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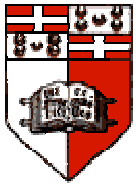

DEPARTMENT OF STATISTICS & OPERATIONS RESEARCH

STATISTICAL FORMULAE BOOKLET

for

Statistics & O.R. Students

This booklet is not to be removed from the examination room or marked in any way.

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CONTENTS

ALGEBRA

Factors

$$a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$$
$$a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$$

Eigenvalues λ of a matrix A

$$\det\left(A-\lambda I_n\right)=0$$

where *n* is the matrix dimension.

Finite Series

$$\sum_{q=0}^{n-1}(a+qd)=\frac{n}{2}[2a+(n-1)d];\qquad \sum_{q=0}^{n-1}ar^q=\frac{a(1-r^n)}{1-r}$$
$$\sum_{r=1}^{n}r=\frac{1}{2}n(n+1);\qquad \sum_{r=1}^{n}r^2=\frac{1}{6}n(n+1)(2n+1);\qquad \sum_{r=1}^{n}r^3=\frac{1}{4}n^2(n+1)^2$$
$$(1+x)^n=1+{}^nC_1x+{}^nC_2x^2+\ldots{}^nC_rx^r+\ldots x^n \quad \text{where } n \text{ is a positive integer}$$
$$(a+x)^n=\sum_{r=1}^{n}{}^nC_ra^{n-r}x^r \quad \text{where } n \text{ is a positive integer}$$

De Moivre's Theorem

If *n* is an integer, $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$.

If *n* is a rational number, $\cos n\theta+i\sin n\theta$ is one of the values of $(\cos\theta+i\sin\theta)^n$.

HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh^{-1} x = \ln\left[x + \sqrt{\left(x^2 + 1\right)}\right]$$

$$\text{Principle value of } \cos h^{-1} x = \ln\left[x + \sqrt{\left(x^2 - 1\right)}\right] \qquad (x \geq 1)$$

$$\tanh^{-1} x = \frac{1}{2}\left|\frac{1+x}{1-x}\right| \qquad (|x| < 1)$$

CIRCULAR FUNCTIONS

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 3A = 3\sin A - 4\sin^3 A; \quad \cos 3A = 4\cos^3 A - 3\cos A$$

$$\sin A + \sin B = 2\sin \tfrac{1}{2}(A+B)\cos \tfrac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos \tfrac{1}{2}(A+B)\sin \tfrac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos \tfrac{1}{2}(A+B)\cos \tfrac{1}{2}(A-B)$$

$$\cos A - \cos B = -2\sin \tfrac{1}{2}(A+B)\sin \tfrac{1}{2}(A-B)$$

$$\sec^2 A = 1 + \tan^2 A; \quad \cos^2 A = \frac{1}{2}(1 + \cos 2A); \quad \sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

$$\sin A = \frac{2t}{1+t^2}; \quad \cos A = \frac{1-t^2}{1+t^2}$$

CALCULUS

1. Infinite Series

Taylor's Formula for functions of 1 variable:

$$f(x) = f(c) + \sum_{k=1}^{n-1} \frac{f^{(k)}(c)}{k!}(x-c)^k + \frac{f^{(n)}(z)}{n!}(x-c)^n$$

Second order Taylor's Formula for functions of *n* variables:

$$f(\mathbf{x}) = f(\mathbf{c}) + \nabla f(\mathbf{c})^T (\mathbf{x}-\mathbf{c}) + (\mathbf{x}-\mathbf{c})^T H(\mathbf{c})(\mathbf{x}-\mathbf{c}) + \|\mathbf{x}-\mathbf{c}\|^2 \alpha(\mathbf{x}-\mathbf{c})$$

for some constant vector **c**.

Other Expansions:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r-1}\frac{x^r}{r} + \ldots \text{ where } -1 < x \le 1$$

* $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots$$

* $$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots$$

* $$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r}}{(2r+1)!} + \ldots$$

* $$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots$$

*These series are valid for all finite x.

2. Derivatives

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$-\sec^2 x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\sec x$	$-\sec x \tan x$
e^x	e^x
$\log_e x \equiv \ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{(\ln a)x}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
uv	$uv' + u'v$
$\frac{u}{v}$	$\frac{(vu' - uv')}{v^2}$

Jacobian Matrix

$$J(\boldsymbol{x}) = D\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} \frac{df_1(\boldsymbol{x})}{dx_1} & \cdots & \frac{df_1(\boldsymbol{x})}{dx_n} \\ \vdots & \ddots & \vdots \\ \frac{df_m(\boldsymbol{x})}{dx_1} & \cdots & \frac{df_m(\boldsymbol{x})}{dx_n} \end{pmatrix}$$

Directional Derivative

$$\nabla_{\boldsymbol{d}} \boldsymbol{f}(\boldsymbol{x}) = \sum_{i=1}^{n} d_i \frac{df(\boldsymbol{x})}{dx_i}$$

Hessian Matrix of Multivariate Functions

$$H(\boldsymbol{x}) = \begin{pmatrix} \frac{d^2 f(\boldsymbol{x})}{dx_1^2} & \cdots & \frac{d^2 f(\boldsymbol{x})}{dx_1 dx_n} \\ \vdots & \ddots & \vdots \\ \frac{d^2 f(\boldsymbol{x})}{dx_n dx_1} & \cdots & \frac{d^2 f(\boldsymbol{x})}{dx_n^2} \end{pmatrix}$$

3. Integrals *(Constants of integration are omitted;* $\ln a \equiv \log_e a$ *)*

$f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left(x+\sqrt{\left(x^2+a^2\right)}\right)$ or $\sinh^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left(x+\sqrt{\left(x^2-a^2\right)}\right)$ or $\cosh^{-1}\left(\frac{x}{a}\right)$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$\ln(\sec x)$
$\cot x$	$\ln(\sin x)$
$\sec x$	$\ln(\sec x+\tan x)$ or $\ln\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)$
$\operatorname{cosec} x$	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$
uv	$uv'+u'v$
$u\frac{dv}{dx}$	$uv-\int v\frac{du}{dx}dx$

4. Approximations

Trapezoidal Rule:

$$\int_a^b y dx \approx \frac{1}{2}h\left[(y_0+y_n)+2(y_1+y_2+...+y_{n-1})\right]$$

Simpson's rule (*n even*)

$$\int_a^b y dx \approx \frac{1}{3}h\left[(y_0+y_n)+4(y_1+y_3+...+y_{n-1})+2(y_2+y_4+...+y_{n-2})\right]$$

Newton's Approximation to a root of $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

SETS

$$X \setminus Y = X \cap Y^c$$

$$\left.\begin{array}{l} X \cap (Y \cap Z) = (X \cap Y) \cap Z = X \cap Y \cap Z \\ X \cup (Y \cup Z) = (X \cup Y) \cup Z = X \cup Y \cup Z \end{array}\right\} \text{ Associative laws}$$

$$\left.\begin{array}{l} X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z) \\ X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z) \end{array}\right\} \text{ Distributive laws}$$

$$\left.\begin{array}{l} (X \cup Y)^c = X^c \cap Y^c \\ (X \cap Y)^c = X^c \cup Y^c \end{array}\right\} \text{ De Morgan's laws}$$

PROBABILITY

Probability Laws

For two events *A* and *B*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If two events *A* and *B* are mutually exclusive then the probability that *A* or *B* occur is

$$P(A \cup B) = P(A) + P(B)$$

If two events *A* and *B* are independent then the probability that *A* and *B* occur is

$$P(A \cap B) = P(A).P(B)$$

If $P(B) \neq 0$ then the probability that event *A* occurs given that event *B* occurs is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Permutations and Combinations

The number of permutations in which *r* objects can be selected from a set on *n* distinct objects is

$${}^{n}P_{r} = \frac{n\ !}{(n-r)\ !}$$

The number of combinations in which *r* objects can be selected from a set of *n* distinct objects is

$${}^{n}C_{r} = \frac{n\ !}{r\ !\ (n-r)\ !}$$

Discrete variable X with probability distribution function $P(X=x)$	**Continuous variable X with probability density function** $f(x)$
Cumulative distribution function $F(x)$ $F(x_0)=P(X\le x_0)=\sum_{x\le x_0}P(X=x)$	$F(x_0)=P(X\le x_0)=\int_{-\infty}^{x_0} f(x)\,dx$
Expectation of *X*, $E(X)$ $E(X)=\mu=\sum xP(X=x)$	$E(X)=\mu=\int xf(x)\,dx$
Expectation of $g(X)$, $E[g(X)]$ $E[g(X)]=\sum g(x)P(X=x)$	$E[g(X)]=\int g(x)f(x)\,dx$
Variance of *X*, $Var(X)$ $Var(X)=\sum(x-\mu)^2P(X=x)$	$Var(X)=\int(x-\mu)^2 f(x)\,dx$
Alternatively, Variance of *X*, $Var(X)$ $Var(X)=E\left(\left[X-E(X)\right]^2\right)=E(X^2)-\left[E(X)\right]^2$	
Covariance of *X* and *Y*, $Cov(X,Y)$ $Cov(X,Y)=E\left(\left[X-E(X)\right]\left[Y-E(Y)\right]\right)$ $=E(XY)-E(X)E(Y)$	
Correlation coefficient of *X* and *Y*, $Corr(X,Y)=\rho_{XY}$ $\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$	

DISTRIBUTIONS

1. Discrete Distributions

If *X* has a Poisson distribution with probability distribution function $P(X=x)$ then for $x=0,1,2,...$

$$P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!} \qquad E(X)=\lambda \qquad Var(X)=\lambda$$

If *X* has a Binomial distribution with probability distribution function $P(X=x)$ then for $x=0,1,2,...,n$

$$P(X=x) = {}^nC_x p^x (1-p)^{n-x} \qquad E(X)=np \qquad Var(X)=np(1-p)$$

If *X* has a Geometric distribution with probability distribution function $P(X=x)$ then for $x=1,2,...$

$$P(X=x)=p(1-p)^{x-1} \qquad E(X)=\frac{1}{p} \qquad Var(X)=\frac{1-p}{p^2}$$

If *X* has a Hypergeometric distribution with probability distribution function $P(X=x)$ then for $x=0,1,2,...,n$

$$P(X=x)=\frac{{}^kC_x \; {}^{N-k}C_{n-x}}{{}^NC_n} \qquad E(X)=\frac{nk}{N} \qquad Var(X)=\frac{nk(N-k)(N-n)}{N^2(N-1)}$$

2. Continuous Distributions

If *X* has a continuous Uniform distribution with probability density function $f(x)$ then for $\alpha < x < \beta$

$$f(x)=\frac{1}{\beta-\alpha} \qquad E(X)=\frac{\alpha+\beta}{2} \qquad Var(X)=\frac{(\beta-\alpha)^2}{12}$$

If *X* has an Exponential distribution with probability density function $f(x)$ then for $x>0$

$$f(x)=\lambda e^{-\lambda x} \qquad E(X)=\frac{1}{\lambda} \qquad Var(X)=\frac{1}{\lambda^2}$$

If *X* has a Standard normal distribution with probability density function $f(x)$ then for $-\infty < x < \infty$

$$f(x)=\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \qquad E(X)=0 \qquad Var(X)=1$$

If *X* has a Normal distribution with probability density function $f(x)$ then for $-\infty < x < \infty$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \qquad E(X) = \mu \qquad Var(X) = \sigma^2$$

If *X* is normally distributed with mean μ and standard deviation σ, then $Z = \frac{X-\mu}{\sigma}$ is normally distributed with mean 0 and standard deviation 1.

If *X* has a Gamma distribution with probability density function $f(x)$ then for $x > 0$

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{\frac{-x}{\beta}} \qquad E(X) = \alpha\beta \qquad Var(X) = \alpha\beta^2$$

If *X* has a Chi-Square distribution with probability density function $f(x)$ then for $x > 0$

$$f(x) = \frac{1}{2^{\frac{v}{2}}\Gamma\left(\frac{v}{2}\right)} x^{\frac{(v-2)}{2}} e^{\frac{-x}{2}} \qquad E(X) = v \qquad Var(X) = 2v$$

If *X* has a Beta distribution with probability density function $f(x)$ then for $0 < x < 1$

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \qquad E(X) = \frac{\alpha}{\alpha+\beta} \qquad Var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

If *X* has a log-normal distribution $\left[\log X \sim N\left(\mu, \sigma^2\right)\right]$ with probability density function $f(x)$ then for $x > 0$

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2\right] \qquad E(X) = e^{\mu+\frac{1}{2}\sigma^2} \qquad Var(X) = e^{\mu+\frac{1}{2}\sigma^2}\sqrt{e^{\sigma^2}-1}$$

If *X* has a Pareto distribution with probability density function $f(x)$ then for $x > 0$, $\alpha > 0$ and $\lambda > 0$

$$f(x) = \frac{\alpha\lambda^{\alpha}}{(\lambda+x)^{\alpha+1}} \qquad E(X) = \frac{\lambda}{\alpha-1} \qquad Var(X) = \frac{\lambda^2}{(\alpha-1)^2(\alpha-2)}$$

If *X* has a generalized Pareto distribution with probability density function $f(x)$ then for $x > 0$

$$f(x) = \frac{\Gamma(\alpha+k)\lambda^{\alpha}x^{k-1}}{\Gamma(\alpha)\Gamma(k)(\lambda+x)^{\alpha+k}} \qquad E(X) = \frac{\lambda k}{\alpha-1} \qquad Var(X) = \frac{\lambda^2 k(k+\alpha-1)}{(\alpha-1)^2(\alpha-2)}$$

If *X* has a Weibull distribution with probability density function $f(x)$ then for $x > 0$

$$f(x) = k\gamma x^{\gamma-1}\exp\left(-kx^{\gamma}\right) \qquad E(X) = \frac{\Gamma\left(1+1/\gamma\right)}{c^{1/\gamma}} \qquad Var(X) = \frac{\Gamma\left(1+2/\gamma\right)}{c^{2/\gamma}} - \left[\frac{\Gamma\left(1+1/\gamma\right)}{c^{1/\gamma}}\right]^2$$

If *X* has a Student's *t* distribution with probability density function $f(x)$ then for $-\infty < x < \infty$

$$f(x) = \frac{K}{\left(1+\frac{x^2}{v}\right)^{(v+1)/2}}$$

where the number of degrees of freedom is $v = n-1$ and the constant $K = \dfrac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi v}\,\Gamma\left(\frac{v}{2}\right)}$.

