Z-Transform

$$X(Z) = \sum_{n=-\infty}^{\infty} x[n] Z^{-n}$$

$$\therefore x[n] \xleftrightarrow{ZT} X(Z).$$

(1) Single sided Z-transform [Causal]

$$X(Z) = \sum_{n=0}^{\infty} x[n] Z^{-n}$$

(2) Double sided Z-transform [non-causal]

$$X(Z) = \sum_{n=-\infty}^{\infty} x[n] Z^{-n}$$

ZT of some signals :-

(1) $x[n] = \delta[n]$

$$\updownarrow ZT$$

$X(Z) = 1$ ROC: entire z-plane

(2) $x[n] = u[n]$

$$\updownarrow ZT$$

$X(Z) = \dfrac{Z}{Z-1}$ ROC: $|Z| > 1$.

Scanned by CamScanner

Properties of Z-Transform:-

① Time shifting property

If $x[n] \xleftrightarrow{ZT} X(z)$

then, $x[n-k] \xleftrightarrow{ZT} z^{-k}X(z)$

$x[n+k] \xleftrightarrow{ZT} z^{k}X(z)$

ROC does not change.

② Time reversal property

If $x[n] \xleftrightarrow{ZT} X(z)$

then $x[-n] \xleftrightarrow{ZT} X(z^{-1})$

ROC gets reversed.

③ Time scaling property

If $x[n] \xleftrightarrow{ZT} X(z)$

then $a^n x[n] \xleftrightarrow{ZT} X[z/a)$

ROC: $|z|>a$.

④ Linearity property

If $x_1[n] \xleftrightarrow{ZT} X_1(z)$ & $x_2[n] \xleftrightarrow{ZT} X_2(z)$

then, $x[n]=ax_1[n]+bx_2[n] \xleftrightarrow{ZT} X[z)=aX_1(z)+bX_2(z)$.

⑤ Differentiation property

If $x[n] \xleftrightarrow{ZT} X(z)$

then, $nx[n] \xleftrightarrow{ZT} -z\frac{d}{dz}X(z)$

ROC will not get changed.

$x[n] = \cos w_0 n \, u[n] \xleftrightarrow{ZT} X(z) = \frac{z^2 - z\cos w_0}{z^2 - 2z\cos w_0 + 1}$

ROC: $|z| > 1$

$x[n] = \sin w_0 n \, u[n] \xleftrightarrow{ZT} X(z) = \frac{z \sin w_0}{z^2 - 2z\cos w_0 + 1}$

ROC: $|z| > 1$

⑥ Convolution Property

If $x_1[n] \xleftrightarrow{ZT} X_1(z)$ & $x_2[n] \xleftrightarrow{ZT} X_2(z)$

then,

$x[n] = x_1[n] * x_2[n] \xleftrightarrow{ZT} X(z) = X_1(z) \cdot X_2(z)$.

$= \sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k]$

Inverse Z-Transform :-

① Long division method.

ⓐ ROC not given

$$X(Z) = \frac{Y(Z)}{H(Z)} = \frac{b_0 + b_1 Z^{-1} + b_2 Z^{-2} + \ldots}{a_0 + a_1 Z^{-1} + \ldots}$$

$$1 + a_1 Z^{-1} + a_2 Z^{-2} \,\big)\, 1 + b_1 Z^{-1} + b_2 Z^{-2} + b_3 Z^{-3} + b_4 Z^{-4} \,(\ldots$$

⋮

ⓑ ROC given

(i) $|Z| > a \rightarrow$ causal signal $\rightarrow$ -ve power of Z.

Eg: $X(Z) = \dfrac{1}{1 - 3/2 Z^{-1} + 1/2 Z^{-2}}$

$$1 - 3/2 Z^{-1} + 1/2 Z^{-2} \,\big)\, 1 \qquad (1 + 3/2 Z^{-1} + 7/4 Z^{-2} + \ldots$$

⋮

(ii) $|Z| < a \rightarrow$ anticausal $\rightarrow$ +ve power of Z

Eg: $X(Z) = \dfrac{1}{1 - 3/2 Z^{-1} + 1/2 Z^{-2}}$

$$1/2 Z^{-2} - 3/2 Z^{-1} + 1 \,\big)\, 1 \qquad (2Z^2 + 6Z^3 + \ldots$$

⋮

Partial Fraction method.

(i) $Z^{-1}\left\{\frac{Z}{Z-P_k}\right\} = \begin{cases} (P_k)^n u[n] \Rightarrow ROC: |Z|>|P_k| \\ \quad [\text{causal signal}] \\ -(P_k)^n u[-n-1] \Rightarrow ROC: |Z|<|P_k| \\ \quad [\text{anticausal signal}] \end{cases}$

(ii) If $A_1 = A_2^*$
$P_1 = P_2^*$

then, $A_k = |A_k| e^{j\alpha_k}$ & $P_k = \gamma_k e^{j\beta_k}$

then,

$$Z^{-1}\left[A_k \cdot \frac{Z}{Z-P_k} - A_k^* \frac{Z}{Z-P_k^*}\right] = 2|A_k| \gamma_k^n \cos(\beta_k n + \alpha_k)\, u[n]$$

(iii) $Z^{-1}\left\{\frac{Z}{(Z-P_k)^n}\right\} = n(P_k)^n u[n].$

One sided ZT :-

$$X^+(Z) = \sum_{n=0}^{\infty} x[n] Z^{-n}$$

$\therefore x[n] \xleftrightarrow{Z^+} X^+(Z).$

Exception of shifting property in one sided ZT:-

(1) Time delay

If $x[n] \xleftrightarrow{Z^+} X^+(Z)$

then,

$$x[n-K] \xleftrightarrow{Z^+} Z^{-K}\left[X^+(Z) + \sum_{n=1}^{K} x[-n] Z^n\right],\ K>0$$

