Resonant states for quantum ring with two infinite leads

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**Abstract.** A scattering model of one dimensional quantum graph  consisting of incoming and outgoing channels with a ring scatterer is studied. Completeness of the resonant states of  in L2 on the ring is proved.

**1. Introduction**

Quantum graph is a widely used model of nanosystem [1-4]. If the graph consists of finite number of edges and all edges have finite lengths then the Hamiltonian has purely discrete spectrum. The system of eigenfunctions is complete in . If the graph contains semi-infinite edges, one has non-empty continuous spectrum and resonances generated by the eigenvalues of the initial Hamiltonian of finite graph. For many applications, it is important to know a space in which the resonant states form a complete system. In the present paper we determine this space for a graph with two infinite leads and one loop using Sz.-Nagy model [5].

**2. Scattering model**

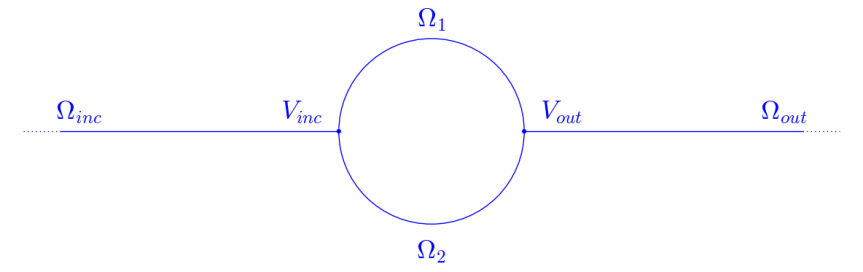


Figure 1. Quantum graph consisting of four edges . and represent the 1D ring connected to lead via the vertex and to lead via .

The wavefunctions on the resonator ring arcs and take the most general form:

We are interested in the scattering of a wave coming from the left lead () to the right one ():

where R is the reflection coefficient, T is the transmission coefficient. Then, we impose boundary conditions at vertices , (so-called -coupling [1]):

where is a real valued coupling constant. With the use of boundary conditions, we obtain system of linear equations which leads to expressions for the reflection and transmission coefficients:

Due to the symmetry of the scattering problem w.r.t. the origin, scattering matrix has the form , and,

We establish the completeness of the resonant states in the space of square integrable functions on the ring. To show this, we have to prove that S is a Blaschke-Potapov product [6], that is,

where is the image of the curve under the map . Substituting , which we calculated above, and estimating the integral, one obtains the desired result.

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