Distribution Analysis – Transformer and Voltage Regulator

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Three-Phase Transformer Models

- Three-phase transformer banks are found in the distribution substation to transform high voltage from the transmission or sub-transmission level to the distribution feeder level.
- In most cases, the substation transformer will be a three-phase unit perhaps with high-voltage off-load tap changers.
- For a four-wire wye feeder, the most common substation transformer connection is the delta–grounded wye. Three-phase transformer banks out on the feeder will provide the final voltage transformation to the customer's load.
- The load can be pure three-phase or a combination of single-phase lighting load and a three-phase load such as an induction motor. In the analysis of a distribution feeder, it is important that the various three-phase transformer connections be modeled correctly.
- Unique models of three-phase transformer banks applicable to radial distribution feeders is developed. Models for the following three-phase connections are included:
  - Delta–grounded wye
  - Ungrounded wye–delta
  - Grounded wye–delta
  - Open wye–open delta
  - Grounded wye–grounded wye
  - Delta–delta
  - Open delta–open delta

Introduction

Fig.1 defines the various voltages and currents for all three-phase transformer banks connected between the source-side node **n** and the load-side node **m**.

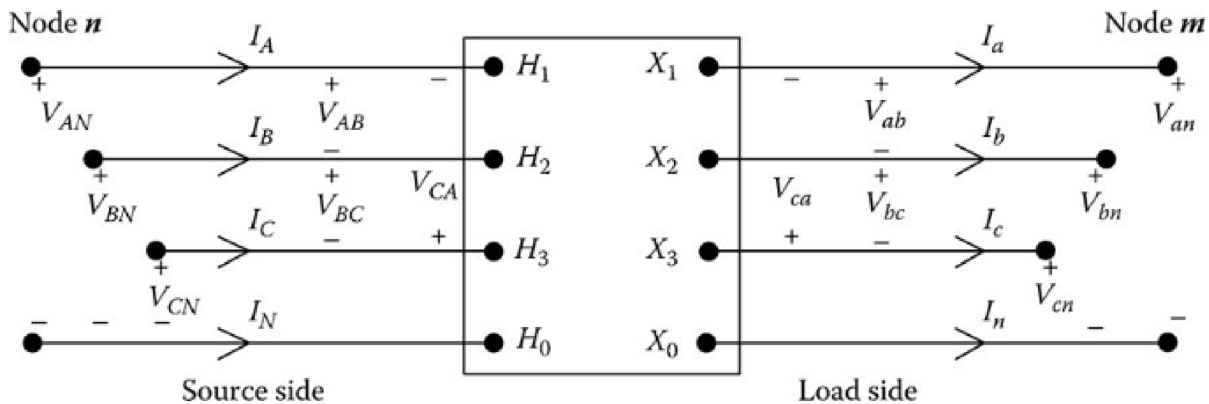


Fig.1 General three-phase transformer bank

It is assumed that all variations of the wye–delta connections are connected in the "American Standard Thirty Degree" connection. The described phase notation and the standard phase shifts for positive sequence voltages and currents are

Step-down connection

$$V_{AB} \text{ leads } V_{ab} \text{ by } 30^\circ \tag{1}$$

$$I_A \text{ leads } I_a \text{ by } 30^\circ \tag{2}$$

Step-up connection

$$V_{ab} \text{ leads } V_{AB} \text{ by } 30^\circ \tag{3}$$

$$I_a \text{ leads } I_A \text{ by } 30^\circ \tag{4}$$

Generalized Matrix

Models to be used in power-flow analysis and short-circuit studies are generalized for the connections in the same form as have been developed for line segments and voltage regulators. In the "forward sweep" of the "ladder" iterative technique, the voltages at node **m** are defined as a function of the voltages at node **n** and the currents at node **m**. The required equation is

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \tag{5}$$

In the "backward sweep" of the ladder technique, the matrix equations for computing the voltages and currents at node **n** as a function of the voltages and currents at node **m** are given by

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \tag{6}$$

$$[I_{ABC}] = [c_t] \cdot [VLN_{abc}] + [d_t] \cdot [I_{abc}] \tag{7}$$

- In Equations (5) through (7), the matrices $[VLN_{ABC}]$ and $[VLN_{abc}]$ represent
  - line-to-neutral voltages for an ungrounded wye connection
  - line-to-ground voltages for a grounded wye connection
  - "equivalent" line-to-neutral voltages for delta connection
  - The current matrices represent the line currents regardless of the transformer winding connection.
- In the modified ladder technique, Equation (5) is used to compute new node voltages downstream from the source using the most recent line currents. In the backward sweep, only Equation (7) is used to compute the source-side line currents using the newly computed load-side line currents.

Transformer Model

$$[VLN_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}] \quad (5)$$

$$[VLN_{ABC}] = [a_t][VLN_{abc}] + [b_t][I_{abc}] \quad (6)$$

$$[I_{ABC}] = [c_t][VLN_{abc}] + [d_t][I_{abc}] \quad (7)$$

	△ − Grounded Y Step-down	**△ − Grounded Y** Step-up	**Ungrounded Y − △** Step-down	**Grounded Y − Grounded Y**
$[a_t]$	$-\frac{n_t}{3}\begin{bmatrix}0 & 2 & 1\\1 & 0 & 2\\2 & 1 & 0\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}2 & 1 & 0\\0 & 2 & 1\\1 & 0 & 2\end{bmatrix}$	$n_t\begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\begin{bmatrix}n_t & 0 & 0\\0 & n_t & 0\\0 & 0 & n_t\end{bmatrix}$
$[b_t]$	$-\frac{n_t}{3}\begin{bmatrix}0 & 2Z_{t_b} & Z_{t_c}\\Z_{t_a} & 0 & 2Z_{t_c}\\2Z_{t_a} & Z_{t_b} & 0\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}2Z_{t_a} & Z_{t_b} & 0\\0 & 2Z_{t_b} & Z_{t_c}\\Z_{t_a} & 0 & 2Z_{t_c}\end{bmatrix}$	$\frac{n_t}{3}\begin{bmatrix}Z_{t_{ab}} & -Z_{t_{ab}} & 0\\Z_{t_{bc}} & 2Z_{t_{bc}} & 0\\-2Z_{t_{ca}} & Z_{t_{ca}} & 0\end{bmatrix}$	$\begin{bmatrix}n_tZ_{t_a} & 0 & 0\\0 & n_tZ_{t_b} & 0\\0 & 0 & n_tZ_{t_c}\end{bmatrix}$
$[c_t]$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$
$[d_t]$	$\frac{1}{n_t}\begin{bmatrix}1 & -1 & 0\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\frac{1}{3n_t}\begin{bmatrix}1 & -1 & 0\\1 & 2 & 0\\-2 & -1 & 0\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
$[A_t]$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & -1\\-1 & 1 & 0\\0 & -1 & 1\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & -1\\0 & 1 & -1\\-1 & 0 & 1\end{bmatrix}$	$\frac{1}{3n_t}\begin{bmatrix}2 & 1 & 0\\0 & 2 & 1\\1 & 0 & 2\end{bmatrix}$	$\frac{1}{n_t}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$
$[B_t]$	$\begin{bmatrix}Z_{t_a} & 0 & 0\\0 & Z_{t_b} & 0\\0 & 0 & Z_{t_c}\end{bmatrix}$	$[Z_{t_{abc}}]$	$\frac{1}{9}\begin{bmatrix}2Z_{t_{ab}} + Z_{t_{bc}} & 2Z_{t_{bc}} - 2Z_{t_{ab}} & 0\\2Z_{t_{bc}} - 2Z_{t_{ca}} & 4Z_{t_{bc}} - Z_{t_{ca}} & 0\\Z_{t_{ab}} - 4Z_{t_{ca}} & -Z_{t_{ab}} - 2Z_{t_{ca}} & 0\end{bmatrix}$	$[Z_{t_{abc}}]$
n_t	$\frac{\text{VLL}_{\text{rated primary}}}{\text{VLN}_{\text{rated secondary}}}$	$\frac{\text{VLL}_{\text{rated primary}}}{\text{VLN}_{\text{rated secondary}}}$	$\frac{\text{VLN}_{\text{rated primary}}}{\text{VLL}_{\text{rated secondary}}}$	$\frac{\text{VLN}_{\text{rated primary}}}{\text{VLN}_{\text{rated secondary}}}$

Dyn11 Step-Down Connection

- Delta–grounded wye (Dyn11) is a popular connection to serve a four-wire wye feeder system and provide single phase load. Because of the wye connection, three single-phase circuits are available thereby making it possible to balance the single-phase loading on the transformer bank.

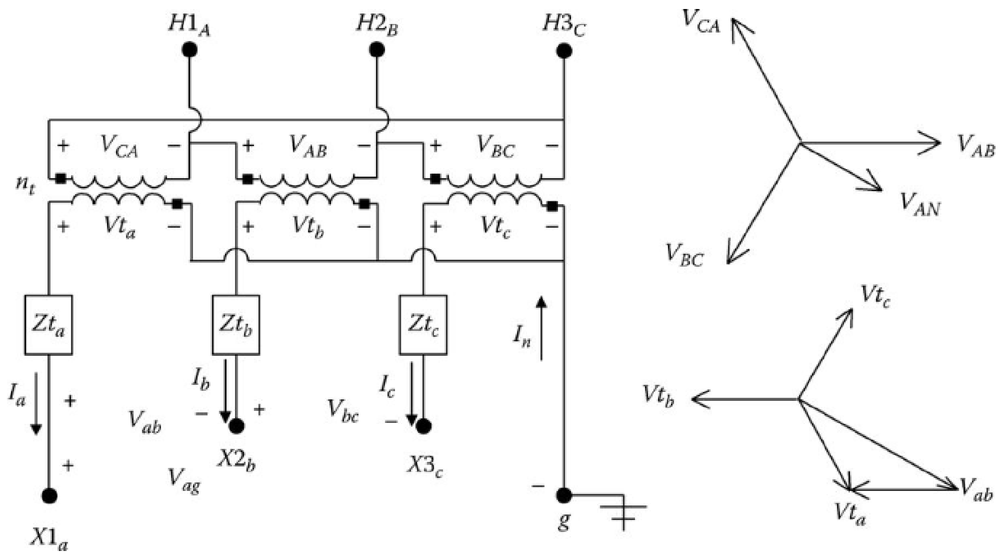


Fig.2 Standard delta-grounded wye connection with voltage

- Care must be taken to observe the polarity marks on the individual transformer windings.
- Observing the polarity markings of the transformer windings, the voltage Vta will be 180° out of phase with the voltage VCA and the voltage Vtb will be 180° out of phase with the voltage VAB. KVL gives the line-to-line voltage between phases a and b as

$$V_{ab} = Vt_a - Vt_b \tag{8}$$

Dyn11 Step Down Connection

- The magnitude changes between the voltages can be defined in terms of the actual winding turns ratio (n_t). With reference to Fig.2, these ratios are defined as follows:

$$n_t = \frac{\mathrm{VLL_{rated\ primary}}}{\mathrm{VLN_{rated\ secondary}}} \tag{9}$$

- With reference to Fig.2, the line-to-line voltages on the primary side of the transformer connection as a function of the ideal secondary side voltages are given by

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = \boxed{n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} \rightarrow [VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{10}$$

- It gives the primary line-to-line voltages at node **n** as a function of the ideal secondary voltages. However, what is needed is a relationship between "equivalent" line-to-neutral voltages at node **n** and the ideal secondary voltages.
- The question is how is the equivalent line-to-neutral voltages determined knowing the line-to-line voltages? One approach is to apply the theory of symmetrical components.
- The known line-to-line voltages are transformed to their sequence voltages by

$$[VLL_{012}] = [A_s]^{-1} \cdot [VLL_{ABC}] \qquad [A_s] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \qquad a_s = 1.0\angle 120 \tag{11}$$

Dyn11 Step Down Connection

- By definition, the zero sequence line-to-line voltage is always zero. The relationship between the positive and negative sequence line-to-neutral and line-to-line voltages is known. These relationships in matrix form are given by

$$\begin{bmatrix} VLN_0 \\ VLN_1 \\ VLN_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t_s^* & 0 \\ 0 & 0 & t_s \end{bmatrix} \cdot \begin{bmatrix} VLL_0 \\ VLL_1 \\ VLL_2 \end{bmatrix} \rightarrow \ [VLN_{012}] = [T] \cdot [VLL_{012}] \qquad t_s = \frac{1}{\sqrt{3}}\angle 30 \qquad (13)$$

- Since the zero sequence line-to-line voltage is zero, the (1,1) term of the matrix [T] can be any value. For the purposes here, the (1,1) term is chosen to have a value of 1. Knowing the sequence line-to-neutral voltages, the equivalent line-to-neutral voltages can be determined.
- The equivalent line-to-neutral voltages as a function of the sequence line-to-neutral voltages are

$$[VLN_{ABC}] = [A_s] \cdot [VLN_{012}] \qquad (14)$$

$$[VLN_{ABC}] = [A_s] \cdot \boxed{[T] \cdot [VLL_{012}]} \qquad (15)$$

$$\boxed{[VLL_{012}]} = [A_s]^{-1} \cdot [VLL_{ABC}] \qquad (11)$$

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] \qquad (16)$$

Redistribution Factor from VLL to VLN:

$$[W] = [A_s] \cdot [T] \cdot [A_s]^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad (17)$$

Dyn11 Step Down Connection

HV(L-N) to HV(L-L):

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] \tag{16}$$

$$[W] = [A_s] \cdot [T] \cdot [A_s]^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \tag{17}$$

HV(L-L) to LV(L-N):

$$[VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{10}$$

- Equation (17) provides a method of computing equivalent line-to-neutral voltages from a knowledge of the line-to-line voltages. This is an important relationship that will be used in a variety of ways as other three-phase transformer connections are studied.
- To continue on, Equation (16) can be substituted into Equation (10):

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] = \boxed{[W] \cdot [AV]} \cdot [Vt_{abc}] = [a_t] \cdot [Vt_{abc}] \tag{18}$$

$$[a_t] = [W] \cdot [AV] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} \tag{19}$$

- Equation (19) defines the generalized $[a_t]$ matrix for the Dyn11 step-down connection.
- The ideal secondary voltages as a function of the secondary line-to-ground voltages and the secondary line currents are

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \tag{20}$$

Dyn11 Step Down Connection

- Ideal secondary voltages as a function of the secondary L-G voltages and the secondary line currents are

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \tag{20}$$

$$[Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \tag{21}$$

- Note in Equation (21) there is no restriction that the impedances of the three transformers be equal.

$$[VLN_{ABC}] = [W] \cdot [VLL] = [W] \cdot [AV] \cdot [Vt_{abc}] = [a_t] \cdot \boxed{[Vt_{abc}]} \tag{18}$$

$$\boxed{[Vt_{abc}]} = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \tag{20}$$

- Substitute Eqt (18) into Eqt (20):

$$[VLN_{ABC}] = [a_t] \cdot ([VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}])$$

$$[VLN_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \tag{22}$$

Effect of Load Current to Primary Voltage:

$$[b_t] = [a_t] \cdot [Zt_{abc}] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 \cdot Zt_b & Zt_c \\ Zt_a & 0 & 2 \cdot Zt_c \\ 2 \cdot Zt_a & Zt_b & 0 \end{bmatrix} \tag{23}$$

Dyn11 Step Down Connection

$$[VLN_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}] \quad (5)$$
$$[VLN_{ABC}] = [a_t][VLN_{abc}] + [b_t][I_{abc}] \quad (6)$$
$$[I_{ABC}] = [c_t][VLN_{abc}] + [d_t][I_{abc}] \quad (7)$$

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} \rightarrow [VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \quad (10)$$

- The generalized matrices $[a_t]$ and $[b_t]$ have now been defined. The derivation of the generalized matrices $[A_t]$ and $[B_t]$ begins with solving Equation (10) for the ideal secondary voltages:

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLL_{ABC}] \quad (24)$$

- The line-to-line voltages as a function of the equivalent line-to-neutral voltages are

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] \quad (25)$$

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (26)$$

- Substitute Equation (25) into Equation (24):

$$[Vt_{abc}] = [AV]^{-1} \cdot [D] \cdot [VLN_{ABC}] = [A_t] \cdot [VLN_{ABC}] \quad (27)$$

$$[A_t] = [AV]^{-1} \cdot [D] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad (28)$$

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \quad (20)$$

Dyn11 Step Down Connection

- Substitute Equation (20) into Equation (27):

$$[Vt_{abc}] = [AV]^{-1} \cdot [D] \cdot [VLN_{ABC}] = [A_t] \cdot [VLN_{ABC}] \tag{27}$$

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \tag{20}$$

$$[VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] = [A_t] \cdot [VLN_{ABC}] \tag{29}$$

- Re-arrange: $$[VLG_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \tag{30}$$

$$[B_t] = [Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \tag{31}$$

$$[VLN_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \tag{22}$$

- Equation (22) is referred to as the "backward sweep voltage equation" and Equations (30) is referred to as the "forward sweep voltage equation." Equations (22) and (30) apply only for the step-down Dyn transformer. Note that these equations are in the same form as those derived in earlier chapters for line segments and step-voltage regulators.

