Exercice 1 : Calcul de coefficients d'ondelettes (1D)

In[40]:= `image[t_] := If[t ≥ 0 && t ≤ 10, 1, 0]`
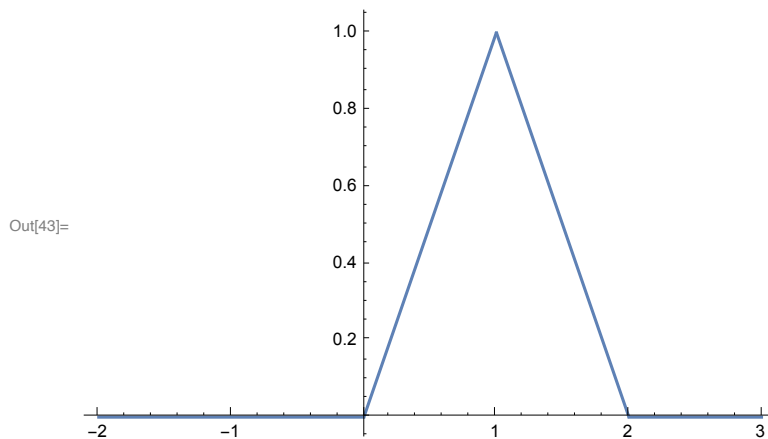
Question 1

In[41]:= `one[t_] := If[t ≥ 0 && t ≤ 1, 1, 0]`

In[42]:= `triangle[t_] = Convolve[one[t], one[t], {t}, {τ}]`

Out[42]= $\begin{cases} 2 - \tau & 1 < \tau < 2 \\ \tau & 0 < \tau \leq 1 \\ 0 & \text{True} \end{cases}$

Question 2

In[43]:= `Plot[triangle[t], {τ, -2, 3}]`

Out[43]=

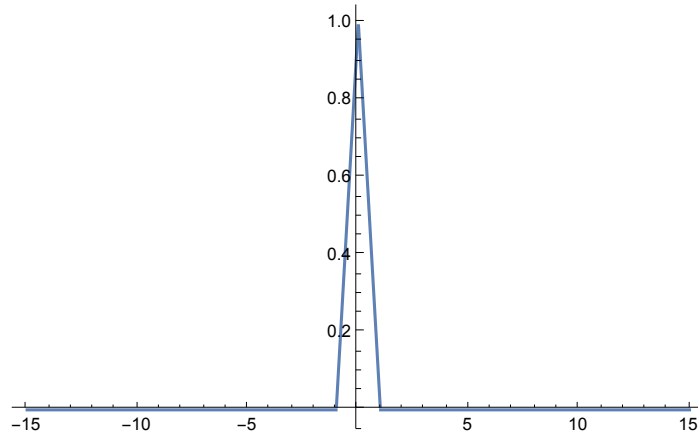## Question 3

In[44]:= `phi[t_] := If[t > 0 && t ≤ 1, 1, 0]`

In[45]:= `atomeL[t_, u_] = phi[t - u]`

Out[45]= $If[t - u > 0 \text{ && } t - u \leq 1, 1, 0]$

In[46]:= `Integrate[one[t] * atomeL[t, u], {t, -∞, ∞}]`
`Plot[%, {u, -15, 15}]`

Out[46]=
$$\begin{cases} 1 - u & 0 \le u < 1 \\ 1 + u & -1 < u < 0 \\ 0 & \text{True} \end{cases}$$

Out[47]=

Question 4

In[48]:= `Integrate[image[t] * atomeL[t, u], {t, -∞, ∞}]`
`Plot[%, {u, -15, 15}]`

Out[48]=
$$\begin{cases} 1 & 0 \le u \le 9 \\ 10 - u & 9 < u < 10 \\ 1 + u & -1 < u < 0 \\ 0 & \text{True} \end{cases}$$

Out[49]=
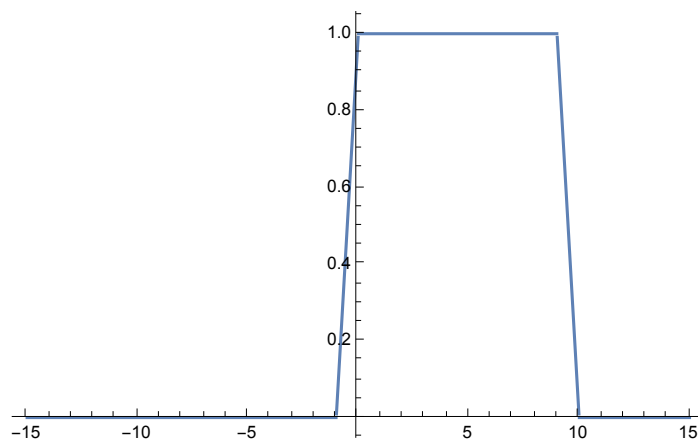
Question 5
psi(t) = phi(2t) - phi(2t-1)

