Current Transformer (CT) Theory & Application – Level 1

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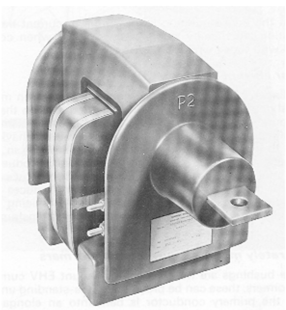
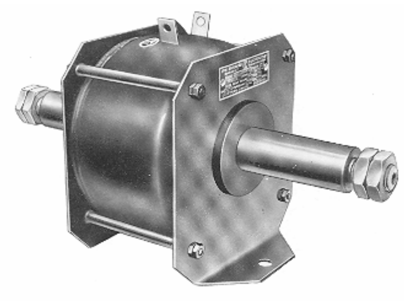
Current Transformer (CT) Introduction

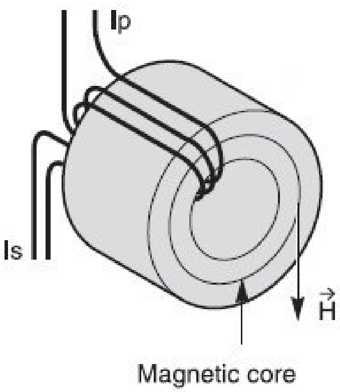
- Function of CT
  - To attenuate power system current (from order of kA to 1A)
  - To insulate secondary circuit from primary system (without using series resistance and measure the voltage across to determine current flowing through)
  - To permit use of standard current rating for secondary equipment (e.g. 1A or 5A relay)
- Measurement CT is required to accurately present load current and protect meters by saturation with fault current.
- Protection CT is required to accurately present fault current smaller than accuracy limiting primary current (= accuracy limiting factor A.L.F. x rated current).
- Burdens are resistance connected at CT to offer a current flow with units of volt-amps (VA) or ohm(Ω). Unless the instrument accuracy is specified at a specific burden, lower burden (i.e. smaller current) usually results in more accurate measurements (i.e. smaller distortion due to voltage drop), based on nameplate ratings. CT are selected to handle a certain amount of burden without exceeding its temperature limit.
- Depending on voltage drop $I_S R_L$ on secondary circuit due to lead resistance and the accuracy requirement, one could select proper CT ratio with rated CT secondary current I_S = 1A or 5A.
- In general, relay in 11kV GIS with all internal wiring would select a 5A relay with CT ratio 400:1, while relay in 132kV protection panel requires CT wiring going from local control panel would use 1A relay with CT ratio 1200:1 or 1600:1, depending on primary capacity.

Types of CT

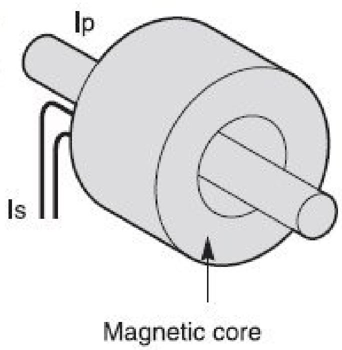
Wound Type Vs Bar Type CT







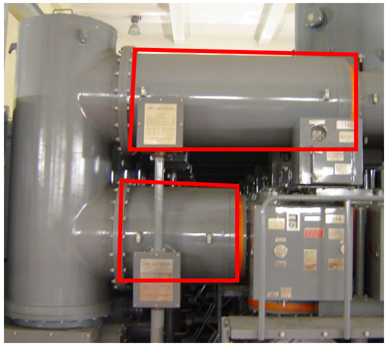
Wound type primary current transformer



Closed core type current transformer

Slip Over Vs Clamp On CT

CT inside GIS



Question –
What is the effect of air-gap in clamp on CT? Why is it not preferred to use clamp on CT in protection purposes?

Types of CT – Remanence and Air Gap

Generally CTs are divided into High, Low and Non-Remanence Type CT.

- High Remanence CTs
  - Magnetic Core without any air gaps
  - Remanent Flux remain for almost infinite time
  - Remanent Flux could be up to 70% to 80%
  - Typical examples are – Class P, Class X and Class TPS / TPX
- Low Remanence CTs
  - Specified limit for remanent flux
  - Small air gaps is provided to reduce remanent flux
  - Remanent Flux up to 10% of Saturation Flux
- Non Remanence CTs
  - Negligible level of remanent flux
  - Large air gap to reduce secondary time constant (to lower needed transient factor)

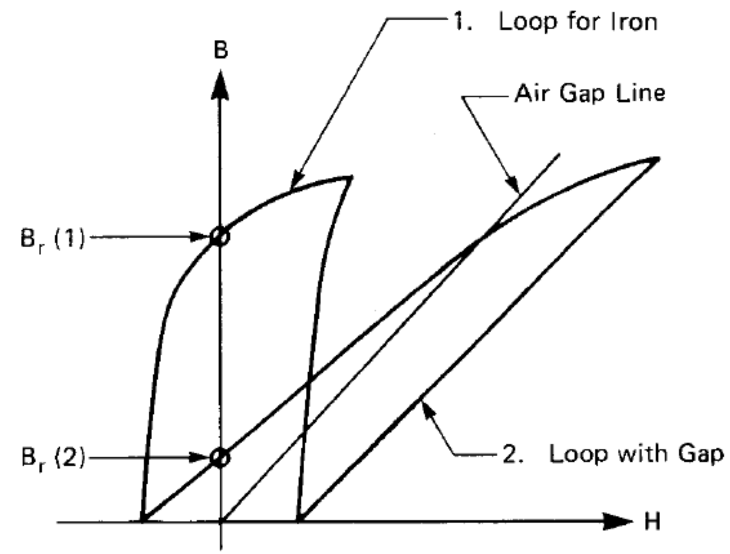
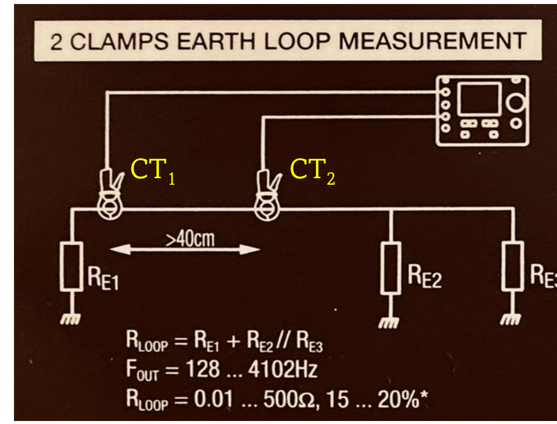


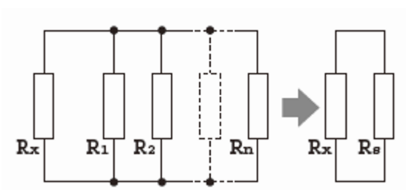
Figure 2. How air gap reduces remnant flux

- Advantage of Gapped Core
  - Reduction of remanent flux with improved transient performance
  - Reduction of time constant allow smaller core cross section (small size)
  - Lower effect of burden power factor on oversizing to prevent saturation
- Disadvantage of Gapped Core
  - Larger magnetizing current leads to higher ratio and phase angle error

Types of CT – Two-Clamp Earth Loop Measurement

- Leakage-to-Frame Busbar Protection requires a switchgear bar (in grey) to have at least 10Ω to earth to ensure the sensitivity for busbar earth fault.
- To measure the earth loop resistance of Leakage-to-Frame Busbar Protection, an Earth & Resistivity Tester (2 Clamp mode) could be used.
- A voltage (V) is applied to the object through voltage injection transformer CT_1 and the current induced (I) is obtained through CT_2. The earth loop resistance could be measured with V / I, given that the resistance in shunt is negligible.



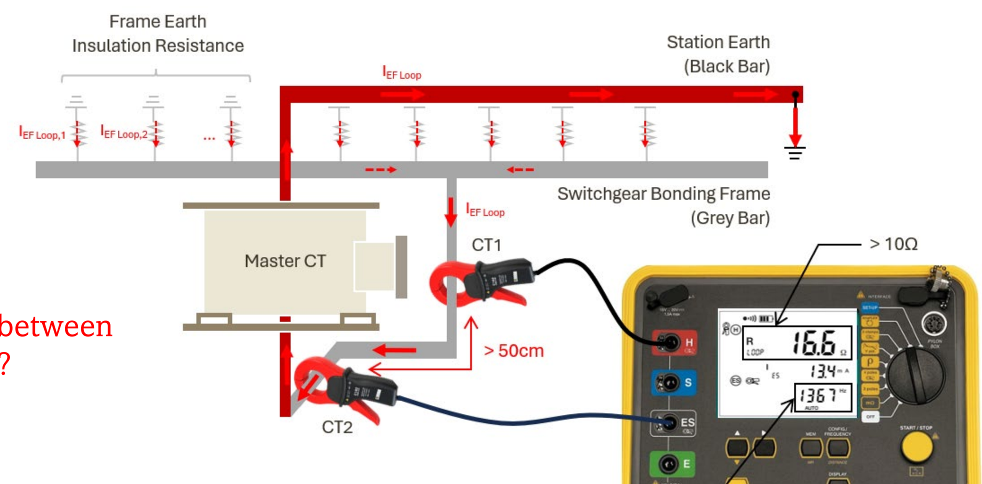


$$\frac{V}{I} = R = R_X + R_S, R_S = \frac{1}{\sum_i \frac{1}{R_i}}$$

If $R_X >> R_S$

$$\frac{V}{I} = R_X$$

Question –
Does the distance between two clamps matter?



CT1 (H): Voltage Injection
CT2 (ES): Current Measurement

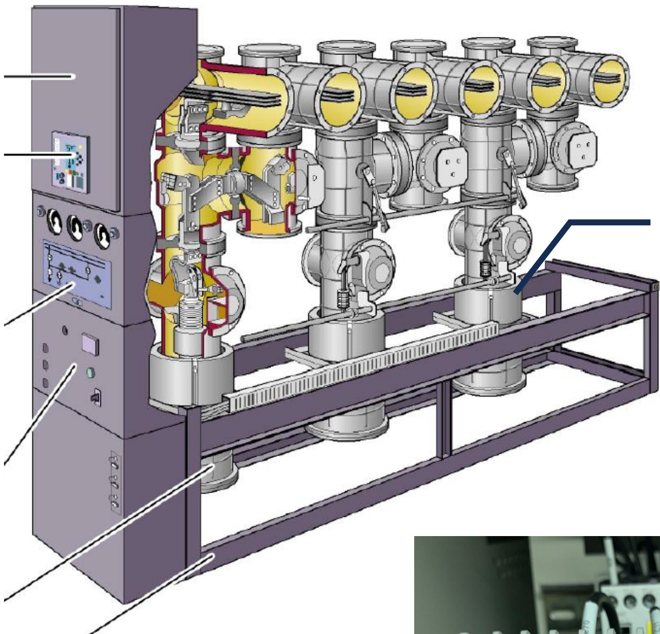
- Similar resistance measurement is used to measure earth mat resistance of substation.

CT with Half Ratio

Question –

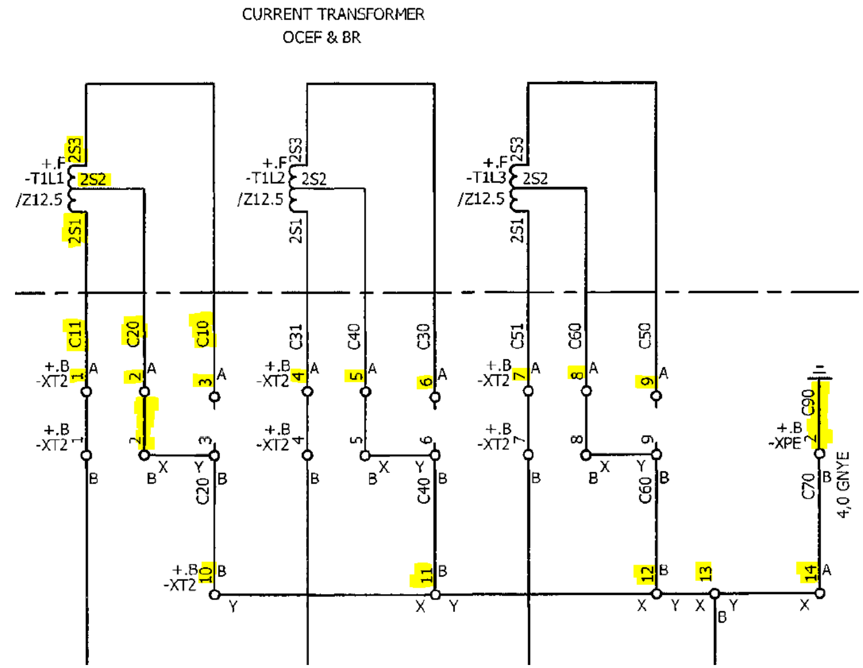
Is there any current output at half ratio (e.g. C11 – C20) if full ratio CT terminal (e.g. C11 and C10) shorted?

A Translay S relay was tripped under through fault with another end (load end, with CT in lower knee point) remain stable. Why? [MTR]



CT designed to be at the external part of switchgear.

Question –
What is the problem for such design?



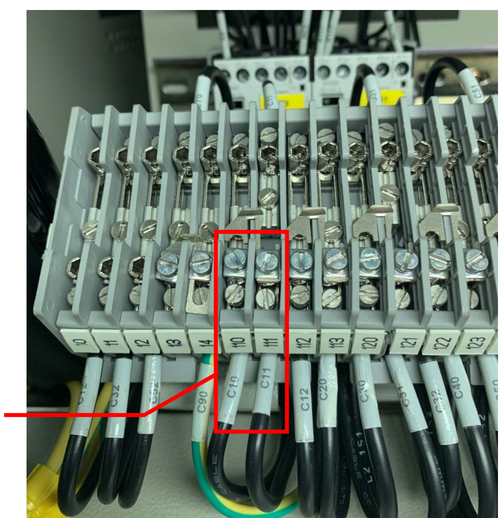
[POS392]

Under a planned outage and during shifting N.O., OCEF protection operates without any voltage dip. It was found that **CT ratio** (200/1 instead of 400/1) on site were **mismatched** with OCEF relay setting. With actual load marginally above 100A, current of around 210A was measured by OCEF relay.

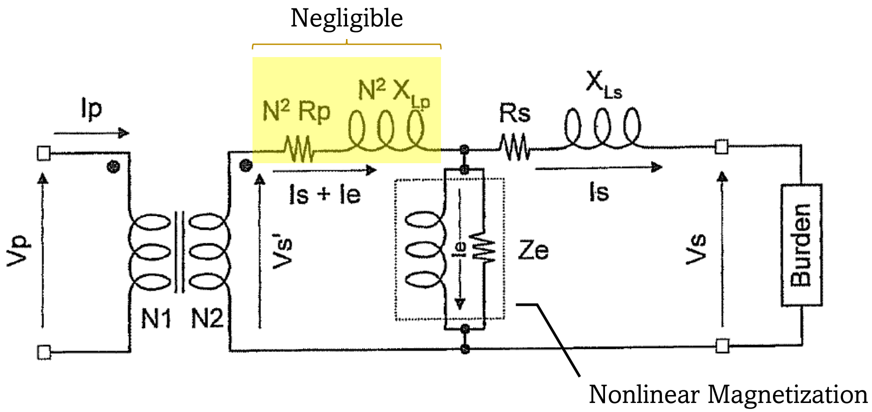
It was found spill current exists during CT balance test (normal condition).

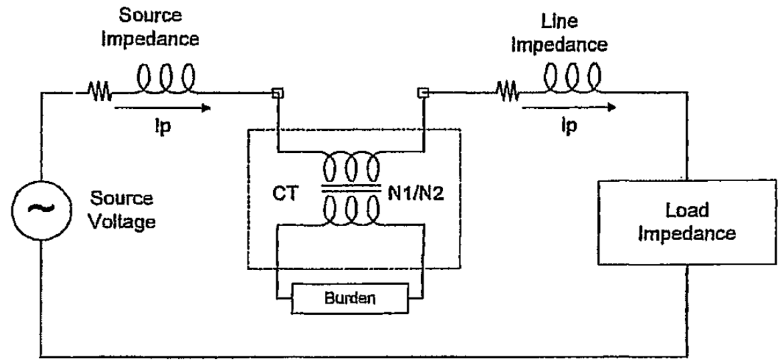
It was found that the wires at 110 and 111 were swopped.

