

# Manual for Package: mesh

## Revision 1:6M

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## 1 @StructuredMesh

### 1.1 StructuredMesh

structured mesh processing  
compatible with Delft3D  
also provides set-up of discretisation matrices

### 1.2 apply\_boundary\_condition

apply boundary condition and the four sides of the domain  
TODO: allow for interior boudaries

### 1.3 bc\_from\_shp

read boundary condition from shape file



## 1.4 `bc_index`

TODO this is deprecated  
generate indices for boundary edges

## 1.5 `bc_isinvalid`

check boundary conditions for stacked domains

## 1.6 `block`

stack multiple meshes to complex domain

## 1.7 `boundary_chain`

return chain of boundary points

## 1.8 `boundary_direction`

return direction of boundary segment

## 1.9 `boundary_indices`

indices of boundary segments  
id : index of boundary point  
jd : index of

## 1.10 `cat`

## 1.11 `centreline`

domain (channel) centreline along chosen dimension

### **1.12 child**

hierarchical mesh generation (for bifurcations)

### **1.13 copy**

### **1.14 corner\_indices**

indices of domain corners

### **1.15 cut\_from\_domain**

cut subdomain

### **1.16 export\_delft3d\_bnd**

export the boundary in delft3d compatible format

### **1.17 export\_delft3d\_dep**

export bathymetry data in Delft3D dep-format

### **1.18 export\_delft3d\_grd**

export mesh in deltares delft3D grd file format

### **1.19 export\_delft3d\_ini**

export delft3D compatible initial condition file

## 1.20 export\_shp

export mesh elements as shape file

## 1.21 extend\_straight\_reach

## 1.22 extract\_elements

element indices from grid

## 1.23 flip\_dimension

flip left and right or top and down

## 1.24 from\_1d\_mesh

convert a 1D mesh to 2D mesh consisting of quadrilaterals

## 1.25 generate\_bifurcation

creates a mesh for bifurcation with bluff, which is required for  
delft3d grids

TODO do not fix indices

TODO determine p individually

bank : bankline shapefile

nn : number of points across branches

ds: spacing along s

p : fraction of right side branch

level : generate hierarchical mesh,

grid points in each branch will be  $2^{n+1}$ ,

and sub meshes until level 1 will be generated

for lower levels the connecting volumes remain narrow,  
as the two volumes left and right of the division line are not  
scaled

-> post smoothing required

```
nn: n=6; for idx=1:5; n(end+1) = 2*(n(end)-3)+3, end
ns: n=18; for idx=1:5; n(end+1) = 2*(n(end)-2)+2, end (should be
    improved to 2*(n-1)+1
```

### 1.26 generate\_disk

generate semicircular domain

### 1.27 generate\_from\_centreline

generate a mesh from a given centreline  
TODO : avoid crossing of inner bed points in sharp bends

### 1.28 generate\_rectangle

discretize a rectangular domain

### 1.29 generate\_structured\_grid

generate a structured mesh consisting of several sub-meshes

### 1.30 grid\_block

mesh a subdomain

### 1.31 improve

improve (smooth) the mesh

### 1.32 interp\_elem2point

interpolate values sampled at element centres to element corners  
TODO allow also interpolation to u and v points

### 1.33 mesh\_polygon

mesh a 1D channel, where boundaries are given as polygon  
TODO, this should better use voronoi-tesselation (see centreline  
class)

### 1.34 orthogonality

orthogonality of elements

### 1.35 orthogonalize

orthogonalize mesh  
set x of point coordinates to 1/2

### 1.36 plot

plot the mesh

### 1.37 plot\_boundary

plot the mesh boundary

### 1.38 plot\_coupling

plot connected vertices, see vertex\_connection\_matrix.m

### 1.39 plot\_orthogonality

plot mesh with edges colored by orthogonality condition

### 1.40 quiver

quiver plot of velocity

#### 1.41 read\_delft3d\_dep

depth in dat file is defined at volume centres (water level point)  
first row, first column and last column are buffer  
but last column is not (only when outflow?)

