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**Boston University Questrom School of Business**

**MF 793 – Fall 2021**

This is a Individual Problem Set 4

Due Sunday November 8th on GradeScope

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* This **individual** problem set will reward you for studying LN9 and LN10.
* It must be handwritten to get a **check**.
* You need to answer **all** the questions
* Answer questions in the space provided

**Problem 1: The theoretical β**

LN 9 computes the optimal β when the linear model y = β x + ϵ, approximates an unknown CEF E(y|x). Generalize this result when the linear model approximation is y = α + β x + ε. I.e., solve for α and β that minimize the MSE, E(ε2) as a function of the true moments of x and y. **10 pts**

一些文字和图片的手机截图

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**Problem 2: The link between standard deviation and option prices**

Don’t even think of zooming into an interview and not knowing the Black-Scholes-Merton model. A Call C (no dividends) trades on a stock of value S. Maturity is τ years, exercise price X. The risk free rate at the time of trading is rF (continuously compounded annual).

a) Write, no proof, the Black-Scholes formula for these known components of the call price. **8 pts**

d1 =

图片包含 室内, 文字, 瓷砖, 房间

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d2 =

白板上写着字

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Δ =

文本, 信件

描述已自动生成

BS(X, S, τ, rF, σ ) =

图片包含 门, 游戏, 一群, 一对

描述已自动生成

**b)** GOOGL trades at $2,960. The 1-month call with exercise price of $2,960 trades at $106. Assume a 0% risk-free rate over that horizon. What is GOOGL’s ***implied standard deviation*** (annualized) for this call. You must use R to answer this problem **8pts**

ISD = 0.3110576

What would be the ISD if the call was trading 10% higher, at $116.60 for the same stock price?

ISD = 0.3421874

What % change in ISD (over the one at $106) does this represent:

% increase in ISD = 0.1000773 ---> 10.008%

How does this % increase compare too the % increase in Call price?

**Answer**: the % increase in ISD is about the same as the % increase in Call price, but there’re still some small differences. 10.008% is a litter higher than 10%, which means the change of call price leads to less change in ISD. In other word, the % increase in ISD might lead to larger % increase in call price in real world.

**c)** Type in the R code you used to find the Implied Standard Deviation? **4pts**

*t <- 1/12*

*S <- 2960*

*X <- 2960*

*callreal <- 116.60 # we can change the parameter here*

*rf <- 0*

*upper <- 1*

*lower <- 0*

*sigma <- 0.5*

*d1 <- (log(S/X)+(rf+0.5\*(sigma\*\*2))\*t)/sigma/(t\*\*0.5)*

*d2 <- d1 - sigma\*(t\*\*0.5)*

*call <- S\*pnorm(d1) - X \* exp(-rf\*t)\*pnorm(d2)*

*call*

*while (abs(call-callreal)>=0.0001){*

*if ((call-callreal)<0){*

*lower<- sigma*

*}*

*if ((call-callreal)>0){*

*upper <- sigma*

*}*

*if ((call-callreal)==0){*

*break*

*}*

*sigma <- (lower+upper)/2*

*d1 <- (log(S/X)+(rf+0.5\*(sigma\*\*2))\*t)/sigma/(t\*\*0.5)*

*d2 <- d1 - sigma\*(t\*\*0.5)*

*call <- S\*pnorm(d1) - X \* exp(-rf\*t)\*pnorm(d2)*

*}*

*call1 <- 0.3110576*

*call2 <- 0.3421874*

*(call2-call1)/call1 #% increase in ISD*

**Problem 3:** Understanding (vector) derivative of inner products and quadratic forms

You need to be comfortable with quadratic forms, inner and outer products, in econometrics and portfolio optimization. This includes their derivatives.

For example, you need to understand the “normal equation”, LN10, p.8.

a) In the Sum of Squares, top of p.8, verify that Z=X’Y has length k. Set k=2, consider a column vector Z=(z1, z2)’, the vector β’ = (β1, β2), and the scalar quantity s=β’Z. Write each component of the 2x1 vector . **6pts**

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Then compare with Z: 白板上的字

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b) Let’s study the second part of the SSE on p.8. Take a column vector β=(β1 , β2)’, and a matrix

.

The scalar quantity Q = β’Hβ is a quadratic form in β. Compute it (write it out) as a function of the h’s and the βs. **4pts**

Q = β’Hβ = 白板上的文字

中度可信度描述已自动生成

**c)** Use your result in b) to write each component of the 2x1 vector ( .

**4pts**

图片包含 图示

描述已自动生成

文本, 日历

描述已自动生成

**d)** Write out each component of the 2x1 vector: 2Hβ in function of the h’s and β’s. **4 pts**

2Hβ = 白板上的字

描述已自动生成

**e)** Compare your results in c) and d):

When is the derivative of β’Hβ with respect to β equal to 2Hβ? **2pts**

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**f)** What kind of matrix verifies this condition? **2pts**

**Answer：** covariance matrix

**Problem4: OLS estimator, The X’X matrix**

You need to understand the structure of (X’X). The OLS estimator is = (X’X)-1X’Y and its variance **V() =** σ2(X’X)-1. For a simple regression with intercept and one X variable, we have **β** = (α , β).

**a)** Write each component of the 2x2 matrix (X’X), with only the sample size T and sample statistics: . **4pts**

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**b)** Invert (X’X). Look at Var(), what leads to a more precisely estimated **slope** coefficient ? **4pts**

Det (X’X) =

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描述已自动生成

σ2 (X’X)-1 =

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**c)** What makes the slope coefficient estimator more precise? **2pts**

**Answer:** from the V() = σ2(X’X)-1, , it’s easy to find that larger size T, larger sample variance of X and smaller variance σ2  makes slope coefficient estimator more precise.

**Problem 5:** **The X’X matrix and the sample covariance of the X variables.**

a) Write X’X in the case of an intercept and 2 variables, X1 and X2, X’X is 3x3. Compute its determinant only as function of the sample statistics of X1 and X2 and the sample size T. **4pts**

X’X =

图示

描述已自动生成

Det = 图示

描述已自动生成

**b)** Write Xd’Xd the 2x2 matrix where X1 and X2 are written in deviation form X1- **1** , X2-, where **1** is a Nx1 vector of ones, so there is no intercept anymore. Compute its determinant. Compare the two determinants. **4pts**

Xd’Xd **= 白板上画着画

低可信度描述已自动生成**

Det =

图片包含 门, 看着, 水, 一群

描述已自动生成

**c)** The two determinants in a) and b) are ….

**Answer：** Obviously, Det(Xd’Xd) ✖️ T = Det(X’X)