

Chapter - 6

Physical Variables and Transducers.

Transducers :-

Basically transducer is defined as a device which converts energy or information from one form to another. A device which converts physical quantities into equivalent electrical quantities is known as electrical transducer. An electrical transducer transforms a physical quantity to be measured directly or with a suitable mechanism into an electrical signal current, voltage or frequency.

* Classification of Transducers :-

1) Primary and Secondary Transducers :-

A transducer in which physical quantities (input signal) is converted into the electrical form directly is known as primary transducer.

Eg :- Thermister used in temperature measurement

A transducer in which input is sensed first by some detector or sensor and then the output of sensor or detector being in the form other than input is converted into electrical form is known as secondary transducer eg: LVDT

2) Active and Passive transducers:-

A self generating type transducer which do not requires external power is known as active transducer. Eg : Thermo couple

A transducer which requires an external power source for energy conversion is known as passive transducer.
Eg: strain gauge.

Analog and Digital transducers

A transducer which converts input signal into output signal which is a continuous function of time is known as analog transducer.
Eg: Strain gauge, LVDT, thermocouple etc.

A transducer which converts input signal into the output signal in the form of pulses is known as digital transducer.

Selection of Transducers :-

The points to be considered in determining a transducer suitability for a specific measurement are:

1) Range :-

The range of the transducer should be large enough to encompass all the expected magnitudes of the measurand.

2) Sensitivity :-

The transducer should give a sufficient output signal per unit of measured input in order to yield meaningful data.

3 Electrical Output Characteristics:

The electrical characteristics such as the output impedance, frequency response, and the response time of the transducer output signal should be compatible with the recording device and the rest of the measuring system equipment.

4 Physical Environment:

The selected transducer must be compatible to withstand the environmental conditions in which measurement performed.

5 Errors:

The errors due to operation of transducer itself or due to environmental condition must be controllable compatible to enough to take meaningful data under measurement.

* Requirement of good transducer:

1) Ruggedness:

It should be capable of withstanding overload and some safety arrangement should be provided for overload protection.

2) Linearity:

Its input-output characteristics should be linear and it should produce these characteristics in symmetrical way.

3 Repeatability:

It should produce same output signal when the same input signal is applied again and again under fixed environmental conditions eg. temperature, pressure, humidity etc.

4 High output signal quality:

The quality of output signal should be good ie. the ratio of the signal to noise should be high and the amplitude of the output signal should be enough.

5 High Reliability and stability:

It should give minimum error in measurement for temperature variations and vibrations, other various change in surrounding.

6 Good dynamic response:

It's output should be faithful to input when taken as a function of time. The effect is analyzed at the frequency response.

7 No hysteresis:

It should not give any hysteresis during measurement while input signal is varied from its low value to high value and vice versa.

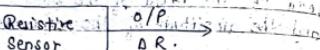
8 Residual Deformation:

There should be no deformation or removal of input signal after long period of application.

* Types of Transducers:

1. Resistive transducers:

In such transducers, resistance between the output terminals of a transducer get varied according to the measurand. Resistive transducers are preferred over other transducers because of their suitable resistance measurement.



Potentiometric Transducers

It is an electromechanical device containing a resistance element that is contacted by a movable slider. The motion of slider result in a resistance change which may be linear, logarithmic, exponential and so depending on the manner of resistance wire winding. This transducer converts linear or rotational displacement into a voltage.

The figure below shows the basic diagram of a potentiometric transducer.

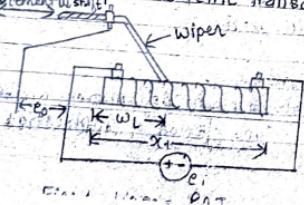
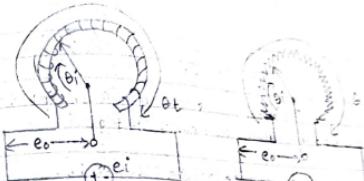


