**Problem Set # 4**

**Problem 1: Covariance Matrix Decomposition:**

1. I choose randomly 100 stocks’ five year daily close price from S&P 500. And here’s a brief view of these data:

图形用户界面, 文本

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1. I compute the log return and get the results:

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1. Here’s the covariance matrix:

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And test the negative or positive characteristic of the eigenvalues:

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I find that all the eigenvalues are **positive**. That’s reasonable. If there’re any negative, maybe my datasets have some errors or the data itself has a strong autocorrelation.

1. The first **2** eigen values account for 50% of the variance. And **43** eigenvalues account for 90% in my datasets.

It makes sense because according to CAPM model, the market risk would account for returns. Although there’re some other factors, they’re not majority. That’s why our first couple of eigenvalues would have larger proportion.

1. Here’s the required plot:

图表

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From the plot we can figure out that the residual is around 0 with a normal distribution.

**Problem 2: Portfolio Construction:**

1. I just use the numpy package, the function np.linalg.inv () to invert this in a stable way. And the invert result is here:



1. I suppose the risk aversion is about 1. And I got the result matrix here, to make is more visible, I also plot it.

图表, 折线图

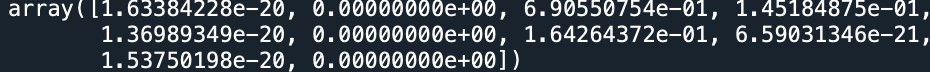
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Unluckily, there’re lots of negative position, which means we must short in daily practice. However, it’s unacceptable in mutual funds. I think we should add a constrain such as making sure all the weight is above zero.

**Problem 3: Portfolio Stability:**

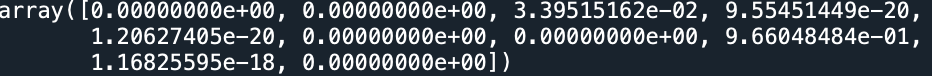
1. Here I guess that the expected return equals the mean of their historical returns.

Then I use scipy.minimize to get the result:



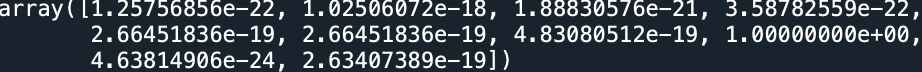
Some of weights are too light so we may ignore them. That is, Sec3: 69.1%, Sec4:14.5%, Sec7:16.4%

1. Using the same method, I got the result:



That is, Sec3:3.40%, Sec8: 96.6%

1. Using the same method, I got the result:



That is, invest all the money on Sec8

1. It would be stable when I change add or minus the same values on all the sec.

From the picture below, we can tell that the majority of weights are stable.

电脑屏幕的照片上有字

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If we just change one of them, the result still reveals stable.

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