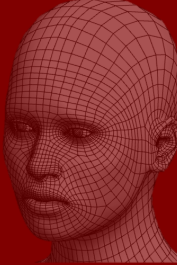
0 Iterations



5 Iterations

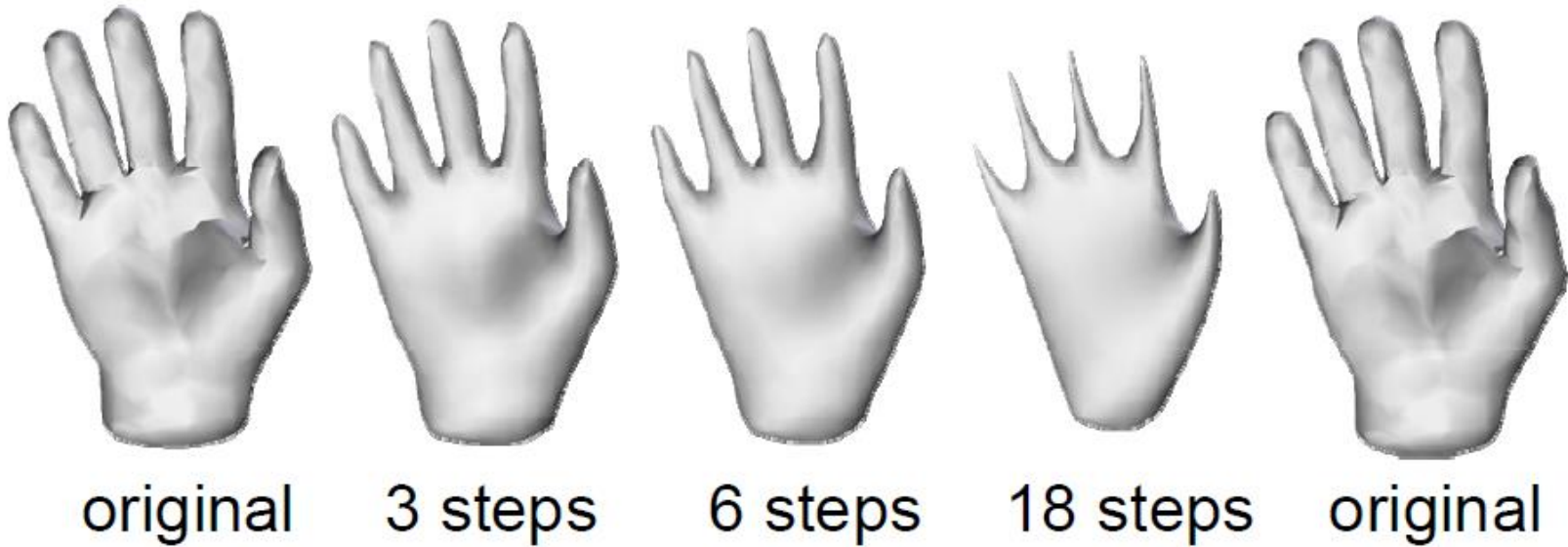


20 Iterations



## Laplacian Smoothing problem: Shrinkage

Repeated iterations of Laplacian smoothing shrinks the mesh







## Taubin Smoothing

Iterate:

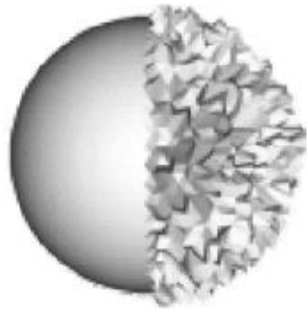
$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda \Delta \mathbf{p}_i$$

Shrink

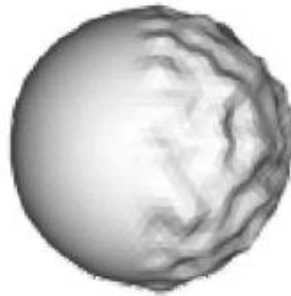
$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \mu \Delta \mathbf{p}_i$$

