Manual for Package: open-channel-flow
Revision 1:7M

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January 27, 2020

Contents

1 @Backwater1D

1.1 Backwater1D

```
solve the gradually varied flow equation (backwater equation)
in one dimension

c.f. Chow, Bresse
```

1.2 backwater_approximation

```
approximation of the backwater curve by an exponential function
note: this is not necessarily a good approximation
in the case of tide, Qt can be given
```

1.3 backwater_curve_iterative

```
analytic solution of the gradually varied flow equation
c.f. Bresse, Chow
```

1.4 backwater_length

```
backwater length
```

1.5 dh_dx

```
change of depth along channel for the backwater equation
beta : momentum coefficient
this is effectively an equation in h^3
```

1.6 dh_dx_

1.7 dzs_dx

```
change of surface elevation along channel
```

1.8 gvf_x_chow

```
analytical solution to the gradually varied flow equation (
    backwater equation)
c.f. Chow, Bresse
```

1.9 invert

```
determine bed level from surface elevation
(inverse backwater equation)
this is ill conditioned, as the surface is smooth for subcritical
    flow,
even if the bed is not smoth

C : chezy
W : width
Q : discharge
S : bed slope
y0 : surface elevation at outflow
lateral inflow
```

1.10 solve

```
solve the gradually varied flow equation (backwater equation)
C : chezy
W : width
Q : discharge
S : bed slope
y0 : surface elevation at outflow
```

1.11 solve_analytic

```
analytical solution to the gradually varied flow equation (bresse
    method)
u_.^(n-m)./(1-u_.^n)
```

1.12 solve_matrix

2 @Lateral_Diversion_Finite_Width

2.1 Jb

2.2 Lateral_Diversion_Finite_Width

2.3 dR

2.4 derive

2.5 evalk

2.6 lateral_outflow_finite_width1

2.7 load_functions

2.8 stagnation_point

```
fdx    = isnan(x);
```

2.9 streamline

2.10 streamline_radius_of_curvature

2.11 u_far

2.12 v_far

2.13 velocity

2.14 velocity_near_bed

3 @Lateral_Diversion_Wide_Channel

3.1 Lateral_Diversion_Wide_Channel

3.2 derive_lateral_outflow

```
derive potential flow solution to lateral outlfow from an
    infinitely
wide main channel
```

3.3 derive_lateral_outflow_finite_width

```
derive coefficients for lateral outflow in the case of potential
    flow
```

3.4 lateral_outflow

```
potential flow solution to the case of lateral outflow from an
    infinitely
wide channel
```

3.5 lateral_outflow_finite_width

```
analytical potential flow solution to lateral outflow from an
    infinitely
wide channel
```

4 @Lateral_Diversion_Wide_Channel_Map

4.1 Lateral_Diversion_Wide_Channel_Map

```
wrapper to store precomputed streamlines of potential flows
```

4.2 streamline

5 @Potential_Flow

5.1 Potential_Flow

```
numerical solution of the potential flow on a curvilinear grid
(not necessarilly curvilinear)
```

5.2 apply_boundary_potential_old

5.3 assemble_discretization_matrix_rectilinear

```
assemble the discretisation matrix
```

5.4 assemble_potential_matrix

```
assemble the discretisation matrix for potential flow
```

5.5 bc_dirichlet

```
apply Dirichlet boundary conditions
```

5.6 boundary_condition_side_outflow

```
apply boundary conditions for side outflow
p*phi + (1-p)*d/db phi = rhs
y : along channel coordinate
```

5.7 boundary_condition_side_outflow_1

```
apply boundary conditions
p*phi + (1-p)*d/db phi = rhs
```

5.8 contour

```
contour plot of the potential flow solution
```

5.9 cut_boundary

```
cut the boundary from the domain
wa : width of inlet to side channel
wb : width of side channel
```

5.10 cut_rectangle

```
cut a rectangle from the domain
TODO, this requires also an adaptation of the derivative matrices
     -> step over to semi-unstructured mesh
```