Dyn11 Step Down Connection

- The thirty degree connection specifies that the positive sequence current entering the H1 terminal will lead the positive sequence current leaving the X1 terminal by 30°. Fig.3 shows the same connection as Fig.2 but with the currents instead of the voltages displayed.

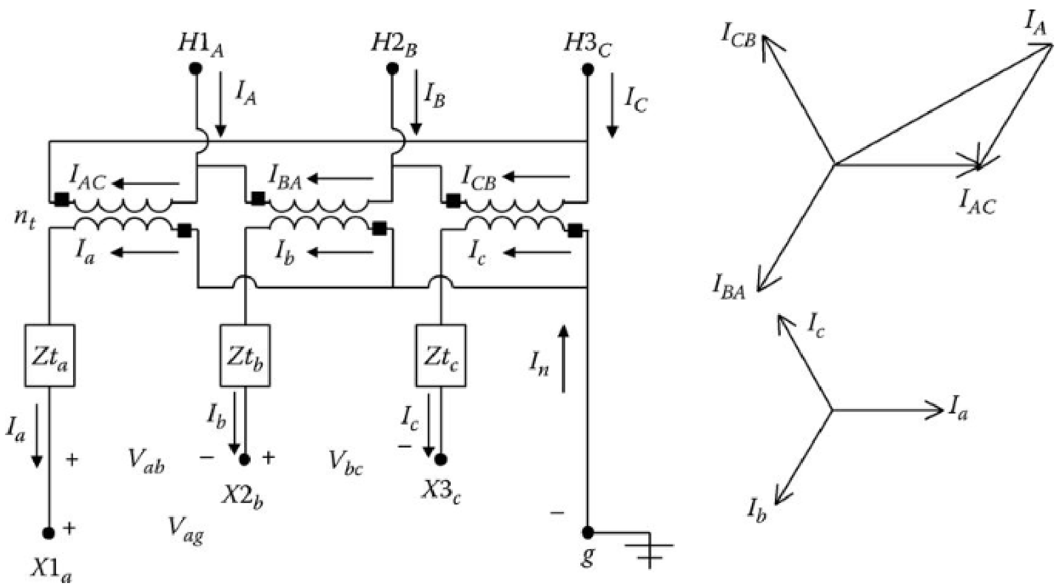


Fig.3 Delta-grounded wye connection with current

- As with the voltages, the polarity marks on the transformer windings must be observed for the currents.
- For example, in Fig.3, the current I_a is entering the polarity mark on the LV winding so the current I_{AC} flowing out of the polarity mark on the high-voltage winding will be in phase with I_a.

Dyn11 Step Down Connection

- The line currents can be determined as a function of the delta currents by applying KCL:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{Ac} \\ I_{BA} \\ I_{CB} \end{bmatrix} \rightarrow [I_{ABC}] = [D] \cdot [ID_{ABC}] \tag{32}$$

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{33}$$

- The matrix equation relating the delta primary currents to the secondary line currents is given by

$$\begin{bmatrix} I_{Ac} \\ I_{BA} \\ I_{CB} \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \rightarrow [ID_{ABC}] = [AI] \cdot [I_{abc}] \tag{34}$$

$$[AI] = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{35}$$

- Substitute Equation (35) into Equation (33):

$$[I_{ABC}] = [D] \cdot [AI] \cdot [I_{abc}] = [c_t] \cdot [VLG_{abc}] + [d_t] \cdot [I_{abc}] \tag{36}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{38}$$

HV current only affected by LV current:

$$[d_t] = [D] \cdot [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{37}$$

Dyn11 Step Down Connection

$$[VLN_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}] \quad (5)$$

$$[VLN_{ABC}] = [a_t][VLN_{abc}] + [b_t][I_{abc}] \quad (6)$$

$$[I_{ABC}] = [c_t][VLN_{abc}] + [d_t][I_{abc}] \quad (7)$$

$$[I_{ABC}] = [D] \cdot [AI] \cdot [I_{abc}] = [c_t] \cdot [VLG_{abc}] + [d_t] \cdot [I_{abc}] \quad (36)$$

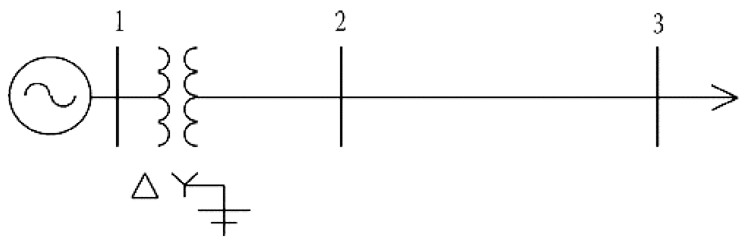
$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (38)$$

$$[d_t] = [D] \cdot [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (37)$$

- Equation (36) (referred to as the "backward sweep current equations") provides a direct method of computing the phase line currents at node **n** knowing the phase line currents at node **m**. Again, this equation is in the same form as that previously derived for three-phase line segments and three-phase step-voltage regulators.
- The equations derived in this section are for the *step-down connection*.

Example 1 – Transformer Feeder in Dyn11

In Figure 4, an *unbalanced* constant impedance load is being served at the end of a 1 mile section of a three-phase line. The 1 mile long line is being fed from a substation transformer rated 5000 kVA, 115 kV delta - 12.47 kV grounded wye with a per-unit impedance of 0.085 ∠ 85.



The phase conductors of the line are 336,400 26/7 ACSR with a neutral conductor 4/0 ACSR. The configuration and computation of the phase impedance matrix are given in Example 4.1. From that example, the phase impedance matrix was computed to be

$$[Zline_{abc}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

The general matrices for the line are

$$[A_{line}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad [B_{line}] = [Zline_{abc}] \qquad [d] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The transformer impedance needs to be converted to per unit referenced to the low-voltage side of the transformer. The base impedance is

$$Z_{base} = \frac{12.47^2 \cdot 1000}{5000} = 31.1\Omega$$

Example 1 – Transformer Feeder in Dyn11

- The transformer impedance referenced to the low-voltage side is

$$Z_t = (0.085\angle 85)31.1 = 0.2304 + \mathrm{j}2.6335\ \Omega$$

- The transformer phase impedance matrix is

$$[Zt_{abc}] = \begin{bmatrix} 0.2304 + j2.6335 & 0 & 0 \\ 0 & 0.2304 + j2.6335 & 0 \\ 0 & 0 & 0.2304 + j2.6335 \end{bmatrix} \Omega$$

- The unbalanced constant impedance load is connected in grounded wye. The load impedance matrix is specified to be

$$[Zload_{abc}] = \begin{bmatrix} 12 + j6 & 0 & 0 \\ 0 & 13 + j4 & 0 \\ 0 & 0 & 14 + j5 \end{bmatrix} \Omega$$

- The unbalanced line-to-line voltages at node 1 serving the substation transformer are given as

$$[VLL_{ABC}] = \begin{bmatrix} 115{,}000\angle 0 \\ 116{,}500\angle -115.5 \\ 123{,}538\angle 121.7 \end{bmatrix} V$$

a. Determine the generalized matrices for the transformer.

- The “transformer turn's” ratio is

$$n_t = \frac{\mathrm{KVLL_{high}}}{\mathrm{KVLN_{low}}} = \frac{115}{12.47/\sqrt{3}} = 15.9732$$

Example 1 – Transformer Feeder in Dyn11

$$\boxed{[a_t] = [W] \cdot [AV] = \frac{-n_t}{3}\begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}} \tag{19}$$

$$[a_t] = \frac{-n_t}{3}\begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -10.6488 & -5.3244 \\ -5.3244 & 0 & -10.6488 \\ -10.6488 & -5.3244 & 0 \end{bmatrix}$$

$$\boxed{[b_t] = [a_t] \cdot [Zt_{abc}] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 \cdot Zt_b & Zt_c \\ Zt_a & 0 & 2 \cdot Zt_c \\ 2 \cdot Zt_a & Zt_b & 0 \end{bmatrix}} \tag{23}$$

$$[b_t] = \begin{bmatrix} 0 & -2.4535 - j28.0432 & -1.2267 - j14.0216 \\ -1.2267 - j14.0216 & 0 & -2.4535 - j28.0432 \\ -2.4535 - j28.0432 & -1.2267 - j14.0216 & 0 \end{bmatrix}$$

$$\boxed{[d_t] = [D] \cdot [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}} \tag{37}$$

$$[d_t] = \begin{bmatrix} 0.0626 & -0.0626 & 0 \\ 0 & 0.0626 & -0.0626 \\ -0.0626 & 0 & 0.0626 \end{bmatrix}$$

Example 1 – Transformer Feeder in Dyn11

$$\boxed{[A_t] = [AV]^{-1} \cdot [D] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}} \tag{28}$$

$$[A_t] = \begin{bmatrix} 0.0626 & 0 & -0.0626 \\ -0.0626 & 0.0626 & 0 \\ 0 & -0.0626 & 0.0626 \end{bmatrix}$$

$$\boxed{[B_t] = [Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_a & 0 \\ 0 & 0 & Zt_a \end{bmatrix}} \tag{31}$$

$$[B_t] = \begin{bmatrix} 0.2304 + j2.6335 & 0 & 0 \\ 0 & 0.2304 + j2.6335 & 0 \\ 0 & 0 & 0.2304 + j2.6335 \end{bmatrix}$$

B. Given the line-to-line voltages at node 1, determine the "ideal" transformer voltages.

$$[AV] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -15.9732 & 0 \\ 0 & 0 & -15.9732 \\ -15.9732 & 0 & 0 \end{bmatrix}$$

$$[Vt_{abc}] = [AV]^{-1} \cdot [D] \cdot [VLN_{ABC}] = \begin{bmatrix} 7734.1\angle -58.3 \\ 7199.6\angle 180 \\ 7293.5\angle 64.5 \end{bmatrix} \text{V}$$

Example 1 – Transformer Feeder in Dyn11

C. Determine the load currents.

- Since the load is modeled as constant impedances, the system is linear and the analysis can combine all of the impedances (transformer, line, and load) to an equivalent impedance matrix. KVL gives

$$[Vt_{abc}] = ([Zt_{abc}] + [Zline_{abc}] + [Zload_{abc}]) \cdot [I_{abc}] = [Zeq_{abc}] \cdot [I_{abc}]$$

$$[Zeq_{abc}] = \begin{bmatrix} 13.0971 + j10.6751 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 14.1141 + j8.6187 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 15.1045 + j9.6507 \end{bmatrix} \Omega$$

- The line current can now be computed:

$$[I_{abc}] = [Zeq_{abc}]^{-1} \cdot [Vt_{abc}] = \begin{bmatrix} 471.7\angle -95.1 \\ 456.7\angle 149.9 \\ 427.3\angle 33.5 \end{bmatrix} A$$

D. Determine the line-to-ground voltages at the load in volts and on a 120 V base:

$$[Vload_{abc}] = [Zload_{abc}] \cdot [I_{abc}] = \begin{bmatrix} 6328.1\angle -68.6 \\ 6212.2\angle 167.0 \\ 6352.6\angle 53.1 \end{bmatrix} V$$

- The load voltages on a 120 V base are

$$[Vload_{120}] = \begin{bmatrix} 105.5 \\ 103.5 \\ 105.9 \end{bmatrix}$$

Example 1 – Transformer Feeder in Dyn11

- The line-to-ground voltage at node 2 are

$$[VLG_{abc}] = [a_{line}] \cdot [Vload_{abc}] + [b_{line}] \cdot [I_{abc}] = \begin{bmatrix} 6965.4\angle-66.0 \\ 6580.6\angle171.4 \\ 6691.4\angle56.7 \end{bmatrix} V$$

E. Using the backward sweep voltage equation, determine the equivalent line-to-neutral voltages and the line-to-line voltages at node 1.

$$[VLN_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] = \begin{bmatrix} 69{,}443\angle-30.3 \\ 65{,}263\angle-147.5 \\ 70{,}272\angle94.0 \end{bmatrix} V$$

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] = \begin{bmatrix} 115{,}000\angle0 \\ 116{,}500\angle-115.5 \\ 123{,}538\angle121.7 \end{bmatrix} V$$

- It is always comforting to be able to work back and compute what was initially given. In this case, the line-to-line voltages at node 1 have been computed and the same values result as given.

