MF 796 Homework 3
Sketch of Solutions

February 18, 2021

Basic Algorithm

1. Construnct $\{x\}_i$ vector which contains a sequence of

$$x_i = \frac{\Delta v}{2} \frac{e^{-\int_0^T r_u du}}{(\alpha + iv)(\alpha + iv + 1)} e^{-i(lnS_0 - \Delta k N/2)} \Phi(v_i - (\alpha+1)i)$$

2. Call built-in FFT function on $\{x\}_i$. For example, in R you should code fft(x), where x is a vector.

3.Calculate

$$C_T(k_i) = \frac{e^{-\alpha[lnS_0 - \Delta k(N/2 - i + 1)]}}{\pi} Re(y_i)$$

(a)

(i) Different prices correspoding to different α with $N = 14$ and $B = 250$ are:

α	0.01	0.25	0.5	1	1.5	5	10	20
C	76.08	21.27	21.27	21.27	21.27	21.27	21.27	21.27

We can see that α's between 0.25-20 are stable.

(ii) The N and B are large enough for an accurate result (not be efficient of course since 2^{14} is such a large number) and the calculation costs 0.3929 second. Choose $\alpha = 1$ and then a suitable combination of N and B should give call price close to 21.27. Use the square error between the price and 21.27 as MSE and seconds it costs as computational effort, we can calculate the computational efficiency. Try N = 9, 10, 11, 12, 13 and B = 10, 50, 100, 200, the efficiency plot should be like as Figure 1. Notice that here "$N = 10$", for example, means "$N = 2^{10}$".

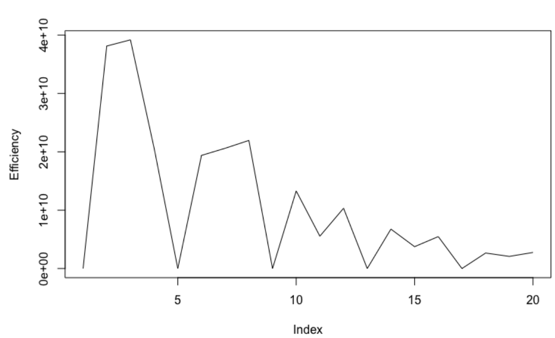

Figure 1: Efficiency

Therefore the most efficient combination is the 3rd one, which is $N = 9$ and $B = 100$.

(iii) Re-calculate the "true value" , with $N = 14$ and $B = 250$. Do the same process as in (ii) and you will find the most efficient combination is the same as (or at least close to) in (ii). So the result seems to be strike-price-invariant.

Q2

Here I use $N = 14$ and $B = 250$.
(i) See Figure 2.

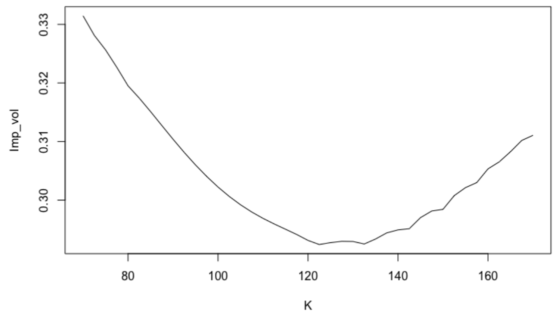

Figure 2: Volatility Skew

(ii) See Figure 4.

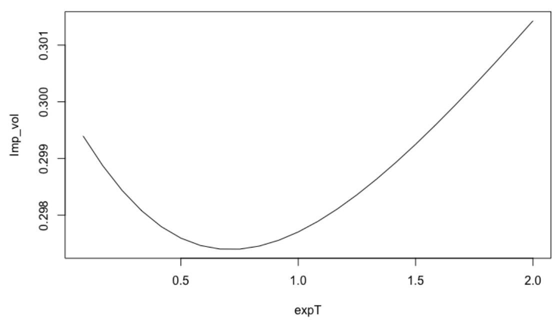

Figure 3: Term Structure

(iii) I will show the effect of some important parameters here.

If we change $\kappa = 0.5$ to $\kappa = 0.1$, i.e. slow down the mean-reverting process, then the implied volatility becomes:

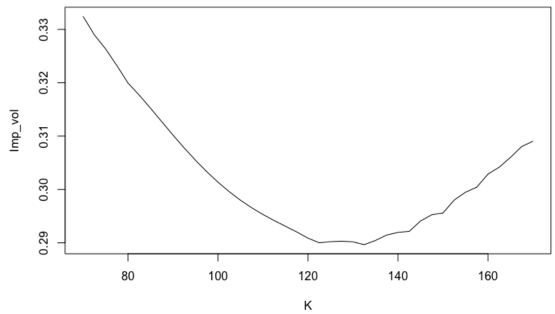

Figure 4: Volatility Skew

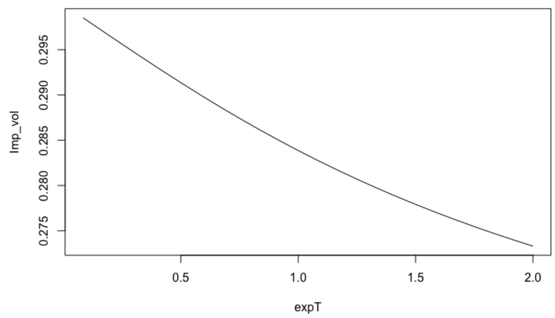

Figure 5: Term Structure

while the volatility skew looks the same, the term structure is completely changed from a "smile" to a "skew".

If we change $\rho = 0.25$ to $\rho = -0.25$, then the implied volatility becomes:

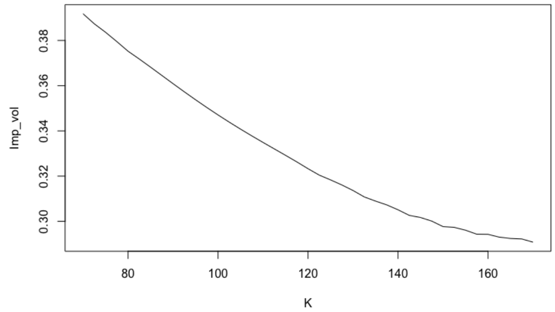

Figure 6: Volatility Skew

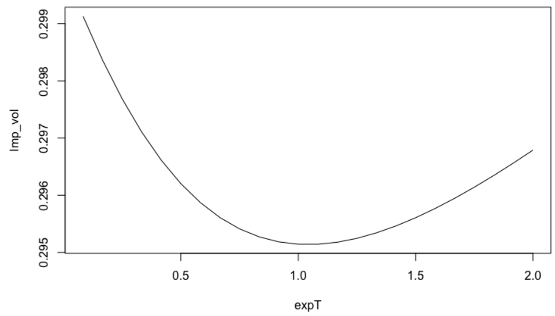

Figure 7: Term Structure

now the volatility skew stops to be like a "smile".