If $x[n]$ is causal,

$$x[n-k] \xleftrightarrow{Z^+} Z^{-k} X^+(Z).$$

② Time advance

If $x[n] \xleftrightarrow{Z^+} X^+(Z)$

then

$$x[n+k] \xleftrightarrow{Z^+} Z^k \left[X^+(Z) - \sum_{n=0}^{K-1} x[n] Z^{-n} \right], \ k>0$$

Discrete Fourier Transform (DFT)

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi nk}{N}} \quad \text{--- (i)}$$

$$W_N = e^{\frac{-j2\pi}{N}}$$
↳ Twiddle factor

& Inverse DFT (IDFT) is

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi}{N} nk} \quad \text{--- (ii)}$$

where, $n = 0, 1, 2, \dots, N-1$
$K = 0, 1, 2, \dots, N-1$

DFT Calculation:-

$$X_K = \begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix}, \quad x_N = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \quad \& \quad W_N = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \dots & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & \dots & W_N^{N-1} \\ \vdots & \vdots & & & \vdots \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \dots & W_N^{(N-1)^2} \end{bmatrix}$$

Hence, $X_K = x_N [W_N]$

& $x_N = \frac{1}{N} X_K [W_N^*]$ ← conjugate matrix of W_N.

Properties of DFT:

① Linearity

If $x_1[n] \xleftrightarrow[N]{DFT} X_1(k)$ & $x_2[n] \xleftrightarrow[N]{DFT} X_2(k)$

then, $x[n] = a_1 x_1[n] + a_2 x_2[n] \xleftrightarrow[N]{DFT} X(k) = a_1 X_1(k) + a_2 X_2(k)$.

② Periodicity

If $x[n] \xleftrightarrow[N]{DFT} X(K)$

then, $x[n+N] = x[n]$

$X(K+N) = X(K)$

③ Time Reversal

If $x[n] \xleftrightarrow[N]{DFT} X(K)$

then,

$x[[-n]]_N = x[N-n] \xleftrightarrow{DFT} X[[-K]]_N = X(N-K)$.

④ Circular shift

If $x[n] \xleftrightarrow[N]{DFT} X(K)$

then,

$$x[n-l] \xleftrightarrow[N]{DFT} X(K) W_N^{kl} = X(K)\, e^{-j\frac{2\pi}{N}\cdot kl}$$

⑤ Circular frequency shift

If $x[n] \xleftrightarrow[N]{DFT} X(K)$

then,

$$x[n] e^{\frac{j2\pi l n}{N}} \xleftrightarrow[N]{DFT} X[(K-l)]_N$$

⑥ Parseval's Theorem

If $x[n] \xleftrightarrow[N]{DFT} X(K)$

then,

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{K=0}^{N-1} |X(K)|^2$$

⑦ Multiplication property

If $x_1[n] \xleftrightarrow[N]{DFT} X_1(K)$ & $x_2[n] \xleftrightarrow[N]{DFT} X_2(K)$

then,

$$x[n] = x_1[n] \cdot x_2[n] \xleftrightarrow[N]{DFT} \frac{1}{N}\left\{ X_1(K) \circledast X_2(K) \right\}$$

⑧ Multiplication of DFTs / Circular Convolution

If $x_1[n] \xleftrightarrow[N]{DFT} X_1(K)$ & $x_2[n] \xleftrightarrow[N]{DFT} X_2(K)$

then

$$y[m] = x_1[n] \circledast x_2[n] \xleftrightarrow[N]{DFT} Y(K) = X_1(K) \cdot X_2(K).$$

$$= \sum_{n=0}^{N-1} x_1[n]\, x_2[m-n]_N.$$

Circular Convolution

↳ To determine the circular convolution, the sequences must be symmetric.

Eg1:- $x[n] = \{0, \underset{\uparrow}{1}, 2, 3\}$ $\longrightarrow$ $x[n] = \{0, \underset{\uparrow}{1}, 2, 3\}$.

$h[n] = \{\underset{\uparrow}{1}, 3, 7\}$ $\qquad$ $h[n] = \{0, \underset{\uparrow}{1}, 3, 7\}$

zero padding

Eg2:- $x[n] = \{1, 2, 3, 4, 5\}$ $\longrightarrow$ $x[n] = \{1, 2, 3, 4, 5, 0, 0, 0\}$

