~~Free~~ probabilities:

probability ~~the~~ at least one server is free $= 1 - P_N$ (P_n

$= 1 - 0.50895$

$p(n < s) = 0.49105$

~~Model II~~

(M/M/1):(N/FIFO)

→ service time distribution (Expo)

→ queue discipline

→ system capacity (FIFO) (finite)

Inter arrival time dist (Exponent)

No. of servers

Assumption

(1) Mean arrival rate : $\lambda_n = \lambda$ for $n = 0, 1, 2 \ldots \ldots N-1$

$= 0$ for $n = N, N+1, N+2 \ldots \infty$

(finite system capacity and single server)

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(v) Mean service rate:

i.e.

$\mu_n = \mu$

Now

Traffic Intensity:

$$\rho = \frac{\lambda}{\mu}$$

1) $P_0 = \frac{1-\rho}{1-\rho^{M+1}}$; for $\rho \neq 1$

$= \frac{1}{M+1}$ for $\rho = 1$

Steady state probability distribution

2) $P_n = \left(\frac{1-\rho}{1-\rho^{M+1}}\right) \cdot \rho^n$

$\boxed{P_n = P_0 \cdot \rho^n}$ for $\rho \neq 1$

3) $\boxed{P_n = \frac{1}{M+1} \text{ for } \rho = 1}$

Now

$L_s = \frac{\rho}{1-\rho} - \frac{(M+1)\rho^{M+1}}{1-\rho^{M+1}}$ for $\rho \neq 1$

$= M/2$ for $\rho = 1$

4) $L_q = L_s - \frac{\lambda_e}{\mu}$

where

λ_e = Effective arrival rate

(when system full no customer can enter the system so adjusted)

where

$\lambda_e = \lambda(1 - P_N) \text{ or } [\mu(1 - P_0)]$

Here,

P_N = Probability the system is full.

$$P_N = \left(\frac{1-\rho}{1-\rho^{N+1}}\right) \times \rho^N$$

Also,

Effective traffic intensity:

$$\rho_e = \frac{\lambda_e}{\mu}$$

5] $W_s = \frac{L_s}{\lambda_e}$

6] $W_q = \frac{L_q}{\lambda_e}$

9] Probability that the potential customers will turn away.

$$\boxed{P_N = P_0 \cdot \rho^N}$$

$M_n = M$

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Numerical

Patients arrive at a single server doctor's clinic according to poisson process at the rate of 30 patients per hour. The waiting room doesn't accomodate more than (N) 9 patients. Examination time per patient is exponentially distributed with mean rate of 20 per hour (μ)

Find:
(i) Probability that an arriving patient will not wait (P_0)
(ii) Find effective arrival rate (λe)
(iii) Average number of patients in the clinic (L_s)
(iv) Expected time a patient spend in the clinic (W_s)
(v) Average number of patients in the queue (L_q)
(vi) Expected waiting time of patients in a queue (W_q)

$P(N)$

$\cancel{P(n=8)}$

(vii) Probability that the potential customer will turn away. (System full)

$\cancel{P(N)}$ The queueing model is $(M/M/1):(N/FIFO)$

Assumptions:

$\lambda_n = \lambda \qquad \mu_n = \mu$

$\lambda = 30$ patients per hours

$N = 9$ patients

$\mu = 20$ per hours

$P_N = P_0 \rho^N$

$\rho = \frac{\lambda}{\mu} = \frac{30}{20} = 3/2$

$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ $\quad \rho < 1$

$= \frac{1-3/2}{1-(3/2)^{10}} = \frac{-1/2}{1-(3/2)^{10}} = 8.823 \times 10^{-3}$

(i) $\cancel{\lambda e}$

$L_s = \frac{\rho}{1-\rho} - \frac{(N+1)\cdot\rho^{N+1}}{1-\rho^{N+1}}$ $\quad$ $\cancel{P_0 = \frac{\rho}{1-\rho} - \frac{(N+1)\cdot\rho^{N+1}}{1-\rho^{N+1}}}$

$L_q = L_s - \frac{\lambda e}{\mu}$ $\quad$ $P_0 = \frac{1-\rho}{1-\rho^{N+1}}$

$\lambda e = \lambda(1-P_N)$

$\lambda e = \mu(1-P_0)$ $\quad$ $P_N = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\cdot \rho^N$

$\lambda e = \mu\left(1 - \frac{1-\rho}{1-\rho^{N+1}}\right)$ $\quad$ $W_s = \frac{L_s}{\lambda e}$

$= 20\left(1 - \frac{3/2}{1-(3/2)^{10}}\right)$ $\quad$ $= \frac{7.17}{1.026}$

$= 1.026$ $\quad$ $L_q = L_s - \frac{\lambda e}{\mu}$

$L_s = \frac{\rho}{1-\rho} - \frac{(N+1)\cdot\rho^{N+1}}{1-\rho^{N+1}}$ $\quad$ $W_q = \frac{L_q}{\lambda e}$

$= \frac{3/2}{1-3/2} - \frac{10\cdot(3/2)^{10}}{1-(3/2)^{10}}$

$= 7.17$ $\quad$ $P(\cancel{N})$.