If *X* has a Fisher's *F* distribution with probability density function $f(x)$ then for $x > 0$

$$f(x) = \frac{Cx^{(v_1/2)-1}}{\left(v_1 x + v_2\right)^{(v_1+v_2)/2}}$$

where $v_1 = n_1 - 1$, $v_2 = n_2 - 1$ and the constant $C = \dfrac{\Gamma\left(\frac{v_1+v_2}{2}\right)}{\Gamma\left(\frac{v_1}{2}\right).\Gamma\left(\frac{v_2}{2}\right)}\left(\frac{v_1}{v_2}\right)^{v_1/2}$.

STATISTICS

For a sample of size *n*:

$$\text{Mean } \bar{x} = \frac{1}{n}\sum_{1}^{n} x_i \qquad \text{Variance } s^2 = \frac{1}{n-1}\sum_{1}^{n}\left(x_i - \bar{x}\right)^2$$

$$\text{Standard deviation } s = \sqrt{Variance}$$

$$\text{Skewness} = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s}\right)^3$$

$$\text{Kurtosis} = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s}\right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)}$$

Sampling Theory

1. The random variable $t = \dfrac{\bar{x} - \mu_{\bar{x}}}{s_{\bar{x}}}$ has a *t* –distribution with $v = n - 1$ degrees of freedom, where:

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

where s^2 is the sample variance.

2. The random variable $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - \mu_{\bar{x}_1 - \bar{x}_2}}{s_{\bar{x}_1 - \bar{x}_2}}$ has a *t* –distribution with $v = n_1 + n_2 - 2$ degrees of freedom, where:

$$s_{\bar{x}_1 - \bar{x}_2} = s_p\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \qquad {s_p}^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

where ${s_p}^2$ is the pooled variance and ${s_1}^2$ and ${s_2}^2$ are the two sample variances.

3. Analysis of Variance: Having observations of *a* independent groups with $m_1, m_2, \ldots, m_a$ replications, where $m = \sum_{j=1}^{a} m_j$ is the total number of observations, $T_{j\cdot} = \sum_{k=1}^{m_j} X_{jk}$ is the *j*th row total and $T = \sum_{j=1}^{a} T_{j\cdot}$ is the grand total, then:

$$V = \sum_{jk} {X_{jk}}^2 - \frac{T^2}{m}$$

$$V_B = \sum_j \frac{1}{m_j}\left(T_{j\cdot}\right)^2 - \frac{T^2}{m}$$

$$V_w = V - V_B$$

4. The sample correlation coefficient r is:

$$r = \frac{n\sum_1^n x_i y_i - \left(\sum_1^n x_i\right)\left(\sum_1^n y_i\right)}{\sqrt{\left[n\sum_1^n x_i^2 - \left(\sum_1^n x_i\right)^2\right]\left[n\sum_1^n y_i^2 - \left(\sum_1^n y_i\right)^2\right]}}$$

FINANCIAL MATHEMATICS

Simple amount when interest is payable per time unit is $S_n = P(1+ni)$

Compound amount when interest is payable per time unit is $S_n = P(1+i)^n$

Nominal rate of interest per unit time payable *p*thly is $i^{(p)} = p\left[(1+i)^{\frac{1}{p}} - 1\right]$

Compound amount when interest is payable *p*thly is $S_n = P\left(1+\frac{i^{(p)}}{p}\right)^{np}$

Nominal rate of discount per unit time payable *p*thly is $d^{(p)} = p\left[1-(1-d)^{\frac{1}{p}}\right]$

The force of interest $\delta(t) = \lim_{h\to 0}\left[\frac{A(t,t+h)-1}{h}\right]$ where $A(t,t+h)$ is the accumulated amount for term *h*

The accumulated amount $A(t_1,t_2) = \exp\left[\int_{t_1}^{t_2}\delta(t)\,dt\right]$

The present value $v(t) = \frac{1}{A(0,t)} = \exp\left[-\int_0^t \delta(s)\,ds\right]$

The present value for discrete cash flows is $\sum_{i=1}^{n} c_i v(t_i)$

The present value for continuous cash flows is $\int_0^T \rho(t)v(t)\,dt$

For a constant force of interest $\delta = \ln(1+i) = -\ln(v) = -\ln(1-d)$

The present value of an immediate annuity-certain is:

$a_{\overline{n}|} = \sum_{k=1}^{n} v^k = \frac{1-v^n}{i}$ if annuities are payable per time unit

$a_n^{(p)} = \sum_{k=1}^{np} \frac{1}{p} v^{\frac{k}{p}} = \frac{ia_n}{i^{(p)}}$ if annuities are payable *p*thly

The present value of an annuity-due is:

$\ddot{a}_{\overline{n}|} = \sum_{k=1}^{n} v^{k-1} = \frac{1-v^n}{d}$ if annuities are payable per time unit

$\ddot{a}_n^{(p)} = \sum_{k=1}^{np} \frac{1}{p} v^{(k-1)/p} = \frac{ia_n}{d^{(p)}}$ if annuities are payable *p*thly

The accumulation of an immediate annuity-certain is:

$s_{\overline{n}|} = \sum_{k=1}^{n} (1+i)^{k-1} = \frac{(1+i)^n - 1}{i}$ if payments are payable per time unit

$s_n^{(p)} = \sum_{k=1}^{np} \frac{1}{p} (1+i)^{(k-1)/p} = \frac{is_n}{i^{(p)}}$ if payments are payable *p*thly

The accumulation of an annuity-due is:

$\ddot{s}_{\overline{n}|} = \sum_{k=1}^{n} (1+i)^{k} = \frac{(1+i)^n - 1}{d}$

$\ddot{s}_n^{(p)} = \sum_{k=1}^{np} \frac{1}{p} (1+i)^{\frac{k}{p}} = \frac{is_n}{d^{(p)}}$ if payments are payable *p*thly

The present value of an increasing annuity-certain is $(Ia)_{\overline{n}|} = \sum_{k=1}^{n} kv^k = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

The present value of an increasing annuity-due is $(I\ddot{a})_{\overline{n}|} = \sum_{k=1}^{n} kv^{k-1} = (1+i)(Ia)_{\overline{n}|}$

The accumulation of an increasing annuity-certain is $(Is)_{\overline{n}|} = \sum_{k=1}^{n} k(1+i)^{n-k} = (1+i)^n (Ia)_{\overline{n}|}$

The accumulation of an increasing annuity-due is $(\ddot{Is})_{\overline{n}|} = \sum_{k=1}^{n} k(1+i)^{n-k+1} = (1+i)^{n+1} (Ia)_{\overline{n}|}$

A loan L is repaid over n time units by a level annuity of L/a_n in arrears. At the t^{th} payment:

Interest content of payment $=1-v^{n-t+1}$

Capital repaid $=v^{n-t+1}$

Remaining loan $=a_{n-t}$

If j_1 is the interest rate of borrowing and j_2 is the interest rate of investing then for a single investment C in return to a series of payments R:

The discounted payback period t satisfies $Ra_t \geq C$

The accumulated profit $P=\left[-C\left(1+j_1\right)^t+Rs_{t\ (j_1)}\right]\left(1+j_2\right)^{n-t}+Rs_{n-t\ (j_2)}$

The net present value with allowance for inflation is $NPV_e = NPV_0\left(\frac{i-e}{1+e}\right)$, where e is the rate of inflation and NPV_0 is the net present value with no allowance

The level annual premium payable in advance for a capital redemption policy with term n years is:

$P_n=\frac{1}{\ddot{s}_n}$ if premiums are payable per time unit

$P_n^{(p)}=\frac{1}{\ddot{s}_n^{(p)}}$ if premiums are payable $p$thly

The policy value at time k with term n years is:

${}_kV_n=\frac{s_k}{s_n}$ if k is an integer and the premium is payable per term

${}_kV_n^{(p)}={}_kV_n$ if k is an integer and the premium is payable $p$thly

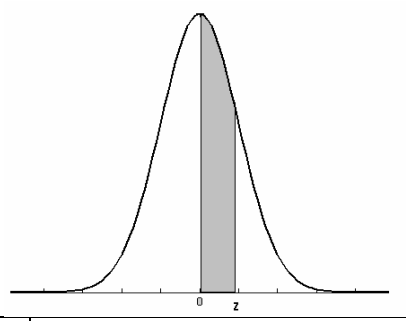
${}_{(r+f)}V_n=\left({}_rV_n+P_n\right)\left(1+i\right)^f$ if $k=r+f$ is not an integer and the premium is payable per term

${}_{(r+f)}V_n^{(p)}=\frac{s_{r+f}}{s_n}$ if $k=r+f$ is not an integer and the premium is payable $p$thly

The paid-up policy value at time k with term n years is:

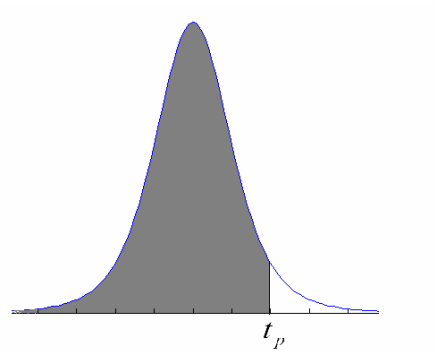
${}_kW_n=\frac{a_k}{a_n}$ if k is an integer and the premium is payable per term

APPENDIX A: TABLES



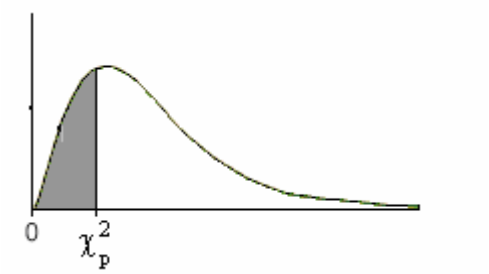
Areas Under The Standard Normal Curve from 0 to z

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1884	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.6078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.2438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000



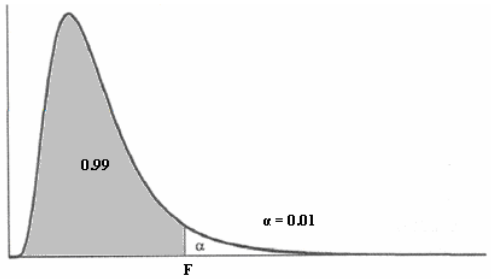
Percentile Values (t_p) for Student's t Distribution with v Degree of Freedom (shaded area = p)

v	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.60}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	2.54	2.09	1.73	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127
22	2.82	2.51	2.07	1.72	1.32	.858	.686	.532	.256	.127
23	2.81	2.50	2.07	1.71	1.32	.858	.685	.532	.256	.127
24	2.80	2.49	2.06	1.71	1.32	.857	.685	.531	.256	.127
25	2.79	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
26	2.78	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
27	2.77	2.47	2.05	1.70	1.31	.855	.684	.531	.256	.127
28	2.76	2.47	2.05	1.70	1.31	.855	.683	.530	.256	.127
29	2.76	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
30	2.75	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
40	2.70	2.42	2.02	1.68	1.30	.851	.681	.529	.255	.126
60	2.66	2.39	2.00	1.67	1.30	.848	.679	.527	.254	.126
120	2.62	2.36	1.98	1.66	1.29	.845	.677	.526	.254	.126
∞	2.58	2.33	1.96	1.645	1.29	.842	.674	.524	.253	.126