$h[n] = \{2, 4, 6, 8, 10\}$ $\qquad$ $h[n] = \{2, 4, 6, 8, 10, 0, 0, 0\}$

Steps for convolution :-

(i) Folding, $h[-k]$

(ii) Shifting, $h[n-k]$

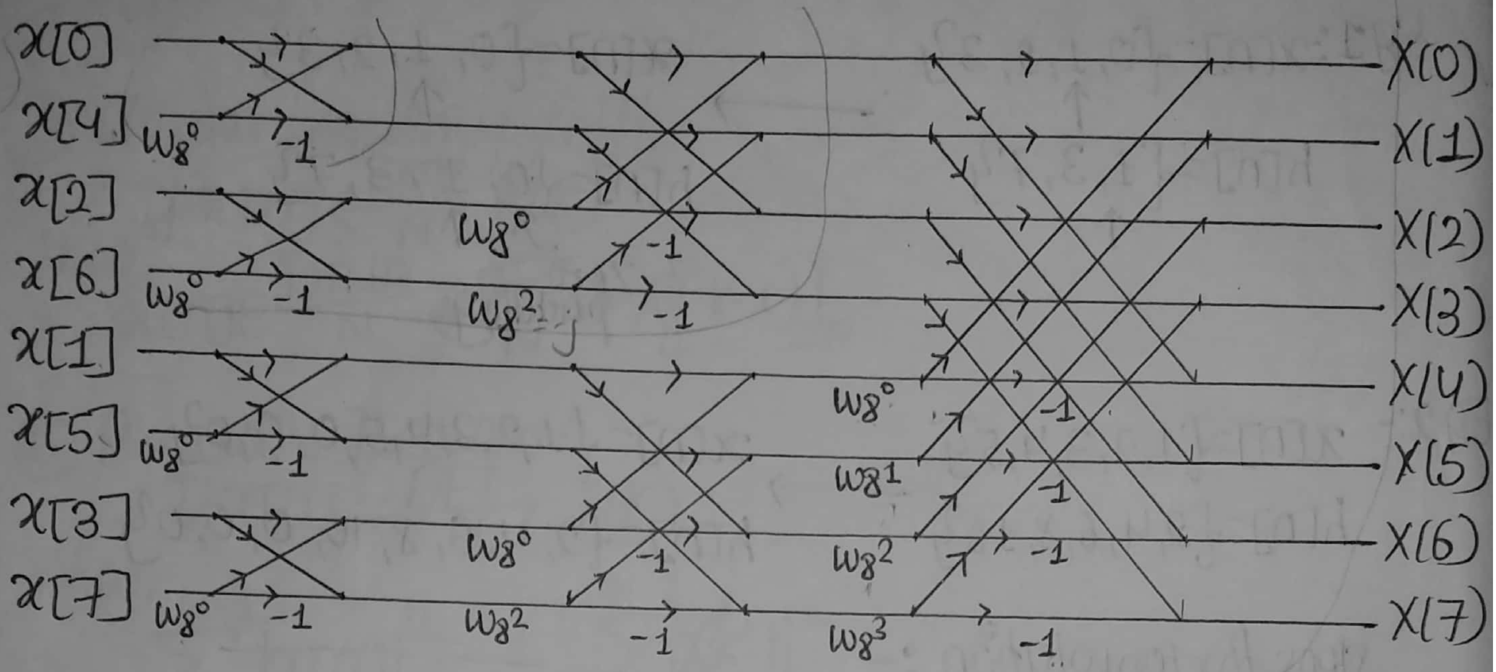
(iii) Multiplication, $x[k]h[n-k]$

(iv) Summation, $\sum x[k]h[n-k]$

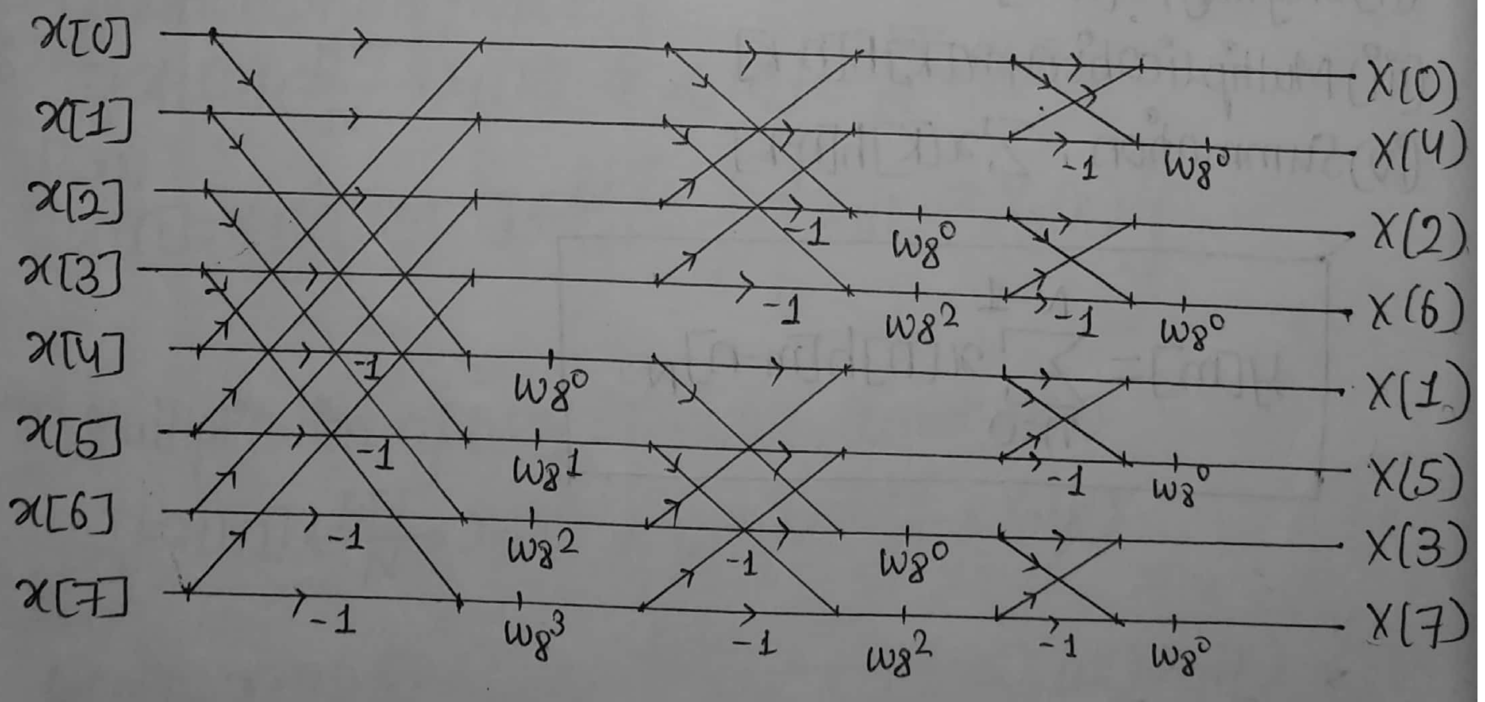
$$\boxed{y[m] = \sum_{n=0}^{N-1} x[n]h[m-n]_N.}$$