5.11 infer_bed_level

```
note: this is pretty much a broken function for the inference of
    stationary
     morphology

Missing:
- rolling down of transverse slope to balance secondary flow in
    bends
- quasi time steippong

at stationary state:
- changes of discharge along the streamlines of discharge are
    balanced
  by a change in depth, to keep the velocity and sediment transport
       constant along the streamline

dz_b/dt = dqs/dx + dqs/dn = 0               (i)
```

```
TODO this only true for infinite bends, as sediment can also move
    to the side
dqs/ds = d/s(q/h) = 1/h dq/ds - q/h^2 dh/ds = 0
TODO this is only true in an ifinite bend (ikeda)
dqs/dn = 0
streamlines along discharge or velocity -> does not matter eq (i)
    is direction independent
```

5.12 infer_bed_level2

```
infer the bed level
```

5.13 infer_bed_level3

5.14 infer_bed_level_loop

```
the bed level does not completely converge but starts to oscillate,
this is presumably due to the non-compact kernel implementation of
    the laplacian oberator
```

5.15 objective_bed_level

```
objective function for determining the bed level
```

5.16 old

5.17 plot

```
surface plot
```

5.18 quiver

5.19 sediment_transport

```
compute the sediment transport
```

5.20 solve_potential

```
solve for the flow potential
```

5.21 streamline

```
compute a streamline
```

5.22 surface_elevation

```
compute surface elevation according to Bernoulli's law
```

5.23 test

5.24 velocity_near_bed

```
determine the velocity near the bed
```

5.25 vertical_velocity

```
determine the vertical velocity from continuity
```

6 @Potential_Flow_Analytic

6.1 Potential_Flow_Analytic

```
analytical solutions to various depth-averaged potential flow
    problems
```

6.2 streamline

```
numerically follow path along streamline by integrating the
    velocity
```

7 @SWE

7.1 SWE

```
Class to solve the (cross sectionally averaged) shallow water
    equation
(st venant equation)
```

7.2 bc_incoming_non_reflecting

```
set non-reflecting boundary condition for the 1D SWE
```

7.3 bc_inflow

```
inflow boundary condition
```

7.4 bc_inflow_low_pass

```
set low frequency Dirichlet, high frequency pass boundary condition
```

7.5 bc_inflow_non_reflecting

```
set non-reflecting boundary condition
```

7.6 bc_level

```
set surface level as Dirichlet boundary condition
```

7.7 bc_level_sommerfeld

```
set surface level as boundary condition by sommerfeld method
```

7.8 bc_nonreflecting

```
set non-reflecting boundary condition
extrapolate 0-order
```

7.9 bc_reflecting

```
set reflecting boundary condition
extrapolate 0-order and invert v
```

7.10 dot

```
time derivative
(only for matlab internal ode-solver)
TODO this is not swe specific
continuity
dA/dt + dQ/dx = I

momentum
dQ/dt + d/dx( Qu + 1/2 gh^2) = gA(S_f - S_b)
S_b = dz_b/dx
S_f = tau_x/rho_w = C_f u|u|
```

7.11 dt_cfl

```
determine time step required by cfl
```

7.12 energy

```
determine total energy as sump of potential and kinetic energy
this is preserved for fricitionless flows
```

7.13 flux

```
st venant's shallow water equation fluw
```

7.14 flux_lin

```
linearised st-venant equation
```

7.15 fluxmateig

```
eigenvalues und vectors of the swe
```

7.16 jacobian

```
Jacobian of the SWE

dq/dt + J dq/dx = sourceterm
note: d/dx(A*q) = J dq/dx
```

7.17 lindot

```
linearised SWE
width variation not included, goes into rhs force term

[      0,  1] [A]   = [   Q]
[ -u^2+gH, 2u] [Q]_dx [Q^2/A+1/2gA^2/w]_dx - 1/2gA^2/w^2 dw/dx
                                          force term
```