$P_N = P_0 \cdot \rho^N$

$= \frac{1-3/2}{1-(3/2)^{10}} \times (3/2)^{9}$

$= 0.332$

$$\cancel{L_q = P_0 \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{s!(1-\rho^2)} \left[1 - \rho^{M-s} \cdot (M-s)\right.}$$

3) $$L_q = P_0 \left(\frac{\lambda}{\mu}\right)^s \cdot \frac{\rho}{s!(1-\rho^2)} \left[1 - \rho^{M-s} - (M-s)(1-\rho) \cdot \rho^{M-s}\right]$$

where

$$\rho = \frac{\lambda}{s\mu}$$

4) $$L_s = L_q + \frac{\lambda_e}{\mu}$$

where,

λ_e = effective arrival rate

$= \lambda(1 - P_M)$

where

P_M = probability that the system is full.

$$\boxed{P_M = \frac{1}{s!\, s^{M-s}} \left(\frac{\lambda}{\mu}\right)^M \cdot P_0}$$ if ~~$s < n < M$~~

Also effective traffic intensity

$$\boxed{\rho_e = \frac{\lambda_e}{s\mu}}$$

5) $W_q = \frac{L_q}{\lambda_e}$

6) $W_s = \frac{L_s}{\lambda_e}$

7) P_W = probability that a customer has to wait

$$\boxed{P_W = 1 - \frac{L_s - L_q}{S}}$$

Q.11

A dispensary has two (M's) doctors and four (S.C) chairs in the waiting room. The patients who arrive at the dispensary leave when all four chairs in the waiting room are occupied. It is known that patients arrive at the rate of 8 per hour and spend an average of 10 minutes for their checkup and medical consultation.

$\boxed{1/\mu = \text{service time}}$

→ ~~Soln:~~
The arrival process is poission and the service time is an exponential random variable. Find

(i) Probability that an arriving patient will not wait (P_0)

(ii) Effective arrival rate (λ_e)

(iii) Expected number of patients at the queue (L_q)

(iv) Expected waiting time of a patient at the queue (W_q)

(v) Expected number of patients at the dispensary (L_s)

(vi) Expected waiting time of patient at the dispensary (W_s)

(vii) OR

~~The~~ Expected time a patient spends at the dispensary.

→ Soln:

The queueing ~~model~~ is

$(M/M/S):(N/FIFO)$

we have,

$\lambda = 8$ per hour

$\frac{1}{\mu} = 10$ minutes

$\frac{1}{\mu} = \frac{10}{60}$ per hour

$\Rightarrow \mu = 6$ per hour

No. of servers $(S) = 2$

system capacity

$N = 4+2 = 6$

Then

Traffic Intensity:

$\rho = \frac{\lambda}{S\mu} = \frac{8}{2 \times 6} = \frac{8}{12} = \frac{2}{3}$

now

$P_0 = \sum_{n=1}^{S-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n$

$M = 4 + 2 = 6$

$$P_0 = \left[\sum_{n=1}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \cdot \sum_{n=s}^{M}\left(\frac{\lambda}{s\mu}\right)^{n-s}\right]^{-1}$$

$$= \left[\frac{1}{1!} + \left(\frac{\lambda}{\mu}\right)^1 + \frac{1}{2!}\left(\frac{\lambda}{\mu}\right)^2 \cdot \sum_{n=2}^{6}\left(\frac{\lambda}{s\mu}\right)^{n-s}\right]^{-1}$$

$$= \left[1 + 8/6 + \frac{1}{2!}(8/6)^2 \cdot \sum_{n=2}^{6}\left(\frac{8}{2\cdot 6}\right)^{n-s}\right]^{-1}$$

$$= \left[1 + 8/6 + \frac{1}{2!}(8/6)^2 \times \left(\left(\frac{8}{12}\right)^1 + (8/12)^2 + (8/12)^3 + (8/12)^4\right)\right]^{-1}$$

$$= \underline{0.2659}$$

$\lambda_e = \lambda(1 - P_M)$

$$P_M = \frac{1}{s!\,(s^{M-s})}\left(\frac{\lambda}{\mu}\right)^M \cdot P_0$$

$$= \frac{1}{2!\,(2^{6-2})} \cdot \left(\frac{8}{6}\right)^6 \times 0.2659$$

$$= \underline{0.0466875}$$

$\lambda_e = \lambda(1 - P_M)$

$= 5.71$ patients per hours

$L_q =$

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$M = 6 + 2 = 26$

$$P_0 = \left[\sum_{n=1}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \cdot \sum_{n=s}^{M}\left(\frac{\lambda}{s\mu}\right)^{n-s}\right]^{-1}$$

$$= \left[\frac{1}{1!} + \left(\frac{\lambda}{\mu}\right)^1 + \frac{1}{2!}\left(\frac{\lambda}{\mu}\right)^2 \cdot \sum_{n=2}^{6}\left(\frac{\lambda}{s\mu}\right)^{n-s}\right]^{-1}$$

$$= \left(1 + 8/6 + \frac{1}{2!}(8/6)^2 \cdot \sum_{n=2}^{6}\left(\frac{8}{2\cdot 6}\right)^{n-s}\right]^{-1}$$

$$= \left(1 + 8/6 + \frac{1}{2!}(8/6)^2 \times \left(\left(\frac{8}{12}\right)^1 + (8/12)^2 + (8/12)^3 + (8/12)^4\right]^{-1}$$