Fast fourier Transform (FFT) Algorithm :

(1) DIT-FFT



(2) DIF-FFT



Here, $W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$

FFT mathematical calculation :-

$N=8=2^{3}$ ⟶ indicates that it requires 3 stages.

$X(0)=x[0]+x[4]+x[2]+x[6]+x[1]+x[5]+x[3]+x[7]$.

$X(4)=x[0]+x[4]+x[2]+x[6]-x[1]-x[5]-x[3]-x[7]$

$X(2)=x[0]+x[4]-x[2]-x[6]+W_8^2x[1]+W_8^2x[5]-W_8^2x[3]-W_8^2x[7]$

$X(6)=$ Complex Conjugate of $X(2)$.

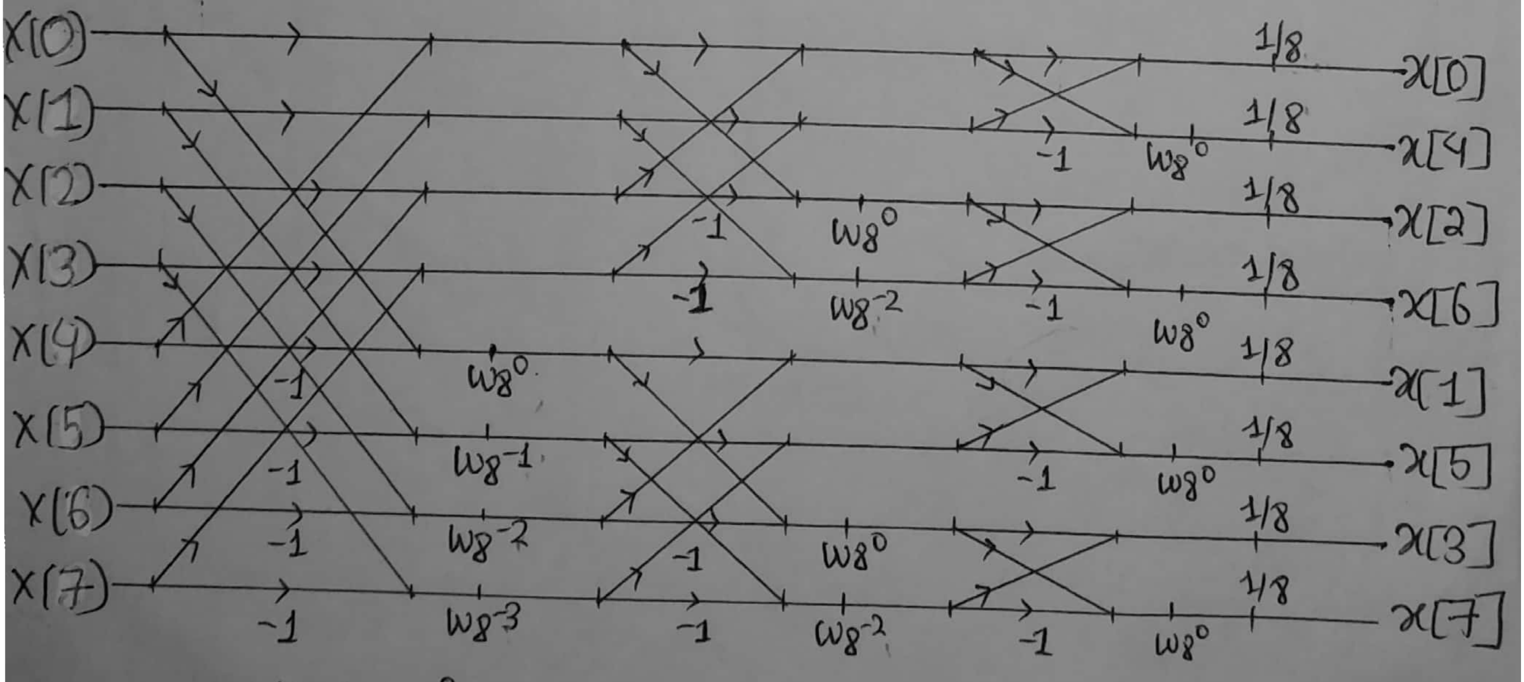
$X(1)=x[0]-x[4]+W_8^2x[2]-W_8^2x[6]+W_8^1x[1]-W_8^1x[5]+W_8^2W_8^1x[3]$
$\quad -W_8^2W_8^1x[7]$.

$X(7)=$ complex conjugate of $X(1)$.

$X(3)=x[0]-x[4]-W_8^2x[2]+W_8^2x[6]+W_8^3x[1]-W_8^3x[5]-W_8^2W_8^3x[3]$
$\quad +W_8^2W_8^3x[7]$

$X(5)=$ complex Conjugate of $X(3)$.

IDFT of sequence using FFT :-



Here, $W_8^0=1$

$W_8^{-1}=0.707+j0.707$

$W_8^{-2}=j$

$W_8^{-3}=-0.707+j0.707$

Conversion of direct form coeff. to Lattice coeff.

