A Symplectic N-Body Engine for Solar System Dynamics: Validation Against JPL Horizons

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Abstract

We present a high-performance N-body gravitational simulator for solar system dynamics, integrating 35 bodies (the Sun, 8 planets, Pluto, and 25 natural satellites) over timescales of up to 249 years. The simulator employs a 4th-order Yoshida symplectic integrator, achieving energy conservation of $\Delta E/E \leq 1.95 \times 10^{-12}$ over 100 years and $\leq 2.34 \times 10^{-12}$ over 249 years with a timestep of $\Delta t = 900\,\mathrm{s}$. A timestep convergence study confirms that $\Delta t = 900\,\mathrm{s}$ lies on the floating-point precision floor; further reduction yields no improvement in energy conservation. Positional accuracy is validated against the NASA JPL Horizons DE441 ephemeris, yielding a mean maximum relative error of 0.05% over 100 years across all 35 bodies. Residual errors are attributed to physics model differences; principally, the absence of post-Newtonian relativistic corrections, solar oblateness J_2, and asteroid perturbations, rather than numerical integration error. The implementation features OpenMP parallelization with SIMD vectorization for large-N scalability, a Structure-of-Arrays memory layout for cache performance, and an automated validation pipeline interfacing directly with the JPL Horizons API.

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1. Introduction

The gravitational N-body problem, that is, computing the trajectories of N mutually interacting masses, is one of the oldest and most fundamental problems in computational physics. Accurate long-duration integration of planetary orbits requires numerical methods that preserve the geometric structure of Hamilton's equations. Non-symplectic integrators exhibit secular energy drift that corrupts orbital elements over extended timescales [2].

This work presents a complete N-body simulation framework for solar system dynamics. The system integrates 35 bodies (the Sun, 8 planets, Pluto, and 25 natural satellites) under pairwise Newtonian gravitation using the 4th-order Yoshida symplectic integrator [1]. We validate the simulator against the NASA JPL Horizons system, which provides state vectors computed from the DE441 planetary ephemeris. DE441 is a comprehensive model incorporating post-Newtonian relativity, solar oblateness, asteroid perturbations from over 300 bodies, and tidal effects [3].

The primary contributions of this work are:

1. Implementation and verification of a 4th-order

Yoshida symplectic integrator achieving $\Delta E/E \leq 10^{-12}$ over multi-century timescales.
2. A systematic timestep convergence study identifying three distinct regimes of integrator behavior: a floating-point precision floor, 4th-order convergence, and dynamical under-resolution.
3. Comprehensive positional validation against JPL DE441 for 35 solar system bodies, with quantitative attribution of residual errors to specific missing physics.
4. An automated validation pipeline that fetches JPL Horizons ephemerides and performs epoch-aligned comparison, eliminating interpolation artifacts.

2. Theoretical Background

2.1. Equations of Motion

The gravitational acceleration of body i due to all other $N-1$ bodies is given by Newton's law of universal gravitation:

$$\mathbf{a}_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{G\, m_j\, (\mathbf{r}_j - \mathbf{r}_i)}{\left(|\mathbf{r}_j - \mathbf{r}_i|^2 + \varepsilon^2\right)^{3/2}} \tag{1}$$

where $G = 6.6743 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ is the gravitational constant, m_j is the mass of body j, and $\mathbf{r}_i$ and $\mathbf{r}_j$ are position vectors. A softening parameter $\varepsilon = 10^{-9}\,\mathrm{m}$ is included to prevent numerical singularity at zero separation. This value is negligible relative to interplanetary distances and does not measurably affect the dynamics; it serves only as a numerical safeguard. The results are insensitive to the choice of ε provided $\varepsilon \ll \min_{i\neq j} |\mathbf{r}_i - \mathbf{r}_j|$.

2.2. Symplectic Integration

Hamiltonian systems possess a symplectic structure: the phase-space flow preserves the canonical 2-form $\sum_i dp_i \wedge dq_i$. Standard Runge-Kutta methods violate this structure, causing secular energy drift proportional to integration time. Symplectic integrators, by construction, exactly preserve a nearby shadow Hamiltonian $\tilde{H} = H + \mathcal{O}(\Delta t^p)$, bounding energy error for all time [2].

