Universidad Mariano Gálvez de Guatemala

Ingeniería en Sistemas de Información y Ciencias de la Computación

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CÁLCULO #1

TAREA DEL PARCIAL FINAL

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TERCER SEMESTRE

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Semana 13

Tarea Semana #13 — Antiderivadas

1. $f(x) = 8x^9 - 3x^6 + 12x^3$

$f(x) = \frac{8x^{9+1}}{9+1} - \frac{3x^{6+1}}{6+1} + \frac{12x^{3+1}}{3+1}$

$f(x) = \frac{8x^{10}}{10} - \frac{3x^7}{7} + \frac{12x^4}{4} + C$

$F(x) = \frac{4x^{10}}{5} - \frac{3x^7}{7} + 3x^4 + C$

R// $\frac{4x^{10}}{5} - \frac{3x^7}{7} + 3x^4 + C$

2. $f(x) = (x+1)(2x-1)$

$f(x) = x(2x-1) + 1(2x-1)$

$2x^2 - x + 2x - 1$

$2x^2 + x - 1$

$(2x^2 + x - 1)\,dx$

$f(x) = 2x^2\,dx + x\,dx - 1\,dx$

$f(x) = \frac{2x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} - x$

$f'(x) = \frac{2x^3}{3} + \frac{x^2}{2} - x + C$

R// $F'(x) = \frac{2x^3}{3} + \frac{x^2}{2} - x + C$

3. $f''(x) = 6x + 12x^2$

$$f''(x) = \frac{6x^{1+1}}{1+1} + \frac{12x^{2+1}}{2+1}$$

$$F''(x) = \frac{6x^2}{2} + \frac{12x^3}{3}$$

$$f''(x) = 3x^2 + 4x^3$$

$$f'(x) = \frac{3x^{2+1}}{2+1} + \frac{4x^{3+1}}{3+1}$$

$$F'(x) = \frac{3x^3}{3} + \frac{4x^4}{4} = F'(x) = x^3 + x^4 + cx + D$$

R// $F'(x) = x^3 + x^4 + cx + D$

4. $f'(x) = 8x^3 + 12x + 3, \quad f(1) = 6$

$$f'(x) = \frac{8x^{3+1}}{3+1} + \frac{12x^{1+1}}{1+1} + 3x + c$$

$$f'(x) = \frac{8x^4}{4} + \frac{12x^2}{2} + 3x + c$$

$$f'(x) = 2x^4 + 6x^2 + 3x + c$$

R// $f'(x) = 2x^4 + 6x^2 + 3x + c$

5. $F''(x) = 2 + x^3 + x^6$

$F''(x) = 2x + \frac{x^{3+1}}{3+1} + \frac{x^{6+1}}{6+1}$

$F''(x) = 2x + \frac{x^4}{4} + \frac{x^7}{7} + C$

$F'(x) = \frac{2x^{1+1}}{1+1} + \frac{x^{4+1}}{4+1} + \frac{x^{7+1}}{7+1} + Cx + D$

$F'(x) = \frac{2x^2}{2} + \frac{x^5}{5} + \frac{x^8}{8} + Cx + D$

$F'(x) = x^2 + \frac{x^5}{5} + \frac{x^8}{8} + Cx + D$

R// $F'(x) = x^2 + \frac{x^5}{5} + \frac{x^8}{8} + Cx + D$

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Tarea Semana # 15

Semana # 15

5.3 - 1-30 Evalue la integral

1. $\int_{-2}^{3} (x^2 - 3)\, dx$

$$\frac{x^{2+1}}{2+1} - 3x = \frac{x^3}{3} - 3x \Big|_{-2}^{3}$$

$$\frac{(3)^3}{3} - 3(3) - \frac{(-2)^3}{3} - 3(-2)$$

$$\left(\frac{27}{3} - 9\right) - \left(\frac{-8}{3} + 6\right)$$

$$(9 - 9) - (-2.66 + 6)$$

$$0 \qquad = -3.34$$

2. $\int_0^2 \left(x^4 - \frac{3}{4}x^2 + \frac{2}{3}x - 7\right) dx$

$$\frac{x^{4+1}}{4+1} - \frac{3}{4}\frac{x^{2+1}}{2+1} + \frac{2}{3}\frac{x^{1+1}}{1+1} - 7x$$

$$\frac{x^5}{5} - \frac{\cancel{3}}{4}\frac{x^3}{\cancel{3}} + \frac{\cancel{2}}{3}\frac{x^2}{\cancel{2}} - 7x$$

$$\frac{x^5}{5} - \frac{x^3}{4} + \frac{x^2}{3} - 7x \Big|_0^2$$

$$\left(\frac{(2)^5}{5} - \frac{(2)^3}{4} + \frac{(2)^2}{3} - 7(2)\right) - \left(\frac{(0)^5}{5} - \frac{(0)^3}{4} + \frac{(0)^2}{3} - 7(0)\right)$$