Question – Why?



Ideal CT Model





Reflect the primary impedance to secondary –

$$\frac{V_p}{V_s} = \frac{N_1}{N_2}, \qquad \frac{I_p}{I_S} = \frac{N_2}{N_1} \quad \rightarrow \quad \frac{V_p/I_p}{V_s/I_s} = \left(\frac{N_1}{N_2}\right)^2 \quad \rightarrow \quad \frac{Z_p}{Z_s} = \left(\frac{N_1}{N_2}\right)^2 = \frac{1}{N^2}$$

From this equation –

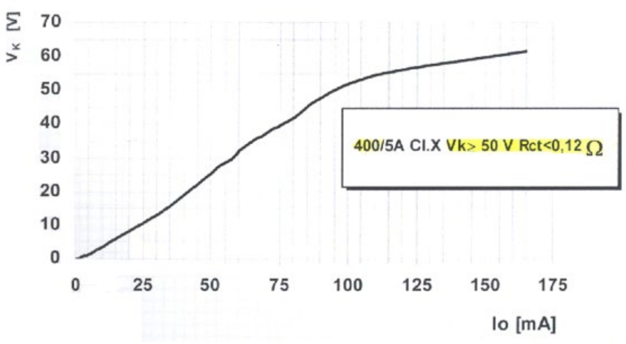
$$Z_p' = N^2 Z_p = N^2(R_p + j\omega L_p)$$

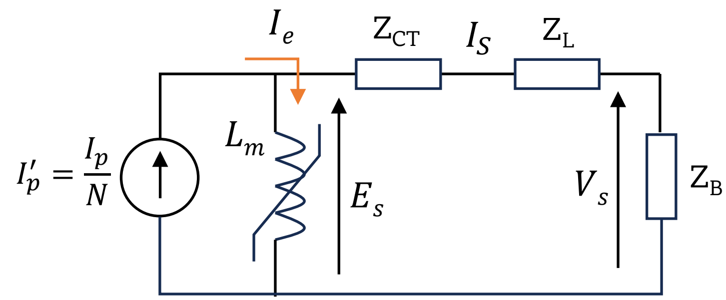
Ideally, CT are constant current device where low leakage impedance is desirable, such that the instrumentation has much less impedance of the circuit being instrumented.

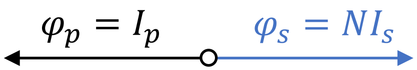
One could perform impedance test with primary current injection to obtain the series impedance Z_{CT} in sum, and obtain the shunt impedance L_m with its excitation curve.

Question – Can CT provides higher output than I_{pri} /N ?

Ideal CT Model







- Primary Ampere Turns = Secondary Ampere Turns $\quad 1 \times I_p = N \times I_S$
- Secondary Voltage –

$$E_S = -N\frac{d\varphi}{dt} = I_S(Z_{CT} + Z_L) + V_S < \boxed{V_K}$$

Knee Point Voltage

- Primary Operating Current (POC) –

$$I_p' = \boxed{I_e} + I_s = \boxed{\frac{E_S}{j\omega L_m}} + \boxed{I_S}$$

Setting Current

Excitation Current to be checked at Magnetizing Curve

- If the CT goes saturation because of large voltage built up with large burden or large input voltage, the shunt branch will have a small saturation inductance and exhibit a shorted behaviour.

Question – What is the maximum primary current allowed such that the CT will not be saturated?

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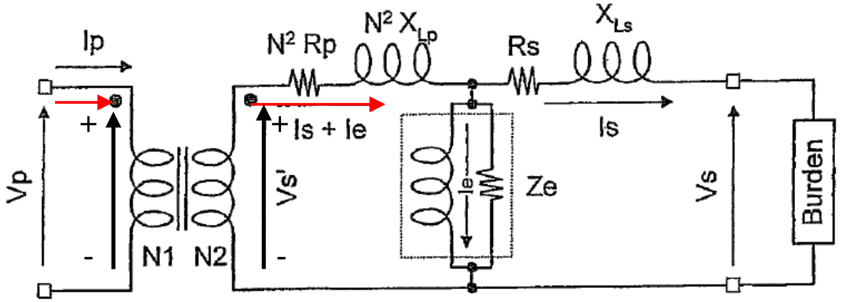
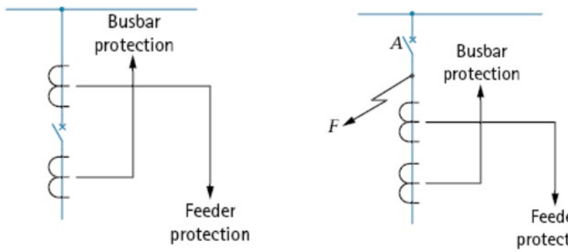
CT Terminology

- Polarity

Polarity designations of instrumentation transformer allow proper phasing. Dot Convention beside the transformer windings in the figure denote the polarity terminals. Convention dictates that primary current into the polarity terminal induces secondary current out of the polarity terminal.





(a) CT's on both sides of circuit breaker (b) CT's on circuit side of circuit breaker

Question – Does it matter if both P1-P2 and S1-S2 are reflected?

- Rated Continuous Thermal Current [kArms]

Value of current in rms can be permitted to flow continuously in the primary winding, the secondary winding being connected to the rated burden without the temperature rise exceeding the value specified.

Question – Does it matter if we use 2000/1000/1 if the primary rated current is 2000A?

- Rated Dynamic Current [kApk]

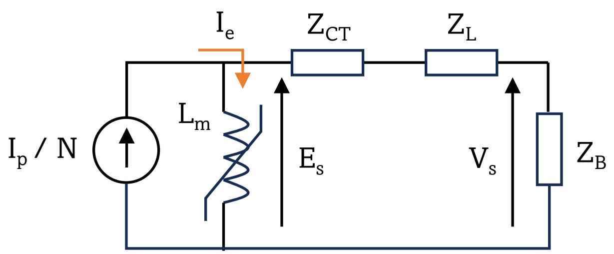
Maximum peak value of the primary current which a transformer will withstand, without being damaged electrically or mechanically by the resulting electromagnetic forces, the secondary winding being short-circuited.

CT Terminology

- Ratio Error

  Error introduced to measurement of current as actual transformation ratio is not equal to rated transformation ratio.

$$\text{Ratio Error [\%]} = \frac{nI_S - I_P}{I_p}$$



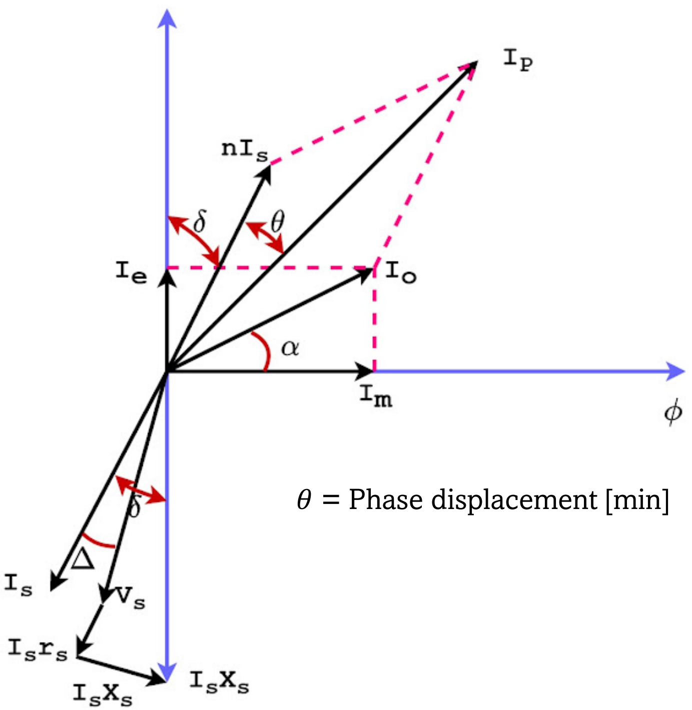
- Composite Error

  Under steady state condition, rms value of the difference between instantaneous values of primary current and that of actual secondary current multiplied by the rated transformation ratio.

$$\text{Composite Error [\%]} = \frac{1}{I_p}\sqrt{\frac{1}{T}\int_0^T (Ni_s - i_p)^2 dt}$$

Another definition used for calculating accuracy limiting factor is

$$|I_0| = \sqrt{I_S^2 + \left(\frac{I_P}{n}\right)^2 - 2(I_S \frac{I_P}{n}\cos\theta)} \qquad \varepsilon_c = \frac{|I_e|}{\frac{I_P}{n}ALF} < 5\%$$



Question –
When is ratio error not applicable in defining its accuracy?

CT Terminology

- Transformation Ratio

$$I_p = \sqrt{I_0^2 + (nI_s)^2 + 2I_0 nI_s \cos(90^o - \alpha - \delta)}$$

$$T = \frac{I_p}{I_s} = \frac{\sqrt{I_0^2 + (nI_s)^2 + 2I_0 nI_s \sin(\alpha + \delta)}}{I_s}$$

As the magnetizing current I_0 is very small compared to primary current I_p. The expression can be simplified as

$$T = \frac{\sqrt{I_0^2 \sin^2(\alpha + \delta) + (nI_s)^2 + 2I_0 nI_s \sin(\alpha + \delta)}}{I_s}$$

$$T = \frac{nI_s + I_0 \sin(\alpha + \delta)}{I_s} = n + \boxed{\frac{I_0}{I_s}\sin(\alpha + \delta)}$$

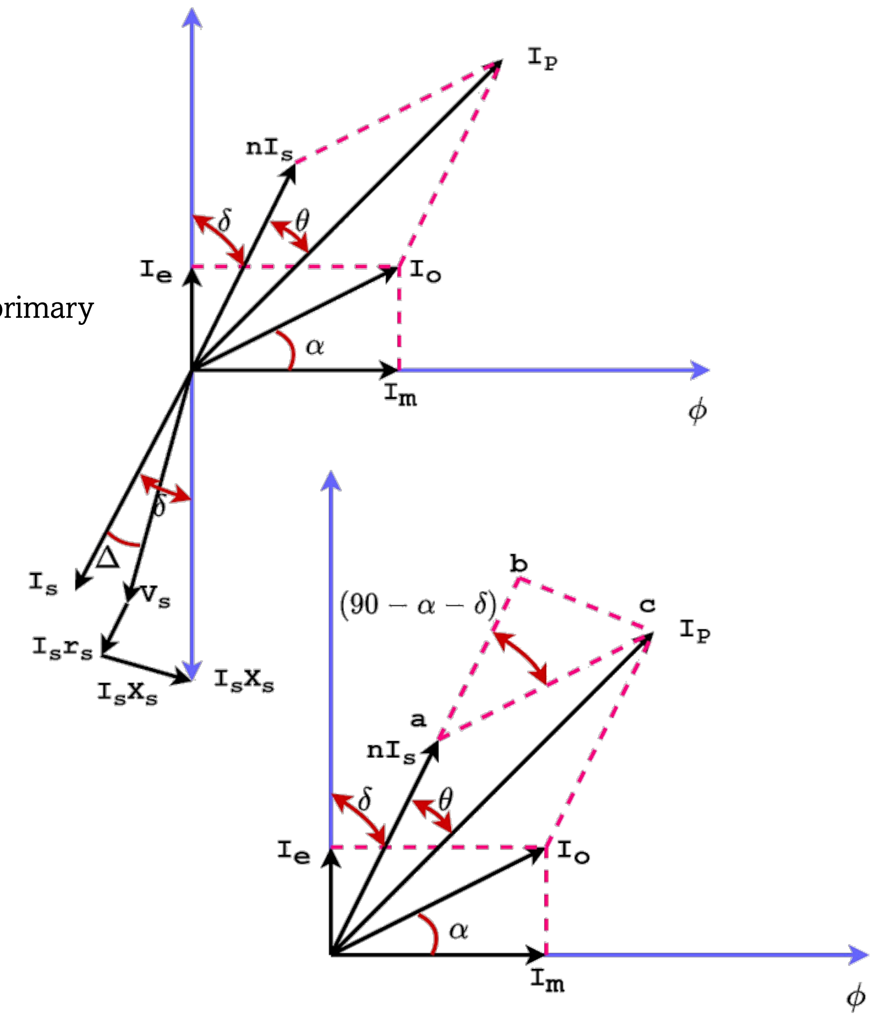
- Phase Displacement d [min]

Given that θ is small and $\tan\theta = \theta$

$$\tan\theta = \theta = \frac{I_0 \cos(\alpha + \delta)}{nI_s + I_0 \sin(\alpha + \delta)} \approx \frac{I_0 \cos(\alpha + \delta)}{nI_s}$$

$$\theta = \frac{I_0 \cos\alpha \cos\delta - I_0 \sin\alpha \sin\delta}{nI_s} = \frac{I_m \cos\delta - I_e \sin\delta}{nI_s}$$

If resistive burden, $\delta = 0$, $\theta = \frac{180^o}{\pi}\frac{I_m}{nI_s}$



CT Terminology

- Error Diagram

Given Overcurrent Factor =

$$K_{sscn} = \frac{I_{psc}}{I_{pn}}$$

where I_{psc} is primary symmetrical short-circuit current.

The error limit listed in IEC60033-1 is the minimum requirement for dimensioning a protection CTs.

The grey area with description 5P20 is the forbidden area for the error of such protective-core CT.

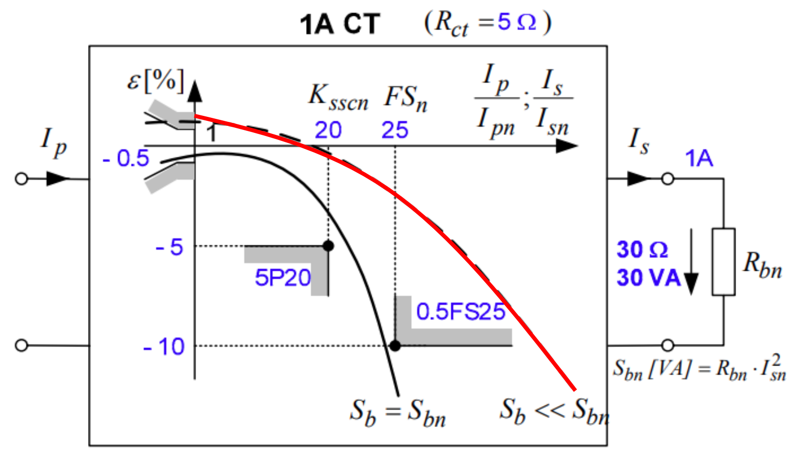
For class P the maximum error current I_{al} as magnetizing current at nominal overcurrent factor K_{sscn}

$$I_{al} = \varepsilon_c K_{sscn} I_{sn} \qquad \text{(1A in the example)}$$

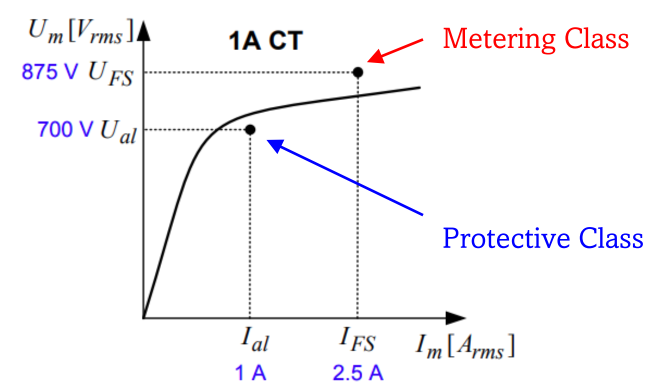
The corresponding calculated accuracy limiting voltage would be

$$U_{al} = K_{sscn} I_{sn} (R_{CT} + R_B) \quad \text{(700V in the example)}$$

The protective class 5P20, 30VA in this example could fulfil a measuring class 0.5FS25, 30VA where instrument security factor FS the error must be at least 10% to against high overcurrent into the meter.