Inflate

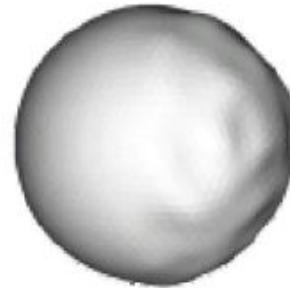
with  $\lambda > 0$  and  $\mu < 0$



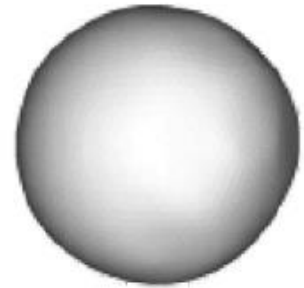
original



10 steps



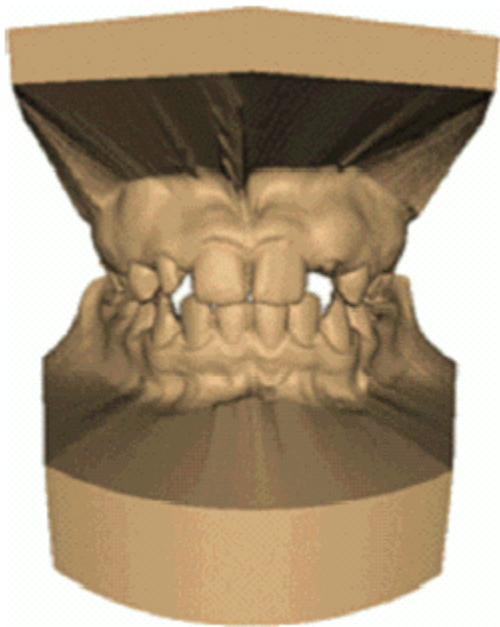
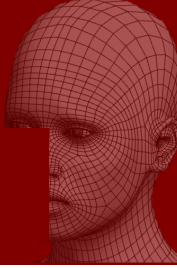
50 steps



