MF731 Team Project Report
Capital charge and Allocation under FRTB

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1. Executive Summary

The Fundamental Review of the Trading Book (FRTB) is a comprehensive suite of capital rule developed by BCBS as a part of Basel III, intended to be applied to banks' wholesale trading activities.

In this project, we propose two allocation methods for capital charges under the FRTB. We focus on risk factor (RF) and liquidity horizon (LH) bucketing. We start our analysis with generate 10000 simulations for 10-day loss of the portfolio. Obtaining the VaR and ES value, we compute the Euler allocation for them. Then, we determine the loss adjusted for the liquidity horizon for the portfolio and each risk position. For each risk factor, we calculate the FRTB estimated deficit for portfolio loss. We also determine the FRTB ES capital charge for the unconstrained portfolio and each risk element as well as the portfolio loss's total capital charge for modellable risk components. We calculate the FRTB Euler allocation of the portfolio's loss adjusted for the liquidity horizon. We verify the connection between the FRTB Euler allocation of the portfolio's loss adjusted for the liquidity horizon and the FRTB projected deficit for loss assigned to each risk component. Ultimately, we get the Euler allocation for IMCC.

2. Methodology and Results

We start our project with considering a portfolio of two risk positions. Each risk position can be thought of as one asset, which has exposure to all different risk factors risk factors RF_i , i = 1, …, 5, and different liquidity horizons LH_j, j = 1,...,5:

$$\{RF_i : 1 \le i \le 5\} = \{CM,CR,EQ,FX,IR\},$$

$$\{LH_j : 1 \le j \le 5\} = \{10,20,40,60,120 \text{ days}\}.$$

Suppose that the 10-day loss of risk position n, attributed to RF_i and LH_j, is denoted by, $\tilde{X}_n(i,j)$and assumed that the correlation inside X_1 is about 0.3 and X_2 is about 0.1:

$$\tilde{X}_1(i,j) \sim N(0.004, 0.04) \text{ and } \tilde{X}_2(i,j) \sim N(0.006, 0.05)$$

For the correlation between X_1 or X_2, we follow the rule below to implement the simulation:

$$\tilde{X}_n(i,j) = \sqrt{\rho} * Z + \sqrt{1-\rho} * Z(i,j)$$

where ρ is the correlation inside each of simulation and Z is a normal random variable. Then the total loss of risk position n and total loss of the portfolio should be:

$$\tilde{X}_n = \sum_{i,j}^{5} \tilde{X}_n(i,j), n = 1,2.$$

$$\tilde{X} = \tilde{X}_1 + \tilde{X}_2$$

Standard VaR and ES: Using the 10-day loss we obtained, we can get the result that $VaR_{0.99} = 1.9575$ and $ES_{0.975} = 1.9543$

VaR and ES allocations: Then we begin to compute Euler allocation for our portfolio. We let $\epsilon = 0.001$, for VaR, we find those with $\tilde{X}_n \in (VaR_{0.99} - \epsilon, VaR_{0.99} + \epsilon)$ and take average for each simulation. For ES, we find those $\tilde{X}_n \geq VaR_{0.975}$ and take average for each simulation.
Therefore, the Euler allocation for VaR is about 1.402 and 0.554, the Euler allocation for ES is about 1.124 and 0.830.

Now we get into deeper to consider about the situation in FRTB. In FRTB, the liquidity horizon adjusted loss for risk position *n* is:

$$\tilde{X}_n(i,j) = \sqrt{\frac{LH_j - LH_{j-1}}{10}} \sum_{k=j}^{5} \tilde{X}_n(i,k), \ 1 \leq i,j \leq 5$$

The liquidity horizon adjusted loss for portfolio is

$$X(i,j) = X_1(i,j) + X_2(i,j)$$

FRTB ES: For each i = 1,…,5, the FRTB expected shortfall for portfolio loss attributed to RF_i is

$$ES^{F,C}(X(i)) = \sqrt{\sum_{j=1}^{5} ES_{0.975}(X(i,j))^2}$$

The table below shows the result we obtained, following the formular:

Risk Factor	**$ES^{F,C}$**
CM	0.9648792
CR	0.9522721
EQ	0.9439267
FX	0.9562566
IR	0.9622664

FRTB ES capital charge: Assume $\frac{ES^{R,S}(X(i))}{ES^{R,C}(X(i))} = 2$ for all i = 1~5. Then the FRTB ES capital charge for risk factors should be

$$IMCC(X(i)) = 2ES^{F,C}(X(i))$$

The table below shows the result we obtained:

Risk Factor	**IMCC**
CM	1.92975842
CR	1.9045442
EQ	1.88785346
FX	1.9125132
IR	1.92453272

