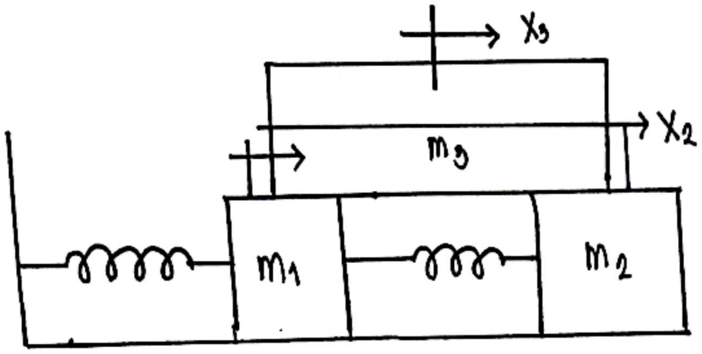



for $m_1$:

$$0 = m_1 x_1 + B_1 x_1 + B_3\dot{x}_1 + k_1 x_1 + k_2 x_1 - B_3\dot{x}_3 - k_2 x_2$$

$$f(t) = m_2\ddot{x}_2 + B_2\dot{x}_2 + B_4\dot{x}_2 + k_2 x_2 - B_4\dot{x}_3 - k_2 x_1$$

$$0 = m_3\ddot{x}_3 + B_3\dot{x}_3 + B_4\dot{x}_3 - B_1\dot{x}_1 - B_2\dot{x}_2$$

| | | |
|---|---|---|
| $x_1 = x_1(t)$ | $x_3 = x_2(t)$ | $x_5 = x_3(t)$ |
| $x_2 = \dot{x}_1(t)$ | $x_4 = \dot{x}_3(t)$ | $x_6 = \dot{x}_3(t)$ |
| $\dot{x}_1 = \dot{x}_1(t) = x_2$ | $\dot{x}_3 = \dot{x}_2(t) = x_4$ | $\dot{x}_5 = \dot{x}_3(t) = x_6$ |
| $\dot{x}_2 = \ddot{x}_1(t)$ | $\dot{x}_1 = \ddot{x}_2(t)$ | $\dot{x}_6 = \ddot{x}_1(t)$ |
| | $U_1 = f(t)$ | |

$$0 = m_1\dot{x}_2 + B_1 x_2 + B_3 x_2 + k_1 x_1 + k_2 x_1 - B_3 x_6 - k_2 x_3$$

$$U_1 = m_2\dot{x}_4 + B_2 x_4 + B_4 x_4 + k_2 x_3 - B_4 x_6 - k_2 x_1$$

$$0 = m_3\dot{x}_6 + B_3 x_6 + B_4 x_6 - B_1 x_2 - B_2 x_4$$

$$x_1 = 0x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 + 0U_1$$

$$\dot{x}_2 = -\left(\frac{k_1+k_2}{m_1}\right)x_1 - \left(\frac{B_4-B_3}{m_1}\right)x_2 + \left(\frac{k_2}{m_1}\right)x_3 + 0x_4 + 0x_5 + \left(\frac{B_3}{m_1}\right)x_6 + 0U_1$$

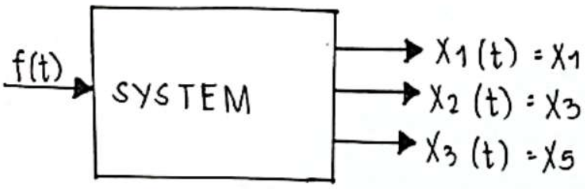
$$\dot{x}_3 = 0x_1 + 0x_2 + 0x_3 + x_4 + 0x_5 + 0x_6 + 0U_1$$

$$\dot{x}_4 = \left(\frac{k_2}{m_2}\right)x_1 + 0x_2 - \left(\frac{k_2}{m_2}\right)x_3 - \left(\frac{B_2+B_4}{m_2}\right)x_4 - 0x_5 + \left(\frac{B_4}{m_2}\right)x_6 + \frac{U_1}{m_2}$$

$$\dot{x}_5 = 0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + x_6 + 0U_1$$

$$\dot{x}_6 = 0x_1 + \left(\frac{B_1}{m_3}\right)x_2 - 0x_3 \left(\frac{B_2}{m_3}\right)x_4 + 0x_5 - \left(\frac{B_3+B_4}{m_3}\right)x_6 + 0U_1$$

$$
\underset{6\times 1}{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix}}
=
\underset{\substack{6\times 6 \\ A}}{
\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-\left(\frac{K_1+K_2}{m_1}\right) & -\left(\frac{B_1+B_3}{m_1}\right) & \left(K_2/m_1\right) & 0 & 0 & \left(B_3/m_1\right) \\
0 & 0 & 0 & 1 & 0 & 0 \\
\left(K_2/m_2\right) & 0 & -\left(K_2/m_2\right) & -\left(\frac{B_2+B_4}{m_2}\right) & 0 & \left(B_4/m_2\right) \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & \left(B_1/m_3\right) & 0 & \left(B_2/m_3\right) & 0 & -\left(\frac{B_3+B_4}{m_3}\right)
\end{bmatrix}
\end{array}}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}
+
\underset{\substack{6\times 1 \\ B}}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \\ 0 \\ 0 \end{bmatrix}} u_1
$$