2.2.1. Velocity Verlet 2nd-Order Integrator

The Velocity Verlet algorithm [4] is a 2nd-order symplectic method widely used in molecular dynamics:

$$\mathbf{r}(t+\Delta t) = \mathbf{r}(t) + \Delta t\,\mathbf{v}(t) + \tfrac{1}{2}\Delta t^2\,\mathbf{a}(t) \tag{2}$$

$$\mathbf{v}(t+\Delta t) = \mathbf{v}(t) + \tfrac{1}{2}\Delta t\,[\mathbf{a}(t) + \mathbf{a}(t+\Delta t)] \tag{3}$$

This requires storing the previous acceleration and one force evaluation per timestep.

2.2.2. Yoshida 4th-Order Integrator

The Yoshida method [1] constructs a 4th-order symplectic scheme by composing three leapfrog sub-steps with carefully chosen coefficients. Define:

$$w_1 = \frac{1}{2 - 2^{1/3}}, \qquad w_0 = -\frac{2^{1/3}}{2 - 2^{1/3}} \tag{4}$$

The position drift and velocity kick coefficients are:

$$c_1 = c_4 = \frac{w_1}{2}, \qquad d_1 = d_3 = w_1 \tag{5}$$

$$c_2 = c_3 = \frac{w_0 + w_1}{2}, \qquad d_2 = w_0 \tag{6}$$

Each timestep proceeds as four position drifts interleaved with three force evaluations ("kicks"):

$$\mathbf{r} \xrightarrow{c_1} \mathbf{v} \xrightarrow{d_1} \mathbf{r} \xrightarrow{c_2} \mathbf{v} \xrightarrow{d_2} \mathbf{r} \xrightarrow{c_3} \mathbf{v} \xrightarrow{d_3} \mathbf{r} \xrightarrow{c_4} \tag{7}$$

Here $\mathbf{r} \xrightarrow{c_k}$ denotes $\mathbf{r} \leftarrow \mathbf{r} + c_k \Delta t\,\mathbf{v}$ and $\mathbf{v} \xrightarrow{d_k}$ denotes $\mathbf{v} \leftarrow \mathbf{v} + d_k \Delta t\,\mathbf{a}(\mathbf{r})$.

The scheme achieves $\mathcal{O}(\Delta t^4)$ local truncation error with only 3 force evaluations per step, compared to 4 for classical RK4. The negative coefficient $w_0 < 0$ (equivalently $d_2 < 0$) introduces a backward sub-step that cancels lower-order error terms. The symplectic structure guarantees that energy errors remain bounded and oscillatory, never exhibiting secular growth.

3. Implementation

3.1. Software Architecture

The simulator is implemented in C++23 with a modular object-oriented architecture. An abstract `Integrator` base class defines the integration interface, with `Yoshida` and `Velocity_Verlet` as concrete implementations. An abstract `Force` base class permits extensible force models; the current implementation provides `Gravity` for pairwise Newtonian interactions. The `Simulation` class orchestrates the time-stepping loop, conservation diagnostics, and binary output.

All simulation parameters (timestep Δt, duration, and output interval) are specified in a single `Config.hpp` header as `constexpr` values. This enables compile-time computation of derived quantities such as total steps and output cadence. A `static_assert` enforces that the output interval is an exact multiple of the timestep.

3.2. Data Layout: Structure of Arrays

Particle data is stored in a Structure-of-Arrays (SoA) layout rather than the conventional Array-of-Structures (AoS). Each physical attribute (position, velocity, acceleration, and mass) is stored in a contiguous heap-allocated array:

$$\texttt{pos\_x}[0..N-1], \quad \texttt{pos\_y}[0..N-1], \quad \ldots$$

This layout maximizes cache-line utilization during the inner force loop. The computation accesses all x-positions contiguously, then all y-positions, and so on. On modern architectures with 64-byte cache lines, this yields 8 contiguous `double` values per cache-line fetch. In contrast, AoS layout scatters a single particle's fields across multiple cache lines, degrading spatial locality.

