Comp 424 - Homework 3
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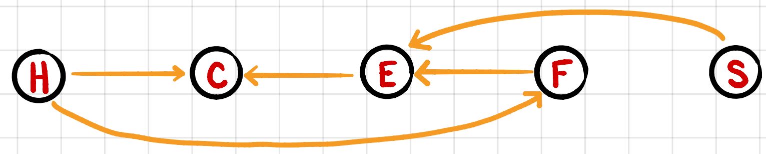
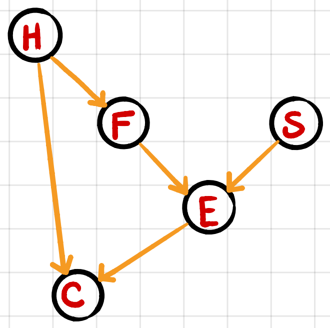
Question 1: Designing a Bayesian Network

H - hot day
C - Marv is content
E - Marv eats ≥ 1 item
F - Marv catches a fish
S - Marv steals a sandwich.



a)



b) This network is not a polytree due to the undirected cycle between the nodes H, F, E and C.

c) $P(F=1 \mid H=1) = x$
$P(F=1 \mid H=0) = y$

P(F | H)

	F=0	F=1
H=0	1-y	y
H=1	1-x	x

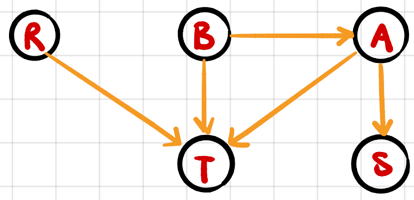
d) $P(E=1 \mid F=1) = 1.0$
$P(E=1 \mid S=1) = 0.5$
$P(E=1 \mid S=0, F=0) = 0$

	E=1	E=0
F=0 S=0	0.0	1.0
F=1 S=0	1.0	0.0
F=1 S=1	1.0	0.0
F=0 S=1	0.5	0.5

e) $P(H=1 \mid C=1) = P(H=1, C=1) \,/\, P(C=1)$
$= \Sigma_{f,e,s} P(H=1, F=f, S=s, E=e, C=1) \,/\, \Sigma_{h,f,e,s} P(H=h, F=f, S=s, E=e, C=1)$

Question 2 : Inference in Bayesian Networks

R : rush hour
B : bad weather
A : accident
T : traffic jam
S : sirens



$P(B=1) = P(b) = 0.4$
$P(R=1) = P(r) = 0.2$

$P(t \mid r, b, a) = 0.98$
$P(t \mid r, \neg b, a) = 0.9$
$P(t \mid r, b, \neg a) = 0.88$
$P(t \mid r, \neg b, \neg a) = 0.85$

$P(t \mid \neg r, b, a) = 0.5$
$P(t \mid \neg r, b, \neg a) = 0.4$
$P(t \mid \neg r, \neg b, a) = 0.6$
$P(t \mid \neg r, \neg b, \neg a) = 0.05$

$P(s \mid a) = 0.95$ $\quad P(s \mid \neg a) = 0.2$

$P(a \mid b) = 0.7$ $\quad P(a \mid \neg b) = 0.2$

Recall the following rules :
- $P(X \mid Y) = P(Y \mid X)\, P(X) / P(Y)$
- $P(Y) = P(Y \mid X) P(X) + P(Y \mid \neg X)\, P(\neg X)$
- $P(X \mid Y) = P(X, Y) / P(Y)$
- $P(X_1, \dots, X_n) = \prod_{i=1}^{n} P(X_i \mid \text{parent}(X_i))$

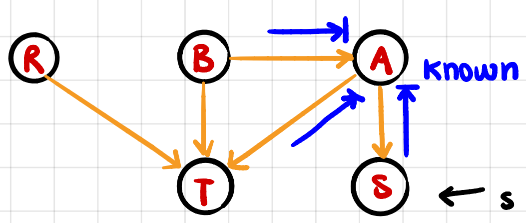
a) $P(a, \neg r) = P(a \mid \neg r) P(\neg r)$
$= P(a) P(r)$

$P(a) = P(a \mid b) P(b) + P(a \mid \neg b) P(\neg b)$
$= (0.7)(0.4) + (0.2)(0.6)$
$= 0.28 + 0.12$
$= 0.4$

$= P(a) P(r) = (0.4)(0.8) = 0.32$

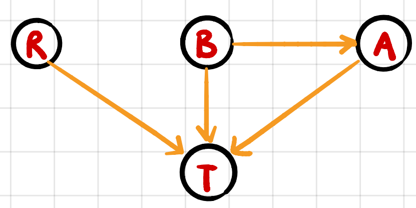
b) $P(b, a) = P(b \mid a) P(a) = P(a \mid b) P(b) = (0.7)(0.4) = 0.28$

c) use Bayes Ball to see what nodes can be pruned.



