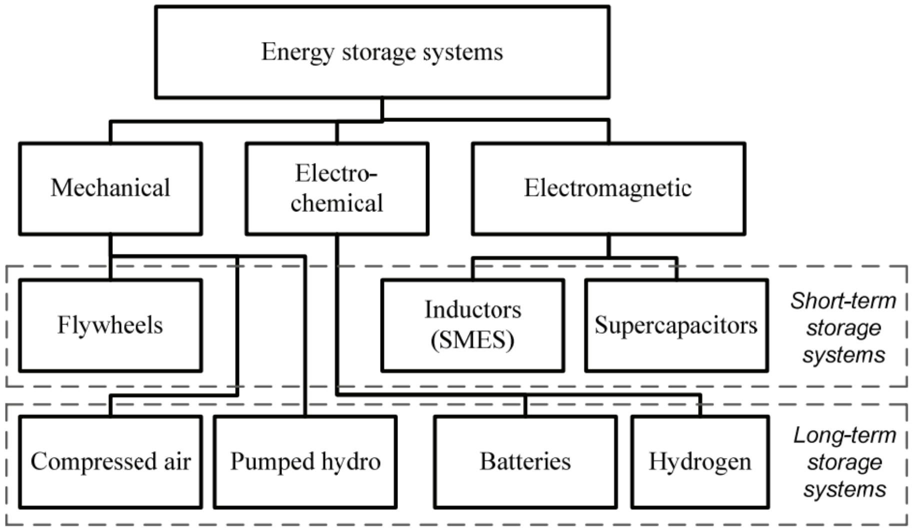
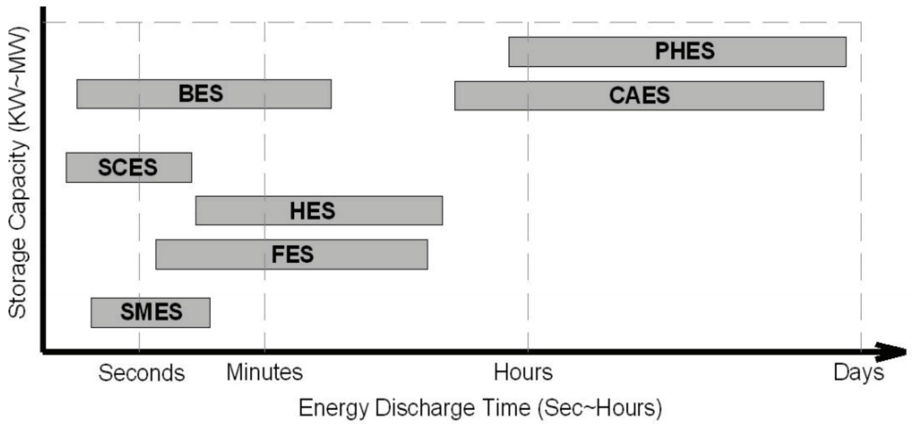
Energy Storage Modelling & Application

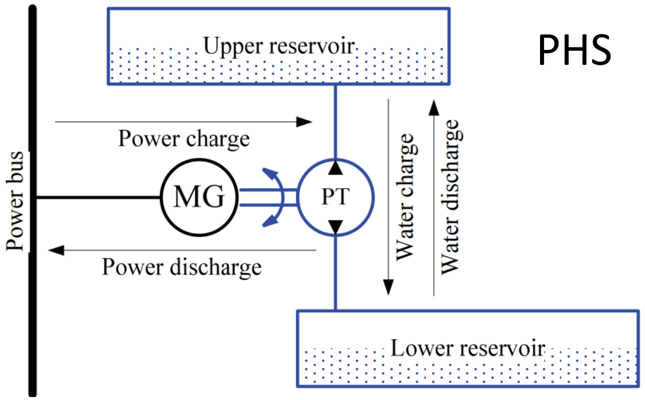
Karl M.H. Lai

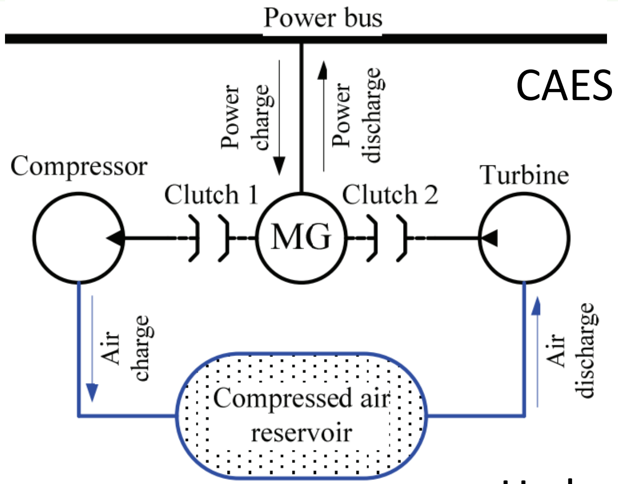
Andrijanovits A. et al (2012), Comparative Review of Long-Term Energy Storage Technologies for Renewable Energy Systems

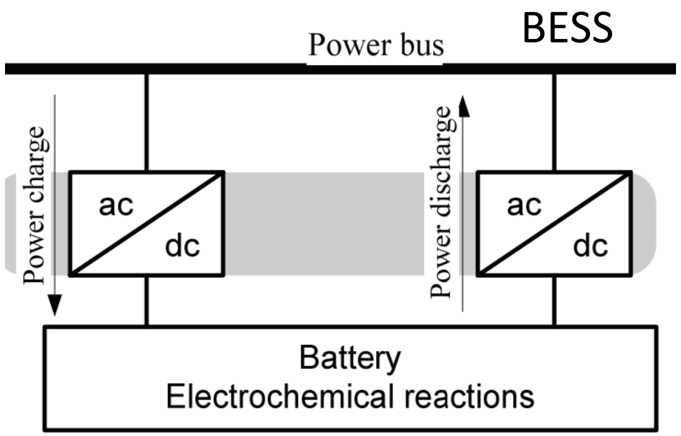
Introduction – Different Types of Energy Storage

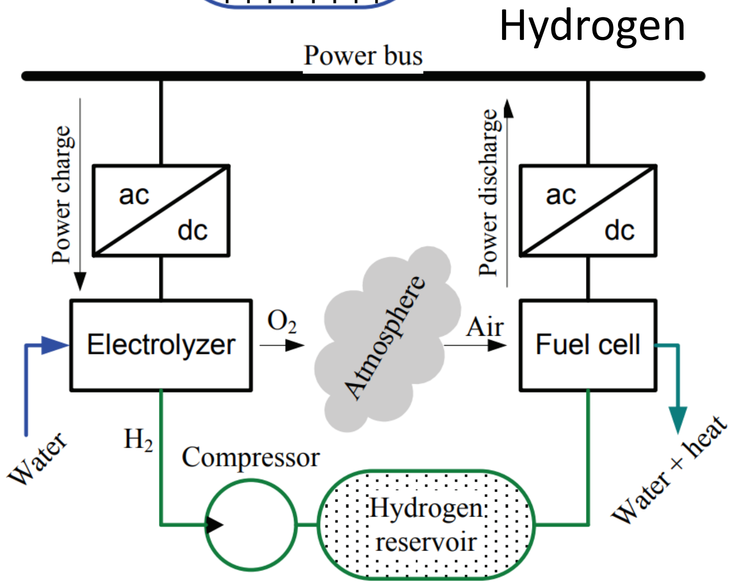




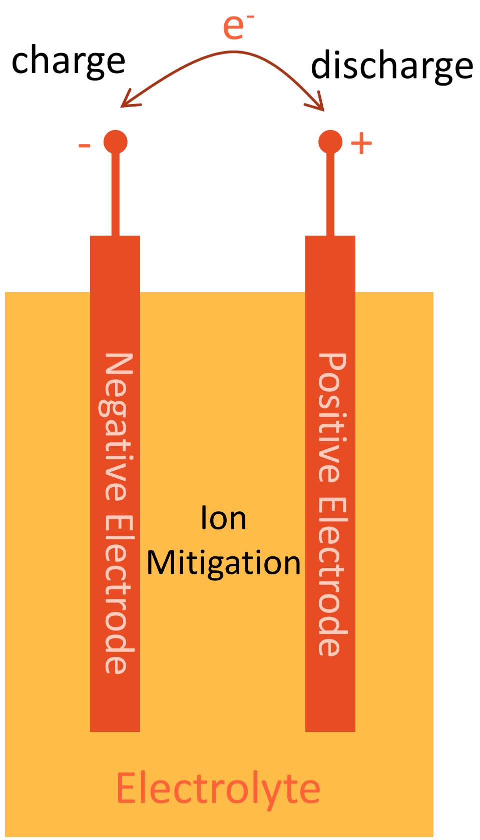








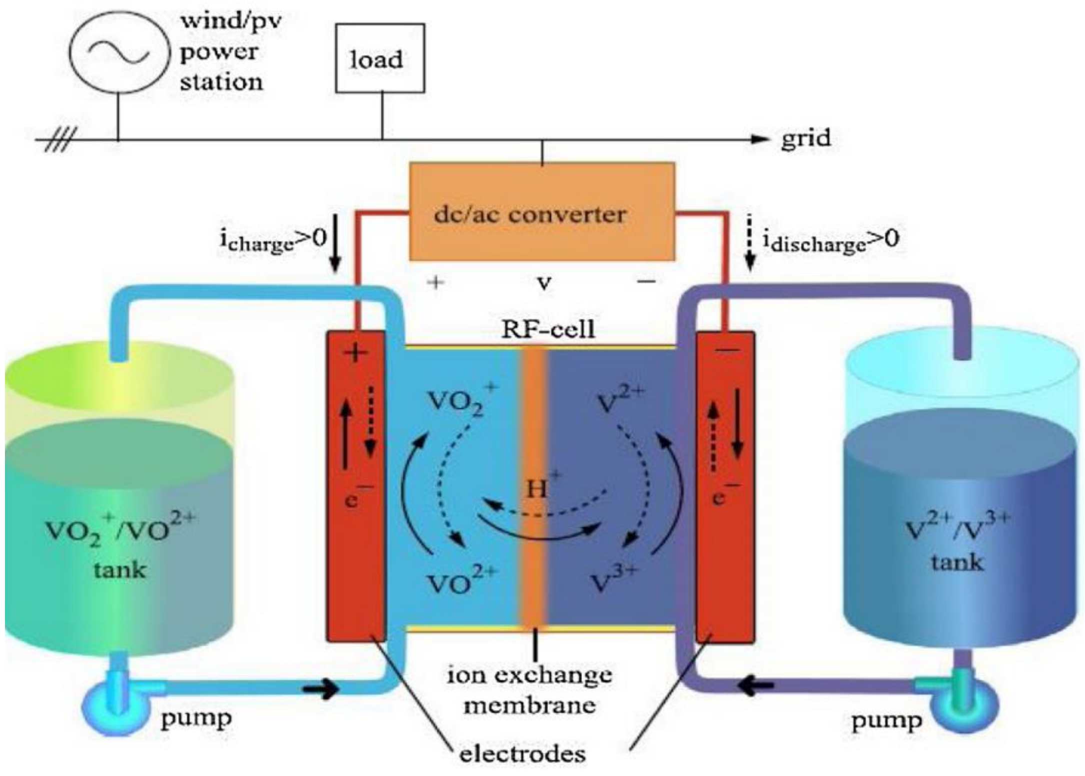
Introduction – Electrochemistry



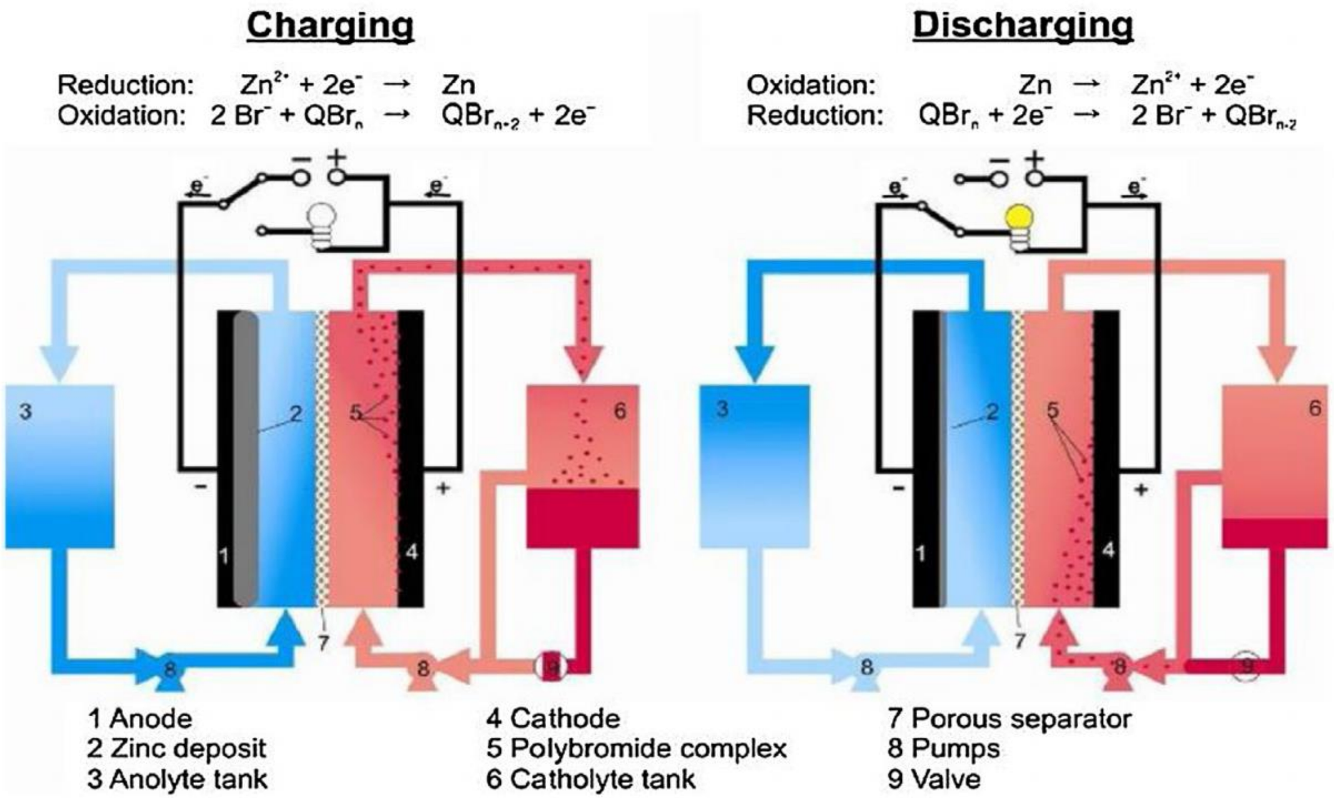
- Chemical Equations in Battery means the relative ratio the chemical needed, and subsequent dynamic equilibrium for material change
- Batteries can **charge** and **discharge**.
- Lead Acid: $Pb + PbO_2 + 2H_2SO_4 \leftrightarrow 2PbSO_4 + 2H_2O$
- NiCd: $Cd + 2NiOOH + 2H_2O \leftrightarrow Cd(OH)_2 + 2Ni(OH)_2$
- NiMH: $MH + NiOOH \leftrightarrow M + Ni(OH)_2$
- Li-ion: $Li_xC + Li_{1-x}MO_2 \leftrightarrow C + Li\,MO_2$
- NaS: $2\,Na + 4S \leftrightarrow Na_2S_4$

Rohit, A.K. et al (2017), An overview of energy storage and its importance Indian renewable energy sector Part I – Technologies and Comparison

Introduction – Electrochemistry



Vanadium Redox Flow Batteries

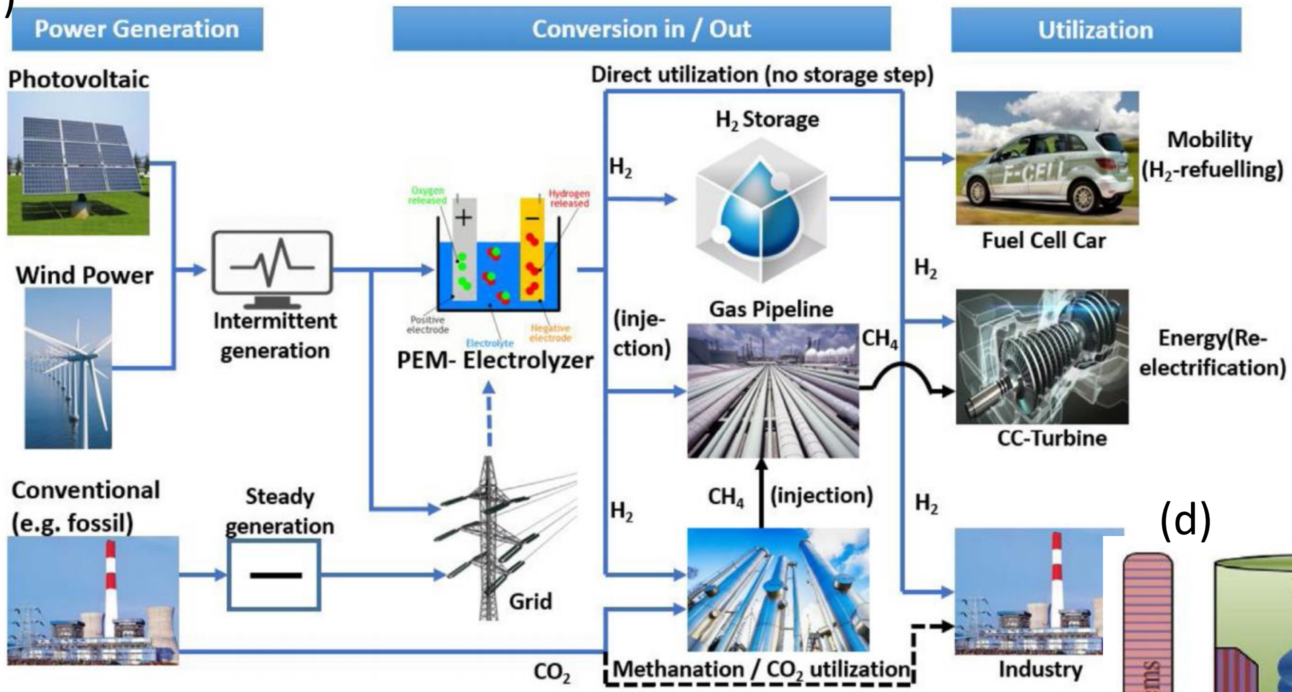


Hybrid Flow Battery (e.g. Zn-Br system)

Rohit, A.K. et al (2017), An overview of energy storage and its importance Indian renewable energy sector Part I – Technologies and Comparison

Introduction – Other Energy Storage

(a)



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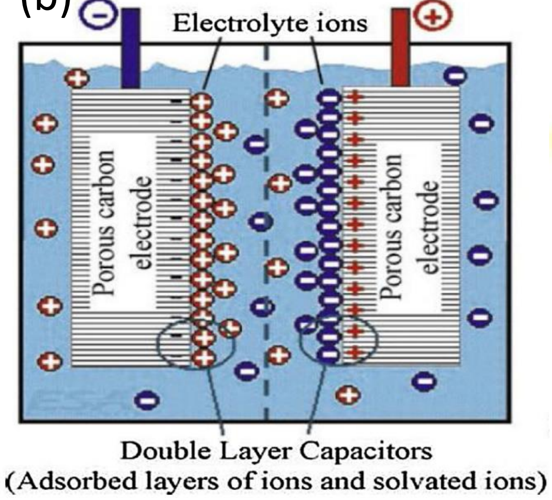

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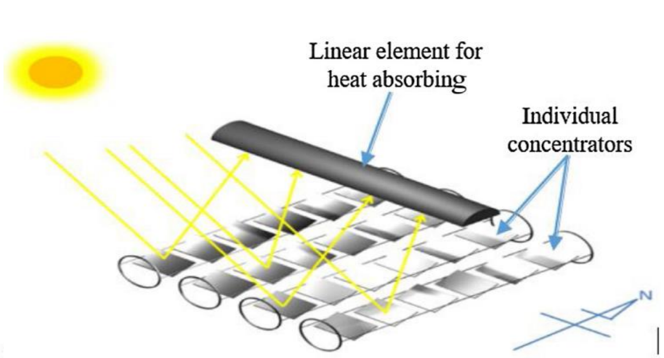

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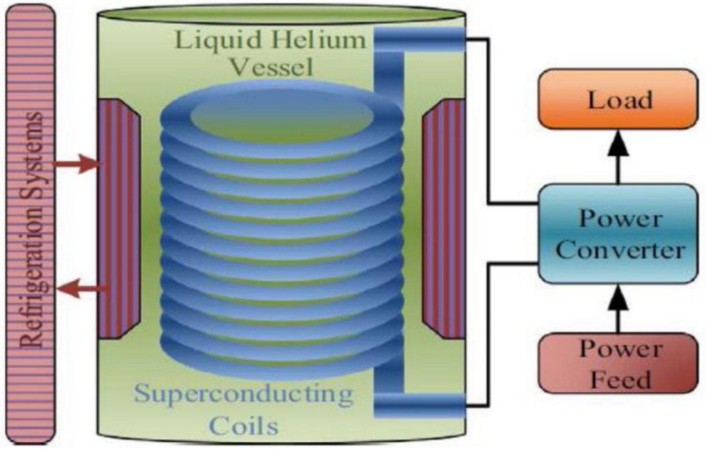

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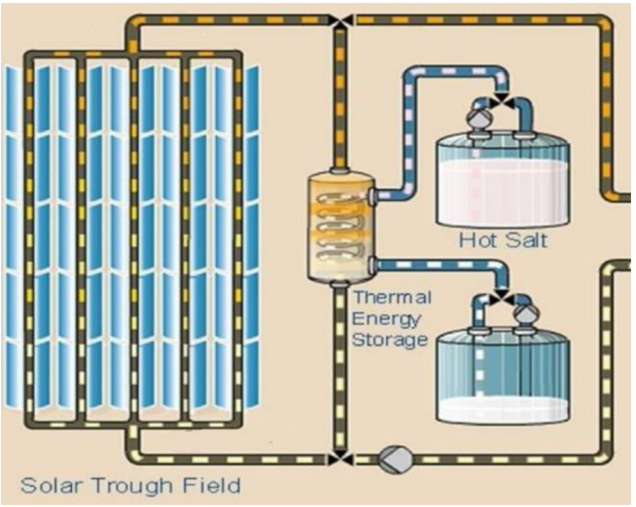
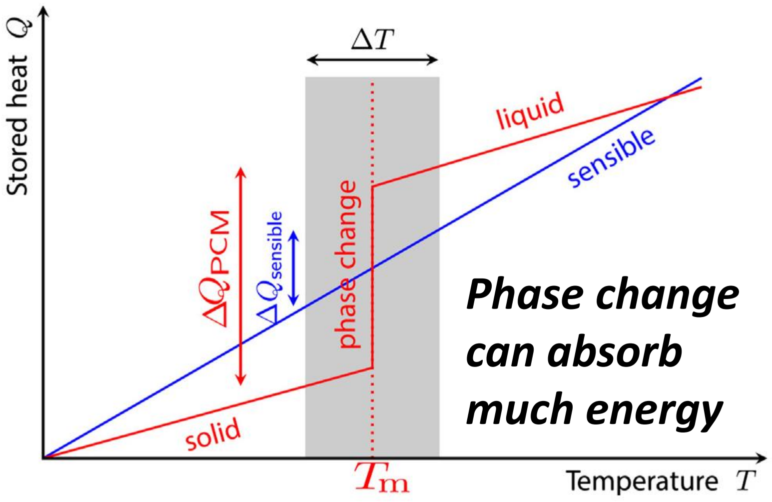


(a) Hydrogen Storage
(b) Supercapacitor (SCES)
(c) Thermal Storage (CL-CSP)
(d) Superconducting Coil (SMES)
(e) Concentrated solar power plant

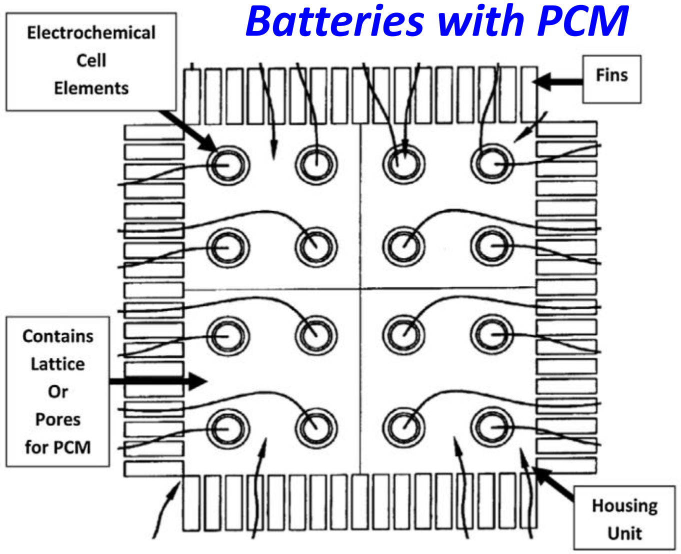
Introduction – Thermal Storage

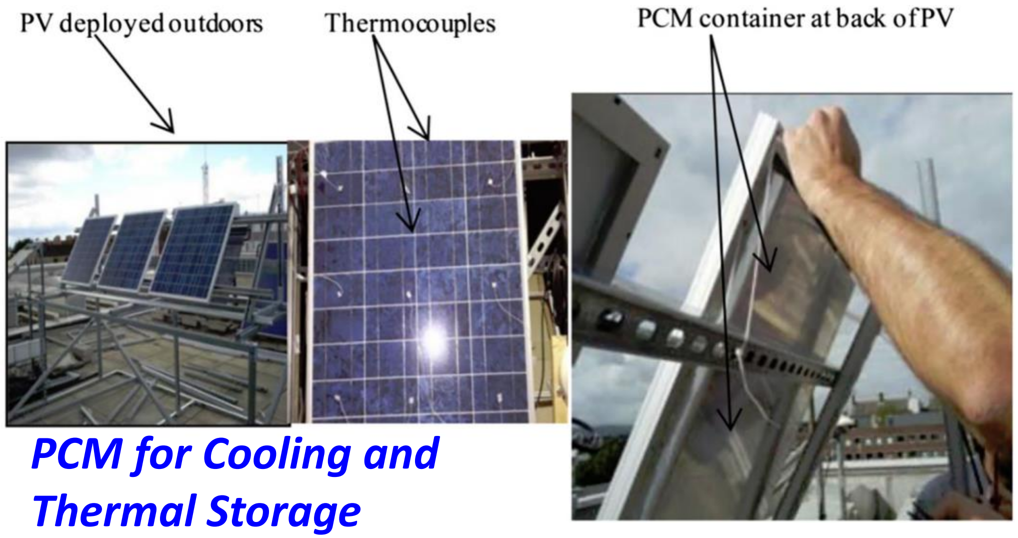
PCM=Phase change material



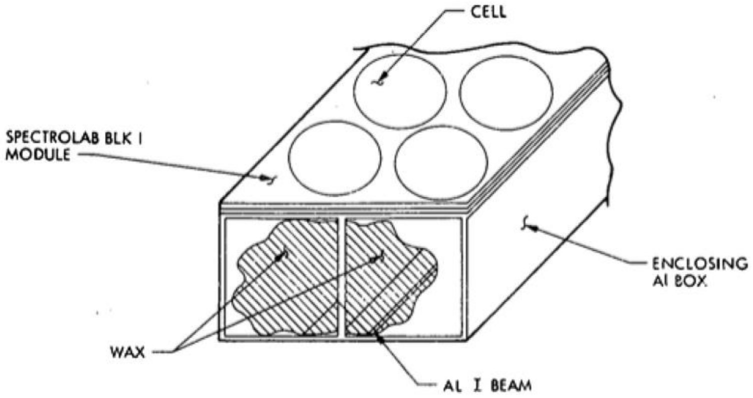
Phase change can absorb much energy

Batteries with PCM



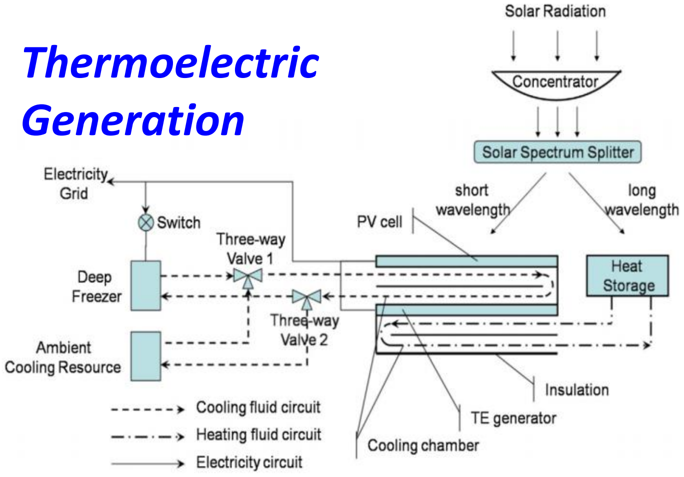


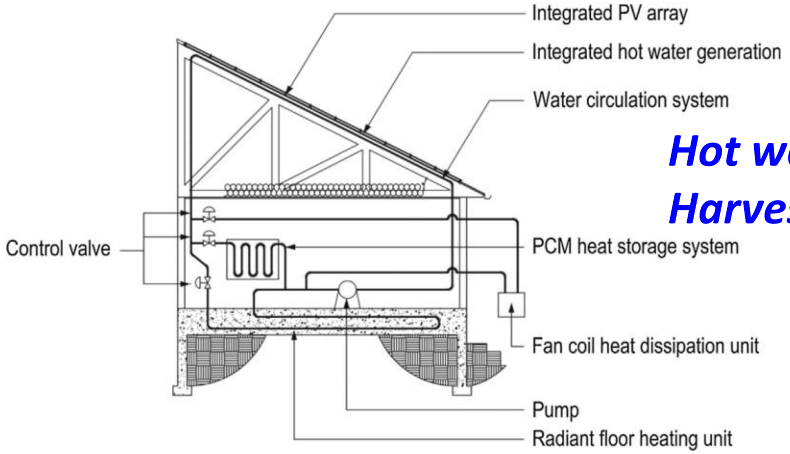
PCM for Cooling and Thermal Storage



PV including PCM

Thermoelectric Generation





Hot water Harvesting

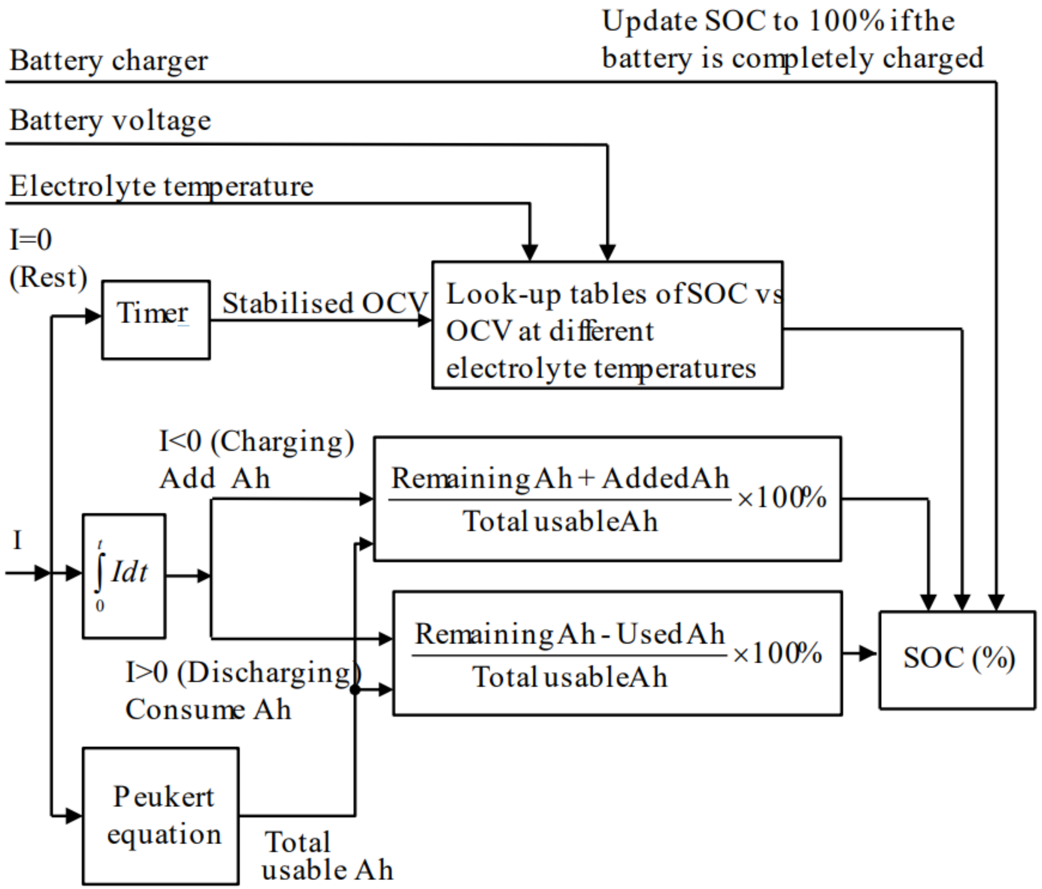
Batteries – terms, state of charge

Chau, K.T. (2018), Electrochemistry, 6th Notes for ELEC3143 Energy Conversion

- State of Charge (SOC) is defined as $\mathbf{C_r/C_t}$, where C_r and C_t are the **residual** and **total usable** coulometric capacity in Ah of the battery.
- SOC is affected by **discharging rate/current** I, **temperature** T and **aging** effect.
- Relationship between capacity and discharging current: (Peukert Equation)

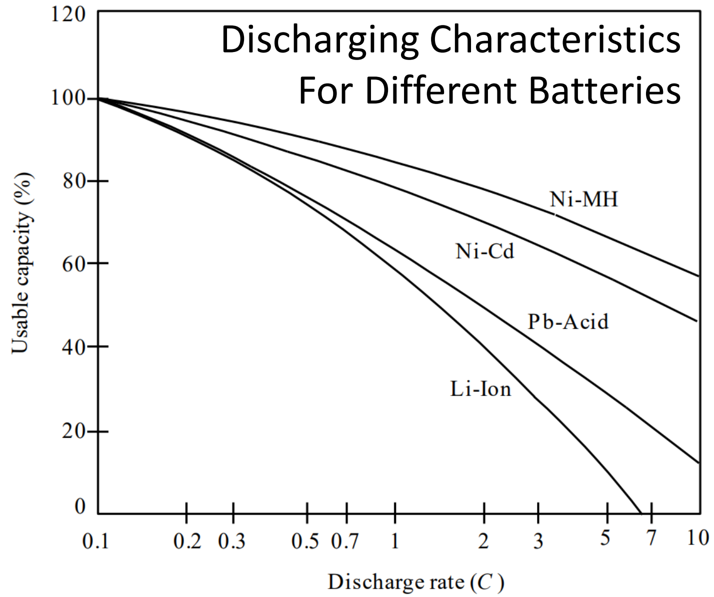
$$\mathbf{C_{tl} = KI^{(1-n)}}$$

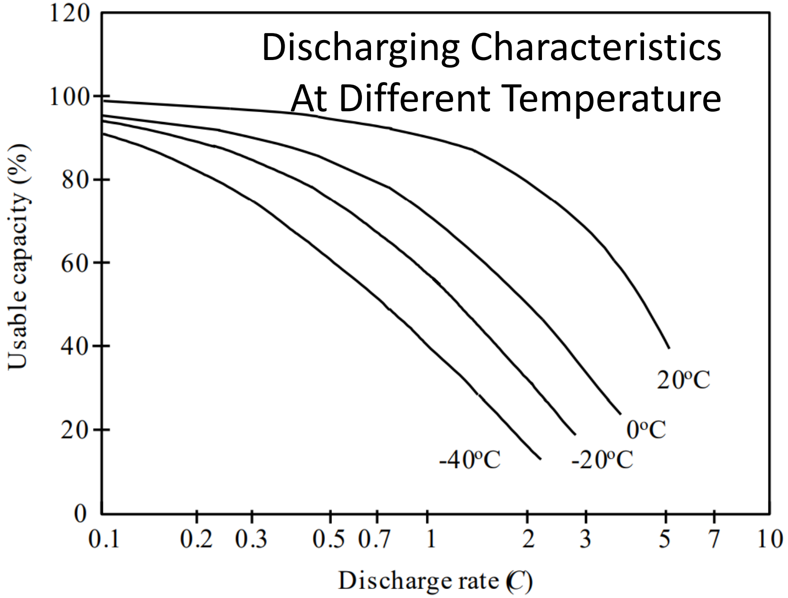
  where C_{tl} is the capacity Ah under particular discharge current I in A and discharge time t in h, while K and n are constants.
- To deduce residual capacity, one can see the open circuit voltage (OCV), with Peukert Equation.

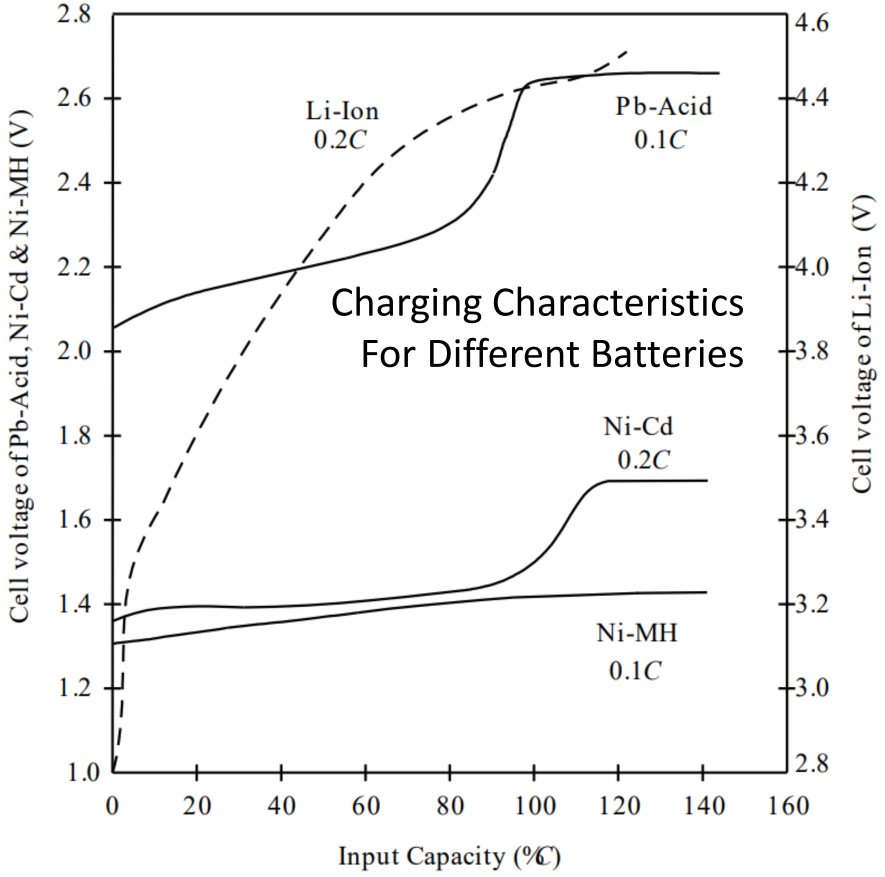


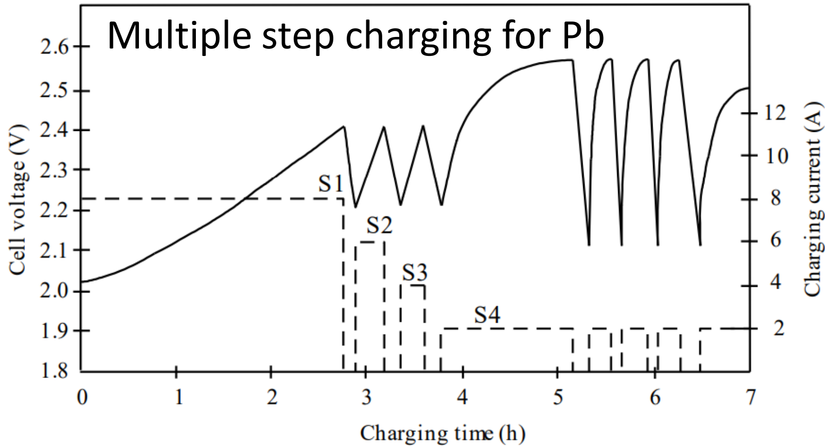
Battery Characteristics

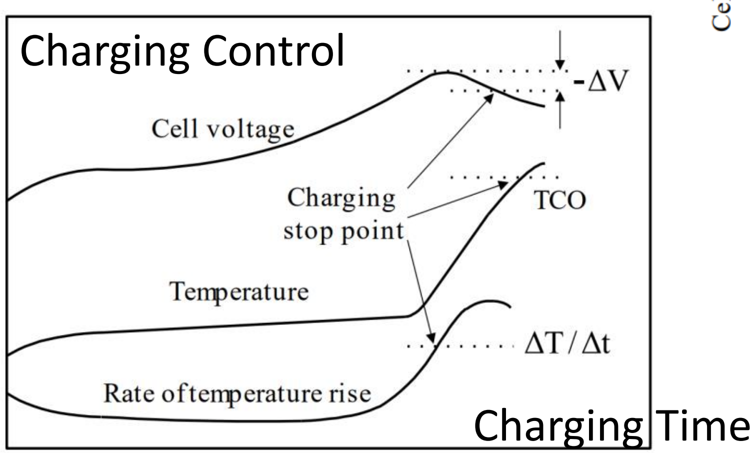
Chau, K.T. (2018), Electrochemistry, 6th Notes for ELEC3143 Energy Conversion











Comparison between Batteries – Numbers

Comparison between different battery technologies.

Diouf B. & Pode R. (2015), Potential of Lithium-ion Batteries in renewable energy

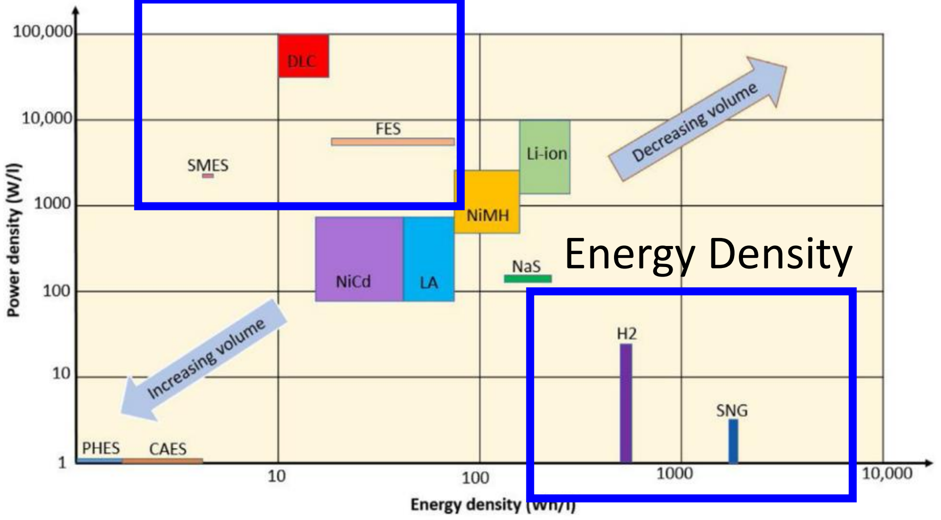
Specifications	Lead acid	NiCd	NiMH	Li-ion		*$LiFePO_4$
	3-5 years			Cobalt	Manganese	Phosphate
Specific energy density (Wh/kg)	30–50	45–80	60–120	150–190	100–135	90–120
Internal resistance (mΩ)	<100 12 V pack	100–300 6 V pack	200–300 6 V pack	150–300 7.2 V	25–75 per cell	25–50 per cell
Cycle life (80% discharge)	200–300	1000	300–500	500–1000	500–1000	1000–2000
Fast charge time	8–16 h	1 h typical	2–4 h	2–4 h	1 h or less	1 h or less
Overcharge tolerance	High	Moderate	Low	Low. Cannot tolerate tickle charge		
Self-discharge/month (room temp.)	5%	20%	30%	<10%		
Cell voltage (nominal)	2 V	1.2 V	1.2 V	3.6 V	3.8 V	3.3 V
Charge cutoff voltage (V/cell)	2.40 Float 2.25	Full charge detection by voltage signature		4.20		3.60
Discharge cutoff voltage (V/cell, 1C)	1.75	1.00		2.50–3.00		2.80
Peak load current	5C	20C	5C	>3C	>30C	>30C
Best result	0.2C	1C	0.5C	>1C	<10C	<10C
Charge temperature	−20 to 50 °C (−4 to 122 °F)	0 to 45 °C (32 to 113 °F)		0 to 45 °C (32 to 113 °F)		
Discharge temperature	−20 to 50 °C (−4 to 122 °F)	−20 to 65 °C (-4 to 49 °F)		−20 to 60 °C (-4 to 140 °F)		
Maintenance requirement	3–6 months (topping charge)	30–60 days (discharge)	60–90 days (discharge)	Not required		
Safety requirements	Thermally stable	Thermally stable, fuse protection common		Protection circuit mandatory		(Explosive)
In use since	Late 1800s	1950	1990	1991	1996	1999
Toxicity	Very high	Very high	Low	Low		
Cost (ratio)	1		1 – 4	4 – 8	Cheaper and Cheaper !!	

5000 cycles
For deep
discharge

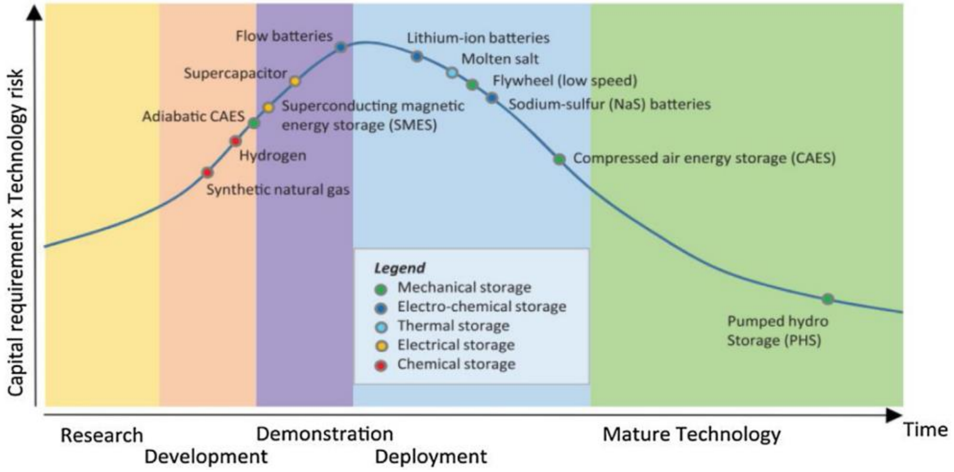
Carbon Footprint:
70kg CO_2 per kWh

Comparison between Energy Storage – Graphs

Power Density



Energy Density



Notes:

1. No single ES can meet all requirements.
2. Among ES, ***HES*** has the **highest energy density**, while for batteries ***NaS and Li-ion*** have **higher energy density**
3. **PHES and CAES** with long discharge duration are fit for ***peak shaving and time shifting***, while **HES, TES and Flow batteries** finds ***applications in medium scale***, **SMES and supercapacitor, flywheels and batteries** can be used for ***power quality control***.
4. **PHES, CAES, HES and FB** have very little ***self-discharge***
5. ***CAES requires lowest capital cost*** than the PHES (second) while flywheel, supercapacitors, SMES and fuel cell are expensive
6. CL-CSP and thermal storage are low cost and more efficient.
7. ***Chemical deterioration*** in batteries shortens their lives.
8. BES is most suitable ES to fulfil rural energy demand as it is portable and scalable
9. ES is important in large RE penetration.