$= \underline{0.2659}$

$\lambda_e = \lambda(1 - P_M)$

$$P_M = \frac{1}{s!\,(s^{M-s})}\left(\frac{\lambda}{\mu}\right)^M \cdot P_0$$

$$= \frac{1}{2!\,(2^{6-2})} \cdot \left(\frac{8}{6}\right)^6 \times 0.2659$$

$= \underline{0.0466875}$

$\lambda_e = \lambda(1 - P_M)$

$= 5.71$ patients per hours

$L_q =$

(S) (S.C)

Q11 A beauty salon has two barbers and 6 chairs to accomodate waiting customers. Potential customers who arrive when all 6 chairs are full, leave the salon immediately. Customers arrive at the salon at the average rate of 10 per hour and spend on average of 10 minutes in the barber's chair. The arrival process is poisson and the service time is an exponential random variable: find

(1) probability of no customer in the beauty salon (P_0)

→ Here $S=2$; $N=6$ ⇒ $M = 6+2 = 8$

$\lambda = 10$ per hr

$\frac{1}{\mu} = 10$ minutes

⇒ $\frac{1}{\mu} = \frac{10}{60}$ per hr

⇒ $\mu = 6$ per hr

$$P_0 = \left[\sum_{n=1}^{S-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^S \cdot \sum_{n=S}^{M} \left(\frac{\lambda}{S\mu}\right)^{n-S}\right]^{-1}$$

Model V

→ Erlang distribution / Gamma distribution (Service time)

$(M/E_k/1):(\infty/FIFO) \to (\infty)$ (SC) infinite

M → Inter arrival time distribution [Exponential distribution].

E_k (Server) → Series Mechanism

Traffic Intensity:

$\boxed{\rho = \lambda/\mu}$

This model consists of a single server in which there are k identical phases in the series for the services.

(Performance Measure) OR

Operating Characteristics of $(M/E_k/1):(\infty/FIFO)$

(1) Expected number of phases (Not customers) in the queue

Queue → k-phases → Service

$$L_q(k) = \frac{k+1}{2} \cdot \frac{\lambda}{\mu(\mu-\lambda)}$$

(2) Expected number of phases in the system i.e.

$$L_s(k) = \frac{k+1}{2} \cdot \frac{\lambda}{(\mu-\lambda)}$$

(3) L_q (customers) i.e.

(not phases)

3) $L_q = \left(\frac{k+1}{2k}\right) \cdot \left(\frac{\lambda^2}{\mu(\mu-\lambda)}\right)$

4) $W_q = L_q/\lambda$

5) $L_s = L_q + \frac{\lambda}{\mu}$ 6) $W_s = \frac{L_s}{\lambda}$

7) ~~Probability that~~
- Most probable time in getting service
$= \frac{k-1}{k\mu}$

Imp

Q4) In a factory, the customers have to pass through 3 counters, the customers buy coupons at the first counter, select and collect the snacks at the 2nd counter and collect Tea ~~at third~~ counter. (phases)
The server at each counter takes on an average of 1.5 minutes although the distribution of ~~service~~ the arrivals is approximately poisson at the rate of 6 per hour. (i) Find the average time of a customer spends waiting in the cafeteria.
(ii) Also find the most probable time in getting service

$\frac{k-1}{k\mu}$ L_q

→ Soln :

$M/3$ $\frac{\cancel{600}}{\cancel{45}}$ $\cancel{120}$ $\cancel{8}$ 40 3 $\frac{\cancel{600}}{\cancel{15}}$ $\cancel{120}$ 40

The given queue model is :

$(M/E_K/1) : (\infty/FIFO)$

we have

$\lambda = 8$ per hour

~~$\frac{1}{\mu} = 1.5$ minutes~~

~~$\Rightarrow \frac{1}{\mu} = \frac{1.5}{60}$ per hour~~

~~$\Rightarrow \mu = \frac{60}{1.5}$ per hour~~

$\frac{1}{\mu} = 3 \times 1.5$ minutes

$\therefore \mu = \frac{60}{3 \times 1.5}$ per hour

$= \cancel{40 \text{ per hour}}$

$= \frac{60}{4.5}$ per hour

$= \frac{40}{3}$ per hour

$= 13.33$ per hour

$L_q(K) = \frac{K+1}{2} \times \frac{\lambda^2}{\mu(\mu-\lambda)}$