# Manual for Package: mesh Revision 1:7M

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#### 1.1 Mesh

#### 1.2 match\_boundary\_condition

match user specified boundary conditions (as line) with boundary of discretized computational domain p\*phi + (1-p)\*d/db phi = rhs

# 2 @StructuredMesh

#### 2.1 StructuredMesh

structured mesh processing compatible with Delft3D also provides set-up of discretisation matrices

# ${\bf 2.2 \quad apply\_boundary\_condition}$

apply boundary condition and the four sides of the domain TODO: allow for interior boudaries

#### 2.3 bc\_from\_shp

read boundary condition from shape file

#### 2.4 bc\_index

TODO this is deprecated generate indices for boundary edges

#### 2.5 bc\_isinvalid

check boundary conditions for stacked domains

#### 2.6 block

stack multiple meshes to complex domain

# 2.7 boundary\_chain

return chain of boundary points

# 2.8 boundary\_direction

return direction of boundary segment

# 2.9 boundary\_indices

```
indices of boundary segments
id : index of boundary point
jd : index of
```

# 2.10 cat

#### 2.11 centreline

domain (channel) centreline along chosen dimension

#### 2.12 child

hierarchical mesh generation (for bifurcations)

# 2.13 copy

# 2.14 corner\_indices

indices of domain corners

#### 2.15 cut\_from\_domain

cut subdomain

#### $2.16 \quad export\_delft3d\_bnd$

export the boundary in delft3d compatible format

# $2.17 \quad export\_delft3d\_dep$

export bathymetry data in Delft3D dep-format

# $2.18 \quad export\_delft3d\_grd$

export mesh in deltares delft3D grd file format

# ${\bf 2.19 \quad export\_delft3d\_ini}$

export delft3D compatible initial condition file

# $2.20 \quad export\_shp$

export mesh elements as shape file

#### 2.21 extend\_straight\_reach

#### 2.22 extract\_elements

element indices from grid

#### 2.23 flip\_dimension

flip left and right or top and down

#### $2.24 \quad from\_1d\_mesh$

convert a 1D mesh to 2D mesh consisting of quadrilaterals

#### 2.25 generate\_bifurcation

```
creates a mesh for bifurcation with bluff, which is required for
   delft3d grids
TODO do not fix indices
TODO determine p individually
bank : bankline shapefile
nn : number of points across branches
ds: spacing along s
p : fraction of right side branch
level : generate hierarchical mesh,
        grid points in each branch will be 2^n+1,
        and sub meshes until level 1 will be generated
for lower levels the connecting volumes remain narrow,
as the two volumes left and right of the division line are not
    scaled
-> post smoothing required
nn: n=6; for idx=1:5; n(end+1) = 2*(n(end)-3)+3, end
ns: n=18; for idx=1:5; n(end+1) = 2*(n(end)-2)+2, end (should be
   improved to 2*(n-1)+1
```

# 2.26 generate\_disk

generate semicircular domain

R1 : inner diameter
R2 : outer diameter
Theta(1) : start angle
Theta(2) : end angle

 $\begin{array}{l} {\tt n(1)} \ : \ {\tt along} \ {\tt channel} \ {\tt number} \ {\tt of} \ {\tt points} \\ {\tt n(2)} \ : \ {\tt across} \ {\tt channel} \ {\tt number} \ {\tt of} \ {\tt points} \end{array}$ 

#### 2.27 generate\_from\_centreline

generate a mesh from a given centreline TODO : avoid crossing of inner bed points in sharp bends

#### 2.28 generate\_meander\_bend

### 2.29 generate\_rectangle

discretize a rectangular domain

#### 2.30 generate\_structured\_grid

generate a structured mesh consisting of several sub-meshes

# ${\bf 2.31 \quad generate\_tidal\_funnel}$

# 2.32 grid\_block

mesh a subdomain

# 2.33 improve

improve (smooth) the mesh

# 2.34 interp\_elem2point

interpolate values sampled at element centres to element corners TODO allow also interpolation to  $\boldsymbol{u}$  and  $\boldsymbol{v}$  points

# $2.35 \quad mesh\_polygon$

mesh a 1D channel, where boundaries are given as polygon
TODO, this should better use voronoi-tesselation (see centreline
 class)

