Manual for Package: tide
Revision 6:13M

Karl Kästner

March 28, 2020

Contents

1 @T_Tide

1.1 T_Tide

```
wrapper for TPXO generated tidal time series
```

1.2 build_index

```
build a structure whose field names contain the index
```

1.3 from_tpxo

```
read TPXO output into tidetable object
```

1.4 get_constituents

```
extract constituents of tpxo object
```

1.5 reorder

```
order constituents as specified by "name"
```

1.6 select

```
select a subsect of constituents
```

1.7 shift_time_zone

```
shift phase according to time zone
```

2 @Tidal_Envelope

2.1 Tidal_Envelope

```
process tidal data to extrac the tidal envelope
```

2.2 init

```
initialize with data
```

3 @Tide_wft

wavelet analysis of tidal data

3.1 Tide_wft

```
wavelet transform of tidal time series
```

3.2 transform

```
wavelet transform tidal time series
input:
time   : [1xn] abszissa of input vector, for example time, must be
    equally spaced
val    : [1xn] signal, input data series (e.g water level or
    velocity)
F      : [1xm] base frequencies, 1, 1, 2, ... for mean level,
    diurnal, semidirunal ...
              base periods from base frequencies T=1/F
n      : [1xm] wavelet window length in multiple of periods
fc, nc : [scalar] low frequency cutoff and window length in periods
winstr : [char] fourier windows (kaiser (recommended), hanning, box
    , etc)
dt_max : [scalar] maximum time to fill gaps in input data series (
    recommended 3/24 for tide)
```

```
output:
tide  : struct with fields
        w_coeff  : [1xn] wavelet coefficients (complex)
        amplitude : amplitude
        phase    : phase
        range    :
        h_tide   :
        h_low    :
        h
```

4 @Tidetable

class for generating tidetable data

4.1 Tidetable

```
Tide table
```

4.2 analyze

```
extract tidal envelope from time series
```

4.3 export_csv

```
export tide table to csv file
```

4.4 generate

```
run TPXO to generate time series
```

4.5 generate_tpxo_input

```
generate tpxo input table
Note: superseeded by perl script
```

4.6 import_tpxo

```
import TPXO data into tidetable object
```

4.7 plot_neap_spring

```
plot average neap and spring tide
```

5 tide

```
analysis prediction of tides in rivers and estuaries by empirical
    and theoretical methods
```

5.1 constituents

5.2 doodson

```
frequency of tidal constituents
method of doodson
source: wikipedia
```

5.3 envelope_amplitude

```
compute envelopes of hw and low water
```

5.4 envelope_slack_water

```
slack water envelope of the tide
```

5.5 interval_extrema

```
times and evelations for high and low water
```

5.6 interval_extrema2

```
mimimum and maximum within intervals of constant length,
intended for periodic functions
```

5.7 interval_zeros

```
times of slack water determined frim velocity u
```

5.8 lunar_phase

```
lunar phase
```

5.9 rayleigh_criterion

```
raleigh criterion for resolving tidal constituents
T > 1/|f1-f2|
```