F. Use the forward sweep voltage equation to verify that the line-to-ground voltages at node 2 can be computed knowing equivalent line-to-neutral voltages at node 1 and the currents leaving node 2.

$$[VLG_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] = \begin{bmatrix} 6965.4\angle-66.0 \\ 6580.6\angle171.4 \\ 6691.4\angle56.7 \end{bmatrix} V$$

Delta-Grounded Wye Step-Up VS Step-Down Connections

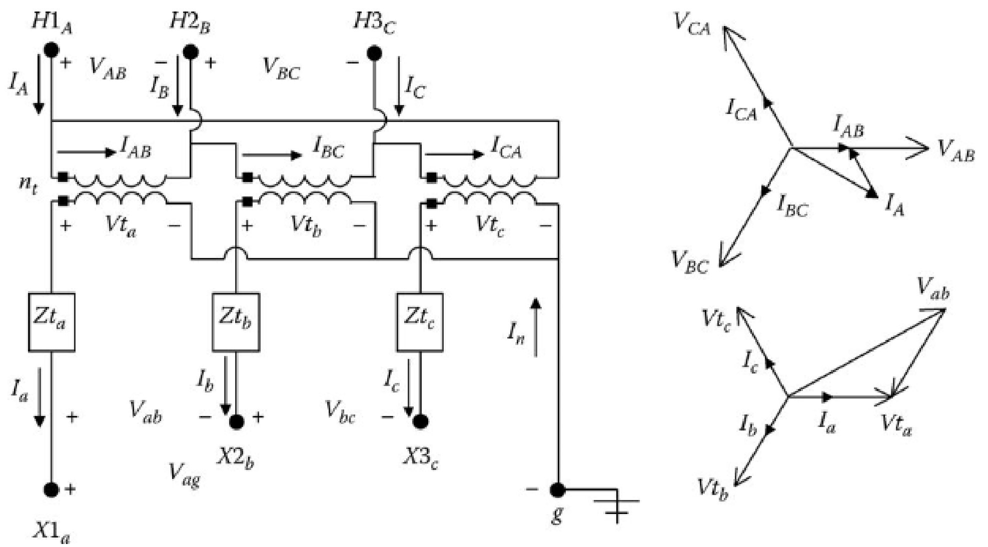


Fig.2 Standard delta-grounded wye Step Down connection with voltage

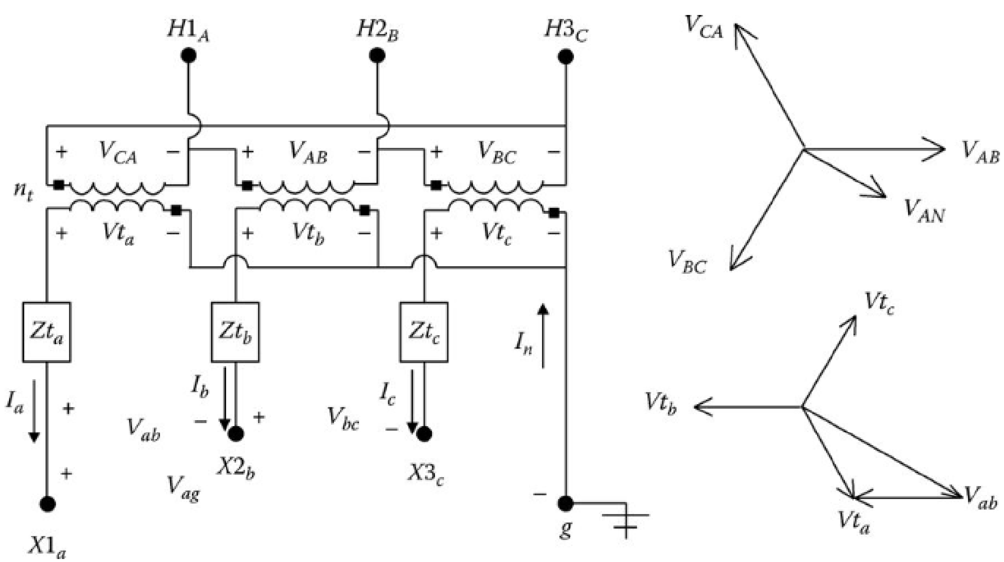


Fig.7 Delta-grounded wye Step-up connection

Ynd05 Step Up Connection

- Phasor diagrams with Connection diagram are shown in Fig.7. Note that the HV_{LL} voltage leads the LV_{LL} voltage and the same can be said for the high- and low-side line currents.

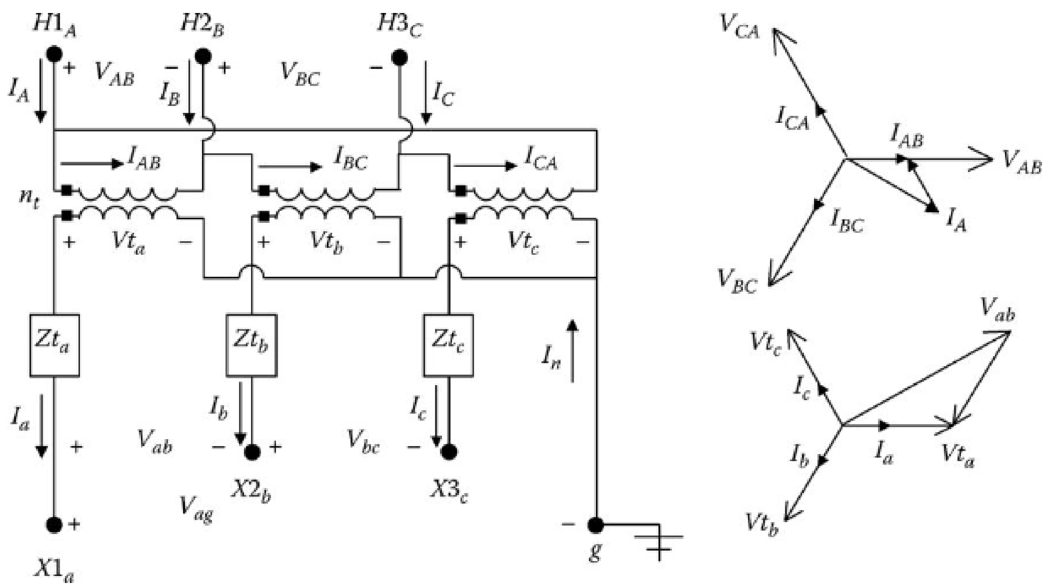


Fig.7 Delta-grounded wye step-up connection

- The primary (low side) line-to-line voltages are given by

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} \rightarrow [VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \quad (39)$$

$$[AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{n_t} = \frac{\mathrm{KVLL_{rated\ Primary}}}{\mathrm{KVLN_{rated\ Secondary}}}$$

Ynd05 Step Up Connection

- The primary delta currents are given by

$$\begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} = \frac{1}{n_t}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \rightarrow [ID_{ABC}] = [AI] \cdot [I_{abc}] \tag{40}$$

$$[AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- The primary line currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} \quad [I_{ABC}] = [DI] \cdot [ID_{ABC}] \tag{41}$$

$$[DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

- Forward sweep matrices are
- Applying Equation (28),

$$[A_t] = [AV]^{-1} \cdot [D] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{42}$$

- Applying Equation (31),

$$[B_t] = [Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \tag{43}$$

Ynd05 Step Up Connection

$$[VLN_{abc}] = [A_t][VLN_{ABC}] - [B_t][I_{abc}] \quad (5)$$

$$[VLN_{ABC}] = [a_t][VLN_{abc}] + [b_t][I_{abc}] \quad (6)$$

$$[I_{ABC}] = [c_t][VLN_{abc}] + [d_t][I_{abc}] \quad (7)$$

- Backward sweep matrices are
- Applying Equation (19),

$$[a_t] = [W] \cdot [AV] = \frac{n_t}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \quad (44)$$

- Applying Equation (23),

$$[b_t] = [a_t] \cdot [Zt_{abc}] = \frac{n_t}{3} \cdot \begin{bmatrix} 2 \cdot Zt_a & Zt_b & 0 \\ 0 & 2 \cdot Zt_b & Zt_c \\ Zt_a & 0 & 2 \cdot Zt_c \end{bmatrix} \quad (45)$$

- Applying Equation (37),

$$[d_t] = [D] \cdot [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (46)$$

- In Delta – ungrounded wye transformer, the grounded wye connection is characterized by
  - The grounded wye provides a path for zero sequence currents for line-to-ground faults upstream from the transformer bank. This causes the transformers to be susceptible to burnouts on the upstream faults.
  - If one phase of the primary circuit is opened, the transformer bank will continue to provide three-phase service by operating as an open wye–open delta bank. However, the two remaining transformers may be subjected to an overload condition leading to burnout.
- Hence, the most common connection is the ungrounded wye–delta. This connection is typically used to provide service to a combination of single-phase “lighting” load and a three-phase “power” load such as an induction motor.

Undergrounded Wye-Delta Step-Down Connection

- Three single-phase transformers can be connected in an ungrounded-wye "standard 30 degree connection" as shown in Fig.8.
- The voltage phasor diagrams in Fig.8 illustrate that the high-side positive sequence line-to-line voltage *leads* the low-side positive sequence line-to-line voltage by 30°. Also, the same phase shift occurs between the high-side line-to-neutral voltage and the low-side "equivalent" line-to-neutral voltage. The negative sequence phase shift is such that the high-side negative sequence voltage will lag the low-side negative sequence voltage by 30°.

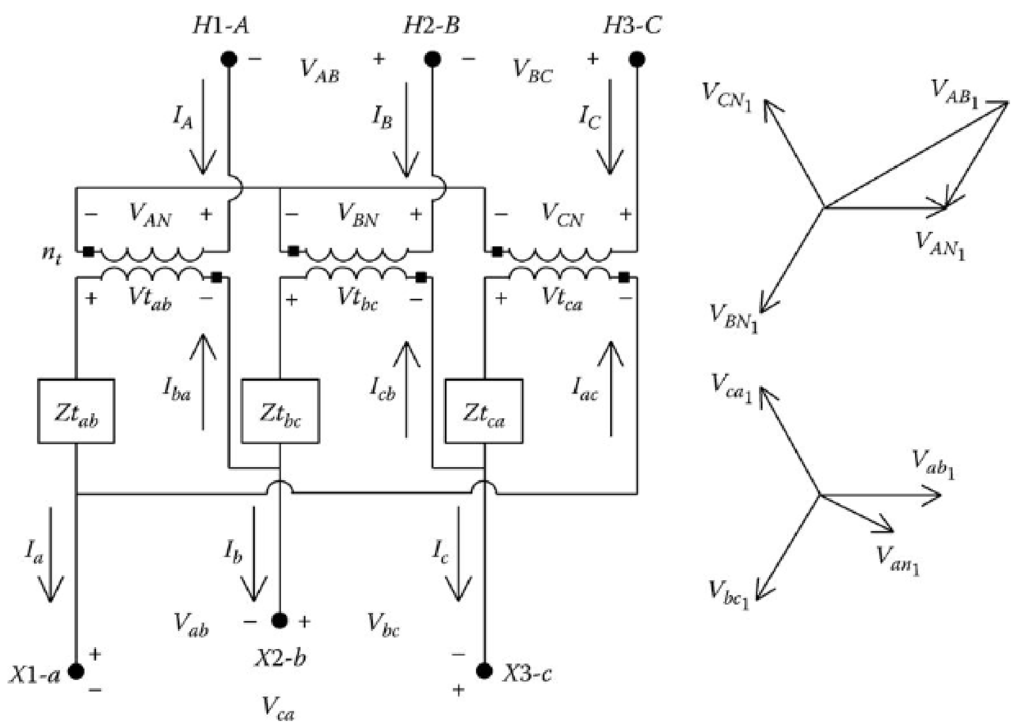


Fig.8 Standard ungrounded wye-delta connection step-down

Undergrounded Wye-Delta Step-Down Connection

- The positive sequence current phasor diagrams for the connection in Fig.8 are shown in Fig.9.
- Fig.9 illustrates that the positive sequence line current on the high side of the transformer (node **n**) leads the low-side line current (node **m**) by 30°. It can also be shown that the negative sequence high-side line current will lag the negative sequence low-side line current by 30°.

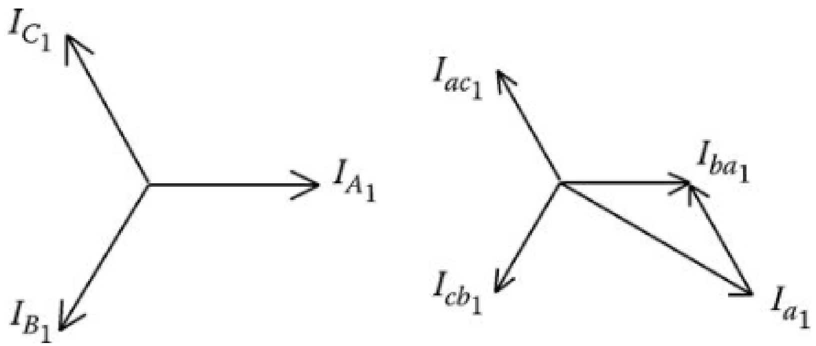


Fig.9 Positive sequence current phasors

- The definition for the "turns ratio n_t" will be the same as Equation (9) with the exception that the numerator will be the line-to-neutral voltage and the denominator will be the line-to-line voltage. It should be noted in Fig.8 that the "ideal" low-side transformer voltages for this connection will be line-to-line voltages. Also, the "ideal" low-side currents are the currents flowing inside the delta.

