

# Parallel Implementation of the A\* Pathfinding Algorithm on a Purely-Functional Programming Language

Prince Bernie B. Colis

Bachelor of Science in Computer Science

John Kenneth S. Lesaba

Bachelor of Science in Computer Science

Jon Ariel N. Maravilla

Bachelor of Science in Computer Science

Karl Frederick R. Roldan

Bachelor of Science in Computer Science

Senior project submitted to the faculty of the

Department of Computer Science

College of Computer Studies, Ateneo de Naga University

in partial fulfillment of the requirements for their respective

Bachelor of Science degrees

---

Project Advisor: Adrian Leo Pajarillo

First Panel Member

Second Panel Member

Third Panel Member

Month Day, 2020

Naga City, Philippines

Keywords: parallel programming, functional programming, graph theory

Copyright 2020, Prince Bernie B. Colis, John Kenneth S. Lesaba, Jon Ariel N. Maravilla, and Karl  
Frederick R. Roldan

The Senior Project entitled

**Parallel Implementation of the A\* Pathfinding Algorithm on a  
Purely-Functional Programming Language**

developed by

**Prince Bernie B. Colis**

Bachelor of Science in Computer Science

**John Kenneth S. Lesaba**

Bachelor of Science in Computer Science

**Jon Ariel N. Maravilla**

Bachelor of Science in Computer Science

**Karl Frederick R. Roldan**

Bachelor of Science in Computer Science

and submitted in partial fulfillment of the requirements of their respective Bachelor of Science degrees  
has been rigorously examined and recommended for approval and acceptance.

**First Panel Member**

Panel Member

Date signed: \_\_\_\_\_

**Second Panel Member**

Panel Member

Date signed: \_\_\_\_\_

**Third Panel Member**

Panel Member

Date signed: \_\_\_\_\_

**Adrian Leo Pajarillo**

Project Advisor

Date signed: \_\_\_\_\_

The Senior Project entitled

**Parallel Implementation of the A\* Pathfinding Algorithm on a  
Purely-Functional Programming Language**

developed by

**Prince Bernie B. Colis**

Bachelor of Science in Computer Science

**John Kenneth S. Lesaba**

Bachelor of Science in Computer Science

**Jon Ariel N. Maravilla**

Bachelor of Science in Computer Science

**Karl Frederick R. Roldan**

Bachelor of Science in Computer Science

and submitted in partial fulfillment of the requirements of their respective Bachelor of Science degrees  
is hereby approved and accepted by the Department of Computer Science, College of Computer  
Studies, Ateneo de Naga University.

**Marianne P. Ang, MS**

Chair, Department of Computer Science

Date signed: \_\_\_\_\_

**Joshua C. Martinez, MIT**

Dean, College of Computer Studies

Date signed: \_\_\_\_\_

# Declaration of Original Work

We declare that the Senior Project entitled

## **Parallel Implementation of the A\* Pathfinding Algorithm on a Purely-Functional Programming Language**

which we submitted to the faculty of the

### **Department of Computer Science, Ateneo de Naga University**

is our own work. To the best of our knowledge, it does not contain materials published or written by another person, except where due citation and acknowledgement is made in our senior project documentation. The contributions of other people whom we worked with to complete this senior project are explicitly cited and acknowledged in our senior project documentation.

We also declare that the intellectual content of this senior project is the product of our own work. We conceptualized, designed, encoded, and debugged the source code of the core programs in our senior project. The source code of third party APIs and library functions used in my program are explicitly cited and acknowledged in our senior project documentation. Also duly acknowledged are the assistance of others in minor details of editing and reproduction of the documentation.

In our honor, we declare that we did not pass off as our own the work done by another person. We are the only persons who encoded the source code of our software. We understand that we may get a failing mark if the source code of our program is in fact the work of another person.