#### 2.36 orthogonality

orthogonality of elements

### 2.37 orthogonalize

orthogonalize mesh set  ${\bf x}$  of point coordinates to 1/2

### 2.38 plot

plot the mesh

# 2.39 plot\_boundary

plot the mesh boundary

#### 2.40 plot\_coupling

plot connected vertices, see vertex\_connection\_matrix.m

# 2.41 plot\_orthogonality

plot mesh with edges colored by orthogonality condition

#### 2.42 quiver

quiver plot of velocity

#### 2.43 read\_delft3d\_dep

depth in dat file is defined at volume centres (water leve point) first row, first column and last column are buffer but nast colum is not (only when outflow?)

#### $2.44 \quad read\_delft3d\_grd$

read mesh in delft3D grd format

#### 2.45 smooth\_cubic

cubically smooth the mesh coordinates

#### 2.46 smooth\_curvilinear

```
smooth the mesh
relax = (10+relax)/11;
relax = min(0.5,relax);
```

# 2.47 smooth\_laplacian

smooth the mesh coordinates

#### 2.48 smooth\_simple

smooth the mesh coordinates

#### $2.49 \quad smooth\_sn$

smooth the mesh coordinates

#### 2.50 snap

snap two meshes that connect at their domain boundaries

#### 2.51 statistic

compute mesh statistics

#### 2.52 to\_UnstructuredMesh

convert to unstructured mesh

#### 2.53 transpose\_dimension

transpose dimensions

# 2.54 vertex\_connection\_matrix

connectivity of neighbouring vertices TODO same for elements

# 3 @UnstructuredMesh

#### 3.1 UnstructuredMesh

class containing some meshing functionality complementary to Mesh\_2d, Mesh\_3d, Tree\_2d and Tree\_3d

#### 3.2 add\_element

add an element with vertex indices, vertices already exist

#### 3.3 add\_vertex

add a vertex

#### 3.4 angle

interior angles of each element

# 3.5 apply\_boundary\_condition

# 3.6 apply\_boundary\_condition\_hermite

- 3.7 assemble\_2d\_dphi\_dphi\_hermite
- ${\bf 3.8}\quad as semble \_2d\_dphi\_dphi\_lagrange$
- 3.9 assemble\_2d\_dphi\_dphi\_morley
- 3.10 assemble\_2d\_dphidn\_phi\_hermite
- 3.11 assemble\_2d\_dphidn\_phi\_lagrange

#### 3.12 assign\_1d

assign coordinatex (x0,y0) to containing element TODO this can fail, if triangulation is not delaunay

#### 3.13 assign\_2d

assign coordinatex (x0,y0) to containing element

#### 3.14 assign\_3d

assign coordinatex (P0,y0) to containing element

#### 3.15 associate\_boundary

#### $3.16 \quad bnd_{-}1d$

left and right end points for 1D meshes

# 3.17 boundary\_1d

convert 1D mesh to 2D mesh

#### 3.18 boundary\_chain2

get chained indices of boundary segments, used for setting up higher order polynomials along the boundary

#### 3.19 boundary\_length\_and\_direction

edge length and direction of boundary segments TODO, this should be just edge length and direction

#### 3.20 cat

concatenate two meshes

#### 3.21 chain\_1d

chain 1D elements (segments)

#### 3.22 check\_dublicate\_elements

check if elements are duplicate elements
TODO, this does not check if elements cover each other, for example
hierarchical meshes or ABC+BCD and ABD+ACD
TODO check overlap by computation of area

# 3.23 check\_edge\_intersection

#### 3.24 clip

clip mesh to polygonal domain TODO only works for triangles

# 3.25 compute\_elem2elem

set up element2element neighbourhood relation

#### 3.26 connect\_1d\_2d

auto merge 1d and 2d mesh
this silently requires that 1d segments consist at least of 3
elements
TODO only implemented for triangles