← s can be pruned from from the graph because its unreachable.

we then obtain the following graph :



d) $P(b \mid a) = P(a \mid b) P(b) / P(a)$

$P(a) = P(a \mid b) P(b) + P(a \mid \neg b) P(\neg b)$
$= (0.7)(0.4) + (0.2)(0.6)$
$= 0.28 + 0.12$
$= 0.4$

$P(b \mid a) = \dfrac{0.7 \times 0.4}{0.4} = 0.7$

Question 3 : variable elimination

$P(T \mid b)$, where T is the query variable
we are given the variable ordering : S, A, R, T

we need to initialize an active factor list with the conditional probability distributions in the Baye's net.
If we look at our graph we obtain: $P(R)\, P(T \mid R, A)\, P(A)\, P(S \mid A)$

1. Pick a variable ordering with *Y* (query variable) at the **end** of the list.
2. Initialize the active factor list with the conditional probability distributions (tables) in the Bayes net.
3. Add to the active factor list the evidence potentials $\delta(e_j, e_j)$ for all evidence variables *E*.
4. For *i=1..n*
   1. Take the next variable X_i from the ordering.
   2. Take all the factors that have X_i as an argument off the active factor list, and multiply them, then sum over all values of X_i, creating a new factor m_{X_i}
   3. Put m_{X_i} on the active factor list.

Step 1 : Eliminate S $\Rightarrow m_S(A) = \sum_s P(s \mid A)$
List : $P(R)\, P(T \mid R, A)\, P(A)\, m_S(A)$

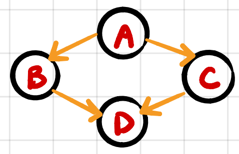
Step 2 : Eliminate A $\Rightarrow m_a(T, R) = \sum_a P(a)\, P(T \mid R, a)\, m_s(a)$
List : $P(R)\, m_a(T, R)$

Step 3 : Eliminate R $\Rightarrow m_r(T) = \sum_r P(r)\, m_A(T, r)$
List : $m_R(T)$

Now T=1 $\Longrightarrow P(T \mid b) = P(t \mid b)\, m_R(t)$
$= P(t \mid b) \sum_r P(r) \sum_a P(a)\, P(t \mid r, a)\, m_s(a)$
$= P(t \mid b) \sum_r P(r) \sum_a P(a)\, P(t \mid r, a) \sum_s P(s \mid A)$
$= P(t \mid b) \sum_r \sum_a P(t \mid r, a)$
$= P(t \mid b) \quad MAP_{r,a}\ P(t = 1 \mid r, b=1, a)$
$= (0.98)(0.57)$
$= 0.56$

Question 4 : Learning with Bayesian Networks

Bernoulli distribution



a) i. $\theta_A = P(A)$
$\theta_B = P(B)$
$\theta_C = P(C)$
$\theta_D = P(D)$

ii Recall that the variables are distributed according to a Bernoulli distribution and thus if we apply MLE to the Bernoulli model we obtain :

the likelihood : $L(\theta|D) = \theta^{x}(1-\theta)^{n-x}$ when n = total # instances
x = instances of query

the log likelihood : $\log L(\theta|D) = x\log\theta + (n-x)\log(1-\theta)$

taking the derivative of the loglikelihood and set it to 0: $\frac{d}{d\theta}\log L(\theta|D) = \frac{x}{\theta} - \frac{(n-x)}{(1-\theta)} = 0$

which then lets us obtain $\theta = x/n$ which we then apply to this problem

$P(A) = 60/144$
$P(B) = 90/144$
$P(C) = 112/144$
$P(D) = 61/144$

iii give the MAP estimate for each parameter after applying Laplace , in Bernoulli modes we use the following :

$\theta = x+1 \,/\, n+2$ as we assume we have seen each possibility at least once.

then we obtain the following

$P(A) = 61/146 = 0.418$
$P(B) = 91/146 = 0.623$
$P(C) = 113/146 = 0.774$
$P(D) = 62/146 = 0.425$

MAP
A = 0
B = 1
C = 1
D = 0

b)

