2013 Fall

1a) In an organization, there are 2 civil engineers, 4 IT officers and 2 accountants. A committee of 4-members is formed. What is the probability that a committee will contain i) one civil engineer and 3 IT officer. ii) No civil engineer iii) atleast one IT officer?

Ans:-

Soln, Given,

2 civil engineers
4 IT officers
2 accountants

Out of 8 members a committee of 4-members is formed is $^8C_4$ ways = 70 ways

(i) Probability that a committee will contain one civil engineer and 3 IT officers $= \frac{^2C_1 \times {^4C_3}}{^8C_4} = \frac{4}{35}$ //

(ii) Probability of No civil engineers $= \frac{^2C_0 \times {^6C_4}}{^8C_4} = \frac{3}{14}$ //

(iii) Probability of atleast one IT officers = 1 − Prob(No IT)

$$= 1 - \frac{^4C_0 \times {^4C_4}}{^8C_4}$$

$$= \frac{69}{70}$$ //

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1b) A consulting firm rents cars from three agencies, 20% from agencies D, 20% from agencies E and 60% from agencies F. It is experienced that 10% of the cars from the D, 12% cars from E and 4% cars from F had bad tyres. What is the probability that the firm will get the tyres? Also Find the probability that a car with bad tyres rented by firm came from agency F.

Ans:-

Soln,

Let,

D = event that rents cars from D

E = event that rents cars from E

F = event that rents cars from F.

N = event that rents cars will had bad tyres.

Now,

$P(D) = 0.2$ $\quad$ $P(N/D) = 0.1$

$P(E) = 0.2$ $\quad$ $P(N/E) = 0.12$

$P(F) = 0.6$ $\quad$ $P(N/F) = 0.04$

i) Probability that the firm will get the bad tyres

$P(N) = P(D \cap N)$ or $P(E \cap N)$ or $P(F \cap N)$

$= P(D) \cdot P(N/D) + P(E) \cdot P(N/E) + P(F) \cdot P(N/F)$

$= 0.2 \times 0.1 + 0.2 \times 0.12 + 0.6 \times 0.04$

$= 0.068$ //

(ii) $P(F/N) = \frac{P(F) \cdot P(N/F)}{P(N)} = \frac{0.6 \times 0.04}{0.068} = 0.35294$ //

2a) The joint probability density function of two dimensional random variable $(x, y)$ is given by;

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & 0<x<1,\ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find marginal probability density function of x and y.
(ii) Find conditional probability density of x given $y=y$.
(iii) Examine whether x and y are independent.

Ans :-

soln, Given,

(i) The marginal p.d.f of x is;

$$f(x) = \int_y f(x,y)\,dy$$

$$= \int_0^1 \frac{2}{3}(x+2y)\,dy$$

$$= \frac{2}{3}\int_0^1 (x+2y)\,dy$$

$$= \frac{2}{3}\left[xy + \frac{2y^2}{2}\right]_0^1$$

$$= \frac{2}{3}(x+1) \quad \text{for } 0<x<1$$ //

The Marginal p.d.f of $y$ is given by;

$$f(y) = \int_x f(x,y)\,dx$$

$$= \int_0^1 \frac{2}{3}(x+2y)\,dx$$

$$= \frac{2}{3}\left[\frac{x^2}{2} + 2xy\right]_0^1$$

$$= \frac{2}{3}\left[\frac{1}{2} + 2y\right] \quad \text{for } 0<y<1$$

ii) The conditional probability density of x given $y=y$

Now,

$$f(x/y=y) = \frac{f(x,y)}{f(y)} = \frac{\frac{2}{3}(x+2y)}{\frac{2}{3}\left(\frac{1}{2}+2y\right)} = \frac{x+2y}{\left(\frac{1}{2}+2y\right)}$$

for $0<x<1$ and $0<y<1$

iii) To check the x and y are independent or not.

then,

or, $f(x) \cdot f(y)$

or, $\frac{2}{3}(x+1) \cdot \frac{2}{3}\left(\frac{1}{2}+2y\right)$

or, $\frac{4}{9}\left(\frac{1}{2}x + 2xy + \frac{1}{2} + 2y\right) \neq f(x,y)$ so,

It is not independent.

2 b) Given that the switchboard of a consultant's office receives on the average of 0.6 calls per Minutes, find the Probabilities that

i) In a given Minute there will be at least 1 call.

ii) In a 4-Minute interval there will be at least 3 calls

iii) Atleast 4 of 12 such tubes (12 Minutes interval)

Ans :-

Soln, Given,

Average number of calls received by switchboard of a consultant's office ($\lambda$) = 0.6 per Minutes.

Let, $X$ = Number of calls arriving at switchboard of a consultant's office.

Now, Here,

$X \sim P(\lambda = 0.6)$ so, the p.m.f of $X$ is

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad ; x = 0, 1, 2, 3, \ldots, \infty.$$

$$\therefore P(X = x) = \frac{e^{-0.6} (0.6)^{x}}{x!} \quad ; x = 0, 1, 2, 3, \ldots, \infty.$$

i) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - \frac{e^{-0.6} (0.6)^{0}}{0!}$

$= 0.451188$ //

(ii) for, 4-Minutes, $\lambda = 4 \times 0.6$
$= 2.4$

so, p.m.f is given by,

$$P(X=x) = \frac{e^{-2.4}(2.4)^x}{x!}$$

Now,

$$P(X \geq 3) = 1 - P(X<3)$$

$$= 1 - \left[P(X=0) + P(X=1) + P(X=2)\right]$$

$$= 1 - e^{-2.4}\left[\frac{(2.4)^0}{0!} + \frac{(2.4)^1}{1!} + \frac{(2.4)^2}{2!}\right]$$

$$= 1 - e^{-2.4}\left[6.28\right]$$

$$= 0.4302912533$$

(iii) for 12 Minutes, $\lambda = 12 \times 0.6$
$= 7.2$

so, p.m.f is given by,

$$P(X=x) = \frac{e^{-7.2}(7.2)^x}{x!}$$

$$P(X \geq 4) = 1 - P(X<4)$$

$$= 1 - \left[P(X=0) + P(X=1) + P(X=2) + P(X=3)\right]$$

$$= 1 - e^{-7.2}\left[\frac{(7.2)^0}{0!} + \frac{(7.2)^1}{1!} + \frac{(7.2)^2}{2!} + \frac{(7.2)^3}{3!}\right]$$

$$= 0.9280828823$$

3a) In an intelligence test administrated on 1000 children, the average was 60 and the standard deviation was 20. Assume that the marks obtained by the children follow a normal distribution, find the number of children who have scored

(i) over 90 marks.

(ii) Below 40 marks.

(iii) Between 50 and 80 marks.

ANS:-

Soln, Given,

Let, $x$ = Number of children who scores different marks.

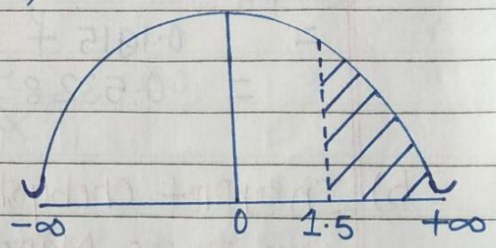
Mean$(\mu) = 60$

$(\sigma)$ SD $= 20$ then,

Now, $x \sim N(60, (20)^2)$;

(i) $P(x > 90) = P\left(Z > \frac{x-\mu}{\sigma}\right)$

$= P\left(Z > \frac{90-60}{20}\right)$



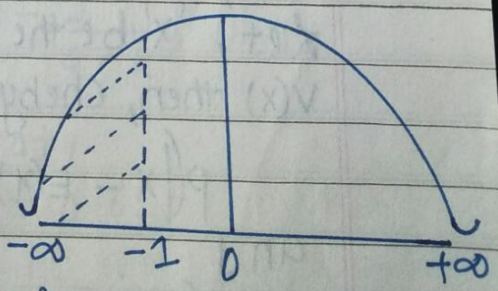

$= P(Z > 1.5)$

$= P(0 < Z < \infty) - P(0 < Z < 1.5)$

$= 0.5 - 0.4332$

$= 0.0668$ //

(ii) $P(x < 40) = P\left(Z < \frac{40-60}{20}\right)$




$= P(Z < -1)$

$= P(-\infty < 0) - P(-1 < Z < 0)$

$= P(0 < Z < \infty) - P(0 < Z < 1)$ } By symmetric }

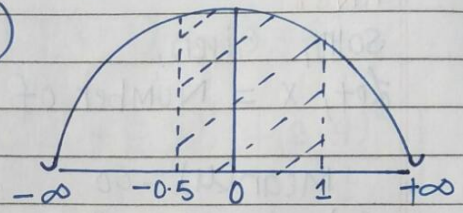
$= 0.5 - 0.3413$
$= 0.1587$ //

(iii) Between 50 and 80 Marks

$P(50 < X < 80)$ Now,

$$P\left(\frac{50-60}{20} < Z < \frac{80-60}{20}\right)$$

$$P(-0.5 < Z < 1)$$




$= P(-0.5 < Z < 0) + P(0 < Z < 1)$
$= P(0 < Z < 0.5) + P(0 < Z < 1)$
$= 0.1915 + 0.3413$
$= 0.5328$ //

3 b) Interpret Chebyshev's inequality. Suppose that a balanced coin tosses Many times. Estimate how Many times Must the coin be tossed so that the probability will be at least 0.90 that the proportion will lie in between 0.4 and 0.6.

Ans:-

Soln,

Let, 'x' be the r.v with finite Mean $E(x)$ and variance $V(x)$ then, Chebyshev's inequality is defined as;

$$P(|x - E(x)| < K\sigma) \geq 1 - \frac{1}{K^2}$$ (lower bound of probability)

and

$$P(|x - E(x)| \geq K\sigma) \leq \frac{1}{K^2}$$ (upper bound of probability)

where, K is a positive constant (i.e. $K>0$)

By question,

$$P\left(0.4 < \frac{X}{n} < 0.6\right) \geq 0.90 \quad \text{eq}^n(i)$$

Now,

$$p = q = 1/2$$

Here, we are given that

$$P\left(\left|\frac{X}{n} - p\right| < K\right) \geq 0.90$$

or, $$P\left(\left|\left(\frac{X}{n} - \frac{1}{2}\right)\right| < K\right) \geq 0.90$$

or, By Bernoulie's law of large numbers (BLLN)

$$P\left(\left|\frac{X}{n} - p\right| < K\right) \geq 1 - \frac{pq}{nK^2} \quad \text{so,}$$

or, $$P\left(\left|\frac{X}{n} - \frac{1}{2}\right| < K\right) \geq 1 - \frac{1}{4nK^2}$$

or, $$P\left(-K < \frac{X}{n} - \frac{1}{2} < K\right) \geq 1 - \frac{1}{4nK^2}$$

or, $$P\left(\frac{1}{2} - K < \frac{X}{n} < \frac{1}{2} + K\right) \geq 1 - \frac{1}{4nK^2}$$

Now, from eq$^n$(i) we know that,

$$\frac{1}{2} - K = 0.4 \qquad \frac{1}{2} + K = 0.6$$

$$K = 0.1$$

$\therefore P\left(0.4 < \frac{x}{n} < 0.6\right) \geq 1 - \frac{1}{4n(0.1)^2}$

comparing it with eqn(i) we get,

or, $1 - \frac{1}{4n(0.1)^2} = 0.90$

or, $n = \frac{1000}{4} = 250$

$\therefore$ The coin Must be tossed $(n = 250)$ times.

4a) Define stochastic process and Markov processes. Why an exponential distribution is called Memoryless? Explain.

Ans:-

Stochastic process :-

- A process which involves time is called a stochastic process. It is one of the Mathematical Model of system which vary in time in a random Manner is called Stochastic process.

- A stochastic process is a family of given probability space index by parameter t. where t, vary over and index set T. the set of all possible values of all individual r.v x(t) are called state process space and values of x(t) are called states.

Markov's process :-

- The process which follows Memoryless property is called a Markov's process. In other words the process in which

the probability of movement of particle from current state to future state is independent of past behaviour of a particle is called a Markov's process.

Memoryless property of exponential distribution

Statement :-

If 'x' be a r.v having exponential distribution with parameter $\theta$ and s and t be any two integers taken by r.v X such that $t>s$, then the Memoryless property of an exponential distribution is given by $P(x>s+t/x>s) = P(x>t)$

ie proof :-

$$P(x>s+t/x>s) = \frac{P\{(x>s+t) \cap (x>s)\}}{P(x>s)}$$

$$= \frac{P\{x>s+t\}}{P(x>s)}$$

Here,

$$P(x>s+t) = \int_{s+t}^{\infty} f(x)\,dx$$

$$= \int_{s+t}^{\infty} \theta e^{-\theta x}\,dx$$

$$= -\frac{\theta}{\theta}\left[e^{-\theta x}\right]_{s+t}^{\infty}$$

$$\therefore P(x>s+t) = e^{-\theta(s+t)}$$

again, $p(x>s) = \int_s^\infty f(x)\,dx$

$$= \int_s^\infty \theta e^{-\theta x}\,dx$$

$$= \theta \int_s^\infty e^{-\theta x}\,dx$$

$$= -\frac{\theta}{\theta}\left[e^{-\theta x}\right]_s^\infty$$

$$\therefore p(x>s) = e^{-\theta s}$$

Now,

$$p\left(x > s+t / x>s\right) = \frac{e^{-\theta(s+t)}}{e^{-\theta s}}$$

$$= e^{-\theta t} \quad LHS //$$

Now, from RHS

$$p(x>t) = \int_t^\infty f(x)\,dx$$

$$= \theta \int_t^\infty e^{-\theta x}\,dx$$

$$\therefore p(x>t) = e^{-\theta t} \quad RHS //$$

$\therefore$ LHS = RHS, Hence Memoryless property is proved.

Here, we have seen that the past has no bearing on its

future behaviour. so this exponential distribution is called the memoryless property.

4 b) Assume that a computer system is in one of three states: busy, idle or undergoing repair, respectively denoted by states 0, 1 and 2. observing its state at 2 P.M each day, we believe that the chain with the transition probability Matrix.

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

prove that the chain is irriducible, and determine steady state probability.

Ans :-

soln, Given

Here, Let, 0 = busy state
1 = idle state
2 = undergoing repair.

1- step tpm is,

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix} \end{array}$$

The 2-step tpm

$$P(2) = P^2 = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0.28 & 0.22 \\ 0.2 & 0.66 & 0.14 \\ 0.6 & 0.12 & 0.28 \end{bmatrix}$$

Since, in 2-state tpm all $P_{ij} > 0$ ie state i and j are communicative with eachother. $i = 0, 1, 2$ and $j = 0, 1, 2$. So, it is irreducible.

Now,

To find steady state probability,

Now, $S = \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} x & y & z \end{bmatrix} \end{array}$ and such that $x + y + z = 1$

$$SP = S$$

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}$$

$$\therefore \quad 0.6x + 0.1y + 0.6z = x$$
$$0.2x + 0.8y + 0z = y$$
$$0.2x + 0.1y + 0.4z = z$$

$$x + y + z = 1 \quad //$$

5a) Cars arrive at a petrol station having only one petrol unit in poisson fashion with an average of 10 cars per hours. The service time is distributed exponentially with a Mean of 3 Minutes. Find

(i) The average number of cars in the system.

(ii) Average time a cars spends in the queue.

(iii) The probability that there will be two cars in the system.

Ans :-

arrival rate $(\lambda)$ = 10 cars per hours

In 3 Minutes 1 car is served
In 1 Minutes $\frac{1}{3}$ car is served

In 60 Minutes $\frac{1}{3} \times 60$ car is served

In 1 hour 20 car is served.
so,
service rate $(\mu) = 20$ cars per hours.

This is M/M/1 $(\infty, FIFO)$

Traffic intensity $(\rho) = \frac{\lambda}{\mu} = \frac{10}{20} = \frac{1}{2} = 0.5$

(i) The average number of cars in a system is $(L_s) = \frac{\rho}{1-\rho}$

$= \frac{0.5}{1-0.5} = 1$ cars.

(ii) Average a time cars spends in a queue $(W_q) = \frac{L_q}{\lambda}$

$= \frac{\rho L_s}{\lambda} = \frac{0.5 \times 1}{10} = 0.05$ hours.

(i) The probability that there will be two cars in the system.
we know $P_n = P_0 \rho^n$

$\therefore \quad P_2 = (1-\rho)\rho^n$
$= (1-0.5)(0.5)^2$
$= 0.125$

5b) A Group of telephone subscribers is observed continously during an 80-Minutes busy hour period. During this time they make 30 calls, with the total conversation time being 4200 seconds. Compute

(i) Average number of telephone subscribers in the system.

(ii) Average time a unit spends in the system.

Ans:-

Soln, Given,

In 80 Minutes 30 calls arrive

In 1 Minutes $\frac{30}{80}$ calls arrive

$\therefore (\lambda) = \frac{3}{8}$ = arrival rate

Also, In 4200 seconds 1 calls is served

and In 1 second $\frac{1}{4200}$ calls is served

In 60 Minutes $\frac{1}{4200} \times 60$ calls is served

In 1 Minutes $= \frac{1}{70}$ calls is served

So,

service rate $(\mu) = \frac{1}{70}$

This is, M/M/1 : (∞, FIFO)

So,

Traffic intensity $(\rho) = \frac{\lambda}{\mu}$

$$= \frac{3/8}{1/70} = 26.25$$

(i) Average number of telephone subscribers in the system.

$$L_s = \left|\frac{\rho}{1-\rho}\right| = \left|\frac{26.25}{1-26.25}\right| = 1.03 \text{ subscribers.}$$

(ii) Average time a unit spends in the system.

$$W_s = \frac{L_s}{\lambda}$$

$$= \frac{1.03}{\frac{3}{8}}$$

$$= 2.75 \text{ Minutes.}$$

6 a) Suppose in a season, days can be modeled as wet and dry. The transition from a wet day to dry is having chance of 20% and that of dry to wet is 50%. If the transitions are considered as homogenous Markov's chain, what is the probability that a day will be dry after five days? Similiarly, what is the probability that a day will be wet after 6 days?

Ans:-

Soln, Given,

Let, 0 = wet state
1 = dry state

The 1-step tpm is given as :-

$$P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.8 & 0.2 \\ 1 & 0.5 & 0.5 \end{array} = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$$

Now,

$p = 0.2$, $q = 0.5$

(i) After 5 days, 5-step tpm is $P(5)$

$$= \frac{1}{p+q}\begin{bmatrix} q & p \\ q & p \end{bmatrix} + \frac{(1-q-p)^n}{p+q}\begin{bmatrix} p & -p \\ -q & q \end{bmatrix}$$

$$= \frac{1}{0.7}\begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.2 \end{bmatrix} + \frac{(1-0.5-0.2)^5}{0.7}\begin{bmatrix} 0.2 & -0.2 \\ -0.5 & 0.5 \end{bmatrix}$$

$$= \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.71 & 0.29 \\ 1 & 0.71 & 0.29 \end{array}$$

$\therefore$ probability that a day will be dray day is
$0.29 = 29\%$

again,

(ii) After 6-days 6-step TPM is $P(6)$.

$$= \frac{1}{0.7}\begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.2 \end{bmatrix} + \frac{(1-0.5-0.2)^6}{0.7}\begin{bmatrix} 0.2 & -0.2 \\ -0.5 & 0.5 \end{bmatrix}$$

$$= \begin{array}{c|cc} & \mathbf{0} & \mathbf{1} \\ \hline \mathbf{0} & 0.71 & 0.29 \\ \mathbf{1} & 0.71 & 0.29 \end{array}$$

$\therefore$ The probability that a day will be wet day is,
$0.71 = 71\%$ //

6 b) During 8:00 – 10:00 AM each Morning, train arrives every 20 Minutes in a yard. The service times have an average of 36 Minutes for each train. If the capacity of the yard is 4 trains only, find

(i) The probability that the yard will be empty.

(ii) The average queue sized.

Ans:-

Soln, Given,

In 20 Minutes 1 train arrives in a yard.
In 1 Minutes $\frac{1}{20}$ train arrives in a yard

$\therefore$ arrival rate $(\lambda) = \frac{1}{20}$ train per Minute.

again,

In 36 Minutes 1 train gets served
In 1 Minute $\frac{1}{36}$ train gets served

$\therefore$ service rate $(\mu) = \frac{1}{36}$ trains per minute

Capacity of the yard $(N) = 4$

Here, This is, $M/M/1\ (N, FIFO)$
so,

Traffic intensity $(\rho) = \frac{\lambda}{\mu}$

$$= \frac{1/20}{1/36} = 1.8$$

(i) The probability that the yard will be empty is,

$$P_0 = \frac{\rho - 1}{\rho^{N+1} - 1} \quad \text{for } \rho > 1$$

$$= \frac{1.8 - 1}{(1.8)^{4+1} - 1} = 0.0447 \,//$$

(ii) The average queue sized

$$L_s = P_0 \sum_{n=0}^{4} n \rho^4$$

$$= P_0 \left[ \rho + 2\rho^2 + 3\rho^3 + 4\rho^4 \right]$$

$$= 0.0447 \left[ 1.8 + 2 \times (1.8)^2 + 3 \times (1.8)^3 + 4 \times (1.8)^4 \right]$$

$$= 3 \,//$$

$\therefore$ 3 trains

7 Write short notes on

a) Bayes Rule.

Let $A_1, A_2, A_3 \ldots A_n$ be n-Mutually exclusive and enhaustive events within a sample space's'. Further let B be any events which can occour either in conjuction with event $A_i$s, then the conditional probability of $A_i$ given that event B has already occoured is given by Baye's theorem.

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum_{n=1}^{n} P(A_i) \cdot P(B/A_i)}$$

example :-

Suppose in a factory Machine A produce 30% of goods. Machine B produces 25% of goods and Machine C produce 45% of goods. It has been found that 1%, 2% and 3% of a goods from Machine A, B, C are deffective respectively then find the probability the item is deffective, that it has been produced from A, B and C?

Soln,

Given,

Let,

A = event that goods are been produced by A.

B = event that goods are been produced by B.

C = event that goods are been produced by C.

D = event that goods are deffective.

$P(A) = 0.30$ $\quad$ $P(D/A) = 0.01$

$P(B) = 0.25$ $\quad$ $P(D/B) = 0.02$

$P(C) = 0.45$ $\quad$ $P(D/C) = 0.03$

By Baye's Rule.

(i) $$p(A/D) = \frac{p(A) \times p(D/A)}{p(A) \times p(D/A) + p(B) \times p(D/B) + p(C) \times p(D/C)}$$

$$= \frac{0.30 \times 0.01}{0.30 \times 0.01 + 0.25 \times 0.02 + 0.45 \times 0.03}$$

$$= \frac{0.30 \times 0.01}{0.0215} = 0.14$$ //

(ii) $$p(B/D) = \frac{p(B) \times p(D/B)}{p(A) \times p(D/A) + p(B) \times p(D/B) + p(C) \times p(D/C)}$$

$$= \frac{0.25 \times 0.02}{0.0215}$$

$$= 0.23$$ //

(iii) $$p(C/D) = \frac{p(C) \times p(D/C)}{p(A) \times p(D/A) + p(B) \times p(D/B) + p(C) \times p(D/C)}$$

$$= \frac{0.45 \times 0.03}{0.0215}$$

$$= 0.63$$ //

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b) Binomial Distribution

A random variable 'x' is said to follow binomial distribution if it assumes only non-negative values and it's proof is given by ;

$$P(X=x) = \binom{n}{x} p^x q^{n-x} ; n = 0, 1, 2, .. n.$$

where,

$n$ = Number of trails
$X$ = Binomial random variable (r.v)
$x$ = Number of success
$p$ = probability of success in a trial.
$q = 1-p$ = probability of faliure in each trial.

# Properties of binomial distribution:-

① It is a discrete probability distribution because it takes only non-negative integral values between (0-n).

② For Binomial distribution Mean = $E(x) = np$ and variance = $V(x) = npq$.

③ For Binomial distribution $E(X) > V(X)$ ie $np > npq$

④ The binomial distribution has two parameters that is n and p so it is denoted by $X \sim B(n,p)$.

# Conditions to apply binomial distribution

① each trial must result 2 mutually-enclusive outcomes. interms of success and failure.

② The probability of success must remain constant in

each trial.

③ Each trial Must be independent with eachother.

examples of binomial distribution.

- A coin toss has only two possible outcomes: either heads or tails and taking a test could result two possible outcomes: pass or fail. and also the probability of success in each trial remains constants i.e. $p(\text{Heads}) = 1/2$.

Deriving Mean and variance of Binomial distribution.

Now, Given,
The probability function for a binomial r.v is

$$P(X=x) = \binom{n}{x} p^x q^{n-x}$$

Now,

$$\text{Mean } E(X) = \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n(n-1)!}{x(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

$$= np \sum_{x=0}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

$$= np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} q^{n-x}$$

$= np\,(p+q)^{n-1}$

$= np\,(p+1-p)^{n-1}$

$= np\,(1)^{n-1}$

$= np$ $\quad \therefore$ Mean $E(X) = np$

again,

$$E\{x(x-1)\} = \sum_{x=0}^{n} x(x-1)\,\frac{n(n-1)(n-2)!}{x(x-1)(x-2)!\,(n-x)!}\,p^{x}\,q^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!}\,p^{x-2}\,q^{n-x}$$

$$= (n^2-n)p^2 \sum_{x=2}^{n} \binom{n-2}{x-2}\,p^{x-2}\,q^{n-x}$$

$$= (n^2p^2 - np^2)(p+q)^{n-2}$$

$$= (n^2p^2 - np^2)(p+1-p)^{n-2}$$

$$= (n^2p^2 - np^2)(1)^{n-2}$$

$$= n^2p^2 - np^2$$

$\therefore\ E(X^2) - E(X) = n^2p^2 - np^2$

$E(X^2) = n^2p^2 - np^2 + np$

Now,

variance $V(X) = E(X^2) - \{E(X)\}^2$

$= n^2p^2 - np^2 + np - n^2p^2$

$= np(1-p)$

$= npq$ //

## (c) Periodicity of a Markov chain.

- If $i \to i$, the greatest common divisor of the set of positive $n$ such that $P_{ii}(n) > 0$ is called the period of state $i$ and is denoted by $d_i$. If state $i$ does not lead to intself, we do not define its period.

- The property of having a period is a class property. State $i$ has period $d$ if the following two conditions hold.

a) $P_{ii}(n) = 0$ unless $n = md$ for some positive integer 'M', and

b) $d$ is the largest integer with property (a).
State $i$ is called aperiodic when $d = 1$.

example
Let $\{X_n\}$ be a Marcov's chain, with state space $\{1,2,3,4\}$ and tpM given below.

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

for the given chain, $P_{11}(n) = 0$ unless $n$ is even. This is so since.

$P_{11}(1) = 0$, $P_{11}(2) = 0$, $P_{11}(3) = 0$, $P_{11}(4) = 1 > 0$

$P_{11}(5) = 0$, $P_{11}(6) = 0$, $P_{11}(7) = 0$, $P_{11}(8) = 1 > 0$

Hence, by $P_{ii}(n) = 0$ unless $n = md$, the period of state 1 is 2, 4, 6 or larger.

Here $P_{ii}(n) = 0$ is satisfied by the even integers 2 and 4 unless $n = md$.

By the largest integer with $P_{ii}(n) = 0$ unless $n = md$, the period of this state is 4. Alternatively, the greatest common divisor of 4 and 8 is 4. Hence, the period of state 1 is 4.

2013 (spring)

1a) A given lot of IC chips contains contains 2 percent defective chips. Each chip is tested before delivery. The tester itself is not totally reliable so that P("Tester says chip is good"/"chip is actually good") = 0.95 and P("Tester says chip is defective"/"chip is actually defective") = 0.94. If a tested device is said to be deffective, what is the probability it is actually defective?

Ans:-

Soln, Given,

Let, $A_1$ = event that chip is actually good.
$A_2$ = event that chip is actually defective.

$B_1$ = event that tester says chip is good.
$B_2$ = event that tester says chip is defective.

Now, D = event to be handled.

$P(B_1/A_1) = 0.95$ $\quad$ $P(B_2/A_2) = 0.94$

$P(A_2) = 0.02$ $\quad$ $P(A_1) = 1 - 0.02 = 0.98$

Now, $P(A_2/B_2) = ?$ $\quad$ $P(B_2/A_1) = 0.05$

Now,

$$P(A_2/B_2) = \frac{P(A_2) \times P(A_2/B_2)}{P(B_2)}$$

$$= \frac{0.02 \times 0.94}{0.98 \times 0.05 + 0.02 \times 0.94}$$

$$= 0.277286$$

(b) It has been claimed that in 60% of all solar heat installations the utility bill is reduced by at least one third. Accordingly, what are the probabilities that the utility will be reduced by atleast one third in

(i) four of five installations.

(ii) atleast four of five installation.

Ans:-

Soln, Given,

No of trials $(n) = 5$

Probability that the utility bill will be reduced by atleast one third $(p) = 0.6$

Probability that the utility bill will be not reduced by one third $(q) = 1-p = 1-0.6 = 0.4$

Let, $X$ = no of installation in which bills will be reduced by 1/3.

Here, $X \sim B(5, 0.6)$ so, the pmf of $X$ is;

$$P(X=x) = {}^nC_x \, p^x q^{n-x} \quad ; x = 0, 1, \ldots n$$

$$= {}^5C_x \, p^x q^{5-x}$$

$$= {}^5C_x \, p^x q^{5-x}$$

$$= {}^5C_x (0.6)^x (0.4)^{5-x} \qquad x = 0, 1, 2 \ldots 5$$

So, Now,

(i) $p(4 \text{ of } 5 \text{ installation}) = p(x=4)$

$$= {}^5C_4 (0.6)^4 (0.4)^{5-4}$$

$$= 0.2592$$

(ii) $p(\text{Atleast } 4 \text{ of } 5 \text{ installation}) = p(x=4) + p(x=5)$

$$= 0.2592 + {}^5C_5 (0.6)^5 (0.4)^0$$

$$= 0.33696$$

2a) A random variable has the probability density function $f(x) = \frac{1}{2} e^{-x/2}$, $x>0$, find Mean and variance.

Soln, Given,

$$f(x) = \frac{1}{2} e^{-x/2} \quad x>0$$

Now,

$$E(x) = \int_0^\infty x f(x)\, dx$$

$$= \int_0^\infty x \frac{1}{2} e^{-x/2}\, dx$$

$$= \frac{1}{2} \int_0^\infty e^{-x/2} x^{2-1}\, dx$$

$$= \frac{\Gamma 2}{\left(\frac{1}{2}\right)^2} \times \frac{1}{2} = 2$$

$$E(x^2) = \int_0^\infty x^2 \frac{1}{2} e^{-x/2}\, dx$$

$$= \frac{1}{2} \int_0^\infty e^{-x/2}\, x^{3-1}\, dx$$

$$= \frac{1}{2} \frac{\gamma 3}{(1/2)^3}$$

$$= \frac{2 \times 2^2}{1}$$

$$= 8$$

Now,

$$V(x) = E(x^2) - \{E(x)\}^2$$

$$= 8 - (2)^2$$

$$= 4 \;//$$

2 b) Let the phase error in a tracking device has probability density

$$f(x) = \begin{cases} \cos x & 0 < x < \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$

find the probability that the phase error is

(i) Between 0 and $\pi/4$.

(ii) Greater then $\pi/3$.

<u>Soln</u>

$$f(x) = \cos x$$

Now, $p(0 < x < \pi/4) = \int_0^{\pi/4} \cos x \, dx$

$= [\sin x]_0^{\pi/4}$

$= 0.85$ //

$p(x > \pi/3) = \int_{\pi/3}^{\pi/2} \cos x \, dx$

$= [\sin x]_{\pi/3}^{\pi/2}$

$= \sin \frac{\pi}{2} - \sin \frac{\pi}{3}$

$= 1 - \frac{\sqrt{3}}{2}$

$= 0.133974$ //

(OR)

$f(x) = \begin{cases} \frac{1}{20} e^{-x/20} & f\ x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

Find the probability that one of these tires will last

i) at Most 10,000 Miles

ii) anywhere from 16,000 to 24,000 Miles.

iii) at least 30,000 Miles.

Ans

Soln, Given,

(i) $p(x \leq 10{,}000) = \int_0^{10000} \frac{1}{20} e^{-x/20}\, dx$

$= \frac{1}{20} \left[ e^{-x/20} \right]_0^{10000} \times \left(-\frac{1}{20}\right)$

$= -\frac{1}{400} \left[ e^{-500} - 1 \right]$

$= \frac{1}{400} \left[ 1 - e^{500} \right]$ //

(ii) $p(16000 \leq x \leq 24000) = \frac{1}{20} \int_{16000}^{24000} e^{-x/20}\, dx$

$= -\frac{1}{400} \left[ e^{-1200} - e^{-800} \right]$

$= \frac{1}{400} \left[ e^{-800} - e^{-1200} \right]$ //

(iii) $p(x > 30000) = \frac{1}{20} \int_{30000}^{\infty} e^{-x/20}\, dx$

$= -\frac{1}{400} \left[ e^{-\infty} - e^{-1500} \right]$

$= \frac{1}{400} e^{-1500}$

$= \frac{1}{400\, e^{1500}}$ //

3a) The marks of 500 candidates in an examination are normally distributed with a mean of 45 marks and standard deviation 20 marks.

(i) Given that the pass marks is 40, estimated the number of candidates who passed the examination.

(ii) If 5% of the candidates obtained distinction by scoring x marks or more, estimate the value of x.