Burden Dependent



CT Terminology

- Burden

  Value of secondary impedance in ohm or in VA at the rated secondary current at the relevant power factor.

$$\text{Burden } [\Omega] = \frac{(\text{Rated VA})}{(\text{Rated I})^2}$$

- Accuracy Limiting Factor

  Ratio of rated accuracy limiting current to the rated current.

$$\text{Accuracy Limiting Current} = \text{ALF } \times \text{Rated I}$$

- Accuracy Limiting Current

  Value of primary current [kA] up to which the CT will comply with the requirement of composite error.

- Knee Point V_K

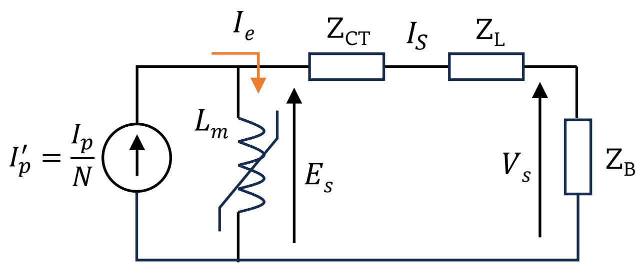
  Sinusoidal e.m.f. of rated frequency applied to the secondary terminals of the transformers, with all other windings being open circuited, which, when increased by 10% causes the exciting current to increase by 50%

- Remanent Flux

  Value of secondary linked flux which would remain in the core 3 minutes after the interruption of a magnetizing current of sufficient magnitude to induce saturation flux

Question – It was known that an engineer mistakenly placed an order to buy a Class P CT for current differential. He wonders if he could use the Class P CT with high enough knee point for CD operation.

Indirect Measurement Method for ALF



1. Calculate the total impedance $Z = \sqrt{(R_b + R_{CT})^2 + X_b^2}$, $R_B = Z_B \cos\varphi$

2. Determine the excitation current I_E and magnetizing voltage E_S where I_E / I_S = 0.05, with I_S = E_S / Z, given that the error current would be 5% for accuracy limit.

3. Calculate the indirect measured ALF (ALFi) with

$$\text{ALFi} = \frac{E_S}{Z\, I_S}$$

4. The ALFi can be verified with

$$\text{ALF (Rated) } I_S \geq \frac{E_S}{Z}$$

Question – Does knee point voltage correlate to accuracy limiting factor (ALF)?

Classification of CT under IEC 60044-1

- Limit of Current Error and Phase Displacement for **Measurement CT**

Class	± percentage current (ratio) error at percentage of rated current shown below				± phase displacement at percentage of rated current shown below in minutes			
	5	**20**	**100**	**120**	**5**	**20**	**100**	**120**
0.1	0.4	0.2	0.1	0.1	15	8	5	5
0.2	0.75	0.35	0.2	0.2	30	15	10	10
0.5	1.5	0.75	0.5	0.5	90	45	30	30
1.0	3.0	1.5	1.0	1.0	180	90	60	60

Class	± percentage current (ratio) error at percentage of rated current shown below	
	50	**120**
3	3	3
5	5	5

- Limit of Current Error and Phase Displacement for **Protection CT**

Class	Current error at rated primary current (%)	Phase displacement at rated primary current (minutes)	Composite error at rated accuracy limit primary current (%)
5P	±1	±60	5
10P	±3	---	10

Class P Protection CT

- Class P Protection CT are often specified in form of 15VA – 5P20 (1A Output)
- It is in form of Rated VA – Accuracy Class – Accuracy Limiting Factor (ALF)
- Recall

$$\text{Rated Burden } [\Omega] = \frac{\text{Rated VA}}{(\text{Rated I})^2} \quad \text{and} \quad \text{Accuracy Limiting Current } [\text{A}] = \text{ALF} \times \text{Rated I}$$

- Output Voltage V_S

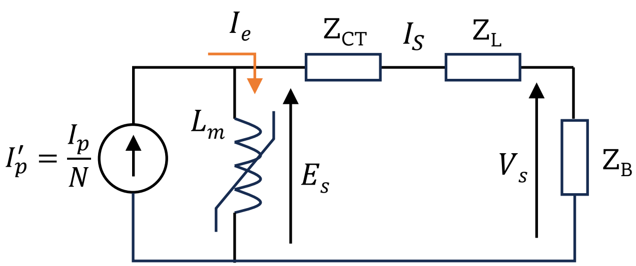
$$V_S = \text{Accuracy Limiting Current} \times \text{Rated Burden}$$

- Output VA

$$\text{Output VA} = V_S \times \text{Accuracy Limiting Current}$$

- Magnetizing Voltage E_S

$$E_S = I_S(Z_{CT} + Z_L) + V_S < E_K$$

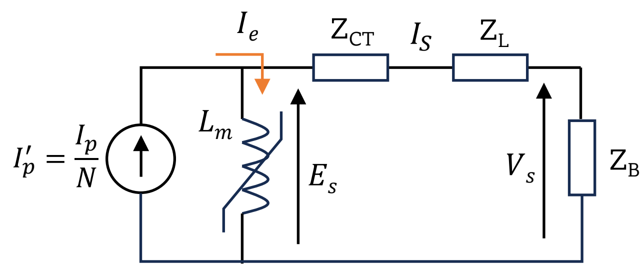


CT Specification	Rated Burden [Ω] = Rated VA / (Rated I)^2	Acc. Limiting Current [A] = ALF x Rated I	Output Voltage [V] = Rated Burden x Acc. Lim Current	Knee Point Voltage Required $E_S < E_K$ = $E_S = I_S Z_{eq} + V_S$
5VA 5P10 (1A)	5 / 1^2 = 5Ω	10 x 1 = 10A	= 10A x 5Ω = 50V	= $10Z_{eq}$ + 50
10VA 5P5 (1A)	10 / 1^2 = 10Ω	5 x 1 = 5A	= 5A x 10Ω = 50V	= $5Z_{eq}$ + 50
10VA 10P10 (1A)	10 / 1^2 = 10Ω	10 x 1 = 10A	= 10A x 10Ω = 100V	= $10Z_{eq}$ + 100
10VA 10P10 (5A)	10 / 5^2 = 0.2Ω	10 x 5 = 50A	= 50A x 0.2Ω = 10V	= $50Z_{eq}$ + 10

Note – Larger core cross-section area A = Larger knee point voltage V_K → 5VA requires a larger core than 10VA

Class P Protection CT

Example 1



Consider a 5VA 5P10 (400:1) CT with 1.27Ω DC resistance and 2Ω burden. Assume excitation current at knee point voltage would be 0.1A

(a) Determine the maximum primary current such that the CT will not be saturated if the CT resistance is measured to be 1.27Ω and the connected burden is 2Ω.

(b) Determine the accuracy limiting factor if the output current is rated.

Solution

(a) Rated Burden = 5 / 1^2 = 5Ω, Accuracy Limiting Current = 10 x 1 = 10A

Output Voltage = 10A x 5Ω = 50V, Magnetizing Voltage = 50 + 1.27(10) = 62.7V < Knee Point Voltage (E_K).

Assume unknown knee point voltage > 62.7V. I_{max} (1.27 + 2) < 62.7V → I_{max} < 19.17A

At 62.7V, the operating current will be $I_{OP} = I_S + I_e$ = 19.17A + 0.1A = 19.27A (= 19.27 x 400 = <u>7.7kA</u>).

(b) E_K = I x ALF x (Rated Burden + CT Resistance) 62.7 = 1 x ALF x (1.27 + 2) → ALF = <u>19.17</u>

Burden and OCEF Setting

Question –

What are the considerations in sizing a CT for OCEF application?

Example 2

Three 100/1 CTs with a linear characteristic as follows:

- Voltage rises from 0V to 20V when the magnetizing current rises from 0 to 0.1A
- Voltage rises from 20V to 24V when the magnetizing current rises from 0.1 to 1A

The CTs are connected in parallel and supply an earth fault overcurrent relay of 1A rating. The relay absorbs 5 VA at setting current. Plot the actual operating current against setting current if the relay setting range is from 20% to 80% of relay rating in steps of 10%. Comment on the effective relay operating current. Assume the CT has no loss and the relay burden is purely reactive.

Solution

At 100% setting, $Z_R = j(5)/(1^2) = j5\Omega$

At 80% setting, $Z_R = j(5)/(0.8^2) = j7.8125\Omega$

Relay voltage, $E_S = 0.8 \times 7.8125 = 6.25\text{V}$

$I_m = 0.1 \times (6.25/20) = 0.03125$ A

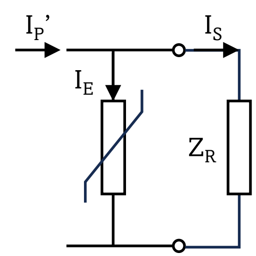
$I_{op} = (0.8 + 3 \times 0.03125) \times 100 = 89.375\text{A}$

Similarly at 60% setting, $I_{op} = 72.5\text{A}$

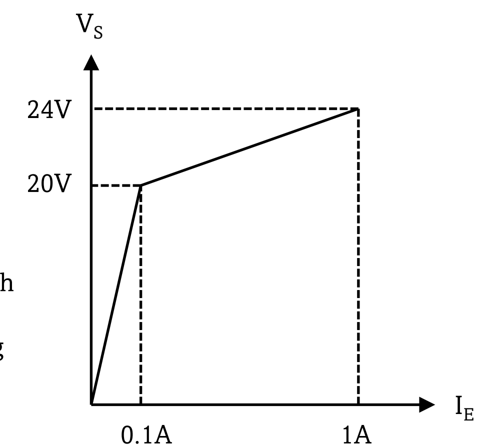
at 40% setting, $I_{op} = 58.75\text{A}$

$\boxed{\text{at 20\% setting, } I_{op} = 387.5\text{A}}$

Due to
Excessive Excitation Current



Goal – tune the burden with setting current to achieve different primary operating current



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- Introduction to Current Transformer (C.T.)
  - Types of CT
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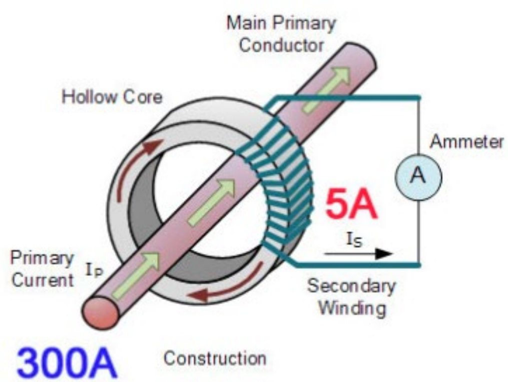
CT Ratio Test and Error

- Phase error is not important for relays which operate on current magnitudes only. It is important to sensitive directional relays.
- Composite error is defined as the r.m.s. value of the difference between the ideal secondary current and the actual secondary current; it includes current and phase errors and the effects of harmonics in the exciting current. It is used to calculate accuracy limiting factor.
- Exciting Current and Core Loss are the main causes of ratio error and phase error of CT. The error can be reduced by –
  - Using a "better" magnetic material
  - Reducing the mean length L of magnetic path
  - Reducing the flux density (B x A) in the iron core

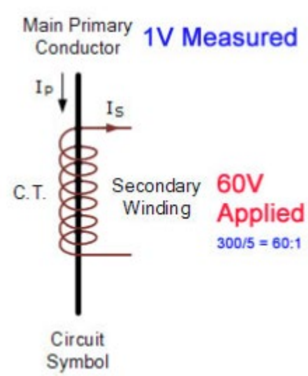
Question –

How do these suggestion help reduce the error?

Is there any problems if such suggestions applies?



300:5 CT Example
Primary current injection ratio test

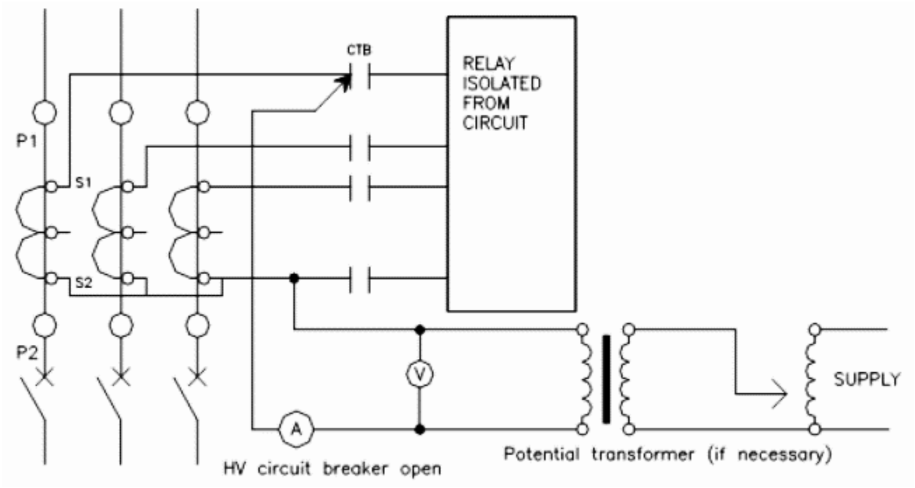


CT Ratio Test
Secondary voltage injection method

Excitation Test (Magnetizing Curve)

- Metering Class CT uses low magnetizing current core (Nickel Iron Alloy) with low core loss. It follows that a high permeability core with low saturation such that meters can be protected at high current.
- Protection Class CT uses core (Grain Oriented Silicon Steel) with high saturation level, even it often requires a higher exciting current, which is a consideration in designing high impedance scheme.

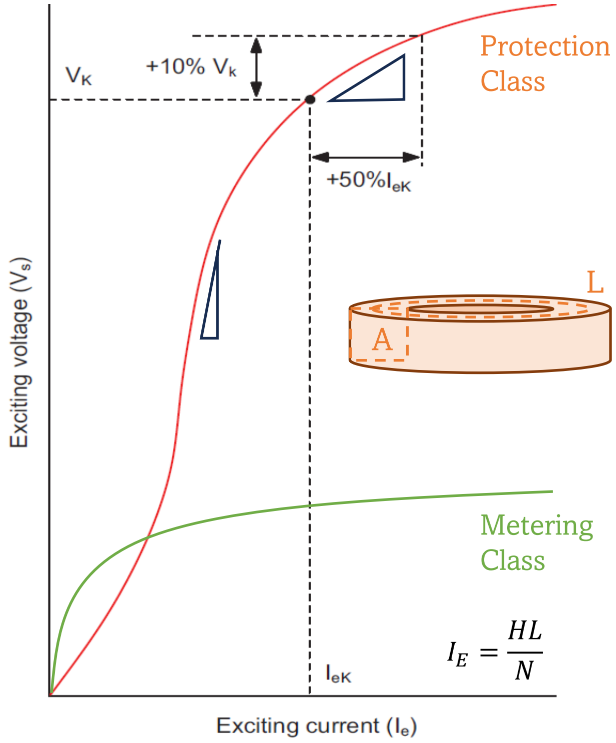
Excitation Curve Test Set



Increase the voltage at secondary and monitor the excitation current up to the CT reaching near to saturation point. Record the reading of voltage and current at several points.

Open Circuit Excitation Curve

$V_s = 4.44fAN \times B = K_V B$



Question –

What if one of the path is still shorted? How does it affect the magnetizing curve?

Excitation Curve by Inductance Model

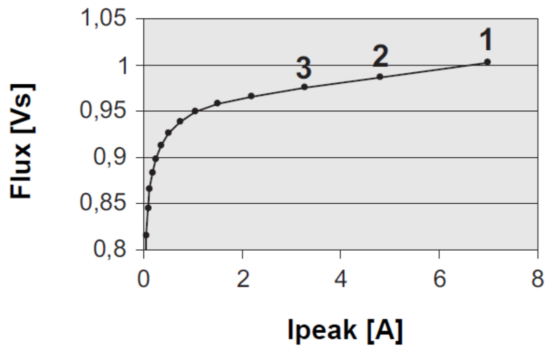
Saturation Inductance

- Measure the flux (V_S) in the main inductivity (i.e. behind winding resistor)
- Determine the saturation inductance by –

$$L_{ij} = \frac{\psi_i - \psi_j}{I_i - I_j}, \qquad i \neq j, \qquad i, j \in 1, 2, 3$$

- If all of the following conditions apply, the saturated inductance can be determined with L_{13}.
  - $I_K < 1A$ AND $I_1 < 5\ I_K$
  - $0.5 < L_{12} / L_{23} < 1.5$
  - $L_{13} > 30mH$.

where I_K is the knee point current.



