Gottfried Wilhelm Leibniz Universität
Hannover, Germany

Studienarbeit

Statistical Monitoring of Image Data

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Examiner: Prof. Dr.
Examiner: Prof. Dr.

July 19, 2021

protocol of statistical monitoring of image data

The purpose of this research is to propose a feasible method to monitor the change image data with the help of statistical method and control chart.

The goal of this thesis is to find out a statik to accuratlly represent to state of image data.

Methodology

Algorithm: statistical based defect detection , e.g., maximun variance of sliding windows, Discrete wavelet transform,etc...
Framework: control chart e.g. Shewhart control chart, χ^2 control chart, Hotelling T^2 control chart.
Programming language: Matlab.

Tasks with Tentative Schedule

- The first and the second months: literature review and data collect
- The second to the forth months: empirical study and hypotheses propose
- In the third month: mid-term reflection
- The forth and the fifth month: test and modify the proposed hypotheses
- The sixth month: writing and presenting thesis
- The end: evaluation of performance (including thesis and presentation)

Statutory Declaration

I, Xilin Huang, declare that this Studienarbeit, and the work indicated herein have been composed by myself, and any sources have not been used other than those specified. All the consulted published or unpublished work of others have been clearly cited. I additionally declare that the work and master's thesis have not been submitted for any other previous degree examinations.

Xilin Huang Hannover, July 19, 2021

Eidesstattliche Erklärung

Ich, Xilin Huang, erkläre hiermit, die Studienarbeit selbstständig verfasst zu haben und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt zu haben. Alle Stellen der Arbeit, die wörtlich oder sinngemäß aus anderen Quellen übernommen wurden, habe ich als solche kenntlich gemacht. Die Arbeit wurde in gleicher oder ähnlicher Form noch keiner Prüfungsbehörde vorgelegt.

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Acknowledgements

Here you can write your acknowledgements.

Abstract

Here write your abstract for your thesis. Please keep it accurate and clear within one page.

Contents

List of Figures

1 Introduction

Here is the introduction. You need to write the following key points for your work. Keep each part concise and short. In total, this chapter should not be more than four pages.

The remainder of this paper is organized as follows. The real-time contrasts framework for image monitoring is introduced in Section 2. Section 3 evaluates the performance of the proposed method using simulations. Section 4 provides an experiment to apply the proposed method in an industrial environment. Finally, the conclusions and directions of future research are presented.

- Background
- Motivation
- Research gap
- Objectives
- Approach (in introduction chapter you do not necessarily need to write the results for your work)
- The structure of your thesis

2 Related Work

Here you need to write your related work and cite properly. For example, one of the most influential researchers in **DL!** (**DL!**) is Yann LeCun with the Nature Article *Deep learning* [**?**]. In total, you should refer to preferably approximately 50 papers (not less than 40 papers).

Here, it is highly recommended to start writing this part as soon as you start reading any papers. It will take you a lot of time to do so and can also help you track the papers that you have been reading.

3 Background

3.1 Discrete wavelet decomposition

The Discrete wavelet transform (DWT) foundations go back to 1976 when Croiser, Esteban, and Galand devised a technique to decompose discrete-time signals. Crochiere, Weber, and Flanagan did similar work on the coding of speech signals in the same year. They named their analysis scheme as subband coding. In 1983, Burt defined a very similar technique to subband coding and named it pyramidal coding, also known as multiresolution analysis. Later in 1989, Vetterli and Le Gall made some improvements to the subband coding scheme, removing the existing redundancy in the pyramidal coding scheme [Polikar et al., 1996].

DWT is any wavelet transform for which the wavelets are discretely sampled. As with other wavelet transforms, a key advantage over Fourier transforms (FT) is temporal resolution: it captures both frequency and location information (location in time).

In Fourier transforms, we cannot know what spectral component exists at any given time instant. The best we can do is to investigate what spectral components exist at any given interval of time. This is a problem of resolution, and it is the main reason we use wavelet transform (WT) instead of FT. FT gives a fixed resolution at all times, whereas WT gives a variable resolution as follows:

Higher frequencies are better resolved in time, and lower frequencies are better resolved in frequency. This means that a particular high-frequency component can be located better in the time domain (with less relative error) than a low-frequency component. On the contrary, a low-frequency component can be better in the frequency domain than a high-frequency component.

we can visualize the resolution problem at figure 3.1:

In figure 3.1 the top row shows that at higher frequencies, we have more samples corresponding to smaller intervals of time. In other words, higher frequencies can be resolved better in time, which mean we can easily find out higher frequency signal in the time domain. As the frequency decrease, there are fewer points to characterize the signal. Therefore, low frequencies have poor resolution in time domain.

In figure 3.2, the time resolution of the signal works the same as shown in figure 3.1, but now, the frequency information has different resolutions at every stage too. We can note that the space between subsequent frequency components increases as frequency increases. Thus lower frequencies are better resolved in frequency, whereas higher frequencies are not.