Raw array pointers are qualified with compiler-specific `__restrict__` annotations, informing the compiler that no aliasing occurs between position, velocity, and acceleration arrays. This enables aggressive SIMD auto-vectorization of the inner loops.

3.3. Force Computation

The gravitational force calculation uses $\mathcal{O}(N^2)$ direct pairwise summation. Self-interaction ($i = j$) is eliminated with a branchless floating-point mask:

$$\texttt{mask}_j = \begin{cases} 0.0 & \text{if } i = j \\ 1.0 & \text{otherwise} \end{cases} \tag{8}$$

The force contribution is multiplied by this mask, avoiding a conditional branch that would disrupt SIMD vectorization. The inner loop is annotated with `#pragma omp simd` with reductions on the acceleration accumulators to enable vectorized execution. For $N \geq 500$, the outer loop is parallelized with OpenMP via `#pragma omp parallel for`.

Note that Newton's third law symmetry ($\mathbf{F}_{ij} = -\mathbf{F}_{ji}$) is not exploited; each body's acceleration is computed independently by summing over all N others. This doubles the floating-point work relative to a symmetric $j > i$ loop but preserves the regular access pattern required for SIMD vectorization and avoids race conditions under thread parallelism.

3.4. Initial Conditions

State vectors for all 35 bodies are sourced from the NASA JPL Horizons system at epoch J1950.0 (January 1, 1950, 00:00 UTC). The reference frame is solar system barycentric (SSB) with the ecliptic reference plane (ICRF). Positions are given in kilometers and velocities in $\mathrm{km\,s^{-1}}$, converted to SI units at initialization. The body catalog comprises the Sun, 8 planets (Mercury through Neptune), Pluto, and 25 natural satellites: the Moon, Phobos, Deimos, the four Galilean moons, Amalthea, 9 Saturnian moons (Mimas through Phoebe), 5 Uranian moons (Ariel through Miranda), Triton, Nereid, and Charon.

The simulation operates in the SSB frame throughout. Because all initial state vectors are sourced from JPL Horizons in this frame, the total momentum of the 35-body subset is not identically zero (the missing solar system bodies carry the complementary momentum). No barycentric correction is applied.

An automated Python pipeline (`jpl_compare.py fetch`) queries the Horizons API, retrieves state vectors at the configured output cadence, and generates both the C++ initial conditions header and the reference ephemeris for validation.

3.5. Build System

The project uses CMake (minimum version 3.20) for cross-platform builds. Compiler optimizations (`-O3 -march=native`) and OpenMP linkage are configured automatically. The build targets GCC, Clang, and MSVC.

4. Validation Methodology

Validation is performed by direct comparison of simulated trajectories against JPL Horizons ephemeris data, supplemented by conservation diagnostics. The pipeline ensures methodological rigor through several design choices.

Epoch-aligned comparison. Both the simulator and the JPL fetch use identical output intervals (`output_hours`), sourced from the same `Config.hpp` file. This eliminates interpolation error in the comparison.

Relative position error. For each body at each epoch, the relative position error is defined as:

$$\epsilon_{\mathrm{rel}}(t) = \frac{|\mathbf{r}_{\mathrm{sim}}(t) - \mathbf{r}_{\mathrm{JPL}}(t)|}{|\mathbf{r}_{\mathrm{JPL}}(t)|} \tag{9}$$

where $|\cdot|$ denotes the Euclidean norm. For each body, we report the maximum of $\epsilon_{\mathrm{rel}}(t)$ over all epochs $t > 0$.

