Chapter 2

Non-Linear Model

Introduction

The linear devices are those devices which follow ohm's law ($V=IR$). In an electric circuit, a linear device is an electrical element with a linear relationship between input current & output voltage. Resistor, inductors, capacitors are example of linear devices.

The VI characteristics of linear device is a straight line.

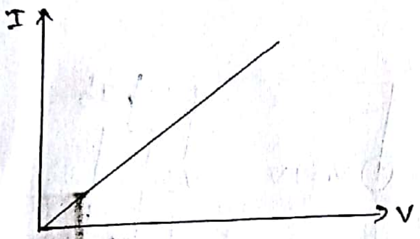


fig: VI characteristics of a linea device

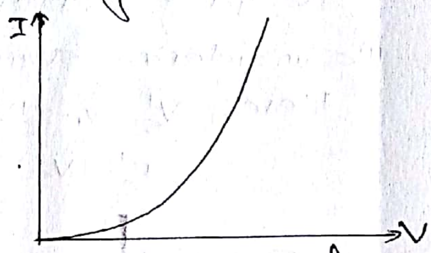


fig: VI characteristics of non-linear devices.

The electronic devices which donot follow ohms law ($V \neq IR$) or whose VI-characteristics is not straight line are called non-linear devices.

Properties of Non-Linear devices

1. They do not follow Ohm's law.
2. Their VI characteristics is not a straight line.
3. They don't follow the network theorem like Thevenins Theorem, Nortons Theorem, superposition theorem etc.
4. The circuits containing non-linear devices can be analyzed by using KVL or KCL.
5. The internal resistance of non-linear device depends on the operating conditions like temperature, current, voltage etc.
6. Examples are: Diode, Transistors, FET etc.

#Non-Linear Circuit Analysis#

Verify with appropriate example that non-linear device doesn't follow principle of superposition.

→ Consider a non-linear device (square-law device) whose VI characteristics eq^n is given by,

$$I_S = A(V_S - V_{TR})^2 \quad \text{---(1)}$$

where, I_S = current through non-linear device

V_S = voltage across non-linear device

V_{TR} = Threshold voltage $= 0$

A = constant $= 1\,mA/V^2$

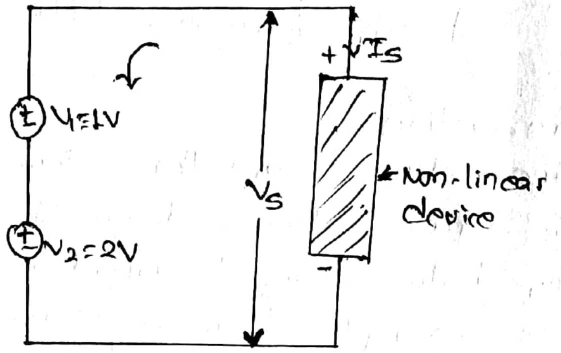
then, $I_S = 1 * (V_S - 0)^2 = V_S^2$ ---(2)

Now, connect the non-linear device in a simple circuit as shown in figure.

Here, using KVL,

$-V_1 - V_2 + V_S = 0$

$\Rightarrow V_S = V_1 + V_2 = 1+2 = 3V$



From eqn (2),

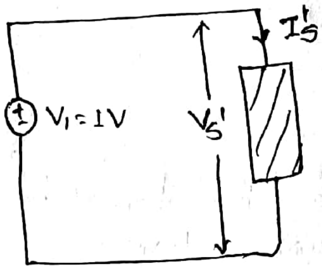
$I_S = V_S^2 = (3)^2 = 9mA$

To apply superposition theorem,

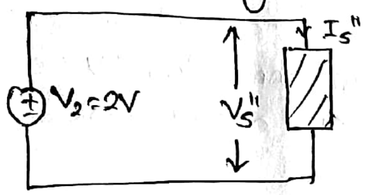
1) Considering source V_1 only,

Here, $V_S' = V_1 = 1V$

$I_S' = (V_S')^2 = 1mA$



2) Considering source V_2 only,



Here, $V_S'' = V_2 = 2V$

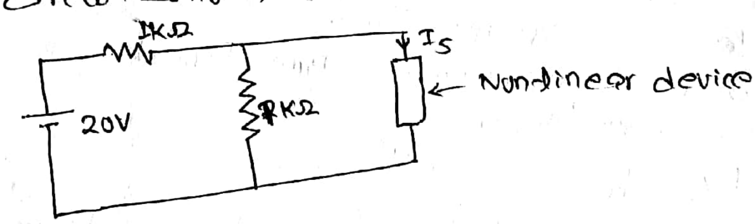
$I_S'' = (V_S'')^2 = (2)^2 = 4mA$

Now, current through a non-linear device according to superposition theorem is,

$I_{S1} = I_S' + I_S'' = 1+4 = 5mA$

Since $I_S \neq I_{S1}$,
we can say that non-linear device doesn't follow principle of superposition.