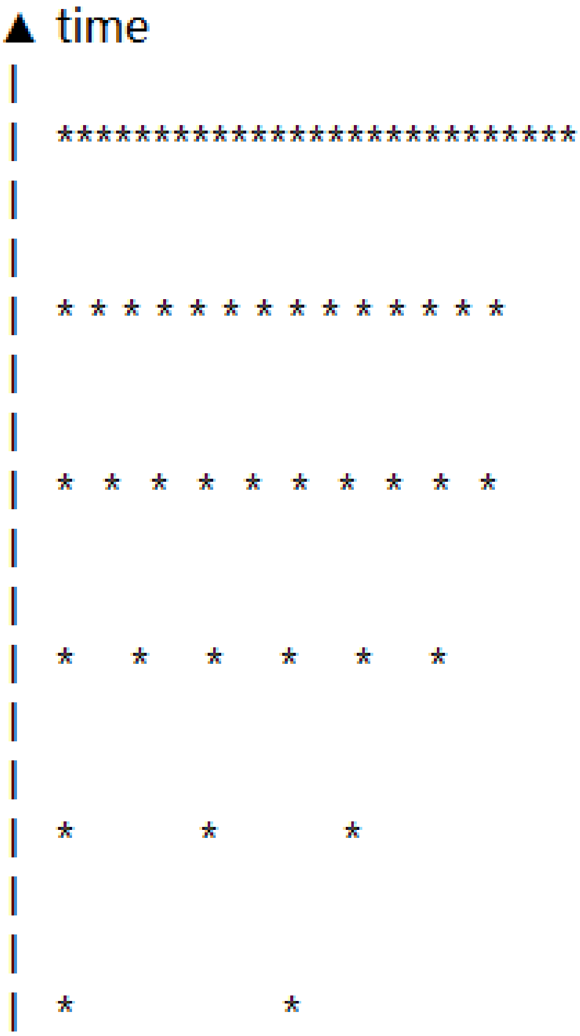

Figure 3.1: High frequency signal is better resolved in time domain

Multiresolution analysis (MRA), also known as Discrete wavelet transform, is a framework proposed to analyzes the signal at different frequencies with different resolutions. MRA is designed to give reasonable time resolution at high frequencies and good frequency resolution at low frequencies. This approach makes sense, especially when the signal at hand has high-frequency components for short durations and low-frequency components for long durations. Fortunately, most signals that are encountered in practical applications are often of this type.

The DWT procedure starts with passing the discrete signal $x[n]$ through a half band digital lowpass filter with impulse response h[n]. Filtering a signal corresponds to the mathematical operation of convolution of the signal with the impulse response of the filter. The convolution operation in discrete time is defined as follows:

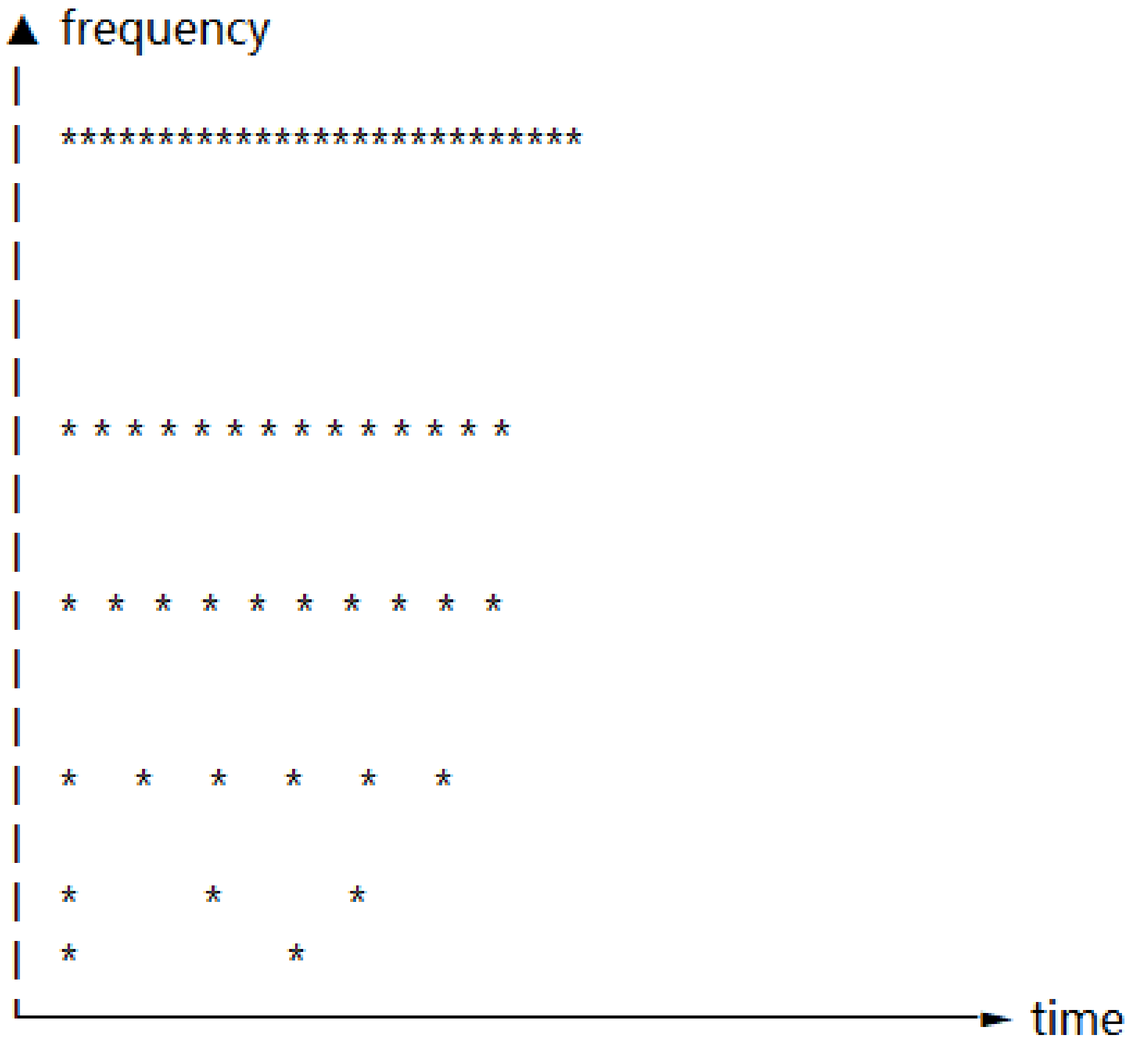

Figure 3.2: Low frequency signal is better resolved in frequency domain

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k] \tag{3.1}$$

After passing the signal through a half-band lowpass filter, Signals with frequencies larger than f/2 have disappeared. Since half the signal frequencies have now been removed, half the samples can be discarded according to Nyquist's rule, which can be achieved by subsampling the signal by two, and the signal will then have half the number of points. The resolution of this signal, which is a measure of the amount of detailed information of signal, is halved by the lowpass filter operation. While the scale, which is the inverse of frequency, of the signal is now doubled.

In order to better illustrate the parameter scale, we can imagine it as the scale used in maps: high scales in case of map correspond to a non-detailed global view (of the signal), and low scales correspond to a detailed local view. Similarly, in terms of frequency, low frequencies (high scales) correspond to global information of a signal (that usually spans the entire signal), whereas high frequencies (low scales) correspond to detailed information of a hidden pattern in the signal.

One thing worth paying particular attention to is that the subsampling operation after filtering does not affect the resolution since removing half of the spectral components from the signal (in which the number of samples stays unchanged) makes half the number of samples redundant. Thus, half the samples can be discarded without any loss of information. In summary, the lowpass filtering halves the resolution but leaves the scale unchanged. The signal is then subsampled by two since half of the number of samples is redundant, which doubled the scale.

This procedure can mathematically be expressed as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[2n-k] \tag{3.2}$$

For the principle of decomposes the signal with DWT, we can draw the following conclusions: The DWT analyzes the signal at different frequency bands with different resolutions by decomposing the signal, which is achieved by filtering the time domain signal successively with highpass and lowpass. The signal filtered by highpass and lowpass constitutes one level of decomposition and can mathematically be expressed as follows:

$$\begin{aligned} y_{\text{low}}[n] &= \sum_{k=-\infty}^{\infty} x[k] g[2n-k] \\ y_{\text{high}}[n] &= \sum_{k=-\infty}^{\infty} x[k] h[2n-k] \end{aligned} \tag{3.3}$$

where $y_{\text{high}}[n]$ and $y_{\text{low}}[n]$ are the outputs of the highpass and lowpass filters after subsampling by two, respectively, which correspond to detail information and coarse approximation.

After decomposition, the time resolution of the signal is halved, while the frequency resolution is doubled, this is because only half of the number of samples can represent the entire signal, and the frequency band of the signal only spans half of the previous frequency band. The above process is also called subband coding and can be repeated for further decomposition. At each level, filtering and subsampling will cause the number of samples to be halved (thus, the time resolution is halved), and the spanned frequency band is halved (thus, the frequency resolution is doubled). This procedure is shown in Figure 3.3, in which x[n] is the original signal, and h[n] and g[n] are lowpass and highpass filters, respectively. f represents the bandwidth of each level.

3.2 Control chart

Statistical process control (SPC) is a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability. SPC is one of the greatest technological developments of the twentieth century because it is based on sound underlying principles, is easy to use, has a significant impact, and can be applied to any process [Montgomery, 2020]. Its seven major tools are

1. Histogram or stem-and-leaf plot
2. Check sheet
3. Pareto chart
4. Cause-and-effect diagram
5. Defect concentration diagram
6. Scatter diagram
7. Control chart

The Control chart is probably the easiest yet effective tool to analyze the process stability of these seven tools. To best understand the concepts of the Control chart, the classification of the Control chart needs to be clarified. Depending on the number of process characteristics to be monitored, there are two primary control charts. The first, referred to as a univariate control chart, is a graphical chart of one quality characteristic, in which one is interested in monitoring changes in the parameter of an underlying univariate distribution over time. The second is a multivariate control chart, a graphical chart of a statistic that fuses more than one quality characteristic to monitor simultaneous changes in the parameter vector of an underlying multivariate distribution over time.

More specifically, the control chart is a graphical display, which plots the value of the quality characteristic that has been measured or calculated from a sample versus the sample number or versus time. Normally there are three lines in a control chart: a centerline corresponding to the mean value for the in-control process. Two other horizontal lines are called the upper control limit (UCL) and the lower control limit (LCL). These control limits are chosen so that almost all data points will fall within these limits as long as the process remains in control. A typical control chart is shown in Fig. 3.4.

3.2.1 Shewhart $\bar{X}$ control chart

In statistical quality control, the $\bar{X}$ and s chart is a type of control chart used to monitor variables data when samples are collected at regular intervals from a business or industrial process [Heckert et al., 2002].

In general, $\bar{X}$ and s control chart has following advantages:

1. The sample size (n) is relatively large ($n > 10$).

2. The sample size is variable.

The chart consists of two individual control charts: One ($\bar{X}$ control chart) to monitor the process mean and the other (s control chart) to monitor the process standard deviation.

