# Parallel Implementation of the A\* Pathfinding Algorithm on a Purely-Functional Programming

### Language

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Bachelor of Science degrees

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### **Declaration of Original Work**

We declare that the Senior Project entitled

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which we submitted to the faculty of the

### Department of Computer Science, Ateneo de Naga University

is our own work. To the best of our knowledge, it does not contain materials published or written by another person, except where due citation and acknowledgement is made in our senior project documentation. The contributions of other people whom we worked with to complete this senior project are explicitly cited and acknowledged in our senior project documentation.

We also declare that the intellectual content of this senior project is the product of our own work. We conceptualized, designed, encoded, and debugged the source code of the core programs in our senior project. The source code of third party APIs and library functions used in my program are explicitly cited and acknowledged in our senior project documentation. Also duly acknowledged are the assistance of others in minor details of editing and reproduction of the documentation.

In our honor, we declare that we did not pass off as our own the work done by another person. We are the only persons who encoded the source code of our software. We understand that we may get a failing mark if the source code of our program is in fact the work of another person.

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### **EXECUTIVE SUMMARY**

To be filled in later. /\*TODO\*/.

I dedicate this research work to all of humanity.

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I thank everyone who helped me finish this thesis.

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### Chapter 1

### Introduction

The A\* pathfinding algorithm is a best-first pathfinding algorithm for graphs commonly used for graph traversal applications such as artificial intelligence, video games, flight paths, and more. However, while most games are written in an imperative and object-oriented language such as C#, C++, and JavaScript, it is possible to write video games in a functional language using a reactive functional programming approach.[6] Likewise, the need for other correct critical software led organizations such as NASA to use Haskell[1], a purely-functional programming language, to be used in systems where high-level assurance and provable programs are a must.[26]

This paper assumes concrete differences between parallel and concurrent where the former is defined to be a hardware feature of having multiple processors or cores to compute a problem whereas the latter is defined to be a software-based approach to decrease the impact of computation bottlenecks by switching between different computations when a computation takes too long.[33] One of the major challenges of parallel programming is controlling the order of execution to prevent race conditions, which can often lead to bugs and are hard to maintain. However, since pure functional languages, such as Haskell, have no mutability and computations lead to the same result regardless of the order, they are a perfect candidate for writing parallel programs.[15] This research aims to find a parallel implementation of the existing A\* pathfinding algorithms using a purely-functional setting with attention to program performance. In turn, this helps in the advancement of different functional programming approaches for parallel graph computations which could eventually lead to critical systems to use a more provable programming language.

### 1.1 Project Context

The current video game industry is dominated by imperative programmers, even though functional programming had been getting more popular recently especially in web-programming due to to languages such as TypeScript and PureScript and frameworks such as ReactJS which favors a reactive functional approach. Thus, most beginners are exposed to imperative algorithms, such as the A\*, due to the popularty of algorithm books with imperative programming in them. [7, 22, 34] However, algorithms designed for imperative programming may not translate as well when implemented in a functional style. The quicksort specification maintains a worst-case space complexity of O(n). However, when the algorithm is written in a pure-function style, it will have to copy every subarray due to its recursive nature, thus yielding a worst-case space complexity of  $O(n^2)$ .

The advantages of functional programming lies with its referential transparency which means that a function definition or a variable will never change its definition throughout the runtime of the program. [20, 15] Hence, mathematically proving functional programs might be easier and can be aided by proof assistants such as Coq or Agda. [4, 35, 9] Likewise, splitting functions into smaller functions and reasoning about those smaller components much like lemmas would mean that functions would be modular and composed of proven subfunctions. [3, 18] Hence, functional programming languages are excellent candidates for parallel programming since the languages do not have mutable states and are therefore, instances of shared variables are abstracted away from the programmer. Similarly, the order of execution of pure functions does not matter as the program will still yield the same results. [20] A parallel and purely-functional approach to the A\* algorithm would eventually lead to more applications such as shortest distance in a map, flight paths, web server searching, and more to use a more provable and type-safe language which could lead to less system failures and high availability.

### 1.2 Purpose and Description

This research aims to utilize the existing parallel A\* pathfinding algorithm [11, 36] and find a way to develop a reasonably-efficient purely-functional implementation of the algorithm using parallel data structures such as STMs or MVars[25].

The A\* Pathfinding algorithm is used heavily in video games, telephone traffic, and other graph traversal problems[17]. This research aims to aid in the development of video games and other uses

where a parallel A\* algorithm would be beneficial using the functional programming paradigm in the future as video game development is dominated by imperative languages.

### 1.3 Objectives

The main objective of the research is to find an efficient parallel purely-functional implementation of the A\* pathfinding algorithm. The research will be done mostly in Haskell with some exceptions. Likewise, concrete comparisons between the number of cores and logical threads will be used to measure the most efficient performance runtime and space complexity of the algorithm.

