Assignment → 2.6

(A) Find the interval, center and radius of convergence of
(Q.1) the of following series:

(i) The Given infinite series is,

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

(Sol) This is an alternating series and its
The general term is, $|u_n| = \left(\frac{x^n}{n}\right)$

and $|u_{n+1}| = \frac{x^{n+1}}{n+1}$

By ratio's Test,

$$\lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|} = \frac{x^{n+1}}{(n+1)} \times \frac{n}{(x^n)}$$

$$= \lim_{n\to\infty} \frac{x}{\left(\frac{n+1}{n}\right)} = \frac{x}{\left(1+\frac{1}{n}\right)}$$

$$= \left(\frac{x}{1+0}\right) = (x)$$

By D'Alembert ratio test, the given series for $|x|<1$ and diverges $|x|>1$ and further test is needed at $|x| = 1$.

~~The general term (u_n)~~ At $|x|=1$, gives $x=1$ and $x=-1$.

At $x=1$, $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

which is alternating series and $(u_{n+1} < u_n)$
General term $(u_n) = \left(\frac{1}{n}\right)$

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \left(\frac{1}{n}\right)$$

$$= (0)$$

By Leibnitz theorem, it is convergent at $x=1$.

convergent = closed [ ]
Divergent = ( )

Again, At $\boxed{x=-1}$ series becomes,

$$= -1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \cdots\cdots$$

$$= -\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots\right)$$

which is divergent by p-test ($p=1$).

Hence, Interval of convergence = $(-1, 1]$

and Centre of convergence $= \frac{-1+1}{2} = \frac{0}{2} = (0)$

Radius $= \frac{1-(-1)}{2} = \left(\frac{2}{2}\right) = (1)$

15. $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$

Sol. General term $(u_n) = \frac{x^n}{n!}$ and $(u_{n+1}) = \frac{x^{n+1}}{(n+1)!}$

By ratio's Test, $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(n+1)!} \times \frac{n!}{x^n}$

$$= \frac{x^1}{(n+1)\cancel{n!}} \times \frac{\cancel{n!}}{1}$$

$$= \lim_{n\to\infty} \frac{x}{(n+1)} = \frac{x}{\infty} = (0)$$

By ratio test for all finite value of 'x' in $(-\infty, \infty)$.

Therefore, the intervals of convergence is $(-\infty, \infty)$

and Centre of convergence is, $\frac{-\infty+\infty}{2} = 0$

~~and~~, Also, Radius of convergence is $\frac{\infty-(-\infty)}{2} = (\infty)$

Thus, the interval is $(-\infty, \infty)$, centre is 0 and radius is ∞ of convergence series.

(16) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

Given that, General term $(u_n) = \frac{x^{2n-1}}{(2n-1)}$

and $u_{(n+1)} = \frac{x^{2(n+1)-1}}{2(n+1)-1}$

$= \frac{x^{2n+2-1}}{(2n+2-1)} = \frac{x^{2n+1}}{(2n+1)}$

Then, By ratio's test,

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{\cancel{x^{2n}} \cdot x}{(2n+1)} \times \frac{(2n-1)}{\cancel{x^{2n}} \cdot x^{-1}}$

$= \lim_{n\to\infty} x^2 \frac{\cancel{n}\left(2-\frac{1}{n}\right)}{\cancel{n}\left(2+\frac{1}{n}\right)}$

$= (x^2)$

By ratio'test, the given series for $x^2 < 1 \Rightarrow |x| < 1$ and is divergent $|x| > 1$. And further test is needed at $|x| = 1$.