#### 3.27 convert\_2d\_to\_1d

#### 3.28 copy

copy constructor

#### 3.29 cross\_section

get cross-sections for 1D elements

#### 3.30 delete\_element

delete an element

#### 3.31 derivative\_matrices

#### 3.32 derivative\_matrices\_1d

first order first derivative discretisation matrix on the 1d mesh

#### 3.33 derivative\_matrices\_2d

first order first derivative discretisation matrix on the mesh

#### 3.34 derivative\_matrix\_2d\_2

second order derivative matrix on a triangulation

#### 3.35 derivative\_matrix\_2d\_hermite

#### 3.36 derivative\_matrix\_3d

first order first derivative discretisation matrix on the mesh

#### 3.37 distance

distance along edges from a point set to all other points

open : id of start point(s)

countflag : if set use number of hops as distance not the euclidean distance  $% \left( 1\right) =\left( 1\right) \left( 1\right)$ 

#### 3.38 dual\_mesh

dual mesh formed by the centre of cicumference the dual mesh consists not only of triangles TODO rename in generate dual mesh

# 3.39 edge\_direction

# 3.40 edge\_length

euclidean edge length

# 3.41 edge\_midpoint

edge mid-points

# 3.42 edges\_from\_elements

edges and boundaries from elements

#### 3.43 eigs

eigenvalues of the lapalcian on the mesh

# 3.44 elem2edge\_

pointer of element to edge

#### 3.45 elem2elem\_matrix

matrix with neighbourhood relations for each element

#### 3.46 element\_area

area of elements
1d elements have zero area and are not processed

#### 3.47 element\_centroid

centroids of lements

# 3.48 element\_midpoint

barymetric centre of elements

# 3.49 elements\_from\_edges

2D elements from edges

# 3.50 eval2pval

element (centroid) value to vertex value TODO, use dual mesh or triangulation

#### $3.51 \quad export\_delft3d\_net$

export into DFLOWFM delft3d net.nc file

#### $3.52 \quad export\_msh$

export mesh in GMSH msh format

### 3.53 export\_pos

export triangles and vertex values to gmsh pos-file format (x,y,z,val) intended for re-meshing with values representing local mesh size

#### $3.54 \quad export\_shp$

```
export edges to GIS shapefile each element as separate polygon with one z-value
```

#### 3.55 facing\_element

```
get triangle ndx that is opposit, e.g. "facing" the vertex vdx of triangle tdx
```

#### 3.56 filter\_neighbour

apply a function on the values on connected vertices

#### 3.57 find\_encroached\_edges

```
find encroached edges in a triangulation,
i.e. edges for which on of the two facing point false into their
   enclosing
circle
```

#### 3.58 flip

```
flip
   for each side
      if (connection between opposit points shorter than
           between edges, swap edge)
      this-> flip
      that-> flip
   end
```

# 3.59 flip\_global

```
recursively flip edges, i.e ABC+BCD -> ABD+ADC,
when new edge (diagonal) is shorter
TODO this is buggy, it cannot be always swapped, only if abcd is
  convex!
```

#### 3.60 flip\_quality

flip edges, when mesh quality constraint improves

#### 3.61 gaussmat\_2d

matrix for gauss integration on a triangulation

#### 3.62 generate\_chews\_first

triangulate domain with chew's first algorithm

#### 3.63 generate\_from\_centreline\_1d

generate a mesh from centreline

#### 3.64 generate\_from\_centreline\_2d

```
generate mesh from centreline
TODO allow number of segments to change
sets up a simple quadrilateral mesh in S-N coordinates
centreline (must be sorted in streamwise direction)
input variables:
{\tt cS} : S (streamwise) coordinates of centreline
cL : N (spanwise) coordinate of left bank
cR : N (spanwise) coordinate of right bank
input variables controlling ouptut resolution:
S : S coordinate of slices in S-direction (diff(S) is element
   width)
      must be sorted in s-direction
n : n number of points per cross section
      (n-1) is number of elements per cross section
output variables:
mesh.\{X,Y,S,N\} : point coordinates
             : point indices of elements (corners of the
    quadrilaterals)
-> make it orthogonal to banks by using a spline along n
```

#### 3.65 generate\_frontal

#### 3.66 generate\_ghost\_elements

```
generate ghost elements, i.e. elements at the domain boundary,
these
elements can overlap

when the project flag set, ghost points are porjected to the
boundary,
the project flag is set for dual mesh generation
the project flag is unset for application of the boundary condition
```