Fig.2: Equivalent circuit of transducer



$\text{Fig.3: Potentiometer. Fig.4: Equivalent circuit}$
 $\text{at total resistance of the potentiometer} = R_p$
 $\text{total length of potentiometer} = x_t$
 $\text{so, resistance per unit length} = R_p/x_t$

$$\text{displacement} = x_i \\ \text{resistance of displacement} = R_{AB} = \frac{R_p}{x_t} \cdot x_i$$

$$= x_i \cdot \frac{R_p}{x_t}$$

$$\therefore R_{AB} = k R_p$$

$$\text{where, } k = \frac{x_i}{x_t} \quad & 0 \leq k \leq 1$$

The ideal O/P voltage is given by:

$$e_o = \frac{R_{AB}}{R_{AB} + R_{BC}} \cdot e_i$$

$$e_o = \frac{k R_p}{R_p + R_{BC}} \cdot e_i$$

$$\therefore e_o = k e_i = \frac{x_i}{x_t} e_i \quad \text{--- (1)}$$

$$\therefore \frac{e_o}{x_i} = \frac{e_i}{x_t} = \text{constant} \quad \text{--- (2)}$$

$$\therefore \frac{e_0}{e_i} = \frac{x_i}{x_t} \quad (\text{III})$$

Thus, there exists a linear relation between output and input as shown in figure.

Sensitivity of the device is given by: $\frac{e_0}{e_i}$

$$S = \text{mag. of } \frac{\partial e_0}{\partial x_i} \text{ with } x_i \neq 0 \\ \text{mag. of } \frac{1}{P_i} \cdot \frac{\partial P_i}{\partial x_i}$$

$$\text{or, } S = \frac{e_0}{e_i} = \frac{x_i}{x_t} = \text{constant}$$

Thus, the sensitivity of the device is also constant. Also, the equation from (I) to (IV) is applicable for rotatory potentiometer if $x_i \rightarrow 0$; and $x_t \rightarrow \theta_t$, i.e.

$$e_0 = k_e i = \frac{\theta_t}{\theta_t} e_i \quad (\text{Ig})$$

$$\frac{e_0}{e_i} = \frac{\theta_t}{\theta_t} = \text{constant} \quad (\text{IIg})$$

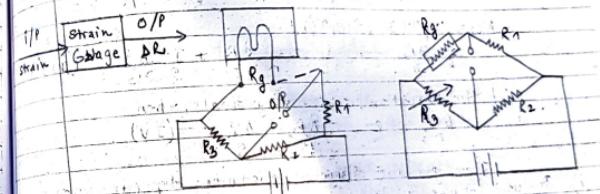
$$\frac{e_0}{e_i} = \frac{\theta_t}{\theta_t} \quad (\text{IIIg})$$

$$S = \frac{e_0}{e_i} = \frac{\theta_t}{\theta_t} \quad (\text{IVg})$$

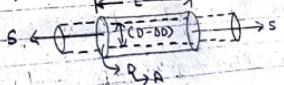
Strain Gauge :-

A strain gauge is a thin, wafer-like device that can be attached to a variety of materials to measure applied strain. It is a passive transducer that converts a mechanical displacement into a change of resistance.

Principle of operation



It is one of the examples of resistive sensor and it is used for measurement of strain.



The unstressed resistance is given by
 $R = \frac{\rho L}{A} \quad (\text{I})$

When a tensile stress 'σ' is applied, there will be change in resistance not only due to the change

in physical dimension, but also due to the change in resistivity. This effect is known as piezo-resistive effect. Thus, piezo-resistive effect is defined as change in resistance due to change in resistivity.

Differentiating eqn (I) partially with respect to s' ,

$$\frac{\partial R}{\partial s} = \frac{2}{s} \left[\frac{s^2}{A} \right]$$

$$\text{or } \frac{\partial R}{\partial s} = \frac{2L}{A^2 s} - \frac{2L}{A^2} \frac{\partial A}{\partial s} + \frac{2}{A} \frac{\partial s}{\partial s} \quad (\text{II})$$

Now, eqn (II) \div eqn (I)

$$\frac{1}{R} \frac{\partial R}{\partial s} = \frac{1}{L} \frac{\partial L}{\partial s} - \frac{1}{A} \frac{\partial A}{\partial s} + \frac{1}{s} \frac{\partial s}{\partial s} \quad (\text{III})$$

If the variation is very very small, then,

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} + \frac{\Delta s}{s} \quad (\text{IV})$$

Thus free unit change in resistance depends upon:

- i) Per unit change in length.
- ii) Per unit change in area of cross-section.
- iii) Per unit change in resistivity.