Energy conservation. Total mechanical energy sampled at regular intervals throughout the integration:

$$T = \sum_{i=1}^{N} \frac{1}{2}\, m_i \left(v_{x,i}^2 + v_{y,i}^2 + v_{z,i}^2\right) \tag{10}$$

$$U = -G \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{m_i\, m_j}{\sqrt{|\mathbf{r}_j - \mathbf{r}_i|^2 + \varepsilon^2}} \tag{11}$$

$$E = T + U \tag{12}$$

Energy conservation is quantified by the peak-to-peak relative variation:

$$\frac{\Delta E}{E} = \frac{|E_{\max} - E_{\min}|}{|E_0|} \tag{13}$$

where $E_{\max}$ and $E_{\min}$ are the extrema over all sampled epochs, and E_0 is the initial energy. Note that this peak-to-peak measure can differ by up to a factor of 2 from the more commonly reported metric $\max_t |E(t) - E_0|/|E_0|$ [2, 7]. The peak-to-peak definition provides a conservative upper bound on the energy variation.

Angular momentum conservation. Total angular momentum is computed as

$$\mathbf{L} = \sum_{i=1}^{N} m_i \left(\mathbf{r}_i \times \mathbf{v}_i\right) \tag{14}$$

and its conservation is quantified by

$$\frac{\Delta L}{L} = \frac{\big||\mathbf{L}|_{\max} - |\mathbf{L}|_{\min}\big|}{|\mathbf{L}_0|} \tag{15}$$

For an isolated Newtonian system, $|\mathbf{L}|$ is exactly conserved; deviations arise solely from floating-point arithmetic. Angular momentum conservation provides an independent diagnostic from energy, as it is sensitive to different classes of numerical error.

Linear momentum conservation. Total linear momentum is computed as

$$\mathbf{P} = \sum_{i=1}^{N} m_i\, \mathbf{v}_i \tag{16}$$

and its drift is quantified analogously:

$$\frac{\Delta P}{P} = \frac{\big||\mathbf{P}|_{\max} - |\mathbf{P}|_{\min}\big|}{|\mathbf{P}_0|} \tag{17}$$

Linear momentum is not expected to be conserved, since the 35-body subset does not constitute a closed system in the barycentric frame. The missing solar system bodies carry the complementary momentum. The drift in $|\mathbf{P}|$ therefore reflects the evolving momentum imbalance of the subset, not a numerical deficiency.

5. Results

5.1. 100-Year Simulation

Table 1 summarizes results for a 100-year integration (1950–2050) with $\Delta t = 360\,\mathrm{s}$ and the Yoshida integrator. The simulation completes in 56.5 s on a single core.

Table 1: 100-year simulation summary (35 bodies, $\Delta t = 360\,\mathrm{s}$, Yoshida 4th-order).

Metric	**Value**
Mean max. relative error	0.053%
Worst-case body	Ganymede (0.290%)
Planet error range	0.0003%–0.173%
Moon error range	0.009%–0.290%
Energy conservation $\Delta E/E$	7.4×10^{-13}
Angular momentum $\Delta L/L$	3.4×10^{-14}
Linear momentum $\Delta P/P$	1.0×10^{-8}
Wall-clock runtime	56.5 s

Table 2 presents the per-body breakdown, grouped by dynamical class.

Table 2: Per-body maximum relative position error (%) for the 100-year simulation.

Body	**Error (%)**	**Body**	**Error (%)**
Sun and Planets			
Saturn	0.0003	Venus	0.022
Uranus	0.0003	Sun	0.071
Neptune	0.0004	Mercury	0.173
Jupiter	0.0009		
Mars	0.006		
Pluto	0.010		
Earth	0.013		
Selected Moons			
Iapetus	0.009	Io	0.114
Miranda	0.010	Titan	0.125
Phobos	0.014	Europa	0.175
Triton	0.016	Callisto	0.188
Moon	0.018	Ganymede	0.290

5.2. 249-Year Simulation

Extending the integration to 249 years (1950–2199) tests the long-term stability of the symplectic integrator. Table 3 compares the two simulation durations, and Table 4 presents the per-body breakdown.

Table 3: Comparison of 100-year and 249-year simulations ($\Delta t = 360\,\mathrm{s}$).