During the 1920s, Dr. Walter A. Shewhart proposed a general model for control charts as follows :

Let s be a sample statistic that measures some continuously varying quality characteristic of interest (e.g., diameter), and suppose that the mean of s is μ_s, with a standard deviation of σ_s. Then the centerline (CL), the upper control limit (UCL), and the lower control limit (LCL) are

$$UCL = \mu_s + k\sigma_s \tag{3.4}$$

$$CL = \mu_s \tag{3.5}$$

$$UCL = \mu_s - k\sigma_s \tag{3.6}$$

K is the distance of the control limits from the centerline, expressed in standard deviation units. When k is set to 3, we speak of 3-sigma control charts. Historically, k=3 has become an accepted standard in the industry.

The centerline is the process mean, which is unknown, so as the σ. We need to replace them with some confident values. e.g., the average of all the data and the average standard deviation, respectively.

There exist two distinct phases of the control chart [Bersimis et al., 2007]:

- Phase I: charts are used for retrospectively testing whether the process was in control when the first subgroups were being drawn. In this phase, the charts are used as aids to the practitioner in bringing a process into a state where it is statistically in control.
- Phase II: control charts are used for testing whether the process remains in control when future subgroups are drawn. In this phase, the charts are used as aids to the practitioner in monitoring the process for any change from an in-control state.

In short, phase I deals with estimating process parameters to ensure process stability using historical data, and phase II pertains to signal any out-of-control condition or shifts in the process parameters.

Suppose in phase I, we have m samples at our disposition and let d_i be the parameter (s) of the ith sample. Then the average of the parameter is

$$\mu_s = \frac{1}{m}\sum_{i=1}^{m} d_i \tag{3.7}$$

If σ^2 is the unknown variance of a probability distribution, then an unbiased estimator of σ^2 is the sample variance is

$$s^2 = \frac{\sum_{i=1}^{m} (d_i - \bar{x})^2}{m-1} \tag{3.8}$$

However, s, the sample standard deviation, is not an unbiased estimator of σ. If the underlying distribution is normal, then s is actually

$$s = c_4 \times \sigma \tag{3.9}$$

where c_4 is

$$c_4 = \sqrt{\frac{2}{m-1}}\frac{\left(\frac{m}{2}-1\right)!}{\left(\frac{m-1}{2}-1\right)!} \tag{3.10}$$

In which m is the sample size.

Finally, the standard deviation of the sample standard deviation is

$$\sigma_s = \sigma\sqrt{1-c_4^2} \tag{3.11}$$

3.2.2 Hotelling T^2 control chart

In fact, most data in the industry (especially in chemistry and manufacturing)are naturally multivariate. Hotelling in 1947 introduced a statistic that uniquely lends itself to plotting multivariate observations. This statistic, appropriately named Hotelling's T^2, is a scalar that combines information from the mean of several variables. We can image this scalar (so-called Hotelling T^2 distance) as a statistic describing the distance away from the mean. The derivation of Hotelling T^2 distance is as follows:

In univariate statistical quality control, we generally use the normal distribution to describe a continuous quality characteristic behavior. The univariate normal probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty \tag{3.12}$$

The mean of the normal distribution is μ and the variance is σ. The term (apart from the minus sign) in the exponent of the normal distribution can be written as follows:

$$\left(x-\mu\right)\left(\sigma^{2}\right)^{-1}\left(x-\mu\right) \tag{3.13}$$

This quantity measures the squared standardized distance from x to the mean μ, whereby the term "standardized" means that the distance is expressed in standard deviation units. This same approach can be employed in the multivariate normal distribution case.

Suppose we have p variables, given by $x_1, x_2, \ldots, x_p$. These variables are arranged in a p-component vector $\mathbf{x}' = [x_1, x_2, \ldots, x_p]$. Let $\mu' = [\mu_1, \mu_2, \ldots, \mu_p]$ be the vector of the means of the x's, and let the variances and covariances of the random variables in $\mathbf{X}$ be contained in a *pp* covariance matrix Σ. The main diagonal elements of Σ are the variances of the x's and the off-diagonal elements are the covariances. Now the squared standardized distance from $\mathbf{x}$ to μ is

$$(\mathbf{x}-\mu)'\Sigma^{-1}(\mathbf{x}-\mu) \tag{3.14}$$

Since we extend the concept of mu and sigma from univariate parameter into multivariate parameters, then our next step is naturally to figure out how to construct sample mean vector ($\overline{\mathbf{x}}$) and sample covariance matrix ($\mathbf{S}$) so that they can achieve the same effect as Shewhart $\bar{X}$ control chart.

Suppose that we have a random sample from a multivariate normal distribution

$$\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n \tag{3.15}$$

where the ith sample vector contains observations on each of the p variables $x_{i1}, x_{i2}, \ldots, x_{ip}$. Then the sample mean vector ($\overline{\mathbf{x}}$) is

$$\overline{\mathbf{x}} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_i \tag{3.16}$$

and the sample covariance matrix ($\mathbf{S}$) is

$$\mathbf{S} = \frac{1}{n-1}\sum_{i=1}^{n}\left(\mathbf{x}_i-\overline{\mathbf{x}}\right)\left(\mathbf{x}_i-\overline{\mathbf{x}}\right)' \tag{3.17}$$

That is, the sample variances on the main diagonal of the matrix S are computed as

$$s_j^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_{ij}-\bar{x}_j\right)^2 \tag{3.18}$$

and the sample covariances are

$$s_{jk} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k) \tag{3.19}$$

Now suppose that $\mathbf{S}$ from equation (3.17) is used to estimate Σ and that the vector $\overline{\mathbf{X}}$ is taken as the in-control value of the mean vector of the process. The Hotelling T^2 statistic is

$$T^2 = n(\overline{\mathbf{x}} - \overline{\overline{\mathbf{x}}})'\mathbf{S}^{-1}(\overline{\mathbf{x}} - \overline{\overline{\mathbf{x}}}) \tag{3.20}$$

In which n is the subgroup size of each sample.

