Boston University Questrom School of Business
MF 793 – Fall 2020

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Problem Set 5

Solutions

Problem 1: *Was the Trump stock market more volatile than Obama's Bootstrapping VR*

"Stock market volatility is back" ... since the 2016 election year. Some say erratic positions on usually consensual economic issues such as trade, inflation and interest rates, the role of the FED, managing the budget deficit during an expansion, do agitate markets. Is stock market volatility up? It's an empirical question!

a) You check this using the market total return (make sure to add back the risk-free rate for what it matters!) from KF's data. You use the files KF-market.csv on the course page. You will compare months: 2012/1-2016/11 to period 2: 2017/1 – 2019/12. You don't include the COVID period for better comparability.
*You use monthly **log-**returns because the well-known fat-tailness of daily log-returns (high kurtosis as is well known) may throw the exact distribution of the Chow test.*
*Recall that KF's data is the discrete return. You compute the **Variance Ratio of Period 2 to Period 1.***

- *What is the theoretical distribution of VR_M under the null of equal variance $\sigma_1 = \sigma_2$, given your monthly sample sizes for periods 1 and 2?*

  The question asked Period 2 to Period 1, so the denominator is Period 2
  An F(ν1 = 36-1=35 , ν2 = 59-1 =58)
  Numerator: period 2
  Denominator: period 1

- *What are the cutoffs for a 2-sided 10% test? How bigger than σ_1 must σ_2 be to reject H_0 with 10% significance? How bigger than σ_2 must σ_1 be to also reject H_0.*

  [0.593 , 1.625]

  - To reject the null by $\sigma_2 < \sigma_1$, we need $\sigma_2 < 0.593\ \sigma_1$ that is $\boldsymbol{\sigma_1 > 1.69\ \sigma_2}$
  - To reject the null by $\sigma_1 < \sigma_2$, we need $\boldsymbol{\sigma_2 > 1.63\ \sigma_1}$**,**

- Why are these 2 ratio cut-offs different? When would they be equal?

  - There is an asymmetry in these cut-off values.
  - It requires a bigger departure from 1 to reject by σ1 > σ2 than by σ1 < σ2 !
  - This is because the F distribution is not symmetric because of the different sample sizes. With $\nu_1 = \nu_2$, the F distribution would be symmetric!

- *Fill in the first three columns of Table 1. Subscript M for monthly returns. Make sure to report annualized standard deviations.*

Table 1: Volatility for periods 1 and 2

	σ_{1M} (ann.)	σ_{2M} (ann.)	$VR_M(2/1)$	σ_{1D} (ann.)	σ_{2D} (ann.)	VR_D
US VW Ret.	10.88	12.63	1.35	13.22	13.19	0.99

- *According to **your** view, how much bigger than σ_1 should σ_2 be to constitute an **economically** significant deviation from equality.*

  Probably a 10% difference in standard deviation is getting notable and way beyond standard transaction costs for liquid transactions. This is not a statistical statement, just a economic view based on the price of volatility. We saw in HW4 that ATM call prices are approximately proportional to standard deviation (as per B-S), so a 10% difference in between two prices is notable. That would amount to a 20% difference in variances. On might argue that a 5% difference is economically relevant

- *Given your view, is the VR_M test is powerful enough to reject H_0, for economically significant deviations from equality?*

  - We have Prob(F(35,58) > 0.8) ≈ 0.25. So one would have to use a 50% significance level to reject $\sigma_2=\sigma_1$ at a 20% difference!!
  - The test has very low power to discriminate a 20% difference

b) *This lack of statistical power is annoying! You decide to redo the VR test with daily returns anyway. With the added power, you figure you can use a 5% test. Fill in the last 3 columns of Table 1.*

- *What are the relevant F statistic and the relevant two-sided 5% cut-off values ?*

  - F(ν_1 = 754-1 , ν_2 = 1258-1)
  - [0.879 , 1.136]
  - A much tighter CI than for monthly data, even though we are now using 5% rather than 10%. A good rule of thumb is to decrease the significance level of the test as the sample size (and in consequence the power of the test) increases

- *Now conclude: Was the Trump market variance higher than Obamas? Statistically and economically?*

  - Looks like the data is playing a trick on us!
  - Using the monthly return, the variance has increased vastly by any economic standard but the VR is not statistically different from 1.
  - Using the daily return, while we know we have a more precisely estimated variance ratio, it is now very close to 1 by any economic standard!

c) Your boss points out that you ignored the well known fact that daily log-returns are much more fat-tailed than a normal distribution. This is so unfair, you knew that all along!
You must simulate the distribution of the VR_D statistic to adjust your confidence cut-offs for non-normality. You do 20000 simulations under H_0 of a sample with N_1 (period 1) and N_2 (period 2) pseudo daily returns, with N_1 and N_2 the sample sizes in b).

