

Problem 2 :

The scaling of error does not reflect scaling discussed in lecture. As shown in the code, the absolute ^{true} error in the differentiation approximation is always zero no matter the choice of step size Δx .

The Central differentiation approximation is given by

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}))}{2\Delta x} - \frac{f'''(x_i)}{3!} (\Delta x)^2 + \dots$$

Where the error is given by the trailing terms

$$\text{Error} = - \frac{f'''(x_i)}{3!} (\Delta x)^2 + \dots$$

However, as the given polynomial $f(x) = x^2 - x + 1$
~~the~~ $f'''(x) = 0$ and any higher order derivatives are
also zero. Thus the error is zero and the central ^{for any Δx}
differentiation approximation will give the exact differential \downarrow
as the order of $f(x)$ is less than 3.