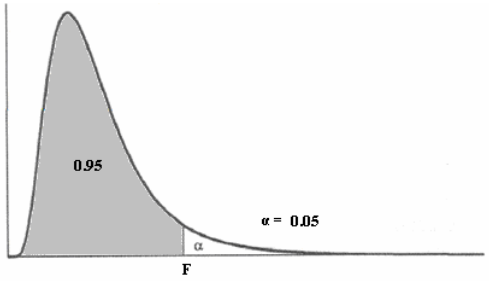
Percentile Values $\left(\chi^2_p\right)$ for the Chi-Square Distribution with *v* Degrees of Freedom (shaded area = *p*)

v	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.75}$	$\chi^2_{.50}$	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$
1	7.88	6.63	5.02	3.84	2.71	1.32	.455	.102	.0158	.0039	.0010	.0002	.0000
2	10.6	9.21	7.38	5.99	4.61	2.77	1.39	.575	.211	.103	.506	.0201	.0100
3	12.8	11.3	9.35	7.81	6.25	4.11	2.37	1.21	.584	.352	.216	.115	.072
4	14.9	13.3	11.1	9.49	7.78	5.39	3.36	1.92	1.06	.711	.484	.297	.207
5	16.7	15.1	12.8	11.1	9.24	6.63	4.35	2.67	1.61	1.15	.831	.554	.412
6	18.5	16.8	14.4	12.6	10.6	7.84	5.35	3.45	2.20	1.64	1.24	.872	.676
7	20.3	18.5	16.0	14.1	12.0	9.04	6.35	4.25	2.83	2.17	1.69	1.24	.989
8	22.0	20.1	17.5	15.5	13.4	10.2	7.34	5.07	3.49	2.73	2.18	1.65	1.34
9	23.6	21.7	19.0	16.9	14.7	11.4	8.34	5.90	4.17	3.33	2.70	2.09	1.73
10	25.2	23.2	20.5	18.3	16.0	12.5	9.34	6.74	4.87	3.94	3.25	2.56	2.16
11	26.8	24.7	21.9	19.7	17.3	13.7	10.3	7.58	5.58	4.57	3.82	3.05	2.60
12	28.3	26.2	23.3	21.0	18.5	14.8	11.3	8.44	6.30	5.23	4.40	3.57	3.07
13	29.8	27.7	24.7	22.4	19.8	16.0	12.3	9.30	7.04	5.89	5.01	4.11	3.57
14	31.3	29.1	26.1	23.7	21.1	17.1	13.3	10.2	7.79	6.57	5.63	4.66	4.07
15	32.8	30.6	27.5	25.0	22.3	18.2	14.3	11.0	8.55	7.26	6.26	5.23	4.60
16	34.3	32.0	28.8	26.3	23.5	19.4	15.3	11.9	9.31	7.96	6.91	5.81	5.14
17	35.7	33.4	30.2	27.6	24.8	30.5	16.3	12.8	10.1	8.67	7.56	6.41	5.70
18	37.2	34.8	31.5	28.9	26.0	21.6	17.3	13.7	10.9	9.39	8.23	7.01	6.26
19	38.6	36.2	32.9	30.1	27.2	22.7	19.3	14.6	11.7	10.1	8.91	7.63	6.84
20	40.0	37.6	34.2	31.4	28.4	23.8	19.3	15.5	12.4	10.9	9.59	8.26	7.43
21	41.4	38.9	35.5	32.7	29.6	24.9	20.3	16.3	13.2	11.6	10.3	8.90	8.03
22	42.8	40.3	36.8	33.9	30.8	26.0	21.3	17.2	14.0	12.3	11.0	9.54	8.64
23	44.2	41.6	38.1	35.2	32.0	27.1	22.3	18.1	14.8	13.1	11.7	10.2	9.26
24	45.6	43.0	39.4	36.4	33.2	28.2	23.3	19.0	15.7	13.8	12.4	10.9	9.89
25	46.9	44.3	40.6	37.7	34.4	29.3	24.3	19.9	16.5	14.6	13.1	11.5	10.5
26	48.3	45.6	41.9	38.9	35.6	30.4	25.3	20.8	17.3	15.4	13.8	12.2	11.2
27	49.6	47.0	43.2	40.1	36.7	31.5	26.3	21.7	18.1	16.2	14.6	12.9	11.8
28	51.0	48.3	44.5	41.3	37.9	32.6	27.3	22.7	18.9	16.9	15.3	13.6	12.5
29	52.3	49.6	45.7	42.6	39.1	33.7	28.3	23.6	19.8	17.7	16.0	14.3	13.1
30	53.7	50.9	47.0	43.8	40.3	34.8	29.3	24.5	20.6	18.5	16.8	15.0	13.8
40	66.8	63.7	59.3	55.8	51.8	45.6	39.3	33.7	29.1	26.5	24.4	22.2	20.7
50	79.5	76.2	71.4	67.5	63.2	56.3	49.3	42.9	37.7	34.8	32.4	29.7	28.0
60	92.0	88.4	83.3	79.1	74.4	67.0	59.3	52.3	46.5	43.2	40.5	37.5	35.5
70	104.2	100.4	95.0	90.5	85.5	77.6	69.3	61.7	55.3	51.7	48.8	45.4	43.3
80	116.3	112.3	106.6	101.9	96.6	88.1	79.3	71.1	64.3	60.4	57.2	53.5	51.2
90	128.3	124.1	118.1	113.1	107.6	98.6	89.3	80.6	73.3	69.1	65.6	61.8	59.2
100	140.2	135.8	129.6	124.3	118.5	109.1	99.3	90.1	82.44	77.9	74.2	70.1	67.3



99th Percentile Values for the *F* Distribution
(v_1 degrees of freedom in numerator)
(v_2 degrees of freedom in denominator)

v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999	5403	5625	5764	5859	5928	5981	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00



95th Percentile Values for the *F* Distribution
(v_1 degrees of freedom in numerator)
(v_2 degrees of freedom in denominator)

v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	199	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Quantiles of the Lilliefors Test Statistic for Normality

	p = *.80*	*.85*	*.90*	*.95*	*.99*
Sample size *n* = 4	.300	.319	.352	.381	.417
5	.285	.299	.315	.337	.405
6	.265	.277	.294	.319	.364
7	.247	.258	.276	.300	.348
8	.233	.244	.261	.285	.331
9	.223	.233	.249	.271	.311
10	.215	.224	.239	.258	.294
11	.206	.217	.230	.249	.284
12	.199	.212	.223	.242	.275
13	.190	.202	.214	.234	.268
14	.183	.194	.207	.227	.261
15	.177	.187	.201	.220	.257
16	.173	.182	.195	.213	.250
17	.169	.177	.189	.206	.245
18	.166	.173	.184	.200	.239
19	.163	.169	.179	.195	.235
22	.160	.166	.174	.190	.231
25	.142	.147	.158	.173	.200
30	.131	.136	.144	.161	.187
Over 30	$\frac{.736}{\sqrt{n}}$	$\frac{.768}{\sqrt{n}}$	$\frac{.805}{\sqrt{n}}$	$\frac{.886}{\sqrt{n}}$	$\frac{1.031}{\sqrt{n}}$

Compound Interest Tables

1 PER CENT

Constants	
Function	Value
i	0.010000
$i^{(2)}$	0.009975
$i^{(4)}$	0.009963
$i^{(12)}$	0.009954
δ	0.009950
$(1+i)$	1.010000
$(1+i)^{1/2}$	1.004988
$(1+i)^{1/4}$	1.002491
$(1+i)^{1/12}$	1.000830
v	0.990099
$v^{1/2}$	0.995037
$v^{1/4}$	0.997516
$v^{1/12}$	0.999171
d	0.009901
$d^{(2)}$	0.009926
$d^{(4)}$	0.009938
$d^{(12)}$	0.009946
$i/i^{(2)}$	1.002494
$i/i^{(4)}$	1.003742
$i/i^{(12)}$	1.004575
i/δ	1.004992
$i/d^{(2)}$	1.007494
$i/d^{(4)}$	1.006242
$i/d^{(12)}$	1.005408

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.01000	0.99010	1.0000	0.9901	0.9901	1
2	1.02010	0.98030	2.0100	1.9704	2.9507	2
3	1.03030	0.97059	3.0301	2.9410	5.8625	3
4	1.04060	0.96098	4.0604	3.9020	9.7064	4
5	1.05101	0.95147	5.1010	4.8534	14.4637	5
6	1.06152	0.94205	6.1520	5.7955	20.1160	6
7	1.07214	0.93272	7.2135	6.7282	26.6450	7
8	1.08286	0.92348	8.2857	7.6517	34.0329	8
9	1.09369	0.91434	9.3685	8.5660	42.2619	9
10	1.10462	0.90529	10.4622	9.4713	51.3148	10
11	1.11567	0.89632	11.5668	10.3676	61.1744	11
12	1.12683	0.88745	12.6825	11.2551	71.8238	12
13	1.13809	0.87866	13.8093	12.1337	83.2464	13
14	1.14947	0.86996	14.9474	13.0037	95.4258	14
15	1.16097	0.86135	16.0969	13.8651	108.3461	15
16	1.17258	0.85282	17.2579	14.7179	121.9912	16
17	1.18430	0.84438	18.4304	15.5623	136.3456	17
18	1.19615	0.83602	19.6147	16.3983	151.3940	18
19	1.20811	0.82774	20.8109	17.2260	167.1210	19
20	1.22019	0.81954	22.0190	18.0456	183.5119	20
21	1.23239	0.81143	23.2392	18.8570	200.5519	21
22	1.24472	0.80340	24.4716	19.6604	218.2267	22
23	1.25716	0.79544	25.7163	20.4558	236.5218	23
24	1.26973	0.78757	26.9735	21.2434	255.4234	24
25	1.28243	0.77977	28.2432	22.0232	274.9176	25
26	1.29526	0.77205	29.5256	22.7952	294.9909	26
27	1.30821	0.76440	30.8209	23.5596	315.6298	27
28	1.32129	0.75684	32.1291	24.3164	336.8212	28
29	1.33450	0.74934	33.4504	25.0658	358.5521	29
30	1.34785	0.74192	34.7849	25.8077	380.8098	30
31	1.36133	0.73458	36.1327	26.5423	403.5817	31
32	1.37494	0.72730	37.4941	27.2696	426.8554	32
33	1.38869	0.72010	38.8690	27.9897	450.6188	33
34	1.40258	0.71297	40.2577	28.7027	474.8599	34
35	1.41660	0.70591	41.6603	29.4086	499.5669	35
36	1.43077	0.69892	43.0769	30.1075	524.7282	36
37	1.44508	0.69200	44.5076	30.7995	550.3324	37
38	1.45953	0.68515	45.9527	31.4847	576.3682	38
39	1.47412	0.67837	47.4123	32.1630	602.8246	39
40	1.48886	0.67165	48.8864	32.8347	629.6907	40
41	1.50375	0.66500	50.3752	33.4997	656.9559	41
42	1.51879	0.65842	51.8790	34.1581	684.6095	42
43	1.53398	0.65190	53.3978	34.8100	712.6412	43
44	1.54932	0.64545	54.9318	35.4555	741.0408	44
45	1.56481	0.63905	56.4811	36.0945	769.7982	45
46	1.58046	0.63273	58.0459	36.7272	798.9037	46
47	1.59626	0.62646	59.6263	37.3537	828.3475	47
48	1.61223	0.62026	61.2226	37.9740	858.1200	48
49	1.62835	0.61412	62.8348	38.5881	888.2118	49
50	1.64463	0.60804	64.4632	39.1961	918.6137	50

Constants	
Function	Value
i	0.020000
$i^{(2)}$	0.019901
$i^{(4)}$	0.019852
$i^{(12)}$	0.019819
δ	0.019803
$(1+i)$	1.020000
$(1+i)^{1/2}$	1.009950
$(1+i)^{1/4}$	1.004963
$(1+i)^{1/12}$	1.001652
v	0.980392
$v^{1/2}$	0.990148
$v^{1/4}$	0.995062
$v^{1/12}$	0.998351
d	0.019608
$d^{(2)}$	0.019705
$d^{(4)}$	0.019754
$d^{(12)}$	0.019786
$i/i^{(2)}$	1.004975
$i/i^{(4)}$	1.007469
$i/i^{(12)}$	1.009134
i/δ	1.009967
$i/d^{(2)}$	1.014975
$i/d^{(4)}$	1.012469
$i/d^{(12)}$	1.010801

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.02000	0.98039	1.0000	0.9804	0.9804	1
2	1.04040	0.96117	2.0200	1.9416	2.9027	2
3	1.06121	0.94232	3.0604	2.8839	5.7297	3
4	1.08243	0.92385	4.1216	3.8077	9.4251	4
5	1.10408	0.90573	5.2040	4.7135	13.9537	5
6	1.12616	0.88797	6.3081	5.6014	19.2816	6
7	1.14869	0.87056	7.4343	6.4720	25.3755	7
8	1.17166	0.85349	8.5830	7.3255	32.2034	8
9	1.19509	0.83676	9.7546	8.1622	39.7342	9
10	1.21899	0.82035	10.9497	8.9826	47.9377	10
11	1.24337	0.80426	12.1687	9.7868	56.7846	11
12	1.26824	0.78849	13.4121	10.5753	66.2465	12
13	1.29361	0.77303	14.6803	11.3484	76.2959	13
14	1.31948	0.75788	15.9739	12.1062	86.9062	14
15	1.34587	0.74301	17.2934	12.8493	98.0514	15
16	1.37279	0.72845	18.6393	13.5777	109.7065	16
17	1.40024	0.71416	20.0121	14.2919	121.8473	17
18	1.42825	0.70016	21.4123	14.9920	134.4502	18
19	1.45681	0.68643	22.8406	15.6785	147.4923	19
20	1.48595	0.67297	24.2974	16.3514	160.9518	20
21	1.51567	0.65978	25.7833	17.0112	174.8071	21
22	1.54598	0.64684	27.2990	17.6580	189.0375	22
23	1.57690	0.63416	28.8450	18.2922	203.6231	23
24	1.60844	0.62172	30.4219	18.9139	218.5444	24
25	1.64061	0.60953	32.0303	19.5235	233.7827	25
26	1.67342	0.59758	33.6709	20.1210	249.3198	26
27	1.70689	0.58586	35.3443	20.7069	265.1380	27
28	1.74102	0.57437	37.0512	21.2813	281.2205	28
29	1.77584	0.56311	38.7922	21.8444	297.5508	29
30	1.81136	0.55207	40.5681	22.3965	314.1129	30
31	1.84759	0.54125	42.3794	22.9377	330.8915	31
32	1.88454	0.53063	44.2270	23.4683	347.8718	32
33	1.92223	0.52023	46.1116	23.9886	365.0393	33
34	1.96068	0.51003	48.0338	24.4986	382.3803	34
35	1.99989	0.50003	49.9945	24.9986	399.8813	35
36	2.03989	0.49022	51.9944	25.4888	417.5293	36
37	2.08069	0.48061	54.0343	25.9695	435.3119	37
38	2.12230	0.47119	56.1149	26.4406	453.2170	38
39	2.16474	0.46195	58.2372	26.9026	471.2330	39
40	2.20804	0.45289	60.4020	27.3555	489.3486	40
41	2.25220	0.44401	62.6100	27.7995	507.5530	41
42	2.29724	0.43530	64.8622	28.2348	525.8358	42
43	2.34319	0.42677	67.1595	28.6616	544.1869	43
44	2.39005	0.41840	69.5027	29.0800	562.5965	44
45	2.43785	0.41020	71.8927	29.4902	581.0553	45
46	2.48661	0.40215	74.3306	29.8923	599.5544	46
47	2.53634	0.39427	76.8172	30.2866	618.0850	47
48	2.58707	0.38654	79.3535	30.6731	636.6388	48
49	2.63881	0.37896	81.9406	31.0521	655.2078	49
50	2.69159	0.37153	84.5794	31.4236	673.7842	50