Non-Saturated Inductance

- Determine the inductance of one point as

$$L_i = \frac{\psi_i}{I_i}, \qquad i = 1, 2, \ldots, n$$

- Determine the total unsaturated inductance as

$$L_m = \frac{1}{n} \sum_{L_{20\%}}^{L_{90\%}} L_i$$

Question –

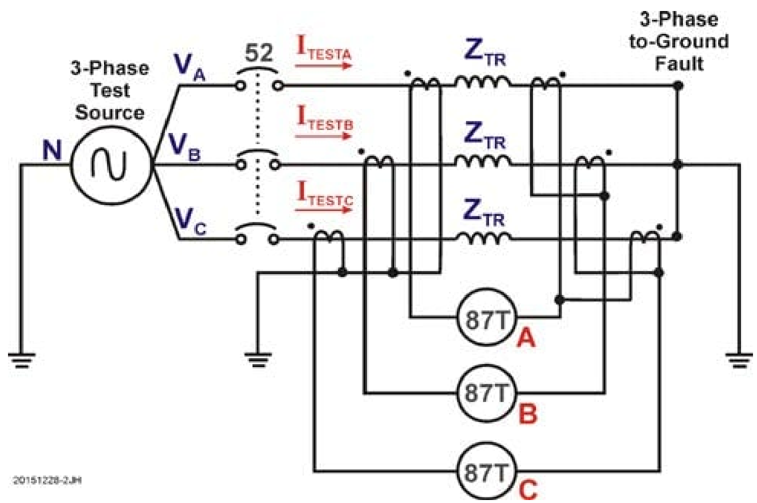
Why determining excitation curve with inductance model approach becomes a more popular method nowadays?

Other CT Tests

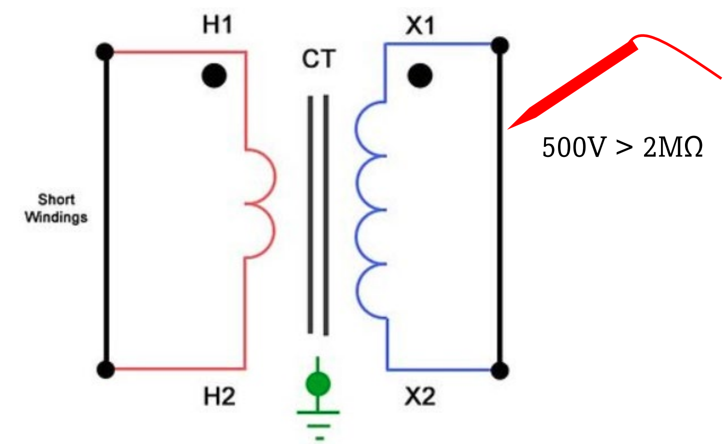
Question – How to arrange the order of performing the CT tests, in terms of its short/open configuration, injection requirement and remanent flux due to DC injection leading to large error?

Question – Why do we need to perform spill test or balance test?

Balance Test (Spill Test)

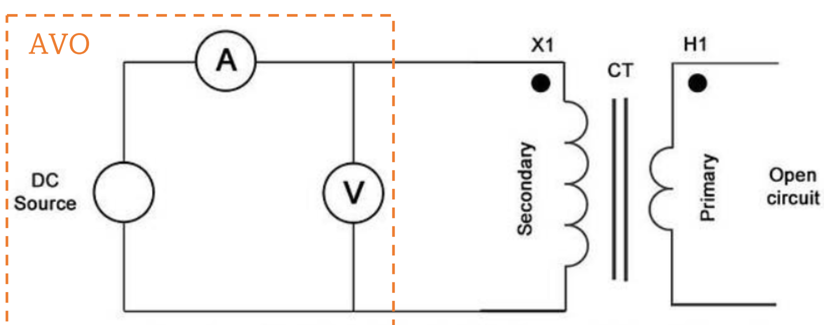


Insulation Resistance Test (Megger Test)



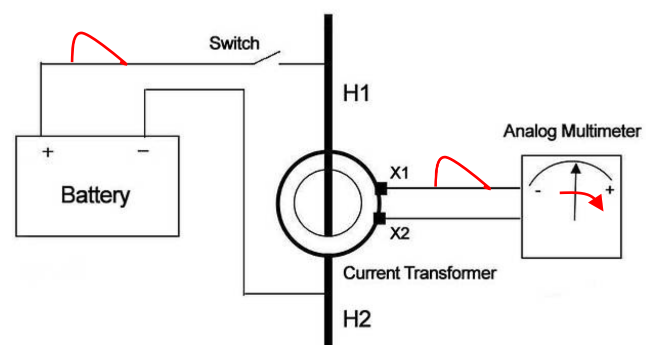
1. Primary to secondary: Checks the condition of the insulation between high to low.
2. Primary to ground: Checks the condition of the insulation between high to ground.
3. Secondary to ground: Checks the condition of the insulation between low to ground.

DC Resistance Test



The resistance of the CT winding is found by dividing the measured voltage with dc current.

Polarity Test (Flick Test)



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Class X CT

Class PX protective CT is a quasi-transient CT with low leakage reactance for which knowledge of the transformer secondary excitation characteristics, winding resistance, burden resistance and turn ratio is sufficient to assess its performance.

Note – IEC60044-1 specifies for inductive CT with steady state symmetrical AC current.

It normally used for cases where accurate current balance is required, e.g. circulating current (CC) / high impedance busbar zone (HZBBZ). The following of a Class PX CT must be specified:

1) Rated primary current; I_P

2) Turns ratio; 1: N

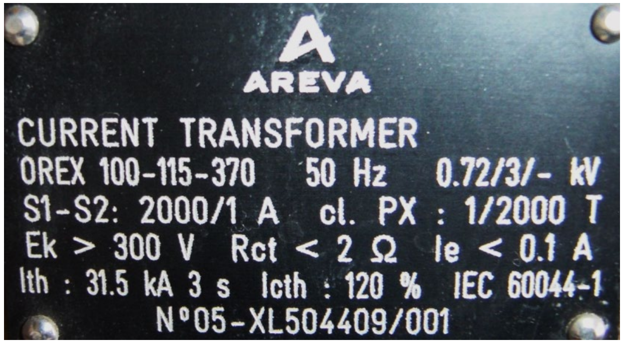
3) Rated knee point e.m.f at maximum secondary turns; V_K

4) Exciting current at the knee point (or some other specified points); I_E

5) Maximum resistance of the secondary winding corrected to 75° C or at the maximum service temperature, whichever is the greater; R_{CT}

6) Turns ratio error should not exceed ±0.25%;

7) Rated resistive burden.



Class TP CT

Class TP CT are specified in IEC60044-6 for current containing exponentially decaying DC of defined time constant.

- Class TPS – Low Leakage Flux Design CT
  - Generally for unit protection (e.g. HZBBZ) where through fault stability is essentially of a transient nature and thus the extent of the unsaturated (or linear) zones is of importance.
  - Under IEC 60044-6 for transient performance with specified rated primary current, turn ratio (error < 0.25%), secondary limiting voltage and resistance of secondary winding
- Class TPX – Closed Core CT for Specified Transient Duty Cycle
  - Similar to TPS except for the error limit prescribed and possible influencing effects for larger size. It has no air gap and hence a high remanent factor (70 – 80% remanence flux).
  - The accuracy limit is defined by the peak instantaneous error during specified transient duty cycle.
  - It is typically for line protection.
- Class TPY – Gapped (Low Remanence) CT for Specified Transient Duty Cycle
  - Core is provided with small air gap to reduce the remanent flux to a level < 10% of saturation flux.
  - It has a higher error in current measurement than TPX during unsaturated operation.
  - It is typically for line protection with auto-reclose.
- Class TPZ – Linear CT (No Remanence)
  - It has negligible remanent flux due to large air gaps in the core.
  - The air gap also minimizes the influence of DC component from primary fault, but it reduces measuring accuracy in unsaturated region.
  - The accuracy limit is defined by peak instantaneous AC component error during single energization with maximum DC offset at specified loop time constant
  - Class TPZ are typically for special application such as generator differential.

CT in Transient State

$$\boxed{L = \mu_0 \mu_r(H) n_s^2 \frac{A_{Fe}}{l_{Fe}}}$$

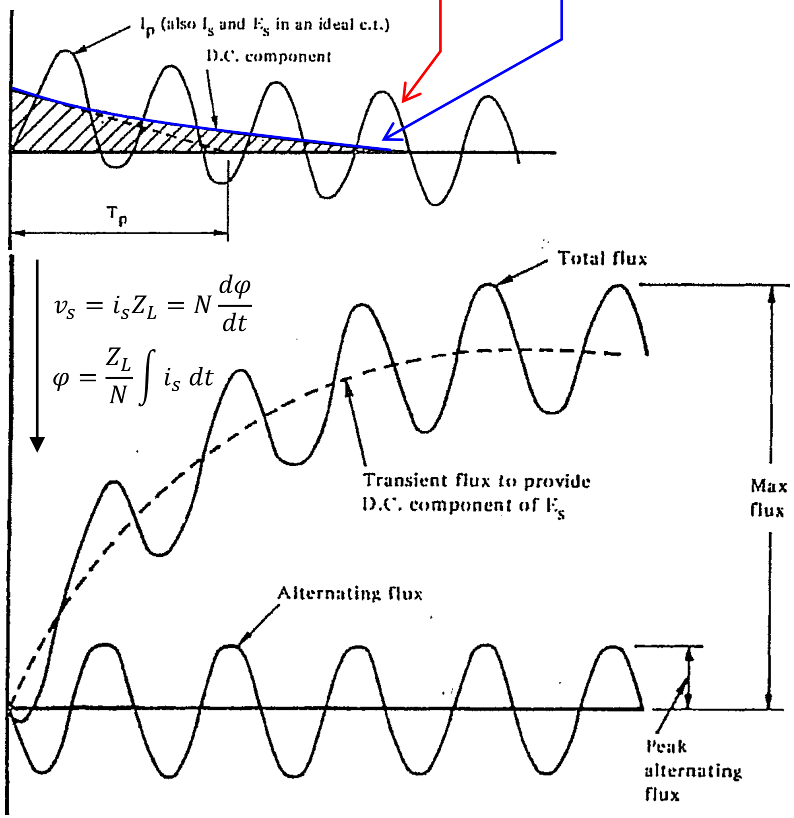
$\mu_r(H)$ is a function of H in B – H curve

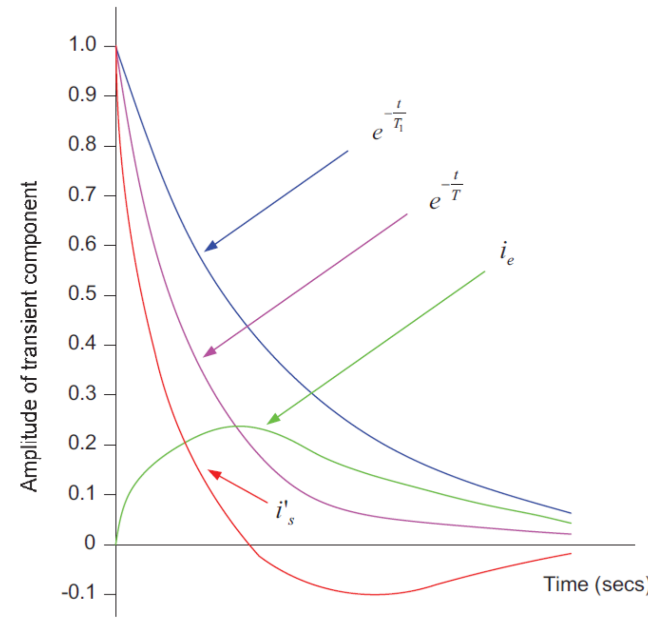
$$\varphi_p = \mu_0 \mu_r \frac{A_{Fe}}{l_{Fe}} n_p I_p \longrightarrow \overset{+}{\bigcirc} \longrightarrow E_s = -N\frac{d\Delta\varphi}{dt} \longrightarrow I_s = \frac{E_s}{Z_B} + I_e$$

$$\varphi_s = \mu_0 \mu_r \frac{A_{Fe}}{l_{Fe}} n_s I_s \quad (-)$$

Given primary fault current could be expressed as

$$i_F = \frac{V_M}{\sqrt{R^2 + (\omega L)^2}} \left(\boxed{\sin(\omega t + \theta - \varphi)} + \boxed{\sin(\theta - \varphi)\, e^{-\frac{R}{L}t}} \right)$$





i_e = Transient exciting current
i'_s = Secondary output current to burden
T = 0.06s
T_1 = 0.12s

Question –

How does the fault time constant (L/R) affect protection operation?

CT in Transient State

Given the primary fault current –

$$i_P = \hat{I}_1\left(\boxed{\sin(\omega t + \theta - \varphi)} + \boxed{\sin(\theta - \varphi)\, e^{-\frac{R}{L}t}}\right)$$

where

$$\hat{I}_1 = \frac{V_M}{\sqrt{R^2 + (\omega L)^2}} \quad \text{and} \quad \varphi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Note that $i_2 = i_1\, N_1/N_2$ and $\hat{I}_2 = \hat{I}_1\, N_1/N_2$

At $\sin(\theta - \varphi) = 1$ or $\theta - \varphi = \pm\pi/2$,

$$i_2 = \hat{I}_2\left[\sin\left(\omega t - \frac{\pi}{2}\right) + e^{-\frac{R_1}{L_1}t}\right] = i_{2DC} + i_{2AC}$$

Assume operating at the linear region with infinite magnetizing inductance (M = ∞) and resistive burden.

$$e = e_{AC} + e_{DC} = N_2\frac{d(\varphi_{AC} + \varphi_{DC})}{dt} = (i_{2AC} + i_{2DC})R_2$$

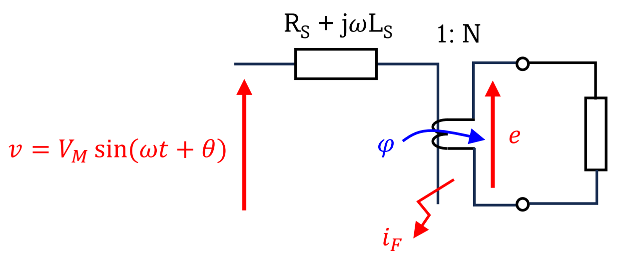
$$e_{AC} = i_{AC}R_2 \rightarrow \varphi_{AC} = \int \frac{\hat{I}_2 R_2}{N_2}\sin\left(\omega t - \frac{\pi}{2}\right)dt = -\frac{\hat{I}_2 R_2}{N_2\omega}\cos\left(\omega t - \frac{\pi}{2}\right)\Big|_0^t$$

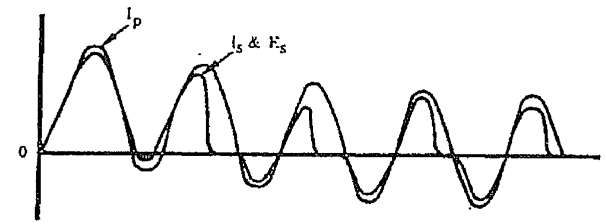
$$\hat{\varphi}_{AC} = \frac{\hat{I}_2 R_2}{N_2\omega}$$

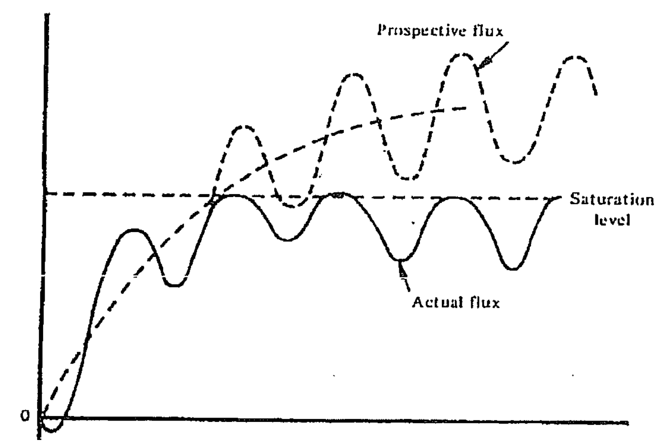
$$\boxed{e_{DC} = i_{DC}R_2} \rightarrow \varphi_{DC} = \int \frac{\hat{I}_2 R_2}{N_2} e^{-\frac{R_1}{L_1}t}dt = \frac{\hat{I}_2 R_2}{N_2}\frac{L_1}{R_1}\left(1 - e^{-\frac{R_1}{L_1}t}\right)$$

Question –

What if L_2 is included?







