Chapter - 5

Instrumentation System and Signal Measurement

* **Instrumentation system :-**

An instrumentation system consists of different components together to perform a measurement and to record the result.
A block diagram of generalized instrumentation system is shown in figure below.

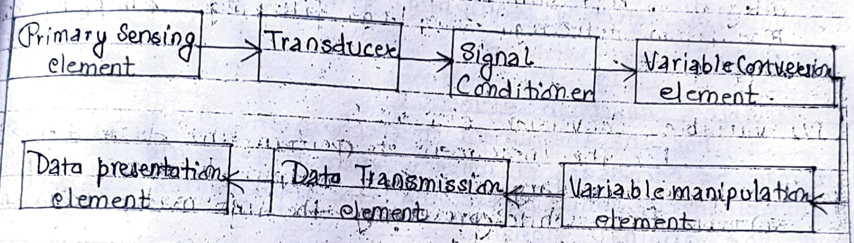


Fig: Block diagram of instrumentation system.

i) **Primary Sensing element :-**

The quantity under measurement makes its first contact with primary sensing element of a measurement system. The primary sensing element first detects the measurands.

ii) **Transducer :-**

In electrical measurement system, transducer is defined as a device that converts any physical quantity into an electrical quantity. It receives the quantity under measurement and delivers a proportional electrical signal to the signal conditioning and processing unit.

iii) Signal Conditioner:

It is an important unit and plays very important role to modify a signal to a format acceptable to the output device signal conditioning and processing unit performs the tasks like amplification, attenuation, integration, differentiation, addition, substraction, modulation, detecting, sampling, filtering, chopping and clipping etc.

iv) Variable Conversion element:

For the instrument to perform the desired function it may be necessary to convert output to some other suitable form while preserving the information content of the original signal.

v) Variable manipulation element:

The function of this element is to manipulate the signal presented to it, preserving the original nature of the signal. It just deals only a change in numerical value of the signal.

vi) Data transmission element:

When components of an instrumentation system are physically seperated then it becomes necessary to transmit data from one to another. The element that performs this function is called a data transmission element.

vii) Data presentation element:-

It converts the information about the quantity under measurement. The choice of the output device depends entirely on the necessity of how we want the signal to be presented at output.

* Units and Standards of measurements:-

The standard measure of each kind of physical quantity is unit. A Unit is a standard physical quantity, defined and adopted by convention, with which other physical quantity of the same kind are compared to express their value. There are two types of unit. They are:

i) Fundamental Unit   ii) Derived Unit

i) Fundamental Unit:-

The physical quantity which donot depend upon any other physical quantity are called fundamental quantities. Fundamental quantities are mass, length, time, temperature, luminous intensity, current and amount of substance contained and their units are called fundamental units.

Seven fundamental Units.

SN	Quantity	Units	Symbol
1)	Length	Meter	M
2)	Mass	Kilogram	Kg
3)	Time	Second	S
4)	Temperature	Kelvin	K

5)	Electric current	Ampere	A
6)	Luminous Intensity	Candela	Cd
7)	Quantity of matter	Mole	Mol.

ii) Derived Unit :

Those physical quantities which depend upon other physical quantity are called derived quantities and their units are called derived units. For example: force, worker or Energy, Power etc are derived quantities and their units Newton(N), Joule (J), watt(w), etc respectively are derived units.

Standards :

Standards are used to determine the value of other physical quantities by comparision method. All standards are preserved at the International Bureau of weight and measures.

There are four catagories of standards :-

i) International Standard
ii) Primary Standard
iii) Secondary Standard
iv) Working standard.

i) International Standard :-

This standard is defined by international agreement. This standard are closest to the accuracy. For example. International ohm, International Ampere etc.

ii) Primary standard :

Primary standards are absolute standard of such high accuracy that they can be used as the ultimate reference standard. They are maintained at the national standard lab. The main function of primary standard is the calibration and verification of secondary standards.

iii) Secondary Standard :

Secondary standards are basic reference standards used by measurement and calibration laboratories in industries. Each industry has it's own secondary standards. Each laboratory periodically sends it's secondary standard to the National standard laboratory for calibration and comparision against the primary standard. After comparision and calibration, the National standards laboratory returns the secondary standards to particular industrial laboratory with a certification of measuring accuracy in terms of a primary standard.

(iv) Working Standard :

This standard is used to checks and calibrate lab instrument for accuracy and performance. For example: Manufacturers of electronic components such as capacitors, ~~resistors~~ resistors and many more use a standard called as working standard for checking the component values being manufactured.

Performance parameters of measuring instruments:

The functional behaviour of any instrument with detailed specifications is termed as performance parameters. It represents the capabilities and limitations of the instrument for particular applications. The performance parameters is classified into two distinct categories:

i) Static performance parameters
ii Dynamic performance parameters

i) Static performance parameters

The functional behaviour of instrument under static condition is termed as static performance parameters. The main static characteristics are:

1) Accuracy
2) Precision
3) Sensitivity
4) Resolution
5) Static error
6) Range or span
7) Bias
8) Linearity
9) Drift
10) Threshold
11) Dead zone
12) Repeatability
13) Hysteresis.

1) Accuracy:

It is the closeness with which an instrument reading approaches the true value of the quantity being measured.

2) Precision:

It is a measure of degree to which successive measurement differ from one another. It is a measure of the reproducibility of the measurement ie for a given fixed value of a quantity, precision is a measure of the degree of agreement within a group of measurements.

3) Sensitivity:-

Sensitivity is the ratio of the magnitude of the output signal or response to the magnitude of input signal or the quantity being measured. It's units are milimetre per microampere, Cunts per volt etc depending upon the types of input and output.

4) Resolution:-

If the input is slowly increased from some arbitary (non-zero) input value, the output will not change until a certain increment is exceeded. This is the smallest change in input to which instrument would just respond is known as resolution or discrimination.

5) Static error:-

It is defined as the difference between the measured value and the true value of quantity ie.

$$\partial A = A_m - A_t \quad \text{---} (i)$$

where, ∂A = error

A_m = measured value of quantity

A_t = true value of quantity.

6) Range or span:

The range or span of an instrument is defined as the difference between the largest and the smallest value of quantity the instrument is designed to measure. The span of an instrument is given by

$$\text{span} = X_{max} - X_{min}$$

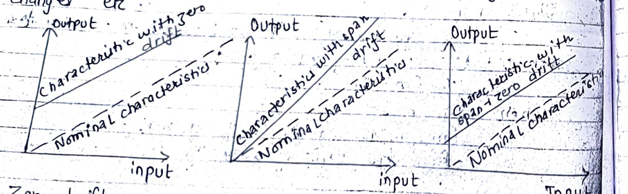
For a thermometer calibrated between 200°c to 500°c, the range is 200°c to 500°c and span = 500 - 200 = 300°c.

7) Bias:

It represents constant error which exists over the full range of measurement of an instrument. By proper calibrate this error can be removed.

8) Drift:

It is the variation in output of measuring system which is not due to any change in input quantity but due to change in operating condition of the components inside the measuring systems like component instability, temperature changes etc.



a) Zero drift

b) Span or Sensitive drift.

c) Zero + Span drift.

