

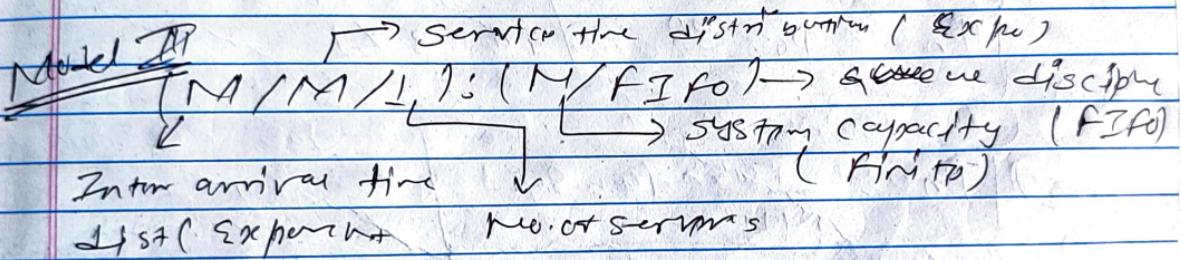
Prob. Probabilities:

Probability that at least one server is

$$\text{prob} = 1 - P_W \quad (P_n)$$

$$= 1 - 0.50895$$

$$P(n < 5) = 0.49105$$



Assumptions

① Mean arrival rate: $\lambda_n = \lambda$ for $n = 0, 1, 2, \dots$

$\lambda_{n+1} = \lambda$ for $n = M, M+1, M+2, \dots, \infty$

(finite system capacity and single server)

(1) Mean service rate:

$$\lambda \cdot n$$

$$M_n = M$$

Mow

traffic intensity:

$$S = \frac{\lambda}{M}$$

$$(1) P_0 = \frac{1 - S}{1 - S^{M+1}}, \text{ for } S \neq 1$$

$$2) \frac{1}{M+1} \quad \text{for } S = 1$$

stationary state probability distribution

$$3) P_n = \left(\frac{1 - S}{1 - S^{M+1}} \right) \cdot S^n$$

$$\boxed{P_n = P_0 \cdot S^n} \quad \text{for } S \neq 1$$

$$3) \boxed{P_n = \frac{1}{M+1} \quad \text{for } S = 1}$$

Mow

$$L_S = \frac{S}{1 - S} = \frac{(M+1) \cdot S^{M+1}}{1 - S^{M+1}}, \text{ for } S \neq 1$$

$$= \frac{M}{2} \quad \text{for } S = 1$$

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Date: _____
Page: _____



$$4) L_q = L_s * \frac{\lambda_e}{\mu}$$

where

λ_e = effective arrival rate
(when system full no customer enter the system so adjusted)

where

$$\lambda_e = (1 - P_{M/F}) [\mu (1 - P_0)]$$

Also,

P_M = probability the system is full.

$$P_M = \left(\frac{1 - S}{1 + S^{M/F}} \right) \times g^M$$

Also,

Effective Traffic Intensity:

$$S_e = \frac{\lambda_e}{\mu}$$

$$5) K_Ls = \frac{L_s}{\lambda_e}$$

$$6) W_q = \frac{L_q}{\lambda_e}$$

7) Probability that the potential customers will turn away.

$$[P_M = P_0 \cdot S^M]$$

M/F: M

numerical

patients arrive at a doctor's clinic according to poisson process at the rate of 30 patients per hour. the waiting room doesn't accommodate more than 10 patients. examination time per patient is exponentially distributed with mean rate of 6 per hour ($\lambda = 6$)

Find:

- (1) probability that an arriving patient will not wait (P_0)
- (2) find effective arrival rate (λ_e)
- (3) average number of patients in the clinic (L_s)
- (4) expected time a patient spends in the clinic (W_s)
- (5) average number of patients in the queue (L_q)
- (6) expected waiting time of patients in a queue (W_q)
- (7) probability that the patient who comes after patient i will turn away. (System full)

p(i+1)

- (7) probability that the patient who comes after patient i will turn away. (System full)

Ans: the queuing model is $(M/M/1):(M/6/10)$

Assumptions:

$$\lambda = \mu \quad \lambda_n = \mu$$

$$\lambda = 30 \text{ patients per hour}$$

$$M = 9 \text{ patients}$$

$$\mu = 6 \text{ per hour}$$

$$P_{i+1} = P_0 \cdot p_{i+1}$$

$$S = \frac{\lambda}{M} = \frac{g_0}{2\pi} = \frac{3}{2}$$

$$P_0 = \frac{1 - g^{1/2}}{1 - g^{1/2} M^{1/2}}$$

$$= \frac{1 - 3/2}{1 - (3/2)^{1/2}} = \frac{-1/2}{1 - (3/2)^{1/2}} = 0.823 > 1/2$$