Rohit, A.K. et al (2017), An overview of energy storage and its importance in Indian renewable energy sector Part II Energy Storage Applications, benefits and market potential

Comparison between Energy Storage – Functions

Applications Technology	Bulk Energy Application		Ancillary Services Application					End-use Energy Application		RE Integration Application	Transport Application	Off grid Application
	Energy Arbitrage	Peak Shaving	Load following	Spining reserve	voltage support	Black start	Frequency Regulation	Power Reliability	Power quality			
Mechanical Energy Storage												
PHES	Y	Y	N	N	N	Y	Y	N	N	Y	N	N
CAES	Y	Y	Y	Y	N	Y	Y	Y	N	Y	N	N
FES	N	N	Y	Y	Y	N	N	Y	Y	N	N	N
Electrochemical Energy Storage												
Lead acid	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
NaS	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Li-ion	N	N	Y	N	Y	Y	Y	Y	Y	Y	Y	Y
Flow Battery	Y	Y	Y	N	Y	Y	Y	Y	Y	Y	N	Y
Electrical Energy Storage												
DLC	N	N	N	N	N	N	N	N	Y	N	Y	N
SMES	N	N	N	N	N	N	N	N	Y	N	N	N
Hydrogen Energy Storage												
Hydrogen Fuel Cell	Y	N	N	N	N	N	N	N	N	N	Y	Y
Thermal Energy Storage												
Thermal	Y	Y	Y	Y	N	N	N	N	N	N	N	N

Comparison between Energy Storage – Functions

- **Energy Arbitrage:** Store energy when it is cheap, sell when it is expensive (**$$**)
- **Peak Shaving:** Cover the peak load, ES installed at customer side
- **Load Following:** Maintain a **balance between generation and load***
- **Spinning Reserve**: Discharge at abnormal situation **with rated parameter values (V & f)**
- **Voltage Support:** Locally regulate the **voltage rise** problem
- **Black Start:** Restart the whole system after unexpected interrupt, balancing
- **Frequency Regulation:** Maintain **balance between generation and demand**
- **Power Quality:** Protect loads **against short-duration events**
- **Time Shifting, Capacity firming, and Energy Smoothing:** to provide energy power during a day even in worst weather without exaggerated fluctuation
- **T&D Upgrade Deferral:** Reduced loading of Tx & Cables
- **Congestion:** Take **heavy load** substations and lines to **relieve power congestion**

Shyam B & Kanakasabapathy P (2018), Large scale electrical energy storage systems in India- current status and future prospects

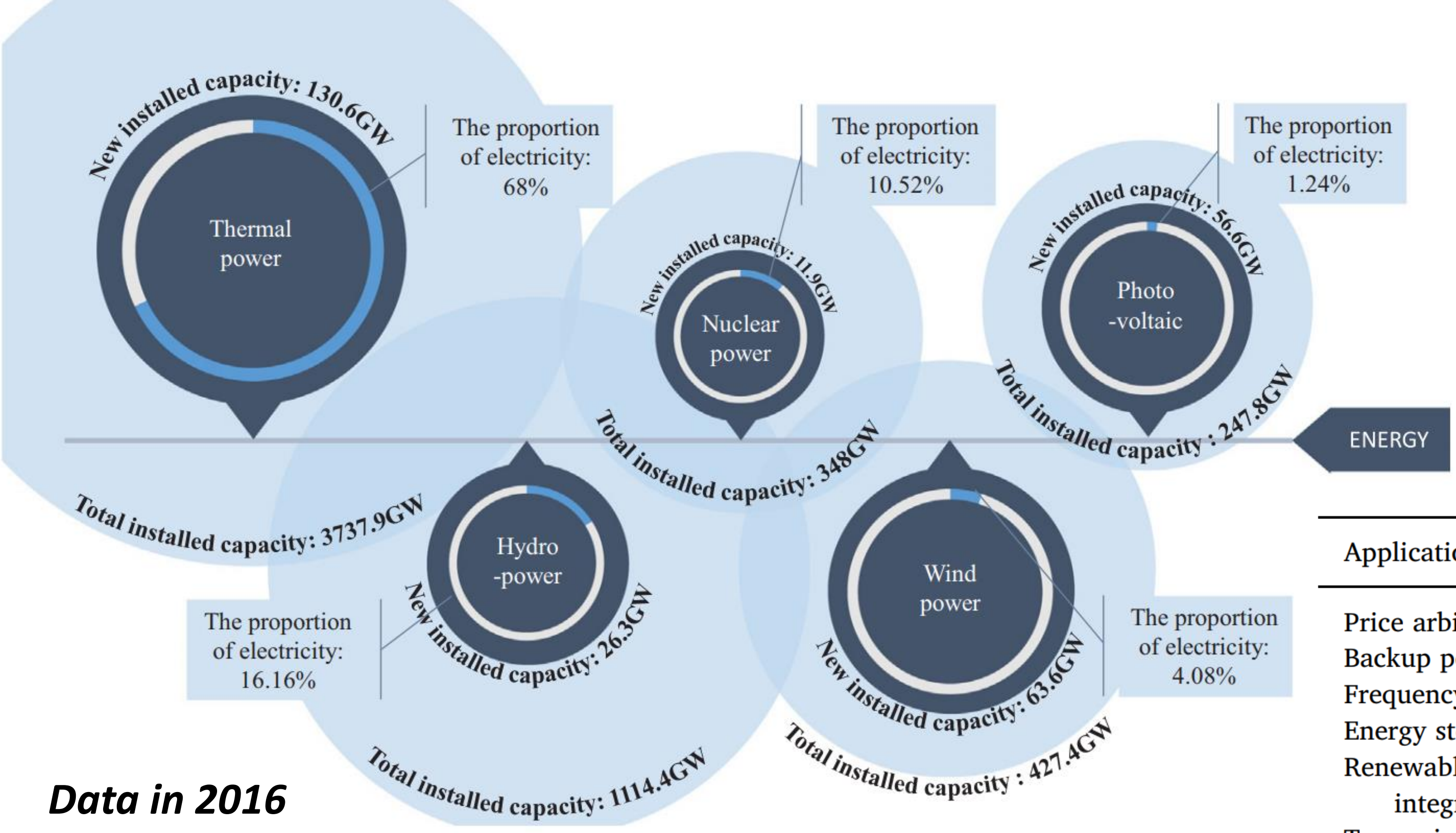
Comparison between Energy Storage – Advantages and Disadvantages

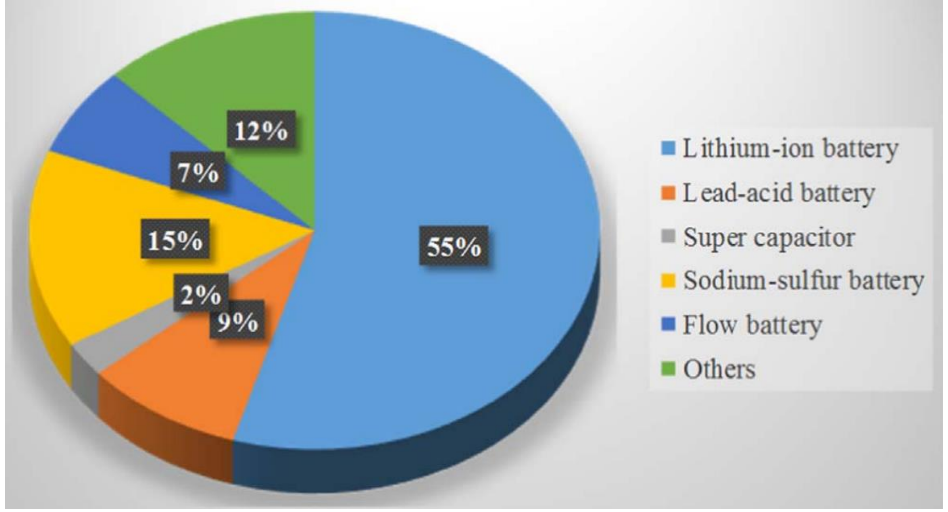
Properties	Technologies	
	BES	PSH
Cycle Efficiency (%)	60–90	70–85
Energy Density (Wh/L)	50–500	0.5–2
Response time	Milli-seconds	Seconds- Minutes
Storage Capacity* (MWh)	Up to 40 MWh	1000s of MWh
Self-discharge (%/day)	0.1–20	~ 0
Cycling times	2000–10,000	–
Period	Medium term	Long term
Status of technology	Mature	Mature
Energy Capital Cost $/kWh	150–1000	5–100

Properties		
	FES	CAES
Cycle Efficiency (%)	Up to 95	40–50
Energy Density (Wh/L)	20–80	0.2–6
Response time	Milli-seconds	1–15 Minutes
Storage Capacity* (MWh)	Up to 5 MWh	1000s of MWh
Self-discharge (%/day)	20–100	~0
Cycling times	105–107	–
Period	Short term	Long term
Status of technology	Early stage	Early stage
Energy Capital Cost $/kWh	1000–5000	2–100

EES System		Advantages	Disadvantages
Battery storage systems	Lead Acid	Fast response time Small daily self – discharge Relatively high cycle efficiencies Low capital cost	Low cycling times Low energy density Low specific energy Performance at low temperature is poor
	Lithium-ion	Fast response time Compact and light weight High cycle efficiency	High cost DoD affect Life *Depth of Discharge* Needs on-board computers
	Sodium-Sulphur	High energy densities Low self-discharges High rated capacity High recyclability	High operating cost Needs temperature control systems *High Temperature ~300°C*
	Nickel-Cadmium	Robust Reliable	Environmental hazards Memory effect
	Vanadium Redox flow Battery	Quick response High Efficiency	High operating cost Low electrolyte stability and solubility Low energy density
	Zinc-Bromine flow Battery	High energy Deep discharge capability and reversibility	Metal corrosion Dendrite formation Low cycle efficiency
Pumped storage hydro		Higher capacity No toxic products Long life Mature and proven technology	Needs geographical specialties Low energy density Long construction time
Flywheel energy storage		Fast response Easy maintenance Relatively high cycle efficiency	Idling loss is present Relatively high self-discharge It cannot serve as a backup system Capital cost is high
Compressed air energy storage		No self-discharge	Relatively high response time Relatively low round trip efficiency Need for geographical specialties
Thermal energy Storage		Low capital cost High energy density Low self-discharge	Low cycle efficiency (with energy conversion)

Energy Storage: Current Situation

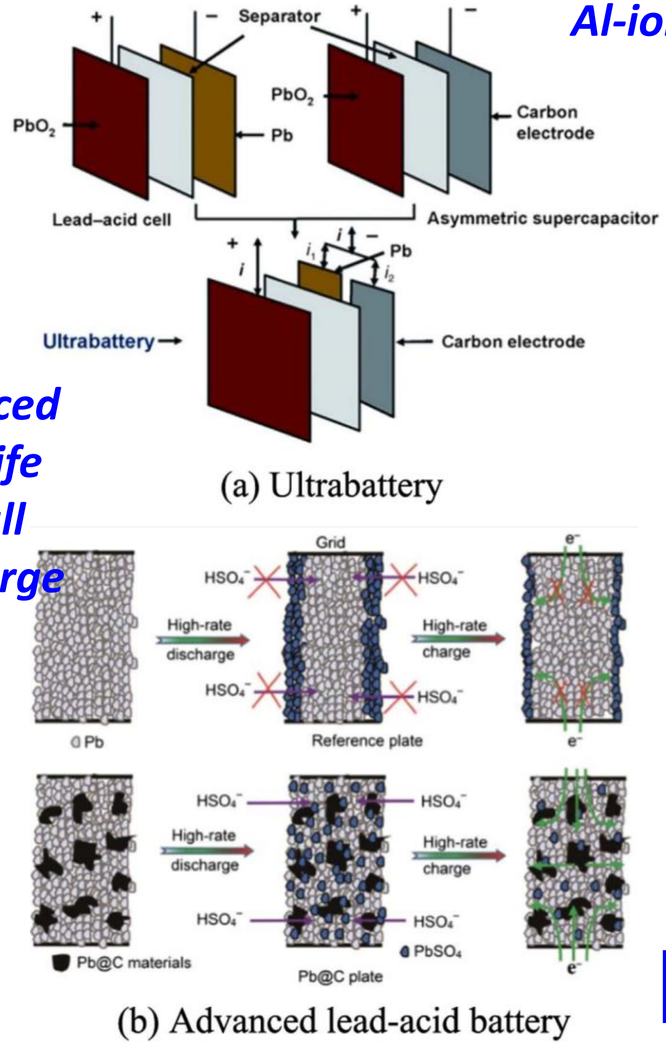




Data in 2016

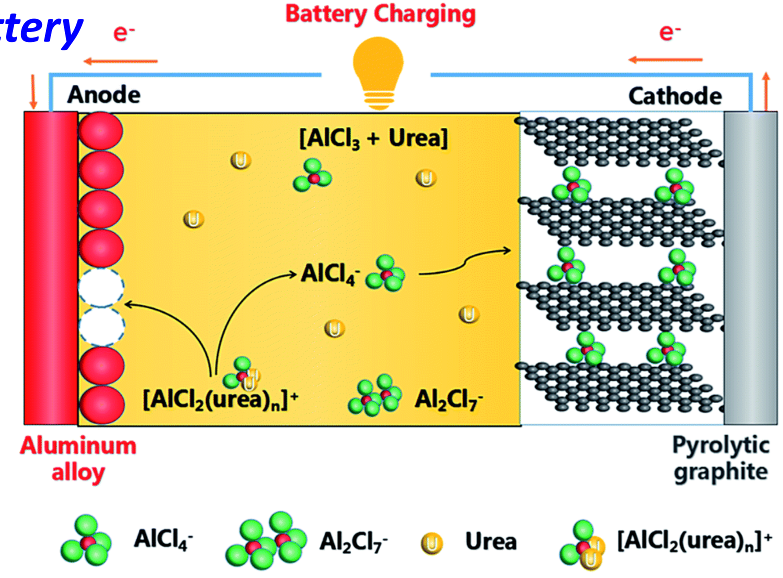
Application mode ***(MW)***	2009	2010	2011	2012	2013	2014	2015
Price arbitrage	258.5	323.2	1.5	8.6	0.9	4.1	27.8
Backup power	135.3	236.5	8.4	15.9	35.4	10.9	64.1
Frequency regulation	11.8	271.4	47.8	185.6	75.5	66.9	651.2
Energy storage	136.5	4.9	60.5	233.8	0.1	113.8	26.7
Renewable energy integration	266.3	470.2	51.3	70.7	290.1	52.0	367.1
Transmission and distribution	195.9	15.6	4.1	1.4	18.2	2.9	60.3
The user side	2.1	0.1	7.9	29.8	2.7	119.2	42.8
Others	335.8	235.0	26.0	62.8	41.1	198.0	147.7

Energy Storage: Current Situation (Highlights)



Enhanced Cycle life in Small Discharge

Al-ion Battery



Research on Battery Power Conversion

1. Energy gap in power electronics device
2. Good understanding but hard to compute real time in devices
3. Multi-port converter topology
4. Modelling of Power Electronics and Non-linear control / analysis

Battery technology	Advantages	Disadvantages	Energy storage applications
Lead-acid battery	Low capital cost	Limited life cycle, long charging time and high self-discharge rate Environmental pollution	Hot spare, frequency control and load adjustment
Lithium-ion battery	High energy densities, high efficiency, and long life cycle	High production cost, requires special charging circuit	Frequency control, load shifting and power quality
Vanadium based flow battery	High power, long life cycle, fast charge and discharge	High production cost, large area	Load shifting, emergency standby and power quality
Sodium-sulfur battery	High power and energy densities, high efficiency	Production cost and safety concerns	Load adjustment and standby power
Aluminum-ion battery (Estimated)	Low capital cost, fast charge and discharge, high efficiency	Under development Low energy densities	N/A

Charge within 1 min

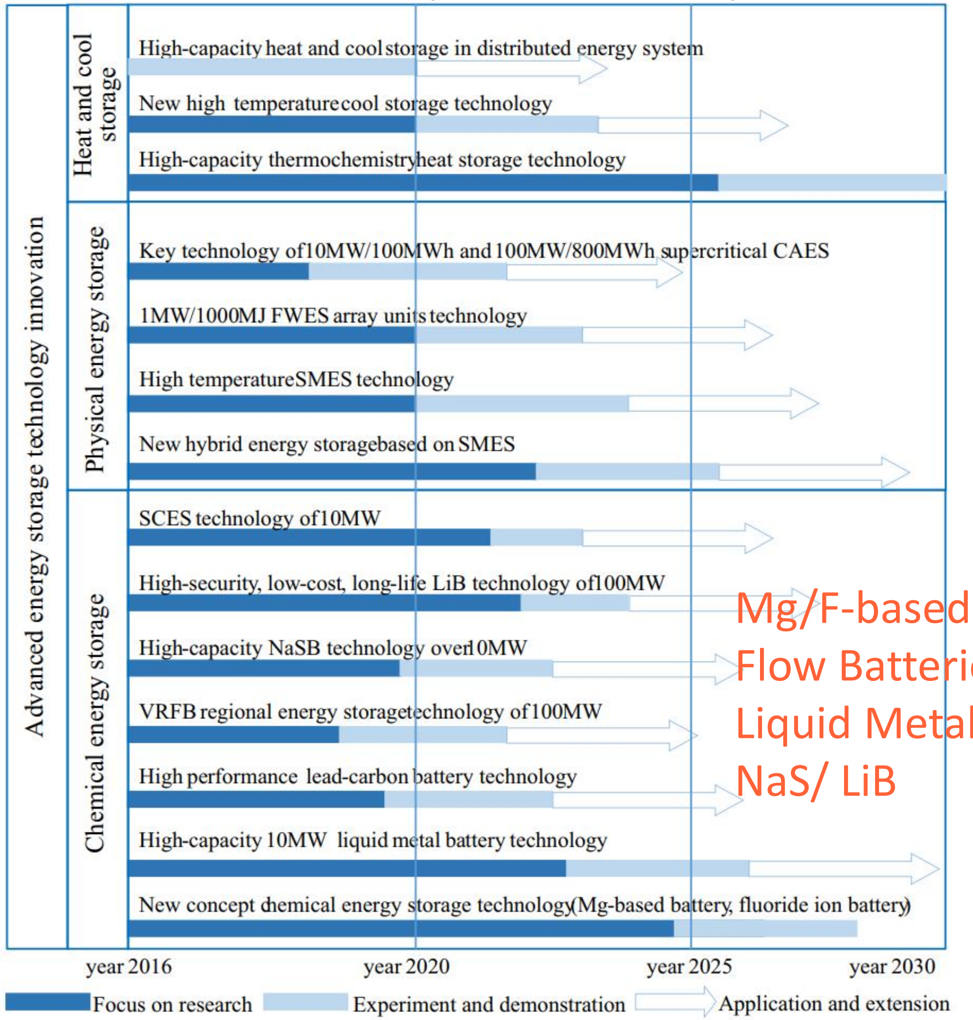
Highlights of China's Energy Storage development (1)

- Facts:
  - ***66%*** of cumulative energy storage is in ***Li-ion Battery***
  - Energy Storage are installed in ***Demand Side***, and most of the ES are DG, or microgrid projects, as high capacity ES in generation is not available
  - In response to ***big peak-valley difference*** (due to high population and growing economy), energy saving, emission reduction, China government has paid attention on demand side response.
  - Need a ***clear technical route*** on development and detailed planning e.g. Germany has road map for LiB, ESEM, SES to promote progress

Yu, H. et al. (2017), China's Energy Storage Industry: Develop Status, Existing Problems and Countermeasures

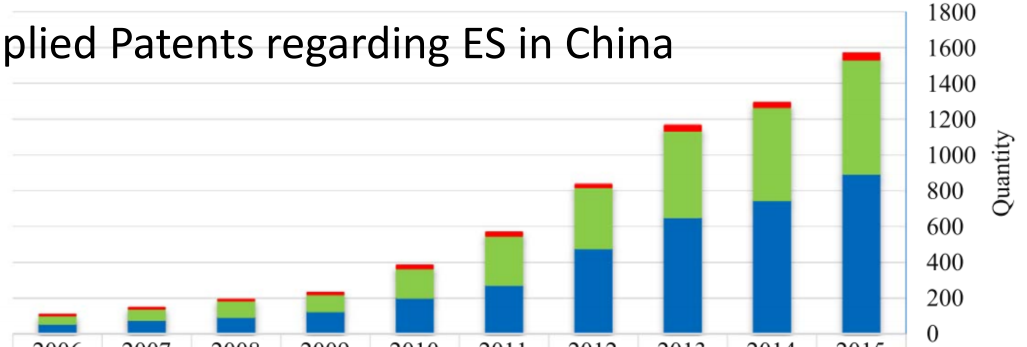
Highlights of China’s Energy Storage development (2)

Development Roadmap for China



Mg/F-based
Flow Batteries
Liquid Metal
NaS/ LiB

Number of Applied Patents regarding ES in China



Year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Appearance design	2	3	2	6	13	17	12	27	21	34
Utility model	47	63	92	95	164	275	342	484	520	639
Invention patent	57	80	97	128	205	275	479	653	749	895

Year	China’s participation in international standards
2011	Undertake the establishment of IEEE P2030.3TM- *Standard for Test Procedures for Electric Energy Storage Equipment and Systems for Electric Power Systems Applications.*
2012	Undertake the work of *Development Roadmap of Nanotechnology (NT)* by IEC Market strategy bureau (MSB) Special work group 4 (SWG4)
2012	take charge of the establishment of white book -*Grid integration of large-capacity renewable energy sources and use of large-capacity electrical energy storage* by IEC MSB
2014	Undertake the establishment of IEC 62932–2–1- *Flow battery systems for stationary applications-part 2–1: performance general requirements & method softest*

Highlights of China's Energy Storage development (3)

- Main Issues/ achievements in **Physical Storage**:
  - For PHS: Pricing Policy
  - For CAES: Compressed air into abandoned miles/ underwater tanks by high pressure → to obtain mixture of natural gas+ air
    → Wind Power can drive compressor directly → $\eta \uparrow$
    → Problems in core technology of gas turbines, lack of oil and gas reserve, or natural caves as gas storage device
  - For FWES: → High Self-Discharging Rate
    → used as UPS, regenerative braking, wind turbine storage, high-power pulse
    → Connect to generator directly

Highlights of China's Energy Storage development (4)

- Main Issues / achievements in **Electromagnetic Storage**:
  - **SMES:** superconducting coil to absorb EM energy
    → Good studies in **superconducting magnet separation, MHD propulsion, nuclear magnetic resonant**
    → Need **current limiter**, e.g. Gansu Province 0.5MVA SMES
  - **SCES:** similar to supercapacitor, based on electric double layer absorption, surface oxidation/reduction, insert ions
    → applied in **elevator energy saving, street lighting, robot energy storage, vehicle start-stop**
    → invented a FeOOH/graphene nanometer to lower cost of SCES

Highlights of China's Energy Storage development (5)

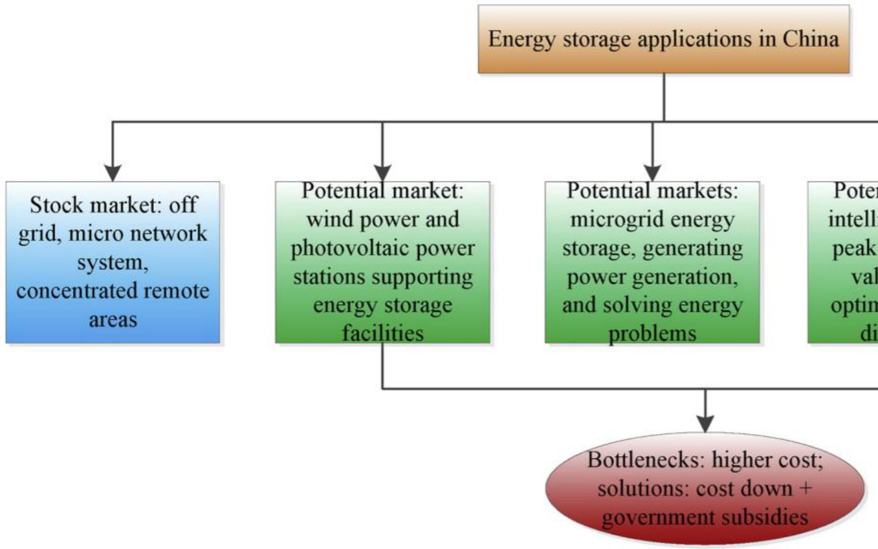
Main Issues / achievements in **Electrochemical Storage**:

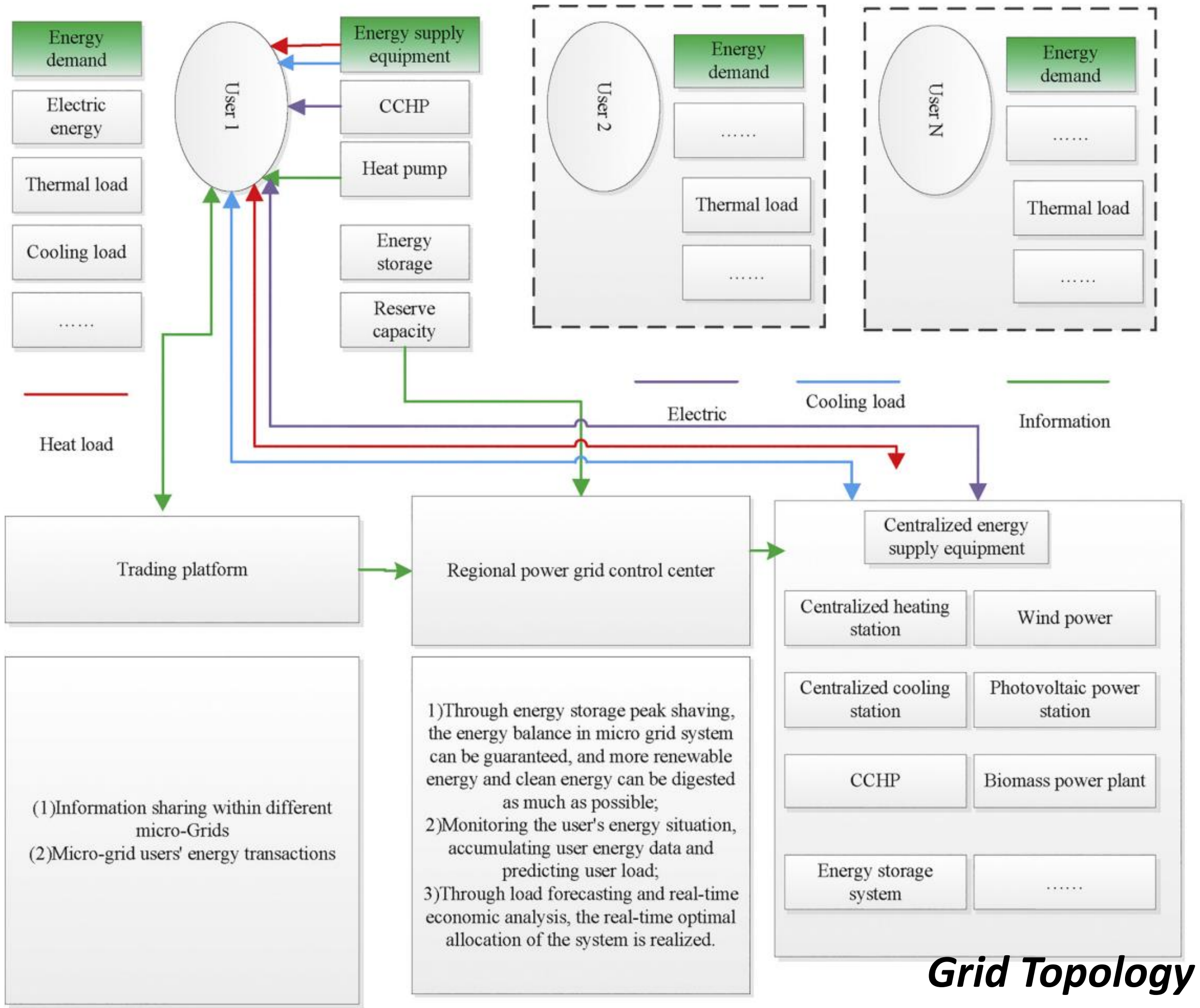
- **PbB** → Lead Fumes, dusts or waste water with Lead
- **LiB:** higher capacity, longer cycle life, no memory effect, with high energy density in weight/volume
- **NaSB:** in theory can store 4-5 times energy of LiB
  for power plant load balancing, UPS, instant power compensation, good in static environment
- **FB** (Flow Battery): VRFB/ ZnBrFB are dominating
  3MW/6MWh in Liaoning Province
  Stable output, dispatch controllability, energy saving, with black start, islanding, harmonic treatment

Highlights of China’s Energy Storage development (6)

Main functions of energy storage equipment in power system.

	Functions
standby powers	To help the power plant black start; to protect the independent grid and Micro-grid operation; to ensure important user power supply
peak shaving and valley filling	Delaying power generation and installation; delaying the upgrading of power transmission and distribution; saving transmission access and congestion costs
Power dispatching	To reduce line loss
Power tracking	Load tracking, frequency modulation, and provide spin backup
Flicker suppression	Avoid flashing effects on electrical equipment





Grid Topology

Highlights of China's Energy Storage development (7)

- SWOT/PEST

analysis of China's energy storage policy weakness.

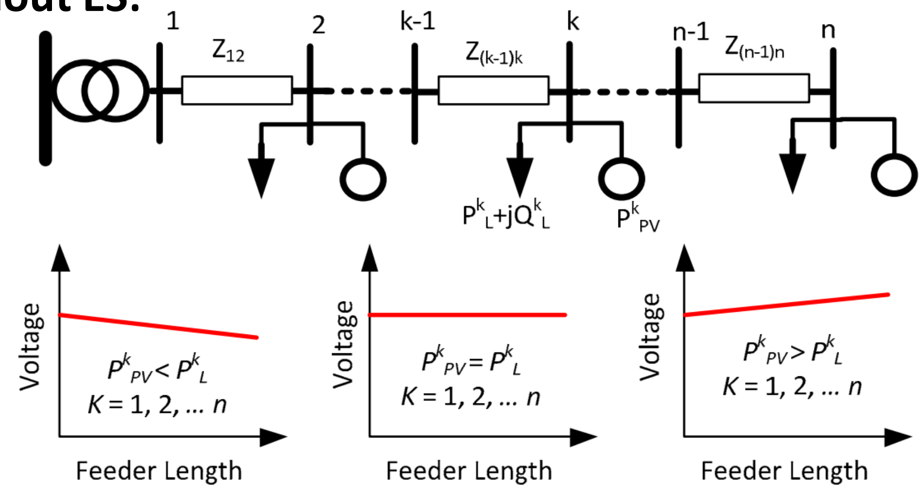
Field	Main content
In demonstration project construction	Extensive policy, the lack of energy storage system program design. The experimental research of energy storage technology lacks continuity and continuity, and there is no definite electricity price and cost accounting and cost recovery plan
In terms of financial subsidies	Lack of energy storage financial plan, and the lack of detailed investment costs, production and operation and maintenance program.
In terms of investment and financing	In promoting the new energy storage industry chain industrialization, engineering application effect is not obvious

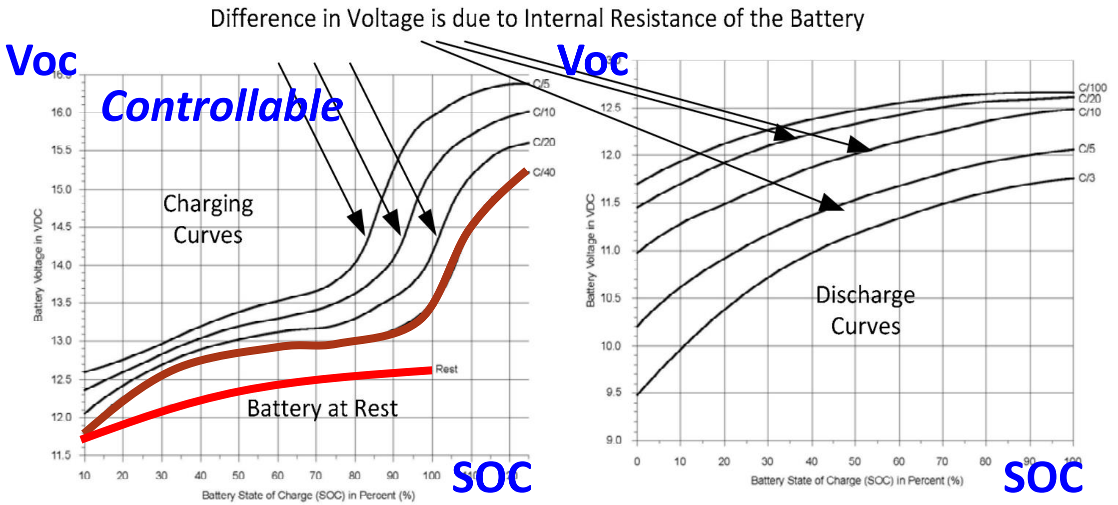
		P **Political**	E **Economy**	S **Society**	T **Topology**
Strength	S	the government clearly supports	Have a good industrial base	Abundant labor resources Abundant resources	The energy storage technology innovation system has been preliminarily constructed
Weakness	W	The legal system of energy is not perfect; Energy storage policy is not matched	The energy storage industry has a low degree of intensification	In terms of energy storage, the total quantity of talents is insufficient and the quality needs to be improved	The overall technological development capability is relatively weak; Lack of core technology
Opportunity	O	Protection of laws and regulations	Economic integration is booming; Opportunities for structural adjustment of energy	Establish various energy storage Industrial Organizations	The development of energy storage battery manufacturing technology has been widely used
Threats	T	The state gradually reduces government support	The energy storage market is becoming more competitive; Energy storage and other high cost, cannot be industrialized	The threat of trade globalization	Energy storage technical barrier

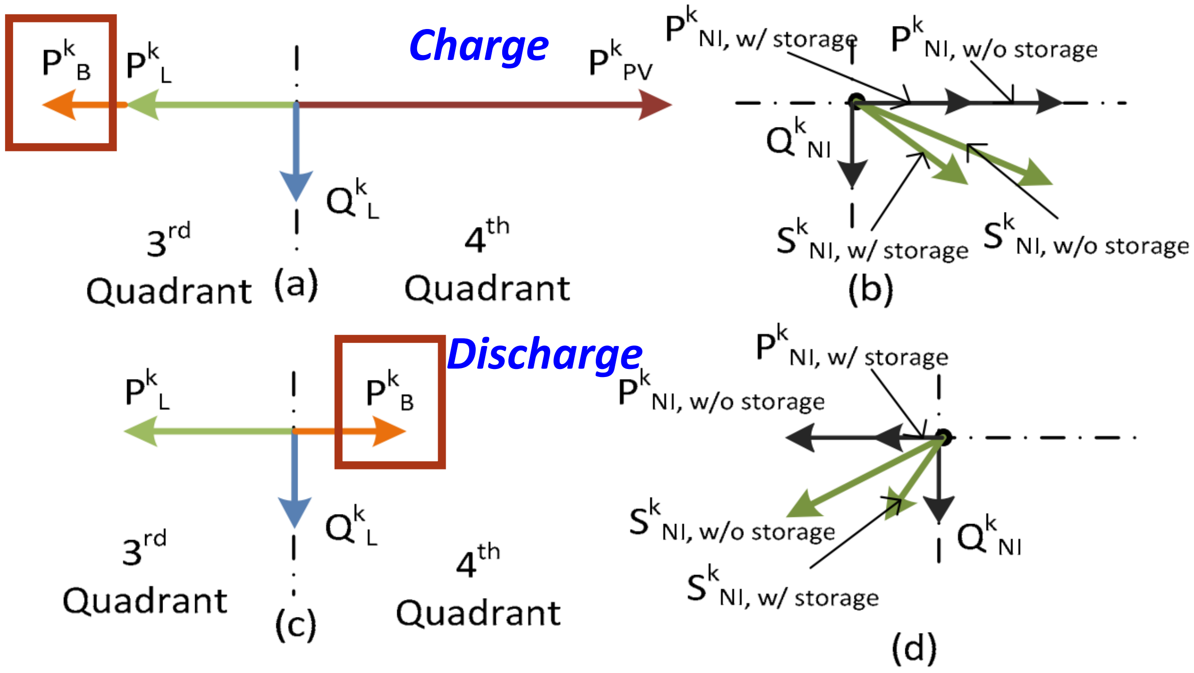
Voltage Rise Issue: Mitigation of Voltage Rise

Alam, M. et al (2012), Distribution Energy Storage for Mitigation of Voltage-Rise Impoact Caused by Rooftop Solar PV

Without ES:





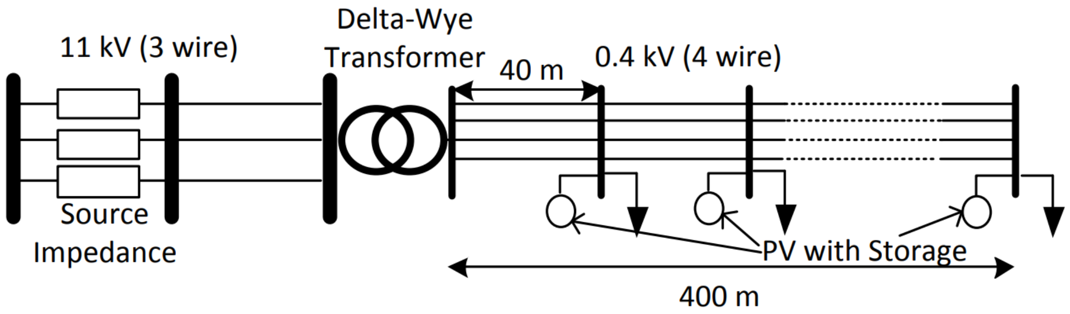


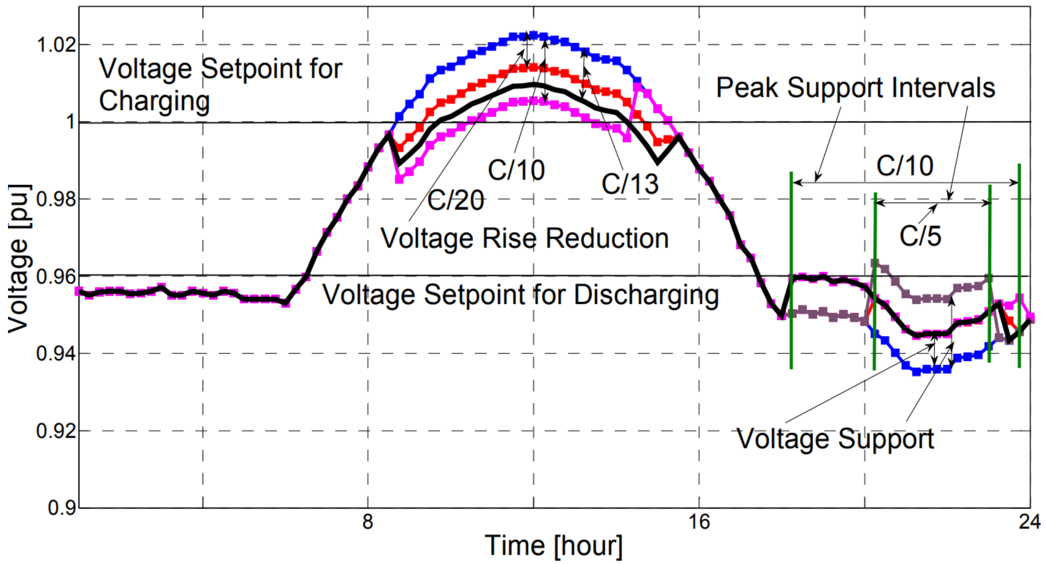
$$P_{chg} = V_{chg} I_{chg}$$
$$V_{chg} = K_1 x^4 + K_2 x^3 + K_3 x^2 + K_4 x + K_5$$
$$x = SOC(t+\Delta t) = SOC(t) + I_{chg}\Delta t$$

Voltage Rise Issue: Mitigation of Voltage Rise

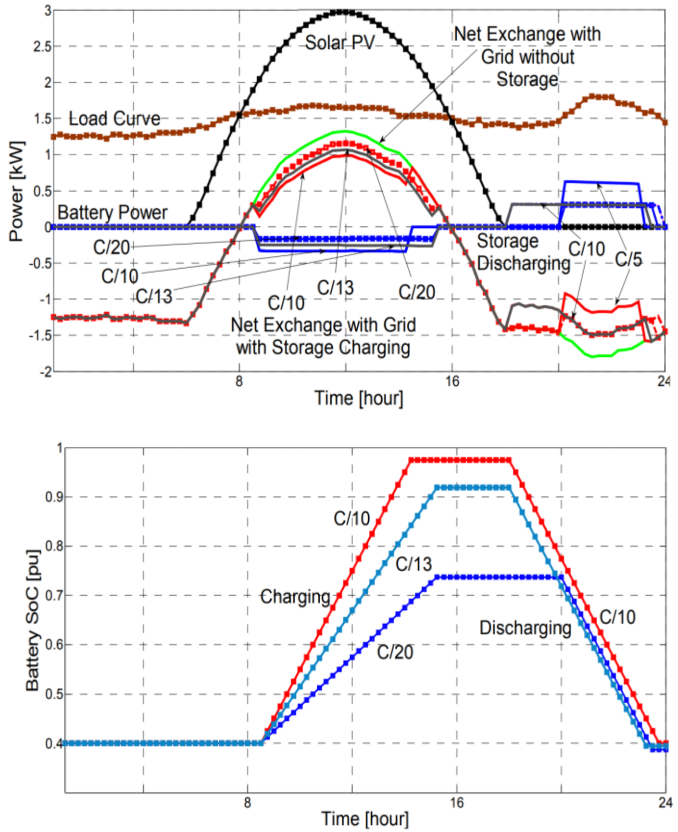
Alam, M. et al (2012), Distribution Energy Storage for Mitigation of Voltage-Rise Impoact Caused by Rooftop Solar PV

Simple LV Test Feeder

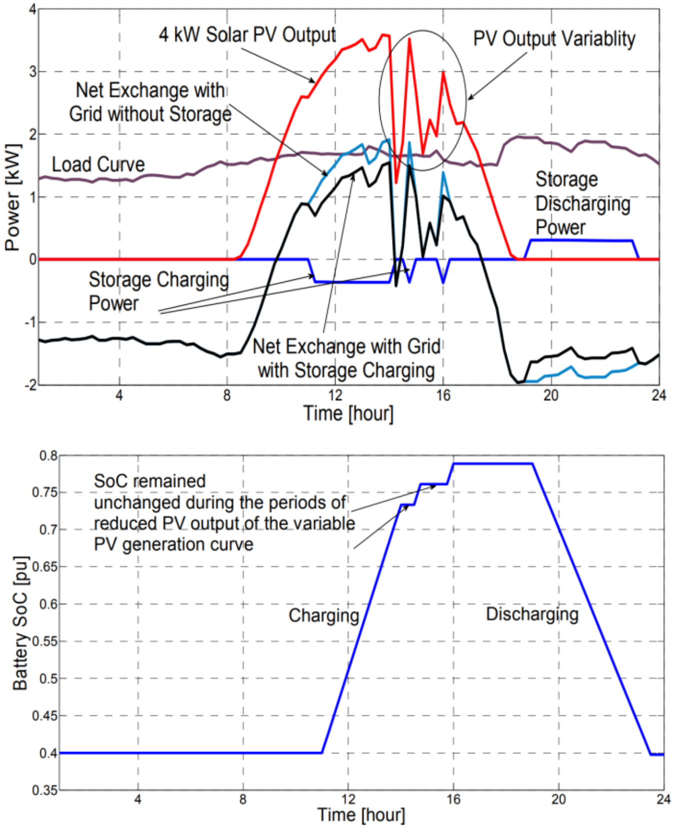




Less Fluctuation

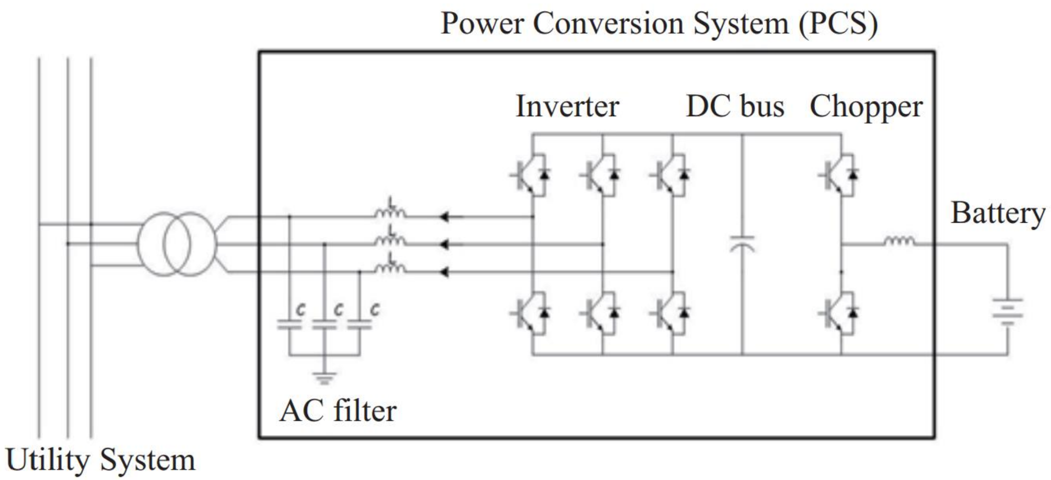


High Fluctuation

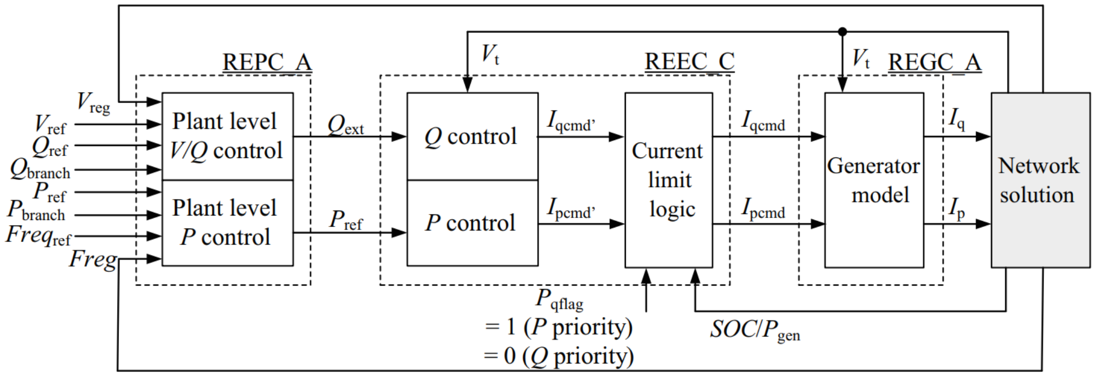


Xu, X. et al. (2016), Application and Modeling of Battery Energy Storage in Power Systems

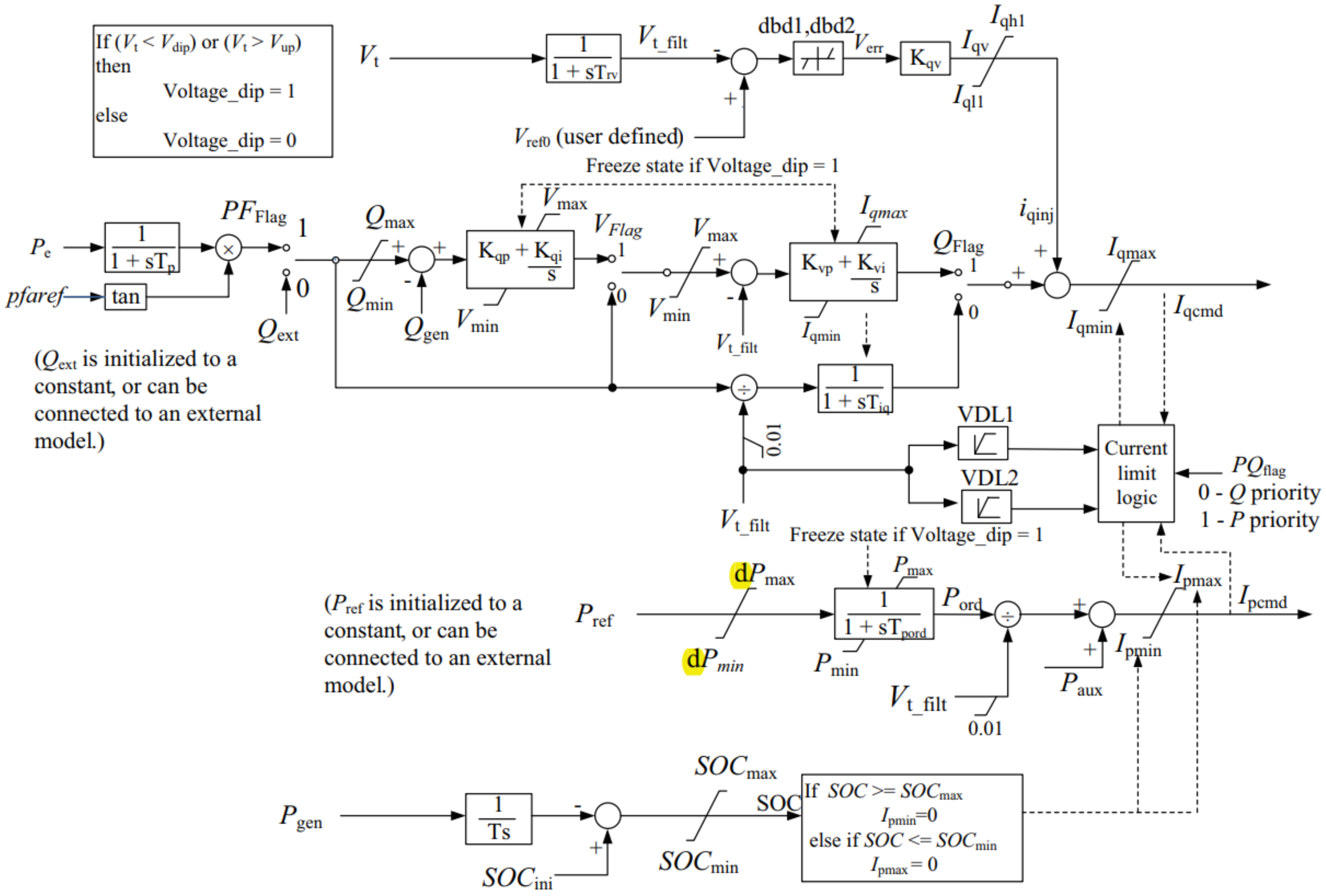
Voltage Rise Issue: Mitigation of Voltage Rise



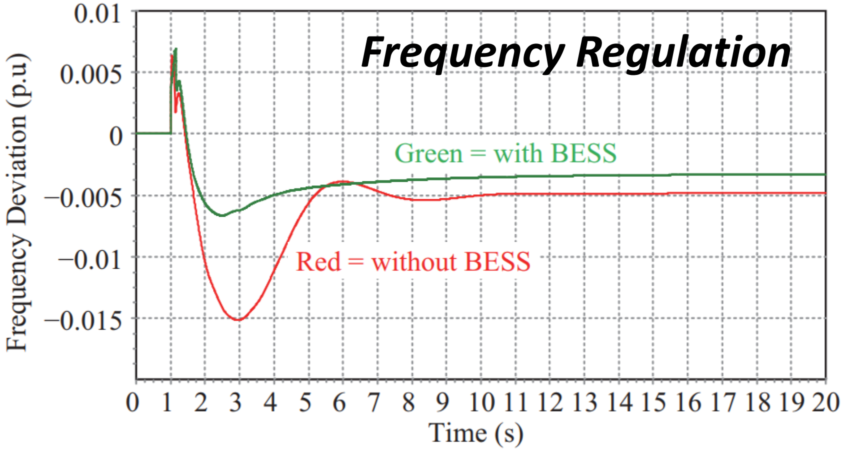
BESS Modelling with ***WECC*** Generic Model

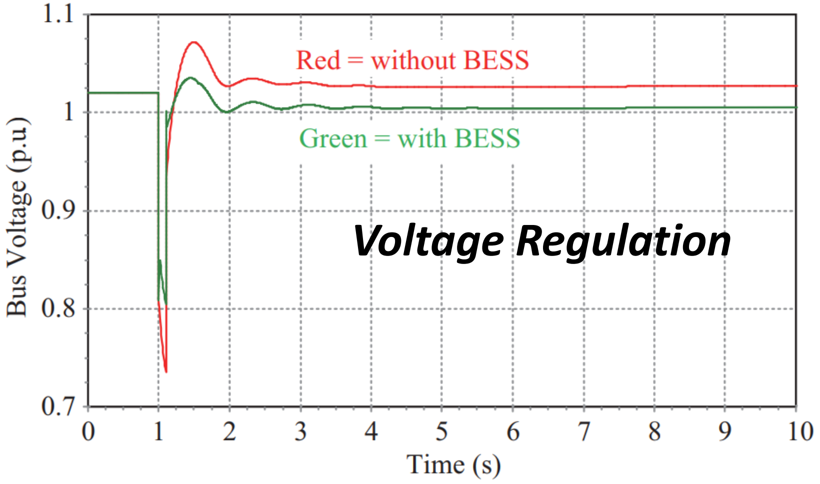


Current Limit Logic

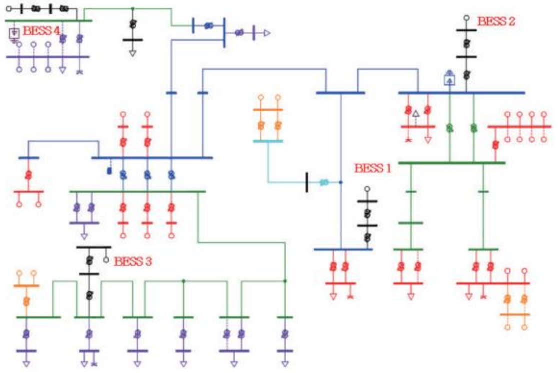


Voltage Rise Issue: Mitigation of Voltage Rise

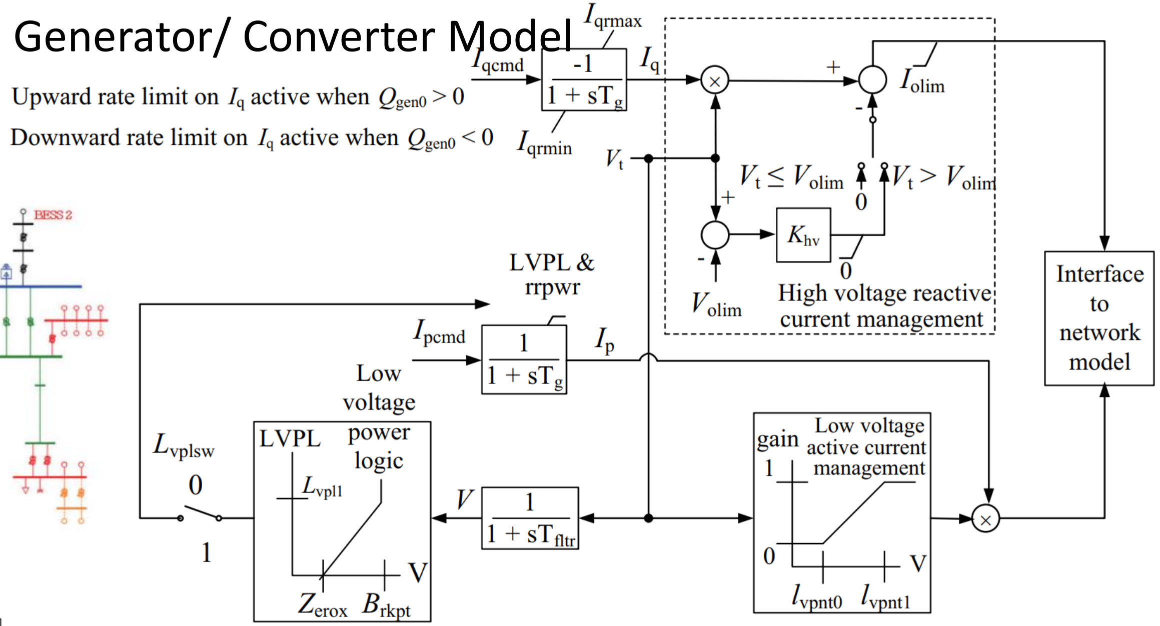


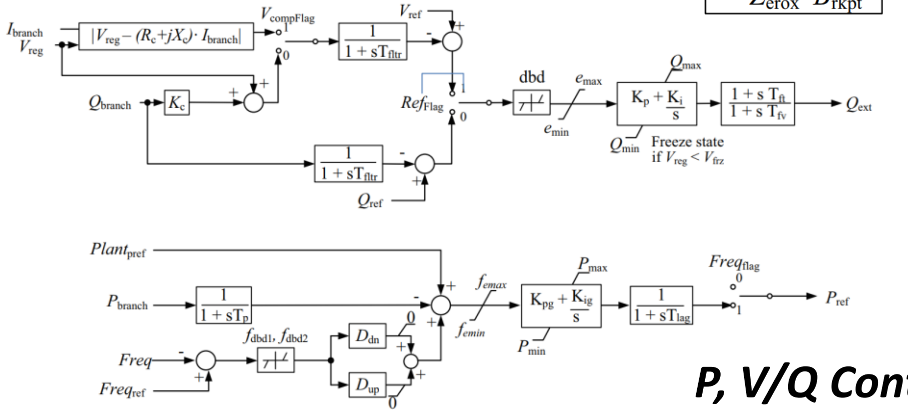


Systems:



Generator/ Converter Model



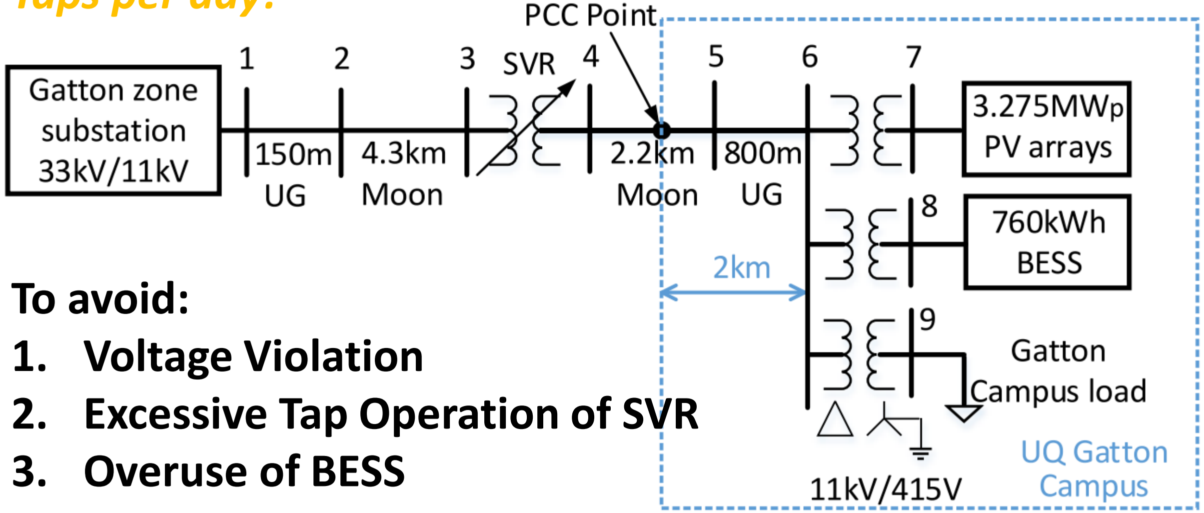


P, V/Q Controller

1) Grid frequency regulation;
2) economic dispatch (arbitrage);
3) time-shifting;
4) output smoothing;
5) output leveling.