Ans:-

Soln

No of candidates $(N) = 500$

$(\mu) = 45$

$(\sigma) = 20$

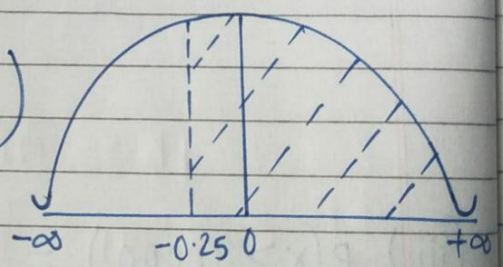
$X$ = Marks of students

Now,

(i) If 40 is pass marks then,

or, $P(X>40) = P\left(Z > \frac{40-45}{20}\right)$

$= P(Z > -0.25)$




$= P(-0.25 < Z < 0) + P(0 < Z < \infty)$

$= P(0 < Z < 0.25) + P(0 < Z < \infty)$ {By symmetric}

$= 0.0987 + 0.5$

$= 0.5987$

$\therefore$ No of passing are $500 \times 0.5987$

$= 299.35$

$\simeq 300$ //

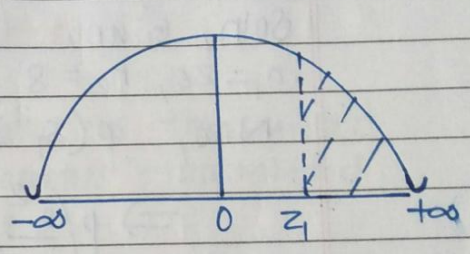
(ii) By question,

$$P(X \geq x) = 5\% = 0.05 \quad \text{eq-(i)}$$

Now,

$$P\left(\frac{X-\mu}{\sigma} \geq \frac{x-\mu}{\sigma}\right) = 0.05$$

$$P\left(Z \geq \frac{x-45}{20}\right) = 0.05$$




$$= P(Z \geq z_1) = 0.05$$

$$\text{or, } P(0 < Z < \infty) - P(0 < Z < z_1) = 0.05$$

$$\text{or, } 0.5 - 0.05 = P(0 < Z < z_1)$$

$$\therefore P(0 < Z < z_1) = 0.45$$

from a standard normal table,

$$z_1 = \frac{1.64 + 1.65}{2} = 1.645$$

Now,

$$z_1 = \frac{x-45}{20}$$

$$1.645 \times 20 + 45 = x$$

$$x = 77.9 \approx 78 \;//$$

3b) Chebsev's inequality repeated.

4a) If two independent samples of size $n_1 = 26$ and $n_2 = 8$ are taken form a normal population, what is the probability that the variance of the first sample will be at least 4 times as large as the variance of the second sample.

Ans :-

Soln, Given,

$n_1 = 26$, $n_2 = 8$ and, $v_1 = n_1 - 1 = 25$, $v_2 = n_2 - 1 = 7$

Now, $p(s_1^2 \geq 4 s_2^2)$

$$\Rightarrow p\left(\frac{s_1^2}{s_2^2} \geq 4\right)$$

$$\Rightarrow p(F(25,7) \geq 4) > 0.05 \text{ (from F table.)}$$

4b) A capacity of a communication channel is 2,000 bits per second. The lines is used to transmit eight bit characters, so the maximum rate is 250 characters per seconds, The application calls for traffic from many devices to be sent on the line with total volume of 12,000 characters per minute. calculate the traffic intensity and average no of characters waiting to be transmitted.

soln, Given,

In 1 minute 12000 characters transmitted

In 1 sec. (In 60 sec) $\frac{12000}{60}$ characters transmitted.

$\therefore$ arrival rate $(\lambda) = 200$ characters per seconds.

service rate $(\mu) = 250$ characters per seconds.

Now,

Traffic intensity $= \frac{\lambda}{\mu} = 0.8$

This is $M/M/1 : (\infty, FIFO)$

Average no of characters waiting to be transmitted is,

$$L_q = \rho L_s$$

$$= \rho\left(\frac{\rho}{1-\rho}\right)$$

$$= \frac{(0.8)^2}{1-0.8} = 3.2$$

$\therefore$ average no of characters waiting to be transmitted is

$L_q = 3.2$ //

5a) A telephone exchange receives on call every 4 minutes and each call last an average of 3 minutes. If the rate of arrival of the call follows poisson distribution and service time follows exponential distribution; calculate

(i) expected waiting time in the queue.

(ii) expected time in the system.

Soln Given,

In 4-Minutes 1 calls is received

In 1-Minute $\frac{1}{4}$ calls is received

$\therefore$ arrival rate $(\lambda) = \frac{1}{4}$ calls per minute.

In 3 Minutes 1 calls is served

In 1 Minute $\frac{1}{3}$ calls is served

Service rate $(\mu) = \frac{1}{3}$ calls per minute.

Now,

This is $M/M/1 : (\infty, FIFO)$

Traffic intensity $(\rho) = \frac{\lambda}{\mu} = \frac{\frac{1}{4}}{\frac{1}{3}} = 0.75$

(i) Expected waiting time in the queue $(W_q) = \frac{L_q}{\lambda}$

$= \frac{\rho L_s}{\lambda}$

$= \frac{0.75 \times 3 \times 4}{1}$

$= 9$ Minutes.

(ii) Expected time in the system $(W_s) = \frac{L_s}{\lambda}$

$$L_s = \frac{\rho}{1-\rho} = \frac{0.75}{0.25} = 3$$

$$\therefore W_s = \frac{3}{\frac{1}{4}} = 12 \text{ Minutes}$$

5b) 'S' super Market has two girls ruging us sales the counters. If the service time for each costomer is exponential with Mean 4 minutes, and if people arrive in a poisson fashion at the rate of 10 an hour, calculate.

(i) probability of having to wait for service.

(ii) Expected percentage of idle time for each girl.

Soln, Given,

In 1 hours 10 is arrived

In 1 Minute $\frac{10}{60}$ is arrived

arrival rate $(\lambda) = \frac{1}{6}$ per Minute

and

4 gets served in 4 minute

$\frac{1}{4}$ gets served in 1 Minute

so, service rate $(\mu) = \frac{1}{4}$ per Minute

No of server $(S) = 2$ this is

$M/M/2 : (\infty, FIFO)$

Traffic intensity $(\rho) = \frac{\lambda}{s\mu} = \frac{1}{3}$.

So, Now,

i) The Probability having wait for the service,
Average no of costumer in a queue.

$$L_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu-\lambda)^2}\right] P_0 \quad \text{eq-(i)}$$

Now,

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{1}{s!}\frac{(s\rho)^s}{1-\rho}\right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{(2\times 1/3)^n}{n!} + \frac{1}{2!}\frac{(2\times 1/3)^2}{2/3}\right]^{-1}$$

$$= \frac{1}{2} \,//$$

$$L_q = \left[\frac{1}{1}\left(\frac{4}{6}\right)^2 \times \frac{\left(\frac{1}{6}\times\frac{1}{4}\right)}{\left(2\times\frac{1}{4}-\frac{1}{6}\right)^2}\right]\frac{1}{2}$$

$$= \left[\frac{4}{9} \times \frac{\frac{1}{24}}{\frac{1}{9}}\right] \times \frac{1}{2}$$

$$= \frac{1}{12} \,//$$

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Probability that a costumer has to wait to service is,

$$P_W = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} P_0$$

$$= \frac{1}{2} \times \frac{16}{36} \times \frac{2 \times \frac{1}{4}}{2 \times \frac{1}{4} - \frac{1}{6}} \times \frac{1}{2}$$

$$= \frac{2}{9} \times \frac{\frac{1}{2}}{\frac{1}{3}} \times \frac{1}{2}$$

$$= \frac{1}{6} //$$

(ii) Expected percentage of idle time for each girl

busy time $(\rho) = \frac{1}{3}$

idle is $= 1 - \frac{1}{3} = \frac{2}{3}$

In percentage,

$$= \frac{2}{3} \times 100\%$$

$$= 66.67\% //$$

6a) Let $\{X_n, n \geq \}$ be a Matrix chain defined on the 3 states 0, 1 and 2. It's transistion probability is given below. Let the initial distribution be.

$P(X_0 = i) = \frac{1}{3} \; \forall i = 0,1,2$ obtain $P(X_2 = 1, X_0 = 0)$

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

Soln, Given,

By question,

$P(X_0 = i) \frac{1}{3} \; \forall i = 0,1,2$ ie.

$P(X_0 = 0) = P(X_0 = 1) = P(X_0 = 2) = \frac{1}{3}$

and

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

Two step tpM is.

$$P^2 = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix} \times \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.625 & 0.3125 & 0.0625 \\ 0.3125 & 0.5 & 0.1875 \\ 0.1875 & 0.5625 & 0.25 \end{bmatrix}$$

we know that,

$$P\left(X_2 = 1 / X_0 = 0\right) = \frac{P\left(X_2 = 01,\ X_0 = 0\right)}{P\left(X_0 = 0\right)}$$

$$\frac{5}{16} = \frac{P\left(X_2 = 01,\ X_0 = 0\right)}{\frac{1}{3}}$$

$$\therefore\ P\left(X_2 = 1,\ X_0 = 0\right) = \frac{5}{48}\ //$$

6b) Repated.

7 all repeated //

2014 (Fall)

1a) An assembly plant receives it's voltage regulators from three different suppliers, 60% from supplier A, 30% from supplier B, and 10% from supplier C. If 95% of the voltage regulators from A, 80% of the voltage regulators from B, and 65% of those from C perform according to the specification given by the suppliers. What is the probability that:

(i) The assembly plant will receive voltage regulator working according to the specification.

(ii) If a selected voltage regulator is found to be not working according to the specification, what is the probability that it will be supplied by supplier C?

Ans:-

Soln, Given,

Let, A = event that voltage is supplied by supplier A.
B = event that voltage is supplied by supplier B.
C = event that voltage is supplied by supplier C.

Let, S = event that performs according to the specification

$P(A) = 0.60$ $\quad$ $P(S/A) = 0.95$

$P(B) = 0.30$ $\quad$ $P(S/B) = 0.80$

$P(C) = 0.10$ $\quad$ $P(S/C) = 0.65$

(i) $P(S) = P(A \cap S) + P(B \cap S) + P(C \cap S)$

$= P(A) \cdot P(S/A) + P(B) \cdot P(S/B) + P(C) \cdot P(S/C)$

$= 0.60 \times 0.95 + 0.30 \times 0.80 + 0.10 \times 0.65$

$= 0.875$

(ii) $P\left(\frac{C}{\bar{S}}\right) = \frac{P(C \cap \bar{S})}{P(\bar{S})} = \frac{P(C) \times P(\bar{S}/C)}{1 - P(S)}$

$= P\left(\frac{\bar{S}}{C}\right) = P(\bar{S}/C) = 1 - P(S/C)$
$= 0.35$

$= \frac{0.10 \times 0.35}{1 - 0.875} = 0.28$ //

1b) The joint density function of x and y is given by
$f(x,y) = Kxy$, if $0 < x < 2$, $0 < y < 4$

Find
(i) The value of the constant K.
(ii) Find $P(X \geq 1, Y \leq 2)$.
(iii) Find the Marginal density function of x and y.
(iv) Check if x and y are independent.

Ans:-
Soln, Given,
$f(x,y) = Kxy$ so,

(i) To find the value of K.

or, $\int_{x=0}^{2} \int_{y=0}^{4} Kxy \, dy \, dx = 1$

or, $\frac{1}{2} \int_0^2 Kx \left[y^2\right]_0^4 dx = 1$

or, $\frac{1}{2} \cdot 16 \int_0^2 Kx \, dx = 1$

or, $8\int_0^2 kx\,dx = 1$

$$\frac{8}{2}K\left[x^2\right]_0^2 = 1$$

$$\frac{32}{2}K = 1$$

$$K = \frac{1}{16}$$

(ii) $\int_{x=1}^{2}\int_{y=0}^{2} \frac{1}{16}xy\,dy\,dx$

$$= \frac{1}{16}\int_1^2 x\left[\frac{y^2}{2}\right]_0^2 dx$$

$$= \frac{1}{32}\cdot 4\int_1^2 x\,dx$$

$$= \frac{1}{8}\left[\frac{x^2}{2}\right]_1^2 dx$$

$$= \frac{1}{16}(2^2-1)$$

$$= \frac{3}{16}$$ //

(iii) $f(x) = \int_y f(x,y)\,dy$

$= \int_0^4 \frac{1}{16} xy\,dy$

$= \frac{1}{16} x \left[\frac{y^2}{2}\right]_0^4$

$= \frac{16}{16 \times 2} x$

$= \frac{1}{2} x$ for $0 < x < 2$

$f(y) = \int_x f(x,y)\,dx$

$= \int_0^2 \frac{1}{16} xy\,dx$

$= \frac{1}{16} y \left[\frac{x^2}{2}\right]_0^2$

$= \frac{4}{16 \times 2} \cdot y$

$= \frac{1}{8} y$ for $0 < y < 4$

(iv) $f(x) \times f(y) = \frac{1}{2}x \times \frac{1}{8}y = \frac{1}{16}xy = f(x,y)$

so, Given function is independent.

2a) It is known that 5% of the screws manufactured by an automatic machine are deffective. If a sample of 20 screws is selected at random. Find the probability that the sample contain:

(i) Enactly 2 defective screws.
(ii) Atleast 2 defective screws.
(iii) At most 2 defective screws.
(iv) No defective screws.

Soln, Given,

Let, $\bar{D}$ be the deffective screws then,
$P(\bar{D}) = 0.05$ and $P(D) = 0.95$

Now,
no of trials $(n) = 20$

let, $x$ = no of deffective articles.

$$\text{Mean } (\lambda) = np = 20 \times 0.05 = 1$$

$X \sim P(\lambda = 2)$ so, PMF is

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

(i) $P(X=2) = \frac{e^{-1} \times (1)^2}{2!} = 0.183939$ //

(ii) $P(X \geq 2) = 1 - P(X<2)$

$= 1 - P(X=0) + P(X=1)$

$= 1 - e^{-1}\left(\frac{(1)^0}{0!} + \frac{(1)^1}{1!}\right)$

$= 1 - \frac{1}{e}[2)$

$= 0.2642411177$ //

(iii) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{2}{e} + 0.183939$

$= 0.919697$ //

(iv) $P(\text{No defective screws}) = 0.95$ //

(b) At a checkout counter customers arrive at the average of 1.5 per minute. Find the probability that

(i) At most 4 will arrive in any given minute.

(ii) At least 3 will arrive during an interval of 2 minutes.

(iii) At most 15 will arrive during an interval of 6 minutes

Ans:-

Soln, Given,

Average no of costomer arriving at checkout counter $(\lambda = 1.5)$

$X \sim P(\lambda = 1.5)$ so, the p.m.f of X is,

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, 2 \ldots \infty.$$

$$P(X=x) = \frac{e^{-1.5}(1.5)^x}{x!} \quad x = 0 \text{ to } \infty$$

i) $P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$$= e^{-1.5}\left[\frac{(1.5)^0}{0!} + \frac{(1.5)^1}{1!} + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!} + \frac{(1.5)^4}{4!}\right]$$

$$= 0.98$$

for 2 min $\lambda = 1.5 \times 2$
$= 3$

PMF is,

$$P(X=x) = \frac{e^{-3}3^x}{x!}$$

(ii) $P(X \ge 3) = 1 - P(X<3)$

$$= 1 - e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right)$$

$$= 0.5768$$

(iii) $\lambda = 1.5 \times 4$
$= 6$

$$P(X=x) = \frac{e^{-6}6^x}{x!}$$

$$P(X=5) = \frac{e^{-6}6^5}{5!} = 0.16$$

3a) The length of time a person speaks over phone follows exponential distribution with mean 6. What is the probability that the person will talk for.

(i) More than 8 minutes.

(ii) Between 4 and 8 minutes.

Soln, Given,

Let, X = Duration of time in a minute that a persons speak in a phone.

$$E(X) = 6\text{Min}, \quad \theta = \frac{1}{6}$$

Here $X \sim Exp(\theta = 1/6)$

$$f(x) = \begin{cases} \frac{1}{6} e^{-x/6} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

(i) $$P(X>8) = \int_8^\infty \frac{1}{6} e^{-x/6} dx$$

$$= \frac{1}{6} \times \left(-\frac{1}{6}\right) \left[e^{-x/6}\right]_8^\infty$$

$$= -\frac{1}{36} \left[0 - e^{-4/3}\right]$$

$$= \frac{1}{36 e^{4/3}}$$

$$= 0.00732214$$ //

(ii) $P(4<x<8) = \int_4^8 f(x)\,dx = \int_4^8 \frac{1}{6} e^{-x/6}\,dx$

$= \int_4^8 \frac{1}{6} e^{-x/6}\,dx$

$= \left(\frac{1}{6}\right) - \frac{1}{6}\left[e^{-x/6}\right]_4^8$

$= \frac{-1}{36}\left[e^{4/3} - e^{2/3}\right]$

$= -0.051275$ //

3b) The actual amount of instant coffee that a filling Machine into '4' ounces jars May be looked upon an a random variable having a normal distribution with standard deviation of 0.04 ounces. If only 2% of the jars are to contain less the 4 ounces. what should be Mean fill to jars?

Ans:-

Soln, (given) then,

Let $X$ = a r.v of coffee filling Machine into ounces jars.

$s.d\ (\sigma) = 0.04$

Now, By question. $X \sim N(\mu, \sigma^2)$

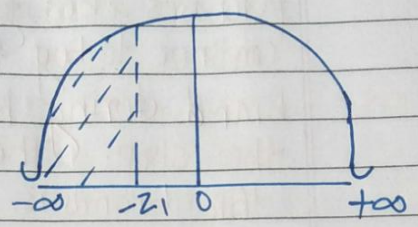
Now,

$P(X<4) = 0.02$

or, $P\left(Z < \frac{X-\mu}{\sigma}\right) = 0.02$

or, $P\left(Z < \frac{X-\mu}{\sigma}\right) = 0.02$

or, $P\left(Z < \frac{4-\mu}{0.04}\right) = 0.02$




or, $P(Z < -z_1) = 0.02$

or, $P(-\infty < Z < 0) - P(-z_1 < Z < 0) = 0.02$

infinity

or, $P(0 < Z < 8) - P(0 < Z < z_1) = 0.02$

or, $0.5 - 0.02 = P(0 < Z < z_1)$

$P(0 < Z < z_1) = 0.48$

$-z_1 = \frac{2.05 + 2.06}{2} = 2.055$

Now,

or, $-z_1 = \frac{X-\mu}{\sigma}$

or, $-2.055 = \frac{4-\mu}{0.04}$

or, $\mu = 4.082$ //

$\therefore$ Mean $(\mu) = 4.082$ //

4A) There are three brands of beauty products A, B and C. in a shop. It has been found that a person when purchases a beauty products of a particular brand, will continue to buy the same brand or switch over to another brand during his next purchase of a beauty product in the shop. The transition probability Matrix with three brands is given below.

$$P = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.6 & 0.1 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

If the initial distribution of purchase of the brands A, B and C is $[0.4 \ 0.4 \ 0.2]$, determine the distribution after two purchase.

Ans:-

Soln, Given,

The 1-step tpM is, given by,

$$P = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.6 & 0.1 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

initial probabiliy vector is,

$$p(0) = S_0 = [0.4 \ 0.4 \ 0.2]$$

After 2-purchase 2-step tpM is.

$$p(2) = P^2 = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.6 & 0.1 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.6 & 0.1 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4 & 0.24 & 0.36 \\ 0.36 & 0.28 & 0.36 \\ 0.36 & 0.24 & 0.4 \end{bmatrix}$$

$$= S_0 P^2 = \begin{bmatrix} 0.4 & 0.4 & 0.2 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.24 & 0.36 \\ 0.36 & 0.28 & 0.36 \\ 0.36 & 0.24 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.376 & 0.256 & 0.368 \end{bmatrix}$$

4b) A counter has single attendent during peak hours 20 per hours. The average served is 24 per hours calculate.

(i) The probability that the cashier is idle.

(ii) The average no of customers in the system.

(iii) There are more than 7 customers in the queue.

(iv) Average waiting time of a customer in the queue waiting for service.

Ans:-

Given,

arrival rate $(\lambda) = 20$ per hour

service rate $(\mu) = 24$ per hours

Traffic intensity $(\rho) = \frac{\lambda}{\mu} = \frac{20}{24} = 0.833$ This is M/M/1 : $(\infty, FIFO)$

(i) $P_0 = 1 - \rho = 1 - 0.833 = 0.17$

(ii) $L_s = \frac{\rho}{1-\rho} = \frac{0.833}{0.17} = 4.9 \approx 5$

(iii) $(\rho)^7 = 0.2783$

(iv) $W_q = \frac{L_q}{\lambda}$

when $L_q = \rho L_s = 0.833 \times 4.9$

$$\therefore W_q = \frac{0.833 \times 4.9}{20} = 0.204085$$

5 a) A petrol pump station has 3 pumps. The service time follows an exponential distribution with mean of 5 minutes and the cars arrive for service in a poission process at the rate of 8 cars per hours. Find

(i) A customer has to wait for service.
(ii) Average no. of customers waiting in the queue.
(iii) Average no of cars in the system.

Soln, Given,

No of server(s) = 3

In one hours 8 cars are arrived
In 1 hours 8 cars are arrived
so, $(\lambda)$ = arrival rate = 8 cars per hours.

In 5 Minutes 1 car is served
In 1 Minutes $\frac{1}{5}$ car is served.

In 60 Minutes $\frac{1}{5} \times 60$ car is served

12 car per hours

service rate $(\mu) = 12$ car per hours.

This is $M/M/S\ (\infty, FIFO)$

Traffic intensity $(\rho) = \frac{\lambda}{\mu s} = \frac{8}{12 \times 3}$

Now, $= 0.22$

(i) $W_q = \frac{L_q}{\lambda} = \frac{0.06204}{8}$

$= 0.007755$

(ii) $L_q = \rho L_s$

$= 0.22 \times 0.2820$

$= 0.06204$

(iii) $L_s = \frac{\rho}{1-\rho} = 0.2820$

5b) A television repairman finds that the time spent on a repair has an exponential distribution with mean of 30 minutes. If he repairs the set in order they come and if the arrival of television set for repair is poission with an average rate of 10 per 8 hour day, what is his expected idle time per day?

Ans:-

soln, Given,

arrival of 10 per 8 hours a day

$$\text{arrival rate } (\lambda) = \frac{10}{8 \times 60} = \frac{1}{48} \text{ per minute}$$

$$(\mu) = \frac{1}{30} \text{ per minute.}$$

The number of hours that the repairman remains busy in working days $= 8 \times \frac{\lambda}{\mu}$

$$= 8 \times \frac{\frac{1}{48}}{\frac{1}{30}} = 5 \text{ hours.}$$

The number of hours that a repairman remains idel in working day is is $8 - 5 = 3$ hours.

$\therefore$ expected idle time per day $= 3$ hours //

6(a) A single-Man barber shop can accommodate Maximum of 5 people at a time (4 waiting and 1 getting haircut). costomers arrive in the shop at the ratio of 8 per hour and the barber takes 6 Minutes for serving each costomer. If the arrival process is poission and service time is an exponential random variable, find.

(i) percentag of time the barber is idle.

(ii) Fraction of potential customers who will be turned away.

Ans :-

soln, given,

$N = 5$

$\lambda = 8$ per hour

$\mu = 10$ per hour.

$\rho = \frac{\lambda}{\mu} = \frac{8}{10} = 0.8$

This is M/M/1 : (N, FIFO)

Now,

(i)

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-0.8}{1-(0.8)^{5+1}}$$

$$= 0.2710$$

$$= 27.10\% \quad //$$

(ii)

$$P(N \geq 5) = \left(\frac{1-\rho}{1-\rho^6}\right)\rho^5 = \frac{0.2 \times (0.8)^5}{1-(0.8)^6}$$

$$= 0.0888$$

$$= 8.88\% \quad //$$

b) Repard //

7) Write short notes on:-

a) Chebyshev's Inequality

Let 'x' be the r.v with finite mean $E(x)$ and variance $V(x)$ then, Chebyshev's inequality is defined as;

$$P(|x - E(x)| < K\sigma) \geq 1 - \frac{1}{K^2}$$ [∵ lower bound of probability]

and

$$P(|x - E(x)| \geq K\sigma) \leq \frac{1}{K^2}$$ [∵ upperbound of probability]

$(K > 0)$

When 'K' is a poitive constant ie $(K > 0)$

if we say $K\sigma = c$, then,

$$P(|x - E(x)| < c) \geq 1 - \frac{1}{\quad}$$

(Lower bound of probability)

Now,

$$P(|x - E(x)| \geq c) \leq \frac{V(x)}{c^2}$$ [∵ upper bound of probability]

example:-

Suppose that 'x' is a random variable such that $E(x) = 13$ and $V(x) = 25$. Using Chebyshev's inequality, give a bound on $P(5 < x < 21)$.

Soln

Given,

$$P(5 < x < 21) = P(5 < x < 21) \quad eq-(i)$$

Now,

By chebyshev's inequality

$$p(|X-13|<8) = 1 - p(|X-13| \geq 8) = p(5<X<21)$$

chebyshev's inequality states that;

$$p(|X-E(X)| \geq K\sigma) \leq \frac{1}{K^2}$$

In this case $\sigma = 5$ because $V(X) = \sigma^2 = 25$ and

$$K\sigma = 8$$

$$K = \frac{8}{5} = 1.6 \quad \text{Now,}$$

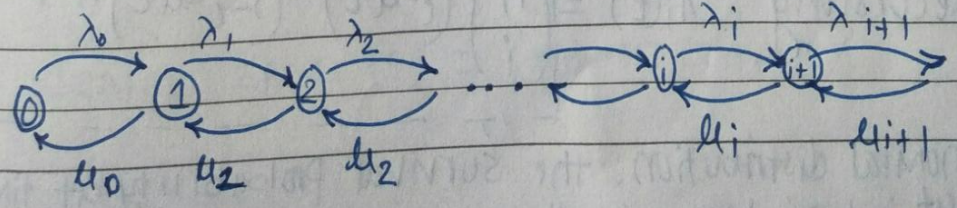
$$p(|X-13| \geq 8) \leq \frac{1}{(1.6)^2} \approx 0.391$$

$$\therefore p(5<X<21) = 1 - p(|X-13| \geq 8)$$
$$= 1 - 0.391$$
$$= 0.609$$

b) Birth and death Process.

A birth- death (BD process) Process refers to a Markov Process with

- a discrete state space.
- the state of which can be enumerated with index $i = 0, 1, 2 \ldots$ such that
- state transistions can occur between neighbouring states, $i \to i+1$ or $i \to i-1$



Transition rates

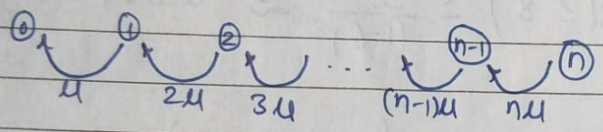
$$q_{ij} = \begin{cases} \lambda_i & \text{when } j = i+1 \\ \mu_i & \text{when } j = i-1 \\ 0 & \text{otherwise} \end{cases} \quad \begin{cases} \text{prob of birth in interval } \Delta t \text{ is } \lambda_i \Delta t \\ \text{prob of death in interval } \Delta t \text{ is } \mu_i \Delta t \\ \text{when the system is in state } i \end{cases}$$

example 1:- Pure death process

$$\begin{cases} \lambda_i = 0 \\ \mu_i = i\mu \end{cases} \quad i = 0, 1, 2 \ldots \qquad \pi_i(0) \begin{cases} 1 & i = n \\ 0 & i \neq n \end{cases}$$

all individual have the same Mortality rate $\mu$

The system starts from state $n$.




state '0' is absorbing state, other states are transient.

$$\begin{cases} \frac{d}{dt}\pi_n(t) = -n\mu\pi_n(t) & \Rightarrow \pi_n(t) = e^{-n\mu t} \\ \frac{d}{dt}\pi_i(t) = (i+1)\mu\pi_{i+1}(t) - i\mu\pi_i(t) & i = 0, 1 \ldots, n-1 \end{cases}$$

$$\frac{d}{dt}\left(e^{i\mu t}\pi_i(t)\right) = (i+1)\mu\pi_{i+1}(t)e^{i\mu t} \Rightarrow \pi_i(t) =$$

$$(i+1)e^{-i\mu t}\mu\int_0^t \pi_{i+1}(t')e^{i\mu t'}dt'$$

$$\pi_{n-1}(t) = ne^{-(n-1)\mu t}\mu\int_0^t \frac{e^{-n\mu t'}e^{(n-1)\mu t'}}{e^{-\mu t'}}dt' =$$

$$ne^{-(n-1)\mu t}\left(1 - e^{-\mu t}\right)$$

Recursively $\pi_i(t) = \boxed{\binom{n}{i}\left(e^{-\mu t}\right)^i\left(1 - e^{-\mu t}\right)^{n-i}}$

Binomial distribution: the survival probability at time $t$ is $e^{-\mu t}$ independent of others.

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## c) Conditional probability :-

- If A and B are events in sample space S and $P(B)>0$, then the conditional probability of A given B is denoted as $P(A/B)$ and

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

- It is also known as probability of an event occuring, given that another event has already occured.
- It is read as "probability of B, given A" if $P(B/A)$.

Properties of conditional probability

- If $B \subset A$, $AB = B$, then,

$$P(A/B) = \frac{P(AB)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

- If $A \subset B$, $AB = A$, then,

$$P(A/B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)} > P(A)$$

example :-
Suppose a coin is tossed twice then. Finding the probability of getting two heads that atleast one head has already occured is as:-

Given, If 2 coins are tossed then,

Sample space (S) = {HH, HT, TH, TT}

Let, A = event of getting exactly two heads.

B = event of getting atleast one head.

So,

No of exactly two heads in a sample space is given by

$n(A) = 1$

and

No of getting atleast one head in a sample space is given by:-

$n(B) = 3$

Now,

we are asked $P(A/B) = ?$

∴ Probability of getting exactly two head that at least one head has already been occured is given by:-

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$$

$$= \frac{n(A \cap B)}{\cancel{n(S)}} \times \frac{\cancel{n(S)}}{n(B)}$$

$$= \frac{1}{3} \quad //$$

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2014 (spring)

1a) There are three switchs in a college network namley A, B and C working independently. This switch should 'on' to transport data for these 1/2, 3/4 and 1/4 respectively. Find the probability that

(i) There will be a successful data transfer.

(ii) There will not be a successfull data transfer.

Ans:-

soln, Given,

$P(A) = 1/2$ $\quad P(\bar{A}) = 1/2$

$P(B) = 3/4$ $\quad P(\bar{B}) = 1/4$

$P(C) = 1/4$ $\quad P(\bar{C}) = 3/4$

Now.

(i) Prob( that there will be successful data transfer ) $= P(A \cap B \cap C)$

$= 1/2 \times 3/4 \times 1/4$

$= 0.09375$

(ii) P( There will not be successfull data transer is ) =

$P(\bar{A}) + P(\bar{B}) + P(\bar{C}) - P(\bar{A} \cap \bar{B}) - P(\bar{B} \cap \bar{C}) - P(\bar{C} \cap \bar{A}) + P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= \frac{1}{2} + \frac{1}{4} + \frac{3}{4} - \frac{1}{2} \times \frac{1}{4} - \frac{1}{4} \times \frac{3}{4} - \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{3}$

$= \frac{29}{32} = 0.906257$ //

$= 0.90$ //

1b)

2a)

2b)

3a)

3b)

4a)

4b)

5a) all repated one.

5b) The average rate of arrival of costumers in a super Market is 10 every 30 minutes. The average time taken by the cashier to list and calculate the customer purchases is 2.5 minutes. suppose that arrivals are assumed to follow poission process and service the follows enponential? calculate Oc of this Model.

Soln, Given,

In 30 Minutes 10 are arived

In 1 Minute $\frac{10}{30}$ are arrived

$\therefore$ arrival rate $(\lambda) = \frac{1}{3}$ per Minute.

In 2.5 Minutes 1 is served

In 1 Minute $\frac{1}{2.5}$ is served.

so, $(\mu) = \frac{1}{2.5}$ per Minute

$\rho = \frac{\lambda}{\mu} = \frac{2.5}{3}$

$= 0.833$

Now, This is $M/M/1 : (\infty, FIFO)$

1) $P_0 = 1 - \rho$

$= 1 - 0.833$

$= 0.167$

2) $P_n = 0.167 \times (0.833)^n$

(3) $L_s = \frac{\rho}{1-\rho} = 4.9 \approx 5$

(4) $W_s = \frac{L_s}{\lambda} = \frac{4.9}{\frac{1}{3}} = 14.7$

(5) $L_q = \rho L_s$

$= 4.0817$

(6) $W_q = \frac{L_q}{\lambda}$

$= 12.2451$

(7) $P_W = \rho = 0.833.$

6a repeated)

6b) The daily consuption of milk in a city is excess of 20,000 gallons, is approximately distributed as a gamma variate with parameters $\alpha = 2$ and $\lambda = \frac{1}{10,000}$. The city has a daily stock of 30,000 gallons. What is the probability that the stock is insufficient on a particular day?
[Note: $\beta = \frac{1}{\lambda}$].

Soln, Given,

Let $x$ = amount of daily consuption of milk in Gallons.

$\alpha = 2$

$\beta = \frac{1}{\lambda}$ and $\lambda = \frac{1}{10000}$ $\therefore \beta = 10000$

By question, $X \sim G(2, 10000)$ The pdf is;

$$f(x) = \frac{e^{-x/\beta} x^{\alpha-1}}{\beta^{\alpha} \Gamma\alpha} \text{ for } (X>0)$$

$$= \frac{e^{-x/10000} x^{2-1}}{(10000)^2 \Gamma 2} = \frac{e^{-x/10000} x^1}{(10000)^2} ; x>0$$

If it has 30000 gallons then probability of insufficient is given by

$$P(X>30000) = \int_{30000}^{\infty} \frac{e^{-x/10000} \cdot x^1}{(10000)^2} dx$$

$$= \frac{1}{(10000)^2} \int_{30000}^{\infty} x e^{-x/10000} \, dx$$

$$= \frac{1}{(10000)^2} \left[ \frac{x e^{-x/10000}}{-\frac{1}{10000}} - \frac{e^{-x/10000}}{\left(-\frac{1}{10000}\right)^2} \right]_{30000}^{\infty}$$

$$= \frac{1}{100000000} \left[ -10000 x e^{-x/10000} - 100000000 e^{-x/10000} \right]_{30000}^{\infty}$$

$$= \frac{1}{100000000} \left[ (-0 - 0) - \left( -10000 \times 30000 \times e^{-3} - 100000000 e^{-3} \right) \right]$$

$$= \frac{e^{-3}(300000000 + 100000000)}{100000000}$$

$$= 0.199$$

7a) Write Short note on

a) central imit theorem.