6 river-tide/@River_Tide

```
predict tide in a backwater affected river with a sloping/varying
    bed

Assumptions and capabilities:
        - tidal dynamics follow the 1D-Shallow-Water-Equation
          (depth and cross-sectionally averaged Navier-Stokes-
              Equation)
        - rectangular cross section
        - width can vary along the channel
        - friction coefficient (cd) constant along channel and over
            time (Chezy)
        - advective accelleration term is considered, but can be
            deactivated
        - vertical profile of streamwise velocity is constant
          (Boussinesq coefficient is unity (1))

Limitations / TODO list:
        - single channel dynamics only (no tidal networks)
        - no wind-shear stress (no storm surges)
        - no tidal flats / intertidal areas (width constant in time)
```

```
        - no flood-plain during high-river flow
        - no stratification or along-channel salinty gradient
        - negligible head loss in channel bends
        - negligible feed-back of the sediment concentration on the
            propagation of the tide
        - low Froude Number (no hydraulic jumps due to cataracts or
            tidal bores)

- At present, only two tidal components are supported
  (either D1 with D2 or D2 with D4, in addition to the mean water
      level z0),
  for mixed diurnal-semidiurnal cases with dominant semidiurnal
      component,
  the class has to be extended to support three components (D1, D2
      and D4)

- At present, the tripel overtide is not computed (D3 for diurnal,
    D6 for semindiurnal tide),
  note that this is the main overtide for the case of low river flow

- At present, the 1/h non-linearity is only included in the
    approximations of
  the backwater curve, but not it's influence on the tidal frequency
       components

Method:

This class calls numerical solvers for second order ordinary
    differential
equation boundary value problems

Tides is represented as exponential series in form of total
    discharge Q = sum Q_i = Q_0 + Q_1 + Q_2,
as discharge is conserved (balanced), the equations are simpler than
     for level z and velocity u,
and the frequency components of z are straight forward determined by
     differentiation of Q

Class and function structure:
        River_Tide :
                computes river tide, provides the ode coefficients to
                    the boundary value solver
        bvp2c, bvp2fdm :
                solve the underlying second order boundary value
                    problem
        River_Tide_Map :
                provides convenient batch runs and processing of
                    River_Tide instances
```

```
Minimum working example, c.f. example_rive_tide.m and
    example_river_tide_map.m

        input:

        Q0    : scalar, river discharge (m^3/s)
        omega : scalar, angular frequency main tidal species in (1/
            seconds)
        x     : 2x1 vector, left and right end of computational
            domain of the river (m)
        w(x)  : function of width along the river (m)
        cd(x) : function of drag coefficient along the river (1)
        zb(x) : function of bed level along the river (m)

        opt   : structure with options
        opt.model_str = 'wave' (other solver are not supported at the
             moment)
        opt.solver = @bvp2c or @bvp2fdm
        opt.nx : number of grid points along channel
        opt.ns : base for logarithmic spacing of grid points, 1 :
            linear spacing

        bc  : structure array of boundary conditions
               r, row 1..2 : left and right end, respectively
               c, column 1 : mean (river) component
                         2..n : condition form column-1 frequency
                             component

               (       q(1)*(p(1) y^-(x0) + p(2) dy^-/dx(x0) ...
                    +  q(2)*(p(1) y^+(x0) + p(2) dy^+/dx(x0) ) = bc(c
                           ,r).val
                            = val(0)

        bc(c,r).var : Quantity, either 'z' or 'Q'
        bc(c,r).val : complex amplitude of chosen variable
                     (c.f. (1 + 0i) [m] for surface elevation
                         amplitude of 1m)
                     (value has to be real for mean component)
                     mean component requires z and Q to be specified
                         at opposit ends
        bc(c,r).p  : factor for Dirichlet p(1) or Neumann p(2)
            condition
                     p = [1,0] : pure Dirichlet
                     p = [0,1] : pure Neumann
                     sum of abs(p) must be nonzero for each end and
                         each frequency component
        bc(c,r).q  : factor for left and right going wave, only
            available for bvp2c
                     q = [1,1] : total water level / discharge
```

```
q = [1,0] : only left going wave
q = [0,1] : only right going wave
q has no meaning for the mean component and is
    ignored
q is only supported by bvp2c,
bvpfdm uses default q = [1,1]
sum of abs(q) for each frequency component must
    be zero
```

6.1 River_Tide

```
river tide in a single 1D channel
TODO split in two classes:
one that stores data (RT_Solve), one that provides equations (
    RT_Analytic)
```

6.2 bc_transformation

6.3 bcfun

```
Robin (mixed) boundary conditions for the river tide,
supplied for each frequency component,
wrapper that copies values are from the member struct "bc"

      q*(p*Q_1^- + (1-p)*dQ_1^-/dx
input :
       x      : coordinate (left or right end)
       id,ccdx : frequency component index
              (1 = 0 omega (mean), 2 : 1 omega, 3 : 2 omega, ...
columns of bc : frequency
rows of bc, left, right boundary
output :
       p : [2x1] linear combination of Dirichlet and Neumann
           boundary condition
         p(1) -> weight Dirichlet boundary condition
           p(2) -> weight Neumann boundary condition
     q linear combination of left and right travelling (incoming and
          outgoing) wave
           q(1) weight left going wave
         q(2) weight right going wave
       rhs = 0 -> homogeneous boundary condition


function [rhs, p, q, obj] = bcfun(obj,x,y,ccdx)
```

6.4 check_continuity

6.5 check_momentum

6.6 d2au1_dx2

```
second derivative of the tidal velocity magnitude

note: this is for finding zeros,
      the true derivative has to be scaled up by z
```

6.7 d2az1_dx2

```
second derivative of the tidal surface elevation

note: this is for finding zeros,
      the true derivative has to be scaled up by z
```

6.8 decompose

```
decompose the tide into a right and left travelling wave,
i.e. into incoming and reflected wave
TODO subtract forcing term
```