Undergrounded Wye-Delta Step-Down Connection

- The basic "ideal" transformer voltage and current equations as a function of the "turns ratio" are

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \rightarrow [VLN_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{47}$$

$$\begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \rightarrow [ID_{abc}] = [AI] \cdot [I_{ABC}] \tag{49}$$

$$n_t = \frac{\mathrm{KVLN}_{\text{rated Primary}}}{\mathrm{KVLL}_{\text{rated Secondary}}}$$

- Solving Equation (47) for the "ideal" delta transformer voltages,

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] \tag{51}$$

- The line-to-line voltages at node **m** as a function of the "ideal" transformer voltages and the delta currents are given by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \tag{52}$$

$$[VLL_{abc}] = [Vt_{abc}] - [Zt_{abc}] \cdot [ID_{abc}] \tag{53}$$

Undergrounded Wye-Delta Step-Down Connection

- Substitute Equations (50) and (51) into Equation (53):

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [ZNt_{abc}] \cdot [I_{ABC}] \tag{54}$$

$$[ZNt_{abc}] = [Zt_{abc}] \cdot [AI] = \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & n_t \cdot Zt_{bc} & 0 \\ 0 & 0 & n_t \cdot Zt_{ca} \end{bmatrix} \tag{55}$$

- The line currents on the delta side of the transformer bank as a function of the wye transformer currents are given by

$$[I_{abc}] = [DI] \cdot [ID_{abc}] \tag{56}$$

$$[DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{57}$$

- Substitute Equation (50) into Equation (56):

$$[I_{abc}] = [DI] \cdot [AI] \cdot [I_{ABC}] = [DY] \cdot [I_{ABC}] \tag{58}$$

$$[DY] = [DI] \cdot [AI] = \boxed{\begin{bmatrix} n_t & 0 & -n_t \\ -n_t & n_t & 0 \\ 0 & -n_t & n_t \end{bmatrix}} \tag{59}$$

Singular Matrix

Undergrounded Wye-Delta Step-Down Connection

$$[I_{abc}] = [DI] \cdot [AI] \cdot [I_{ABC}] = [DY] \cdot [I_{ABC}] \tag{58}$$

$$[DY] = [DI] \cdot [AI] = \boxed{\begin{bmatrix} n_t & 0 & -n_t \\ -n_t & n_t & 0 \\ 0 & -n_t & n_t \end{bmatrix}} \tag{59}$$

Singular Matrix

- Because the matrix [DY] is singular, it is not possible to use Equation (58) to develop an equation relating the wye-side line currents at node **n** to the delta-side line currents at node **m**.
- To develop the necessary matrix equation, three independent equations must be written. Two independent KCL equations at the vertices of the delta can be used. Because there is no path for the high-side currents to flow to ground, they must sum to zero and, therefore, so must the delta currents in the transformer secondary sum to zero. This provides the third independent equation. The resulting three independent equations in matrix form are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{60}$$

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 1 \\ -2 & -1 & 1 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} \tag{61}$$

$$[ID_{abc}] = [L0] \cdot [I_{ab0}] \tag{62}$$

Undergrounded Wye-Delta Step-Down Connection

$$[ID_{abc}] = [L0] \cdot [I_{ab0}] \tag{62}$$

- Equation (62) can be modified to include the phase c current by setting the third column of the [L0] matrix to zero:

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \rightarrow [ID_{abc}] = [L] \cdot [I_{abc}] \tag{63}$$

(64)

- Solve Equation (50) for [IABC] and substitute into Equation (64):

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \tag{50}$$

$$[I_{ABC}] = [AI]^{-1} \cdot [L] \cdot [I_{abc}] = [d_t] \cdot [I_{abc}] \tag{65}$$

$$[d_t] = [AI]^{-1}[L] = \frac{1}{3 \cdot n_t}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix} \tag{66}$$

- Equation (66) defines the generalized constant matrix $[d_t]$ for the ungrounded wye–delta transformer connection. In the process of the derivation, a very convenient equation (Equation (63)) evolved that can be used anytime the currents in a delta need to be determined knowing the line currents. However, it must be understood that this equation will only work when the delta currents sum to zero, which means an ungrounded neutral on the primary.
- The generalized matrices $[a_t]$ and $[b_t]$ can now be developed. Solve Equation (54) for $[VLN_{ABC}]$:

$$[VLN_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [ZNt_{abc}] \cdot [I_{ABC}] \tag{67}$$

Undergrounded Wye-Delta Step-Down Connection

$$[I_{ABC}] = [AI]^{-1} \cdot [L] \cdot [I_{abc}] = [d_t] \cdot [I_{abc}] \tag{65}$$

$$[VLN_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [ZNt_{abc}] \cdot [I_{ABC}] \tag{67}$$

- Substitute Eqt (65) into (67):

$$[VLN_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [ZNt_{abc}] \cdot [d_t] \cdot [I_{abc}]$$

$$[VLL_{abc}] = [D] \cdot [VLN_{abc}]$$

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[VLN_{ABC}] = [AV] \cdot [D] \cdot [VLN_{abc}] + [AV] \cdot [ZNt_{abc}] \cdot [d_t] \cdot [I_{abc}]$$

$$[VLN_{abc}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \tag{68}$$

$$[a_t] = [AV] \cdot [D] = n_t \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{69}$$

$$[b_t] = [AV][ZNt_{abc}][d_t] = \frac{n_t}{3} \begin{bmatrix} Zt_{ab} & -Zt_{ab} & 0 \\ Zt_{bc} & 2 \cdot Zt_{bc} & 0 \\ -2 \cdot Zt_{ca} & -Zt_{ca} & 0 \end{bmatrix} \tag{70}$$

Undergrounded Wye-Delta Step-Down Connection

- The generalized constant matrices have been developed for computing voltages and currents from the load toward the source (backward sweep). The forward sweep matrices can be developed by referring to Equation (54), which is repeated here for convenience:

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [ZNt_{abc}] \cdot [I_{ABC}] \quad (71)$$

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] \quad (16)$$

- Equation (16) is used to compute the equivalent line-to-neutral voltages as a function of the line-to-line voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] \quad (72)$$

- Substitute Equation (71) into Equation (72):

$$[VLN_{abc}] = [W] \cdot [AV]^{-1} \cdot [VLN_{ABC}] - [W] \cdot [ZNt_{abc}] \cdot [d_t] \cdot [I_{ABC}] \quad (73)$$

$$[A_t] = [W] \cdot [AI]^{-1} = \frac{1}{3 \cdot n_t}\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \quad (74)$$

$$[B_t] = [W] \cdot [ZNt_{abc}] \cdot [d_t] = \frac{1}{9}\begin{bmatrix} 2 \cdot Zt_{ab} + Zt_{bc} & 2 \cdot Zt_{bc} - Zt_{ab} & 0 \\ 2 \cdot Zt_{bc} - 2 \cdot Zt_{ca} & 4 \cdot Zt_{bc} - Zt_{ca} & 0 \\ Zt_{ab} - 4 \cdot Zt_{ca} & -Zt_{ab} - 2 \cdot Zt_{ca} & 0 \end{bmatrix} \quad (75)$$

- The generalized matrices have been developed for the ungrounded wye–delta transformer connection. The derivation has applied basic circuit theory and the basic theories of transformers. The result of the derivations is to provide an easy method of analyzing the operating characteristics of the transformer connection.

Example 3

Fig.10 shows three single-phase transformers in an ungrounded wye–delta connection serving a combination of single-phase and three-phase load in a delta connection. The voltages at the load are balanced three phase of 240 V line to line. The net loading by phase is

$$S_{ab} = 100 \text{ kVA at } 0.9 \text{ lagging power factor}$$

$$S_{bc} = S_{ca} = 50 \text{ kVA at } 0.8 \text{ lagging power factor}$$

The transformers are rated as follows:

Phase AN: 100 kVA, 7200–240 V, Z = 0.01 + j0.04 per unit
Phases BN and CN: 50 kVA, 7200–240 V, Z = 0.015 + j0.035 per unit

Determine the following:

a.) currents in the load, b.) secondary line currents, c.) equivalent line-to-neutral secondary voltages, d.) primary line-to-neutral and line-to-line voltages, e.) primary line currents

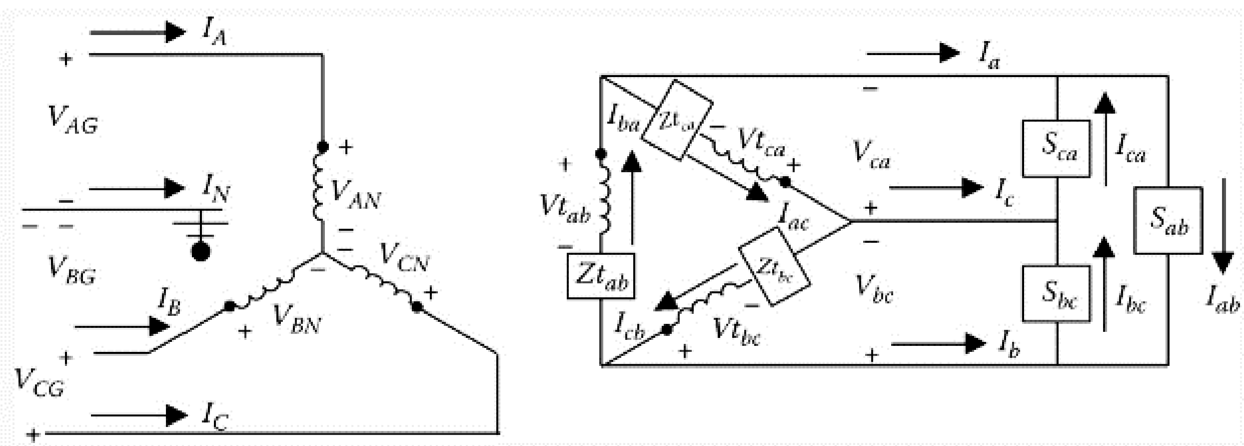


Fig.10 Undergrounded wye-delta step-down with unbalanced load

Example 3

Before the analysis can start, the transformer impedances must be converted to actual values in Ohms and located inside the delta-connected secondary windings.

"Lighting" transformer:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{100} = 0.576 \qquad Zt_{ab} = (0.01 + j0.4) \cdot 0.576 = 0.0058 + j0.023\ \Omega$$

"Power" transformers:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{50} = 1.152 \qquad \begin{aligned} Zt_{bc} = Zt_{ca} &= (0.015 + j0.35) \cdot 1.152 \\ &= 0.0173 + j0.0403\ \Omega \end{aligned}$$

The transformer impedance matrix can now be defined:

$$[Zt_{abc}] = \begin{bmatrix} 0.0058 + j0.023 & 0 & 0 \\ 0 & 0.0173 + j0.0403 & 0 \\ 0 & 0 & 0.0173 + j0.0403 \end{bmatrix} \Omega$$

The turn's ratio of the transformers is n_t = 7200/240 = 30.

Define all of the matrices

$$[W] = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad [D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad [DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[a_t] = n_t \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 30 & -30 & 0 \\ 0 & 30 & -30 \\ -30 & 0 & 30 \end{bmatrix}$$

Example 3

$$[b_t] = \frac{n_t}{3} \cdot \begin{bmatrix} Zt_{ab} & -Zt_{ab} & 0 \\ Zt_{bc} & 2Zt_{bc} & 0 \\ -2Zt_{ca} & -Zt_{ca} & 0 \end{bmatrix} = \begin{bmatrix} 0.0576 + j0.2304 & -0.576 - j0.2304 & 0 \\ 0.1728 + j0.4032 & 0.3456 + j0.8064 & 0 \\ -0.3456 - j0.8064 & -0.1728 - j0.4032 & 0 \end{bmatrix}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[d_t] = \frac{1}{3n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.0111 & -0.0111 & 0 \\ 0 & 0.0111 & -0.0111 \\ -0.0111 & 0 & 0.0111 \end{bmatrix}$$

$$[A_t] = \frac{1}{3n_t} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0.0222 & 0.0111 & 0 \\ 0 & 0.0222 & 0.0111 \\ 0.0111 & 0 & 0.0222 \end{bmatrix}$$

$$[B_t] = \frac{1}{9} \begin{bmatrix} 2Zt_{ab} + Zt_{bc} & 2Zt_{bc} - Zt_{ab} & 0 \\ 2Zt_{bc} - 2Zt_{ca} & 4Zt_{bc} - Zt_{ca} & 0 \\ Zt_{ab} - 4Zt_{ab} & -Zt_{ab} - 2Zt_{ab} & 0 \end{bmatrix} = \begin{bmatrix} 0.0032 + j0.0096 & 0.0026 + j0.0038 & 0 \\ 0 & 0.0058 + j0.0134 & 0 \\ -0.007 - j0.0154 & -0.0045 - j0.0115 & 0 \end{bmatrix}$$

Define the line-to-line load voltages: $[VLL_{abc}] = \begin{bmatrix} 240\angle 0 \\ 240\angle -120 \\ 240\angle 120 \end{bmatrix}$

Define the load:

$$[SD_{abc}] = \begin{bmatrix} 100\angle \cos^{-1}(0.9) \\ 50\angle \cos^{-1}(0.8) \\ 50\angle \cos^{-1}(0.8) \end{bmatrix} = \begin{bmatrix} 90 + j43.589 \\ 30 + j30 \\ 30 + j30 \end{bmatrix} \text{kVA}$$

Example 3

- Calculate the delta load current

$$ID_i = \left(\frac{SD_i \cdot 1000}{VLL_{abc_i}}\right)^* \ A \qquad [ID_{abc}] = \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix} = \begin{bmatrix} 416.7\angle -25.84 \\ 208.3\angle -156.87 \\ 208.3\angle 83.13 \end{bmatrix} A$$

- Compute the secondary line current:

$$[I_{abc}] = [DI] \cdot [ID_{abc}] = \begin{bmatrix} 522.9\angle -47.97 \\ 575.3\angle -119.06 \\ 360.8\angle 53.13 \end{bmatrix} A$$

- Compute the equivalent secondary line-to-neutral voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] = \begin{bmatrix} 138.56\angle -30 \\ 138.56\angle -150 \\ 138.56\angle 90 \end{bmatrix} V$$

- Use the generalized constant matrices to compute the primary line-to-neutral voltages and line-to-line voltages:

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] = \begin{bmatrix} 7367.6\angle 1.4 \\ 7532.3\angle -119.1 \\ 7406.2\angle 121.7 \end{bmatrix} V$$

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] = \begin{bmatrix} 12{,}9356\angle 31.54 \\ 12{,}8845\angle -88.95 \\ 12{,}8147\angle 151.50 \end{bmatrix} kV$$

Example 3

- The high primary line currents are

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] = \begin{bmatrix} 11.54\angle - 28.04 \\ 8.95\angle - 166.43 \\ 7.68\angle 101.16 \end{bmatrix} A$$

- It is interesting to compute the operating kVA of the three transformers. Taking the product of the transformer voltage times the conjugate of the current gives the operating kVA of each transformer.

$$ST_i = \frac{VLN_{ABC_i} \cdot \left(I_{ABC_i}\right)^*}{1000} = \begin{bmatrix} 85.02\angle 29.46 \\ 67.42\angle 47.37 \\ 56.80\angle 20.58 \end{bmatrix} kVA$$

- The operating power factors of the three transformers are

$$[PF] = \begin{bmatrix} \cos(29.46) \\ \cos(47.39) \\ \cos(20.58) \end{bmatrix} = \begin{bmatrix} 87.1 \\ 67.7 \\ 93.6 \end{bmatrix} \%$$

- Note that the operating kVAs do not match very closely the rated kVAs of the three transformers.
- Transformer on phase A did not serve the total load of 100 kVA that is directly connected its terminals. That transformer is operating below rated kVA, while the other two transformers are overloaded.
- Transformer connected to phase B is operating 35% above rated kVA. Because of this overload, the ratings of the three transformers should be changed so that the phase B and phase C transformers are rated 75 kVA.
- Finally, the operating power factors of the three transformers bare little resemblance to the load power factors.