As we know $A = \frac{\pi D^2}{4}$ \rightarrow (V)

$$\therefore \frac{\partial A}{\partial s} = \frac{\pi 2D}{4} \frac{dD}{ds} \quad (\text{VI})$$

Now, eqn (VI) \div (V)

$$\frac{1}{A} \frac{\partial A}{\partial s} = \frac{2}{D} \frac{\partial D}{\partial s} \quad (\text{VII})$$

From eqn (VII) and (IV) we get,

$$\frac{1}{R} \frac{\partial R}{\partial s} = \frac{1}{L} \frac{\partial L}{\partial s} + \frac{2}{D} \frac{\partial D}{\partial s} + \frac{1}{s} \frac{\partial s}{\partial s} \quad (\text{VIII})$$

If the variation is very very small, then

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{2\Delta D}{D} + \frac{\Delta s}{s} \quad (\text{IX})$$

Poisson's ratio is denoted by,

$\mu = \frac{1}{2}$ lateral strain.

longitudinal strain,

$$\mu = - \frac{(\Delta D/L)}{(\Delta L/L)}$$

$$\text{or } \frac{-\Delta D}{D} = \mu \frac{\Delta L}{L}$$

$$\frac{-\Delta D}{D} = \frac{1}{2} \mu E \quad (\text{X})$$

where, $E = \frac{\Delta L}{L}$ = strain.

From eqn (IX) and (X)

$$\frac{\Delta R}{R} = E + 2\mu E + \frac{\Delta s}{s} \quad (\text{XI})$$

Gauge Factor (G):

It is defined as ratio of free unit change in resistance to free unit change in length.

$$\therefore G = \frac{(\Delta R/R)}{(\Delta L/L)}$$

$$\text{or } \frac{\Delta R}{R} = G \frac{\Delta L}{L} = GE \quad (\text{XII})$$

From eqn (XI) and (XII)

$$GE = E + 2\mu E + \frac{\Delta s}{s}$$

$$G = 1 + 2\mu + \frac{\Delta s/E}{E}$$

If the effect of resistivity is neglected, then

$$G = 1 + 2\mu \quad (\text{XIV})$$

Though, strain is a unitless quantity, it is expressed in microstrain.

$$E = \frac{\Delta L}{L}$$

$$1 \text{ micro-strain} = \frac{1 \mu\text{m}}{1 \text{ m}}$$

$$\therefore E = 8 \text{ micro-strain} = 8 \times 10^{-6}$$

→ Tensile strain is taken as positive where compressive strain is taken as negative.

A compressive strain of 5 micro-strain is applied,

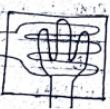
$$E = -5 \text{ micro-strain}$$

$$= -5 \times 10^{-6}$$

Strain gauge is available in variety of configurations such as uniaxial, biaxial or multidirectional, based on strain to be measured.



a) Uniaxial strain gauge.



b) Two element strain gauge.



c) Three different element resistor

Application of strain gauge:-

- 1) It is extensively used for analyzing the dynamic strains in complex structures, such as stress and strain bridges, automobiles, roads etc.
- 2) It is used for measuring tensile torque, force, stresses in structures.
- 3) It is used for measurement of force by strain produced in load rings.

Resistance Thermometers:-

Resistance thermometers often referred as resistance-temperature detector (RTD) operate upon the fact that almost all pure metals have the property of varying their resistance with temperature, and change in resistance is almost directly proportional to the change in temperature. Electrical resistance with temperature, for most metallic materials, can be represented by an equation of the form

$$R_t = R_0 [1 + \alpha(t - t_0) + \beta t^2 + \gamma t^3 + \dots + \omega t^n] \quad (i)$$

where, R_t = Resistance at temperature t .

R_0 = Resistance at 0°C

$$\alpha, \beta, \gamma, \dots, \omega = \text{temperature coefficients}$$

In the narrow range of operation, β and higher order coefficients are negligibly small and, therefore the expression for electrical resistance with temperature becomes:

$$R_t = R_0 (1 + \alpha t) \quad (ii)$$

The RTD does not generate its own voltage, so a voltage source is required to be incorporated into the measuring circuit.

RTD is applicable for measurements of small temperature differences as well as for wide ranges of temperature. The main drawback of an RTD is in its large size and sophisticated instrumentation.