6.9 discharge2level

```
tidal component of surface elevation determined from tidal
    discharge

by continuity :

dz/dt + dq/dx = 0
=> i o z = - dq/dx
=>     z = -1/(io) dq/dx
=>     z = 1i/o dq/dx
```

```
TODO allow Q as input
TODO rename into Q1_to_z1
Mon  7 Oct 19:04:14 PST 2019 : added correction for change of width
```

6.10 dkq_dx

```
along-channel derivative of the wave number of the discharge
neglects width variation

TODO, rederive with g as variable
```

6.11 dkz_dx

```
along channel derivative of the wave number of the tidal surface
    elevation
ignores width variation dh/dx and second order depth variation (d^2
    h/dx^2)
TODO rederive with g symbolic
```

6.12 even_overtide_analytic

6.13 friction_coefficient_dronkers

```
friction coefficient according to Dronkers

the coefficients are semi-autogenerated

c.f. dronkers 1964
c.f. Cai 2016

p = [p0,p1,p2,p3];
alpha = Ur/Ut = river velocity / tidal velocity amplitude = (umax+
    umin)/(umax-umin)

function p = friction_coefficient_dronkers(alpha,order)
```

6.14 friction_coefficient_godin

```
friction coefficient according to Godin
these coefficients are identical to Dronker's for U_R = phi = 0

function G = friction_coefficient_godin(obj,phi)
```

6.15 friction_coefficient_lorentz

```
friction coefficient according to Lorent'z
identical to Dronker's coefficient for zero river flow
and a single frequency component
c.f. Cai
c.f. Dronkers

function L = friction_coefficient_lorentz(obj,phi)
```

6.16 friction_dronkers

```
friction determined by Dronker's method

input :
       u    : velocity time series
         Umid : arithmetic mean of mininmum and maximum velocity
              (not the mean of the velocity, usually non-zero even
                  without river flow)
       Uhr  : half-range of the velocity, less than the sum of
              the frequency amplitudes, except at perigean spring
                  tides

function [uau_sum uau p] = friction_dronkers(u,Umid,Uhr,order)
```

6.17 friction_exponential_dronkers

```
friction coefficicients for the frequency components computed by
    Dronkers method
c.f. Dronker's 1964 eq 8.2 and 8.4
Note: Cai dennominates alpha as phi

function [c uau uau_ p] = friction_trigonometric_dronkers(u,dp,Umid
    ,Uhr,order,psym)
```

6.18 friction_godin

```
compute friction with the method of Godin
```

6.19 friction_lorentz

6.20 friction_quadratic

```
friction determined by Dronker's method
```

6.21 friction_trigonometric_dronkers

```
friction computed by the method of Dronkers
expressed as coefficients for the frequency components
c.f. dronkers 1964 eq 8.2 and 8.4
Note: Cai dennominates alpha as phi
```

6.22 friction_trigonometric_godin

```
friction computed by the method of Godin
expressed as coefficients of the frequency components (
    trigonometric form)

 function [c, uau] = friction_trigonometric_godin(obj,u,dp,Umax)
```

6.23 friction_trigonometric_lorentz

```
friction computed by the method of Lorent'z
expressed as coefficients of the frequency components (
    trigonometric form)
```

6.24 generate_delft3d

6.25 init

```
provide initial condition by solving the backwater equation for
    surface level
TODO this should not be solved as a ivp but included in the bvp
    iteration
TODO generate the mesh here and precompute fixed values instead of
    passing functions
TODO Q0 should not be a function
function obj = init(obj, Xi)
```

6.26 mwl_offset

```
offset of the tidally averaged surface elevation caused by tidal
    friction
Linear estimate of the mean water level offset (ignoring feed-back
    of tide)
```

6.27 mwl_offset_2

6.28 mwl_offset_analytic

6.29 odefun

```
coefficients of the backwater and wave equation for river-tides
```

6.30 odefun0

```
coefficients of the backwater equation for the river tide
TODO merge with backwater
```

6.31 odefun_advective_acceleration

6.32 odefun_advective_acceleration_old

6.33 odefun_friction

6.34 odefun_ghof

6.35 odefun_swe_jacobian

6.36 odefun_width

6.37 odefunk

```
coefficients of the ordinary differential equation of the k-th
    freqeuncy
component of the tide

f1 Q'' + f2 Q' + f3 Q + f4 = 0

TODO rename f into c
TODO better pass dzb_dx instead of dz0_dx
TODO aa, oh and gh terms are not tested for width ~= 1
```