1. $a_m(0) = 1$
2. $a_m(m) = k_m$
3. $a_{m-1}(k) = \dfrac{a_m(k) - a_m(m) \cdot a_m(m-k)}{1 - a_m^2(m)}$

m	k
3	1
	2
2	1

For FIR filter,

$$H(Z) = a_m(0) + a_m(1)Z^{-1} + a_m(2)Z^{-2} + \dots$$

$$\boxed{a_m(0) = 1}$$

For IIR filter,

$$H(Z) = \frac{A}{a_m(0) + a_m(1)Z^{-1} + a_m(2)Z^{-2} + \dots}$$

$$\boxed{a_m(0) = 1}$$

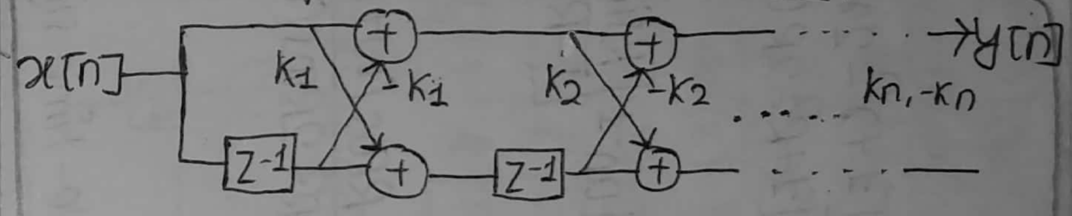
1

Conversion of Lattice coeff. to direct form coeff.

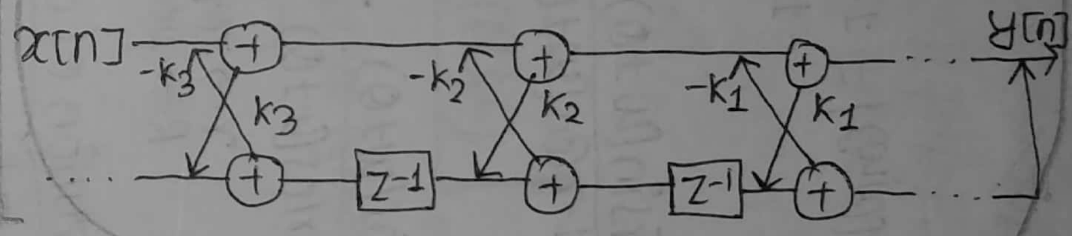
1. $a_m(0) = 1$
2. $a_m(m) = k_m$
3. $a_m(k) = a_{m-1}(k) + a_m(m) \cdot a_{m-1}(m-k)$

m	k
3	1
	2
2	1

For FIR filter / all zero lattice structure,



For IIR filter / pole-zero filter / all pole lattice str.



5

condition for stability of the system :-

$$|K_m| < 1, \quad m = 1, 2, 3, \dots$$

Lattice-Ladder structure :-

$$H(Z) = \frac{B_M(Z)}{A_N(Z)}$$

For ladder,

$$C_m = b_m - \sum_{i=m+1}^{M} a_i(i-m) \cdot C_i, \quad m = M, M-1, M-2, \dots$$

$\therefore A_N(Z) \longrightarrow$ gives Lattice structure. $(D \xrightarrow{conv.} L)$

$\therefore B_M(Z) \longrightarrow$ gives Ladder structure.

Frequeny Response :-

For magnitude plot,

$$\text{magnitude}, 20\log|H(e^{j\omega})| = 10\log[1+r^2-2r\cos(\omega\pm\theta)] - 10\log[1+R^2-2R\cos(\omega\pm\theta)]$$

For phase plot,

$$\measuredangle H(e^{j\omega}) = \tan^{-1}\left[\frac{r\sin(\omega\pm\theta)}{1-r\cos(\omega\pm\theta)}\right] - \tan^{-1}\left[\frac{R\sin(\omega\pm\theta)}{1-R\cos(\omega\pm\theta)}\right]$$

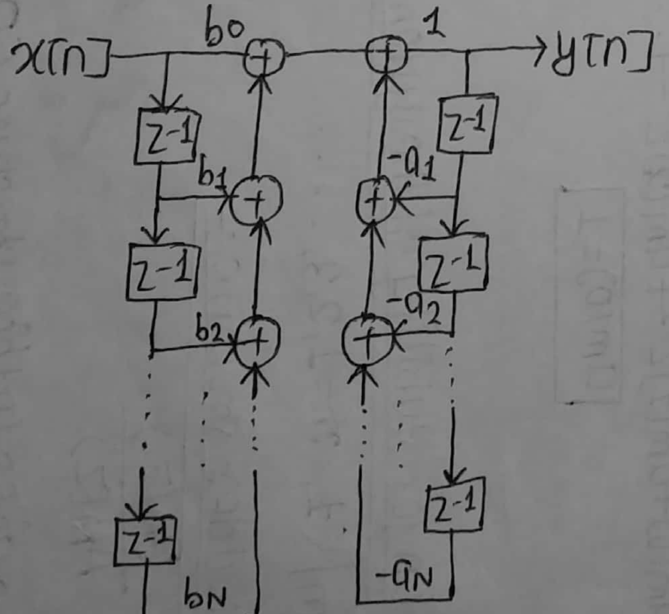
For group delay,

$$grd[H(e^{j\omega})] = \frac{r^2-r\cos(\omega\pm\theta)}{1+r^2-2r\cos(\omega\pm\theta)} - \frac{R^2-R\cos(\omega\pm\theta)}{1+R^2-2R\cos(\omega\pm\theta)}$$