- Let $x \sim N(\mu, \sigma^2)$, then the sampling distribution of Sample Mean $(\bar{x})$ also i.e. named $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Also, sum of Samples $S_n = \Sigma x \sim N(n\mu, n\sigma^2)$

- The central limit theorem is the Most important single result, which clarifies the role of nomal distribution, and has vital implications for statistics.

- It states that the Mean of the distribution of sample Means is almost identical to the Mean of the underlying population. on the other hand, the variability of sample Means is farless than that of the population.

- The distribution of sample Means is approximately bell shaped and symmetrical, when the original population distribution is skewed. It is one of the most remarkable features of mathematical statistics, and is called the central limit theorem.

eg:- Suppose a population of 29 years-old Male has a Mean salary of \$29,321 with a standard deviation of \$2120. If a sample of 100 Men is taken, what is the probability their Mean salaries will be less than \$29,000?

Ans:-

Given,

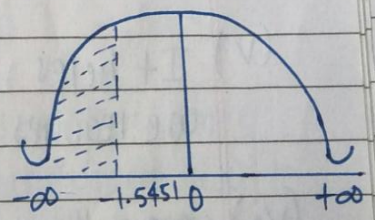
Average sallary $(\mu) = \$29321$

Sd $(\sigma)$ \$2120

Sample size $(n) = 100$ $\quad P(\bar{x} < 29000) = ?$

Now, By central limit theorem we can say that,

$\bar{x} \sim N(\mu, \sigma^2/n)$




$$P\left\{\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) < \frac{29000-29321}{2120/10}\right\}$$

or, $P(Z < -1.5451)$

or, $P(-\infty < Z < 0) - P(-1.5451 < Z < 0)$

or, $P(0 < Z < \infty) - P(0 < Z < 1.5451)$

or, $0.5 - 0.4406$

$= 0.0594$ $\quad$ That is 5.94%. //

b) Application of queue theory in computer system.

- Queueing theory is the Mathematical study of waiting lines, or queues.
- A queuing Model is constructed so that queue lengths and waiting time can be predicted.

It has the Great application in computer system. It is follows :-

(i) It helps to view networks as collections of queues such as FIFO data-structures, FCFS data-structure and SIRO data-structure also other data-structure too.

(ii) It helps in determining the sequence of computer operations and predicting computer performance.

(iii) Telecommunication and Traffic control is one of the application of queue theory.

(iv) Data computation and process in queue with better and high performance Measure.

(v) It helps in cpu sheduling, Disk scheduling when we requires the Muliple Processes and synchronization.

(vi) Queue theory helps in determining average time in a queue while processing data also Average length of it.

(vii) It helps the user to easily interpret data by looking at different scenarious quickly, accurately, and easily.

## (c) Random variable

A Random variable is that kind of variable which is used to denote the various outcomes within a sample space 's'. In otherwords, a r.v is a real valued function of samplepoints. The random variable are denoted by capital letters X, Y, Z etc.

eg:- Let, Two coins are tossed simultenously then, the sample space 's' is {HH, HT, TH, TT

Let, X = no. of heads where,
X is a r.v taking values 0, 1 and 2 and

$P(X=0) = 1/4$ $P(X=1) = 2/4$ and $P(X=2) = 1/4$

| No of heads ($x$) | 0 | 1 | 2 |
|---|---|---|---|
| $P(X=x)$ or $P(x)$ | 1/4 | 2/4 | 1/4 |

This tabular representation is called probability distribution of r.v X.

Types of random variable are:-
(i) Discrete random variable and (ii) continous random variable

(i) Discrete random variable:-
A discrete r.v is that random variable which assumes only integral values within a sample space. In other words discrete random variables are those random variable which assumes only counting numbers. it is generated from discrete sample space.

eg:- No of heads, no of coins thrown, no of boys, No of pens, No of cows in a farm, No of girls, No of tables and No of seeds etc.

(ii) continous random variable

It is said to be a continous random variable if it assumes fractional as well as non-fractional values within the interval $(-\infty, \infty)$. In other words a continous r.v is a measurable r.v. It is generated from continous sample space.

eg:- Age, weight, height, etc.

# probability mass function (pmf)

Let, 'x' be a discrete r.v then assuming, the discrete values $x_1, x_2, x_3 \dots x_n$ with respective probabilities $p(x_1)$, $p(x_2)$ and $p(x_n)$ then a function $p(X=x) = p(x)$ is called a pmf of 'x' if it satisfies.

(i) $p(x) \geq 0$

(ii) $\sum p(x) = 1$

# probability density function (p.d.f)

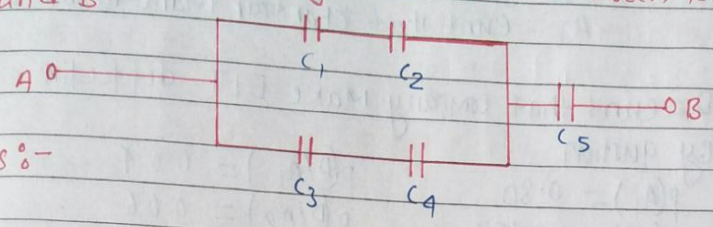
Let 'x' be a continous random variable which assumes values in the interval $(-\infty, \infty)$ then, a function $f(x)$ is called a p.d.f of 'x' which gives the probability $(x < x < x + dx)$ if it is satisfies

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

2015-Fall

1a) For a Given circuit each relay is 0.6 independently find current probability that exist between terminal A and B.




Ans:-

Let, $A_1$ = event in upper terminal
$A_2$ = event in lower terminal

$$P(A_1 \cup B_1) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$
$$= P(C_1 C_2 C_3) + P(C_3 C_4 C_5) - P(C_1 C_2 C_3 C_4 C_5)$$
$$= (0.6)^3 + (0.6)^3 - (0.6)5$$
$$= 0.35424$$

1b) State Baye's theorem. An ELT is a device desgined to transmit a signal in the case of a crash. The Altigauge Manufacturing company makes 80% of the ELTs the Braynt makes 15% of them, and the chartair company makes the other 5%. The ELTs have a 6% rate of defects, the chartair ELTs have 9% rate of defects.

(i) If a randomly selected ELT is tested to be deffective, find the probability it was made by the Altigauge Manufaction company.

(ii) If it is deffective. find probabability that it has may been Bryant Manufacturing company.

Ans:- Let,

$A_1$ = event that Altigauge Manufactures ELT

$A_2$ = event that Bryant Manufactures ELT.

$A_3$ = event that Chartair Manufacture ELT.

D = event that company Make ELT defective.

By qustion

$P(A_1) = 0.80$ $\quad$ $P(D/A_1) = 0.04$

$P(A_2) = 0.15$ $\quad$ $P(D/A_2) = 0.06$

$P(A_3) = 0.05$ $\quad$ $P(D/A_3) = 0.09$

Now,

$$P(D) = P(A_1 \cap D) + P(A_2 \cap D) + P(A_3 \cap D)$$
$$= 0.80 \times 0.04 + 0.15 \times 0.06 + 0.05 \times 0.09$$
$$= 0.0455$$

(i) $$P(A_1/D) = \frac{P(A_1 \cap D)}{P(D)} = \frac{0.80 \times 0.04}{0.0455} = 0.703296$$

(ii) $$P(A_2/D) = \frac{P(A_2) \times P(\bar{D}/A_2)}{1 - P(D)}$$

$$\left[ P(\bar{D}/A_2) = 1 - P(D/A_2) = 1 - 0.06 = 0.94 \right]$$

$$= \frac{0.15 \times 0.94}{1 - 0.0455}$$

$$= 0.1477213$$

2(a) $f(x) = \begin{cases} 720x^{-2}, & 400 \le x \le 900 \\ 0, & \text{otherwise.} \end{cases}$

(i) Probability that the printer ink has lifetime not more than 550 hours.

(ii) Probability it has more 700 given that more then 600 hours.

(iii) Expected life time of a printer ink.

Soln, Given.

i) $P(X>550) = \int_{550}^{900} f(x)\,dx = \frac{28}{55} = 0.5090$

(ii) $P(X \ge 700 / X \ge 600) = \frac{P(X>700 \cap X>600)}{P(X>600)} = \frac{P(X>700)}{P(X>600)}$

$= \frac{8/35}{2/5}$

$= 4/7 = 0.5714$ //

(iii) $E(X) = \int_{400}^{900} x\, f(x)\,dx$

$= \int_{400}^{900} \frac{1}{x} 720\,dx$

$= 253.571$ //

3(a) An information source emits a 10 digit Message into a channel in binary code. probability of 1's is 0.3. Find the probability that the Message contains

(i) Three ones

(ii) Between 2 and 4 ones.

(iii) No less then 2 zeros.

Soln, Given,

No of trials $(n) = 10$

Let, $X$ = event that 1's has occured.

Probability that 1's has occured $(p) = 0.3$

Probability that 0's has occured $(q) = 0.7$

Here, This is $X \sim B(10, 0.3)$ so, pmf is

$$p(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x} \qquad n = 0 \text{ to } 10, \quad x = 0 \text{ to } n$$

$$p(x) = {}^{10}C_{x}\, p^{x} q^{10-x}$$

Now,

(i) $p(X=3) = {}^{10}C_{3}\, p^{3} q^{7}$

$= {}^{10}C_{3} (0.3)^{3} (0.7)^{7}$

$= 0.266827932$

(ii) $p(2 \le x \le 4) = {}^{n}C_{2}(0.3)^{2}(0.7)^{8} + p(X=3) + {}^{10}C_{4}(0.3)^{4}(0.7)^{6}$

$= 0.23347 + 0.26682 + 0.20012$

$= 0.70041$

(iii) $p(x \le 8) = 1 - p(x > 8)$

$= 1 - [p(x=9) + p(x=10)]$

$= 1 - [{}^{10}C_9 (0.3)^9 (0.7)^{10-9} + {}^{10}C_{10} (0.3)^{10} (0.7)^{10-10}]$

$= 1 - [1.37781 \times 10^{-4} + 5.9049 \times 10^{-6}]$

$= 0.999856$ //

3b) An office switch board receives an average of 4 emergency calls in a 10 minutes interval. what is the probability that i) there is no call in 10 minutes interval, (ii) there is at most 2 calls in 5 minutes interval?

Ans:-

Soln, Given

In 10 minutes 4 calls

In 1 minutes $\frac{4}{10}$ calls

Now,

Average calls received by switch board is $\frac{4}{10}$ calls

$\lambda = \frac{4}{10}$ calls per minute.

$X \sim P(\lambda = \frac{2}{5})$ then pmf is given by

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0, 1, 2 \ldots \infty.$$

$$P(x) = \frac{e^{-2/5} (2/5)^x}{x!} \quad x = 0, 1, 2 \ldots \infty$$

for 10 min $\lambda = 4$

(i) $P(X=0) = \frac{e^{-4} \cdot (4)^0}{0!} = \frac{1}{e^4} = 0.018315$ //

for 5 Minute $\lambda = \frac{4}{10} \times 5 = 2$

Now,

(ii) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-2}\left[\frac{(2)^0}{0!} + \frac{(2)^1}{1!} + \frac{(2)^2}{2!}\right]$

$= 0.676676$ //

3(c) suppose that the life time (in Months) of a large number of transistors is a gamma variable with $\alpha = 6$ and $\beta = 4$. Find the probability that a transistor will last at Most 24 Months. Also, find Mean life of these transistors.

soln, Given,

Let, $X$ = Lifetime of transistor in a Months.

$\alpha = 6 \quad \beta = 4 \quad \beta = \frac{1}{\lambda}$ or, $4 = \frac{1}{\lambda} \therefore \lambda = \frac{1}{4}$

By question, $X \sim G(\alpha, \beta)$

$X \sim G(6, 4)$ The Pdf is,

$$f(x) = \frac{e^{-x/\beta}\, x^{\alpha-1}}{\beta^{\alpha}\, \Gamma\alpha} \quad x > 0$$

$$= \frac{e^{-x/4}\, x^{6-1}}{(4)^6 \times \Gamma 6}$$

$$= \frac{e^{-x/4} \times x^5}{491520} \quad \text{for } x > 0$$

$$P(x \le 24) = \int_0^{24} \frac{e^{-x/4}\, x^5}{491520}\, dx$$

$$= 0.59 //$$

Now,

$$\left[ \frac{x^5 e^{-x/4}}{(-1/4)} - \frac{5x^4 e^{-x/4}}{(-1/4)^2} + \frac{20x^3 e^{-x/4}}{(-1/4)^3} - \frac{60x^2 e^{-x/4}}{(-1/4)^4} + \frac{120x e^{-x/4}}{(-1/4)^5} - \frac{120 e^{-x/4}}{(-1/4)^6} \right]_0^{24}$$

$$= \frac{-\left[199357.352 - 491520\right]}{491520}$$

$$= \frac{292162.648}{491520}$$

$$= 0.5944 //$$

(ii) $E(x) = \int_0^\infty x f(x)\, dx$

$$= \int_0^\infty x \frac{x^5 e^{-x/4}}{491520}\, dx$$

$$= \frac{1}{491520}\left[\frac{x^6 e^{-x/4}}{(-1/4)} - \frac{6x^5 e^{-x/4}}{(-1/4)^2} + \frac{30x^4 e^{-x/4}}{(-1/4)^3} - \frac{120x^3 e^{-x/4}}{(-1/4)^4}\right.$$

$$\left. + \frac{360x^2 e^{-x/4}}{(-1/4)^5} - \frac{720x\, e^{-x/4}}{(-1/4)^6} + \frac{720 e^{-x/4}}{(-1/4)^7}\right]_0^\infty$$

$$= \frac{1}{491520}\left[(0-0+0-0+0-0+0) - (0-0+0-0+0-0-11796480)\right]$$

$$= \frac{11796480}{491520}$$

$$= 24$$

$$\therefore E(x) = 24 \quad //$$

4a) In a test administered to 1000 students, the average score was 60 and standard deviation 10. Find,

(i) The number of students exceeding a score 50.

(ii) The number of students lying between 44 and 74.

(iii) The value of score exceeded by the top 100 students.

Soln, Given,

No. of students $(N) = 1000$

Mean $(\mu) = 60$

S.d. $(\sigma) = 10$

Let, $X$ = Marks of student

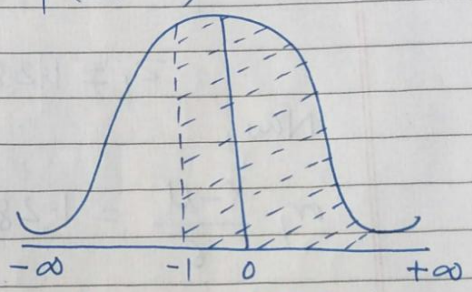
$X \sim N(60, (10)^2)$ Now,

i) $P(X \geq 50) = P\left(Z > \frac{50-60}{10}\right) = P(Z > -1)$

$= P(-1 < Z < 0) + P(0 < Z < \infty)$

$= 0.3413 + 0.5$

$= 0.8413$




Now,

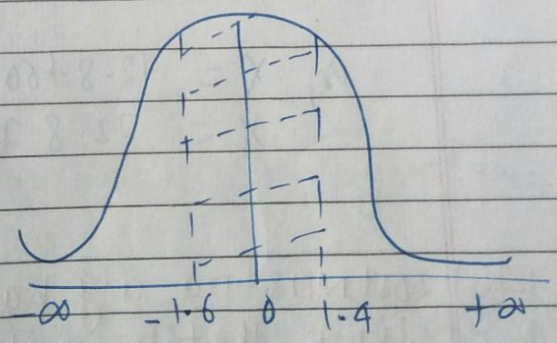
The number of student who exceeding a score 50 is,

$0.8413 \times 1000 = 841.3 \approx 841$

(ii) $P(-1.6 < Z < 1.4)$

$= 0.4452 + 0.4192$

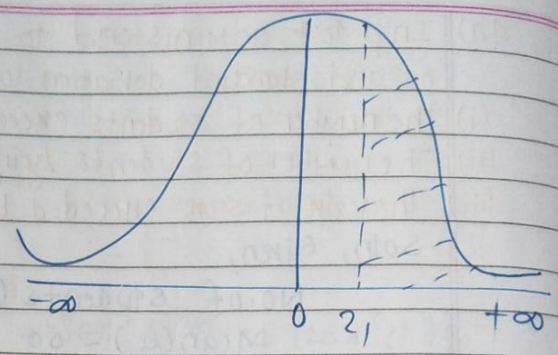
$= 0.8644$




(n) = 864.4 //

(iii)

or, $P\left(Z > \frac{X-\mu}{\sigma}\right) = 0.1$




$P\left(Z > \frac{X-60}{10}\right) = 0.1$

or, $P(Z > z_1) = 0.1$

or, $P(0 < Z < \infty) - P(0 < Z < z_1) = 0.1$

or, $0.5 - 0.1 = P(0 < Z < z_1)$

$P(0 < Z < z_1) = 0.4$

$z_1 = 1.28$

Now,

or, $\frac{X-\mu}{\sigma} = 1.28$

or, $\frac{X-60}{10} = 1.28$

or, $X = 12.8 + 60$

$X = 72.8 \approx 73$ //

4b) Costumers in a city has switch use brand 'A' next week is 0.5. that he switches to brand B is 0.2 and switches to brand 'C' is 0.3. Brand B 0.6 and C is 0.6 and Assume that.

(i) Find the probability a customer now using brand A will be using Brand B in three weeks.

(ii) If the % of customer now using brand A is 30% B is 20% C = 50%, find the percentage of customer using brand C in three weeks.

(iii) Find the perentage of customers using each brand soap in the long run.

Ans:-

Soln, Given.

State are A B C so,

The one step tpm is given by.

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.0 & 0.6 & 0.4 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} \end{array}$$

Now,

(i) After three weeks, 3-step tpm is given by $P^3$ then,

$$P^2 = P \cdot P = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.0 & 0.6 & 0.4 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.0 & 0.6 & 0.4 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.31 & 0.34 & 0.35 \\ 0.08 & 0.52 & 0.4 \\ 0.18 & 0.44 & 0.38 \end{bmatrix}$$

$$P^3 = P^2 \times P$$

$$= \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.225 & 0.406 & 0.369 \\ 0.12 & 0.488 & 0.392 \\ 0.166 & 0.452 & 0.382 \end{bmatrix} \end{array}$$

i) probability that a costumer now using brand A will be using brand B in three weaks is $P^3_{01} = 0.406$

$= 40.6\%$ //

(ii) if given, $S = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.30 & 0.20 & 0.50 \end{bmatrix} \end{array}$

In three weeks

$= SP^3$

$$= \begin{bmatrix} 0.30 & 0.20 & 0.50 \end{bmatrix} \times \begin{bmatrix} 0.225 & 0.406 & 0.369 \\ 0.12 & 0.488 & 0.392 \\ 0.166 & 0.452 & 0.382 \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.1745 & 0.4454 & 0.3801 \end{bmatrix} \end{array}$$

using brand $c = 38.01\%$ //

(iii) Let $s = [x, y\ z]$ for a long run steady state Matrix be, such that $x+y+z=1$

we know,

$sp = s$

or, $[x\ y\ z] \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.0 & 0.6 & 0.4 \\ 0.2 & 0.4 & 0.4 \end{bmatrix} = [x\ y\ z]$

$0.5x + 0.2y = x$ eqn(i) $\quad 0.3x + 0.4y + 0.4z = z$ eqn(iii)

$0.2x + 0.6y + 0.4z = y$ eqn(ii) $\quad x+y+z = 1$ eqn(iv)

on solving we get

$x = 2/11$

$y = 5/11$

$z = 4/11$ so,

The Matrix is, probability that each be is

$s = \begin{bmatrix} 2/11 & 5/11 & 4/11 \end{bmatrix}$

A = 18.18% B = 45.45% C = 36.36%

5a) A city has water supplied from a dam. It was found in the begining of summer the dam wass full. the probability it would be full in next summer is 0.9 and if the begining dam was not full the probability that it would not full in the beging is 0.4. What is the transition probability matrix? What is the probability matrix of this problem? after three year dam was full or not find of not full.

Soln, Given,

Let, A = 0 = dam was not full in the begining of the summer.

1 = dam was full in the bigining of the summer.

The tpm for the proble is

$$P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.4 & 0.6 \\ 1 & 0.1 & 0.9 \end{array}$$

Now,

Three step tpm is,

$$P^3 = P^2 \times P$$

$$P^2 = \begin{bmatrix} 0.4 & 0.6 \\ 0.1 & 0.9 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.6 \\ 0.1 & 0.9 \end{bmatrix}$$

$$= \begin{bmatrix} 0.22 & 0.78 \\ 0.13 & 0.87 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.22 & 0.78 \\ 0.13 & 0.87 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.6 \\ 0.1 & 0.9 \end{bmatrix}$$

$$P^3 = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.166 & 0.834 \\ 1 & 0.139 & 0.861 \end{array}$$

The probability that of finding it full after 3 years given that in the begining summer the dam was not full. $P^3_{01} = 0.834$ //

5b) A self serrie store employs one cashier at its counter. Nine customer arrive on an average in every 5 minutes while can serve 10 customers ins minutes Find.

(i) The average no of costumer in the queue.
(ii) Average time spent in a system.
(iii) Average time spent by a costumer in a queue.

Soln, Given,

In 5 Minutes 9 costumer arrives
In 1 Minutes $\frac{9}{5}$ costumer arrives.

arrival rate $(\lambda) = 9/5$
In 5 Minutes 10 costumer is served
In 1 Minuts $\frac{10}{5}$ costumer is served
service rate $(\mu) = \frac{10}{5}$

Traffic intensity $= \rho = \frac{\lambda}{\mu} = 0.9$

Now, This is $M/M/1 : (\infty, FIFO)$

so,

(i) $L_q = \rho L_s$

$= \rho\left(\frac{\rho}{1-\rho}\right) = \frac{\rho^2}{1-\rho} = 8.1$

(ii) $W_s = \frac{L_s}{\lambda} = \frac{9}{\frac{9}{5}} = \frac{1}{\frac{1}{5}}$ per minute $= 5$ Minute

(iii) $W_q = \frac{L_q}{\lambda} = \frac{8.1}{\frac{9}{5}} = 4.5$ Minute.

6(a) Repeated

6b) In a factory, pass three canteen. The server at each counter takes on an average 1.5 Minute although the average rate 6 per hour.

(i) Find the average time a costumer spends waiting in cafteria.

(ii) Also find the Most probable time spent in getting the service.

Ans:- soln, Given,

arrival rate ($\lambda$) = 6 per hours

1.5 Minutes takes to serve 1 customer

1 Minutes $\longrightarrow \frac{1}{1.5}$ customer

in 60 minutes $\longrightarrow \frac{1}{1.5} \times 60$

for 3 customer $(1.5 \times 3)$ so,

$\Rightarrow \frac{1}{(1.5) \times 3} \times 60 = 13.33$ per hour

$K = 3$.

This is $M/E_K/1 : (\infty, FIFO)$ so,

(i) The average time a customer spends waiting in the cafeteria is;

$$W_q = \frac{K+1}{2K} \frac{\lambda}{\mu(\mu-\lambda)} = \frac{3+1}{2 \times 3} \times \frac{6}{13.33(13.33-6)}$$

$= 2.43$ Minutes or, $\frac{9}{220}$ hours

Average time spent in getting service $= \frac{1}{\mu} = \frac{1}{13.34}$ hours

or, 4.50 Minutes.

(ii) The Most probable time Spent is $\frac{K-1}{K\mu} = \frac{3-1}{3 \times 13.33}$

$= \frac{1}{20}$ hours or, 3 Minutes //

7 a) Write short notes on :-

a) Law of Large numbers :-

Let 'x' be a r.v having two catagories then, the Bernaulli's law of large number states that,

$$P\left(\left|\frac{x}{n} - p\right| < K\right) \geq 1 - \frac{pq}{nK^2}$$ (lower bound of probability) (atleast)

and

$$P\left(\left|\frac{x}{n} - p\right| \geq K\right) \leq \frac{pq}{nK^2}$$ (upper bound of probability) (atMost)

where,

$\frac{x}{n}$ = Relative frequency of success (i.e. sample probability of success.

$p$ = Theoritical probability of success.

$q$ = Theoritical probability of failure $= 1-p$

$n$ = Sample size.

According to the law of large number, if we repeat many trials of a random phenomenon and take the average $\bar{x}$ of the outcomes, the more trials we repeat the closer this average $\bar{x}$ will get to the expected value ($\mu$).

eg:- a fair coin toss is a Bernoulli trial. when a fair coin is flipped once, the theortical probability that the outcomes will be heads is equal to $1/2$. Therefore, according to law of large number, the proportion of heads in a large number of coins flip should be roughly $1/2$.

- The Law of Large number says that the long-run relative frequency of repeated (event) independent events get closer to a single valued function.

- If an experiement is repeated again and again, the probability of an event obtained from the relative frequency approaches the actual or theoritical probability.

6) Moment generating Function.

A Moment Generating function (mgf) generates Moments of a rv. using a mgf, the Moments of a probability distribution can be found. Moreover given a set of Moments, it is possible to determine the probability function from which these Moment came and also to verify if this is the only one probability function that generates these particular Moments.

Let, 't' be an arbitary real constant with respect to the rv X. The mgf of X, denoted by $M_X(t)$ is defined as:

$$M_X(t) = E(e^{tX}).$$

$$\text{or, } M_X(t) = \sum_{k=0}^{\infty} \frac{\mu_k'}{k!} t^k$$

Where, $\mu_k'$ is the $k^{th}$ raw Moment about the origin $x=0$.

Thus, if $M_X(t)$ for any distribution is expanded by algebric or other Methods in a power series the coefficient of $\frac{t^k}{k!}$ will be $\mu_k'$ and hence the name Moment generating function. The mgf exists only if the series $1 + \frac{t}{1!}\mu_1' + \frac{t^2}{2!}\mu_2' + \cdots$ is convergent for some t.

We may also define a mgf to generate moments about the mean. The expected value of $e^{t(x-\mu)}$ which is a function of $t$ is called the moment generating function about the mean of $x$. symbolically,

$$M_{x-\mu}(t) = E\left(e^{t(x-\mu)}\right) = 1 + \frac{t}{1!}\mu_1 + \frac{t^2}{2!}\mu_2 + \cdots$$

We see that $\mu_k$, the $k^{th}$ central moment about the mean is the coefficient of $\frac{t^k}{k!}$. That is to say, the various central moments $\mu_1, \mu_2$, are obtained as the coefficients of $\frac{t}{1!}, \frac{t^2}{2!} \ldots$ if $M_{x-\mu}(t)$ exists and is convergent for some $t$.

2015 Spring

1a) In a certain group of computer personnel, 65% have insufficient knowledge of hardware, 45% have inadequate idea of software, and 70% are either one or both of the two categories. what is the percentage of people who knows software among those who have a sufficient knowledge of hardware?

Soln, Given,

Let, A = event that have insufficient Knowlage of hardwar.

B = event that have inadequate idea of software.

then,

$P(A) = 0.65 \quad P(B) = 0.45$

$P(A \cup B) = 70\% = 0.70$

P( who know software / who have sufficient Knowladge of hardware ) = ?

ie $P(\bar{B}/\bar{A}) = ?$

$$P(\bar{B}/\bar{A}) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})}$$

$$= \frac{P(\overline{A \cup B})}{P(\bar{A})} \quad \text{[By demorgans thorem}$$

$$= \frac{1 - P(A \cup B)}{1 - P(A)}$$

$$= \frac{1 - 0.70}{1 - 0.65} = \frac{6}{7}$$

$\% = 85.71\%$ //

1b) A factory has three machines A, B, and C which produce 40% 35% and 25% of the products respectively out of their output, 8%, 10% and 5% are defective respectively. An item is chosen at random from the total output and tested. What is the probability that it is good? If it is good, find the probability that is was manufactured by machine

soln, Given,

$A_1$ = event producted by A
B = event produced by B
C = event produced by C.
D = diffective

Given,

$P(A) = 0.40$ $\quad$ $P(D/A) = 0.08$
$P(B) = 0.35$ $\quad$ $P(D/B) = 0.10$
$P(C) = 0.25$ $\quad$ $P(D/C) = 5\% = 0.05$

Now,

(a) find $P(D) = 0.0795$
Good are $P(\bar{D}) = 1 - P(D) = 0.9205$ //

(i) $P(A/\bar{D}) = \dfrac{P(A) \times P(\bar{D}/A)}{P(\bar{D})} = 0.4$

(ii) $P(B/\bar{D}) = \dfrac{P(B) \times P(\bar{D}/B)}{P(\bar{D})} = 0.3422$ //

(iii) $P(C/\bar{D}) = \dfrac{P(C) \times P(\bar{D}/C)}{P(\bar{D})} = 0.013579$ //

2 a) Assume that telephone calls come to a hotel switch board at an average rate of 12 calls per minute. Determine the probability of receiving in a 30 second interval, exactly 4 calls, or at least 4 calls.

Soln, Given,

Let, 'X' = r.v that denote the number of incomming calls.

$\lambda = 6$ calls per ~~minute~~ seconds.

$\therefore$ $X \sim P(\lambda = 6)$ then, pmf is given by

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{for } x = 0 \text{ to } \infty$$

Now,

Probability of receiving 4 calls in 30 seconds is.

(i) $$P(X=4) = \frac{e^{-6}(6)^4}{4!} = 0.1339$$

(ii) $$P(X \le 4) = \frac{1}{e^6}\left[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}\right]$$

$$= 0.2849$$

(ii) $$P(X \ge 4) = 1 - P(X < 4)$$
$$= 1 - (0.0024 + 0.0148 + 0.0446 + 0.0892)$$
$$= 0.8454$$

//

2b)

(i) $\int_{x=0}^{2}\int_{y=0}^{2} kxy \, dy\, dx = 1 \quad \therefore K = \frac{1}{16}$

(ii) $E(X) = \int_0^2\int_0^4 \frac{1}{16} x^2 y \, dy\, dx = \int_0^2 \frac{1}{16} x^2 y \, dy\, dx = \frac{8}{6}$

(ii) $E(Y) = \int_0^2\int_0^4 \frac{1}{16} x^2 y^2 \, dy\, dx = \frac{32}{9}$

(ii) $V(x) = E(x^2) - \{E(x)\}^2$ and $V(y) = E(y^2) - \{E(y)\}^2$ //

(iii) $f(x) = \int_0^4 \frac{1}{16} xy \, dy = \frac{1}{16} x \times \left[\frac{y^2}{2}\right]_0^4$

$= \frac{1}{16} \times x \times \frac{4^2}{2} = \frac{1}{2} x$ for $0 < x < 2$

$f(y) = \frac{1}{16}\int_0^2 xy \, dx = \frac{1}{16}\left[\frac{x^2}{2}\right]_0^2 y = \frac{\cancel{4}\, y}{\cancel{16}_{4} \times 2} = \frac{1}{8} y$

for $0 < y < 4$

$f(x) \times f(y) = \frac{1}{2} x \times \frac{1}{8} y = \frac{1}{16} xy = f(x, y)$

$\therefore$ The function is independent //

3(a) Repeated.

3b) The heights of boys in a particular age. Follows a normal distribution with mean 150.3 cm, and standard deviation 5cm find the probability that a boy picked at random from this age group has height

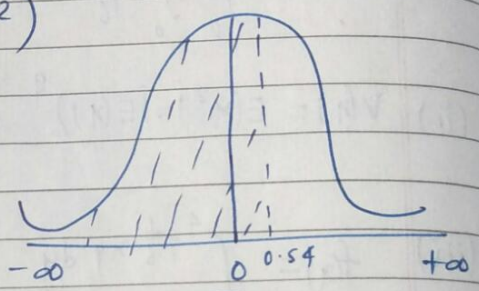
Soln, Given, Let, $x$ = height of body

Mean($\mu$) = 150.3 CM

S.d ($\sigma$) = 5CM

By question $X \sim N(150.3, 5^2)$

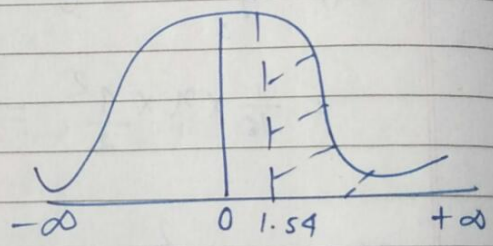
(i) $P(X < 153) = P(Z < 0.54)$




$= P(0 < Z < \infty) + P(0 < Z < 0.54)$

$= 0.7054$

(ii) $P(X > 158) = P(Z > 1.54)$

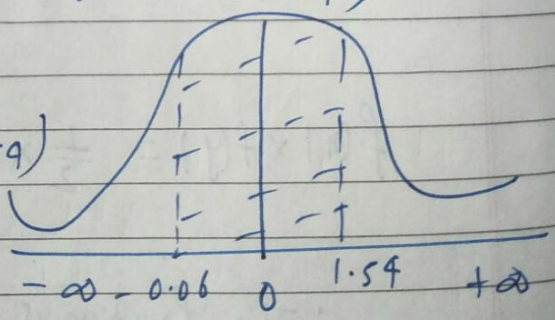



$= P(0 < Z < \infty) - P(0 < Z < 1.54)$

$= 0.5 - 0.4382$

$= 0.0618$

(iii) $P(150 < X < 158) = P(-0.06 < Z < 1.54)$




$= P(0 < Z < 0.06) + P(0 < Z < 1.54)$

$= 0.0239 + 0.4382$

$= 0.4621$ //

4a)

5

A railway reservation center has a single booking counter. During the rush hour, customers arrive at the rate of 30 customers per hour. The average number of customers that can be attended by the booking attendant is 40 per hour, Assuming that the arrivals are poisson and the service time is exponentially distributed. Find.

soln, Given,

arrival rate $(\lambda) = 30$ customer per hour

service rate $(\mu) = 40$ per hour.