**Prince Bernie B. Colis**

3 - Bachelor of Science in Computer Science

**John Kenneth S. Lesaba**

3 - Bachelor of Science in Computer Science

**Jon Ariel N. Maravilla**

3 - Bachelor of Science in Computer Science

**Karl Frederick R. Roldan**

3 - Bachelor of Science in Computer Science

This declaration is witnessed by:

**Adrian Leo Pajarillo**

Project Advisor

# **Parallel Implementation of the A\* Pathfinding Algorithm on a Purely-Functional Programming Language**

by

Prince Bernie B. Colis, John Kenneth S. Lesaba, Jon Ariel N. Maravilla, and Karl Frederick R.

Roldan

Project Advisor: Adrian Leo Pajarillo

Department of Computer Science

## **EXECUTIVE SUMMARY**

To be filled in later. /\*TODO\*/.

I dedicate this research work to all of humanity.

# ACKNOWLEDGEMENTS

I thank everyone who helped me finish this thesis.



# TABLE OF CONTENTS

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Project Context . . . . .	2
1.2	Purpose and Description . . . . .	2
1.3	Objectives . . . . .	3
1.4	Scope and Limitations . . . . .	3
<b>2</b>	<b>Review of Related Systems and Related Literature</b>	<b>4</b>
2.1	The A* Algorithm . . . . .	4
2.2	Parallel A* Algorithm . . . . .	5
2.2.1	Bidirectional A* Search Algorithms . . . . .	5
2.2.2	Parallelism by Hashing . . . . .	7
2.3	Real-time Performance of Functional Programs . . . . .	8
<b>3</b>	<b>Technical Background</b>	<b>9</b>
<b>A</b>	<b>Code Listing</b>	<b>11</b>

# LIST OF FIGURES

2.1	A directional multigraph . . . . .	6
3.1	SCS Decision Problem . . . . .	10

# LIST OF TABLES

# Chapter 1

## Introduction

The A\* pathfinding algorithm is a best-first pathfinding algorithm for graphs commonly used for graph traversal applications such as artificial intelligence, video games, flight paths, and more. However, while most games are written in an imperative and object-oriented language such as C#, C++, and JavaScript, it is possible to write video games in a functional language using a reactive functional programming approach.[6] Likewise, the need for other correct critical software led organizations such as NASA to use Haskell[1], a purely-functional programming language, to be used in systems where high-level assurance and provable programs are a must.[28]

This paper assumes concrete differences between *parallel* and *concurrent* where the former is defined to be a hardware feature of having multiple processors or cores to compute a problem whereas the latter is defined to be a software-based approach to decrease the impact of computation bottlenecks by switching between different computations when a computation takes too long.[35] One of the major challenges of parallel programming is controlling the order of execution to prevent *race conditions*, which can often lead to bugs and are hard to maintain. However, since pure functional languages, such as Haskell, have no mutability and computations lead to the same result regardless of the order, they are a perfect candidate for writing parallel programs.[15] This research aims to find a parallel implementation of the existing A\* pathfinding algorithms using a purely-functional setting with attention to program performance. In turn, this helps in the advancement of different functional programming approaches for parallel graph computations which could eventually lead to critical systems to use a more provable programming language.

## 1.1 Project Context

The current video game industry is dominated by imperative programmers, even though functional programming had been getting more popular recently especially in web-programming due to languages such as TypeScript and PureScript and frameworks such as ReactJS which favors a reactive functional approach. Thus, most beginners are exposed to imperative algorithms, such as the A\*, due to the popularity of algorithm books with imperative programming in them.[7, 23, 36] However, algorithms designed for imperative programming may not translate as well when implemented in a functional style. The quicksort specification maintains a worst-case space complexity of  $O(n)$ . However, when the algorithm is written in a pure-function style, it will have to copy every subarray due to its recursive nature, thus yielding a worst-case space complexity of  $O(n^2)$ .

The advantages of functional programming lies with its *referential transparency* which means that a function definition or a variable will never change its definition throughout the runtime of the program.[21, 15] Hence, mathematically proving functional programs might be easier and can be aided by proof assistants such as Coq or Agda.[4, 37, 9] Likewise, splitting functions into smaller functions and reasoning about those smaller components much like lemmas would mean that functions would be modular and composed of proven subfunctions.[3, 19] Hence, functional programming languages are excellent candidates for parallel programming since the languages do not have mutable states and are therefore, instances of shared variables are abstracted away from the programmer. Similarly, the order of execution of pure functions does not matter as the program will still yield the same results.[21] A parallel and purely-functional approach to the A\* algorithm would eventually lead to more applications such as shortest distance in a map, flight paths, web server searching, and more to use a more provable and type-safe language which could lead to less system failures and high availability.