Question 6

In[50]:= `psi[t_] := If[t > 0 && t ≤ 1 / 2, 1, If[t > 1 / 2 && t ≤ 1, -1, 0]]`

In[51]:= **atomeH[t_, u_] = psi[t - u]**

Out[51]= $\text{If}\left[t - u > 0 \,\&\&\, t - u \le \dfrac{1}{2},\ 1,\ \text{If}\left[t - u > \dfrac{1}{2} \,\&\&\, t - u \le 1,\ -1,\ 0\right]\right]$

In[52]:= **coeffH[u_] = Integrate[image[t] * atomeH[t, u], {t, -∞, ∞}]**
Plot[coeffH[u], {u, -5, 15}]

Out[52]= $\begin{cases} -\dfrac{1}{2} & u == -\dfrac{1}{2} \\[6pt] \dfrac{1}{2} & u == \dfrac{19}{2} \\[6pt] -1 - u & -1 < u < -\dfrac{1}{2} \\[6pt] 10 - u & \dfrac{19}{2} < u < 10 \\[6pt] -9 + u & 9 < u < \dfrac{19}{2} \\[6pt] u & -\dfrac{1}{2} < u < 0 \\[6pt] 0 & \text{True} \end{cases}$
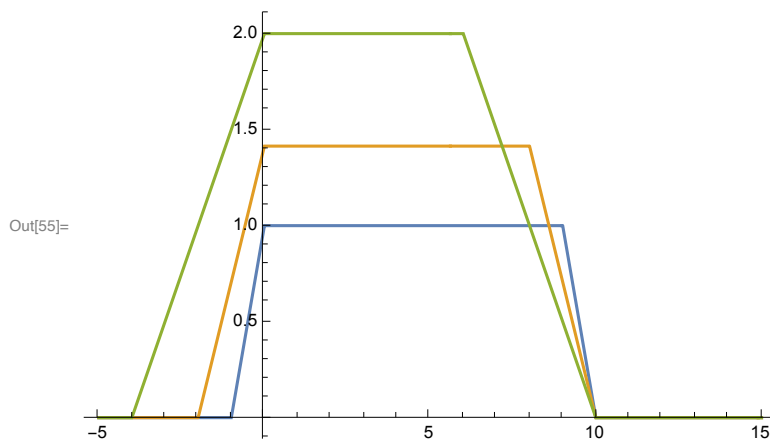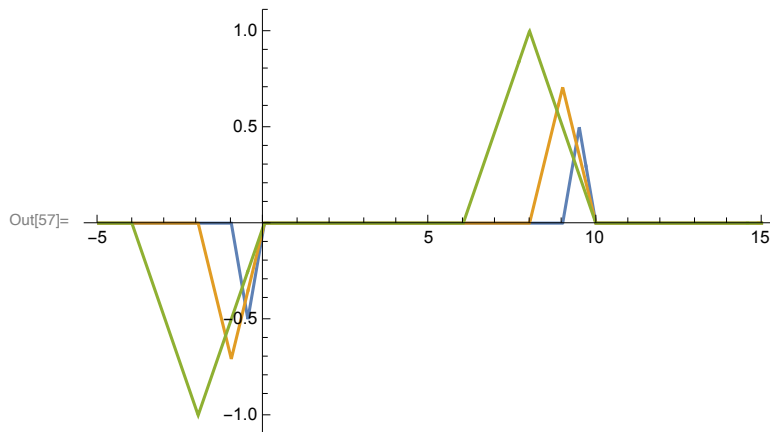
Out[53]=
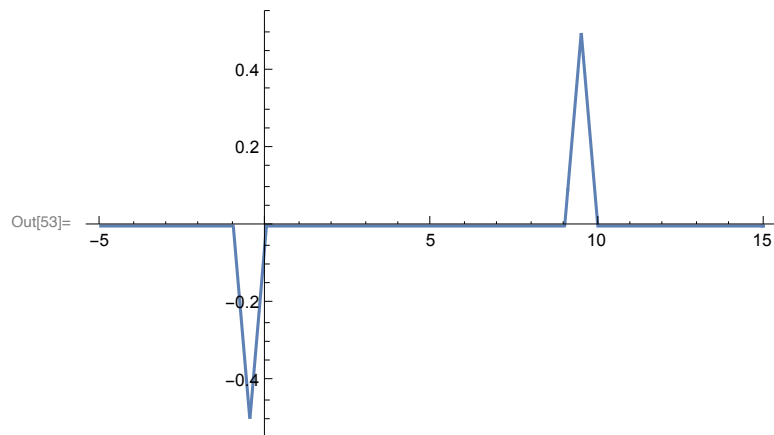
Observer que les coefficients de la fonction d'échelle font apparaître les BFs quand ceux des ondelettes font apparaître les HF (variations), et tout cela de façon LOCALE.