9) Linearity:

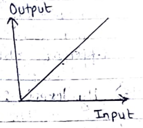
It represent that output of an instruments is linearly proportional to the quantity being measured. The linear relationship is expressed by $y = mx + c$

where, y = output measure

m = slope

x = Variable to be measure

c = Intercept on y-axis

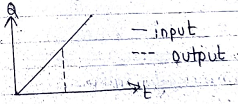


10) Threshold:

It is the minimum value of input below which no output can be detected.

11) Dead zone:

It is defined as the largest change of input quantity for which there is no output of the instrument. It is also known as dead band or dead space. It occured due to static friction, back lash or Hysteresis.

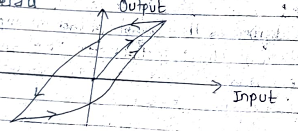


12) Repeatability:

Repeatability and reproducibility are a measure of closeness with which a given input may be measured over and over again. Repeatability represents the closeness of output reading when same input applied repetitively over a short period of time, with same instrument same measurement condition and same observer. Whereas reproducibility represents the closeness of output reading when same input applied repetitively over a short period of time with same instrument with different measurement condition and different observer.

13) Hysteresis :-

The non coincidence between the loading and unloading curves is known as hysteresis. It occurs due to friction, backlash, or characteristics of magnetized materials. It can be minimized by proper design and selection of mechanical components, introducing greater flexibility and providing suitable heat treatment to the materials.



ii) Dynamic performance parameter :-

The functional behaviour of instrument under dynamic condition is referred to as dynamic performance parameter. The main dynamic characteristics are:

1) Speed of response
2) Response time
3) Dynamic error
4) Fidelity

1) Speed of response :

It is a speed with which an instrument responds to the suddenly change in the quantity to be measured.

2) Response time :

It is the time required by an instruments or measurement system to settle down to it's steady state position after the application of input. A smaller setting time indicate higher speed of response.

3) Measuring lag :

It represents the delay in response of an instrument to a change in the measurement. It is quite small in magnitude, but for high speed measurement it becomes quite significant.

4) Fidelity :

It is ability of an instrument or system to reproduce output in the same form as the inputs.

5) Dynamic Error :-

It is the difference between true value of quantity changing with time and the value indicating by instrument under the dynamic condition assuming static error zero.

i.e $\partial A = A_t - A_i$

where,

∂A = Error

A_t = True value

A_i = indicated value of quantity

* Indicating Instruments:
In these instruments there must be a pointer attached to moving over a calibrated scale. The pointer is attached to the system that is pivoted on jewelled bearing. The torques that are mandatory for the satisfactory operation of these ~~instrument~~ instrument:

i) Deflecting torque
ii) Controlling torque
(iii) Damping torque

i) Deflecting torque :-
The deflecting torque is produced by making use of one of the following effects of current and voltage. The effects of current and voltages are:

a) Magnetic b) Heating c) Chemical d) Electrostatic
e) Electromagnetic.

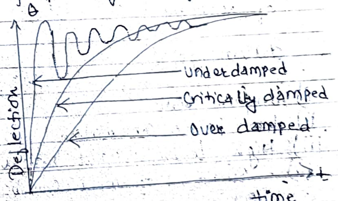
When the instrument is connected to the electrical circuit to measure the electrical quantity, deflecting torque moves the moving system of the instrument from it's zero position.

ii) Controlling torque:-
Controlling torque always opposes the deflecting torque and is increased with the increase in the deflection of the system. If the controlling torque is absent in the instrument the pointer will swing over it's maximum deflection position irrespective of the size of measured quantity and will not come back to the zero position on removing the measured quantity. There are two types of controlling torque:

a) Spring Control b) Gravity Control

iii) Damping torque:
The function of damping is to absorb energy from the oscillating system and to bring it rest promptly in it's equilibrium position so that it's indication may be observes.
The damping torque should be such that the pointer quickly comes to it's final steady position, without overshooting. If the instrument is underdamped, the moving system will oscillate about the final steady position with a decreasing amplitude and will take some time before it comes to rest. If the instrument is overdamped, the moving system moves slowly to it's final steady position in a lethargic fashion. So the instrument should be critically damped to get the rapid and smooth movement of moving system to it's final steady position.
There are various method for producing damping torque which are:-

i) Air friction damping
ii) Fluid friction damping
iii) Eddy Current damping
iv) Electromagnetic damping



1) Permanent Magnet Moving Coil (PMMC):

Permanent magnet moving coil Instrument is the most accurate type for dc measurement. It consist of a V shaped permanent magnets having soft iron pole pieces. The moving coils mounted on a rectangular aluminium former moves freely in the field of permanent magnet. The moving coil is pivoted on jewelled bearings. Two phosphor broze hair springs are provided for controlling torque. These spring is also serve to lead current in and out of the coil. Damping torque is produced by movement of the aluminium former moving in the magnetic field of the permanent magnet. The pointer carried by spindle moves over a graduated scale.

The working principle of these instruments is same as d'Arsonval galvanometer. When current passes through a coil which is kept in the field of permanent magnet, the coil moves and the direction of movement is determined by the direction of current flowing through the coils & the magnitude of the current.

Advantages:-

i) Uniform Scale
ii) Low power consumption
iii) High torque weight ratio which provides high accuracy
iv) Single instrument can be used to measure both current and voltage.
v) No effect of ~~strag~~ stray magnetic field.
vi) No hysterisis loss

Disadvantages:-

i) Not useful for A.C measurement.
ii) Costlier than moving iron instrument due to delicate construction and assembly of various parts.
iii) Errors introduces due to friction, spring temperature, magnet, etc.

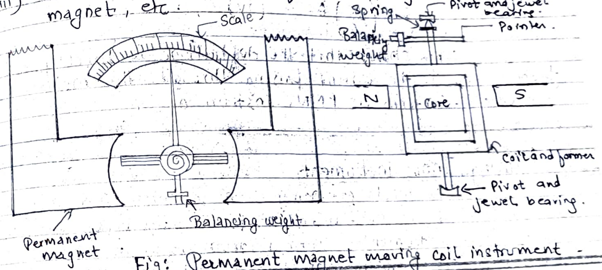


Fig: Permanent magnet moving coil instrument.

Torque equation:

The torque for a moving coil instrument is given by deflecting torque $(T_d) = BINA$ ——(i).

where,

B = flux density in the air gap, wb/m^2
I = Current through moving coil, A
N = Number of ~~time~~ turns in coil
A = Area of the rectangular coil, m^2

$\therefore T_d = GI$ —— ii)

where, $G = BNA$ = Constant

Also,

The controlling torque provided by control spring is given by controlling torque $(T_c) = K\theta$ —— (iii)

where, K = Constant

θ = deflection

For final steady deflection, $T_d = T_c$

$GI = K\theta$

$\therefore I = \left(\frac{K}{G}\right)\theta$ —— (iv)

This shows that the deflection is directly proportional to the current passing through the meter. So the scale is uniform in PMMC instrument.

2) Moving Iron instrument

The most commonly used ammeter and voltmeter at laboratories or switch board at power frequencies are moving iron instruments. These instruments can be constructed to measure current and voltage to an accuracy needed in most engineering works and is also cheap as compared to other types of a.c instrument of same accuracy and ruggedness.