## 1.2 Purpose and Description

This research aims to utilize the existing parallel A\* pathfinding algorithm [11, 38] and find a way to develop a reasonably-efficient purely-functional implementation of the algorithm using parallel data structures such as STMs or MVars[26].

The A\* Pathfinding algorithm is used heavily in video games, telephone traffic, and other graph traversal problems[18]. This research aims to aid in the development of video games and other uses

where a parallel A\* algorithm would be beneficial using the functional programming paradigm in the future as video game development is dominated by imperative languages.

### 1.3 Objectives

The main objective of the research is to find an efficient parallel purely-functional implementation of the A\* pathfinding algorithm. The research will be done mostly in Haskell with some exceptions. Likewise, concrete comparisons between the number of cores and logical threads will be used to measure the most efficient performance runtime and space complexity of the algorithm.

The researchers aim to complete the following specific tasks:

- Write a *generator* that will generate an arbitrary-sized maze. The maze should be relatively hard to solve without the aid of computers in a short amount of time.[5]
- And a *solver* program that will be written in Haskell, a lazy purely-functional programming language, for translating the output of the generator to a graph.[1]
- The solver program should have a web-based user interface where the researchers can view the maze and how the solver program was able to solve it correctly.
- Performance of the solution shall be measured by using ThreadScope to monitor the thread and core activities while the program is being run.[2]

### 1.4 Scope and Limitations

The research will only cover solvable-mazes as the A\* algorithm does not halt when there is no reachable end goal (e.g, the start vertex and end vertex lie on different components of the graph).[18] Likewise, there will be no generality and all programs will be written in Haskell. Translation to other functional programming languages is not a priority and, thus, lambda notation will not be used. Other concurrent data structures besides MVar and Software Transactional Memory will not be utilized.

The concrete implementation and analysis is planned to be tested only on four CPUs such as Intel Core i7-9750H and AMD Ryzen 5 3500x. Other CPU architectures are not planned to be tested on.

## Chapter 2

# Review of Related Systems and Related Literature

Due to the trend of parallelization in the modern computing era, [32, 38, 11, ?] several researchers developed more efficient and faster implementations of the A\* algorithm by parallizing the algorithm in various ways.

### 2.1 The A\* Algorithm

The A\* pathfinding algorithm, first described by Hart, Nilsson, and Raphael, is a *best-first search* algorithm for finding a path between two vertices in a directed weighted graph.[18] The A\* search algorithm can be seen as a variant of Dijkstra's algorithm with an added heuristic function to guide its search for an optimal solution. That is, the A\* search functions the same as Dijkstra's algorithm if the heuristic function  $h(n) = 0$  for any  $n$ . [13]

Hart, Nilsson, and Raphael had proven that the A\* algorithm is *admissible* whenever the heuristic function  $h$  is admissible. That is, A\* is guaranteed to return an optimal solution. The optimality of the path generated by the A\* algorithm depends on its heuristic function which will be discussed further in Chapter 3 of this paper.

## 2.2 Parallel A\* Algorithm

Zaghloul, Al-Jami, Bakalla, et al. had shown that an implementation of a parallel A\* algorithm had a speed up of 3-4 times on problem sizes of 2000x2000 to 4000x4000.[11] The implementation took advantage of reducing the problem into subgraphs where the neighbors of a vertex in a graph will have threads assigned to them and a neighbor  $n_i$  will be the new start vertex of the A\* search for that given thread with the same goal vertex. By Ahmdal's Law, runtime efficiency will decrease when the number of threads are increased due to the overhead of garbage collection and thread creation.