6.38 odefunk_old

```
coefficients of the ordinary differential equation of the k-th
    freqeuncy
component of the tide
```

```
f1 Q'' + f2 Q' + f3 Q + f4 = 0

TODO rename f into c
TODO better pass dzb_dx instead of dz0_dx
TODO aa, oh and gh terms are not tested for width ~= 1
```

6.39 solve

```
call stationary or non-stationary solver respectively

function obj = solve(obj)
```

6.40 solve_swe

```
determine river tide by the fully non-stationary FVM and then
    extract the tide
 this is experimental and not yet fully working
```

6.41 solve_wave

```
solve for the oscillatory (tidal) componets

function obj = solve_wave(obj)
```

6.42 wave_number_analytic

```
analytic expression of the wave number

valid for both tidally, river dominated and low friction conditions
and converging channels

k      : complex wave number in a reach with constant width and bed
    slope
im(k)  : damping modulus (rate of amplitude change)
re(k)  : actual wave number (rate of phase change)

c.f. derive_wave_number
```

6.43 wave_number_approximation

```
approximate wave number of the left and right traveling wave for
    variable coefficients

TODO merge with wave_number_analytic

function [k, k0, dk0_dx_rel, obj] = wave_numer_aproximation(obj)
```

7 river-tide/@River_Tide_Cai

Prediction of river tide by the method of Cai
c.f. Cai 2013, Cai 2015

7.1 Gamma

```
Gamma parameter for tidal propagation
c.f. Cai 2014
```

7.2 River_Tide_Cai

```
prediction of river tide by the method of Cai (2014)
```

7.3 river_tide_cai_

```
determine the surface amplitude of the river-tide
c.f. Cai
```

7.4 rt_quantities

```
determine the quantities that determine the tidal propagation
c.f. Cai

Note: this computes 4 unknowns following Cai, however,
lambda, mu and epsilon can be substituted
making it an equation in one unknown (delta) only
```

8 river-tide/@River_Tide_Empirical

```
Empirical fit to measurement and prediction (from tide at sea and
    river discharge)
of the river tide
```

8.1 River_Tide_Empirical

```
class for fitting models to at-a-station time series of tidal
    elevation
```

8.2 fit_amplitude

```
fit the oscillatory components
```

8.3 fit_mwl

```
fit the tidally averaged water level
```

8.4 fit_phase

```
fit the phase of the oscillatory components
```

8.5 fit_range

```
fit the tidal range
```

8.6 predict_amplitude

```
predict the oscillatory components
```

8.7 predict_mwl

```
predict the mean water level
```

8.8 predict_phase

```
predict tidal phase
```

8.9 predict_range

```
predict the tidal range
```

8.10 rt_model

```
select the model for fitting
```

9 river-tide/@River_Tide_JK

```
empirical analysis and prediction of river tides by the method of
    Jay and Kukulka
```

9.1 River_Tide_JK

9.2 damping_modulus

```
damping modulus of the river tide
c.f. Jay and Kukula
function r = damping_modulus(obj,h0,b,Qr)
```

9.3 mean_level

```
tidally averaged surface elevation
c.f. Jay and Kukulka
```

9.4 rivertide_predict

```
predict river tide by the method of jay and kukulka
TODO rename
```

9.5 rivertide_regress

```
Regression of tidal coefficients according to Jay & Kulkulka

coefficients of the r-regression factor 2 apart for specis (jay C7)
this can be repeated for each tidal species (diurnal, semidiurnal)
```

9.6 tidal_discharge

```
tidal discharge
c.f. Jay and Kukulka
function Qt = tidal_discharge(obj,x,R0,h0,b,Qr)
```

9.7 tidal_range

```
predict tidal range
```

10 river-tide/@River_Tide_Map

```
hash container for a set of River_Tide predictions for different
   boundary conditions
```

10.1 River_Tide_Map

```
container class to store individual river tide scenarios
```

10.2 fun

```
compute river tide for a scenario with specific boundary conditions
     and store it in the hash,
or retrive the scenario, if it was already computed
```

10.3 key

```
key for storing a scenario

function [key obj] = key(obj,varargin)
```

10.4 plot

```
quick plot of scenario result

function obj = plot(obj,Xi,Q0,W0,S0,z1_downstream,cd,zb_downstream,
    omega,q,opt)
```

11 river-tide/@River_Tide_Network

predict tides in a fluvial channel network

11.1 River_Tide_Network

```
tide in a fluvial delta channel network, extension of 1D river tide
the network is a directed graph
TODO convert from trig-to exponential form
```