At $x = 1$, the given series is, $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

This is an alternative series, whose general positive term is, $(u_n) = \frac{1}{(2n-1)}$

$\lim_{n\to\infty} (u_n) = \lim_{n\to\infty} \frac{1}{(2n-1)}$

$= \frac{1}{\infty} = 0.$

By, Leibnitz theorem it is convergent at $x = 1$

Again, At $x = -1$, series becomes, $\left(-1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} + \cdots\right)$

$= -\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$

Which is alternating series and $(u_{n+1}) < u_n$. Then,

$$(u_n) = \frac{1}{(2n-1)}$$

$$\lim_{n\to\infty}(u_n) = \lim_{n\to\infty}\frac{1}{n(2-\frac{1}{n})} = \frac{1}{\infty} = (0)$$

By Leibnitz, It is convergent at $x=1$

Hence interval of convergence $[-1,1]$

$\therefore$ Radius of convergence $= \frac{1-(-1)}{2} = (1)$

Centre of convergence $= \left(\frac{-1+1}{2}\right) = (0)$

Given series is,

$$x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \cdots$$

This is an alternative series, Then,

$\therefore$ General term $|u_n| = \frac{x^n}{n^2}$

and $|u_{n+1}| = \frac{x^{(n+1)}}{(n+1)^2} = \frac{x^n.x}{(n+1)^2}$

By ratio's Test, $\lim_{n\to\infty}\frac{|u_{n+1}|}{|u_n|} = \lim_{n\to\infty}\frac{\cancel{x^n}.x}{(n+1)^2} \times \frac{n^2}{\cancel{x^n}}$

$$= \lim_{n\to\infty}\frac{n^2.(x)}{(n+1)^2}$$

$$= \lim_{n\to\infty}\frac{\cancel{n^2}(x)}{\cancel{n^2}(1+\frac{1}{n})^2} = (x)$$

Here, By ratio test the given series is convergent for $|x|<1$ and is diverges for $|x|>1$. And further test is needed at $|x|=1$.

At $x=1$ then the series, $\left(1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\right)$

This is an alternative series, whose

$|u_n| = \frac{1}{n^2}$

Here, the series is convergent by p-test (p>2).
at $x = 1$.

Now, At $x = -1$, the series becomes, $-1 - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - \cdots$

$$= -\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)$$

Then,

$$(u_n) = \left(\frac{1}{n^2}\right)$$

Clearly, the series is convergent by p-test (p>2) at $x = -1$.

Then,

interval of convergence is $[-1, 1]$

Centre of the convergence $= \left(\frac{-1+1}{2}\right) = 0$

Also, Radius of convergence $= \frac{1-(-1)}{2}$

$$= \frac{1+1}{2} = (1)$$

Find the interval of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{x^n}{(n+4)}$$

Here, Given series is, $\sum_{n=0}^{\infty} \left(\frac{x^n}{n+4}\right)$

General term $(u_n) = \left(\frac{x^n}{n+4}\right)$ and $u_{n+1} = \left(\frac{x^{n+1}}{n+5}\right)$

Then, $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{n+5} \times \frac{(n+4)}{x^n} = \lim_{n\to\infty} \frac{x \cdot n(1+\frac{4}{n})}{n(1+\frac{5}{n})}$

$$= (x)$$

By ratio test, the given series is convergent for $|x|<1$ and is divergent $|x|>1$ and further test is needed at $|x| = 1$.

At $x=1$ the series becomes.

$$\text{General term } (u_n) = \left(\frac{1}{n+4}\right) = \frac{1}{n(1+4/n)}$$

$$\lim_{n\to\infty} u_n = \frac{1}{\infty} = (0) \quad (\text{it may be convergent})$$

Let us choose another infinite series,

$$(V_n) = \left(\frac{1}{n}\right). \text{ Then,}$$

$$\therefore \lim_{n\to\infty}\left(\frac{u_n}{V_n}\right) = \frac{1}{n(1+4/n)} \times \frac{n}{1} = \left(\frac{1}{1+\frac{1}{\infty}}\right)$$

$$= \left(\frac{1}{1+0}\right) = (1) \text{ (which is non-zero and finite)}$$

which is $V_n = \left(\frac{1}{n}\right)$ is divergent p-test. $(p \le 1)$.

Since the series ΣV_n is divergent and therefore the series $\Sigma u_n = \Sigma\left(\frac{1}{n+4}\right)$ is also divergent.