Metric	**100 yr**	**249 yr**
Mean max. error	0.053%	0.115%
Worst body	Ganymede (0.29%)	Sun (1.34%)
Worst planet	Mercury (0.17%)	Mercury (0.42%)
$\Delta E/E$	7.4×10^{-13}	1.5×10^{-12}
$\Delta L/L$	3.4×10^{-14}	4.3×10^{-13}
$\Delta P/P$	1.0×10^{-8}	7.0×10^{-8}
Runtime	56.5 s	171.6 s

Table 4: Per-body maximum relative position error (%) for the 249-year simulation.

Body	**Error (%)**	**Body**	**Error (%)**
Sun and Planets			
Saturn	0.0007	Earth	0.031
Uranus	0.0007	Venus	0.055
Neptune	0.0009	Mercury	0.417
Jupiter	0.002	Sun	1.336
Mars	0.015		
Pluto	0.034		
Selected Moons			
Miranda	0.010	Io	0.115
Triton	0.016	Nereid	0.149
Umbriel	0.019	Europa	0.183
Iapetus	0.022	Titan	0.184
Phobos	0.022	Hyperion	0.226
Moon	0.045	Ganymede	0.290
Amalthea	0.050	Callisto	0.411

Several observations follow from these results.

Energy and angular momentum conservation remain bounded. The energy drift increases only from 7.4×10^{-13} to 1.5×10^{-12} when the integration duration grows by a factor of 2.5. This sub-linear growth is characteristic of symplectic integrators, which preserve a shadow Hamiltonian $\tilde{H}$ and exhibit only oscillatory energy variations, never secular drift. Angular momentum conservation follows the same pattern, with $\Delta L/L$ remaining at $\sim 10^{-13}$ over 249 years.

Linear momentum drift is physical, not numerical. The linear momentum drift grows from

1.0×10^{-8} to 7.0×10^{-8} over $2.5\times$ the integration time. This $\sim 7\times$ increase over a $\sim 25\times$ longer integration is consistent with the slow evolution of the momentum imbalance carried by the missing solar system bodies, not a numerical deficiency.

Positional errors grow sub-linearly. The mean error roughly doubles (0.053% to 0.115%) over $2.5\times$ the integration time. This is consistent with slow phase drift accumulation, not exponential divergence. The outer planets (Saturn, Uranus, Neptune) remain below 10^{-3}% even at 249 years, reflecting the dominance of solar gravitation in their dynamics and the absence of significant missing physics at their orbital distances.

The Sun becomes the worst-case body at 249 years. This is a coordinate artifact, not a simulation failure. In the solar system barycentric frame, the Sun's position is determined by the gravitational influence of the planets (predominantly Jupiter and Saturn). The Sun's barycentric displacement is only $\sim 10^6$ km, roughly one solar radius, while outer planets orbit at $\sim 10^9$ km. A given absolute position error therefore produces a relative error roughly 10^3–$10^4\times$ larger for the Sun than for a giant planet. At 100 years the Sun's relative error is 0.07%; by 249 years, accumulated absolute error yields 1.34% relative error. In absolute terms, this remains physically negligible.

Callisto overtakes Ganymede at 249 years. Callisto's error grows from 0.188% to 0.411%, surpassing Ganymede (0.290% at both durations). This is consistent with the longer orbital period of Callisto (16.7 d vs. 7.15 d), which couples to slightly different phase drift accumulation rates over multi-century timescales. Hyperion (0.226%) also emerges as a notable outlier among Saturnian moons, reflecting its chaotic rotation and irregular orbit.

5.3. Sources of Residual Error

The residual errors are dominated by physics model differences between the simulator (Newtonian gravity, 35 bodies) and JPL DE441. The principal missing contributions are described below.