#### 1.42 read\_delft3d\_grd

read mesh in delft3D grd format

#### 1.43 smooth\_cubic

cubically smooth the mesh coordinates

#### 1.44 smooth\_curvilinear

smooth the mesh  
relax = (10+relax)/11;  
relax = min(0.5,relax);

#### 1.45 smooth\_laplacian

smooth the mesh coordinates

better than before, but causes dn in inner bends to be narrower  
than in outer bends  
(straightens the lines)  
better smooth p: i.e. fractional distance from left to right,  
this is complicated at the bif  
better: two neighbour smooth: smooth dn and ds with left/right, top  
bottom only

#### 1.46 smooth\_simple

smooth the mesh coordinates

### 1.47 `smooth_sn`

smooth the mesh coordinates

### 1.48 `snap`

snap two meshes that connect at their domain boundaries

### 1.49 `statistic`

compute mesh statistics

### 1.50 `to_unstructured_mesh`

convert to unstructured mesh

### 1.51 `transpose_dimension`

transpose dimensions

### 1.52 `vertex_connection_matrix`

connectivity of neighbouring vertices  
TODO same for elements

## 2 `@UnstructuredMesh`

### 2.1 `UnstructuredMesh`

class containing some meshing functionality  
complementary to `Mesh_2d`, `Mesh_3d`, `Tree_2d` and `Tree_3d`

## 2.2 add\_element

add an element with vertex indices, vertices already exist

## 2.3 add\_vertex

add a vertex

## 2.4 angle

interior angles of each element

## 2.5 assign\_1d

assign coordinates  $(x_0, y_0)$  to containing element  
TODO this can fail, if triangulation is not delaunay

## 2.6 assign\_2d

assign coordinates  $(x_0, y_0)$  to containing element

## 2.7 assign\_3d

assign coordinates  $(P_0, y_0)$  to containing element

## 2.8 bnd\_1d

left and right end points for 1D meshes

## 2.9 boundary\_1d

convert 1D mesh to 2D mesh



## 2.10 boundary\_chain2

get chained indices of boundary segments,  
used for setting up higher order polynomials along the boundary

## 2.11 boundary\_length\_and\_direction

edge length and direction of boundary segments  
TODO, this should be just edge length and direction

## 2.12 cat

concatenate two meshes

## 2.13 chain\_1d

chain 1D elements (segments)

## 2.14 check\_duplicate\_elements

check if elements are duplicate elements  
TODO, this does not check if elements cover each other, for example  
hierarchical meshes or ABC+BCD and ABD+ACD  
TODO check overlap by computation of area

## 2.15 check\_edge\_intersection

## 2.16 clip

clip mesh to polygonal domain  
TODO only works for triangles

## 2.17 `compute_elem2elem`

set up element2element neighbourhood relation

## 2.18 `connect_1d_2d`

auto merge 1d and 2d mesh

this silently requires that 1d segments consist at least of 3  
elements

TODO only implemented for triangles

## 2.19 `convert_2d_to_1d`

## 2.20 `copy`

copy constructor

## 2.21 `cross_section`

get cross-sections for 1D elements

## 2.22 `delete_element`

delete an element

## 2.23 `derivative_matrix_1d`

first order first derivative discretisation matrix on the 1d mesh

## 2.24 `derivative_matrix_2d`

first order first derivative discretisation matrix on the mesh

## 2.25 derivative\_matrix\_2d\_2

second order derivative matrix on a triangulation

## 2.26 derivative\_matrix\_3d

first order first derivative discretisation matrix on the mesh

## 2.27 distance

distance along edges from a point set to all other points

open : id of start point(s)  
countflag : if set use number of hops as distance not the euclidean  
distance

## 2.28 dual\_mesh

dual mesh formed by the centre of circumference  
the dual mesh consists not only of triangles  
TODO rename in generate dual mesh