① ~~Ansatz~~

$$L_S = \frac{S}{1-g} = (M+1) \cdot S^{1/2} M^{1/2}$$

$$P_M = \frac{S}{1-g} = (M+1) \cdot 2 \cdot S^{1/2} M^{1/2}$$

$$L_g = L_S = \frac{\lambda e}{M}$$

$$P_M = \frac{1 - g}{1 - g^{1/2} M^{1/2}}$$

$$\Delta e = \lambda (1 - P_M)$$

$$\Delta e = M (1 - P_0)$$

$$P_M = \left(\frac{1 - g}{1 - g^{1/2} M^{1/2}} \right) \cdot P_0 g^{1/2}$$

$$\Delta e = 1/4 \left(1 - \frac{1 - g}{1 - g^{1/2} M^{1/2}} \right)$$

$$= 20 \left(1 - \frac{3/2}{1 - (3/2)^{1/2}} \right)$$

$$= \underline{1.026}$$

$$W_S = \frac{L_S}{\Delta e}$$

$$= \frac{2 + 1/2}{1.026}$$

$$L_g = L_S + \frac{\Delta e}{M}$$

$$L_S = \frac{S}{1-g} = (M+1) \cdot S^{1/2} M^{1/2}$$

$$= \frac{3/2}{1 - 3/2} = \frac{10 \cdot (3/2)^{1/2}}{1 - (3/2)^{1/2}}$$

$$= \underline{2.97}$$

$$L_g = \frac{L_S}{\Delta e}$$

$$P(M)$$

$$P_M = P_0 \cdot \frac{S^{1/2}}{1 - g^{1/2} M^{1/2}} = P_0 \cdot \frac{S^{1/2}}{1 - 3/2}$$

$$= \underline{2.033}$$

$$L_q = P_0 \left(\frac{\lambda}{\mu} \right)^s \cdot \frac{s}{s! (s+1)} \left[1 - \frac{1 - s^{M-s}}{(M-s)} \right]$$

$$3) L_q = P_0 \left(\frac{\lambda}{\mu} \right)^s \cdot \frac{s}{s! (s+1)} \left[1 - s^{M-s} - (M-s)(1-s) \cdot s^{M-s} \right]$$

where

$$S = \frac{\lambda}{\mu}$$

$$4) L_s = L_q + \frac{\lambda e}{\mu}$$

where,

$$\lambda_e = \text{effective arrival rate}$$

$$= \lambda (1 - P_M)$$

where

P_M = probability that the system is full.

$$P_M = \frac{1}{s! (s+1)} \left(\frac{\lambda}{\mu} \right)^{s+1} \cdot P_0$$

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Also effective traffic intensity

$$\lambda_e = \frac{\lambda}{s+1}$$

$$5) W_q = \frac{L_q}{\lambda_e}$$

$$6) W_s = \frac{L_s}{\lambda_e}$$

7) P_W = probability that a customer has to wait

$$P_W = 1 - \frac{L_s + L_q}{S}$$

SOL

A dispensary has two doctors and four chairs in the waiting room. The patients who arrive at the dispensary leave when all four chairs in the waiting room are occupied. It is known that patients arrive at the rate of 8 per hour and spend an average of 10 minutes for their checkup and medical consultation.

8) $\lambda = \text{service rate}$

SOL:

The arrival process is Poisson and the service time is an exponential random variable. Find

(i) Probability that an arriving patient will not wait (P_0)

(ii) Effective arrival rate (λ_e)

(iii) Expected number of patients at the queue (L_q)

(iv) Expected waiting time of a patient at one queue (W_q)

- Date _____
Page _____
- (V) Expected number of patients at the dispensary (L_s)
 - (W) Expected waiting time of patient at the dispensary (W_s)
 - (M) OR
Expected time a patient spends at the dispensary.

