Capital Charge and Allocation under FRTB

October 17, 2022

1 Project Description

1. Consider a portfolio of TWO risk positions. Each risk position can be thought as one asset, which has exposure to all different *risk factors* RF_i, $i = 1, \dots, 5$, and different *liquidity horizons* LH_j, $j = 1, \dots, 5$:

$$\{\mathrm{RF}_i : 1 \leq i \leq 5\} = \{\mathrm{CM}, \mathrm{CR}, \mathrm{EQ}, \mathrm{FX}, \mathrm{IR}\},$$
$$\{\mathrm{LH}_j : 1 \leq j \leq 5\} = \{10, 20, 40, 60, 120 \text{ days}\}.$$

2. Suppose that the 10 days loss of risk position n, attributed to RF_i and LH_j, is denoted by $\tilde{X}_n(i,j)$, for $n = 1, 2$, $1 \leq i, j \leq 5$. Assume that

$$\tilde{X}_1(i,j) \sim N(0.004, 0.04) \quad \text{and} \quad \tilde{X}_2(i,j) \sim N(0.006, 0.05), \tag{1}$$

where 0.04 and 0.05 are the standard deviation of these normal distributions. The correlation between any two different $\tilde{X}_1(i,j)$ and $\tilde{X}_2(k,\ell)$ is assumed to be 0.3. The correlation between any two different $\tilde{X}_2(i,j)$ and $\tilde{X}_2(k,\ell)$ is assumed to be 0.1. $\tilde{X}_i(i,j)$ and $\tilde{X}_2(k,\ell)$ are assumed to be independent.

3. The total loss of the risk position n is

$$\tilde{X}_n = \sum_{i,j}^{5} \tilde{X}_n(i,j), \quad n = 1, 2. \tag{2}$$

The total loss of the portfolio is

$$\tilde{X} = \tilde{X}_1 + \tilde{X}_2. \tag{3}$$

4. (**Standard VaR and ES**) Use the simulation method to calculate the $\mathrm{VaR}_{0.99}$ and $\mathrm{ES}_{0.975}$ for the 10 days loss of the portfolio.

To simulate different $\tilde{X}_1(i,j)$ with pair correlation 0.2, we can set

$$\tilde{X}_1(i,j) = 0.3Z + \sqrt{1 - 0.3^2}Z(i,j), \tag{4}$$

where Z and $\{Z_{i,j}\}_{1\le i,j\le 5}$ are all independent standard normal random variables. Similar method can be used to simulate $\tilde{X}_2(i,j)$.

5. (**VaR and ES allocations**) For each simulation of $\tilde{X}_n(i,j)$, we have a simulation of $\tilde{X}_1$, $\tilde{X}_2$, and $\tilde{X}$. Using these simulations to compute Euler allocations for VaR and ES:

   - Among all simulations, find those with $\tilde{X} \in (\mathrm{VaR}_{0.99} - \epsilon, \mathrm{VaR}_{0.99} + \epsilon)$, for a small ϵ. Take average of $\tilde{X}_1$ and $\tilde{X}_2$ for these simulations. They are Euler allocations of $\mathrm{VaR}_{0.99}$ for the risk position 1 and 2, respectively.
   - Among all simulation, find those with $\tilde{X} \ge \mathrm{VaR}_{0.975}$. Take average of $\tilde{X}_1$ and $\tilde{X}_2$ for these simulations. They are Euler allocations of $\mathrm{ES}_{0.975}$ for the risk position 1 and 2, respectively.

6. In FRTB, the liquidity horizon adjusted loss for risk position n is

$$X_n(i,j) = \sqrt{\frac{\mathrm{LH}_j - \mathrm{LH}_{j-1}}{10}} \sum_{k=j}^{5} \tilde{X}_n(i,k), \quad 1 \le i,j \le 5. \tag{5}$$

   The liquidity horizon adjusted loss for the portfolio is

$$X(i,j) = X_1(i,j) + X_2(i,j), \quad 1 \le i,j \le 5. \tag{6}$$

7. (**FRTB ES**) For each $i = 1,\dots,5$, the FRTB expected shortfall for portfolio loss attributed to RF_i is

$$\mathrm{ES}(X(i)) = \sqrt{\sum_{j=1}^{5} \mathrm{ES}_{0.975}(X(i,j))^2}. \tag{7}$$

   Denote the previous expected shortfall as $\mathrm{ES}^{\mathrm{F,C}}(X(i))$.