CT in Transient State

$$\hat{\varphi}_{DC} = \frac{\hat{I}_2 R_2}{N_2}\frac{L_1}{R_1}\Big|_{t\to\infty}$$

Total Flux with given assumption -

$$\varphi_{\Sigma} = \varphi_{AC} + \varphi_{DC} = \frac{\hat{I}_2 R_2}{N_2\omega}\left[-\cos\left(\omega t - \frac{\pi}{2}\right) + \frac{X_1}{R_1}\left(1 - e^{-\frac{R_1}{L_1}t}\right)\right]$$

With peak flux

$$\hat{\varphi} = \hat{\varphi}_{AC} + \hat{\varphi}_{DC} = \frac{\hat{I}_2 R_2}{N_2\omega}\left(1 + \boxed{\frac{X_1}{R_1}}\right) = \frac{\hat{I}_2 R_2}{N_2\omega}(1 + \omega T_p)$$

If secondary inductance L_2 is also considered (from burden or CT leakage)

$$\varphi_{\Sigma} = \frac{\hat{I}_2 R_2}{N_2\omega}\left[\frac{\omega T_1 T_2}{T_2 - T_1}\left(e^{-\frac{t}{T_2}} - e^{-\frac{t}{T_1}}\right) - \sin\omega t\right]$$

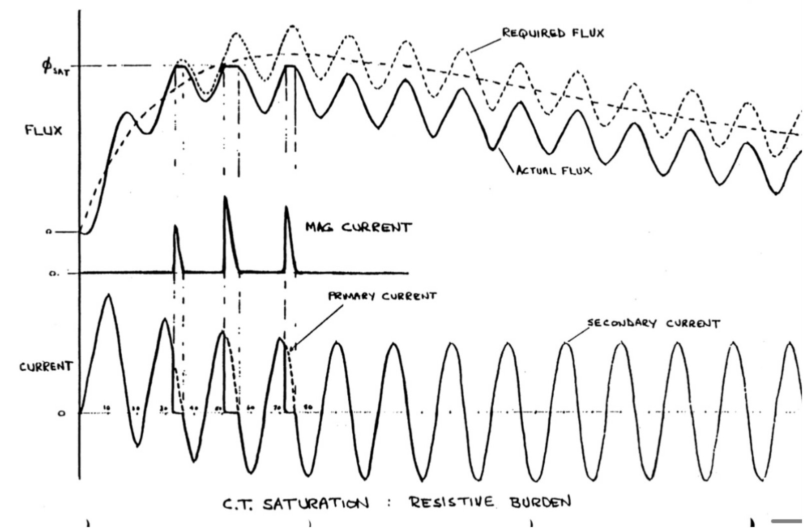
It is the required flux to generate i_2 without saturation.

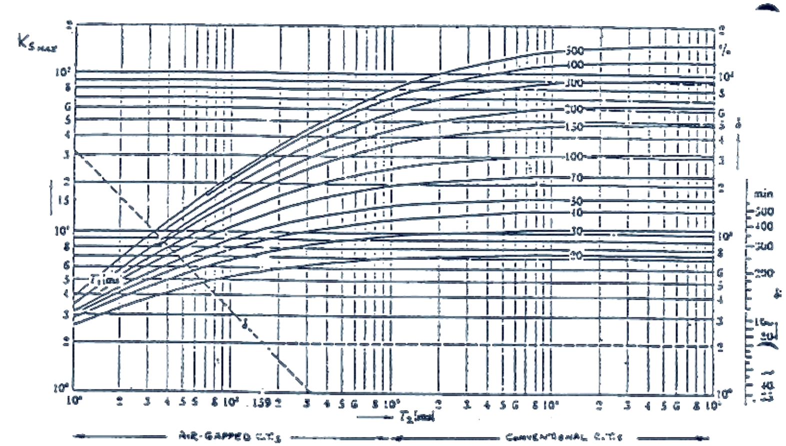
Consider V_K the C.T. rms saturation voltage $\rightarrow$ $\sqrt{2}V_K = \omega N_2 \varphi_{AVAIL}$

$$\varphi_{AVAIL} > \varphi_{req} \rightarrow \frac{\sqrt{2}V_K}{\omega N_2} > \frac{\hat{I}_2 R_2}{N_2\omega}\left[\frac{\omega T_1 T_2}{T_2 - T_1}\left(e^{-\frac{t}{T_2}} - e^{-\frac{t}{T_1}}\right) - \sin\omega t\right]$$

Saturation Factor

$$\frac{V_K}{\hat{I}_2 R_2} = \left[\frac{\omega T_1 T_2}{T_2 - T_1}\left(e^{-\frac{t}{T_2}} - e^{-\frac{t}{T_1}}\right) - \sin\omega t\right]$$





Practical Applications

Practical condition differs from the model for the following reasons –

1. Secondary Leakage or Burden Inductance NOT in Consideration

   Primary transient current converted to secondary without considering the effect of leakage and burden inductance on the maximum flux, given that the effect is small as compared to system source R and L. [Effect – Underestimate peak flux]

2. Iron Loss is NOT in Consideration

   The equivalent iron loss resistance is variable, depending upon both sine and exponential terms. Hence, it cannot be included in linear theory, and it is complicated to include.
   [Effect – Overestimate the response rate / Reduced time constant]

3. Assumption on Linear Excitation

   The linear excitation assumption only valid up to the knee point of the excitation curve. Precise solution for non-linearity is not practicable. Better solution could be provided with linearization of excitation curve into more pieces. The above theory is sufficient to give good insight into the problem and to most practical problems to be decided. [Effect – Incorrect estimation to the shunt inductance (linear region and saturation region]

4. Effect of Hysteresis and Remanent Flux

   Hysteresis makes the effective shunt inductance a variable, so as the secondary time constant. Remanent flux in addition to the transient flux could be positive or negative, such that the result could be saturation or not.

Knee Point Voltage Dimensioning in Transient

- ## Short Circuit Factor

This factor accounts for the symmetrical fault current magnitude under the worst case of the fault. This is given by the ratio of the maximum secondary symmetrical short circuit current to the secondary nominal current

$$K_{SC} = \frac{I_{sc,max}}{I_n}$$

- ## Remanent Flux Factor

CT cores with air gap can retain remanent flux for a very long time. Thus, if the increases in flux during fault is in the same direction of the remanent flux, then the CT core can reach a saturation value faster it would have if the remanent flux was zero. Thus, K_{REM} accounts for dimensioning the CT to take care of remanent flux

$$K_{REM} = \frac{1}{1 - \frac{\psi_{rem}}{\psi_{sat}}}$$

- ## Asymmetric Transient Factor

The transient over dimensioning factor is given by the ratio of the secondary linked flux in the CT due to the total fault current to the flux linked due to the AC component of the fault current.

$$K_{TF} = \frac{\omega T_p T_s}{T_p - T_s} \cos\theta \left(e^{-\frac{t}{T_p}} - e^{-\frac{t}{T_s}}\right) + \sin\theta \; e^{-\frac{t}{T_s}} - \sin(\omega t + \theta)$$

Knee Point Voltage Dimensioning in Transient

where:

T_p is the primary system constant L/R; T_s is CT secondary time constant $\{L_m/(R_{CT}+R_B)\}$;

Θ is the difference between point on wave angle and fault current phase angle (switching angle);
t t is the time since fault.

It is known that the maximum DC offset in the fault current occurs when θ=0° , thus the transient dimensioning factor becomes:

$$K_{TF} = \frac{\omega T_p T_s}{T_p - T_s}\left(e^{-\frac{t}{T_p}} - e^{-\frac{t}{T_s}}\right) - \sin(\omega t) = \boxed{1 + X/R}$$

Given that $T_S >> T_P$ for high remanent CT

The value of K_{TF} used in dimensioning the CT, thus depends on the minimum duration for which CT is required to operate unsaturated. This duration is termed as time to saturation of the CT.

The knee point voltage requirement of the CT can be written as

$$V_{K,max} > K_{TF} K_{SC} K_{REM}\, I_N (R_B + R_{CT})$$

Given that the actual time to saturation [s] –

$$t_s = -T_p\left[\ln\left(1 - \frac{K_s - 1}{T_p \omega}\right)\right]$$

where

$$K_S = \text{saturation factor} = \frac{V_K}{\frac{I_{F,sym}}{N}\,(R_{CT} + R_B)}$$

Saturation Factor and Time to Saturate

- If practical, the effects of saturation can be avoided by sizing the CT to have a knee-point voltage above that required for the maximum expected fault current and CT secondary burden, with suitable allowance for possible DC component and remanence.

Only AC Saturation

$$V_K > I_S\,(Z_{CT} + Z_B)$$

$\longrightarrow$ DC + AC Saturation

$$V_K > I_S\,(Z_{CT} + Z_B)\left(1 + \frac{X}{R}\right)$$

$\longrightarrow$ DC + AC Saturation + Inductive Burden

$$V_K > I_S\,(Z_{CT} + Z_B)\left(1 + \frac{X}{R} \times \frac{R_{CT} + R_B}{Z_B}\right)$$

$\downarrow$

DC + AC Saturation
+ Inductive Burden
+ Remanent Flux

$$V_K > I_S \frac{(Z_{CT} + Z_B)\left(1 + \frac{X}{R} \times \frac{R_{CT} + R_B}{Z_B}\right)}{1 - \frac{\psi_{REM}}{\psi_S}}$$

- Saturation Factor K_S is the ratio of saturation voltage to excitation voltage applied. It is an index of how close/ how far a CT is to saturation. It is used to calculate time-to-saturate under transient condition.
- Time-to-Saturation is important in design and application of protection relay. A CT should be capable of accurately replicating offset primary current for one or two cycles before CT saturate.
- It depends on –
  - Degree of Fault Current Offset – System X/R Ratio and Fault Incident Angle determines the amount of DC current contributes to increase of flux.
  - Fault Current Magnitude – The magnitude of offset current is proportional to magnitude of sinusoidal current.

Saturation Factor and Time to Saturate

- It also depends on
  - Remanent Flux in CT core – It could be additive or subtractive to the flux produced by other mechanism. When the remanent flux results in an increase, the time-to-saturation is shortened.
  - Secondary Circuit Impedance – High burden demands a high voltage at a given current, and the flux is proportional to the voltage. Hence, high burden CT could saturate faster. An inductive burden (lower pF) will give a longer time-to-saturation because the inductance has a low impedance to DC offset current reducing burden voltage drop and associate flux.
  - Saturation Voltage / Knee Point – The secondary excitation impedance of a CT depends on the quantity and quality of the iron core. The larger the cross-section of CT core, the more flux is required to saturate it. It results in higher knee point and time-to-saturation will be longer.
  - Turn Ratio – measure of CT saturation is the degree that flux density exceeds the saturation flux density level. For a given core area and primary current, increasing the turns ratio of a CT decreases the flux and, thereby, reduces the flux density. The reduction in flux may be visualized as the result of two effects –
    1) reduced flux to produce a reduced secondary EMF
    2) reduced current in secondary leads to smaller secondary voltage

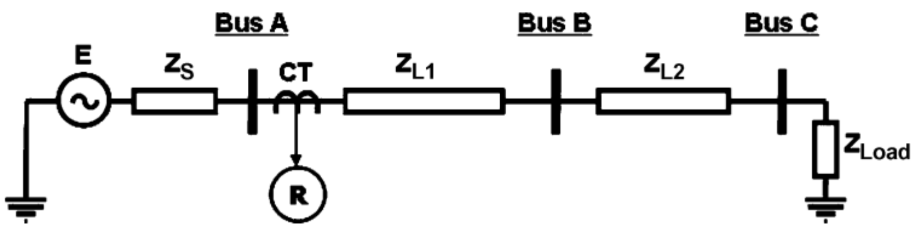
    Note – Ohmic burden of secondary circuit will increase if CT ratio is increased.
    1) Winding resistance proportional to number of turns
    2) The use of higher turn ratio may require a more sensitive, higher burden relay or relay tap (particularly if EM relay is employed)

Question – Why is time-to-saturation important to protection scheme design?

How does reactive burden affect protection operation?

Knee Point Voltage Dimensioning in Transient

Example 3



The values of different system parameters are –

E = system voltage = 400kV

Z_S = source impedance = 60 $\underline{/88^o}$ Ω

Z_{L1} = line impedance for line 1 = 30 $\underline{/80^o}$ Ω

Z_{L2} = line impedance for line 2 = 15 $\underline{/80^o}$ Ω

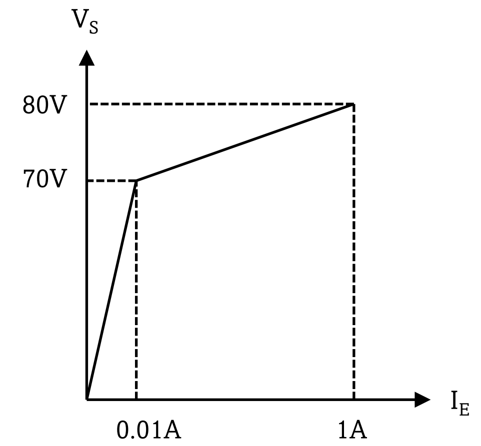
Z_{load} = load impedance = 190.42 $\underline{/3.37^o}$ Ω (minimum value)

I_L = 1000A (max value)

CT ratio = 1000/1

Relay burden R_B = 2.25 Ω

CT secondary winding resistance R_{CT} = 2Ω



Determine the dimensioning factor when through fault stability (faulted at B) is concerned.

Knee Point Voltage Dimensioning in Transient

Through Fault Current:

$$I_F = \frac{E/\sqrt{3}}{Z_S + Z_{L1}} = \frac{400/\sqrt{3}}{(60\angle 88^o + 30\angle 80^o)} = 2.57\text{kA} \ (= 2.57\text{Asec})$$

Short Circuit Factor:

$$K_{SC} = \frac{I_F}{I_L} = \frac{2.57}{1.00} = 2.57$$

Primary Time Constant:

$$T_p = \frac{L_s}{R_s} = \frac{\text{Im}(Z_s + Z_{L1})}{\omega \ \text{Re}(Z_s + Z_{L1})} = \frac{\text{Im}(60\angle 88^o + 30\angle 80^o)}{2\pi 60 \ \text{Re}(60\angle 88^o + 30\angle 80^o)} = 32.5\text{ms}$$

Secondary Time Constant:

$$T_s = \frac{L_m}{R_{CT} + R_B} = \frac{\left(\frac{\frac{70}{0.01}}{2\pi(60)}\right)}{2.25 \times 2} = 4.13\text{s} \gg \text{T}_\text{p}$$

Transient Factor:

$$K_{TF} = 1 + \frac{X}{R} = 1 + \omega T_p = 1 + 2\pi(60) \times 32.5\text{m} = 13.25$$

Neglecting the remanent flux factor, the knee point voltage required will be –

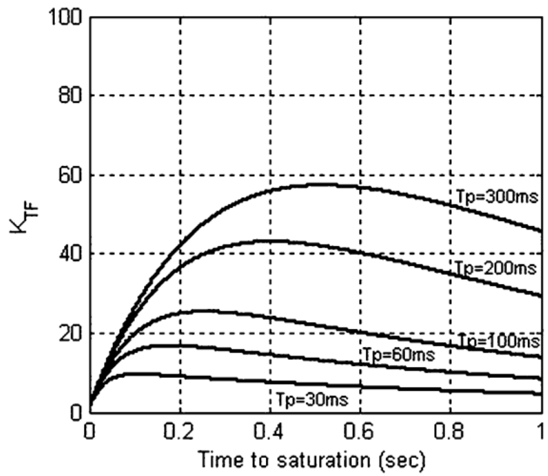
$$V_K = K_{TF} K_{SC} I_F (R_{CT} + R_B) = 13.25 \times 2.57 \times 2.57(2.25 + 2) = 372V$$

Question – How to ensure protection stability for feeder current differential with auto-reclose?