→ Solution:

The queuing model is

(M/M/1/∞): (M/FIFO)

we have,

$\lambda = 8 \text{ per hour}$

$\lambda_M = 10 \text{ minutes}$

$\lambda_M = \frac{10}{60} \text{ per hour}$

2) $M = 6 \text{ per hour}$

then

Traffic Intensity:

No. of servers (S)

System capacity

$$M = \underline{\underline{24+2}} = \underline{\underline{6}}$$

$$\rho = \frac{\lambda}{\mu M} = \frac{8}{12 \times 6} = \frac{8}{72} = \frac{2}{18} = \frac{1}{9}$$

now $S=1$

$$P_{02} = \sum_{n=1}^{\infty} \frac{\rho^n}{n!} = \frac{1}{9}$$

$$M = \lambda + 2 \frac{28}{5}$$

 Date: _____
 Page: _____

$$\begin{aligned}
 P_0 &= \left(\sum_{n=1}^{s-1} \frac{\lambda^n}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{\lambda^s}{s!} \left(\frac{\lambda}{\mu} \right)^s \cdot \sum_{n=s}^{\infty} \left(\frac{\lambda}{\mu} \right)^{n-s} \right)^{-1} \\
 &= \left[\left(\frac{\lambda}{\mu} \right) + \left(\frac{\lambda}{\mu} \right)^2 + \frac{\lambda^2}{2!} \left(\frac{\lambda}{\mu} \right)^2 \cdot \sum_{n=2}^{\infty} \left(\frac{\lambda}{\mu} \right)^{n-s} \right]^{-1} \\
 &= \left(1 + \frac{8}{6} + \frac{1}{2!} \left(\frac{8}{6} \right)^2 \cdot \sum_{n=2}^{\infty} \left(\frac{8}{6} \right)^{n-s} \right)^{-1} \\
 &= \left(1 + \frac{8}{6} + \frac{1}{2!} \left(\frac{8}{6} \right)^2 \times \left(\left(\frac{8}{12} \right)^2 + \left(\frac{8}{12} \right)^2 + \left(\frac{8}{12} \right)^4 \right) \right)^{-1} \\
 &= 0.2655
 \end{aligned}$$

$$\Delta e = \lambda (1 - P_M)$$

$$\begin{aligned}
 P_M &= \frac{1}{s! (s\mu - s)} \left(\frac{\lambda}{\mu} \right)^s \cdot P_0 \\
 &= \frac{1}{2! (2^6 - 2)} \cdot \left(\frac{8}{6} \right)^6 \times 0.2655 \\
 &= 0.0466875
 \end{aligned}$$

$$\begin{aligned}
 \Delta e &= \lambda (1 - P_M) \\
 &= 5.71 \text{ passengers per hour}
 \end{aligned}$$

$$Lg =$$



Date _____
Page _____

$$M = \lambda + 2 - S$$

Date _____
Page _____

$$\begin{aligned} P_0 &= \left(\sum_{n=1}^{\infty} \frac{\lambda^n}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{\lambda}{\mu} \left(\frac{\lambda}{\mu} \right)^0 \cdot \sum_{n=s}^{\infty} \left(\frac{\lambda}{\mu} \right)^{n-s} \right)^{-1} \\ &= \left[\left(\frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu} \right)^1 + \frac{\lambda}{\mu} \left(\frac{\lambda}{\mu} \right)^2 \cdot \sum_{n=2}^{\infty} \left(\frac{\lambda}{\mu} \right)^{n-s} \right]^{-1} \right. \\ &\quad \left. \times \left(1 + \frac{8}{6} + \frac{1}{2!} \left(\frac{8}{6} \right)^2 \cdot \sum_{n=2}^{\infty} \left(\frac{8}{6} \right)^{n-s} \right)^{-1} \right. \\ &\quad \left. \times \left(1 + \frac{8}{6} + \frac{1}{2!} \left(\frac{8}{6} \right)^2 \times \left(\left(\frac{8}{12} \right)^1 + \left(\frac{8}{12} \right)^2 + \left(\frac{8}{12} \right)^3 \right) \right. \right. \\ &\quad \left. \left. + \left(\frac{8}{12} \right)^4 \right) \right]^{-1} \\ &= 0.2655 \end{aligned}$$

$$\lambda_e = \lambda(1 - P_M)$$

$$\begin{aligned} P_M &= \frac{1}{S_0(S^{M-s})} \left(\frac{\lambda}{\mu} \right)^M \cdot P_0 \\ &= \frac{1}{2! (2^6 - 1)} \cdot \left(\frac{8}{6} \right)^6 \times 0.2655 \\ &= 0.0466875 \end{aligned}$$

$$\begin{aligned} \lambda_e &= \lambda(1 - P_M) \\ &= 5.71 \text{ patients per hour} \end{aligned}$$

$$L_q =$$

(S)