For the risk position *n*, the unconstrained portfolio with LH_j is

$$X_n(6,j) = \sum_{i=1}^{5} X_n(i,j)$$

For the portfolio, the unconstrained portfolio with LH_j is

$$X(6,j) = X_1(6,j) + X_2(6,j)$$

FRTB capital charge for modellable risk factors: Using all the information we have, we obtained IMCC(6) = 7.0310. Then, for the portfolio loss X, its aggregate charge for modellable risk factors is:

$$IMCC(X) = 0.5IMCC\big(X(6)\big) + 0.5\sum_{i=1}^{5} IMCC\big(X(i)\big) \approx 8.2951$$

FRTB Euler allocation of X(i,j): Using the similar method as we did in VaR and ES allocation before. We denote Euler allocation of X(i,j) as $ES\big(X_1(i,j)|\, X(i,j)\big)$ and $ES\big(X_2(i,j)|\, X(i,j)\big)$, then

$$ES\big(X_1(i,j)|\, X(i,j)\big) = \begin{pmatrix} 0.2365 & 0.1987 & 0.2162 & 0.1566 & 0.1670 \\ 0.2365 & 0.1976 & 0.2127 & 0.1540 & 0.1569 \\ 0.2269 & 0.1922 & 0.2084 & 0.1530 & 0.1601 \\ 0.2338 & 0.1929 & 0.2103 & 0.1454 & 0.1496 \\ 0.2492 & 0.2019 & 0.2184 & 0.1518 & 0.1491 \end{pmatrix}$$

$$ES\big(X_2(i,j)|\, X(i,j)\big) = \begin{pmatrix} 0.2571 & 0.2207 & 0.2640 & 0.2014 & 0.2239 \\ 0.2570 & 0.2139 & 0.2554 & 0.1975 & 0.2320 \\ 0.2507 & 0.2112 & 0.2552 & 0.2019 & 0.2389 \\ 0.2576 & 0.2206 & 0.2548 & 0.2095 & 0.2501 \\ 0.2455 & 0.2155 & 0.2592 & 0.2051 & 0.2400 \end{pmatrix}$$

Then for each i ~[1,5], we calculate

$$ES^{F,C}\big(X_n(i,j)|\, X(i)\big) = \frac{ES\big(X(i,j)\big)}{ES\big(X(i)\big)} ES\big(X_n(i,j)|\, X(i,j)\big)$$

we get the result below

$$ES^{F,C}\big(X_1(i,j)|\, X(i)\big) = \begin{pmatrix} 0.1210 & 0.0863 & 0.1076 & 0.0580 & 0.0677 \\ 0.1225 & 0.0854 & 0.1046 & 0.0568 & 0.0641 \\ 0.1148 & 0.0821 & 0.1024 & 0.0575 & 0.0677 \\ 0.1201 & 0.0834 & 0.1023 & 0.0539 & 0.0625 \\ 0.1281 & 0.0876 & 0.1084 & 0.0563 & 0.0603 \end{pmatrix}$$

$$ES^{F,C}\big(X_2(i,j)|\, X(i)\big) = \begin{pmatrix} 0.1315 & 0.0959 & 0.1314 & 0.0747 & 0.0901 \\ 0.1331 & 0.0925 & 0.1256 & 0.0729 & 0.0948 \\ 0.1269 & 0.0903 & 0.1254 & 0.0759 & 0.1010 \\ 0.1323 & 0.0954 & 0.1239 & 0.0777 & 0.1046 \\ 0.1262 & 0.0935 & 0.1287 & 0.0761 & 0.0970 \end{pmatrix}$$

$$For\ check\text{: } ES\big(X(i)\big) = \begin{pmatrix} 0.9649 & 0.9523 & 0.9439 & 0.9563 & 0.9623 \end{pmatrix}$$

Then, follow the same method above, we can calculate that

$$ES^{F,C}\big(X_1(6,j)|\, X(6)\big) = \begin{pmatrix} 0.6425 & 0.4241 & 0.4882 & 0.2334 & 0.2042 \end{pmatrix}$$

$$ES^{F,C}\big(X_2(6,j)|\,X(6)\big) = \begin{pmatrix}0.4461 & 0.3057 & 0.3738 & 0.1883 & 0.2092\end{pmatrix}$$

Euler allocation of IMCC: Use the assumption in FRTB ES capital charge, we get

$$IMCC\big(X_n(i,j)|X(i)\big) = 0.5\,\frac{ES^{R,S}\big(X(i)\big)}{ES^{R,C}\big(X(i)\big)}\,ES^{F,C}\big(X_n(i,j)|\,X(i)\big)$$