For the circuit shown, find I_S & V_S.

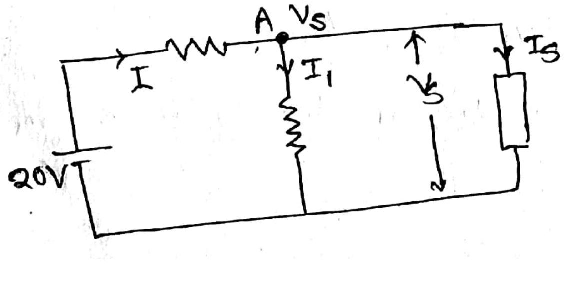


Soln: We know that,

$I_S = A(V_S - V_{TR})^2$, where,

$A = 1\,mA/V^2$ & $V_{TR} = 0$.

$\therefore I_S = 1 \times (V_S - 0)^2 = V_S^2$ ——— (1)



Applying KCL at node A,

$$I = I_1 + I_s$$

or, $\frac{20-V_s}{1} = \frac{V_s}{1} + I_s$

or, $20 - V_s = V_s + I_s$

or, $20 - V_s = V_s + V_s^2$ $\quad$ ($\because$ from eqn (1), $I_s = V_s^2$)

or, $V_s^2 + 2V_s - 20 = 0$

which is quadratic in V_s.

On solving we get,

$V_s = 3.58V$ or $-5.58V$

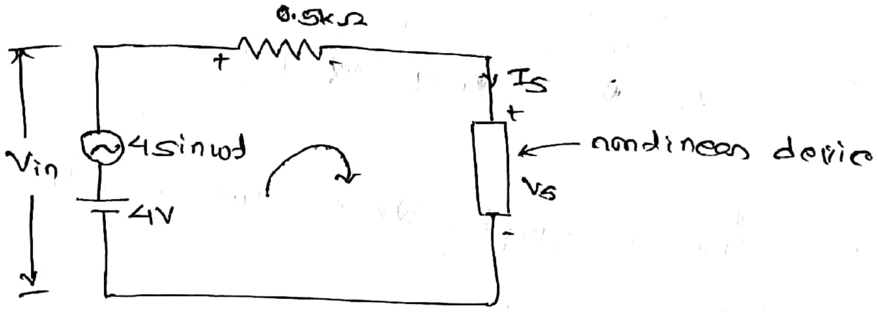
Neglecting -ve value of V_s, we get

$\therefore V_s = 3.58V$

and, $I_s = V_s^2 = (3.58)^2 = 12.82 mA$

Graphical Analysis of Circuit with non-linear Devices:

Find the operating point (or value of I_s & V_s) of non-linear device shown in circuit below. Assume $I_s = A(V_s - V_{TR})^2$ where, $A = 1 mA/V^2$ & $V_{TR} = 0$.



Soln: Given, $I_s = A(V_s - V_{TR})^2 = 1 * (V_s - 0)^2 = V_s^2$

$\Rightarrow I_s = V_s^2$ ——(1)

To plot eqn (1) in a graph,

V_s	0	1	2	3	4
I_s	0	1	4	9	16

Applying KVL in given circuit,

$4 + 4\sin\omega t - 0.5 * I_s - V_s = 0$

or, $4 + 4\sin\omega t = 0.5 I_s + V_s$

$\Rightarrow 8 + 8\sin\omega t = I_s + 2V_s$ ——(2)

Take $\omega t = \pi/2$,

Then, $8 + 8\sin\pi/2 = I_s + 2V_s$

or, $8 + 8 = I_s + 2V_s$

$\Rightarrow 16 - 2V_s = I_s$ ——(3)