### 2.2.1 Bidirectional A\* Search Algorithms

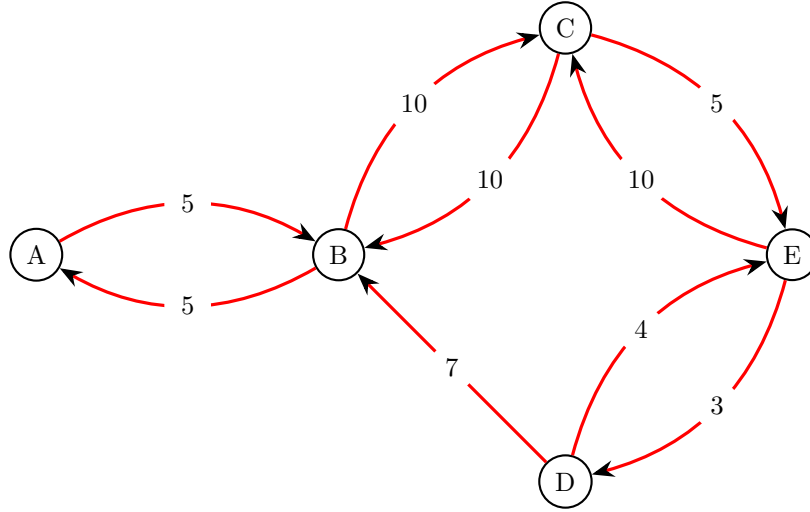
Bidirectional search is said to be possible when there is exactly one start vertex and exactly one goal vertex. Since the computer cannot differentiate between the two if they are reversible, it is possible to start searching from the goal  $g$  to the start  $s$  and vice versa.[20] Bidirectional search is useful when implemented on a unidirectional multigraph(2.1) where there can be an efficient path between from one vertex to another in one direction but the same path cannot exist in the other direction.

The BHPA algorithm is a bidirectional search employing two A\* algorithms running sequentially with a sequential computer switching directions multiple times. The BHPA algorithm terminates whenever two search paths from both directions meet and returns the minimum  $f$  cost of either direction or by adding the  $g$  costs of the two directions. However, Kaïndl and Kaïnz had proven that the best case performance of BHPA\* is not significantly better than the unidirectional A\* algorithm due to the cost of the BHPA's termination condition.[20] Kaïndl and Kaïnz developed a more efficient implementation of a non-tradition bidirectional heuristic search that is effective on machines with limited memory. The algorithm instantiates by assigning memory to a unidirectional approach and uses A\* to find a path from the start to the end vertices. If there is such a path given the memory, then the algorithm terminates. Otherwise, a reverse search from the end vertex to an intermediate vertex from the first search.

The New Bidirectional A\* Search (NBA\*) uses the same technique as the BHPA in the sense that the execution of each search direction alternates.[29] However, NBA\* employs the reverse search on a reverse graph. That is, let  $V(G)$  and  $E(G)$  be the vertex set and edge set of graph  $G$  and if  $u, v \in V(G)$  and  $uv \in E(G)$  but  $vu \notin E(G)$ , then for the reverse graph  $\text{rev}(G)$  with  $u, v \in V(\text{rev}(G))$



Figure 2.1: A directional multigraph



and  $vu \in E(\text{rev}(G))$  but  $uv \notin E(\text{rev}(G))$ . The edges  $uv$  in  $E(G)$  and  $vu$  in  $E(\text{rev}(G))$  have the same weights. Likewise, if  $s_1$  and  $t_1$  are the start and goal vertices of graph  $G$ , then  $s_2$  and  $t_2$  are the start and goal vertices of graph  $\text{rev}(G)$  with  $s_2 = t_1$  and  $t_2 = s_1$ . As a corollary, the vertex set of  $\text{rev}(G)$  is equal to the vertex set of  $G$  denoted by  $V$  from this point.