The researchers aim to complete the following specific tasks:

- Write a *generator* that will generate an arbitrary-sized maze. The maze should be relatively hard to solve without the aid of computers in a short amount of time.[5]
- And a *solver* program that will be written in Haskell, a lazy purely-functional programming language, for translating the output of the generator to a graph.[1]
- The solver program should have a web-based user interface where the researchers can view the maze and how the solver program was able to solve it correctly.
- Performance of the solution shall be measured by using ThreadScope to monitor the thread and core activities while the program is being run.[2]

### 1.4 Scope and Limitations

The research will only cover solvable-mazes as the A\* algorithm does not halt when there is no reachable end goal (e.g., the start vertex and end vertex lie on different components of the graph).[17] Likewise, there will be no generality and all programs will be written in Haskell. Translation to other functional programming languages is not a priority and, thus, lambda notation will not be used. Other concurrent data structures besides MVar and Software Transactional Memory will not be utilized.

The concrete implementation and analysis is planned to be tested only on four CPUs such as Intel Core i7-9750H and AMD Ryzen 5 3500x. Other CPU architectures are not planned to be tested on.

### Chapter 2

# Review of Related Systems and Related Literature

Due to the trend of parallelization in the modern computing era, [30, 36, 11, ?] several researchers developed more efficient and faster implementations of the A\* algorithm by parallizing the algorithm in various ways.

### 2.1 The A\* Algorithm

The A\* pathfinding algorithm, first described by Hart, Nilsson, and Raphael, is a best-first search algorithm for finding a path between two vertices in a directed weighted graph.[17] The A\* search algorithm can be seen as a variant of Dijkstra's algorithm with an added heuristic function to guide its search for an optimal solution. That is, the A\* search functions the same as Dijkstra's algorithm if the heuristic function h(n) = 0 for any n.[13]

Hart, Nilsson, and Raphael had proven that the  $A^*$  algorithm is *admissible* whenever the heuristic function h is admissible. That is,  $A^*$  is guaranteed to return an optimal solution. The optimality of the path generated by the  $A^*$  algorithm depends on its heuristic function which will be discussed further in Chapter 3 of this paper.

### 2.2 Parallel A\* Algorithm

Zaghloul, Al-Jami, Bakalla, et al. had shown that an implementation of a parallel A\* algorithm had a speed up of 3-4 times on problem sizes of  $2000 \times 2000$  to  $4000 \times 4000$ .[11] The implementation took advantage of reducing the problem into subgraphs where the neighbors of a vertex in a graph will have threads assigned to them and a neighbor  $n_i$  will be the new start vertex of the A\* search for that given thread with the same goal vertex. By Ahmdal's Law, runtime efficiency will decrease when the number of threads are increased due to the overhead of garbage collection and thread creation.

### 2.2.1 Bidirectional A\* Search Algorithms

Bidirectional is said to be possible when there is exactly one start vertex and exactly one goal vertex. Since the computer cannot differentiate between the two if they are reversible, it is possible to start searching from the goal g to the start s and vice versa.[19] Bidirectional search is useful when implemented on a unidirectional multigraph(2.1) where there can be an efficient path between from one vertex to another in one direction but the same path cannot exist in the other direction.

The BHPA algorithm is a bidirectional search employing two  $A^*$  algorithms running sequentially with a sequential computer switching directions multiple times. The BHPA algorithm terminates whenever two search paths from both directions meet and returns the minimum f cost of either direction or by adding the g costs of the two directions. However, Kaindl and Kainz had proven that the best case performance of BHPA\* is not significantly better than the unidirectional  $A^*$  algorithm due to the cost of the BHPA's termination condition.[19] Kaindl and Kainz developed a more efficient implementation of a non-tradition bidirectional heuristic search that is effective on machines with limited memory. The algorithm instantiates by assigning memory to a unidirectional approach and uses  $A^*$  to find a path from the start to the end vertices. If there is such a path given the memory, then the algorithm terminates. Otherwise, a reverse search from the end vertex to an intermediate vertex from the first search.

The New Bidirectional A\* Search (NBA\*) uses the same technique as the BHPA in the sense that the execution of each search direction alternates. [27] However, NBA\* employs the reverse search on a reverse graph. That is, let V(G) and E(G) be the vertex set and edge set of graph G and if  $u, v \in V(G)$  and  $uv \in E(G)$  but  $vu \notin E(G)$ , then for the reverse graph  $vva \in V(G)$  with  $va \in V(va \in V(G))$ 

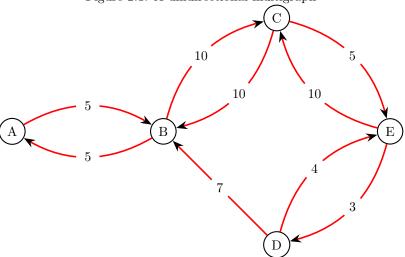


Figure 2.1: A unidirectional multigraph

and  $vu \in E(rev(G))$  but  $uv \notin E(rev(G))$ . The edges uv in E(G) and vu in E(rev(G)) have the same weights. Likewise, if  $s_1$  and  $t_1$  are the start and goal vertices of graph G, then  $s_2$  and  $t_2$  are the start and goal vertices of graph rev(G) with  $s_2 = t_1$  and  $t_2 = s_1$ . As a corollary, the vertex set of rev(G) is equal to the vertex set of G denoted by V from this point.