7.18 roe_average

```
roe average for the SWE
```

7.19 solve_analytic

```
linearised analytic solution of the swe
```

7.20 solve_stationary

```
stationary solution to the SWE
```

7.21 source_bed_level

```
source term of the SWE caused by a change of the bed level

Note: this term causes splitting and averaging methods to fail to
      give accurate predictions of the smooth surface at steps of
         the bed
```

7.22 source_friction

```
friction source term of the SWE
```

7.23 source_width

```
source term (reaction term) for channels with variable width
```

7.24 swe_geometry

```
predefined functions to set up channel geometry
```

7.25 swe_ic

```
predefined functions of channel geometries
```

8 @SWE_2d

8.1 SWE_2d

```
Dynamic solution of the shallow water equation (depth average, 2D)
```

8.2 apply_boundary_condition_stationary

```
apply boundary condition for stationary flow
```

8.3 assemble_stationary

```
TODO, g should be replaced by gx,gy,gz, see chaudhri
assemble discretisation matrix for stationary flow
```

8.4 solve_stationary

```
solve SWE for statinary flow (dU/dt = dQ/dt = 0)
```

9 @Side_Weir

9.1 Side_Weir

```
side weir, analytical solution to (critical) lateral outflow
```

9.2 dzs_dx

```
side weir, along channel surface gradient
```

9.3 surface_elevation

```
along-channel surface elevation for (critical) lateral outflow over
     a side-weir
```

10 open-channel-flow

```
functions for open channel flow, sub modules:

@Backwater1D
       gradually varied flow in 1D (backwater)
@Potential_Flow
```

```
       depth averaged potential flow, numerical solution
@Potential_Flow_Analytic
       depth averaged potential flow, analytical solution
rating-curve
       empirical rating curves
@Side_Weir
       analytical solution to lateral outflow over a side weir
@SWE
       dynamical solution of the shallow water equation (saint-
           venant-equation)
       in 1D
@SWE_2d
       dynamical solution of the shallow water equation (saint-
           venant-equation)
       in 2D
velocity-profile
       vertical and transverse velocity profiles of the streamwise
           velocity
```

10.1 Lateral_Diversion_Finite_Width_Map

10.2 chezy2manning

10.3 chezy2z0

10.4 diffusion_wave

```
propagation of a diffusion wave (flood wave), c.f. ponce
advection
diffusion
where is the bed slope?
friction slope
eddy slope
chow 1988
d(A+A0)/dt + dQ/dx = q
dQ/dt + d/dx betaQ^2/A + gA(dh/dx + Sf + Se) - beta q_i v_i + Wf B
    = 0
A0 ignored
inflow and wind shear ignored
```

10.5 friction_slope

```
friction slope (surface slope) for uniform stationary flow
```

10.6 linear_wave

```
linear wave routing (linearised kinematic wave)
```

10.7 manning2chezy

10.8 manning2drag

10.9 manning2z0

11 meander-bend/@Equilibrium_Bend

11.1 Equilibrium_Bend

```
Transverse profile of the bed level and bed material grain size in
an equilibrium (infintely long) meander bend
```

11.2 bed_profile

```
predict transverse bed profile of an equilibrium meander bend
```

11.3 bed_profile_uniform

```
transverse profile of the bed level of an equilibrium meander bend
with uniform grain size
```

11.4 calibrate

```
calibrate bend geometry to given profile
```

11.5 dD_dr

11.6 dh_dr

```
across channel derivative of flow depth for a meandering river
```

11.7 dh_dr_uniform

```
transverse gradient of the bed level of an equilibrium meander bend
for the case of uniform bed material
```

11.8 grain_size_profile

```
transverse (across channel) profile of the bed material grain size
in a river meander
```

12 meander-bend

12.1 Kinoshita

```
% Public properties
% Public get properties
% Private properties
% Constructor
% Setters and getters
% generic methods
```

12.2 bend_transverse_velocity

```
transverse velocity profile in a meander bend
```