Constants	
Function	Value
i	0.030000
$i^{(2)}$	0.029778
$i^{(4)}$	0.029668
$i^{(12)}$	0.029595
δ	0.029559
$(1+i)$	1.030000
$(1+i)^{1/2}$	1.014889
$(1+i)^{1/4}$	1.007417
$(1+i)^{1/12}$	1.002466
v	0.970874
$v^{1/2}$	0.985329
$v^{1/4}$	0.992638
$v^{1/12}$	0.997540
d	0.029126
$d^{(2)}$	0.029341
$d^{(4)}$	0.029450
$d^{(12)}$	0.029522
$i/i^{(2)}$	1.007445
$i/i^{(4)}$	1.011181
$i/i^{(12)}$	1.013677
i/δ	1.014926
$i/d^{(2)}$	1.022445
$i/d^{(4)}$	1.018681
$i/d^{(12)}$	1.016177

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.03000	0.97087	1.0000	0.9709	0.9709	1
2	1.06090	0.94260	2.0300	1.9135	2.8561	2
3	1.09273	0.91514	3.0909	2.8286	5.6015	3
4	1.12551	0.88849	4.1836	3.7171	9.1554	4
5	1.15927	0.86261	5.3091	4.5797	13.4685	5
6	1.19405	0.83748	6.4684	5.4172	18.4934	6
7	1.22987	0.81309	7.6625	6.2303	24.1850	7
8	1.26677	0.78941	8.8923	7.0197	30.5003	8
9	1.30477	0.76642	10.1591	7.7861	37.3981	9
10	1.34392	0.74409	11.4639	8.5302	44.8390	10
11	1.38423	0.72242	12.8078	9.2526	52.7856	11
12	1.42576	0.70138	14.1920	9.9540	61.2022	12
13	1.46853	0.68095	15.6178	10.6350	70.0546	13
14	1.51259	0.66112	17.0863	11.2961	79.3102	14
15	1.55797	0.64186	18.5989	11.9379	88.9381	15
16	1.60471	0.62317	20.1569	12.5611	98.9088	16
17	1.65285	0.60502	21.7616	13.1661	109.1941	17
18	1.70243	0.58739	23.4144	13.7535	119.7672	18
19	1.75351	0.57029	25.1169	14.3238	130.6026	19
20	1.80611	0.55368	26.8704	14.8775	141.6761	20
21	1.86029	0.53755	28.6765	15.4150	152.9647	21
22	1.91610	0.52189	30.5368	15.9369	164.4463	22
23	1.97359	0.50669	32.4529	16.4436	176.1002	23
24	2.03279	0.49193	34.4265	16.9355	187.9066	24
25	2.09378	0.47761	36.4593	17.4131	199.8468	25
26	2.15659	0.46369	38.5530	17.8768	211.9028	26
27	2.22129	0.45019	40.7096	18.3270	224.0579	27
28	2.28793	0.43708	42.9309	18.7641	236.2961	28
29	2.35657	0.42435	45.2189	19.1885	248.6021	29
30	2.42726	0.41199	47.5754	19.6004	260.9617	30
31	2.50008	0.39999	50.0027	20.0004	273.3613	31
32	2.57508	0.38834	52.5028	20.3888	285.7881	32
33	2.65234	0.37703	55.0778	20.7658	298.2300	33
34	2.73191	0.36604	57.7302	21.1318	310.6755	34
35	2.81386	0.35538	60.4621	21.4872	323.1139	35
36	2.89828	0.34503	63.2759	21.8323	335.5351	36
37	2.98523	0.33498	66.1742	22.1672	347.9295	37
38	3.07478	0.32523	69.1594	22.4925	360.2881	38
39	3.16703	0.31575	72.2342	22.8082	372.6024	39
40	3.26204	0.30656	75.4013	23.1148	384.8647	40
41	3.35990	0.29763	78.6633	23.4124	397.0675	41
42	3.46070	0.28896	82.0232	23.7014	409.2038	42
43	3.56452	0.28054	85.4839	23.9819	421.2671	43
44	3.67145	0.27237	89.0484	24.2543	433.2515	44
45	3.78160	0.26444	92.7199	24.5187	445.1512	45
46	3.89504	0.25674	96.5015	24.7754	456.9611	46
47	4.01190	0.24926	100.3965	25.0247	468.6762	47
48	4.13225	0.24200	104.4084	25.2667	480.2922	48
49	4.25622	0.23495	108.5406	25.5017	491.8047	49
50	4.38391	0.22811	112.7969	25.7298	503.2101	50

Constants	
Function	Value
i	0.040000
$i^{(2)}$	0.039608
$i^{(4)}$	0.039414
$i^{(12)}$	0.039285
δ	0.039221
$(1+i)$	1.040000
$(1+i)^{1/2}$	1.019804
$(1+i)^{1/4}$	1.009853
$(1+i)^{1/12}$	1.003274
v	0.961538
$v^{1/2}$	0.980581
$v^{1/4}$	0.990243
$v^{1/12}$	0.996737
d	0.038462
$d^{(2)}$	0.038839
$d^{(4)}$	0.039029
$d^{(12)}$	0.039157
$i/i^{(2)}$	1.009902
$i/i^{(4)}$	1.014877
$i/i^{(12)}$	1.018204
i/δ	1.019869
$i/d^{(2)}$	1.029902
$i/d^{(4)}$	1.024877
$i/d^{(12)}$	1.021537

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.04000	0.96154	1.0000	0.9615	0.9615	1
2	1.08160	0.92456	2.0400	1.8861	2.8107	2
3	1.12486	0.88900	3.1216	2.7751	5.4776	3
4	1.16986	0.85480	4.2465	3.6299	8.8969	4
5	1.21665	0.82193	5.4163	4.4518	13.0065	5
6	1.26532	0.79031	6.6330	5.2421	17.7484	6
7	1.31593	0.75992	7.8983	6.0021	23.0678	7
8	1.36857	0.73069	9.2142	6.7327	28.9133	8
9	1.42331	0.70259	10.5828	7.4353	35.2366	9
10	1.48024	0.67556	12.0061	8.1109	41.9922	10
11	1.53945	0.64958	13.4864	8.7605	49.1376	11
12	1.60103	0.62460	15.0258	9.3851	56.6328	12
13	1.66507	0.60057	16.6268	9.9856	64.4403	13
14	1.73168	0.57748	18.2919	10.5631	72.5249	14
15	1.80094	0.55526	20.0236	11.1184	80.8539	15
16	1.87298	0.53391	21.8245	11.6523	89.3964	16
17	1.94790	0.51337	23.6975	12.1657	98.1238	17
18	2.02582	0.49363	25.6454	12.6593	107.0091	18
19	2.10685	0.47464	27.6712	13.1339	116.0273	19
20	2.19112	0.45639	29.7781	13.5903	125.1550	20
21	2.27877	0.43883	31.9692	14.0292	134.3705	21
22	2.36992	0.42196	34.2480	14.4511	143.6535	22
23	2.46472	0.40573	36.6179	14.8568	152.9852	23
24	2.56330	0.39012	39.0826	15.2470	162.3482	24
25	2.66584	0.37512	41.6459	15.6221	171.7261	25
26	2.77247	0.36069	44.3117	15.9828	181.1040	26
27	2.88337	0.34682	47.0842	16.3296	190.4680	27
28	2.99870	0.33348	49.9676	16.6631	199.8054	28
29	3.11865	0.32065	52.9663	16.9837	209.1043	29
30	3.24340	0.30832	56.0849	17.2920	218.3539	30
31	3.37313	0.29646	59.3283	17.5885	227.5441	31
32	3.50806	0.28506	62.7015	17.8736	236.6660	32
33	3.64838	0.27409	66.2095	18.1476	245.7111	33
34	3.79432	0.26355	69.8579	18.4112	254.6719	34
35	3.94609	0.25342	73.6522	18.6646	263.5414	35
36	4.10393	0.24367	77.5983	18.9083	272.3135	36
37	4.26809	0.23430	81.7022	19.1426	280.9825	37
38	4.43881	0.22529	85.9703	19.3679	289.5433	38
39	4.61637	0.21662	90.4091	19.5845	297.9915	39
40	4.80102	0.20829	95.0255	19.7928	306.3231	40
41	4.99306	0.20028	99.8265	19.9931	314.5345	41
42	5.19278	0.19257	104.8196	20.1856	322.6226	42
43	5.40050	0.18517	110.0124	20.3708	330.5849	43
44	5.61652	0.17805	115.4129	20.5488	338.4189	44
45	5.84118	0.17120	121.0294	20.7200	346.1228	45
46	6.07482	0.16461	126.8706	20.8847	353.6951	46
47	6.31782	0.15828	132.9454	21.0429	361.1343	47
48	6.57053	0.15219	139.2632	21.1951	368.4397	48
49	6.83335	0.14634	145.8337	21.3415	375.6104	49
50	7.10668	0.14071	152.6671	21.4822	382.6460	50

Constants	
Function	Value
i	0.050000
$i^{(2)}$	0.049390
$i^{(4)}$	0.049089
$i^{(12)}$	0.048889
δ	0.048790
$(1+i)$	1.050000
$(1+i)^{1/2}$	1.024695
$(1+i)^{1/4}$	1.012272
$(1+i)^{1/12}$	1.004074
v	0.952381
$v^{1/2}$	0.975900
$v^{1/4}$	0.987877
$v^{1/12}$	0.995942
d	0.047619
$d^{(2)}$	0.048200
$d^{(4)}$	0.048494
$d^{(12)}$	0.048691
$i/i^{(2)}$	1.012348
$i/i^{(4)}$	1.018559
$i/i^{(12)}$	1.022715
i/δ	1.024797
$i/d^{(2)}$	1.037348
$i/d^{(4)}$	1.031059
$i/d^{(12)}$	1.026881

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.05000	0.95238	1.0000	0.9524	0.9524	1
2	1.10250	0.90703	2.0500	1.8594	2.7664	2
3	1.15763	0.86384	3.1525	2.7232	5.3580	3
4	1.21551	0.82270	4.3101	3.5460	8.6488	4
5	1.27628	0.78353	5.5256	4.3295	12.5664	5
6	1.34010	0.74622	6.8019	5.0757	17.0437	6
7	1.40710	0.71068	8.1420	5.7864	22.0185	7
8	1.47746	0.67684	9.5491	6.4632	27.4332	8
9	1.55133	0.64461	11.0266	7.1078	33.2347	9
10	1.62889	0.61391	12.5779	7.7217	39.3738	10
11	1.71034	0.58468	14.2068	8.3064	45.8053	11
12	1.79586	0.55684	15.9171	8.8633	52.4873	12
13	1.88565	0.53032	17.7130	9.3936	59.3815	13
14	1.97993	0.50507	19.5986	9.8986	66.4524	14
15	2.07893	0.48102	21.5786	10.3797	73.6677	15
16	2.18287	0.45811	23.6575	10.8378	80.9975	16
17	2.29202	0.43630	25.8404	11.2741	88.4145	17
18	2.40662	0.41552	28.1324	11.6896	95.8939	18
19	2.52695	0.39573	30.5390	12.0853	103.4128	19
20	2.65330	0.37689	33.0660	12.4622	110.9506	20
21	2.78596	0.35894	35.7193	12.8212	118.4884	21
22	2.92526	0.34185	38.5052	13.1630	126.0091	22
23	3.07152	0.32557	41.4305	13.4886	133.4973	23
24	3.22510	0.31007	44.5020	13.7986	140.9389	24
25	3.38635	0.29530	47.7271	14.0939	148.3215	25
26	3.55567	0.28124	51.1135	14.3752	155.6337	26
27	3.73346	0.26785	54.6691	14.6430	162.8656	27
28	3.92013	0.25509	58.4026	14.8981	170.0082	28
29	4.11614	0.24295	62.3227	15.1411	177.0537	29
30	4.32194	0.23138	66.4388	15.3725	183.9950	30
31	4.53804	0.22036	70.7608	15.5928	190.8261	31
32	4.76494	0.20987	75.2988	15.8027	197.5419	32
33	5.00319	0.19987	80.0638	16.0025	204.1377	33
34	5.25335	0.19035	85.0670	16.1929	210.6097	34
35	5.51602	0.18129	90.3203	16.3742	216.9549	35
36	5.79182	0.17266	95.8363	16.5469	223.1705	36
37	6.08141	0.16444	101.6281	16.7113	229.2547	37
38	6.38548	0.15661	107.7095	16.8679	235.2057	38
39	6.70475	0.14915	114.0950	17.0170	241.0224	39
40	7.03999	0.14205	120.7998	17.1591	246.7043	40
41	7.39199	0.13528	127.8398	17.2944	252.2508	41
42	7.76159	0.12884	135.2318	17.4232	257.6621	42
43	8.14967	0.12270	142.9933	17.5459	262.9384	43
44	8.55715	0.11686	151.1430	17.6628	268.0803	44
45	8.98501	0.11130	159.7002	17.7741	273.0886	45
46	9.43426	0.10600	168.6852	17.8801	277.9645	46
47	9.90597	0.10095	178.1194	17.9810	282.7091	47
48	10.40127	0.09614	188.0254	18.0772	287.3239	48
49	10.92133	0.09156	198.4267	18.1687	291.8105	49
50	11.46740	0.08720	209.3480	18.2559	296.1707	50