Thermistors:

Thermistor or thermal resistors are semiconductor device having negative temperature coefficient of resistance. Its resistance changes with temperature i.e. their resistance decreases with the increase in temperature. It is very small sensitive to temperature and will suited to precision measurement, control and compensation. Thermistor has non-linear characteristics. The resistance of thermistor at any temperature can be expressed as:

$$R_t = R_0 e^{B \left(\frac{1}{T} - \frac{1}{T_0} \right)} \quad (iii)$$

Where,

R_t = Resistance at temperature T .

R_0 = Resistance at temperature T_0 .

B = experimentally determined constant or curve fitting constant for given thermistor material.

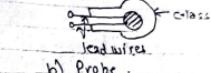
Thermistors are composed of a sintered mixture of metallic oxides. Their resistance ranges from 0.5 to 75 MΩ and they are available in variety of shape and sizes.



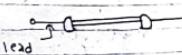
a) Bead.



c) Disc



b) Probe.



d) Rod.

Fig: Commercial form of thermistor

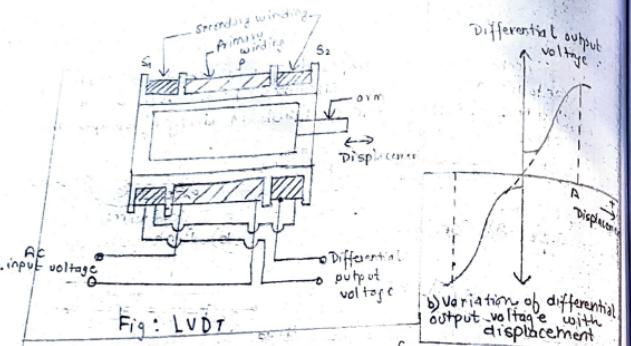
Inductive Transducers:

These are analog passive transducers. These transducers operate generally upon one of the following three principles:

- i) Variation of self inductance of the coil.
- ii) Variation of mutual inductance of coil.
- iii) Production of eddy current.

Linear variable Differential Transformer (LVDT):-

This is the most widely used inductive transducer for translating the linear motion into an electrical signal. The basic construction of an LVDT is shown in figure below:



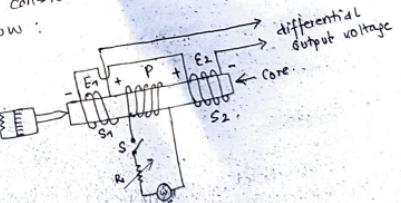
LVDT is a differential transformer consisting of one primary winding 'P' and two identical secondary windings S_1 and S_2 , wound over a hollow bobbin of non-magnetic and insulating material. The secondary windings S_1 and S_2 , having equal numbers of turns are arranged concentrically and placed either side of the primary winding 'P'. A soft iron core in the shape of rod or cylinder which is attached to the sensing element slides freely in the hollow portion of the bobbin. Primary winding is connected to an AC source.

When the core is moved inside the bobbin, it varies coupling of primary winding to secondary windings S_1 and S_2 . In position of the core in central position, coupling of primary winding to both of the secondary windings are equal and so output voltage induced in secondary

windings are equal and

windings S_1 and S_2 are equal. As the core moves towards right from its null position, the magnetic linkage to secondary winding S_2 increase and the secondary winding S_1 decrease because the portion of the core inside secondary winding S_2 increases and that inside secondary winding S_1 decreases. Therefore, output voltage induced in secondary winding S_2 increased whereas the output voltage induced in the secondary winding S_1 decreased. The movement of the core to the left will have opposite effect. As secondary windings S_1 and S_2 are connected in series opposition, the difference of output voltages of secondary windings give the measurement of displacement.

Let us consider the different case in the figure given below:



Case 1: When core is at centre
Differential output $(E_1 - E_2) = 0$. [$\because E_1 = E_2$].

Case II: When core is towards left
Differential output $(E_1 - E_2) = \text{ve}$, $(\because E_1 > E_2)$

Case III: When core is towards right
Differential output $(E_1 - E_2) = \text{ve}$, $(\because E_2 > E_1)$

Hence, the output voltage shows both magnitude and direction of displacement.