Post-Newtonian relativity. The Einstein–Infeld–Hoffmann (EIH) equations contribute $\sim 43\,\mathrm{arcsec\,century}^{-1}$ of precession to Mercury's orbit [5]. This is the dominant error source for Mercury (0.17% at 100 yr, 0.42% at 249 yr), as the Newtonian model cannot capture this effect. Implementation of 1PN corrections was attempted; however, the velocity-dependent post-Newtonian potential breaks the separability assumption ($H = T(\mathbf{p}) + V(\mathbf{q})$) required by the Yoshida integrator. This degraded energy conservation by ~ 4 orders of magnitude (from 10^{-13} to 10^{-9}), producing worse overall accuracy than the purely Newtonian model. Implicit symplectic splitting schemes [6] or mixed-variable symplectic methods [5] offer potential paths to incorporating these corrections without sacrificing symplecticity.

Solar oblateness (J_2). The Sun's quadrupole moment produces a small additional precession, particularly affecting Mercury's perihelion advance.

Asteroid perturbations. JPL DE441 includes gravitational contributions from over 300 asteroids (principally Ceres, Pallas, and Vesta). These bodies are absent from the 35-body model.

Moon phase drift. Satellites with short orbital periods accumulate phase errors over many orbits. Io orbits Jupiter every 1.77 d and completes $\sim$20,600 orbits in 100 years. At $\Delta t = 360\,\mathrm{s}$, each orbit is resolved by ~ 425 timesteps. This is adequate for 4th-order per-orbit accuracy, but small per-orbit phase errors compound over thousands of orbits. Reducing Δt or implementing adaptive timestepping would improve satellite accuracy at the cost of runtime.

Tidal and non-gravitational effects. Tidal dissipation, solar radiation pressure, and planetary ring interactions are included in DE441 but absent from the simulator.

5.4. Timestep Convergence Study

To distinguish integration error from physics model error, a systematic convergence study was performed over a range of timesteps spanning two orders of magnitude. Table 5 presents the peak-to-peak energy drift $\Delta E/E$ for the 249-year simulation at each Δt.

Table 5: Energy conservation vs. timestep (249-year simulation, 35 bodies, Yoshida 4th-order).

Δt (s)	$\Delta E/E$
60	3.74×10^{-12}
180	3.20×10^{-12}
360	1.46×10^{-12}
600	1.22×10^{-12}
900	2.08×10^{-12}
1800	2.40×10^{-11}
3600	4.79×10^{-10}
7200	4.34×10^{-8}
9600	7.55×10^{-9}

The data reveal three distinct regimes of integrator behavior.

Regime A: Precision floor ($\Delta t \leq 900$ s). For Δt from 60 s to 900 s, the energy drift remains at $\sim 10^{-12}$ with no systematic dependence on timestep. Reducing Δt below 900 s does not reduce the energy error; the integration error has fallen below the floating-point roundoff accumulation floor. This floor arises from finite-precision arithmetic in the force summation and long-time energy evaluation, not from the integrator itself. The production timestep $\Delta t = 900$ s is at the edge of this converged regime.

Regime B: 4th-order convergence ($900 \leq \Delta t \leq 3600$ s). In this range, the energy error scales approximately as $\mathcal{O}(\Delta t^4)$, consistent with the expected behavior of the Yoshida integrator. From $\Delta t = 900$ s to 1800 s (a factor of 2), the energy drift increases by a factor of ~ 11.5 (expected: $2^4 = 16$). From 1800 s to 3600 s, the drift increases by a factor of ~ 20 (expected: 16). Both ratios are in good agreement with 4th-order scaling; the deviation from the theoretical factor of 16 is expected when measuring peak-to-peak variation over a long integration rather than single-step local truncation error.

Regime C: Dynamical under-resolution ($\Delta t \geq 7200$ s). At $\Delta t = 7200$ s (~ 2 hr), the energy drift exceeds the 4th-order prediction by a large factor: the ratio from 3600 s to 7200 s is ~ 90, far exceeding the expected 16. At this timestep, the fastest-orbiting body (Io, period 1.77 d) is resolved by only ~ 21 timesteps per orbit. The symplectic splitting approximation degrades when the timestep becomes comparable to the dynamical timescale of the system. The non-monotonic behavior at $\Delta t = 9600$ s ($\Delta E/E$ decreasing relative to 7200 s) is consistent with the oscillatory energy error characteristic of symplectic integrators: the peak-to-peak variation depends on the resonance between the timestep and the orbital frequencies, and can fluctuate non-monotonically at large Δt [2].