## 2.29 edge\_length

euclidean edge length

## 2.30 edge\_midpoint

edge mid-points

## 2.31 edges\_from\_elements

edges and boundaries from elements

### 2.32 eigs

eigenvalues of the lapalcian on the mesh

### 2.33 elem2edge\_

pointer of element to edge

### 2.34 elem2elem\_matrix

matrix with neighbourhood relations for each element

### 2.35 element\_area

area of elements  
1d elements have zero area and are not processed

### 2.36 element\_centroid

centroids of lements

### 2.37 element\_midpoint

barymetric centre of elements

### 2.38 elements\_from\_edges

2D elements from edges

### 2.39 eval2pval

element (centroid) value to vertex value  
TODO, use dual mesh or triangulation

## 2.40 export\_delft3d\_net

export into DFLOWFM delft3d net.nc file

## 2.41 export\_msh

export mesh in GMSH msh format

## 2.42 export\_pos

export triangles and vertex values to gmsh pos-file format (x,y,z,  
val)  
intended for re-meshing with values representing local mesh size

## 2.43 export\_shp

export edges to GIS shapefile  
each element as separate polygon with one z-value

## 2.44 facing\_element

get triangle ndx that is opposit, e.g. "facing" the vertex vdx of  
triangle tdx

## 2.45 filter\_neighbour

apply a function on the values on connected vertices

## 2.46 find\_encroached\_edges

find encroached edges in a triangulation,  
i.e. edges for which on of the two facing point false into their  
enclosing  
circle

## 2.47 flip

```
flip edges between two triangles
  flip
  for each side
    if (connection between opposit points shorter than
        between edges, swap edge)
      this-> flip
      that-> flip
  end
```

## 2.48 flip\_global

```
recursively flip edges, i.e ABC+BCD -> ABD+ADC,
when new edge (diagonal) is shorter
TODO this is buggy, it cannot be always swapped, only if abcd is
convex!
```

## 2.49 flip\_quality

```
flip edges, when mesh quality constraint improves
```

## 2.50 gaussmat\_2d

```
matrix for gauss integration on a triangulation
```

## 2.51 generate\_chews\_first

```
triangulate domain with chew's first algorithm
```

## 2.52 generate\_from\_centrelines

```
generate a mesh from centrelines
```

## 2.53 generate\_from\_centreline\_2d

generate mesh from centreline  
TODO allow number of segments to change

sets up a simple quadrilateral mesh in S-N coordinates  
centreline (must be sorted in streamwise direction)  
input variables:  
cS : S (streamwise) coordinates of centreline  
cL : N (spanwise) coordinate of left bank  
cR : N (spanwise) coordinate of right bank  
input variables controlling output resolution:  
S : S coordinate of slices in S-direction (diff(S) is element  
width)  
must be sorted in s-direction  
n : n number of points per cross section  
(n-1) is number of elements per cross section  
output variables:  
mesh.{X,Y,S,N} : point coordinates  
mesh.T : point indices of elements (corners of the  
quadrilaterals)  
-> make it orthogonal to banks by using a spline along n

## 2.54 generate\_frontal

## 2.55 generate\_ghost\_elements

generate ghost elements, i.e. elements at the domain boundary,  
these  
elements can overlap

when the project flag set, ghost points are projected to the  
boundary,  
the project flag is set for dual mesh generation  
the project flag is unset for application of the boundary condition

## 2.56 generate\_gmsh

generate a mesh from a polygon using gmsh

`inshp` : file name of shape file of preloaded shape file  
containing a polygon  
`obase` : base of output file name  
`resolution` : struct containing default mesh resolution settings  
`resfile_C` : file names of shape files, defining local resolution in  
polygonal regions  
`opt` : options, see below  
  
this is a Static function

## 2.57 `generate_hierarchical`

generate a hierachical mesh by recursively splitting elements  
containing boundary points

## 2.58 `generate_triangle`

generate a mesh from a polygon using the programme "Triangle"