Voltage Rise Issue: Real time voltage control

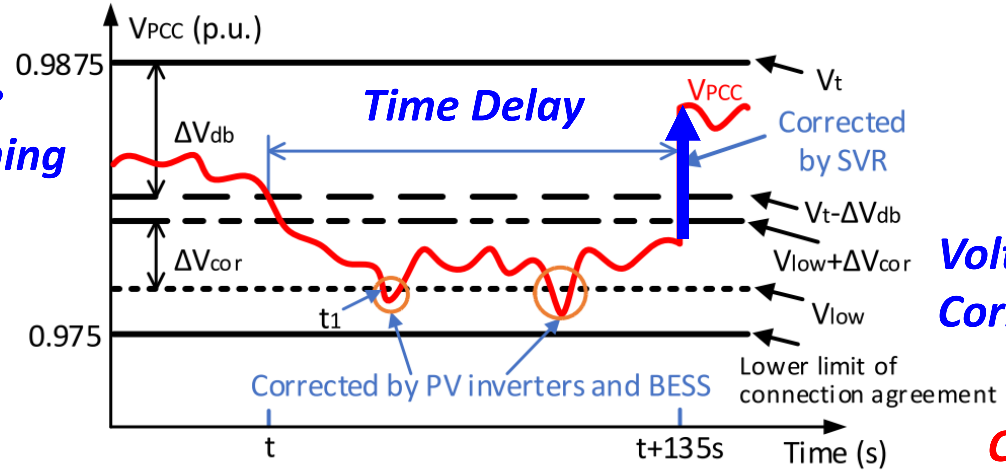
10 Taps per day!



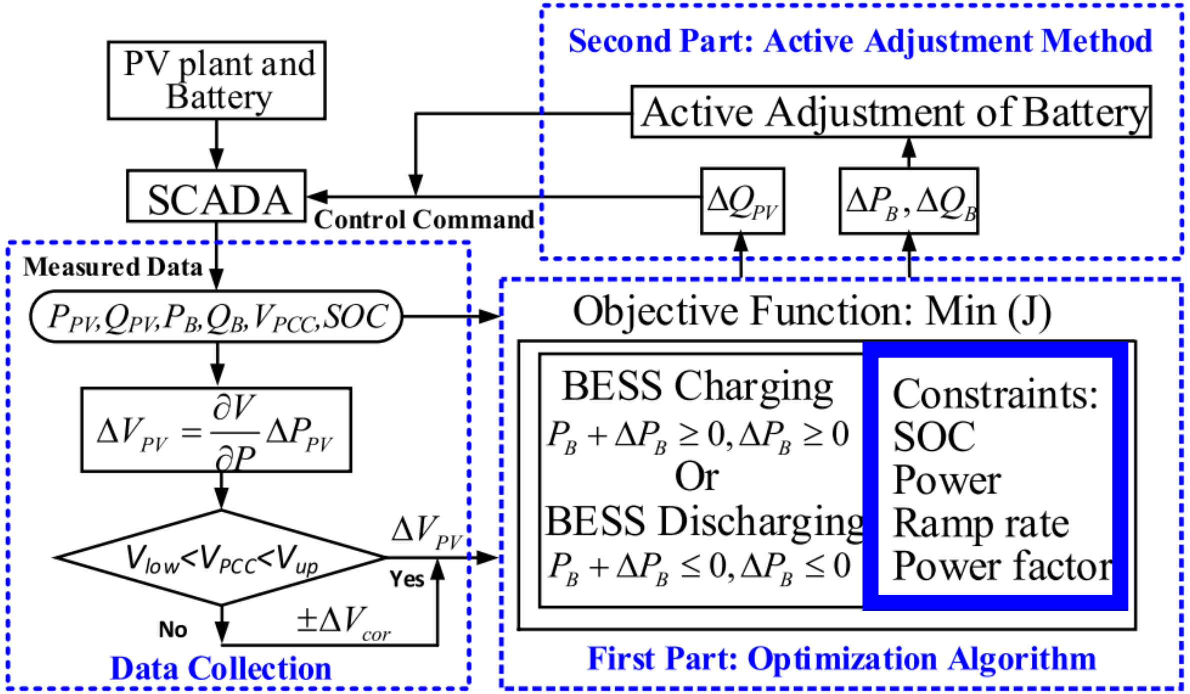
To avoid:

1. **Voltage Violation**
2. **Excessive Tap Operation of SVR**
3. **Overuse of BESS**

Voltage Smoothing



Voltage Correction



Objective Function:

$$J = \left| \Delta V_{PV} + \frac{\partial V}{\partial P} \Delta P_B + \frac{\partial V}{\partial Q} (\Delta Q_B + \Delta Q_{PV}) \right| + \alpha |P_B + \Delta P_B|$$

Voltage violation ***Power of BESS***

Constraint: Ramp Rate, Power Limit, Battery Capacity, Power Factor

Voltage Rise Issue: Voltage Control with P-Q

Clarke Transform: $\begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} = \begin{pmatrix} i_L\angle\varphi \\ i_L\angle\varphi + \frac{\pi}{2} \end{pmatrix}$

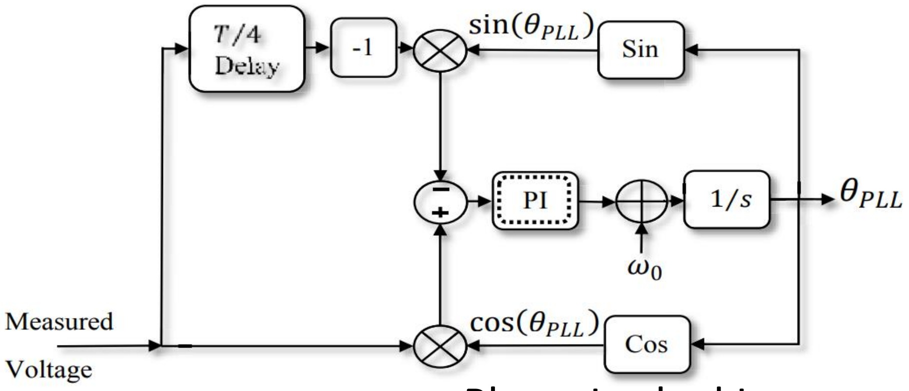
V_q = 0 during synchronization

$i_d^* = \frac{2P^*}{V_d}$ $\quad i_q^* = -\frac{2Q}{V_d}$

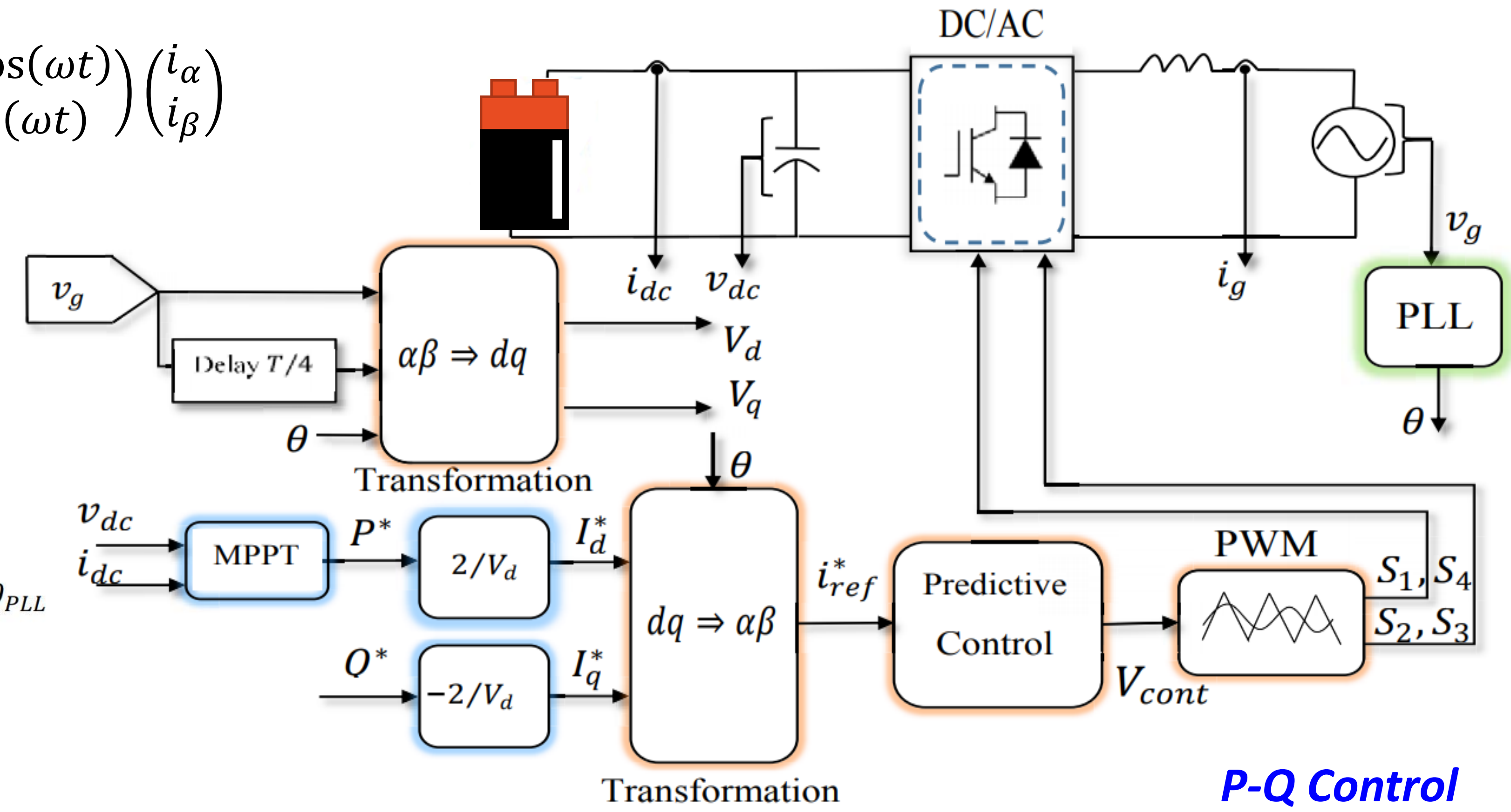
Park Transform: $\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} \sin(\omega t) & -\cos(\omega t) \\ \cos(\omega t) & \sin(\omega t) \end{pmatrix} \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}$

$$P = \frac{1}{2}(v_d i_d + v_q i_q)$$

$$Q = \frac{1}{2}(-v_d i_q + v_q i_d)$$



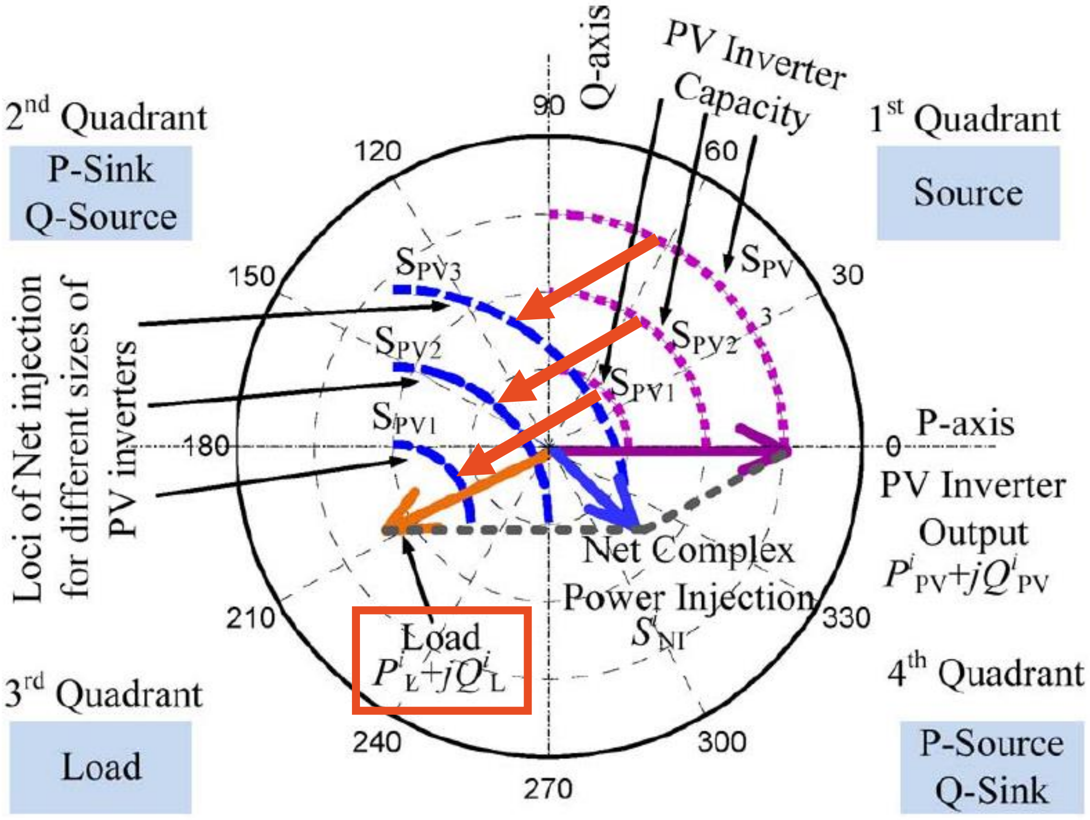
Phase Locked Loop



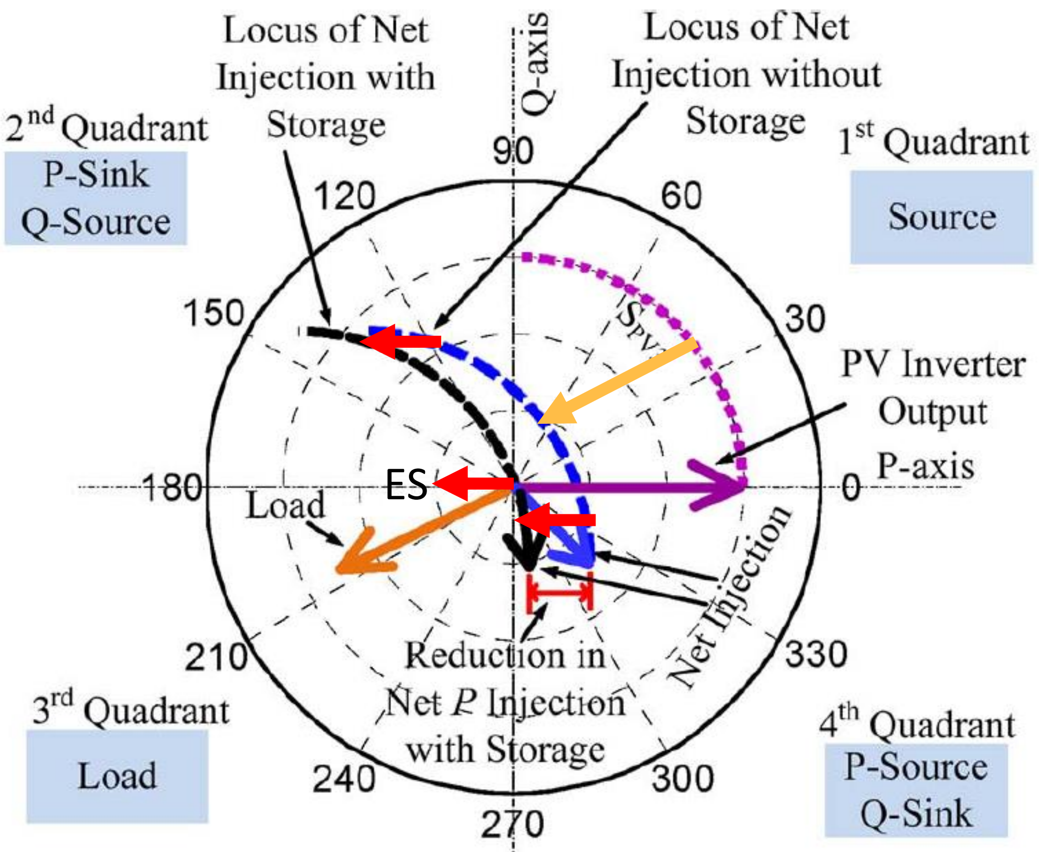
P-Q Control

Alam M. et al. (2013) Mitigation of Rooftop Solar PV Impacts and Evening Peak Support by Managing Available Capacity of Distributed Energy Storage Systems

Voltage Rise Issue: Managing Available Capacity



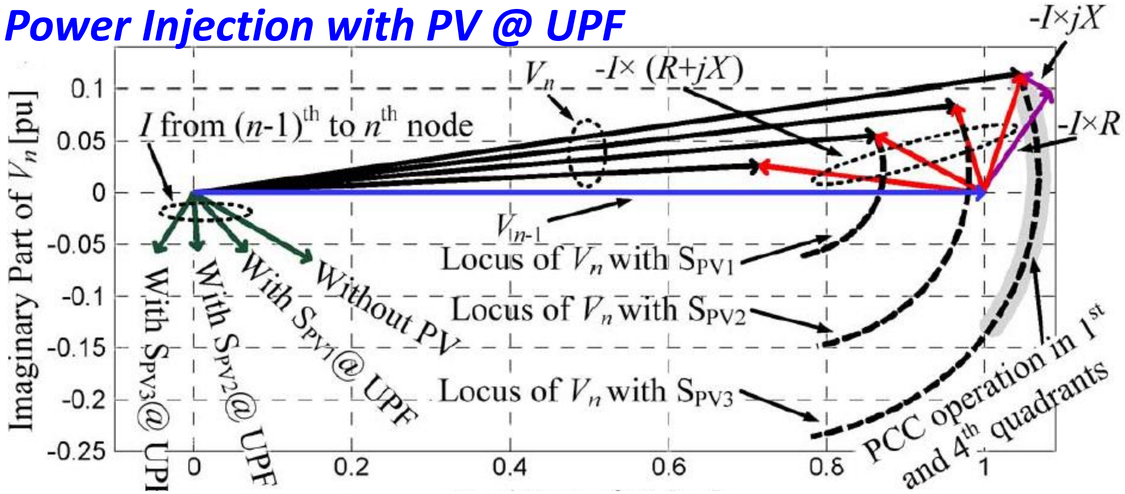
Loci of net power injection and voltage: Net power injection with different PV capacity



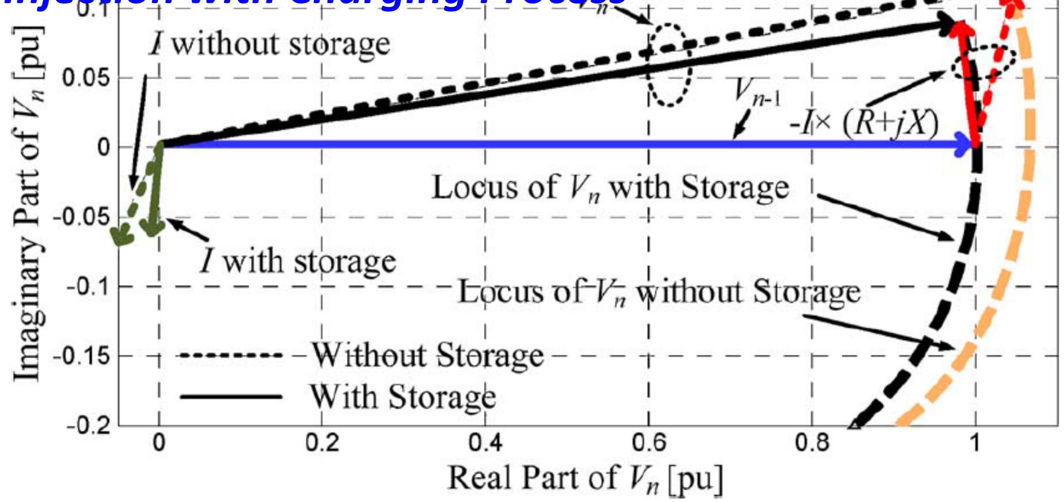
Four quadrant net power injection diagram with storage: Net injection in charging operation

Voltage Rise Issue: Managing Available Capacity

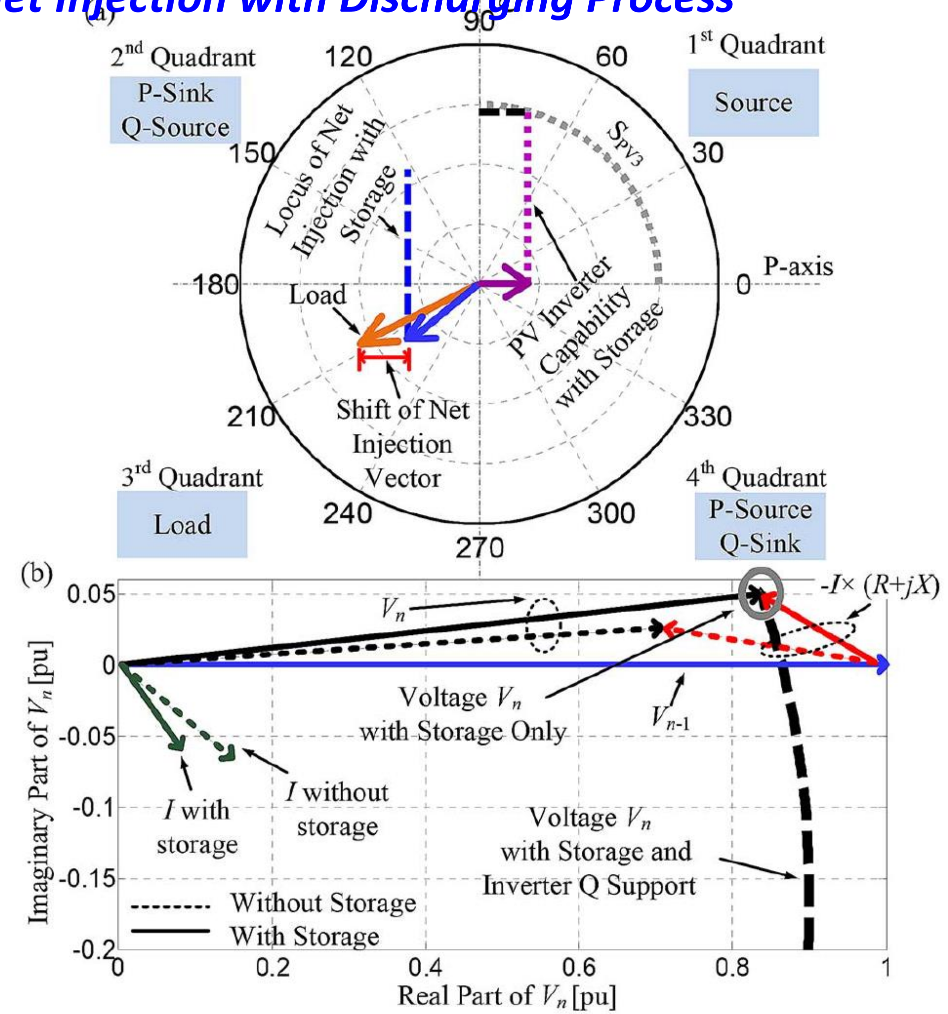
Net Power Injection with PV @ UPF



Net Injection with Charging Process



Net Injection with Discharging Process



Voltage Rise Issue: Managing Available Capacity

Battery Voltage:

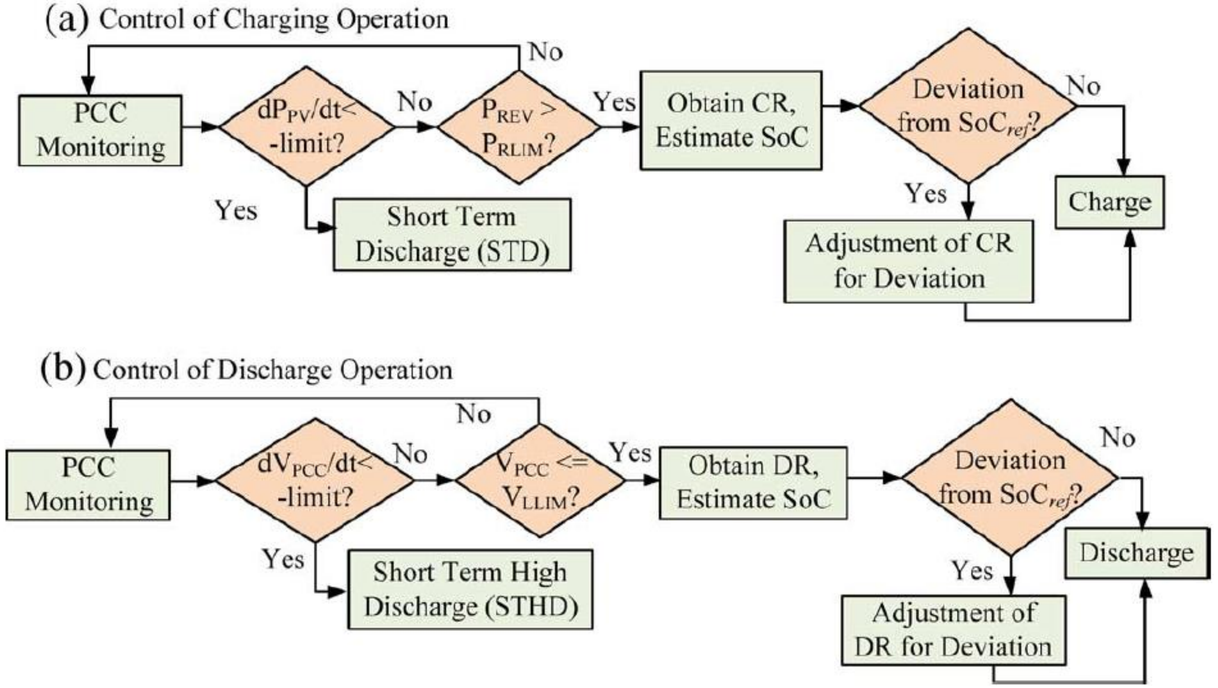
$$V_B = \begin{cases} V_{B0} + I_B\,(R_{BD} - \rho \times \mathrm{SoC}) \\ V_{B0} - I_B\,(R_{BC} + \rho \times \mathrm{SoC}) \end{cases}$$

(Voc) $$V_{B0} = V_{BC} - K \times \frac{C_B}{C_B - \mathrm{SoC}} + Ae^{-B \times \mathrm{SoC}}$$

Maximum Charging Rate:

$$\mathrm{CR}_{\max} = \left| \frac{-(P_{L\,\min} - \eta_{inv} P_{PV\,\max})}{\eta_{inv} \times V_B^{\mathrm{Full}}} \right|$$

where $P_{L\,min}$ = min load power, $P_{PV\,max}$ = max PV power, η_{inv} = efficiency of invertor, V_B^{Full} = Voc at 100% SOC



SCR = slope of charging rate

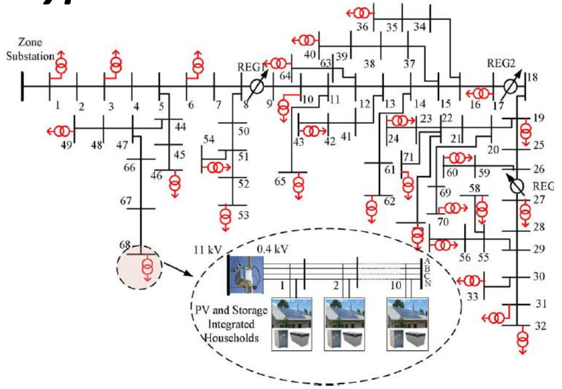
Charging Rate: $$\mathrm{CR}(k) = \begin{cases} \mathrm{CR}(k-1) + \mathrm{SCR}, \text{if } \mathrm{SoC}(k-1) \le \mathrm{ToS}_1 \\ \mathrm{CR}_{\mathrm{Sat}}, \text{if } \mathrm{ToS}_1 < \mathrm{SoC}(k-1) < \mathrm{ToS}_2 \\ \mathrm{CR}(k-1) - \mathrm{SCR}, \text{if } \mathrm{SoC}(k-1) \ge \mathrm{ToS}_2 \\ 0, \text{if } \mathrm{SoC}(k-1) \ge \mathrm{SoC}_{\max}. \end{cases}$$

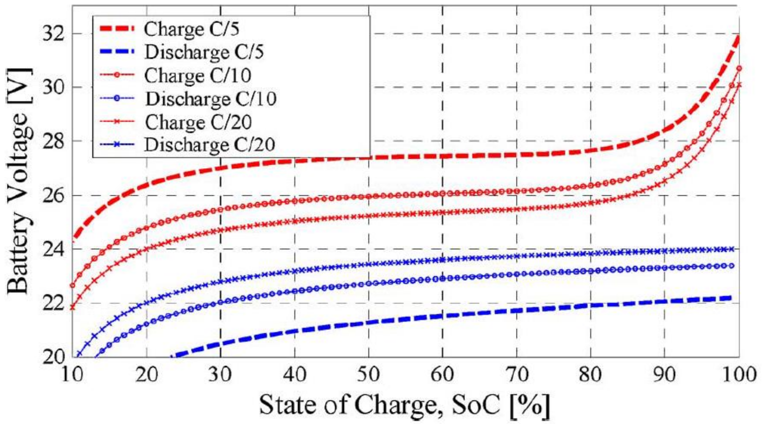
Discharging Rate:

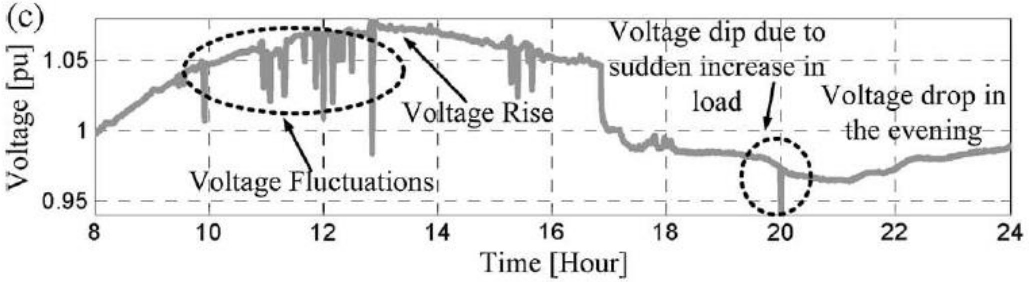
$$\mathrm{DR}(k) = \begin{cases} \mathrm{DR}(k-1) + \mathrm{SDR}, \text{if } \mathrm{SoC}(k-1) \ge \mathrm{ToS}_2 \\ \mathrm{DR}_{\mathrm{Sat}}, \text{if } \mathrm{ToS}_2 > \mathrm{SoC}(k-1) > \mathrm{ToS}_1 \\ \mathrm{DR}(k-1) - \mathrm{SDR}, \text{if } \mathrm{SoC}(k-1) \le \mathrm{ToS}_1 \\ 0, \text{if } \mathrm{SoC}(k-1) \le \mathrm{DoD}_{\max}. \end{cases}$$

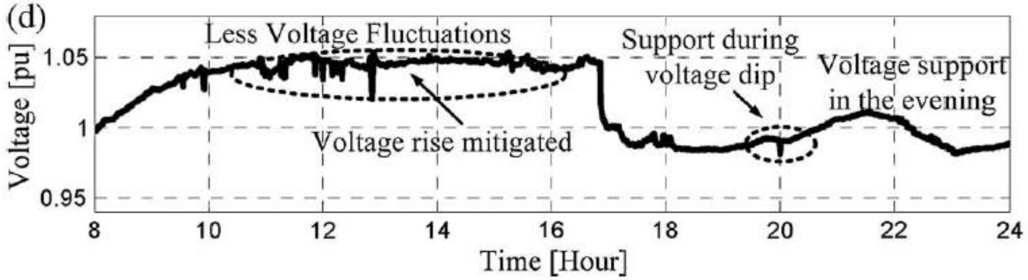
Voltage Rise Issue: Managing Available Capacity

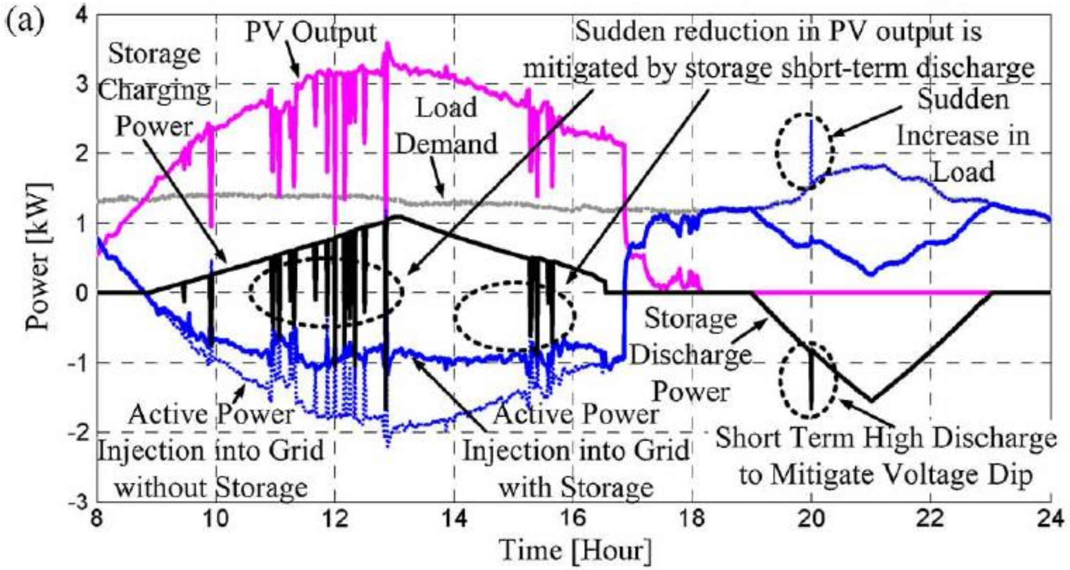
Typical Feeder in Australia

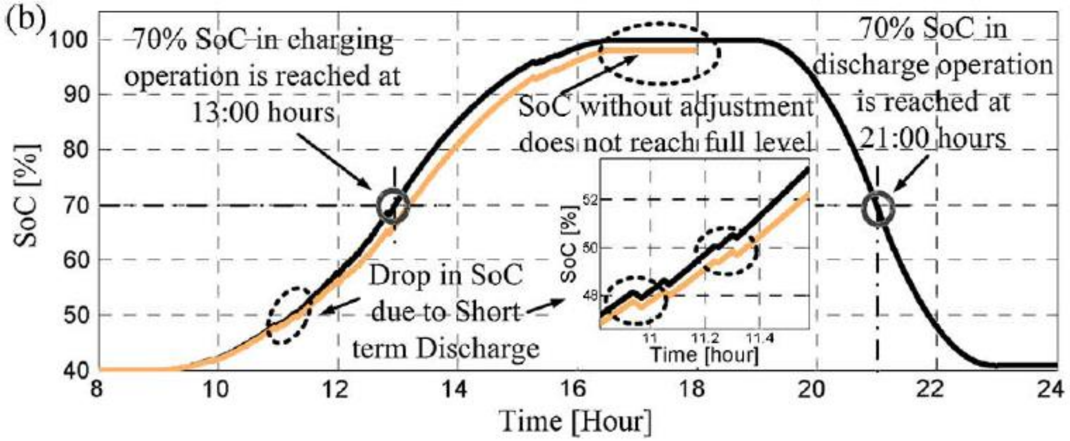






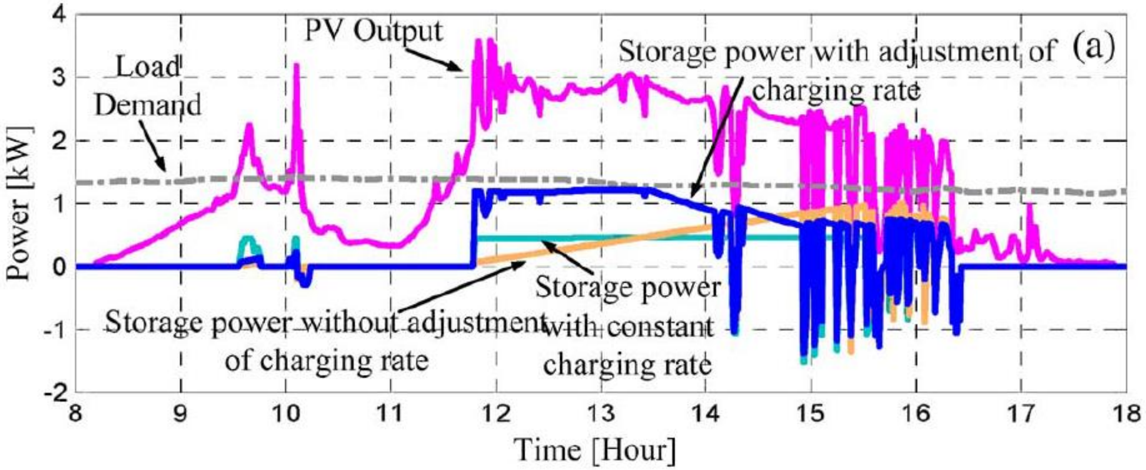


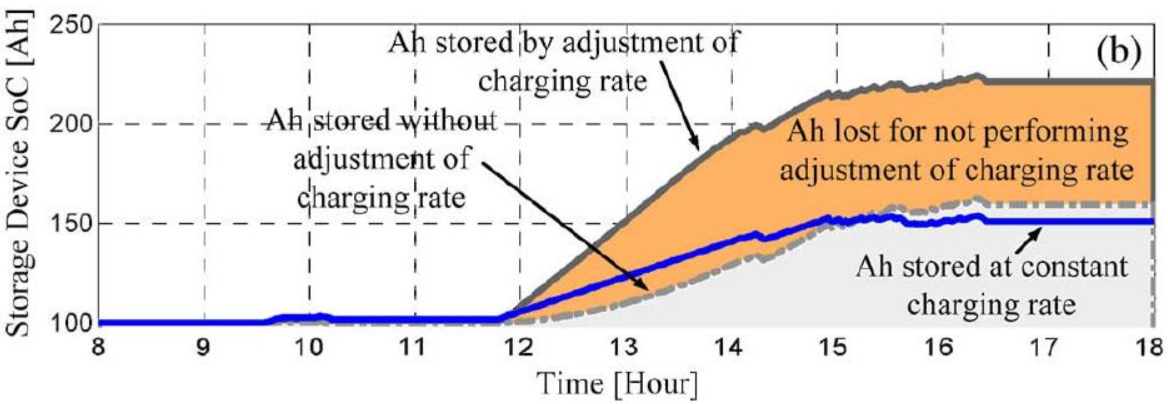




Voltage Rise Issue: Managing Available Capacity

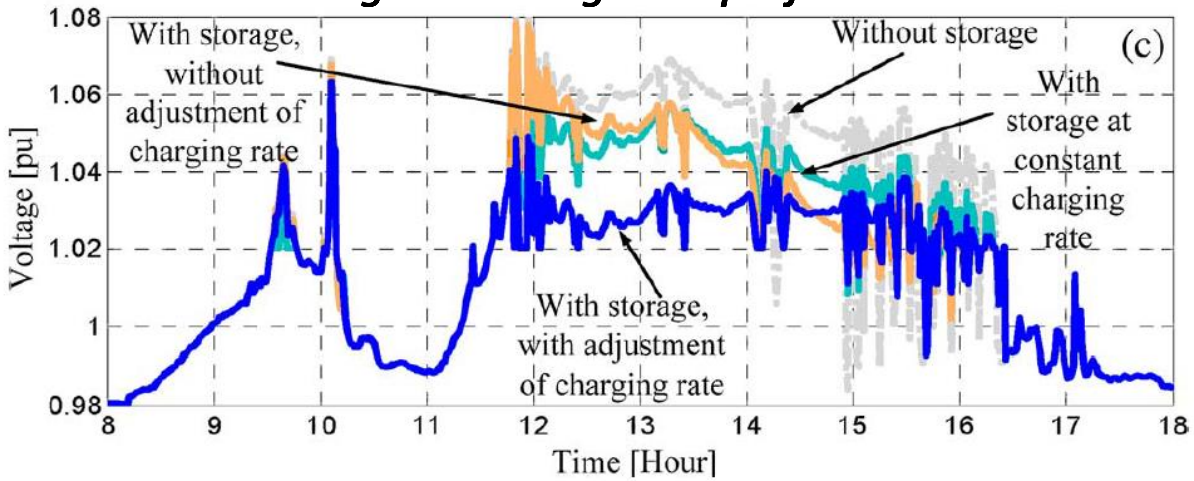
Effect on power consumed by storage in charging operation

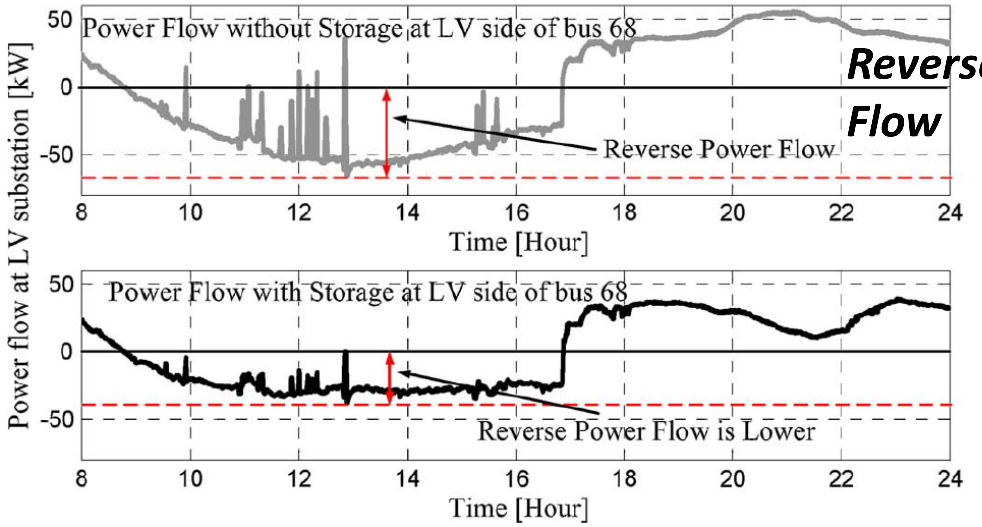




Utilization of battery capacity.