12.3 bend_velocity_near_bed

```
near-bed-velocity in a meander bend
```

12.4 kinoshita_

12.5 random_meander

```
generate a pseudo random meander
```

12.6 test_rozovskii

13 old

13.1 UniformFlow

14 rating-curve

14.1 ChezyRatingCurve

```
rating curve, Chezy formalism
```

14.2 DynamicKeuleganRC

```
Dynamic Rating Curve, Keulegan roughness formulation
(dynamic = correction for hysteresis loop)
```

14.3 DynamicManningRC

```
Dynamic Rating Curve, Manning roughness formulation
(dynamic = correction for hysteresis loop)
```

14.4 DynamicPowerRC

```
Dynamic Power Law Rating curve
(dynamic = correction for hysteresis loop)
```

14.5 KeuleganRatingCurve

14.6 ManningRatingCurve

14.7 PolyRatingCurve

14.8 PowerRatingCurve

```
stationary rating curve, power law
```

14.9 PowerRatingCurveOffset

```
stationary rating curve, stage-discharge follows power law
```

14.10 RatingCurve

```
Fri Feb 13 10:02:52 CET 2015
rating curve superclass
```

14.11 csarea

```
predict cross sectional area from transverse bed level profile
and surface elevation
```

14.12 csdischarge

```
compute discharge
```

14.13 csperimeter

```
compute wetted perimeter
```

14.14 csradius

```
compute hydraulic radius of the cross section
```

14.15 cswidth

```
determine cross section width
```

14.16 test_PowerRatingCurve

14.17 wfunc

```
determine channel width
```

15 open-channel-flow

```
functions for open channel flow, sub modules:

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       in 1D
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       dynamical solution of the shallow water equation (saint-
           venant-equation)
       in 2D
velocity-profile
       vertical and transverse velocity profiles of the streamwise
           velocity
```

15.1 shear_velocity

15.2 surface_slope

```
 surface slope for uniform stationary flow
```

15.3 sw_reflection

```
 reflection coefficients of shallow water waves at a sudden change
     of the
 cross section (sudden change of admittance)
 c.f. lighthill, ippen-harleman
```

15.4 sw_reflection_stepwise

```
time passes and phase shifts
transmission and reflection coefficient depend on direction !
iterative (recursive) reflection and transmission
```

16 test/test_Backwater1D

16.1 test_bw1d_solve_matrix

17 open-channel-flow

```
functions for open channel flow, sub modules:

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       gradually varied flow in 1D (backwater)
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       depth averaged potential flow, numerical solution
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       dynamical solution of the shallow water equation (saint-
           venant-equation)
       in 1D
@SWE_2d
       dynamical solution of the shallow water equation (saint-
           venant-equation)
       in 2D
velocity-profile
       vertical and transverse velocity profiles of the streamwise
           velocity
```

17.1 test_inverse_backwater_curve

17.2 test_normal_flow

17.3 test_nse_nz

18 uniform-stationary-flow

18.1 chezy2drag

18.2 critical_flow_depth

```
critical flow depth in uniform stationary flow
```

18.3 drag2chezy

```
convert drag coefficient to chezy coefficient
g dz_s/dx + cd w u^2/h = 0 (swe formalism)
     - S  +  1/C^2 U^2/H = 0 (chezy formalism)
```

18.4 normal_flow_depth

```
normal flow depth for uniform stationary flow
function H = normal_flow_depth(Q,W,C,S)
```

18.5 normal_flow_depth_

```
normal flow depth in uniform stationary flow
```

18.6 normal_flow_discharge

```
normal flow discharge for uniform stationary flow
```

18.7 normal_flow_slope

```
normal flow slope in uniform stationary flow
```

18.8 normal_flow_velocity

```
normal flow velocity in uniform stationary flow
```

18.9 normal_shear_velocity

19 velocity-profile/@Log_profile

19.1 Log_profile

```
logarithmic profile of the streamwise velocity
```

19.2 df_dh

```
sensitivity of profile with respect to depth
```

19.3 df_dh_

```
sensitivity of profile with respect to depth
```

19.4 df_dln_z0

```
sensitivity of velocity profile with respect to roughness length
```