11.2 discharge_amplitude

```
discharge amplitude
```

11.3 mean_water_level

```
predict the mean water level
```

11.4 plot_mean_water_level

```
plot tidally averaged water level
```

11.5 plot_water_level_amplitude

```
plot surface elevation amplitude
```

11.6 solve

```
solve for the tide in a fluvial chanel network

boundary condition at end points not connected to junctions
      [ channel 1 id, endpoint id (1 or 2), s0, c0
      ...
       channel n id, endpoint id (1 or 2), s0, c0]

conditions at junctions are specified as cells
       each cell contains an nx2 array
       n : number of connecting channels
       [channel id1, endpoint id (1 or 2), ...
       channel idn, endpoint id (1 or 2)]

every tidal species for each channel has 4 unknowns
these are 2x2 unknowns for the sin + cos of left and right going
    wave
```

11.7 water_level_amplitude

```
predict the surface elevation amplitude
```

12 river-tide

```
analysis and prediction of river tides

Sub-Classes:

@River_Tide
       - prediction of river tide in a backwater affected river with
            a sloping bed
@River_Tide_Cai
       - prediction of river tide, method of Cai
@River_Tide_Empirical
       - prediction of river tide, empirical
@River_Tide_JK
       - prediction of river tide, empirical after Jay and Kukulka
@River_Tide_Map
       - mulitple-scenaria container for River_Tide
@River_Tide_Network
       - extension of River_Tide to networks
```

12.1 damped_wave_bvp

```
solved damped wave equation
z'' + a z = 0
z(0) = z0, z(L) = 0
```

12.2 damped_wave_ivp

```
linearly damped wave in rectangular channel
solve tide as initial value problem
damped wave approximation

z'' + a z = 0

x_t = Ax + b
```

12.3 damping_modulus_river

```
damping modulus of the tidal wave for river flow only
```

12.4 rdamping_to_cdrag_tide

```
converts damping rate to drag coefficient
c.f. friedrichs, ippen harleman
```

12.5 river_tide_godin

```
analytic solution to the river tide formulated as boundary value
    problem
in a river with finite length

c.f. Godin 1986
```

12.6 rt_celerity

```
celerity of the tidal wave
```

12.7 rt_quasi_stationary_complex

```
quasi-stationary solution of the SWE
TODO staggered grid does not help: q1' needed
```

12.8 rt_quasi_stationary_trigonometric

```
quasi statinary form of the SWE
```

12.9 rt_reflection_coefficient_gradual

```
reflection coefficient for gradual varying cross section geometry
without damping
```

12.10 rt_wave_equation

```
solve river tide as boundary value problem

input:
omega : [nfx1] angluar frequency of tidal component, zero for mean
    flow
reach : [nrx1] struct
.L    : [1x1] length of reaches
        .width(x,h)   width
        .bed(x,h)     bed level
        .surface(x,h) surface elevation
        .Cd(x,h)      drag coefficient
.bc   : [nd,nf] boundary/junction conditions
         bc(id,if).type : {surface, velocity, discharge} (dirichlet)
         bc(id,if).val : value
opt   : [1x1] struct
       - constant surface elevation
       - deactivative advective acceleration
       .dx : spatial resolution

dimensions:
       nr : nurmber or reaches
       nd : upstream/downstream index
       nf : frequency index
```

12.11 rt_z2q

```
determine tidal discharge from water level for tidal wave
```

13 river-tide/test/test

13.1 test_bvp2c_sym

13.2 test_celerity

13.3 test_characteristic_rate_of_change

13.4 test_dronkers_compound

13.5 test_friction_dronkers

13.6 test_friction_dronkers2

13.7 test_fv_compare_schemes

13.8 test_fv_convergence

13.9 test_power_series

13.10 test_reflection_coefficient_gradual

13.11 test_ricatti

13.12 test_river_tide_models

13.13 test_rt_reflection

13.14 test_rt_zs0

13.15 test_swe

13.16 test_utm2latlon

13.17 test_wave_twopass

14 river-tide/test

14.1 test_bvp2c2

14.2 test_complex_even_overtide

14.3 test_fourier_power_exp

14.4 test_friction

14.5 test_reflection

14.6 test_rt_wave_number

14.7 test_tidal_river_network

14.8 test_tidal_river_network_z0

14.9 test_tide_slack_exp

14.10 test_wave_number_godin

14.11 test_wave_numer_aproximation

15 river-tide

```
analysis and prediction of river tides

Sub-Classes:

@River_Tide
       - prediction of river tide in a backwater affected river with
            a sloping bed
@River_Tide_Cai
       - prediction of river tide, method of Cai
@River_Tide_Empirical
       - prediction of river tide, empirical
@River_Tide_JK
       - prediction of river tide, empirical after Jay and Kukulka
@River_Tide_Map
       - mulitple-scenaria container for River_Tide
@River_Tide_Network
       - extension of River_Tide to networks
```

