MF850: Advanced Computational Methods

Problem Set 3

If you run into problems, try to verify each component of your code on simple cases to make sure they work independently.

Due date: Thursday November 10, 2022, at noon Boston time.

Instructions: You submit on Blackboard. You may solve this assigment in groups of two. A submission is constituted by answers to the problems along with the code used. A file called `hw3.jl` should contain your code, or your entry point if you separate your code into multiple files. This file should run without errors from a fresh Julia instance/REPL. The code must be formatted by loading the package `JuliaFormatter` and running `format` on the submission files. In other words, submissions in notebook format are not accepted (but you may of course develop in them before creating the submission).
Hint: Running `format(".")` runs the formatter on every `.jl` file recursively in the current directory.

Please contact the instructor or a TA if you have questions regarding these instructions or if you find the problem formulation unclear.

Problem 3.1 The problem of acquiring shares with a price limit can be modeled as follows.

Denote by ν_t the trading speed at time t. The midprice is modeled as

$$\mathrm{d}S_t^\nu = b\nu_t \,\mathrm{d}t + \sigma \,\mathrm{d}W_t,$$

where $b\nu_t$ ($b \geq 0$) is the *permanent market impact* caused by the agent. The execution price is

$$\hat{S}_t^\nu = S_t^\nu + (\tfrac{1}{2}\Delta + k\nu_t),$$

where $\Delta \geq 0$ is the bid-ask spread and $k\nu_t$ ($k > 0$) is the *temporary market impact.*

The agent trades until time τ when either the desired number of shares have been aquired or the price has reached the price limit $\overline{S}$. If $\overline{S}$ is reached, the agent makes an instantaneous purchase of the remaining units Y_τ^ν, with the market impact αY_τ. With the remaining shares to be acquired evolving as

$$\mathrm{d}Y_t^\nu = -\nu_t \,\mathrm{d}t,$$

the optimization problem can be written as[1]

$$V(s,y) = \sup_\nu \mathbb{E}\left[\int_0^\tau (S_t + k\nu_t)\,\mathrm{d}t + Y_\tau(S_\tau + \alpha Y_\tau) + \phi \int_0^\tau Y_t^2 \,\mathrm{d}t \,\middle|\, S_0 = s, Y_0 = y\right].$$

Here, the last term is an inventory penalty and is not monetary.

The dynamic programming equation for this problem is

$$\tfrac{1}{2}\sigma^2 \partial_{ss}^2 V + \phi y^2 + \min_\nu \big\{ -\nu\partial_y V + b\nu\partial_s H + (s + k\nu)\nu \big\} = 0,$$

with the boundary condition $V(\overline{S}, y) = (\overline{S} + \alpha y)$.

Assume there is no permanent price impact, i.e., $b = 0$.

(a) Show that the maximizer $\nu^* = \nu^*(s,y) = -\frac{1}{2k}(s - \partial_y V(s,y))$.

[1] We assume the bid-ask spread to be constant, so it cannot be optimized and need not be included below.

(b) Show that the ansatz $V(s, y) = ys + y^2 h(s)$ leads to the equation

$$\tfrac{1}{2}\sigma^2 h'' + \phi + \min_{\nu} \big\{ -2\nu h + k\nu^2 \big\} = 0 \tag{1}$$

and $\nu^* = \frac{1}{k} h(s)$.

(c) To solve the problem numerically, one must introduce a lower boundary $\underline{S} > 0$ and a boundary condition. It can be shown that a natural choice of boundary condition is $h(\underline{S}) = \sqrt{(}k\phi)$, if $\underline{S} \ll \overline{S}$.

Solve (1) using policy iteration with $\sigma = 0.1$, $k = 10.0^{-}4$, $\phi = 10 * k$, $\alpha = 100 * k$, $\underline{S} = 20$, and $\overline{S} = 20.1$.

Plot the optimal strategy as a function of s.

Hint: For the policy updating step, use the explicit solution to the optimization problem, like that found in (a) and (b).

More details on this type of problem can be found in [CJP15], in particular Chapter 7.

References

[CJP15] Álvaro Cartea, Sebastian Jaimungal, and José Penalva. *Algorithmic and high-frequency trading*. Cambridge University Press, 2015.