- *The mean is irrelevant so you simulate zero mean. For H_0: $\sigma=\sigma_1=\sigma_2$, explain why the standard deviation is also irrelevant and you can simulate $\sigma = 1$.*
    - Variances are computed by subtracting the sample mean, so it does not matter what it is.
    - Also, we only care about the equality of the two variances and their ratio. So the level of variance does not matter. The only thing that matters is if / by what fraction, one of the two variances is bigger than the other.

- *You fill in rows 1 and 2 of Table 2. Row 1: Normal log-returns, Row 2: Student-t(dof=6.) log-returns. Row 1 is just there to check your code, you should match the theory. Row 2 checks the effect of non-normality with a specific departure t(6).*

Table 2: Theoretical and simulated Chow Test, 40000 simulations

	Theory with Normal returns				Simulation		
	ν_1	ν_2	$F_{0.025}$	$F_{0.975}$	% reject	$VR_{0.025}$	$VR_{0.975}$
Normal	753	1257	0.879	1.136	0.0496	0.878	1.135
t(6)					0.159	0.835	1.198
Bootstrap					0.214	0.816	1.220

% reject is the % times you reject when using the theory F cut-offs.

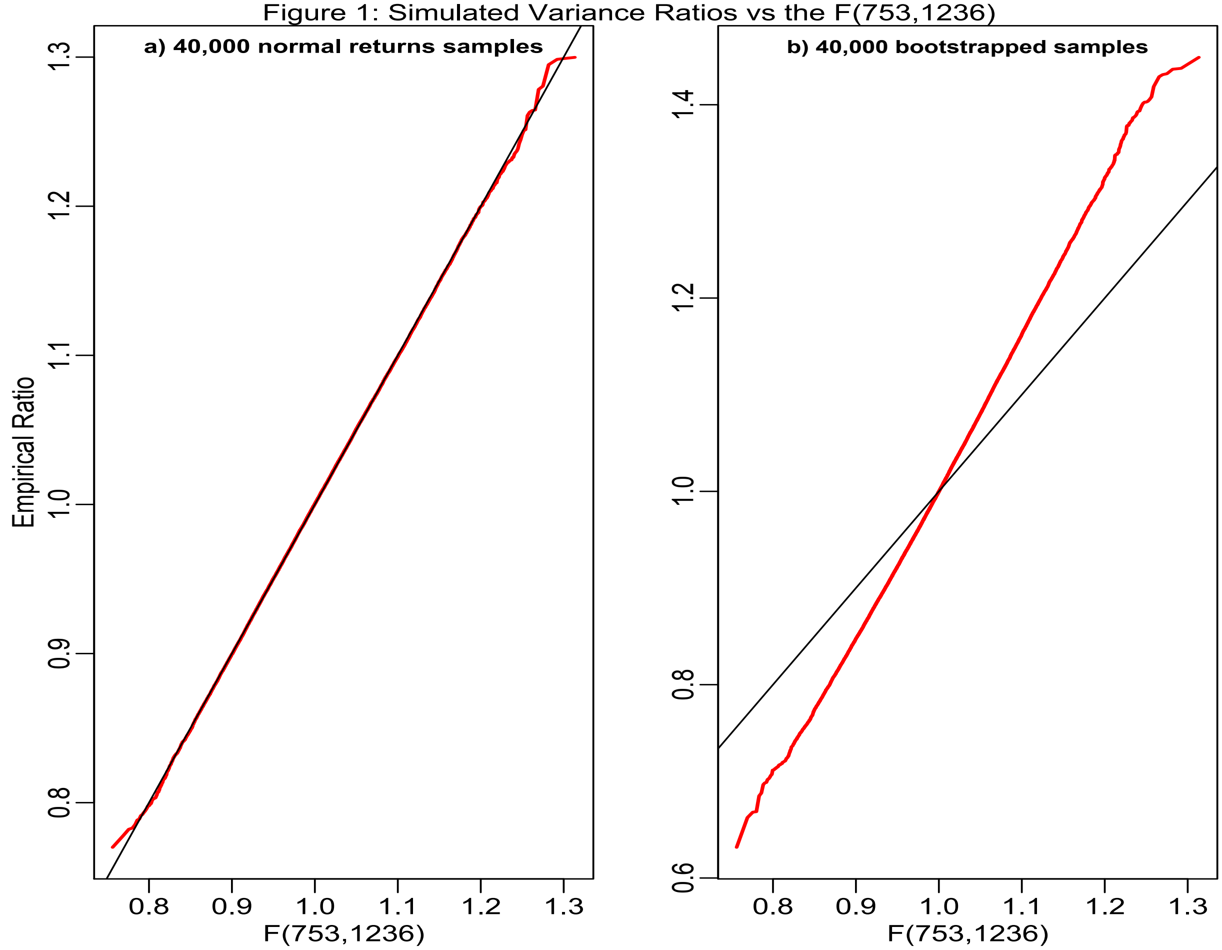
Row 1 (right) shows that our simulation works, with normal returns we produce the actual theoretical distribution of the VR.
However, when we simulate Student-t (ν =6) returns, the actual distribution of the VR is much wider, 5% of it is below 0.835 or above 1.2. This would lead us to reject the null 16% of the time.