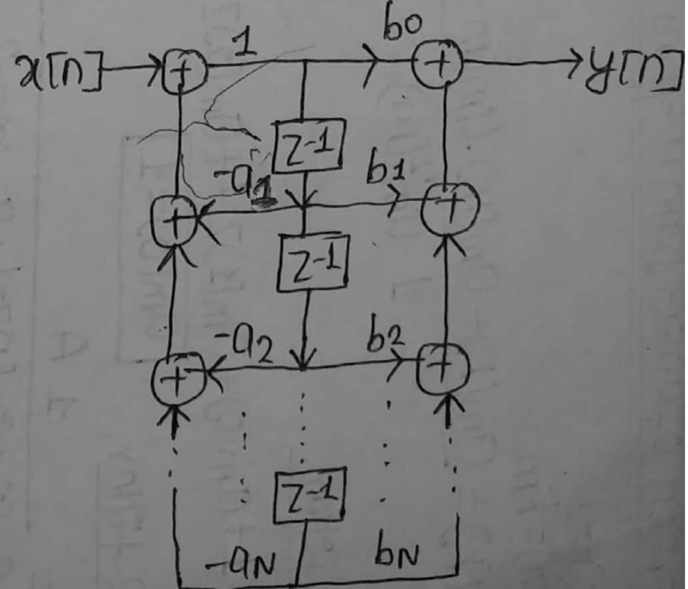
$r \rightarrow$ zeros
$R \rightarrow$ poles

Direct-form Realization of discrete-time signal :-

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots}{1+a_1z^{-1}+a_2z^{-2}+\cdots}$$



Direct-form I



Direct-form II

Cascade Realization

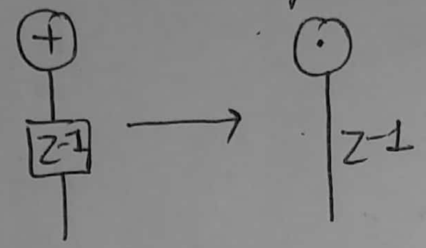
↳ In cascade realization of IIR filter, the T.F. $H(z)$ is split into a product of TFs of $H_1(z), H_2(z), \ldots H_k(z)$

$$H(z) = \frac{B_M(z)}{A_N(z)}$$

M = no. of zeroes
N = no. of poles

$$\boxed{m = \text{int}\left(\frac{M+1}{2}\right)} \quad \& \quad \boxed{n = \text{int}\left(\frac{N+1}{2}\right)}$$

Note :- For signal flow diagram, replace summation by

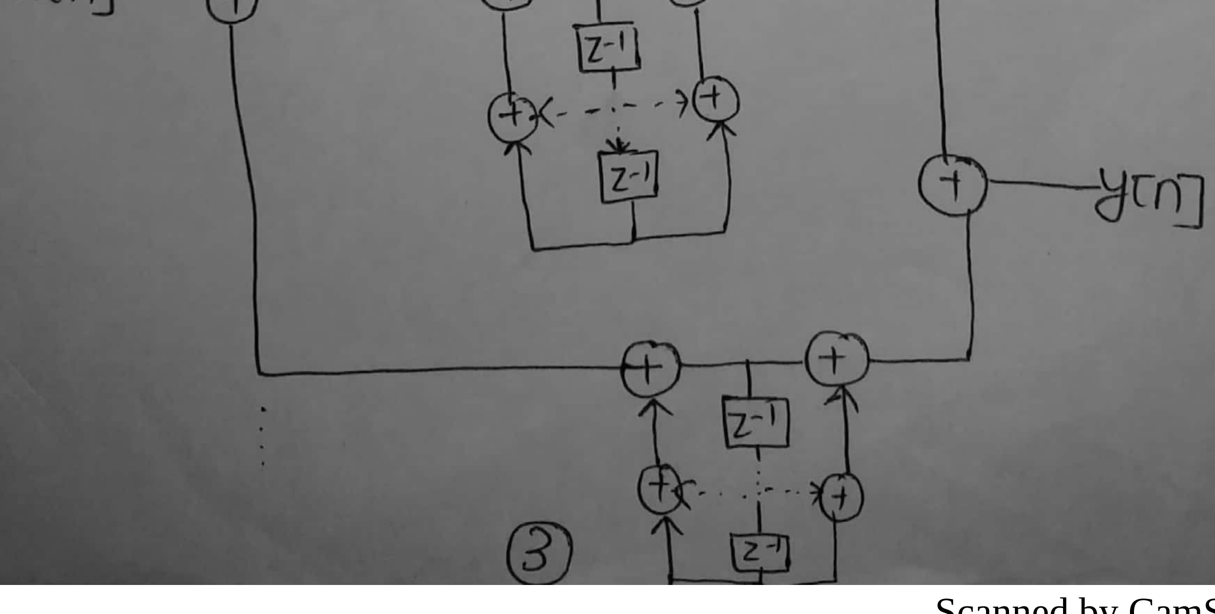


Parallel Form Realization

↳ By using partial fraction expansion, the transfer fn of IIR filter can be realized in parallel form (addition)

(In order to realize IIR filter, the TF should be in proper form).

$$H(z) = \frac{B_M(z)}{A_N(z)} \qquad N > M$$



Steps to design Butterworth filter :-

① obtain equivalent analog domain frequencies in PB & SB

IIM method

$$\Omega = \frac{\omega}{T}$$

$$\Omega_p = \frac{\omega_p}{T} \;\&\; \Omega_s = \frac{\omega_s}{T}$$

BLT method

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) \;\&\; \Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right)$$