In some industrial settings (e.g., chemical and process industries) the subgroup size is naturally $n = 1$. Then the Hotelling T^2 statistic in equation (3.20) becomes

$$T^2 = (\overline{\mathbf{x}} - \overline{\overline{\mathbf{x}}})'\mathbf{S}^{-1}(\overline{\mathbf{x}} - \overline{\overline{\mathbf{x}}}) \tag{3.21}$$

The phase II control limits for this statistic are

$$\begin{aligned} \text{UCL} &= \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha,p,m-p} \\ \text{LCL} &= 0 \end{aligned} \tag{3.22}$$

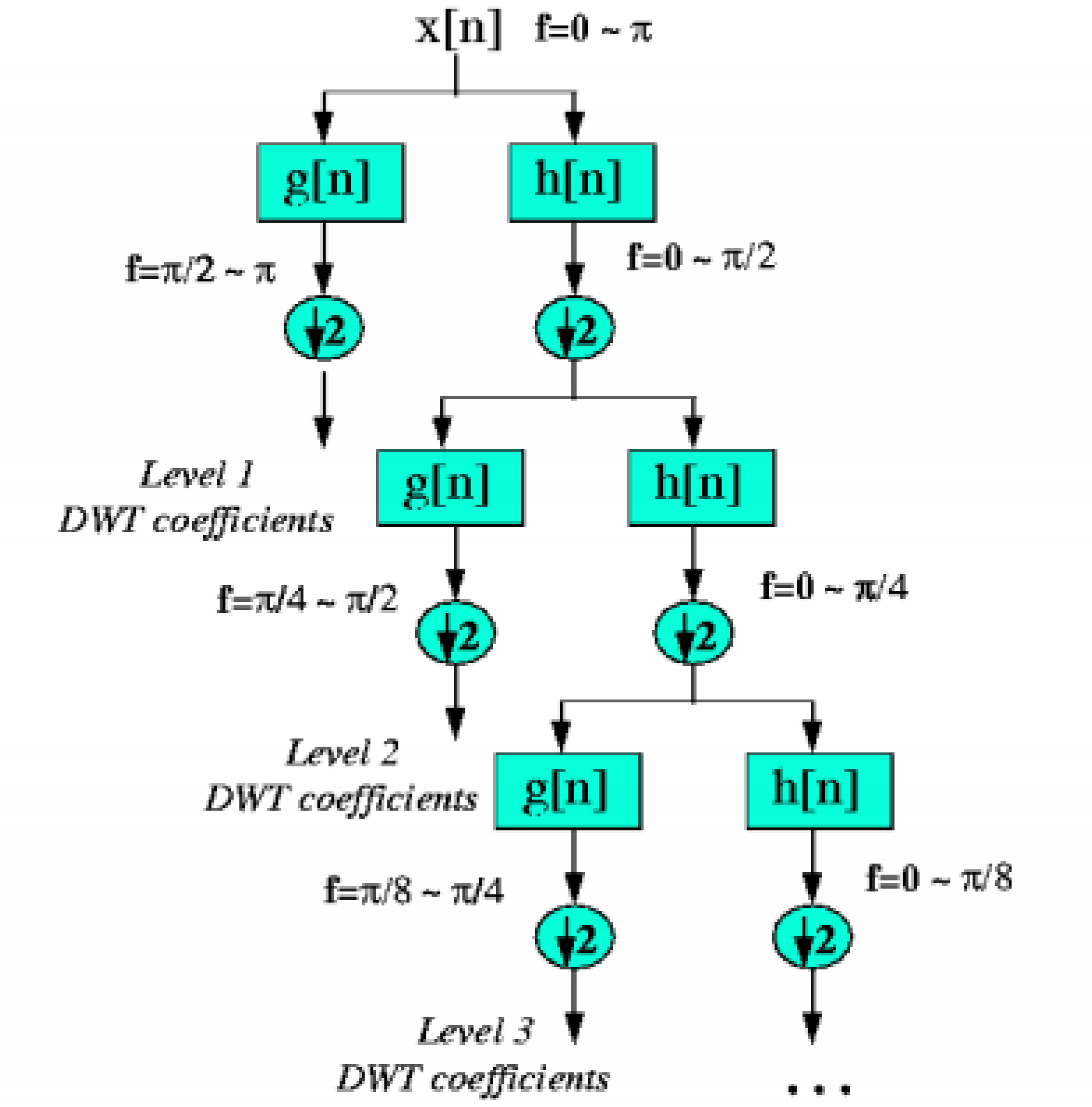

Figure 3.3: DWT decomposition algorithms

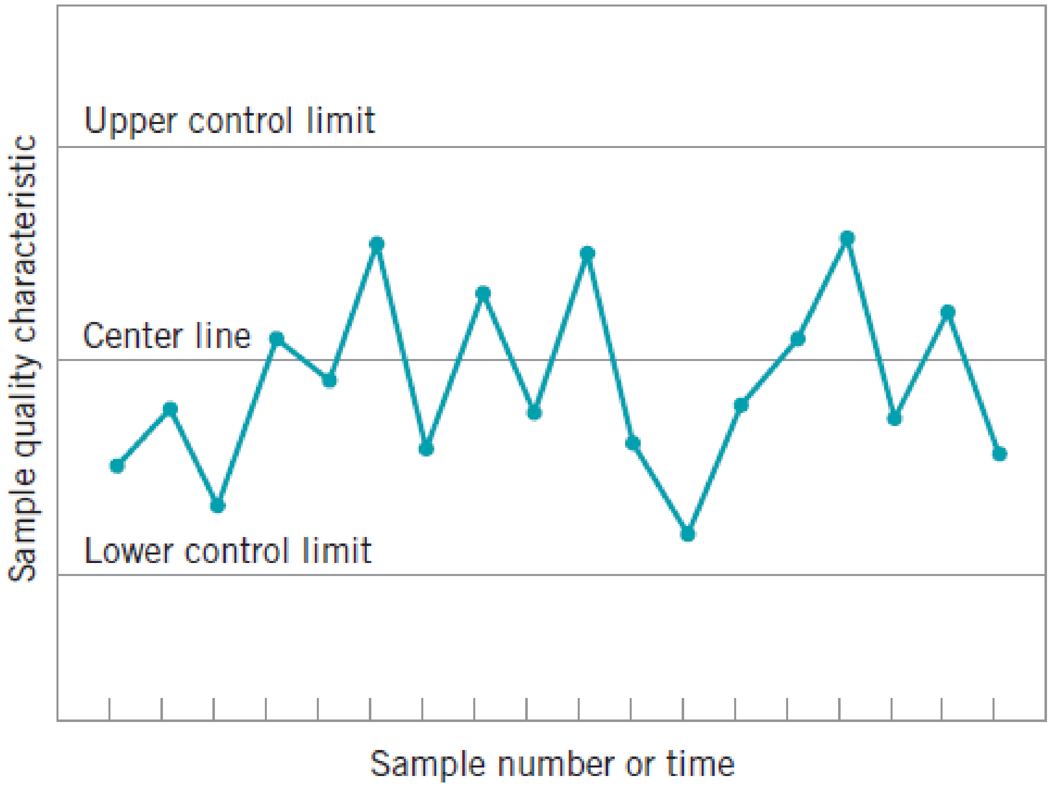

Figure 3.4: a center line that correspond to the mean value for the in-control process. Two other horizontal lines, called the upper control limit (UCL) and the lower control limit (LCL). Figure is adapted from [Montgomery, 2020]

4 Methodology

The proposed framework combines feature extract methods and statistical process control (SPC) monitoring techniques to gradually infer a high-level statistic value in the cover of mobile phone image from the low-level representation of the scratches and stains and monitor the image base on statistic value. Firstly, the surface texture properties such as scratch and stain are decomposed into so-called statistical characteristics by mean of sliding-window method 4.1 and the wavelet transform 4.3. Then statistical approach, i.e., Shewhart control chart and Hotelling T^2 control chart, are utilized respectively to monitor mean statistic value and the mean statistic vector of a univariate and multivariate process, which can be used to judge the existence of scratch defects in the sample image. The performance comparison of two combinations will be illustrated in chapter 5.

4.1 Maximum variance of sliding-windows

We assume that the pixels on the surface of the phone's cover are homogeneous. Therefore, an idea of using the variance of pixel values to extract features representing the state of the surface is proposed.

The control charts are used spatially by moving a mask (or window) across the image and then calculating and plotting a statistic each time the mask is moved. The size of the mask depends on the expected size of the defects to be detected, with smaller defective regions requiring smaller mask sizes [Megahed et al., 2011]. Inspired by this view, we move a ten by ten window (The size of the window is obtained by measuring the smallest size of defective region) across the image and calculate the variance of the pixel value each time the window is moved. The value with the largest variance among all windows in this image is taken as the desired statistic (maximum-variance) describing this image.