d) Your boss is decidedly on a micro-managing rampage these days. "How do you know stock returns are t(6), what if they are not? Use the Empirical distribution of the data rather than making a likely wrong assumption! Add a bootstrap row to that table and bring me the results yesterday morning". You have so had it you feel like turning your video off so you can make faces

- *To add Row 3, you bootstrap the actual returns. That is. you randomly select N_1 returns* ***with replacement*** *from your returns series, all N_1+N_2, daily returns for this. Similarly you draw N_2 random returns with replacement from the entire series. Then you compute VR. You do this 20000 times, and compute the relevant statistics for the 3rd row. Use the R command "sample", don't forget "replace = T"*

- *In* ***Figure 1****, show an* ***F-probability plot*** *of the 20,000 VRs against the theoretical F, Figure 1a, for the normal monthly case, Figure 1b for the bootstrapped case. How does the distributions departs from the theoretical F.*

Figure 1: Simulated Variance Ratios vs the F(753,1236)



- ○ Tables 2 and Figure 1 show that the empirical variance ratio does not follow the theoretical density when the underlying return is fat tail. It is more spread out than the F(753,1236).
- ○ As the F-probability is a straight line, the empirical ratio might still be well approximated by an F, but with lower degrees of freedom.

- *Given Table 2, do you reject too much or not enough or about appropriately when using the theoretical F for daily returns?*

  We reject too much

- *Revise, if necessary, your conclusion from Table 1 VR_D, by using the proper cutoffs.*

  In our case, we did not reject even with the wrong (theoretical) cutoffs. In general the proper method is to use the cutoffs from our simulation with bootstrapped data in order to reject the null hypothesis.

These R commands will help to construct an F-probability plot

To do Figure 2, generalizing qqnorm to any distribution not just a normal: Combine the commands ppoints and qqplot

qqplot(x1,x2) sorts x1 and x2 and does a scatter plot, it is comparison of two empirical densities, x1 and x2 have the same length.
qqnorm(x2), is just like qqplot(x1,x2) where x1 is the theoretical quantiles of the normal for as many points as x2.
To generalize it, we can use qqplot and replace x1 by the quantiles of the theoretical distribution we want to compare with x2. The command ppoints will help.
ppoints(nsim) will generate evenly spaced points on [0,1], like nsim evenly spread quantiles of a density. Recall the inverse transform theorem: The inverse CDF gives us the theoretical values that generated these quantiles:
qf(ppoints(1000),dof1,dof2)) gives 1000 sorted values according to the F.
qqplot(qf(ppoints(1000),dof1,dof2), mydata) # for a 1000 long mydata
plots a F-probability plot of mydata