The moving element of MI instrument consists of a plate or vane of soft iron or of high permeability steel which is so situated that it can move in a magnetic field produced by a stationary coil. The coil is excited by the current or voltage under measurement. When the coil is excited, it becomes an electromagnet and the iron vane moves in such a way that it increases the flux of the electromagnet. This is because the vane tries to occupy a position of minimum reluctance. Thus the force produced is always in such direction. So as to increase the inductance of coil.

Torque equation:

When there is a small increment in current supplied to the instrument, there will be a small deflection $d\theta$ and some mechanical work will be done. If T_d be the deflecting torque, then mechanical workdone = ~~$T_d \cdot d\theta$~~ $T_d \cdot d\theta$ —— (i)

If the current increases by dI then the deflection changes by $d\theta$ and the inductance by dL. Hence the increase in the applied voltage is given by

$$e = \frac{d}{dt}(LI) = I\frac{dL}{dt} + L\frac{dI}{dt} \quad \text{—— (ii)}$$

Now, The electrical energy supplied $eIdt = I^2dL + LIdI$ —— (iii)

As the stored energy changes from $\frac{1}{2}LI^2$ to $\frac{1}{2}(L+dL)(I+dI)^2$ the change in

$$\text{stored energy} = \frac{1}{2}(L+dL)(I+dI)^2 - \frac{1}{2}LI^2$$

$$= \frac{1}{2}(L+dL)(I^2+2IdI+dI^2) - \frac{1}{2}LI^2$$

$$= \frac{1}{2}\left[LI^2 + 2LIdI + LdI^2 + I^2dL + 2IdI\,dL + dL\cdot dI^2\right] - \frac{1}{2}LI^2$$

Now, neglecting second and higher order terms, we get

Change in stored energy $= \frac{1}{2}LI^2 + LIdI + \frac{1}{2}I^2dL + \frac{1}{2}LI^2$

$= LIdI + \frac{1}{2}I^2dL$ —— (iv)

Now, from the principle of conservation of energy.

Electrical energy supplied = increase in stored energy + mechanical work done.

$I^2dL + LIdI = LIdI + \frac{1}{2}I^2dL + T_d\,d\theta$

$\frac{1}{2}I^2dL = T_d\,d\theta$.

$T_d = \frac{1}{2}I^2\frac{dI}{d\theta}$.

∴ Deflecting torque $(T_d) = \frac{1}{2}I^2\frac{dL}{d\theta}$ —— (v)

where,

I = current in Ampere.

L = inductance in Henry

θ = deflection in radian.

T_d = Torque in Newton.

Also, controlling torque provided by an spring is $T_c = K\theta$ —— (vi)

where,

K = Spring constant

θ = deflection in radian.

for final steady deflection, $T_d = T_c$

$\frac{1}{2}I^2\frac{dL}{d\theta} = K\theta$

∴ deflection $(\theta) = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$ —— (vii)

Hence the deflecting torque is proportional to the square of the rms value of operating current. Therefore the deflecting torque is unidirectional and scale of the instrument is non uniform.

There are two types of MI instrument, they are:

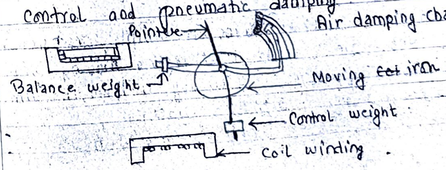
i) Attracted type.

ii) Repulsion type.

i) Attracted type :-

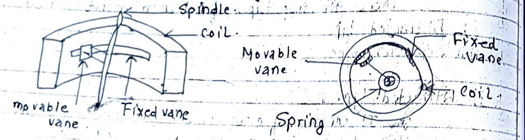
It consist of a coil or solenoid and an oval-shaped iron which is pivoted in such a way that it can move in or out of the solenoid. A pointer is attached to this iron which deflects along with the moving iron over a graduated scale.

When the current to be measured is passed through the solenoid, magnetic field will set up inside the solenoid which will magnetize the iron and is attracted towards the coil. This instrument generally have spring control and pneumatic damping.



ii) Repulsion type :-

In this type of instrument there are two pieces of iron, one is fixed and other is movable. Moving iron is mounted on a shoot short arm fixed to the spindle of instrument. These two iron pieces lies in the magnetic field due to a coil. When the current to be measured is passed through the coil, magnetic field is set up which magnetizes the two iron pieces in the same direction and a repulsive force is set up between the two iron pieces. Hence the moving iron piece is repelled by the fixed iron piece causing the deflection of the pointer.



Advantages of MI instrument:

i) Use in both A.C and D.C circuit
ii) High operating torque
iii) Simple Construction
iv) It can withstand overloaded momentarily
v) Less friction error
vi) The instruments are robust, owing to simple construction and also that there are no current carrying moving parts.
vii) These are capable of giving an accuracy within the limits of both precision and industrial grades.

Disadvantages :-

i) Scale are not uniform.
ii) Higher power consumption for low voltage range
iii) The spring stiffness decreases with increase in temperature
iv) It is subjected to serious error due to frequency change, hysteresis and stray magnetic fields.
v) Instrument need to calibrate for frequency at which these are to be used.

* DC Ammeters :-

i) Ammeter Shunts :-

The basic movement of dc-ammeter is a PMMC d'Arsonval galvanometer. Since the coil winding of a basic movement is small and light, it can carry very small current. So, when heavy currents are to be measured, the major part of the current is bypassed through a low resistance called a shunt as shown in figure below.

Here,
R_{sh} = Resistance of shunt, Ω
R_m = internal resistance of movement (coil) Ω
I_{sh} = Shunt Current, A
I = Current to be measured, A
I_m = full scale deflection current of movement, A

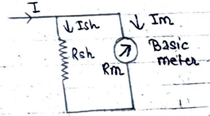


Fig: Basic ammeter Circuit.

Since the shunt resistance is in parallel with meter movement, the voltage drop across shunt and movement must be the same,

$\therefore I_{sh} R_{sh} = I_m R_m$

$R_{sh} = \frac{I_m}{I_{sh}} R_m$ ——(i).

Since $I_{sh} = I - I_m$

$\therefore R_{sh} = \left(\frac{I_m}{I - I_m}\right) R_m$

As the ratio of total current to the current in the movement is called multiplying power of shunt ie $m = I/I_m$

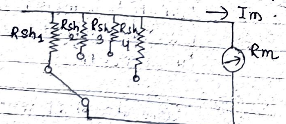
Now,

$R_{sh} = \left(\frac{1}{\frac{I}{I_m} - 1}\right) R_m$

$\therefore R_{sh} = \frac{R_m}{m-1}$ —— (ii)

ii) Multirange Ammeter :-

In multirange ammeter, the current range of a d.c ammeter can be further extended by a number of shunts, selected by a range switch. The figure below shown as multirange ammeter consisting of four shunts R_{sh1}, R_{sh2}, R_{sh3} and R_{sh4} to give four different current ranges I_1, I_2, I_3 and I_4. If m_1, m_2, m_3 and m_4 be the shunt multiplying power for current I_1, I_2, I_3 and I_4 respectively then,



$R_{sh1} = \frac{R_m}{m_1 - 1}$

$R_{sh2} = \frac{R_m}{m_2 - 1}$ $R_{sh3} = \frac{R_m}{m_3 - 1}$

Fig: multi-range Ammeter

$R_{sh4} = \frac{R_m}{}$

Universal or Ayrton Shunt :-

This eliminates the possibility of having the meter in the circuit without shunt. The universal or Ayrton shunt is shown in figure below. Consider that the meter ranges have to be extended to I_1, I_2 and I_3.