19.5 df_dln_z0_

```
sensitivity of profile with respect to roughness length
```

19.6 profile

```
vertical profile of the streamwise velocity
```

19.7 profile_

```
scale of velocity at instrument depth to depth average velocity
roughness length and associated standard error can change in time,
i.e. may be passed as vectors

zs    : [1xn] water surface level
zb    : [1x1] bottom level
za    : [1xn] or [1x1]
        level of velocity measurement,
        i.e. level of HADCP beam bin centre, coincides with
            instrument level,
        if the HADCP is horizontally aligned
        only needs to be passed as vector if instrument is
            redeployed or
        becomes misaligned
ln_z0 : [1xn] or [1x1]
        natural logarithm of the roughness length
s     : [1xn] or [1x1]
        standard error of ln_z0
function [fz_mu fz_s fz_sp fz_bias fz_eps] = log_profile(zs,zb,za,
    ln_z0,s,sp,e)
```

19.8 profile_bias

19.9 regmtx

```
regression matrix
```

19.10 ubar

```
depth averaged velocity
```

20 velocity-profile/@Log_profile_with_bend_correction

20.1 Log_profile_with_bend_correction

```
vertical velocity profile corrected for bend flow
```

20.2 df_dc

```
sensitivity of the velocity profile with respect to the bend
    correction
parameter c
```

20.3 df_dc_

20.4 du_dz

20.5 fit

```
fit the vertical velocity profile
```

20.6 profile_

```
vertical velocity profile
```

20.7 regmtx

```
regression matrix
```

20.8 u

```
streamwise velocity
```

20.9 u_

```
streamwise velocity
```

21 velocity-profile/@Log_profile_with_cubic_wake

21.1 Log_profile_with_cubic_wake

```
log profile with cubic wake
```

21.2 df_dc

```
sensitivity of profile with respect to wave parameter
```

21.3 df_dc_

```
sensitivity of profile with respect to wake parameter
```

21.4 profile_

```
vertical velocity profile
```

21.5 regmtx

```
regression matrix
```

22 velocity-profile/@Log_profile_with_dip

22.1 Log_profile_with_dip

```
Logarithmic profile with dip
```

22.2 fit

```
fit the vertical velocity profile
```

23 velocity-profile/@Log_profile_with_linear_bend_correction

23.1 Log_profile_with_linear_bend_correction

```
log profile with linear bend correction
```

23.2 df_dc

```
sensitivity of profile with respect to wake parameter
```

23.3 df_dc_

```
sensitivity of velocity profile with respect to wave parameter
```

23.4 du_dz

```
velocity shear along vertical
```

23.5 profile_

```
velocity profile
```

23.6 regmtx

```
regression matrix
```

24 velocity-profile/@Log_profile_with_wake

24.1 Log_profile_with_wake

```
logarithmic velocity profile with wake correction
c.f. coles
```

24.2 df_dc

```
sensitivity of profile with respect to wake parameter
```

24.3 df_dc_

```
sensitivity of velocity profile with respect to wake parameter
```

24.4 du_dz

```
velocity shear
```

24.5 profile_

```
predict velocity profile
```

24.6 regmtx

```
log law with wake
u = us/k ln(z) - us/k ln(z0) + us/k (2/H^2 z - 3/H^3 z^2)
```

25 velocity-profile/@VP

25.1 VP

```
velocity profile
```

25.2 process_joint

25.3 process_transverse_profile

```
process the transverse velocity profile
```

25.4 process_vertical_profile

```
predict vertical profile error distribution parameter for HADCP
    error estimate
```