Problem 2: How precisely can we estimate stock and portfolio betas?

a) *Consider N (I = 1 , N) stocks with market betas β_I, and a portfolio of the N stocks with weight vector w. Write (prove) the portfolio beta β_P, as a function of the N β_Is. It should show that the porfolio β is a weighted average of the stock betas.*

b) *Clearly your result also applies to estimates of the betas. Consider the $\widehat{\beta}_i$ standard deviation $s_{\beta i}$ (you estimate the β_Is from simple regressions on the market return).*

a) $$\beta_P = \frac{Cov(\tilde{R}_P, \tilde{R}_M)}{\sigma_M^2} = \frac{Cov\left(\sum_{i=1}^{N} w_i \tilde{R}_i, \tilde{R}_M\right)}{\sigma_M^2}$$

$$\beta_P = \frac{\sum w_i \, Cov(\tilde{R}_i, \tilde{R}_M)}{\sigma_M^2} = \sum w_i \beta_i$$

b) Simplifying assumption just makes calculus easier, it does not change the fundamental result.

$$Var(\hat{\beta}_P) = Var\left(\sum_i \frac{1}{N} \hat{\beta}_i\right)$$

$$= \frac{1}{N^2} \sum_i v(\hat{\beta}_i) + \sum_i \sum_{j \neq i} \frac{1}{N} \frac{1}{N} cov(\hat{\beta}_i \hat{\beta}_j)$$

$$= \frac{1}{N^2} N v(\hat{\beta}) + \frac{N(N-1)}{N^2} \rho_b v(\hat{\beta})$$

$$var(\hat{\beta}_P) = var(\hat{\beta}) \left[\frac{1}{N} + \left(1 - \frac{1}{N}\right)\rho_b\right]$$

Assume the individual stock beta estimates are unrelated. Write the standard deviation $s_{\beta P}$ of $\hat{\beta}_P$ as a function of the $s_{\beta i}$s. Assume an equal-weighted portfolio to simplify the calculus. At what rate does the standard deviation of $\hat{\beta}_P$ decrease as the portfolio size increases?

If the β_i's are unrelated, i.e., $\rho_b = 0$ in the formula above, the standard deviation of $\hat{\beta}_P$ decreases at rate $\sqrt{N}$.

c) *Consider the classic market regression $R_{it} = \alpha + \beta R_{Mt} + \varepsilon_{it}$, $t = 1, \ldots T$. You can run it on monthly or daily returns. Say daily returns have negligible autocorrelation. Going from Monthly to Daily returns, what happens to the standard errors of $\hat{\alpha}$ and $\hat{\beta}$?*
I don't ask how α and β change with the horizon but that's obvious, right?

$\hat{\alpha}$ is a sample mean (in excess of βR_{Mt}), so there is no gain in precision.

$\hat{\beta}$ is a ratio of covariance over variance, both become more precise as we sample more often (at rate $\sqrt{N}$).

β is a ratio of 2 quantities which "shrink" at rate $\sqrt{N}$ when we sample more frequently, so β itself is not likely to change with the data sampling frequency.

α is a mean, so it shrinks at the rate N at which the data sampling frequency increases.

d) Reality check. Use the available stock files to get the estimates for Table 1, period 2011-2017. For both daily and monthly returns, regress the 4 assets on the market, report the α and β and their standard errors. You can do 4 regressions and save these 4 quantities in one shot by using the method (sapply(?, '[', c(??))) seen in class, see R lecture note and code help below.
Also write in the **annualized** standard deviations for each stock σ_D and σ_M.
Make sure to *appropriately* transform the α's so they are comparable between daily and monthly regression.

Table 1: Market regressions from monthly and daily returns. Period 2011-2017

		Monthly Regression on US_{VW}					Daily Regression on US_{VW}			
	σ_D	α ann	t_α	β	s_β	σ_M	α ann	t_α	β	s_β
Biogen	0.xx	0.152	1.50	0.99	0.23		0.134	1.42	1.08	0.047
GE		-0.076	-1.23	1.18	0.14		-0.070	-1.47	1.03	0.024
Pepsi		0.067	1.74	0.43	0.09		0.055	1.46	0.51	0.019
Valero		0.102	1.02	1.58	0.23		0.103	1.15	1.42	0.045
EW Port				1.04	0.09				1.01	0.017

Average over the stocks: 1.04 0.17 1.01 0.034

It looks like this below is a lot, but these are one or two sentence or one or two word answers, as I am walking you through the story step by step. Enjoy.

d1) Compare the 4 monthly and daily βs.