4.2 Sliding-windows based $\bar{X}$ control chart

In this section we combine Shewhart $\bar{X}$ control chart with sliding-window approach to perform statistical monitoring of image data. In section 4.1 a statistic "maximum-variance" is retrieved from a image. Thus in Phase I n standard sample images are used as Phase I data to retrieve the mean value ($\overline{\mathbf{m}}$)

$$\overline{\mathbf{m}} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{m}_i \tag{4.1}$$

and sample standard deviation (σ_s) of maximum-variance according to equation(3.8)-(3.11)

$$\sigma_s = \frac{s}{c_4}\sqrt{1 - c_4^2} \tag{4.2}$$

of the samples' statistic (maximum variance). In phase II, The UCL

$$UCL = \mu_m + 3\sigma_s \tag{4.3}$$

and LCL

$$UCL = \mu_m - 3\sigma_s \tag{4.4}$$

will form a qualified range for the maximum-variance of the sample. we will monitor whether the maximum-variance of the incoming sample is within this range. If it is the case, this sample will be considered qualified. Otherwise, it is unqualified.

4.3 Discrete Wavelet transform decomposition

The continuous wavelet transform is computed by changing the scale of the analysis window, shifting the window in time, multiplying by the signal, and integrating overall times. While in discrete wavelet transform (DWT) case, filters of different cutoff frequencies are used to analyze the signal at different scales. The signal is passed through a series of high pass filters to analyze the high frequencies, and it is passed through a series of low pass filters to analyze the low frequencies [Polikar et al., 1996].

The DWT [Fig. 4.1] analyzes the signal at different frequency bands with different resolutions by decomposing the signal into a coarse approximation and detail information, which is associated with low-pass and high-pass filters, respectively. In our case, we use Haar discrete wavelet transform as the basic function to perform signal decomposition so that an original image is decomposed into four coefficients: one low-pass filtering coefficients (approximation coefficients) and three high-pass filtering coefficients (detail coefficients, containing the horizontal (h), vertical (v), and diagonal (d) detail coefficients) at each level. The number of coefficients for approximation coefficients and detail coefficients is halved each time the level increase.