### 3.67 generate\_gmsh

```
generate a mesh from a polygon using gmsh
```

inshp : file name of shape file of preloaded shape file
 containing a polygon

obase : base of output file name
resolution : struct containing default mesh resolution settings

resfile\_C: file names of shape files, defining local resolution in polygonal regions

opt : options, see below

this is a Static function

#### 3.68 generate\_hierarchical

generate a hierachical mesh by recursively splitting elements containing boundary points

#### 3.69 generate\_triangle

generate a mesh from a polygon using the programme "Triangle"

#### 3.70 generate\_uniform\_1d

generate a uniformly spaced 1D mesh

### 3.71 generate\_uniform\_quadrilateral

generate a uniform 2D mesh

#### 3.72 generate\_uniform\_tetra

uniformly tesselate a rhombic domain in 3D into tetrahedra

### 3.73 generate\_uniform\_triangulation

uniformly tesselate a rectangular (2d) domain into triangles

# 3.74 get\_facing\_and\_shared\_vertices

for a pairwise list (array) of triangles, determine there common and facing edges

#### 3.75 grid2tri

topologically split a uniform mesh on a rectangular domain into triangles

#### 3.76 import\_delft3d\_net

import mesh from Delft3d file ( {filanme}\_net.nc )

#### $3.77 \quad import\_msh$

import mesh from {filename}.msh files as generated by GSMH

#### 3.78 import\_triangle

import a mesh generated with triangle (ele and node)

#### 3.79 improve\_iterative\_relocate\_insert

### 3.80 improve\_iterative\_relocate\_uniform

### 3.81 improve\_relocate\_global1

iteratively improve angles to remove obtuse triangles

#### 3.82 improve\_relocate\_global2

improve mesh globally

#### 3.83 improve\_relocate\_global\_3

improve mesh quality globally

#### 3.84 improve\_relocate\_local

iteratively improve angles to remove obtuse triangles

# 3.85 improve\_relocate\_local\_old

iteratively improve angles to remove obtuse triangles

# 3.86 improve\_topology

improve mesh topology

#### 3.87 insert\_mid\_points

insert mid points into the mesh
the new mesh is of much lower quality, but if all edges are flipped
,
this leads to the sqrt(2) refinement

#### 3.88 insert\_steiner\_points

refine mesh by inserting steiner points (centre of circumference) for elements specified by  $\ensuremath{\mathsf{tdx}}$ 

#### 3.89 integrate\_1d

integrate a quantity val across the mesh

#### 3.90 integrate\_discharge

integrate discharge

#### 3.91 interp\_1d

interpolate on a 1D mesh

#### 3.92 interp\_2d

interpolate on a 2D mesh

# 3.93 interp\_fourier

interpolate values on the mesh using fourier methods

#### 3.94 interp\_tikhonov\_1d

interpolation with Tikhonov regularisation

#### 3.95 interp\_tikhonov\_2d

interpolation wiht Tikhonov regularisation in 2D

# 3.96 interp\_tikhonov\_3d

# 3.97 interpolate\_from\_boundary

interpolate interior values from the boundary

# 3.98 interpolate\_point

interpolate from samples to mesh points by IDW method

# 3.99 interpolation\_error\_1d

estimate interpolation error in 1D

# 3.100 interpolation\_error\_2d

interpolate error in 2D

# 3.101 interpolation\_error\_3d

estimate interpolation error in 3D

# 3.102 interpolation\_matrix\_1d

linear interpolation matrix from mesh points to arbitrary coordinates  $\ensuremath{\text{PO}}$ 

# ${\bf 3.103 \quad interpolation\_matrix\_2d}$

# 3.104 interpolation\_matrix\_3d

interpolation matrix for interpolation in 3D

#### 3.105 isacute

determine acute triangles

#### 3.106 isobtuse

determine obtuse triangles

#### 3.107 iterate\_smooth2

iteratively improve the mesh by smoothing

# 3.108 limit\_by\_distance

max edge length
minimum distance
TODO, this will always be zero

#### 3.109 make\_elements\_ccw

make all 2D elements clock wise (such that their area is positive)