When switch is at position 1,

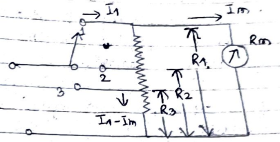


Fig: Universal or Ayrton Shunt

$I_m R_m = (I_1 - I_m) R_1$

$R_1 = \left(\frac{I_m}{I_1 - I_m}\right) R_m$

$= \frac{1}{\left(\frac{I_1}{I_m} - 1\right)} R_m$

$\therefore R_1 = \frac{R_m}{(m_1 - 1)}$ ——(i).

When switch is at position 2:

$I_m(R_m + R_1 - R_2) = (I_2 - I_m) R_2$

or, $I_m R_m + I_m R_1 - I_m R_2 = I_2 R_2 - I_m R_2$

or, $R_2 = \frac{I_m}{I_2}(R_m + R_1)$

$\therefore R_2 = \frac{R_m + R_1}{m_2}$ ——(ii).

When switch is at position 3,

$I_m(R_m + R_1 - R_3) = (I_1 - I_m) R_3$

or, $I_m R_m + I_m R_1 - I_m R_3 = I_1 R_3 - I_m R_3$

$R_3 = \frac{I_m}{I_1}(R_m + R_1)$

$\therefore R_3 = \frac{R_m + R_1}{m_3}$ ——(iii)

D.C. voltmeter

i) Voltmeter multipliers:

A d'Arsonval basic meter movement is converted into a voltmeter by connecting a series resistance, known as multiplier, with it. The multiplier limits the current through the meter so that it doesn't exceed the value for full scale deflection and thus prevents the movement from being damaged.

Here,

I_m = full scale deflection current of meter.

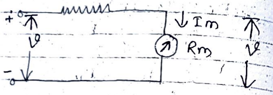


Fig: Basic voltmeter circuit.

R_m = internal resistance of meter movement.

R_s = multiplier resistance

v = full range voltage of instrument.

Now, from circuit,

$$V = I_m (R_s + R_m)$$

or, $$V = I_m R_s + I_m R_m$$

$$R_s = \frac{V}{I_m} - \frac{I_m R_m}{I_m}$$

$$\therefore R_s = \frac{V}{I_m} - R_m \quad \text{——(i)}$$

Since multiplying factor $(m) = \frac{V}{v}$

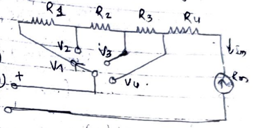
So, $$\frac{R_s}{R_m} = \frac{V}{I_m R_m} - 1$$

$$R_s = R_m(m-1) \quad \text{——(ii)}$$

(ii) Multirange Voltmeter:

The multirange voltmeter can be obtained by providing additional number of multiplier with range switch as shown in figure below. It consists of a four position switch, four multipliers R_1, R_2, R_3 & R_4 for voltage range of V_1, V_2, V_3 & V_4 respectively.

Now from figure,



$$V_1 = I_m(R_1+R_2+R_3+R_4+R_m) \quad \text{—(i)}$$

$$V_2 = I_m(R_2+R_3+R_4+R_m) \quad \text{—(ii)}$$

$$V_3 = I_m(R_3+R_4+R_m) \quad \text{—(iii)}$$

$$V_4 = I_m(R_4+R_m) \quad \text{—(iv)}$$

Now,

$$V_4 = I_m R_4 + I_m R_m$$

$$R_4 = \frac{V_4}{I_m} - \frac{I_m R_m}{I_m}$$

$$\frac{R_4}{R_m} = \frac{V_4}{I_m R_m} - 1 \Rightarrow \frac{R_4}{R_m} = \frac{V_4}{v} - 1$$

$$\therefore R_4 = (m_4 - 1) R_m \quad \text{——(v)}$$

From eqn (iii)

$$R_3 = \frac{V_3}{I_m} - R_4 - R_m$$

$$\frac{R_3}{R_m} = \frac{V_3}{I_m R_m} - \frac{R_4}{R_m} - 1$$

$$\frac{R_3}{R_m} = \frac{V_3}{I_m R_m} - \frac{R_4}{R_m} - 1$$

$$\frac{R_3}{R_m} = m_3 - m_4 + 1 - 1$$

$$\therefore R_3 = (m_3 - m_4) R_m \quad \text{——(vi)}$$

From eq^n (ii)

$R_2 = \frac{V_2}{I_m} - R_3 - R_4 - R_m$

$\frac{R_2}{R_m} = \frac{V_2}{I_m R_m} - \frac{R_3}{R_m} - \frac{R_4}{R_m} - 1$

$= m_2 - \frac{(m_3 - m_4)R_m}{R_m} - \frac{(m_4 - 1)R_m}{R_m} - 1$

$= m_2 - m_3 + m_4 - m_4 + 1 - 1$

$\therefore R_2 = (m_2 - m_3) R_m$ —— (vii)

from eq^n (I)

$R_1 = \frac{V_1}{I_m} - R_2 - R_3 - R_4 - R_m$

$\frac{R_1}{R_m} = \frac{V_1}{I_m R_m} - \frac{R_2}{R_m} - \frac{R_3}{R_m} - \frac{R_4}{R_m} - 1$

$= m_1 - \frac{(m_2 - m_3)R_m}{R_m} - \frac{(m_3 - m_4)R_m}{R_m} - \frac{(m_4 - 1)R_m}{R_m} - 1$

$= m_1 - m_2 + m_3 - m_3 + m_4 - m_4 + 1 - 1$

$\therefore R_1 = (m_1 - m_2) R_m$ —— (viii)

Errors :-

It is the deviation from the true value of the measured variable. Measurement done in laboratory or at some other place always involve error. No measurement is free from errors. Thus it becomes very important to find out the actual accuracy and how different error get entered into measurement. The study of error help to reduce them as well as allow to determine the accuracy of result. Errors may arise from different sources and are usually classified into three catagories.

1) Gross Error 2) Systematic Error 3) Random Error

---

1) Gross Error :-

It occurs due to human mistake in reading instrument recording and calculating measured result. Some of the important causes of gross error are:

i) Misreading of instrument
ii) Computational mistake
iii) Recording mistake
iv) Incorrect adjustment. eg: zero adjustment.

Although complete elimination of gross ~~eve~~ error is probably impossible, we should try to anticipate and correct them. Gross error may be of any amount and therefore their mathematical analysis is impossible. However they can be avoided by adopting two means they are:

a) Taking the great care in reading and recording the data.
b) Taking two, three or even more reading for the quantity under measurement.

2) Systematic Errors :-

The systematic errors are divided into three categories

i) Instrumental Error

ii) Environmental Error

iii) Observational Error

i) Instrumental Error :-

These errors arise due to three main reasons

a) due to inherent short coming in the instrument

b) Due to miss use of instrument

description

a) Due to inherent short comings in the instrument :- These errors are inherent in instruments because of their mechanical structure such as bearing, spring, calibration, function, gear and other parts, hysteresis, stretching of coil, worn parts, over loading of instrument, etc. This error can be reduce by :-

1) Planning the procedure of measurement carefully.

2) Applying correction factors after determining the instrumental error.