$\boxed{\text{At } x=-1}$ then,

$$\text{General term } (u_n') = \frac{(-1)^n}{(n+4)}$$

$$|u_n| = \left(\frac{1}{n+4}\right)$$

$$\text{and } |u_{n+1}| = \left(\frac{1}{n+5}\right)$$

$$\text{and } \lim_{n\to\infty}\frac{|u_{n+1}|}{|u_n|} = \lim_{n\to\infty}\frac{1}{(n+5)} \times \frac{(n+4)}{1} = \frac{\cancel{n}(1+\frac{4}{n})}{\cancel{n}(1+\frac{5}{n})}$$

$$= \left(\frac{1}{1}\right) = (1)$$

which is convergent at $x=-1$.

OR method

At $x = -1$,

General term $(u_n) = \frac{(-1)^n}{(n+4)}$

$$= \frac{-1}{5} + \frac{1}{6} - \frac{1}{7} + \frac{1}{8} + \cdots$$

This is alternating series and then $u_{n+1} < u_n$.

$$(u_n) = \left(\frac{1}{n+4}\right)$$

$$\lim_{n\to\infty} u_n = \frac{1}{n(1+\frac{4}{n})} = (0)$$

By Leibnitz's theorem it is convergent.

Hence,

$$\text{interval of convergence} = [-1, 1)$$

$$\text{Centre} = \frac{-1+1}{2} = 0$$

$$\text{Radius} = \frac{1-(-1)}{2} = \frac{2}{2} = (1)$$

$$\sum_{n=0}^{\infty} \frac{n^2}{2^n} x^n$$

Given, series is, $(u_n) = \left(\frac{n^2}{2^n} x^n\right)$

and $(u_{n+1}) = \frac{(n+1)^2}{2^{(n+1)}} x^{(n+1)}$

By ratio's Test,

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+1)^2}{2^{(n+1)}} x^{(n+1)} \times \frac{2^n}{n^2 \cdot x^n}$$

$$= \lim_{n\to\infty} \frac{x}{2} \frac{\cancel{n^2}(1+\frac{1}{n})^2}{\cancel{n^2}} = \left(\frac{x}{2}\right)$$

By ratio's test, by given series is convergent for $\left|\frac{x}{2}\right| < 1$ and $\left|\frac{x}{2}\right| > 1$ is divergent and further test is needed at $\left|\frac{x}{2}\right| = 1 \Rightarrow |x| = 2$.

At $x=2$,

$$\lim_{n\to\infty} u_n = \left(\frac{n^2 \cdot 2^n}{2^n}\right) = \lim_{n\to\infty} \frac{n^2 \cdot (2)^n}{(2^n)} = (\infty) \neq 0$$

So, the given series is divergent at $x=2$.

$\boxed{\text{At } x=-2}$

$$\lim_{n\to\infty} (u_n) = \frac{n^2(-2)^n}{2^n} = (-1)^n n^2$$

series becomes, $-1+4-9+16-\cdots$

This given series is alternative series, ~~let us choose another series infinite series,~~

~~$\Sigma v_n = n^2$~~

~~The $\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{n^2}{n^2} = (1)$ which is non-zero & finite.~~

~~Here The $\Sigma |u_n| = 1^2+2^2+3^2+\cdots$ So, is denoted by $t=-2$.~~

$$\sum_{n=1}^{\infty} (-1)^n n^2 = (-1^2+2^2-3^2+4^2-5^2+\cdots)$$

$$= -(1^2+2^2+3^4+4^2+\cdots)$$

Here, the terms are $u_{n+1} \nless u_n$. The Leibnitz theorem is not applicable.

which is divergent.

Thus, the given series is convergent for $|x|<2$ and diverges for $|x| \geq 2$. That is the interval of convergence is $(-2, 2)$.

(20) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{\sqrt{n}}$

Soln General term $(u_n) = \left(\frac{x^n}{\sqrt{n}}\right)$. Then, $(u_{n+1}) = \left(\frac{x^{n+1}}{\sqrt{n+1}}\right)$

By ratio Test,

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{\sqrt{n+1}} \times \frac{\sqrt{n}}{x^n} = \lim_{n\to\infty} \frac{x}{\sqrt{\left(1+\frac{1}{n}\right)}}$$

$$= (x)$$

By ratio test, the given series is convergent $|x|<1$ and divergent $|x|>1$ and further test is needed at $|x|=1$.