15.1 tidal_ellipse

```
 tidal ellipse, numerical ode solution
```

15.2 tide_slack_exp

15.3 wave_number_tide

```
 wave number of the tide without river flow
 c.f. friedrichs, ippen harleman
 output :
 k : wave number, such that
```

```
               z(t,x) = z1(t,0) exp(1i*(omega*t-k*x))

 re(k) : rate of phase change
-im(k) : damping rate

function [k k_low k_high] = damping_modulus_tide(omega,cd,h0,az1)
```

15.4 wavetrainz

```
determine river tide by iterated integration of the surface
    elevation
```

15.5 wavetwopassz

```
two pass solution for the linearised wave equation, for surface
    elevation
```

16 test/river-tide

16.1 example_river_tide

16.2 example_river_tide_map

16.3 river_tide_test

16.4 river_tide_test_01

16.5 river_tide_test_02

16.6 river_tide_test_03

16.7 river_tide_test_04

16.8 river_tide_test_05

16.9 river_tide_test_06

16.10 river_tide_test_07

16.11 river_tide_test_08

```
hold on;
plot(x,abs(z),'--');
hold on;
plot(x,angle(z),'--');
```

16.12 river_tide_test_09

16.13 river_tide_test_10

16.14 river_tide_test_11

16.15 river_tide_test_12

16.16 river_tide_test_13

16.17 river_tide_test_plot

17 test

17.1 test_tidal_harmonic_analysis

18 tide

```
analysis prediction of tides in rivers and estuaries by empirical
    and theoretical methods
```

18.1 tidal_constituents

18.2 tidal_energy_transport_1d

```
 energy transport of a tidal wave
```

18.3 tidal_envelope

```
 envelope of the tide

 input : t time in days
         f surface elevation
 ouput : tl time of low water
         vl surface elevation at low water
         ldx index of low water
```

```
th time of high water
vh surface elevation at high water
hdx index of high water
ndx neap index
sdx spring index
dmax:
drange: range per day
```

18.4 tidal_envelope2

```
surface levelation envelope of the tide
low water, high water and tidal range for lunar each day


input:
      time  :
      L     : surface elevation
      order : interpolation order (default 2)
ouput:
      timei : vector eqispaced
      lmini : minimum level
      lmaxi : maximum level
      rangei : range
      midrangei : (min + max)/2, usually different from mean
      phii  : pseudo phase

Note: the pseudo phase phi jumps, this is because if the tide is
    semidiurnal,
     sometimes the lower hw becomes the next day higher then than
        the
      current high water, e.g. there is no smooth transition by
     51min but a jump by 12h
```

18.5 tidal_harmonic_analysis

```
tidal_harmonic analysis
```

18.6 tidal_range_exp

18.7 tidal_range_tri

19 tide-savenije

19.1 savenije_phase_lag

```
phase lag of high and low water

phi : u_river/u_tide < 1

delta_eps_hw = omega*(t_hws - t_hw)
delta_eps_hw = omega*(t_lws - t_lw)

c.f. savenije
```

19.2 savenije_tidal_range

```
tidal range

based on Savenije 2012

x    : distance to river mouth
eta  : range
eta0 : range at river mouth
hbar : mean water depth
phi  : velocity ratio u_tide/u_river
       note: this varies in strongly convergent estuaries
K    : mannings coefficient
I    : residual surface slope I
```

19.3 savenije_tidal_range1

```
tidal range

based on Horrevoets/Savenije, 2004

H0   : tidal range at river mouth
h0   : initial water depth
v    : velocity scale
b    : convergence length
sine : phase lag
K    : Mannings coefficient
Q_r   : river discharge
```

19.4 savenije_timing_hw_lw

```
time of high water and low water
c.f. savenije 2012
```

19.5 tide-savenije

20 tide

```
analysis prediction of tides in rivers and estuaries by empirical
    and theoretical methods
```

20.1 tide_low_high_exp

20.2 tide_low_high_tri