Question 7

In[54]:= `Integrate[image[t] * 1 / Sqrt[s] * phi[(t - u) / s],`
`{t, -∞, ∞}, Assumptions → {s > 0}];`
`Plot[{% /. {s → 1}, % /. {s → 2}, % /. {s → 4}}, {u, -5, 15}]`

Out[55]=

In[56]:= `Integrate[image[t] * 1 / Sqrt[s] * psi[(t - u) / s],`
`{t, -∞, ∞}, Assumptions → {s > 0}];`
`Plot[{% /. {s → 1}, % /. {s → 2}, % /. {s → 4}}, {u, -5, 15}]`

Out[57]=

Exercice 2 : Calcul de coefficients d'ondelettes (2D)

=> sans calcul, intuiter le résultat

In[58]:= `image2D[x_, y_] := If[x ≥ 0 && x ≤ 10, 1, 0] * If[y ≥ 0 && y ≤ 10, 1, 0]`

In[59]:= `atomeLL[x_, y_] := phi[x] * phi[y]`

In[60]:= `atomeLH[x_, y_] := phi[x] * psi[y]`

In[61]:= `atomeHL[x_, y_] := psi[x] * phi[y]`

In[62]:= `atomeHH[x_, y_] := psi[x] * psi[y]`
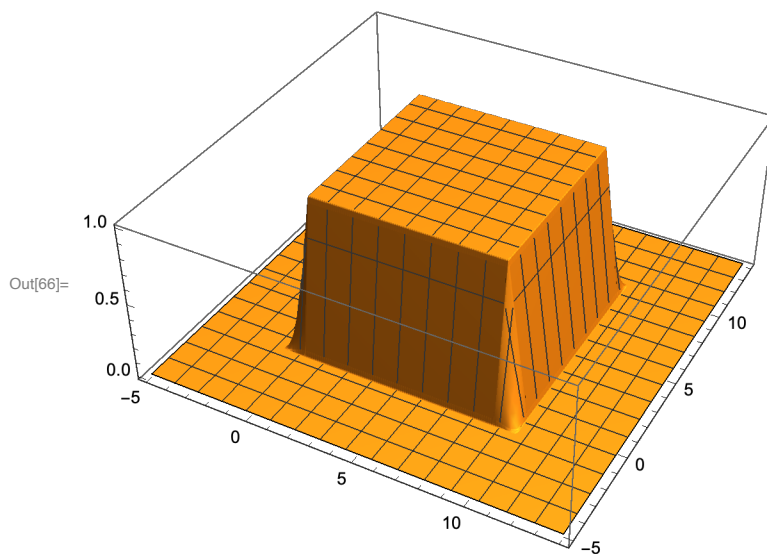
Observer séparabilité atomes et image ! (simplifiera les calculs !)
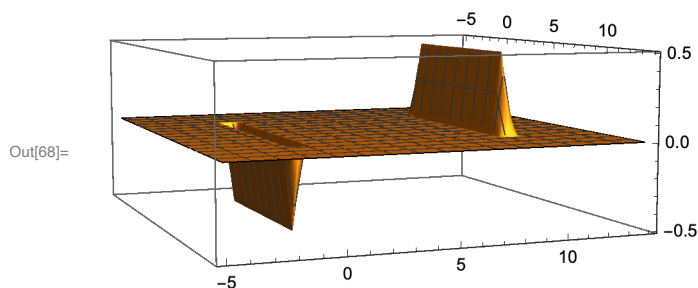
In[63]:= `xmin = -5; xmax = 14;`
`ymin = -5; ymax = 14;`

In[65]:= `coeffLL[u_, v_] =`
`    Integrate[image2D[x, y] * atomeLL[x - u, y - v], {x, -∞, ∞}, {y, -∞, ∞}];`
`Plot3D[%, {u, xmin, xmax}, {v, ymin, ymax}, PlotPoints → 100,`
` ExclusionsStyle → Automatic]`