At $\boxed{x=1}$, the given series,

$$= \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots\right)$$

This given ~~series~~ is aliternative series, and $(u_{n+1} < u_n)$. So, $(u_n) = \left(\frac{1}{\sqrt{n}}\right)$

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n^{\frac{1}{2}}} = \left(\frac{1}{\infty}\right) = (0)$$ (it satisfy the leibnitz)

Then, it is convergent.

$\boxed{\text{At } x=-1}$ Then, series becomes,

$$-\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} - \cdots$$

$$= -\left(\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots\right)$$

$$(u_n) = \frac{1}{\sqrt{n}} = \frac{1}{(n^{\frac{1}{2}})}$$

which is divergent p-test $\left(P = \frac{1}{2} < 1\right)$.

Hence interval of convergence $= (-1, 1]$

1) $\sum_{n=2}^{\infty}\left(\frac{n}{n^2+1}\right)x^n$

Given that General form $(u_n) = x^n\left(\frac{n}{n^2+1}\right)$

and $(u_{n+1}) = x^{n+1}\frac{(n+1)}{(n+1)^2+1}$

By ratio's Test,

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n} = \lim_{n\to\infty}\frac{x^{n+1}(n+1)}{(n+1)^2+1}\times\frac{(n^2+1)}{n\cdot x^n}$$

$$= \lim_{n\to\infty} x\cdot\frac{\cancel{n^3}\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n^2}\right)}{\cancel{n^3}\left[\left(1+\frac{1}{n}\right)^2+\frac{1}{n^2}\right]}$$

$$= \left(\frac{x}{1}\right) = (x)$$

By ratio test, the given series is convergent for $|x|<1$ and divergent $|x|>1$. And further test is needed at $|x|=1$.

At $x=1$ the given series is,

$$= \frac{1}{2}+\frac{2}{5}+\frac{3}{10}+\cdots$$

$$(u_n) = \left(\frac{n}{n^2+1}\right)$$

$$\lim_{n\to\infty}(u_n) = \lim_{n\to\infty}\frac{n}{(n^2+1)} = \lim_{n\to\infty}\frac{n}{n^2\left(1+\frac{1}{n^2}\right)} = \frac{1}{n\left(1+\frac{1}{n^2}\right)}$$

$$= \frac{1}{\infty} = (0) \text{ (it may be convergent.)}$$

= Let us choose another infinite series,

$\sum v_n = \left(\frac{1}{n}\right)$ which is divergent p-test.

and $\lim_{n\to\infty}\frac{u_n}{v_n} = \lim_{n\to\infty}\frac{\left(\frac{1}{n}\right)\times\frac{1}{\left(1+\frac{1}{n^2}\right)}}{\left(\frac{1}{n}\right)} = (1)$ (non-zero) & finite quantity

Hence the given series divergent.

At $x=-1$, The given series becomes,

$$\sum_{n=2}^{\infty}\left(\frac{n}{n^2+1}\right)(-1)^n = \frac{2}{5}-\frac{3}{10}+\frac{4}{17}-\frac{5}{26}+\ldots$$

The given series is alternative series. and $(u_{n+1} > u_n)$

Then,

$$(u_n) = \left(\frac{n}{n^2+1}\right)$$

$$\lim_{n\to\infty}(u_n) = \lim_{n\to\infty}\left(\frac{n}{n^2+1}\right) = \frac{n}{n^2\left(1+\frac{1}{n^2}\right)}$$

$$= \lim_{n\to\infty}\frac{1}{n\left(1+\frac{1}{n^2}\right)} = \frac{1}{\infty} = (0)$$

It satisfies the Leibnitz condition. Then it is convergent.

Thus, The given series is convergent for $|x|<1$ and divergies for $|x|>1$. ~~Thus~~ Then,

Interval of convergence of series $[-1,1)$.