② Evaluate order of the filter 'N'

(i) specifications are not in dB

$$N = \frac{\log\left[\frac{(1/A_s^2 - 1)}{(1/A_p^2 - 1)}\right]}{2\log(\Omega_s/\Omega_p)}$$

(ii) specifications are in dB

$$N = \frac{\log\left[\frac{10^{0.1A_s} - 1}{10^{0.1A_p} - 1}\right]}{2\log(\Omega_s/\Omega_p)}$$

③ Evaluate cut-off freq. in analog domain, (Ω_c)

(i) Specifications are not in dB

$$\Omega_c = \frac{\Omega_p}{(1/A_p^2 - 1)^{1/2N}}$$

(ii) specifications are in dB

$$\Omega_c = \frac{\Omega_p}{(10^{0.1A_p} - 1)^{1/2N}}$$

④ Determine pole position

$$P_i = \pm \Omega_c\, e^{j(N+2i+1)\frac{\pi}{2N}} \quad \text{where } i = 0, 1, 2, \dots, (N-1)$$

If poles are complex conjugate, organize complex conjugate pair as S_1, S_1^*, S_2, S_2^*.

⑤ Determine $H_a(S)$

$$H_a(s) = \frac{\Omega_c^N}{(S-P_1)(S-P_2)\dots}$$

But if poles are complex conjugate pair then

$$H_a(s) = \frac{\Omega_c^N}{(S-P_1)(S-P_1^*)(S-P_2)(S-P_2^*)}$$

⑥ Finally, we design digital filter using either IIM or BLT method.

[To realize the digital filter, we use direct-II form]

Steps to design chebyshev filter

① Determine ripple parameters

(i) Specifications are not in dB

$\epsilon = \left(\frac{1}{A_p^2} - 1\right)^{1/2}$

$\lambda = \left(\frac{1}{A_s^2} - 1\right)^{1/2}$

(ii) Specifications are in dB

$\epsilon = \left(10^{0.1A_P} - 1\right)^{1/2}$

$\lambda = \left(10^{0.1A_S} - 1\right)^{1/2}$

② Determine order of filter, N

$$N \geq \frac{\cosh^{-1}(\lambda/\epsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)}$$

③ Determine pole position

$$\theta_i = \frac{\pi}{2} + \frac{(2i+1)\pi}{2N} \quad \text{where } i = 0, 1, 2, \ldots (N-1)$$

poles of chebyshev filter lies on ellipse.

so, $x_i = r\cos\theta_i$ & $y_i = R\sin\theta_i$

r = minor axis of ellipse $= \Omega_p\left(\frac{\beta^2-1}{2\beta}\right)$

R = major axis of ellipse $= \Omega_p\left(\frac{\beta^2+1}{2\beta}\right)$

where, $\beta = \left[\frac{\sqrt{1+\epsilon^2}+1}{\epsilon}\right]^{1/N}$

∴ pole position, $S_p = r\cos\theta_i + jR\sin\theta_i = x_i + jy_i$

④ Determine Ha(s)

$$Ha(s) = \frac{i}{(s-s_0)(s-s_1)(s-s_2)\ldots} = \frac{i}{s^N + b_{N-1}s^{N-1} + b_{N-2}s^{N-2} + \ldots + b_0}$$

where, $i = \begin{cases} b_0 & \text{for } N = \text{odd} \\ \frac{b_0}{\sqrt{1+\epsilon^2}} & \text{for } N = \text{even} \end{cases}$

⑤ we determine digital filter using either BLT or IIM method.

[To realize a digital filter, we use Direct form-II]

Frequency Transformation in digital domain:-

S.N	Types of Transformation	Transformation	Parameters
1.	LPF	$z^{-1} \longrightarrow \dfrac{z^{-1}-a}{1-az^{-1}}$	$a = \dfrac{\sin\left(\frac{\omega_p-\omega_p'}{2}\right)}{\sin\left(\frac{\omega_p+\omega_p'}{2}\right)}$; ω_p' = new PB edge freq.
2.	HPF	$z^{-1} \longrightarrow -\dfrac{z^{-1}+a}{1+az^{-1}}$	$a = \dfrac{\cos\left(\frac{\omega_p+\omega_p'}{2}\right)}{\cos\left(\frac{\omega_p-\omega_p'}{2}\right)}$
3.	BPF	$z^{-1} \longrightarrow -\dfrac{z^{-2}-a_1z^{-1}+a_2}{a_2z^{-2}-a_1z^{-1}+1}$	$a_1 = \dfrac{2\alpha K}{K+1}$, $a_2 = \dfrac{K-1}{K+1}$ ω_u = Upper freq. ω_L = Lower freq. $\alpha = \dfrac{\cos\left(\frac{\omega_u+\omega_L}{2}\right)}{\cos\left(\frac{\omega_u-\omega_L}{2}\right)}$ $K = \cot\left(\frac{\omega_u-\omega_L}{2}\right)\tan\left(\frac{\omega_p}{2}\right)$
4.	BSF	$z^{-1} \longrightarrow \dfrac{z^{-2}-a_1z^{-1}+a_2}{a_2z^{-2}-a_1z^{-1}+1}$	$a_1 = \dfrac{-2\alpha K}{K+1}$, $a_2 = \dfrac{1-K}{1+K}$ $\alpha = \dfrac{\cos\left(\frac{\omega_u+\omega_L}{2}\right)}{\cos\left(\frac{\omega_u-\omega_L}{2}\right)}$ $K = \tan\left(\frac{\omega_u-\omega_L}{2}\right)\cot\left(\frac{\omega_p}{2}\right)$