8. (**FRTB ES capital charge**) Assume that $\mathrm{ES}^{\mathrm{R,S}}(X(i))/\mathrm{ES}^{\mathrm{R,C}}(X(i)) = 2$ for all $1 \le i \le 5$. (In practice, this ratio is calculated using the loss data in the stress period and in the current 12 months.) The FRTB ES capital charge for RF_i is

$$\mathrm{IMCC}(X(i)) = \frac{\mathrm{ES}^{\mathrm{R,S}}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))} \mathrm{ES}^{F,C}(X(i)), \quad 1 \le i \le 5. \tag{8}$$

9. For the risk position n, the unconstrained portfolio with LH_j is

$$X_n(6,j) = \sum_{i=1}^{5} X_n(i,j), \quad 1 \le j \le 5. \tag{9}$$

   For the portfolio, the unconstrained portfolio with LH_j is

$$X(6,j) = X_1(6,j) + X_2(6,j). \tag{10}$$

   $\mathrm{IMCC}(X(6))$ is calculated similarly as in item 7 and 8 with $i = 6$.

10. (**FRTB capital charge for modellable risk factors**) For the portfolio loss X, its aggregate capital charge for modellable risk factors is

$$\mathrm{IMCC}(X) = 0.5\mathrm{IMCC}(X(6)) + 0.5\sum_{i=1}^{5}\mathrm{IMCC}(X(i)). \tag{11}$$

11. Use the simulation in item 2 and the procedure introduced in item 6-10, calculate $\mathrm{IMCC}(X)$.

12. (**FRTB Euler allocation of** $X(i,j)$) For each i,j, use the simulations of $\tilde{X}_n(i,j)$ in item 2 to simulate $X_1(i,j)$, $X_2(i,j)$, and $X(i,j)$ in item 6. Calculate $\mathrm{VaR}_{0.975}(X(i,j))$. Find all simulations with $X(i,j) \geq \mathrm{VaR}_{0.975}(X(i,j))$. Among all these simulations, calculate the average of $X_1(i,j)$ and $X_2(i,j)$. They are Euler allocation of $X(i,j)$. We denote them as $\mathrm{ES}(X_1(i,j)|X(i,j))$ and $\mathrm{ES}(X_2(i,j)|X(i,j))$.

13. For each $1 \leq i \leq 5$, use the results in item 7 and 12 to calculate

$$\mathrm{ES}(X_n(i,j)|X(i)) = \frac{\mathrm{ES}(X(i,j))}{\mathrm{ES}(X(i))}\mathrm{ES}(X_n(i,j)|X(i,j)), \quad n = 1,2, j = 1,\dots,5. \tag{12}$$

    Check that

$$\sum_{n=1}^{2}\sum_{j=1}^{5}\mathrm{ES}(X_n(i,j)|X(i)) = \mathrm{ES}(X(i)). \tag{13}$$

    Denote $\mathrm{ES}(X_n(i,j)|X(i))$ as $\mathrm{ES}^{\mathrm{F,C}}(X_n(i,j)|X(i))$.

14. Follow the same method as in item 13, calculate $\mathrm{ES}^{F,C}(X_n(6,j)|X(6))$.

15. (**Euler allocation of IMCC**) Use the assumption in item 8, calculate

$$\mathrm{IMCC}(X_n(i,j)|X(i)) = 0.5\frac{\mathrm{ES}^{R,S}(X(i))}{\mathrm{ES}^{\mathrm{R,C}}(X(i))}\mathrm{ES}^{\mathrm{F,C}}(X_n(i,j)|X(i)), \tag{14}$$

    for $i = 1,\dots,6$ and $j = 1,\dots,5$.

16. For each $i = 1,\dots,6$, calculate

$$\mathrm{IMCC}(\tilde{X}_n(i,k)|X_n(i,j)) = \frac{1}{5-j+1}\mathrm{IMCC}(X_n(i,j)|X(i)), \quad k \geq j. \tag{15}$$

    Finally, the Euler allocation of IMCC is

$$\mathrm{IMCC}(\tilde{X}_n(i,k)|X(i)) = \sum_{j=1}^{k}\mathrm{IMCC}(\tilde{X}_n(i,k)|X_n(i,j)). \tag{16}$$

17. Report $\mathrm{IMCC}(\tilde{X}_n(i,k)|X(i))$, $i = 1,\dots,6$ and $k = 1,\dots,5$, following the procedure in item 12 - 16.

2 Team

This is a team project. Each team is allowed to have maximum of 4 members. Everyone needs to write part of the codes.

3 Report

1. Submit a research report including an executive summary, presentation of your results and methodology.

2. Submit a Zip file of your Mathlab script, Python notebook, or R script with sufficient comments.

3. A short paragraph discussion tasks accomplished by each team members.

4 Grading Criteria

Quality of the write up, quality of data analysis, and discussion of results. Equally-contributing team members will get the same grade on the project. Team members who do not contribute sufficiently will be marked separately.

This is the first part of the final project. This part determines 25% of the final grade.