TODO ???

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**Abstract.** A scattering model of one dimensional quantum graph  consisting of incoming and outgoing channels with a ring scatterer is studied. Completeness of the resonant states of  in L2 on the ring is proved.

1. Introduction

TODO some motivation behind the problem?

The graph consists of four edges , where and represent the 1D ring connected to via the vertex and to via .

1. Scattering model

TODO picture!

The graph consists of four edges , where and represent the 1D ring connected to via the vertex and to via .

We assume the wavefunctions on the resonator arcs 1 and 2 take the most general form:

Since we are interested in the scattering from the left case, in the left (x < -d) and right (x > d) asymptotic regions:

, where R is the reflection coefficient, T is the transmission coefficient.

Then, we impose boundary conditions at vertices , :

, where is a real valued coupling constant.

1. Solution of the scattering problem

With the use of boundary conditions, we obtain system of six linear equations in six variables ( and being symbolic parameters)

, solving it yields:

1. Scattering matrix

Due to the symmetry of the scattering problem w.r.t. the origin, scattering matrix has the form , that is,

1. Showing completeness

To establish the completeness, we have to prove that S is a Blaschke-Potapov product (TODO reference?), that is,  
, or, mapping it to the complex upper half-plane via the Cayley transformation:

Substituting det S which we calculated in section 4 and estimating the integral indeed yields the desired result. TODO Should I insert the calculations here?

1. Acknowledgments

TODO ???

References

TODO ??? 5.1. For papers:

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