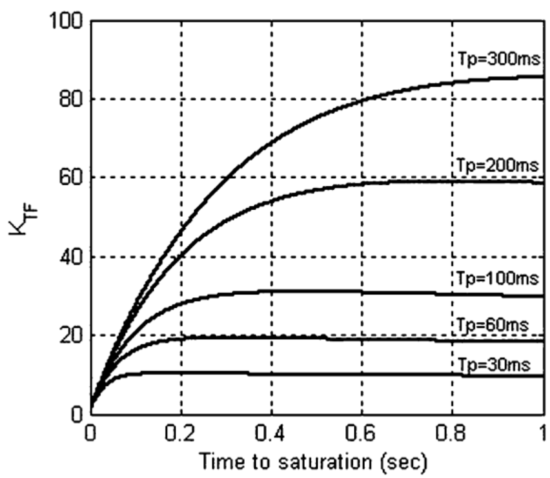
Knee Point Voltage Dimensioning in Transient

Transient Dimensioning Factor

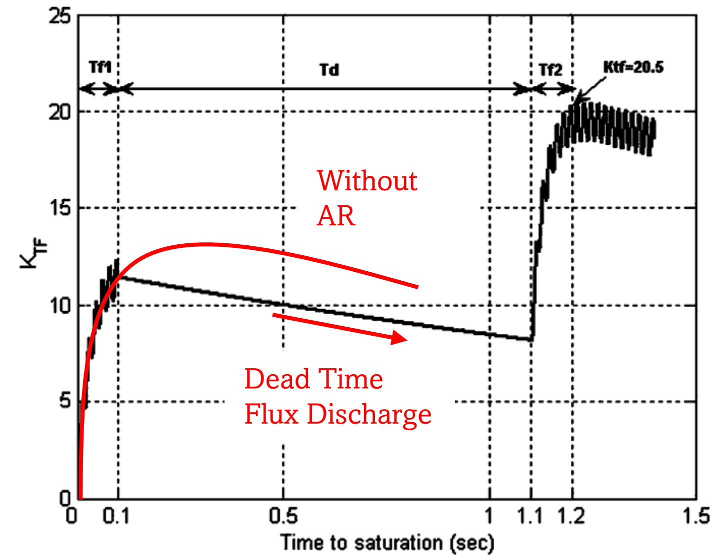
$T_S = 1s$



$T_S = 10s$



K_{TF} with Auto-Reclose Function



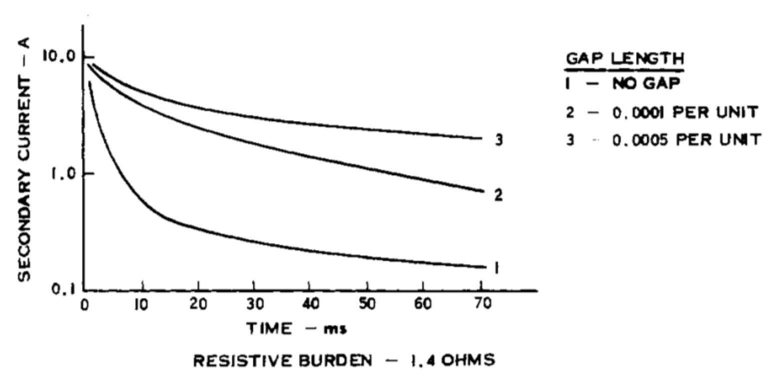
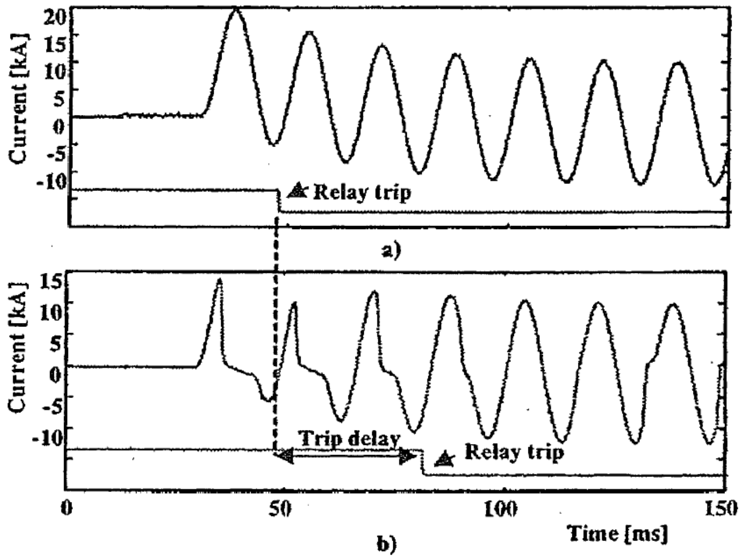


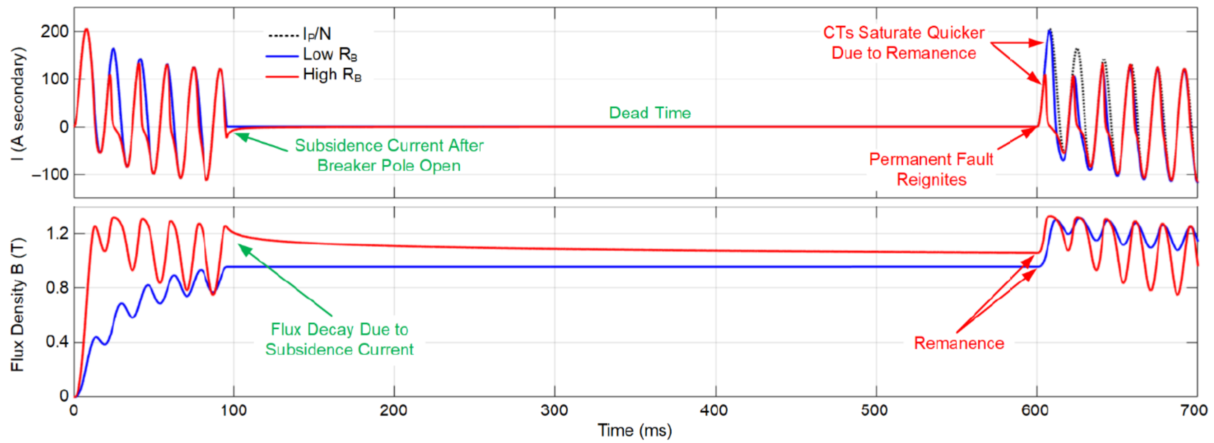
FIGURE 2: CT SUBSIDENCE CURRENT

Effects of CT Saturation on Relay Operation

- Most protective relays make operating decisions based on the RMS value of fault current, such as OCEF element.
- If the signal supplied by the CT is distorted by saturation, the RMS value sensed will be much lower than the actual fault current.
- Other than reduced RMS, CT saturation could also lead to un-stabilized direction element, as the CT output is multiplied with a factor of
$$a\angle-\theta$$
where $0<a<1,\ 0<\theta<180^{o}$.



- CT saturation at one end, could lead to maloperation with differential scheme. Subsidence current with different remanent flux could lead to delayed auto-reclose time.



Effects of CT Saturation on Relay Operation

Most relays employ Discrete Fourier Transform (DFT) to transform instantaneous value to phasor quantity.

$$i[n] = I_M \sin\left(\frac{2\pi}{N}n + \theta\right) \rightarrow \; I[n] = \frac{2}{N}\sum_{k=0}^{N-1} i[n-k]e^{-j\frac{2\pi}{N}k}$$

(Other possible algorithms are half-cycle DFT, cosine filter)

$a\angle -\theta$

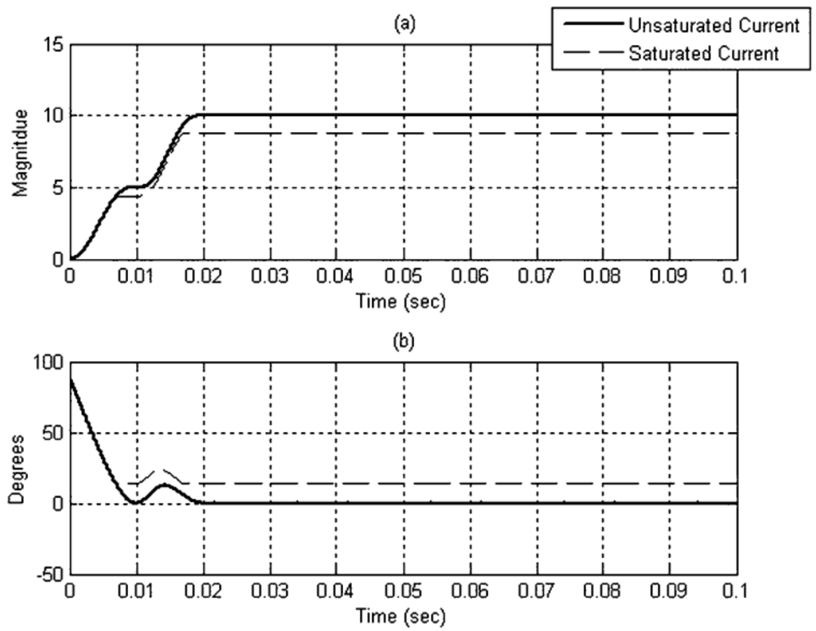


Fig. 6. Estimated phasor magnitude and phase angle of the unsaturated and saturated current signal. (a) Phasor magnitude. (b) Phasor angle.

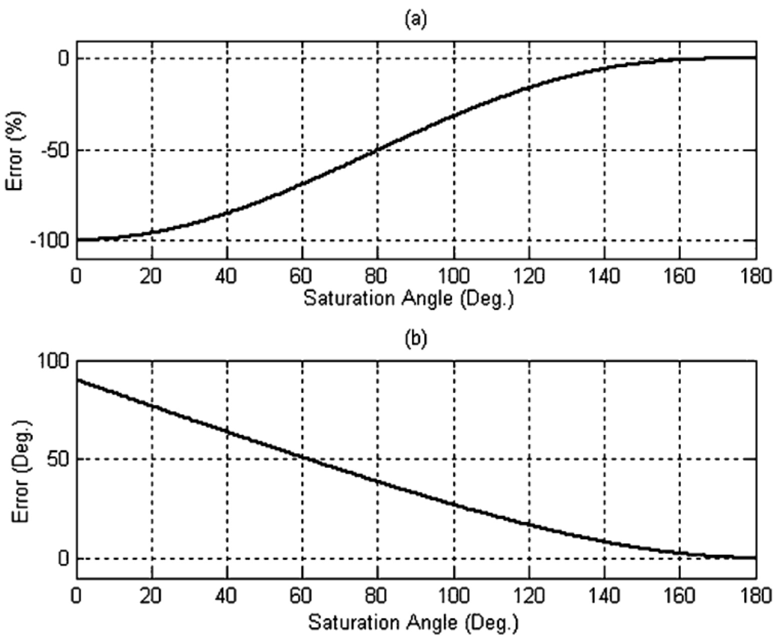
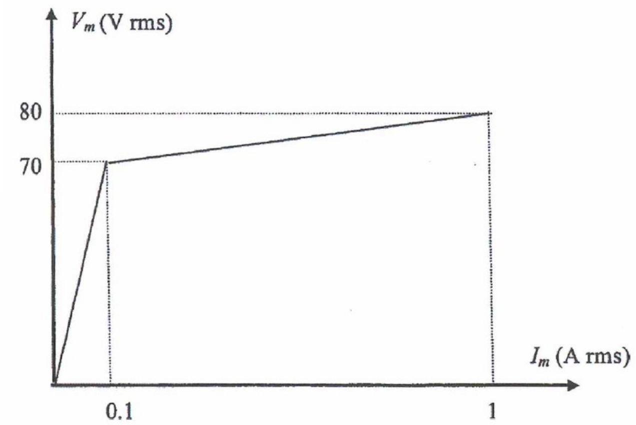


Fig. 7. Magnitude and phase error for different saturation angles. (a) Phasor magnitude error versus saturation angle. (b) Phasor angle error versus saturation angle.

Effects of CT Saturation on Relay Operation

Example 4

A 1000/1 C.T. having a simplified magnetization characteristic as shown in the figure is connected with a resistive burden of 15Ω. If the primary current is 5000A (rms), sketch the estimated C.T secondary current waveform for at least a cycle.



Solution

$$I_f = \frac{5000}{1000} \times \sqrt{2} = 7.07\,A_{pk}$$

Without saturation, $X_{LM} = j\frac{70}{0.1} = j700\,\Omega$

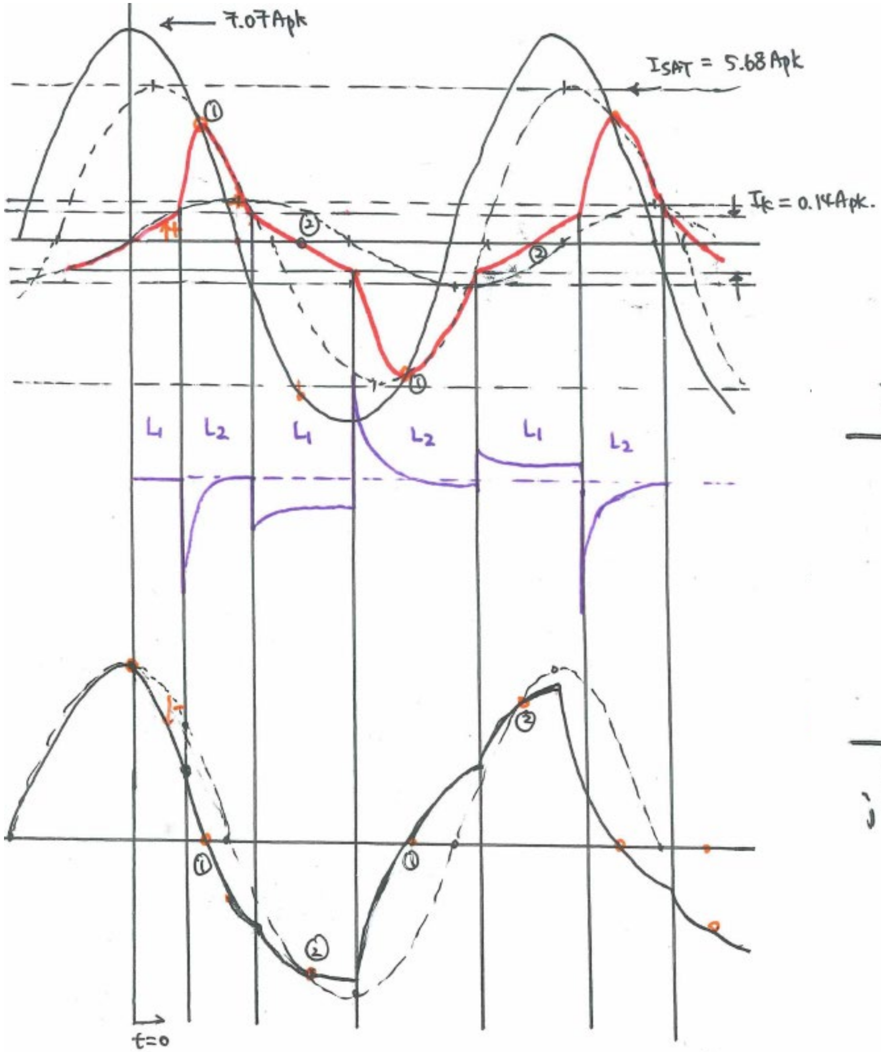
$$\rightarrow I_{E,pk} = 7.07 \times \frac{15}{(15+j700)} = 0.15\angle{-88.8^\circ}\,A_{pk}$$

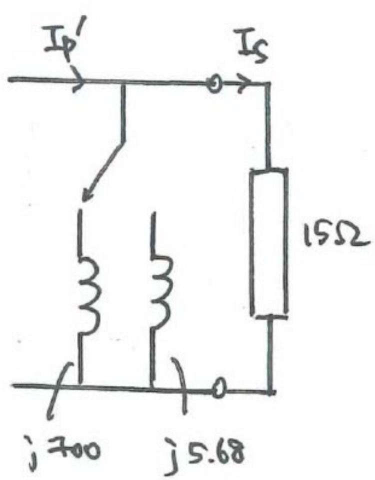
During saturation, $X_{L,SAT} = j\frac{80-70}{1-0.1} = j11.11\,\Omega$

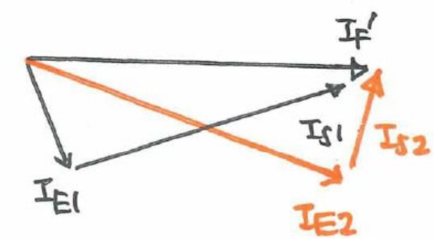
$$\rightarrow I_{E,SAT} = 7.07 \times \frac{15}{15+j11.1} = 5.68\angle{-36.5^\circ}\,A_{pk}$$

if $i > 0.1\sqrt{2} = 0.14A$ → saturation occurs.