Voltage rise mitigation performance



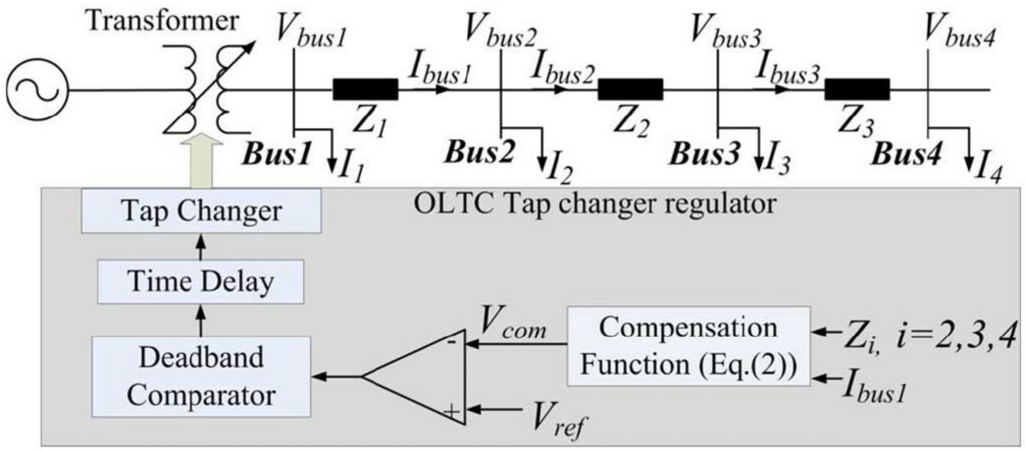


Reverse Power Flow

Liu X. et al. (2012) Coordinated Control of Distributed Energy Storage System With Tap Changer Transformers for Voltage Rise Mitigation Under High Photovoltaic Penetration

Voltage Rise Issue: Control with OLTC

Traditional Method:



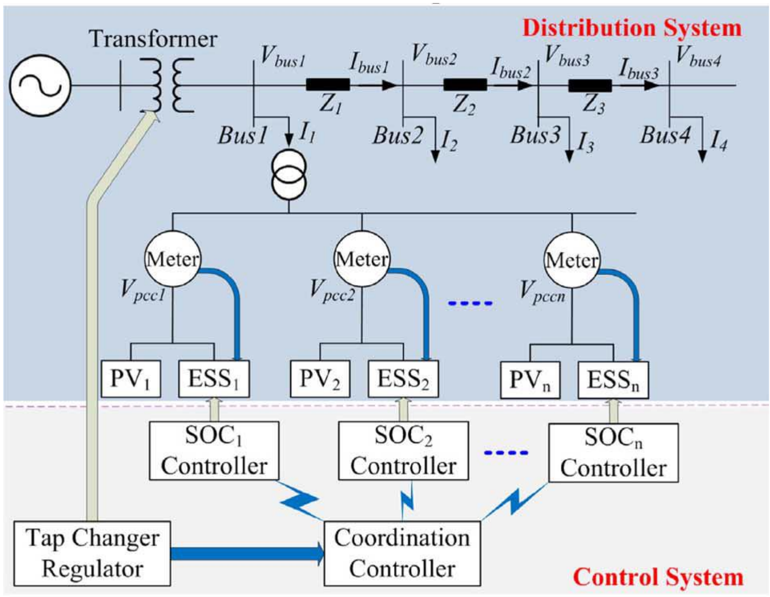
Generator Voltage:

$$V_G \approx V_2 + R(P_G - P_L) + X(Q_G - Q_L) \qquad (1)$$

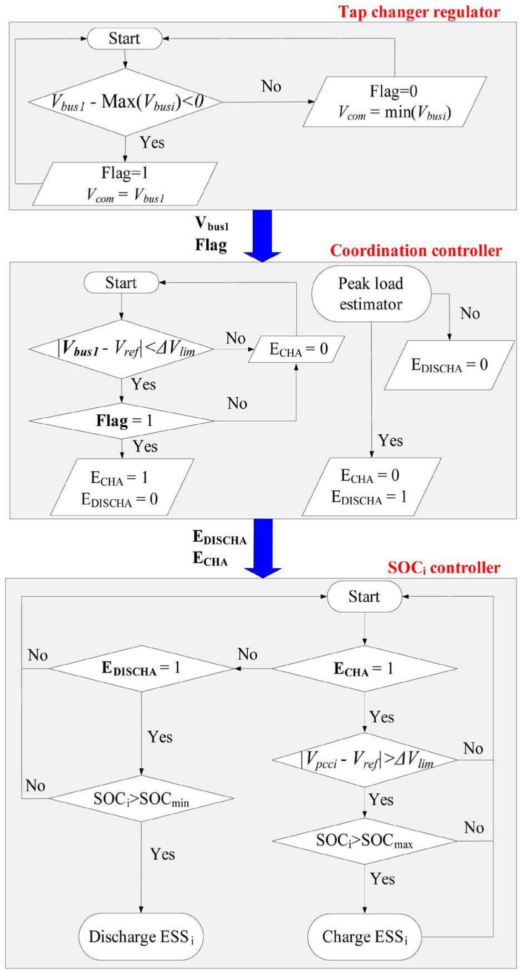
Compensation Voltage for last bus (i.e. bus 4):

$$V_{\text{com}} = V_{\text{bus1}} - Z_1 \cdot I_2 - (Z_1 + Z_2) \cdot I_3 - (Z_1 + Z_2 + Z_3) \cdot I_4. \qquad (2)$$

With ESS:

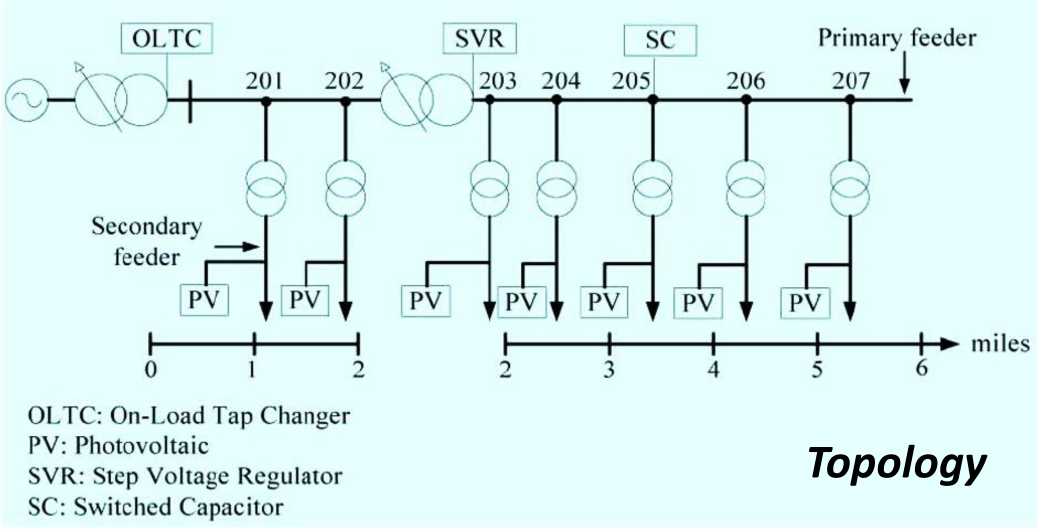


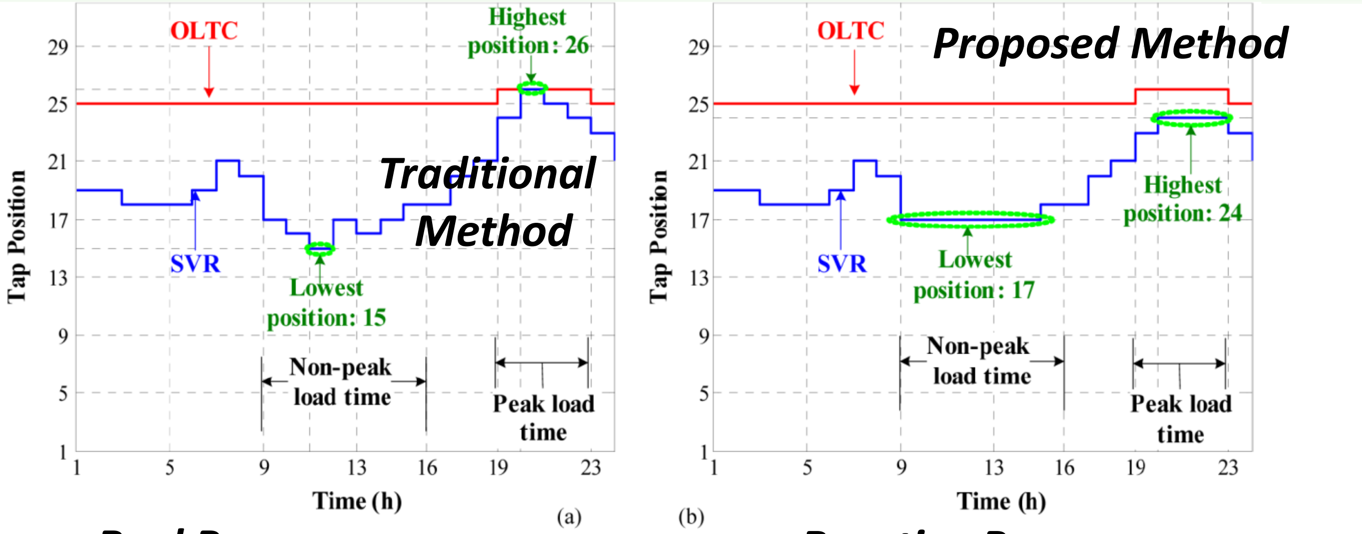
Control Strategy:



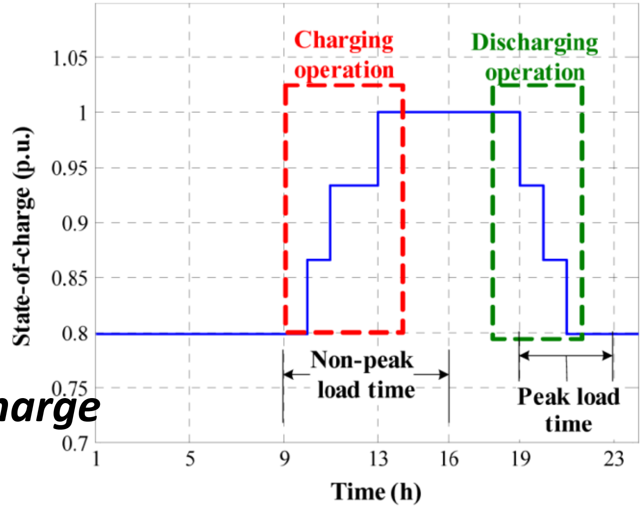
(b)

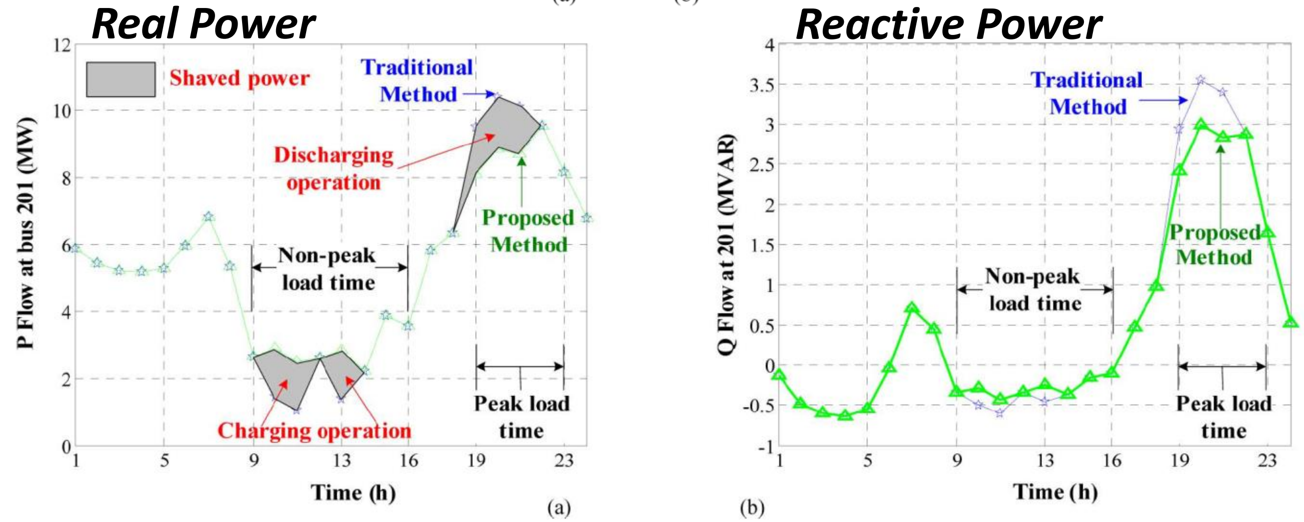
Voltage Rise Issue: Control with OLTC





Daily State of Charge





Power Fluctuation Issue: Ramp Rate Control

Moving Average of Ramp Rate PV output

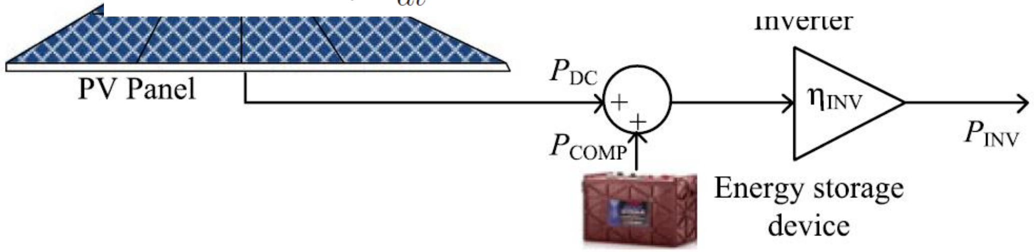
$$MoRR(k) = \frac{1}{w} \times \frac{\sum_{i=0}^{w-1} P_{\mathrm{DC}}(k-i) - \sum_{i=0}^{w-1} P_{\mathrm{DC}}(k-1-i)}{t(k) - t(k-1)} = \frac{1}{w} \times \frac{P_{\mathrm{DC}}(k) - P_{\mathrm{DC}}(k-w)}{t(k) - t(k-1)}$$

Ramp rate of inverter output

$$\frac{dP_{\mathrm{INV}}}{dt} = \eta_{\mathrm{INV}} \times \left[\frac{dP_{\mathrm{DC}}}{dt} + \frac{dP_{\mathrm{COMP}}}{dt}\right]$$

Dead Band Function to force P_{DC} = 0 within limit

$$f\left(\frac{dP_{\mathrm{DC}}}{dt}\right) = \begin{cases} 0, & \text{if } \eta_{\mathrm{INV}} \times \left|\frac{dP_{\mathrm{DC}}}{dt}\right| \leq \left.\frac{dP_{\mathrm{INV}}}{dt}\right|_{\mathrm{des}} \\ \frac{dP_{\mathrm{DC}}}{dt}, & \text{otherwise.} \end{cases}$$



Switching Function

$$S = \begin{cases} 0, & \text{if } |P_{\mathrm{COMP}}| < \left.\frac{dP_{\mathrm{INV}}}{dt}\right|_{\mathrm{des}} \text{ and } f\left(\frac{dP_{\mathrm{DC}}}{dt}\right) = 0 \\ 1, & \text{otherwise.} \end{cases}$$

Compensated Power/ Energy Storage Output

$$\frac{dP_{\mathrm{COMP}}}{dt} = S \times \left[\frac{1}{\eta_{\mathrm{INV}}} \times \left\{\left.\frac{dP_{\mathrm{INV}}}{dt}\right|_{\mathrm{des}} - \eta_{\mathrm{INV}} \times f\left(\frac{dP_{\mathrm{DC}}}{dt}\right)\right\}\right]$$

Compensated Power Moving Average

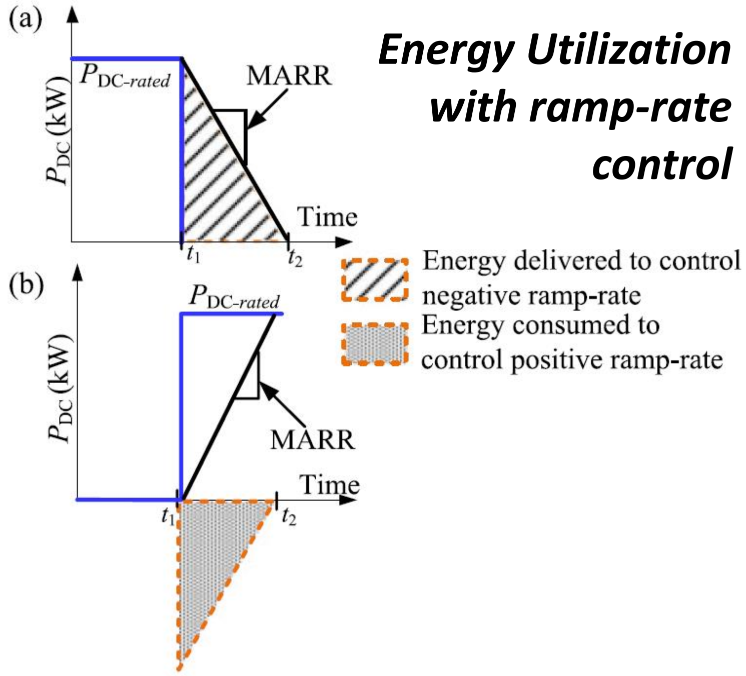
$$P_{\mathrm{COMP}}(k) = \frac{\sum_{i=0}^{w-1} P_{\mathrm{DC}}(k-i)}{w} - P_{\mathrm{DC}}(k)$$

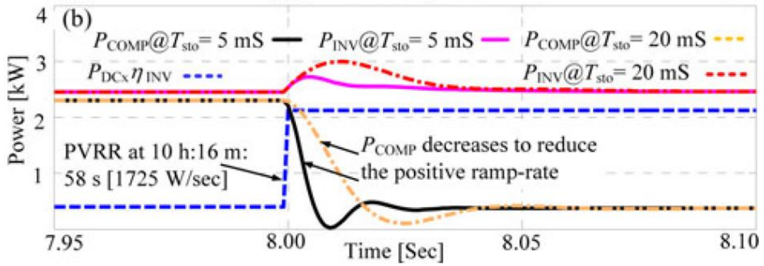
Maximum Allowable Ramp Rate with Moving Average and SOC droop

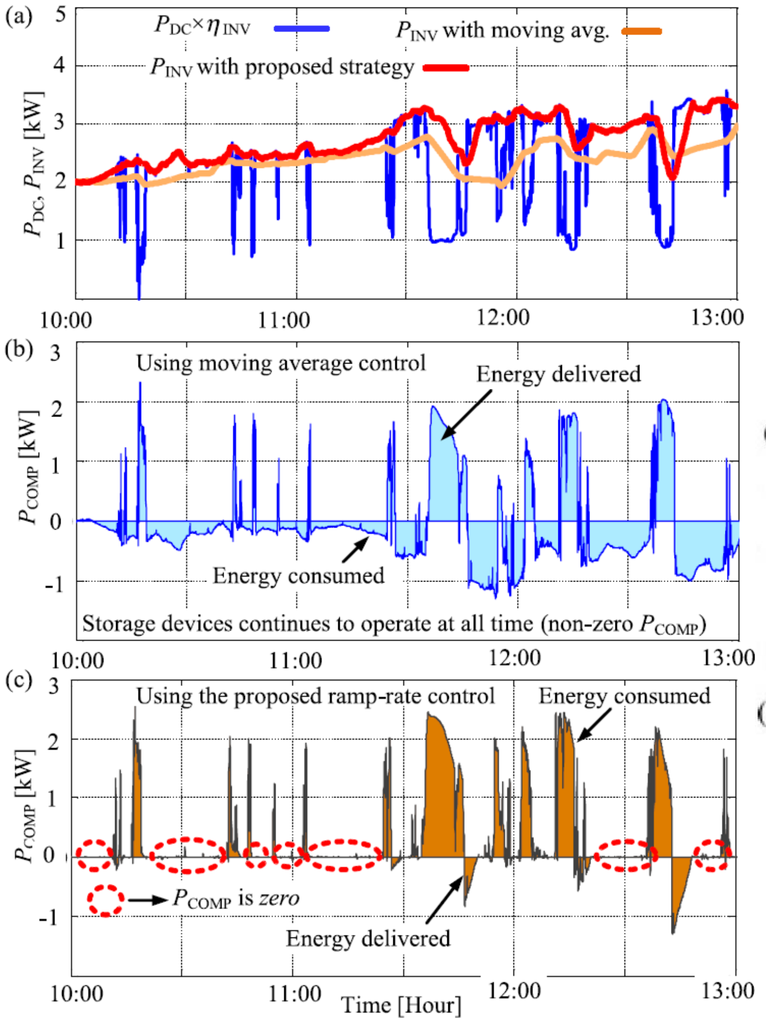
$$\mathrm{MARR}_{\sigma}(k) = \begin{cases} \mathrm{MARR}_{\min}, & \text{if } |\Delta\mathrm{SoC}(k)| < \mathrm{SoC}_{\mathrm{LB}} \\ \mathrm{MARR}_{\min} + \frac{DB_{\mathrm{MARR}}}{DB_{\mathrm{SoC}}} \times [|\Delta\mathrm{SoC}(k)| - \mathrm{SoC}_{\mathrm{LB}}] & \\ & \text{if } \mathrm{SoC}_{\mathrm{LB}} \leq |\Delta\mathrm{SoC}(k)| \leq \mathrm{SoC}_{\mathrm{UB}} \\ \mathrm{MARR}_{\max}, & \text{if } |\Delta\mathrm{SoC}(k)| > \mathrm{SoC}_{\mathrm{UB}}. \end{cases}$$

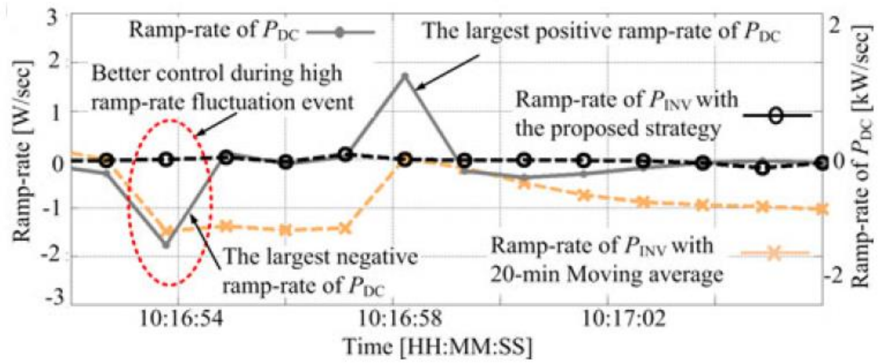
Power Fluctuation Issue: Ramp Rate Control

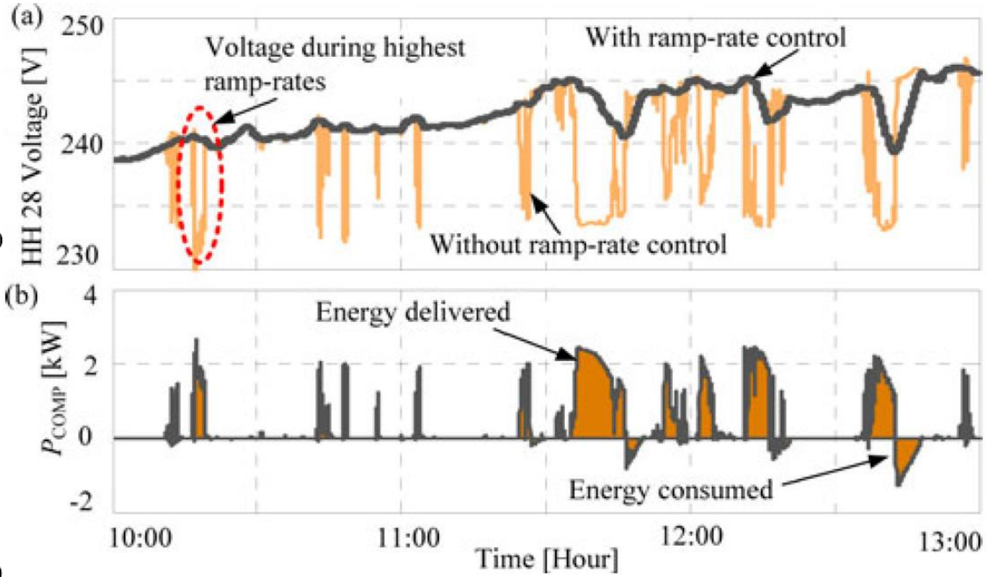
Energy Utilization with ramp-rate control











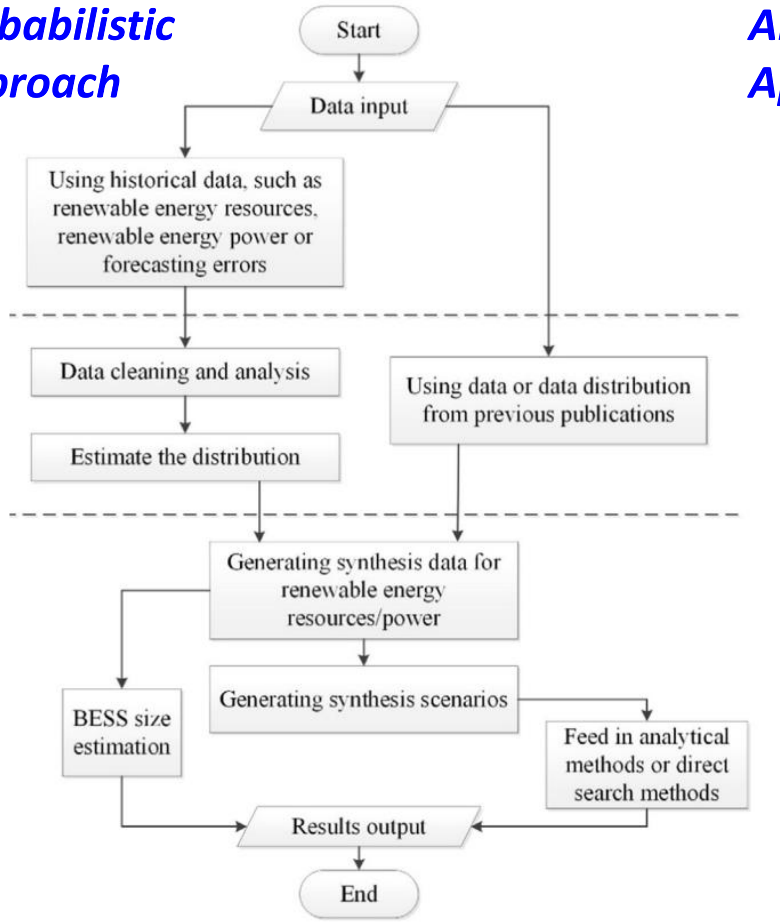
Energy Storage: Sizing Methods

Indicator to be considered:

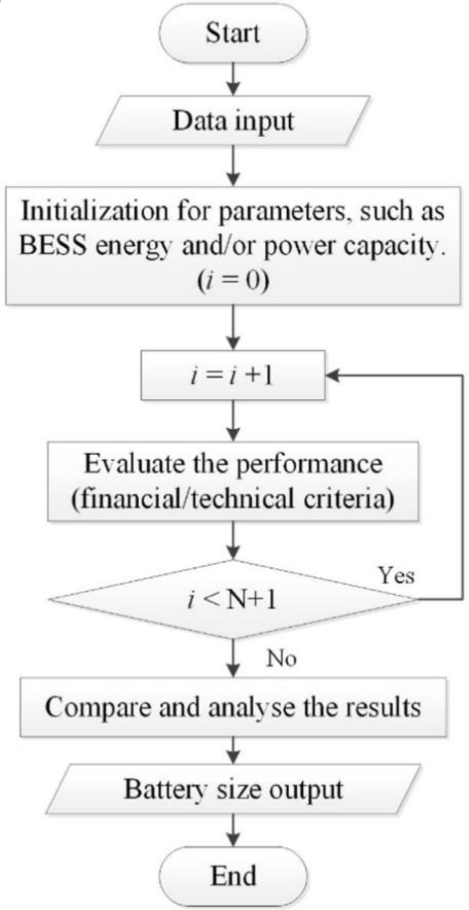
1. **Financial Indicators**
   Overall cost, upfront capital cost, Operation/Maintenance cost, fuel cost, difference of op. cost w/ wo PV, disposal & recycling cost, feed in tariff
2. **Technical Indicators**
   Voltage stability, frequency stability, energy curtailment, reliability, const. power output, islanded power use, depth of discharge, charge/discharge rate, state of health (SOH), round trip efficiency

Sizing Techniques:

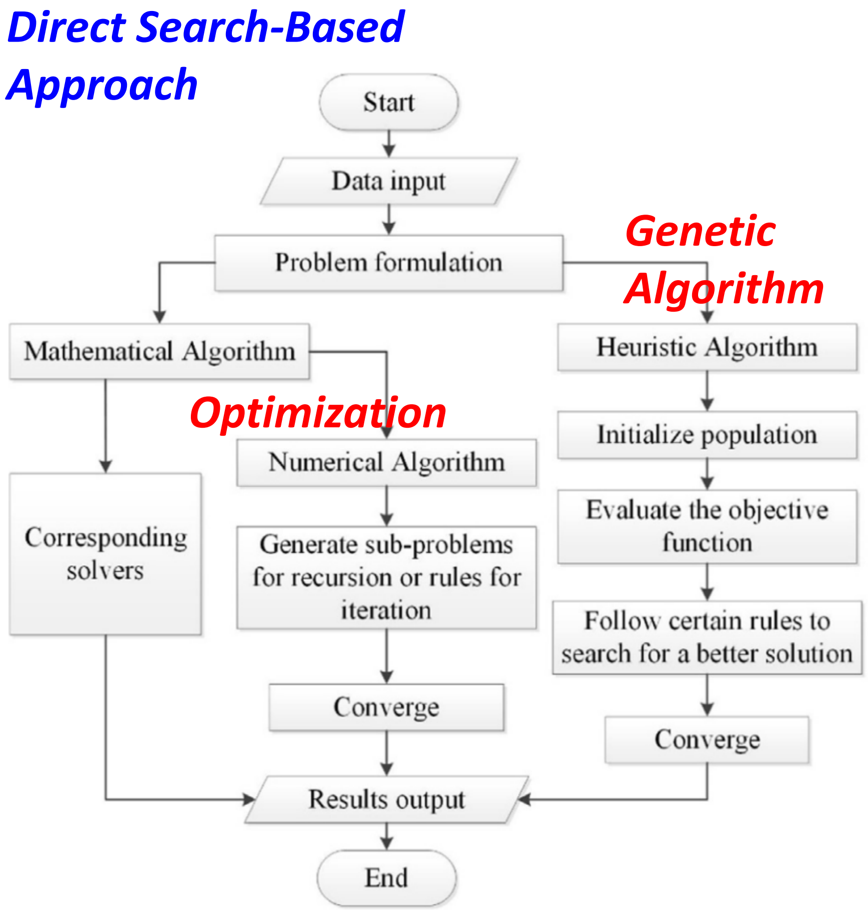
Probabilistic Approach



Analytical Approach



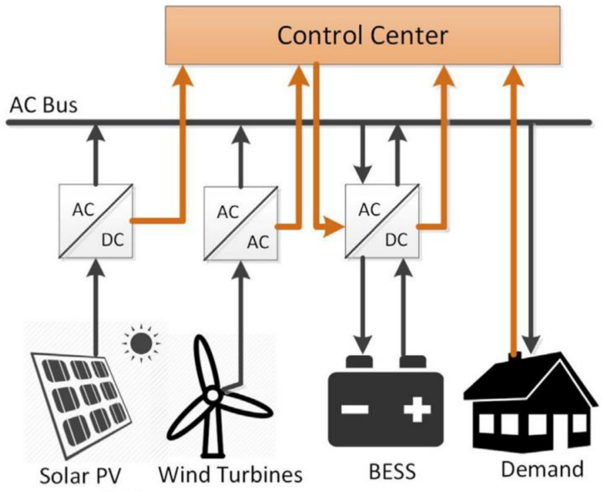
Energy Storage: Sizing Methods

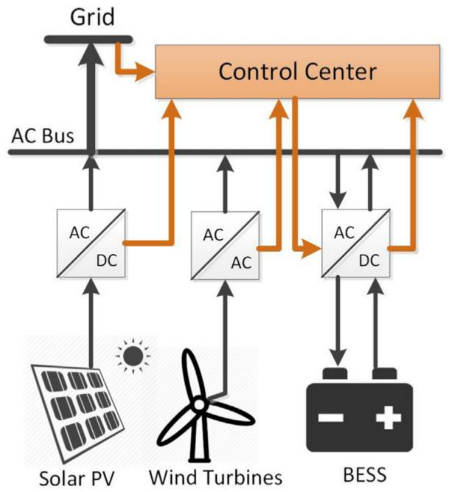


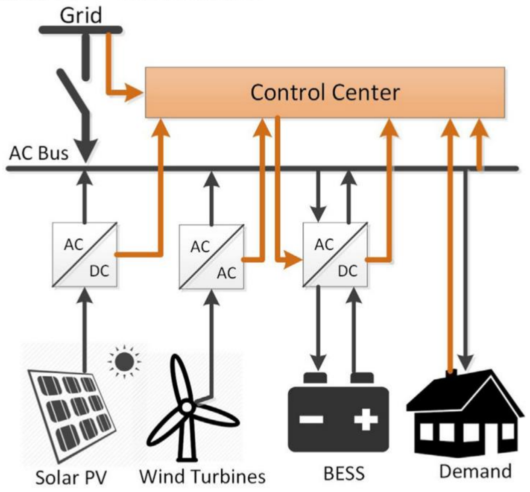
Summary of pros and cons of BESS Sizing Techniques

Technique (Time Horizon)	Implementation	Pros	Cons
Probabilistic (depends on data resolution, more likely intra-hour, hourly data for long duration)	• Generates synthetic weather resources and PV/wind power generation data • Generates synthetic scenarios for stochastic optimisation	• Overcomes the restriction of limited data availability • Gives results with confidence levels	• Accuracy relies on the availability of historical data • May require computational extensive resources
Analytical (Applications for optimisation horizons ranging from repeated intra-hour simulations to several years assessment)	• Direct calculation based on intuitive criteria • Repeated computation/simulation with fixed intervals • Sensitivity analysis	• Better visualization with the change of battery sizes • Strong flexibility for all criteria and simulation environments	• Computational intensive • May miss global optimum if the data resolution is not high enough
Mathematical optimisation (Applications for hourly, intra-day or daily optimisation)	• Linear, mixed-integer, quadratic programming problems • Problems that can be linearized • Problems that can be solved by numerical methods	• Strong capability to find the global optimum • Fast convergence and high robustness for linear problems	• High efficiency limited to linear/ mixed-integer/quadratic programming problems • Linearization may require extra derivations • Explicit mathematical formulation required
Heuristic (Applications for hourly, intra-day or daily optimisation)	• Non-linear optimisation problems • Apply nature-inspired algorithms such as GA, PSO, Tabu search and Bat Algorithms	• Strong flexibility to solve all optimisation problems • Avoid complicated derivatives • Use less computational resources • Simple implementation • Large assortment of algorithms	• May converge in local optimum instead of global optimum • Less robustness and accuracy for linear problems
Hybrid (Applications for hourly, intra-day or daily optimisation)	• Decoupled methods combined sequentially • Hybridisation of different methods in a coupled way	• Combines strengths of different methods • Improves robustness and ensures global optimum found	• Likely to increase the complexity • May require high computational resources than heuristic methods

Energy Storage: Sizing Methods







Different grid topology has different expectation on the BESS's function

Optimal Sizing of BESS also depends on:

- Avoid voltage/ power/ frequency fluctuation (Microgrid)
- Absorb energy and Release Energy to reduce electricity cost
- Serve as constant voltage/frequency provider at PCC
- Reliability, Continuity and Stability of Supply (Standalone)
- Smoothing power output, load management (Power Plant)
- Avoid power curtailment, Compensate forecasting error
- Optimal Place of BESS (to avoid network congestion, upgrade, mitigate working stress of voltage regulating device)
- Application scale of Microgrid
- Auxiliary Technology (e.g. batteries, other ESS, ultracapacitors) ratio of battery to supercapacitor, expected function
- What batteries to be used? Vanadium Flow decouples energy density & power density, which has different capital cost; Zn/Br has minimized annual cost; Li-ion has less replacement;
- Selection range, Partial fulfilment, Environmental concerns, Hybrid Optimization

Sizing of ES: Frequency Control

Rate of change of frequency: $\bar{f}\dfrac{d\bar{f}}{dt} = \dfrac{\bar{P}_{\mu G,g} - \bar{P}_{\mu G,d}}{2 \cdot H_{\mu G}} = \dfrac{\Delta \bar{P}_{\mu G}}{2 \cdot H_{\mu G}}$

Voltage Stability: $\Delta V_1 = V_S - V_{min} \cong \dfrac{P_{R,1}R + Q_{R,1}X}{V_{min}}$

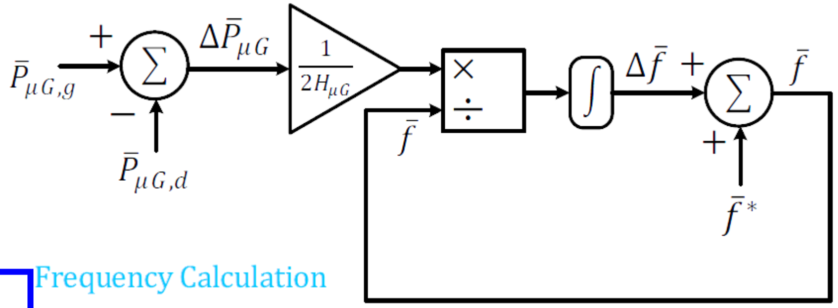
Equivalent Inertia: $H_{\mu G} = \displaystyle\sum_{i=1}^{N} \frac{H_{gen,i} \cdot S_{gen,i}}{S_{\mu G}}$

Primary Inertia Deficiency: $H_{def,1} = \dfrac{\Delta \bar{P}_{\mu G} \cdot \Delta t_1}{\bar{f}_2^2 - \bar{f}_1^2} - H_{\mu G}$

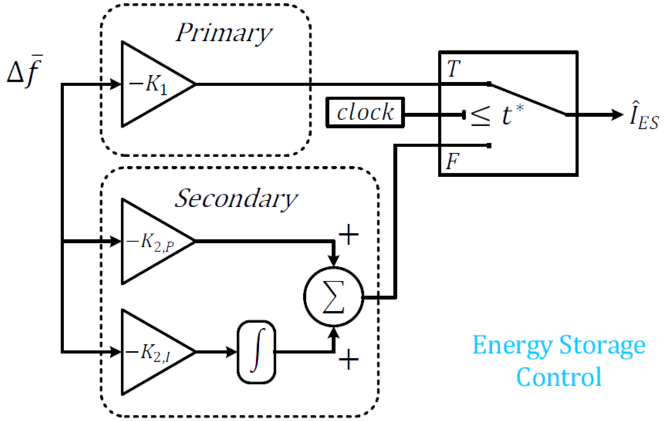
$$\boxed{E_{storage,1} \geq \frac{H_{def,1} \cdot P_{\mu G,g} \cdot |\Delta \bar{P}_{\mu G}| \cdot \Delta t_1}{(H_{def,1} + H_{\mu G})(3.6 \times 10^6)}}$$

Secondary Inertia Deficiency: $H_{def,2} = \dfrac{\Delta \bar{P}_{\mu G} \cdot \Delta t_2}{\bar{f}_2^2 - \bar{f}_1^2} - H_{\mu G}$

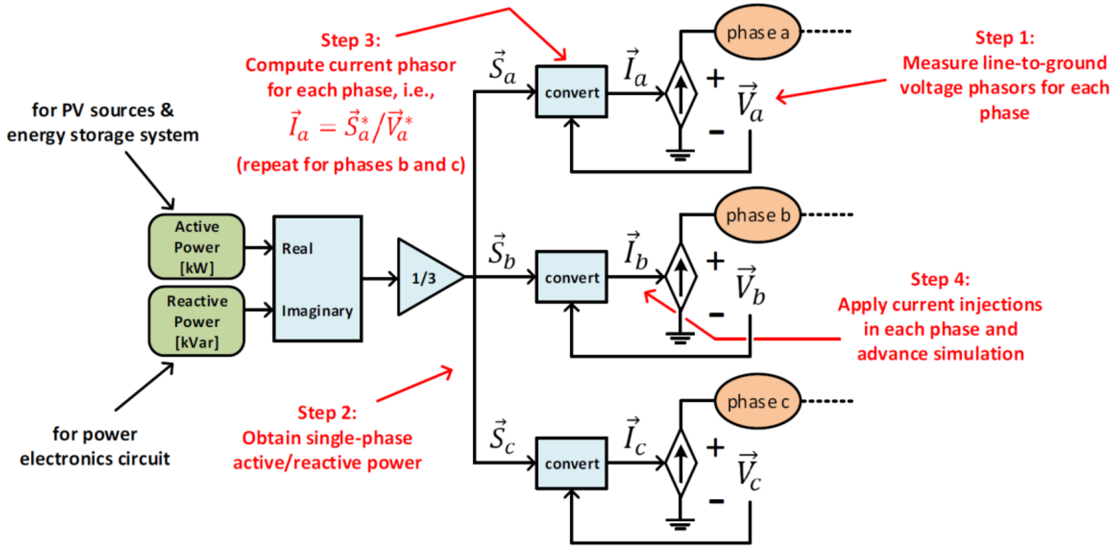
$$\boxed{E_{storage,2} \geq \frac{H_{def,2} \cdot P_{\mu G,g} \cdot |\Delta \bar{P}_{\mu G}| \cdot \Delta t_2}{(H_{def,2} + H_{\mu G})(3.6 \times 10^6)}}$$



Frequency Calculation

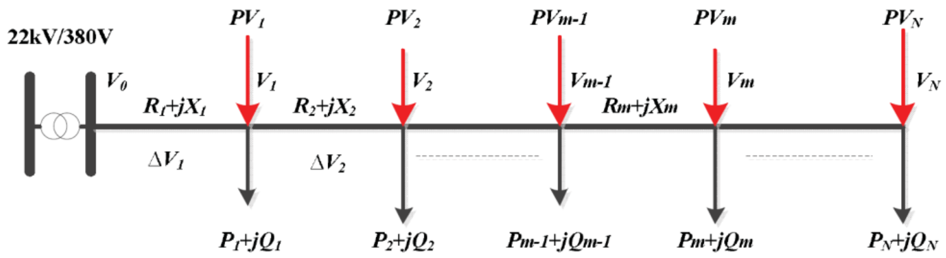


Energy Storage Control



Chaiyatham T. & Ngamroo I. (2012), Bee Colony Optimization of Battery Capacity and Placement for Mitigation of Voltage Rise by PV in Radial Distribution Network

Sizing of ES: Bee Colony Optimization



$$\boxed{\begin{array}{l} Minimize: PB + dV \\ \text{Subject to } V_{f\min} < V_m < V_{f\max} \qquad dV = \left| V_{b\max} - V_{b\min} \right| \end{array}} \quad (*)$$

(Voltage Before PV Position)

$$V_m = V_0 - \sum_{k=1}^{m} \frac{((\sum_{n=k}^{N} P_n) - PV).rl_k + ((\sum_{n=k}^{N} Q_n)).xl_k}{V_{k-1}}$$

(Voltage at PV Position)

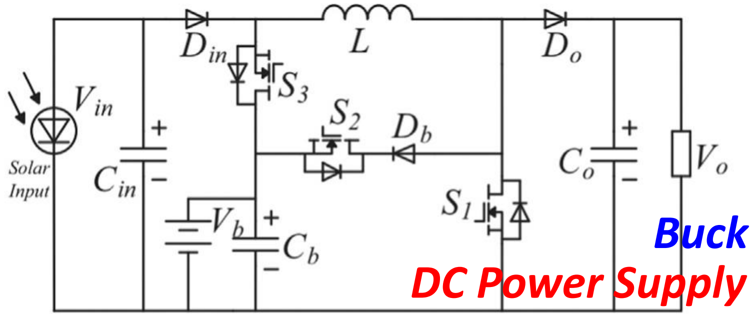
$$V_p = V_0 - \sum_{k=1}^{p} \frac{((\sum_{n=k}^{N} P_n) - PV).rl_k + ((\sum_{n=k}^{N} Q_n)).xl_k}{V_{k-1}}$$

(Voltage after PV Position)

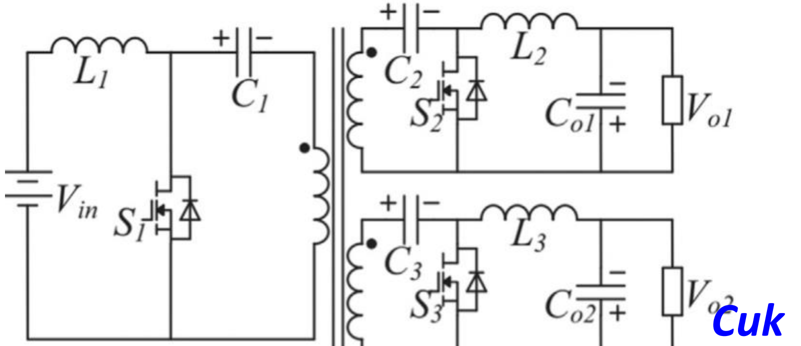
$$V_m = V_0 - \sum_{k=1}^{p} \frac{((\sum_{n=k}^{N} P_n) - PV).rl_k + (\sum_{n=k}^{N} Q_n).xl_k}{V_{k-1}} - \sum_{k=p+1}^{m} \frac{(\sum_{n=k}^{N} P_n).rl_k + (\sum_{n=k}^{N} Q_n).xl_k}{V_{k-1}}$$

Step 1: Generate randomly the initial populations of ne scout bees for the parameter. These initial populations must be feasible candidate solutions that satisfy the constraints. Set NC = 0.
Step 2: Represent the value of parameter from each population.
Step 3: Evaluate the fitness value of the initial populations by (*).
Step 4: Select m best sites for neighborhood search. Separate the m best sites to two groups, the first group has e best sites and another group has m-e best sites.
Step 5: Determine the size of neighborhood search of each best size (patch size, ngh).
Step 6: Recruit bees of ne employed bees for selected sites (more bees for the best e sites).
Step 7: Represent the value of parameter from each employed bee.
Step 8: Select the fittest bees from each patch.
Step 9: Check the stopping criterion. If satisfied, terminate the search, else NC=NC+1.
Step 10: Assign the n-m remaining bees to random search. Go to Step 2.

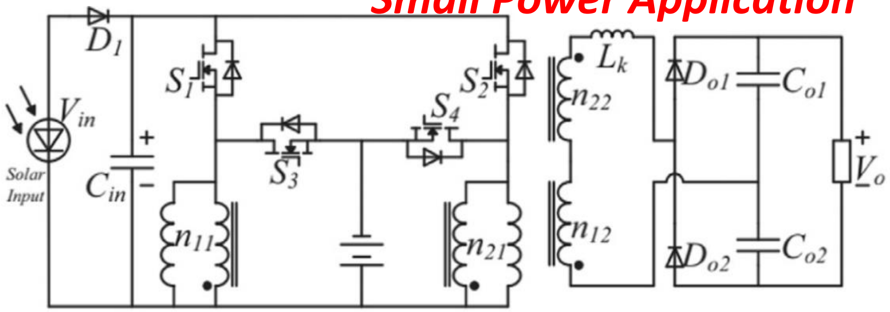
Power Electronics Topology and Analysis



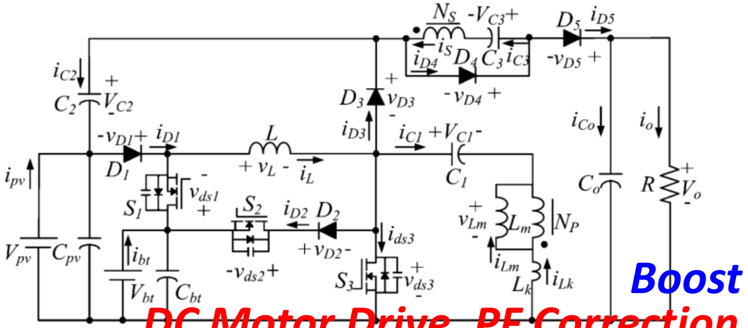
Buck
DC Power Supply



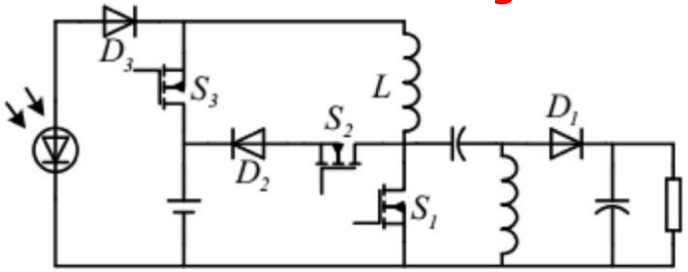
Cuk
Reverse Switching Power Supply



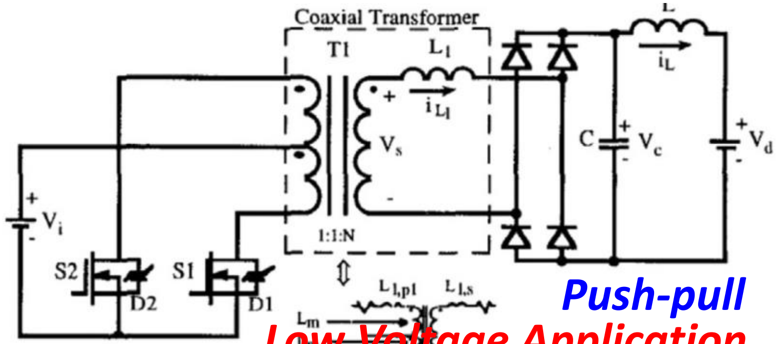
flyback
Small Power Application



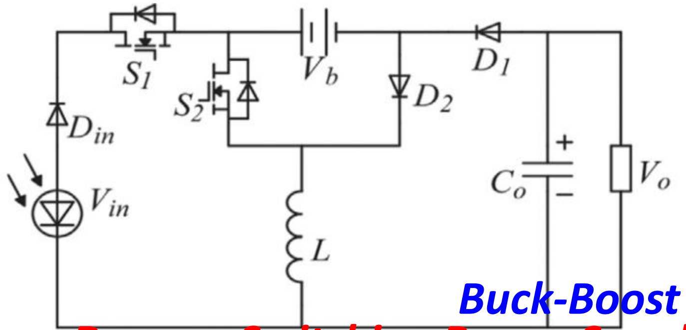
Boost
DC Motor Drive, PF Correction



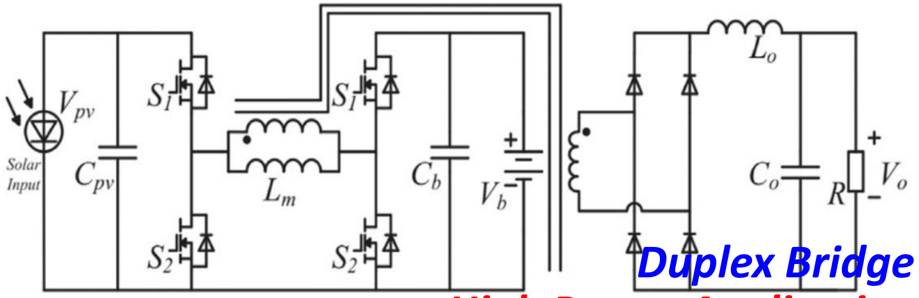
Sepic
Power Factor Correction



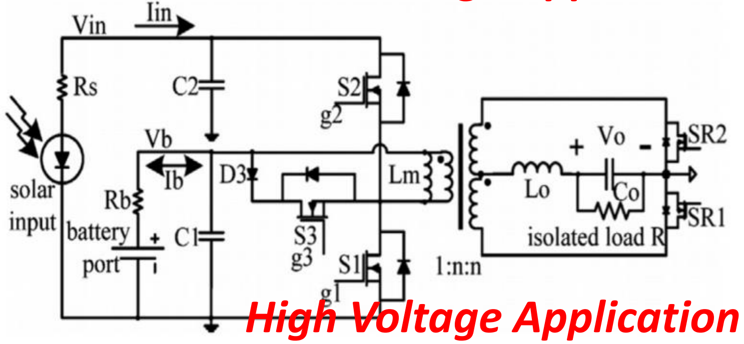
Push-pull
Low Voltage Application



Buck-Boost
Reverse Switching Power Supply



Duplex Bridge
High Power Application



High Voltage Application
Half-bridge Buck Boost

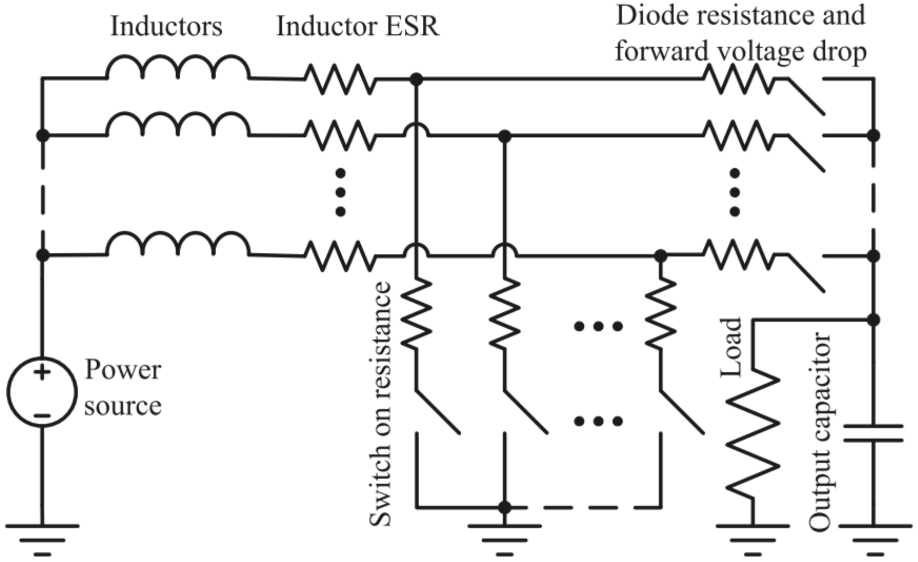
Power Electronics Topology and Analysis

- Differential Equation Method

$$\left(L_{l_{ij}} + L_{m_i}\right)\frac{di_{L_{ij}}(t)}{dt} - L_{m_i}\sum_{k=1,k\neq j}^{N}\frac{di_{L_{ik}}(t)}{dt}$$

$$= v_g(t) - \left[1 - s_{ij}(t)\right]v_c(t) - \left[1 - s_{ij}(t)\right]v_D$$
$$-\left\{R_{L_{ij}} + s_{ij}(t)R_{ON_{ij}} + \left[1 - s_{ij}(t)\right]R_{D_{ij}}\right\}i_{L_{ij}}(t)$$

Multi-phase Boost Converter



$$C_f\frac{dv_c(t)}{dt} = \sum_{i=1}^{M}\sum_{j=1}^{N}\left[1 - s_{ij}(t)\right]i_{L_{ij}}(t) - \frac{v_c(t)}{R} - i_o(t)$$

while $s_{ij}(t) = \begin{cases} 0 & \text{switch ON} \\ 1 & \text{switch OFF} \end{cases}$

Disturbance input: $w = [v_g(t), i_o(t), v_D]$

Inductor current & cap. voltage: $x = [i_{L_{11}}(t), \ldots, i_{L_{MN}}(t), v_c(t)]$

State Equation: $K\dot{x} = A[S(t)]x + B[S(t)]w$

Power Electronics Topology and Analysis

- Differential Equation Method (cont')

Switch Period Averaging: $\sum_{\alpha=1}^{n} t_\alpha = T \quad \sum_{\alpha=1}^{n} \frac{t_\alpha}{T} s_{ij_\alpha} = d_{ij}(t)$

Denote $\sum_{\alpha=1}^{n} \frac{t_\alpha}{T} = \Gamma$

The averaged switching period model:

$$K\dot{x} = \left[\frac{1}{T}\sum_{\alpha=1}^{n} t_\alpha A(S_\alpha)\right] x + \left[\frac{1}{T}\sum_{\alpha=1}^{n} t_\alpha B(S_\alpha)\right] w$$

Define Duty Cycle Vector:

$$D(t) = [d_{11}(t), \dots, d_{1N}(t), d_{21}(t), \dots, d_{MN}(t)]^T$$

With linearity of circuit differential equations,

$$\Gamma A(S_\alpha) = A[D(t)] \qquad \Gamma B(S_\alpha) = B[D(t)]$$

Average State Equation: $K\dot{x} = A[D(t)]x + B[D(t)]w$

$L_{l_{ij}}$	leakage inductance of phase j, group i;
L_{m_i}	mutual inductance of group i;
$i_{L_{ij}}(t)$	inductor current of phase j, group i;
$v_g(t)$	power source voltage;
v_D	diode forward voltage drop;
$v_c(t)$	voltage of output capacitor;
$R_{D_{ij}}$	diode resistance of phase j, group i;
$R_{ON_{ij}}$	switch conduct resistance of phase j, group i;
$R_{L_{ij}}$	inductor ESR of phase j, group i;
R	load resistance;
C_f	output capacitance;
i_o	output current;
$s_{ij}(t)$	switching state of phase j, group i; and

$$s_{ij}(t) = \begin{cases} 0 \text{ switch turned OFF} \\ 1 \text{ switch turned ON.} \end{cases}$$

Power Electronics Topology and Analysis

- Differential Equation Method (cont')

For steady state analysis, $D(t) \rightarrow D$ and $\dot{x} = 0$

$$x = -A(D)^{-1}Bw$$

With elementary row transformation we can obtain the inverse of A(D).