# 3.110 merge\_duplicate\_points

merge duplicate points

# $3.111 \quad merge\_facing\_blunt\_triangles$

merge blunt triangles that face each other

#### 3.112 mesh1

mesh in 1D

#### $3.113 \quad mesh\_1d$

extract the 1d mesh

#### $3.114 \quad \text{mesh\_2d}$

extract the 1d mesh

# 3.115 mesh\_junctions

 $\begin{array}{c} \text{mesh junctions of a channel network} \\ \text{hold on} \end{array}$ 

# 3.116 nearest\_boundary

determine nearest boundary segment for each input coordindate

3.117 nedge\_

#### 3.118 nonobtuse\_refinement

nonobtuse refinement according to Korotov not feasible for most obtuse triangles

# 3.119 objective\_A

one objective function value per angle

# 3.120 objective\_T

wrapper for mesh optimisation objective functions univariate in triangles

# 3.121 objective\_angle

objective function for iterative angle improvement

#### 3.122 optimum\_angle

optimum angle for each vertex =  $360^{deg}$  / number of connected edges

# ${\bf 3.123} \quad orthogonality\_quadrilaterals$

orthogonality condition for quadrilaterals

# 3.124 path

path along edges

# 3.125 plot

plot the mesh (and a discretised function) as a surface and net

# 3.126 plot1d

plot 1D mesh

# 3.127 plot3

plot mesh and values

# **3.128** plotcs

plot cross section

# 3.129 project\_to\_boundary

project a point to the boundary

# 3.130 pval2eval

vertex to element value

# 3.131 quad2tri

quadrilaterals to triangles

#### **3.132** quiver

# 3.133 raster\_boundary

#### 3.134 recover\_edges

recover (boundary) edges

#### **3.135** refine

refine by splitting marked triangles

# 3.136 refine\_edge\_halving

mesh refinement by longest edge bisection

# 3.137 remove\_empty\_triangles

remove degenerated triangles with zero area

#### 3.138 remove\_isolated\_vertices

remove points that are not part of the mesh (gmsh leaves sometimes spurious points in the msh file)

# 3.139 remove\_points

remove points and associated elements

# 3.140 remove\_quartered\_triangles

point has connectivity 4 and is not on the boundary

#### 3.141 remove\_small\_islands

delft3D requires islands to have at least 7 edges this functions splits edges surrounding small islands

#### 3.142 remove\_triply\_connected\_boundary\_vertices

remove boundary vertices that are connected only to three vertices

#### 3.143 remove\_trisected\_triangles

remove trisected trianges point has connectivity 3 and is not on the boundary

#### 3.144 renumber\_point\_indices

renumber vertex indices

#### 3.145 resolve\_8\_vertices

improve mesh by removing one edge from vertices with 8-edges
(an interior vertex in a regular triangulation has 6 neighbours,
and unstructured meshes with local refinement are possible with
5 and 7 neighbours, 4,3, or 8 and more connected vertices are not
necessary

#### 3.146 restore\_acuteness

restore acuteness

Laplacian smoothing may at some places decrease the mesh quality, this locally restores acute elements

### 3.147 retriangulate

retriangulate the mesh

### 3.148 ruppert

refine the mesh using ruppert's algorithm

### 3.149 scale\_to\_boundary

scale hierarchical mesh to match boundary coordinates experimental  $% \left( 1\right) =\left( 1\right) \left( 1\right) +\left( 1\right) \left( 1\right) \left( 1\right) +\left( 1\right) \left( 1\right$ 

### 3.150 scatterplot

scatterplot of data on mesh

#### 3.151 section

### 3.152 segment

segment the mesh into parts according to laplacian eigenvalues

### 3.153 smooth2

Laplacian smoothing of vertex coordinates, replace every point by the average coordinate of its neibghbours

## $3.154 \quad smooth\_1d$

smoothes values in each reach does not smooth the values at the connection points

#### $3.155 \quad smooth\_val$

smooth values on the mesh
TODO allow for smooting boundary only along boundary

#### 3.156 smoothness

### 3.157 split3

split those triangles that contain a boundary point in three pieces , for hierarchical mesh generation