Advantages:

- 1) It has linear characteristics.
- 2) It has high sensitivity.
- 3) It consumes very less power (less than 1 watt).
- 4) It can be used on high frequency upto 20KHz.
- 5) It has low hysteresis, hence good repeatability can achieve.
- 6) It is rugged in construction thus can tolerate shocks and vibration.
- 7) Small in size, less in weight and more reliable device.
- 8) It has high output which eliminates the need of amplification.

Disadvantages:

- 1) It is sensitive to stray magnetic field.
- 2) Its performance get effected by vibrations and temperature.
- 3) Its dynamic response get limited mechanically by the mass of core and electrically by frequency of

Applied voltage
It requires large displacement for differential output

Capacitive Transducer:

The variable capacitance transducer consists of a capacitor whose capacitance is varied by the non-electrical quantity being measured. The capacitance of a parallel plate capacitor is given by

$$C = \frac{\epsilon_0 \epsilon_r A}{d} \text{ farads}$$

where;

$$\epsilon_0 = \text{permittivity}$$

$$\epsilon_r = \text{relative permittivity of free space} = 8.85 \times 10^{-12} \text{ F/m}$$

$$A = \text{area of plates}$$

$$d = \text{distance between two plates}$$

As the capacitance of a capacitor depends upon the area of its electrodes, the distance between them and the relative permittivity of the dielectric material, such transducers can be used for measurement of non-electrical quantities by affecting any one of the above mentioned parameters.

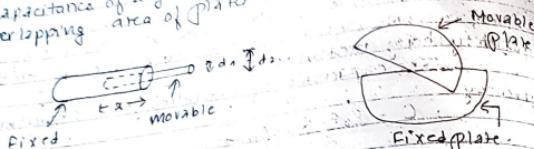
Capacitive transducers are analog passive transducers in which capacitance is varied by any one of the following three methods.

By varying overlapping area of plates, A.

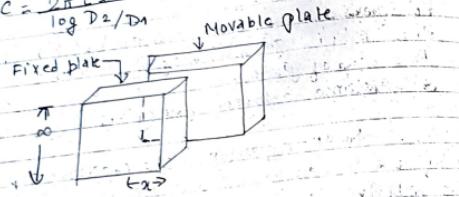
By varying distance between plates, d.

By varying relative permittivity of dielectric material between two plates.

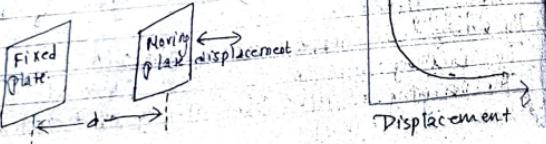
i). By varying overlapping area of plates. A.C. Such transducer operates on the fact that capacitance of any capacitor is proportional to the overlapping area of plates.



$$C = \frac{2\pi\epsilon_0}{\log D_2/D_1}$$



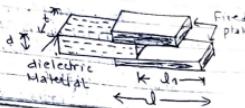
ii). By varying distance between the plates. Such transducer operates upon the fact that the capacitance of any capacitor is inversely proportional to the distance between the plates.



iii) By varying relative permittivity of dielectric material between two plates.

In this arrangement a dielectric material of relative permittivity ϵ_r moves between the two fixed parallel plates according to the displacement under measurement.

$$C = \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d} (\epsilon_r - 1)$$



Advantages :-

- 1) It has high impedance thus loading effect becomes minimum.
- 2) It is extremely sensitive.
- 3) It has excellent frequency response (up to 50 kHz). Thus used for measurement of both static & dynamic phenomena.
- 4) It is not effected by stray magnetic field.
- 5) It can achieve a resolution of about 2.5 micrometres.
- 6) It requires small force and power to operate.

Disadvantages :-

- 1) Measuring circuit becomes complicated due to high output impedance.
- 2) Error introduces in measurement due to insulation resistance get effected by physical condition such as temperature, humidity.
- 3) It needs electrostatic screening.

- 4) Capacitance get effected by moisture, dust, temperature, stray magnetic field etc.

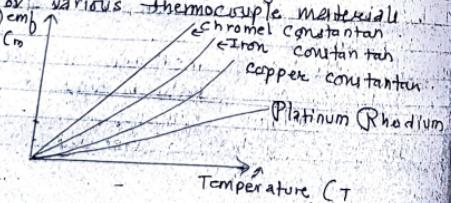
Thermocouples:

It consists of two dissimilar metal wire A & B insulated from each other but welded or brazed together at their ends forming two junctions as shown in figure.