- Are the monthly and daily βs markedly different? **No**
- Are their standard errors markedly different?

  Daily more precise factor of 5

  Note: sqrt(251/12) = 4.6
- So, which frequency seems better as an estimations strategy? **Daily, for precision.**

d2) Compare monthly with daily α's.

- *Are the monthly and daily $\widehat{\alpha}$ very different (after appropriate transformations)?*

  Similar after annualization
- *What "appropriate" transformation?*

  Annualization, as the alphas are means – after beta adjustments

  The Jensen's α is the excess mean return of the stock after accounting for β.
- *Are the αs estimated more precisely with daily than monthly returns? Explain why or why not in one sentence*

  No !

  We know that means can not be more precisely estimated by using higher frequency returns

d3) Portfolio vs stock regression precision. Make an equal weighted portfolio "EW Port" of the four stocks and regress it on the US market return. Fill in the β estimates and std. deviations in Table 1

- What is the average s_β for the stock regressions? 0.17
- What is the standard error $s_{\beta P}$ for the EW Port regression? 0.09
- Given your result in b), do the cross-correlation of the 4 $\widehat{\beta}$ look close to 0? **Yes !!**
- Get the 4 residual vectors (of the 4 stock regressions) in one data matrix. Compute the 4x4 correlation matrix, and compute the average of the 6 cross-correlations. Make sure to remove the ones! What are they (for monthly and daily regressions)?

Average cross-correlations:

	Daily	Monthly
Stocks	0.32	0.26
Residuals	-0.028	-0.01

<u>This done in class will help (see your R lecture notes):</u>

Do not loop around lm for several regressions. You do them all with:

```
mymod <- lm( stkrets~rm)       # creates the N  regressions if stkrets has the N  stocks
modsum<-summary(mymod)  # creates summary object with all the good stuff in it.
coefficients(mymod)            # also works to print the output with estimates and std.errs.
```

To run in multivariate mode (several regressions), lm wants a matrix or ts object. When we read data by read.csv they appear as a list to lm. This needs to be modified. I just do this:

```
zstocks<-read.csv("zfile.csv",header=T)
strkets<-as.matrix(zstocks)              # step to select the correct columns for the y variables
```

mymod and modsum are now **multivariate** linear model object, lists where each regression is an item in the list, then each regression is itself a list in the list. It makes it hard to retrieve in a vector for example, all the standard errors. To see the problem, do names(mymod), names(modsum). Some commands now don't work. You can't do things like: confint(mymod)

What we want (estimates and their stddevs, maybe the ts) is now 2 steps down in the list hierarchy. You can extract it with the **lapply** (list apply) and **sapply** commands, similar to **apply** but extract components of lists. In coefficients(mymod), you see in what order the output is. Now try this for example, you will see what it does:

lapply(modsum, coefficients) is exactly the same as coefficients(modsum). But is is now a list with 12 items. This below will takes what we need from it:

```
sapply(lapply(modsum,coefficients),   '[',     c(1,3) )
```

coefficients output for all regressions	'[' says to go down a level	c(1,3) takes elements 1 and 3

Recall that R, like all languages stores column wise, so find the order of the intercept and slope, and standard errors or whatever you want in coefficients(modsum), and use that in c(?,?,?...)

Problem 3: Autocorrelation and heteroskedasticity in daily regressions

Pick the **daily** GE regression and use the methods shown in class and in the R lecture note to compute OLS robust standard errors and confidence intervals. Fill in Table 1.

Table 1: Sandwich estimates of the slope standard error for the Biogen daily regression

	$\hat{\beta}_{OLS}$	$s_{\hat{\beta}}$	5%	95%
OLS – iid				
HC – White				
HAC - Andrews				

- From these results, does there seem to be a lot of heteroskedasticity in the daily residuals? a lot of autocorrelation?