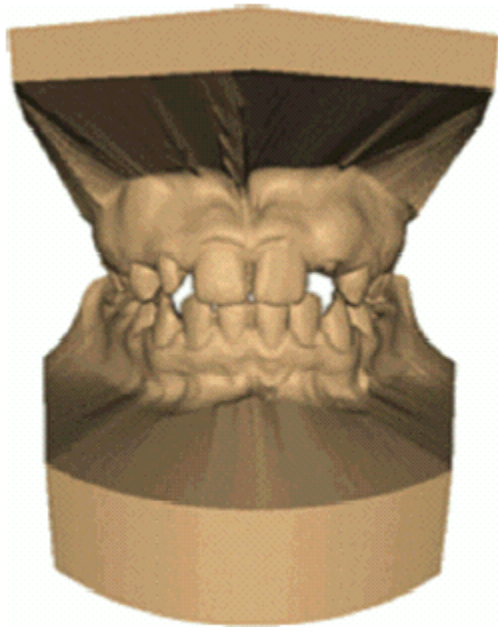
200 steps

From Taubin, Siggraph 1995

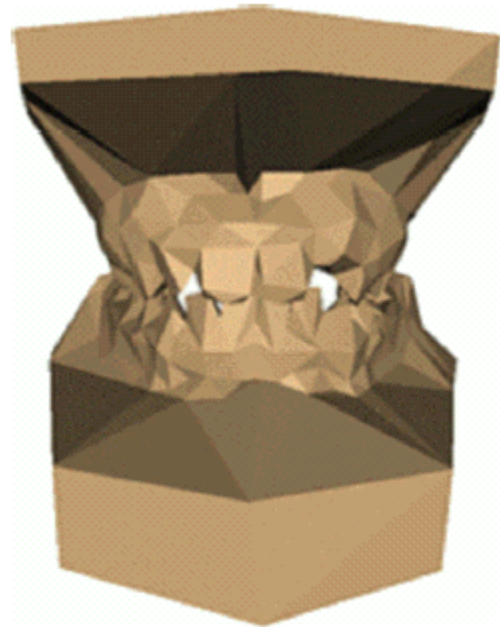
## Mesh Simplification



Full Resolution

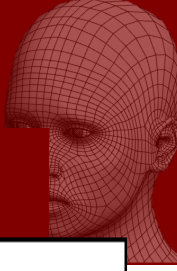


60,000 triangles

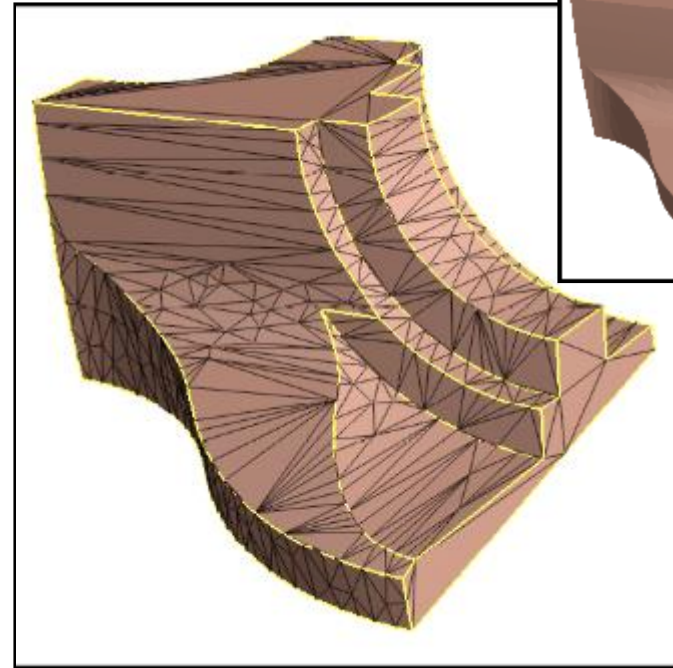


1000 triangles

## Application – Mesh compression



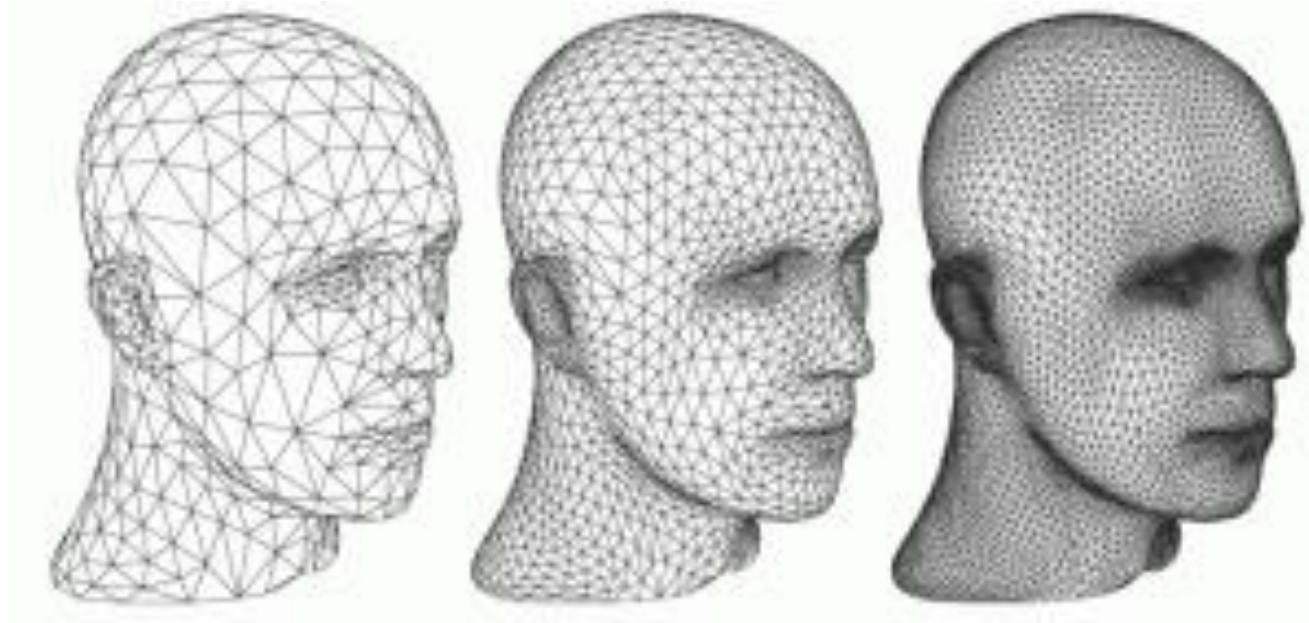
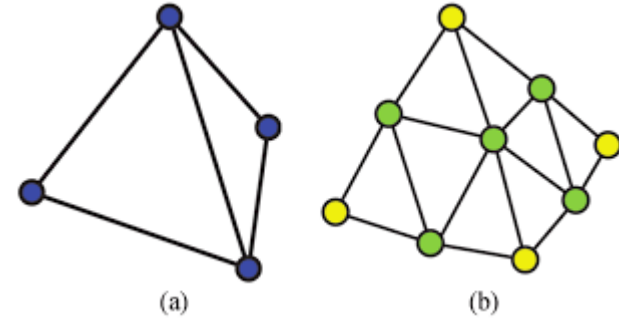
12964 faces