Constants	
Function	Value
i	0.060000
$i^{(2)}$	0.059126
$i^{(4)}$	0.058695
$i^{(12)}$	0.058411
δ	0.058269
$(1+i)$	1.060000
$(1+i)^{1/2}$	1.029563
$(1+i)^{1/4}$	1.014674
$(1+i)^{1/12}$	1.004868
v	0.943396
$v^{1/2}$	0.971286
$v^{1/4}$	0.985538
$v^{1/12}$	0.995156
d	0.056604
$d^{(2)}$	0.057428
$d^{(4)}$	0.057847
$d^{(12)}$	0.058128
$i/i^{(2)}$	1.014782
$i/i^{(4)}$	1.022227
$i/i^{(12)}$	1.027211
i/δ	1.029709
$i/d^{(2)}$	1.044782
$i/d^{(4)}$	1.037227
$i/d^{(12)}$	1.032211

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.06000	0.94340	1.0000	0.9434	0.9434	1
2	1.12360	0.89000	2.0600	1.8334	2.7234	2
3	1.19102	0.83962	3.1836	2.6730	5.2422	3
4	1.26248	0.79209	4.3746	3.4651	8.4106	4
5	1.33823	0.74726	5.6371	4.2124	12.1469	5
6	1.41852	0.70496	6.9753	4.9173	16.3767	6
7	1.50363	0.66506	8.3938	5.5824	21.0321	7
8	1.59385	0.62741	9.8975	6.2098	26.0514	8
9	1.68948	0.59190	11.4913	6.8017	31.3785	9
10	1.79085	0.55839	13.1808	7.3601	36.9624	10
11	1.89830	0.52679	14.9716	7.8869	42.7571	11
12	2.01220	0.49697	16.8699	8.3838	48.7207	12
13	2.13293	0.46884	18.8821	8.8527	54.8156	13
14	2.26090	0.44230	21.0151	9.2950	61.0078	14
15	2.39656	0.41727	23.2760	9.7122	67.2668	15
16	2.54035	0.39365	25.6725	10.1059	73.5651	16
17	2.69277	0.37136	28.2129	10.4773	79.8783	17
18	2.85434	0.35034	30.9057	10.8276	86.1845	18
19	3.02560	0.33051	33.7600	11.1581	92.4643	19
20	3.20714	0.31180	36.7856	11.4699	98.7004	20
21	3.39956	0.29416	39.9927	11.7641	104.8776	21
22	3.60354	0.27751	43.3923	12.0416	110.9827	22
23	3.81975	0.26180	46.9958	12.3034	117.0041	23
24	4.04893	0.24698	50.8156	12.5504	122.9316	24
25	4.29187	0.23300	54.8645	12.7834	128.7565	25
26	4.54938	0.21981	59.1564	13.0032	134.4716	26
27	4.82235	0.20737	63.7058	13.2105	140.0705	27
28	5.11169	0.19563	68.5281	13.4062	145.5482	28
29	5.41839	0.18456	73.6398	13.5907	150.9003	29
30	5.74349	0.17411	79.0582	13.7648	156.1236	30
31	6.08810	0.16425	84.8017	13.9291	161.2155	31
32	6.45339	0.15496	90.8898	14.0840	166.1742	32
33	6.84059	0.14619	97.3432	14.2302	170.9983	33
34	7.25103	0.13791	104.1838	14.3681	175.6873	34
35	7.68609	0.13011	111.4348	14.4982	180.2410	35
36	8.14725	0.12274	119.1209	14.6210	184.6596	36
37	8.63609	0.11579	127.2681	14.7368	188.9440	37
38	9.15425	0.10924	135.9042	14.8460	193.0951	38
39	9.70351	0.10306	145.0585	14.9491	197.1142	39
40	10.28572	0.09722	154.7620	15.0463	201.0031	40
41	10.90286	0.09172	165.0477	15.1380	204.7636	41
42	11.55703	0.08653	175.9505	15.2245	208.3978	42
43	12.25045	0.08163	187.5076	15.3062	211.9078	43
44	12.98548	0.07701	199.7580	15.3832	215.2962	44
45	13.76461	0.07265	212.7435	15.4558	218.5655	45
46	14.59049	0.06854	226.5081	15.5244	221.7182	46
47	15.46592	0.06466	241.0986	15.5890	224.7572	47
48	16.39387	0.06100	256.5645	15.6500	227.6851	48
49	17.37750	0.05755	272.9584	15.7076	230.5048	49
50	18.42015	0.05429	290.3359	15.7619	233.2192	50

Constants	
Function	Value
$i^{(2)}$	0.070000
$i^{(4)}$	0.068816
$i^{(12)}$	0.068234
δ	0.067850
	0.067659
$(1+i)^{1/2}$	1.070000
$(1+i)^{1/4}$	1.034408
$(1+i)^{1/12}$	1.017059
	1.005654
$v^{1/2}$	0.934579
$v^{1/4}$	0.966736
$v^{1/12}$	0.983228
	0.994378
$d^{(2)}$	0.065421
$d^{(4)}$	0.066527
$d^{(12)}$	0.067090
	0.067468
$i/i^{(4)}$	1.017204
$i/i^{(12)}$	1.025880
	1.031691
	1.034605
$i/d^{(4)}$	1.052204
$i/d^{(12)}$	1.043380
i	1.037525

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.07000	0.93458	1.0000	0.9346	0.9346	1
2	1.14490	0.87344	2.0700	1.8080	2.6815	2
3	1.22504	0.81630	3.2149	2.6243	5.1304	3
4	1.31080	0.76290	4.4399	3.3872	8.1819	4
5	1.40255	0.71299	5.7507	4.1002	11.7469	5
6	1.50073	0.66634	7.1533	4.7665	15.7449	6
7	1.60578	0.62275	8.6540	5.3893	20.1042	7
8	1.71819	0.58201	10.2598	5.9713	24.7602	8
9	1.83846	0.54393	11.9780	6.5152	29.6556	9
10	1.96715	0.50835	13.8164	7.0236	34.7391	10
11	2.10485	0.47509	15.7836	7.4987	39.9652	11
12	2.25219	0.44401	17.8885	7.9427	45.2933	12
13	2.40985	0.41496	20.1406	8.3577	50.6878	13
14	2.57853	0.38782	22.5505	8.7455	56.1173	14
15	2.75903	0.36245	25.1290	9.1079	61.5540	15
16	2.95216	0.33873	27.8881	9.4466	66.9737	16
17	3.15882	0.31657	30.8402	9.7632	72.3555	17
18	3.37993	0.29586	33.9990	10.0591	77.6810	18
19	3.61653	0.27651	37.3790	10.3356	82.9347	19
20	3.86968	0.25842	40.9955	10.5940	88.1031	20
21	4.14056	0.24151	44.8652	10.8355	93.1748	21
22	4.43040	0.22571	49.0057	11.0612	98.1405	22
23	4.74053	0.21095	53.4361	11.2722	102.9923	23
24	5.07237	0.19715	58.1767	11.4693	107.7238	24
25	5.42743	0.18425	63.2490	11.6536	112.3301	25
26	5.80735	0.17220	68.6765	11.8258	116.8071	26
27	6.21387	0.16093	74.4838	11.9867	121.1523	27
28	6.64884	0.15040	80.6977	12.1371	125.3635	28
29	7.11426	0.14056	87.3465	12.2777	129.4399	29
30	7.61226	0.13137	94.4608	12.4090	133.3809	30
31	8.14511	0.12277	102.0730	12.5318	137.1868	31
32	8.71527	0.11474	110.2182	12.6466	140.8585	32
33	9.32534	0.10723	118.9334	12.7538	144.3973	33
34	9.97811	0.10022	128.2588	12.8540	147.8047	34
35	10.67658	0.09366	138.2369	12.9477	151.0829	35
36	11.42394	0.08754	148.9135	13.0352	154.2342	36
37	12.22362	0.08181	160.3374	13.1170	157.2612	37
38	13.07927	0.07646	172.5610	13.1935	160.1665	38
39	13.99482	0.07146	185.6403	13.2649	162.9533	39
40	14.97446	0.06678	199.6351	13.3317	165.6245	40
41	16.02267	0.06241	214.6096	13.3941	168.1833	41
42	17.14426	0.05833	230.6322	13.4524	170.6331	42
43	18.34435	0.05451	247.7765	13.5070	172.9772	43
44	19.62846	0.05095	266.1209	13.5579	175.2188	44
45	21.00245	0.04761	285.7493	13.6055	177.3614	45
46	22.47262	0.04450	306.7518	13.6500	179.4084	46
47	24.04571	0.04159	329.2244	13.6916	181.3630	47
48	25.72891	0.03887	353.2701	13.7305	183.2286	48
49	27.52993	0.03632	378.9990	13.7668	185.0085	49
50	29.45703	0.03395	406.5289	13.8007	186.7059	50

Constants	
Function	Value
i	0.080000
$i^{(2)}$	0.078461
$i^{(4)}$	0.077706
$i^{(12)}$	0.077208
δ	0.076961
$(1+i)$	1.080000
$(1+i)^{1/2}$	1.039230
$(1+i)^{1/4}$	1.019427
$(1+i)^{1/12}$	1.006434
v	0.925926
$v^{1/2}$	0.962250
$v^{1/4}$	0.980944
$v^{1/12}$	0.993607
d	0.074074
$d^{(2)}$	0.075499
$d^{(4)}$	0.076225
$d^{(12)}$	0.076715
$i/i^{(2)}$	1.019615
$i/i^{(4)}$	1.029519
$i/i^{(12)}$	1.036157
i/δ	1.039487
$i/d^{(2)}$	1.059615
$i/d^{(4)}$	1.049519
$i/d^{(12)}$	1.042824

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.08000	0.92593	1.0000	0.9259	0.9259	1
2	1.16640	0.85734	2.0800	1.7833	2.6406	2
3	1.25971	0.79383	3.2464	2.5771	5.0221	3
4	1.36049	0.73503	4.5061	3.3121	7.9622	4
5	1.46933	0.68058	5.8666	3.9927	11.3651	5
6	1.58687	0.63017	7.3359	4.6229	15.1462	6
7	1.71382	0.58349	8.9228	5.2064	19.2306	7
8	1.85093	0.54027	10.6366	5.7466	23.5527	8
9	1.99900	0.50025	12.4876	6.2469	28.0550	9
10	2.15892	0.46319	14.4866	6.7101	32.6869	10
11	2.33164	0.42888	16.6455	7.1390	37.4046	11
12	2.51817	0.39711	18.9771	7.5361	42.1700	12
13	2.71962	0.36770	21.4953	7.9038	46.9501	13
14	2.93719	0.34046	24.2149	8.2442	51.7165	14
15	3.17217	0.31524	27.1521	8.5595	56.4451	15
16	3.42594	0.29189	30.3243	8.8514	61.1154	16
17	3.70002	0.27027	33.7502	9.1216	65.7100	17
18	3.99602	0.25025	37.4502	9.3719	70.2144	18
19	4.31570	0.23171	41.4463	9.6036	74.6170	19
20	4.66096	0.21455	45.7620	9.8181	78.9079	20
21	5.03383	0.19866	50.4229	10.0168	83.0797	21
22	5.43654	0.18394	55.4568	10.2007	87.1264	22
23	5.87146	0.17032	60.8933	10.3711	91.0437	23
24	6.34118	0.15770	66.7648	10.5288	94.8284	24
25	6.84848	0.14602	73.1059	10.6748	98.4789	25
26	7.39635	0.13520	79.9544	10.8100	101.9941	26
27	7.98806	0.12519	87.3508	10.9352	105.3742	27
28	8.62711	0.11591	95.3388	11.0511	108.6198	28
29	9.31727	0.10733	103.9659	11.1584	111.7323	29
30	10.06266	0.09938	113.2832	11.2578	114.7136	30
31	10.86767	0.09202	123.3459	11.3498	117.5661	31
32	11.73708	0.08520	134.2135	11.4350	120.2925	32
33	12.67605	0.07889	145.9506	11.5139	122.8958	33
34	13.69013	0.07305	158.6267	11.5869	125.3793	34
35	14.78534	0.06763	172.3168	11.6546	127.7466	35
36	15.96817	0.06262	187.1021	11.7172	130.0010	36
37	17.24563	0.05799	203.0703	11.7752	132.1465	37
38	18.62528	0.05369	220.3159	11.8289	134.1868	38
39	20.11530	0.04971	238.9412	11.8786	136.1256	39
40	21.72452	0.04603	259.0565	11.9246	137.9668	40
41	23.46248	0.04262	280.7810	11.9672	139.7143	41
42	25.33948	0.03946	304.2435	12.0067	141.3718	42
43	27.36664	0.03654	329.5830	12.0432	142.9430	43
44	29.55597	0.03383	356.9496	12.0771	144.4317	44
45	31.92045	0.03133	386.5056	12.1084	145.8415	45
46	34.47409	0.02901	418.4261	12.1374	147.1758	46
47	37.23201	0.02686	452.9002	12.1643	148.4382	47
48	40.21057	0.02487	490.1322	12.1891	149.6319	48
49	43.42742	0.02303	530.3427	12.2122	150.7602	49
50	46.90161	0.02132	573.7702	12.2335	151.8263	50

Constants	
Function	Value
i	0.090000
$i^{(2)}$	0.088061
$i^{(4)}$	0.087113
$i^{(12)}$	0.086488
δ	0.086178
$(1+i)$	1.090000
$(1+i)^{1/2}$	1.044031
$(1+i)^{1/4}$	1.021778
$(1+i)^{1/12}$	1.007207
v	0.917431
$v^{1/2}$	0.957826
$v^{1/4}$	0.978686
$v^{1/12}$	0.992844
d	0.082569
$d^{(2)}$	0.084347
$d^{(4)}$	0.085256
$d^{(12)}$	0.085869
$i/i^{(2)}$	1.022015
$i/i^{(4)}$	1.033144
$i/i^{(12)}$	1.040608
i/δ	1.044354
$i/d^{(2)}$	1.067015
$i/d^{(4)}$	1.055644
$i/d^{(12)}$	1.048108

