MF796 Homework3
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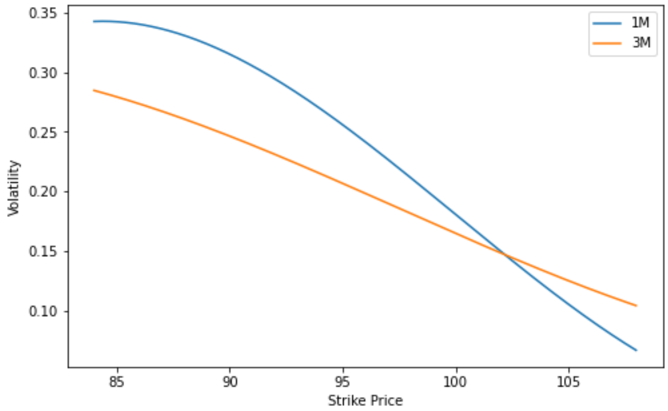
Problem Set # 3

1. Implementation of Breeden-Litzenberger:

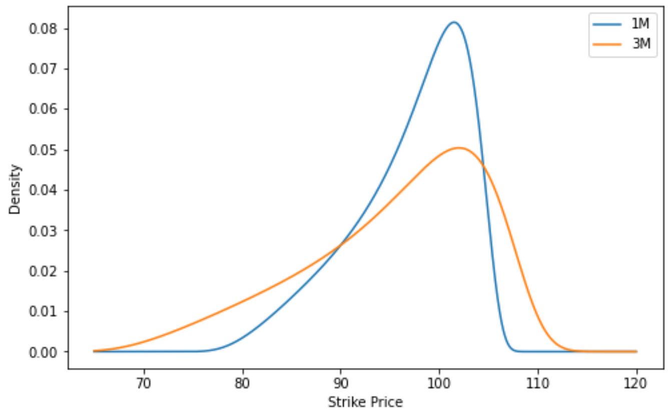
(a) According to the BS formular, I got the strike prices below:

Delta	1M	3M	Put or Call	1M_Strike	3M_Strike
0.10	0.3225	0.2836	put	89.138758	84.225674
0.25	0.2473	0.2178	put	95.542103	93.470685
0.40	0.2012	0.1818	put	98.705646	98.127960
0.50	0.1824	0.1645	call	100.138720	100.338826
0.40	0.1574	0.1462	call	101.262273	102.141761
0.25	0.1370	0.1256	call	102.783751	104.532713
0.10	0.1148	0.1094	call	104.395838	107.422225

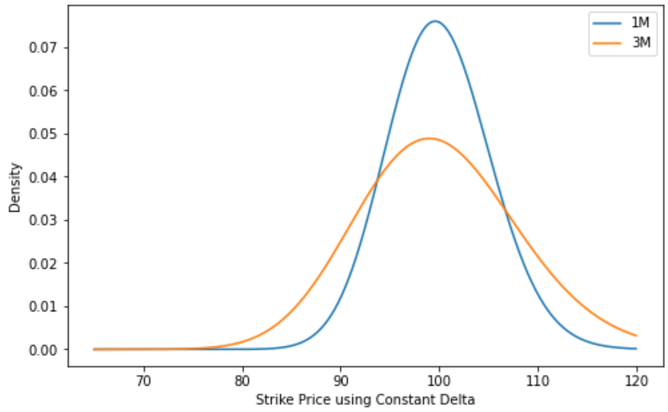
(b) Using np.polyfit, I got the volatility function for all strikes, $\sigma(K)$:



(c) Here's the plot the neutral density:



(d) Using a constant volatility, I can get the new density plot below:



(e):

(i) 1M European Digital Put Option with Strike 110 is about: **1.00**
(ii) 3M European Digital Call Option with Strike 105 is about: **0.36**
(iii) 2M European Call Option with Strike 100 is about: **2.76**

2. Calibration of Heston Model:

(a) There're three main methods to detect the arbitrage. First, call(put) prices are monotonically decreasing(increasing). Second, call(put) prices whose rate of change is greater than 0 (-1) and less than 1(0). Third, Call and put prices that are convex with respect to changes in strike.
After checking, the options have no chances for arbitrage.

(b)(c) In a form of **[kappa, theta, sigma, rho, v0]**, by changing bounds and start value, I got the result below:

Initial Value	Lower bound	Upper bound	Final Value	Least Squares
[0,0.2,0.2,0,0.2]	[0.01, 0.01, 0.0, -1, 0]	[2.5, 1, 1, 0.5, 0.5]	[1.26997742, 0.08847748, 1, -0.82384662, 0.03426619]	33.494880284930886
[0.2, 0.3, 0.3, 0.1, 0.3]	[0.01, 0.01, 0.0, -1, 0]	[2.5, 1, 1, 0.5, 0.5]	[1.27005383, 0.08847459, 1, -0.82384995, 0.03426606]	33.49488031669151
[0.2, 0.3, 0.3, 0.1, 0.3]	[0.001, 0, 0.0, -1, 0]	[5, 2, 2, 1, 1]	[4.1604416, 0.0590171, 1.67455619, -0.809526, 0.03980849]	32.17558690782599

From this form, we can conclude that the least squares nearly have significant connection with the bounds and the start value because it is around 33. For the final value, they vary from with each other.
All of these parameters seem not to be influenced by start value. However, kappa and theta would be influenced by the bounds apparently. For example, we can see that by

changing upper bound from 2.5 to 5, kappa changes from 1.27 to 4.16 , that's not a small shift comparing to other parameters.

(d) By using the unequal function, repeat what I did above and get the form below:

Initial Value	Lower bound	Upper bound	Final Value	Least Squares
[0,0.2,0.2,0,0.2]	[0.01, 0.01, 0.0, -1, 0]	[2.5, 1, 1, 0.5, 0.5]	[2.43971485 ,0.05886434 ,1, -0.78507201,0.03284587]	253.5842246586348
[0.2, 0.3, 0.3, 0.1, 0.3]	[0.01, 0.01, 0.0, -1, 0]	[2.5, 1, 1, 0.5, 0.5]	[2.43966924, 0.0588646, 1, -0.78507271,0.03284602]	253.58422485269378
[0.2, 0.3, 0.3, 0.1, 0.3]	[0.001, 0, 0.0, -1, 0]	[5, 2, 2, 1, 1]	[3.33259696, 0.05420142, 1.16166716, -0.77654707, 0.03376776]	251.62993375165848

Using the unequal weighted function, we can see that the least squares results are much larger than the previous ones. But it still stay in stable, around 250. For the final value, there're some small changes in values but still have the same tendency as we talked in (c).

3. **Hedging Under Heston Model:**

I use the parameters calibrated in problem 2.b to calculate.

```
ka = 3.333
theta = 0.054
sigma = 1.161
rho = -0.776
v0 = 0.034
```

(a)(i) Delta of models:
Delta computed by Heston: **0.47967255696321764**
Delta computed by BS: **0.3412312268682058**
Obviously, they're not same. Heston model take the volatility effect in to consideration while BS model take the volatility constant. So I think Heston Model calculated through FFT is better.

(ii) To reach delta neutral, we need to short 0.48 unit of underlying asset.
(b)(i) Vega of models:
Vega computed by Heston: **131.79138380102174**
Vega computed by BS: **45.57996112892058**

(ii) They're different. Heston model take the Rho into consideration which would affect Vega while BS model doesn't.