## 2.59 `generate_uniform_1d`

generate a uniformly spaced 1D mesh

## 2.60 `generate_uniform_quadrilateral`

generate a uniform 2D mesh

## 2.61 `generate_uniform_tetra`

uniformly tessellate a rhombic domain in 3D into tetrahedra

## 2.62 `generate_uniform_triangulation`

uniformly tessellate a rectangular (2d) domain into triangles



### 2.63 `get_facing_and_shared_vertices`

for a pairwise list (array) of triangles, determine there common  
and facing edges

### 2.64 `grid2tri`

topologically split a uniform mesh on a rectangular domain into  
triangles

### 2.65 `import_delft3d_net`

import mesh from Delft3d file ( {filename}\_net.nc )

### 2.66 `import_msh`

import mesh from {filename}.msh files as generated by GSMH

### 2.67 `import_triangle`

import a mesh generated with triangle (ele and node)

### 2.68 `improve_iterative_relocate_insert`

iteratively improve the mesh by inserting vertices and smoothing  
fprintf('Iteration %d, %d elements, %d vertices, %d  
obtuse elements (%g%%)\n', iter, obj.nelem, obj.np  
, nobtuse, nobtuse./obj.nelem);

### 2.69 `improve_iterative_relocate_uniform`

improve mesh by smoothing following by uniform refinement  
fprintf('Iteration %d, %d elements, %d vertices, %d  
obtuse elements (%g%%)\n', iter, obj.nelem, obj.np  
, nobtuse, nobtuse./obj.nelem);

## **2.70   improve\_relocate\_global1**

iteratively improve angles to remove obtuse triangles

## **2.71   improve\_relocate\_global2**

improve mesh globally

## **2.72   improve\_relocate\_global\_3**

improve mesh quality globally

## **2.73   improve\_relocate\_local**

iteratively improve angles to remove obtuse triangles

## **2.74   improve\_relocate\_local\_old**

iteratively improve angles to remove obtuse triangles

## **2.75   improve\_topology**

improve mesh topology

## **2.76   insert\_mid\_points**

insert mid points into the mesh  
the new mesh is of much lower quality, but if all edges are flipped  
,  
this leads to the  $\sqrt{2}$  refinement

## **2.77 insert\_steiner\_points**

refine mesh by inserting steiner points (centre of circumference)  
for elements specified by tdx

## **2.78 integrate\_1d**

integrate a quantity val across the mesh

## **2.79 integrate\_discharge**

integrate discharge

## **2.80 interp\_1d**

interpolate on a 1D mesh

## **2.81 interp\_2d**

interpolate on a 2D mesh

## **2.82 interp\_fourier**

interpolate values on the mesh using fourier methods

## **2.83 interp\_tikhonov\_1d**

interpolation with Tikhonov regularisation

## **2.84 interp\_tikhonov\_2d**

interpolation with Tikhonov regularisation in 2D

## 2.85 `interp_tikhonov_3d`

## 2.86 `interpolate_from_boundary`

interpolate interior values from the boundary

## 2.87 `interpolate_point`

interpolate from samples to mesh points by IDW method

## 2.88 `interpolation_error_1d`

estimate interpolation error in 1D

## 2.89 `interpolation_error_2d`

interpolate error in 2D

## 2.90 `interpolation_error_3d`

estimate interpolation error in 3D

## 2.91 `interpolation_matrix_1d`

linear interpolation matrix from mesh points to arbitrary  
coordinates P0

## 2.92 `interpolation_matrix_2d`

### **2.93 interpolation\_matrix\_3d**

interpolation matrix for interpolation in 3D

### **2.94 isacute**

determine acute triangles

### **2.95 isobtuse**

determine obtuse triangles

### **2.96 iterate\_smooth2**

iteratively improve the mesh by smoothing

### **2.97 limit\_by\_distance**

max edge length  
minimum distance  
TODO, this will always be zero

### **2.98 make\_elements\_ccw**

make all 2D elements clock wise (such that their area is positive)