### 3.158 split\_edge

split an edge

# ${\bf 3.159}\quad {\bf split\_edge\_perpendicular}$

split edge perpendicularly

### $3.160 ext{ split_elem_1d}$

split a 1d element

### 3.161 split\_encroached\_edges

recursively split encroached edges

### 3.162 split\_obtuse

split obtuse elements

# 3.163 split\_unsmooth\_edges

split unsmooth edges

### 3.164 statistics

compute mesh statistics

### 3.165 streamwise\_derivative\_matrix

streamwise derivative matrix

### 3.166 thalweg

thalweg (deepest point along channel)

### 3.167 to\_single

TODO, also with indices

#### 3.168 uncross\_elements

make sure, that 4 point elements span an area, and do not form a
 cross
a call to this function should be succeeded by make\_ccw
this operator is idempotent

### ${\bf 3.169} \quad uncross\_quadrilaterals$

make sure, that 4 point elements span an area, and do not form a
 cross
a call to this function should be succeeded by make\_ccw
this operator is idempotent

#### 3.170 vertex\_distance

connectivity of directly connected vertices

### $3.171 \quad vertex\_to\_edge$

connectivity matrix between vertices and adjacent edges

#### 3.172 vertex\_to\_element

connectivity matrix between vertices and elements

#### 3.173 vertex\_to\_vertex

connectivity matrix between vertices

#### 3.174 vertices\_1d

### 3.175 weighed\_laplacian\_smoothing

weighed Laplacian smoothing

### $3.176 \quad xy2xys$

for boundary points: convert XY coordinate into a 1Dparametric coordinate, applied in mesh optimization, where movement of boundary points is constrained on the boundary

## $3.177 ext{ xys}2xy$

convert parametric 1D coordinate of boundary point back to cartesian XYc oordinate

#### 4 mesh

mesh generation, manipulation, analysis, refinement and optimization

### 4.1 add\_to\_rhs

### 4.2 append2buffer\_asym

#### 4.3 append2buffer\_dphi\_dphi

# 5 grid/@Grid1

#### 5.1 Grid1

lump spatiotemporal data into a 1-dimensional grid

#### 5.2 binop

operate function fun on data val within the context of a grid cell (for fitting grid cell values from sampled values)

#### 5.3 build\_index

compute the grid-cell index for samples sampled at points X1

name : name of the index field
X1 : coordinate of source points

R : cut off radius (if not supplied ident to mesh width)

#### 5.4 fit

lump (fit) sampled values into the corresponding grid cell

### 5.5 predict

interpolate from lumped data to specified location

# 6 grid/@Grid2

#### 6.1 Grid2

lump spatiotemporal data into a 2-dimensional grid

#### 6.2 binop

operate function fun on data val within the context of a grid cell (for fitting grid cell values from sampled values)

#### 6.3 build\_index

compute the grid-cell index for samples sampled at points X1

X1 : coordinate along first dimension
X2 : coordinate along second dimension

**6.4** plot

### 6.5 predict

interpolate from lumped data to specified location

# 7 grid/@Grid3

### 7.1 Grid3

lump spatiotemporal data into a 3-dimensional grid

### 7.2 build\_index

compute the grid-cell index for samples sampled at points X1

X1 : coordinate along first dimension
X2 : coordinate along second dimension
X3 : coordinate along third dimension

- 8 mesh1d
- 8.1 dxspace
- 8.2 dxspace2
- 8.3 dzmesh
- 8.4 mesh1

8.5 mesh1d
8.6 nlogstep
9 mesh
mesh generation, manipulation, analysis, refinement and optimization ${\bf 9.1}  {\bf nxfun}$
10 optimization 10.1 improve_smooth_insert
10.2 objective0_angle1_barycentric
10.3 objective0_angle2_barycentric
10.4 objective0_angle2_barycentric9
$10.5$ objective $0_{\text{angle}}$ $2_{\text{cartesian}}$

- $10.6 \quad objective 0\_angle\_inf\_cartesian$
- 10.7 objective0\_barycentric9
- 10.8 objective0\_pythagoras1\_barycentric9
- $10.9 \quad objective 0\_pythagoras 1\_cartesian$
- $10.10 \quad objective 0\_pythagoras 2\_barycentric 9$
- $10.11 \quad objective 0\_pythagoras 2\_cartesian$
- $10.12 \quad objective\_3\_angle$
- 10.13 objective\_A\_bnd
- 10.14 objective\_P\_angle
- 10.15 objective\_P\_angle\_scaled