Effects of CT Saturation on Relay Operation







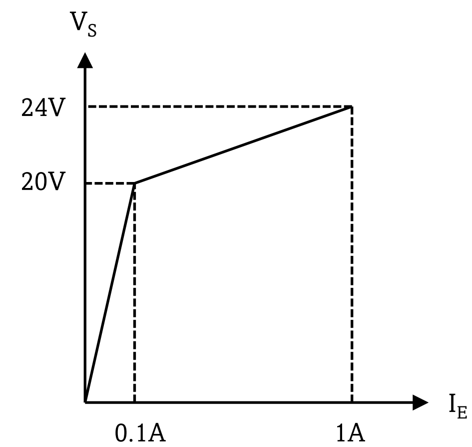
Exercise

Q1

a) Explain the considerations required in choosing a suitable protection current transformer (CT). Describe the physical requirements to have a CT high knee point, low excitation current and low reactance.

b) A 100/1 CT having simplified magnetization characteristics as shown in the figure is connected to a resistive burden of 10Ω. If the primary current is 2000A (rms), sketch the estimated CT secondary current waveform for at least half a cycle. There is no need to plot the waveform to scale but the numerical values should be indicated. List the assumption made.

c) Three 100/1 CTs with the same magnetizing characteristics as in the figure are connected in parallel and supply an earth fault relay of rating 1A. The relay absorbs 5VA at setting current. Plot the actual operating current against setting current (referred to the primary) if the relay setting range is from 20% to 80% in step of 20%. Comment on the effective relay operating current in this case. Assume the relay burden is purely reactive and the relay impedance is inversely proportional to the square of its current setting.



Q2

a) Explain, with the aid of appropriate diagrams, the flux, secondary emf and current conditions of a CT when a fault current containing a transient DC component passes through it given that
i) the CT is gapped and the excitation current is negligible;
ii) Core saturation takes place.

b) Describe how the line reactance and resistance will affect the peak value of the flux in the ideal CT and the effect of core saturation to the operation of protection relays.

Content

- Introduction to Current Transformer (C.T.)
  - Types of CT
  - Half Ratio
  - CT Equivalent Circuit
- Terminology
  - Capacity – Thermal Capacity (r.m.s.) vs Dynamic Capacity (pk)
  - Errors – Ratio Error, Transformation Error, Composite Error,
    Phase Displacement
  - Accuracy Limiting Factor
  - Class P CT
  - Relationship between Burden, Knee Point and OCEF Setting
- CT Tests
- Transient Performance of CT → Time to Saturation & CT Over-Dimensioning Factor
  - Class X and Class TP CT
  - Transient Dimensioning (1 + X/R)
  - Saturation Factor and Time-to-Saturation
  - Effect of CT Saturation to Protection Operation

An introduction to level 2

Time to Saturation & CT Over-Dimensioning Factor

- Short Circuit at **Primary**

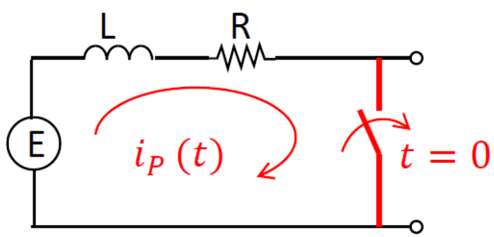
$$E_m = \frac{\sqrt{2}V_{LL}}{\sqrt{3}} \text{ [peak value]} \qquad E(t) = E_m \sin(\omega t + \theta)$$

$$I_m = \frac{E_m}{Z} = \frac{E_m}{\sqrt{R^2 + (\omega L)^2}}$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$\angle Z = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{X}{R}$$

$$T_P = \frac{L}{R} \text{ [sec, time constant at primary]}$$



$$\boxed{i_P(t) = -I_m \sin(\theta - \varphi)\, e^{-\frac{R}{L}t} + I_m \sin(\omega t + \theta - \varphi) \qquad i_P(0) = 0} \qquad \ldots(1)$$

- Parameter in **Secondary**

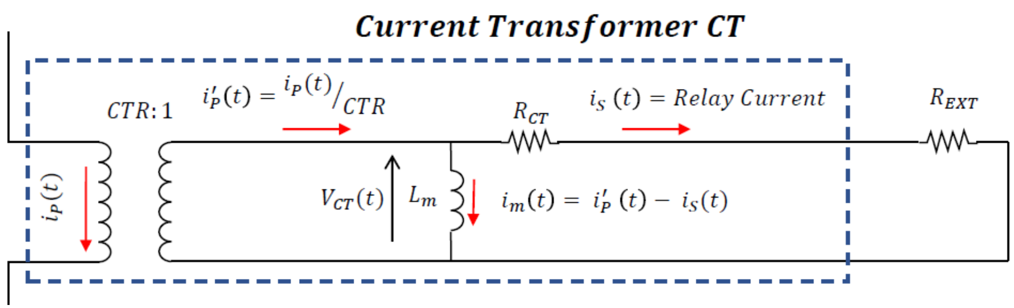
CTR: CT Ratio

R_{CT} = CT Resistance

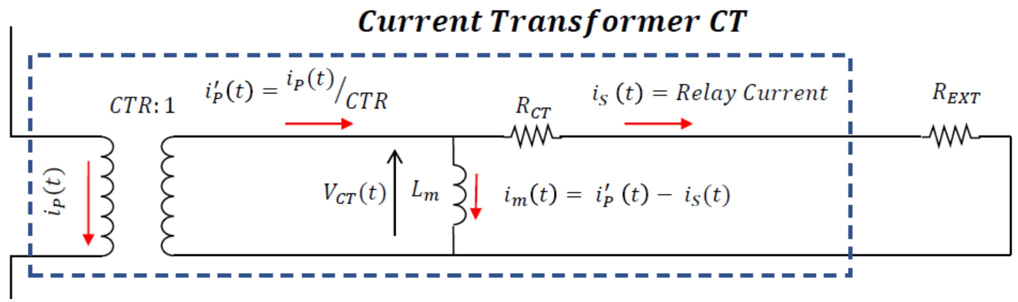
R_{EXT} = External Resistance

$R_b = R_{CT} + R_{EXT}$
= CT total burden

$$T_S = \frac{L_m}{R_b} \text{ [sec, time constant at secondary]}$$



Time to Saturation & CT Over-Dimensioning Factor



- CT Secondary Voltage

$$V_{CT}(t) = i_s(t)R_b = L_m\frac{di_m(t)}{dt} = L_m\left(\frac{di_P'(t)}{dt} - \frac{di_s(t)}{dt}\right) \rightarrow \frac{di_s(t)}{dt} + \frac{R_b}{L_m}\,i_s(t) = \frac{di_p'(t)}{dt}$$

$$\boxed{\frac{di_s(t)}{dt} + \frac{1}{T_s}\,i_s(t) = \frac{di_p'(t)}{dt}} \qquad \ldots(2)$$

$$i_P(t) = -I_m\sin(\theta - \varphi)\,e^{-\frac{t}{T_P}} + I_m\sin(\omega t + \theta - \varphi) \qquad i_P'(t) = \frac{i_p(t)}{\mathrm{CTR}}$$

$$i_P{}'(t) = -\frac{I_m}{\mathrm{CTR}}\sin(\theta - \varphi)\,e^{-\frac{t}{T_P}} + \frac{I_m}{\mathrm{CTR}}\sin(\omega t + \theta - \varphi) \qquad \text{Define}\quad I_m'(t) = \frac{I_m(t)}{\mathrm{CTR}}$$

$$i_P{}'(t) = -I_m{}'\sin(\theta - \varphi)\,e^{-\frac{t}{T_P}} + I_m{}'\sin(\omega t + \theta - \varphi)$$

$$\boxed{\frac{di_P'^{(t)}}{dt} = -\frac{I_m'}{T_p}\sin(\theta - \varphi)\,e^{-\frac{t}{T_P}} + \omega I_m{}'\cos(\omega t + \theta - \varphi)} \qquad \ldots(3)$$

Time to Saturation & CT Over-Dimensioning Factor

$$\frac{di_s(t)}{dt} + \frac{1}{T_s}\, i_s(t) = \frac{di_p'(t)}{dt} \quad \ldots(2)$$

$$\frac{di_P{}'^{(t)}}{dt} = -\frac{I_m'}{T_p}\sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + \omega I_m{}'\cos(\omega t+\theta-\varphi) \quad \ldots(3)$$

Substitute (2) into (3),

$$\frac{di_s(t)}{dt} + \frac{1}{T_s}\, i_s(t) = -\frac{I_m'}{T_p}\sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + \omega I_m{}'\cos(\omega t+\theta-\varphi) \quad \ldots(4)$$

$$\boxed{i_s(t) = \frac{T_P}{T_s - T_P} I_m' \sin(\theta-\varphi)\, e^{-\frac{t}{T_S}} - \frac{T_s}{T_s - T_P} I_m' \sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + I_m{}'\sin(\omega t+\theta-\varphi)} \quad \ldots(5)$$

Calculate V_{CT} (t)

$$\boxed{V_{CT}(t) = R_b i_s(t) = R_b\left[\frac{T_P}{T_s - T_P} I_m' \sin(\theta-\varphi)\, e^{-\frac{t}{T_S}} - \frac{T_s}{T_s - T_P} I_m' \sin(\theta-\varphi)\, e^{-\frac{t}{T_P}} + I_m' \sin(\omega t+\theta-\varphi)\right]} \ldots(6)$$

Calculate CT Flux $\phi_{CT}(t)$

$$V_{CT}(t) = R_b i_s(t) = -\frac{d\phi_{CT}(t)}{dt} \;\rightarrow\; \phi_{CT}(t) = \int -V_{CT}(t)\, dt$$

Time to Saturation & CT Over-Dimensioning Factor

$$V_{CT}(t) = R_b i_s(t) = -\frac{d\phi_{CT}(t)}{dt} \quad \rightarrow \quad \phi_{CT}(t) = \int -V_{CT}(t)\, dt$$

$$V_{CT}(t) = R_b i_s(t) = R_b\left[\frac{T_P}{T_s - T_P} I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_S}} - \frac{T_s}{T_s - T_P} I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}} + I'_m \sin(\omega t + \theta - \varphi)\right] \qquad \ldots(6)$$

$$\phi_{CT}(t) = -R_b \int \left[\frac{T_P}{T_s - T_P} I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_S}} - \frac{T_s}{T_s - T_P} I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}} + I'_m \sin(\omega t + \theta - \varphi)\right] dt$$

$$\phi_{CT}(t) = \underbrace{\frac{T_P T_s}{T_s - T_P} I'_m R_b \sin(\theta - \varphi)\left(e^{-\frac{t}{T_S}} - e^{-\frac{t}{T_P}}\right)}_{\text{Transient Term}} + \underbrace{\frac{I'_m R_b}{\omega} \cos(\omega t + \theta - \varphi)}_{\text{Steady State Term}} + \underbrace{M e^{-\frac{t}{T_s}}}_{\text{Residual Term}}$$

Transient Term
Excited by difference
in Primary and Secondary Circuit

Steady State Term

Residual Term
Due to initial or remanent flux
$\phi_{CT}(0)$

Note
Remanent Flux ϕ_R is normally shown as per unit of knee point flux ϕ_K. (e.g. $\phi_R = 0.3\phi_K$)

Definition of K_R

$$\phi_{CT}(0) = K_R \phi_K = \frac{I'_m R_b}{\omega} \cos(\theta - \varphi) + M$$

$$M = K_R \phi_K - \frac{I'_m R_b}{\omega} \cos(\theta - \varphi)$$

Time to Saturation & CT Over-Dimensioning Factor

$$\phi_{CT}(t) = \frac{I'_m R_b}{\omega}\left[\frac{\omega T_P T_s}{T_s - T_P}\sin(\theta-\varphi)\left(e^{-\frac{t}{T_S}} - e^{-\frac{t}{T_P}}\right) + \cos(\omega t + \theta - \varphi)\right] + Me^{-\frac{t}{T_s}}$$

$$= \frac{I'_m R_b}{\omega}\left[\frac{\omega T_P T_s}{T_s - T_P}\sin(\theta-\varphi)\left(e^{-\frac{t}{T_S}} - e^{-\frac{t}{T_P}}\right) + \cos(\omega t + \theta - \varphi) + \frac{M}{\frac{I'_m R_b}{\omega}}e^{-\frac{t}{T_s}}\right] \qquad M = K_R\phi_K - \frac{I'_m R_b}{\omega}\cos(\theta-\varphi)$$

$$= \frac{I'_m R_b}{\omega}\left[\frac{\omega T_P T_s}{T_s - T_P}\sin(\theta-\varphi)\left(e^{-\frac{t}{T_S}} - e^{-\frac{t}{T_P}}\right) + \cos(\omega t + \theta - \varphi) + \left[K_R\left(\frac{\phi_K}{\frac{I'_m R_b}{\omega}}\right) - \cos(\theta-\varphi)\right]e^{-\frac{t}{T_s}}\right]$$

Definition of $\phi_{AC,max}$

$$\phi_{CT}(t) = \phi_{AC,max}\left[\frac{\omega T_P T_s}{T_s - T_P}\sin(\theta-\varphi)\left(e^{-\frac{t}{T_S}} - e^{-\frac{t}{T_P}}\right) + \cos(\omega t + \theta - \varphi) + \left[K_R\boxed{\left(\frac{\phi_K}{\phi_{AC,max}}\right)} - \cos(\theta-\varphi)\right]e^{-\frac{t}{T_s}}\right]$$

At knee point, define $K_s = \dfrac{\phi_K}{\phi_{AC,max}} = \dfrac{V_K}{V_{AC,max}} = \dfrac{V_K}{R_B I'_m}$

Multiply ϕ_K/ϕ_K to $\phi_{CT}(t)$,

$$\phi_{CT}(t) = \frac{\phi_K}{\phi_K/\phi_{AC,max}}\left[\frac{\omega T_P T_s}{T_s - T_P}\sin(\theta-\varphi)\left(e^{-\frac{t}{T_S}} - e^{-\frac{t}{T_P}}\right) + \cos(\omega t + \theta - \varphi) + \left[K_R\left(\frac{\phi_K}{\phi_{AC,max}}\right) - \cos(\theta-\varphi)\right]e^{-\frac{t}{T_s}}\right]$$

$$\phi_{CT}(t) = \frac{\phi_K}{K_s}\left[\frac{\omega T_P T_s}{T_s - T_P}\sin(\theta-\varphi)\left(e^{-\frac{t}{T_S}} - e^{-\frac{t}{T_P}}\right) + \cos(\omega t + \theta - \varphi) + [K_R K_s - \cos(\theta-\varphi)]e^{-\frac{t}{T_s}}\right] \qquad \ldots(7)$$

Time to Saturation & CT Over-Dimensioning Factor

Time to Saturation t_S is the first instant that CT flux or CT voltage reach to ϕ_K or V_K

$$\phi_{CT}(t_S) = \phi_K = \frac{\phi_K}{K_s}\left[\frac{\omega T_P T_s}{T_s - T_P}\sin(\theta-\varphi)\,(e^{-\frac{t_S}{T_S}} - e^{-\frac{t_S}{T_P}}) + \cos(\omega t_S + \theta - \varphi) + [K_R K_s - \cos(\theta-\varphi)]e^{-\frac{t_S}{T_s}}\right]$$

or at t_S,

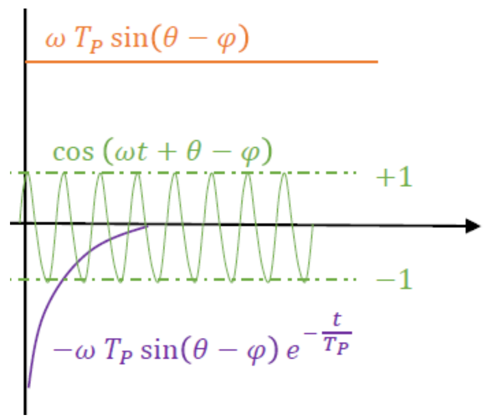
$$\boxed{K_S = \frac{\omega T_P T_s}{T_s - T_P}\sin(\theta-\varphi)\,(e^{-\frac{t_S}{T_S}} - e^{-\frac{t_S}{T_P}}) + \cos(\omega t_S + \theta - \varphi) + [K_R K_s - \cos(\theta-\varphi)]e^{-\frac{t_S}{T_s}}} \quad \ldots(8)$$