IIM method

Mapping of poles:-

① $\frac{1}{s-p_i} \Rightarrow \frac{1}{1-e^{p_i T} z^{-1}}$

② $\frac{s+a}{(s+a)^2+b^2} \Rightarrow \frac{1-e^{-aT}\cos bT z^{-1}}{1-2e^{-aT}\cos bT z^{-1}+e^{-2aT}z^{-2}}$

③ $\frac{b}{(s+a)^2+b^2} \Rightarrow \frac{e^{-aT}\sin bT z^{-1}}{1-2e^{-aT}\cos bT z^{-1}+e^{-2aT}z^{-2}}$

BLT method

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}*\frac{1-z^{-1}}{1+z^{-1}}}$$

PB edge = f_p
SB edge = f_s

PB freq. = ω_p
SB freq. = ω_s

Steps to design FIR filter using Kaiser Window (T+D) (7)

(T = Theory, D = Design)

① Determine optimum value of ripple

$\delta = \min(\delta_s, \delta_p)$ where, $\delta_s = 10^{-0.05A_s}$, $\delta_p = \dfrac{10^{0.05A_p}-1}{10^{0.05A_p}+1}$

② Calculate the attenuation in dB

$A = -20\log(\delta)$.

③ Determine kaiser parameter (α).

$$\alpha = \begin{cases} 0.1102(A-8.7) & ; A>50 \\ 0.5842(A-21)^{0.4}+0.07866(A-21) & ; 21 \le A \le 50 \\ 0 & ; A<21 \end{cases}$$

④ The length of the filter is

$L = M+1$ where, $M = \dfrac{A-8}{2.285\Delta W}$; $\Delta W = W_s - W_p$

⑤ we select desired impulse response depending upon type of filter (LPF, HPF, BPF, BSF) and the window function (kaiser) is defined as

$$W_n[n] = \begin{cases} \dfrac{I_0\left[\alpha\sqrt{1-\left(\frac{2n}{M-1}\right)^2}\right]}{I_0(\alpha)} & ; |n| \le \dfrac{M-1}{2} \quad \left(\tau = \dfrac{M-1}{2}\right) \\ 0 & ; \text{otherwise} \end{cases}$$

where, $I_0(\alpha) = 1 + \dfrac{0.25\alpha^2}{(1!)^2} + \dfrac{(0.25\alpha^2)^2}{(2!)^2} + \dfrac{(0.25\alpha^2)^3}{(3!)^2} + \ldots$

⑥ Finally, impulse response of FIR filter is

$h[n] = h_d[n] \cdot w[n]$.

$h_d[n]$:- (Practical)

(i) LPF $\longrightarrow$ $h_d[n] = \begin{cases} 2f_c \dfrac{\sin[w_c(n-\tau)]}{w_c(n-\tau)} & , n \ne \tau \\ 2f_c & , n = \tau \end{cases}$

(ii) HPF $\longrightarrow$ $h_d[n] = \begin{cases} -2f_c \dfrac{\sin[w_c(n-\tau)]}{w_c(n-\tau)} & ; n \ne \tau \\ 1-2f_c & , n = \tau \end{cases}$

(iii) BPF $\rightarrow$ $h_d[n] = \begin{cases} 2f_{c_2} \dfrac{\sin[w_u(n-\tau)]}{w_u(n-\tau)} - 2f_{c_1} \dfrac{\sin[w_L(n-\tau)]}{w_L(n-\tau)} & , n \ne \tau \\ 2f_{c_2} - 2f_{c_1} & , n = \tau \end{cases}$

(iv) BSF $\rightarrow$ $h_d[n] = \begin{cases} 2f_{c_1} \dfrac{\sin[w_L(n-\tau)]}{w_L(n-\tau)} - 2f_{c_2} \dfrac{\sin[w_u(n-\tau)]}{w_u(n-\tau)} & , n \ne \tau \\ 1-(2f_{c_2}-2f_{c_1}) & , n = \tau. \end{cases}$

(1)

Design of FIR filter using Windowing Technique.

SN	Window Type	Window length	Window function for $0 \le n \le \frac{M-1}{2}$	Mainlobe width $(\omega_s-\omega_p)$	SB attenuation A_s, (dB)
1.	Rectangular window	$M = \frac{0.9}{\Delta f}$	$W_R[n] = 1$	$\frac{4\pi}{M}$	-21 dB
2.	Triangular/ Bartlett window	—	$W_T[n] = 1 - \frac{2\lvert n \rvert}{M-1}$	$\frac{8\pi}{M}$	-25 dB
3.	Raised Cosine window	—	$W_{RC}[n] = \alpha - (1-\alpha)\cos\left(\frac{2\pi n}{M-1}\right)$		
(a)	Hanning window	$M = \frac{3.1}{\Delta f}$	$W_{Han}[n] = 0.5 - 0.5\cos\left(\frac{2\pi n}{M-1}\right)$	$\frac{8\pi}{M}$	-44 dB
(b)	Hamming window	$M = \frac{3.3}{\Delta f}$	$W_{Ham}[n] = 0.56 - 0.44\cos\left(\frac{2\pi n}{M-1}\right)$	$\frac{8\pi}{M}$	-53 dB
(c)	Blackmann window	$M = \frac{5.5}{\Delta f}$	$W_B[n] = 0.42 - 0.50\cos\left(\frac{2\pi n}{M-1}\right) + 0.08\cos\left(\frac{4\pi n}{M-1}\right)$	$\frac{12\pi}{M}$	-74 dB