Implications. The convergence study confirms that residual positional errors are physics-limited rather than numerics-limited. In Regime A, the integration error has converged to the floating-point floor, yet positional errors against JPL remain unchanged. Furthermore, the error hierarchy matches the expected importance of missing physics: Mercury (relativity) > Galilean moons (phase drift) > outer planets (minimal missing effects). The optimal operating point is $\Delta t \approx 900$ s, where the integration error has just converged to the precision floor; this is adopted as the production timestep.

6. Performance

6.1. Computational Complexity

The force computation scales as $\mathcal{O}(N^2)$ per timestep due to direct pairwise summation. The Yoshida integrator requires 3 force evaluations per step, yielding a total cost of $\mathcal{O}(3N^2 \cdot T/\Delta t)$ for a simulation of duration T.

For the 35-body solar system with $\Delta t = 900$ s over 249 years, this corresponds to $3 \times 35^2 \times 8{,}735{,}580 \approx 3.2 \times 10^{10}$ pairwise interaction evaluations, completed in ~ 79 s on a single core.

6.2. Parallelization

The outer loop of the force computation is parallelized with OpenMP for particle counts exceeding a configurable threshold ($N \geq 500$). The inner loop uses `#pragma omp simd` with reductions to enable SIMD auto-vectorization. For the 35-body solar system, the overhead of thread management exceeds the parallelization benefit, so OpenMP threading is automatically disabled. SIMD vectorization of the inner loop remains active regardless of N.

To validate the parallelization at scale, the solar system body catalog was replicated to produce a 1,050-body test case. On a 6-core CPU, OpenMP threading (56,811 ms) yielded a 3.96× speedup over the single-threaded baseline (224,860 ms), demonstrating strong parallel scaling. This is expected: the $\mathcal{O}(N^2)$ pairwise force loop is embarrassingly parallel over the outer index, and at $N = 1{,}050$ the computation-to-synchronization ratio is large enough to saturate all cores.

6.3. Memory Efficiency

The SoA layout stores 10 scalar fields per particle (3 positions, 3 velocities, 3 accelerations, and 1 mass) plus 3 additional acceleration fields retained for the Velocity Verlet integrator, totaling $13N$ doubles or $104N$ bytes. For $N = 35$, the entire particle dataset occupies $\sim 3.6\,\mathrm{KB}$, fitting comfortably within L1 cache (typically 32–64 KB). The `__restrict__` qualifiers permit the compiler to assume no pointer aliasing, enabling store-to-load forwarding optimizations.

7. Conclusion

We have presented a symplectic N-body simulator that achieves near-machine-precision energy conservation ($\Delta E/E \leq 10^{-12}$) and angular momentum conservation ($\Delta L/L \leq 10^{-13}$) over multi-century integration of the solar system. Positional accuracy against JPL Horizons DE441 is 0.05% mean maximum relative error over 100 years across 35 bodies. The Yoshida 4th-order integrator provides an excellent balance between accuracy and computational cost, requiring only 3 force evaluations per timestep while delivering 4th-order convergence and guaranteed bounded energy error.

A systematic timestep convergence study identifies $\Delta t \approx 900\,\mathrm{s}$ as the optimal operating point, where the integration error has converged to the floating-point precision floor. Below this timestep, no further improvement in energy conservation is achievable with double-precision arithmetic.

The residual positional errors are conclusively attributed to physics model limitations rather than numerical integration error. This is demonstrated by the convergence of energy conservation to the machine-precision floor while positional errors remain unchanged, and by the agreement of the error hierarchy with the expected importance of missing physics.

Future work includes implementing post-Newtonian corrections via an implicit or mixed-variable symplectic scheme, adding solar oblateness perturbations, and incorporating adaptive timestepping for improved resolution of fast-orbiting satellites.

Code Availability

The source code for the simulator and the validation pipeline are open source and available on GitHub.

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