$$\frac{32}{5} - \frac{8}{4} + \frac{4}{3} - 2$$

$$6.4 - 2 + 1.33 - 2 = \underline{3.73}$$

3. $\int_0^1 \left(1 + \frac{1}{2}u^4 - \frac{2}{5}u^9\right) du$

$$1u + \frac{1}{2}\frac{u^{4+1}}{4+1} - \frac{2}{5}\frac{u^{9+1}}{9+1}$$

$$1u + \frac{1}{2}\frac{u^5}{5} - \frac{2}{5}\frac{u^{10}}{10}$$

$$1u + \frac{u^5}{10} - \frac{2u^{10}}{50} \int_0^1$$

$$\left(1(1) + \frac{(1)^5}{10} - \frac{2(1)^{10}}{50}\right) - \cancel{\left(1(0) + \frac{(0)^5}{10} - \frac{2(0)^{10}}{50}\right)}$$

$$\left(1 + \frac{1}{10} - \frac{2}{50}\right)$$

$$1 + 0.1 - 0.04 = \underline{1.06}$$

4. $\int_1^2 (1+2y)^2 \, dy$

$(1+2y)^2 = 1+4y+4y^2$

$\int (1+4y+4y^2)\,dy = y+2y^2+\frac{4y^3}{3} \Big]_1^2$

$= \left((2)+2(4)+\frac{4(8)}{3}\right) - \left((1)+2(1)^2+\frac{4(1)^3}{3}\right)$

$\left(2+8+\frac{32}{3}\right) - \left(1+2+\frac{4}{3}\right)$

$= 10 + 10.66 - 3 + 1.33$

$= 20.66 - 4.33$

$= \underline{16.33}$

5. $\int_1^9 \frac{x-1}{\sqrt{x}}\,dx$

$$\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}} = \frac{x}{x^{1/2}} - \frac{1}{x^{1/2}}$$

$$= x^{1/2} - x^{-1/2} \int_1^9$$

$$= \int_4^9 \left(x^{1/2} - x^{-1/2}\right) dx$$

$$\frac{x^{1/2+1}}{1/2+1} - \frac{x^{-1/2+1}}{-1/2+1} = \frac{x^{3/2}}{3/2} - \frac{x^{1/2}}{1/2} \int_1^9$$

$$= \frac{2x^{3/2}}{3} - \frac{2x^{1/2}}{1} \int_4^9$$

$x^{m/n} = \sqrt[n]{x^m}$

$x^{3/2} = \sqrt[2]{x^3}$

$\sqrt{x^2 \cdot x} = \sqrt{x^2} \cdot \sqrt{x}$

$= x\sqrt{x}$

$$\frac{2x\sqrt{x}}{3} - 2\sqrt{x} \int_1^9$$

$$\left(\frac{2(9)\sqrt{9}}{3} - 2(9)\right) - \left(\frac{2(1)\sqrt{1}}{3} - 2(1)\right)$$

$$\left(\frac{2(9)(3)}{3} - 18\right) - \left(\frac{2}{3} - 2\right) = \left(\frac{2(27)}{3} - 18\right) - \frac{(2-6)}{3} = -\frac{4}{3}$$

(1.33)

$$\frac{54-18}{12} - 1.33 = \frac{36}{12} - 1.33$$

$$3 - 1.33 = \underline{1.67}$$

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Tarea Semana # 15

Semana #15

5.4 - 7-18 ; la Primera parte del Teorema Fundamental de calculo.

7. $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$

$g'(x) = \left(\frac{1}{x^3+1}\right)$

$\left(\frac{1}{x^3+1}\right) * 1 - \cancel{\left(\frac{1}{x^3+1}\right) * 0}$

$g'(x) = \left(\frac{1}{x^3+1}\right)$

R// $g'(x) = \frac{1}{x^3+1}$

2. $g(x) = \int_3^x e^{t^2-t}\,dt$

$\left(e^{x^2-x}\right)$

$= \left(e^{x^2-x}\right) \times 1 - \cancel{\left(e^{x^2-x}\right) \times 0}$

$g'(x) = e^{x^2-x}$

D// $g'(x) = e^{x^2-x}$

3. $g(y) = \int_{2}^{y} t^2 \, \text{Sen}\, t \, dt$

$= (y^2 \ \text{Sen}\ y)$

$= (y^2 \ \text{Sen}\ y) \times 1 - \cancel{((2)^2 \ \text{Sen}\ y) \times 0}$

$g'(y) = y^2 \ \text{Sen}\ y$

R// $g'(y) = y^2 \ \text{Sen}\ y$

4. $g(r) = \int_0^r \sqrt{x^2+4}\,dx$

$= \sqrt{r^2+4}$

$= (\sqrt{r^2+4}) \times \underset{1}{(r)'} - \cancel{(\sqrt{r^2+4}) \times \underset{0}{(0)'}}$