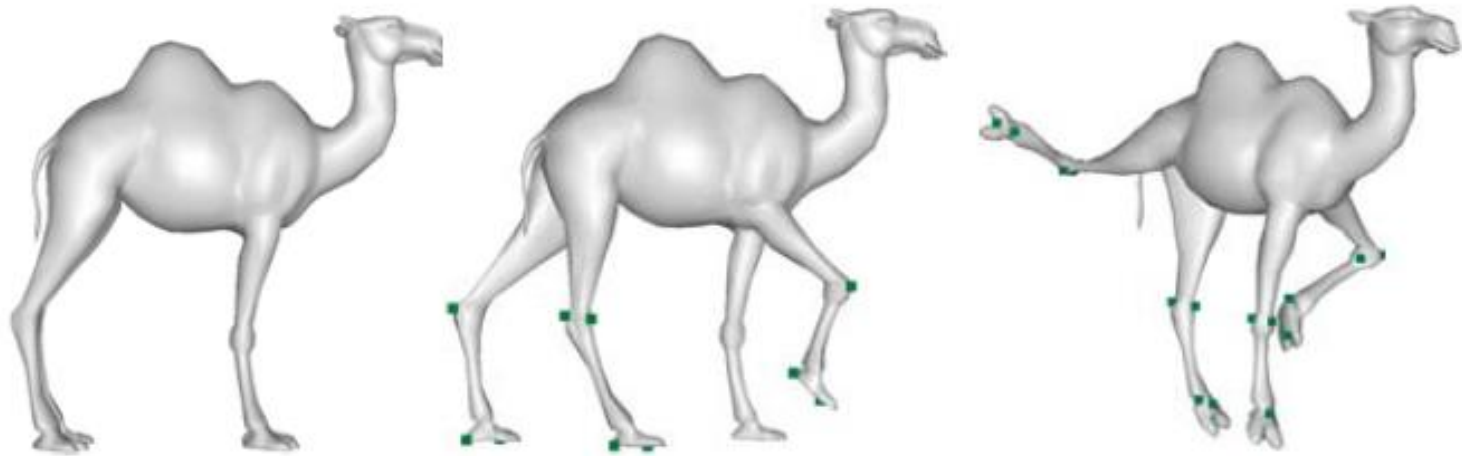
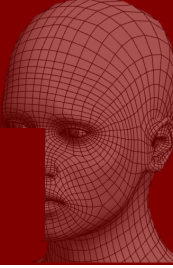
1000 faces