$$\boxed{\begin{aligned} I_{L_{ij}} &= \frac{W-P\left(1-d_{ij}\right)}{WR_{ij}\left(d_{ij}\right)} v_g + \frac{1-d_{ij}}{WR_{ij}\left(d_{ij}\right)} i_0 - \frac{1-d_{ij}}{WRR_{ij}(dij)} v_D \\ V_c &= \frac{P}{W} v_g - \frac{1}{W} i_o - \left(1 - \frac{1}{RW}\right) v_D \end{aligned}}$$

where W and P are defined as

$$W = \frac{1}{R} + \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{\left(1-d_{ij}\right)^2}{R_{ij}\left(d_{ij}\right)}$$

$$P = \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{1-d_{ij}}{R_{ij}\left(d_{ij}\right)}$$

For Dynamic Analysis, $\begin{aligned} D(t) &= \overline{D} + \widetilde{D}(t) \\ w(t) &= w + \widetilde{w}(t) \end{aligned}$

Similarly,

$$K\frac{d\tilde{x}}{dt} = \tilde{A}(t)\bar{x}(t) + \bar{A}(t)\tilde{x}(t) + \tilde{B}(t)\overline{w}(t) + \bar{B}(t)\widetilde{w}(t)$$

$$\tilde{A}(t)\bar{x}(t) + \tilde{B}(t)\overline{w}(t) = \overline{H}\widetilde{D}(t)$$

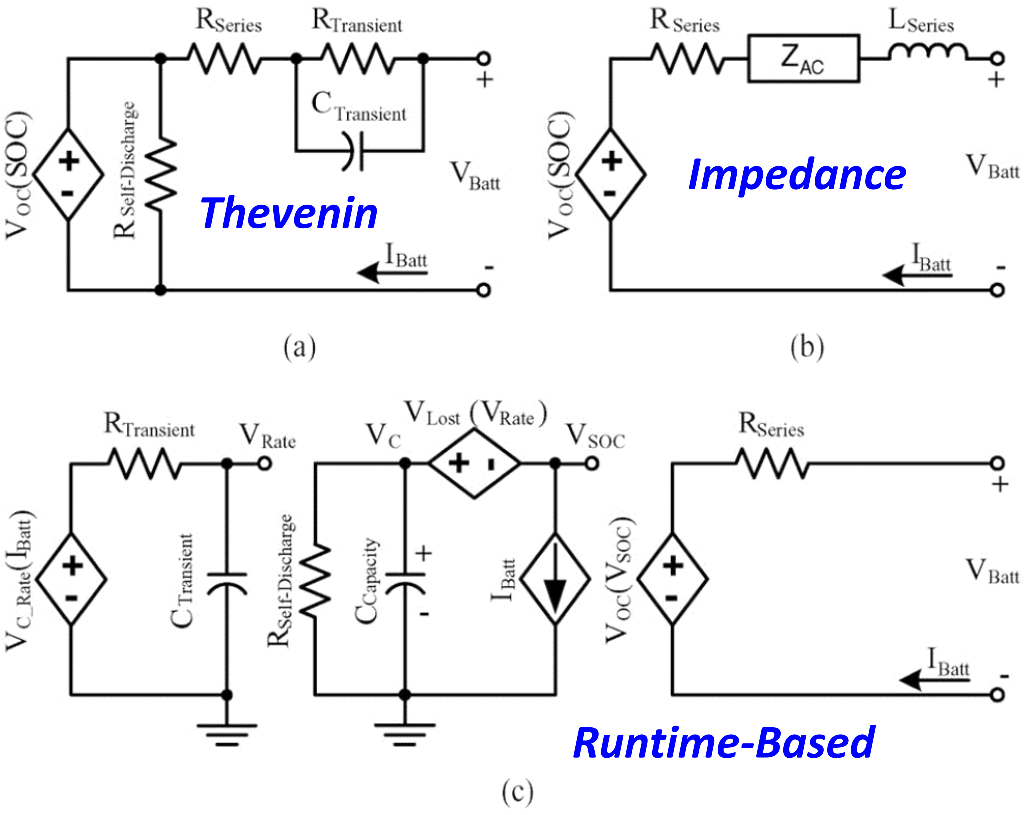
$$H = \left(\frac{\delta\left(V_{ij}\right)}{\left[-I_{L_{ij}}\right]^T}\right) \quad \text{and} \quad V_c - \left(R_{ON_{ij}} - R_{D_{ij}}\right) I_{L_{ij}} + v_D$$

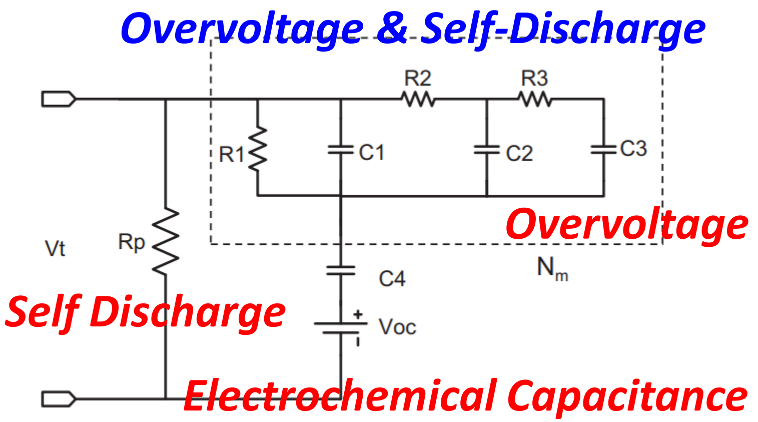
$$K\frac{d\tilde{x}}{dt} = \bar{A}(t)\tilde{x}(t) + \overline{H}\widetilde{D}(t) + \bar{B}(t)\widetilde{w}(t)$$

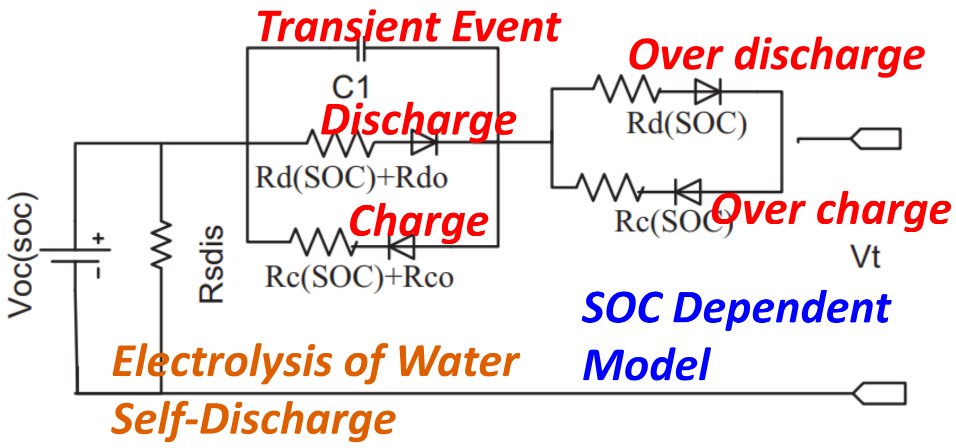
If x(0)=0, $\quad Ks\tilde{x}(s) = \bar{A}(s)\tilde{x}(s) + \overline{H}\widetilde{D}(s) + \bar{B}(s)\widetilde{w}(s)$

$$\boxed{\tilde{x}(s) = \left(Ks - \bar{A}\right)^{-1} \overline{H}\widetilde{D}(s) + \left(Ks - \bar{A}\right)^{-1} \bar{B}\widetilde{w}(s)}$$

Battery Models: (Integer) Equivalent Circuit



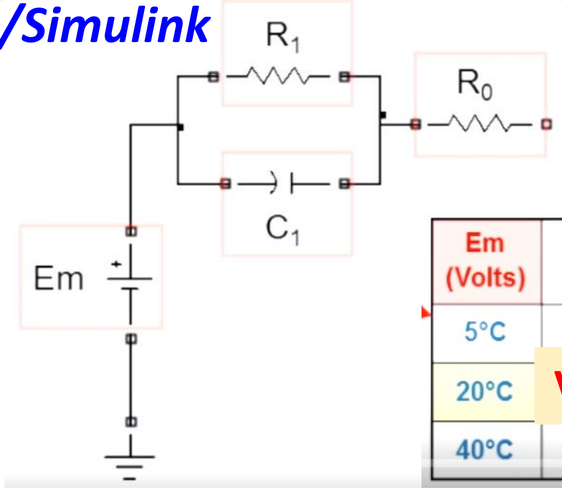




Mousavi S.M. & Nikdel M. (2014), Various Battery Models for Various Simulation Studies and Applications

Battery Models: (Integer) Equivalent Circuit

Matlab/Simulink

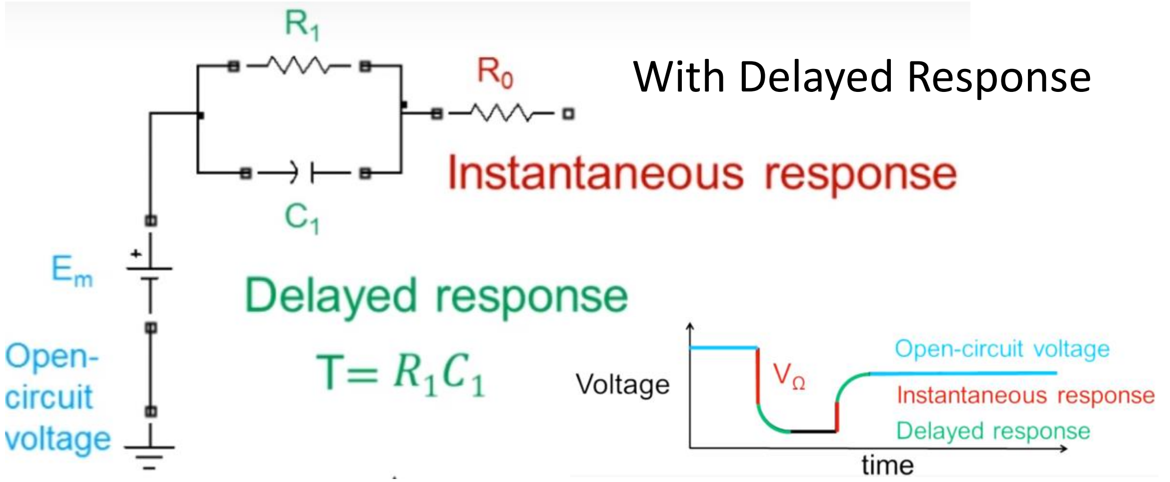


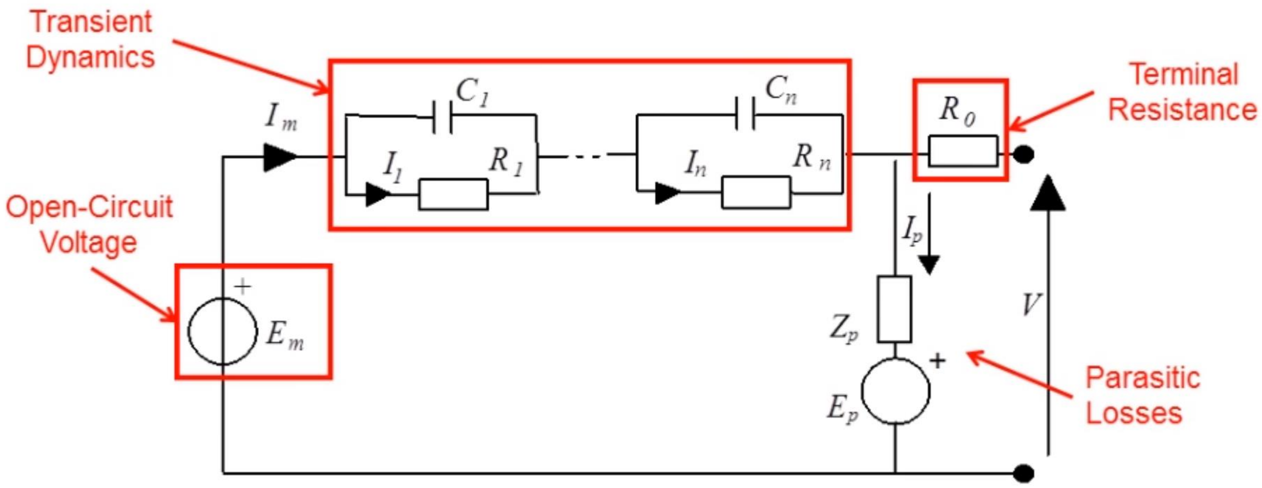
Simple RC Network

Em (Volts)	SOC 1	SOC 0.9	SOC 0.8	…	SOC 0
5°C					
20°C					
40°C					

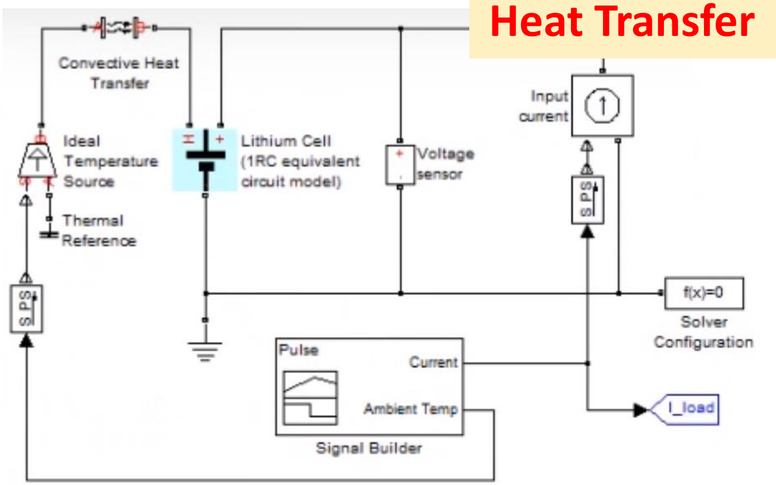
Variable Em with SOC/T

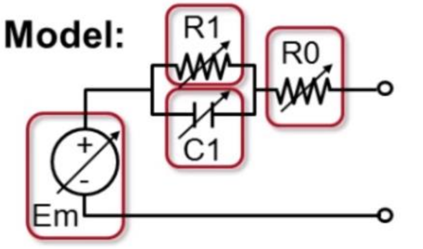
With Delayed Response





Heat Transfer



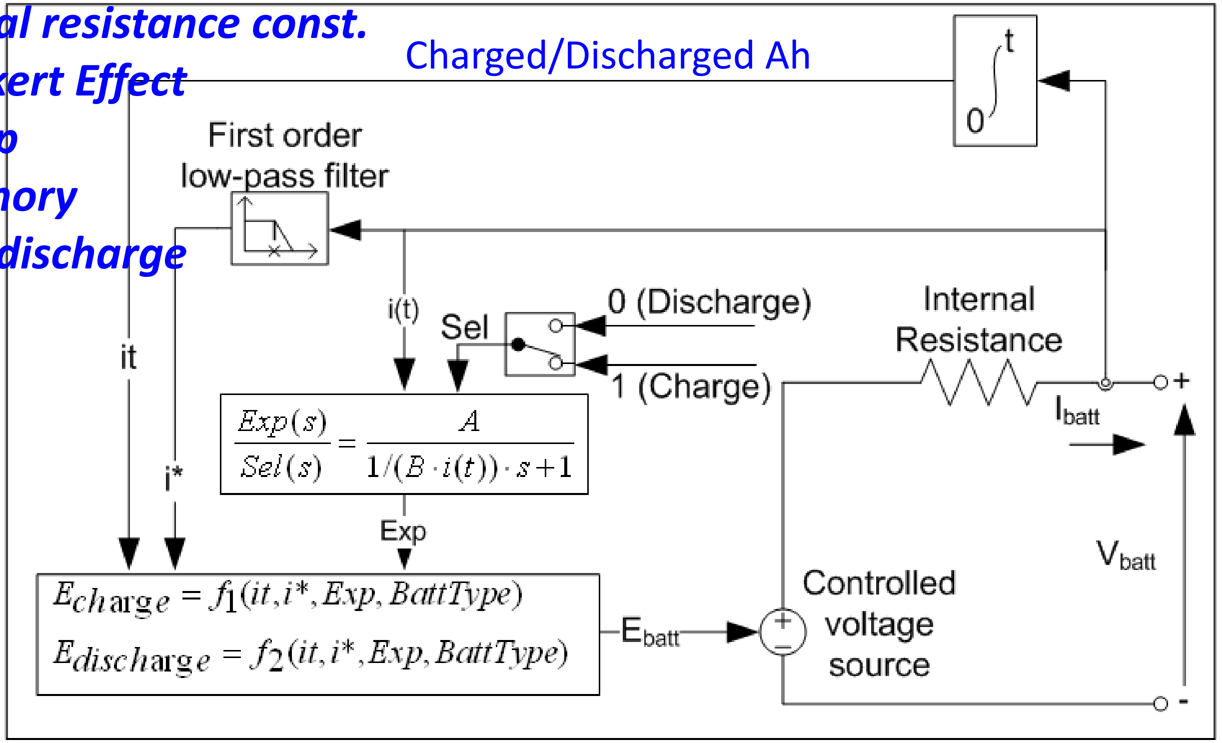


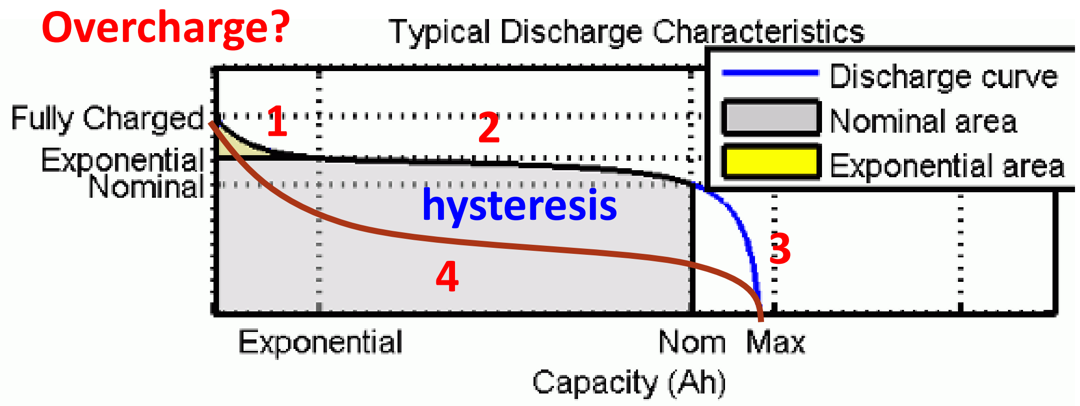
Behavior varies with SOC, temperature, etc.

Battery Models: (Integer) Equivalent Circuit

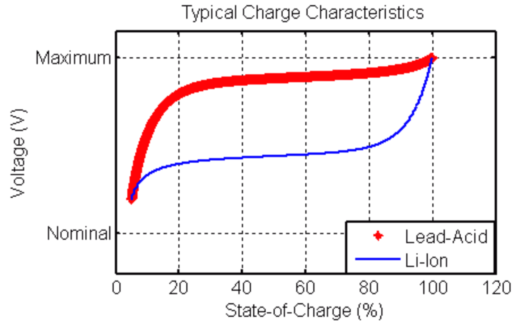
Assumption:

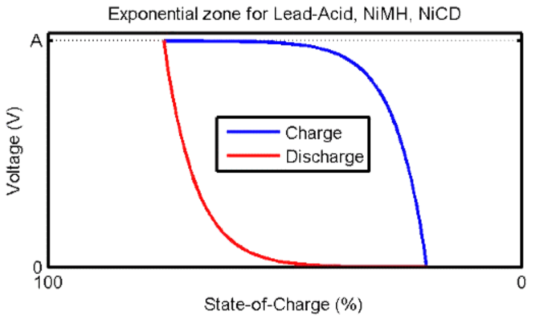
- ***Internal resistance const.***
- ***X Peukert Effect***
- ***X Temp***
- ***X memory***
- ***X self-discharge***





1. Exponential Voltage Drop when the battery is charged
2. Charge can be extracted from the battery until voltage drop below nominal voltage
3. Total discharge of battery, when voltage drops rapidly
4. Battery current negative, battery recharges.





For Li-ion Cell:

Discharge Model (i* >0): $f_1 = E_0 - k\frac{Q}{Q-it}i^* - K\frac{Q}{Q-it}it + Ae^{-B\,it}$

Charge Model (i*<0): $f_2 = E_0 - K\frac{Q}{it+0.1Q}i^* - K\frac{Q}{Q-it}it + Ae^{-B\,it}$

Battery Models: (Integer) Equivalent Circuit

For Li-ion Battery: (With **Temperature Effect**)

Discharge Model (i*>0)

$$f_1 = E_0(T) - K(T)\frac{Q(T_a)}{Q(T_a)-it}\,(i^* + it) + Ae^{-B\,it} - C\,it$$

$$V_{batt}(T) = f_1 - R(T)i$$

Charge Model (i*<0)

$$f_2 = E_0(T) - K(T)\frac{Q(T_a)}{it+0.1Q(T_a)}\,i^* - K(T)\frac{Q(T_a)}{Q(T_a)-it}it + \,Ae^{-B\,it} - C\,it$$

$$V_{batt}(T) = f_2 - R(T)i$$

with

$$E_0(T) = E_0|_{T_{ref}} + \frac{\partial E}{\partial T}(T - T_{ref})$$

$$K(T) = K|_{T_{ref}}e^{\alpha\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)}$$

$$Q(T_a) = Q|_{T_a} + \frac{\partial Q}{\partial T}(T_a - T_{ref})$$

$$R(T) = R|_{T_{ref}}e^{\beta\left(\frac{1}{T}-\frac{1}{T_{ref}}\right)}$$

under cell temperature, T, at time, t

$$T(t) = L^{-1}\left\{\frac{P_{loss}R_{th} + T_a}{1 + st_c}\right\}$$

$$P_{loss} = \big(E_0(T) - V_{batt}(T)\big)i + \frac{\partial E}{\partial T}iT$$

where

T_{ref} = nominal ambient temperature

T = cell temperature

T_a = ambient temperature

E/T= reversible voltage temperature coefficient

α = Arrhenius rate constant for polarization resistance

β = Arrhenius rate constant for internal resistance

$\partial Q/\partial T$ = maximum capacity temp coefficient

C = nominal discharge curve slope in V/ Ah

t_c = thermal time constant

Obtained in: https://www.mathworks.com/help/physmod/sps/powersys/ref/battery.html

Battery Models: (Integer) Equivalent Circuit

Cyclic **Ageing Effect** for Li-ion Cell: with n = kT_h

Capacity: $Q(n) = \begin{cases} Q_{BOL} - \varepsilon(n)(Q_{BOL} - Q_{EOL}) & if\ k/2 \neq 0 \\ Q(n-1) & otherwise \end{cases}$

Internal Resistance: R$(n) = \begin{cases} R_{BOL} - \varepsilon(n)(R_{EOL} - R_{BOL}) & if\ k/2 \neq 0 \\ R(n-1) & otherwise \end{cases}$

where
T_h = half-cycle duration in s
Q_{BOL} = battery max capacity in Ah, at beginning
Q_{EOL} = battery max capacity in Ah, at end of life
R_{BOL} = Internal resistance at BOL, nominal T
R_{EOL} = Internal resistance at EOL, nominal T
ε = battery aging factor (=0 at BOL, =1 at EOL)

Battery Aging Factor: $\varepsilon(n) = \begin{cases} \varepsilon(n-1) + \frac{0.5}{N(n-1)}\left(2 - \frac{DOD(n-2)+DOD(n)}{DOD(n-1)}\right) & if\ \frac{k}{2} \neq 0 \\ \varepsilon(n-1) & otherwise \end{cases}$

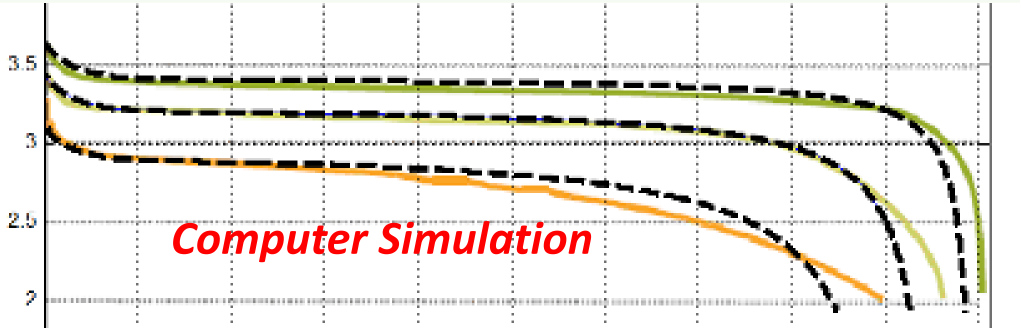
where
DOD = depth of discharge (%) after half-cycle

$$N(n) = H\left(\frac{DOD(n)}{100}\right)^{-\xi} \exp\left(-\Psi\left(\frac{1}{T_{ref}} - \frac{1}{T_a(n)}\right)\right)\left(I_{dis_{ave}}(n)\right)^{-\gamma_1}\left(I_{ch_{ave}}(n)\right)^{-\gamma_2}$$

where: H is cycle number constant, ξ= exponential factor for DOD, Ψ= Arrhenius rate constant for cycle number,
I_{dis_ave} = average discharge current (A) during half-cycle, I_{ch_ave} = average charge current (A) during half-cycle
γ_1 = exponent factor for the discharge current and γ_2 = exponent factor for the charge current.

Battery Models: (Integer) Equivalent Circuit

Assumption:
- Internal Resistance = constant
- Parameter for charge = that of discharge
- Capacity does not change with current (X Peukert Effect)
- No Self-Discharge / Memory Effect



Extracting Information from Data Sheet:

Rated Capacity

Thermal Resistance

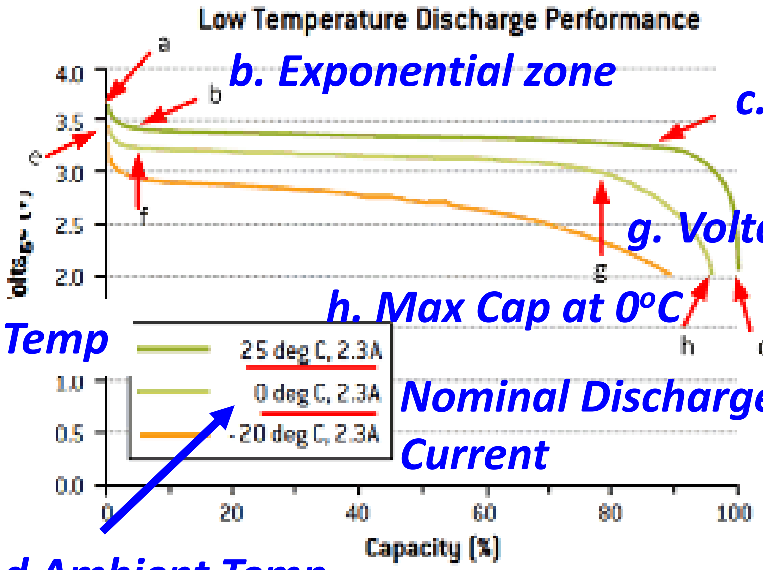
Specifications

Nominal capacity and voltage	2.3 Ah, 3.3 V
Internal impedance (1kHz AC)	8 mΩ typical
Internal resistance (10A, 1s DC)	10 mΩ typical
Recommended standard charge method	
Recommended fast charge current	
Maximum continuous discharge	
Pulse discharge at 10 sec	120A
Cycle life at 10C discharge, 100% DOD	
Recommended pulse charge/discharge cutoff	
Operating temperature range	-30°C to +60°C
Storage temperature range	-50°C to +60°C
Core cell weight	70 grams

e. Initial discharge voltage

Nominal Ambient Temp

a. Fully charged voltage



b. Exponential zone

c. Nominal Voltage

g. Voltage at 90% max cap

h. Max Cap at 0°C

d. Max Capacity

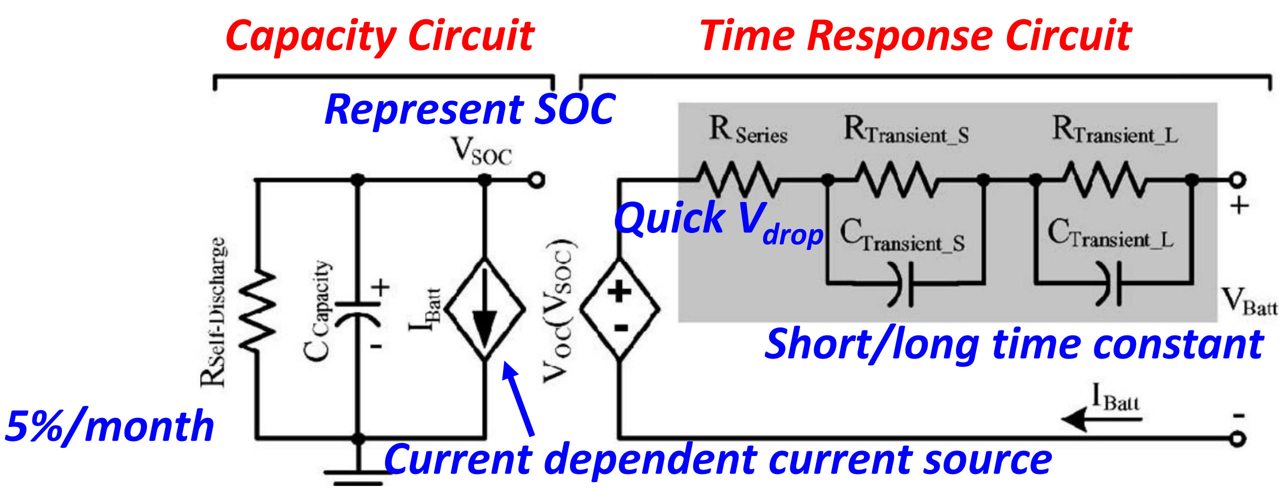
Nominal Discharge Current

Second Ambient Temp

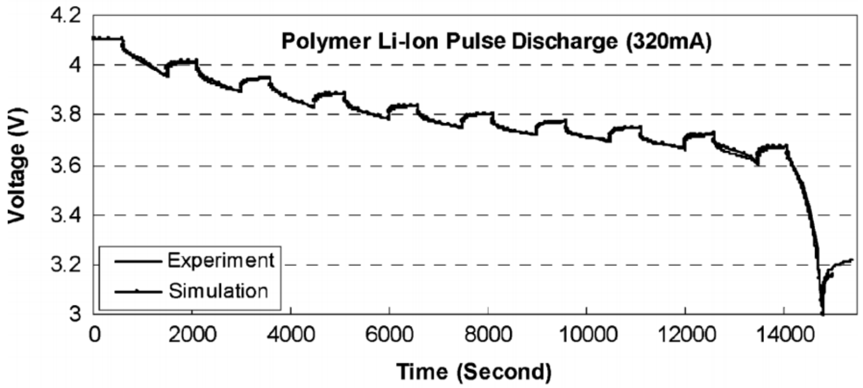
Chen M & Rincon-Mora G.A. (2006) Accurate Electrical Battery Model Capable of Predicting Runtime and I-V Performance

Battery Models: (Integer) Equivalent Circuit

MapleSim Equivalent Circuit Model



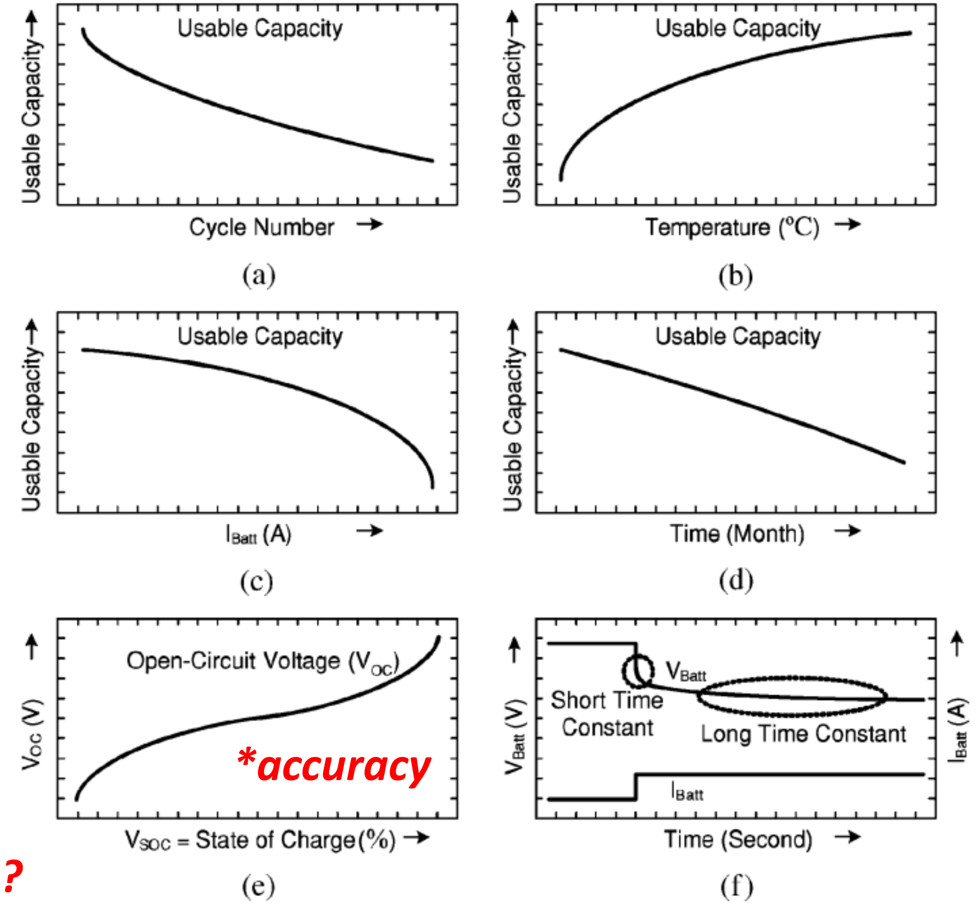
Parameters are temp, SOC, current and cycle dependent!



Max Error Voltage (mV)	Runtime Error * (%)
15	0.039%
17	0.118%
18	0.020%
21	0.029%

What will be the runtime?

Usable Cap ↓ when cycle number, discharge current, storage time ↑ and temperature ↓



Battery Models: (Fractional) Equivalent Circuit

- **Fractional Calculus** suits for infinite dynamic processes, such as movement of charge carriers through electrolytes, charge-transfer reactions, solid-phase diffusion, to stray storage and cyclic operation and ageing process. It takes into account the entire past trajectory, with a long memory property.

Impedance of Capacitance: $Z_{CPE} = \frac{1}{C_\alpha (j\omega)^\alpha}$ for $\alpha \in (0,1)$ CPE = constant phase element, C_α= pseudo cap

Fractional Order Derivative of the CPE is given as:

$$ {}_0D_t^\alpha = \frac{d^\alpha(.)}{d(.)^\alpha} = \lim_{T \to 0} \frac{1}{T^\alpha} \sum_{k=0}^{t/T} (-1)^k \langle \alpha, k \rangle f(t - kT) $$

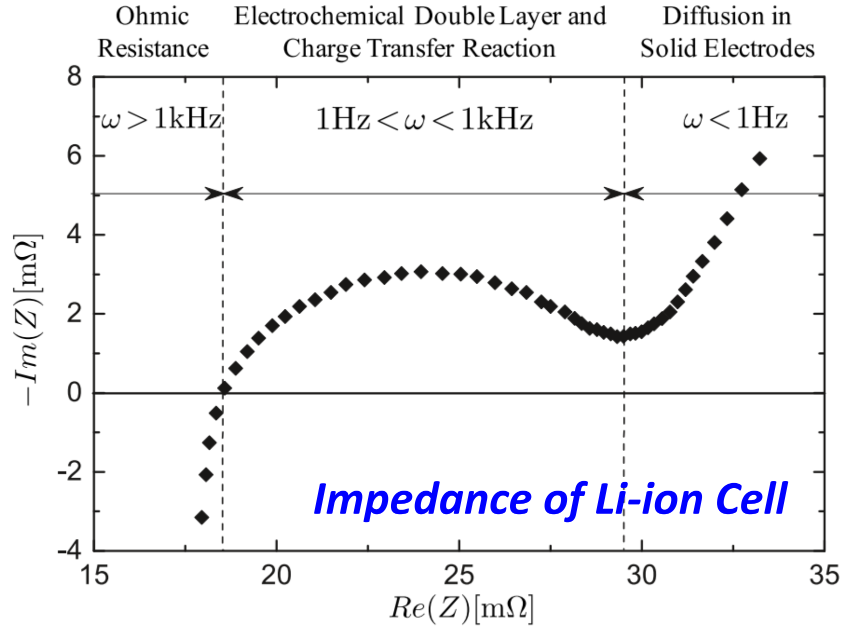
for Newton-Binomial coefficient:

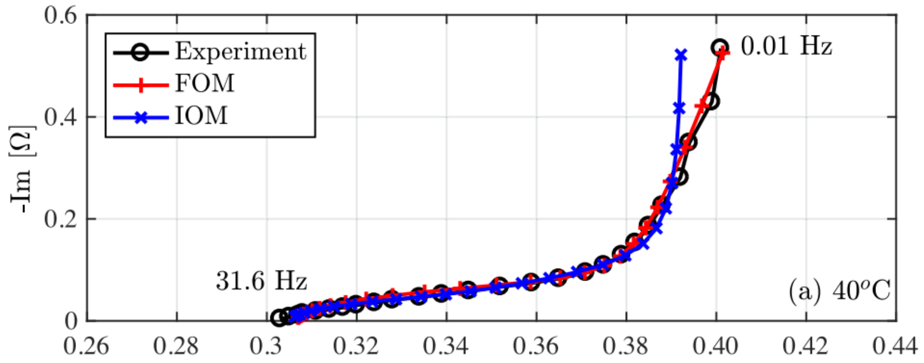
$$ \langle \alpha, k \rangle = \frac{\Gamma(\alpha + 1)}{\Gamma(k+1)\Gamma(\alpha - k + 1)} $$

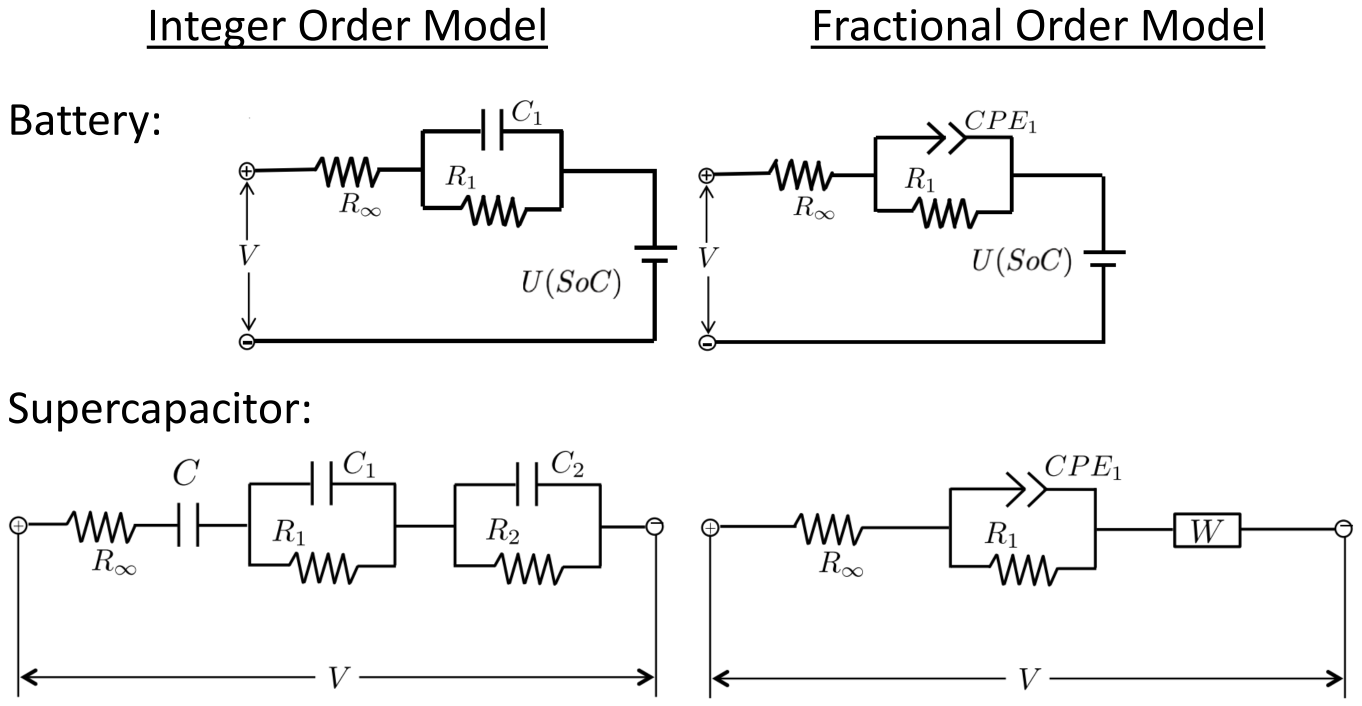
and Gamma Function:

$$ \Gamma(\alpha) = \int_0^\infty \xi^{\alpha - 1} e^{-\xi} d\xi $$

Battery Models: (Fractional) Equivalent Circuit







Newman J. & Tiedemann W. (1975), Porous Electrode Theory with Battery Applications

Battery Model: Electrochemical Modelling

Porous Electrode Theory

Governing Equations

Diffusive Flow in Liquid Phase

$$\varepsilon_i \frac{\partial c}{\partial t} = \nabla \cdot D_{eff,i} \nabla c + a_p (1 - t_+) j_i$$

KCL + Ohm's Law

$$-\nabla\cdot[\sigma_{eff}\nabla\Phi_1]-\nabla\cdot[\kappa_{eff}\nabla\Phi_2]+\nabla\cdot\left[\frac{2\kappa_{eff}RT}{F}(1-t_+)\nabla \ln c\right]=0$$

$$\sigma_{eff,i}\nabla^2\Phi_1 = a_i F j_i$$

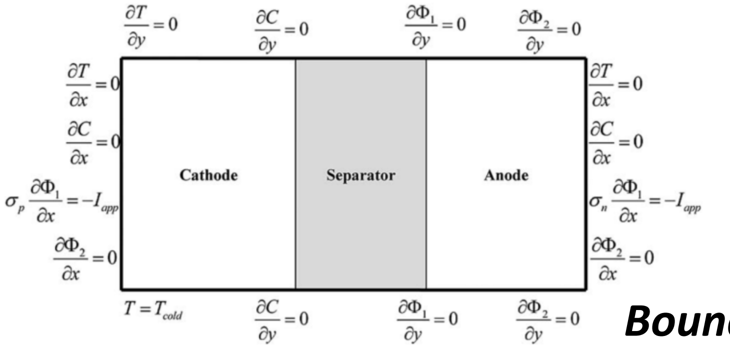
Charge Conservation

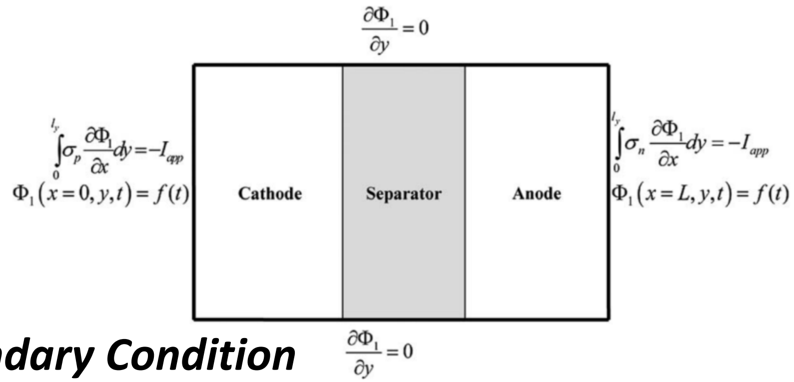
$$\frac{\partial c_p^s}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 D_p^s \frac{\partial c_p^s}{\partial r}\right]$$

Diffusive Flow in Solid State

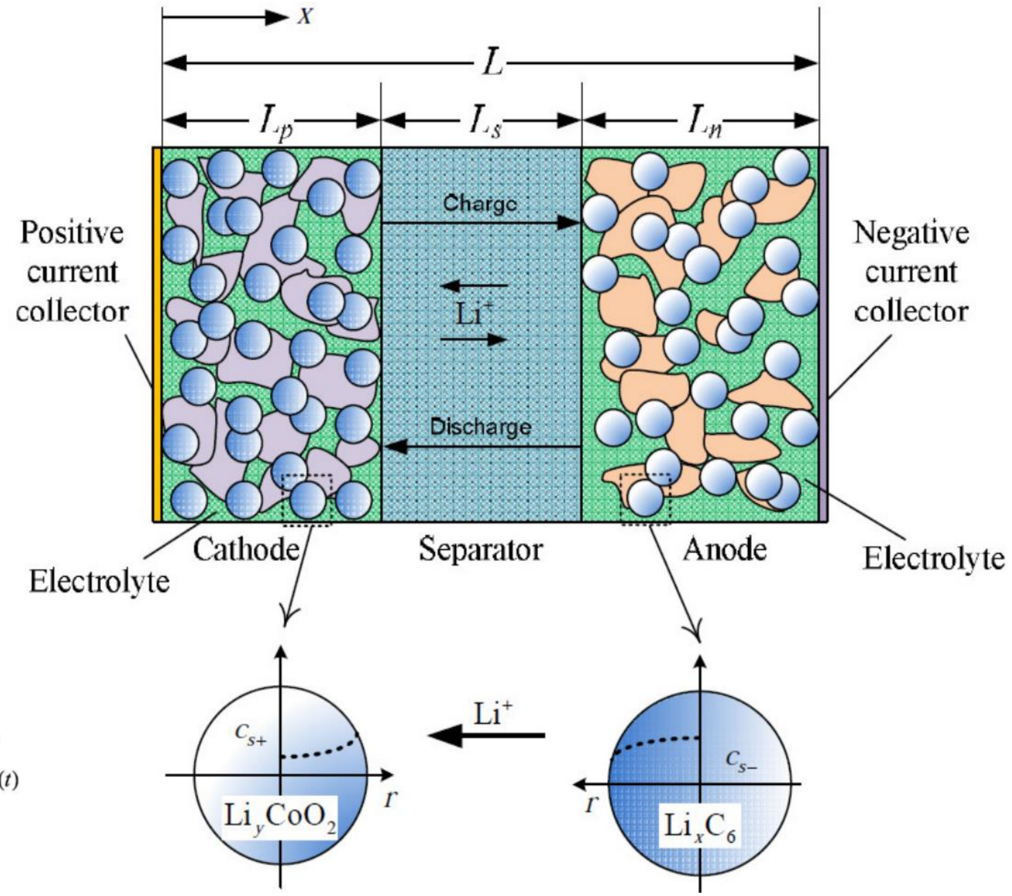
$$\rho_i C_{p,i}\frac{dT_p}{dt} = \nabla\cdot\lambda_i \nabla T + Q$$

Heat Transfer





Boundary Condition



Anatomy of a lithium ion cell

Battery Model: Electrochemical Modelling

- Single Particle Model

(Current density, conc., over-potential)

Butler-Volmer Equation: $j = k\left(c_{s,max} - c_{s,surf}\right)^{0.5}\left(c_{s,surf}\right)^{0.5} c_e^{0.5} \sinh\left(\frac{0.5F\mu}{RT}\right)$

$$V(t) = (U_p(t) - U_n(t)) - (\phi_e(L_c, t) - \phi_e(0, t)) - \left(\eta_p(t) - \eta_n(t)\right)$$
$$- (R_r(t) + R_c)I(t) - V_h(t) \quad \text{(Electrical)}$$

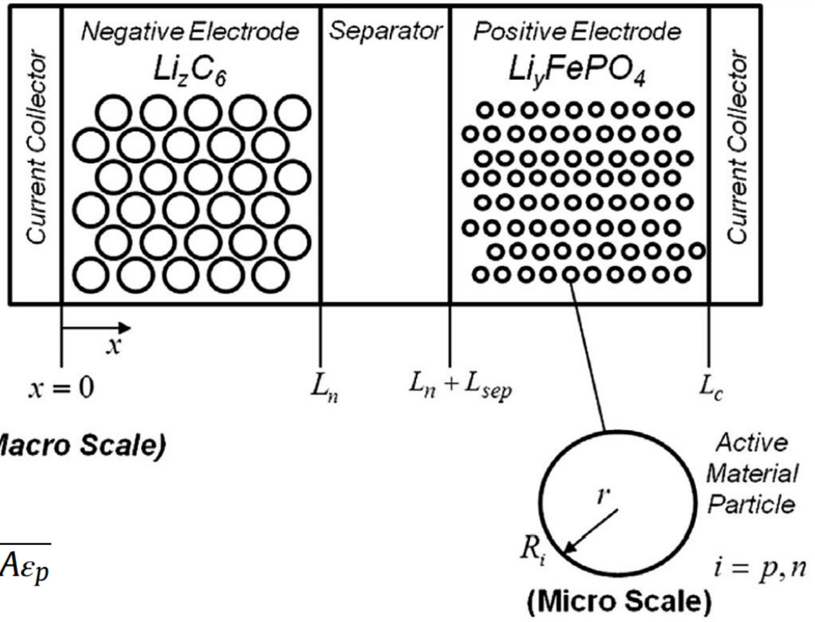
$$\frac{dV_h}{dt} = |I|\tau_h(T)(H(T, SOC) - V_h) \quad \text{(Hysteresis)}$$

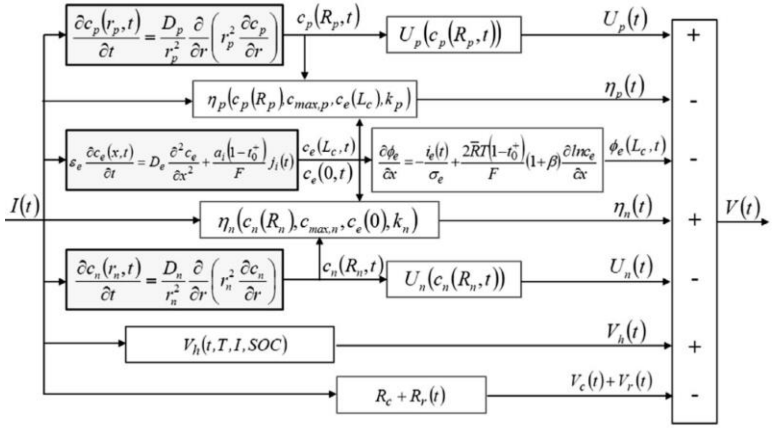
$$R_r(t) = \frac{R_p L_{cond}(t)}{3L_p A \varepsilon_p}\frac{1}{\sigma_p} \quad \text{(Increasing R with depth of discharge)} \quad \frac{dL_{cond}}{dt} = \frac{\pi R_p |I|}{F c_{max,p} L_p A \varepsilon_p}$$

$$\frac{\partial c_i}{\partial t} = \frac{D_i}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial c_i}{\partial r}\right) \quad \text{Mass diffusion equation (overpotential)}$$

$$\varepsilon_e \frac{\partial c_e}{\partial t} = D_e \frac{\partial^2 c_e}{\partial x^2} + \frac{a_i\left(1 - t_0^+\right)}{F} j_i(t) \quad \text{(Ohmic Transport in Liquid state)}$$

$$\frac{\partial \phi_e}{\partial x} = -\frac{i_e(t)}{\sigma_e} + \frac{2\bar{R}T\left(1 - t_0^+\right)}{F}(1 + \beta)\frac{\partial \ln(c_e)}{\partial x} \quad \text{(potential-concentration relation)}$$