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.09000	0.91743	1.0000	0.9174	0.9174	1
2	1.18810	0.84168	2.0900	1.7591	2.6008	2
3	1.29503	0.77218	3.2781	2.5313	4.9173	3
4	1.41158	0.70843	4.5731	3.2397	7.7510	4
5	1.53862	0.64993	5.9847	3.8897	11.0007	5
6	1.67710	0.59627	7.5233	4.4859	14.5783	6
7	1.82804	0.54703	9.2004	5.0330	18.4075	7
8	1.99256	0.50187	11.0285	5.5348	22.4225	8
9	2.17189	0.46043	13.0210	5.9952	26.5663	9
10	2.36736	0.42241	15.1929	6.4177	30.7904	10
11	2.58043	0.38753	17.5603	6.8052	35.0533	11
12	2.81266	0.35553	20.1407	7.1607	39.3197	12
13	3.06580	0.32618	22.9534	7.4869	43.5600	13
14	3.34173	0.29925	26.0192	7.7862	47.7495	14
15	3.64248	0.27454	29.3609	8.0607	51.8676	15
16	3.97031	0.25187	33.0034	8.3126	55.8975	16
17	4.32763	0.23107	36.9737	8.5436	59.8257	17
18	4.71712	0.21199	41.3013	8.7556	63.6416	18
19	5.14166	0.19449	46.0185	8.9501	67.3369	19
20	5.60441	0.17843	51.1601	9.1285	70.9055	20
21	6.10881	0.16370	56.7645	9.2922	74.3432	21
22	6.65860	0.15018	62.8733	9.4424	77.6472	22
23	7.25787	0.13778	69.5319	9.5802	80.8162	23
24	7.91108	0.12640	76.7898	9.7066	83.8499	24
25	8.62308	0.11597	84.7009	9.8226	86.7491	25
26	9.39916	0.10639	93.3240	9.9290	89.5153	26
27	10.24508	0.09761	102.7231	10.0266	92.1507	27
28	11.16714	0.08955	112.9682	10.1161	94.6580	28
29	12.17218	0.08215	124.1354	10.1983	97.0405	29
30	13.26768	0.07537	136.3075	10.2737	99.3017	30
31	14.46177	0.06915	149.5752	10.3428	101.4452	31
32	15.76333	0.06344	164.0370	10.4062	103.4753	32
33	17.18203	0.05820	179.8003	10.4644	105.3959	33
34	18.72841	0.05339	196.9823	10.5178	107.2113	34
35	20.41397	0.04899	215.7108	10.5668	108.9258	35
36	22.25123	0.04494	236.1247	10.6118	110.5437	36
37	24.25384	0.04123	258.3759	10.6530	112.0692	37
38	26.43668	0.03783	282.6298	10.6908	113.5066	38
39	28.81598	0.03470	309.0665	10.7255	114.8600	39
40	31.40942	0.03184	337.8824	10.7574	116.1335	40
41	34.23627	0.02921	369.2919	10.7866	117.3311	41
42	37.31753	0.02680	403.5281	10.8134	118.4566	42
43	40.67611	0.02458	440.8457	10.8380	119.5137	43
44	44.33696	0.02255	481.5218	10.8605	120.5061	44
45	48.32729	0.02069	525.8587	10.8812	121.4373	45
46	52.67674	0.01898	574.1860	10.9002	122.3105	46
47	57.41765	0.01742	626.8628	10.9176	123.1291	47
48	62.58524	0.01598	684.2804	10.9336	123.8960	48
49	68.21791	0.01466	746.8656	10.9482	124.6143	49
50	74.35752	0.01345	815.0836	10.9617	125.2867	50

Constants	
Function	Value
i	0.100000
$i^{(2)}$	0.097618
$i^{(4)}$	0.096455
$i^{(12)}$	0.095690
δ	0.095310
$(1+i)$	1.100000
$(1+i)^{1/2}$	1.048809
$(1+i)^{1/4}$	1.024114
$(1+i)^{1/12}$	1.007974
v	0.909091
$v^{1/2}$	0.953463
$v^{1/4}$	0.976454
$v^{1/12}$	0.992089
d	0.090909
$d^{(2)}$	0.093075
$d^{(4)}$	0.094184
$d^{(12)}$	0.094933
$i/i^{(2)}$	1.024404
$i/i^{(4)}$	1.036756
$i/i^{(12)}$	1.045045
i/δ	1.049206
$i/d^{(2)}$	1.074404
$i/d^{(4)}$	1.061756
$i/d^{(12)}$	1.053378

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.10000	0.90909	1.0000	0.9091	0.9091	1
2	1.21000	0.82645	2.1000	1.7355	2.5620	2
3	1.33100	0.75131	3.3100	2.4869	4.8159	3
4	1.46410	0.68301	4.6410	3.1699	7.5480	4
5	1.61051	0.62092	6.1051	3.7908	10.6526	5
6	1.77156	0.56447	7.7156	4.3553	14.0394	6
7	1.94872	0.51316	9.4872	4.8684	17.6315	7
8	2.14359	0.46651	11.4359	5.3349	21.3636	8
9	2.35795	0.42410	13.5795	5.7590	25.1805	9
10	2.59374	0.38554	15.9374	6.1446	29.0359	10
11	2.85312	0.35049	18.5312	6.4951	32.8913	11
12	3.13843	0.31863	21.3843	6.8137	36.7149	12
13	3.45227	0.28966	24.5227	7.1034	40.4805	13
14	3.79750	0.26333	27.9750	7.3667	44.1672	14
15	4.17725	0.23939	31.7725	7.6061	47.7581	15
16	4.59497	0.21763	35.9497	7.8237	51.2401	16
17	5.05447	0.19784	40.5447	8.0216	54.6035	17
18	5.55992	0.17986	45.5992	8.2014	57.8410	18
19	6.11591	0.16351	51.1591	8.3649	60.9476	19
20	6.72750	0.14864	57.2750	8.5136	63.9205	20
21	7.40025	0.13513	64.0025	8.6487	66.7582	21
22	8.14027	0.12285	71.4027	8.7715	69.4608	22
23	8.95430	0.11168	79.5430	8.8832	72.0294	23
24	9.84973	0.10153	88.4973	8.9847	74.4660	24
25	10.83471	0.09230	98.3471	9.0770	76.7734	25
26	11.91818	0.08391	109.1818	9.1609	78.9550	26
27	13.10999	0.07628	121.0999	9.2372	81.0145	27
28	14.42099	0.06934	134.2099	9.3066	82.9561	28
29	15.86309	0.06304	148.6309	9.3696	84.7842	29
30	17.44940	0.05731	164.4940	9.4269	86.5035	30
31	19.19434	0.05210	181.9434	9.4790	88.1186	31
32	21.11378	0.04736	201.1378	9.5264	89.6342	32
33	23.22515	0.04306	222.2515	9.5694	91.0550	33
34	25.54767	0.03914	245.4767	9.6086	92.3859	34
35	28.10244	0.03558	271.0244	9.6442	93.6313	35
36	30.91268	0.03235	299.1268	9.6765	94.7959	36
37	34.00395	0.02941	330.0395	9.7059	95.8840	37
38	37.40434	0.02673	364.0434	9.7327	96.8999	38
39	41.14478	0.02430	401.4478	9.7570	97.8478	39
40	45.25926	0.02209	442.5926	9.7791	98.7316	40
41	49.78518	0.02009	487.8518	9.7991	99.5551	41
42	54.76370	0.01826	537.6370	9.8174	100.3221	42
43	60.24007	0.01660	592.4007	9.8340	101.0359	43
44	66.26408	0.01509	652.6408	9.8491	101.6999	44
45	72.89048	0.01372	718.9048	9.8628	102.3172	45
46	80.17953	0.01247	791.7953	9.8753	102.8910	46
47	88.19749	0.01134	871.9749	9.8866	103.4238	47
48	97.01723	0.01031	960.1723	9.8969	103.9186	48
49	106.71896	0.00937	1057.1896	9.9063	104.3778	49
50	117.39085	0.00852	1163.9085	9.9148	104.8037	50

Constants	
Function	Value
i	0.110000
$i^{(2)}$	0.107131
$i^{(4)}$	0.105733
$i^{(12)}$	0.104815
δ	0.104360
$(1+i)$	1.110000
$(1+i)^{1/2}$	1.053565
$(1+i)^{1/4}$	1.026433
$(1+i)^{1/12}$	1.008735
v	0.900901
$v^{1/2}$	0.949158
$v^{1/4}$	0.974247
$v^{1/12}$	0.991341
d	0.099099
$d^{(2)}$	0.101684
$d^{(4)}$	0.103010
$d^{(12)}$	0.103908
$i/i^{(2)}$	1.026783
$i/i^{(4)}$	1.040353
$i/i^{(12)}$	1.049467
i/δ	1.054044
$i/d^{(2)}$	1.081783
$i/d^{(4)}$	1.067853
$i/d^{(12)}$	1.058634

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.11000	0.90090	1.0000	0.9009	0.9009	1
2	1.23210	0.81162	2.1100	1.7125	2.5241	2
3	1.36763	0.73119	3.3421	2.4437	4.7177	3
4	1.51807	0.65873	4.7097	3.1024	7.3526	4
5	1.68506	0.59345	6.2278	3.6959	10.3199	5
6	1.87041	0.53464	7.9129	4.2305	13.5277	6
7	2.07616	0.48166	9.7833	4.7122	16.8994	7
8	2.30454	0.43393	11.8594	5.1461	20.3708	8
9	2.55804	0.39092	14.1640	5.5370	23.8891	9
10	2.83942	0.35218	16.7220	5.8892	27.4109	10
11	3.15176	0.31728	19.5614	6.2065	30.9011	11
12	3.49845	0.28584	22.7132	6.4924	34.3311	12
13	3.88328	0.25751	26.2116	6.7499	37.6788	13
14	4.31044	0.23199	30.0949	6.9819	40.9268	14
15	4.78459	0.20900	34.4054	7.1909	44.0618	15
16	5.31089	0.18829	39.1899	7.3792	47.0745	16
17	5.89509	0.16963	44.5008	7.5488	49.9582	17
18	6.54355	0.15282	50.3959	7.7016	52.7090	18
19	7.26334	0.13768	56.9395	7.8393	55.3249	19
20	8.06231	0.12403	64.2028	7.9633	57.8056	20
21	8.94917	0.11174	72.2651	8.0751	60.1522	21
22	9.93357	0.10067	81.2143	8.1757	62.3669	22
23	11.02627	0.09069	91.1479	8.2664	64.4528	23
24	12.23916	0.08170	102.1742	8.3481	66.4137	24
25	13.58546	0.07361	114.4133	8.4217	68.2539	25
26	15.07986	0.06631	127.9988	8.4881	69.9781	26
27	16.73865	0.05974	143.0786	8.5478	71.5911	27
28	18.57990	0.05382	159.8173	8.6016	73.0981	28
29	20.62369	0.04849	178.3972	8.6501	74.5043	29
30	22.89230	0.04368	199.0209	8.6938	75.8148	30
31	25.41045	0.03935	221.9132	8.7331	77.0347	31
32	28.20560	0.03545	247.3236	8.7686	78.1693	32
33	31.30821	0.03194	275.5292	8.8005	79.2233	33
34	34.75212	0.02878	306.8374	8.8293	80.2017	34
35	38.57485	0.02592	341.5896	8.8552	81.1090	35
36	42.81808	0.02335	380.1644	8.8786	81.9498	36
37	47.52807	0.02104	422.9825	8.8996	82.7282	37
38	52.75616	0.01896	470.5106	8.9186	83.4485	38
39	58.55934	0.01708	523.2667	8.9357	84.1145	39
40	65.00087	0.01538	581.8261	8.9511	84.7299	40
41	72.15096	0.01386	646.8269	8.9649	85.2982	41
42	80.08757	0.01249	718.9779	8.9774	85.8226	42
43	88.89720	0.01125	799.0655	8.9886	86.3063	43
44	98.67589	0.01013	887.9627	8.9988	86.7522	44
45	109.53024	0.00913	986.6386	9.0079	87.1630	45
46	121.57857	0.00823	1096.1688	9.0161	87.5414	46
47	134.95221	0.00741	1217.7474	9.0235	87.8897	47
48	149.79695	0.00668	1352.6996	9.0302	88.2101	48
49	166.27462	0.00601	1502.4965	9.0362	88.5048	49
50	184.56483	0.00542	1668.7712	9.0417	88.7757	50

Constants	
Function	Value
i	0.120000
$i^{(2)}$	0.116601
$i^{(4)}$	0.114949
$i^{(12)}$	0.113866
δ	0.113329
$(1+i)$	1.120000
$(1+i)^{1/2}$	1.058301
$(1+i)^{1/4}$	1.028737
$(1+i)^{1/12}$	1.009489
v	0.892857
$v^{1/2}$	0.944911
$v^{1/4}$	0.972065
$v^{1/12}$	0.990600
d	0.107143
$d^{(2)}$	0.110178
$d^{(4)}$	0.111738
$d^{(12)}$	0.112795
$i/i^{(2)}$	1.029150
$i/i^{(4)}$	1.043938
$i/i^{(12)}$	1.053875
i/δ	1.058867
$i/d^{(2)}$	1.089150
$i/d^{(4)}$	1.073938
$i/d^{(12)}$	1.063875