3) Re-calibrating the instrument carefully.

b) Miss use of instrument

A good instrument used in an unintelligent way may give erroneous results. Miss use of instrument may be failure to adjust the zero of instrument, poor initial adjustments, use of too high resistive leads etc.

c) Due to loading effect of instruments :-

Improper use of an instrument leads to the loading effect. For example, a well calibrated voltmeter may give a misleading voltage reading when connected across a high resistance circuit.

ii) Environmental Errors :-

These errors are due to conditions external to the measuring device including conditions in the area surrounding the instrument. These may be effect of temperature, pressure, humidity, dust, vibrations or of external magnetic or electrostatic field. The corrective measures employed to eliminate or to reduce these undesirable effects are :-

1) Performing measurement under ideal environmental condition.

2) By proper use of electrostatic and magnetic shilding.

3) Adopting policy of new calibration in new condition.

4) Hermitically equipment.

5) By use of instrument that not get effected greatly by environmental change.

(iii) Observational Errors :

These errors are due to conditions external to increasing device including conditions in the area surrounding the instrument. These may be effect of temperature, Pressure, humidity, dust, vibrations or of external magnetic or electrostatic field. The corrective measures employed to eliminate or to reduce those undesirable effects are :

1) Performing measurement under ideal environmental condition
2) By proper use of electrostatic and magnetic shilding.
3) Adopting policy of new calibration in new condition.
4) Herr

(iii) Observational Errors :-

These error are due to sensing capabilities of observer and their position.

3) Random error :-

It occurs due to unknown causes or due to causes which are very difficult to find out. They occurs even when all systematic errors are avoided. Those errors are due to a multitude of small factors which changes or fluctuate from one measurement to another. Random errors can be minimize by taking many reading and using statistical approach, such as mean, average deviation, standard deviation etc.

i) Mean

$$\bar{X} = \frac{X_1 + X_2 + \cdots X_n}{n} = \frac{\Sigma X}{n}$$

ii) Deviation from mean :

$d_1 = X_1 - \bar{X}$ $\quad d_2 = X_2 - \bar{X}$ $\quad d_n = X_n - \bar{X}$

iii) Average deviation

$$D = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n}$$

iv) Standard deviation

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + \cdots + d_n^2}{n}}$$ for infinite observation

v) Variance :

$(V) = \sigma^2$.

Gaussian Distribution of Errors :

To understand the concept of Gaussian distribution curve. Let us assume that experiment performed to measure voltage at certain interval of time. The nominal value of the measured voltage was 100V. When we take number of measurement at small interval of time it is observed that most of the readings are close to the actual value. There are only few readings which deviate much from the actual value there readings are very less in numbers.

Assume that readings has taken 50 no. of times to measure same 100V. When no. of observed readings are plotted against voltage, the nature of graph obtained is illustrated in figure.

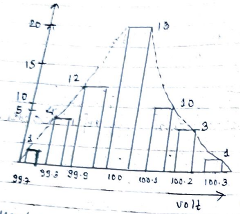


Fig: Histogram of No. of reading

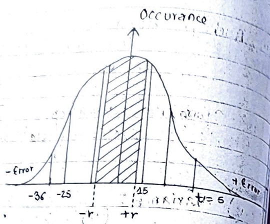


Fig: Normal probability Curve

This bell shaped curve is known as a Gaussian curve. The Gaussian or ~~norm~~ normal law of error forms the basis of analytical study of random effects. Gaussian curve rise to Gaussian law of error as:-

i) The observations include small disturbing facts called random errors.
ii) Random errors can be positive or negative.
iii) There is an equal probability of positive and negative random errors.

As measurement includes both plus and minus errors the total error becomes very small and also mean value becomes true value of measured variable.

The possible error distribution curve stated as:-

a) Large errors are very improbable.
b) Small errors are more probable than large errors
c) There is an equal probability of plus and minus errors so that probability error become about zero value.

Probable error :-

S.N	Voltage	Reading
1	99.7	1
2	99.5	4
3	99.9	12
4	10	19
5	100.1	10
6	100.2	3
7	100.3	1

Deviation $\pm/\sigma$	Fraction of total area Included.
0.67456	0.50000
1.0	0.6625
2.0	0.9546
3.0	0.9972

Let us consider two points $-r$ and r. These points are so located that the area bounded by the curve, the X-axis, and the ordinates erected at $\sigma = \pm r$ is equal to half of the total area under the curve.

A convenient measure of precision is the quantity 'r'. The quantity 'r' is termed as probable errors. Thus probable error is defined as

$$r = \pm 0.67456$$

This value is probable in the sense that there is a even chance that any one observation will have a random error no greater than $\pm r$.

Limiting error :-

It is defined as limit of deviation from specified value. Circuit Components are guaranted within a certain percentage of their rated value.

Eg: If resistance value is specified as $r = 100\Omega \pm 5\%$.

It means that the value of resistance falls between the limit 95Ω and 105Ω.

Similarly, indicating instruments accuracy is guaranted to a certain percentage of full scale reading.

Eg: A 0-100V voltmeter has a guaranted accuracy of 1% of the full scale deflection (reading). It means that the value of voltage fall between the limits 99 and 100V. The voltmeter would never exceed the specified limit.

Here, the manufacturer will not specify the standard deviation or a problem error, but guaranted that the error would not exceed the set limit at any cost.

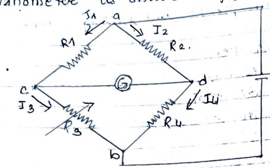
Example :-

1 Ten measurements for currents from ammeter yields values as 500.1 A, 500.3 A, 500.5 A, 500.0 A, 500.7 A, 500.3 A, 500.5 A, 500.2 A, 500.4 A and 500.5 A. Assume that only random error are present. Analytically compute

a) The arithmetic mean
b) The average deviation
c) The standard deviation
d) The probable error.

1) **Wheatstone bridge :-**

It is used to determine the value of unknown resistance. It consist of four resistive arms, together with a source of emf and a null detector, usually a galvanometer as shown in figure.



Here the current through the galvanometer depends on the potential difference between point 'c' and 'd'. The bridge is said to be balance when the potential difference across the galvanometer is 0V, so that there is no current through the galvanometer. This condition occurs when the voltage from point 'c' to 'a' equals the voltage from 'd' to 'a'. so at balance condition.

$$R_1 I_1 = I_2 R_2 \quad \text{—— (i)}$$

If the current through galvanometer is zero, then,

$$I_1 = I_3 = \frac{V}{R_1 + R_3} \quad \text{—— (ii) and.}$$

$$I_2 = I_4 = \frac{V}{R_2 + R_4} \quad \text{—— (iii)}$$

from eq^n (i), (ii) & (iii)

$$\frac{VR_1}{R_1 + R_3} = \frac{VR_2}{R_2 + R_4}$$

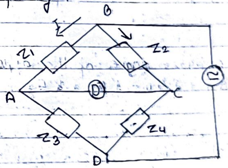
or, $VR_1R_2 + VR_1R_4 = VR_2R_1 + VR_2R_3$

$$R_1R_4 = R_2R_3.$$

$$R_4 = \frac{R_2R_3}{R_1} \quad \text{—— (iv)}$$

2 **AC Bridge :**

An AC bridge consist of four bridge arms, a source of excitation and a null detector. The power source supplied an ac voltage to the bridge at the desired frequency.