Practical Simplifications

In closed core CT: $T_S \gg T_P \;\rightarrow\; T_S - T_P \approx T_S$

$$K_S = \frac{\omega T_P T_s}{T_s}\sin(\theta-\varphi)\,(e^{-\frac{t_S}{T_S}} - e^{-\frac{t_S}{T_P}}) + \cos(\omega t_S + \theta - \varphi) + [K_R K_s - \cos(\theta-\varphi)]e^{-\frac{t_S}{T_s}}$$

$$K_S = \omega T_P \sin(\theta-\varphi)\,(e^{-\frac{t_S}{T_S}} - e^{-\frac{t_S}{T_P}}) + \cos(\omega t_S + \theta - \varphi) + [K_R K_s - \cos(\theta-\varphi)]e^{-\frac{t_S}{T_s}}$$



For typical values of T_S (e.g. 10 sec), the magnitude of $e^{-\frac{t_S}{T_S}} \approx 1$ during the first few cycles (< 100ms).

$$K_S = \omega T_P \sin(\theta-\varphi)\,(1 - e^{-\frac{t_S}{T_P}}) + \cos(\omega t_S + \theta - \varphi) + K_R K_s - \cos(\theta-\varphi)$$

Worse Case Scenario: $\cos(\omega t_S + \theta - \varphi) = 1$

$$K_S = \omega T_P \sin(\theta-\varphi)\,(1 - e^{-\frac{t_S}{T_P}}) + 1 + K_R K_s - \cos(\theta-\varphi)$$

$$e^{-\frac{t_S}{T_P}} = 1 - \frac{K_S(1-K_R) + \cos(\theta-\varphi) - 1}{\omega T_P \sin(\theta-\varphi)}$$

Time to Saturation & CT Over-Dimensioning Factor

Practical Simplifications

$$e^{-\frac{t_S}{T_P}} = 1 - \frac{K_S(1-K_R) + \cos(\theta-\varphi) - 1}{\omega T_P \sin(\theta-\varphi)}$$

$$\boxed{t_S = -T_P \ln\left[1 - \frac{K_S(1-K_R) + \cos(\theta-\varphi) - 1}{\omega T_P \sin(\theta-\varphi)}\right]}$$

With $\omega T_P = X/R$,

$$t_S = -\frac{X}{\omega R} \ln\left[1 - \frac{K_S(1-K_R) + \cos(\theta-\varphi) - 1}{\frac{X}{R}\sin(\theta-\varphi)}\right] \quad \ldots(9)$$

$$K_S = \frac{\boxed{1} - \cos(\theta-\varphi) + \frac{X}{R}\sin(\theta-\varphi)\left(1 - e^{-\frac{\omega R}{X}t_S}\right)}{1-K_R} \quad \ldots(10)$$

Same sign as $\sin(\theta-\varphi)$

Worse Case Scenario
$\sin(\theta-\varphi) = 1$
$\cos(\theta-\varphi) = 0$

$$t_S = -\frac{X}{\omega R} \ln\left[1 - \frac{K_S(1-K_R) - 1}{\frac{X}{R}}\right]$$

$$K_S = \frac{1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X}t_S}\right)}{1-K_R}$$

$$K_S = \frac{V_K}{R_b I'_{SC}}$$

where

I'_{SC} = the maximum AC symmetrical short circuit current divided by CT ratio,

V_K = CT knee point voltage,

$R_b = R_{CT} + R_{EXT}$,

K_R = the per unit value of remanence flux compared to the max flux (knee point flux),

θ = voltage angle at the instant of short circuit,

φ = angle of Thevenin equivalent impedance at the point of short circuit,

Time to Saturation & CT Over-Dimensioning Factor

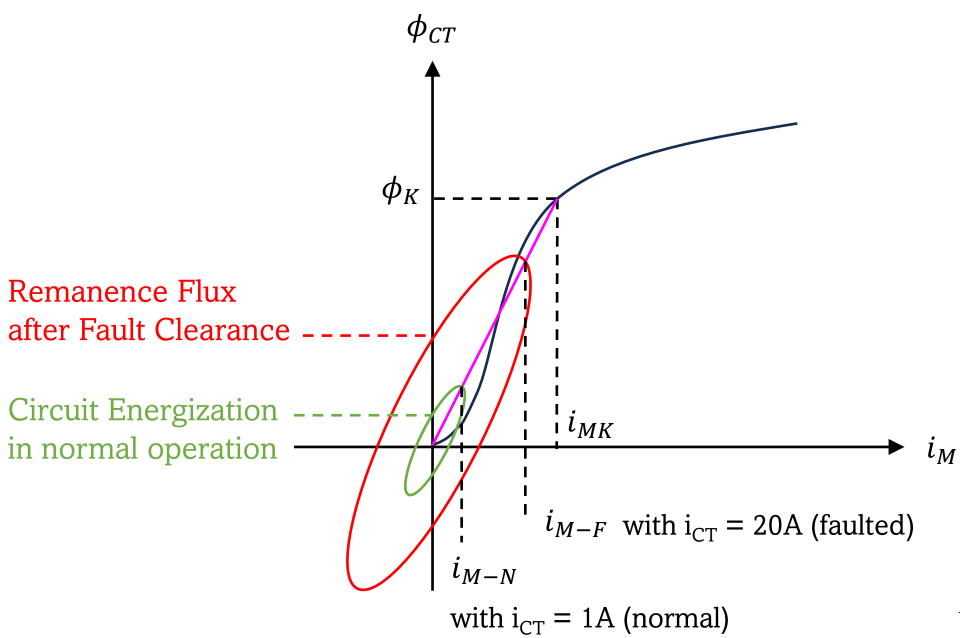
Practical Simplifications - CT Dimensioning

As a convenient selection of time to saturation, put relay operating time or relay pick-up time $t_{PK} = t_S$

$$K_S = \frac{1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X}t_S}\right)}{1 - K_R} \xrightarrow{t_S = t_{PK}} K_S = \frac{1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X}t_{PK}}\right)}{1 - K_R}$$

$$K_S = \frac{1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X}t_S}\right)}{1 - K_R} \xrightarrow{t_S = \infty} \boxed{K_S = \frac{1 + \frac{X}{R}}{1 - K_R}} \qquad \ldots(11)$$

Conversative value for K_S,
a K_S that the CT will never saturate at highest fault.



In most applications, after fault clearance with CB tripped, the circuit is not re-energized immediately.

At such condition, the remanence flux in normal operation is very low compared to the CT knee flux.

Ignore CT remanence flux (if there is no auto-reclosing),

$$K_S = \frac{1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X}t_{PK}}\right)}{1 - K_R} \xrightarrow{K_R = 0} \boxed{K_S = 1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X}t_{PK}}\right)} \qquad \ldots(12)$$

Time to Saturation & CT Over-Dimensioning Factor

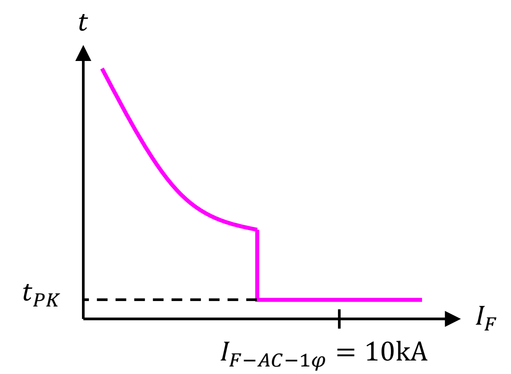
Example 5 – Effect of Relay Operation Time on CT Dimension Factor

Given that CT = 250/1, $I_{F\text{-}AC\text{-}1\varphi}$ = 10kA, X/R = 10, R_b = 4Ω.

t_{PK1} = 0.5 cycles = 10ms; t_{PK2} = 1.5 cycles = 30ms

Determine the CT dimensioning factor, and hence the CT knee point voltage required for both operation time.

$$\boxed{K_S = 1 + \frac{X}{R}\left(1 - e^{-\frac{\omega R}{X}t_{PK}}\right)}$$



t_{PK1} = 0.5 cycles = 10ms

$$K_{S1} = 1 + 10\left(1 - e^{-\frac{2\pi 50(0.01)}{10}}\right) = 3.7 \longrightarrow V_{K1} = K_{S1} R_b \frac{I_F\,[\text{A}]}{\text{CTR}}$$

$$= 3.7 \times 4 \times \frac{10000}{250} = 592\text{V}$$

t_{PK2} = 1.5 cycles = 30ms

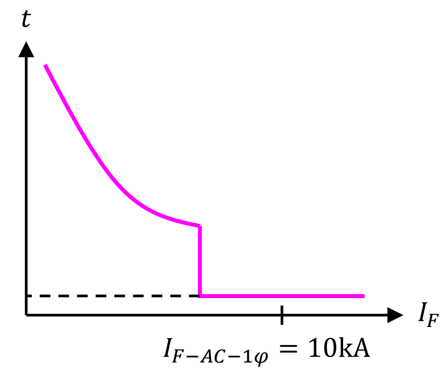
$$K_{S2} = 1 + 10\left(1 - e^{-\frac{2\pi 50(0.03)}{10}}\right) = 7.1 \longrightarrow V_{K2} = K_{S1} R_b \frac{I_F\,[\text{A}]}{\text{CTR}}$$

$$= 7.1 \times 4 \times \frac{10000}{250} = 1136\text{V}$$

NOTE – Minimum required knee point voltage of relay with 10ms operation time is less than that with 30ms operation time by half.

Time to Saturation & CT Over-Dimensioning Factor

Example 6 – Effect of Relay Operation Time on CT Dimension Factor



Given that CT = 250/1, $I_{F\text{-}AC\text{-}1\varphi}$ = 10kA, X/R = 10, R_b = 4Ω.

t_{PK} = 1.5 cycles = 30ms

Determine the CT knee point voltage under a 3-phase fault.

$$\boxed{K_S = 1 - \cos(\theta - \varphi) + \frac{X}{R}\sin(\theta - \varphi)\left(1 - e^{-\frac{\omega R}{X}t_{PK}}\right)}$$

Assume worse case switching instant at Phase A.

$$\theta_A - \varphi = 90^o \rightarrow \sin(\theta_A - \varphi) = 1, \quad \cos(\theta_A - \varphi) = 0$$

$$\theta_B - \varphi = 240^o + 90^o = 330^o \rightarrow \sin(\theta_B - \varphi) = -0.5, \quad \cos(\theta_B - \varphi) = 0.866$$

$$\theta_C - \varphi = -240^o + 90^o = 210^o \rightarrow \sin(\theta_C - \varphi) = -0.5, \quad \cos(\theta_C - \varphi) = -0.866$$

$$\text{A: } K_{SA} = 1 - 0 + 10 \times 1 \times \left(1 - e^{-\frac{2\pi 50(0.03)}{10}}\right) = 7.1 \qquad V_{KA} = 7.1 \times 4 \times \frac{10000}{250} = 1136\text{V}$$

$$\text{B: } K_{SB} = -1 - 0.866 + 10 \times -0.5 \times \left(1 - e^{-\frac{2\pi 50(0.03)}{10}}\right) = 4.92 \longrightarrow V_{KB} = 4.92 \times 4 \times \frac{10000}{250} = 787.2\text{V}$$

$$\text{C: } K_{SC} = -1 + 0.866 + 10 \times -0.5 \times \left(1 - e^{-\frac{2\pi 50(0.03)}{10}}\right) = 3.2 \qquad V_{KC} = 3.2 \times 4 \times \frac{10000}{250} = 512\text{V}$$

NOTE – Although the knee point voltage experienced by other phases under extreme fault at Phase A is much smaller than that of A, it is required to consider the worse case at all phase.

Time to Saturation & CT Over-Dimensioning Factor

As the end of this part, it is demonstrated in how to solve the secondary current $i_s(t)$.

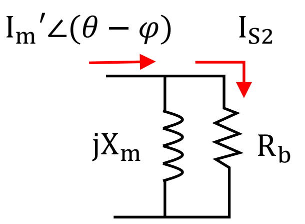
$$\boxed{\frac{di_s(t)}{dt} + \frac{1}{T_s}\, i_s(t)} = \boxed{-\frac{I'_m}{T_p}\sin(\theta - \varphi)\, e^{-\frac{t}{T_P}}} + \boxed{\omega I_m{}'\cos(\omega t + \theta - \varphi)} \qquad \ldots(4)$$

Put $\quad i_s(t) = i_{s0}(t) + i_{s1}(t) + i_{s2}(t) = \boxed{K_0 e^{-\frac{t}{T_s}}} + \boxed{K_1 e^{-\frac{t}{T_p}}} + \boxed{K_2 \sin(\omega t + \beta)}$

Consider $i_{s1}(t)$:

$$\frac{d}{dt}\left(K_1 e^{-\frac{t}{T_p}}\right) + \frac{1}{T_s}\left(K_1 e^{-\frac{t}{T_p}}\right) = -\frac{I'_m}{T_p}\sin(\theta - \varphi)\, e^{-\frac{t}{T_P}}$$

$$i_{s1}(t) = -\frac{T_S}{T_S - T_P}\, I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}}$$



Consider $i_{s2}(t)$: [by phasor method] $\quad I_m \sin(\omega t + \theta - \varphi) = I_m \angle(\theta - \varphi)$

$=1\ (X_m \gg R_b)$

$$I_{S2} = I_m \angle(\theta - \varphi)\frac{jX_m}{R_b + jX_m} = I_m \angle(\theta - \varphi)\, \frac{X_m\angle 90^o}{|R_b + jX_m|\angle \tan^{-1}\frac{X_m}{R_b}} = \frac{I_m X_m}{\sqrt{R_b^2 + X_m^2}} \angle\left(\theta - \varphi + 90^o - \tan^{-1}\frac{X_m}{R_b}\right)$$

$= 90^o\ (X_m \gg R_b)$

$$I_{S2} = I_m \angle(\theta - \varphi)$$

$$i_{s2}(t) = I'_m \sin(\omega t + \theta - \varphi)$$

Time to Saturation & CT Over-Dimensioning Factor

Consider $i_{s0}(t)$ and $i_s(t)$:

$$i_s(t) = i_{s0}(t) + i_{s1}(t) + i_{s2}(t) = K_0 e^{-\frac{t}{T_S}} + K_1 e^{-\frac{t}{T_P}} + K_2 \sin(\omega t + \beta)$$

$$= K_0 e^{-\frac{t}{T_S}} - \frac{T_S}{T_S - T_P} I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}} + I'_m \sin(\omega t + \theta - \varphi)$$

With boundary condition $i_s(0) = 0$,

$$i_s(0) = 0 = K_0 - \frac{T_S}{T_S - T_P} I'_m \sin(\theta - \varphi) + I'_m \sin(\theta - \varphi)$$

$$K_0 = \frac{T_S}{T_S - T_P} I'_m \sin(\theta - \varphi) + I'_m \sin(\theta - \varphi)$$

$$= \frac{T_P}{T_S - T_P} I'_m \sin(\theta - \varphi)$$

Hence,

$$\boxed{i_s(t) = \underbrace{\frac{T_P}{T_S - T_P} I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_S}}}_{\text{Natural Response of the differential equation}} - \underbrace{\frac{T_S}{T_S - T_P} I'_m \sin(\theta - \varphi)\, e^{-\frac{t}{T_P}}}_{\text{Excited by Transient Response of primary circuit}} + \underbrace{I'_m \sin(\omega t + \theta - \varphi)}_{\text{Excited by Steady State Response of primary circuit}}}$$