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.12000	0.89286	1.0000	0.8929	0.8929	1
2	1.25440	0.79719	2.1200	1.6901	2.4872	2
3	1.40493	0.71178	3.3744	2.4018	4.6226	3
4	1.57352	0.63552	4.7793	3.0373	7.1647	4
5	1.76234	0.56743	6.3528	3.6048	10.0018	5
6	1.97382	0.50663	8.1152	4.1114	13.0416	6
7	2.21068	0.45235	10.0890	4.5638	16.2080	7
8	2.47596	0.40388	12.2997	4.9676	19.4391	8
9	2.77308	0.36061	14.7757	5.3282	22.6846	9
10	3.10585	0.32197	17.5487	5.6502	25.9043	10
11	3.47855	0.28748	20.6546	5.9377	29.0665	11
12	3.89598	0.25668	24.1331	6.1944	32.1467	12
13	4.36349	0.22917	28.0291	6.4235	35.1259	13
14	4.88711	0.20462	32.3926	6.6282	37.9906	14
15	5.47357	0.18270	37.2797	6.8109	40.7310	15
16	6.13039	0.16312	42.7533	6.9740	43.3410	16
17	6.86604	0.14564	48.8837	7.1196	45.8169	17
18	7.68997	0.13004	55.7497	7.2497	48.1576	18
19	8.61276	0.11611	63.4397	7.3658	50.3637	19
20	9.64629	0.10367	72.0524	7.4694	52.4370	20
21	10.80385	0.09256	81.6987	7.5620	54.3808	21
22	12.10031	0.08264	92.5026	7.6446	56.1989	22
23	13.55235	0.07379	104.6029	7.7184	57.8960	23
24	15.17863	0.06588	118.1552	7.7843	59.4772	24
25	17.00006	0.05882	133.3339	7.8431	60.9478	25
26	19.04007	0.05252	150.3339	7.8957	62.3133	26
27	21.32488	0.04689	169.3740	7.9426	63.5794	27
28	23.88387	0.04187	190.6989	7.9844	64.7518	28
29	26.74993	0.03738	214.5828	8.0218	65.8359	29
30	29.95992	0.03338	241.3327	8.0552	66.8372	30
31	33.55511	0.02980	271.2926	8.0850	67.7611	31
32	37.58173	0.02661	304.8477	8.1116	68.6126	32
33	42.09153	0.02376	342.4294	8.1354	69.3966	33
34	47.14252	0.02121	384.5210	8.1566	70.1178	34
35	52.79962	0.01894	431.6635	8.1755	70.7807	35
36	59.13557	0.01691	484.4631	8.1924	71.3894	36
37	66.23184	0.01510	543.5987	8.2075	71.9481	37
38	74.17966	0.01348	609.8305	8.2210	72.4604	38
39	83.08122	0.01204	684.0102	8.2330	72.9298	39
40	93.05097	0.01075	767.0914	8.2438	73.3596	40
41	104.21709	0.00960	860.1424	8.2534	73.7531	41
42	116.72314	0.00857	964.3595	8.2619	74.1129	42
43	130.72991	0.00765	1081.0826	8.2696	74.4418	43
44	146.41750	0.00683	1211.8125	8.2764	74.7423	44
45	163.98760	0.00610	1358.2300	8.2825	75.0167	45
46	183.66612	0.00544	1522.2176	8.2880	75.2672	46
47	205.70605	0.00486	1705.8838	8.2928	75.4957	47
48	230.39078	0.00434	1911.5898	8.2972	75.7040	48
49	258.03767	0.00388	2141.9806	8.3010	75.8939	49
50	289.00219	0.00346	2400.0182	8.3045	76.0669	50

Constants	
Function	Value
i	0.130000
$i^{(2)}$	0.126029
$i^{(4)}$	0.124104
$i^{(12)}$	0.122842
δ	0.122218
$(1+i)$	1.130000
$(1+i)^{1/2}$	1.063015
$(1+i)^{1/4}$	1.031026
$(1+i)^{1/12}$	1.010237
v	0.884956
$v^{1/2}$	0.940721
$v^{1/4}$	0.969908
$v^{1/12}$	0.989867
d	0.115044
$d^{(2)}$	0.118558
$d^{(4)}$	0.120369
$d^{(12)}$	0.121597
$i/i^{(2)}$	1.031507
$i/i^{(4)}$	1.047509
$i/i^{(12)}$	1.058269
i/δ	1.063676
$i/d^{(2)}$	1.096507
$i/d^{(4)}$	1.080009
$i/d^{(12)}$	1.069102

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.13000	0.88496	1.0000	0.8850	0.8850	1
2	1.27690	0.78315	2.1300	1.6681	2.4512	2
3	1.44290	0.69305	3.4069	2.3612	4.5304	3
4	1.63047	0.61332	4.8498	2.9745	6.9837	4
5	1.84244	0.54276	6.4803	3.5172	9.6975	5
6	2.08195	0.48032	8.3227	3.9975	12.5794	6
7	2.35261	0.42506	10.4047	4.4226	15.5548	7
8	2.65844	0.37616	12.7573	4.7988	18.5641	8
9	3.00404	0.33288	15.4157	5.1317	21.5601	9
10	3.39457	0.29459	18.4197	5.4262	24.5059	10
11	3.83586	0.26070	21.8143	5.6869	27.3736	11
12	4.33452	0.23071	25.6502	5.9176	30.1421	12
13	4.89801	0.20416	29.9847	6.1218	32.7962	13
14	5.53475	0.18068	34.8827	6.3025	35.3257	14
15	6.25427	0.15989	40.4175	6.4624	37.7241	15
16	7.06733	0.14150	46.6717	6.6039	39.9880	16
17	7.98608	0.12522	53.7391	6.7291	42.1167	17
18	9.02427	0.11081	61.7251	6.8399	44.1113	18
19	10.19742	0.09806	70.7494	6.9380	45.9745	19
20	11.52309	0.08678	80.9468	7.0248	47.7102	20
21	13.02109	0.07680	92.4699	7.1016	49.3229	21
22	14.71383	0.06796	105.4910	7.1695	50.8181	22
23	16.62663	0.06014	120.2048	7.2297	52.2015	23
24	18.78809	0.05323	136.8315	7.2829	53.4789	24
25	21.23054	0.04710	155.6196	7.3300	54.6564	25
26	23.99051	0.04168	176.8501	7.3717	55.7402	26
27	27.10928	0.03689	200.8406	7.4086	56.7361	27
28	30.63349	0.03264	227.9499	7.4412	57.6502	28
29	34.61584	0.02889	258.5834	7.4701	58.4879	29
30	39.11590	0.02557	293.1992	7.4957	59.2549	30
31	44.20096	0.02262	332.3151	7.5183	59.9562	31
32	49.94709	0.02002	376.5161	7.5383	60.5969	32
33	56.44021	0.01772	426.4632	7.5560	61.1816	33
34	63.77744	0.01568	482.9034	7.5717	61.7147	34
35	72.06851	0.01388	546.6808	7.5856	62.2004	35
36	81.43741	0.01228	618.7493	7.5979	62.6424	36
37	92.02428	0.01087	700.1867	7.6087	63.0445	37
38	103.98743	0.00962	792.2110	7.6183	63.4099	38
39	117.50580	0.00851	896.1984	7.6268	63.7418	39
40	132.78155	0.00753	1013.7042	7.6344	64.0431	40
41	150.04315	0.00666	1146.4858	7.6410	64.3163	41
42	169.54876	0.00590	1296.5289	7.6469	64.5640	42
43	191.59010	0.00522	1466.0777	7.6522	64.7885	43
44	216.49682	0.00462	1657.6678	7.6568	64.9917	44
45	244.64140	0.00409	1874.1646	7.6609	65.1756	45
46	276.44478	0.00362	2118.8060	7.6645	65.3420	46
47	312.38261	0.00320	2395.2508	7.6677	65.4925	47
48	352.99234	0.00283	2707.6334	7.6705	65.6285	48
49	398.88135	0.00251	3060.6258	7.6730	65.7513	49
50	450.73593	0.00222	3459.5071	7.6752	65.8623	50

Constants	
Function	Value
i	0.140000
$i^{(2)}$	0.135416
$i^{(4)}$	0.133198
$i^{(12)}$	0.131746
δ	0.131028
$(1+i)$	1.140000
$(1+i)^{1/2}$	1.067708
$(1+i)^{1/4}$	1.033299
$(1+i)^{1/12}$	1.010979
v	0.877193
$v^{1/2}$	0.936586
$v^{1/4}$	0.967774
$v^{1/12}$	0.989140
d	0.122807
$d^{(2)}$	0.126828
$d^{(4)}$	0.128905
$d^{(12)}$	0.130316
$i/i^{(2)}$	1.033854
$i/i^{(4)}$	1.051067
$i/i^{(12)}$	1.062649
i/δ	1.068472
$i/d^{(2)}$	1.103854
$i/d^{(4)}$	1.086067
$i/d^{(12)}$	1.074316

n	$(1+i)^n$	v^n	S_n	a_n	$(Ia)_n$	n
1	1.14000	0.87719	1.0000	0.8772	0.8772	1
2	1.29960	0.76947	2.1400	1.6467	2.4161	2
3	1.48154	0.67497	3.4396	2.3216	4.4410	3
4	1.68896	0.59208	4.9211	2.9137	6.8094	4
5	1.92541	0.51937	6.6101	3.4331	9.4062	5
6	2.19497	0.45559	8.5355	3.8887	12.1397	6
7	2.50227	0.39964	10.7305	4.2883	14.9372	7
8	2.85259	0.35056	13.2328	4.6389	17.7417	8
9	3.25195	0.30751	16.0853	4.9464	20.5092	9
10	3.70722	0.26974	19.3373	5.2161	23.2067	10
11	4.22623	0.23662	23.0445	5.4527	25.8095	11
12	4.81790	0.20756	27.2707	5.6603	28.3002	12
13	5.49241	0.18207	32.0887	5.8424	30.6671	13
14	6.26135	0.15971	37.5811	6.0021	32.9030	14
15	7.13794	0.14010	43.8424	6.1422	35.0045	15
16	8.13725	0.12289	50.9804	6.2651	36.9707	16
17	9.27646	0.10780	59.1176	6.3729	38.8033	17
18	10.57517	0.09456	68.3941	6.4674	40.5054	18
19	12.05569	0.08295	78.9692	6.5504	42.0814	19
20	13.74349	0.07276	91.0249	6.6231	43.5367	20
21	15.66758	0.06383	104.7684	6.6870	44.8770	21
22	17.86104	0.05599	120.4360	6.7429	46.1088	22
23	20.36158	0.04911	138.2970	6.7921	47.2383	23
24	23.21221	0.04308	158.6586	6.8351	48.2723	24
25	26.46192	0.03779	181.8708	6.8729	49.2170	25
26	30.16658	0.03315	208.3327	6.9061	50.0789	26
27	34.38991	0.02908	238.4993	6.9352	50.8640	27
28	39.20449	0.02551	272.8892	6.9607	51.5782	28
29	44.69312	0.02237	312.0937	6.9830	52.2271	29
30	50.95016	0.01963	356.7868	7.0027	52.8159	30
31	58.08318	0.01722	407.7370	7.0199	53.3496	31
32	66.21483	0.01510	465.8202	7.0350	53.8329	32
33	75.48490	0.01325	532.0350	7.0482	54.2701	33
34	86.05279	0.01162	607.5199	7.0599	54.6652	34
35	98.10018	0.01019	693.5727	7.0700	55.0220	35
36	111.83420	0.00894	791.6729	7.0790	55.3439	36
37	127.49099	0.00784	903.5071	7.0868	55.6341	37
38	145.33973	0.00688	1030.9981	7.0937	55.8955	38
39	165.68729	0.00604	1176.3378	7.0997	56.1309	39
40	188.88351	0.00529	1342.0251	7.1050	56.3427	40
41	215.32721	0.00464	1530.9086	7.1097	56.5331	41
42	245.47301	0.00407	1746.2358	7.1138	56.7042	42
43	279.83924	0.00357	1991.7088	7.1173	56.8579	43
44	319.01673	0.00313	2271.5481	7.1205	56.9958	44
45	363.67907	0.00275	2590.5648	7.1232	57.1195	45
46	414.59414	0.00241	2954.2439	7.1256	57.2305	46
47	472.63732	0.00212	3368.8380	7.1277	57.3299	47
48	538.80655	0.00186	3841.4753	7.1296	57.4190	48
49	614.23946	0.00163	4380.2819	7.1312	57.4988	49
50	700.23299	0.00143	4994.5213	7.1327	57.5702	50

Power of the F -test: $\pi(\phi) = P\left(F_{\nu_1,\nu_2,\phi} > F_{\nu_1,\nu_2,\alpha}\right)$

$\nu_1 = 1, \alpha = 0.05$

	ϕ										
ν_2	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33
5	0.21	0.33	0.48	0.62	0.75	0.85	0.92	0.96	0.98	0.99	1.00
6	0.22	0.36	0.51	0.66	0.78	0.88	0.94	0.97	0.99	1.00	1.00
7	0.23	0.37	0.53	0.68	0.81	0.90	0.95	0.98	0.99	1.00	1.00
8	0.24	0.38	0.54	0.70	0.82	0.91	0.96	0.98	0.99	1.00	1.00
9	0.24	0.39	0.56	0.71	0.83	0.92	0.96	0.99	1.00	1.00	1.00
10	0.25	0.40	0.57	0.72	0.84	0.92	0.97	0.99	1.00	1.00	1.00
12	0.26	0.41	0.58	0.74	0.86	0.93	0.97	0.99	1.00	1.00	1.00
15	0.26	0.42	0.60	0.75	0.87	0.94	0.98	0.99	1.00	1.00	1.00
20	0.27	0.43	0.61	0.77	0.88	0.95	0.98	0.99	1.00	1.00	1.00
30	0.28	0.45	0.63	0.78	0.89	0.95	0.98	1.00	1.00	1.00	1.00
60	0.29	0.46	0.64	0.79	0.90	0.96	0.99	1.00	1.00	1.00	1.00
1000	0.29	0.47	0.65	0.81	0.91	0.96	0.99	1.00	1.00	1.00	1.00