### **2.99 merge\_duplicate\_points**

merge duplicate points

### **2.100 merge\_facing\_blunt\_triangles**

merge blunt triangles that face each other

## **2.101 mesh1**

mesh in 1D

## **2.102 mesh\_1d**

extract the 1d mesh

## **2.103 mesh\_2d**

extract the 1d mesh

## **2.104 mesh\_junctions**

mesh junctions of a channel network  
hold on

## **2.105 nearest\_boundary**

determine nearest boundary segment for each input coordinate

## **2.106 nedge\_**

## **2.107 nonobtuse\_refinement**

nonobtuse refinement according to Korotov  
not feasible for most obtuse triangles

## **2.108 objective\_A**

one objective function value per angle

## **2.109 objective\_T**

wrapper for mesh optimisation objective functions univariate in triangles

## **2.110 objective\_angle**

objective function for iterative angle improvement

## **2.111 optimum\_angle**

optimum angle for each vertex =  $360^\circ / \text{number of connected edges}$

## **2.112 orthogonality\_quadrilaterals**

orthogonality condition for quadrilaterals

## **2.113 path**

path along edges

## **2.114 plot**

plot the mesh (and a discretised function) as a surface and net

## **2.115 plot1d**

plot 1D mesh

## **2.116 plot3**

plot mesh and values

### **2.117   plotcs**

plot cross section

### **2.118   project\_to\_boundary**

project a point to the boundary

### **2.119   pval2eval**

vertex to element value

### **2.120   quad2tri**

quadrilaterals to triangles

### **2.121   raster\_boundary**

### **2.122   recover\_edges**

recover (boundary) edges

### **2.123   refine**

refine by splitting marked triangles

### **2.124   refine\_edge\_halving**

mesh refinement by longest edge bisection



### **2.125 remove\_empty\_triangles**

remove degenerated triangles with zero area

### **2.126 remove\_isolated\_vertices**

remove points that are not part of the mesh  
(gmsh leaves sometimes spurious points in the msh file)

### **2.127 remove\_points**

remove points and associated elements

### **2.128 remove\_quartered\_triangles**

point has connectivity 4 and is not on the boundary

### **2.129 remove\_small\_islands**

delft3D requires islands to have at least 7 edges  
this functions splits edges surrounding small islands

### **2.130 remove\_triply\_connected\_boundary\_vertices**

remove boundary vertices that are connected only to three vertices

### **2.131 remove\_trisected\_triangles**

remove trisected triangles  
point has connectivity 3 and is not on the boundary

### **2.132 renumber\_point\_indices**

renumber vertex indices

### 2.133 resolve\_8\_vertices

improve mesh by removing one edge from vertices with 8-edges  
(an interior vertex in a regular triangulation has 6 neighbours,  
and unstructured meshes with local refinement are possible with  
5 and 7 neighbours, 4,3, or 8 and more connected vertices are not  
necessary

### 2.134 restore\_acuteness

restore acuteness  
Laplacian smoothing may at some places decrease the mesh quality,  
this locally restores acute elements

### 2.135 retriangulate

retriangulate the mesh

### 2.136 ruppert

refine the mesh using ruppert's algorithm

### 2.137 scale\_to\_boundary

scale hierarchical mesh to match boundary coordinates  
experimental

### 2.138 scatterplot

scatterplot of data on mesh

### 2.139 section

## 2.140 segment

segment the mesh into parts according to laplacian eigenvalues

## 2.141 smooth2

Laplacian smoothing of vertex coordinates,  
replace every point by the average coordinate of its neighbours

## 2.142 smooth\_1d

smoothes values in each reach  
does not smooth the values at the connection points

## 2.143 smooth\_val

smooth values on the mesh  
TODO allow for smoothing boundary only along boundary

## 2.144 smoothness

mesh smoothness as ratio of maximum edge length and minimum edge  
length

## 2.145 split3

split those triangles that contain a boundary point in three pieces  
,  
for hierarchical mesh generation