# Mesh Subdivision

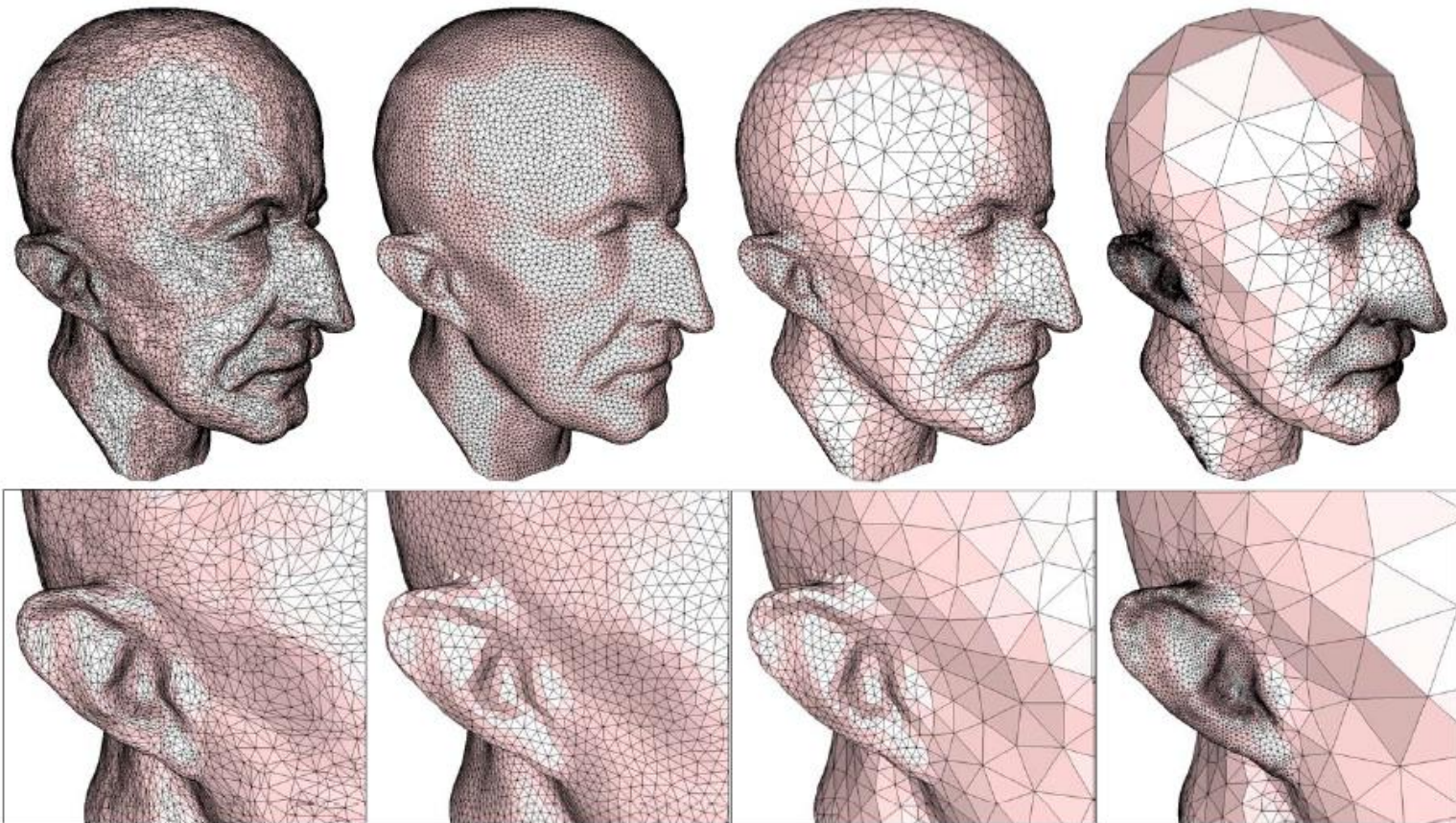
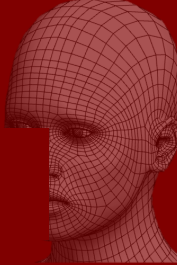