In  $\text{NBA}^*$ , all vertices  $v \in V$  are initially labeled  $g(v) = \infty$  and the set  $\mathcal{S} = \emptyset$  is the set of vertices with permanent labels. Likewise, there are shared variables  $\mathcal{L} = \infty$  and  $\mathcal{R} = \emptyset$  which will hold the the best cost so far and the rejected vertices, respectively. During the runtime of the algorithm, a vertex is either expanded if its heuristics or label satisfies the necessary conditions with respect to  $\mathcal{L}$  or rejected and thus added in the set  $\mathcal{R}$ . The algorithm halts when there is one search side with no more candidates to be expanded.

### Parallel New Bidirectional A\* (PNBA\*)

PNBA\* was proposed by Rios and Chaimowicz by parallelizing the NBA\* algorithm.[32] PNBA\* leverages on a shared memory model and runs both search processes in parallel. Due to this, the PNBA\* algorithm is faster than the NBA\* and thus has a better runtime performance. The implementation of PNBA\* has a variable  $\mathcal{S}$ , like the NBA\*, shared by both processes and contains the vertices, say  $v$ , with finite labels  $g(v)$ . However, since both  $\mathcal{L}$  and  $\mathcal{S}$  are updated by both processes in parallel, it is possible that there would be a race condition and some vertex in one

thread may be expanded due to the race conditions. However, this does not affect the admissibility of PNBA\*. While this may incur some overhead due to expanding additional vertices, parallelism makes up for the additional overhead costs.[32]

### 2.2.2 Parallelism by Hashing

The Hash Distributed A\* Algorithm (HDA\*) builds on the the hash-based work distribution of PRA\* and asynchronous communication by TDA.[22, 12, 33] The HDA\* search introduces *ownership* where each processor has their own local closed and open lists and the search space is divided among processors. Unlike the implementation by Zaghloul, when a vertex is expanded by a processor  $P_1$  in HDA\*,  $P_1$  will not immediately own the generated vertices but instead send those vertices to the message queue to be hashed by available processors which can then be processed. A pseudo-random hash function is important to make sure work is distributed equally among available processors. If the hash function is static, that is it only assigns every generated vertex to the same processor that generated said vertices, then the algorithms performs the same way as the classic A\* algorithm. Due to the parallel nature of HDA\* and having local open and closed sets per processor, there is no guarantee that an expanded vertex will no longer be expanded.

The simplest approach for the termination condition of HDA\* is the *barrier method* wherein a processor waits for other processors when its open set is empty. This incurs some overhead and is not viable. Weinstock and Holladay describes the *sum flags method* where a binary flag is raised by a processor if its open set is empty.[38] Let  $p_i$  be processors for all  $i \in 1..n$  where  $n$  is the number of processors in the computer. The algorithm will terminate if  $p_1 \wedge p_2 \wedge \dots \wedge p_n = True$ .

The original paper by Kishimoto, Fukunaga, and Botea showed that HDA\* performs significantly better than A\*, WSA\*<sup>1</sup>, and PRA\* when using message passing on a number of problems. When the HDA\* is run using 4 CPU cores, the speedup averages 3 times and is shown to be very efficient compared to the other parallel implementations and the sequential A\* algorithm. Likewise, the speedup is even more significant at an average of 5 times when using 8 cores. Kishimoto, Fukunaga, and Botea expounded further that the HDA\* is also efficient in a distributed computing environment by showing a speedup of an average of 30-60 times in a 128-core distributed computer.[22]

---

<sup>1</sup>Another parallel implementation of the A\* algorithm that involves work stealing

## 2.3 Real-time Performance of Functional Programs

The dominant programming languages used for time-critical software systems such as GPS systems and flight guidance softwares are written in mid-level programming languages that has direct hardware access such as C or C++. Frame and Coffey were concerned that the correctness of C and C++ were refutable and thus benchmarked and compared the real-time performance of the same program written in C++ and in Haskell, the latter being a programming language known to its correctness due to its strict typing rules.[14] Frame and Coffey hypothesized that imperative and functional programming may differ in their implementations but both are powerful enough to take on math-intensive problems.