Undergrounded Wye-Delta Step-up Connection

- The connection diagram for the step-up connection is shown in Fig.11.
- The only difference between the step-up and step-down connections are the definitions of the turns ratio n_t, [AV], and [AI]. For the step-up connection,

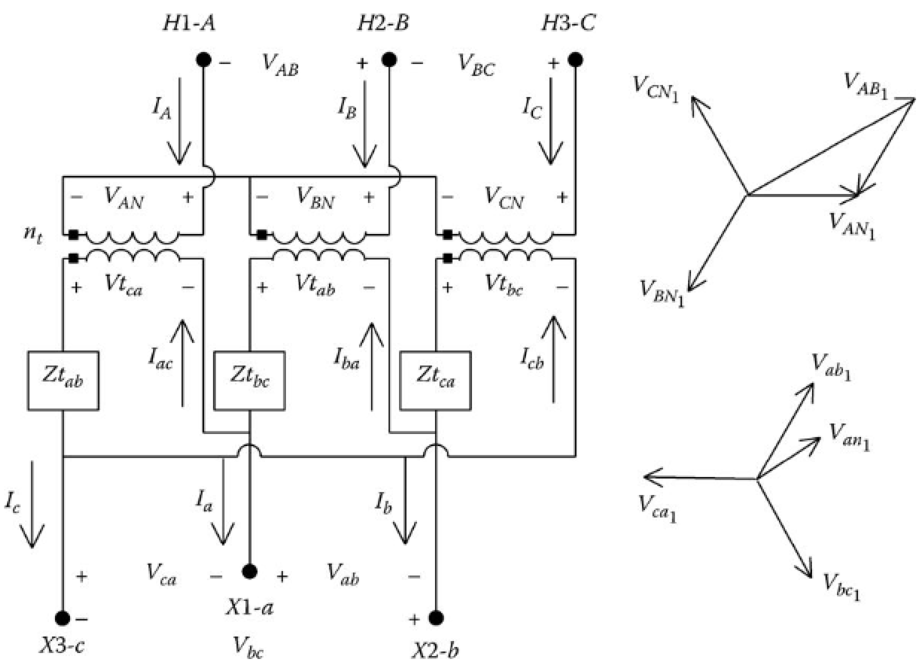


Fig.11 Undergrounded wye-delta step-up connection

$$n_t = \frac{\mathrm{KVLN}_{\text{rated Primary}}}{\mathrm{KVLL}_{\text{rated Secondary}}} \tag{76}$$

$$[AV] = n_t \cdot \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \tag{77}$$

$$[AI] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \tag{78}$$

Grounded Wye-Delta Step-Down Connection

The connection diagram for the standard 30 degree grounded wye (high)–delta (low) transformer connection grounded through an impedance of Zg is shown in Fig.12. Note that the primary is grounded through an impedance Zg.

Basic transformer equations

The turn's ratio is given by

$$n_t = \frac{\text{KVLN}_{\text{rated Primary}}}{\text{KVLL}_{\text{rated Secondary}}} \tag{79}$$

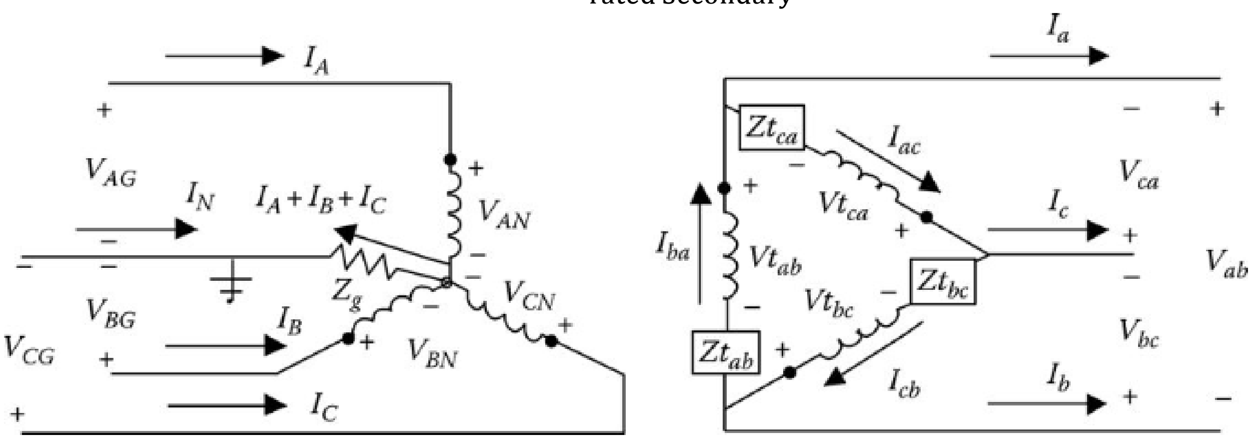


Fig.12 The grounded wye-delta connection

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \to [VLN_{ABC}] = [AV] \cdot [Vt_{abc}] \qquad [AV] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{80}$$

$$\begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \to [ID_{abc}] = [AI] \cdot [I_{ABC}] \qquad [AI] = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{81}$$

Grounded Wye-Delta Step-Down Connection

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \tag{80}$$

Solving Equation (80) for the "ideal" transformer voltages,

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] \tag{82}$$

The line-to-neutral transformer primary voltages as a function of the system line-to-ground voltages are given by

$$\begin{aligned} V_{AN} &= V_{AG} - Z_g \cdot (I_A + I_B + I_C) \\ V_{BN} &= V_{BG} - Z_g \cdot (I_A + I_B + I_C) \rightarrow \\ V_{CN} &= V_{CG} - Z_g \cdot (I_A + I_B + I_C) \end{aligned} \qquad \begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} - \begin{bmatrix} Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$[VLN_{ABC}] = [VLG_{ABC}] - [ZG] \cdot [I_{ABC}]$$

where $$[ZG] = \begin{bmatrix} Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \end{bmatrix}$$

The line-to-line voltages on the delta side are given by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \rightarrow [VLL_{abc}] = [Vt_{abc}] - [Zt_{abc}] \cdot [ID_{abc}] \tag{84}$$

Grounded Wye-Delta Step-Down Connection

The line-to-line voltages on the delta side are given by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \rightarrow \ [VLL_{abc}] \ = [Vt_{abc}] - [Zt_{abc}] \ \cdot [ID_{abc}]$$

Substitute Equation (82) into Equation (84):

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [Zt_{abc}] \ \cdot [ID_{abc}] \tag{85}$$

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \tag{81}$$

Substitute Equation (81) into Equation (85):

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - ([Zt_{abc}] \cdot [AI]) \ \cdot [I_{ABC}] \tag{86}$$

$$[VLN_{ABC}] = [VLG_{ABC}] - [ZG] \ \cdot [I_{ABC}] \tag{83}$$

Substitute Equation (83) into Equation (86):

$$\begin{aligned} [VLL_{abc}] &= [AV]^{-1} \cdot ([VLG_{ABC}] - [ZG] \ \cdot [I_{ABC}]) - ([Zt_{abc}] \cdot [AI]) \ \cdot [I_{ABC}] \\ [VLL_{abc}] &= [AV]^{-1} \cdot [VLG_{ABC}] - ([AV]^{-1} \cdot [ZG] + [Zt_{abc}] \cdot [AI]) \ \cdot [I_{ABC}] \end{aligned} \tag{87}$$

Equation (81) gives the delta secondary currents as a function of the primary wye-side line currents. The secondary line currents are related to the secondary delta currents by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \rightarrow [I_{abc}] \ = [DI] \cdot [ID_{abc}] \tag{88}$$

Grounded Wye-Delta Step-Down Connection

- The real problem of transforming currents from one side to the other occurs for the case when the line currents on the delta secondary side [Iabc] are known and the transformer secondary currents [IDabc] and primary line currents on the wye side [IABC] are needed.
- The only way a relationship can be developed is to recognize that the sum of the line-to-line voltages on the delta secondary of the transformer bank must add up to zero. Three independent equations can be written as follows:

$$I_a = I_{ba} - I_{ac}$$
$$I_b = I_{cb} - I_{ba} \tag{89}$$

- KVL around the delta secondary windings gives

$$Vt_{ab} - Zt_{ab} \cdot I_{ba} + Vt_{bc} - Zt_{bc} \cdot I_{cb} + Vt_{ca} - Zt_{ca} \cdot I_{ac} = 0 \tag{90}$$

- Replacing the "ideal" secondary delta voltages with the primary line-to-neutral voltages,

$$\frac{V_{AN}}{n_t} + \frac{V_{BN}}{n_t} + \frac{V_{CN}}{n_t} = Zt_{ab} \cdot I_{ba} + Zt_{bc} \cdot I_{cb} + Zt_{ca} \cdot I_{ac} \tag{91}$$

- Multiply both sides of the Equation (91) by the turns ratio nt:

$$V_{AN} + V_{BN} + V_{CN} = n_t \cdot Zt_{ab} \cdot I_{ba} + n_t \cdot Zt_{bc} \cdot I_{cb} + n_t \cdot Zt_{ca} \cdot I_{ac} \tag{92}$$
$$[VLN_{ABC}] = [VLG_{ABC}] - [ZG] \cdot [I_{ABC}] \tag{83}$$

- Determine the left side of Equation (92) as a function of the line-to-ground voltages using Equation (83):

$$V_{AN} + V_{BN} + V_{CN} = V_{AG} + V_{BG} + V_{CG} - 3 \cdot \frac{1}{n_t} \cdot Z_g \cdot (I_{ba} + I_{cb} + I_{ac}) \tag{93}$$

Grounded Wye-Delta Step-Down Connection

$$V_{AN} + V_{BN} + V_{CN} = n_t \cdot Zt_{ab} \cdot I_{ba} + n_t \cdot Zt_{bc} \cdot I_{cb} + n_t \cdot Zt_{ca} \cdot I_{ac} \tag{92}$$

$$V_{AN} + V_{BN} + V_{CN} = V_{AG} + V_{BG} + V_{CG} - 3 \cdot \frac{1}{n_t} \cdot Z_g \cdot (I_{ba} + I_{cb} + I_{ac}) \tag{93}$$

- Substitute Eqt (92) into (93):

$$V_{AG} + V_{BG} + V_{CG} - \frac{3}{n_t} \cdot Z_g \cdot (I_{ba} + I_{cb} + I_{ac}) = n_t \cdot Zt_{ab} \cdot I_{ba} + n_t \cdot Zt_{bc} \cdot I_{cb} + n_t \cdot Zt_{ca} \cdot I_{ac}$$

$$V_{sum} = (n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g) \cdot I_{ba} + (n_t \cdot Zt_{bc} + \frac{3}{n_t} \cdot Z_g) \cdot I_{cb} + (n_t \cdot Zt_{ca} + \frac{3}{n_t} \cdot Z_g) \cdot I_{ac} \tag{94}$$

$$V_{sum} = V_{AG} + V_{BG} + V_{CG}$$

- Equations (88), (89), and (94) can be put into matrix form:

$$\begin{bmatrix} I_a \\ I_b \\ V_{sum} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g & n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g & n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{95}$$

$$[X] = [F] \cdot [ID_{abc}] \tag{96}$$

$$[ID_{abc}] = [F]^{-1} \cdot [X] = [G] \cdot [X] \tag{97}$$

Grounded Wye-Delta Step-Down Connection

- Equations (97) in full form is

$$[ID_{abc}] = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ V_{AG} + V_{AG} + V_{AG} \end{bmatrix}$$

$$[ID_{abc}] = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{98}$$

- Equations (98) in short form is

$$[ID_{abc}] = [G1] \cdot [VLG_{ABC}] + [G2] \cdot [I_{abc}] \tag{99}$$

- Substitute Equation (81) into Equation (98):

$$[I_{ABC}] = [AI]^{-1} \cdot [ID_{abc}] = [AI]^{-1} \cdot ([G1] \cdot [VLG_{ABC}] + [G2] \cdot [I_{abc}])$$

$$[I_{ABC}] = [x_t] \cdot [VLG_{ABC}] + [d_t] \cdot [I_{abc}] \tag{100}$$

$$[x_t] = [AI]^{-1} \cdot [G1]$$

$$[d_t] = [AI]^{-1} \cdot [G2]$$

- Equation (100) is used in the “backward” sweep to compute the primary currents based upon the secondary currents and primary LG voltages.

Grounded Wye-Delta Step-Down Connection

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLG_{ABC}] - ([AV]^{-1} \cdot [ZG] + [Zt_{abc}] \cdot [AI]) \ \cdot [I_{ABC}] \tag{87}$$

$$[I_{ABC}] = [x_t] \cdot [VLG_{ABC}] + [d_t] \cdot [I_{abc}] \tag{100}$$

The “forward” sweep equation is determined by substituting Equation (100) into Equation (87):

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLG_{ABC}] - \boxed{([AV]^{-1} \cdot [ZG] + [Zt_{abc}] \cdot [AI])} \ \cdot ([x_t] \cdot [VLG_{ABC}] + [d_t] \cdot [I_{abc}])$$

Define $[X1] = [Zt_{abc}] \cdot [AI] + [AV]^{-1} \cdot [ZG]$

$$[VLL_{abc}] = ([AI] - [X1] \cdot [x_t]) \cdot [VLG_{ABC}] - [X1] \cdot [d_t] \cdot [I_{abc}]$$

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}]$$

$$[VLN_{abc}] = [W] \cdot (([AV]^{-1} - [X1] \cdot [x_t]) \cdot [VLG_{ABC}] - [X1] \cdot [d_t] \cdot [I_{abc}])$$

$$[VLN_{abc}] = [W] \cdot ([AV]^{-1} - [X1] \cdot [x_t]) \cdot [VLG_{ABC}] - [W] \cdot [X1] \cdot [d_t] \cdot [I_{abc}] \tag{101}$$

The final form of Equation (101) gives the equation for the forward sweep:

$$[VLN_{abc}] = [A_t] \cdot [VLG_{ABC}] - [B_t] \cdot [I_{abc}] \tag{102}$$

$$[A_t] = [W] \cdot ([AV]^{-1} - [X1] \cdot [x_t])$$

$$[B_t] = [W] \cdot [X1] \cdot [d_t]$$

Open Wye-Open Delta

- When the three-phase load is small compared to the single-phase load, the open wye–open delta connection is commonly used. The open wye–open delta connection requires only two transformers, but the connection will provide three-phase line-to-line voltages to the combination load.
- Fig.13 shows the open wye–open delta connection and the primary and secondary positive sequence voltage phasors.
- With reference to Fig.13, the basic "ideal" transformer voltages as a function of the "turn's ratio" are

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \qquad (103)$$

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}]$$

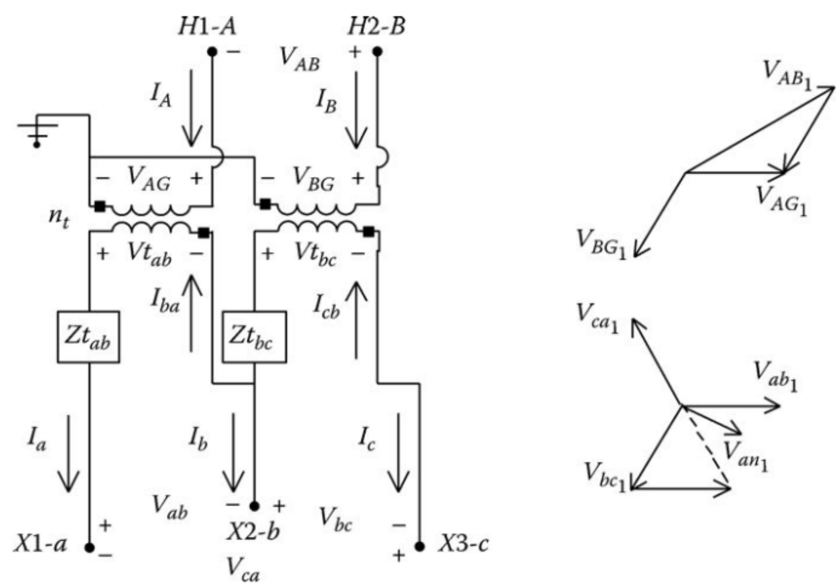


Fig.13 Open wye-open delta connection

Open Wye-Open Delta

- The currents as a function of the turn's ratio are given by

$$I_{ba} = n_T \cdot I_A = I_a$$
$$I_{cb} = n_T \cdot I_B = -I_c \quad \rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ -n_t & n_t & 0 \\ 0 & -n_t & 0 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \rightarrow [I_{abc}] = [AI] \cdot [I_{ABC}] \tag{104}$$
$$I_b = -(I_a + I_c) \tag{105}$$