$\nu_1 = 1, \alpha = 0.01$

	ϕ										
ν_2	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33	4.67	5.00
5	0.18	0.27	0.38	0.50	0.61	0.72	0.80	0.87	0.92	0.95	0.97
6	0.21	0.31	0.44	0.57	0.69	0.79	0.87	0.92	0.96	0.98	0.99
7	0.23	0.35	0.48	0.62	0.74	0.84	0.91	0.95	0.98	0.99	1.00
8	0.25	0.38	0.52	0.66	0.78	0.87	0.93	0.97	0.99	0.99	1.00
9	0.26	0.40	0.55	0.69	0.81	0.89	0.95	0.98	0.99	1.00	1.00
10	0.28	0.42	0.57	0.71	0.83	0.91	0.96	0.98	0.99	1.00	1.00
12	0.30	0.45	0.61	0.75	0.86	0.93	0.97	0.99	1.00	1.00	1.00
15	0.32	0.48	0.64	0.78	0.88	0.94	0.98	0.99	1.00	1.00	1.00
20	0.34	0.51	0.67	0.81	0.90	0.96	0.98	1.00	1.00	1.00	1.00
30	0.36	0.54	0.71	0.84	0.92	0.97	0.99	1.00	1.00	1.00	1.00
60	0.39	0.57	0.74	0.86	0.94	0.98	0.99	1.00	1.00	1.00	1.00
1000	0.41	0.60	0.76	0.88	0.95	0.98	1.00	1.00	1.00	1.00	1.00

$\nu_1 = 2, \alpha = 0.05$

	ϕ										
ν_2	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33
5	0.20	0.32	0.46	0.61	0.75	0.85	0.92	0.96	0.98	0.99	1.00
6	0.21	0.34	0.50	0.66	0.79	0.89	0.95	0.98	0.99	1.00	1.00
7	0.22	0.37	0.53	0.70	0.83	0.91	0.96	0.99	1.00	1.00	1.00
8	0.23	0.38	0.56	0.72	0.85	0.93	0.97	0.99	1.00	1.00	1.00
9	0.24	0.40	0.58	0.74	0.87	0.94	0.98	0.99	1.00	1.00	1.00
10	0.25	0.41	0.59	0.76	0.88	0.95	0.98	1.00	1.00	1.00	1.00
12	0.26	0.43	0.62	0.78	0.90	0.96	0.99	1.00	1.00	1.00	1.00
15	0.27	0.45	0.64	0.81	0.91	0.97	0.99	1.00	1.00	1.00	1.00
20	0.28	0.47	0.67	0.83	0.93	0.98	0.99	1.00	1.00	1.00	1.00
30	0.29	0.49	0.69	0.85	0.94	0.98	1.00	1.00	1.00	1.00	1.00
60	0.31	0.51	0.71	0.87	0.95	0.99	1.00	1.00	1.00	1.00	1.00
1000	0.32	0.53	0.73	0.88	0.96	0.99	1.00	1.00	1.00	1.00	1.00

$v_1 = 2, \alpha = 0.01$

	ϕ										
v_2	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33	4.67	5.00
5	0.16	0.25	0.36	0.48	0.60	0.70	0.80	0.87	0.92	0.95	0.97
6	0.20	0.30	0.43	0.57	0.69	0.80	0.88	0.93	0.96	0.98	0.99
7	0.22	0.35	0.50	0.64	0.76	0.86	0.92	0.96	0.98	0.99	1.00
8	0.25	0.39	0.54	0.69	0.81	0.90	0.95	0.98	0.99	1.00	1.00
9	0.27	0.42	0.58	0.73	0.85	0.92	0.97	0.99	1.00	1.00	1.00
10	0.29	0.45	0.62	0.76	0.87	0.94	0.98	0.99	1.00	1.00	1.00
12	0.32	0.49	0.67	0.81	0.91	0.96	0.99	1.00	1.00	1.00	1.00
15	0.35	0.54	0.71	0.85	0.93	0.98	0.99	1.00	1.00	1.00	1.00
20	0.39	0.58	0.76	0.88	0.96	0.99	1.00	1.00	1.00	1.00	1.00
30	0.43	0.63	0.80	0.91	0.97	0.99	1.00	1.00	1.00	1.00	1.00
60	0.47	0.68	0.84	0.94	0.98	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.51	0.72	0.87	0.96	0.99	1.00	1.00	1.00	1.00	1.00	1.00

$v_1 = 3, \alpha = 0.05$

	ϕ										
v_2	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33
5	0.19	0.31	0.46	0.61	0.75	0.86	0.93	0.97	0.99	0.99	1.00
6	0.21	0.35	0.51	0.67	0.81	0.90	0.96	0.98	0.99	1.00	1.00
7	0.22	0.37	0.55	0.72	0.85	0.93	0.97	0.99	1.00	1.00	1.00
8	0.24	0.40	0.58	0.75	0.87	0.95	0.98	0.99	1.00	1.00	1.00
9	0.25	0.41	0.60	0.77	0.89	0.96	0.99	1.00	1.00	1.00	1.00
10	0.25	0.43	0.63	0.79	0.91	0.97	0.99	1.00	1.00	1.00	1.00
12	0.27	0.45	0.66	0.82	0.93	0.98	0.99	1.00	1.00	1.00	1.00
15	0.28	0.48	0.69	0.85	0.94	0.98	1.00	1.00	1.00	1.00	1.00
20	0.30	0.51	0.72	0.87	0.96	0.99	1.00	1.00	1.00	1.00	1.00
30	0.32	0.54	0.75	0.90	0.97	0.99	1.00	1.00	1.00	1.00	1.00
60	0.34	0.57	0.78	0.92	0.98	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.36	0.60	0.81	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00

$v_1 = 3, \alpha = 0.01$

	ϕ										
v_2	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33	4.67	5.00
5	0.16	0.25	0.36	0.48	0.60	0.71	0.80	0.87	0.92	0.95	0.98
6	0.20	0.31	0.44	0.58	0.71	0.81	0.89	0.94	0.97	0.99	0.99
7	0.23	0.36	0.52	0.66	0.79	0.88	0.94	0.97	0.99	1.00	1.00
8	0.26	0.41	0.57	0.72	0.84	0.92	0.96	0.99	1.00	1.00	1.00
9	0.29	0.45	0.62	0.77	0.88	0.95	0.98	0.99	1.00	1.00	1.00
10	0.31	0.48	0.66	0.81	0.91	0.96	0.99	1.00	1.00	1.00	1.00
12	0.35	0.54	0.72	0.86	0.94	0.98	0.99	1.00	1.00	1.00	1.00
15	0.39	0.59	0.77	0.90	0.96	0.99	1.00	1.00	1.00	1.00	1.00
20	0.44	0.65	0.83	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00
30	0.49	0.71	0.87	0.96	0.99	1.00	1.00	1.00	1.00	1.00	1.00
60	0.55	0.77	0.91	0.97	0.99	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.60	0.82	0.94	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00

$v_1 = 4, \alpha = 0.05$

	ϕ										
v_2	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33
5	0.19	0.31	0.46	0.62	0.76	0.86	0.93	0.97	0.99	1.00	1.00
6	0.21	0.35	0.52	0.69	0.82	0.91	0.96	0.99	1.00	1.00	1.00
7	0.23	0.38	0.56	0.73	0.86	0.94	0.98	0.99	1.00	1.00	1.00
8	0.24	0.41	0.60	0.77	0.89	0.96	0.99	1.00	1.00	1.00	1.00
9	0.25	0.43	0.63	0.80	0.91	0.97	0.99	1.00	1.00	1.00	1.00
10	0.26	0.45	0.65	0.82	0.93	0.98	0.99	1.00	1.00	1.00	1.00
12	0.28	0.48	0.69	0.85	0.95	0.98	1.00	1.00	1.00	1.00	1.00
15	0.30	0.51	0.73	0.88	0.96	0.99	1.00	1.00	1.00	1.00	1.00
20	0.32	0.54	0.76	0.91	0.97	0.99	1.00	1.00	1.00	1.00	1.00
30	0.34	0.58	0.80	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00
60	0.37	0.62	0.83	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.39	0.65	0.86	0.96	0.99	1.00	1.00	1.00	1.00	1.00	1.00

$v_1 = 4, \alpha = 0.01$

	ϕ										
v_2	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33	4.67	5.00
5	0.16	0.25	0.36	0.48	0.60	0.71	0.80	0.88	0.92	0.96	0.98
6	0.20	0.32	0.45	0.59	0.72	0.83	0.90	0.95	0.97	0.99	1.00
7	0.24	0.38	0.53	0.68	0.81	0.89	0.95	0.98	0.99	1.00	1.00
8	0.27	0.43	0.60	0.75	0.86	0.94	0.97	0.99	1.00	1.00	1.00
9	0.30	0.47	0.65	0.80	0.90	0.96	0.99	1.00	1.00	1.00	1.00
10	0.33	0.51	0.70	0.84	0.93	0.97	0.99	1.00	1.00	1.00	1.00
12	0.38	0.58	0.76	0.89	0.96	0.99	1.00	1.00	1.00	1.00	1.00
15	0.43	0.64	0.82	0.93	0.98	0.99	1.00	1.00	1.00	1.00	1.00
20	0.49	0.71	0.87	0.96	0.99	1.00	1.00	1.00	1.00	1.00	1.00
30	0.55	0.78	0.92	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
60	0.62	0.83	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.69	0.88	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

$v_1 = 5, \alpha = 0.05$

	ϕ										
v_2	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33
5	0.19	0.31	0.47	0.62	0.76	0.87	0.93	0.97	0.99	1.00	1.00
6	0.21	0.35	0.53	0.70	0.83	0.92	0.97	0.99	1.00	1.00	1.00
7	0.23	0.39	0.58	0.75	0.88	0.95	0.98	0.99	1.00	1.00	1.00
8	0.24	0.42	0.62	0.79	0.90	0.97	0.99	1.00	1.00	1.00	1.00
9	0.26	0.44	0.65	0.82	0.93	0.98	0.99	1.00	1.00	1.00	1.00
10	0.27	0.46	0.67	0.84	0.94	0.98	1.00	1.00	1.00	1.00	1.00
12	0.29	0.50	0.72	0.88	0.96	0.99	1.00	1.00	1.00	1.00	1.00
15	0.31	0.54	0.76	0.90	0.97	0.99	1.00	1.00	1.00	1.00	1.00
20	0.34	0.58	0.80	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00
30	0.36	0.62	0.84	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00
60	0.40	0.66	0.87	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.43	0.71	0.90	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00

$v_1 = 5, \alpha = 0.01$

	ϕ										
v_2	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33	4.67	5.00
5	0.16	0.25	0.36	0.48	0.61	0.72	0.81	0.88	0.93	0.96	0.98
6	0.20	0.32	0.46	0.60	0.73	0.84	0.91	0.95	0.98	0.99	1.00
7	0.24	0.39	0.55	0.70	0.82	0.91	0.96	0.98	0.99	1.00	1.00
8	0.28	0.44	0.62	0.77	0.88	0.95	0.98	0.99	1.00	1.00	1.00
9	0.31	0.49	0.68	0.82	0.92	0.97	0.99	1.00	1.00	1.00	1.00
10	0.35	0.54	0.72	0.86	0.94	0.98	0.99	1.00	1.00	1.00	1.00
12	0.40	0.61	0.79	0.91	0.97	0.99	1.00	1.00	1.00	1.00	1.00
15	0.46	0.68	0.85	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00
20	0.53	0.76	0.91	0.97	0.99	1.00	1.00	1.00	1.00	1.00	1.00
30	0.60	0.82	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
60	0.68	0.88	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.76	0.93	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

$v_1 = 6, \alpha = 0.05$

	ϕ										
v_2	1.00	1.33	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33
5	0.19	0.31	0.47	0.63	0.77	0.87	0.94	0.97	0.99	1.00	1.00
6	0.21	0.36	0.53	0.70	0.84	0.92	0.97	0.99	1.00	1.00	1.00
7	0.23	0.39	0.59	0.76	0.88	0.95	0.98	1.00	1.00	1.00	1.00
8	0.25	0.43	0.63	0.80	0.91	0.97	0.99	1.00	1.00	1.00	1.00
9	0.26	0.45	0.66	0.83	0.94	0.98	1.00	1.00	1.00	1.00	1.00
10	0.28	0.48	0.69	0.86	0.95	0.99	1.00	1.00	1.00	1.00	1.00
12	0.30	0.52	0.74	0.89	0.97	0.99	1.00	1.00	1.00	1.00	1.00
15	0.32	0.56	0.78	0.92	0.98	1.00	1.00	1.00	1.00	1.00	1.00
20	0.35	0.60	0.82	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00
30	0.39	0.65	0.87	0.97	0.99	1.00	1.00	1.00	1.00	1.00	1.00
60	0.42	0.70	0.90	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.47	0.75	0.93	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00

$v_1 = 6, \alpha = 0.01$

	ϕ										
v_2	1.67	2.00	2.33	2.67	3.00	3.33	3.67	4.00	4.33	4.67	5.00
5	0.16	0.25	0.36	0.49	0.61	0.72	0.81	0.88	0.93	0.96	0.98
6	0.20	0.33	0.47	0.61	0.74	0.84	0.91	0.96	0.98	0.99	1.00
7	0.25	0.39	0.56	0.71	0.83	0.91	0.96	0.98	0.99	1.00	1.00
8	0.29	0.46	0.63	0.79	0.89	0.95	0.98	0.99	1.00	1.00	1.00
9	0.33	0.51	0.70	0.84	0.93	0.97	0.99	1.00	1.00	1.00	1.00
10	0.36	0.56	0.74	0.88	0.95	0.98	1.00	1.00	1.00	1.00	1.00
12	0.42	0.64	0.82	0.93	0.98	0.99	1.00	1.00	1.00	1.00	1.00
15	0.49	0.71	0.88	0.96	0.99	1.00	1.00	1.00	1.00	1.00	1.00
20	0.57	0.79	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00
30	0.65	0.86	0.96	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
60	0.74	0.92	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1000	0.81	0.96	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00