# Mesh Deformation



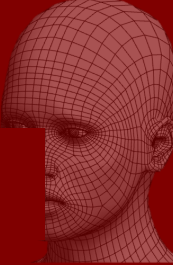


# Remeshing

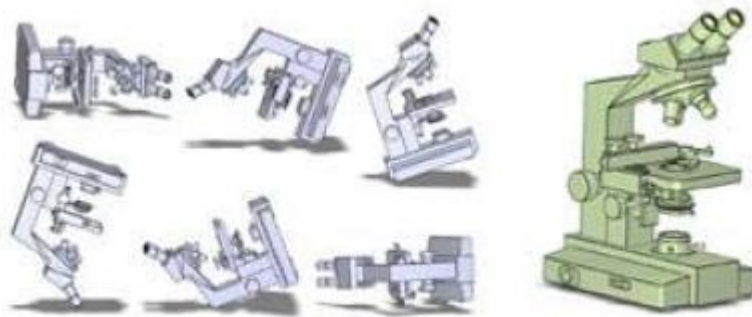




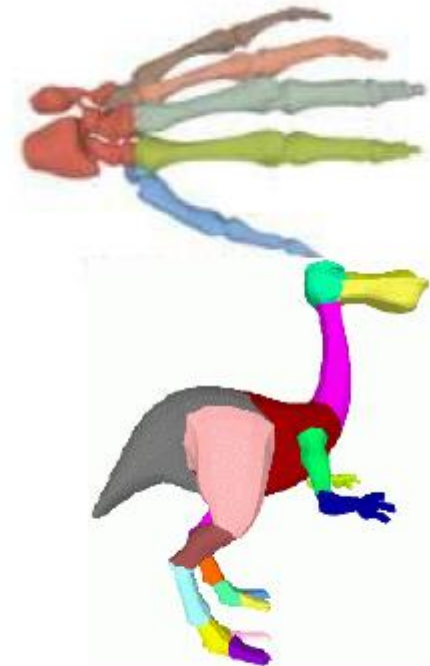
# Mesh Analysis



Matching



Orientation/View Selection



Segmentation



## Resources

OpenMesh:

[OpenMesh web page](#)

[OpenMesh documentation](#)

Mesh manipulation:

[MeshLab](#)

[Graphite](#)

Models:

[AIM@SHAPE Repository](#)

[Princeton Segmentation Database \(82M\)](#)

Library

[CGAL](#)

[PCL](#)



## References

- **Book**

“Polygon Mesh Processing”

*by Mario Botsch, Leif Kobbelt, Mark Pauly, Pierre Alliez, Bruno Levy*

- **Eurographics 2008 course notes**

“Geometric Modeling Based on Polygonal Meshes”

*by Mario Botsch, Mark Pauly, Leif Kobbelt, Pierre Alliez, Bruno Levy, Stephan Bischoff, Christian Rössl*

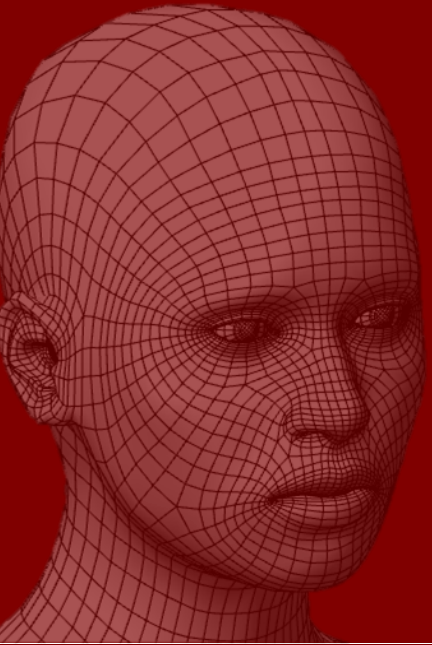
- **Tutorials and papers**

“Polygonal Mesh –Data Structure and Processing” *by Chiew-Lan Tai*



## Play with 3D mesh

- **Download Meshlab** <http://www.meshlab.net/#download>
- Download Meshes from learning platform
- Fun begin, play with meshes !!



**Thank you for your attention**