- The ideal voltages in the secondary can be determined by

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{103}$$

$$Vt_{ab} = V_{ab} + Zt_{ab} \cdot I_a$$
$$Vt_{bc} = V_{bc} - Zt_{bc} \cdot I_c \tag{106}$$

$$\longrightarrow \quad V_{AG} = n_t \cdot Vt_{ab} = n_t \cdot V_{ab} + n_t \cdot Zt_{ab} \cdot I_a$$
$$V_{BG} = n_t \cdot Vt_{bc} = n_T \cdot V_{bc} - n_t \cdot Zt_{bc} \cdot I_c \tag{107}$$

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} + \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & 0 & -n_t \cdot Zt_{bc} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{108}$$
$$[VLG_{ABC}] = [AV] \cdot [VLL_{abc}] + [b_t] \cdot [I_{abc}]$$

- The secondary line-to-line voltages in Equation (108) can be replaced by the equivalent line-to-neutral secondary voltages:

$$[VLG_{ABC}] = [AV] \cdot [D] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}]$$
$$[VLG_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \tag{109}$$
$$[a_t] = [AV] \cdot [D] \qquad [b_t] = \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & 0 & -n_t \cdot Zt_{bc} \\ 0 & 0 & 0 \end{bmatrix}$$

Open Wye-Open Delta

- The source-side line currents as a function of the load-side line currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & 0 & -\frac{1}{n_t} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad [d_t] = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & 0 & -\frac{1}{n_t} \\ 0 & 0 & 0 \end{bmatrix} \tag{110}$$

$$[I_{ABC}] = [d_t] \cdot [I_{abc}]$$

- Equations (109) and (110) give the matrix equations for the backward sweep. The forward sweep equation can be determined by solving Equation (107) for the two line-to-line secondary voltages:

Forward Sweep:

$$[VLG_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \tag{109}$$

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] \tag{110}$$

$$V_{ab} = \frac{1}{n_t} \cdot V_{AG} - Zt_{ab} \cdot I_a \qquad V_{bc} = \frac{1}{n_t} \cdot V_{BG} - Zt_{bc} \cdot I_c \tag{111}$$

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & \frac{1}{n_t} & 0 \\ -\frac{1}{n_t} & -\frac{1}{n_t} & 0 \end{bmatrix} \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & Zt_{bc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{112}$$

Backward Sweep: $[VLL_{ab}] = [BV] \cdot [VLG_{ABC}] - [Zt_{abc}] \cdot [I_{abc}]$

Open Wye-Open Delta

- The equivalent secondary line-to-neutral voltages are then given by

$$[VLN_{abc}] = [W][VLL_{ABC}] = [W] \cdot [BV] \cdot [VLG_{ABC}] - [W] \cdot [Zt_{abc}] \cdot [I_{abc}] \tag{113}$$

- The forward sweep equation is given by

$$[VLN_{abc}] = [A_t] \cdot [VLG_{ABC}] - [B_t] \cdot [I_{abc}] \tag{114}$$

$$[A_t] = [W] \cdot [BV] = \frac{1}{3 \cdot n_t} \cdot \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ -1 & -2 & 0 \end{bmatrix}$$

$$[B_t] = [W] \cdot [Zt_{abc}] = \frac{1}{3} \cdot \begin{bmatrix} 2 \cdot Zt_{ab} & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & 2 \cdot Zt_{bc} \end{bmatrix}$$

- The terms “leading” and “lagging” connection are also associated with the open wye–open delta connection.
- When the *lighting transformer* is connected across the leading of the two phases, the connection is referred to as the “leading” connection. Similarly, when the lighting transformer is connected across the lagging of the two phases, the connection is referred to as the “lagging” connection.

Grounded Wye - Grounded Wye Connection

- The grounded wye–grounded wye connection is primarily used to supply single-phase and three-phase loads on four-wire multigrounded systems. The grounded wye–grounded wye connection is shown in Fig.14.
- Unlike the delta–wye and wye–delta connections, there is *no phase shift* between the voltages and the currents on the two sides of the bank. This makes the derivation of the generalized constant matrices much easier. The ideal transformer equations are

$$n_t = \frac{\text{KVLN}_{\text{rated Primary}}}{\text{KVLL}_{\text{rated Secondary}}} \tag{115}$$

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}$$

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{116}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$[I_{abc}] = [AI] \cdot [I_{ABC}] \tag{117}$$

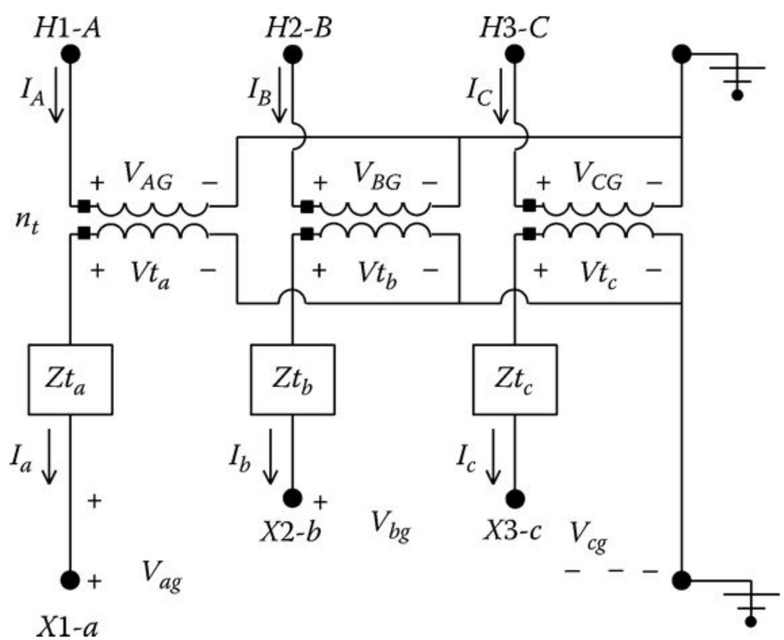


Fig.14 Grounded wye-grounded wye connection

Grounded Wye - Grounded Wye Connection

- With reference to Fig.14, the ideal transformer voltages on the secondary windings can be computed by

$$\begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} + \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \rightarrow [Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \tag{118}$$

- Substitute Equation (118) into Equation (116):

$$[VLG_{ABC}] = [AV] \cdot ([VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}])$$

$$[VLG_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \tag{119}$$

$$[a_t] = [AV] = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \tag{120}$$

$$[b_t] = [AV] \cdot [Zt_{abc}] = \begin{bmatrix} n_t \cdot Zt_a & 0 & 0 \\ 0 & n_t \cdot Zt_b & 0 \\ 0 & 0 & n_t \cdot Zt_c \end{bmatrix} \tag{121}$$

- Equation (119) is the backward sweep equation.
- The primary line currents as a function of the secondary line currents are given by

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] \qquad [d_t] = [AI]^{-1} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{122}$$

Grounded Wye - Grounded Wye Connection

- The forward sweep equation is determined solving Equation (119) for the secondary line-to-ground voltages:

$$[VLG_{abc}] = [AV]^{-1} \cdot [VLG_{ABC}] - [Zt_{abc}] \cdot [I_{abc}]$$

$$[VLG_{abc}] = [A_t] \cdot [VLG_{ABC}] - [B_t] \cdot [I_{abc}] \tag{123}$$

$$[A_t] = [AV]^{-1} \qquad [B_t] = [Zt_{abc}]$$

- The modeling and analysis of the grounded wye–grounded wye connection does not present any problems. Without the phase shift there is a direct relationship between the primary and secondary voltages and currents as had been demonstrated in the derivation of the generalized constant matrices.

Delta-Delta Connection

- The delta–delta connection is primarily used on three-wire delta systems to provide service to a three-phase load or a combination of three-phase and single-phase loads. Three single-phase transformers connected in a delta–delta are shown in Fig.15.
- The basic ideal transformer voltage and current equations as a function of the "turns ratio" are

$$n_t = \frac{\text{VLL}_{\text{rated Primary}}}{\text{VLL}_{\text{rated Secondary}}} \tag{124}$$

$$\begin{bmatrix} VLL_{AB} \\ VLL_{BC} \\ VLL_{CA} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix}$$

$$[VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{125}$$

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ca} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix}$$

$$[ID_{abc}] = [AI] \cdot [ID_{ABC}] \tag{126}$$

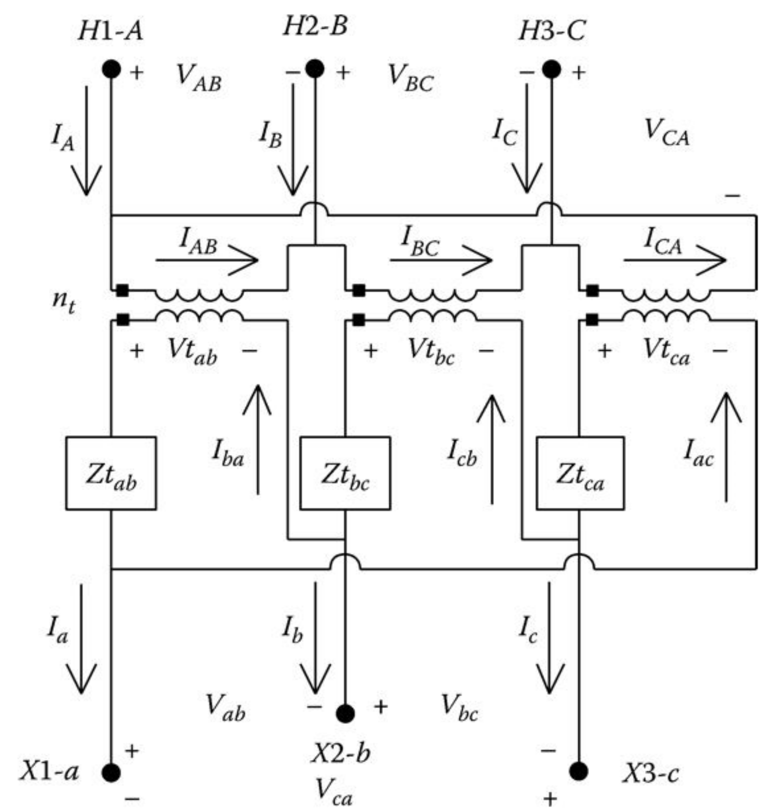


Fig.15 Delta – delta connection

Delta-Delta Connection

- Solve Equation (126) for the source-side delta currents:

$$[ID_{abc}] = [AI] \cdot [ID_{ABC}] \tag{126}$$

- The line currents as a function of the delta currents on the source side are given by

$$[ID_{ABC}] = [AI]^{-1} \cdot [ID_{abc}]$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} \tag{127}$$

- Substitute Equation (127) into Equation (128)

$$[I_{ABC}] = [DI] \cdot [ID_{ABC}] \qquad [DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{128}$$

$$[I_{ABC}] = [DI] \cdot [AI]^{-1} \cdot [ID_{abc}] \tag{129}$$

- Since [AI] is a diagonal matrix, Equation (129) can be rewritten as

$$[I_{ABC}] = [AI]^{-1} \cdot [DI] \cdot [ID_{abc}] \tag{130}$$

- The load-side line currents as a function of the load-side delta currents:

$$[I_{abc}] = [DI] \cdot [ID_{abc}] \tag{131}$$

- Applying Equation (131), Equation (130) becomes

$$[I_{ABC}] = [AI]^{-1} \cdot [I_{abc}] \tag{132}$$

Delta-Delta Connection

- Turn Equation (132) around to solve for the load-side line currents as a function of the source-side line currents:

$$[I_{abc}] = [AI] \cdot [I_{ABC}] \tag{133}$$

- Equations (132) and (133) merely demonstrate that the line currents on the two sides of the transformer are in phase and differ only by the turn's ratio of the transformer windings. In the per-unit system, the per-unit line currents on the two sides of the transformer are exactly equal.

- The ideal delta voltages on the secondary side as a function of the line-to-line voltages, the delta currents, and the transformer impedances are given by

$$[Vt_{abc}] = [VLL_{abc}] + [Zt_{abc}] \cdot [ID_{abc}] \qquad [Zt_{abc}] = \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \tag{134}$$

- Substitute Equation (134) into Equation (125)

$$[VLL_{abc}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [Zt_{abc}] \cdot [ID_{abc}] \tag{135}$$

- Solve Equation (135) for the load-side line-to-line voltages:

$$[VLL_{abc}] = [AI]^{-1} \cdot [VLL_{ABC}] - [Zt_{abc}] \cdot [ID_{abc}] \tag{136}$$

- The delta currents [IDabc] in Equations (135) and (136) need to be replaced by the secondary line currents [Iabc].

Delta-Delta Connection

- To develop the needed relationship, three independent equations are needed. The first two come from applying KCL at two vertices of the delta connected secondary:

$$I_a = I_{ba} - I_{ac}$$
$$I_b = I_{cb} - I_{ba} \tag{137}$$

- The third equation comes from recognizing that the sum of the primary line-to-line voltages and therefore the secondary ideal transformer voltages must sum to zero. KVL around the delta windings gives

$$Vt_{ab} - Zt_{ab} \cdot I_{ba} + Vt_{bc} - Zt_{bc} \cdot I_{cb} + Vt_{ca} - Zt_{ca} \cdot I_{ac} = 0 \tag{138}$$

- Replacing the "ideal" delta voltages with the source-side line-to-line voltages,

$$\frac{V_{AB}}{n_t} + \frac{V_{BC}}{n_t} + \frac{V_{CA}}{n_t} = Zt_{ab} \cdot I_{ba} + Zt_{bc} \cdot I_{cb} + Zt_{ca} \cdot I_{ac} \tag{139}$$

- Since the sum of the line-to-line voltages must equal zero (KVL) and the turn's ratios of the three transformers are equal, Equation (139) is simplified to

$$0 = Zt_a \cdot I_{ba} + Zt_b \cdot I_{cb} + Zt_c \cdot I_{ac} \tag{140}$$

- Note in Equation (140) that if the three transformer impedances are equal, then the sum of the delta currents will add to zero, meaning that the zero sequence delta currents will be zero.

