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MF850 Problem Set 3

Problem 3.1

(a)&(b) The answer to the first two questions are shown below:

Problem 3.1 (a) we have $\min_{v}\{-v\partial_y V + bv\partial_s H + (s+kv)v\}$

$$FOC \Rightarrow (kv^2 + (s-\partial_y V)v)' = 0$$

$$\Rightarrow 2kv = -(s - \partial_y V(s,y))$$

$$\Rightarrow v^* = \frac{1}{-2k}(s - \partial_y V(s,y))$$

(b) we have $V(s,y) = ys + y^2 h(s)$

$$\begin{cases} \partial_s V(s,y) = y^2 + y^2 h' \\ \partial_{ss} V(s,y) = y^2 h'' \\ \partial_y V(s,y) = s + 2yh \end{cases} \Rightarrow \frac{1}{2}\sigma^2 y^2 h'' + \phi y^2 + \min_{v}\{-vs - 2yh\cdot v + sv + kv^2\} = 0$$

$$y^2\left(\frac{1}{2}\sigma^2 h'' + \phi\right) + \min_{v}\{-2yhv + kv^2\} = 0, \quad v^* = \frac{-2yh}{-2k}$$

$$\Rightarrow y^2\left(\frac{1}{2}\sigma^2 h'' + \phi - \frac{h^2}{k}\right) = 0, \quad y^2 \neq 0$$

$$\Rightarrow \frac{1}{2}\sigma^2 h'' + \phi - \frac{h^2}{k} = 0$$

$$\Rightarrow \frac{1}{2}\sigma^2 h'' + \phi + \min_{v}(-2vh + kv^2) = 0, \quad \text{where } v^* = \frac{1}{k}h(s)$$

And $h(\bar{S}) = \alpha$

(c) The plot is shown below: (for the detail, please check the code file.)