**Boston University Questrom School of Business**

**MF 793 – Fall 2021**

Eric Jacquier

**Problem Set 3**

**Not Due Sunday October 24rd at 1pm Boston Time**

This Problem Set is intended to help you further prepare for the midterm. It will not be graded. A purely numerical solution with no explanation and no R code will be posted on Sunday October 24th at 1pm Boston time.

**Problem 1:**

The file Vanguard.csv contains the weekly NAV of two Vanguard funds for 5 years. Compute the log-returns.

1. Find out what these funds are trying to do. Describe in a line for each

VSIAX:

This low-cost index fund offers exposure to small-capitalization U.S. value stocks. Value stocks are those that may be temporarily undervalued by investors. These companies typically grow at a slower pace than the typical company. The fund seeks to track a value-style index of small-sized companies. One of the fund’s primary risks is its focus on the small-cap arena, which is an often-volatile segment of the market. Investors looking to add a passively managed, small-cap value allocation to an already diversified portfolio may wish to consider this fund.

VSGAX:

This low-cost index fund offers exposure to small-capitalization U.S. growth stocks, which tend to grow more quickly than the broader market. The fund seeks to track a growth-style index of small-sized companies. One of the fund’s primary risks is its focus on the small-cap arena, which is an often-volatile segment of the market.  Investors looking to add a passively managed, small-cap growth allocation to an already diversified portfolio may wish to consider this fund.

1. Estimate the means and std. devs. of the two funds. Make the usual assumption for the aggregation formulas to work. Assume 52 weeks per year. Fill in Table 1 with the 95% confidence intervals for these four quantities for both funds and show your work below.
2. Fill in skewness and kurtosis estimates in Table 1. Also fill in Table 1 with in an asymptotic 95% confidence intervals for the skewness estimate and a **90%**  asymptotic confidence interval (**CI**)for the kurtosis estimate.
3. Can you reject the null of zero skewness, of kurtosis equal to 3, at the 5% level? Assume a two-sided alternative for skewness and a one-sided alternative for Kurtosis (i.e. true kurtosis can’t be smaller than 3)
4. Learn what the Jarque-Bera test of normality is, write its formula and its asypmtotic distribution under the null of normality:

JB = JB

Fill in the JB estimates and ***p-values*** in Table 1

Given these, do you reject the null of Normality?

1. Write a 95% confidence interval (annualized) for the difference between the two means. Fill in Table 1 for VSIAX – VSGX. Can you reject the null hypothesis that the two funds have equal returns?
2. Compute the classic Variance Ratio statistic, and test (See the F distribution) the hypothesis that the two funds have equal variance at the 5% level.

VR = VR ∼

Test cut-off value:

Table 1: Estimates, CI’s in brackets

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Weekly | | Annualize | |  |  |  |
|  | μ | σ | μann | σann | Sk | Ku | JB |
| VSIAX |  |  |  |  |  |  | Estimate |
|  | [ , ] ] | [ , ] | [ , ] | [ , ] | [ , ] | [ , ] | p-value |
| VSGX |  |  |  |  |  |  | Estimate |
|  | [ , ] ] | [ , ] | [ , ] | [ , ] | [ , ] | [ , ] | p-value |
| VSIAX-VSGX |  |  |  |  | -- | -- | -- |
|  | [ , ] ] | [ , ] | [ , ] | [ , ] | -- | -- | -- |

**Problem 2: Gains (?) from higher frequency data.**

The stk-11 files have daily and monthly returns on 11 stocks. Consider GE for the entire period. To be precise, convert these discrete returns into log-returns.

1) Fill in Table 2

Table 2: Estimates and CI’s for GE

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Daily data | | | |  | Monthly data | | |
|  | μD | σD | ρD(1) | ρD(2) | Kurtosis | μM | σM | Kurtosis |
| Raw Data | x.xx | x.xx | 0.xx | 0.xx | x.xx | x.xx | x.xx | x.xx |
|  | [ , ] ] | [ , ] | [ , ] | [ , ] | [ , ] | [ , ] | [ , ] | [ , ] |
| *Monthly*-ized |  |  |  |  |  |  |  |  |
|  | [ , ] ] | [ , ] ] |  |  |  |  |  |  |

\* In each box, the top-row is the estimate, the bottom row [ , ] is a 95% CI, The approximate variance of an autocorrelation estimate is 1/T.

\*\* All means and standard deviations in %, i.e., 1.5 is 1.5% or 0.015

2) Using monthly returns, write the t-statistic for the null hypothesis H0: μM = 0.01

tM =

3) From daily returns, write the t-statistic for the null hypothesis on μD which corresponds exactly to the H0 tested in 2).

H0: μD =

td =

4) Using the monthly returns, write a 95% CI for σM **both ways**, first using the approximate distribution of sM, second using its exact distribution under the assumption that the log-returns are normally distributed

Approximate:

Exact:

Are these CIs very different?

5) Do the same thing for the daily data.

Approximate:

Exact:

Are these CIs very different (more or less than for monthly data), conclude?

**Problem 3: Another way to gauge the effect of Fat-tails**

The kurtosis should be 3 if the distribution is normal. So, is 4.5 very different from 3? Hard to have any intuition for this. Risk managers care about the frequency of extreme ***ly bad!*** values. What Philip Sun showed during his presentation is precisely what prompted researchers to investigate how to model the non-normality of financial series: There is about no chance that one would encounter the extremely large negative shocks such as October 19th 1987 with a normal or lognormal distribution. You now verify this yourself in a slightly different way.

Use the daily KF factor file, compute the total daily market log-returns from 1941-2020 included. Count the exact number of days.

Under the assumption of lognormal returns with constant mean and variance, how many days over the 80 years would one expect to find a return more than (10, 5, 4, 3, 2, 1.96, 1.644 ) standard deviations below the mean.

Compare with the actual stock returns, i.e., how many days do we actually have a return more than (10, 5, 4, 3, 2, 1.96, 1.644 ) standard deviations below the mean.

Another food for thought, is the negative skewness driven mostly by a very small number of extreme negative days or is it more pervasive in the 5% tail range.