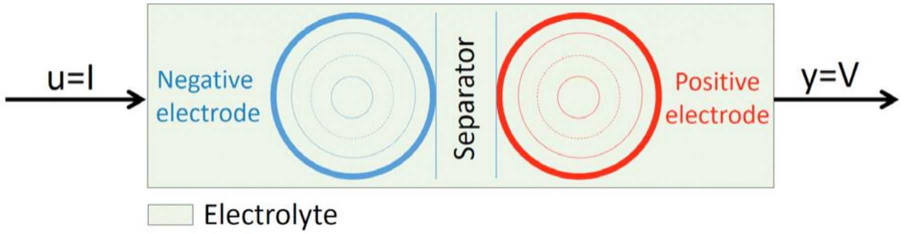
Marcicki, J. et al. (2013) Design and parametrization analysis of a reduced-order electrochemical model of graphite/LiFePO4 cells for SOC/SOH estimation

Klein R. et al. (2013), Electrochemical model based observer design for a lithium-ion battery

Battery Model: Electrochemical Modelling

- Observer Design

Let x be a vector of concentration and u be a vector of current I.



$$\frac{dc_n}{dt} = \frac{S_{n-1}}{r_n - r_{n-1}} \frac{D_s}{V_n} c_{n-1} - \left(\frac{S_{n-1}}{r_n - r_{n-1}} + \frac{S_n}{r_{n+1} - r_n} \right) \frac{D_s}{V_n} c_n + \frac{S_n}{r_{n+1} - r_n} \frac{D_s}{V_n} c_{n+1},$$

$$\frac{dc_{N_s}}{dt} = \frac{S_{N_s-1}}{r_{N_s} - r_{N_s-1}} \frac{D_s}{V_{N_s}} (c_{N_s-1} - c_{N_s}) + K_I^s I.$$

$$c_1^{neg} = K + \sum_{i=2}^{N_{neg}} \beta_i^{neg} c_i^{N_{neg}} + \sum_{i=1}^{N_{pos}} \beta_i^{pos} c_i^{N_{pos}}$$

$$K := \frac{Q}{\alpha_{neg} V_1^{neg}}$$

$$\beta_i^{neg} := -\frac{V_i^{neg}}{\alpha_{neg} V_1^{neg}} \qquad \beta_i^{pos} := -\frac{\alpha_{pos} V_i^{pos}}{\alpha_{neg} V_1^{neg}}.$$

(Voc) $y := OCV_{pos}(c_{pos}^{surf}) - OCV_{neg}(c_{neg}^{surf}) + g(u)$

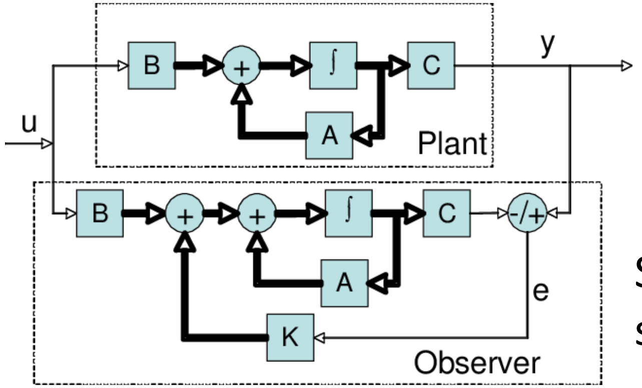
g(u) is current dependent

State Equation: $\begin{cases} \dot{x} = Ax + Bu + K + Ew \\ y = h(x) + g(u) + Dz, \end{cases}$

Ew is perturbation
Dz is measurement noise

State Observer: $\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + K + L(y - \hat{y}) \\ \hat{y} = h(\hat{x}) + g(u), \end{cases}$

Estimation error: $\dot{e} = Ae + Ew - L(h(x) - h(\hat{x})) - LDz.$



State space analysis and state observer

Bizeway, A. et al. (2015), Lithium-ion battery thermal-electrochemical model-based state estimation using orthogonal collocation and a modified extended Kalman filter

Battery Model: Electrochemical Modelling

- Kalman Filter

State-Space Eqt: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{z}, u)$ $\quad \mathbf{x} = [\bar{\mathbf{c}}_s, \mathbf{c}_e, T]^T$

$0 = \mathbf{g}(\mathbf{x}, \mathbf{z}, u)$ $\quad \mathbf{z} = \left[\mathbf{j}^{Li}, \phi^0_{s,c}, \phi^0_{s,a}\right]^T$

c_s = solid-phase conc

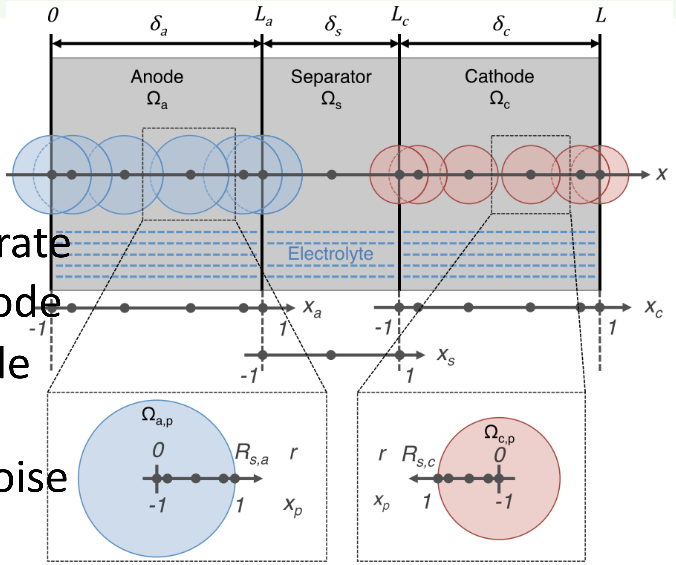
c_e = electrolyte conc

T = bulk temperature

j = volumetric reaction rate

$\phi^0_{s,c}$ = potential at cathode

$\phi^0_{s,a}$ = potential at anode



y= [V T]

V = voltage

T = temp

$$\mathbf{y} = [H_x \quad H_z] \begin{bmatrix} \mathbf{x} \\ \mathbf{z} \end{bmatrix} + H_u u$$

H_x, H_z, H_u is derived from electrochemical eqts

For a linear State-Space: $\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + \mathbf{w}(t)$

$$\mathbf{y}_k = C\mathbf{x}_k + \mathbf{v}_k$$

w(t)= white Gaussian Noise with covariance Q

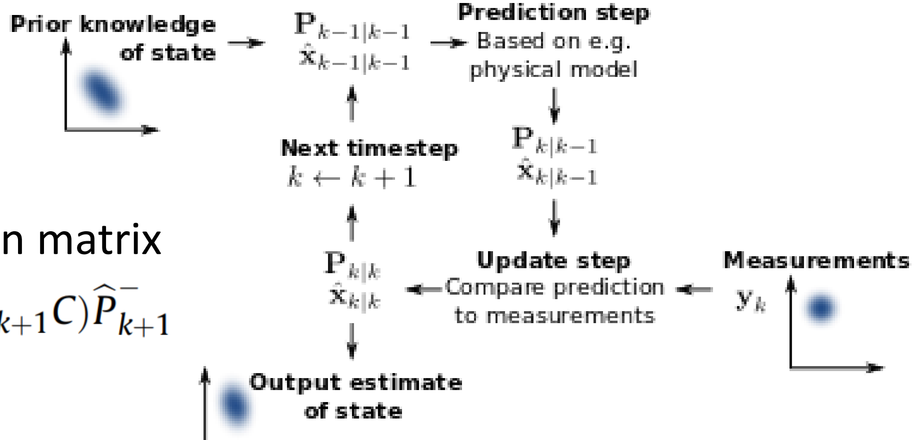
$\mathbf{v}_k$ = measurement noise with covariance R

Time Update: $\widehat{\mathbf{x}}^-_{k+1} = \Phi\widehat{\mathbf{x}}_k + \int_{t_k}^{t_{k+1}} \Phi B\mathbf{u}(\tau)d\tau$

Measurement Update: $\widehat{P}^-_{k+1} = \Phi\widehat{P}_k + \widehat{P}_k\Phi^T + Q$ $\quad$ $\boldsymbol{\Phi}$ = state transition matrix

$$K_{k+1} = \widehat{P}^-_{k+1}C^T\left(C\widehat{P}^-_{k+1}C^T + R\right)^{-1} \qquad \widehat{P}_{k+1} = (I - K_{k+1}C)\widehat{P}^-_{k+1}$$

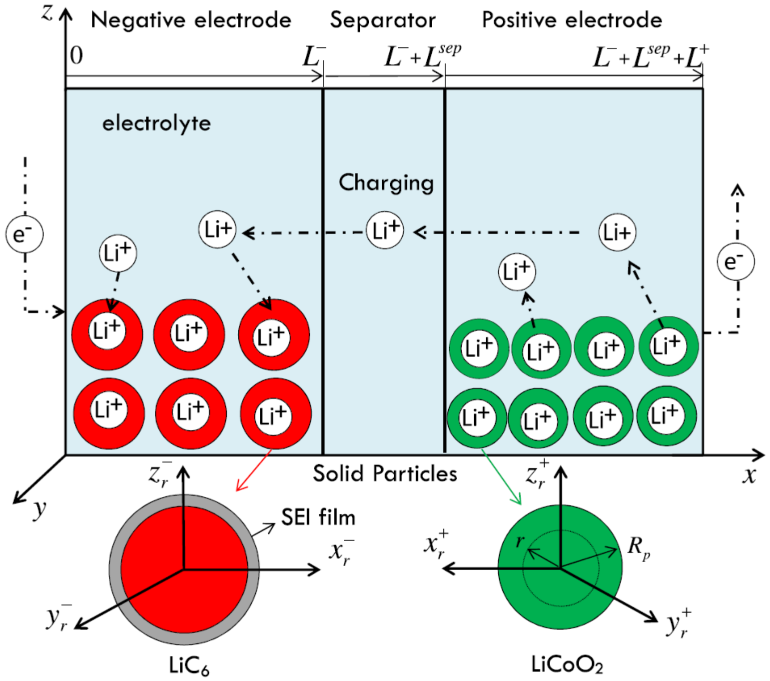
$$\widehat{\mathbf{x}}_{k+1} = \widehat{\mathbf{x}}^-_{k+1} + K_{k+1}\left(\mathbf{y}_{k+1} - \widehat{\mathbf{y}}^-_{k+1}\right)$$

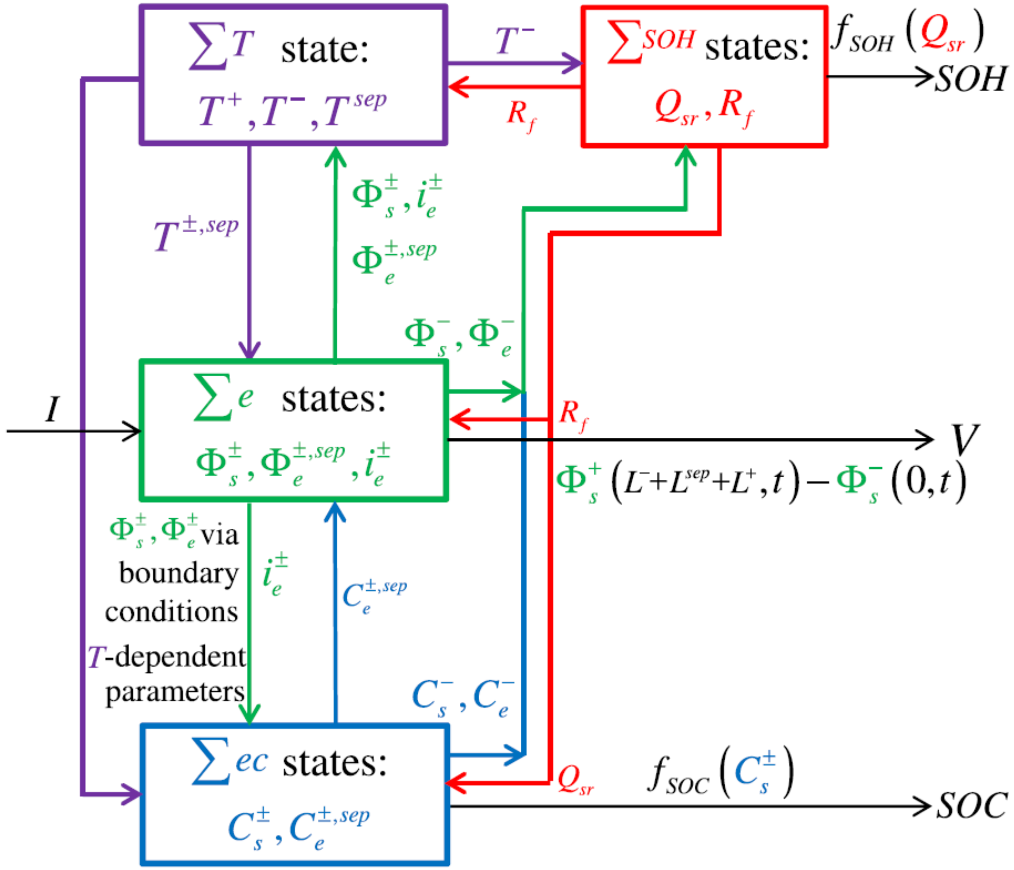


State time update -> linearization -> error time update -> linearization -> measurement update

Battery Model: Electrochemical Modelling

- 3D Model





Transformation to Hilbert to obtain infinite D and easier separate solution for short time and long time.

State estimation is needed for charging strategy evaluation, residual capacity, lifetime estimation, fast charging control…

Battery Model: Electrochemical Modelling

Latest Development:

- Extended Single Particle Model (ESPM)
- Multiple Particle Model (MP)
- Pade Approximation Method
- Orthogonal Decomposition

Detailed model is needed for:

- Cycling
- Capacity Fade
- Hysteresis
- Memory Effect
- Recovery Effect
- Endothermic Effect

* Stochastic Model can only tackle a specific characteristics

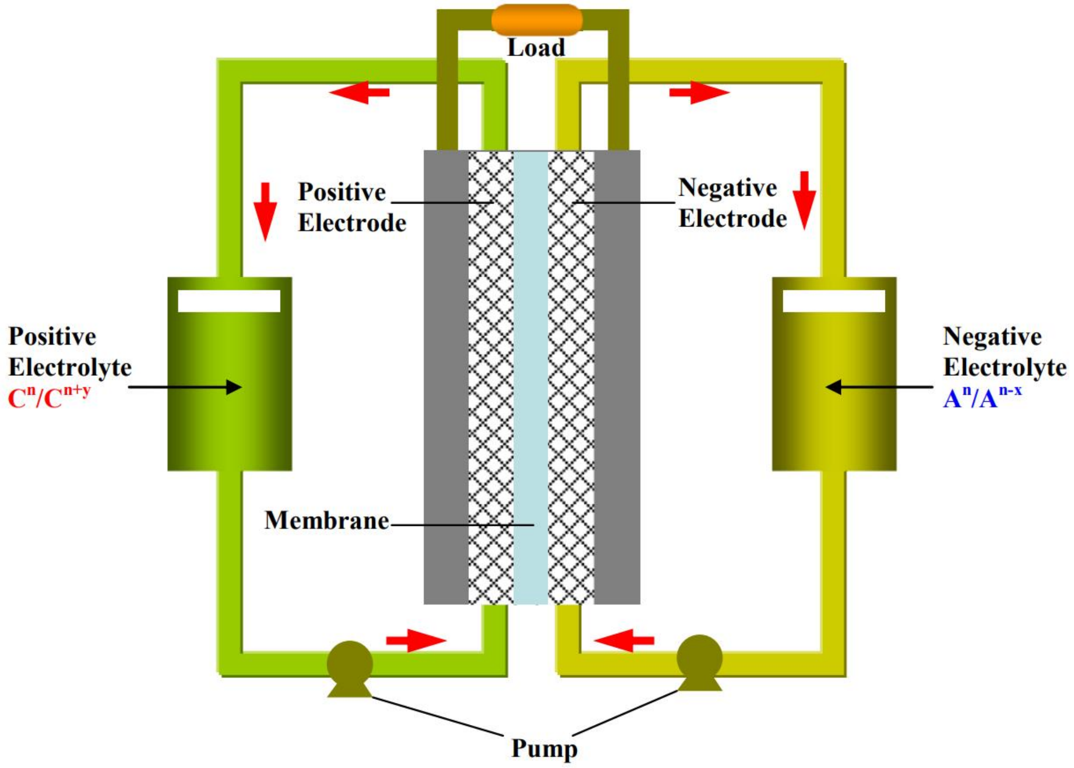
Further studies:

1. Rank the **most influential parameters** in Li-ion cell
2. Higher-order polynomial / non-polynomial **profile**
3. **Coupled Electrochemical and Thermal** Equations for real-time
4. Develop simplified P2D model with **aging** phenomena
5. Real time model for internal properties Li-ion cell based on **inverse heat transfer methods**
6. Simplified model compatible with **$LiFePO_4$ cathode** materials (different batteries, different structures, different equations)
7. **Multiple Cells** with series and parallel connection studies
8. **Bifurcation studies** and **fractional calculus method** for memory effect with **simplificatio**n in solving are needed
9. Real time transient model with fluctuating load is lacked

Mohammed YS et al. (2014), Hybrid renewable energy systems for off-grid electric power: Review of substantial issues
Suresh, R. et al. (2016), Modeling of Rechargeable Batteries
Jokar A. et al. (2016), Review of simplified P2D models of lithium ion batteries

Battery Model: Electrochemical Modelling

- Flow Battery



Vanadium Redox Flow Battery (VRFB):

$$\text{Positive electrode}: \quad VO^{2+} + H_2O - e^- \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} VO_2^+ + 2H^+$$

$$\text{Negative electrode}: \quad V^{3+} + e^- \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} V^{2+}$$

$$\text{Overall reaction}: \quad VO^{2+} + V^{3+} + H_2O \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} VO_2^+ + V^{2+} + 2H^+$$

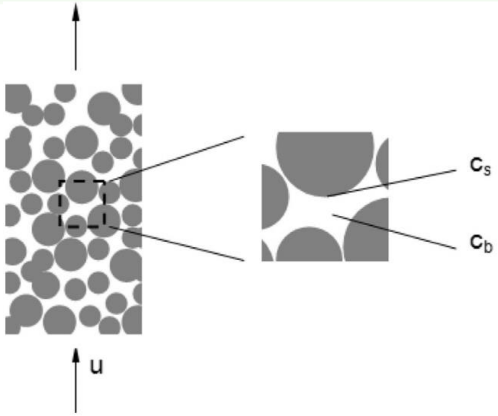
Zinc/Cerium Redox Flow Cell (Zn/Ce FB)

$$\text{Positive electrode}: \quad Ce^{3+} - e^- \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} Ce^{4+}$$

$$\text{Negative electrode}: \quad Zn^{2+} + 2e^- \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} Zn$$

$$\text{Overall reaction}: \quad Zn^{2+} + 2Ce^{3+} \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} Zn + 2Ce^{4+}$$

Battery Model: Electrochemical Modelling



$$\frac{\mu}{K}\vec{u} = -\nabla p \quad \text{(Darcy's Law)}$$

$$\nabla \cdot \vec{u} = 0 \quad \text{(Mass Balance)}$$

$$p_c = p_g - p_l = \sigma \cos \theta_c (\varepsilon / K)^{0.5} J(s) \quad \text{(Capillary Pressure)}$$

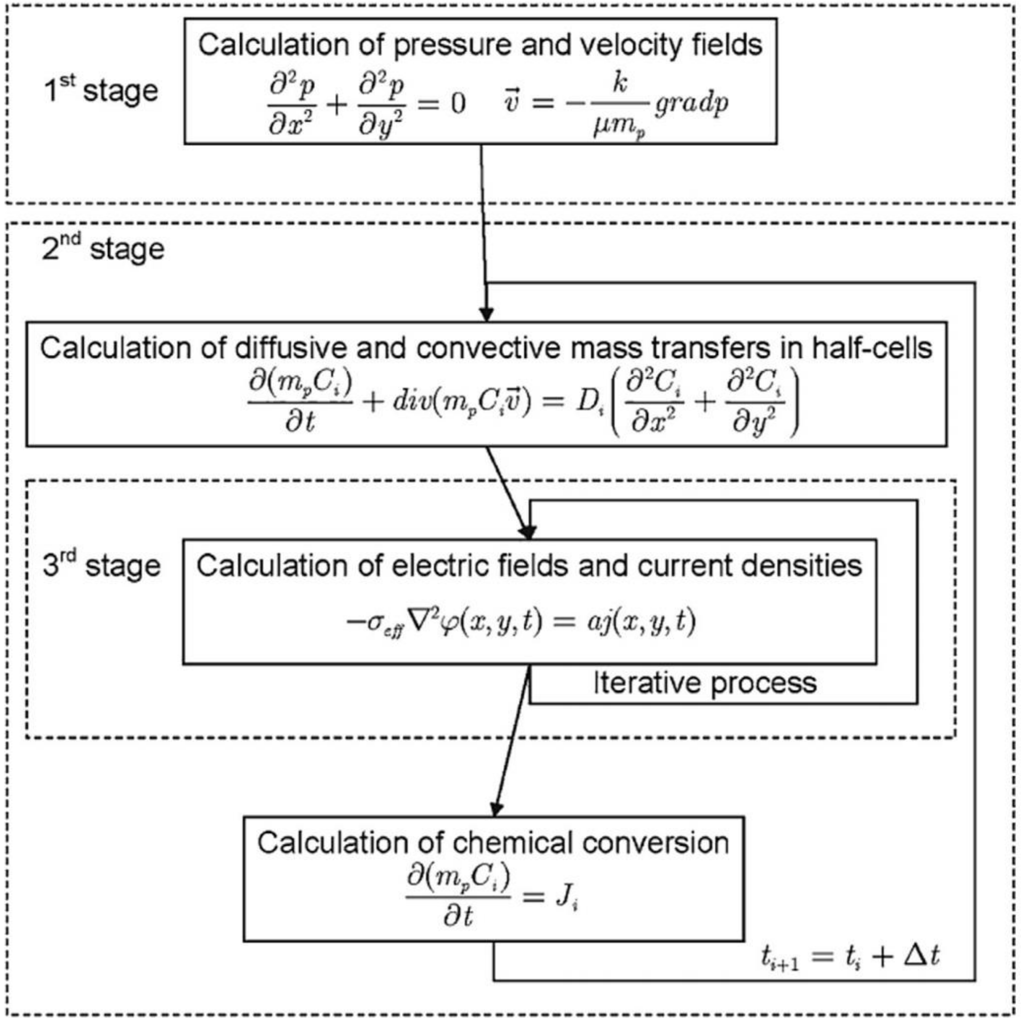
$$\vec{u}_{slip} = \frac{d_b^2}{18\mu_l}\nabla p \quad \text{(Slip Velocity)}$$

(Mass Transport) $\frac{\partial c_i}{\partial t} = \nabla \cdot \left(D_i^{eff} \nabla c_i\right) + \frac{F z_i}{RT}\nabla \cdot \left(D_i^{eff} c_i \nabla \phi_s\right) - \nabla \cdot (\vec{u} c_i) + S_i$

(Energy Transport) $\frac{\partial}{\partial t}\left(\overline{\rho C_p T}\right) + \nabla \cdot \left(\vec{u}\,\overline{\rho C_p T}\right) = \bar{\lambda}\nabla^2 T + \sum_k Q_k$

$$j_1 = \varepsilon A_v F k_1 \left(c^s_{V(III)}\right)^{\alpha_{-,1}} \left(c^s_{V(II)}\right)^{\alpha_{+,1}} \left\{\exp\left(\frac{\alpha_{+,1} F \eta_1}{RT}\right) - \exp\left(-\frac{\alpha_{-,1} F \eta_1}{RT}\right)\right\}$$

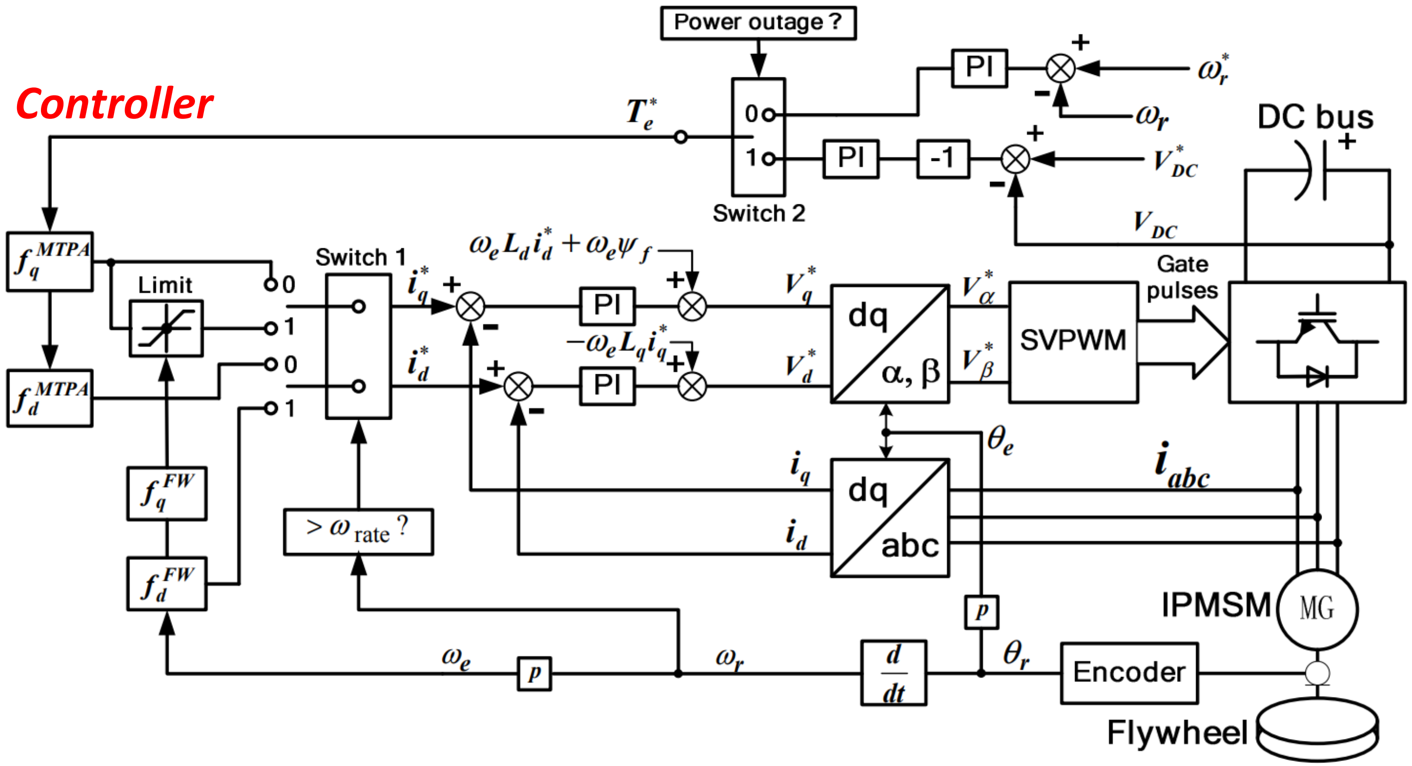
(Butler-Volmer Eqt)

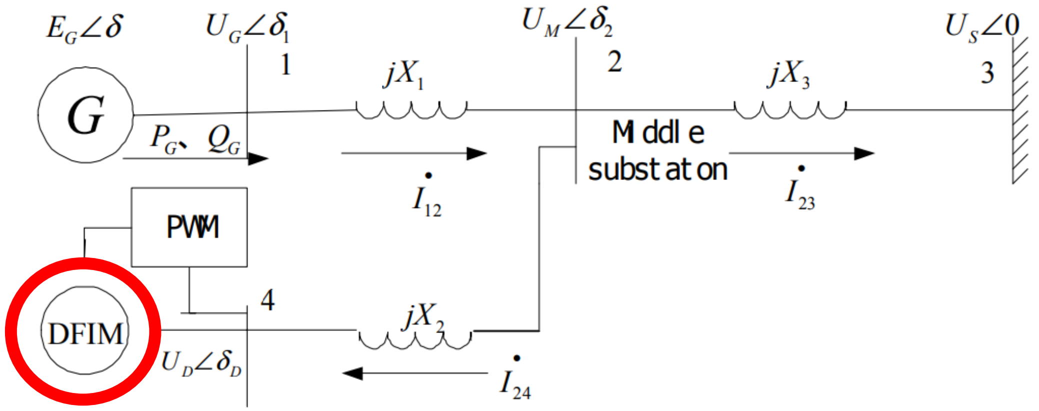


Zhou, L. & Qi, Z. (2009), Modeling and Control of a Flywheel Energy Storage system for Uninterruptible Power Supply

Shi L. et al. (2008), P-H Model of Power Systems Installed with a Flywheel Energy Storage for the Dynamic Analysis

Flywheel Model





Flywheel = Doubly-Fed Induction Motor

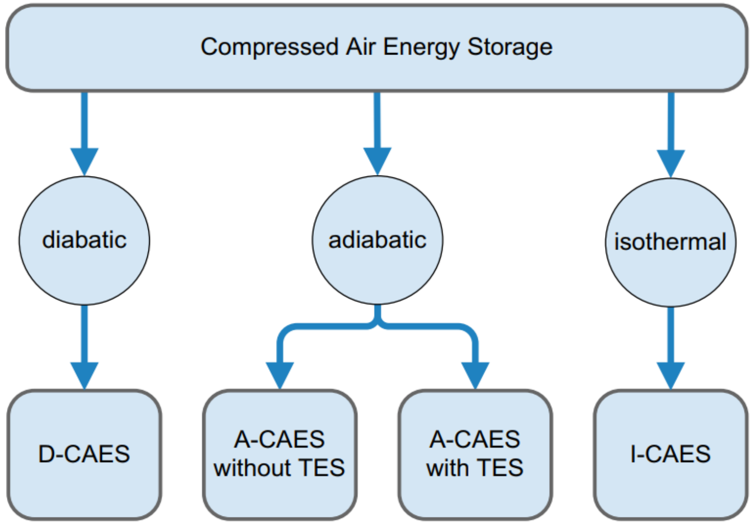
$$U_{ds} = E_d' - X' I_{qs}$$
$$U_{qs} = E_q + X' I_{ds}$$

$$\frac{dE_d'}{dt} = -\frac{\omega_0 x_m}{X_{rr}} U_{qr} - \frac{1}{T_{d0}'} E_d' + s\omega_0 E_q' - \frac{x_{ss} - X'}{T_{d0}'} I_{qs}$$
$$\frac{dE_q'}{dt} = \frac{\omega_0 x_m}{X_{rr}} U_{dr} - \frac{1}{T_{d0}'} E_q' - s\omega_0 E_d' + \frac{x_{ss} - X'}{T_{d0}'} I_{ds}$$
$$\frac{ds}{dt} = \frac{1}{T_j}(-T_e) \quad s = \frac{\omega_0 - \omega_r}{\omega_0}$$

$$E_d' = -\frac{x_m}{X_{rr}} \Psi_{qr} \qquad X_{rr} = x_r + x_m; x_{ss} = x_s + x_m; X' = x_{ss} - \frac{x_m^2}{X_{rr}}$$
$$E_q' = \frac{x_m}{X_{rr}} \Psi_{dr} \qquad T_{d0}' = \frac{X_{rr}}{\omega_0 r_r}; T_e = \Psi_{qr} I_{dr} - \Psi_{dr} I_{qr} = E_d' I_{ds} + E_q' I_{qs}$$

Compressible Air Energy Storage Model

	Diabatic		Adiabatic		Isothermal	
Cycle efficiency (AC to AC)	Today 0.54	Goal 0.6	Today –	Goal 0.7	Today 0.38	Goal 0.8
Energy density (per m^3 of CAS)	2–15 kW h/m^3		0.5–20 kW h/m^3		1–25 kW h/m^3	
Start-up time	10–15 min		5–15 min		<1 min	
Power range	5 MW–1 GW		1 MW–1 GW		5 kW–1 GW	
Development status	Application/Demonstration		Research/Demonstration		Research/Demonstration	



How heat is handled during compression and prior to expansion of the air?

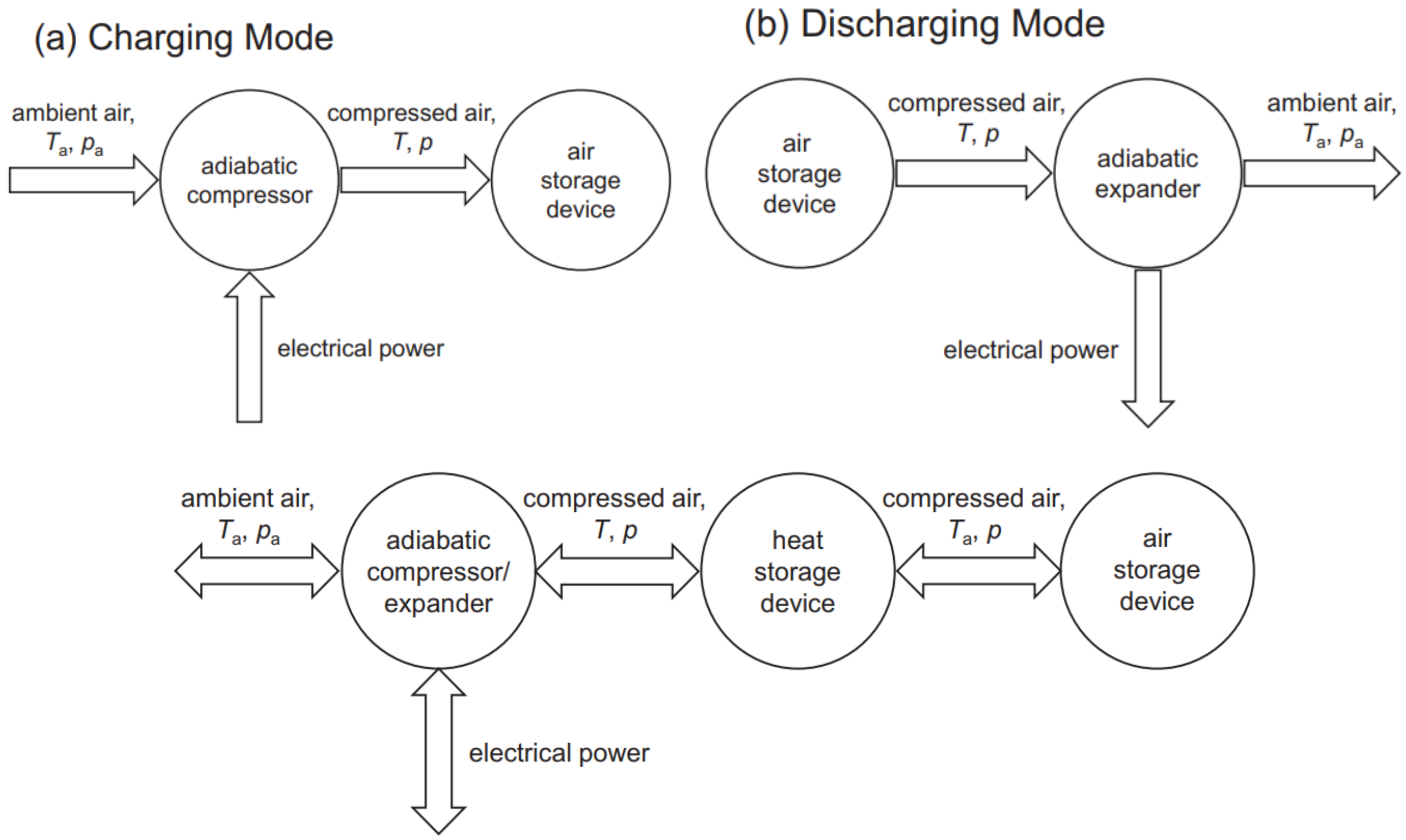
D-CAES: heat resulting from air compression is wasted to ambient, external heat source is needed to prevent condensation during expansion process

A-CAES: heat of compression is captured in additional TES devices and is utilized for expansion

I-CAES: heat of compression is minimized and prevented in I-CAES

Compressible Air Energy Storage Model

For A-CAES:



First Law: $P_{el} = \dot{m}(h(T,p) - h(T_a,p_a))$

Assuming adiabatic and relatively const. c_P

$$h^0(T,p) - h^0(T_a,p_a) = c_p^0(T - T_a)$$

$$s^0(T,p) - s^0(T_a,p_a) = c_p^0 \ln\left(\frac{T}{T_a}\right) - R_L \ln\left(\frac{P}{P_a}\right)$$

Exergy: P_{el}

$$= \dot{m}\left[T_a c_p^0 \left(\frac{T}{T_a} - 1 - \ln\left(\frac{T}{T_a}\right)\right) + T_a R_L \ln\left(\frac{p}{p_a}\right)\right]$$

For adiabatic compressor:

$$\frac{T}{T_a} = \left(\frac{p}{p_a}\right)^{\frac{k-1}{k}}$$

Heat flow $Q_{storage}$ becomes power supplied to compressor.

$$Q_{storage} = P_{el} = \dot{m}c_p^0(T - T_a)$$

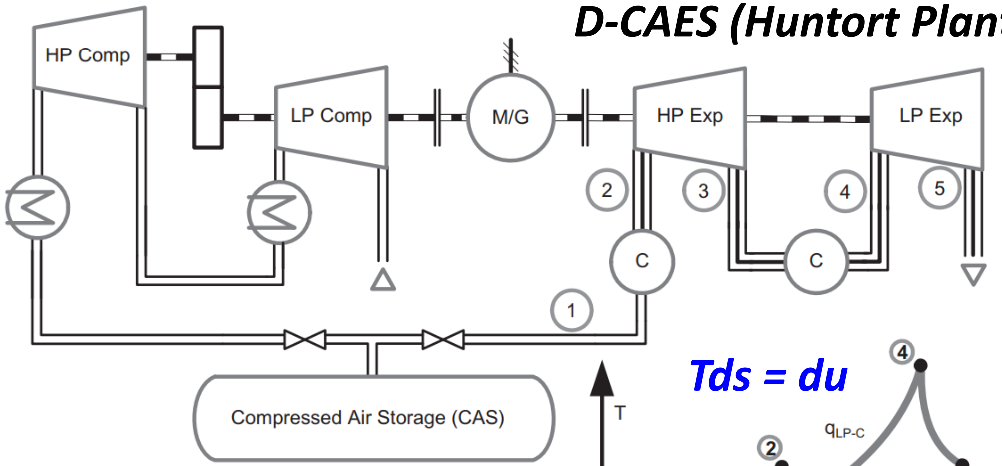
Efficiency:

$$\eta_{cyc} = \frac{E_{out}}{E_{in}}$$

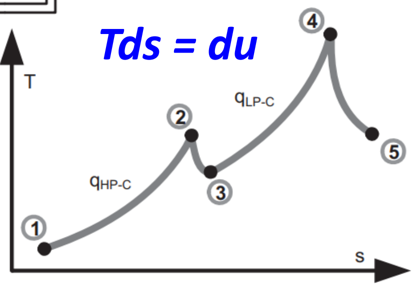
Budt M. et al. (2016), A review on compressed air energy storage: Basic principles, past milestones and recent developments

Compressible Air Energy Storage Model

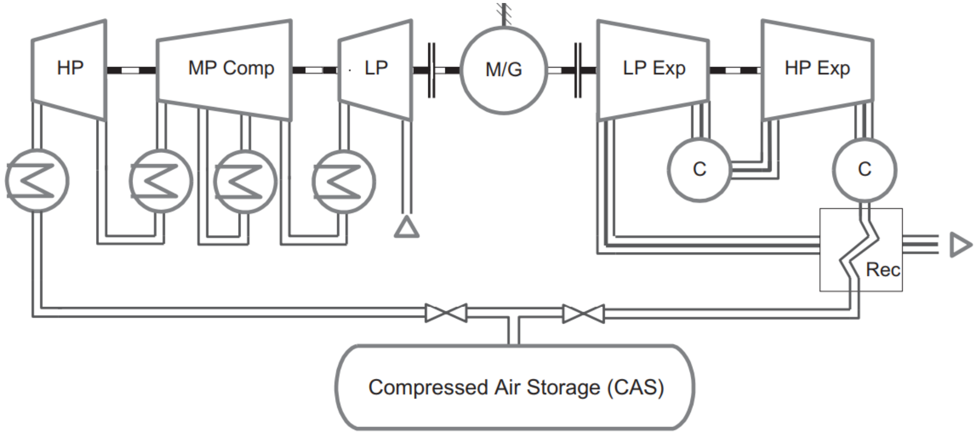
D-CAES (Huntort Plant)



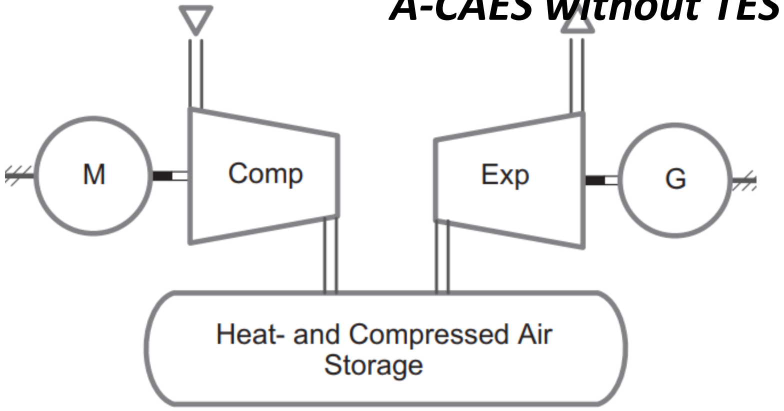
Tds = du

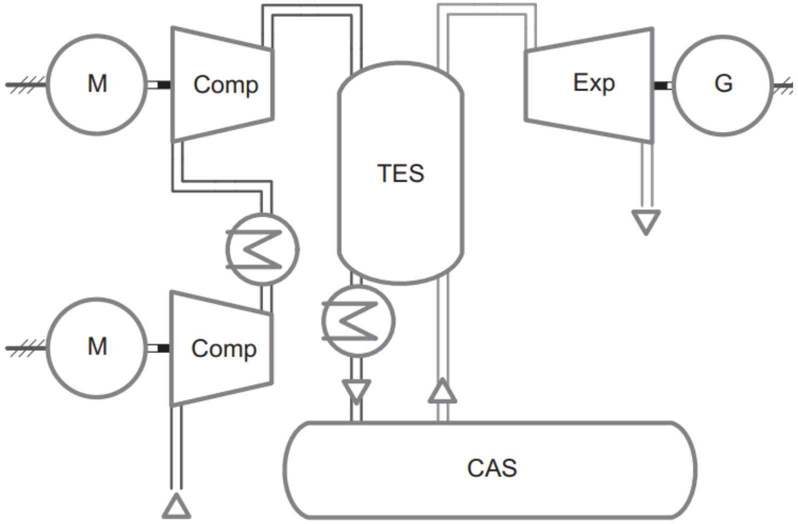


D-CAES (McIntosh Plant)



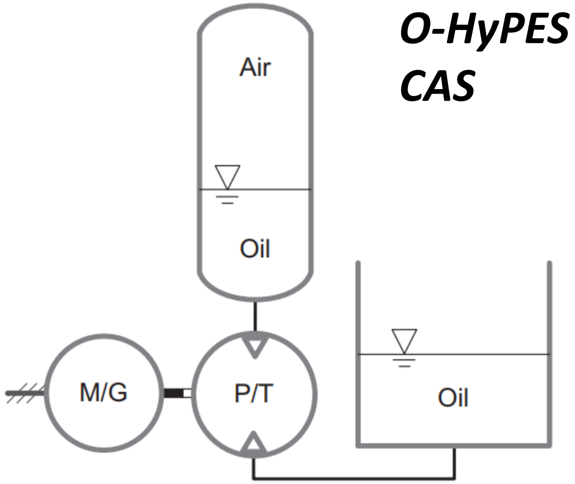
A-CAES without TES

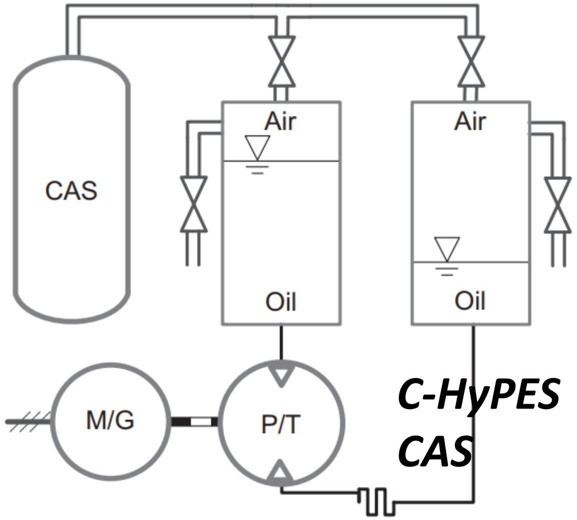




High-Temp A-CAES

O-HyPES CAS

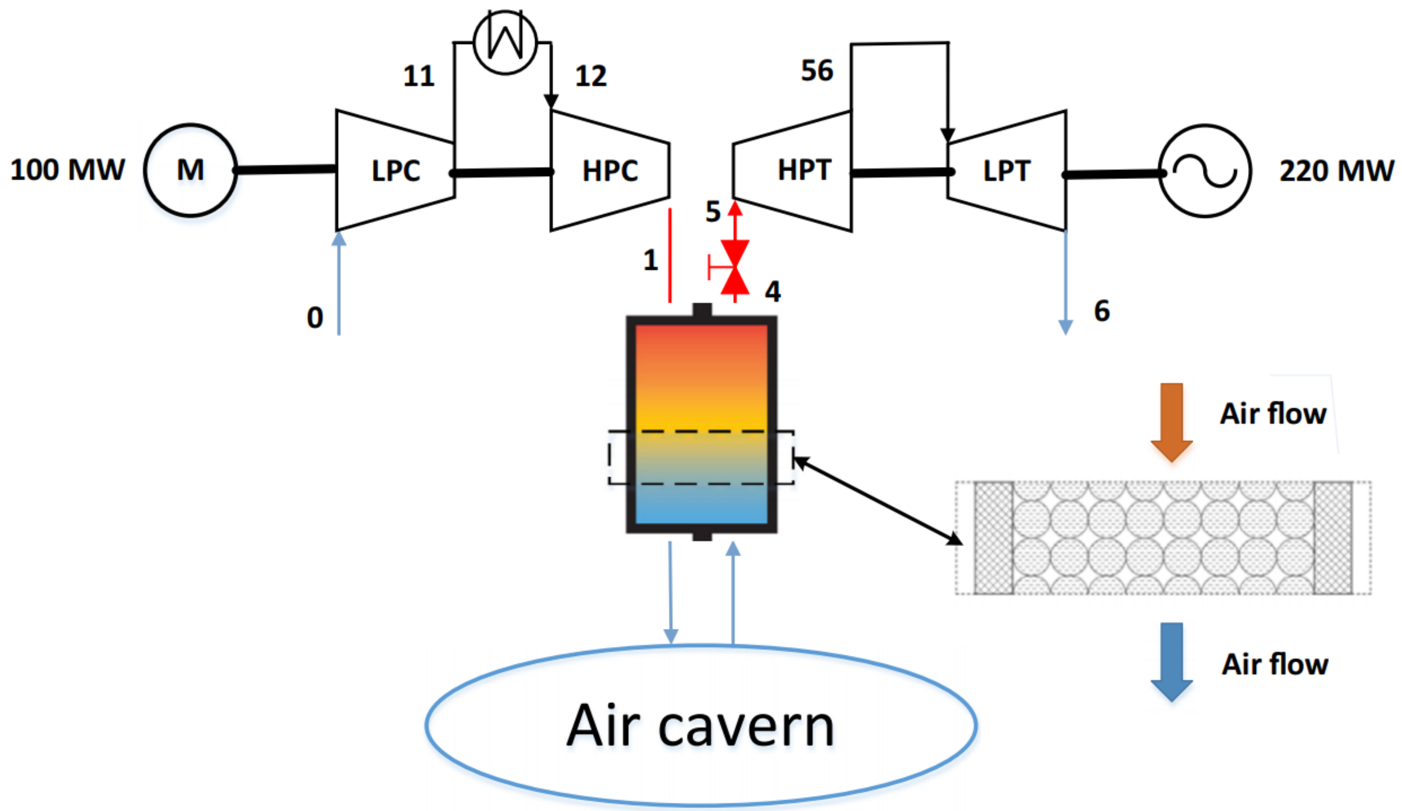




C-HyPES CAS

Compressible Air Energy Storage Model

Typical A-CAES Cycle and Typical Data



Quantity	Value
Ambient temperature	293.15 K
Ambient pressure	1.01325 bar
Expansion train rated power	220 MW
HP turbine design inlet temperature	905.15 K
HP turbine design inlet pressure	46 bar
HP turbine design expansion ratio	4.18
LP turbine design inlet temperature	655.15 K
LP turbine design inlet pressure	11 bar
LP turbine design expansion ratio	11
Turbines design efficiency	88%
Compression train rated power	100 MW
HP compressor design inlet temperature	480.15 K
HP compressor design compression ratio	8.4
LP compressor design compression ratio	8.4
Cavern volume	230,000 m^3
Cavern min/max pressure	46/72 bar
Cavern wall heat transfer coefficient	$0.02356 + 0.0149\|\dot{m}_{in} - \dot{m}_{iout}\|^{0.8}$

A-CAES performance for full load charging/discharging.

Quantity	Value
Number of cycles (–)	30
Round trip efficiency η_{cycle} (–)	74%[a]
Total output energy (MW h_e)	22,100
Charge time (h)	9.1
Discharge time (h)	3.3
Thermal energy stored (MW h_{th})	940[a]
Thermal energy storage efficiency η_{th} (–)	93%

[a] Averaged value over 30 cycles.

Sciacovelli A. et al. (2017), Dynamic simulation of Adiabatic Compressed Air Energy Storage (A-CAES) plant with integrated thermal storage – Link between components performance and plant performance

Compressible Air Energy Storage Model

Compressor:

$$T_{c,out}^{isen} = T_{c,in}\,\beta^{\frac{k-1}{k}}$$

$$\eta_c = \frac{T_{c,out}^{isen} - T_{c,in}}{T_{c,out} - T_{c,in}}$$

where β is compression ratio.