At balance condition,

$$\vec{E}_{BA} = \vec{E}_{BC}$$

$$\vec{I_1} \times \vec{Z_1} = \vec{I_2} Z_2 \quad \text{—— (i)}$$

for zero detective current, eq^n (i) becomes,

$$\frac{E\vec{Z_1}}{\vec{Z_1} + \vec{Z_3}} + \frac{E\vec{Z_2}}{\vec{Z_2} + \vec{Z_4}}$$

$$\vec{Z_1}\vec{Z_2} + \vec{Z_1}\vec{Z_4} = \vec{Z_1}\vec{Z_2} + \vec{Z_2}\vec{Z_3}$$

$\vec{Z_1}\vec{Z_4} = \vec{Z_2}\vec{Z_3}$ —— (ii)

If the impedance is written in the form $\vec{Z} = Z\angle\theta$ where Z represents the magnitude and θ be the phase angle of the complex impedance equation (ii) can be written as:

$(Z_1\angle\theta_1)(Z_4\angle\theta_4) = (Z_2\angle\theta_2)(Z_3\angle\theta_3)$ —— (iii)

$(Z_1 Z_4)\angle(\theta_1+\theta_4) = (Z_2 Z_3)\angle(\theta_2+\theta_3)$ —— (iv)

This equation (iv) shows that two conditions must be meet simultaneously when balancing an ac. bridge. They are:

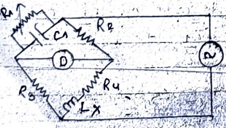
i) The product of magnitude of the opposite arms must be equal ie $Z_1Z_4 = Z_2Z_3$

ii) The sum of the phase angles of the opposite arms must be equal $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

3) Maxwell's Bridge :-

Maxwell bridge is used to measure an unknown inductance in terms of a known capacitance. The schematic diagram of Maxwell bridge is shown in figure below.

One of the ratio arm has a resistance and capacitance in parallel.



Here, $Y = (1/R_1) + j\omega C_1$

$Z_1 = R_2$ $\quad Z_4 = R_4 + j\omega L_x$

$Z_3 = R_3$

At balance condition,

$Z_1 Z_4 = Z_2 Z_3$

$Z_4 = Z_2 Z_3 Y_1$

$R_4 + j\omega L_x = R_2R_3[(1/R_1) + j\omega C_1]$

$R_4 + j\omega L_x = \frac{R_2R_3}{R_1} + j\omega C_1 R_2 R_3$ [c in

Comparing real & imaginary parts.

$R_4 = \frac{R_2R_3}{R_1}\ \Omega$ & $C_x = C_1R_2R_3$ Henry

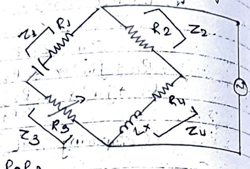
The usual procedure for balancing the maxwell bridge is by first adjusting R_3 for inductive balance and then adjusting R_1 for resistive balance.

Limitation :

The maxwell bridge is limited to the measurement of medium-Q coils $(1 < Q \leq 10)$. It is not suitable in the inductance measurement of coil will less Q and high Q.

Hay's Bridge

The schematic diagram of a Hay bridge is shown in the figure. One of the ratio arm consist of a capacitor in series with resistor. The Hay bridge is more convenient for measuring high Q coils.



At balance condition,

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{—(i)}$$

$$\left(R_1 - \frac{1}{\omega C_1}\right)\left(R_4 + j\omega L_x\right) = R_2 R_3$$

or, $R_1 R_4 + j\omega L_x R_1 - \frac{jR_4}{\omega C_1} + \frac{L_x}{C_1} = R_2 R_3$

or, $R_1 R_4 + \frac{L_x}{C_1} + j\left(\omega L_x R_1 - \frac{R_4}{\omega C_1}\right) = R_2 R_3$

Comparing real & imaginary parts,

$$R_1 R_4 + \frac{L_x}{C_1} = R_2 R_3 \quad \text{—(ii)}$$

$$\omega L_x R_1 = \frac{R_4}{\omega C_1} \quad \text{—(iii)}$$

Solving eq^n (ii) & (iii) simultaneously, we get

$R_4 = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 C_1^2 R_1^2}$ and $L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2}$

and,

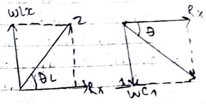
$$\tan\theta_L = \frac{X_L}{R_4} = \frac{\omega L_x}{R_4} = Q$$

$$\tan\theta_C = \frac{X_C}{R_1} = \frac{1}{\omega C_1 R_1}$$

When two phase angles are equal

$\tan\theta_L = \tan\theta_C$ or $\theta = \frac{1}{\omega C_1 R_1}$

Now, $L_x = \frac{R_2 R_3 C_1}{1 + (1/Q)^2}$



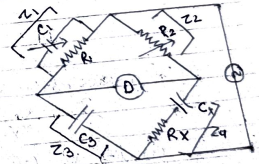
For value of Q greater than ie, $\left(\frac{1}{Q}\right) << 1$, so neglecting $\left(1/Q\right)^2$, we get,

$L_x = R_2 R_3 C_1$ which is same as maxwell equation.

But for the inductor with Q less than 10, $1/Q^2$ is not neglectable so this bridge is not suited for measurement of coils having Q less than 10.

A commercial bridge measure from 1 μH – 100 H with ±2% error.

Schering Bridge :-



Schering bridge is one of the important ac bridge which is extensively used for the measurement of capacitance. Although, the schering bridge is used for capacitance measurements in a general sense; it is particularly useful for measuring insulating properties, ie for phase angle very nearly 90°. The schematic diagram of schering bridge is shown in figure below. The arm 1 contains a parallel combination of a resistor and capacitor, and the standard arm contains only a capacitor. The standard capacitor is usually a high quality mica capacitor for general measurement or an air capacitor for insulation measurement.

Now, at balance condition,

$$\vec{Z_1}\vec{Z_4} = \vec{Z_2}\vec{Z_3}$$
$$\vec{Z_4} = \vec{Z_2}\vec{Z_3}\vec{Y_1}$$

$$R_x - \frac{j}{\omega C_x} = R_2\left(\frac{-j}{\omega C_3}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x - \frac{j}{\omega C_x} = -\frac{jR_2}{\omega C_3} + \frac{R_2 C_1 \omega}{C_3}$$

Comparing real and imaginary part,

$$R_x = \frac{R_2 C_1}{C_3} \text{ --- (2)} \qquad C_x = \frac{R_1 C_3}{R_2} \text{ --- (3)}$$

The power factor of series R-C combination

$$PF = \frac{R_Y}{Z_Y} \quad \text{but for phase angle} \simeq 90°$$

$$PF = \frac{R_Y}{X_Y} = \omega C_Y R_X \text{ --- (4)}$$

and dissipation factor of series RC circuit is

$$D = \frac{R_X}{X_X} = \omega C_Y R_Y \text{ --- (5)}$$

Quality of coil $(Q_c) = \frac{X_L}{R_L}$

$$\therefore D = \frac{1}{Q}$$

From equation (2), (3) & (4) we get

$$D = \omega \frac{R_1 C_3}{R_2} \cdot \frac{R_2 C_1}{C_3}$$

$$\boxed{D_x = \omega_1 R_1 C_1}$$

---

* A ~~para~~ permanent magnet moving coil instrument has a coil of dimensions 15mm × 12mm. The flux density in the gap is $1.8\times10^{-3}\ wb/m^2$ and the spring constant is 0.14×10^{-6} Nm/rad. Determine the number of turns required to produce an angular deflection of 90° when a current of 5mA is flowing through the coil.