## 2.146 split\_edge

split an edge

### **2.147 split\_edge\_perpendicular**

split edge perpendicularly

### **2.148 split\_elem\_1d**

split a 1d element

### **2.149 split\_encroached\_edges**

recursively split encroached edges

### **2.150 split\_obtuse**

split obtuse elements

### **2.151 split\_unsmooth\_edges**

split unsmooth edges

### **2.152 statistics**

compute mesh statistics

### **2.153 streamwise\_derivative\_matrix**

streamwise derivative matrix

### **2.154 thalweg**

thalweg (deepest point along channel)

## 2.155 to\_single

TODO, also with indices

## 2.156 uncross\_elements

make sure, that 4 point elements span an area, and do not form a  
cross  
a call to this function should be succeeded by make\_ccw  
this operator is idempotent

## 2.157 uncross\_quadrilaterals

make sure, that 4 point elements span an area, and do not form a  
cross  
a call to this function should be succeeded by make\_ccw  
this operator is idempotent

## 2.158 vertex\_distance

connectivity of directly connected vertices

## 2.159 vertex\_to\_edge

connectivity matrix between vertices and adjacent edges

## 2.160 vertex\_to\_element

connectivity matrix between vertices and elements

## 2.161 vertex\_to\_vertex

connectivity matrix between vertices

## 2.162 vertices\_1d

## 2.163 weighed\_laplacian\_smoothing

weighed Laplacian smoothing

## 2.164 xy2xys

for boundary points: convert XY coordinate into a 1Dparametric coordinate,  
applied in mesh optimization, where movement of boundary points is constrained on the boundary

## 2.165 xys2xy

convert parametric 1D coordinate of boundary point back to cartesian XYc oordinate

# 3 grid/@Grid1

## 3.1 Grid1

lump spatiotemporal data into a 1-dimensional grid

## 3.2 binop

operate function fun on data val within the context of a grid cell  
(for fitting grid cell values from sampled values)

### 3.3 build\_index

compute the grid-cell index for samples sampled at points X1  
name : name of the index field  
X1 : coordinate of source points  
R : cut off radius (if not supplied ident to mesh width)

### 3.4 fit

lump (fit) sampled values into the corresponding grid cell

### 3.5 predict

interpolate from lumped data to specified location

## 4 grid/@Grid2

### 4.1 Grid2

lump spatiotemporal data into a 2-dimensional grid

### 4.2 binop

operate function fun on data val within the context of a grid cell  
(for fitting grid cell values from sampled values)

### 4.3 build\_index

compute the grid-cell index for samples sampled at points X1  
X1 : coordinate along first dimension  
X2 : coordinate along second dimension

## 4.4 plot

## 4.5 predict

interpolate from lumped data to specified location

# 5 grid/@Grid3

## 5.1 Grid3

lump spatiotemporal data into a 3-dimensional grid

## 5.2 build\_index

compute the grid-cell index for samples sampled at points X1  
X1 : coordinate along first dimension  
X2 : coordinate along second dimension  
X3 : coordinate along third dimension

