1. An oil company purchased an option on land in Alaska. Preliminary Geological studies assigned the following probabilities:
P(high-quality oil) = 0.5
P(medium-quality oil) = 0.2
P(no oil) = 0.3

a) What is the probability of finding oil?
b) After 200 feet of drilling on the first well, a soil test is made. The probabilities of finding the particular type of soil identified by the test are
P(Soil | high quality oil) = 0.2
P(Soil | medium-quality oil) = 0.8
P(Soil | no oil) = 0.2
How should the firm interpret the soil test? What are the revised probabilities, and what is the new probability of finding oil?

$P(H)=0.5 \quad P(M)=0.2 \quad P(NO)=0.3$

a) $P(H \cup M) = P(H) + P(M) - \cancel{P(H \cap M)}^{0} = 0.5 + 0.2 = \boxed{0.7}$

b) $P(S|H)=0.2$, $P(S|M)=0.8$, $P(S|NO)=0.2$

$P(H|S)=?$, $P(M|S)=?$, $P(NO|S)=?$

$$P(H|S) = \frac{P(S|H)\cdot P(H)}{P(S|H)\cdot P(H)+P(S|M)\cdot P(M)+P(S|NO)\cdot P(NO)}$$

$$= \frac{0.2 * 0.5}{0.2*0.5 + 0.8 * 0.2 + 0.2 * 0.3} = \frac{5}{16}$$

$$P(M|S) = \frac{0.8 * 0.2}{0.2*0.5+0.8*0.2+0.2*0.3} = \frac{8}{16}$$

$$P(NO|S) = 1 - \left(\frac{5+8}{16}\right) = \frac{3}{16}$$

$$\boxed{P(oil|S) = \frac{13}{16}}$$

2. M.D. Computing (vol, no. 5, 1991) describes the use of Bayes' theorem and conditional probability in medical diagnosis. Prior probabilities of diseases are based on the physician's assessment of factors such as geographic location, seasonal influence, and occurrence of epidemics. Assume that a patient is believed to have one of two diseases, denoted D1 and D2, with P(D1)_ = 60% and P(D2) = 40% and that medical research shows a probability associated with each symptom that may accompany the diseases. Suppose that given diseases D1 and D2, the probabilities a patient will have suptoms S1, S2, or S3 are as follows:

	S1	S2	S3
D1	15%	10%	15%
D2	80%	15%	3% → (P(S3 | D1)

After finding that a certain symptom is present, the medical diagnosis may be aided by finding the revised probabilities the patient has each particular disease. Compute the probability of each disease for the following medical findings.

a) The patient has symptom S1.
b) The patient has symptom S2.
c) The patient has symptom S3.
d) For the patient with symptom S1 in part (a), suppose that symptom S2, also is present. What are the revised probabilities for D1 and D2? (Bonus Question)

a) $P(D_1|S_1) = \dfrac{P(S_1|D_1)\cdot P(D_1)}{P(S_1|D_1)\cdot P(D_1) + P(S_1|D_2)\cdot P(D_2)} = \dfrac{0.15*0.6}{0.15*0.6 + 0.8*0.4} = 21.95$

$P(D_2|S_1) = 1 - 21.95\% = 78.05\%$

b) $P(D_1|S_2) = \dfrac{P(S_2|D_1)\cdot P(D_1)}{P(S_2|D_1)\cdot P(D_1) + P(S_2|D_2)\cdot P(D_2)} = \dfrac{0.1*0.6}{0.1*0.6 + 0.15*0.4} = 0.5$

$P(D_2|S_2) = 1 - P(D_1|S_2) = 1 - 0.5 = 0.5$

c) $P(D_1|S_3) = \dfrac{P(S_3|D_1)\cdot P(D_1)}{P(S_3|D_1)\cdot P(D_1) + P(S_3|D_2)\cdot P(D_2)} = \dfrac{0.15*0.6}{0.15*0.6 + 0.03*0.4} = 88.24$

$P(D_2|S_3) = 1 - P(D_1|S_3) = 1 - 0.8824 = 11.76\%$

$\boxed{P(D_1^{Revised}) = P(D_1|S_1}$

d) $P(D_1^R|S_2) = \dfrac{P(S_2|D_1)\cdot P(D_1^{Revised})}{P(S_2|D_1)\cdot P(D_1^{Revised}) + P(S_2|D_2)\cdot P(D_2^{Revised})}$

$= \dfrac{0.1*0.2195}{0.1*0.2195 + 0.15*.7805} = 0.157 \rightarrow P(D_2^R|S_2) = 0.843$

$\boxed{R \equiv \text{Revise}}$

Alternatively one can define $P(D_1^{Revised})$ as $P(D_1|S_2)$

$P(D_1^R|S_1) = \dfrac{P(S_1|D_1)\cdot P(D_1^{Revised})}{P(S_1|D_1)\cdot P(D_1^{Revised}) + P(S_1|D_2)\cdot P(D_2^R)} = 0.157$

3. The ABC Manufacturing company purchases a certain part from Supplier A, B, and C. Supplier A supplies 60% of the parts, B 30%, and C 10%. The quality of parts varies among the suppliers, with A, B, and C parts having 0.25%, 1%, and 2% defective rates, respectively. The parts are used in one of the company's major products.

a) What is the probability that the company's major product is assembled with a defective part?

b) When a defective part is found, which supplier is the likely source?

$P(A) = 0.6$, $P(B) = 0.3$, $P(C) = 0.1$

$P(D|A) = 0.0025$, $P(D|B) = 0.01$, $P(D|C) = 0.02$

Part a) $P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D) = P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)$

$= 0.6 * 0.0025 + 0.3 * 0.01 + 0.1 * 0.02 = \boxed{0.65\%}$

Part b) $P(A|D) = \dfrac{P(D|A).P(A)}{P(D)} = \dfrac{0.6 * 0.0025}{0.0065} = \boxed{23\%}$

$P(B|D) = \dfrac{D(D|B).D(B)}{P(D)} = \dfrac{0.3 * 0.01}{0.0065} = \boxed{46\%}$ More likely Source

$P(C|D) = 1 - (23\% + 46\%) = \boxed{31\%}$

4. Companies that do business over the Internet can often obtain probability information about web sites visited. For instance, the Q.M. in action, Internet marketing, described how clickstream data on Websites visited could be used in conjunction with a Baysian updating scheme to determine the probability that a Website visitor was female. Par Fore created a Website to market golf equipment and apparel. Management would like a certain offer to appear for female visitors and a different offer to appear for male visitors. A sample of pas Website visits indicates that 60% of the visitors to ParFare.com are male and 40% are female.

a) What is your prior probability that the next visitor to the Website will be female?

b) Suppose you know that the current visitor previously visited the Dillards Website and that women are three times as likely to visit this Website as men. What is your revised probability that the visitor is female? Should you display the offer that has more appeal to female visitors, or the one that has more appeal to male visitors

a) $P(M \mid Parfare) = 0.6$, $P(F \mid Parfare) = 0.4$

or we can write $P(M) = 0.6$, $P(F) = 0.4$

b)

$$P(F \mid Dillards) = \frac{P(Dillards \mid F) \cdot P(F)}{P(Dillards \mid F) \cdot P(F) + P(Dillards \mid M) \cdot P(M}$$

$$= \frac{3x * 0.4}{3x * 0.4 + x * 0.6} = \frac{12}{18} = 2/3 = \boxed{66.67\%}$$

$P(M \mid Dillards) = \boxed{33.33\%}$

So weshould show ads assuming that the visitor is more likely a woman.