Scott and Frame also tacked the ease of use of the programming language to the programmers and they concluded that the functional programming language may seem unfamiliar to an imperative language user at first but may be easier to understand and maintain in the long run.[14] This is especially true in static typing languages such as Haskell where programmers will be able to make less typing errors such as adding a `uint_8` to a `bool` which may cause an overflow.[16] The paper by Scott and Frame showed that functional programs are less verbose and are more expressive than their imperative counterparts for the same problem, however it still lacks in performance power. However, despite the slower runtime performance of functional programs, mathematically provable functional programs are still capable in being written as runtime verification systems in ultra-critical systems, as demonstrated by the National Aeronautics and Space Administration (NASA).[28] Due to this, it is important to develop more performant, perhaps parallel, algorithms for functional programs especially in the area of combinatorial optimization, such as the problem posed in this paper.

Harris, Marlow, and Jones evaluated the parallel performance of Haskell and overhead costs when using parallel data structures. Since Haskell is a garbage collected programming language, there would be some performance cost whenever data is being garbage collected. However, due to the lazy-nature of Haskell, unnecessary data that will never be used in any computation will not be evaluated. The parallel implementation of the Glasgow Haskell Compiler (GHC) does not really state explicit parallelism. Instead, a programmer may annotate (called *sparkling*) whether a computation can be parallelized or not with different strategies. It is up to the Haskell runtime system whether the annotation will be assigned to a different processor or thread. This abstraction enables the programmer to be worry-free about how parallelism will be implemented and diverts it to other matters such as performance and how many sparks will be created.[17, 27]

## Chapter 3

# Technical Background

A superstring is simply a string over some alphabet for which given a set of string from the same alphabet, the latter's members are all substrings of the former. To understand the SCS problem, it is best to assume some important concepts related to the problem itself. First, let us assume some set  $R = \{x_1, \dots, x_n\}$  as a set of strings (or blocks) over some alphabet  $\Sigma$ . Formally, we define a *superstring* of  $R$  as a string  $w$  containing each  $x_i \in R$ , as a substring.

Following are the elementary operations associated with the construction of a superstring. The *overlap*( $u, v$ ) of two strings  $u$  and  $v$  is the maximum overlap between  $u$  and  $v$ . That is to say, the longest suffix of  $u$  (in terms of characters) that is a prefix of  $v$ . The *prefix*( $u, v$ ) of  $u$  and  $v$  is the prefix of  $u$  obtained by removing its overlap with  $v$ . Lastly, *merge*( $u, v$ ) is the concatenation of  $u$  and  $v$  with the overlap appearing only once.

As an example, say we have,  $\Sigma = \{a, b, c\}$  and  $R = \{cbcaca, cacac\}$ . The string *cacacbcaca* is a superstring of  $R$  while the string *cbcacac* is the shortest common superstring. The following relations also hold:

$$\text{overlap}(cbcaca, cacac) = caca,$$

$$\text{overlap}(cacac, cbcaca) = c,$$

$$\text{prefix}(cbcaca, cacac) = cb,$$

$$\text{merge}(cbcaca, cacac) = cbcacac.$$

A *superstring*  $w$  is also defined as the string  $\text{prefix}(x_1, x_2) \cdot \text{prefix}(x_2, x_3) \cdots \text{prefix}(x_n, x_1) \cdot \text{overlap}(x_n, x_1)$ . That is, a *superstring* is simply a concatenation of all the strings “minus” the overlapping duplicates. Apparently, each superstring of a set of strings defines a permutation of

the set's elements. Conversely, every permutation of the set's elements corresponds to a single superstring

Our interest however lies on defining the SCS problem. Essentially, the SCS problem is: Given a set of strings  $R$  and a positive integer  $K$ , does  $R$  have a superstring of length  $K$ ?

Figure 3.1 shows the SCS Decision Problem following the template provided by Garey and Johnson [?].

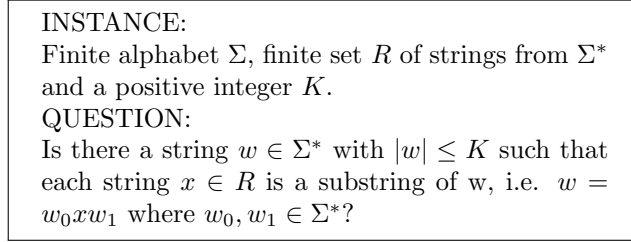


Figure 3.1: SCS Decision Problem

In the Compendium of NP Optimization problems published by [?] and in the list of NP-Complete Problems published by [?] the SCS problem appears under under Storage and Retrieval (SR).