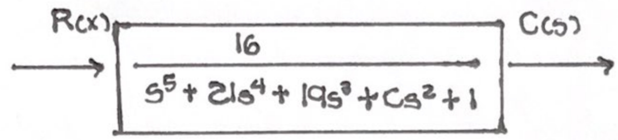

$y_1 = x_1 + 0u_1$

$y_2 = x_3 + 0u_1$

$y_3 = x_5 + 0u_1$

$$
y = \underset{\substack{3\times 6 \\ C}}{
\begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}
+
\underset{\substack{3\times 1 \\ D}}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} u_1
$$

2.)




$$\frac{C(s)}{R(s)} = \frac{16}{s^5 + 21s^4 + 19s^3 + 6s^2 + 1}$$

$$C(s)\left[ s^5 + 21s^4 + 19s^3 + 6s^2 + 1 \right] = 16R(s)$$

$$\mathcal{L}^{-1}\left\{ s^5C(s) + 21s^4C(s) + 19s^3C(s) + 6s^2C(s) + C(s) = 16R(s) \right\}$$

$$\underset{\dot{x}_5}{\overset{\cdot\cdot\cdot\cdot\cdot}{C}} + 21\underset{x_5}{\overset{\cdot\cdot\cdot\cdot}{C}} + 19\underset{x_4}{\dddot{C}} + 6\underset{x_3}{\ddot{C}} + \underset{x_1}{C} = 16\underset{u_1}{r}$$

| | | |
|---|---|---|
| $x_1 = C$ | $\dot{x}_1 = \dot{C} = x_2$ | $u_1 = r$ |
| $x_2 = \dot{C}$ | $\dot{x}_2 = \ddot{C} = x_3$ | |
| $x_3 = \ddot{C}$ | $\dot{x}_3 = \dddot{C} = x_4$ | |
| $x_4 = \dddot{C}$ | $\dot{x}_4 = \overset{\cdot\cdot\cdot\cdot}{C} = x_5$ | |
| $x_5 = \overset{\cdot\cdot\cdot\cdot}{C}$ | $\dot{x}_5 = \overset{\cdot\cdot\cdot\cdot\cdot}{C}$ | |

$$\dot{x}_1 = 0x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 + 0u_1$$
$$\dot{x}_2 = 0x_1 + 0x_2 + x_3 + 0x_4 + 0x_5 + 0u_1$$
$$\dot{x}_3 = 0x_1 + 0x_2 + 0x_3 + x_4 + 0x_5 + 0u_1$$
$$\dot{x}_4 = 0x_1 + 0x_2 + 0x_3 + 0x_4 + x_5 + 0u_1$$
$$\dot{x}_5 = -x_1 + 0x_2 - 6x_3 - 19x_4 - 21x_5 + 16u_1$$

STATE SPACE EQUATION

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & -6 & -19 & -21 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{array}{c} u_1 \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 16 \end{bmatrix} \end{array} u_1$$

$$y = C = x_1$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + [0]\,u_1$$

Problem 3

$$R(s) \longrightarrow \boxed{\frac{s^2+3s+3}{s^3+2s^2+11s+6}} \xrightarrow{C(s)}$$

$$R(s) \longrightarrow \boxed{\frac{1}{s^3+2s^2+11s+6}} \xrightarrow{X_1(s)} \boxed{s^2+3s+3} \xrightarrow{C(s)}$$

$$\frac{X_1(s)}{R(s)} = \frac{1}{s^3+2s^2+11s+6}$$

$$X_1(s)\left[s^3+2s^2+11s+6\right] = R(s)$$

$$s^3X_1(s) + 2s^2X_1(s) + 11sX_1(s) + 6X_1(s) = R(s)$$

$$\mathcal{L}^{-1}\left\{s^3X_1(s) + 2s^2X_1(s) + 11sX_1(s) + 6X_1(s) = R(s)\right\}$$

$$\underset{\dot{x}_3}{\dddot{x}_1} + 2\underset{x_3}{\ddot{x}_1} + 11\underset{x_2}{\dot{x}_1} + 6\underset{x_1}{x_1} = \underset{v_1}{r}$$

$x_1 = x_1$ $\quad \dot{x}_1 = \dot{x}_1 = x_2$ $\quad v_1 = r$

$x_2 = \dot{x}_1$ $\quad \dot{x}_2 = \ddot{x}_1 = x_3$

$x_3 = \ddot{x}_1$ $\quad \dot{x}_3 = \dddot{x}_1$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -6x_1 - 11x_2 - 2x_3 + v_1$$

STATE EQUATION:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}v_1$$

OUTPUT EQUATION:

$$X_1(s) \longrightarrow \boxed{s^2+3s+3} \xrightarrow{C(s)}$$

$$\frac{C(s)}{X_1(s)} = s^2+3s+3$$

$$C(s) = s^2X_1(s) + 3sX_1(s) + 3X_1(s)$$

CONTINUATION:

$$\mathcal{L}^{-1}\left\{s^2X_1(s) + 3sX_1(s) + 3X_1(s)\right\}$$

$$c = \ddot{x}_1 + 3\dot{x}_1 + 3x_1$$
$$y = \ddot{x}_1 + 3\dot{x}_1 + 3x_1$$
$$y = x_3 + 3x_2 + 3x_1$$

$$y = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix}3 & 3 & 1\end{bmatrix} \end{array} \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}v_1$$