25.5 profile_prediction_error

```
input :
U     : [nbin x nens]
        - values for each bin (or across section) and ensemble (or
            reference measurement)
        this are estimates estimates of the discharge or the cross
            sectional averaged
        velocity from the raw values
        - the profile should be limited to the effective profiling
            range,
        abobj 75-100m for a 600kHz ADCP

dn    : distance between HADCP bins
width : cross section width

objput:
      sd_n : expected standard deviation for increasing profiling
          range
function [s_rel s_err s_dat rho res m2 u_pred fdx] =
    velocity_variation(U)
hadcp_prediction_error
TODO take scales and unscaled velocity to do combine with harmmean
    estimate
note: previus versions:
       residual was computed with respect to the predicted local
           velocity
       mse was not upscaled to cs, as profile was expected to cover
           entire cs
       finite width of cs was not considered
parametric estimate from moments, objliers should be filtered
    beforehand
Note that the median absolute deviation is not a good estimate,
because it may excludes rare events like reverse flow of floods
thus, the only acceptible more robust estimate would be mean
    absolute deviation
```

26 velocity-profile/@Vertical_profile

26.1 Vertical_profile

```
vertical profile of the streamwise velocity, superclass
```

26.2 fit

```
fit vertical velocity profile parameter

function obj = fit(obj,U,S,h,binmask)
```

26.3 u

```
predict velocity along the vertical based on profile
```

27 velocity-profile

27.1 fit_displacement_profile

```
fit the log profile to the vertical profile of the streamwise
    velocity
```

27.2 lateral_division_method

```
transverse (across channel) profile of the streamwise velocity
in a straight channel
numerical solution
the eps seems incorrect, use better stationary_1d_swe

rho g h S - beta q^2 f / (8 h^2) + d/dy(eps_t dq/dy) = 0
rho g h S - beta q^2 g / (C^2 h^2) + d/dy(eps_t dq/dy) = 0
```

27.3 test_law_of_the_wall_fit

27.4 transverse_profile_parameter

27.5 transverse_velocity_profile

```
transverse profile of the streamwise velocity
c.f. shiono knight
```

27.6 transverse_velocity_profile_olesen

```
transverse profile of the streamwise velocity in a meander bend
```

27.7 transverse_velocity_profile_rozovskii

```
transversal velocity distribution in a bend
Rososkii,
as in the book central differences along the radius and euler
    forward in space
are used, note that since the advent of the computer more advanced
    schemes
could be used (see build in solvers)
cfl condition is not explicitely checked
Rosovsky assumes a constant water level, e.g. does not consider
    superelevation

I_theta = -1/r dz/d_theta   (p. 22)
d_theta = 1/R ds|_R
=> I_theta = -R/r dz/ds = -R/r I_0
It : (1.32) drop of level per unit angle, identical across section
```

27.8 transverse_velocity_profile_shiono_knight

```
transverse profile of the streamwise velocity, determined
    analytically
by the method of shiono and knight
shape of velocity profile only dependent on lambda, f, H, not slope
```

27.9 transverse_velocity_profile_tidal_channel

27.10 transverse_velocity_profile_with_slope

```
stationary 1D shallow water equation across a river section
0 = - g h S0 - tau_b/rho + d/dn (nu h du/dn)
0 = - g h S0 + g u^2/C^2 + d/dn (nu h du/dn)
includes tranvese gradient term

note that shiono/knight 1991 provide an _analytic_ solution,
which takes the form of an expontially decaying side wall effect
```

27.11 vertical_profile_of_velocity_vriend

```
vertical profile of the streamwise velocity, method of de vriend
```

27.12 vertical_velocity_profile

```
vertical profile of the streamwise velocity in non-uniform flow
```

27.13 z2s_rational

28 wrapper

28.1 discharge2stage

```
wrapper function
```

28.2 stage2discharge

29 open-channel-flow

```
functions for open channel flow, sub modules:

@Backwater1D
       gradually varied flow in 1D (backwater)
@Potential_Flow
       depth averaged potential flow, numerical solution
@Potential_Flow_Analytic
       depth averaged potential flow, analytical solution
rating-curve
       empirical rating curves
@Side_Weir
       analytical solution to lateral outflow over a side weir
@SWE
       dynamical solution of the shallow water equation (saint-
           venant-equation)
       in 1D
@SWE_2d
       dynamical solution of the shallow water equation (saint-
           venant-equation)
       in 2D
velocity-profile
       vertical and transverse velocity profiles of the streamwise
           velocity
```

29.1 z0tochezy