(b) $\sum_{n=0}^{\infty}\left(\frac{n+1}{10^n}\right)(x-4)^n$

The given series,

$$\text{General term } (u_n) = \frac{(n+1)}{(10^n)}(x-4)^n$$

$$\text{and } (u_{n+1}) = \frac{(n+2)}{10^{(n+1)}}(x-4)^{n+1}$$

By ratio test, $\lim_{n\to\infty}\frac{u_{n+1}}{u_n} = \lim_{n\to\infty}\frac{\cancel{(x-4)^n}\cdot(x-4)(n+2)}{10^{(n+1)}}\times\frac{(10^n)}{(n+1)\cancel{(x-4)^n}}$

$$= \lim_{n\to\infty}\frac{(x-4)(n+2)}{10(n+1)} = \lim_{n\to\infty}\frac{n\left(1+\frac{2}{n}\right)(x-4)}{n\left(1+\frac{1}{n}\right)\times 10}$$

$$= \frac{(x-4)}{10}$$

By ratio test, the given series is convergent for $\left|\frac{x-4}{10}\right| < 1 \Rightarrow |x-4| < 10$ and divergent for $|x-4| > 10$.

And further test needed at $|x-4| = 10$.

At $(x-4) = 10$

$$(u_n) = \frac{(n+1)}{(10^n)} \times (10)^n = (n+1)$$

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} (n+1) = (\infty \neq 0)$$

The given series is divergent at $\underline{x = 14}$.

$\boxed{(x-4) = -10}$

The general term of given series is $(u_n) = \frac{(n+1)}{10^n} \times (-10)^n$

$$= (-1)^n (n+1)$$

The given series becomes,

$$= -2 + 3 - 4 + 5 - \cdots$$

The given series is ~~divergent~~ alternative and $(u_{n+1} \ngtr u_n)$

$$\lim_{n\to\infty} (u_n) = \lim_{n\to\infty} (n+1) \Rightarrow (\infty) \neq 0.$$

Then, The given series is divergent.

Thus, the given series is convergent for $|x-4| < 10$ and is divergent for $|x-4| \geq 10$.

interval of convergence of given series is,

$$-10 < (x-4) < 10$$

$$\Rightarrow -10+4 < x < 10+4$$

$$\Rightarrow -6 < x < 14 \quad \text{i.e } (-6, 14).$$

OR Method, By ratio test, it is convergent for $\left|\frac{x-4}{10}\right| < 1$ and divergent for $\left|\frac{x-4}{10}\right| > 1$. further test is needed at $\left|\frac{x-4}{10}\right| = 1$.

$$\therefore \left|\frac{x-4}{10}\right| = 1 \Rightarrow \left(\frac{x-4}{10}\right) = (-1) \text{ and } \left(\frac{x-4}{10}\right) = (1)$$

$\frac{x-4}{10} = -1$ and $\frac{x-4}{10} = 1$

$\Rightarrow (x-4) = -10$ $\Rightarrow x-4 = 10$

$\Rightarrow \boxed{x = -6}$ $\therefore \boxed{x = 14}$

$\boxed{\text{At } x = 14}$

$$= \sum_{n=1}^{\infty} \left(\frac{n+1}{10^n}\right)(x-4)^n \Rightarrow \left(\frac{n+1}{(10^n)}\right)(10)^n$$

$$(2+3+4+5+\cdots)$$

$$\lim_{n\to\infty}(u_n) = \lim_{n\to\infty}\left(\frac{n+1}{10^n}\right)(10)^n$$

$$= \lim_{n\to\infty}(n+1) \Rightarrow (\infty \neq 0)$$

It is divergent at $x = 14$.

$\boxed{\text{At } x = -6}$

$$\sum_{n=1}^{\infty} \frac{(n+1)}{(10)^n}(-10)^n = \sum_{n=0}^{\infty}(n+2)(-1)^n$$

$$= (-2+3-4+5-6+\cdots)$$

Since the series is alternative series ~~and~~ but $(u_{n+1} \nless u_n)$. Then, Leibnitz condition can not be used.

$$\therefore \lim_{n\to\infty}|u_n| = \lim_{n\to\infty}(n+1)$$

$$= (\infty \neq 0).$$

The given series is divergent.