Since the analyzed images are in the form of 2-D, we need to perform the 2-D Haar wavelet transform by applying 1-D wavelet transform first on rows and then on columns. There is a built-in function in MATLAB Haart2, which performs a 2-D Haar wavelet transform. The lowest level (L_l) that can be obtained by the Haart2 transformation is

$$L_l = \log_2(min(r, c)) \tag{4.5}$$

If the row or column dimension of data is even, but not a power of two, the lowest level (L_l) that can be obtained by the Haart2 transformation is

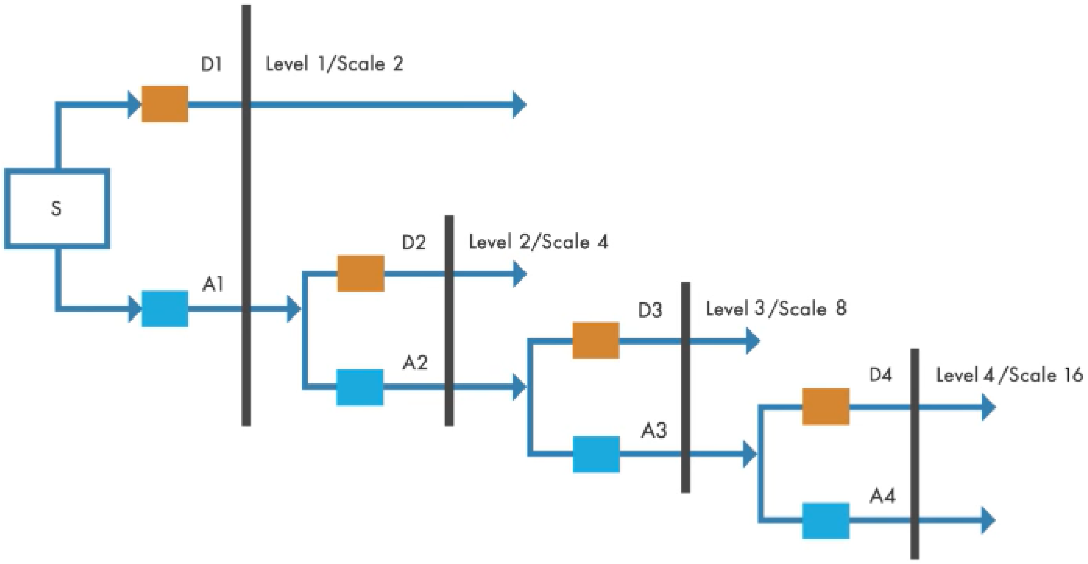

Figure 4.1: A general structure of DWT, the orange cube represent high-pass filter, the blue cube represent low-pass filter. Figure is adapted from MATLAB Tech Talks.

$$L_l = \lfloor \log_2(\frac{min(r,c)}{2}) \rfloor \tag{4.6}$$

which r, c is the row and column dimension of image, respectively, and the symbol $\lfloor t \rfloor$ rounds element t to the nearest number.

4.4 Wavelet decomposition based Hotteling T^2 control chart

In order to monitor the surface quality of the mobile phone's cover, feature statistics are needed to characterize the quality of the cover. Lin (2007) used wavelets and multivariate statistical approaches, including the Hotelling T2 control charts, to detect ripple and other types of defects in electronic components, particularly surface barrier layer (SBL) chips of ceramic capacitors. The specific approach first divides a grayscale image of (256×256) pixels into a set of equal-size sub-images of (4×4) pixels. The DWT is used to retrieved quality characteristics from image data since it decomposes the image data into some detailed coefficients, which contain horizontal (h), vertical (v), and diagonal(d) coefficients, further the multivariate statistic (e.g. T^2) integrates the multiple wavelet characteristics (h,d,v) into a statistic value for each sub-image. This statistic value can be regarded as a distance value of the sub-image. The larger the statistic value, the larger the difference between the region and the normal area, the more that region can be judged as a defect region. Finally, a T^2 spatial distribution map of the image combined with T^2 statistic of sub-images is generated. The localization of the defect of the product can be achieved by finding the high value of the T^2 spatial distribution map.

Since our goal is to monitor whether an RGB image with three frames: Red (R), Green (G), and Blue (B) frame is qualified or not, the location of the defect is not our concern. We apply Haar wavelet transform on each frame simultaneously. The coefficients of the final level (L_l) have L times filtered by the high pass filter, which means the amplitude of coefficients contains the information of the high-frequency signal in the original image. The higher the coefficients, the more likely the signal will be abrupt. The abrupt in the signal then correspond to the defects in the monitored products.

The first step is to decompose an image of ($M \times N$) pixels by haar wavelet transform

$$[AHVD] = \text{HAAR-WAVELET-TRANSFORM[IMAGE]} \tag{4.7}$$

we get horizontal (H), vertical (V), diagonal (D), and approximation (A) coefficient matrix ($S \times S$) for each frame at final level (L_l), each coefficient matrix have S^2 coefficients,an image sample can have $4 \times S^2$ coefficients.

Montgomery (2020) present tables indicating the recommended number of quality characteristics p = 2, 3, 4, 5, 10, and 20. Here we use p = 3 characteristics retrieved from diagonal coefficient matrix, since the coefficients of the diagonal coefficient matrix can best reflect the surface defects of various shapes by experiment 5. Thus we absolute all coefficients of D matrix

$$[D] = \text{ absolute[D]} \tag{4.8}$$

which turn negative coefficients into positive coefficients without changing the value itself.