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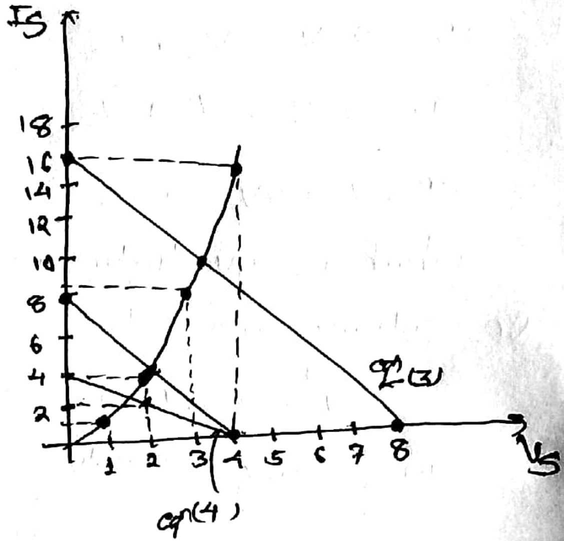
To plot eqn (3) on graph,

I_s	14	12	10	8	6	16	0
V_s	1	2	3	4	5	0	8



Again, take $\omega t = \pi$

Then, $8 + 8\sin\pi = I_s + 2V_s$

or, $8 = I_s + 2V_s$

or, $I_s = 8 - 2V_s$ ——— (4)

To plot eqn (4) on graph,

I_s	8	6	4	2	0
V_s	0	1	2	3	4

Take $\omega t = 3\pi/2$

Then, $8 + 8\sin(3\pi/2) = I_s + 2V_s$

or, $8 + 8(-1) = I_s + 2V_s$

$\Rightarrow I_s = -2V_s$ ——— (5)

To plot eqn (5) in graph,

I_s	0	-2	2	-4	4	-6	6
V_s	0	1	-1	2	-2	3	-3

Now, from a graph, Q_2 is the operating point.

Thus, $V_s = 2V$

& $I_s = 2mA$.

Diode as a Non-linear Device

The device which doesnot follow Ohm's law ($V = IR$) or whose VI characteristic curve is not a straight line is called a non-linear device.

For a diode, the voltage-current relation is given by

$$I = I_s(e^{V/\eta V_T} - 1)$$

where, I_s = reverse (leakage) saturation current

η = constant $(1 \le \eta \le 2)$

V_T = thermal voltage

Diode is a non-linear device because diode does not have a linear relationship betⁿ current & voltage. Diode are called non-linear devices because they possesses the properties of non-linear devices. (which are described earlier).

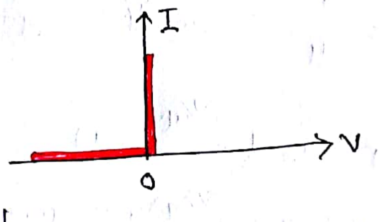
Piecewise Linear Model of a Diode

Ideal Model

Forward biased diode → short circuit (closed switch),

Reverse biased diode → open circuit (open switch)

Barrier potential $=0$



Piecewise Linear model

The V-I relationship for a diode is given by

$$I = I_S\left(e^{V/\eta V_T} - 1\right) \quad \text{—— (1)}$$

Where, I_S = reverse saturation current

η = constant $(1 \le \eta \le 2)$

If we plot above eqⁿ, we get a non-linear characteristics as shown in figure below

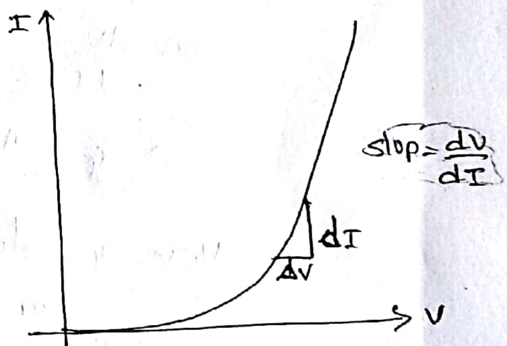


Fig: VI characteristics of a diode.

In piecewise linear modeling of the diode, we represent the above non-linear characteristics by two straight lines OA & AB as shown in figure below.

Equation of line OA:

$$I = 0 \quad \text{for } V < V_f$$

Equation of line AB:

$$I = \frac{V - V_f}{r_f} \quad \text{for } V > V_f$$

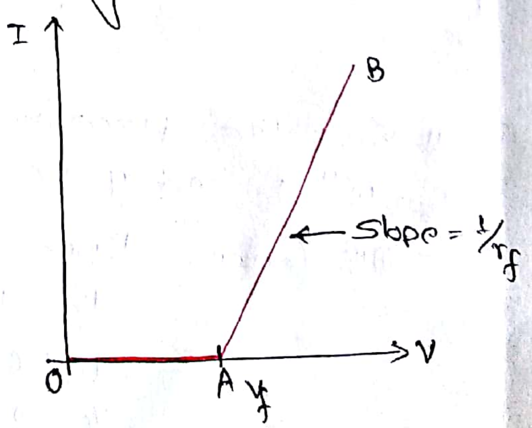


Fig: piecewise linear model of a diode.