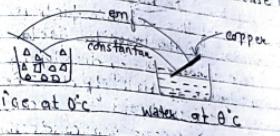
The operation of a thermocouple is based on Seebeck effect, i.e. if two wires of different metals are joined together at each end and form a complete electric circuit then current flows in the circuit when the two junctions are kept at different temperatures. This current is caused by an emf, called the thermo-electric emf. This thermo-electric emf is a function of temperature difference of two junctions. The relation between the thermo emf set up and temperature difference of hot and cold junction is given by

$$E = \alpha(T - T_0) + \beta(T^2 - T_0^2) \dots \dots \dots (1)$$

The figure given below represents the relationship between Seebeck voltage and temperature for various thermocouple materials.



There are several methods of joining the two dissimilar metals, for example joining of the two dissimilar produces a voltage on the order of a few tens of millivolts with the positive potential at copper side. An increase in temperature causes increase in voltage.



Laws governing thermocouple:

- 1 Law of intermediate temperatures
- 2 Law of intermediate metals

Advantages:

- 1 Lesser accuracy than RTD or thermistor.
- 2 It needs periodical checking when employed at elevated temperature.
- 3 Reference junction compensation is required.
- 4 It needs compensating leads.
- 5 It must protected in open or close metal protecting tubes to ensure long life.

Hall effect Transducers:

When a conductor is kept perpendicular to the magnetic field and a direct current is passed through it, it results in an electric field perpendicular to the direction of both the magnetic field and current. The magnitude of this electric field is proportional to the product of the magnetic field strength and current. This phenomenon of developing voltage across a conductor is known as Hall effect. The voltage developed is very small and it becomes difficult to detect it but in some semiconductors such as germanium, this voltage becomes sufficient enough for measurement with a sensitive measuring coil instrument.

The elements of hall effect transducer is illustrated in figure below. Here a slab of conducting material is connected to a battery so that current (I) flows in slab and potential difference created between top and bottom of the slab.

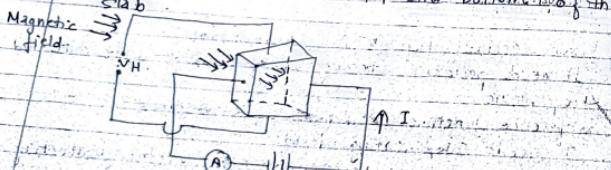


Fig: Hall effect

The magnitude of this induced voltage is proportional to the product of magnetic field strength and current via slab and is given by the expression

$$V_H = K_H I B \frac{t}{d} \quad (i)$$

where -

K_H = Hall effect on efficiency

I = Current flowing through slab

B = Flux density of magnetic field

t = thickness of slab

Piezo electric Transducers:

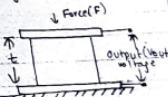
In some crystalline or ceramic materials, a potential difference appears across the opposite faces of the material as a result of dimensional change when a mechanical force is applied to it. This is called piezoelectric effect and such material is called piezoelectric material. Emf produced across crystal mainly depends upon three parameters - magnitude of applied pressure, voltage sensitivity and thickness of crystal.

$$\therefore V_{lt} = g t p$$

Where g = Voltage sensitivity

t = thickness of crystal

p = pressure applied



These transducers are widely employed for measurement of dynamic pressure, force and shock or

vibratory motion

- * A resistance strain gage with a gauge factor of 2 is fastened to a steel member subjected to a stress of 1250 kg/cm^2 . The modulus of elasticity of steel is approximately $2.1 \times 10^6 \text{ kg/cm}^2$. Calculate the change in resistance of strain gage elements due to applied stress and also compute poisson's ratio. Here,

$$\text{Gauge factor (K)} = 2$$

$$\text{Stress (P)} = 1250 \text{ kg/cm}^2$$

$$\text{Modulus of elasticity (E)} = 2.1 \times 10^6 \text{ kg/cm}^2$$

Now,

As we know,

$$\frac{\text{Stress}}{\text{strain}} = E$$

strain

$$\text{Strain} = \frac{\text{Stress (P)}}{E} = \frac{1250}{2.1 \times 10^6}$$

$$\therefore \text{Strain} = \left(\frac{\Delta l}{l} \right) = 59.5 \cdot 238 \times 10^{-6}$$