Here,

Area of coil (A) = $(15\times12)\ mm^2$
$= 180\ mm^2$

flux density (B) = $1.8\times10^{-3}\ wb/m^2$

Spring constant (K) = 0.14×10^{-6} Nm/rad

Angular deflection (θ) = 90°
$= \pi/2$ rad

No. of turns (N) = ?

At balance condition, $T_d = T_c$

$$BINA = K\theta$$

$$\text{or, } N = \frac{K\theta}{BIA}$$

$$= \frac{0.14\times10^{-6}\times\pi/2}{1.8\times10^{-3}\times5\times10^{-3}\times180\times10^{-6}}$$

$$= 135.75 = 136$$

∴ Number of turns = 136 Ans

A moving coil voltmeter with a resistance of 20Ω gives a full scale deflection of $100°$ when potential difference of $100mV$ is applied across it. The moving coil has dimension of $30mm \times 25mm$ and is wound with 100 turns. The control spring constant is 0.375×10^{-6} Nm/deg. Find ~~above~~ also the the flux density in the air gap. Find the diameter of copper wire of coil. Winding if 30% of instrument resistance is due to coil winding. The specific resistance of copper $= 1.7 \times 10^{-3}\, \Omega m$.

Here,

Resistance of meter $(R) = 20\Omega$

Full scale deflection $(\theta) = 120$

Potential difference $(V) = 100mV$

dimension of coil $(A) = 30mm \times 25mm$

No. of turns $(N) = 100$

Spring constant $(K) = 0.375 \times 10^{-6}$ Nm/deg

flux density $(B) = ?$

diameter of copper wire $(d) = ?$

Current in instrument for full scale deflection

$$I = \frac{V}{R} = \frac{100 \times 10^{-3}}{20} = 5 \times 10^{-3} A$$

for steady position $T_d = T_c$

$BINA = K\theta$.

$$B = \frac{K\theta}{INA} = \frac{0.375 \times 10^{-6} \times 120}{5 \times 10^{-3} \times 100 \times 30 \times 25 \times 10^{-6}}$$

$= 0.12\ wb/m^2$

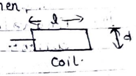
Resistance of coil $R_e = 0.3 \times 20\Omega = 6\Omega$ [Since resistance of coil is 30% of resistance of Instrument]

length of mean turn $l_{mt} = 2(l + d)$

$= 2(80 + 25) = 110mm$

If 'A' be the cross-sectional area of wire then,

Resistance of coil $= \frac{\rho l_{mt}}{A} \times N$



$$R = \frac{\rho l_{mt} \times N}{\frac{\pi d^2}{4}} = \frac{4 \rho l_{mt} \times N}{\pi d^2}$$

diameter of coil $(d) = \sqrt{\frac{4 \rho l_{mt} N}{R \pi}}$

$$= \sqrt{\frac{4 \times 1.7 \times 10^{-5} \times 110 \times 10^{-3} \times 100}{\pi \times 6}}$$

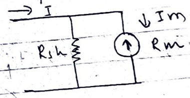
$= 1.992 \times 10^{-4}\ m$

$= 0.2\ mm$ Ans.

---

3 A 1mA meter d' Arsonval movement with an internal resistance of 100Ω is to be converted into a 0-100mA ammeter. Calculate the shunt resistance required.

Here,



$I_m = 1mA$

$R_m = 100\Omega$.

$I = 100mA$

$R_{sh} = ?$

$I_m R_m = (I - I_m) R_{sh}$

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{1 \times 10^{-3} \times 100}{(100 - 1) \times 10^{-3}} = 1.01\Omega \text{ Ans.}$$

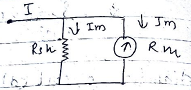
4. A moving coil ammeter has fixed shunt of $0.02\,\Omega$, with a coil resistance of $R_m = 1000\,\Omega$ and a potential difference of 500mV across it, full scale deflection is obtained.

i) Find shunt current at full scale deflection.

ii) Calculate the value of R_m to give full scale deflection when shunted current is 10A.

iii) Find value of R_m for 40% deflection with shunted current of 100A.

Here



i)
$R_{sh} = 0.02\,\Omega$
$R_m = 1000\,\Omega$
$V = 500\,mV$

$$I_m = \frac{V}{R_m} = \frac{500 \times 10^{-3}}{1000} = 0.5 \times 10^{-3} = 0.5\,mA$$

From figure,

$$I_{sh} R_{sh} = I_m R_m$$

$$I_{sh} = \frac{0.5 \times 10^{-3} \times 1000}{0.02} = 25\,A \text{ Ans}$$

ii)
$I_{sh} = 10\,A$
$I_{sh} R_{sh} = 10 \times 0.02 = 0.2\,V$

$$\therefore R_m = \frac{0.2}{0.5 \times 10^{-3}} = 400\,\Omega$$

iii) For 40% deflection $I_m = 0.5 \times 10^{-3} \times 0.4$
$= 0.2 \times 10^{-3}\,A$

Now,

$I_{sh} R_{sh} = R_m' \times I_m'$

$100 \times 0.02 = 0.2 \times 10^{-3} \times R_m'$

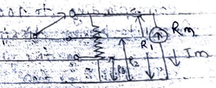
$\therefore R_m' = 10{,}000\,\Omega$ Ans

5) Design a multi-range d.c milli-ammeter using a basic movement with an internal resistance $R_m = 50\,\Omega$ and a full scale deflection current $I_m = 1\,mA$. The range required are 0-10mA, 0-150mA, 0-100mA and 0-500mA.

Ans: $R_{sh_1} = 5.55\,\Omega$; $R_{sh_2} = 1.03\,\Omega$
$R_{sh_3} = 0.506\,\Omega$, $R_{sh_4} = 0.1\,\Omega$.

6. Design an Ayrton shunt to provide an ammeter with current ranges of 1A, 5A and 10A. A basic meter with an internal resistance of $50\,\Omega$ and a full scale deflection current of 1mA is to be used.

Here,



$I_m = 1\,mA$, $R_m = 50\,\Omega$
$I_1 = 1\,A$, $I_2 = 5\,A$, $I_3 = 10\,A$

We have,

$$m = \frac{I}{I_m}$$

$$\therefore m_1 = \frac{I_1}{I_m} = \frac{1}{1 \times 10^{-3}} = 1000$$

$$m_2 = \frac{I_2}{I_m} = \frac{5}{1 \times 10^{-3}} = 5000$$

$$m_3 = \frac{I_3}{I_m} = \frac{10}{1 \times 10^{-3}} = 10{,}000$$

Now,

$$R_1 = \frac{R_m}{m_1 - 1} = \frac{50}{1000 - 1} = 0.05\,\Omega$$

$$R_2 = \frac{R_m + R_1}{m_2} = \frac{50 + 0.05}{5000} \approx 0.01\,\Omega$$

$R_2 = \frac{R_1 + R_m}{m_3} = \frac{0.05 + 50}{10,000} = 0.005\,\Omega$

$\therefore R_1 - R_2 = 0.05 - 0.01 = 0.040\,\Omega$

$R_2 - R_3 = 0.01 - 0.005 = 0.005\,\Omega$

$R_3 = 0.005\,\Omega$

7. A moving coil instrument gives a full scale deflection of 10mA when the potential difference across it's terminals is 100mV. Calculate

a) the shunt resistance for a full scale deflection corresponding to 100A

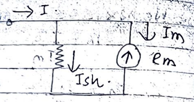
b) the series resistance for a full scale deflection corresponding to 1000V