Hence, the interval of convergence $= (-6, 14)$

$\therefore$ Centre $= \frac{-6+14}{2} = \frac{8}{2} = (4)$ Ans

Radius $= \frac{14-(-6)}{2} = \left(\frac{20}{2}\right) = (10)$ Ans

(3) $\sum_{n=0}^{\infty} \frac{n!}{100^n} x^n$

The general term $(u_n) = \frac{n!}{(100)^n} x^n$

and $(u_{n+1}) = \frac{(n+1)!}{(100)^{n+1}} x^{n+1}$

By ratio's Test,

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+1)!\, x^{n+1}}{(100)^n.(100)} \times \frac{(100)^n}{x^n . n!}$$

$$= \lim_{n\to\infty} \frac{(n+1)\cancel{(n!)}\cancel{x^n}.x \times \cancel{(100)^n}}{\cancel{(100)^n}.(100)\ \cancel{x^n}\cancel{(n!)}}$$

$$= \lim_{n\to\infty} \frac{x(n+1)}{100} = (\infty).$$

It is divergent. ∞

It is converges only for $x = 0$.

(4) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(-4)^n}$

Ans. General term $|u_n| = \frac{x^{(2n+1)}}{(4)^n}$

and $|u_{n+1}| = \frac{x^{2(n+1)+1}}{(4)^{n+1}} = \frac{x^{(2n+3)}}{(4)^{n+1}}$

By ratio test, $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{x^{(2n+3)}}{(4)^{n+1}} \times \frac{(-4)^n}{x^{(2n+1)}}$

$$= \frac{x^2}{(4)}$$

By ration test,

It is convergent $\left|\frac{x^2}{4}\right| < 1$ and divergent $\left|\frac{x^2}{4}\right|$

and further test is needed at $\left|\frac{x^2}{4}\right| = 1$.

$\therefore \left|\frac{x^2}{4}\right| = 1 \Rightarrow \left(\frac{x^2}{4}\right) = 1$ and $\frac{x^2}{4} = -1$

and ~~[illegible]~~ and $x^2 = -4$

$\therefore x^2 = 4$ $\quad\therefore x^2 = (-4)$

$\therefore x = 2$ $\quad\therefore (x = \pm 2)$

$\boxed{\text{At } x=2}$

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(-4)^n} = \sum_{n=0}^{\infty} \frac{(2)^{2n+1}}{(-4)^n}$$

$$= \sum_{n=0}^{\infty} \frac{(2)^{2n+1}}{(-4)^n} = 2 - 2 + 2 - 2 + 2 - \cdots$$

The given series alternative series, but $u_{n+1} \not< u_n$.
Then, can not be Leibnitz condition.
~~It is divergent~~

$\therefore \lim_{n\to\infty} |u_n| = \lim_{n\to\infty} |2| \neq 0$

$= (2)$. It is divergent.

Again,

At $x = -2$, $\sum_{n=0}^{\infty} \frac{(-2)^{2n+1}}{(-4)^n}$

$= (-2 + 2 - 2 + 2 - \cdots)$

$\lim_{n\to\infty} (u_n) = (2) \neq 0$.

It is divergent.

Hence the interval of convergence $= (-2, 2)$

(25) $\sum_{n=0}^{\infty} \frac{2^n}{(2n)!} \cdot x^{2n}$

The general term $(u_n) = \frac{2^n \cdot x^{2n}}{(2n)!}$ and,

$$(u_{n+1}) = \frac{2^{n+1} \cdot x^{2(n+1)}}{2(n+1)!}$$

By ratio test,

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{2^{n+1} \cdot x^{2n+2}}{(2(n+1))!} \times \frac{(2n)!}{2^n \cdot x^{2n}}$$

$$= \lim_{n\to\infty} \frac{2x^2}{(2n+2)(2n+1)} = \lim_{n\to\infty} \frac{2x^2}{n^2(2+\frac{2}{n})(2+\frac{1}{n})}$$

$$= \frac{2x^2}{\infty} = (0)$$

It is convergent for every finite value of x.

interval of convergence $(-\infty, \infty)$

$\therefore$ centre $= 0$.