$\underline{g'(r) = \sqrt{r^2+4}}$ //

R// $g'(r) = \sqrt{r^2+4}$

5. $F(x) = \int_x^{\pi} \sqrt{1 + \sec t}\, dt$

$$\int_x^{\pi} \sqrt{1+\sec t}\, dt = -\int_{\pi}^{x} \sqrt{1+\sec t}\, dt$$

$$= \sqrt{1+\sec x}$$

$$= \cancel{\left(\sqrt{1+\sec x}\right) * \underbrace{(\pi)'}_{*\,0}} - \left(\sqrt{1+\sec x}\right) \underbrace{(x)'}_{1}$$

$$= \underline{\sqrt{1+\sec x}}$$

$$F'(x) = \sqrt{1+\sec x}$$

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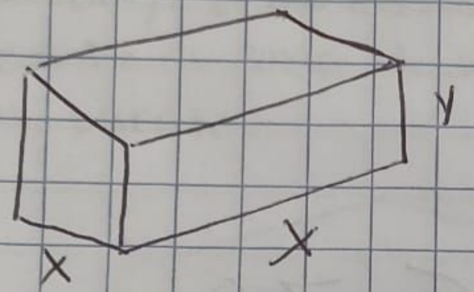
Tarea Semana #17 — Problemas de Optimización.

Semana #17

7) Una caja con una base cuadrada, abierta en la parte superior, debe tener un volumen de 32,000 cm^3. Encuentre las dimensiones de la caja que minimicen la cantidad de material que se ha de utilizarse.



$$y = \frac{32,000}{x^2}$$

Volumen = largo * ancho * alto $= x x y = x^2 y = 32,000$

Establecer la ecuación de optimización
Minimizar la cantidad de material del area.

$A = 4xy + x^2$

$$A(x) = 4x \frac{32,000}{x^2} + x^2$$

$$= \frac{128,000}{x} + x^2 \quad \text{Dominio; } x > 0$$

$$A'(x) = -\frac{128,000}{x^2} + 2x \rightarrow -\frac{128,000}{x^2} + 2x = 0$$

$$2x = \frac{128,000}{x^2} = x^3 = 64,000 \quad x = 40$$

Numero critico

$$A'(x) = -128{,}000\,x^{-2} + 2x$$

$$A''(x) = 256{,}000\,x^{-3} + 2$$

$$= \frac{256{,}000}{x^3} + 2$$

$$A''(40) = \frac{256{,}000}{40^3} + 2 = 6 > 0$$

Mínimo en $x = 40$.

$$y = \frac{32{,}000}{40^2} = 20$$

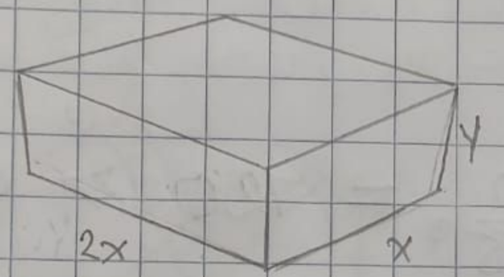
$$A = 4xy + x^2 =$$

$$= 4(40)(20) + (40)^2$$

$$= 4{,}800 \text{ cm}^2$$

R// $4{,}800 \text{ cm}^2$

② Un recipiente rectangular con tapa abierta, ha de tener un Volumen de $10\ m^3$. La longitud de su base es el doble de su ancho. El material para la base cuesta \$10 por metro cuadrado; el de los costados cuesta \$6 por metro cuadrado. Encuentre el costo de materiales para hacer el recipiente mas barato.



Volumen = largo × ancho × alto $= (2x)xy = 2x^2y = 10 \rightarrow y = \frac{10}{2x^2}$

Establecer la ecuación de optimización

Minimizar el costo : Costo = Costo de la base + Costo de los costados

$C = 10 \times$ Area de la base $+ 6 \times$ Area de las paredes.