$$= ES^{F,C}\big(X_n(i,j)|\,X(i)\big)$$

$$IMCC^{F,C}\big(X_1(i,j)|\,X(i)\big) = \begin{pmatrix}0.1210 & 0.0863 & 0.1076 & 0.0580 & 0.0677\\ 0.1225 & 0.0854 & 0.1046 & 0.0568 & 0.0641\\ 0.1148 & 0.0821 & 0.1024 & 0.0575 & 0.0677\\ 0.1201 & 0.0834 & 0.1023 & 0.0539 & 0.0625\\ 0.1281 & 0.0876 & 0.1084 & 0.0563 & 0.0603\\ 0.6425 & 0.4241 & 0.4882 & 0.2334 & 0.2042\end{pmatrix}$$

$$IMCC^{F,C}\big(X_2(i,j)|\,X(i)\big) = \begin{pmatrix}0.1315 & 0.0959 & 0.1314 & 0.0747 & 0.0901\\ 0.1331 & 0.0925 & 0.1256 & 0.0729 & 0.0948\\ 0.1269 & 0.0903 & 0.1254 & 0.0759 & 0.1010\\ 0.1323 & 0.0954 & 0.1239 & 0.0777 & 0.1046\\ 0.1262 & 0.0935 & 0.1287 & 0.0761 & 0.0970\\ 0.4461 & 0.3057 & 0.3738 & 0.1883 & 0.2092\end{pmatrix}$$

For each i = 1,…,6 calculate

$$IMCC\left(\tilde{X}_n(i,k)|X_n(i,j)\right) = \frac{1}{5-j+1}\,IMCC\big(X_n(i,j)|X(i)\big)$$

We have,

$$IMCC\left(\tilde{X}_1(i,k)|X_n(i,j)\right) = \begin{pmatrix}0.0202 & 0.0173 & 0.0268 & 0.0194 & 0.0338\\ 0.0204 & 0.0171 & 0.0261 & 0.0189 & 0.0320\\ 0.0191 & 0.0164 & 0.0256 & 0.0192 & 0.0338\\ 0.0200 & 0.0167 & 0.0256 & 0.0179 & 0.0312\\ 0.0214 & 0.0175 & 0.0271 & 0.0188 & 0.0301\\ 0.1071 & 0.0848 & 0.1221 & 0.0778 & 0.1020\end{pmatrix}$$

$$IMCC\left(\tilde{X}_2(i,k)|X_n(i,j)\right) = \begin{pmatrix}0.0793 & 0.0733 & 0.1274 & 0.1008 & 0.1966\\ 0.0817 & 0.0744 & 0.1198 & 0.1015 & 0.1940\\ 0.0799 & 0.0707 & 0.1253 & 0.1006 & 0.1941\\ 0.0807 & 0.0739 & 0.1251 & 0.1031 & 0.1962\\ 0.0789 & 0.0690 & 0.1181 & 0.1010 & 0.2136\\ 0.2854 & 0.2329 & 0.3455 & 0.2452 & 0.3843\end{pmatrix}$$

Finally the Euler allocation of IMCC is

$$IMCC\left(\tilde{X}_n(i,k)|X_n(i)\right) = \sum_{j=1}^{k} IMCC\left(\tilde{X}_n(i,k)|X_n(i,j)\right)$$

We obtain,

$$IMCC\left(\tilde{X}_1(i,k)|X(i)\right) = \begin{pmatrix} 0.0106 & 0.1904 & 0.3237 & 0.4162 & 0.5783 \\ 0.0108 & 0.1949 & 0.3343 & 0.4280 & 0.5978 \\ 0.0969 & 0.1806 & 0.3136 & 0.4123 & 0.5853 \\ 0.1048 & 0.1913 & 0.3270 & 0.4206 & 0.5933 \\ 0.1043 & 0.1918 & 0.3265 & 0.4188 & 0.5596 \\ 0.5253 & 0.9381 & 1.5676 & 1.9621 & 2.4770 \end{pmatrix}$$

$$IMCC\left(\tilde{X}_2(i,k)|X(i)\right) = \begin{pmatrix} 0.0219 & 0.0411 & 0.0739 & 0.0988 & 0.1442 \\ 0.0222 & 0.0407 & 0.0721 & 0.0964 & 0.1437 \\ 0.0211 & 0.0392 & 0.0705 & 0.0958 & 0.1463 \\ 0.0221 & 0.0411 & 0.0721 & 0.0980 & 0.1503 \\ 0.0210 & 0.0397 & 0.0719 & 0.0973 & 0.1458 \\ 0.0743 & 0.1355 & 0.2289 & 0.2917 & 0.3963 \end{pmatrix}$$

3. Task Division

This project is accomplished with the efforts of the whole team, but different member plays different roles.

For the code part, Xueyi Wang participates in the whole process including solving all of these 17 problems and debugging. Xuyang Liu participates in question 1~12, Pei Zhu participates in question 13~17 and Haoran Chen participates in question 16~17.

For the report part, Haoran Chen collects the results from question 1~12, Pei Zhu collects the results from question 13~17. Xueyi Wang edits the Latex and comments in the notebook. And Xuyang Liu edits and sums up all of these results and writes this final report.