$\therefore$ radius of convergence tends to infinity.

(26) $\sum_{n=0}^{\infty} \frac{3^{2n} (x-2)^n}{(n+1)}$

General term $(u_n) = \frac{3^{2n}(x-2)^n}{(n+1)}$ and

$$\therefore (u_{n+1}) = \frac{3^{2n+2} \cdot (x-2)^{n+1}}{(n+2)}$$

By ratio's test, $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{3^{(2n+2)}(x-2)^{n+1}}{(n+2)} \times \frac{(n+1)}{(3^{2n})(x-2)^n}$

$$= \lim_{n\to\infty} \frac{3^2 \times (x-2)(n+1)}{(n+2)}$$

$= \lim_{n\to\infty} \dfrac{9(x-2)\ \cancel{n}(1+\frac{1}{n})}{\cancel{n}(1+\frac{2}{n})}$

$= \lim \ 9(x-2)$

$= 9x-18$

(2)

By ratio test, it is convergent for $|9x-18|<1$ and divergent for $|9x-18|>1$. Further test is needed at $|9x-18|=1$

$\therefore 9x-18=1$ and $\therefore 9x-18=-1$

$\Rightarrow x = \left(\frac{19}{9}\right)$ $\Rightarrow 9x = 17$

$\therefore x = \left(\frac{17}{9}\right)$

$\boxed{\text{At } x = \frac{19}{9}}$

$\sum_{n=0}^{\infty} \dfrac{3^{2n}(x-2)^n}{(n+1)} = \sum_{n=0}^{\infty} \dfrac{3^{2n}\left(\frac{1}{9}\right)^n}{(n+1)} = \sum_{n=0}^{\infty} \dfrac{(1)^n}{(n+1)}$

$= 1 + \frac{1}{2} + \frac{1}{3} + \cdots$

$\therefore (u_n) = \left(\frac{1}{n+1}\right) \Rightarrow \lim_{n\to\infty} u_n = \lim_{n\to\infty} \dfrac{1}{n(1+\frac{1}{n})} = \frac{1}{\infty} = (0)$

Let $v_n = \left(\frac{1}{n}\right)$, $\lim_{n\to\infty} \dfrac{u_n}{v_n} = \dfrac{\cancel{n}(1+\frac{1}{n})}{\cancel{n}} = (1)$ (non zero finite)

~~$\lim_{n\to\infty}(v_n) = \lim_{n\to\infty}\left(\frac{1}{n}\right) = (0)$ it may be convergent~~

It is divergent series by ~~[illegible]~~ p-test.

At $\left(x=\frac{17}{9}\right)$, $= \sum_{n=0}^{\infty} \dfrac{3^{2n}\left(\frac{17}{9}-2\right)^n}{(n+1)} = \dfrac{3^{2n}\left(-\frac{1}{9}\right)^n}{(n+1)}$

$(u_n) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

This given series is alternative series: and $(u_{n+1}) > u_n$.

$\lim_{n\to\infty}(u_n) = \lim_{n\to\infty}\frac{1}{(n+1)} = \frac{1}{\infty} = (0)$. It is convergent.

(Leibnitz satisfy)

interval of convergence = $\left[\frac{17}{9}, \frac{19}{9}\right)$.

(27) $\sum_{n=0}^{\infty} \frac{n^2}{2^{3n}} (x+4)^n$

Sol^n: General term $(u_n) = \frac{n^2}{2^{3n}} (x+4)^n$ and

$$(u_{n+1}) = \frac{(n+1)^2}{2^{3n+3}} (x+4)^{n+1}$$

and $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+1)^2 (x+4)^{n+1}}{2^{3n+3}} \times \frac{2^{3n}}{n^2 (x+4)^n}$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}\left(1+\frac{1}{n}\right)^2 (x+4)}{\cancel{n^2} \cdot 8}$$

$$= \frac{(x+4)}{8}$$

By ratio test the given series is convergent for $\left|\frac{x+4}{2}\right| < 1$ and is divergent for $\left|\frac{x+4}{8}\right| > 1$. and further test needed at $\left|\frac{x+4}{2}\right| = 1$ $\Rightarrow$ $(x+4) = 8$ and $(x+4) = -8$