- $10.16 \quad objective\_P\_angle\_scaled\_area$
- $10.17 \quad objective\_P\_midpoint$
- 10.18 objective\_angle
- $10.19 \quad objective\_angle2\_barycentric$
- 10.20 objective\_angle\_p
- $10.21 \quad objective\_angle\_scaled\_area$
- $10.22 \quad objective\_angle\_scaled\_circumference$
- 10.23 objective\_cosa
- 10.24 objective\_cosa\_p
- 10.25 objective\_cosa\_scaled\_side\_length

10.26	$objective\_distance\_edge\_centre$
10.27	$objective\_distance\_edge\_centre\_perpendicular$
10.28	$objective\_distance\_orthocentre\_excentre$
10.29	$objective\_incentre\_excentre$
10.30	$objective\_length\_min\_max$
10.31	$objective\_length\_var$
10.32	$objective\_thales$
10.33	$objective\_thales\_difference$

 $10.34 \quad test\_objective\_cosa\_p$ 

### 11 mesh

mesh generation, manipulation, analysis, refinement and optimization

#### 11.1 preload\_msh

# 12 sparsemesh/@SparseMesh1

### 12.1 SparseMesh1

lump time series of sampled spatial data in one dimension (
 projected)

#### 12.2 assign

assign (lump) data "v0" sampled at sample times/location to field "field"

### 12.3 assignS

lump sequentially sampled data "v0" and assign to field "field"

#### 12.4 init

initialize, segment sampling locations/times into blocks the
 sampled
data is lumped to

#### 12.5 interp

interpolate data stored in field "field" to coordinates Xi ingnore invalid data TODO, check if convex

#### 12.6 interpS

interpolate data stored in field "field" to coordinates  ${\tt Xi}$ , do not ignore invalid data

### 12.7 rmse\_interp

```
interpolation part of the error :
e ~ 1/2*d^2v/dx^2 * dx^2 + higher order terms
  ~ 1/2*d^2 v
the other part of the error is the sampling error (gaussian noise)
the mesh is optimal, when e_nois ~ e_interp
```

# 13 sparsemesh/@SparseMesh2

### 13.1 SparseMesh2

```
lump time series of sampled spatial data (track recordings) along
    two dimensions,
e.g 1 projected spatial dimension and one for time time
TODO : better blocks (all neighbours within mahalanobis distance)
TODO : do not use simple mean, but allow for least squares
    regression
TODO : precompute the least squares weights for accummarray
```

#### 13.2 assign

assign (lump) data "v0" sampled at sample times/location to field "
 field"

### 13.3 assignS

lump sequentially sampled data "v0" and assign to field "field"

### 13.4 init

initialize, segment sampling locations/times into blocks the
 sampled
data is lumped to

#### 13.5 interp

interpolate data stored in field "field" to coordinates Xi
ingnore data outside of the domain (convex interpolation)

#### 13.6 interpS

interpolate data stored in field "field" to coordinates Xi, extrapolate beyond domain

#### 13.7 rmse\_interp

```
interpolation part of the error :
e ~ 1/2*d^2v/dx^2 * dx^2 + higher order terms
    ~ 1/2*d^2 v
the other part of the error is the sampling error (gaussian noise)
the mesh is optimal, when e_nois ~ e_interp
TODO this is e ~ f', not f''
```

### 14 sparsemesh

lumping and interpolation of spatio-temporal data into a "mesh" that
 is spaced
optimally for the local density of sample points

allows for processing of large data sets with lower memory consumption and run time

intended for ADCP data processing

Overcomes the limitation of gridding, where some grid cells can have an insufficient number of samples

# 14.1 SparseMesh

SparseMesh superclass

- 15 test
- 15.1 test\_MMesh\_segment
- 15.2 test\_derivative\_matrix\_curvilinear\_2

# 16 mesh

 ${\tt mesh\ generation,\ manipulation,\ analysis,\ refinement\ and\ optimization}$ 

- 16.1 test\_nxfun
- 16.2 tidal\_funnel\_idealized
- $16.3 trimesh_fast$