$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = 0.75$ Now, M/M/1 : $(\infty, FIFO)$

(i) Probability that operator is idle $(P_0) = 1-\rho = 0.25$

(ii) Average no of costumer in a system $(L_s) = \frac{\rho}{1-\rho} = 3$

(iii) Average time for costumer in a system $(W_s) = \frac{L_s}{\lambda} = \frac{3}{30} = 0.1$

(iv) Average no of costumer in a queue $(L_q) = \rho L_s = 2.25$

4b) A petrol pump station has 4 pumps. The service time follows an exponential distribution with a mean of 6 minutes, and vehicles arrive for serive in a poisson process at the rate of 30 per hour.

soln, Given,

$S = 4$ pumps. arrival rate $(\lambda) = 30$ per hour

In 6 min 1 is served

In 60 min $\frac{1}{60} \times 60$ is served $(\mu) = 60$ per hour.

M/M/S : $(\infty, FIFO)$

$$\rho = \frac{\lambda}{\mu s} = \frac{30}{40 \times 4} = 0.1875.$$

(i) That an arrival has to wait for the service $(P_w) =$

$$\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} P_0$$

Now,

$$P_0 = \left[\sum_{n=0}^{3} \frac{(4 \times 0.1875)^n}{n!} + \frac{1}{4!} \frac{(4 \times 0.1875)^s}{1 - 0.1875}\right]^{-1}$$

$$= 1664/4577 \;//$$

$$= \frac{1}{4!} \times \left(\frac{30}{60}\right)^4 \times \frac{4 \times 60}{(4 \times 60 - 30)} \times \frac{1664}{4577}$$

$$= \frac{1}{384} \times \frac{240}{210} \times \frac{1664}{4577}$$

$$= 0.0010820146 \;//$$

(ii) Average waiting time in a queue $(W_q) = \frac{L_q}{\lambda}$ or,

$$= \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu}{(s\mu - \lambda)^2}\right] P_0$$

$$= \left[\frac{1}{3!} \times (0.5)^4 \frac{40}{(4 \times 40 - 30)^2}\right] \times 0.001082 =$$

$$= 2.67 \times 10^{-8}$$

(iii) idel is $P_0 = 0.001082$ 5a) repeated

5b) There are two food stores A and B in a certain area. An investigation of the preferences of costomers revealed that with probability 0.15 a costomer of store A one week would go over to store B next week and with probability 0.10 that a customer of store B would go over to store A. If initially, 60 % of the ~~co~~ customers were with store A, and remaining with B, what percentage of customer will remain with A and B, after 4 weeks

Soln, Let 0 = Store A, 1 = Store B.

By question 1-step tpM is $P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.85 & 0.15 \\ 1 & 0.10 & 0.90 \end{array}$ $p = 0.15$, $q = 0.10$

then,

After 4-weeks 4th step tpM is,

$$P^4 = \frac{1}{p+q}\begin{bmatrix} q & p \\ q & p \end{bmatrix} + \frac{(1-q-p)^n}{p+q}\begin{bmatrix} p & -p \\ -q & q \end{bmatrix}$$

$$P^4 = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.59 & 0.41 \\ 1 & 0.27 & 0.73 \end{array}$$

Now,

$$S_0 = \begin{array}{cc} 0 & 1 \\ \hline 0.6 & 0.4 \end{array}$$

after 4 weeks

$$S_0P^4 = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}\begin{bmatrix} 0.59 & 0.41 \\ 0.27 & 0.71 \end{bmatrix} = \begin{bmatrix} 0.46 & 0.54 \end{bmatrix}$$

At store A 46% and at B = 54%.

6a) A barber shop has two barbers and three chairs for customers. Assume that the customers arrive in a poisson fashion at a rate of 5 per hour and each barber serves a customer, the service time being exponentially distributed with a mean of 15 minutes Find

Soln, Given,

$S = 2$, $N = 3$ $\lambda = \frac{5}{60}$, $\mu = \frac{1}{15}$

$\rho = 5/8$ Now, M/M/S : (N, FIFO)

(i) $$P_0 = \left[\sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \sum_{n=s}^{N} \frac{1}{s!\, s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$= 0.2841$

(ii) Expected number of customer in the shop.

$$L_s = L_q + s - P_0 \sum_{n=0}^{s-1} \frac{s-n}{n!} \left(\frac{\lambda}{\mu}\right)^n$$

$$L_q = \frac{(s\rho)^s}{s!\,(1-\rho)^2} \left[ 1 - \rho^{N-s+1} - (1-\rho)(N-s+1)\rho^{N-s} \right] \rho P_0$$

$$L_q = 0.13872$$

$$L_s = 1.226$$

6b) Determine steady state probabilities of a homogeneous markov chain given by the following transition probability matrix:

$$\begin{matrix} 0.1 & 0.6 & 0.3 \\ 0.5 & 0.1 & 0.4 \\ 0.1 & 0.2 & 0.7 \end{matrix}$$

Ans Given,

$$\begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.5 & 0.1 & 0.4 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

Let, steady state Matrix be

$$S = [x \;\; y \;\; z] \quad \therefore \Rightarrow x+y+z = 1$$

and $SP = S$

$$[x \;\; y \;\; z] \begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.5 & 0.1 & 0.4 \\ 0.1 & 0.2 & 0.7 \end{bmatrix} = [x \;\; y \;\; z]$$

$$\left[\begin{array}{l} 0.1x + 0.5y + 0.1z = x \\ 0.6x + 0.1y + 0.2z = y \\ 0.3x + 0.4y + 0.7z = z \\ x + y + z = 1 \end{array}\right]$$

//

7(a) repeated (b) repeated

(c) stochastic process :-

- A process which involves time is called a stochastic process. Mathematical Model of system which vary in time in a random manner is called stochastic process.

- A stochastic process is a family of given probability space index by parameter t. when t, vary over and index set T. The set of all possible values of all individual r.v x(t) are called state space and values of x(t) are called states.

- A stochastic or random process can be defined as a collection of r.v that is indexed by some mathematical model set, Meaning that each random variable of a stochastic process is uniquely associated with an element in the set.

- A stochastic process is a probabillistic (non-deterministic) system that evolves with time via random changes to a collection of variables. eg:- simple random walk.

Some of the type of stochastic process are

(*) Discrete time, discrete state space stochastic process (DTDS).
- such a process involves discrete time scale. We can see the state space is discrete.

eg:- average annual temperature of a place in various years, number of persons in intervals of 5 minutes etc.

(*) Discrete time, continous state space stochastic process (DTCS)
- such a process involves continous rvs in discrete time scale and is referred to as a continous stochastic process with a discrete parameter or a stochastic sequence.

eg:- volume of water in a dam at various points of time, stock of petrol in a bunker every morning at a fueling stations, etc.

(*) continous time, discrete state space stochastic process (CTDS)
- such a process that involves discrete rvs in a continous time scale. The process is called a discrete valued continous parameter stochastic process.

eg:- would be a number of person waiting at a reve for service at any time of the day etc.

(*) continous time, continous state space stochastic process (CTCS)
- such a process involves continous rvs in continous time scale. The process is also simply referred to as a continous stochastic process.

eg:- volume of water in a reservior observed over a period of time, movement of particles suspended in a fluid under the random impact of neighbouring particles etc.

2016 Fall

1a) A binary communication channel carries data as one of two types of signals denoted by 0 and 1. Owing to the noise. For a given channel, assume a probability of 0.94 that a transmitted '0' is correctly received as '0' and a probability that a transmitted 1 is received as a 1. Further assume a probability of 0.45 of transmitting a 0. If a signal is sent, determine:

(i) Probability that a 1 is received.

(ii) Probability that a 0 is received.

(iii) Probability that a 1 was transmitted, given that 1 was received.

(iv) Probability that a 0 was transmitted, given that 0 was received.

(v) Probability of error.

Ans:-

Soln,

Let, $R_0$ = event that 0 is received.

$R_1$ = event that 1 is received.

$T_0$ = event that 0 is transmitted.

$T_1$ = event that 1 is transmitted.

By question,

$p(R_0/T_0) = 0.94$ $\quad$ $p(R_1/T_0) = 0.06$

$p(R_1/T_1) = 0.91$ $\quad$ $p(R_0/T_1) = 0.09$

$p(T_0) = 0.45$

$p(T_1) = 0.55$

(i) Probability that a 1 is received $P(R_1)$ is given by :-

$$P(R_1) = P(T_0 \cap R_1) + P(T_1 \cap R_1)$$
$$= P(T_0) \cdot P(R_1/T_0) + P(T_1) \cdot P(R_1/T_1)$$
$$= 0.45 \times 0.06 + 0.55 \times 0.91$$
$$= 0.5275$$

(ii) Probability that a '0' is received $P(R_0)$ is given by :-

$$P(R_0) = 1 - P(R_1)$$
$$= 1 - 0.5275$$
$$= 0.4725$$

(iii) Probability that a 1 was transmitted, given that 1 was received.

$$P(T_1/R_1) = \frac{P(T_1) \cdot P(R_1/T_1)}{P(R_1)} = \frac{0.55 \times 0.91}{0.5275} = 0.94881$$

(iv) Probability that a 0 was transmitted, given that '0' was transmitted to be received.

$$P(T_0/R_0) = \frac{P(T_0) \cdot P(R_0/T_0)}{P(R_0)} = \frac{0.45 \times 0.94}{0.4725} = 0.89523.$$

(v) Probability of error.

$$= P(R_0 \cap T_1) + P(R_1 \cap T_0)$$
$$= P(R_0) \times P(T_1/R_0) + P(R_1) \times P(T_0/R_1)$$
$$= 0.4725 \times 0.10477 + 0.5275 \times 0.05119$$
$$= 0.0765 //$$

b) The joint probability density function of two random variables x and y are given by :-

$$f(x,y) = \frac{9(1+x+y)}{2(1+x)^4(1+y)^4} \quad \text{for } x \geq 0, y \geq 0$$

Find the marginal distribution of x and y and check the independence.

Ans :-

Soln, Given,

$$f(x,y) = \frac{9(1+x+y)}{2(1+x)^4(1+y)^4} \quad \text{then}$$

To find the marginal probability,

$$f(x) = \int_y f(x,y)\,dy$$

$$= \int_0^\infty \frac{9}{2} \frac{(1+x+y)}{(1+x)^4(1+y)^4}\,dy$$

Let, $(1+y)=a$ then $dy = da$

$y=0$ then, $a=1$

$y=\infty$ then, $a=\infty$

$$= \int_0^\infty \frac{9}{2} \frac{(a+x)}{(1+x)^4\, a^4}\,dy$$

$$= \frac{9}{2} \times \frac{1}{(1+x)^4} \times \frac{(3+2x)}{6}$$

$$= \frac{9}{12} \frac{(3+2x)}{(1+x)^4} \quad \text{for } x \geq 0$$

$$f(y) = \int_0^\infty \frac{9}{2} \frac{(1+x+y)}{(1+x)^4 (1+y)^4} \, dx$$

Similiarly

$$f(y) = \frac{9}{12} \frac{(3+2y)}{(1+y)^4}$$

Now,

To check the independence

$= f(x) \cdot f(y)$

$$= \frac{9}{12} \frac{(3+2y)}{(1+y)^4} \times \frac{9}{12} \frac{(3+2x)}{(1+x)^4}$$

$\neq f(x,y)$

$\therefore f(x) \times f(y) \neq f(x,y)$ so, this is not independent.

2(a) The time required to repair a machine is exponentially distributed with parameter 1/2. What is the probability that a repair time takes atleast 10 hours given that its duration exceeds 8 hours? What is the probability that a repair time exceeds 2 hours?

Ans:-

Soln, Given, Let $X$ = r.v that denotes the time to repair a machine.

$\theta$ = parameter $(\lambda) = 1/2$ so, pdf of $x$ is given by

$$f(x) = \theta e^{-\theta x} \quad ; \quad x > 0$$

$$f(x) = \frac{1}{2} e^{-x/2} \quad ; \quad x > 0$$

So, $X \sim Exp(1/2)$

Probability that a repair time exceeds 2 hours is given by

$$p(X>2) = \int_2^{\infty} \frac{1}{2} e^{-x/2}\, dx$$

$$= \frac{1}{2}(-2)\left[e^{-x/2}\right]_2^{\infty}$$

$$= -\left[0 - 0.367879\right]$$

$$= 0.367879$$

$$p(X \geq 10 / X>9) = p(X \geq 9+1 / X>9) = p(X>1)$$

[∵ By a Memoryless property]

So,

$$p(X>1) = \int_1^{\infty} \frac{1}{2} e^{-x/2}\, dx$$

$$= -\left[e^{-x/2}\right]_1^{\infty} = 0.606530$$

b) The life time in hours of a certain electrical equipment has the normal distribution with Mean 80 hours and standard deviation 16 hours.

i) What is the probability that equipment lasts atleast 100 hours?

ii) If the equipment has already lasted 88 hours, what is the conditional probability that it will last atleast another 12 hours.

Ans:-

Soln, Given,

Mean ($\mu$) = 80

S.d ($\sigma$) = 16

Let, X = rv that denotes the lifetime of a certain electrical equipment.

$$X \sim N(80, 16^2)$$

Then,

(i) probability that equipment lasts atleast 100 hours.

$$P(X \geq 100) = P\left(\frac{X-\mu}{\sigma} > \frac{X-\mu}{\sigma}\right) = P\left(Z > \frac{100-80}{16}\right)$$

$$= P(Z > 1.25)$$

$$= P(0 < Z < \infty) - P(0 < Z < 1.25)$$

$$= 0.5 - 0.3944$$

$$= 0.1056$$

(ii) $$P(X \geq 12 / X > 88) = \frac{P(X \geq 12 \cap X > 88)}{P(X > 88)}$$

$$= \frac{P(X > 88)}{P(X > 88)}$$

$$= \frac{P(Z > 0.5)}{P(Z > 0.5)}$$

$$= \frac{P(0 < Z < \infty) - P(0 < Z < 0.5)}{P(0 < Z < \infty) - P(0 < Z < 0.5)}$$

$$= \frac{0.5 - 0.1915}{0.5 - 0.1915} = 1$$ //

3a) How many times would you have to roll a fair dice in order to be at least 99% sure that the relative frequency of having a six come up is within 0.02 of the theoritical probability 1/6.

Ans:-

Soln, Given, By question

Theoritical probability of success (p) = 1/6 $\quad q = 5/6$

Now,

$$P\left(\left|\frac{x}{n} - P\right| \leq 0.02\right) \geq 0.99 \quad eqn(i)$$

Lower bound probability of BLLN is given by:-

$$P\left(\left|\frac{x}{n} - P\right| \leq K\right) \geq 1 - \frac{1}{nK^2} \times Pq$$

comparing it with eqn(i) we get,

$$\text{or, } 1 - \frac{Pq}{nK^2} = 0.99$$

$$\text{or, } 0.01 = \frac{Pq}{nK^2}$$

$$\text{or, } n = \frac{\frac{1}{6} \times \frac{5}{6}}{0.01 \times (0.02)^2}$$

$$\therefore n = 34722.2 \approx 347223$$ //

b) A discrete r.v takes values $-1, 0, 1$ with probability $1/8, 3/4, 1/8$ respectively. Evaluate $p|x-E(x)| \geq 1$ and compare it with probability of chebyshev's inequality.

Ans:-

Soln,

The probability distribution of x is,

| X | -1 | 0 | 1 |
|---|---|---|---|
| $p(x=x)$ | 1/8 | 3/4 | 1/8 |

Now,

$$E(x) = \sum_{x=-1}^{1} x\, p(x) = 0 \qquad E(x^2) = \sum_{x=-1}^{1} x^2 p(x) = \frac{1}{4}$$

so, $V(x) = E(x^2) - \{E(x)\}^2$

$= \frac{1}{4}$ $\quad \therefore V(x) = \sigma^2$ so,

$\sigma = \frac{1}{2}$

Now,

$$\begin{aligned} p(|x-E(x)| \geq 1) &= p(|x-0| \geq 1) \\ &= p(x \geq 1) \\ &= 1 - p(|x|<1) \\ &= 1 - p(-1<x<1) \\ &= 1 - p(x=0) \\ &= 1 - 3/4 \\ &= 1/4 \end{aligned}$$

//

comparing it with upper band of probability

$$P(|X - E(X)| \geq K\sigma) \leq \frac{1}{K^2}$$

Now,

$$P\left(|X - E(X)| \geq \frac{1}{2}K\right) \leq \frac{1}{K^2}$$

K must be 2 so,

$$P(|X - E(X)| \geq 1) \leq \frac{1}{4} \quad //$$

4(a) Find the characterstic function of uniform distribution.

$$f(x) = \begin{cases} \left(\frac{1}{b-a}\right) & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$$

Also

Fid Mean and variance

Ans:-

Soln, Given,

$$f(x) = \frac{1}{b-a} \quad \text{if } a < x < b \quad \text{so,}$$

The characterstics function is given as :-

$$\phi_X(t) = E(e^{itX}) = \int_a^b e^{itx} \frac{1}{b-a} \, dx$$

$$= \frac{1}{b-a} \left[\frac{e^{itx}}{it}\right]_a^b$$

$$= \frac{1}{it(b-a)} \left(e^{ibt} - e^{iat}\right)$$

$$= \frac{e^{ibt} - e^{iat}}{it(b-a)}$$

$\therefore$ The characterstics function are :-

$$\phi_x(t) = E(e^{itx}) = \begin{cases} \frac{e^{ibt} - e^{iat}}{it(b-a)} & t \neq 0 \\ 1 & t = 0 \end{cases}$$

Now,

$$\text{Mean } E(x) = \int_a^b x\, f(x)\, dx$$

$$= \int_a^b x \frac{1}{(b-a)}\, dx$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{2(b-a)} \left[x^2\right]_a^b$$

$$= \frac{(b^2 - a^2)}{2(b-a)} = \frac{(b+a)\cancel{(b-a)}}{2\cancel{(b-a)}}$$

$$= \frac{b+a}{2} \quad //$$

$$E(x^2) = \int_a^b x^2 \frac{1}{(b-a)}\, dx$$

$$= \frac{1}{(b-a)} \left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{3(b-a)} (b^3 - a^3)$$

$$= \frac{(b-a)(b^2+ab+a^2)}{3(b-a)}$$

$$= \frac{b^2+ab+a^2}{3}$$

Variance $V(x) = E(x^2) - \{E(x)\}^2$

$$= \frac{b^2+ab+a^2}{3} - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{(4b^2+4ab+4a^2-3b^2-6ab-3a^2)}{12}$$

$$= \frac{a^2-2ab+b^2}{12}$$

$$= \frac{(a-b)^2}{12} \quad //$$

5a) Traffic to a Message switching centre for one of the outgoing communication lines arrives in a random pattern at an average rate of 240 Message per Minute. The line has transmission rate of 800 characters per seconds. The message length distribution is approximately exponential with an average length of 176 characters. Calculate and assuming that a very large number of message buffers are provided.

(i) Average number of Message in the system.

(ii) Average number of Messages in the queue waiting to be transmitted.

(iii) Average time a Message waits for transmission.

(iv) probability that 10 or More Message are waiting to be transmitted.

Ans:-

Arrival rate ($\lambda$) = 240 Message per Minute

$$\text{service rate } (\mu) = \frac{\text{Average Message length}}{\text{line speed}}$$

$$= \frac{176}{800}$$

$$= 0.22 \text{ sec/Message}$$

$$= \frac{1}{0.22} \text{ Message/sec}$$

$$= 4.55 \text{ Message/sec}$$

In one second 4.55 Message is transmitted

In 60 seconds 4.55×60 Message is transmitted

∴ In 1 Minute 273 Message is transmitted.

Service rate ($\mu$) = 273 Message per Minute.

This is M/M/1:($\infty$, FIFO)

$\rho = \frac{\lambda}{\mu} = \frac{240}{273} = 0.88$

(i) $L_s = \frac{\rho}{1-\rho} = \frac{0.88}{1-0.88} = 7.33$ Minutes.

(ii) $L_q = \rho L_s = 0.88 \times 7.33$
$= 6.4504$ (Message)

(iii) $W_s = \frac{L_s}{\lambda} = \frac{7.33}{240} = 0.030541$ minute

(iv) $\rho^n = (0.88)^{10} = 0.2785$ //

(v) 90th percentile waiting time in a queue.

$$\pi_q(90) = W_s \ln[(100-90)\rho]$$
$$= W_s \ln(10\rho)$$
$$= 0.066 \text{ (Minute)}$$
$$= 3.98 \text{ (Sec)}$$

b) patient arrives for a Physical examination according to a poission process at the rate of one per hour. The physical examination requires the stage of three with service time of 15 Minutes. Determine the average number of delayed patient i.e. Average length of queue ($L_q$) for the system. Also Find expected time a patient spend in physical examination centre.

Ans:-

Soln, Given,

$K = 3$

arrival rate $(\lambda) = 1$ per hour $= \frac{1}{60}$ per minute

$\mu = \frac{1}{(15+15+15)} = \frac{1}{45}$ per minute $\quad \rho = \frac{\lambda}{\mu} = \frac{3}{4}$,

This is $M/E_K/1$ :

Now,

(i) Average number of delayed patients and average length of queue $(L_q) = \frac{K+1}{2K} \frac{\lambda^2}{\mu(\mu-\lambda)}$

$$= \frac{3+1}{2\times 3} \times \frac{\left(\frac{1}{60}\right)^2}{\frac{1}{45}\left(\frac{1}{45} - \frac{1}{60}\right)}$$

$$= \frac{4}{6} \times \frac{8100}{3600}$$

$$= 1.5 \,//$$

(ii) $W_s = \frac{K+1}{2K} \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu}$

$$= \frac{3+1}{6} \times \frac{1/60}{\frac{1}{8100}} + \frac{1}{\frac{1}{45}}$$

$$= 135 \text{ minutes}$$

$$= 2.25 \text{ hours} \,//$$

2016 (spring)

1a) repeated

b) Show that for 1000000 flips of a balanced coin the probability is atleast 0.99 that the portion of heads will fall between 0.495 and 0.505.

Ans:-

Soln,

$n = 1000000$

$p = \frac{1}{2}$, $q = \frac{1}{2}$ so,

By BLLN; Lower bound of probability is given by

$$P\left(\left|\frac{x}{n} - p\right| < K\right) \geq 1 - \frac{pq}{nK^2}$$

By question.

we have to show this, so,

$$P\left(0.495 < \frac{x}{n} < 0.505\right) \geq 0.99$$

or, $$P\left(\left|\frac{x}{n} - p\right| < K\right) \geq 1 - \frac{pq}{nK^2}$$

or, $$P\left(-K < \frac{x}{n} - \frac{1}{2} < K\right) \geq 1 - \frac{pq}{nK^2}$$

or, $$P\left(\frac{1}{2} - K < \frac{x}{n} < \frac{1}{2} + K\right) \geq 1 - \frac{pq}{nK^2}$$

$\frac{1}{2} - K = 0.495$ $\quad$ $\frac{1}{2} + K = 0.505$

$\therefore K = \frac{1}{200}$

$$P\left(\frac{1}{2}-\frac{1}{200} < \frac{x}{n} < \frac{1}{2}+\frac{1}{200}\right) \geq 1-\frac{pq}{nk^2}$$

$$P\left(0.495 < \frac{x}{n} < 0.505\right) \geq 1-\frac{\frac{1}{2}\cdot\frac{1}{2}}{1000000 \times \left(\frac{1}{200}\right)^2}$$

$$P\left(0.495 < \frac{x}{n} < 0.505\right) \geq 1-0.01$$

$$\therefore P\left(0.495 < \frac{x}{n} < 0.505\right) \geq 0.99$$ Hence proved //

2a) Among the 24 invoices prepared by billing department, 4 contain errors while the others do not. If we randomly check 2 of three invoices, what is the probability that
(i) both will contain errors.
(ii) neither will contain errors. iii) Atleast one contain error.

Ans:-

Soln

Total invoices (T) = 24

4 contain error then

(24-4) = 20 doesn't contain error.

first first draw probability of error is $\frac{4}{24}$

for second draw probability of error is $\frac{3}{23}$

we have randomly checked 2 of this invoices so,

(i) $P(\text{both will contain errors}) = \frac{4}{24} \times \frac{3}{23} = \frac{1}{46}$

$= 0.0217$ //

for first draw probability of no error is $\frac{20}{24}$

and for second draw probability of no error is $= \frac{19}{(24-1)}$

so,

(ii) $P(\text{Neigther will contain errors}) = \frac{20}{24} \times \frac{19}{(24-1)}$

$= \frac{95}{138}$

$= 0.6884$

(iii) $P(\text{atleast one contain error}) = 1 - P(\text{No error})$

$= 1 - \frac{95}{138}$

$= \frac{43}{138}$

$= 0.311594$ //

2b) repeated

3a) A r.v has normal distribution with $\mu = 62.4$. Find it's standard deviation if the probability is 0.20 that it will take on a value greater than 79.2.

Soln,

$\mu = 62.4$

$S.d(\sigma) = ?$ then

By question

$P(X > 79.2) = 0.20$

$P\left(Z > \frac{79.2 - 62.4}{\sigma}\right) = 0.20$

$P(Z > Z_1) = 0.20$

$= P(0<z<\infty) - P(0<z<z_1) = 0.20$

or, $0.5 - 0.20 = P(0<z<z_1)$

$\therefore P(0<z<z_1) = 0.3$

$\therefore z_1 = 0.84$

Now, Given,

$$z_1 = \frac{79.2 - 62.4}{\sigma}$$

$$0.84 = \frac{16.8}{\sigma}$$

$\therefore sd(\sigma) = 20$ //

3b) repeated.

4a) If 2 independent random samples of size $n_1 = 9$ and $n_2 = 16$ are taken from a normal population, what is the probability that the variance of the second sample will be atleast 2.4 times the variance of the 1st sample.

5b) Repeated

5 a) Telephone calls arrive at a booth according to poission distribution with Mean time of 9 Minutes between two consecutive calls. The length of a call is assumed to be enponentially distributed with the Mean of 3 Minutes. calculate

i) Probability that a person will have to wait to Make a call is.

(ii) The average queue length.

Ans :-

soln, Given, In 9 Minutes 1 call is arrind

In 1 Minute $\frac{1}{9}$ call is arrived

$\therefore$ arrival rate $(\lambda) = \frac{1}{9}$ call per Minute

again,

In 3 Minutes 1 call is arrived to serve

In 1 Minute $\frac{1}{3}$ calls are served so,

Service rate $(\mu) = \frac{1}{3}$ calls per Minute

$\therefore$ This is M/M/1 $(\infty, FIFO)$ so,

Traffic intensity $(\rho) = \frac{\lambda}{\mu} = \frac{3}{9} = \frac{1}{3}$

Now,

(i) probability that a person have to wait to Make a call is $(P_W)$

$\therefore P_W = \rho = 0.33$

(ii) The average queue length $(L_q) = L_q = \rho L_s = \rho \times \frac{\rho}{1-\rho}$

$= \frac{1}{3} \times \frac{1}{3(1-\frac{1}{3})} = \frac{1}{6} = 0.167$ //

5b) The arrival of large jobs at a computer center forms a poission process with rate 2 per hour. The service time of such jobs are exponentially distributed with a mean of 20 minutes. only 4 large jobs can be accomodated in a system of a time.

i) Determine the probability that a large job will be turned away because of lack of storage.

ii) What is the probability that the arriving costumer has to wait?

Ans:-

Soln, Given,

arrival rate $(\lambda) = 2$ per hour

In 20 minutes 1 is served

In 1 minute $\frac{1}{20}$ is served

In 60 minute $\frac{1}{20} \times 60$ is served.

In 1 hour 3 is served.

service rate $(\mu) = 3$ per hour.

Traffic inensity $(\rho) = \frac{\lambda}{\mu} = \frac{2}{3}$

$N = 4$

(i) probability that a large job will be turned away

$P_4 = ?$

$P_W = P_0 \rho^n$

Now,

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = 0.3838$$

$P_4 = P_0 \rho^4$
$= 0.3838 \left(\frac{2}{3}\right)^4$
$= 0.07$

$P_W = P_0 \rho^4$
$= 0.07$

6b) Consider a casacade of binary communication channel as given by transition probability matrix.

$$P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

compute the n step transition probability matrix.

Ans:-

Given, $P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$ $\begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$

$p = 1/4 \quad q = 1/2$

The nth step tpm is given as:-

$$= \frac{1}{p+q}\begin{bmatrix} q & p \\ q & p \end{bmatrix} + \frac{(1-q-p)^n}{p+q}\begin{bmatrix} p & -p \\ -q & q \end{bmatrix}$$

$$= \frac{1}{\frac{1}{4}+\frac{1}{2}}\begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} + \frac{\left(1-\frac{1}{2}-\frac{1}{4}\right)^n}{\frac{1}{4}+\frac{1}{2}}\begin{bmatrix} 1/4 & -1/4 \\ -1/2 & +1/2 \end{bmatrix}$$

$$= \frac{4}{3}\begin{bmatrix} 1/2 & 1/4 \\ 1/2 & 1/4 \end{bmatrix} + \frac{4}{3}(0.25)^n\begin{bmatrix} 1/4 & -1/4 \\ -1/2 & 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 2/3 & 1/3 \\ 2/3 & 1/3 \end{bmatrix} + (0.25)^n\begin{bmatrix} 1/3 & -1/3 \\ -2/3 & 2/3 \end{bmatrix}$$

2017 spring

1a) A group of finding has five printes in a sample consists of three good, labeled $d_1$ and two defective, labeled $d_1$ and $d_2$. If three printers are selected at random from this group, what is the probability of the event E = "Two of these three selected printers are good"?

Ans:-

soln, Given.

There are 5 printers and 3 are selected

3 are select from 5 printers ie $^5C_3 = 10$

Good are $g_1, g_2, g_3$ deffective are $d_1$ and $d_2$

Probability that Two of 3 selected printers are good is given by:- $\frac{^3C_2 \cdot {^2C_1}}{^5C_3} = \frac{6}{10} = 0.6$ //

b) The joint probability density function of two dimensional r.v (x, y) is given by:-

$f(x) = 4xy \qquad 0 \le x \le 1, \; 0 \le y \le 1$

(i) Find the Marginal PDF of x and y.

(ii) Find the conditional probability x given y=y

(iii) Examine whether x and y are indpendence.

Soln, Given,

(i) Marginal of x and y are

$$f(x) = \int_0^1 4xy\,dy$$

$$= 4x\left[\frac{y^2}{2}\right]_0^1$$

$$= 4x\,\frac{1}{2}$$

$$= 2x \quad \text{for } 0 \le x \le 1$$

$$f(y) = \int_0^1 4xy\,dx$$

$$= 4y\left[\frac{x^2}{2}\right]_0^1$$

$$= 2y \quad \text{for } 0 \le y \le 1$$

(ii) conditional probability x given y=y

Probability,

$$f\left(x/y=y\right) = \frac{f(x,y)}{f(y)} = \frac{4xy}{2y} = 2x \quad \text{for } 0 \le x \le 1$$

(iii) To check the independence.

$$f(x) \times f(y) = 2x \times 2y = 4xy = f(x,y)$$

$f(x)\cdot f(y) = f(x,y)$ is independent.

2a) A r.v X has the following Probability distribution.

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| P(X) | a | a/2 | a/3 | a/4 | a/5 | a/6 |

Find the value of

i) value of a. ii) $P(X<3)$ iii) $(P X \geq 3)$ iv) $P(0<X<5)$

Ans:-

(i) The value of a

or, $\sum_{x=0}^{5} P(x) = 1$

or, $P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = 1$

or, $a + \frac{a}{2} + \frac{a}{3} + \frac{a}{4} + \frac{a}{5} + \frac{a}{6} = 1$

or, $a\frac{49}{20} = 1$ $\therefore a = \frac{20}{49}$

$\therefore$ Probability distribution be like

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| P(x) | $\frac{20}{49}$ | $\frac{10}{49}$ | $\frac{20}{147}$ | $\frac{5}{49}$ | $\frac{4}{49}$ | $\frac{10}{147}$ |

(ii) $P(X<3) = P(0) + P(1) + P(2)$

$= \frac{20}{49} + \frac{10}{49} + \frac{20}{147}$

$= \frac{110}{147}$

(iii) $P(X \geq 3) = 1 - P(X<3)$

$= 1 - \frac{110}{147}$

$= \frac{37}{147}$ //

(iv) $P(0<x<5) = P(1)+P(2)+P(3)+P(4)$ or $1-[P(0)+P(5)]$

$$= \frac{10}{49} + \frac{20}{147} + \frac{5}{49} + \frac{4}{49}$$

$$= \frac{11}{21} //$$

b) The probability that the noise level of a band pass amplifier will exceed 2 db is 0.1. Find the probability that among 100 such amplifiers the noise level of

(i) one will exceed 2dB;
(ii) At Most two will exceed 2 dB;
(iii) Two or More will exceed 2 dB;

Ans :-

Soln, Given,

No of trails $(n) = 100$

Let, $x$ be a r.v that denotes the noise level of band pass amplifier will exceed 2dB

then,

$p = 0.1 \qquad q = 1-p = 1-0.1 = 0.9$

Now, $\lambda = np = 100 \times 0.1 = 10$

Here $x \sim P(\lambda = 10)$ so, $pmf = P(x=x) = \frac{e^{-10}(10)^x}{x!}$ $x = 0,1,\infty$

(i) $P(x \geq 1) = 1 - P(x<1)$

$= 1 - P(x=0)$

$= 1 - \frac{e^{-10}(10)^0}{0!}$

$= 0.99 //$

$P(x=1)$ is only correct so,

$P(x=1) = \frac{e^{-10}(10)^1}{1!}$

$= 0.00045399 //$

(ii) $p(x \leq 2) = p(x=0) + p(x=1) + p(x=2)$

$$= e^{-10}\left[\frac{(10)^0}{0!} + \frac{(10)^1}{1!} + \frac{(10)^2}{2!}\right]$$

$$= \frac{1}{e^{10}}\left[1 + 10 + 50\right]$$

$$= 0.0027693$$

(iii) $p(x \geq 2) = 1 - p(x < 2)$

$$= 1 - \left[p(x=0) + p(x=1)\right]$$

$$= 1 - \left[e^{-10}\left(\frac{10^0}{0!} + \frac{10^1}{1!}\right)\right.$$

$$= 0.999500 \;//$$

3 a) A Man leaving for work every Morning is equally likely to step out of his door at any time between 8:00 to 8:12. If he works for 220 days each year, compute the probability that the average time he leaves for works lies between 8:04 to 8:08.