After that, we take the maximal coefficient among the diagonal matrix and consider them the desired statistical characteristics (d1,d2,d3) of the corresponding frame.

$$\text{[d1,d2,d3]} = \text{max[D]} \tag{4.9}$$

Further, three coefficients retrieved from three frames compose the multiple wavelet characteristic vector $\mathbf{x}$.

$$\mathbf{x} = \text{[d1,d2,d3]} \tag{4.10}$$

These procedures are illustrated in Figure 4.2. Finally, the Hotelling T^2 statistic

$$T^2 = (\mathbf{x} - \overline{\mathbf{x}})'\mathbf{S}^{-1}(\mathbf{x} - \overline{\mathbf{x}}) \tag{4.11}$$

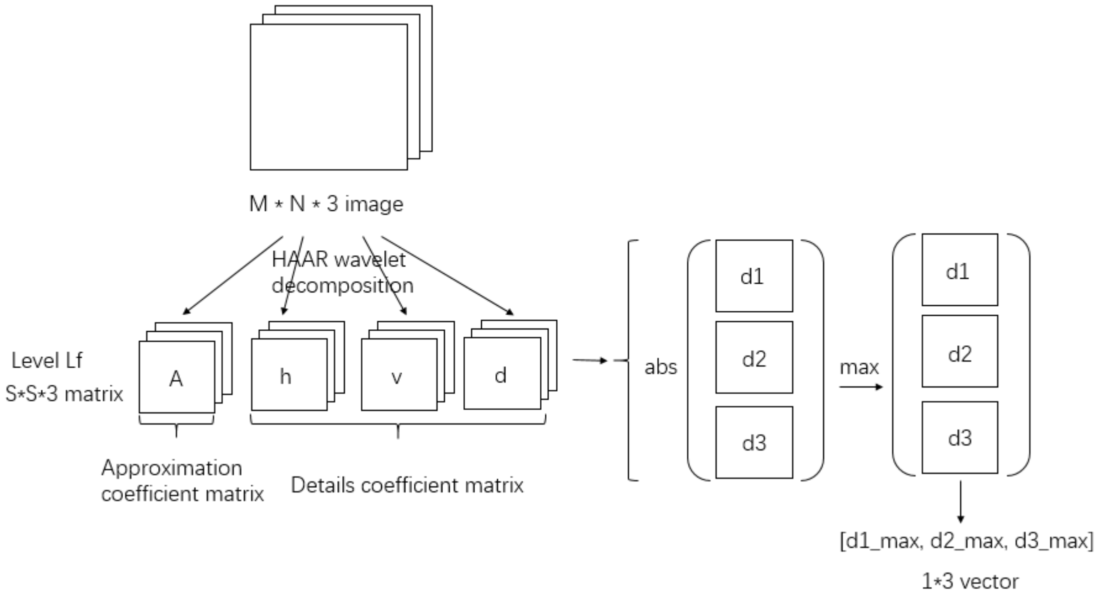

Figure 4.2: Decomposition of RGB image into 1×3 wavelet characteristic vector

in which $\overline{\mathbf{x}}$ and $\mathbf{S}$ be the sample mean vector and covariance matrix, respectively, of these observations, integrates the multiple wavelet characteristics $\mathbf{x}$ into a statistic value T^2 for each sample image. If this statistic value is larger than Upper Control Limit(UCL)

$$\mathrm{UCL} = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha,p,m-p} \tag{4.12}$$

in which m is the sample size, p is the number of quality characteristics, α is confident level, we are $(1-\alpha)$ confident that this sample is out of control. Vice versa, we are $(1-\alpha)$ confident that this sample is in control.

The output of the phase I (the sample mean vector $\overline{\mathbf{x}}$ and covariance matrix $\mathbf{S}$ of standard images as well as UCL) is used as the input of phase II. The images in phase II are first decomposed by Haar wavelet transform into 3×1 characteristic vector $\mathbf{x}$. Then by using Equation 4.11 we can calculate the Hotelling T^2 statistic of this sample and compare it with UCL to judge if the sample is in control.

5 Dataset and Experiments

6 Results and Discussion

Please write down the findings for the experiments that you have been doing. It is also better to describe the results in figures or tables than only plain text. But again, you also need to have adequate text to explain the underlying meaning for the results.

Bibliography

[Bersimis et al., 2007] Bersimis, S., Psarakis, S., and Panaretos, J. (2007). Multivariate statistical process control charts: an overview. *Quality and Reliability engineering international*, 23(5):517–543.

[Heckert et al., 2002] Heckert, N. A., Filliben, J. J., Croarkin, C. M., Hembree, B., Guthrie, W. F., Tobias, P., and Prinz, J. (2002). Handbook 151: Nist/sematech e-handbook of statistical methods.

[Lin, 2007] Lin, H.-D. (2007). Automated visual inspection of ripple defects using wavelet characteristic based multivariate statistical approach. *Image and Vision Computing*, 25(11):1785–1801.

[Megahed et al., 2011] Megahed, F. M., Woodall, W. H., and Camelio, J. A. (2011). A review and perspective on control charting with image data. *Journal of Quality Technology*, 43(2):83–98.

[Montgomery, 2020] Montgomery, D. C. (2020). *Introduction to statistical quality control*. John Wiley & Sons.

[Polikar et al., 1996] Polikar, R. et al. (1996). The wavelet tutorial.