# 6 mesh1d

## 6.1 dxspace

## 6.2 dxspace2

## 6.3 dzmesh

## 6.4 mesh1



## 6.5 mesh1d

## 6.6 nlogstep

# 7 mesh

mesh generation, manipulation, analysis, refinement and optimization

## 7.1 nxfun

# 8 optimization

## 8.1 improve\_smooth\_insert

## 8.2 objective0\_angle1\_barycentric

## 8.3 objective0\_angle2\_barycentric

## 8.4 objective0\_angle2\_barycentric9

## 8.5 objective0\_angle\_2\_cartesian

8.6 objective0\_angle\_inf\_cartesian

8.7 objective0\_barycentric9

8.8 objective0\_pythagoras1\_barycentric9

8.9 objective0\_pythagoras1\_cartesian

8.10 objective0\_pythagoras2\_barycentric9

8.11 objective0\_pythagoras2\_cartesian

8.12 objective\_3\_angle

8.13 objective\_A\_bnd

8.14 objective\_P\_angle

8.15 objective\_P\_angle\_scaled

8.16 objective\_P\_angle\_scaled\_area

8.17 objective\_P\_midpoint

8.18 objective\_angle

8.19 objective\_angle2\_barycentric

8.20 objective\_angle\_p

8.21 objective\_angle\_scaled\_area

8.22 objective\_angle\_scaled\_circumference

8.23 objective\_cosa

8.24 objective\_cosa\_p

8.25 objective\_cosa\_scaled\_side\_length

8.26 objective\_distance\_edge\_centre

8.27 objective\_distance\_edge\_centre\_perpendicular

8.28 objective\_distance\_orthocentre\_excentre

8.29 objective\_incentre\_excentre

8.30 objective\_length\_min\_max

8.31 objective\_length\_var

8.32 objective\_thales

8.33 objective\_thales\_difference

8.34 test\_objective\_cosa\_p

## 9 mesh

mesh generation, manipulation, analysis, refinement and optimization

### 9.1 preload\_msh

## 10 sparsemesh/@SparseMesh1

### 10.1 SparseMesh1

lump time series of sampled spatial data in one dimension (  
projected)

### 10.2 assign

assign (lump) data "v0" sampled at sample times/location to field "  
field"

### 10.3 assignS

lump sequentially sampled data "v0" and assign to field "field"

### 10.4 init

initialize, segment sampling locations/times into blocks the  
sampled  
data is lumped to

### 10.5 interp

interpolate data stored in field "field" to coordinates Xi  
ignore invalid data  
TODO, check if convex

## 10.6 interpS

interpolate data stored in field "field" to coordinates Xi,  
do not ignore invalid data

## 10.7 rmse\_interp

interpolation part of the error :  
$$e \sim \frac{1}{2} d^2 v / dx^2 * dx^2 + \text{higher order terms}$$
$$\sim \frac{1}{2} d^2 v$$
the other part of the error is the sampling error (gaussian noise)  
  
the mesh is optimal, when  $e_{\text{nois}} \sim e_{\text{interp}}$

## 11 sparsemesh/@SparseMesh2

### 11.1 SparseMesh2

lump time series of sampled spatial data (track recordings) along  
two dimensions,  
e.g 1 projected spatial dimension and one for time time  
TODO : better blocks (all neighbours within mahalanobis distance)  
TODO : do not use simple mean, but allow for least squares  
regression  
TODO : precompute the least squares weights for accummarray

### 11.2 assign

assign (lump) data "v0" sampled at sample times/location to field "  
field"

### 11.3 assignS

lump sequentially sampled data "v0" and assign to field "field"

## 11.4 init

initialize, segment sampling locations/times into blocks the  
sampled  
data is lumped to

## 11.5 interp

interpolate data stored in field "field" to coordinates Xi  
ignore data outside of the domain (convex interpolation)

## 11.6 interpS

interpolate data stored in field "field" to coordinates Xi,  
extrapolate beyond domain

## 11.7 rmse\_interp

interpolation part of the error :  
$$e \sim \frac{1}{2} d^2 v / dx^2 * dx^2 + \text{higher order terms}$$
$$\sim \frac{1}{2} d^2 v$$
the other part of the error is the sampling error (gaussian noise)  
  
the mesh is optimal, when  $e_{\text{nois}} \sim e_{\text{interp}}$   
  
TODO this is  $e \sim f'$ , not  $f''$

## 12 sparsemesh

lumping and interpolation of spatio-temporal data into a "mesh" that  
is spaced  
optimally for the local density of sample points

allows for processing of large data sets with lower memory  
consumption and run time

intended for ADCP data processing

Overcomes the limitation of gridding, where some grid cells can have  
an insufficient  
number of samples

## 12.1 SparseMesh

SparseMesh superclass

## 13 test

### 13.1 test\_MMesh\_segment

### 13.2 test\_derivative\_matrices\_curvilinear

## 14 mesh

mesh generation, manipulation, analysis, refinement and optimization

### 14.1 test\_nxfun

### 14.2 trimesh\_fast