Ans:-

Soln, Given.

at 8:00 $\rightarrow a=0$ at 8:12 $\rightarrow b=12$

Here $a=0$, $b=12$ Minutes This follows uniform distribution

i.e. $f(x) = \frac{1}{b-a} = \frac{1}{12-0} = \frac{1}{12}$; $0<x<12$

Now;

The average time he leaves for works lies between 8:04 to 8:08 probability is given by,

$$P(4<X<8) = \int_4^8 f(x)\,dx$$

$$= \int_4^8 \frac{1}{12}\,dx$$

$$= \frac{1}{12}[x]_4^8$$

$$= \frac{4}{12} = \frac{1}{3}$$

No of days out of 220 days $= \frac{1}{3} \times 220$

$= 73.33$ days //

3 b)

3b) At a rail station, 3 passengers arrive per minute on the average. What is the probability that exactly 30 customers will arrive during on 5 minute span?

Ans:-

Soln, Given,

Mean $(\lambda) = 3$ passenger per minute.

Now,

$X \sim P(\lambda = 3)$ so, pmf for this is given by;

$$P(X = x) = \frac{e^{-3}(3)^x}{x!} \quad x = 0 \text{ to } \infty$$

for 5 minutes,

$\lambda = 3 \times 5 = 15$ passenger per minute.

so, pmf is given as:-

$$P(X = x) = \frac{e^{-15}(15)^x}{x!} \quad \text{for } x = 0 \text{ to } \infty.$$

$$P(X = 30) = \frac{e^{-15}(15)^{30}}{30!}$$

$$= \frac{(15)^{30}}{e^{15} \cdot 30!}$$

$$= 0.0002211365$$

$$\therefore P(X = 30) = 0.0002211365$$

5b) In a university canteen it was observed that there were only two servers who takes on an average 4 Minutes to serve a student. If student arrive in the canteen at an average of 30 per hour, how much time is a student expected to be in the queue at any time.

Ans:-

soln, Given,

$S = 2$

$\lambda = 30$ per hours $\quad \mu = 15$ per hours.

Now, $\rho = \frac{\lambda}{\mu S} = \frac{30}{2 \times 15} = 1$ $\quad M/M/S : (\infty, FIFO)$

$$W_q = \left[\frac{1}{(S-1)!}\left(\frac{\lambda}{\mu}\right)^S \frac{\mu}{(S\mu - \lambda)^2}\right] P_0$$

Now,

$$P_0 = \left[\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} + \frac{1}{S!}\frac{(S\rho)^S}{1-\rho}\right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{(2 \times 1)^n}{n!} + \frac{1}{2!}\frac{(2)^2}{1-1}\right]^{-1}$$

$$= \frac{1}{\infty}$$

$$= 0$$

$W_q = 0$

$$W_S = W_q + \frac{1}{\mu}$$

$$= 0 + \frac{1}{15}$$

$$= 0.067$$ //

6b) At TIA, it takes 8 minutes for a plane to land. Although incomming planes have scheduled arrival times the wide variability in arrival time produces an effect making the incomming planes appear to arrive in poission fashion at an average rate of 5 planes an hour. A pilot enhort. Calculate the average time that the plane has to circle the valley in the air.

Ans:-

Soln, Given,

Given,

$\lambda$ = 5 planes an hour

$\mu$ = 7.5 plane an hours

$K = \infty$

$$W_q = \lim_{K \to \infty} \frac{K+1}{2K} \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \lim_{K \to \infty} \frac{1}{2}\left(1+\frac{1}{K}\right) \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \frac{1}{2}\left(1+\frac{1}{\infty}\right) \frac{5}{7.5(7.5-5)}$$

$$= 0.13333 \text{ hours or } 8 \text{ Minutes}$$

4b) Normal curve explain in copy

5a) 6a) repeated

7a) exponential distribution

7b c) repeated.

2018 (spring)

1a) b) 2a) b) 3a) b) all repeated

4a) A random sample size is taken 100 from an infinite population having Mean($\mu$) = 76 and $\sigma^2 = 256$. What is the probability that sample Mean ($\bar{x}$) will be between 75 and 78?

Soln, Given,

$\mu = 76$

$\sigma = \sqrt{256} = 16$ so,

$n = 100$

$P(75 < \bar{x} < 78) = ?$ By central limit theorem.

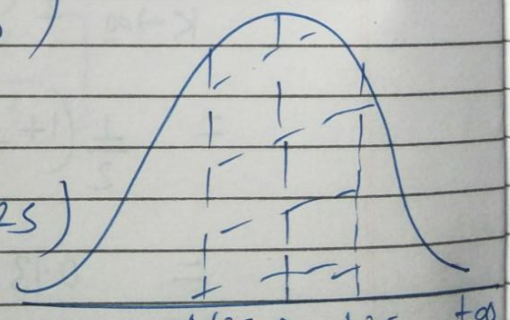
so, $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ then,

$$P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) = P(z) \text{ so,}$$

$$P\left(\frac{75-76}{16/10} < z < \frac{78-76}{16/10}\right)$$

$$P(-0.625 < z < 1.25)$$

$$= P(0 < z < 0.625) + P(0 < z < 1.25)$$




$$= 0.2357 + 0.3944$$

$$= 0.6301$$

5a) A telephone has two long distance operator. The telephone company finds that during the peak load, long distance calls arrives in a poission fashion at an average rate of 15 per hour. The length of service on these calls is approximately enponential distributed with mean length 5 minutes. What is the probablity that a subscriber will have to wait for his long-distance call during peak hour of a day?

Ans:-

Soln, Given, $\lambda = \frac{1}{4}$ per minute, $\mu = \frac{1}{5}$ per minute

$S = 2$ so,

$$\rho = \frac{\lambda}{\mu s} = \frac{5}{8}$$

$$p_0 = \left[\sum_{n=0}^{1} \frac{1}{n!}\left(\frac{5}{4}\right)^n + \frac{1}{2!}\left(\frac{5}{4}\right)^2 \frac{2/5}{(2/5 - 1/4)}\right]^{-1}$$

$$= 3/13$$

$$W_q = \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu}{(s\mu - \lambda)^2} = 3.2 \text{ minutes.}$$

b) A self-service store employs one chasier at its counter. Nine customers arrive on average in every 5 minutes while the cashier can serve 10 customers in 5 minutes. Find.

(i) The average number of customers in the queue.

(ii) Average time spent by a customer in the system.

(iii) Average time spent by a customer in the queue.

Ans:-

Soln, Given,

$\lambda = 1.8$ customer per minute

$\mu = 2$ customer per minute.

$M/M/1 (\infty, FIFO)$ $\rho = \frac{1.8}{2} = 0.9$

$L_q = \rho L_s$

$= \rho \times \frac{\rho}{1-\rho}$

$= 0.9 \times \frac{0.9}{0.1} = 8.1$

$W_s = \frac{L_s}{\lambda} = \frac{9}{1.8} = 5$ //

$W_q = \frac{L_q}{\lambda} = \frac{8.1}{1.8} = 4.5$ //

6 a, b 7 a, b, c all repeated //

2019 - Fall

1a) The chance that a doctor A will diagnose a disease x correctly is 60%. The chances that a patient will die by his treatment after correct diagonsis is 40% and the chance of death by wring diagonisis is 70%. If a patient of doctor A, who has disease x died. What is the chance that his disease was diagonosed correctly?

Ans :-

Soln, Given,

Let,

$E_1$ = event has the disease x is diagonesed correctly by doctor A.

$E_2$ = event that disease x is not diagnosed correctly by doctor A.

B = A patient (of doctor A) who has disease x dies.

Then, we are given,

$P(E_1) = 0.6$

$P(E_2) = 0.4 = 1 - 0.6 = 0.4$

and $P(B/E_1) = 0.4\%$ $\quad P(B/E_2) = 0.7$

By Bay's theorem.

$$P(E_1/B) = \frac{P(E_1) \times P(B/E_1)}{P(B)}$$

$$= \frac{0.6 \times 0.4}{0.6 \times 0.4 + 0.4 \times 0.7} = \frac{6}{13} //$$

1b) 2a) 2b) all same type //

3a) A blindfolded marksman finds that on the average he hits the target 4 times out of 5. If he fins 4 shorts, what is the probability of

(i) More than 2 hits?

(ii) Atleast 3 Misses?

Ans:-

No of trials $(n) = 4$,

$p = \frac{4}{5} = 0.8 \quad q = 0.2$

Let, $X$ = r.v that denotes the numer of hits.

Here, $X \sim B(4, 0.8)$

Now, so, pmf is $^4C_x (0.8)^x (0.2)^{4-x}$

(i) $p(X>2) = p(X=3) + p(X=4)$

$= {}^4C_3 (0.8)^3 (0.2)^1 + {}^4C_4 (0.8)^4 (0.2)^0$

$= 0.4096 + 0.4096$

$= 0.8192$ //

(ii) Atleast 3 Misses Means 1 hits and 4 Misses Means 0 hits.

$P(X \leq 1) = p(X=0) + p(X=1)$

$= {}^4C_0 (0.8)^0 + (0.2)^{4-0} + {}^4C_1 (0.8)^1 (0.2)^3$

$= 0.0272$ //

3b) The time taken by a Milkman to deliver Milk is normally distributed with Mean 12 Minutes and sd = 2 Min. (days 365 = 1 year)

(i) longer than 17 Minutes

(ii) less than 10 minutes.

(iii) between 9 and 13 Minutes.

Soln,

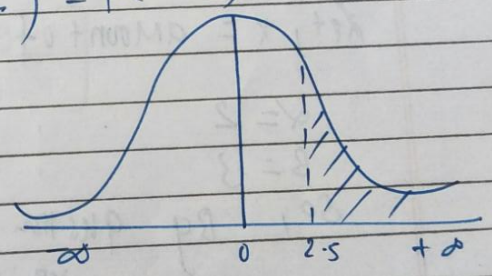
Let, 'x' = r.v that denotes the time taken by a delivery Man to deliver a Milk.

Now,

$\mu = 12 \quad \sigma = 2$ so,

$X \sim N(12, 4)$ then,

(i) $P(X > 17) = P\left(Z > \frac{17-12}{2}\right) = P(Z > 2.5)$

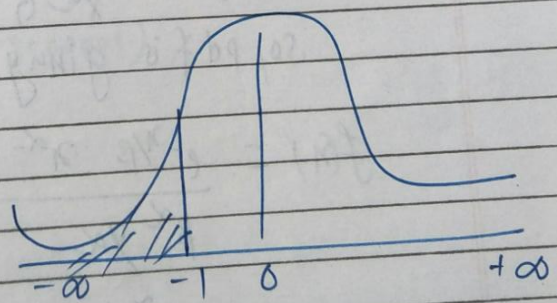



$= P(0 < Z < \infty) - P(0 < Z < 2.5)$

$= 0.5 - 0.4938$

$= 0.0062$ //

(ii) $P(X < 10) = P(Z < -1)$




$= P(-\infty < Z < \infty) - P(0 < Z < 1)$

$= 0.5 - 0.3413$

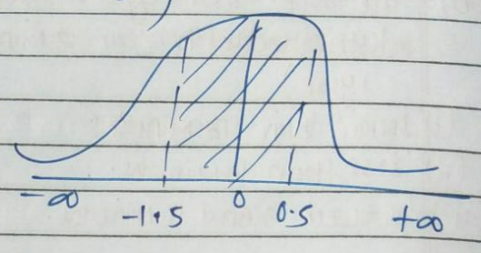
$= 0.1587$ //

(iii) $P(9 < X < 13) = P(-1.5 < Z < 0.5)$

$\therefore P(-1.5 < Z < 1.5) + P(0 < Z < 0.5)$

$= 0.4332 + 0.1915$

$= 0.6247$ //




4b) In a certain city the daily consumption of water follows gamma variate with $\alpha = 2$ and $\beta = 3$. If the daily capacity of this city is 9 million Gallons of water, what is the probability that on any given day the water supply is inadequate?

Soln, Given,

Let, $X$ = amount of daily consuption of water in Gallons.

$\alpha = 2$ $\quad \beta = \frac{1}{\lambda} \quad \lambda = \frac{1}{3}$

$\beta = 3$

So, By qustion.

$X \sim G(2, 3)$

So, pdf is ginny by

$$f(x) = \frac{e^{-x/\beta}\, x^{\alpha-1}}{\beta^{\alpha}\, \gamma\alpha} \quad x > 0 \quad \text{so,}$$

$$= \frac{e^{-x/3}\, x^{1}}{(3)^2 \times \gamma 2} = \frac{1}{9}\, x\, e^{-x/3}$$

if it has only 9000000 gallons the probability of inadequate is given by;

$$P(X > 9000000) = \frac{1}{9}\int_{9000000}^{\infty} x e^{-x/3}\, dx$$

$$= \frac{1}{9}\left[\frac{x e^{-x/3}}{(-1/3)} - \frac{e^{-x/3}}{(-1/3)^2}\right]_{9000000}^{\infty}$$

$$= \frac{-1}{9}\left[(0+0) - \left(\frac{3 \times 9000000}{e^{9000000}} + \frac{9}{e^{9000000}}\right)\right]$$

$$= \frac{1}{9}\left[\frac{27000009}{e^{9000000}}\right]$$

$$= \frac{3000001}{e^{9000000}}$$

$$\therefore \frac{1}{e^{9000000}}(3000001) \;//$$

All repeated.

2019 spring :-

1a) soln

Let,

A = event that patient are prescribed narcotic pain killers

B = event that clinic patient are addicted to narcotic.

~~C = event that patients are addict~~

Given,

$P(A) = 0.10$ $\quad P(\bar{A}) = 0.90$

$P(B) = 0.05$ $\quad P(\bar{B}) = 0.95$

$P(B/A) = 0.08$

$$P(A/B) = \frac{P(A) \times P(B/A)}{P(B)}$$

$$= \frac{P(A) \times P(B/A)}{}$$

$$= \frac{0.10 \times 0.08}{0.05} = 0.16$$

(b) soln

9 items taken from 21 items is ${}^{21}C_9 = 293930$

faults = 5 $\quad$ Non-faulty = 16

Choosing 0 faulty item is. ${}^{5}C_0 \times {}^{16}C_9$

Choosing 1 faulty item is ${}^{5}C_1 \times {}^{16}C_8$

Chossing 2 faulty item is ${}^{5}C_2 \times {}^{16}C_7$

choosing 3 faulty item is ${}^{5}C_3 \times {}^{16}C_6$

probability that box is accepted is given by :-

$$= \frac{{}^5C_0 \times {}^{16}C_9 + {}^5C_1 \times {}^{16}C_8 + {}^5C_2 \times {}^{16}C_7 + {}^5C_3 \times {}^{16}C_6}{{}^{21}C_9}$$

$$= 0.91950464 \,//$$

20) soln

Let, $X$ = r.v that insurance sellsman sells some policies.

$\lambda = 3$ per week

$X \sim Poi(\lambda = 3)$ so, pmf is

$$P(X = x) = \frac{e^{-3}(3)^x}{x!} \quad \text{for } x = 0, 1, 2, \ldots \infty$$

(i) $P(X \geq 1) = 1 - P(X \leq 1)$

$$= 1 - P(X = 0)$$

$$= 1 - \frac{e^{-3}(3)^0}{0!}$$

$$= 0.95021 \,//$$

(ii) $P(2 \leq X < 5) = P(X = 2) + P(X = 3) + P(X = 4)$

$$= \frac{1}{e^3}\left[\frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}\right]$$

$$= \frac{1}{e^3}\left[\frac{9}{2} + \frac{27}{6} + \frac{81}{24}\right]$$

$$= 0.61611 \,//$$

assuming 5 working days per week that

(iii) In 5 working days 3 life insurance policies is sold

In 1 " " $\frac{3}{5}$ " " " " "

$\therefore \lambda = 3/5$

pmf is given by

$$p(x=x) = \frac{e^{-\lambda}(\lambda)^x}{x!} \quad x = 0, 1, \ldots \infty.$$

$$= \frac{e^{-3/5}(3/5)^x}{x!}$$

$$p(x=1) = \frac{e^{-3/5}(3/5)^1}{(1)!}$$

$$= 0.329286 //$$

(9a) Soln

$X$ be a r.v that denotes mean salary

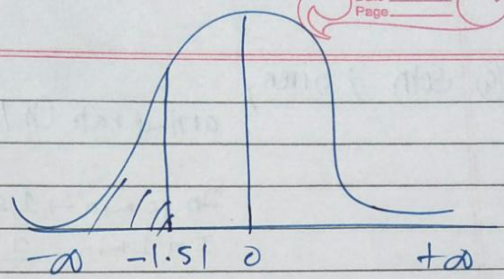
then,

$n = 100$ men. $\mu = 29321$

$\sigma = 2120$

$$p(\bar{X} < 29000) = p\left(z < \frac{29000 - 29321}{2120/\sqrt{10}}\right)$$

$$p(z < -1.514)$$




$p(0<z<\infty) - p(0<z<1.51)$

$= 0.5 - 0.4345$

$= 0.0655$ //

4 (b) Soln

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1/4 & 1/2 & 1/2 \\ 2 & 1/3 & 0 & 2/3 \\ 3 & 1/2 & 1/2 & 0 \end{array}$$

$$p(x_1=3) = 1 - [p(x_1=1) + p(x_1=2)]$$

$$p(x_1=3) = 1 - \left[\frac{1}{4} + \frac{1}{4}\right]$$

$$= \frac{1}{2}$$

we can write

$$p(x_1=3, x_2=2, x_3=1) = p(x_1=3) \cdot p_{32} \cdot p_{21}$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{1}{12} \quad //$$

5(a) Soln ; Given,

arrival rate ($\lambda$) = 30 train per day

In 36 min → 1 served

In 1 min $\frac{1}{36}$ "

In 60 minute $\frac{60}{36}$

M/M/1 : ($\infty$, FIFO) In 1 hr $\frac{60}{36}$

In 24 hr $\frac{60}{36} \times 24$ = 40 train per day

$$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = 0.75$$

Now,

(i) $L_q = \rho L_s = 0.75 \left(\frac{0.75}{1-0.75}\right)$

$= 2.25$ //

(ii) $(\rho)^n = (0.75)^{10} = 0.0563135197$ //

(iii) $E(x) =$ find it $\qquad V(x) =$ find it

$\int_{-\infty}^{\infty} x f(x) dx \qquad V(x) = E(x^2) - \{E(x)\}^2$

Now,

VVI

(8) A r.v X has p.df

$$f(x) = \begin{cases} \theta e^{-\theta x} & ; x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find Mean and variance

Now,

$$E(x) = \int_0^{\infty} x f(x) dx$$

$$= \int_0^{\infty} x \theta e^{-\theta x} dx$$

$$= \theta \int_0^{\infty} x e^{-\theta x} dx$$

$$= \theta \int_0^{\infty} e^{-\theta x} x^{2-1} dx$$

$$= \theta \frac{\gamma 2}{\theta^2} \qquad \because \gamma n = (n-1)!$$

$$= \frac{1}{\theta} (2-1)! = \frac{1}{\theta}$$

$$V(x) = E(x^2) - \{E(x)\}^2$$

Now,

$$E\{x^2\} = \theta \int_0^\infty x^2 e^{-\theta x} dx$$

$$= \theta \int_0^\infty e^{-\theta x} x^{3-1} dx$$

$$= \theta \frac{\gamma 3}{\theta^3}$$

$$= \frac{1}{\theta^2} (3-1)!$$

$$= \frac{2}{\theta^2}$$

Formula

$$\int_0^\infty e^{-x} x^{\alpha-1} dx = \gamma \alpha$$

$$\int_0^\infty e^{-\beta x} x^{\alpha-\beta=1} dx = \frac{\gamma \alpha}{\beta^\alpha}$$

$$\gamma n = (n-1)!$$

$$V(x) = \frac{2}{\theta^2} - \frac{1}{\theta^2}$$

$$= \frac{1}{\theta^2}$$

(f)

Soln

Let,

$A_1$ = event that Mr X is a principle

$A_2$ = event that Mr Y is a principle

$A_3$ = event that Mr Z is a Principle

Now,

N = New academic program in the college

Now,

$P(A_1) = 0.5$, $P(A_2) = 0.25$, $P(A_3) = 0.25$

$P(N/A_1) = 0.01$, $P(N/A_2) = 0.05$, $P(N/A_3) = 0.02$

(i) P(that introducing new academic program) = $P(A_1 \cap N)$ or $P(A_2 \cap N)$ or $P(A_3 \cap N)$

$= 0.5 \times 0.01 + 0.25 \times 0.05 + 0.25 \times 0.02$

$= 0.0225$

(ii) By question,

$$P(X \geq x) = 5\% = 0.05 \quad \text{eqn(i)}$$

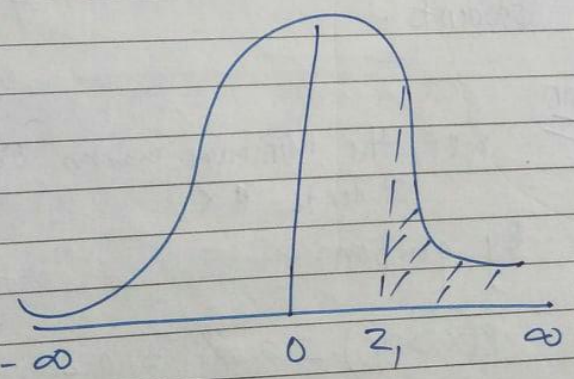
for Here,

$$P(X \geq x) = 0.05$$

$$P\left(\frac{X-\mu}{\sigma} \geq \frac{x-\mu}{\sigma}\right) = 0.05$$

$$P\left(Z > \frac{x-45}{20}\right) = 0.05$$

$$P(Z > z_1) = 0.05$$




$$P(0 < Z < \infty) - P(0 < Z < z_1) = 0.05$$

$$0.5 - 0.05 = P(0 < Z < z_1)$$

$$P(0 < Z < z_1) = 0.45$$

(iii) $E(x) =$ find it $\quad V(x) =$ find it

$$\int_{-\infty}^{\infty} x f(x)\,dx \qquad V(x) = E(x^2) - \{E(x)\}^2$$

Now,

VVI

(8) A r.v X has p.d.f

$$f(x) = \begin{cases} \theta e^{-\theta x} & ; x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find Mean and variance

Now,

$$E(x) = \int_0^{\infty} x f(x)\,dx$$

$$= \int_0^{\infty} x\, \theta e^{-\theta x}\,dx$$

$$= \theta \int_0^{\infty} x e^{-\theta x}\,dx$$

$$= \theta \int_0^{\infty} e^{-\theta x} x^{2-1}\,dx$$

$$= \theta \frac{\gamma 2}{\theta^2} \qquad \because \gamma n = (n-1)!$$

$$= \frac{1}{\theta} (2-1)! = \frac{1}{\theta}$$

(i) $\sum_{0}^{4} \frac{3x}{K(K+1)} = 1$

$\therefore K = -6$ or $K = 5$

$$p(x) = \begin{cases} x/10, & x = 0,1,2,3,4 \\ 0, & \text{otherwise.} \end{cases}$$

(ii) p.d is.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $p(x)$ | 0 | 1/10 | 2/10 | 3/10 | 4/10 |

(iii) a) $p(x=1) = 1/10$

b) $p(x \geq 3) = p(x=3) + p(x=4)$

$= 3/10 + 4/10$

$= 7/10$

c) $p(0 < x < 3) = 3/10$

d) $p(x > 2) = 7/10$

(i) $p(R_1) = p(T_0 \cap R_1)$ or $p(T_1 \cap R_1)$

$= p(T_0) \times p(R_1/T_0) + p(T_1) \times p(R_1/T_1)$

$= 0.45 \times 0.06 + 0.55 \times 0.01$

$= 0.5275$

(ii) $p(R_0) = 1 - p(R_1)$

$= 1 - 0.5275$

$= 0.4725$

(iii) $p(T_1/R_1) = \dfrac{p(T_1) \times p(R_1/T_1)}{p(R_1)} = \dfrac{0.55 \times 0.01}{0.5275} = 0.95$

(iv) $p(T_0/R_0) = \dfrac{p(T_0) \times p(R_0/T_0)}{p(R_0)} = \dfrac{0.45 \times 0.94}{0.5275} = 0.8952$

(v) probability of error $= p(R_0 \cap T_1)$ or $p(R_1 \cap T_0)$

$= p(R_0) \times p(T_1/R_0) + p(R_1) \times p(T_0/R_1)$

Now,

$p(T_1/R_0) = 1 - p(T_0/R_0)$ , $p(T_0/R_1) = 0.05$

$= 0.1048$

probability of error $= 0.075893$ //

# Bernaulli's law of large Numbers (BLLN)

Let, 'x' be a r.v having two catogories then, the Bernaulli's law of large number states that,

$$P\left(\left|\frac{x}{n} - p\right| < K\right) \geq 1 - \frac{pq}{nK^2}$$ (Lower bound) (at least)

and

$$P\left(\left|\frac{x}{n} - p\right| \geq K\right) \leq \frac{pq}{nK^2}$$ (upper bound) atmost.

where,

$\frac{x}{n}$ = Relative frequency of success (i.e. sample prob of success)

$p$ = Theoretical prob. of "

$q$ = Theoretical prob of " of failure = $1-p$

$n$ = sample size.

(4) exponential distribution.

# A r.v is said to follow exponential distribution if parameter $\theta$.

if it's p.d.f is given by;

$$f(x) = \begin{cases} \theta e^{-\theta x} ; & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Mean $= \frac{1}{\theta}$ and variance $= \frac{1}{\theta^2}$

IMP frequently asked

# Memoryless property of exponential distribution (Markovian property)

proof:-

Statement:-

If 'x' be a r.v having exponential distribution with parameter $\theta$ and s and t be any two integers taken by r.v X. such that $t > s$, then the memoryless property of exponential distribution is given by

$$P(X > s+t / X > s) = P(X > t)$$

proof:-

LHS

$$P(X > s+t / X > s) = \frac{P\{(X > s+t) \cap (X > s)\}}{P(X > s)}$$

$$= \frac{P\{X > s+t\}}{P(X > s)}$$

(ii) $P_n = \begin{cases} \frac{\rho^n}{n!} P_0 \; ; \; n \le s & = \frac{(0.3)^n}{n!} \times 0.532 \\ \frac{\rho^n}{s!\, s^{n-s}} P_0 \; ; \; n > s & \frac{(0.3)^n}{2!\, 2^{n-2}} \times 0.532 \end{cases}$

(iii) $L_q = \left[ \frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu - \lambda)^2} \right] P_0$

again, the upper bond of probability obtained by, the seve's inequality is,

$$P(|X - E(X)| \geq K\sigma) \leq \frac{1}{K^2}$$

or, $P\left(|X - 0| \geq \frac{1}{2}K\right) \leq \frac{1}{K^2}$

if $K = 2$

$$\therefore P((X-0) \geq 1) \leq \frac{1}{4} \quad //$$

HW

# A. r.v. 'X' has p.d.f of $e^{-x}$ for $x > 0$. show that the chebyshev's inequality gives $P(|X-1| > 2) \leq \frac{1}{4}$ and show that actual probability is $e^{-3}$.

soln

PTO

ii) $P(A_1/N) = \frac{0.5 \times 0.01}{0.0225} = 0.2$ //

$P(A_3/N) = \frac{0.25 \times 0.02}{0.0225} = 0.2$ //

(g)

Soln

Let,

$A_1$ = event that the chips are deffective.

$A_2$ = event that the chips are non-deffective (ie good)

$B$ = event that the tester say not good (deffective)

$\bar{B}$ = event that the tester say non deffective (good)

By question.

$P(A_1) = 0.02$ $\quad P(A_2) = 1 - P(A_1)$
$= 0.98$

Now,

$P(\bar{B}/A_2) = 0.95$ , $P(B/A_1) = 0.94$

$P(A_1/B) = ?$

$P(B/A_2) = 0.05$ , $P(\bar{B}/A_1) = 0.06$

Here,

$$p(x>s+t) = \int_{s+t}^{\infty} f(x)\,dx$$

$$= \int_{s+t}^{\infty} \theta e^{-\theta x}\,dx$$

$$= \theta \left[\frac{e^{-\theta}}{\theta}\right]_{s+t}^{\infty}$$

$$\therefore p(x>s+t) = e^{-\theta(s+t)}$$

again,

$$p(x>s) = \int_{s}^{\infty} f(x)\,dx$$

$$= \theta \int_{s}^{\infty} e^{-\theta x}\,dx$$

$$p(x>s) = e^{-\theta s} \quad //$$

$s = [x^0 \; y^1 \; z^2]$ $\quad x+y+z=1$

$sp = s$

$$[x \; y \; z]\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix} = [x \; y \; z]$$

$$\left[\begin{array}{l} 0.6x + 0.1y + 0.6z = x \\ 0.2x + 0.8y = y \\ 0.2x + 0.1y + 0.4z = z \\ x+y+z = 1 \end{array}\right]$$

4

2013 spring (6a)

Let,

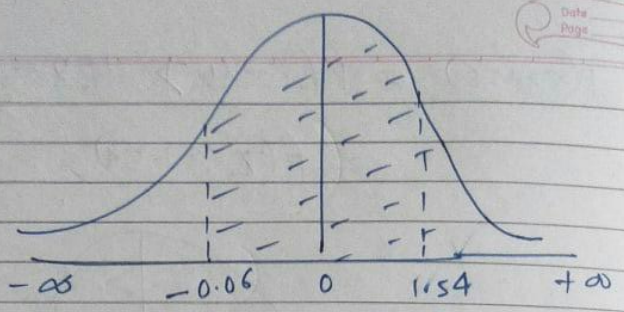
2. If is

Soln:

two-step tpm is

$P^2 = \begin{bmatrix} 3/4 \\ 1/4 \\ 0 \end{bmatrix}$

$P^2 = \begin{bmatrix} 5/8 \\ 5/16 \\ 3/16 \end{bmatrix}$




$$P(0 < Z < 0.06) + P(0 < Z < 1.54)$$

$$= 0.0239 + 0.4382$$
$$= 0.4621$$

(*) The time taken by a delivery man to deliver vegetable is normally distributed with Mean 12 Minutes and standard deviation 2 Minutes. He delivers vegetable every day. estimate the number of days in a year (take 1 year = 365 days). when he takes

(i) longer then 17 min.
(ii) between 9 and 13 min.

soln

Let,
X = Time taken to deliver vegetables in minutes.

$$= \begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix}$$

(iii) The initial probability vector is,

$$S_0 = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}$$

after 4 weeks.

$$= S_0 P^4$$

$$= \begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 0.59 & 0.41 \\ 0.27 & 0.71 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 0.4 & 0.6 \\ 0.4 & 0.6 \end{bmatrix}}$$

$$= \begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 0.59 & 0.41 \\ 0.27 & 0.71 \end{bmatrix}$$

$$= \begin{bmatrix} 0.46 & 0.54 \end{bmatrix}$$

Expected queue size.

(i) $L_q = \rho L_s$
$= 2.25$ trains per days

(ii) Probability the queue size exceed 10
$= q^n$
$= (0.75)^{10}$
$= 0.0563$ //

(iii) Again, if $\lambda = 33$ per day.
$\mu = 40$ per day.

$\rho = \frac{33}{40} = 0.825$

---

(7) Soln

In 9 min = 1 call arrive
In 1 min $\frac{1}{9}$ calls arrive

$\lambda = \frac{1}{9}$

and 3 min is 1 call
1 min is $\frac{1}{3}$ call

$\mu = \frac{1}{3}$ per calls.

(i) $(\rho_w) = \rho = \frac{\lambda}{\mu} = 0.33$

(ii) $(L_q) = \rho L_s = \rho\left(\frac{\rho}{1-\rho}\right)$

$p(-1.5 < x < 1.5 \cap 1 < x < 3)$

$p(|x| < 1.5)$

$$p(-1.5 < x < 1.5) = \int_1^2 \frac{1}{3}(3-x)\,dx + \int_2^3 \frac{1}{4}\,dx$$

$$= \frac{1}{3}\left[3x - \frac{x^2}{2}\right]_1^2 + (3-2)\frac{1}{4}$$

$$= \frac{1}{3}\left[6 - \frac{4}{2} - \frac{1}{1}\times 3 + \frac{1}{2}\right] + \frac{1}{4}$$

$$= \frac{1}{3}\left[\frac{3}{2}\right] + \frac{1}{4}$$

$$= \frac{3}{4}$$

(iii) $p(A/B) = p\{(-1.5 < x < 1.5)/(1 < x < 3)\}$

$$= \frac{p(0<x\ 1)}{p(1<x<3)} = p\{(-1.5 < x < 1.5)/(1 < x < 3)\}$$

$$= \frac{15/32\ p(1 < x < 1.5)}{p(1 < x < 3)}$$

Now

$$p(1 < x < 1.5) = \int_1^{1.5} f(x)\,dx =$$

$$p(1 < x < 3) = \int_1^3 f(x)\,dx = \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx$$

$$\frac{1}{1000 \times 1000} \int_{3000}^{\infty} x \, e^{-x/1000} \, dx$$

$$= \frac{1}{1000000} \left[ x \frac{e^{-x/1000}}{-\frac{1}{1000}} - \frac{e^{-x/1000}}{\left(-\frac{1}{1000}\right)^2} \right]_{3000}^{\infty}$$

$$= \frac{1}{1000000} \left[ -1000 \, e^{-x/1000} - e^{-x/1000} \times (1000)^2 \right]_{3000}^{\infty}$$

$$= \frac{1}{1000000} \left[ e^{-\frac{3000}{1000}} - e^{-3000/1000} \times (1000)^2 \right]$$

$$= 0.199$$

PU 2019 Spring Q No 6(b)

Gamma

Soln

Let $x$ = amount daily consumption of milk in Gallons.

$\alpha = 2$

$\beta = \frac{1}{\lambda}$ and $\lambda = \frac{1}{1000}$

$\Rightarrow \beta = 1000$

By question.