In NBA\*, all vertices  $v \in V$  are initially labeled  $g(v) = \infty$  and the set  $\mathcal{S} = \emptyset$  is the set of vertices with permanent labels. Likewise, there are shared variables  $\mathcal{L} = \infty$  and  $\mathcal{R} = \emptyset$  which will hold the the best cost so far and the rejected vertices, respectively. During the runtime of the algorithm, a vertex is either expanded if its heuristics or label satisfies the necessary conditions with respect to  $\mathcal{L}$  or rejected and thus added in the set  $\mathcal{R}$ . The algorithm halts when there is one search side with no more candidates to be expanded.

#### Parallel New Bidirectional A\* (PNBA\*)

PNBA\* is proposed by Rios and Chaimowicz by parallelizing the NBA\* algorithm.[30] PNBA\* leverages on a shared memory model and runs both search processes in parallel. Due to this, the PNBA\* algorithm is faster than the NBA\* and thus has a better runtime performance. The implementation of PNBA\* has a variable S, like the NBA\*, shared by both processes and contains the vertices, say v, with finite labels g(v). However, since both  $\mathcal{L}$  and S are updated by both processes in parallel, it is possible that there would be a race condition and some vertex in one

Colis, Lesaba, Maravilla, and Roldan Chapter 2. Review of Related Systems and Related Literature 7 thread may be expanded due to the race conditions. However, this does not affect the admissibility of PNBA\*. While this may incur some overhead due to expanding additional vertices, parallelism makes up for the additional overhead costs.[30]

### 2.2.2 Parallelism by Hashing

The Hash Distributed A\* Algorithm (HDA\*) builds on the the hash-based work distribution of PRA\* and asynchronous communication by TDA.[21, 12, 31] The HDA\* search introduces ownership where each processor has their own local closed and open lists and the search space is divided among processors. Unlike the implementation by Zaghloul, when a vertex is expanded by a processor  $P_1$  in HDA\*,  $P_1$  will not immediately own the generated vertices but instead send those vertices to the message queue to be hashed by available processors which can then be processed. A pseudo-random hash function is important to make sure work is distributed equally among available processors. If the hash function is static, that is it only assigns every generated vertex to the same processor that generated said vertices, then the algorithms performs the same way as the classic A\* algorithm. Due to the parallel nature of HDA\* and having local open and closed sets per processor, there is no guarantee that an expanded vertex will no longer be expanded.

The simplest approach for the termination condition of HDA\* is the barrier method wherein a processor waits for other processors when its open set is empty. This incurs some overhead and is not viable. Weinstock and Holladay describes the sum flags method where a binary flag is raised by a processor if its open set is empty.[36] Let  $p_i$  be processors for all  $i \in 1..n$  where n is the number of processors in the computer. The algorithm will terminate if  $p_1 \wedge p_2 \wedge \cdots \wedge p_n = True$ .

The original paper by Kishimoto, Fukunaga, and Botea showed that HDA\* performs significantly better than A\*, WSA\*1, and PRA\* when using message passing on a number of problems. When the HDA\* is run using 4 CPU cores, the speedup averages 3 times and is shown to be very efficient compared to the other parallel implementations and the sequential A\* algorithm. Likewise, the speedup is even more significant at an average of 5 times when using 8 cores. Kishimoto, Fukunaga, and Botea expounded further that the HDA\* is also efficient in a distributed computing environment by showing a speedup of an average of 30-60 times in a 128-core distributed computer.[21]

<sup>&</sup>lt;sup>1</sup>Another parallel implementation of the A\* algorithm that involves work stealing

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### 2.3 Real-time Performance of Functional Programs

In this section, we compare the usefulness of functional programs in real-time situations, that is, in applications were time is critical such as GPS systems and flight guidance software. [14] Since we're primarily concerned with developing a fast-enough parallel program for a correct software that would be needed for critical systems, we have to compare the performance of our program with recent imperative implementations. This should be stated here.

Likewise, we make use of Tim Harris, Simon Marlow, and Simon Peyton Jones' research on the parallel performance of Haskell and overhead costs when using parallel data structures.[16]

Appendix A

Code Listing

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