HW Problems for Assignment 4

1. (50 Points) Stress Test for a Market-Cap Weighted Portfolio of Microsoft and Apple Stocks. In this exercise you will perform the stress test shown in class, except that we will assume a negative shock to both stocks, not just one, with one negative shock dependent upon the other.

We have a market cap weighted portfolio of Microsoft and Apple stocks, where the caps are computed as of $9/1/16$. We have daily log return data from $9/1/11 - 9/1/16$. We shock the portfolio by assuming a large negative daily log return for Apple, which leads to a large negative log return for Microsoft. We want to see how this shock affects our losses over a K day horizon. In particular, we wish to estimate the shocked K day Value at Risk, and the frequency at which losses exceed the regulatory capital requirement estimated prior to the shock.

To perform the stress test:

(1) With the historical log return data, estimate the mean vector $\mu_{t+\Delta}$ and covariance matrix $\Sigma_{t+\Delta}$ as of time $t = 9/1/16$ using EWMA. You can start off with initial values of 0 for both and then update through time via the recursive equations

$$\mu_{s+\Delta} = \lambda\mu_s + (1-\lambda)X_s;$$
$$\Sigma_{s+\Delta} = \theta\Sigma_s + (1-\theta)(X_s - \mu_s)(X_s - \mu_s)^{\mathrm{T}}.$$

(2) At t assume $X_{t+\Delta} \overset{\mathcal{F}_t}{\sim} N\left(\mu_{t+\Delta}, \Sigma_{t+\Delta}\right)$. Estimate $\mathrm{VaR}_\alpha\left(L_{t+\Delta}^{lin}\right)$, and using the square root of time rule, estimate $\mathrm{VaR}_\alpha\left(L_{t+K\Delta}^{lin}\right)$ as well as the regulatory capital change $3 \times \mathrm{VaR}_\alpha\left(L_{t+K\Delta}^{lin}\right)$.

(3) Shock the system by assuming a large negative return for Apple, where below (1) corresponds to Microsoft, and (2) corresponds to Apple. Specifically:

   (i) Set $X_{t+\Delta,(2)} = x_{(2)} = \mu_{t+\Delta,(2)} - 5 \times \sigma_{t+\Delta,(2)}$.
   (ii) To obtain $X_{t+\Delta,(1)}$, first use the conditioning result which showed that given $X_{t+\Delta,(2)} = x_{(2)}$, $X_{t+\Delta,(1)}$ is normally distributed with

$$\widetilde{\mu}_{t+\Delta,(1)} = \mu_{t+\Delta,(1)} + \frac{\rho_{t+\Delta,(1)}\sigma_{t+\Delta,(1)}}{\sigma_{t+\Delta,(2)}}\left(x_{(2)} - \mu_{t+\Delta,(2)}\right);$$
$$\widetilde{\sigma}_{t+\Delta,(1)}^2 = \sigma_{t+\Delta,(1)}^2\left(1 - \rho_{t+\Delta}^2\right).$$

   Then, assume a -5 sigma (conditional) shock for Microsoft by setting $X_{t+\Delta,(1)} = x_{(1)} = \widetilde{\mu}_{t+\Delta,(1)} - 5\widetilde{\sigma}_{t+\Delta,(1)}$. This part varies from the lecture slides, and makes the stress test (with high probability) more stressful.
   (iii) Use $x_{(1)}, x_{(2)}$ to update $\mu_{t+2\Delta}, \Sigma_{t+2\Delta}$ via EWMA.

(4) For days $k = 2, \ldots, K$ sample $X_{t+k\Delta} \sim N(\mu_{t+k\Delta}, \Sigma_{t+k\Delta})$ and update $\mu_{t+(k+1)\Delta}, \Sigma_{t+(k+1)\Delta}$ via EWMA (the mean and covariance need not be updated for $k = K$).

(5) Estimate the linearized K day portfolio loss by assuming the shares the stocks were held constant: i.e. by setting $L_{t+K\Delta}^{lin} = -\theta_t^T \left(\sum_{k=1}^{K} X_{t+k\Delta} \right)$ where $\theta_t = S_t \lambda_t$ is the dollar position in the stocks at time t.

You will then repeat steps $(3) - (5)$ above $\hat{M}$ times to get the shocked losses

$$\ell_m = L_{t+K\Delta}^{Lin,m} = -\theta_t^T \left(\sum_{k=1}^{K} X_{t+k\Delta}^m \right).$$

Using the losses $\{\ell_m\}_{m=1}^{\hat{M}}$ compute

(a) The average K-day portfolio loss.

(b) An estimate of the K-day VaR_α.

(c) The frequency with which the losses exceeded the initial K day Value at Risk found using the square root of time rule.

(d) The frequency with which the losses exceed the regulatory capital found using the square root of time rule.

To obtain the frequencies, take the number of exceedances and multiply by $100/\hat{M}$ to get the percentage of days the losses exceeded the given values.

The log return data is in the file

"MSFT_AAPL_Log_Returns.csv".

This file has three columns: the date in numeric format sorted oldest to newest; the daily log return for MSFT; the daily log return for APPL. In order to help out the grader, you MUST write your program to use this data file, in this specific format. The market caps as of 9/1/16 are

MSFT: 448.77(b); AAPL: 575.11(b).

For the confidence use $\alpha = .95$. For the number of days use $K = 10$. For the portfolio value use $\$1,000,000$. For the initial days of data use $M = 100$. For the EWMA procedure use $\lambda = \theta = .97$. For the number of paths to run in your simulation use $\hat{M} = 50,000$.