## Appendix A

### Code Listing

# REFERENCES

- [1] *Haskell programming language*. <https://haskell.org>.
- [2] *Threadscope*. <https://github.com/haskell/ThreadScope>.
- [3] A. ABEL, M. BENKE, A. BOVE, J. HUGHES, AND U. NORELL, *Verifying haskell programs using constructive type theory*, 01 2005, pp. 62–73.
- [4] J. BREITNER, A. SPECTOR-ZABUSKY, Y. LI, C. RIZKALLAH, J. WIEGLEY, AND S. WEIRICH, *Ready, set, verify! applying hs-to-coq to real-world haskell code*, 2018.
- [5] J. BUCK, *Mazes for Programmers*, The Pragmatic Programmers, 2015.
- [6] M. H. CHEONG, *Functional programming and 3d games*, (2006).
- [7] T. H. CORMEN, C. E. LEISERSON, R. L. RIVEST, AND C. STEIN, *Introduction to Algorithms, Third Edition*, The MIT Press, 3rd ed., 2009.
- [8] L. DHULIPALA, G. E. BLELLOCH, AND J. SHUN, *Theoretically efficient parallel graph algorithms can be fast and scalable*, in Proceedings of the 30th on Symposium on Parallelism in Algorithms and Architectures, SPAA '18, New York, NY, USA, 2018, Association for Computing Machinery, p. 393–404.
- [9] Y. EL BAKOUNY, T. CROLARD, AND D. MEZHER, *A coq-based synthesis of scala programs which are correct-by-construction*, Proceedings of the 19th Workshop on Formal Techniques for Java-like Programs, (2017).
- [10] M. ERWIG, *Inductive graphs and functional graph algorithms*, Journal of Functional Programming, 11 (2001), pp. 467–492.
- [11] Z. ET AL., *Parallelizing a\* path finding algorithm*, International Journal Of Engineering And Computer Science, 6 (2017), pp. 22469–22476.
- [12] M. EVETT, J. HENDLER, A. MAHANTI, AND D. NAU, *Pra\*: Massively parallel heuristic search*, Journal of Parallel and Distributed Computing, 25 (1995), pp. 133–143.
- [13] D. FERGUSON, M. LIKHACHEV, AND A. STENTZ, *A guide to heuristic-based path planning*, in Proceedings of the international workshop on planning under uncertainty for autonomous systems, international conference on automated planning and scheduling (ICAPS), 2005, pp. 9–18.