$$C = 10(2x^2) + 6[2(2xy) + 2xy)]$$

$$C = 20x^2 + 36xy\frac{10}{2x^2}$$

$$C(x) = 20x^2 + \frac{180}{x} \quad \text{Dominio } x > 0$$

$$C'(x) = 40x + \frac{180}{x^2} \rightarrow 40x - \frac{180}{x^2} = 0$$

$$40x^3 = 180 \rightarrow \frac{180}{40} = x^3 = 4.5$$

$$x = \frac{4.5}{3} = 1.65$$

Numero critico

$$C'(x) = 40x - \frac{180}{x^2}$$

$$C'(x) = 40x + 180x^{-2}$$

$$C''(x) = 40 - 360x^{-3}$$

$$C''(x) = 40 + \frac{360}{x^3}$$

$$C''(1,65) = 40 + \frac{360}{1.65^3} = 120 > 0$$

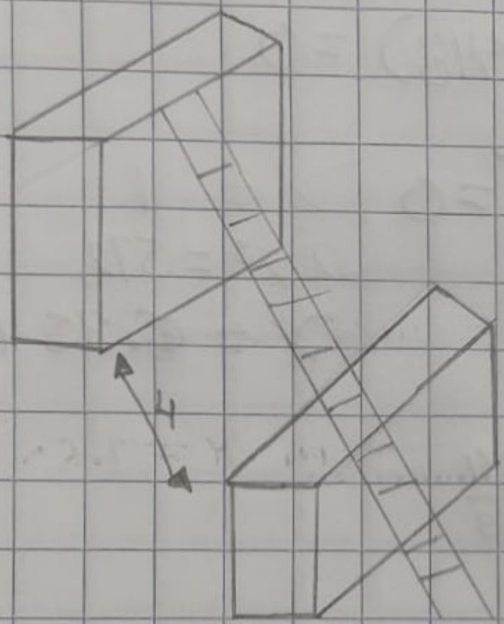
Minimo en $x = 1.65$

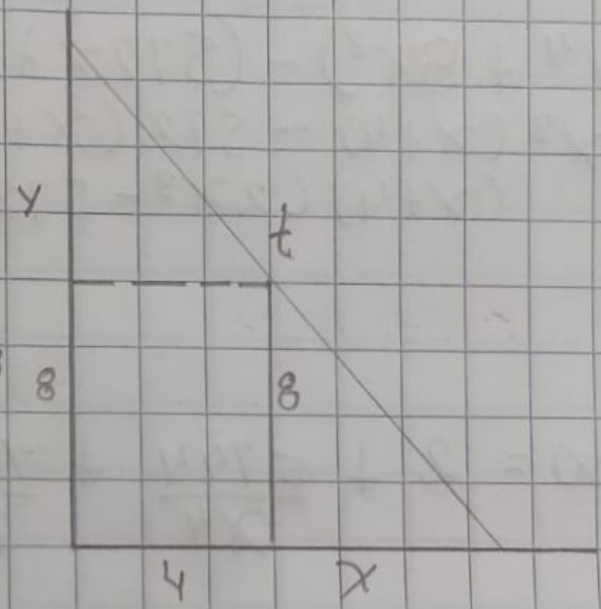
$$y = \frac{10}{2(1.65)^2} = \underline{1.84}$$

$$C = 20(1.65)^2 + 36(1.65)(1.83)$$
$$= 20(2.72) + 36(3.01)$$
$$54.4 + 108.36$$
$$\underline{\$162.76}$$

El costo de materiales es $ 162.76

3) Una valla de 8ft de alto corre paralela a un edificio alto a una distancia de 4ft del edificio ¿cual es la longitud de la escalera más corta que llegue del suelo y pase sobre la cerca hasta la pared del edificio?





Establecer la ecuación de optimización
Minimizar la longitud de la escalera: $t^2 = (x+4)^2 + (y+8)^2$

$$\frac{y}{4} = \frac{y+8}{4+x} = 4y + xy = 4y + 32 \rightarrow y = \frac{32}{x}$$

$$f(x) = (x+4)^2 + \left(\frac{32}{x} + 8\right)^2 \quad \text{Dominio } x > 0$$

$$f'(x) = 2(x+4) + 2\left(\frac{32}{x} + 8\right)\left(-\frac{32}{x^2}\right)$$

$$f'(x) = 2x + 8 - \frac{2048}{x^3} - \frac{512}{x^2}$$

$$F'(x) = \frac{2x^4 + 8x^3 - 572x - 2048}{x^3} = 0$$

$$2x^4 + 8x^3 - 572x - 2048 = 0$$

$$(2x^4 + 8x^3) - (572x + 2048) = 0$$

$$2x^3(x+4) - 572(x+4)$$

$$(x+4)(2x^3 - 572) = 0$$

$$2x^3 = 572$$

$x = 6.35$ numero critico

$$F''(x) = 2 + \frac{6744}{x^4} + \frac{7024}{x^3}$$

$$F''(6.35) = 2 + \frac{6744}{6.35^4} + \frac{7024}{6.35^3}$$

$F''(6.35) = 9.70 > 0$ minimo en $x = 6.35$

$$y = \frac{32}{x} = \frac{32}{6.35} = 5.04$$

$t^2 = (6.35+4)^2 + (5.04+8)^2 \rightarrow t^2 = 277.7647$

$t = \underline{16.65 \text{ pies}}$

La escalera de menor longitud, Colocada en el suelo, pasando sobre la barda, para que alcanze la pared del edificio es de:

$\underline{t = 16.65 \text{ Ft}}$