Thus we represent the non-linear characteristics of diode in piecewise linear model as,

$$I = \begin{cases} 0 & \text{for } V < V_f \\ \dfrac{V - V_f}{r_f} & \text{for } V > V_f \end{cases}$$

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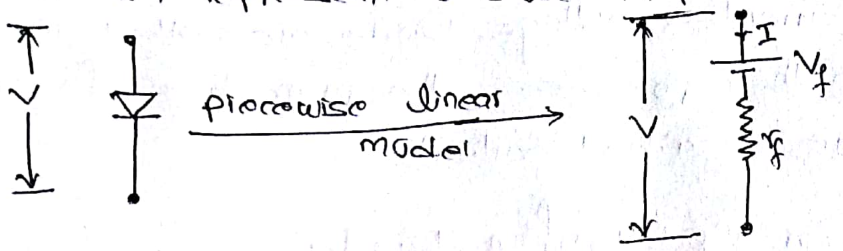
So, we can represent a diode in piecewise linear model as,



we know that,

$$\text{slope} = \frac{1}{r_f}$$

or, $\frac{dI}{dV} = \frac{1}{r_f}$ $\Rightarrow r_f = \frac{dV}{dI}$ ——— (1)

we have,

$$I = I_S\,(e^{V/\eta V_T} - 1)$$

or, $I = I_S\, e^{V/\eta V_T}$ (∵ neglecting 1)

on differentiating w.r.to V

$$\frac{dI}{dV} = I_S \cdot \frac{1}{\eta V_T} \cdot e^{V/\eta V_T} = \frac{I_S\, e^{V/\eta V_T}}{\eta V_T} = \frac{I}{\eta V_T}$$

or, $\frac{dV}{dI} = \frac{\eta V_T}{I}$

$$\Rightarrow r_f = \frac{\eta V_T}{I}$$

Here, r_f = forward diode resistance or air resistance or dynamic resistance.

Also, from above fig,

$$V = I \cdot r_f + V_f$$

$$\therefore V_f = V - I \cdot r_f$$

Find the piecewise linear model of a diode (silicon) with $I_S = 10^{-11}$ A and $\eta = 1.6$ in the vicinity of operating point, I = 1mA.

Soln: Given, Diode current (I) = 1 mA = 1×10^{-3} A

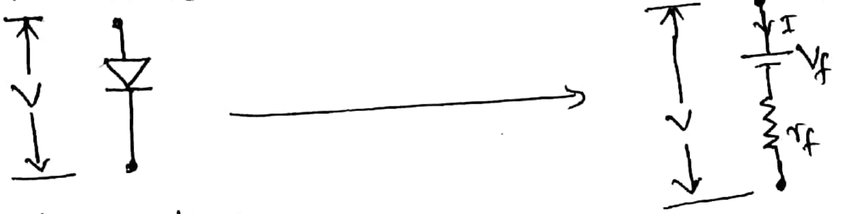
$I_S = 10^{-11}$ A

$\eta = 1.6$

$r_f = ?$

$V_f = ?$

The piecewise linear model of a diode is,



We know that,

forward diode resistance, $r_f = \frac{\eta V_T}{I} = \frac{1.6 \times 0.026}{1\times10^{-3}} = 41.6\Omega$

Also, $V_f = V - I * r_f$ —— (1)

We know that,

$I = I_S \left(e^{V/\eta V_T} - 1 \right)$

or, $\frac{I}{I_S} = e^{V/\eta V_T} - 1$

or, $\frac{1\times10^{-3}}{10^{-11}} = e^{V/(1.6\times0.026)} - 1$

or, $10^8 + 1 = e^{V/0.0416}$

or, $\ln(10^8+1) = \frac{V}{0.0416}$

or, $V = 0.0416 * 18.42$

$\therefore V = 0.766V$

Again from eqn (1),

$V_f = V - I * r_f = 0.766 - 10^{-3} * 41.6 = 0.766 - 0.0426.$

$\therefore V_f = 0.723V$

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