$X \sim G(2, 1000)$

The pdf is

$$f(x) = \frac{e^{-x/\beta}\, x^{\alpha-1}}{\beta^{\alpha}\, \Gamma\alpha} \quad x > 0$$

$$= \frac{e^{-x/1000}\, x^{2-1}}{(1000)^2\, \Gamma 2}$$

$$f(x) = \frac{e^{-x/1000}\, x^{1}}{(1000)^2} \quad ; x > 0$$

if it has only 3000 Gallons the probab it insufficients is 13 given by

$$P(X > 3000) = \int_{3000}^{\infty} \frac{e^{-x/1000}\, x}{(1000)^2}\, dx$$

Now, distribution function is,

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| F(x) | 1/8 | 4/8 | 7/8 | 1 |

2)

Soln

(i) $\sum_{x=-3}^{3} p(x) = 1$

or, $p(-3) + p(-2) + p(-1) + p(0) + p(1) + p(2) + p(3) = 1$

$K = 0.1$

(ii) a) $P(X \leq 2) = 1 - P(X > 2)$

$= 1 - P(X = 3)$

$= 1 - 0.1$

$= 0.9$

b) $P(X > -1) = 0.1 + 0.1 + 0.1 + 0.2$

$= 0.5$ //

$$= \begin{array}{ccc} 0 & 1 & 2 \\ \end{array}$$
$$= \begin{bmatrix} 0.349 & 0.273 & 0.398 \end{bmatrix}$$

2016 fall:

(b) Soln

irreducible (regular mc)

Here, 0 = state busy
1 = state idle.
2 = Undergoing repair.

1-step tpm is,

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix} \end{array}$$

The 2-step tpm

$$P(2) = P^2 = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix} \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

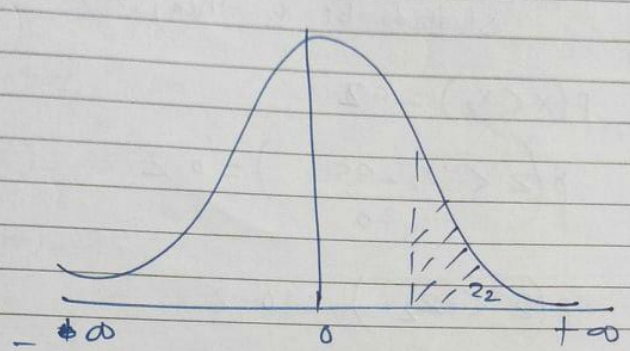
$$= \begin{bmatrix} 0.5 & 0.28 & 0.22 \\ 0.2 & 0.66 & 0.14 \\ 0.6 & 0.12 & 0.28 \end{bmatrix}$$

Since, in 2-state tpm all $P_{ij} > 0$ i.e state i and j are communicative with each other.
$i = 0, 1, 2$ and $j = 0, 1, 2$.

So, it is irreducible.

a, $P(Z > z_2) = 0.2$

a,




$$p(0 < z < \infty) - p(0 < z < z_2) = 0.2$$

$$0.5 - 0.2 = p(0 < z < z_2)$$

$$p(0 < z < z_2) = 0.3$$

Now,

$$z_2 = 0.84$$

$$0.84 = \frac{x_1 - 45}{20}$$

$$x_1 = 61.9$$

$$= 62$$

$$P\{(x > s+t)/P(x>s)\} = e^{-\theta s - \theta t + \theta s}$$

$$= e^{-\theta t} \quad LHS$$

Now, from RHS

$$P(x>t) = \int_t^{\infty} f(x)\,dx$$

$$\theta \int_t^{\infty} e^{-\theta x}\,dx$$

$$= -\frac{\theta}{\theta}\left[e^{-\theta}\right]_t^{\infty}$$

$$P(x>t) = e^{-\theta t} \quad RHS$$

$\therefore$ LHS = RHS, Hence memory less property is proved

# There are two food stores A and B in a certain Area. In an investigation of the preferences of customers revealed that with prob 0.15. A customer at prob 0.15. A customer at (A) go over to store B. next week and with prob 0.1 a customer at store B would go over to store A.

(i) Find what % of the customers will go from store A to B and from store B to A after 4 weeks.

(ii) Find what % of the customers will go to store B from A and to A from store B after sufficiently long time.

(iii) If initially 60% of the customers were going to store A and 40% of the customers were going to store B, find what % of the customers will be going to store A and B after 4 weeks.

<u>Soln</u>

Let $0 \Rightarrow$ store A

$1 \Rightarrow$ store B.

By question by one-step tpm is,

$$P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.85 & 0.15 \\ 1 & 0.10 & 0.90 \end{array} = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$$

(i) After 4 weeks 4th step tpm is $P(4)$

$$= \frac{1}{p+q}\begin{bmatrix} p & -p \\ -q & q \end{bmatrix} + \frac{(-q-p)^n}{p+q}\begin{bmatrix} q & p \\ q & p \end{bmatrix}$$

# Higher order tpm

The one-step tpm only gives the conditional probability of movement of particle in one-state. but there may arise different condition we need to find prob. for more than one step. in this case we need ?.

Let, $P$ be a one-step tpm. the $n^{th}$ step tpm is given by,

$$P(n) = P^n = P \cdot P \cdot P \cdots \quad (n\text{-times})$$

# Two State markov's chain (MC).

⇒ The markov's chain having only two states 0, and 1. is called a two state markov's chain. for eg:-
Let we have only two states 0 and 1.

one-step tpm for a two state markov's chain is,

$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix} \end{array}$$

~~for 2×2~~ for 2-state mc, the $n^{th}$ step is step tpm is,

$$P(n) = \frac{1}{p+q}\begin{bmatrix} p & -p \\ -q & q \end{bmatrix} + \frac{(1-q-p)^n}{p+q} \cdot \begin{bmatrix} q & p \\ q & p \end{bmatrix}$$

and in long run i.e $n \to \infty$ the tpm is

$$P(n) = \frac{1}{p+q}\begin{bmatrix} p & -p \\ -q & q \end{bmatrix}$$

# Q If $Z \sim N(0,1)$ Then find

(i) $P(0 < Z < 1.4)$

(ii) $P(0 < Z < 2.69)$

(iii) $P(Z > 1.84)$

(iv) $P(Z \leq -2.6)$

(v) $P(Z > -3.1)$

(vi) $P(Z < 1.26)$

(vii) If $(P(Z > a)) = 0.8$, find value of $a$.

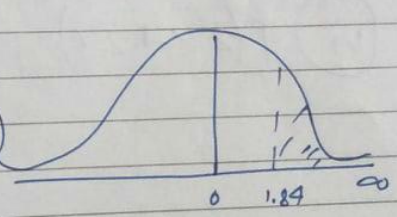
(i) $P(0 < Z < 1.4)$

$= 0.4192$

(ii) $P(0 < Z < 2.69)$

$= 0.4964$

(iii) $P(Z > 1.84)$

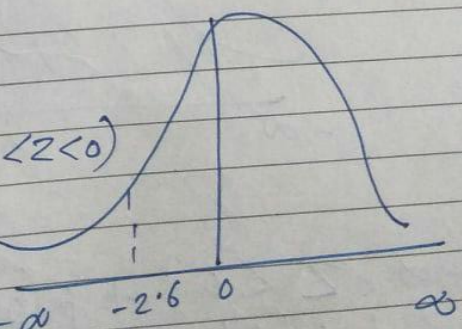



$= P(0 < Z < \infty) - P(0 < Z < 1.84)$

$= 0.5 - 0.4671$

$= 0.0329$

(iv) $P(Z \leq -2.6)$




$P(-\infty < Z < 0) - P(-2.6 < Z < 0)$

$0.5 - P(0 < Z < 2.6)$

$= 0.5 - 0.4953$

$= 0.0047$

Q No 1) Let x be the average life time of a special kind of an electric bulb. Let it has an exponential distribution with mean of 100hrs. find the probability that a bulb randomly chosen has the life

i) less then 90hrs.

ii) Atleast 90hrs.

iii) Find the If the bulb has already lasted for 10hrs what is the probability that the bulb will last atleast 40hrs.

By question.

Average life of electric bulbs = 100hrs.

Let $X$ = life time of electric bulb in hours.

$E(X) = 100$

$\frac{1}{\theta} = 100$ since $X \sim Exp(\theta = 0.01)$ we have.

$\theta = 0.01$

so, pdf of $X$ is

$$f(x) = \theta e^{-\theta x} \; ; x > 0$$

$$f(x) = \frac{1}{100} e^{-x/100} \; ; x > 0$$

# Initial probability vector ($S_0$) :-

⇒ The Matrix representing the initial probability distribution of the states is called a initial probability vector it has only one row. Let, 0 and 1 are two states then the initial probability vector is $S_0 = [\overset{0}{x} \ \overset{1}{y}]$ such that ; $x+y = 1$

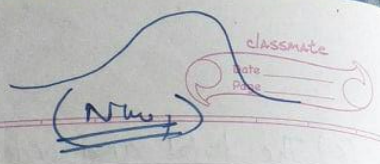
where $x$ = initial prob of state 0
$y$ = " " " " " 1.

# Steady state probability(s) :- (Long run prob.).

for a regular mc, the steady state probability vector (S) is given by, $sp = s$ ~ (i) where,

$$s = [\overset{0}{x}, \overset{1}{y}, \overset{2}{z}] \quad \text{such that } x+y+z = 1$$

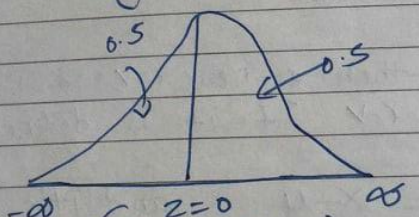
$$P = \text{one step tpm} = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & p_{00} & p_{01} & p_{02} \\ 1 & p_{10} & p_{11} & p_{12} \\ 2 & p_{20} & p_{21} & p_{22} \end{array}$$




So, $Z \sim N(0,1)$

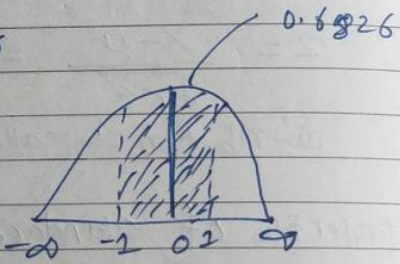
- It is symmetrical about $z = 0$ //

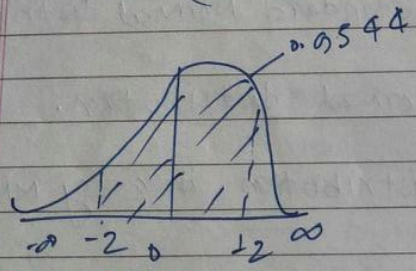
③ Area property of standard Normal curve.

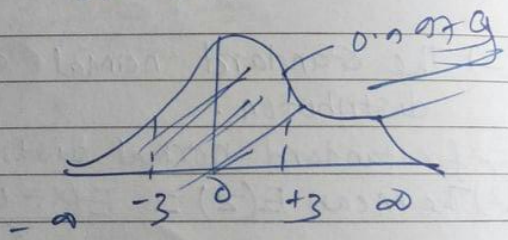



$P(-\infty < Z < \infty) = 1$

$P(x > \mu + \sigma)$ or $P(Z > 1)$




$P(-\infty < Z < 0) = 0.5$
$P(0 < Z < \infty) = 0.5$ } Symmetric property.

$P(-1 < Z < 1) = 0.6826$
$P(-2 < Z < 2) = 0.0544$
$P(-3 < Z < 3) = 0.0979$ //

# Ram Hari and shyam are playing basketball. Ram always throws to Hari and Hari always throws to shyam. shyam throws to Ram. 2/3 of the time and to hari 1/3 of time.

(i) Find 1 step tpm.

(ii) In long run, what % of the time do each of the players have basketball.

soln

Let, 0 → state Ram.
1 → State Hari
2 → State Shyam.

By question,

(i) 1 step tpm is

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 2 & 2/3 & 1/3 & 0 \end{array}$$

Here, Let, for long run the steady state Matrix be

$S = [x, y, z]$ such that

$x + y + z = 1$

$S = ?$

We know, $SP = S$

$$\text{or, } [x\ y\ z] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2/3 & 1/3 & 0 \end{bmatrix} = [x\ y\ z]$$

$$\text{or, } \left[\frac{2z}{3}\ \ x + \frac{z}{3}\ \ y\right] = x\ y\ z$$

$$V(x) = 3.7 - (-0.1)^2$$
$$= 3.69$$

3)

Soln

(i) or, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

or, $\int_{0}^{2} K(2x - x^2)\,dx = 1$

or, $K\left[2\frac{x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$

or, $K = \frac{3}{4}$

pdf is $f(x) = \begin{cases} \frac{3}{4}x(2-x); & 0 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$

VVI

# Addition and Multiplication of probability.

Let, A and B be any two events with in a sample spaces. Then the Probability of happening any one of the event either A or B is given by,

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$ [∵ If A and B are non-Mutually exclusive event]

and $P(A \cup B) = P(A) + P(B)$ [For Mutually, exclusive event]

If A, B, C then

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$ for Mutually exclusive event

NOTE:- Use Addition theorem union [atleast, atMost, either or, anyone of them, neither, nor].

) Multiplicative:-

Let, A and B be any two events within a sample space. Then, the probability of happening both the events A and B simultaneously is given by,

$P(A \text{ and } B) = P(AB) = P(A \cap B) = P(A) \cdot P(B/A)$ or, $P(B) \cdot P(A/B)$ for dependent events.

## Poission distribution

# At a checkout counter costumer arrive at an average no of 1.5 per Minute. find the probability that
i) At Most 4 will arrive in any given Minute
ii) Atleast 3 will arrive during an interval of 2 Mintes
iii) exactly 5 will arrive in an interval of 4 Minutes.

**soln**

Average no of costumer arriving at checkout counter($\lambda$) = 1.5 per Minute.

Let, $x$ = no of costumer arriving at checkout counter.

$x \sim p(\lambda = 1.5)$, so, the p.m.f of $x$ is

$$p(x) = \frac{e^{-\lambda} \lambda^x}{x!} \; ; \; x = 0, 1, 2 \ldots \infty$$

$$p(x = x) = \frac{e^{-1.5} (1.5)^x}{x!} \quad x = 0, 1, 2, \infty$$

(i) $p(x \leq 4) = p(x=0) + p(x=1) + p(x=2) + p(x=3) + p(x=4)$

$$= \left[ \frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)}{(1)!} + \frac{e^{-1.5}(1.5)^2}{(2)!} + \frac{e^{-1.5}(1.5)^3}{3!} + \frac{e^{-1.5}(1.5)^4}{4!} \right]$$

$$= e^{-1.5} \left[ 1 + 1.5 + 1.125 + 0.5625 + \frac{27}{128} \right]$$

$$= 0.98 \quad //$$

$$\gamma\alpha = \int_0^\infty e^{-x} x^{\alpha-1}\, dx$$

(5) Gamma distribution. $f(x) = \frac{e^{-x} x^{\alpha-1}}{\gamma\alpha}$

A random variable 'x' is said to follow Gamma distribution with parameter if it's p.d.f is given by,

Mean $E(x) = \int_0^\infty x f(x)\, dx$

$$= \int_0^\infty \frac{x e^{-x} x^{\alpha-1}}{\gamma\alpha}\, dx$$

$$= \frac{1}{\gamma\alpha} \int_0^\infty e^{-x} x^{\alpha+1-1}\, dx$$

$$= \frac{1}{\gamma\alpha} \gamma\alpha+1$$

$$= \frac{\alpha \cancel{\gamma\alpha}}{\cancel{\gamma\alpha}}$$

$$\therefore E(x) = \alpha$$

Now,

$$E(x^2) = \int_0^\infty x^2 f(x)\, dx$$

(2015-spring)

3 (b) Soln

Let $x$ = height of boys

Mean ($\mu$) = 150.3 CM

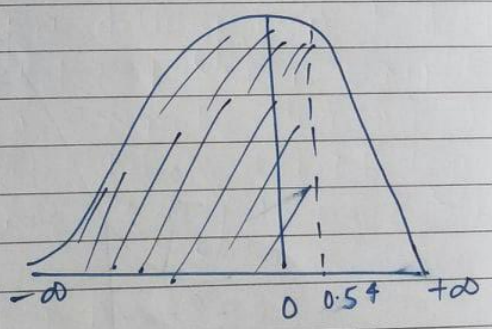
s.d ($\sigma$) = 5 CM

By, question $x \sim N(150.3, 5^2)$

(i) Probability of boy hight less the 153 is

$$p(x < 153) = p\left(\frac{x-\mu}{\sigma} < \frac{153-\mu}{\sigma}\right)$$

$$= p(z < 0.54)$$




$$p(z < 0.54) = p(-\infty < z < 0) + p(0 < z < 0.54)$$

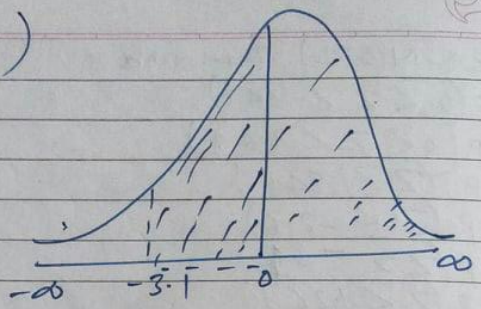
$$= p(0 < z < \infty) + p(0 < z < 0.54)$$

$$= 0.5 + \cancel{0.1736}\ 0.2054$$

$$= \cancel{0.6736}$$

$$= 0.7054$$

(v) $P(Z > -3.1)$



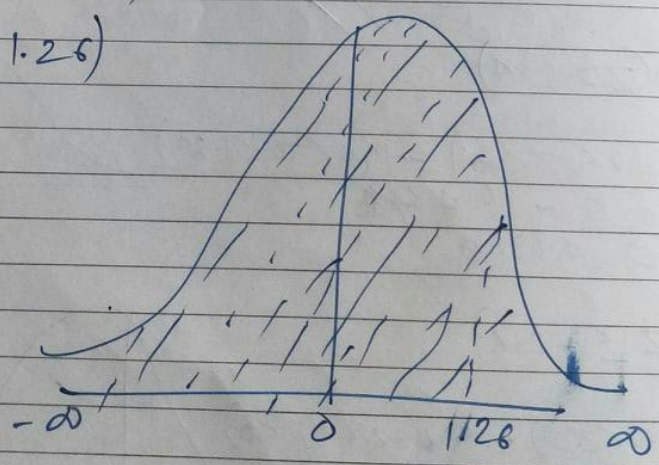

$= P(-3.1 < Z < 0) + P(0 < Z < \infty)$

$= P(0 < Z < 3.1) + P(0 < Z < \infty)$

$= 0.4990 + 0.5$

$= 0.9990$ //

(vi) $P(Z < 1.26)$




$P(-\infty < Z < 0) + P(0 < Z < 1.26)$

$= 0.5 + 0.3962$

$= 0.8962$ //

# How many times would you have to toss a fair coin in order to be atleast 95% sure that the relative frequency of turning of a head is within 0.01 of the theoretical prob $\frac{1}{2}$.

**Soln**

By question;

Theoritical prob of success
ie (head) $p = \frac{1}{2}$

Theoritical prob of Tail $= 1-p = \frac{1}{2}$

and

or, $P\left(\left|\frac{x}{n} - p\right| \leq 0.01\right) \geq 0.95$ ~ (i)

or, and the lower bond of prob by BLLN is

$$P\left(\left|\frac{x}{n} - p\right| \leq K\right) \geq 1 - \frac{1}{nK^2}$$

$$\Rightarrow P\left(\left|\frac{x}{n} - p\right| \leq 0.01\right) \geq 1 - \frac{1}{n(0.01)^2} \quad \text{--- (ii)}$$

$$0.95 = 1 - \frac{pq}{n(0.01)^2}$$

$$\frac{pq}{n(0.01)^2} = 1 - 0.95$$

$$n = 50000$$

$$\therefore \; P(8 < X < 16) \geq 1 - \frac{1}{(5/3)^2}$$

$$\therefore \; P(8 < X < 16) \geq \frac{\left(\frac{5}{3}\right)^2 - 1}{\left(\frac{5}{3}\right)^2} = \frac{16}{25} \; //$$

(ii) The upper bound of prob is,

$$P\left(|X - E(X)| \geq c\right) \leq \frac{V(X)}{c^2}$$

$$P\left(|X - 11| \geq c\right) \leq \frac{9}{c^2} \quad \text{eq-(i)}$$

By question,

$$P\left(|X - 11| \geq c\right) \leq 0.09 \quad \text{eq-(ii)}$$

$$\frac{9}{c^2} = 0.09$$

$$c^2 = \frac{9}{0.09} \quad c = 10 \; //$$

③ soln

arival rate ($\lambda$) = 3 trucks per hour.

service rate ($\mu$) = 4 trucks per hours.

This is M/M/1 ($\infty$, FIFO)

Traffic intensity ($\rho$) = $\frac{\lambda}{\mu} = \frac{3}{4} = 0.75$

# varias of operating characteristics (oc) (M/M/1) ($\infty$, FIFO)

(i) probability that there are 0 costumer in a queue system ($P_0$) = $1-\rho$
$= 1-0.75$
$= 0.25$

(ii) probability that there are n costumer in a queue system ($P_n$) = $P_0 \rho^n$
$= 0.25 \times (0.75)^n$

(iii) $P_w = \rho = 0.75$

(iv) $L_s = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = \frac{0.75}{0.25} = 3$ trucks.

(v) $L_q = \rho L_s = 0.75 \times 3 = 2.25$ trucks.

(vi) $W_s = \frac{L_s}{\lambda} = \frac{3}{3} = 1$ hours

(vii) = $W_q = \frac{L_q}{\lambda} = \frac{2.25}{3} = 0.75$ hours.

(iii) $L_q = 0.5 \times 1 = 0.5$

(iii) $P_2 = ?$

$$P_n = P_0(\rho)^n$$
$$= (1-0.5)(0.5)^n$$

$$P_2 = (0.5)(0.5)^2$$
$$= 0.125$$

IU

(8) soln

arrival rate $(\lambda) = 30$ trains per days.

36 min 1 train is served

1 min $\frac{1}{36}$ " "

60 min $\frac{1}{36} \times 60$ can

24 hours $\frac{1}{36} \times 60 \times 24$

1 day 40 trains

$(\mu) = 40$

$M/M/1 (\infty, FIFO)$

$\rho = \frac{\lambda}{\mu} = 0.75$

$$\frac{1.64 + 1.65}{2} \quad 1.$$

from Standard normal table,

$$z_1 = 1.645$$

$$z_1 = \frac{x - 45}{20}$$

$$x = 77.9$$

$$x = 78$$

(iii) Find the Minimum Marks obtained by top 20% of students and Maximum Marks obtained by below 20% of the students.

(iv) Find the limits of Marks of Middle 40% of the students.

(iii) soln

Let, the Minimum marks obtainted top 20% of the students $x_1$

By qustion.

$$P(X > x_1) = 20\% = 0.2$$

$$P\left(Z > \frac{x_1 - \mu}{\sigma}\right) = 0.2$$

$x = \frac{2z}{3}$ eq (i) $\quad x + \frac{z}{3} = y \sim$ (ii)

$y = z \sim$ (iii)

$x + y + z = 1 \sim$ (iv)

$x + y + z = 1$

$\frac{2z}{3} + z + z = 1$

$z = 3/8$

$y = 3/8$

$x = \frac{2 \times 3/8}{3}$

$= 1/4$ //

So, The steady state probabilities are.

$x = 1/4 \quad y = 3/8 \quad z = 3/8$

$S = \left[ 1/4, 3/8, 3/8 \right]$

or, $P\left(-K < \frac{X}{n} - \frac{1}{2} < K\right) \geq 1 - \frac{1}{4nK^2}$

or, $P\left(\frac{1}{2} - K < \frac{X}{n} < K + \frac{1}{2}\right) \geq 1 - \frac{1}{4nK^2}$

$\frac{1}{2} - K = 0.4$ $\quad K + \frac{1}{2} = 0.6$

$K = (0.1)^2$

$\therefore P\left(0.4 < \frac{X}{n} < 0.6\right) \geq 1 - \frac{1}{4 \times n \times (0.1)^2}$ ~ (ii)

$1 - \frac{1}{4n(0.1)^2}$

Now, By question

$P\left(\left|\frac{X}{n} - p\right| < K\right) \geq 0.90$

or, $P\left(\left|\frac{X}{n} - \frac{1}{2}\right| < 0.1\right) \geq 0.90$ eq ~ (iii)

$1 - \frac{1}{4 \times n \times (0.1)^2} = 0.90$

$1 - 0.90 = \frac{1}{n \times 0.04}$

$n = 250$

# Markov's chain (MC).

The Markov's chain is the process having finite no of states. A stachastic process $\{X_n ; n = 0, 1, 2 \ldots\}$ is called a Markov's chain if for $\underbrace{i_0, i_1, i_2, i_3, i_4, i_5}_{\text{states}} \in I$ (state space), then for any time 'n' if

$$P\{x_{n+1} = i_5 / x_n = i_4, x_{n-1} = i_3, \ldots x_0 = i_0\} =$$

$$P(x_{n+1} = i_5 / x_n = i_4)$$, then the process is called the Markov's chain.

NOTE:- A Markov's chain for which the probability $P\{X_{n+1} = i_5 / x_n = i_n\}$ is same for all $n > 0$ is a time homogenous or stationery Markov's chain.

# Transition probability Matrix. (tpm):-

Consider a Markov's chain with finite state space $i = 0, 1, 2, 3$ then the transistion probabilities $p_{ij}$ arranged in a matrix in the form.

future state

$$P_{ij} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} \\ p_{10} & p_{11} & p_{12} & p_{13} \\ p_{20} & p_{21} & p_{22} & p_{23} \\ p_{30} & p_{31} & p_{32} & p_{33} \end{bmatrix} \end{array}$$

current state

(i) $p_{ij} > 0$

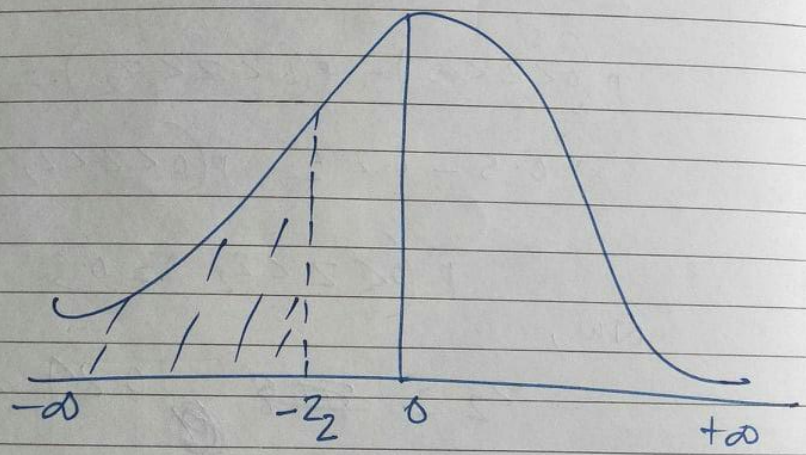
(ii) sum ~~taken elements of~~ each row is unity (1).

Let, the maximum marks obtained below 20% of students be $x_2$ then,

$$p(x < x_2) = 0.2$$

$$p\left(z < \frac{x_2 - 45}{20}\right) = 0.2$$

$$p(z < -z_2) = 0.2$$




$$p(-\infty < z < 0) - p(-z_2 < z < 0) = 0.2$$

$$0.5 - 0.2 = p(0 < z < z_2)$$

$$p(0 < z < z_2) = 0.3$$

$$z_2 = -0.84$$

$$-z_1 = \frac{45-\mu}{\sigma}$$

$$\sigma, -0.5 = \frac{45-\mu}{\sigma} \quad eq-(*)$$

again, from eq-(ii) we get,

$$P(X>64) = 0.08$$

Now,

for $P(X>64)$, $Z = \frac{X-\mu}{\sigma} = \frac{64-\mu}{\sigma} = z_2$ (let)

$$P(X>64) = P(Z>z_2) = 0.08 \text{ (let)}$$

$$P(0<Z<\infty) - P(0<Z<z_2) = 0.08$$

$$0.5 - P(0<Z<z_2) = 0.08$$

$$P(0<Z<z_2) = 0.42$$

$$z_2 = 1.41$$

$$\therefore \quad 1.41 = \frac{64-\mu}{\sigma} \quad \text{--}(**)$$

(14) $\lambda = 1/20$

$\mu = \frac{1}{36}$

M/M/1 (N, FIFO)

$\rho = \frac{\lambda}{\mu} = \frac{1}{20} \times \frac{36}{1} = 1.8 > 1$

$P_0 = \frac{\rho - 1}{\rho^{N+1} - 1} = \frac{1.8}{(1.8)^5 - 1} = \frac{1.8}{17.89568}$

$= 0.10058$ //

(15) PU

$\lambda = 2$ per hours

20 min · 1

1 min $= \frac{1}{20}$

60m $= \frac{60}{20} = 3$

$\mu = 3$

$N = 4$

M/M/1 (N, FIFO)

(i) $P_4 = ?$ (ii) $P_W$ //

$$P(A_1/B) = \frac{P(A_1) \times P(B/A_1)}{P(A_1 \cap B) \text{ or } P(A_2 \cap B)}$$

$$= \frac{0.02 \times 0.04}{0.02 \times 0.04 + 0.98 \times 0.05}$$

$$= 0.277286$$

## CHAPTER :- 2 (Random variable and probability distribution)

A Random variable is that kind of variable which is used to denote the various outcomes within a sample space 'S'. In otherwords, A r.v is a real valued function of sample points. The r.v are denoted by capital letters like X, Y, Z etc.

Let, Two coins are tossed simultenously then, the sample space 'S' is $\{\underset{2}{HH}, \underset{1}{HT}, \underset{1}{TH}, \underset{0}{TT}\}$

Let, X = no of heads. Here, X is r.v taking values 0, 1 and 2. and

$P(X=0) = 1/4$ , $P(X=1) = 2/4$ , $P(X=2) = 1/4$

| No of heads (X) | 0 | 1 | 2 | |
|---|---|---|---|---|
| $P(X=x)$ or $P(x)$ | 1/4 | 2/4 | 1/4 | = 1 |

This, tabular representation is called probability distribution of r.v X.

$e^{3}$

$v(x) = 2 - 1$
$= 1$

$v(x) = \sigma^2$

$\sigma = 1$

$P(|X - E(X)| \geq k\sigma) \leq \frac{1}{k^2}$

$\therefore \quad k\sigma = \sigma$
$k = 6$ so,

$P(|X-1| \geq 6) \leq \frac{1}{36}$

$E(X) = 1$
$v(X) = 1$

$P(|X-1| \geq 2) \leq 1/4$

$\because$ v BD

$P(|X - E(X)| > k\sigma) \leq \frac{1}{k^2}$

$P(|X-1| > 2) \leq \frac{1}{4} \leq \frac{1}{4}$

$P(|X-1| > 2)$

$= P(-2 < X-1 < 2)$

$= P(-1 < X < 3)$

$= \int_{-1}^{3} f(x)\,dx$

$= \int_{0}^{3} e^{-x}\,dx$

$= \left[-e^{-x}\right]_{0}^{3}$

$= e^{-3}$

$$V(X) = \cancel{\alpha^2\beta^2} + \alpha\beta^2 - \cancel{\alpha^2\beta^2}$$

$$= \alpha\beta^2 \;//$$

⑥ Beta Distribution.

A r.v $x$ is said to follow 'β' Beta distribution with parameter M and N if its p.d.f is given by

$$f(x) = \begin{cases} \dfrac{x^{m-1}(1-x)^{n-1}}{\beta(m,n)} & ; 0 < x < 1 \\ 0 & ; \text{otherwise} \end{cases}$$

It is denoted as $X \sim \beta(m,n)^{\alpha,\beta}$.