$\boxed{x = 4}$ and $\boxed{x = -12}$

$\boxed{\text{At } x = 4}$

$$\sum_{n=0}^{\infty} \frac{n^2}{(2^{3n})} (8)^n = (1 + 4 + 9 + 16 + \cdots$$

$\boxed{\because 8^n = (2^3)^n = 2^{3n}}$

$\therefore (u_n) = n^2$

$\lim_{n\to\infty} u_n = (n^2) \Rightarrow \infty \; (\infty \neq 0)$

It is divergent

$\boxed{\text{At } x = -12}$. $\sum_{n=0}^{\infty} \frac{n^2}{2^{3n}} (-8)^n = n^2 (-1)^n$

$$= -1 + 4 - 9 + 16 + \cdots$$

The given series is alternative but ($u_{n+1} \nless u_n$)
Then, Leibnitz can not be applicable.

$$|u_n| = n^2$$

$$\lim_{n\to\infty} |u_n| = \lim_{n\to\infty} (n^2) = (\infty) \text{ (divergent)}$$

$\therefore$ Hence, the interval of convergence $= (-11, 4)$.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (2x-1)^n}{n \cdot 6^n}$$

General term $|u_n| = \dfrac{(2x-1)^n}{n \cdot 6^n}$.

and $|u_{n+1}| = \dfrac{(2x-1)^{n+1}}{(n+1)\, 6^{(n+1)}}$

By ratio test,

$$\lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|} = \lim_{n\to\infty} \frac{(2x-1)^{n+1}}{(n+1)\, 6^{(n+1)}} \times \frac{n \cdot 6^n}{(2x-1)^n}$$

$$= \lim_{n\to\infty} \frac{(2x-1)\, \cancel{n}}{\cancel{n}(1+\frac{1}{n}) \times 6}$$

$$= \frac{(2x-1)}{6}$$

By ratio's test, it is convergent for $\left|\frac{2x-1}{6}\right| \leq 1$ and divergent for $\left|\frac{2x-1}{6}\right| \geq 1$. Further test is needed.

at $\left|\frac{2x-1}{6}\right| = 1$.

$\left(\frac{2x-1}{6} = 1\right)$ & $\left(\frac{2x-1}{6} = -1\right)$

or $2x = 7$ & $2x = -6+1$

$\therefore (x = 7/2)$ $\quad \therefore \boxed{x = -5/2}$

At $x = \left(\frac{7}{2}\right)$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (6)^n}{n \cdot (6)^n} \Rightarrow -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots$$

This is alternative series and $(u_{n+1} < u_n)$. Then, it satisfy the Leibnitz's condition.

$\lim_{n\to\infty} (u_n) = \lim_{n\to\infty} \left(\frac{1}{n}\right) = (0)$ is convergent.

$\boxed{\text{At} = -\frac{5}{2}}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (-6)^n}{n (6)^n} = \sum_{n=1} \frac{(-1)^{2n}}{n}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

$\therefore$ it is divergent by p-test.

Thus, interval of convergence $= \left(-\frac{5}{2}, \frac{7}{2}\right]$

$$\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n!} (x-4)^n$$

Soln Given as, $|u_n| = \frac{3^n}{n!} (x-4)^n$

and $|u_{n+1}| = \frac{3^{n+1}}{(n+1)!} (x-4)^{n+1}$

By ratio test, $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{|u_{n+1}|}{|u_n|} = \frac{3^{n+1}}{(n+1)!} (x-4)^{n+1} \times \frac{n!}{3^n (x-4)^n}$

$$= \lim_{n\to\infty} \frac{3(x-4)}{(n+1)}$$

$$= (0)$$

It is convergent for every finite value of x.

interval of convergence $= (-\infty, \infty)$