i. $\theta_A = N_{A=1} / 144$
$= 60 / 144$

$\theta_{B|A=0} = N_{B=1,A=0} / N_{A=0}$
$= 57/84$

$\theta_{B|A=1} = N_{B=1,A=1} / N_{A=1}$
$= 33/60$

$\theta_{C|A=0} = N_{C=1,A=0} / N_{A=0}$
$= 62/84$

$\theta_{C|A=1} = N_{C=1,A=1} / N_{A=1}$
$= 50/60$

$\theta_{D|B=0,C=0} = N_{D=1,C=0,B=0} / N_{C=0,B=0}$
$= 4/7$

$\theta_{D|B=1,C=0} = N_{D=1,C=0,B=1} / N_{B=1,C=0}$
$= 11/25$

$\theta_{D|B=1,C=1} = N_{D=1,C=1,B=1} / N_{B=1,C=1}$
$= 20/65$

$\theta_{D|B=0,C=1} = N_{D=1,C=1,B=0} / N_{B=0,C=1}$
$= 26/47$

E-step

$w_{B_{S1}=0} = P(B_{S1}=0 \mid A_{S1})$
$= P(B=0 \mid A=1)$
$= 27/60$
$= 0.45$

$w_{B_{S1}=1} = P(B_{S1}=1 \mid A_{S1})$
$= P(B=1 \mid A=1)$
$= 33/60$
$= 0.55$

$w_{D_{S1}=0} = P(D_{S1}=0 \mid B_{S1}, C_{S1})$
$= P(D=0 \mid B=0, C=0)\, w_{B_{S1}=0} + P(D=0 \mid B=0, C=0)\, w_{B_{S1}=1}$
$= (21/47)(0.45) + (45/65)(0.55$
$= 0.58$

$w_{D_{S1}=1} = P(D_{S1}=1 \mid B_{S1}, C_{S1})$
$= P(D=1 \mid B=0, C=1)\, w_{B_{S1}=0} + P(D=1 \mid B=0, C=1)\, w_{B_{S1}=1}$
$= (26/47)(0.45) + (20/65)(0.55)$
$= 0.42$

ii <u>M-Step</u>

$$\theta_A = N_A(1{:}146)/146$$
$$= 62/146$$

$$\theta_{B|A=0} = N_{B=1,A=0}(1{:}144,146) + w_{B_{S1}=1} \;/\; N_{A=0}(1{:}146)$$
$$= 57.55/48$$

$$\theta_{B|A=1} = N_{B=1,A=1}(1{:}144,146) + w_{B_{S1}=1} / N_{A=1}(1{:}146)$$
$$= 34.55/62$$

$$\theta_{C|A=0} = N_{C=1,A=0}(1{:}146) / N_{A=0}(1{:}146)$$
$$= 62/84$$

$$\theta_{C|A=1} = N_{C=1,A=1}(1{:}146) / N_{A=1}(1{:}146)$$
$$= 51/62$$

$$\theta_{D|B=0,C=0} = N_{D=1,B=0,C=0}(1{:}144,146) + w_{B_{S1}=0} + w_{D_{S1}=1} / N_{B=0,C=0}(1{:}144,146) + w_{B_{S1}=0}$$
$$= 5.03/7.45$$

$$\theta_{D|B=1,C=0} = N_{D=1,B=1,C=0}(1{:}144,146) + w_{B_{S1}=1} + w_{D_{S1}=1} / N_{B=1,C=0}(1{:}144,146) + w_{B_{S1}=1}$$
$$= 12.13/26.55$$

$$\theta_{D|B=0,C=1} = N_{D=1,B=0,C=1}(1{:}144,146) + w_{B_{S1}=0} + w_{D_{S1}=1} / N_{B=0,C=1}(1{:}144,146) + w_{B_{S1}=0}$$
$$= 27.03/47.45$$

$$\theta_{D|B=1,C=1} = N_{D=1,B=1,C=1}(1{:}144,146) + w_{B_{S1}=1} + w_{D_{S1}=1} / N_{B=1,C=1}(1{:}144,146) + w_{B_{S1}=1}$$
$$= 21.13/65.55$$

iii <u>E-Step</u> now estimate using new weights & obtain new weights

$$w_{B_{S1}=0} = P(B_{S1}=0 \mid A_{S1})$$
$$= P(B=0 \mid A=1)$$
$$= 0.44$$
$$w_{B_{S1}=1} = P(B_{S1}=1 \mid A_{S1})$$
$$= P(B=1 \mid A=1)$$
$$= 0.56$$

$$w_{D_{S1}=0} = P(D_{S1}=0 \mid B_{S1}, C_{S1})$$
$$= P(D=0 \mid B=0, C=1)\, w_{B_{S1}=0} + P(D=0 \mid B=1, C=1)\, w_{B_{S1}=1}$$
$$= (0.43)(0.44) + (0.68)(0.56)$$
$$= 0.57$$
$$w_{D_{S1}=1} = P(D_{S1}=1 \mid B_{S1}, C_{S1})$$
$$= P(D=1 \mid B=0, C=1)\, w_{B_{S1}=0} + P(D=1 \mid B=1, C=1)\, w_{B_{S1}=1}$$
$$= (0.57)(0.44) + (0.32)(0.56)$$
$$= 0.43$$