Here, $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

$$\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$$

(i) $p(X<90) = \int_0^{90} f(x)\,dx$

$= \int_0^{90} \frac{1}{100} e^{-x/100}\,dx$

$= \cancel{100} - \frac{1}{\cancel{100}} \left[ e^{-x/100} \right]_0^{90}$

$= -\left[ e^{-90/100} - e^{0} \right]$

$= e^{0} - e^{-90/100}$

$= 0.593$ //

(ii) $p(X \geq 40) = \int_{40}^{\infty} \frac{1}{100} e^{-x/100}\,dx$

$= \frac{-\cancel{100}\,1}{\cancel{100}} \left[ e^{-x/100} \right]_{40}^{\infty}$

$= -\left[ e^{-\infty} - e^{-40/100} \right]$

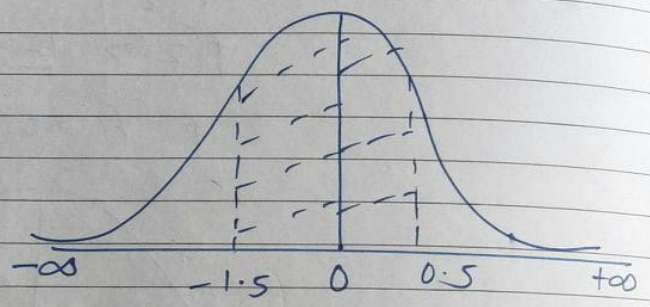
$= e^{-40/100}$

$= \frac{1}{e^{0.4}} = 0.67$ //

(ii) P(between 9 and 13 min)

or, $P\left(\frac{9-12}{2} < z < \frac{13-12}{2}\right)$

or, $P(-1.5 < z < 0.5)$




$P(0 < z < 1.5) + P(0 < z < 0.5)$

$0.4332 + 0.1915$

$0.6247$

between 9 to 13.

days = 228 days //

$$V(x) = E(x^2) - \{E(x)\}^2$$

Now,

$$E\{x^2\} = \theta \int_0^\infty x^2 e^{-\theta x} dx$$

$$= \theta \int_0^\infty e^{-\theta x} x^{3-1} dx$$

$$= \theta \frac{\gamma 3}{\theta^3}$$

$$= \frac{1}{\theta^2} (3-1)!$$

$$= \frac{2}{\theta^2}$$

Formula

$$\int_0^\infty e^{-x} x^{\alpha-1} dx = \gamma \alpha$$

$$\int_0^\infty e^{-\beta x} x^{\alpha-\beta=1} dx = \frac{\gamma \alpha}{\beta^\alpha}$$

$$\gamma n = (n-1)!$$

$$V(x) = \frac{2}{\theta^2} - \frac{1}{\theta^2}$$

$$= \frac{1}{\theta^2}$$

$1-p = 0.85$

$1-0.85 = p$

$p = 0.15.$

$q = 0.10$

Now,

(i) $$= \frac{1}{p+q}\begin{bmatrix} q & p \\ q & p \end{bmatrix} + \frac{(1-q-p)^n}{p+q}\begin{bmatrix} p & -p \\ -q & q \end{bmatrix}$$

$$= \frac{1}{0.15+0.10}\begin{bmatrix} 0.10 & 0.15 \\ 0.10 & 0.15 \end{bmatrix} + \frac{(1-0.10-0.15)^4}{0.15+0.10}\begin{bmatrix} 0.15 & -0.15 \\ -0.10 & 0.10 \end{bmatrix}$$

$$= \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.59 & 0.41 \\ 1 & 0.27 & 0.73 \end{array}$$

% of customers moving from A to B after four weeks is 41%

B to A = 27%

(ii) $\lim_{n\to\infty}$ ;

$$\lim_{n\to\infty} = \frac{1}{p+q}\begin{bmatrix} q & p \\ q & p \end{bmatrix}$$

$$= \frac{1}{0.25}\begin{bmatrix} 0.10 & 0.15 \\ 0.10 & 0.15 \end{bmatrix}$$

(2013 spring)

2ⓐ soln

$$f(x) = \begin{cases} \frac{1}{2} e^{-x/2} & ; x > 0 \\ 0 & \text{otherwise.} \end{cases}$$ //

$$E(x) = \int_0^\infty x \frac{1}{2} e^{-x/2} \, dx$$

$$= \frac{1}{2} \int_0^\infty e^{-x/2} \, x^{2-1} \, dx$$

$$= \frac{1}{2} \frac{\gamma 2}{(1/2)^2}$$

$$2$$

$$E(x^2) = \frac{1}{2} \int_0^\infty e^{-x/2} \, x^{3-1} \, dx$$ //

$$= \frac{\gamma 3}{(1/2)^3} \times (1/2)$$

$$2 \times 4$$

$$8$$

$$V(x) = 8 - (2)^2$$

$$= 4$$ //

Q 1)

soln

If a coin is tossed three times.

$S = \{HH, HT, TH, TT\} \{H, T\}$

$= \{HHH, HTH, THH, TTH, HHT, HTT, THT, TTT\}$

Let, $x$ = no of heads then, $x = 0, 1, 2, 3$

and

$P(x=0) = 1/8$, $P(x=1) = 3/8$, $P(x=2) = 3/8$

$P(x=3) = 1/8$

so, probability distribution is

| No of heads $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X=x)$ $P(x)$ | 1/8 | 3/8 | 3/8 | 1/8 |

Probability that there are 5 trucks in a system.

$= 0.25 \times (0.75)^5$

$= 0.05932617$ //

④ soln

arrival rate $(\lambda) = 10$ cars per hours

~~service rate $(\mu)$ = 20~~

Here, 3 min one car is served

1 min " $\frac{1}{3}$ car served.

60 min " " $\frac{1}{3} \times 60$

20 cars per hour.

$\therefore$ service $(\mu) = 20$

This is $M/M/1(\infty, FIFO)$

Traffic intensity $(\rho) = \frac{\lambda}{\mu} = \frac{10}{20} = 0.5$

(i) $L_s = \frac{\rho}{1-\rho} = \frac{0.5}{1-0.5} = 1$

$L_q = \rho L_s = 0.5 \times 1$

$= 0.5$

(ii) $W_q = \frac{L_q}{\lambda} = \frac{0.5}{10} = 0.05$ hours.

(iii) $L_q =$

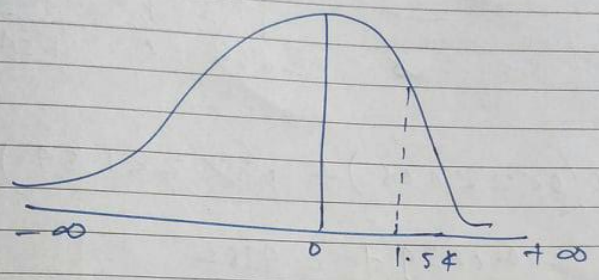
(iii) $P_2 =$

IU

⑧ soln

(ii) $P(X > 158) = P\left(\frac{X-\mu}{\sigma} > \frac{158 - 150.3}{5}\right)$

$= P(Z > 1.54)$




$P(Z > 1.54) = P(0 < Z < \infty) - P(0 < Z < 1.54)$

$= 0.5 - 0.4382$

$= 0.0618$ //

between 150 and 158cm then,

$P\left(\frac{150 - 150.3}{5} < Z < \frac{158 - 150.3}{5}\right)$

$P(-0.06 < Z < 1.54)$

(IMP)

# In a distribution exactly normal 31% of the items are under 45 and 8% of the items 64 and find the Mean and s.d of distribution

Let, 'x' be the normal random variable (r.v)
By question;

$p(x < 45) = 31\%$ ~ (i)
$p(x > 64) = 8\%$ ~ (ii) then,

Mean ($\mu$) = ?
s.d ($\sigma$) = ?

Now,
from eq ~ (i) $p(x < 45) = 31\% = 0.31$

for $(x < 45)$, $z = \frac{x-\mu}{\sigma} = \left(\frac{45-\mu}{\sigma}\right) = z_1$ (let)

so,

$p(x < 45) = p(z < -z_1) = 0.31$

or, $p(-\infty < z < 0) - p(-z_1 < z < 0) = 0.31$

or, $p(0 < z < \infty) - p(0 < z < z_1) = 0.31$

$0.5 - 0.31 = p(0 < z < z_1)$

$p(0 < z < z_1) = 0.19$

$z_1 = 0.5$ so,

Hw

# If the Mean no of sucides per Month in a large country is 1. then find the probability that In the current year there will be

$\lambda = 1 \times 12$

$\lambda = 12$

(i) At least 10 sucides
(ii) At Most 5 "
(iii) No sucides
(iv) exactly 8 sucides.

(Possion approximiation of Binomial distribution)

# There are 2% deffective items in a lot of certian articles. if 100 items are randomly selected from the lot find the probability that the sample contains
(i) exactly 5 deffective items.
(ii) No deffective items.
(iii) At Most 2 deffective items.

Soln

probability of deffective articles $(p) = 0.02$

no of trials $(n) = 100$

Now, $\lambda = np = 2$

Let $x$ = no of deffective articles.

$x \sim p(\lambda)$

$x \sim p(\lambda = 2)$ pmf of $x$ is

$$P(x = n) = \frac{e^{-\lambda} \lambda^{n}}{n!}$$

$$E(x^2) = \sum_{x=-1}^{1} x^2 p(x)$$

$$= (-1)^2 \times 1/8 + 0 + (1)^2 \times 1/8$$

$$= \frac{1}{4}$$

$$\text{or, } V(x) = E(x^2) - (E(x))^2$$

$$V(x) = \sigma^2$$

$$= \frac{1}{4}$$

$$\sigma = \frac{1}{2}$$

$$P(|x - E(x)| \geq 2\sigma) = P(x - 0 \geq 2 \times 1/2)$$

$$= P(|x| \geq 1)$$

$$= 1 - P(|x| < 1)$$

$$= 1 - P(-1 < x < 1)$$

$$= 1 - P(x = 0)$$

$$= 1 - 3/4 = 1/4 \quad //$$

again, th
che

HW

# A. r.v. 'x' ha
the cu

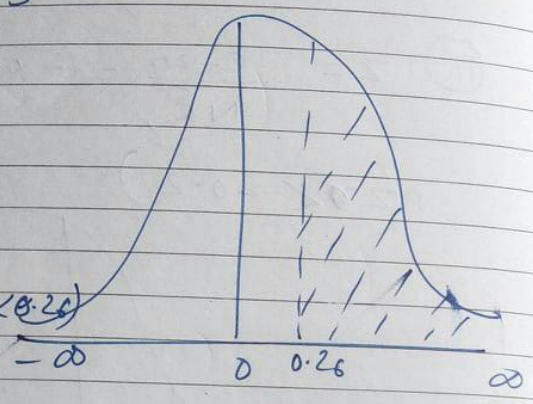
and show

soln

(iii) $p(x > 21.3)$

$$z = \frac{21.3 - 20}{5} = 0.26$$

$p(z > 0.26)$




$p(0 < z < \infty) - p(0 < z < 0.26)$

$= 0.5 - 0.1026$

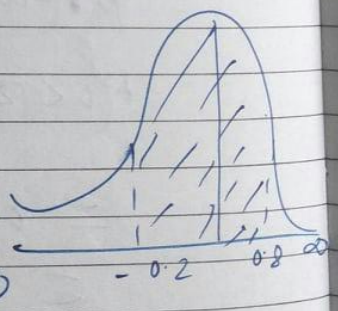
$= 0.3974$ //

(iv) $p(19 < x < 24)$

for $x = 19$

$$z = \frac{19 - 20}{5} = -0.2$$

$$z = \frac{24 - 20}{5} = 0.8$$




$= p(-0.2 < z < 0.8)$

$= p(0 < z < 0.2) + p(0 < z < 0.8)$

HW (K = 10 ans)

# What MUST be the value of K in Chebyshev's inequality for which the prob that a r.v takes a value between $\mu - K\sigma$ and $K\sigma + \mu$ is at least 0.99.

$$P(\mu - K\sigma < X < \mu + K\sigma) \geq 0.99$$

$$P(|X - E(X)| < K\sigma) \geq 0.99 \quad \sim (i)$$

$$P(|X - E(X)| < K\sigma) \geq 1 - \frac{1}{K^2} \quad \sim (ii)$$

comparing eqn (i) and (ii) then,

$$1 - \frac{1}{K^2} = 0.99$$

$$1 - 0.99 = \frac{1}{K^2} \qquad K^2 = 100 \qquad K^2 = \pm 100 \qquad \therefore K = 10$$

PU 9

# A discrete r.v 'X' takes the values $-1, 0, 1$ with prob $\frac{1}{8}, \frac{3}{4}, \frac{1}{8}$. Evaluate probabilities $P(|X - E(X)| \geq 2\sigma)$ and compare it with upper bound of probability obtained by Chebyshev's inequality.

Soln

The prob distribution of X is,

| X | -1 | 0 | 1 |
|---|---|---|---|
| P(x) | 1/8 | 3/4 | 1/8 |

$$E(X) = \sum_{x=-1}^{1} x\,p(x) = -1 \times \frac{1}{8} + 0 + 1 \times \frac{1}{8}$$

$$E(X) = 0$$

$= 0.0793 + 0.2881$

$= 0.3674$ //

(v) $P(X < x_1) = 0.1$ find the value of $x_1$

$P(X < x_1) = 0.1$ then,

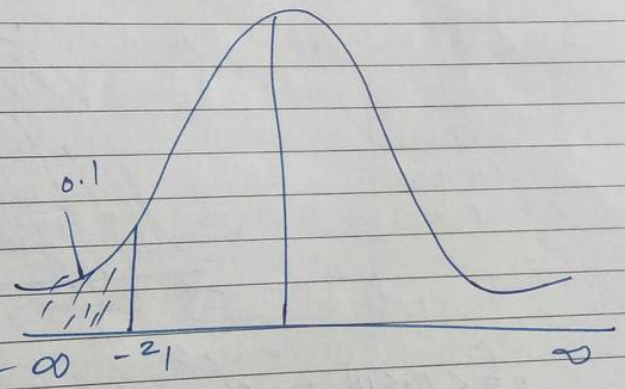
for $X = x_1$

$$z = \frac{x_1 - 20}{5} = z_1$$

$$-1.28 = \frac{x_1 - 20}{5}$$

$$x_1 = 13.6$$ //

$$P(X < x_1) = P(Z < -z_1) = 0.1$$




$$P(0 < Z < \infty) - P(0 < Z < z_1) = 0.1$$

$$0.5 - P(0 < Z < z_1) = 0.1$$

$$P(0 < Z < z_1) = 0.4$$

$$1.2 + 0.08$$

$$1.28$$ //

$P(Z < -1) = P(-\infty < Z < 0) - P(-1 < Z < 0)$

$= P(0 < Z < \infty) - P(0 < Z < 1)$

$= 0.5 - 0.3413$

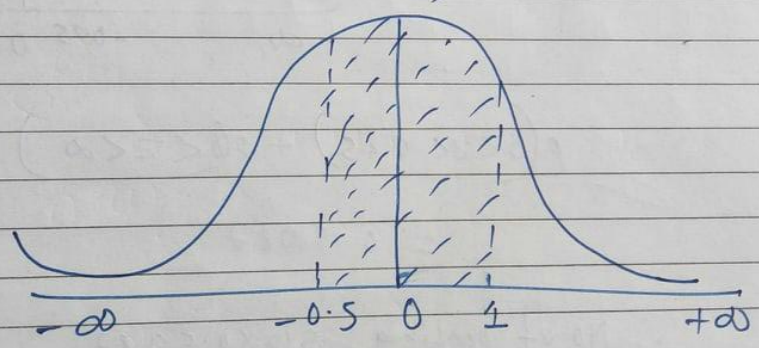
$= 0.1587$ //

(iii) Between 50 and 80 Marks

$P(50 < X < 80)$ Now,

$$P\left(\frac{50-60}{20} < Z < \frac{80-60}{20}\right)$$

$$P(-0.5 < Z < 1)$$




$= P(-0.5 < Z < 0) + P(0 < Z < 1)$

$= P(0 < Z < 0.5) + P(0 < Z < 1)$

$= 0.1915 + 0.3413$

$= 0.5328$ //

$$p(X<0.2) = 6\int_0^{0.2} (x - x^2)\,dx$$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{0.2}$$

$$= 6\left[\frac{0.04}{2} - \frac{0.008}{3}\right]$$

$$=$$

(iii) $$p(0.2 < X < 0.5) = 6\int_{0.2}^{0.5} (x - x^2)\,dx$$

(iv) $$E(X) = \frac{m}{m+n}$$

$$V(X) = \frac{mn}{m+n}$$

Problem set 4

3(a) soln

in min.

Let $X$ = duration of time that a person spends in a phone.

$E(X) = 6$ min.

$\theta = \frac{1}{6}$

Here $X \sim Exp(\theta = 1/6)$

$$f(x) = \begin{cases} \frac{1}{6} e^{-x/6} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$$

(i) $P(X > 8) = \int_8^{\infty} f(x)\,dx$

(ii) $P(4 < X < 8) = \int_4^8 f(x)\,dx$

2013 spring (6a)

Imp

Let, $\{X_n, n \geq 0\}$, be a mc defined on state 0, 1 and 2. It is given that $P(X_0 = i) = 1/3$ for $i = 0, 1, 2$.

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

obtain, $P(X_2 = 1, X_0 = 0)$

Soln:

By question,

$P(X_0 = i) = 1/3$ for $i = 0, 1, 2$ i.e.

$P(X_0 = 0) = P(X_0 = 1) = P(X_0 = 2) = 1/3$

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

two-step tpm is

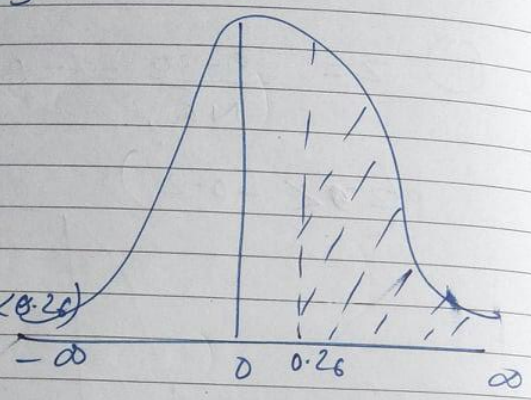
$$P^2 = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix} \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 5/8 & 5/16 & 1/16 \\ 5/16 & 9/16 & 3/16 \\ 3/16 & 9/16 & 4/16 \end{bmatrix}$$

(iii) $p(x > 21.3)$

$$z = \frac{21.3 - 20}{5} = 0.26$$

$p(z > 0.26)$




$p(0 < z < \infty) - p(0 < z < 0.26)$

$= 0.5 - 0.1026$

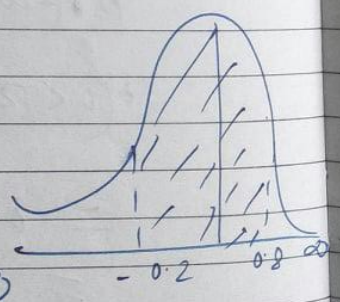
$= 0.3974$ //

(iv) $p(10 < x < 24)$

for $x = 19$

$$z = \frac{19 - 20}{5} = -0.2$$

$$z = \frac{24 - 20}{5} = 0.8$$




$= p(-0.2 < z < 0.8)$

$= p(0 < z < 0.2) + p(0 < z < 0.8)$

$\therefore$ No of trials $(n) = 6$

Probability of Meassage containing 1 $(p) = 0.3$
Probability of containing 0's $(q) = 0.7$ //

Let,
$x$ = no of 1's
Now,
Here, $X \sim B(6, 0.3)$, so, Probability mass fxn of $X$ is given by

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \quad ; x = 0, 1, 2 \ldots n$$

$$= {}^6C_x (0.3)^x (0.7)^{n-x} \quad x = 0, 1 \ldots 6$$

(a) The no of 1's will follow Binomial distribution with parameter $n = 6$ and $p = 0.3$

(b) $$P(X=3) = {}^6C_3 (0.3)^3 (0.7)^{6-3}$$

$$= 20 \times 0.027 \times 0.343$$

$$= 0.18522$$ //

Here, Mean $E(x) = \int_0^1 x\, f(x)\, dx$

$$= \int_0^1 x \cdot \frac{x^{m-1}(1-x)^{n-1}}{\beta(m,n)}\, dx$$

$$\frac{1}{\beta(m,n)} \int_0^1 x^{m+1-1} (1-x)^{n-1}\, dx$$

$$= \frac{1}{\beta(m,n)} \beta(m+1, n)$$

$$= \frac{\gamma m+1 \; \cancel{\gamma n}}{\gamma m+n+1} \; \frac{\gamma m+n}{\gamma m \; \cancel{\gamma n}}$$

$$= \frac{m \cancel{\gamma m} \; \cancel{\gamma m+n}}{m+n \; \cancel{\gamma m+n} \; \cancel{\gamma m}}$$

$$= \frac{m}{m+n} \;//$$

$$V(x) = \frac{mn}{(m+n)^2 (m+n+1)} \;//$$

Soln

Let,

$R_0$ = event that '0' is received.

$R_1$ = event that '1' is received.

$T_0$ = event that '0' is transmitted.

$T_1$ = event that '1' is transmitted.

By question,

$P(R_0/T_0) = 0.94$ , $P(R_1/T_1) = 0.01$

$P(R_1/T_0) = 0.06$ $P(R_0/T_1) = 0.09$

Now,

$P(T_0) = 0.45$

$P(T_1) = 0.55$

again,

$$\frac{1}{\beta^{\alpha}\Gamma\alpha}\,\frac{\alpha\Gamma\alpha}{\frac{1}{\beta^{\alpha+1}}}$$

$$E(X) = \alpha\beta \;//$$

again,

$$E(X^2) = \int_0^{\infty} x^2 \frac{e^{-x/\beta}\, x^{\alpha-1}}{\beta^{\alpha}\Gamma\alpha}\, dx$$

$$= \frac{1}{\beta^{\alpha}\Gamma x}\int_0^{\infty} e^{-x/\beta}\, x^{\alpha+2-1}\, dx$$

$$= \frac{1}{\beta^{\alpha}\Gamma x}\,\frac{\Gamma\alpha+2}{\left(\frac{1}{\beta}\right)^{\alpha+2}}$$

$$= \frac{\Gamma+1\;\Gamma(\alpha+1)}{\beta^{\alpha}\Gamma x\;\frac{1}{\beta^{\alpha}}\cdot\frac{1}{\beta^{2}}}$$

$$\left\{\frac{(\alpha+1)\,\alpha\,\Gamma\alpha}{\Gamma\alpha}\right\}\beta^2$$

$$= (\alpha^2+\alpha)\,\beta^2$$

$$= \alpha^2\beta^2+\alpha\beta^2$$

(c) $P(\text{At least 2 ones}) = P(X=2)+P(X=3)+P(X=4)+P(X=5)+P(X=6)$

or,

$= 1 - P(X<2)$

$= 1 - [P(X=0)+P(X=1)]$

or, $= 1 - [{}^6C_0(0.3)^0(0.7)^6 + {}^6C_1(0.3)^1(0.7)^5]$

$= 1 - (0.117649 + 0.302526)$

$= 0.579825$ //

(d) ~~$P(X=2)$~~

$P(\text{At most 4 1's}) = P(X \le 4)$

$= 1 - P(X>4)$

$= 1 - [P(X=5)+P(X=6)]$

$= 1 - [{}^6C_5(0.3)^5(0.7)^1 + {}^6C_6(0.3)^6(0.7)^0]$

$= 0.868021$

(e) mean $(n \times p) =$

variance $(npq) =$

2013 spring
Q No 1 b)

It has been
installation
third. Acc
the utility

(i) 4 of 5
(ii) At lea

soln

no
probability
at least one th

probability
"

Let $X =$
reduced

Here,

$P(X=$

Mean ($\mu$) = 12 min

sd ($\sigma$) = 2 min

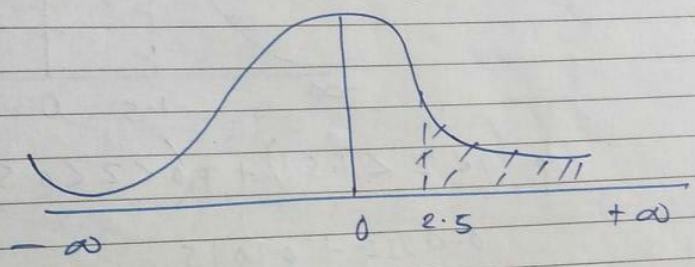
ie $X \sim N(13, 2^2)$

Total no of days in a year (N) = 365

① $P(X > 17 \text{ mins}) = P\left(\frac{X-\mu}{\sigma} > \frac{17-\mu}{\sigma}\right)$

$P\left(Z > \frac{17-12}{2}\right)$

$P(Z > 2.5)$




$P(0 < Z < \infty) - P(0 < Z < 2.5)$

$= 0.5 - 0.4938$

$= 0.0062$ //

∴ No of days in a year when he takes longer than 17 min in a year = $N \times 0.0062$

2.262

≈ 2 days //

(e) $P\{(3/2 < x < 7/2) / x > 2\}$

$$= \frac{P\{(3/2 < x < 7/2) \cap (x > 2)\}}{P(x > 2)}$$

$$= \frac{P(x = 1, 2, 3 \cap x = 3, 4)}{P(x > 2)}$$

$$= \frac{P(x = 3)}{P(x > 2)} = \frac{3}{7}$$

(f) $P(x \leq \eta) > 0.5$

Here, if $\eta = 0$ $\quad P(x \leq 0) = 0 < 0.5$

if $\eta = 1$ $\quad P(x \leq 1) = 0.1 < 0.5$

$\eta = 3$ $\quad P(x \leq 3) = 0.6 > 0.5$

$\therefore$ $x = 3$ is the smallest

(iv)

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $F(\eta)$ | 0 | 3/30 | 9/30 | 18/30 | 1 |

probability function

$$F(x \geq 3) = 1 - F(x < 3)$$
$$= 1 - F(x \leq 2)$$
$$= 1 - F(2)$$
$$= 1 - 9/30 = 21/30$$

Operating Characteristics /(OC)/

(i) $P_0$ = prob that there are 0 customer in queuing system.

(ii) $P_n$ = prob that there are n " " " " ,

(iii) $P_w$ = " " arriving customer has to wait for the service

(iv) $L_s$ = Length of system. (average expected no of customers in the system)

(v) $L_q$ = (length of queue) (average or expected no of queue)

(vi) $W_s$ = Average or expected time that a customer spends in a system

(vii) $W_q$ = Average or " " " " in a queue.

$L_s \uparrow \quad L_q$

Some Modals are:-

(1) M/M/1 : (∞, FIFO)

(2) M/M/1 : (∞, SIRO)

(3) M/M/1 : (N, FIFO)

(4) M/M/S : (∞, FIFO)

(5) M/M/S : (N, FIFO)

(6) M/M/1 : M/G/D

(7) M/$E_K$/1 → $E_K$

$$\Rightarrow \frac{e^{-2}(2)^x}{2!} \quad x = 0 \text{ to } \infty.$$

## # Normal distribution

A r.v 'X' is said to follow normal distribution if it's p.d.f is given by;

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \; ; -\infty < x < \infty$$

$$-\infty \; \mu < \infty$$

$$\sigma > 0$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \text{ then,}$$

Here; $\mu$ and $\sigma^2$ are two parameters of normal distribution. It is denoted as $X \sim N(\mu, \sigma^2)$ //

## # Properties of Normal distribution

(1) Normal distribution is a continous probability distribution because it takes the values from $(-\infty$ to $+\infty)$

(2) Normal distribution has 2-parameters i.e. $\mu$ and $\sigma^2$.

(3) For Normal distribution Mean $= E(X) = \mu$ and variance $V(X) = \sigma^2$

(4) Normal distribution is a symmetrical distribution ie (Mean = Medium = mode) $= \mu$

2013 spring
Q No 1 b)

It has been claimed that in 60% of all solar heat installation the utility bill is reduced by at least one third. Accordingly, what are the probabilities that the utility bill will be reduced by at least 1/3 in

(i) 4 of 5 installation
(ii) At least four of 5 installation.

soln

no of trials (n) :- 5

Probability that the utility bill will be reduced by atleast one third $(p) = 0.6$

Probability that the " " " not be " " " " " $(q) = 0.4$

Let $X$ = no of installation in which no of bill will be reduced by 1/3.

Here, $X \sim B(5, 0.6)$ so, the p.m.f of $X$ is,

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x} \quad n = (0, 1 .. n)$$

$$= {}^{5}C_{x} (0.6)^{x} (0.4)^{n-x} \quad (x = 0, .. 5)$$

PTO

$$E(x) = \int_0^\infty e^{-x}\, dx$$

$$= -\left[e^{-x}\right]_0^\infty$$

$$= -\left[e^{-\infty} - e^{-0}\right]$$

$$= 1$$

$\cancel{E(x^2)}$ $$E(x) = \int_0^\infty x e^{-x}\, dx$$

$$= \int_0^\infty e^{-x} x^{2-1}\, dx$$

$$= \frac{\gamma 2}{(1)^2}$$

$$= 1$$

$$E(x^2) = \int_0^\infty x^2 e^{-x}\, dx$$

$$= \int_0^\infty e^{-x} x^{3-2}\, dx$$

$$= \gamma 3$$

$$= (3-1)!$$

$$= 2 \;//$$

(iii) $P\left(X \geq 40 / X > 10\right) = ?$

$$P\left(X \geq 30+10 / X > 10\right) = P(X > 30)$$

By memoryless property

$$= \int_{30}^{\infty} \frac{1}{100} e^{-x/100} \, dx$$

$$= \frac{-100}{100} \left[ e^{-x/100} \right]_{30}^{\infty}$$

$$= \frac{1}{e^{0.3}}$$

$$= 0.7408 \text{ hrs}$$

Central Limit theorem

# Chebyshev's inequality.

Let, 'x' be the r.v with finite Mean $E(x)$ and variance $V(x)$ then, Chebyshev's inequality is defined as,

$$P(|X - E(x)| < k\sigma) \geq 1 - \frac{1}{k^2}$$ (lower bound of probability)

and

$$P(|X - E(x)| \geq k\sigma) \leq \frac{1}{k^2}$$ (upper bound of prob)

When, K is a positive constant (i.e. $K > 0$)

if we say $k\sigma = c$, then,

$$P(|X - E(x)| < c) \geq 1 - \frac{1}{}$$

$$P(|X - E(x)| \geq c) \leq \frac{V(x)}{c^2}$$ [upperbond of prob

$$= \int_0^\infty x^2 \frac{e^{-x} x^{\alpha-1}}{\gamma\alpha} dx$$

$$= \frac{1}{\gamma\alpha} \int_0^\infty e^{-x} x^{\alpha+2-1} dx$$

$$= \frac{1}{\gamma\alpha} \gamma\alpha+2$$

$$= \frac{(\gamma+1) \cdot \gamma(\alpha+1)}{\gamma\alpha}$$

$$= \frac{(\alpha+1)\alpha\cancel{\gamma\alpha}}{\cancel{\gamma\alpha}}$$

$$= \alpha^2+\alpha$$

$$V(x) = \alpha^2+\alpha-\alpha^2$$

$$\therefore V(x) = \alpha$$

(18) Soln

no of server $(S) = 2$

arrival rate $(\lambda) = 10$ per hour,

4 min 1 can serve

1 min $\frac{1}{4}$

1 hour $\frac{60}{4} = 15$

$(\mu) = 15$.

This is, $M/M/S(\infty, FIFO)$

Traffic intensity $(\rho) = \frac{\lambda}{\mu S}$

$= \frac{10}{15 \times 2} = 0.33$

(i) $P_0 = \left[ \sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} + \frac{1}{S!} \frac{(S\rho)^S}{(1-\rho)} \right]^{-1}$

$= \left[ \sum_{n=0}^{2-1} \frac{(2 \times 0.3)^n}{n!} + \frac{(2 \times 0.3)^2}{2!(1-0.3)} \right]^{-1}$

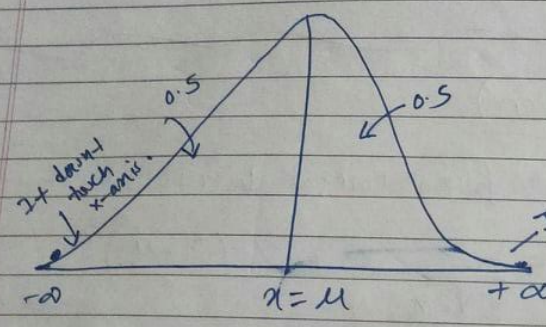
$= \left[ 1 + 2 \times 0.3 + \frac{(2 \times 0.3)^2}{2 \; 0.7} \right]^{-1}$

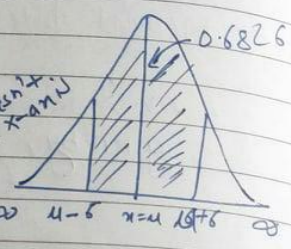
$= \frac{7}{13} = 0.538$ //

(ii) $P_n =$

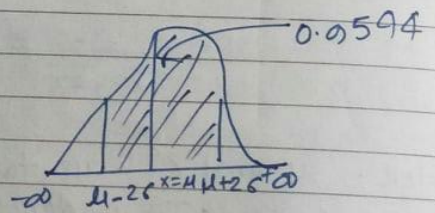
(iii) $L_q =$

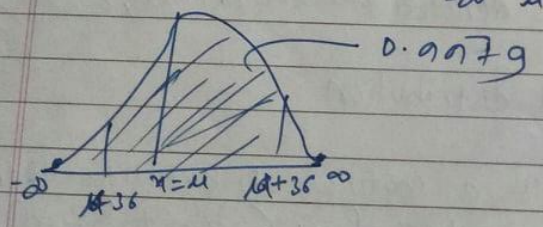
(5) Area property of Normal distribution.




