MF728 Homework 4
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Problem Set # 4

1. **Swaption Pricing and Risk Management under the SABR Model:**

(a) Here's the result of these forward rates:

	Instantaneous Forward Rates
1Y	0.011711
2Y	0.012024
3Y	0.013259
4Y	0.015147
5Y	0.017112

(b) Here's the results of these annuities:

	Annuity Value
1Y	4.813959
2Y	4.780274
3Y	4.726896
4Y	4.653985
5Y	4.571786

(c) The premium of each swaptions shown below:

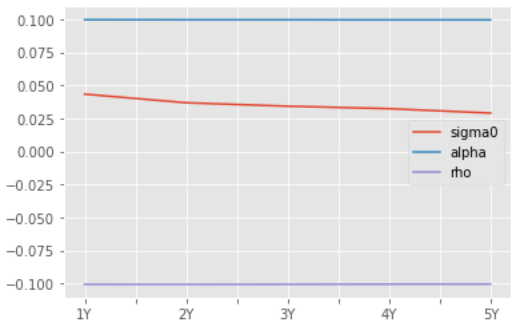
	ATM-50	ATM-25	ATM-5	ATM+5	ATM+25	ATM+50
1Y	0.038894	0.028657	0.022388	0.019749	0.012430	0.006789
2Y	0.036035	0.026528	0.019595	0.017009	0.011019	0.005765
3Y	0.033570	0.025254	0.019595	0.015442	0.009978	0.005025
4Y	0.032842	0.023788	0.017348	0.014719	0.009216	0.004493
5Y	0.031973	0.022901	0.016541	0.013950	0.008648	0.004120

(d) Using the package scipy.optimize in python, I got the result of these parameters:

Out[411]:

	sigma0	alpha	rho
1Y	0.043408	0.099742	-0.100629
2Y	0.036871	0.099701	-0.100659
3Y	0.034284	0.099676	-0.100584
4Y	0.032398	0.099659	-0.100492
5Y	0.029064	0.099641	-0.100437

(e) Here's a plot of these parameters:



From the plot we can see that sigma0 and alpha have a trend in rising while the rho is decreasing as the expiries become longer. But for the degree, the change of sigma0 is obviously larger than the others, alpha and rho are not so sensitive to the change of expiry.

(f) Here's the results of volatilities and prices with strikes equal to ATM - 75 and ATM + 75:

	ATM-75	ATM+75
1Y	0.003643	0.005218
2Y	0.003226	0.004514
3Y	0.003268	0.004380
4Y	0.003424	0.004382
5Y	0.003363	0.004150

	ATM-75	ATM+75
1Y	0.039893	0.008829
2Y	0.038508	0.006403
3Y	0.038181	0.005907
4Y	0.037980	0.005823
5Y	0.037153	0.005023

(g) The equivalent Black volatilities are:

Out[417]:

	ATM-50	ATM-25	ATM-5	ATM+5	ATM+25	ATM+50
1Y	0.716403	0.521184	0.446667	0.421488	0.327186	0.268785
2Y	0.594870	0.454323	0.375674	0.355493	0.293245	0.242608
3Y	0.455283	0.381635	0.342317	0.295750	0.250889	0.209620
4Y	0.375330	0.306216	0.264058	0.250357	0.211473	0.178296
5Y	0.315302	0.258944	0.224921	0.213909	0.182496	0.155073

(h) The delta under BS model:

Out[418]:

	ATM-50	ATM-25	ATM-5	ATM+5	ATM+25	ATM+50
1Y	4.208608	3.776969	3.400283	3.202427	2.602305	1.858270
2Y	4.096886	3.677080	3.254801	3.037263	2.467138	1.703434
3Y	3.943788	3.538688	3.154188	2.874792	2.313117	1.548550
4Y	3.789823	3.383974	2.966234	2.735274	2.167889	1.413052
5Y	3.654206	3.256646	2.841590	2.609987	2.050851	1.310141

(i)The adjust delta are calculated below:

	ATM-50	ATM-25	ATM-5	ATM+5	ATM+25	ATM+50
1Y	3.542022	3.280922	2.962026	2.783207	2.340878	1.728937
2Y	3.521693	3.220868	2.878540	2.676120	2.206438	1.562082
3Y	3.531889	3.166127	2.801877	2.594809	2.106288	1.444923
4Y	3.458855	3.110183	2.732247	2.513842	2.013278	1.345728
5Y	3.373223	3.026406	2.643726	2.422102	1.916887	1.249282

From the data we can conclude that SABR delta is larger than BS's delta. However, they have the same tendency with the respect with expiry and strike. To be specific, longer expiry leads to lower delta and lower strike leads to lower delta.