BU Questrom School of Business, Fall 2022
Prof. Max Reppen — Dapeng Shang

MF850: Advanced Computational Methods

Problem Set 1

One component of part (b) uses definitions presented in Lecture 2.

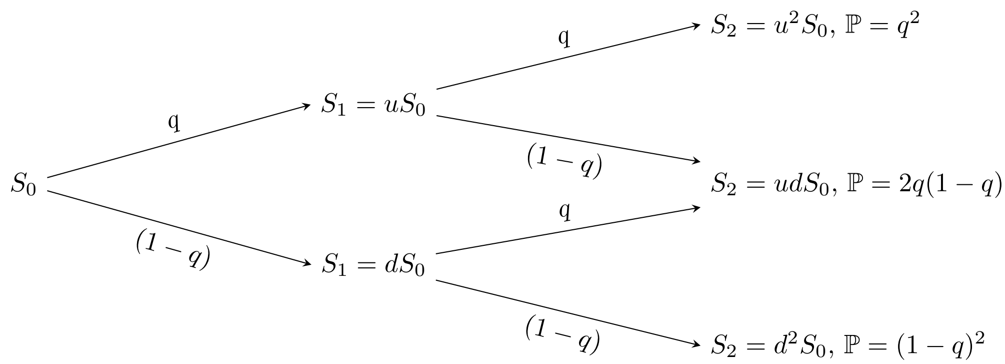
Due date: Thursday September 19, 2022, at noon.

Instructions: You submit on Blackboard. You may solve this assigment in groups of two. A submission is constituted by answers to the problems along with the code used. A file called `hw1.jl` should contain your code, or your entry point if you separate your code into multiple files. This file should run without errors from a fresh Julia instance/REPL. The code must be formatted by loading the package `JuliaFormatter` and running `format` on the submission files. In other words, submissions in notebook format are not accepted (but you may of course develop in them before creating the submission).
Hint: Running `format(".")` runs the formatter on every `.jl` file recursively in the current directory.

Please contact the instructor or a TA if you have questions regarding these instructions or if you find the problem formulation unclear. Remember to check the QuestromTools calendar for office hours.

Problem 1.1 Consider a Black–Scholes market with one risky asset $(S_t)_{t\geq 0}$ with volatility $\sigma = 10\%$, and $S_0 = 100$. Let the risk-free interest rate be $r = 2\%$.

Consider a binomial approximation with N steps following pattern in the following diagram, with $u = 1/d = e^{\sigma\sqrt{1/N}}$ and risk neutral probability $q = \frac{e^{r/N}-d}{u-d}$.



(a) Find an approximation to the price of a European put option on S with strike $K = 95$ and maturity $T = 1$ by constructing a binomial tree and computing the price backwards in a dynamic programming fashion.

Hint: If you are unsure of how to represent the tree, you may use a two-dimensional array. A more elegant solution would use a custom struct to hide the implementation in an interface.

(b) Plot and analyze the convergence of the binomial tree approximation to the Black–Scholes price in (a) as the number of steps increases. Solve for at least $\{1, \ldots, 200\}$ steps. Explain the result and comment on the convergence and convergence *rate*.

Hint: Solving for even more steps might help in explaining this.

(c) Now consider an American put option. Provide an approximation to the price at time zero and comment on the convergence.

This value can be computed using dynamic programming and in each step setting the value to the maximum of the next-step-expectation and the current payoff—i.e., the maximum of waiting and exercising.