(6) ~~The total area under~~




fm ~~$\mu$~~ $(1 - 0.9979)$
$= (0.0027)$ outer of $\mu-3\sigma$ and $\mu+3\sigma$

as

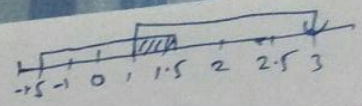
$P(-\infty < x < \infty) = 1$

$P(-\infty < x < \mu) = P(\mu < x < \infty) = 0.5$

$P(\mu-\sigma < x < \mu+\sigma) = 0.6826$

$P(\mu-2\sigma < x < \mu+2\sigma) = 0.9544$

$P(\mu-3\sigma < x < \mu+3\sigma) = 0.9977$




7)

Soln

(i) $\int_0^1 2Kx\,dx + \int_1^2 K(3-x)\,dx + \int_2^3 K\,dx + \int_3^4 K(4-x)\,dx = 1$

$K = \frac{1}{4}$

(ii) $P(X \geq 3) = \int_3^4 \frac{1}{4}(4-x)\,dx$

$= \frac{1}{8}$

(ii) $P(X<2) = P(X=0) + P(X=1)$ 3

$= e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!}\right)$

$= (1+3)e^{-3}$

$= 0.10014$

No $= 0.10014 \times 1000$

$= 100.14 \approx 100$

(iii) $P(X=5) = e^{-3}\left(\frac{3^5}{5!}\right)$

$= 0.100$

No $= 0.100 \times 1000$

$= 100$ //

(iv) $P(X \geq 2) = 1 - P(X<2)$

$= 1 - P(X=0) + P(X=1)$

$= 1 - 0.10014$

$= 0.8000$

No $= 0.8000 \times 1000$

$= 800$ //

Now, $\lambda = 1.5$ for (iv)

$1 - e^{-1.5}\left(\frac{1.5^0}{0!} + \frac{(1.5)^1}{1!}\right)$

$0.4421 \times 1000$

$= 442$ //

(i) $P(X>3) = 1 - P(X \leq 3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$= 1 - e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right)$

$= 1 - e^{-3}\left(1 + 3 + 9 + \frac{9}{2}\right)$

$= 0.128726$

$\therefore$ No of taxi driver with more than 3 accident

$= N \times P(X>3)$

$= 0.1287 \times 1000$

$= 128.7$

$$\left(\frac{a-45}{20}\right) = -0.685$$

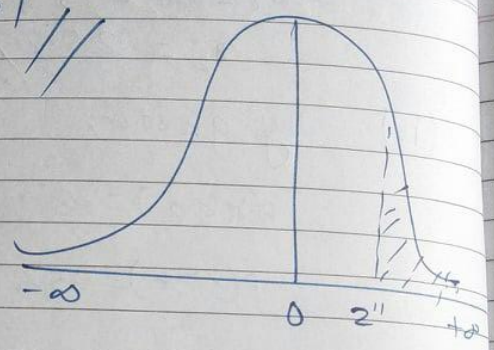
$$a = -13.7 + 45$$

$$a = 31.3 \quad // \qquad a = 31 \quad //$$

again,

$$p(x > b) = 0.25$$

$$p\left(z > \frac{b-45}{20}\right) = 0.25$$




Now,

$$p(z > z'') = 0.25$$

$$p(0 < z < z'') = 0.25$$

so,

$$z'' = 0.685$$

Now,

$$z'' = \frac{b-45}{20}$$

$$0.685 \times 20 + 45 = b$$

$$b = 58.7 \quad //$$

$$\therefore b = 59 \quad //$$

$L_q = L_s - \rho$

$= 2 - 0.67 = 1.33$

and $P_N = P_0 \times (0.67)^{14}$

$= 0.33 \times (0.67)^{14}$

Now, $= 0.00121$ //

$W_s = \dfrac{L_s}{\lambda(1-P_N)} = \dfrac{2}{20(1-0.00121)}$

$= \dfrac{2}{19.9758}$

$= 0.100121\,1466$ //

(vi) $W_q = \dfrac{L_q}{\lambda(1-P_N)} = \dfrac{1.33}{20(1-0.00121)}$

$= 0.06658$ //

(vii) $P_n = P_0 \rho^n$

$= 0.33 \times (0.67)^n$ //

b) $P(A_2/D) = \frac{P(A_2) \times P(D/A_2)}{P(D)} = \frac{0.35 \times 0.04}{0.0225} = 0.62$ //

c) $P(A_3/D) = \frac{P(A_3) \times P(D/A_3)}{P(D)} = \frac{0.45 \times 0.01}{0.0225} = 0.2$

iii) $\bar{D}$ = probability of non-deffective

$P(A_3) = 0.45$ NOW,

$P(A_3/\bar{D}) = \frac{P(A_3) \times P(\bar{D}/A_3)}{P(\bar{D})}$ Here,

$P(\bar{D}/A_3) = 0.99$

$= \frac{0.45 \times 0.99}{1 - 0.0225}$

$= \frac{891}{1955}$ //

(e) probability.

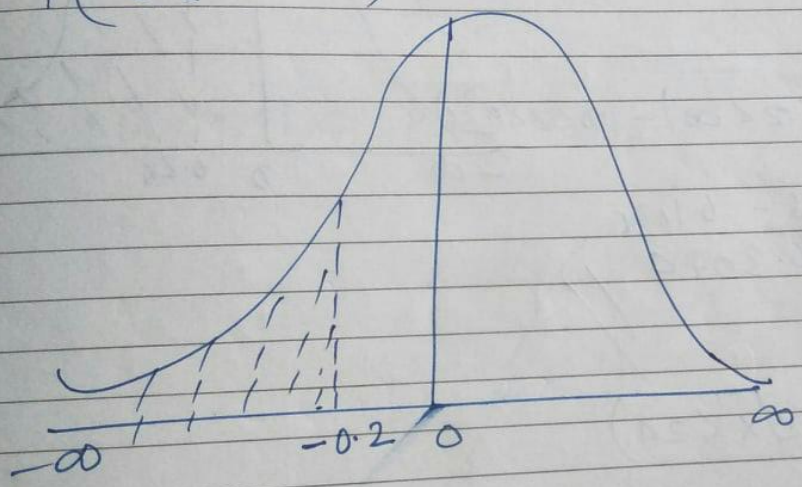
$p(0 < z < 0.4) + p(0 < z < \infty)$ By.

$= 0.1554 + 0.5$

$= 0.6554$ //

(ii) $z = \frac{19-20}{N\xi_0} = -0.2$

$p(z < -0.2)$




$p(-\infty < z < 0) - p(-0.2 < z < 0)$

$p(0 < z < \infty) - p(0 < z < 0.2)$ By symmetry

$= 0.5 - 0.0793$

$= 0.4207$ //

(6)

Soln

(i) $\int_0^1 x\,dx + \int_1^2 (2-x)\,dx = 1$

$c = 2, 2$

(ii) $P(0.8 < X < 0.6 \times 2)$

$P(0.8 < X < 1.2)$

$= \int_{0.8}^{1} x\,dx + \int_{1}^{1.2} (2-x)\,dx$

$= \left[\frac{x^2}{2}\right]_{0.8}^{1} + \left[2x - \frac{x^2}{2}\right]_{1}^{1.2}$

$= 0.18 + 2.4 - \frac{1.44}{2} - 2 + \frac{1}{2}$

$= 0.36$ //

# In a group of register voters 80% of those who voted in the last election will also vote in the next election and 30% of those who didn't vote in last election will vote in next election. In long run, what % of the voters are expected to vote in a given election.

Soln Let state.

0 = voting

1 = Non-voting then,

By question

1' step tpm is,

$$P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.8 & 0.2 \\ 1 & 0.30 & 0.70 \end{array}$$

In long run let steady state matrix of the vector be

$$S = \begin{array}{cc} 0 & 1 \\ \end{array}$$
$$S = \begin{bmatrix} x & y \end{bmatrix}$$

$S = ?$ such that $x+y = 1$ then,

$$SP = S$$

$$\begin{bmatrix} x, y \end{bmatrix} \begin{bmatrix} 0.8 & 0.2 \\ 0.30 & 0.70 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$$\begin{bmatrix} 0.8x + 0.30y & 0.2x + 0.70y \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$\therefore x = 0.8x + 0.30y$

$y = 0.2x + 0.7y$

$x + y = 1$

$x = 3/5$

$y = 2/5$ //

(i) exactly 3 heads

$$P(X=3) = {}^5C_3 \times \left(\frac{1}{2}\right)^5$$

$$= 0.3125$$

(ii) Atleast one head $= 1 - P(X<1)$

$$= 1 - P(X=0)$$

$$= 1 - {}^5C_0 \left(\frac{1}{2}\right)^5$$

$$= 1 - \frac{1}{32} = \frac{31}{32}$$ //

(iii) AtMost 2 heads $= P(X=0) + P(X=1) + P(X=2)$

$$= \left({}^5C_0 + {}^5C_1 + {}^5C_2\right) \frac{1}{32}$$

$$= (1 + 5 + 10) \times \frac{1}{32}$$

$$= \frac{1}{2} = 0.5$$ //

~~n = 4 trials and one head~~

(iv) P(1 head and 4 tails) $= P(X=1)$

$$= {}^5C_1 \times \frac{1}{32}$$

$$= \frac{5}{32}$$ //

(v) (some head) $= \frac{31}{32}$ //

ii) a) $P(x \leq 0.5)$

$$= \frac{3}{4}\int_0^{0.5}(2x - x^2)\,dx$$

$$= \frac{5}{32}$$

b) $P(x > 0.8)$

$$= \frac{3}{4}\int_{0.8}^{2}(2x - x^2)\,dx$$

$$= 0.648 \,//$$

c) $P(x \leq 1.5)$

$$= \frac{3}{4}\int_0^{1.5}(2x - x^2)\,dx$$

$$= \frac{27}{32} \,//$$

(iii) $E(x) = \int_0^2 \frac{3}{4}x(2x - x^2)\,dx$

$$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{2(2)^3}{3} - \frac{(2)^4}{4}\right]$$

$$= \frac{3}{4}\left[\frac{4}{3}\right] = 1$$

c)

Soln

1st box

white balls = 5

black balls = 8

i) a) p(of both white balls) = $p(w_1)$ and $p(w_2)$

$= \frac{5}{13} \times \frac{8}{13} = \frac{40}{169}$ //

b) p(one white and one black ball) = $p(w_1 \cap B_1)$ or $p(B_1 \cap w_2)$

$= \frac{5}{13} \times \frac{8}{13} + \frac{5}{13} \times \frac{8}{13}$ Not allowed

so,

$= \frac{5}{13} \times \frac{5}{13} + \frac{8}{13} \times \frac{8}{13}$

$= \frac{89}{169}$

(vi) $P(\text{some tails}) = 1 - P(\text{No tails})$

$= 1 - P(\text{all heads})$

$= 1 - P(X = 5)$

$= 1 - {}^5C_5 \left(\frac{1}{2}\right)^5$

$= 1 - \frac{31}{32}$

[(*) PU 2019]

# An information shows/emits a 6 digit message into a channel in binary code. Each digit is choosen independently of other and is a one(1) with probability 0.3

(a) Which distribution is applied to the no of digits transmitted as a 1.

(b) Find the probability that message contains three 1's

(c) Find the probability that message contains at least two ones.

(d) Find the probability that message contains no less then two 1's

(e) find mean and varience of no of 1's.

* A r.v 'x' has certain probability distribution with expected valn of 'x' = 11 and v(x) = 9. using chesebyev's inequality find the following

i) A lowerbound for $P(6 < X < 16)$

ii) Find the value of 'c' such that $P(|X-11| \geq c) \leq 0.09$.

Soln

Here;

$E(X) = 11$; $V(X) = 9$

$v(x) = \sigma^2 = \sqrt{9} = 3$

$\sigma = 3$.

By using the sebysher's inequality, the lower bound of probablity is

i)

$$P(|X - E(X)| < K\sigma) \geq 1 - \frac{1}{K^2}$$

or, $$P(|X - 11| < 3K) \geq 1 - \frac{1}{K^2}$$

or, $$P(-3K < X - 11 < 3K) \geq 1 - \frac{1}{K^2}$$

or, $$P(11 - 3K < X < 3K + 11) \geq 1 - \frac{1}{K^2}$$

$P(6 < x < 16)$

$11 - 3K = 6$ $\quad$ $3K + 11 = 16$

$K = 5/3$.

Also, A r.v $x$ is said to follow Gamma distribution with ~~par~~ two parameters $\alpha$ and $\beta$ if it's p.d.f is given by,

$$f(x) = \frac{e^{-x/\beta} x^{\alpha-1}}{\beta^{\alpha} \Gamma\alpha} ; x > 0$$

It is denoted by $x \sim G(\alpha, \beta)$

when,

$$\left[ \frac{\Gamma\alpha}{\beta^{\alpha}} = \int_0^{\infty} e^{-\beta x} x^{\alpha-1} dx \right]$$

Here,

Mean $E(x) = \int_0^{\infty} x f(x) dx$

$$= \int_0^{\infty} x \frac{e^{-x/\beta} x^{\alpha-1}}{\beta^{\alpha} \Gamma\alpha} dx$$

$$\frac{1}{\beta^{\alpha} \Gamma\alpha} \frac{\Gamma(\alpha+1)}{\left(\frac{1}{\beta}\right)^{\alpha+1}}$$

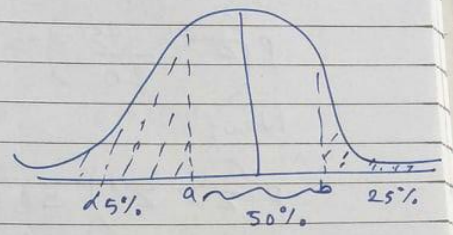
$-16.8 = x_2 - 45$

$x_2 = 28.2$ //

(iv) By question,

$P(X < a) = 25\%$

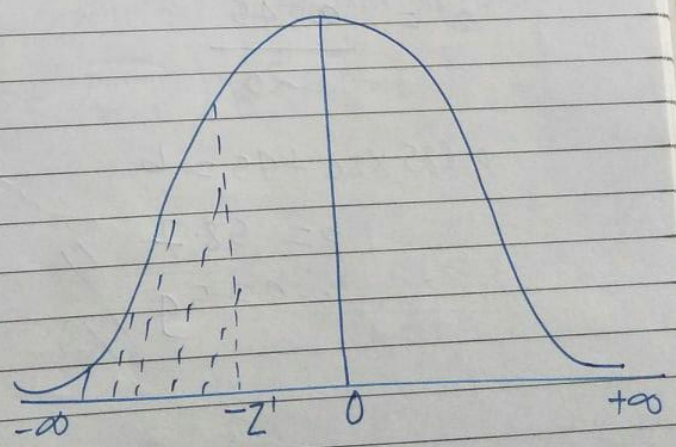
$P(X > b) = 25\%$




$$P\left(Z < \frac{a-45}{20}\right) = 0.25$$

$$P(Z < Z') = 0.25$$

$$P(Z < -Z') = 0.25$$




$$P(0 < Z < Z') = 0.5 - 0.25$$

$$P(0 < Z < Z') = 0.25$$

$$Z' = \frac{0.68 + 0.69}{2} = -0.685$$ //

$x \sim N(\mu, \sigma^2)$

If $X \sim N(20, 25)$ then find

(i) $p(X > 18)$
(ii) $p(X < 19)$
(iii) $p(X > 21.3)$
(iv) $p(19 < X < 24)$
(v) $p\,p(X < x_1) = 0.1$, find the value of $x_1$

Soln

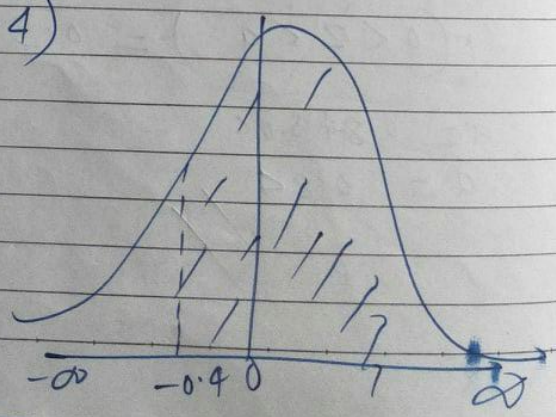
Here, Mean $(\mu) = 20$
variance $\sigma = 5$

$$z = \frac{x - \mu}{\sigma}$$

(i) $p(x > 18)$

$x = 18 \quad z = \frac{18 - 20}{5}$
$= -0.4$

$p(z > -0.4)$



c) $p[(-2<x<2)/x>0] = \dfrac{p\{(-2<x<2) \cap (x>0)\}}{p(x>0)}$

$= \dfrac{p\{(x=-1,0,1) \cap (x=1,2,3)\}}{p(x>0)}$

$= \dfrac{p(x=1)}{p(x>0)}$

$= \dfrac{0.1}{0.4} = 0.25$

d) $F(-1) = 0.5, \quad F(0) = 0.6$ //

(iii) a) Mean $E(x) = \sum_{x=-3}^{3} x\, p(x)$

$= -0.1$

b) variance $V(x) = E(x^2) - \{E(x)\}^2$

Now,

$E(x^2) = \sum_{x=-3}^{3} x^2 p(x)$

$= 9\times0.1 + 4\times0.2 + 1\times0.2 + 0 + 1\times0.1 + 4\times0.2 + 9\times0.1$

$= 3.7$

$$= \frac{(0.33)^2}{1-0.33}$$

$$= 0.16253$$ //

(13) Soln

arrival rate $(\lambda) = 20$ patient per hours.

$(\mu) = 30$ per hour.

size of waiting room $(N) = 4$

This is, $M/M/1 : (N/FIFO)$

traffic intensity $(\rho) = \frac{\lambda}{\mu} = 0.67$

(i) $$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$$

$$= \frac{1-0.67}{1-(0.67)^{15}}$$

$$= 0.33$$

(ii) $$P_n = 0.33 \times (0.67)^n$$

(iii) $$L_s = \frac{0.67}{1-0.67} - \frac{15 \times (0.67)^{15}}{1-(0.67)^{15}}$$

$$= 2.030 - \frac{0.03691}{0.96308411}$$

$$= 1.922$$ //

Queing Theory.

stachastic process.

⇒ A process which involvs time is called a stachastic process. Mathematical Model of system which vary in time in a random manner is called stachastic process.

A stachastic process is a family of given probability space index by parameter t. When t, vary over and index set T. the set of all possible values of all individual r.v x(t) are called state space and values of x(t) are called states.

Type of stachastic process.

Markov's process

1. Discrete time Discrete state space stochastic process (DIDS)
2. DICS :- eg:- volume of water in a dam in a certain point of time.
3. CIDS :- eg:- No of customer in a queue waiting for a service in a bank in certain interval of time. (queue process)
4. CICS :- eg:- Max temp in a day in a certain interval of time.

# Markov's process.

The process which follows memoryless property is called a Markov's process. In other words the process in which the probability of movement of a particle from current state to future state is independent of past behaviour of a particle is called a Markov's process.

# Binomial distribution Numericals.

(*) A coin is tossed five times. find the probability of getting
(i) exactly 3 heads (ii) Atleast one head (iii) Atmost 2 heads
iv) 1 head and 4 tails (v) some heads (vi) some tails.

Soln

no of trial $(n) = 5$

Probability of getting head $= \frac{1}{2}$

Probability of getting a tail $(q) = 1-p = \frac{1}{2}$

Let, $x$ = No of heads

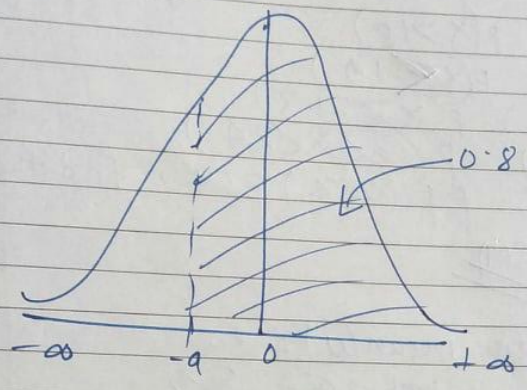
Here, $x \sim B(n = 5)$ so, the p.m.f of $x$ is

$$P(x=x) = \binom{n}{x} p^x q^{n-x} \quad x = 0,1,2 \ldots 5$$

$$P(x=x) = \binom{5}{x}\left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x} \quad x = 0, \ldots 5$$

$$= 5_{C_x} \left(\frac{1}{2}\right)^5$$

(vii) $p(Z > a) = 0.8$ find the value of a.




$p(Z > -a) = 0.8$

or, $p(-a < Z < 0) + p(0 < Z < \infty) = 0.8$

or, $p(0 < Z < a) + p(0 < Z < \infty) = 0.8$

$p(0 < Z < a) + 0.5 = 0.8$

$p(0 < Z < a) = 0.3$

$a = 0.8 + 0.04$

$a = 0.84$ //

where,

$P(A/B)$ = A condition probability of event A given that B has already occured.

$P(B/A)$ = A condition probability of event B given that A has already occured.

$P(B) > 0$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)}$$

← Joint Probability

← Marginal Probability

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) > 0$$

If A and B are independependent events then,

$P(A/B) = P(A)$ and $P(B/A) = P(B)$ so,

$P(A \cap B) = P(A) \cdot P(B)$.

# Baye's Theorem.

Let, $A_1, A_2, A_3, \ldots, A_n$ be n-Mutually enclusive and enhaustive events with in a sample space S. Further let B be any events which can occur either in conjuction with event $A_i$'s, then, the conditional probability of $A_i$ given that event B has already occured is given by Baye's theorem.

(5)

Soln

a) $\sum_{n=1}^{\infty} K\left(\frac{1}{2}\right)^n = 1$

$K\left[\frac{1}{2} + \frac{2/2}{2\times 2} + \frac{3/3}{2\times 2\times 2} + \dots\right] = 1$

$K\,\frac{1}{2}\left(\frac{1-r^n}{1-r}\right) = 1$

$K = 1$ //

| $x$ | 1 | 2 | 3 | $\infty$ |
|---|---|---|---|---|
| $P(x)$ | $1/2$ | $1/4$ | $1/8$ | 0 |
| $F(x)$ | $1/2$ | $3/4$ | $7/8$ | 1 |

PU 2016 Fall

6(a) soln

The 2-step tpm is, given by,

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix} \end{array}$$

initial probablity vector.

$$p(0) = S_0 = [0.4 \quad 0.3 \quad 0.3]$$

After 2 message 2-step tpm is,

$$p(2) = P^2 = P \cdot P$$

$$= \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix} \times \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.37 & 0.27 & 0.36 \\ 0.46 & 0.28 & 0.26 \\ 0.21 & 0.27 & 0.52 \end{bmatrix}$$

$$\underset{S_0}{} \; P^2$$

$$[0.4 \quad 0.3 \quad 0.3] \times \begin{bmatrix} 0.37 & 0.27 & 0.36 \\ 0.46 & 0.28 & 0.26 \\ 0.21 & 0.27 & 0.52 \end{bmatrix}$$

$-0.56 = 45 - \mu$

$\mu = 45 + 0.56$

Now,

$1.41 = \frac{64 - 45 - 0.56}{\sigma}$

$1.41\sigma = 19 - 0.56$

$\sigma = \frac{19}{1.91}$

$\sigma = 9.947$ //

$\mu = \cancel{45+}\ 49.97$ //

[PU(2014 spring)] Q 3(a)

Soln

X = No. of children who scored diff marks.

Mean ($\mu$) = 60

S.d ($\sigma$) = 20

Now,

$X \sim N(60, (20)^2)$;

Now,

(i) $P(X > 90) = P\left\{Z > \left(\frac{X-\mu}{\sigma}\right)\right\}$

$= P\left(Z > \frac{90-60}{20}\right)$

$= P(Z > 1.5)$

[3 4 7 2 3]

PU

$n = 34722.2$ //

# How many times would you have to toss a fair die in order to be atleast 99% sure that the relative frequency of having a 6 come up is within 0.02 of the theoritical prob $1/6$.

PU

# Determine How many times a fair coin must be tossed in order that the probability will be atleast 0.9 that the proportion of success will lie between 0.4 and 0.6.

Soln

By question, $P\left(0.4 < \frac{x}{n} < 0.6\right) \geq 0.90$

$p = q = 1/2$

Here, we are given that

$$P\left(\left|\frac{x}{n} - p\right| < k\right) \geq 0.90$$

or, $P\left(\left(\frac{x}{n} - \frac{1}{2}\right) < k\right) \geq 0.90$ eqn (i)

and

or, By BLLN;

$$P\left(\left|\frac{x}{n} - p\right| < k\right) \geq 1 - \frac{pq}{nk^2}$$

$$P\left(\left|\frac{x}{n} - \frac{1}{2}\right| < k\right) \geq 1 - \frac{1/4}{nk^2}$$

## Beta

# Let 'X' has a Beta dist$^n$, with $m = n = 2$. Then find the probability that 'X' bring not greater than 0.2. Also find the probability that random variable 'X' lies between 0.2 and 0.5 and find mean and variance of Beta distribution.

Give that, $X \sim \beta(m=2, n=2)$ The pdf

$$f(x) = \frac{x^{m-1}(1-x)^{n-1}}{\beta(m,n)}$$

$$= \frac{x^{2-1}(1-x)^{2-1}}{\beta(2,2)}$$

$$= \frac{x(1-x)}{\frac{\gamma_2 \gamma_2}{\gamma_4}}$$

$$= \frac{x(1-x)}{\frac{1}{6}}$$

$$= 6(x - x^2) \therefore 0 < x < 1$$

vvt

# Central Limit theorem (CLT) :-

Let $X \sim N(\mu, \sigma^2)$, then the sampling distribution of dist of sample mean ($\bar{x}$) also, ie mean $\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$

Also, sum of samples $S_n = \Sigma x \sim N(n\mu, n\sigma^2)$

2010 spring 4(a)

Soln :-

Average sallary ($\mu$) = \$29,321

sd ($\sigma$) = \$2120

sample size ($n$) = 100

~~Sample Mean ($\bar{x}$)~~

$P(\bar{x} < 29000) = ?$

$P(\bar{x} < 29000)$, By central limit theorem, we can say that,

$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$

$$P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{29000 - 29321}{\frac{2120}{10}}\right)$$

$$P(Z < -1.5451)$$

Solve by normal table

$$V(x) = E(x^2) - \{E(x)\}^2$$

Now,

$$E(x^2) = \int_0^2 \frac{3}{4}(2x^3 - x^4)\,dx$$

$$= \frac{3}{4}\left[2\frac{x^4}{4} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{32}{4} - \frac{32}{5}\right]$$

$$= \frac{3}{4} = 0.75$$

Now,

$$V(x) = 0.75 - 1$$

$$= 0.25$$

(4)

Soln

Let,

$A_1$ = event that the article was produced by Machine A.

$A_2$ = event that the article was produced by Machine B.

$A_3$ = event that the article was produced by Machine C.

D = event that are deffective,

By question,

$P(A_1) = 0.2$ , $P(A_2) = 0.35$ , $PA(3) = 0.45$

Now,

$P(D/A_1) = 0.02$, $P(D/A_2) = 0.04$ , $P(D/A_3) = 0.01$

Now,

i) $P(\text{all deffective}) = P(A_1 \cap D)$ or $P(A_2 \cap D)$ or $P(A_3 \cap D)$

$$= P(A_1) \cdot P(D/A_1) + P(A_2) \cdot P(D/A_2) + P(A_3) \cdot P(D/A_3)$$

$$= 0.2 \times 0.02 + 0.35 \times 0.04 + 0.45 \times 0.01$$

$$= 0.0225$$

) a) $P(A_1/D) = ?$ By Baye's rule.

$$P(A_1/D) = \frac{P(A_1) \times P(D/A_1)}{P(d)} = \frac{0.2 \times 0.02}{0.0225} = \frac{8}{45}$$

we know,

$$P\left(x_2 = 1 / x_0 = 0\right) = \frac{P(x_2 = 0, x_0 = 0)}{P(x_0 = 0)}$$

or,

$$\frac{5}{16} = \frac{P(x_2 = 0 \; x_0 = 0)}{1/3}$$

$$P(x_2 = 0 \; x_0 = 0) \; \frac{5}{48}$$

$$\boxed{P(x_3 = 2, x_0 = 1) = ? \text{ HW}}$$

iii) $1^{st}$ red and $2^{nd}$ black $= \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

Case II :-

(i) $P(\text{Both red balls}) = P(R_1 \cap R_2) = P(R_1) \times P(R_2/R_1)$

$= \frac{3}{8} \times \frac{2}{7} = \frac{3}{23}$

(ii) $P(\text{one and one black balls}) = P(R_1 \cap B_1)$ or $P(B_1 \cap R_2)$

$= P(R_1) \cdot P(B_1/R_1) + P(B_1) + P(R_2/B_1)$

$= \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7}$

$= \frac{15}{28}$ //

iii) $P(R_1 \cap B_1) = P(R_1) \times P(B_1/R_1)$

$= \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$ //

For 2 Min $\lambda = 1.5 \times 2$
$= 3$

so,
PMf is given by

$$P(X=x) = \frac{e^{-3} 3^x}{x!}$$

(ii) $P(X \geq 3) = 1 - P(X<3)$
$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$$= 1 - e^{-3}\left[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right]$$

$$= 1 - e^{-3}[1 + 3 + 4.5]$$

$$= 0.5768$$

(iii) $\lambda = 1.5 \times 4$
$= 6$

$$P(X=x) = \frac{e^{-6} 6^x}{x!}$$

$$p(x=5) = \frac{e^{-6} 6^5}{5!}$$

$$= 0.16$$ //

# Q The no of accident in a year attributed to taxi driver in a city is a poission distribution with Mean 3 out of 1000 taxidriver find approximately the no of drivers with

(i) More than 3 accidents in a year
(ii) less than 2 " " " "
(iii) exactly 5 " " " "
(iv) Atleast 2 " in 6 Months of time

Soln

Let,

$x$ = no of accidents.

Average no of accidents $(\lambda) = 3$

Total no of taxi driver $(N) = 1000$

Given that

$x \sim P(\lambda = 3)$ so, pmf of $x$ is,

$$P(x=x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad x = 0 \text{ to } \infty$$

$$\Rightarrow \frac{e^{-3} 3^x}{x!} \quad ; x = 0 \text{ to } \infty$$ //

2013 Spring

3(a) No of candidates $(N) = 500$

$\mu = 45$

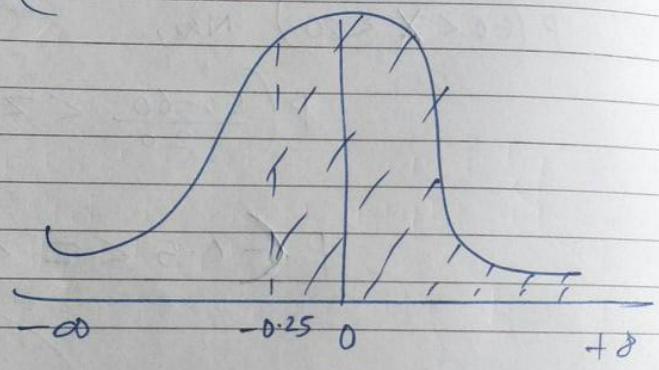
$\sigma = 20$

Now, $X$ = Marks of students.

(i) If 40 is marks then,

or, $P(X > 40) = P\left(Z > \frac{40-45}{20}\right)$

$= P(Z > -0.25)$




$P(0 < Z < 0.25) + P(0 < Z < \infty)$

$= 0.5987$

$\therefore$ No of passing $500 \times 0.5987$

$= 299.35$

$\approx 300$ //

# Theory of queue

⇒ The queue theory deals with problems that involves waiting or queuing. Some examples are :- no of customers waiting in a canteen, No of Messages waiting to be delivered, No of cars waiting to fill the fuel in petrol pump. etc.

(*) Some notation and symbols.

$n$ = no. of customers

$\lambda$ = Arrival time Average no of customers arriving to join a queue in per unit time

$\mu$ = service rate (Average no of " that can be served in per unit time)

$s$ = no of servers

$N$ = size of waiting room.

$qd$ = queue disciple (Ex, FIFO, LIFO, SIRO) (service in random order)

Short notes

Kendals notation :-

The kendal's notation is defined as.

$$A/B/s = (N, qd)$$

where, $A$ = Inter-arrival time distribution.

$B$ = service time distribution.

$s$ = no. of servers.

$N$ = size of waiting room.

$q.d$ = Queue disciple (FIFO, LIFO / SIRO)

b)

Soln

No of Red balls $n(R) = 3$
No of black balls $R(B) = n(B) = 5$

Total balls = 8
Now,

Case I :- with replecement

) probability that both balls are red $= P(R_1) \cdot P(R_2)$

$$= \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$$

) probabitity that one red one blue $= P(R_1 \cap B_1)$ or $P(B_1 \cap R_2)$

$$= \frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8}$$

$$= \frac{30}{64}$$

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum_{n=1}^{n} P(A_i) \cdot P(B/A_i)}$$

a)

Soln

If 2 coins are tossed then,

$S = \{HH, HT, TH, TT\}$

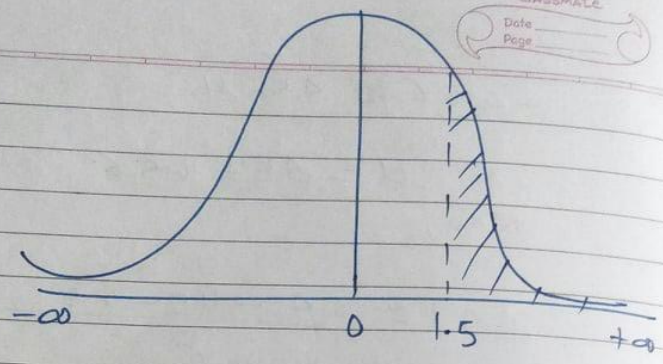
Let, A = event of getting two heads
B = event of getting atleast one head.

So, $n(A) = 1$ $n(B) = 3$

$P(A/B) = ?$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{n(A \cap B)}{n(S)} \times \frac{n(S)}{n(B)}$$

$$= \frac{1}{3}$$




Now,

$= p(0 < z < \infty) - p(0 < z < 1.5)$

$= 0.5 - 0.4332$

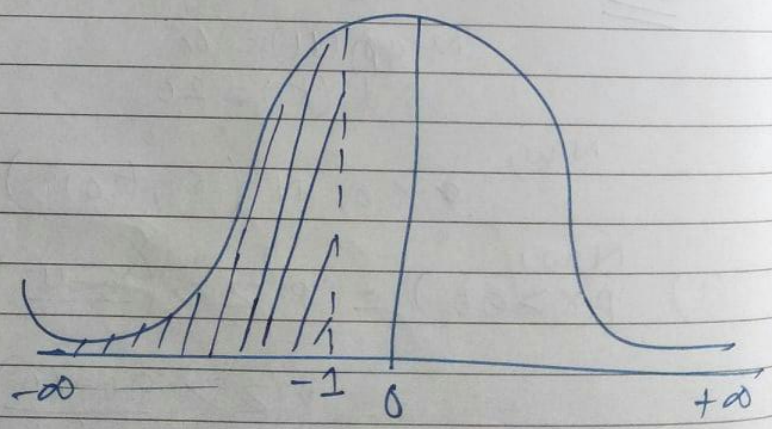
$= 0.0668$ //

(ii) Below 40 Marks

$p(x < 40) = p\left(z < \frac{40-60}{20}\right)$

$= p(z < -1)$

Now,



(6) The total area under normal curve is always 1 (one).

(7) The normal curve is symmertical about the line $x=\mu$ and it is bell shaped.

# Standard Normal distributions.

If $x \sim N(\mu, \sigma^2)$, then a new r.v $Z$ is called a standard normal r.v if it is defined as,

$$Z = \frac{X-\mu}{\sqrt{\sigma^2}} = \frac{X-\mu}{\sigma}$$ and it's probability distribution is called standard normal distribution.

# properties of standard normal distribution.

(1) The standard normal distribution is a symmetrical distribution.

(2) for standard normal distribution

The Mean $E(Z) = E\left(\frac{X-\mu}{\sigma}\right) = \frac{E(X)-\mu}{\sigma}$

$$= \frac{\mu-\mu}{\sigma}$$

$$E(Z) = 0$$

variance $V(Z) = V\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2} V(X)$

$$= \frac{1}{\cancel{\sigma^2}} \times \cancel{\sigma^2}$$

$$= 1$$