Delta-Delta Connection

- Equations (137) and (140) can be put into matrix form:

$$\begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ Zt_{ab} & Zt_{bc} & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \rightarrow [I0_{abc}] = [F] \cdot [ID_{abc}] \tag{141}$$

- Solve Equation (141) for the load-side delta currents:

$$[ID_{abc}] = [F]^{-1} \cdot [I0_{abc}] = [G] \cdot [I0_{abc}] \tag{142}$$

- Writing Equation (142) in matrix form gives

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} \tag{143}$$

- From Equations (142) and (143), it is seen that the delta currents are a function of the transformer impedances and just the line currents in phases a and b. Equation (143) can be modified to include the line current in phase c by setting the last column of the [G] matrix to zeros:

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad [ID_{abc}] = [G1] \cdot [I_{abc}] \qquad [G1] = \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \tag{144}$$

Delta-Delta Connection

$$[VLL_{abc}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [Zt_{abc}] \cdot [ID_{abc}] \quad (135)$$

$$[ID_{abc}] = [G1] \cdot [I_{abc}] \quad (144)$$

- When the impedances of the transformers are equal, the sum of the delta currents will be zero meaning that there is no circulating zero sequence current in the delta windings.
- Substitute Equation (144) into Equation (135):

$$[VLL_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV] \cdot [Zt_{abc}] \cdot [G1] \cdot [I_{abc}] \quad (145)$$

- The generalized matrices are defined in terms of the line-to-neutral voltages on the two sides of the transformer bank. Equation (145) is modified to be in terms of equivalent line-to-neutral voltages:

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] = [W] \cdot [AV] \cdot [D] \cdot [VLN_{abc}] + [W] \cdot [Zt_{abc}] \cdot [G1] \cdot [I_{abc}] \quad (146)$$

- Equation (146) is in the general form

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}]$$

$$[a_t] = [W] \cdot [AV] \cdot [D] \quad (147)$$

$$[b_t] = [AV] \cdot [W] \cdot [Zt_{abc}] \cdot [G1]$$

- Equation (133) gives the generalized equation for currents:

$$[I_{ABC}] = [AI]^{-1} \cdot [I_{abc}] = [d_t] \cdot [I_{abc}] \quad (148)$$

$$[d_t] = [AI]^{-1}$$

Delta-Delta Connection

- The forward sweep equations can be derived by modifying Equation (136) in terms of equivalent line-to-neutral voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{ABC}] = [W] \cdot [AV]^{-1} \cdot [D] \cdot [VLN_{ABC}] - [W] \cdot [Zt_{abc}] \cdot [G1] \cdot [I_{abc}] \quad (149)$$

- The forward sweep equation is

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \quad (150)$$

$$[A_t] = [W] \cdot [AV]^{-1} \cdot [D] \qquad [B_t] = [W] \cdot [Zt_{abc}] \cdot [G1]$$

- The forward and backward sweep matrices for the delta–delta connection have been derived. Once again it has been a long process to get to the final six equations that define the matrices. The derivation provides an excellent exercise in the application of basic transformer theory and circuit theory. Once the matrices have been defined for a particular transformer connection, the analysis of the connection is a relatively simple task.

Example 8

Fig.16 shows three single-phase transformers in a delta–delta connection serving an unbalanced three-phase load connected in delta.

The source voltages at the load are balanced three phase of 240 V line to line:

$$[VLL_{abc}] = \begin{bmatrix} 12{,}470\angle 0 \\ 12{,}470\angle -120 \\ 12{,}470\angle 120 \end{bmatrix} V$$

Loading: Sab = 100 kVA at 0.9 lagging power factor, Sbc = Sca = 50 kVA at 0.8 lagging power factor

Transformer Rating:

Phase AB: 100 kVA, 12,470–240 V, Z = 0.01 + j0.04 per unit

Phases BC and CA: 50 kVA, 12,470–240 V, Z = 0.015 + j0.035 per unit

Determine the following:

1. The load line-to-line voltages
2. The secondary line currents
3. The primary line currents
4. The load currents
5. Load voltage unbalance

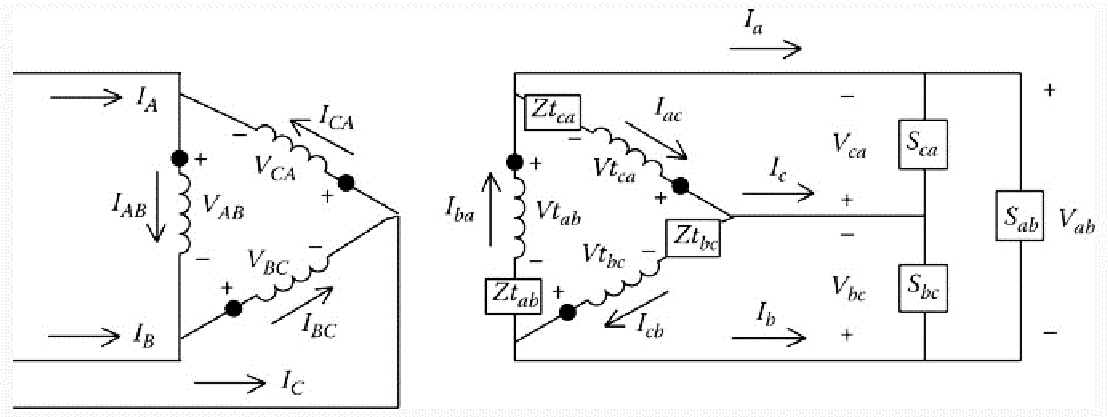


Fig.16 Delta-delta bank serving an unbalanced delta connected load

Example 8

Before the analysis can start, the transformer impedances must be converted to actual values in Ohms and located inside the delta connected secondary windings.

Phase ab transformer:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{100} = 0.576\ \Omega \qquad Zt_{ab} = (0.01 + j0.04) \cdot 0.576 = 0.0058 + j0.023\ \Omega$$

Phase bc and ca transformers:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{50} = 1.152\ \Omega \qquad Zt_{bc} = Zt_{ca} = (0.015 + j0.035) \cdot 1.152 = 0.0173 + j0.0403\ \Omega$$

The transformer impedance matrix can now be defined

$$[Zt_{abc}] = \begin{bmatrix} 0.0058 + j0.23 & 0 & 0 \\ 0 & 0.0173 + j0.0403 & 0 \\ 0 & 0 & 0.0173 + j0.0403 \end{bmatrix} \Omega$$

The turn's ratio of the transformers is $n_t = 12{,}470/240 = 51.9583$.

Define all of the matrices:

$$[W] = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \qquad [D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \qquad [DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Example 8

$$[AV]=[AI]=n_t\cdot\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}51.9583&0&0\\0&51.9583&0\\0&0&51.9583\end{bmatrix}$$

$$[F]=\begin{bmatrix}1&0&-1\\-1&1&0\\0.0058+j0.023&0.0173+j0.0403&0.0173+j0.0404\end{bmatrix}$$

$$[G]=[F]^{-1}=\begin{bmatrix}0.3941-j0.0134&-0.3941+j0.0134&3.2581-j8.378\\0.3941-j0.0134&0.6059+j0.0134&3.2581-j8.378\\-0.6059-j0.0134&-0.3941+j0.0134&3.2581-j8.378\end{bmatrix}$$

$$[G1]=\begin{bmatrix}0.3941-j0.0134&-0.3941+j0.0134&0\\0.3941-j0.0134&0.6059+j0.0134&0\\-0.6059-j0.0134&-0.3941+j0.0134&0\end{bmatrix}$$

$$[a_t]=[W]\;\cdot[AV]\cdot[D]=\begin{bmatrix}34.6489&-17.3194&-17.3194\\-17.3194&34.6489&-17.3194\\-17.3194&-17.3194&34.6489\end{bmatrix}$$

$$[b_t]=[AV]\cdot[W]\;\cdot[Zt_{abc}]\;\cdot[G1]=\begin{bmatrix}0.2166+j0.583&0.0826+j0.1153&0\\0.0826+j0.1153&0.2166+j0.583&0\\-0.2993-j0.6983&-0.2993-j0.6983&0\end{bmatrix}$$

$$[d_t]=[AI]^{-1}=\begin{bmatrix}0.0192&0&0\\0&0.0192&0\\0&0&0.0192\end{bmatrix}$$

Example 8

$$[A_t] = [W]\ \cdot [AV]^{-1} \cdot [D] = \begin{bmatrix} 0.0128 & -0.0064 & -0.0064 \\ -0.0064 & 0.0128 & -0.0064 \\ -0.0064 & -0.0064 & 0.0128 \end{bmatrix}$$

$$[B_t] = [W]\ \cdot [Zt_{abc}]\ \cdot [G1] = \begin{bmatrix} 0.0042 + j0.0112 & 0.0016 + j0.0022 & 0 \\ 0.0016 + j0.0022 & 0.0042 + j0.0112 & 0 \\ -0.0058 - j0.0134 & -0.0058 - j0.0134 & 0 \end{bmatrix}$$

After six iterations, the results are

$$[VLL_{abc}] = \begin{bmatrix} 232.9\angle 28.3 \\ 231.0\angle -91.4 \\ 233.1\angle 148.9 \end{bmatrix}$$

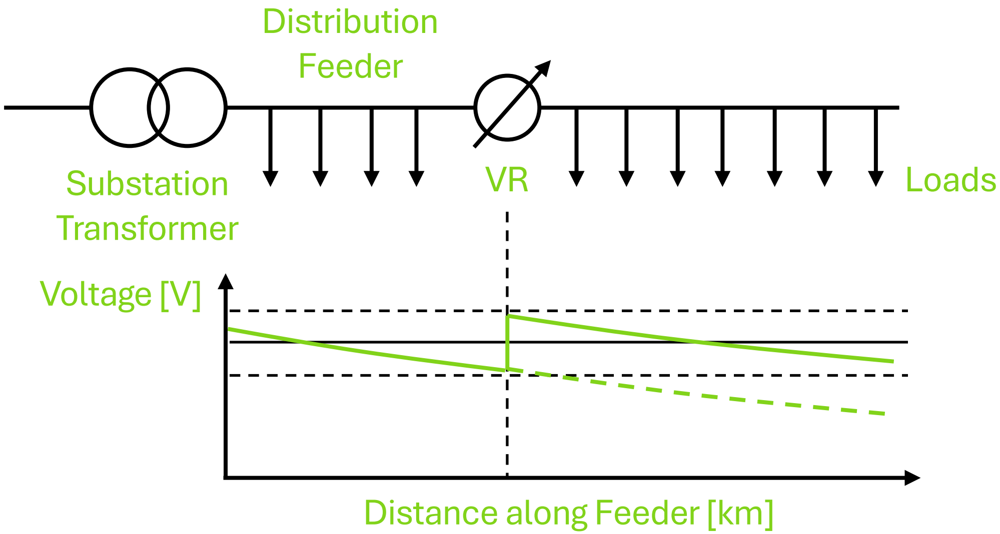
$$\mathrm{V}_{\mathrm{unbalance}} = 0.59\%$$

$$[I_{ABC}] = \begin{bmatrix} 10.4\angle -19.5 \\ 11.4\angle -161.5 \\ 7.2\angle 81.7 \end{bmatrix}$$

$$[I_{abc}] = \begin{bmatrix} 540.3\angle -19.5 \\ 593.6\angle -161.5 \\ 372.8\angle 81.7 \end{bmatrix}$$

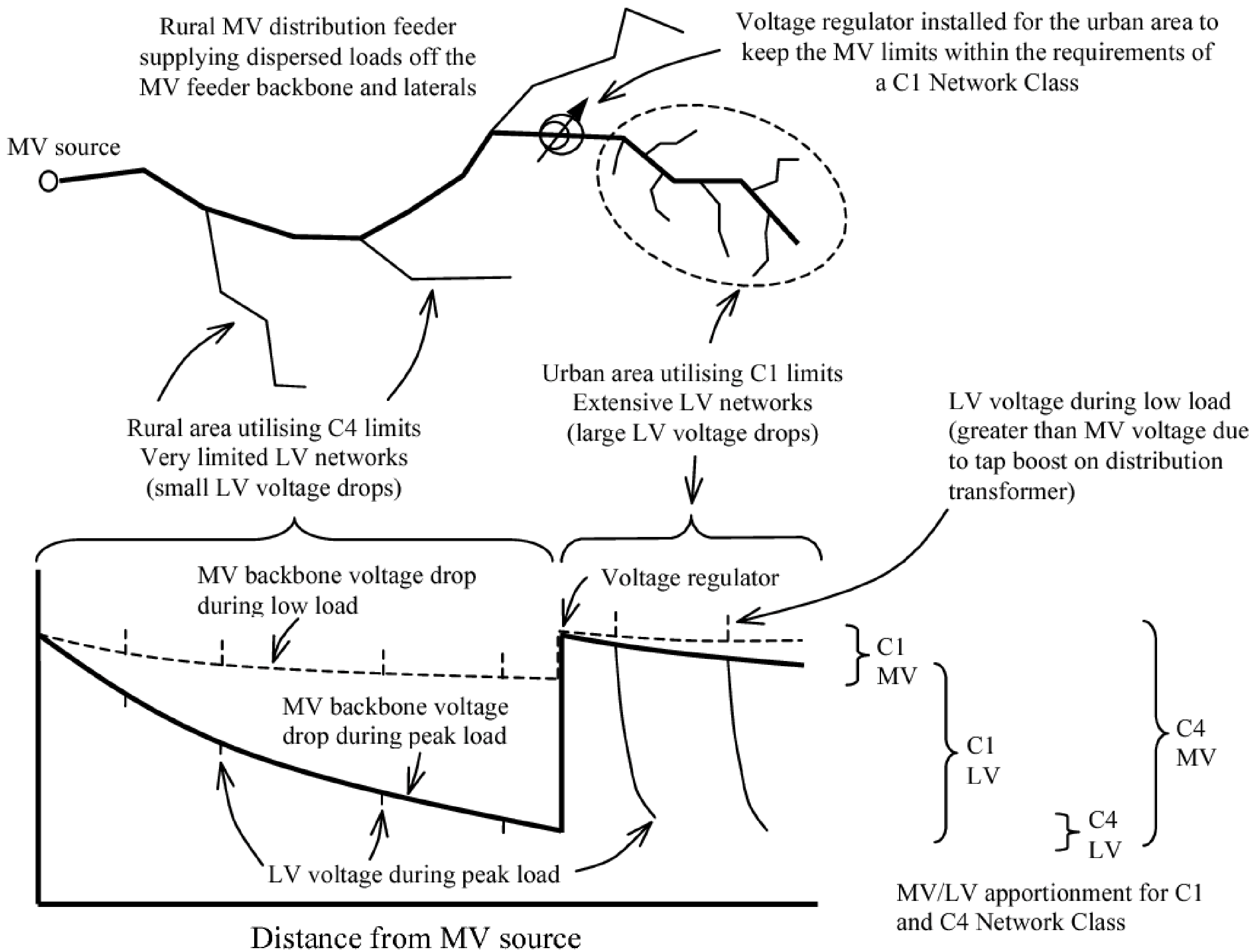
$$[ID_{abc}] = \begin{bmatrix} 429.3\angle 2.4 \\ 216.5\angle -128.3 \\ 214.5\angle 112.0 \end{bmatrix}$$

Voltage Regulation



- Voltage regulation is an important function on a distribution feeder. Utilities are required to maintain supply voltage within limit in supply rules.
- As the loads on the feeders *vary*, there must be some means of voltage regulation to remain voltage at all connection points within an acceptable level.
- Common methods of regulating the voltage are the application of step-type voltage regulators, load tap changing (LTC) transformers, and shunt capacitors.
- A step-voltage regulator is basically an autotransformer with a LTC mechanism on the "series" winding. The voltage change is obtained by changing the number of turns (tap changes) of the series winding of the autotransformer.