Power of Compressor:

$$W_c = \dot{m}_c(h_{c,out} - h_{c,in})$$

Heat Exchanger:
(Energy Balance and ε-NTU Method)

$$\varepsilon = \frac{1 - \exp(-NTU(1-\chi))}{1 - \chi\exp(-NTU(1-\chi))}$$

$$NTU = \frac{UA}{C_{min}} \text{ and } \chi = \frac{C_{min}}{C_{max}}$$

Charge process effectiveness:

$$\Phi = \varepsilon C_{min}(T_{in,h} - T_{in,c})$$

Turbine:

$$T_{t,out}^{isen} = T_{t,in}/(\pi_i)^{\frac{k-1}{k}}$$

$$\eta_t = \frac{T_{t,in} - T_{t,in}}{T_{t,out} - T_{t,in}^{isen}}$$

where $\pi_i = p_{in}/p_{out}$

Power output:

$$W_t = \dot{m}_t(h_{t,out} - h_{t,in})$$

Flugel Formula: (off-design)

$$\frac{\dot{m}_t}{\dot{m}_{t0}} = \alpha\sqrt{\frac{T_{t0,in}}{T_{t,in}}}\sqrt{\frac{\pi_t^2 - 1}{\pi_{t0}^2 - 1}}$$

$$\frac{\eta_t}{\eta_{t0}} = (1 - t(1-\dot{n}_t)^2)\left(\frac{\dot{n}_t}{\dot{G}_t}\right)(2 - \frac{\dot{n}_t}{\dot{G}_t})$$

Compressed Air Reservoir:

$$\frac{dT_r}{dt} = \frac{1}{m_r}[\left(1 - \frac{1}{k}\right)(m_{in}T_{in} - m_oT_r) + \frac{h_wA_w(T_w - T_r)}{c_{p,a}}]$$

$$\frac{dm_r}{dt} = m_{in} - m_o$$

Thermal Energy Storage Packed Bed:

$$\varepsilon\rho_a c_{p,a}\frac{\partial T_a}{\partial t} + \varepsilon\rho_a c_{p,a}u_a\frac{\partial T_a}{\partial x} = k_a\frac{\partial^2 T_a}{\partial x^2} - h_v(T_a - T_s) - U_w(T_a - T_0)$$

$$(1-\varepsilon)\rho_s c_{p,s}\frac{\partial T_s}{\partial t} = k_s\frac{\partial^2 T_s}{\partial x^2} + h_v(T_s - T_a)$$

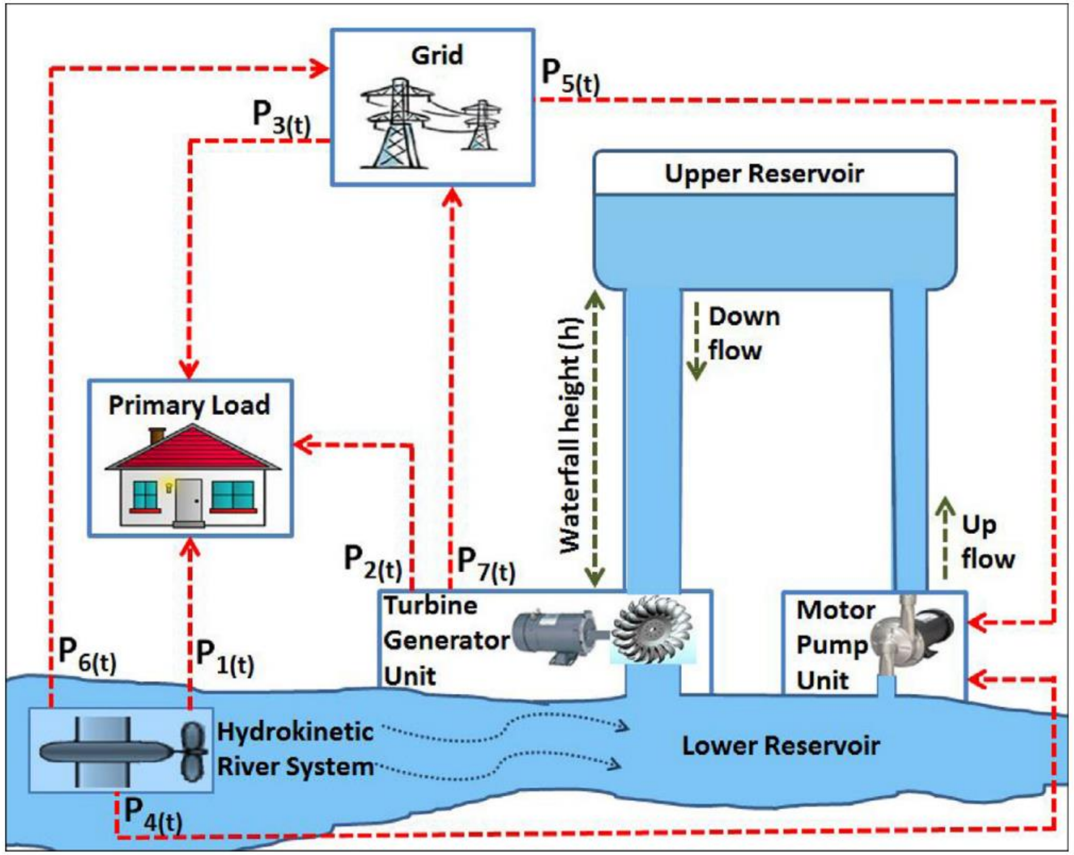
Compressible Air Energy Storage Model

Concerns

- CAES shows lower cycle efficiency than PHES or batteries
- No off-the-shelf machinery available that is suitable for efficient CAES plants, especially for high temperature adiabatic ones
- Low flexibility in short start-up times and fast ramping
- Site independence and low cost air reservoir are needed.
- Heat storage devices with high power and energy densities are needed for A-CAES
- (Turbo)machinery capable of being used as compressor or turbine to increase efficiency
- Tools for detailed simulations including humidity, part-load and dynamic operation of machinery needs to be developed.

Koko S.P. et al. (2018) Optimal power dispatch of a grid-interactive micro-hydrokinetic -pumped hydro storage system

Pumped Hydro Storage Model



Energy Pumped up to certain height (H):

$$P_{M:P} = \frac{\rho_W \times g \times H \times Q_{M:P}}{\eta_{M:P}}$$

Power Generated in Turbine Generator:

$$P_{T:G} = \rho_W \times g \times H \times Q_{T:G} \times \eta_{T:G}$$

Cost Function:

$$F = \sum_{j=1}^{N} C_j \cdot (P_{3(j)} + P_{5(j)}) \cdot \Delta t - C_j \cdot R_a \sum_{j=1}^{N} (P_{6(j)} + P_{7(j)}) \cdot \Delta t$$

Power Constraint: $P_{Load\ (j)} = P_{1(j)} + P_{2(j)} + P_{3(j)} \ (1 \le j \le N)$

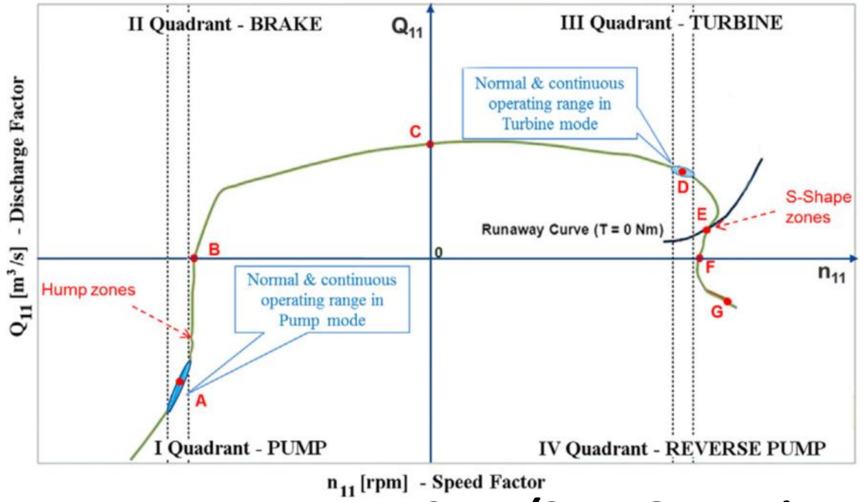
Storage Constraint: $Cap^{\min} \le Cap_{(j)} \le Cap^{\max}$

Final State Constraint: $\sum_{j=1}^{N} (P_{4(j)} + P_{5(j)}) - \sum_{j=1}^{N} (P_{2(j)} + P_{7(j)}) = 0$

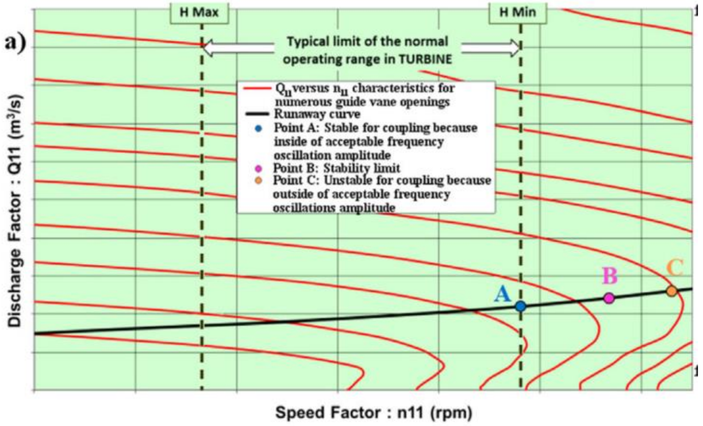
Cavazzini G. et al. (2018) Unstable behaviour of pump-turbines and its effects on power regulation capacity of pumped-hydro energy storage plants

Pumped Hydro Storage Model

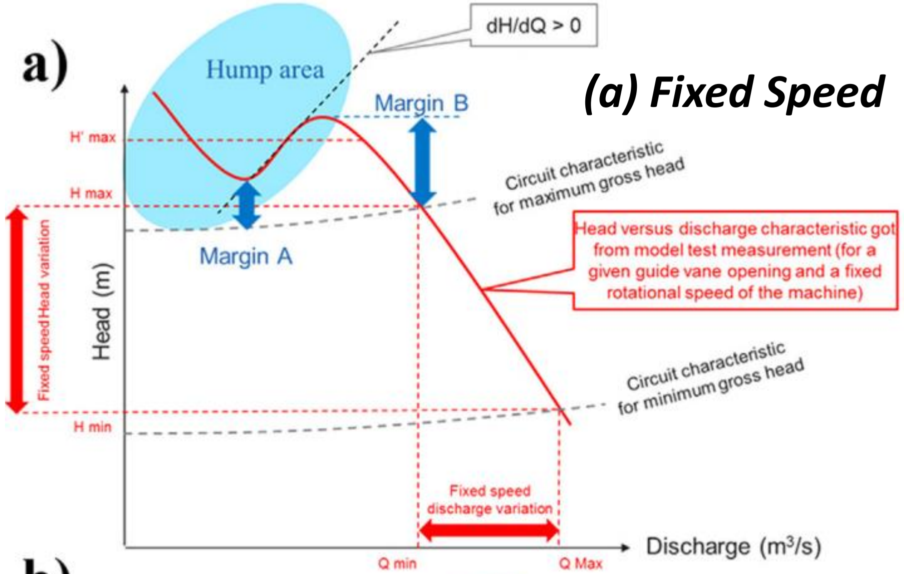
Four Quadrants of Operation



Discharge factor Vs speed factor

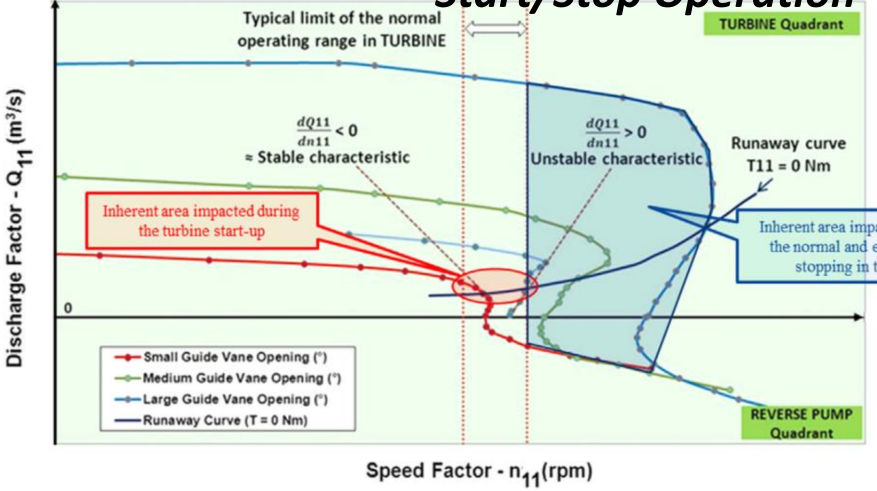


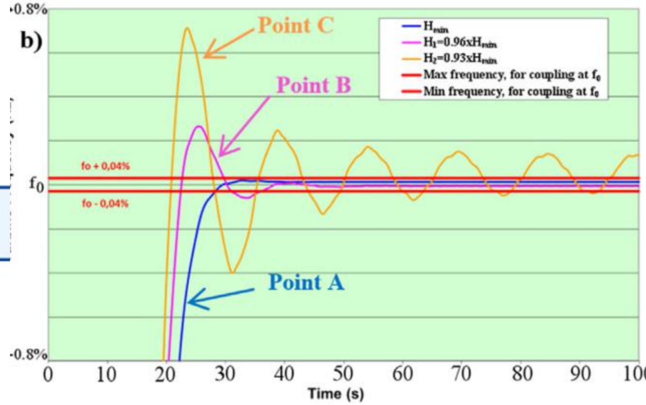
Operation Domain



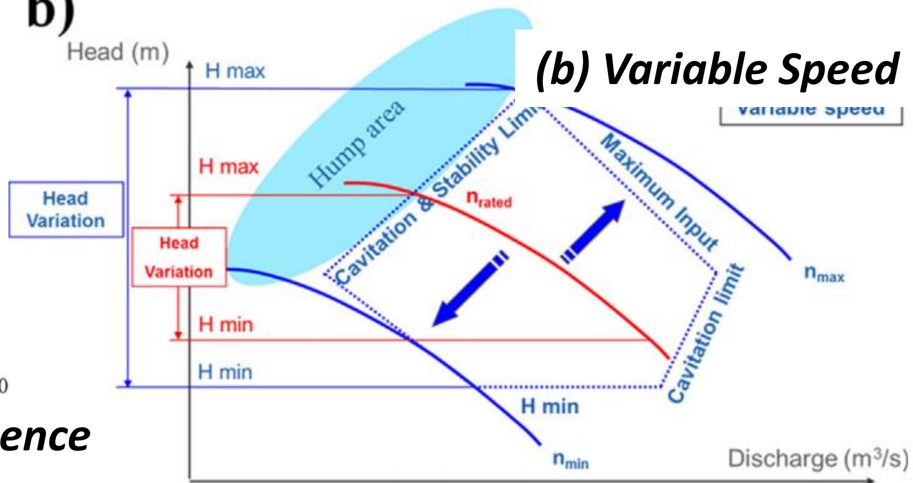
(a) Fixed Speed

Start/Stop Operation





frequency Vs time at start-up sequence

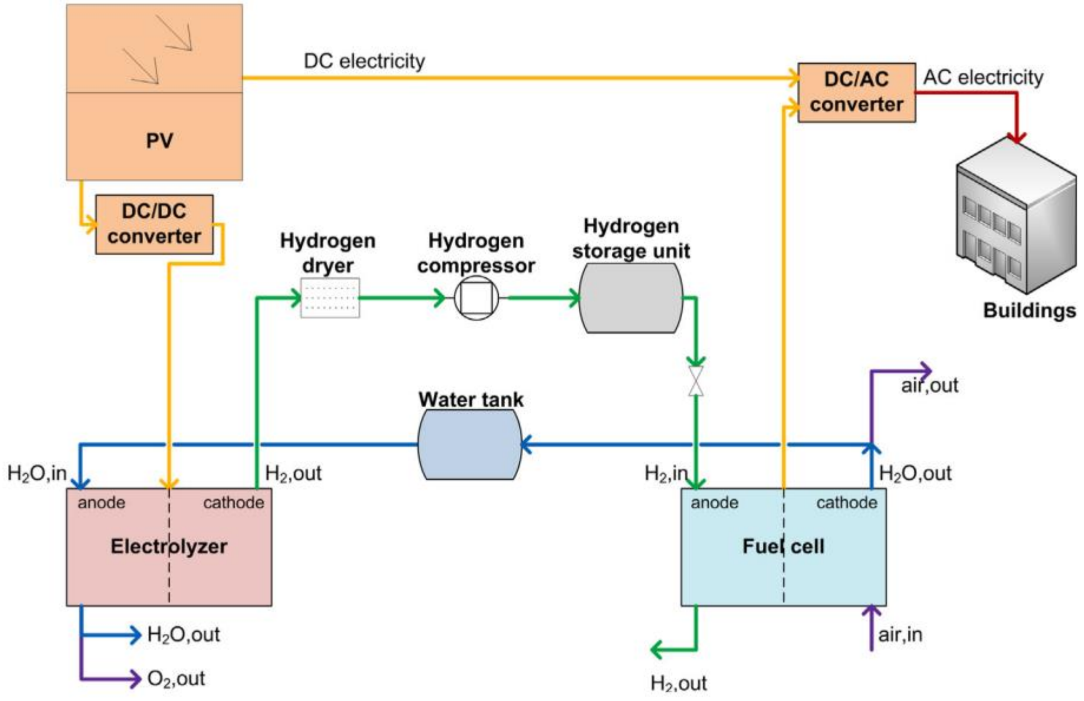


(b) Variable Speed

Arsalis A. et al. (2018), Thermoeconomic modeling of a completely autonomous, zero-emission photovoltaic system with hydrogen storage for residential applications

Hydrogen Storage Model

Suggested book: Nand, K. & Jesus F.A. (2017) Modeling and Dynamic Behaviour of Hydropower Plants



Electrolyzer – hydrogen storage – fuel cell subsystem

Anode, Cathode and Ohmic Overpotential:

$$\eta_a = \left(\frac{R \cdot T_{fc}}{\alpha \cdot F}\right) \cdot arcsinh\left(\frac{i}{2 \cdot k_{eh} \cdot \theta_{H_2}}\right)$$

$$\eta_c = \left(\frac{R \cdot T_{fc}}{\alpha \cdot F}\right) \cdot ln\left(\frac{i}{i_{oc}}\right)$$

$$\eta_{ohmic} = i \cdot R_{ohmic}$$

Electrolyzer voltage:

$$V_{cell,ec} = V_0 + (\eta_a + \eta_c + \eta_{ohmic})$$

Surface Coverage Differential for oxidation part:

$$\frac{d\theta_{H_2}}{dt} = \frac{k_{fh} \cdot y_{H_2} \cdot p_{ec} \cdot (1 - \theta_{H_2} - \theta_{CO} - \theta_{O_2})^2 - b_{fh} \cdot k_{fh} \cdot \theta_{H_2}^2 - i - k_{oh} \cdot \theta_{H_2}^2 \cdot \theta_{O_2}}{\rho} = 0$$

Hydrogen (initial) mass flow rate:

$$\dot{m}_{H_2,s} = \dot{m}_{H_2,s0} + \dot{m}_{H_2,ecs} - \dot{m}_{H_2,fcs}$$

Fuel Cell Efficiency:

$$\eta_{fc} = \frac{\dot{P}_{fc}}{\dot{m}_{H_2,cons} \cdot HHV_{H_2}}$$

Electrolyzer Efficiency:

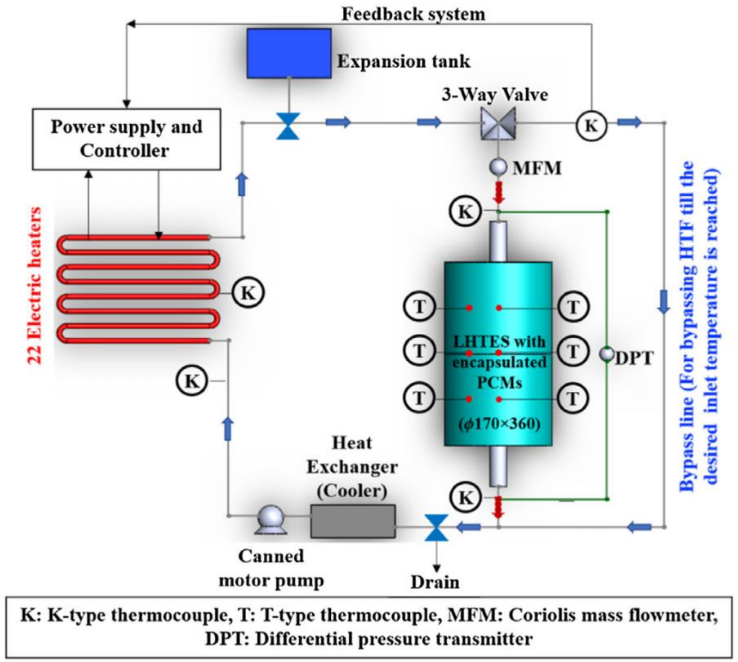
$$\eta_{ec} = \frac{\dot{m}_{H_2,gen} \cdot HHV_{H_2}}{\dot{P}_{ec}}$$

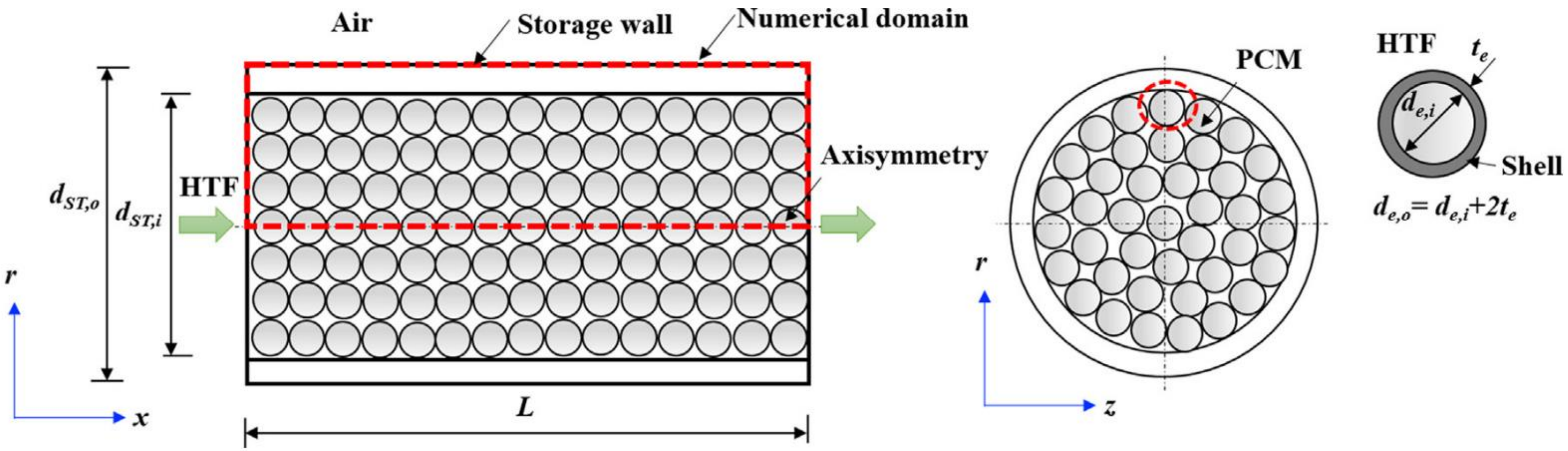
HHV: Higher heating value

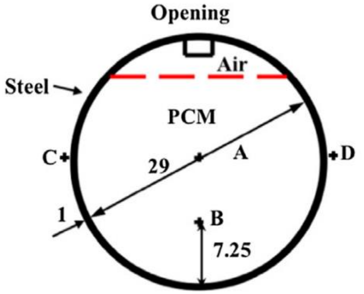
Raul, A. et al. (2018) Modelling and experimental study of latent heat thermal energy storage with encapsulated PCMs for solar thermal applications

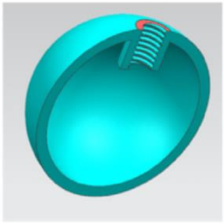
Thermal Storage Modelling

For a PCM Thermal Storage:









Solid model

Photograph of a capsule

Heat Transfer Fluid: $$\varphi\rho_F c_{p,F}\frac{\partial T_F}{\partial t} + \frac{\partial(\rho_F c_{p,F} T_F u)}{\partial x} = \frac{\partial^2(\varphi k_F T_F)}{\partial x^2} + h_{conv,F}\mathscr{A}_e(T_S - T_F) + h_{conv,F-ST}\mathscr{A}_{ST,i}(T_{ST} - T_F)$$

Phase Change Material: $$(1-\varphi)\rho_S c_{p,S}\frac{\partial T_S}{\partial t} = \frac{\partial^2[(1-\varphi)k_S T_s]}{\partial x^2} + h_{conv,F}\mathscr{A}_e(T_F - T_S) + S2$$

(Latent Heat Content): $S2 = -(1-\varphi)\rho_S \frac{d\Delta E}{dt}$

Suggested Book: Cabeza L. (2015) Advances in Thermal Energy Storage Systems *Methods and Applications*

Thermal Storage Modelling

Latent Heat change:

$$[\Delta E_P]_{n+1} = [\Delta E_P]_n + \lambda c_{p,S}\frac{a_p}{a_p^0}([T_{PCM}]_n - T_m)$$

$$[\Delta E_P]_n = 0 \quad \text{If}[\Delta E_P]_{n+1} < 0$$

$$[\Delta E_P]_n = L_{lat} \quad \text{If}[\Delta E_P]_{n+1} > L_{lat}$$

Energy Storage for wall:

$$\rho_{ST} c_{p,ST} \forall \frac{\partial T_{ST}}{\partial t} = h_{conv,F-ST}\mathscr{A}_{ST,i}(T_F - T_{ST}) - h_{conv,amb}\mathscr{A}_{ST,o}(T_{ST} - T_{amb})$$

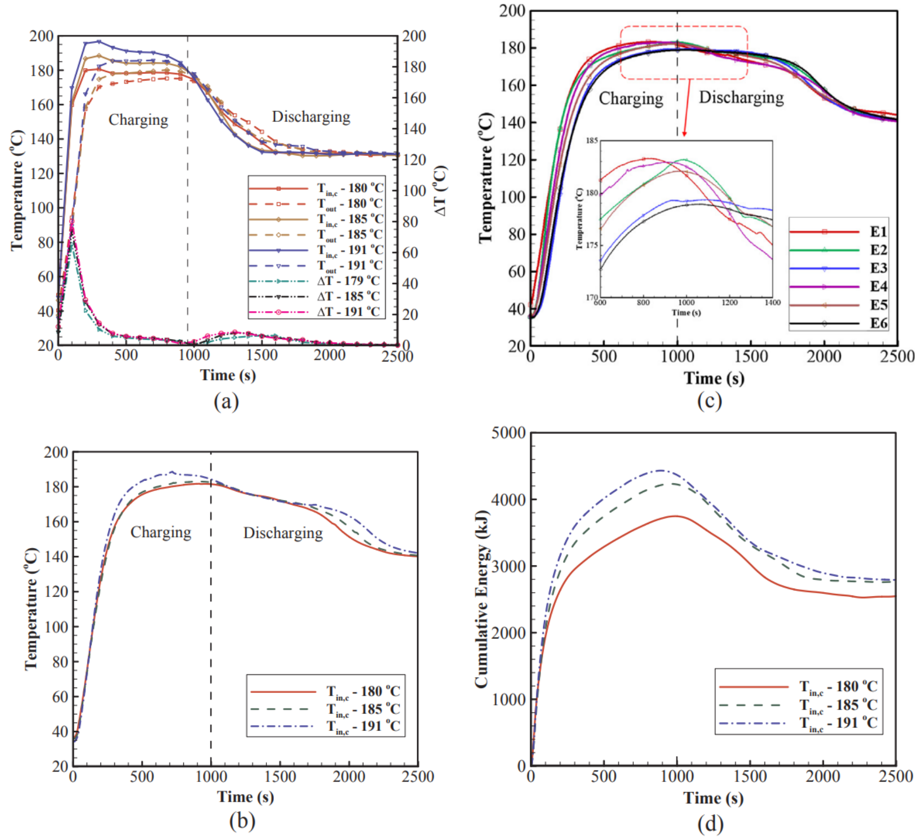
Storage wall volume: $\forall = \left[\frac{V_{ST,o} - V_{ST,i}}{V_{ST,i}} = \left(\frac{d_{ST,o}}{d_{ST,i}}\right)^2 - 1\right]$

Packed void fraction: $\varphi = 0.4272 - 4.516 \times 10^{-3}\left(\frac{d_{ST,i}}{d_{e,o}}\right) + 7.881 \times 10^{-5}\left(\frac{d_{ST,i}}{d_{e,o}}\right)^2$

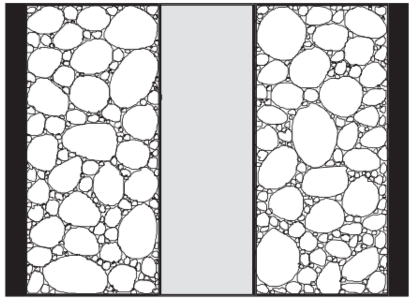
Convection coeff: $h_{conv,Fo} = \frac{6(1-\varphi)(2.2Re^{0.6}Pr^{1/3})k_F}{d_{e,o}^2}$

Pressure Change: $\frac{\Delta P}{L} = 0.061\left(\frac{1-\varphi}{\varphi^3}\right)\left(\frac{d_{ST,i}}{d_{e,o}}\right)^{0.2}(1000Re^{-1} + 60Re^{-0.5} + 12)\left(\frac{\rho_F}{d_{e,o}}\right)U^2$

Temperature Profile



Supercapacitor Modelling – Physical Model



- Charge conservation across the double layer

$$aC\frac{\partial(\phi_1-\phi_2)}{\partial t}=\sigma\frac{\partial^2\phi_1}{\partial\chi^2}$$

- Elctrolyte diffusion

$$\epsilon\frac{\partial c}{\partial t}=D\frac{\partial^2 c}{\partial\chi^2}-\frac{aC}{F}\left(t_-\frac{dq_+}{dq}+t_+\frac{dq_-}{dq}\right)\frac{\partial(\phi_1-\phi_2)}{\partial t},$$

- Ohm's law

$$\kappa\left(\frac{RT(t_+-t_-)}{F}\right)\frac{\partial}{\partial\chi}\ln(c)+\sigma\frac{\partial(\phi_1-\phi_2)}{\partial\chi}+\left(\kappa\frac{\partial}{\partial\chi}+\sigma\frac{\partial}{\partial\chi}\right)\phi_2+i=0$$

State-Space Form:

$$\begin{bmatrix}\epsilon & \frac{aC}{F}(t_-\frac{dq_+}{dq}+t_+\frac{dq_-}{dq}) & 0\\ 0 & aC & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{c}\\ \phi_1-\phi_2\\ \phi_2\end{bmatrix}=\begin{bmatrix}D\frac{\partial^2}{\partial\chi^2} & 0 & 0\\ 0 & \sigma\frac{\partial^2}{\partial\chi^2} & \sigma\frac{\partial^2}{\partial\chi^2}\\ 0 & \sigma\frac{\partial}{\partial\chi} & \kappa\frac{\partial}{\partial\chi}+\sigma\frac{\partial}{\partial\chi}\end{bmatrix}\begin{bmatrix}c\\ \phi_1-\phi_2\\ \phi_2\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{\kappa RT(t_+-t_-)}{F}\frac{\partial}{\partial\chi}\end{bmatrix}\ln(c)+\begin{bmatrix}0\\0\\1\end{bmatrix}i$$

$$\begin{bmatrix}\epsilon & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot{c}\\ \phi_2\end{bmatrix}=\begin{bmatrix}D\frac{\partial^2}{\partial\chi^2} & 0\\ 0 & \kappa\frac{\partial}{\partial\chi}\end{bmatrix}\begin{bmatrix}c\\ \phi_2\end{bmatrix}+\begin{bmatrix}0\\ \kappa(\frac{t_+-t_-}{f})\frac{\partial}{\partial\chi}\end{bmatrix}\ln(c)+\begin{bmatrix}0\\1\end{bmatrix}i.\qquad y=V=\phi_1|_{x=0}-\phi_1|_{x=L}.$$

Convert DAE into ODE for $\boldsymbol{\phi_2}$:

$$\sigma\hat{\mathbf{D}}_{\phi_1}(\boldsymbol{\phi_1}-\boldsymbol{\phi_2})+\left(\kappa\hat{\mathbf{D}}_{\phi_2}+\sigma\hat{\mathbf{D}}_{\phi_1}\right)\boldsymbol{\phi_2}$$
$$\kappa\left(\frac{t_+-t_-}{f}\right)\hat{\mathbf{D}}_{\ln c}\ln(\mathbf{c})+i=0$$

$$\longrightarrow$$

$$\boldsymbol{\phi_2}=-\left(\kappa\hat{\mathbf{D}}_{\phi_2}+\sigma\hat{\mathbf{D}}_{\phi_1}\right)^{-1}[\sigma\hat{\mathbf{D}}_{\phi_1}(\boldsymbol{\phi_1}-\boldsymbol{\phi_2})$$
$$\kappa(\frac{t_+-t_-}{f})\hat{\mathbf{D}}_{\ln c}\ln(\mathbf{c})+i].$$

State-Space Representation:

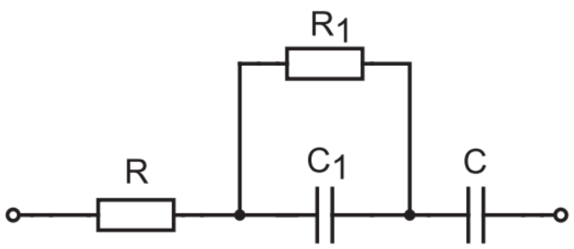
$$M\dot{x}=Ax+\tilde{B}_1\ln(c)+\tilde{B}_2 i$$
$$y=Cx+\tilde{D}_1\ln(c)+\tilde{D}_2 i$$

$$x:=[\mathbf{c}^T,\ \ \boldsymbol{\phi_1}^T-\boldsymbol{\phi_2}^T]^T$$

Devillers N. et al. (2014), Review of characterization methods for supercapacitor modelling

Supercapacitor Modelling – Equivalent Circuit

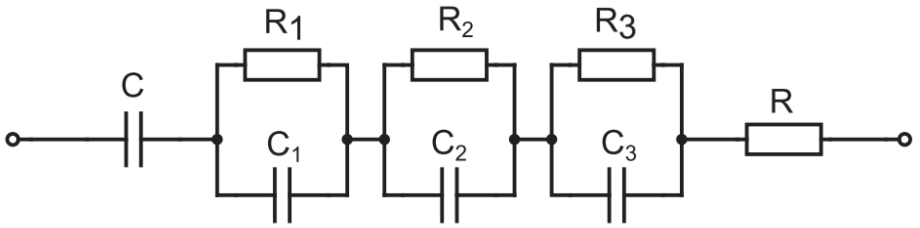
Classic Topology



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{R_1C_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \frac{1}{C_1} \end{bmatrix} i$$

$$V = x_1 + x_2 + Ri.$$

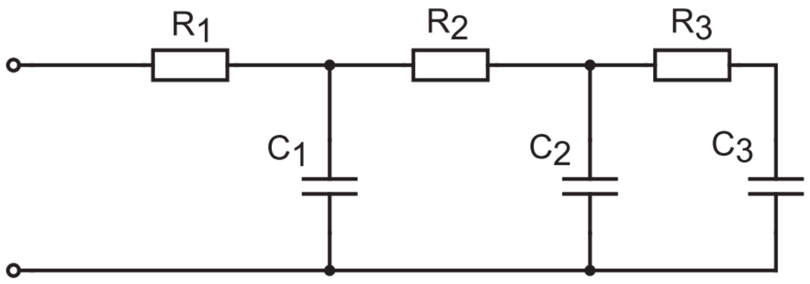
Dynamic Topology



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_1C_1} & 0 & 0 \\ 0 & 0 & -\frac{1}{R_2C_2} & 0 \\ 0 & 0 & 0 & -\frac{1}{R_3C_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \\ \frac{1}{C_3} \end{bmatrix} i$$

$$V = x_1 + x_2 + x_3 + x_4 + R_s i.$$

Ladder Topology

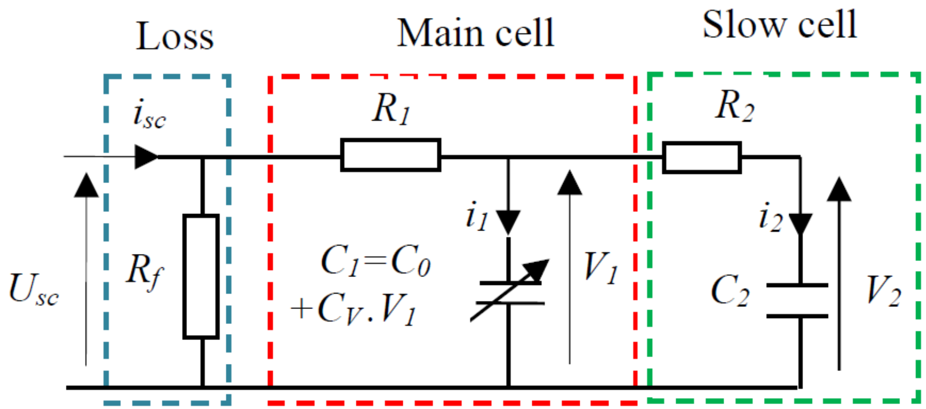


$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_2C_1} & \frac{1}{R_2C_1} & 0 \\ \frac{1}{R_2C_2} & -\frac{R_2+R_3}{R_3R_2C_2} & \frac{1}{R_3C_2} \\ 0 & \frac{1}{R_3C_3} & -\frac{1}{R_3C_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} \\ 0 \\ 0 \end{bmatrix} i$$

$$V = x_1 + R_1 i.$$

Supercapacitor Modelling – Equivalent Circuit

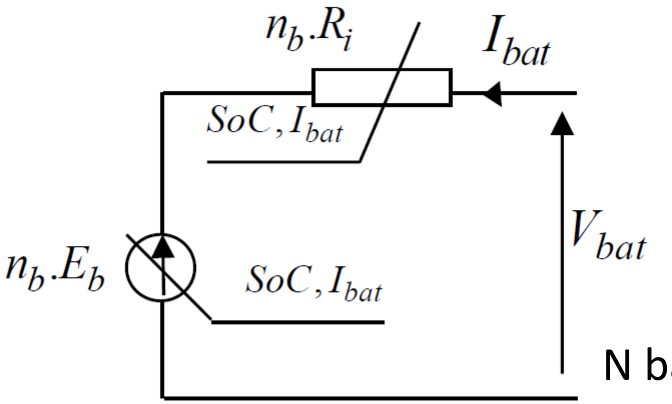
Zubieta and Bonert model



$$U_{SC} = N_{Ssc} v_{sc} = N_{Ssc}\left(v_1 + R_1 \frac{I_{SC}}{N_{Psc}}\right)$$

$$v_2 = \frac{1}{C_2}\int i_2\, dt = \frac{1}{C_2}\int \frac{1}{R_2}(v_1 - v_2)\, dt.$$

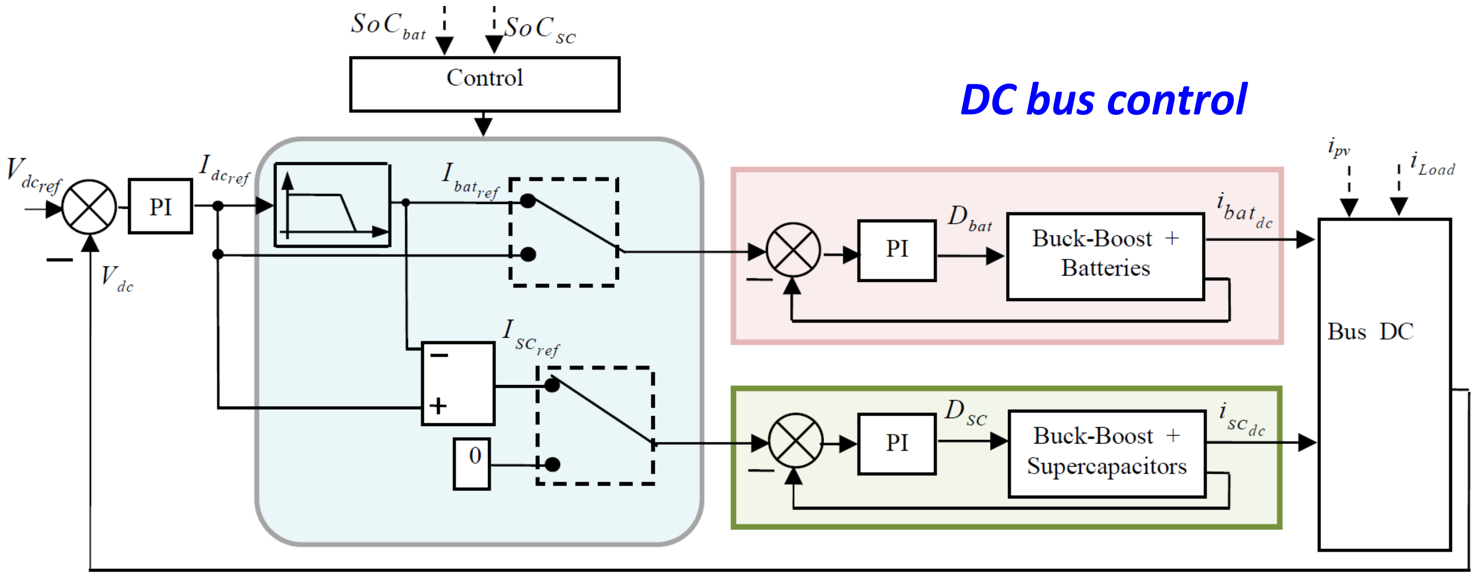
$$i_1 = C_1 \frac{dv_1}{dt} = \frac{dQ_1}{dt} = (C_0 + C_v v_1)\frac{dv_1}{dt}$$



N batteries in series

$$V_{bat} = n_b.E_b + n_b.R_i.I_{bat}$$

DC bus control



Energy Storage – Testing and Commissioning

No.	Test items	Type tests	Production tests	Commissioning tests	Periodic tests
1	**Temperature stability**	√			
2	State of charge (SOC)	√			√
3	Conversion efficiency	√			
4	Response time	√			√
5	Ramp-rate	√			√
6	Synchronization	√	√	√	
7	Reconnection after abnormal condition	√	√		
8	Harmonics	√			√
9	Flicker	√			
10	Voltage unbalance	√			
11	Open phase	√			
12	Overcurrent	√	√		
13	DC injection	√			
14	Response to abnormal voltage condition	√	√		√
15	Response to abnormal frequency condition	√	√		√
16	Unintentional islanding	√	√	√	
17	Low-voltage ride through (LVRT)	√	√		
18	Interconnection integrity	√			
19	Continuous operation	√	√		
20	Stop charging/discharging	√		√	

Note:
This standard allows **simulated Electrical Power System (EPS)** with Energy Storage Systems (ESS)

However, the following should be satisfied.

Requirement	Allowance
THD	50% as in EPS
Steady State	1% U_n fluctuation
Voltage Deviation	$\pm 3\%$ of U_n
Frequency Deviation	$\pm 0.01 Hz$
Unbalance of 3 phases	$\pm 3\%$ of U_n $\pm 3^o$ of phase angle

https://www.simulationx.com/industries/applications/energy-storages.html

SimulationX – Consideration in designing ES

System Level:
- How much energy must the energy storage be able to store and release over a certain period of time?
- How much storage may be needed in average and how much for peak loads?
- How long must energy be stored in the storage?
- Which storage solution (electro-chemical, mechanical, thermal etc.) keeps losses between the generating and the consuming devices at a minimum?
- What losses are to be expected during transmission and conversion?

Reliability:
- How does a failing component impact the entire system?
- How can I prevent a component or system breakdown?
- How does an alternative system layout impact the system's reliability and safety?

CAES
- Determine and optimize energy efficiency
- Specify system components to meet the given requirements
- Develop and test the optimum control strategy for your compressed air energy storage
- Master thermal effects and increase the efficiency by making use of occurring heat

Flywheel
- Simulate the interactions between all assemblies including flywheel mass, magnetic bearing, generator, vacuum pump and cooling system
- Dimension your components precisely already during the conceptual phase through conceptual simulations
- Identify undesired vibrations and examine eigenfrequencies with torsional vibration analyses

SimulationX – Consideration in designing ES

Batteries:

•Master the thermal behavior of batteries through simulation with respect to material and the geometrical, mechanical and electrical properties of the cells and their interactions with each other

•Develop **optimized** battery management systems (BMS) in regard to the battery's **state of charge** (SOC) and **state of health** (SOH)

•Ensure a sustained usage of the battery system

•Make reliable predictions about a battery's range and aging effects

•Make profound estimations about a battery's lifespan by simulating calendric and cyclical aging

•Channel waste heat of batteries effectively by dimensioning the cooling system optimally through simulation

Power to Gas/ Gas to Power:

•Analyze the behavior of valves for inlet and outlet under high pressure

•Simulate the fluid's properties for variable boundary conditions. Which fluid phase occurs when and where?

•Design a cooling system fast and reliably

•Prove the safety of the pressure system with automated fault tree analyses (FTA) and failure mode and effect analyses (FMEA)

Thermal Storage:

•Design heat and cold storage systems through detailed simulation:

•Investigate temperature distribution and heat transfer

•Analyze the behavior of fluids for phase changes (e.g. in latent heat storage systems)

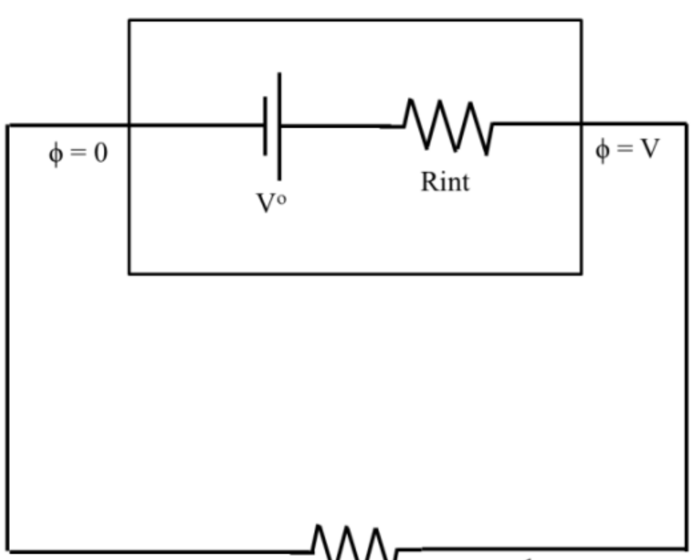
•Specify compressors, condensers, pipes, heat exchangers etc. for efficient functionality throughout the entire system

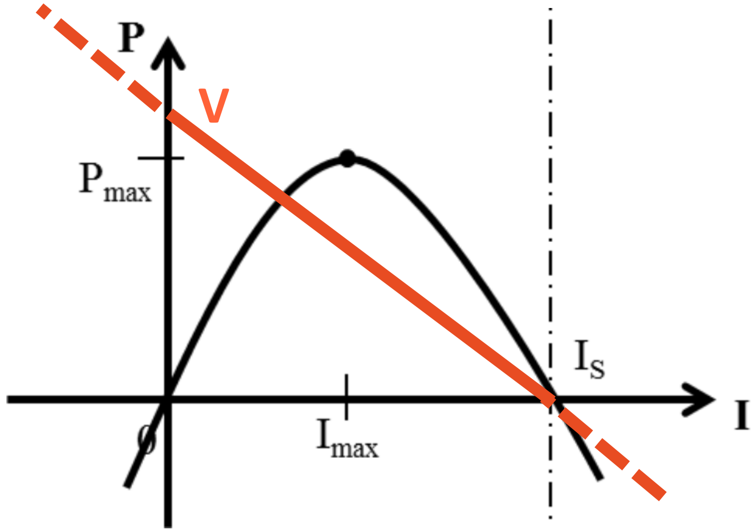
•Select the appropriate cold or heat storage on the basis of the simulation results for the whole system including heat sources, heat sinks, storages and pipes

Appendix: Basic Physics of Cells

1. Cell Voltage $V(I, Q, \ldots) = \Delta G / ne$
   = [Free energy difference of net reaction] / [charge transferred]

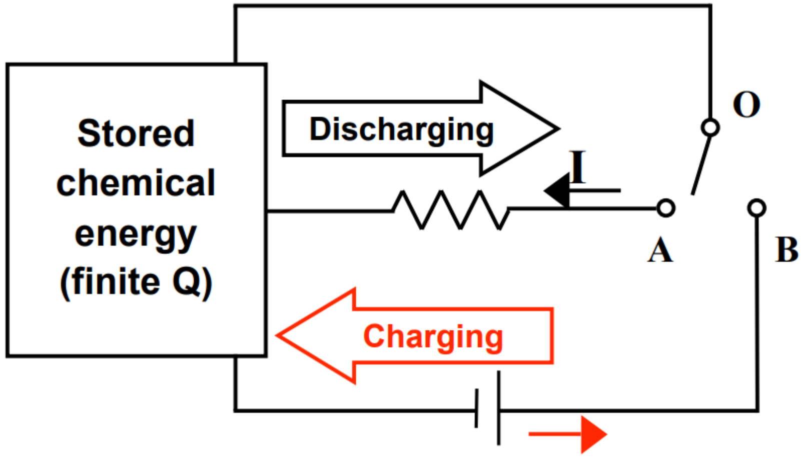
2. Consider the equivalent circuit.

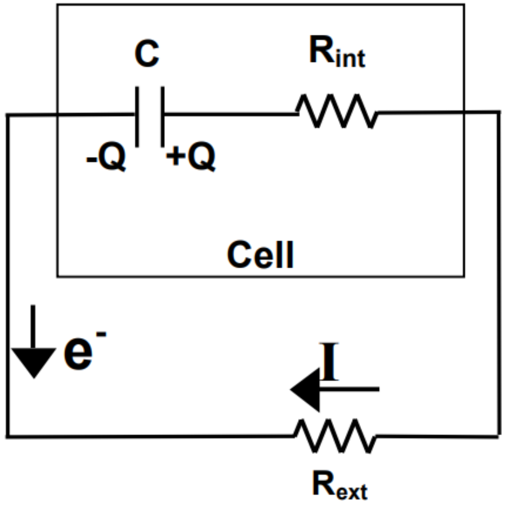




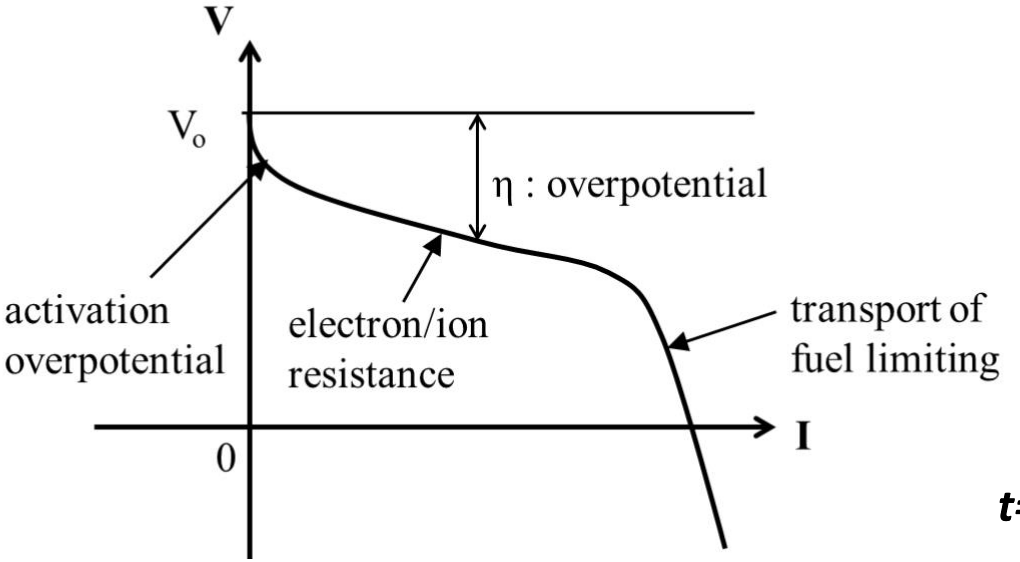
Electrolytic Regime	Galvanic Regime	Super Galvanic Regime
$P < 0$ $I < 0$ $V > V_o$	$P > 0$ $I > 0$ $0 < V < V_o$	$P < 0$ $I > I_S$ $V < 0$
Electrical Energy → Chemical Energy Storage	Chemical Energy → Electrical Energy + Heat Loss	Chemical + Electrical Energy → Heat Loss
e.g. charging a Li-ion battery by applying a reverse voltage	e.g. discharging a battery through an external load	e.g. forcing the battery to discharge faster than I_S by applying a voltage externally

Appendix: Basic Physics of Cells





Resistance are normally current dependent and there are limiting factors in charge / ion transport.



