

Problem Set # 5

Problem 1: Numerical PDEs:

- (a) This instrument isn't same as hold a real American option, because for a American option, which can be executed at any time, we can't choose a suitable time. And also, when we short a position, it's not our right to execute the option.
- (b) I search it on Federal Reserve Economic Data and the riskless rate is about: **0.5**, I think FRED is a really official website where the data is reliable.
- (c) We can calculate the implied volatility from its history, we use the std of SPY the past year and then compute the volatility. The volatility is about **0.148**
- (d) I choose Smax as **550** because I think the price won't be higher that 550 during this period. I choose hs as **275**, ht as **1000**. Because I want to more discretization of time T to make the result more accurate.
- (e) Here's the brief view of eigen Value of A:

```
In [214]: egv
Out[214]:
array([[ 0.99999727,  0.9999939 ,  0.99998763,  0.99997813,  0.99996507,
         0.99994815,  0.99992709,  0.99990163,  0.99987149,  0.99983644,
         0.99979623,  0.99975063,  0.9996994 ,  0.99964232,  0.99957916,
         0.9995097 ,  0.99943373,  0.99935103,  0.99926138,  0.99916456,
         0.99906038,  0.99894861,  0.99882904,  0.99870147,  0.99856568,
         0.99842147,  0.99826862,  0.99810693,  0.99568422,  0.99595716,
         0.99621892,  0.99646971,  0.99670975,  0.99693926,  0.99715844,
         0.99793618,  0.99736751,  0.99756668,  0.99775617,  0.99539988,
         0.99510393,  0.99479614,  0.99056522,  0.99101806,  0.99447629,
         0.99145656,  0.99344217,  0.99379953,  0.99307187,  0.99268838,
         0.99229149,  0.99188096,  0.99414416,  0.98326361,  0.98392501,
         0.98456886,  0.98519541,  0.99009781,  0.98580492,  0.98639764,
         0.98697384,  0.98911831,  0.98860574,  0.98807764,  0.98753375,
         0.98961558,  0.98188707,  0.98043713,  0.98117141,  0.98258438,
         0.97968395,  0.97891161,  0.97730832,  0.97811982,  0.9700718 ,
         0.97105148,  0.9720091 ,  0.97294496,  0.9764768 ,  0.975625 ,
         0.97385937,  0.97475262,  0.96906976,  0.96804506,  0.95251472,
         0.95386369,  0.96699737,  0.95518568,  0.95648103,  0.9577501 ,
         0.96483179,  0.96371325,  0.96257046,  0.95899324,  0.96140309,
         0.96021079,  0.96592638,  0.95113841,  0.94830231,  0.94684178,
         0.93578965,  0.93746002,  0.93909923,  0.94535244,  0.94070769,
         0.9438339 ,  0.9497344 ,  0.94228578,  0.93235379,  0.93058746,
         0.92878831,  0.92695591,  0.93408771,  0.92508983,  0.92318961,
         0.92125483,  0.91523852,  0.91727974,  0.91928503,  0.88261272,
         0.88520944,  0.88776303,  0.89027404,  0.91316088,  0.89274304,
         0.89990336,  0.90220967,  0.89755717,  0.89517057,  0.90447663,
         0.90889447,  0.90670472,  0.91104636,  0.87728758,  0.87997229,
         0.86021584,  0.86317891,  0.86609403,  0.86896184,  0.87178296,
         0.87455801,  0.85720419,  0.83807644,  0.85414328,  0.85103247,
         0.84139376,  0.84787107,  0.8446584 ,  0.83470572,  0.83128087,
         0.82780114,  0.82426577,  0.82067399,  0.78981191,  0.79388441,
         0.79789381,  0.81702502,  0.80184101,  0.80572689,  0.80955229,
         0.81331805,  0.78567541,  0.74540876,  0.75016497,  0.75945954,
         0.76400003,  0.76847072,  0.77287261,  0.77720671,  0.67640311,
         0.68231731,  0.68361282,  0.68357888,  0.68112818,  0.75101821])
```

```
In [216]: sum(abs(egv)>1)
Out[216]: 0
```

And we check the absolute value: `Out[216]: 0`, which means all the eigen value is less than 1. Therefore, the stability condition is met.

- (f) As our discretization follows the linear relation, so we can use the interpolation of linear method to choose today's price. In python, I use `np.interp()`, then I get today's price is about **2.1**
- (g) Using the same methods, but follow the American style, the price is about: **4.17**
- (h) The premium is about: **2.1**. I think this result is reasonable. American option (right of early exercise) offers people more benefits. That's why the price is more expensive to cover the benefit.