Calculate power dissipation in each case.

a) $I = 100\,A$

$I_m = 10\,mA$

$V = 100\,mV$



$I_{sh} R_{sh} = I_m R_m$

$R_{sh} = \frac{100 \times 10^{-3}}{(100 - 10 \times 10^{-3})} = 0.001$

Power dissipation $= VI = 100 \times 100 \times 10^{-3} = 10\,W$

b) $V = 1000$

$V - I_m (R_s + R_m) = 0$

$R_s = \frac{V}{I_m} - R_m$

$= \frac{1000}{10 \times 10^{-3}} - \frac{V}{I_m}$

$R_s = \frac{1000}{10 \times 10^{-3}} - \frac{100}{10} = 99990\,\Omega$

Power dissipation $= VI_m = 1000 \times 10 \times 10^{-3} = 10\,W$ #

9. A basic d' Arsonval meter movement with an internal resistance $R_m = 100\Omega$ and full scale current of $I_m = 1mA$ is to be converted into a multi-range dc voltmeter with range of 0-10V, 0-50V, 0-250V and 0-500V. Find the value of various resistance using the potential divider arrangement.

Ans: $R_1 = 9900\Omega$, $R_2 = 40k\Omega$, $R_3 = 200k\Omega$, $R_4 = 250k\Omega$

9. Three resistance have the following ratings:

$R_1 = 37\Omega \pm 5\%$ $R_2 = 75\Omega \pm 5\%$ and $R_3 = 50\Omega \pm 5\%$

Determine the magnitude and limiting error in ohm and in percentage of the resistance of these resistances connected in series.

Here, the value of resistances are,

$R_1 = 37\Omega \pm 37 \times \frac{5}{100} = 37 \pm 1.85\,\Omega$

$R_2 = 75\Omega \pm 75 \times \frac{5}{100} = 75 \pm 3.75\,\Omega$

$R_3 = 50\Omega \pm 50 \times \frac{5}{100} = 50 \pm 2.50\,\Omega$

$\therefore$ The limiting value of resultant resistance,

$R = (37 \pm 1.85 + 75 \pm 3.75 + 50 \pm 2.50)$

$= (37 + 75 + 50) \pm (1.8 + 3.75 + 2.50)$

$\therefore R = 162 \pm 8.10\,\Omega$

So magnitude of resistance $= 162\,\Omega$

error in ohm $= \pm 8.1\,\Omega$

Now,

percent limiting error of series combination of resistance

$= \pm \frac{8.1}{162} \times 100 = \pm 5\%$

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10) The resistance of a circuit is found by measuring current flowing and the power fed into the circuit. Find the limiting error in the measurement of resistance when the limiting errors in the measurement of power and current are respectively $\pm 1.5\%$ and $\pm 1.0\%$.

Here,

Resistance $(R) = \frac{Power}{(current)^2} = \frac{P}{I^2} = PI^{-2}$

$\therefore$ Relative limiting error in measurement of resistance is.

$$\frac{\partial R}{R} = \pm\left(\frac{\partial P}{P} + \frac{2\partial I}{I}\right)$$

$$= \pm(1.5 + 2 \times 1.0)$$

$$= \pm 3.5\%.$$

11) The solution of an unknown resistance for a wheatstone bridge is $R_x = \frac{R_2 R_3}{R_1}$.

where $R_1 = 100 \pm 0.5\%\,\Omega$. $R_2 = 1000 \pm 0.5\%\,\Omega$.

$R_3 = 842 \pm 0.5\%\,\Omega$.

Determine the magnitude of the unknown resistance and the limiting error in percent and in ohm for the unknown resistance R_x.

Here,

Unknown resistance $(R_x) = \frac{R_2 R_3}{R_1} = \frac{1000 \times 842}{100}$

$= 8420\,\Omega$.

Relative limiting error of unknown resistance is

$$\frac{\partial R_x}{R_x} = \pm\left(\frac{\partial R_2}{R_2} + \frac{\partial R_3}{R_3} + \frac{\partial R_1}{R_1}\right)$$

$$= \pm(0.5 + 0.5 + 0.5)$$

$$= \pm 1.5\%$$

Limiting error in ohm $= \pm \frac{1.5}{100} \times 8420 = \pm 126.3\,\Omega$

Gauranted value of resistance are between $8420 - 126.3 = 8293.7$ and $8420 + 126.3 = 8546.3\,\Omega$.

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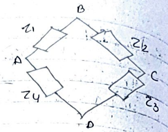
12) An AC bridge is in balance with the following constant. arm AB, $R = 400\,\Omega$; BC, $R = 300\,\Omega$ in series with $C = 0.047\,\mu F$; arm CD - unknown arm, arm DA, $R = 200\,\Omega$ in series with $L = 15.9\,mH$. The oscillator frequency is 1kHz. Find the constant of arm CD.

$Z_1 = 400 \angle 0$

$Z_2 = 300 + \frac{j}{2\pi f C} = 300 - j3356.27)\ \Omega$



$Z_2 = 3399.53 \angle -54.94$

$Z_3 = ?$

$Z_4 = 200 + j\omega L$

$= 200 + j(2\pi f L)$

$= (200 + j99.9)\ \Omega$

$= (223.56 \angle 26.54)$

At balanced condition,

$Z_1 Z_3 = Z_2 Z_4$

$Z_3 = \frac{Z_2 Z_4}{Z_1} = \frac{(3399.53 \angle -84.93)(223.56 \angle 26.54)}{(400 \angle 0)}$

$= \frac{759998 \angle -58.39}{(400 \angle 0)}$

$= 1899.995 \angle -58.39$

$= 1900 \angle -58.39$

$= 995.85 + j(-1618.10)$

$= (995.85 - j1618.10)\ \Omega$

This unknown branch contains resistors and capacitor in series

Resistance (R) = $995.85\ \Omega$

$\therefore X_C = \frac{1}{2\pi f C} = 1615.10$

$C = \frac{1}{2\pi \times 1000 \times 1618.10}$

$C = 9.836 \times 10^{-8}$ F

$= 98.36$ nF Ans.

13) A maxwell bridge is used to measure an inductive impedance. The bridge constants at balance are $C_1 = 0.01\ \mu F$, $R_1 = 470\ k\Omega$, $R_2 = 5.1\ k\Omega$ and $R_3 = 100\ k\Omega$. Find the series equivalent of the unknown impedance.

Ans: $L_x = 5.24$, $R_x = 1.085\ k\Omega$.

14) In a balanced network, arm AB contains a resistance of $500\ \Omega$ in series with an inductor of 0.18 H, BC and DA are non-inductive resistances of $1000\ \Omega$ each and CD arm consists of a resistance R in series with a capacitor C. A potential difference of 5 V at a frequency $\frac{5000}{2\pi}$ is applied between points A & C. Determine the values of R and C.

Ans: $C = 0.23\ \mu F$, $R = 471.44\ \Omega$.