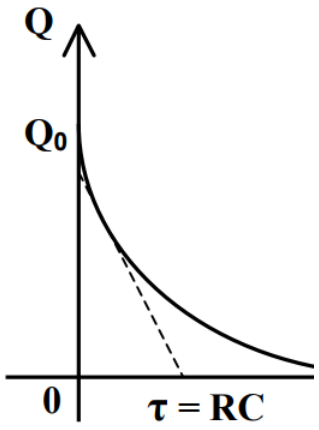
$$I = -\frac{dQ}{dt} = -C\frac{dV_C}{dt}$$

$$V = \frac{Q}{V_C} - IR_{int} = IR_{ext}$$

$$Q = -RC\frac{dQ}{dt}$$

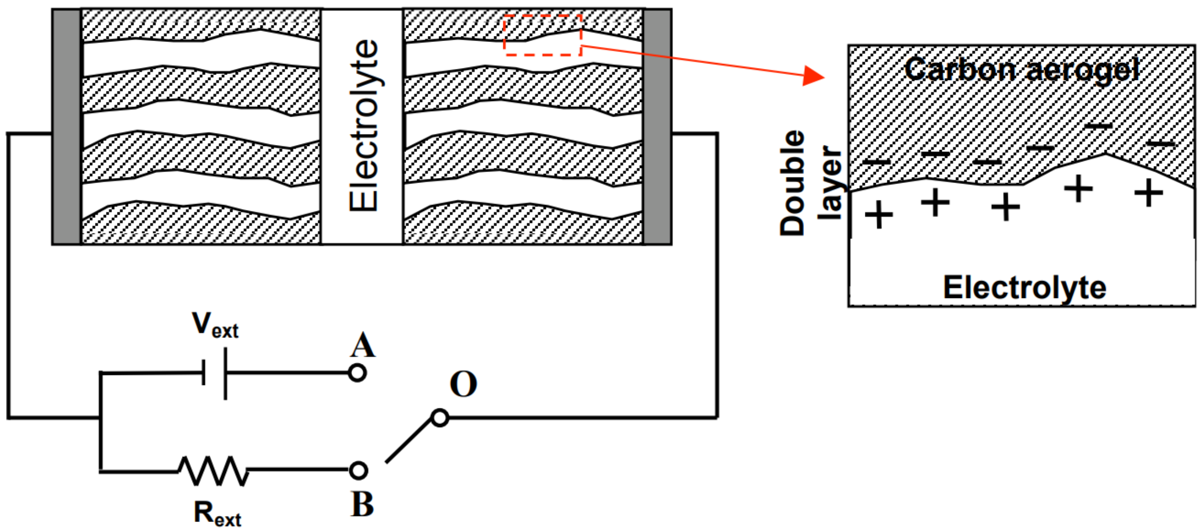
t= RC as time constant

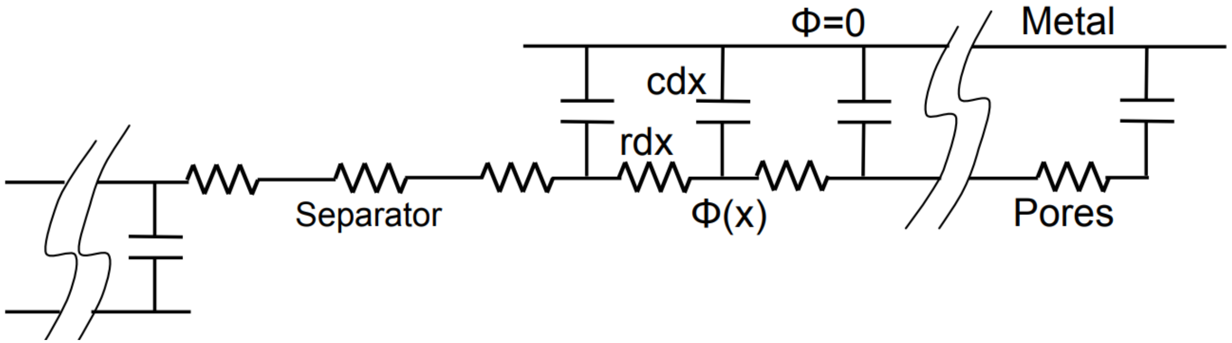
$$Q = Q_0 e^{-\frac{t}{RC}}$$



Appendix: Basic Physics of Cells

Supercapacitor





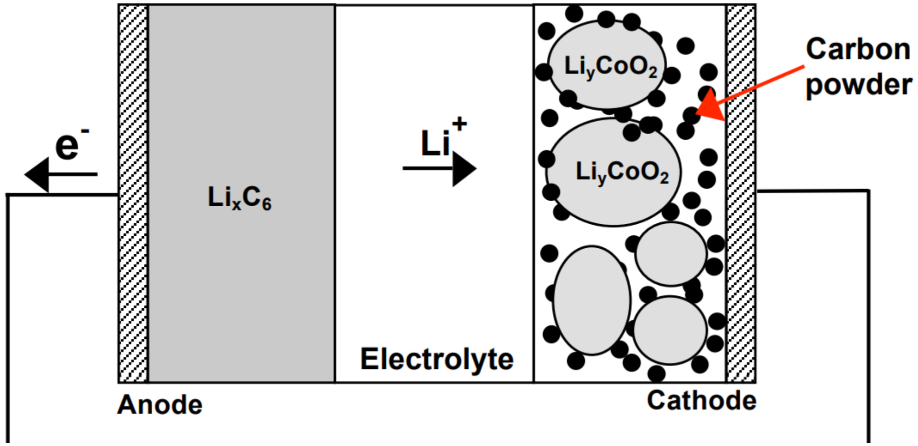
Consider two identical porous electrodes with thickness L, and electrolyte-filled pore space has constant volume-average resistance per length r and cap per length c. The mean potential satisfies diffusion eqt:

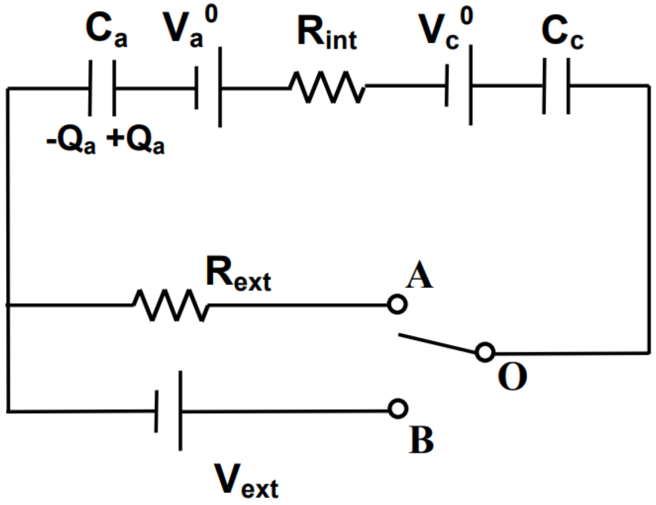
$$rc\frac{\partial\varphi}{\partial t}=\frac{\partial^2\varphi}{\partial x^2}$$

For a sudden change of voltage at x = 0,

$$I\sim\begin{cases} V\sqrt{\dfrac{c}{rt}} & ,t<<rcL^2 \\ \dfrac{V}{rL}e^{-\frac{t}{rcL^2}} & ,t>>rcL^2 \end{cases}$$

Appendix: Basic Physics of Cells





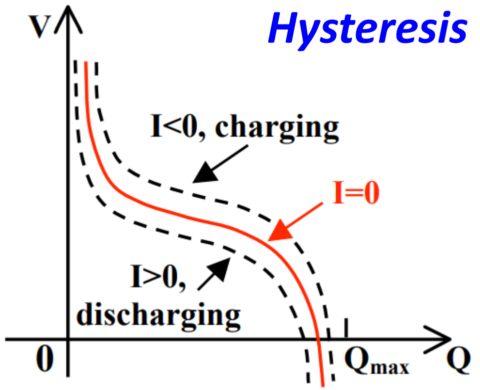
1) Discharging (connect OA in Figure):

$$\begin{cases} I = -\dfrac{dQ_a}{dt} = \dfrac{dQ_c}{dt} = \dfrac{V_a^0(Q_a) + V_c^0(Q_c)}{R} \\ Q_a(t=0) = Q_a^{\ 0} \\ Q_c(t=0) = 0 \end{cases}$$

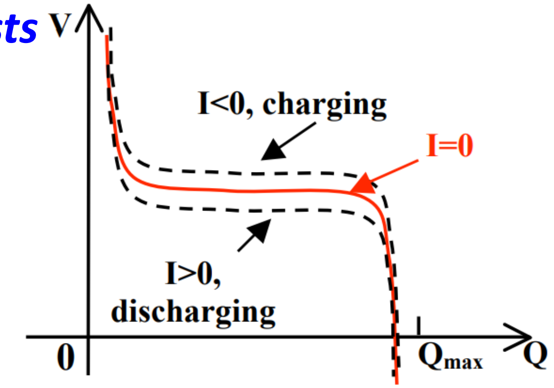
2) Charging (connect OB in Figure):

$$I = \frac{V_a^0(Q_a) + V_c^0(Q_c) - V_{ext}}{R} < 0$$

Hysteresis exists

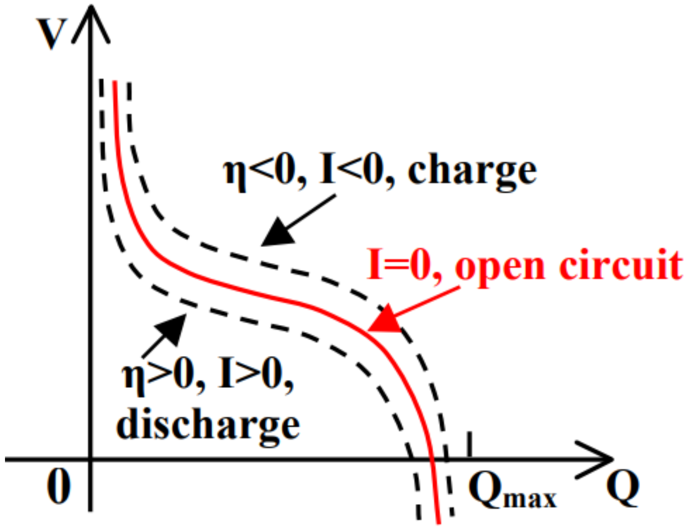


(a) Li_xCoO_2



(b) Li_xFePO_4

Appendix: Basic Physics of Cells



$$V = V_o(Q) - \eta(I,Q) = IR_{ext}$$

Linear Response: Assume V = V(Q, I)

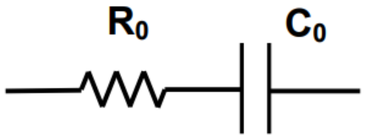
$$\Delta V = \left.\frac{\partial V}{\partial Q}\right|_{Q_0,I_0} \Delta Q + \left.\frac{\partial V}{\partial I}\right|_{Q_0,I_0} \Delta I$$

Define differential internal resistance: $R_0 = \left.\frac{\partial V}{\partial I}\right|_{Q_0,I_0}$ and differential cell cap:

$$\frac{1}{C_0} = \left.\frac{\partial V}{\partial Q}\right|_{Q_0,I_0} = \left(\frac{dV_O}{dQ} - \frac{\partial \eta}{\partial Q}\right)_{Q_0,I_0}$$

$$\Delta V = \frac{1}{C_0}\Delta Q + R_0 \frac{d\Delta Q}{dt}$$

Equivalent Circuit:



Appendix: Basic Physics of Cells

Constant Load:

$$V(t) = V_o(Q) - \eta(\frac{dQ}{dt}, Q) = \frac{dQ}{dt} R_{\text{ext}}$$

$$V = V_O(Q) - \frac{dQ}{dt} R_{int}(Q) = \frac{dQ}{dt} R_{ext}$$

$$\frac{dQ}{dt} = \frac{V_O(Q)}{R_{ext} + R_{\text{int}}(Q)}$$

Assume the overpotential is linearly dependent on current

$$\int_{Q_0}^{Q} \frac{(R_{ext} + R_{\text{int}}(Q))dQ}{V_o(Q)} = -\int_0^t dt = -t$$

Constant Current:

$$Q = Q_0 + It$$

$$V(t) = V_o(Q_0 + It) - \eta(I, Q_0 + It) = IR_{\text{ext}}(t)$$

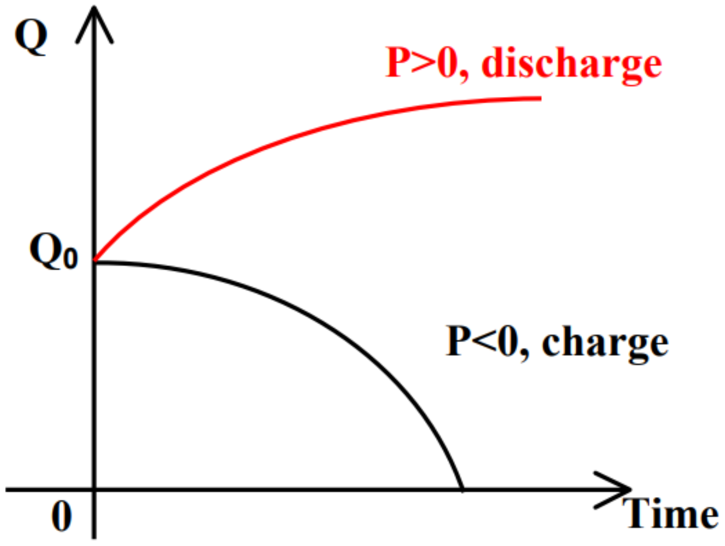
$R_{ext}(t)$ should be continuously varying to suit for constant current.

Appendix: Basic Physics of Cells

Constant Power:

$$P = IV = \text{constant}$$

$$P = \left[V_0(Q) - \eta(\frac{dQ}{dt}, Q) \right] \left(\frac{dQ}{dt} \right)$$



C rate:

$$\text{C rate} = \frac{\text{total capacity}}{\text{n hours}} = \frac{\text{C}}{\text{n}}$$

C/60 = batteries discharge total capacity (Ah) in 60 hours.
60C = batteries discharge total capacity (Ah) in 1 minutes.

Frequency Response:

$$\mathbf{V} \simeq \frac{\mathbf{Q}}{\mathrm{C_{ref}}} + \mathrm{R_{ref}} i\omega \mathbf{Q} = \frac{\mathbf{I}}{i\omega \mathrm{C_0}} + \mathrm{R_0}\mathbf{I}$$

Complex Impedance: $\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}}{|\mathbf{I}|} e^{-i\varphi}$

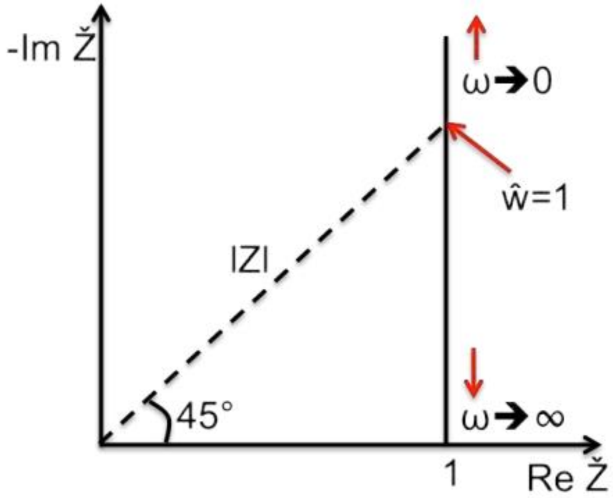
Complex Capacitance: $\mathbf{C} = \frac{\mathbf{Q}}{\mathbf{V}}$

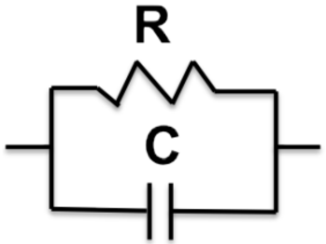
Appendix: Basic Physics of Cells

$$\mathbf{Z} = R_{ref} + \frac{1}{i\omega C_{ref}} \qquad \tilde{\mathbf{Z}} = 1 + \frac{1}{i\tilde{\omega}}$$

$$\mathbf{Y} = \frac{i\omega C_{ref}}{1+i\omega R_{ref}C_{ref}} \qquad \tilde{\mathbf{Y}} = \frac{i\tilde{\omega}}{1+i\tilde{\omega}}$$

$$\mathbf{C} = \frac{C_{ref}}{1+i\omega R_{ref}C_{ref}} \qquad \tilde{\mathbf{C}} = \frac{1}{1+i\tilde{\omega}}$$

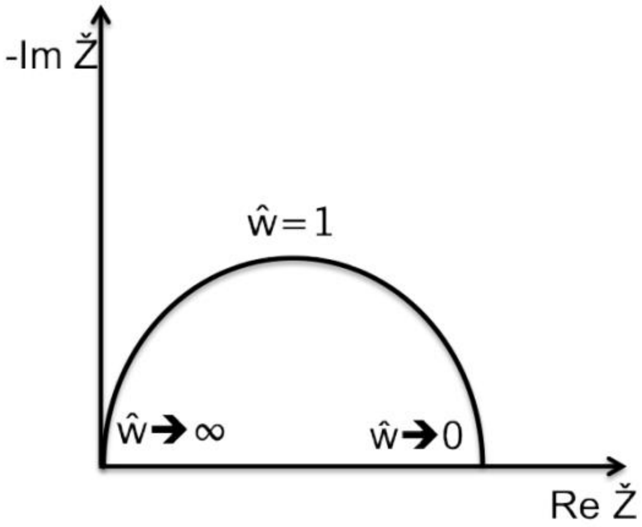




$$\tilde{\mathbf{Z}} = \frac{1}{1+i\tilde{\omega}}$$

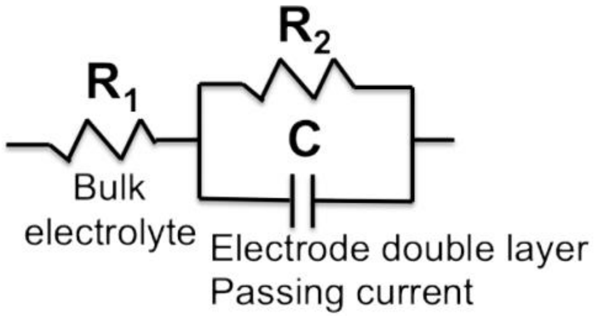
$$\tilde{\mathbf{Y}} = 1 + i\tilde{\omega}$$

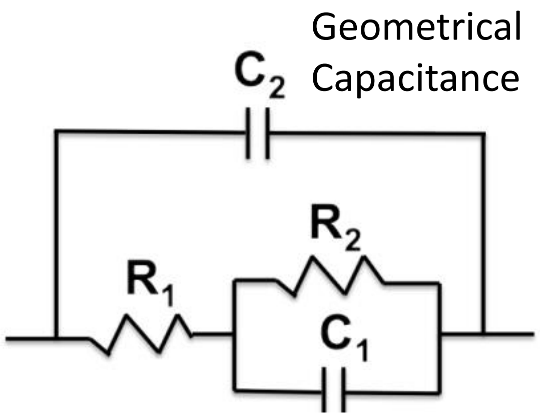
$$\tilde{\mathbf{C}} = 1 + \frac{i}{\tilde{\omega}}$$



Dimensionless form: $\tilde{\omega} = \omega R_{ref}C_{ref}$, $\tilde{\mathbf{Z}} = \frac{Z}{R_{ref}}$, $\tilde{\mathbf{Y}} = YR_{ref}$, $\tilde{\mathbf{C}} = \frac{C}{C_{ref}}$

Appendix: Basic Physics of Cells

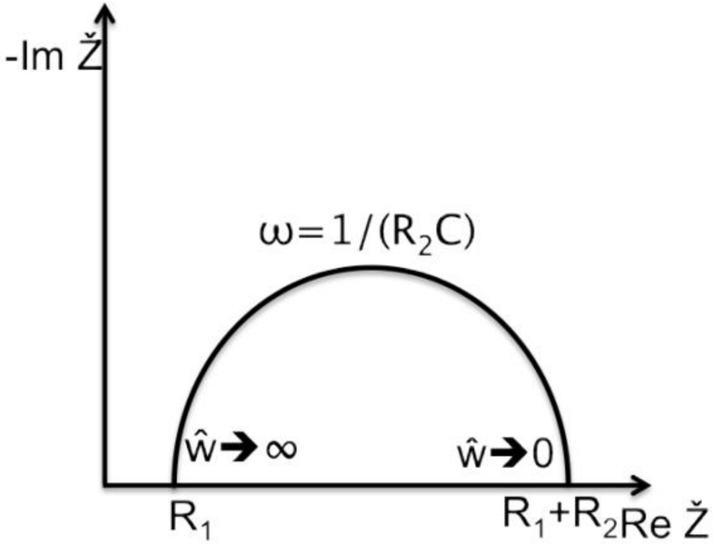


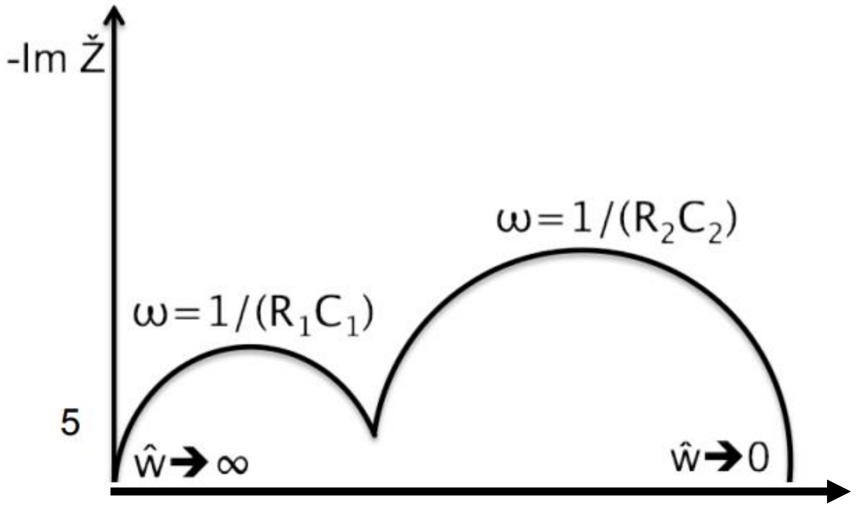


For an R-C parallel network, one can obtain a linear transformation for further analysis:

$$\tilde{Z} = \frac{1}{1 + i\tilde{\omega}}$$

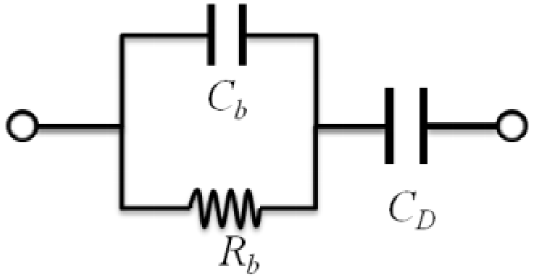
$$\bar{Z} = 2\tilde{Z} - 1 = \frac{1 - i\tilde{\omega}}{1 + i\tilde{\omega}}$$





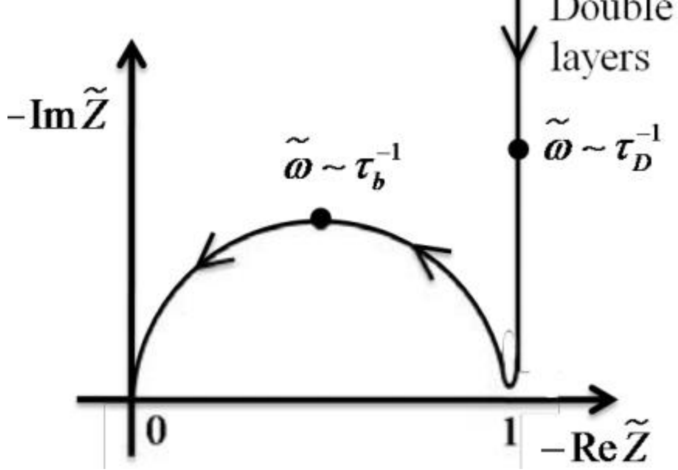
Appendix: Basic Physics of Cells

Ideal Blocking Electrode:

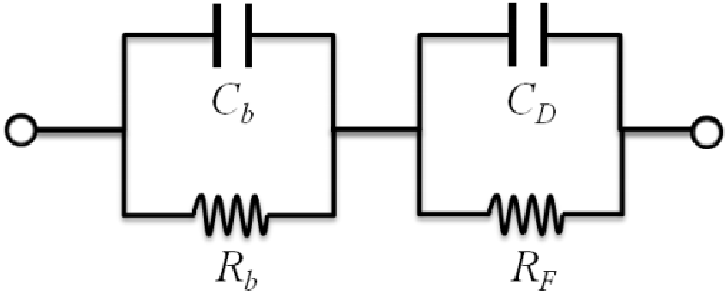


$$Z = \frac{1}{i\omega C_D} + \frac{1}{i\omega C_b + 1/R_b}$$

$$\tilde{Z} = \frac{1}{i\omega\tau_D} + \frac{1}{1 + i\omega\tau_b}$$

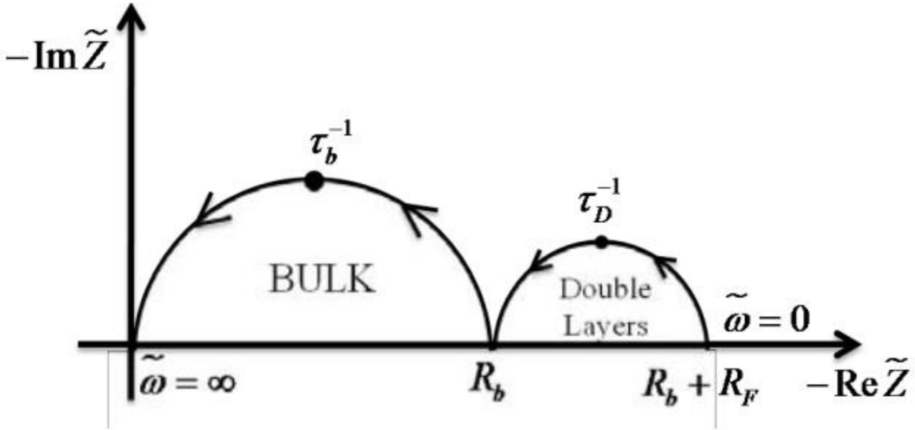


Partially Polarizing Electrode:



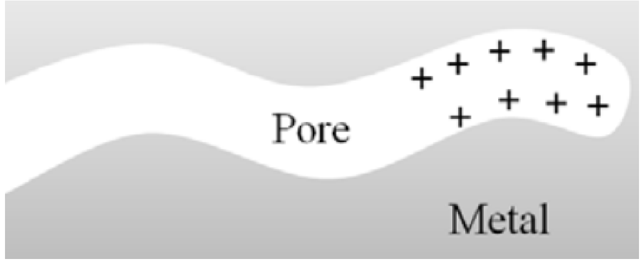
$$Z = \frac{1}{i\omega C_D + 1/R_F} + \frac{1}{i\omega C_b + 1/R_b}$$

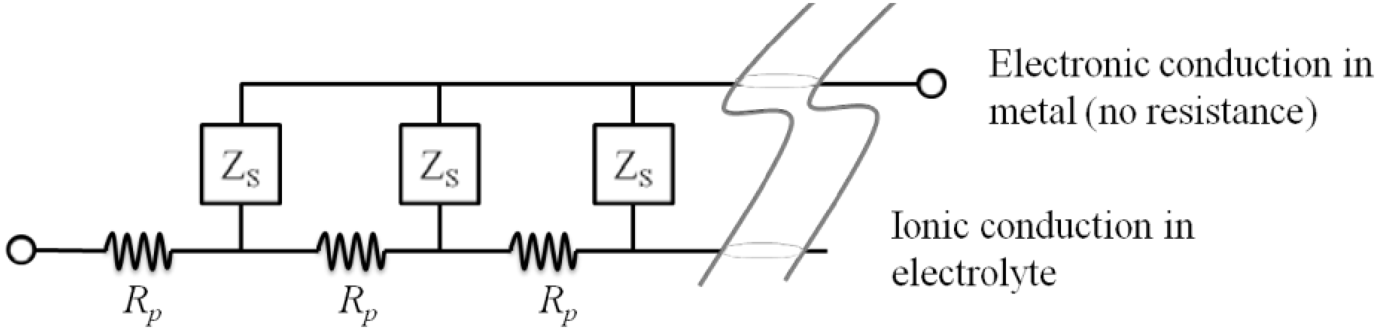
$$\tilde{Z} = \frac{1}{i\omega\tau_D + R_b/R_F} + \frac{1}{1 + i\omega\tau_b}$$



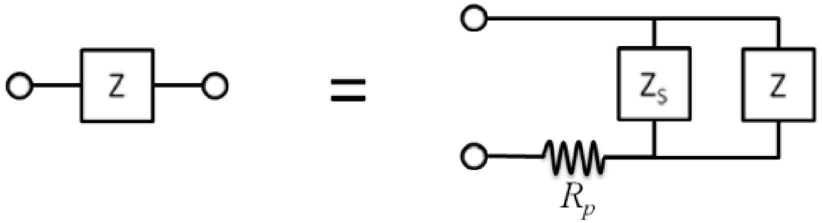
Appendix: Basic Physics of Cells

Homogeneous Microstructures:





Solving the equivalent impedance equation:
(Assuming infinite long with pattern)

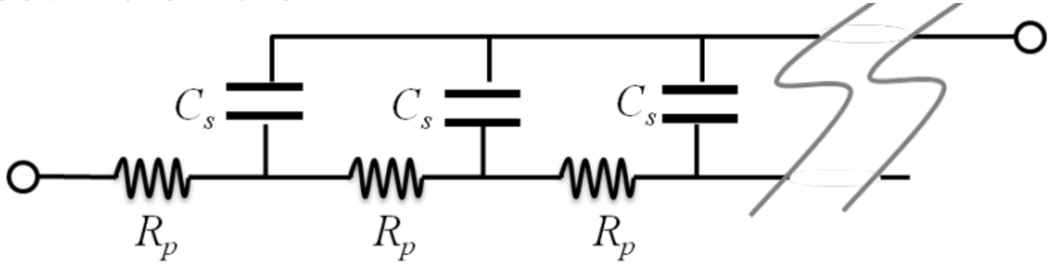


$$\tilde{Z} = \frac{1+\sqrt{1+4\tilde{Z}_s}}{2} \sim \begin{cases} 1 & \text{for } \tilde{Z}_s \ll 1 \\ \sqrt{\tilde{Z}_s} & \text{for } \tilde{Z}_s \gg 1 \end{cases}$$

Where, $\tilde{Z}_s = {Z_s}/{R}$ and $\tilde{Z} = {Z}/{R}$

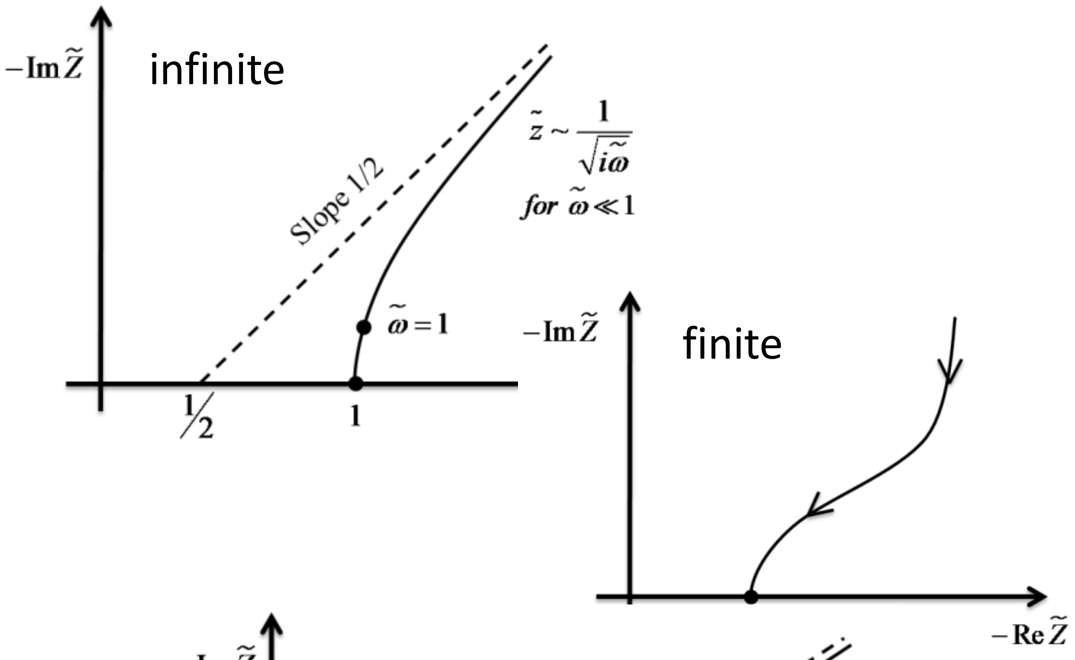
Appendix: Basic Physics of Cells

Smooth Pore Walls:

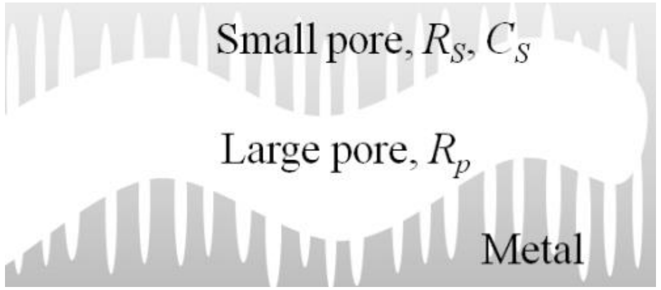


$$\tilde{Z}_s = {}^{1}\!/_{i\omega RC_s} = {}^{1}\!/_{i\tilde{\omega}}$$

$$\tilde{Z} = \frac{1+\sqrt{1+{}^{4}\!/_{i\tilde{\omega}}}}{2} \sim \begin{cases} \dfrac{1}{\sqrt{i\tilde{\omega}}} & \text{for } \tilde{\omega} \ll 4 \\ 1 & \text{for } \tilde{\omega} \ll 4 \end{cases}$$

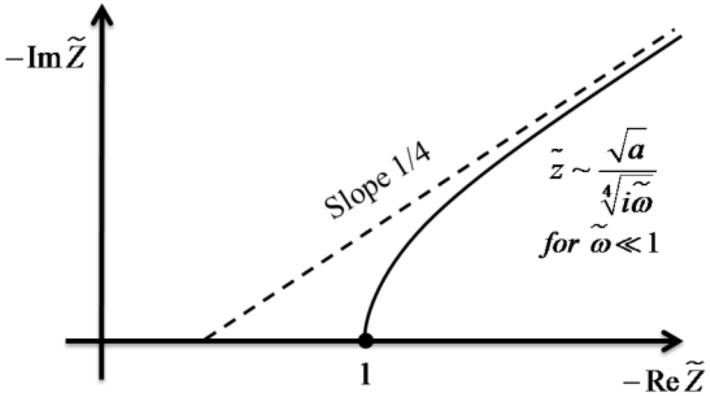


Porous Pore Wall:



$$\tilde{Z}_s = \frac{1+\sqrt{1+{}^{4}\!/_{i\tilde{\omega}}}}{2}$$

$$\tilde{Z} = \sqrt{\frac{1+\sqrt{1+{}^{4}\!/_{i\tilde{\omega}}}}{2}} \sim \frac{1}{(i\tilde{\omega})^{1/4}} \quad \text{for} \quad \tilde{\omega} \to 0$$



Appendix: Warburg Element

Suppose $\Delta V \sim \frac{k_B T}{ne}\ln\left(\frac{C_0+\Delta C}{C_0}\right) \sim \frac{k_B T}{ne}\frac{\Delta C}{C_0}$ for linear response (e.g. Nernst equation, $\Delta C << C_0$) and $\Delta I \sim -neAD\frac{\partial C}{\partial x}(x=0)$

Also, assume quasi-equilibrium reactions at $x=0$ and linear diffusion.

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

Alternating current: $\Delta V = \mathrm{Re}[Ve^{iwt}] \overset{\mathrm{Real}\,V}{=} V\cos(wt)$

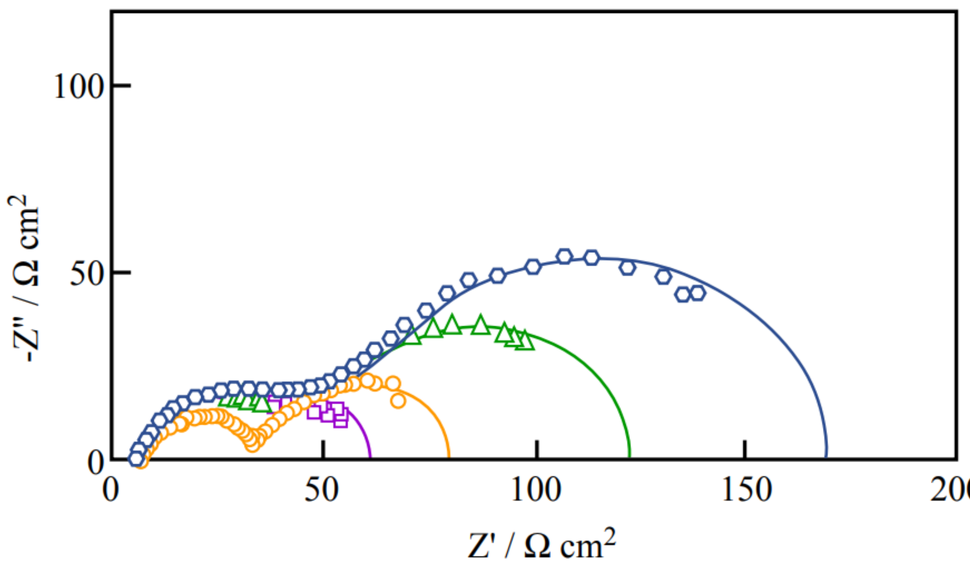
$$\Delta I = \mathrm{Re}[Ie^{iwt}] = |I|\cos(wt+\varphi) \quad I = |I|e^{i\varphi}$$

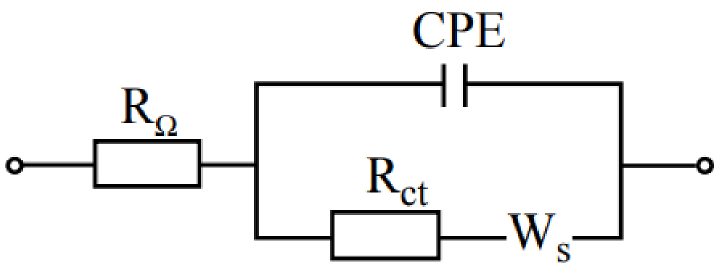
$$\Delta C = C(x,t) - C_0 = \mathrm{Re}[Ce^{iwt}]$$

Therefore, $iwC = DC''$

Hence, $C = C(x=0)e^{-\sqrt{\frac{iw}{D}}x}$ $(C \to 0 \text{ as } x \to \infty)$

$$\frac{C(x=0)}{-C'(x=0)} = \sqrt{\frac{D}{iw}}$$





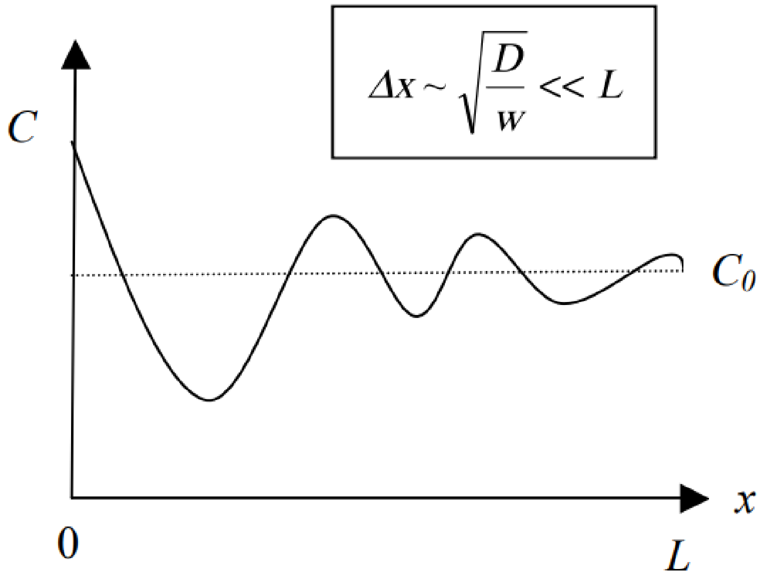
Appendix: Warburg Element

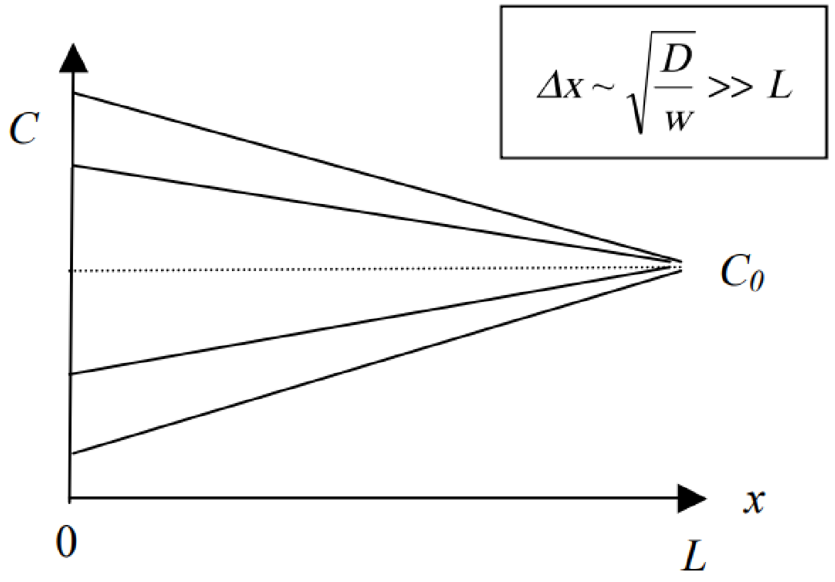
Thus, impedance is

$$Z = \frac{\Delta V}{\Delta I} = \frac{k_B T}{(ne)^2 ADC_0} \frac{C(x=0)}{-C'(x=0)} = \frac{A}{\sqrt{iw}}, \quad A = \frac{k_B T}{(ne)^2 AC_0 \sqrt{D}}$$

For oscillating diffusion layer, the characteristic length is $\Delta x \sim \sqrt{Dt} = \sqrt{\frac{D}{w}}$.

(Fuel Cell)





Appendix: Warburg Element

Now, $C=C_0$ is imposed on $x=L$, not at the infinity. At low frequency ($w \to 0$), FLW acts like a resistor. This situation is depicted in above figure.

Solve $iwC = DC''$ with $C=0$ at $x=L$.

The general solution is $C = A\sinh\left[\sqrt{\frac{iw}{D}}(L-x)\right] + B\cosh\left[\sqrt{\frac{iw}{D}}(L-x)\right]$.

$B=0$ due to $C(x=L)=0$. Thus,

$$C = A\sinh\left[\sqrt{\frac{iw}{D}}(L-x)\right], \quad C' = -\sqrt{\frac{iw}{D}}A\cosh\left[\sqrt{\frac{iw}{D}}(L-x)\right]$$

$$\frac{C(x=0)}{-C'(x=0)} = \sqrt{\frac{D}{iw}}\tanh\left(\sqrt{\frac{iw}{D}}L\right)$$

Hence, the impedance is,

$$Z = \frac{\Delta V}{\Delta I} = \frac{k_B T}{(ne)^2 ADC_0}\frac{C(x=0)}{-C'(x=0)} = Z_0 \frac{1}{L}\sqrt{\frac{D}{iw}}\tanh\left(\sqrt{\frac{iw}{D}}L\right), \quad Z_0 = \frac{k_B TL}{(ne)^2 ADC_0}$$

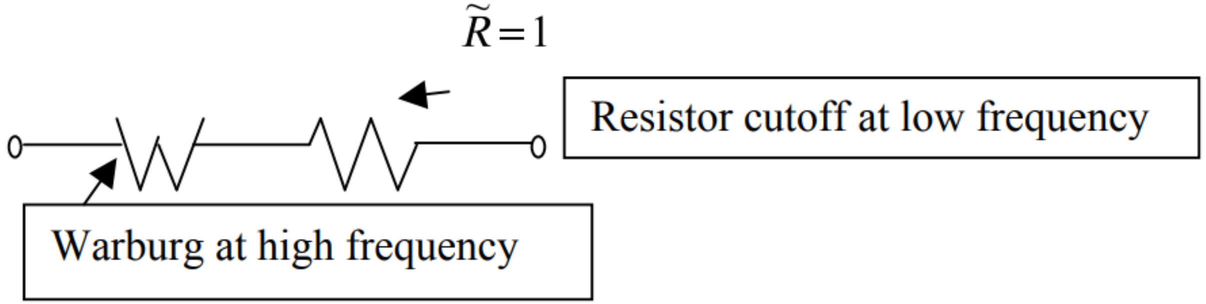
Appendix: Warburg Element

$$\tilde{Z} = \frac{Z}{Z_0} \text{ and } \tilde{w} = \frac{wL^2}{D}$$

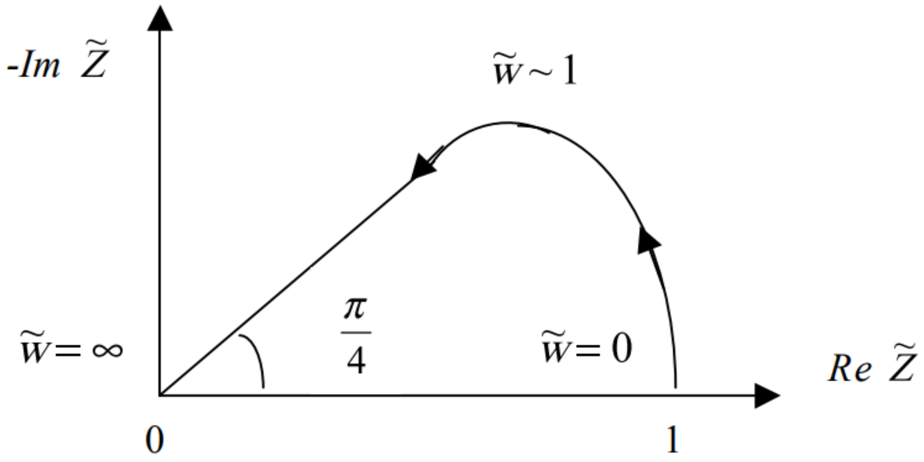
Then, dimensionless impedance is

$$\tilde{Z} = \frac{1}{\sqrt{i\tilde{w}}}\tanh\left(\sqrt{i\tilde{w}}\right) \sim \begin{cases} \dfrac{1}{\sqrt{i\tilde{w}}} (\tilde{w} >> 1) \\ 1 - \dfrac{i\tilde{w}}{3} + \ldots (\tilde{w} << 1) \end{cases}$$

Hence, this is similar to the following circuit, except in the transition region:



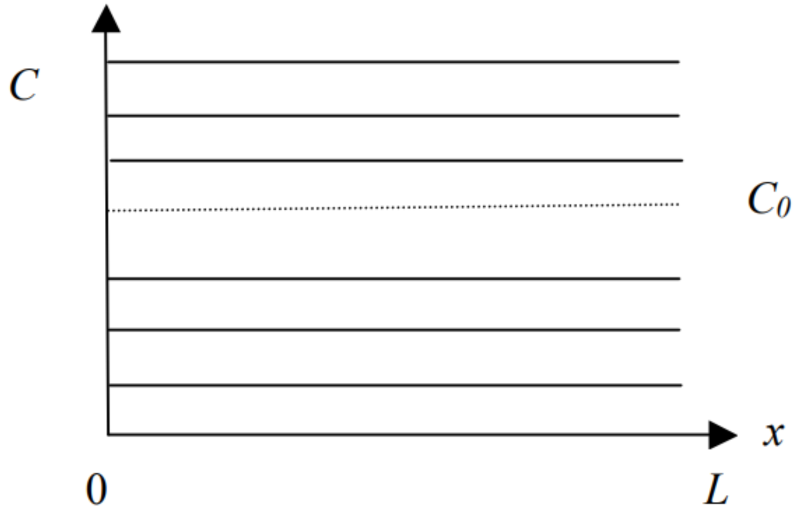
The Nyquist plot is,



Appendix: Warburg Element

Now apply a zero flux Neuman BC at $x=L$ instead of Dirichlet BC, to represent the finite end of intercalation particle. At higher frequency, this system has the same Warburg impedance as before. However, this system acts as a capacitor at low frequency at this time. Schematic figure is as follows.

(Li-ion Cell)



$$\Delta x \sim \sqrt{\frac{D}{w}} >> L$$

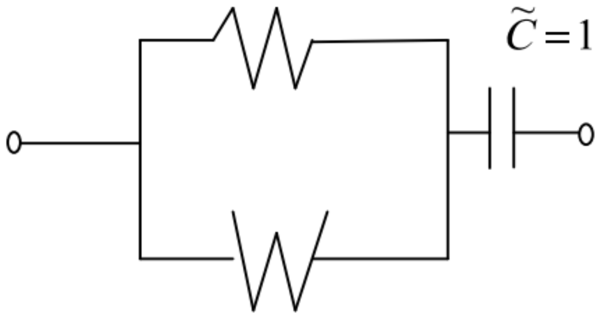
Still, general solution is $C = A\sinh\left[\sqrt{\frac{iw}{D}}(L-x)\right] + B\cosh\left[\sqrt{\frac{iw}{D}}(L-x)\right]$.

Appendix: Warburg Element

Still, general solution is $C = A\sinh\left[\sqrt{\frac{iw}{D}}(L-x)\right] + B\cosh\left[\sqrt{\frac{iw}{D}}(L-x)\right]$.

With $C'(x=L)=0$, $A=0$. $C = B\cosh\left[\sqrt{\frac{iw}{D}}(L-x)\right]$, $C' = -\sqrt{\frac{iw}{D}}B\sinh\left[\sqrt{\frac{iw}{D}}(L-x)\right]$

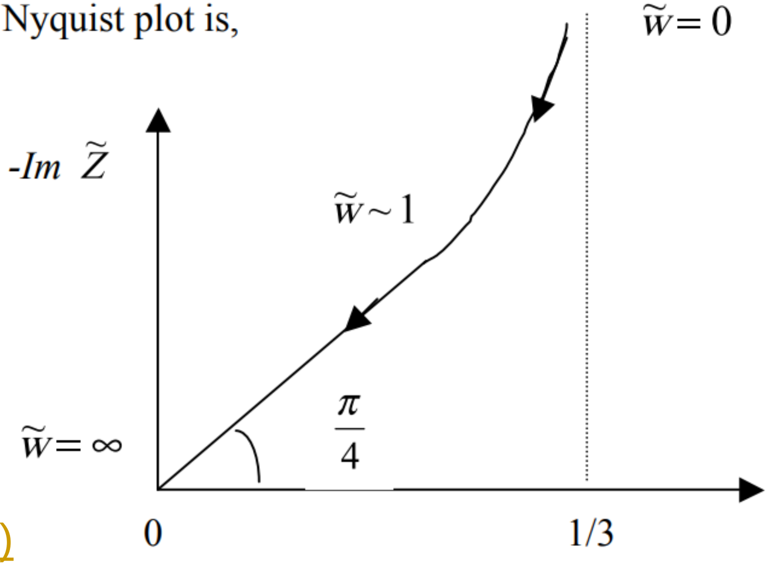
$$\frac{C(x=0)}{-C'(x=0)} = \sqrt{\frac{D}{iw}}\coth\left(\sqrt{\frac{iw}{D}}L\right)$$



Use the same notation as before. Then, dimensionless impedance is

$$\tilde{Z} = \frac{1}{\sqrt{i\tilde{w}}}\coth\left(\sqrt{i\tilde{w}}\right) \sim \begin{cases} \frac{1}{\sqrt{i\tilde{w}}} \; (\tilde{w} >> 1) \\ \frac{i\tilde{w}}{1} + \frac{1}{3} \ldots (\tilde{w} << 1) \end{cases}$$

The Nyquist plot is,



Materials taken from [10.626 Electrochemical Energy Systems (Spring 2011)]