Q1 A beauty salon has two barbers and 6 chairs to accommodate waiting customers. Potential customers who arrive when all 6 chairs are full, leave the salon immediately. Customers arrive at the salon at the average rate of 10 per hour and spend on average of 10 minutes in the barbers chair. The arrival process is poisson and the service time is an exponential random variable. Find

(1) probability of no customer in the beauty salon (P_0)

$$\rightarrow \text{Arr. } S = 2; N = 6 \Rightarrow M = 6 + 2 = 8$$

$$\lambda = 10 \text{ per hr}$$

$$\frac{\lambda}{M} = 10 \text{ minutes}$$

$$\Rightarrow \lambda = 60 \text{ per hr}$$

$$\Rightarrow N = 6 \text{ per hr}$$

$$P_0 = \left(\sum_{n=1}^{S-1} \frac{1}{n!} \left(\frac{\lambda}{M}\right)^n + \frac{1}{S!} \left(\frac{\lambda}{M}\right)^S \cdot \sum_{n=S}^M \left(\frac{\lambda}{M}\right)^{n-S} \right)^{-1}$$

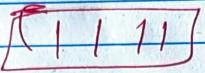
~~M/M/1~~

Erlang distribution / homogeneous distribution
(Service times are equal)

$$(M/E_K/1) \rightarrow (\infty/RIFO) \rightarrow Q.D) \\ (S = \text{Infinite})$$

Series mechanism

Interarrival time distribution
(Exponential distribution).



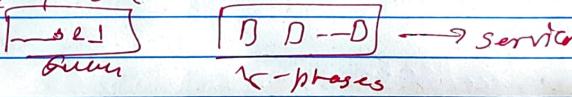
Traffic pattern:
 $S = \lambda/\mu$

This model consists of a single server in which there are K identical phases in the series for the services.

(Performance Measure) OR

Operating Characteristics of $(M/E_K/1)^S (\lambda/RIFO)$

- ① Expected number of phases (wait customers)
In the queue



$$L_q(K) = \frac{K+1}{2} \cdot \frac{\lambda}{\mu(\mu-\lambda)}$$

- ② Expected number of phases in the system

$$L_s(K) = \frac{K+1}{2} \cdot \frac{\lambda}{\mu(\mu-\lambda)}$$

- ③ L_q (customers) \downarrow \uparrow
(wait phases)

Date _____
Page _____

$$\textcircled{9} \quad L_q = \left(\frac{k+1}{2k} \right) \cdot \left(\frac{\lambda^2}{\mu(\mu-\lambda)} \right)$$

$$\textcircled{10} \quad W_q = \frac{L_q}{\lambda}$$

$$\textcircled{11} \quad L_s = L_q + \frac{\lambda}{\mu} \quad \textcircled{12} \quad W_s = \frac{L_s}{\lambda}$$

\textcircled{13} Probability that

- Most probable time in getting service

$$= \frac{k+1}{k\mu}$$

W_q

- \textcircled{14} In a factory, the customers have to pass through 3 counters, the customers buy Coupons at the first counter, select and collect the snacks at the 2nd counter and collect Tea at the 3rd counter. The server at each counter takes on an average of 1.5 minutes although the distribution of service arrivals is approximately Poisson at the rate of 6 per hour. (i) Find the average time a customer spends waiting in the system and (ii) Also find the most probable time in getting service

Given:

$$\downarrow \frac{k+1}{k\mu}$$

$$\downarrow g(1.5)$$

$$\text{WIP} \quad \frac{600}{15} \quad \frac{100}{8} \quad \frac{40}{3}$$

$$= 40 \quad = 12.5 \quad = 13.33$$

Done
Page

The given queue rules is:

$$(M/Ec/1) : (Q/FIFO)$$

We have

$$A = 8 \text{ per hour}$$

$$\begin{aligned} \frac{1}{M} &= 1.5 \text{ minutes} \\ \text{or } \frac{1}{M} &= \frac{1.5}{60} \text{ per hour} \\ \Rightarrow M &= \frac{60}{1.5} \text{ per hour} \\ \frac{1}{M} &= 40 \text{ minutes} \end{aligned}$$

$$\Rightarrow M = \frac{60}{40} \text{ per hour}$$

$$= \frac{60}{40} \text{ per hour}$$

$$= \frac{60}{40} \text{ per hour}$$

$$= 1.5 \text{ per hour}$$

$$= 13.33 \text{ per hour}$$

$$Lq(15) = \frac{15+1}{2} \times \frac{1}{M(M-1)}$$