Now,

$$\text{Gauge factor (K)} = \frac{(DR/R)}{(\Delta l/l)}$$

$$\frac{\Delta R}{R} = K * \frac{\Delta l}{l}$$

$$= 2 \times 59.5 \cdot 238 \times 10^{-6}$$

$$= 1.190 \times 10^{-5}$$

$$\therefore \frac{\Delta R}{R} = 0.119 \%$$

Also we have,

$$K = (1 + 2\mu)$$

$$2 = 1 + 2\mu$$

$$\mu = \frac{1}{2} = 0.5 \text{ Ans}$$

Physical variables:-

The variables that exists in physical environments are physical variables. The physical variable to be measured (either pressure, displacement, temperature, light intensity, force etc.) are applied to input device or transducer. Suitable transducers are used for measurement of these physical quantities.

Measurement of various physical variables are presented below:

1 Pressure Measurement

- i Variable Capacitance pressure gauge
- ii Piezo electric pickup

2 Displacement Measurement

- i Potentiometric device
- ii Linear variable differential transformer (LVDT)

3 Temperature Measurement

- i Thermocouple
- ii Resistance Thermometer
- iii Thermistor

- 4 Light intensity measurement
 i) Photo-voltaic cell
 ii) Photo-emissive cell

- 5 Magnetic flux / magnetic field:
 i) Hall effect transducer

- 6 Force, Torque:

- 7 Strain gauge:

* An LVDT is used for measuring the deflection of a bellows. The sensitivity of LVDT is 50 V/mm . The bellows is deflected by 0.145 mm by a pressure of $10 \times 10^5 \text{ N/m}^2$. Determine the sensitivity of LVDT in V/mm^2 and pressure when voltage output for LVDT becomes 4.5 V .

Here,

$$V_{out} = 50 \times 0.145 = 7.25 \text{ V}$$

Hence sensitivity of LVDT = $\frac{V_{out}}{P}$:

$$\begin{aligned} &= \frac{7.25}{10 \times 10^5} = 7.25 \times 10^{-6} \\ &= 7.25 \times 10^{-6} \text{ V per } \text{N/m}^2 \end{aligned}$$

Again,

Pressure for output voltage of 4.5 V

$$(F) = 4.5$$

$$7.25 \times 10^{-6}$$

$$= 6.20 \times 10^{-5} \text{ N/m}^2 \text{ approx}$$

A strain gauge is bonded to a beam 0.1 m long and has a cross-sectional area 4 cm^2 . Young modulus for steel is 207 GN/m^2 . The strain gauge has unstrained resistance of $20 \text{ k}\Omega$ and a gauge factor of 2.2 . When a load is applied, the resistance of gauge changes by $0.013 \text{ }\Omega$. Calculate the change in length of the steel beam and the amount of force applied to the beam.

Here,
 Unstrained resistance (R_0) = $20 \text{ k}\Omega$
 Change in resistance (ΔR) = $0.013 \text{ }\Omega$
 Gauge factor (k) = 2.2

Initial length of wire (l_0) = 0.1 m
 Young Modulus of Steel (E) = 207 GN/m^2
 Cross sectional area of wire (a) = 4 cm^2
 $= 4 \times 10^{-6} \text{ m}^2$

$$\begin{aligned} \text{Strain (}\epsilon\text{)} &= \frac{\Delta L}{L} = \frac{\Delta R/R_0}{k} \\ &= \frac{0.013/200}{2.2} = 0.0000246 \\ \Delta L &= 0.1 \times 0.0000246 \\ &= 2.46 \times 10^{-6} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Stress (}\sigma\text{)} &= E \times \text{strain} \\ &= 207 \times 10^9 \times 0.0000246 \\ &= 5.0922 \times 10^6 \text{ N/m}^2 \end{aligned}$$

$$\begin{aligned} \text{Force applied (F)} &= \sigma \times A \\ &= 5.0922 \times 10^6 \times 4 \times 10^{-6} \\ &= 20.3688 \text{ N approx} \end{aligned}$$