Out[66]=

In[67]:= `coeffLH[u_, v_] =`
`    Integrate[image2D[x, y] * atomeLH[x - u, y - v], {x, -∞, ∞}, {y, -∞, ∞}];`
`Plot3D[%, {u, xmin, xmax}, {v, ymin, ymax}, PlotPoints → 100,`
` ExclusionsStyle → Automatic]`

Out[68]=
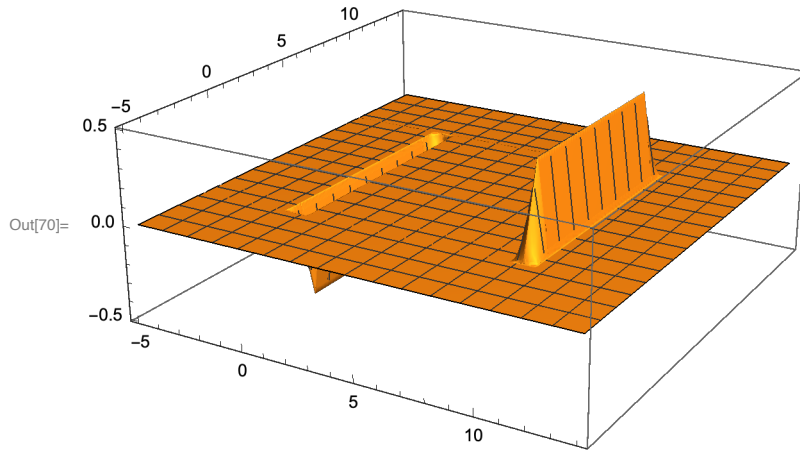
```
In[69]:= coeffHL[u_, v_] =
          Integrate[image2D[x, y] * atomeHL[x - u, y - v], {x, -∞, ∞}, {y, -∞, ∞}];
        Plot3D[%, {u, xmin, xmax}, {v, ymin, ymax}, PlotPoints → 100,
         ExclusionsStyle → Automatic]
```

Out[70]=


```
In[71]:= coeffHH[u_, v_] =
          Integrate[image2D[x, y] * atomeHH[x - u, y - v], {x, -∞, ∞}, {y, -∞, ∞}];
        Plot3D[%, {u, xmin, xmax}, {v, ymin, ymax}, PlotPoints → 100,
         ExclusionsStyle → Automatic]
```
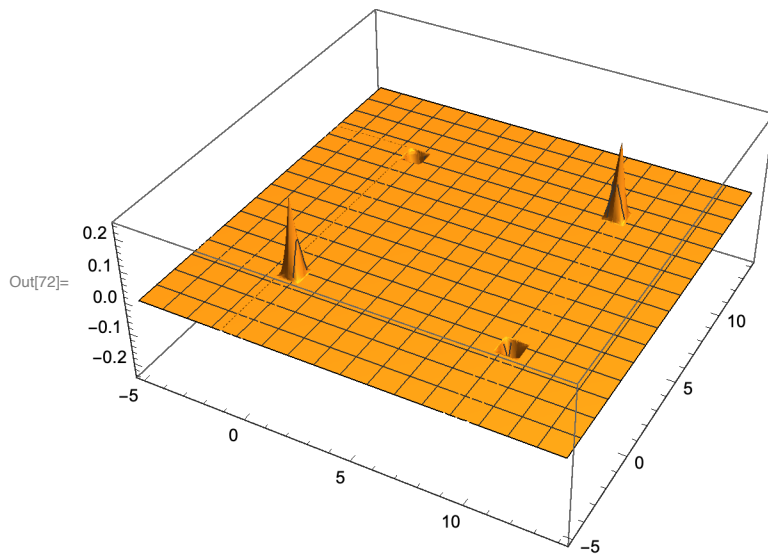
Out[72]=


```
In[73]:= pas = 0.1;
        COEFFSLL = Table[coeffLL[u, v], {u, xmin, xmax, pas}, {v, ymin, ymax, pas}];
        COEFFSLH = Table[coeffLH[u, v], {u, xmin, xmax, pas}, {v, ymin, ymax, pas}];
        COEFFSHL = Table[coeffHL[u, v], {u, xmin, xmax, pas}, {v, ymin, ymax, pas}];
        COEFFSHH = Table[coeffHH[u, v], {u, xmin, xmax, pas}, {v, ymin, ymax, pas}];
```

In[78]:=
```
GraphicsRow[{Image[COEFFSLL] // ImageAdjust, Image[COEFFSLH] // ImageAdjust,
    Image[COEFFSHL] // ImageAdjust, Image[COEFFSHH] // ImageAdjust}]
```

Out[78]=

Attention, où sont les détails verticaux ? et horizontaux ? (penser 1D !)