- [14] S. FRAME AND J. W. COFFEY, *A comparison of functional and imperative programming techniques for mathematical software development*, Journal of Systemics, Cybernetics and Informatics, 12 (2014), pp. 1–10.
- [15] K. HAMMOND, *Why parallel functional programming matters: Panel statement*, in Reliable Software Technologies - Ada-Europe 2011, A. Romanovsky and T. Vardanega, eds., Berlin, Heidelberg, 2011, Springer Berlin Heidelberg, pp. 201–205.
- [16] S. HANENBERG, S. KLEINSCHMAGER, R. ROBBES, É. TANTER, AND A. STEFIK, *An empirical study on the impact of static typing on software maintainability*, Empirical Software Engineering, 19 (2014), pp. 1335–1382.
- [17] T. HARRIS, S. MARLOW, AND S. P. JONES, *Haskell on a shared-memory multiprocessor*, in Proceedings of the 2005 ACM SIGPLAN Workshop on Haskell, 2005, pp. 49–61.
- [18] P. E. HART, N. J. NILLSON, AND R. BETRAM, *A formal basis for the heuristic determination of minimum cost paths*, IEEE Transactions of Systems Science and Cybernetics, SSC-4 (1968).
- [19] J. HUGHES, *Why Functional Programming Matters*, Computer Journal, 32 (1989), pp. 98–107.
- [20] H. KAINDL AND G. KAINZ, *Bidirectional heuristic search reconsidered*, Journal of Artificial Intelligence Research, 7 (1997).
- [21] M. KESSELER, *The Implementation of Functional Languages on Parallel Machines with Distributed Memory*, PhD thesis, Radboud University Nijmegen, 1996.
- [22] A. KISHIMOTO, A. FUKUNAGA, AND A. BOTEÁ, *Scalable, parallel best-first search for optimal sequential planning*, Association for the Advancement of Artificial Intelligence, (2009).
- [23] D. E. KNUTH, *The Art of Computer Programming, Volume 1 (3rd Ed.): Fundamental Algorithms*, Addison Wesley Longman Publishing Co., Inc., USA, 1997.
- [24] H.-W. LOIDL, F. RUBIO, N. SCAIFE, K. HAMMOND, S. HORIGUCHI, U. KLUSIK, R. LOOGEN, G. J. MICHAELSON, R. PEÑA, S. PRIEBE, Á. J. REBÓN, AND P. W. TRINDER, *Comparing parallel functional languages: Programming and performance*, Higher-Order and Symbolic Computation, 16 (2003), pp. 203–251.
- [25] A. LUMSDAINE, D. P. GREGOR, B. HENDRICKSON, AND J. W. BERRY, *Challenges in parallel graph processing*, Parallel Process. Lett., 17 (2007), pp. 5–20.
- [26] S. MARLOW, *Parallel and Concurrent Programming in Haskell*, O’Reilly Media, Inc., 2013.
- [27] S. MARLOW, S. PEYTON JONES, AND S. SINGH, *Runtime support for multicore haskell*, in Proceedings of the 14th ACM SIGPLAN international conference on Functional programming, 2009, pp. 65–78.
- [28] I. PEREZ, F. DEDDEN, AND A. GOODLOE, *Copilot 3*, Tech. Rep. 20200003164, National Aeronautics and Space Administration (NASA), May 2020.
- [29] W. PIJLS AND H. POST, *A new bidirectional search algorithm with shortened postprocessing*, European Journal of Operational Research, 198 (2009), pp. 363–369.



- [30] R. C. PRIM, *Shortest Connection Networks And Some Generalizations*, Bell System Technical Journal, 36 (1957), pp. 1389–1401.
- [31] F. RABHI AND G. LAPALME, *Algorithms; A Functional Programming Approach*, Addison-Wesley Longman Publishing Co., Inc., USA, 1st ed., 1999.
- [32] L. H. O. RIOS AND L. CHAIMOWICZ, *Pnba\*: A parallel bidirectional heuristic search algorithm*, in ENIA VIII Encontro Nacional de Inteligência Artificial, 2011.
- [33] J. ROMEIN, A. PLAAT, H. BAL, AND J. SCHAEFFER, *Transposition driven work scheduling in distributed search*, in AAAI National Conference, 1999, pp. 725–731.
- [34] V. SANZ, A. DE GIUSTI, AND M. NAIOUF, *Improving hash distributed a\* for shared memory architectures using abstraction*, in International Conference on Algorithms and Architectures for Parallel Processing, Springer, 2016, pp. 431–439.
- [35] A. SILBERSCHATZ, P. B. GALVIN, AND G. GAGNE, *Operating System Concepts*, Wiley Publishing, 10th ed., 2018.
- [36] S. S. SKIENA, *The Algorithm Design Manual*, Springer, London, 2008.
- [37] A. SPECTOR-ZABUSKY, J. BREITNER, C. RIZKALLAH, AND S. WEIRICH, *Total haskell is reasonable coq*, Proceedings of the 7th ACM SIGPLAN International Conference on Certified Programs and Proofs, (2018).
- [38] A. WEINSTOCK AND R. HOLLADAY, *Hash distributed a\* distributed termination detection path reconstruction*, 2016.

# VITA

`/*TODO*/` are BS Computer Science student of the Department of Computer Science at the Ateneo de Naga University.