Voltage Regulation



Standard Voltage Ratings

The American National Standards Institute (ANSI) standard ANSI C84.1-1995 for "Electric Power Systems and Equipment Voltage Ratings (60 Hertz)" provides the following definitions for system voltage terms [1]:

- **System voltage**: The root mean square (rms) phasor voltage of a portion of an alternating current electric system. Each system voltage pertains to a portion of the system that is bounded by transformers or utilization equipment.
- **Nominal system voltage**: The voltage by which a portion of the system is designated and to which certain operating characteristics of the system are related. Each nominal system voltage pertains to a portion of the system bounded by transformers or utilization equipment.
- **Maximum system voltage**: The highest system voltage that occurs under normal operating conditions, and the highest system voltage for which equipment and other components are designed for satisfactory continuous operation without derating of any kind.
- **Service voltage**: The voltage at the point where the electrical system of the supplier and the electrical system of the user are connected.
- **Utilization voltage**: The voltage at the line terminals of utilization equipment.
- **Nominal utilization voltage**: The voltage rating of certain utilization equipment used on the system.

[1] *American Nation Standard for Electric Power—Systems and Equipment Voltage Ratings (60 Hertz)*, ANSI C84.1-1995, National Electrical Manufacturers Association, Rosslyn, VA, 1996.

Standard Voltage Ratings

- These ANSI standards give the distribution engineer a range of "normal steady-state" voltages (range A) and a range of "emergency steady-state" voltages (range B) that must be supplied to all users.
- In addition to the acceptable voltage magnitude ranges, the ANSI standard recommends that the "electric supply systems should be designed and operated to limit the maximum voltage unbalance to 3% when measured at the electric-utility revenue meter under a no-load condition." Voltage unbalance is defined as

$$\text{VBF} = \frac{\text{Max. deviation from average voltage}}{\text{Average voltage}} \cdot 100\% \tag{1}$$

- Step-voltage regulators can be single phase or three phase. Single-phase regulators can be connected in wye, delta, or open delta, in addition to operating as a single-phase device. The regulators and their controls allow the voltage output to vary as the load varies.

Two-Winding Transformer Theory

- In Fig.1, the high-voltage transformer terminals are denoted by H_1 and H_2, and the low-voltage terminals are denoted by X_1 and X_2. The standards for these markings are such that at no load the voltage between H_1 and H_2 will be in phase with the voltage between X_1 and X_2. Under a steady-state load condition, the currents I_1 and I_2 will be in phase.

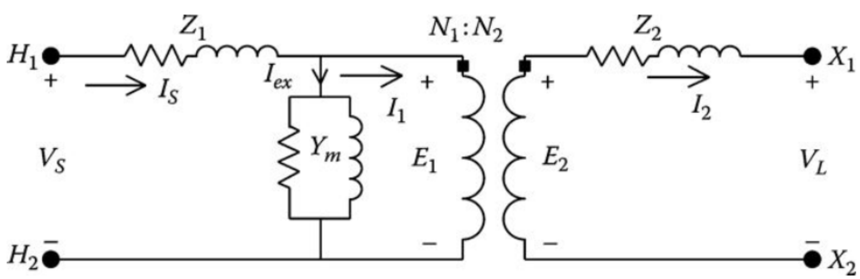


Fig.1 Two-winding transformer: exact equivalent circuit

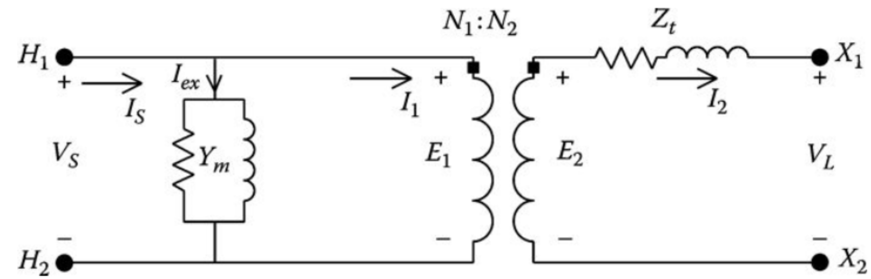


Fig.2 Two-winding transformer: approx. equivalent circuit

- Without introducing a significant error, the exact equivalent circuit of Fig.1 is modified by referring the primary impedance (Z_1) to the secondary side as shown in Fig.2. Referring to Fig.2, the total "leakage" impedance of the transformer is given by

$$Z_t = n_t^2 \cdot Z_1 + Z_2 \tag{2}$$

$$n_t = \frac{N_2}{N_1} \tag{3}$$

- Referring to Fig.2, the equations for the ideal transformer become

$$E_2 = \frac{N_2}{N_1} \cdot E_1 = n_t \cdot E_1 \qquad I_1 = \frac{N_2}{N_1} \cdot I_2 = n_t \cdot I_2$$

(4)

(5)

Two-Winding Transformer Theory

- Apply KVL in the secondary circuit,

$$E_2 = V_L + Z_t \cdot I_2$$
$$V_s = E_1 = \frac{1}{n_t} \cdot E_2 = \frac{1}{n_t} \cdot V_L + \frac{Z_t}{n_t} \cdot I_2 \qquad (6)$$

- In general form, equation (6) can be written as

$$\boxed{V_s = a \cdot V_L + b \cdot I_2} \qquad (7)$$

$$a = \frac{1}{n_t} \qquad b = \frac{Z_t}{n_t} \qquad (8)\ (9)$$

- The input current to the two-winding transformer is given by

$$I_s = Y_m \cdot V_s + I_1 \qquad (10)$$

- Substitute Equations (5) and (6) into Equation (10)

$$I_s = Y_m \cdot \frac{1}{n_t} \cdot V_L + Y_m \cdot \frac{Z_t}{n_t} \cdot I_2 + n_t \cdot I_2$$
$$I_s = \frac{Y_m}{n_t} \cdot V_L + \left(\frac{Y_m \cdot Z_t}{n_t} + n_t \right) \cdot I_2 \qquad (11)$$

- In general form, Equation (11) can be written as

$$\boxed{I_s = c \cdot V_L + d \cdot I_2} \qquad (12)$$

$$c = \frac{Y_m}{n_t} \qquad d = \frac{Y_m \cdot Z_t}{n_t} + n_t \qquad (13)\ (14)$$

Two-Winding Transformer Theory

Single Phase Model derived:

$$V_s = a \cdot V_L + b \cdot I_2 \quad (7)$$

$$I_s = c \cdot V_L + d \cdot I_2 \quad (12)$$

Three Phase Model:

$$[VLG_{abc}]_n = [a] \cdot [VLG_{abc}]_m + [b] \cdot [I_{abc}]_m \quad (7.a)$$

$$[I_{abc}]_n = [c] \cdot [VLG_{abc}]_m + [d] \cdot [I_{abc}]_m \quad (12.a)$$

- Equations (7) and (12) are used to compute the input voltage and current to a two-winding transformer with known load voltage and current.
- These two equations are of the same form as Equations (7.a) and (12.a) for the three-phase line models. The only difference at this point is that only a single-phase two-winding transformer is being modeled. Later, in this chapter, the terms a, b, c, and d are expanded to 3 × 3 matrices for all possible three-phase regulator connections.
- Sometimes, particularly in the ladder iterative process, the output voltage needs to be computed knowing the input voltage and the load current. Solving Equation (7) for the load voltage yields

$$V_L = \frac{1}{a} \cdot V_s - \frac{b}{a} \cdot I_2 \quad (15)$$

$$a = \frac{1}{n_t} \quad (8) \qquad b = \frac{Z_t}{n_t} \quad (9)$$

- Substituting Equations (8) and (9) into Equation (15) results in

$$V_L = A \cdot V_s - B \cdot I_2 \quad (16)$$

$$A = n_t \quad (17) \qquad B = Z_t \quad (18)$$

Example 1

A single-phase transformer is rated 75 kVA, 2400–240 V. The transformer has the following impedances and shunt admittance:

Z1 = 0.612 + j1.2 Ω	(high-voltage winding impedance)
Z2 = 0.0061 + j0.0115 Ω	(low-voltage winding impedance)
Ym = $1.92 \times 10^{-4} - j8.52 \times 10^{-4}$ S	(referred to the high-voltage winding)

Determine the generalized a, b, c, and d constants and the A and B constants.

The transformer "turns ratio" is $n_t = \frac{\mathrm{N}_2}{\mathrm{N}_1} = \frac{\mathrm{V}_{\mathrm{rated\_2}}}{\mathrm{V}_{\mathrm{rated\_1}}} = \frac{240}{2400} = 0.1$

The equivalent transformer impedance referred to the low-voltage side:

$$Z_t = Z_2 + n_t^2 \cdot Z_1 = 0.0122 + j0.0235$$

$$V_s = a \cdot V_L + b \cdot I_2 \qquad (7)$$

$$I_s = c \cdot V_L + d \cdot I_2 \qquad (12)$$

$$V_L = A \cdot V_s - B \cdot I_2 \qquad (16)$$

The generalized constants are

$$a = \frac{1}{n_t} = \frac{1}{0.1} = 10 \qquad c = \frac{Y_m}{n_t} = 0.0019 - j0.0085$$

$$b = \frac{Z_{\mathrm{t}}}{0.1} = 0.1222 + j0.235 \qquad d = \frac{Y_m \cdot Z_{\mathrm{t}}}{n_t} + n_t = 0.1002 - j0.0001$$

$$A = n_t = 0.1 \qquad B = Z_t = 0.01222 + j0.0235$$

Example 1

Assume that the transformer is operated at rated load (75 kVA) and rated voltage (240 V) with a power factor of 0.9 lagging. Determine the source voltage and current using the generalized constants.

$$V_L = 240\angle 0$$

$$I_2 = \frac{75 \cdot 1000}{240}\angle -\cos^{-1}(0.9) = 312.5\angle -25.84$$

Applying the values of the *a*, *b*, *c*, and *d* parameters computed earlier:

$$V_s = a \cdot V_L + b \cdot I_2 = 2466.9\angle 1.15 \text{ V}$$

$$I_s = c \cdot V_L + d \cdot I_2 = 32.67\angle -28.75 \text{ V}$$

Using the computed source voltage and the load current, determine the load voltage:

$$\begin{aligned} V_L &= A \cdot V_s - B \cdot I_s \\ &= (0.1) \cdot (2466.9\angle 1.15) - (0.0122 + j0.0235) \cdot (312.5\angle -25.84) \\ &= 240\angle 0 \end{aligned}$$

For future reference, the per-unit impedance of the transformer is computed by

$$Z_{base} = \frac{kV_{ratd\_2}^2 \cdot 1000}{kVA} = \frac{0.240^2 \cdot 1000}{75} = 0.768\ \Omega$$

$$Y_{base} = \frac{\text{kVA}}{kV_1^2 \cdot 1000\text{kVA}} = \frac{75}{2.4^2 \cdot 1000} = 0.013\ S$$

$$Z_{pu} = \frac{Z_t}{Z_{base}} = \frac{0.0122 + j0.0115}{0.768} = 0.0345\angle 62.5 \text{ pu}$$

$$Y_{pu} = \frac{Y_m}{Y_{base}} = 0.0147 - j0.0654 \text{ pu}$$

Two-Winding Autotransformer

A two-winding transformer can be connected as an autotransformer. Connecting the high-voltage terminal H_1 to the low-voltage terminal X_2 as shown in Fig.3 can create a "step-up" autotransformer. The source is connected to terminals H_1 and H_2, while the load is connected between the X_1 terminal and the extension of H_2.

$$E_2 = \frac{N_2}{N_1} \cdot E_1 = n_t \cdot E_1 \qquad I_1 = \frac{N_2}{N_1} \cdot I_2 = n_t \cdot I_2 \qquad (4) \quad (5)$$

Generalized constants similar to those of the two-winding transformer can be developed for the autotransformer. The total equivalent transformer impedance is referred to the "series" winding. The "ideal" transformer Equations (4) and (5) still apply.

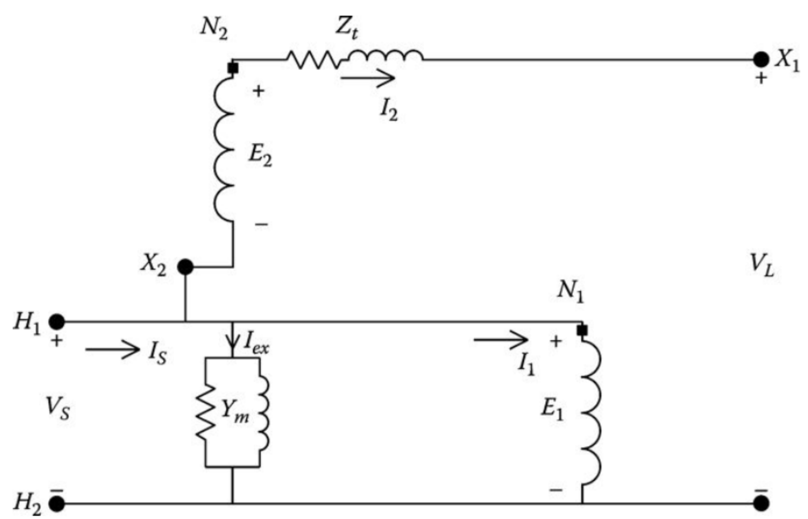


Fig.3 Step-up autotransformer

Two-Winding Autotransformer

Applying KVL in the secondary circuit,

$$E_1 + E_2 = V_L + Z_t \cdot I_2 \tag{19}$$

Using the "ideal" transformer relationship of Equation (5),

$$E_1 + n_t \cdot E_1 = (1 + n_t) \cdot E_1 = V_L + Z_t \cdot I_2 \tag{20}$$

Since the source voltage V_S is equal to E_1 and I_2 is equal to I_L, Equation (20) can be modified to

$$V_s = \frac{1}{1 + n_t} \cdot V_L + \frac{Z_t}{1 + n_t} \cdot I_2 \tag{21}$$

$$V_s = a \cdot V_L + b \cdot I_2 \tag{22}$$

$$a = \frac{1}{1 + n_t} \qquad b = \frac{Z_t}{1 + n_t} \qquad (23)\ (24)$$

Applying KCL at input node H_1,

$$I_s = I_1 + I_2 + I_{ex}$$

$$I_s = (1 + n_t) \cdot I_2 + Y_m \cdot V_s \tag{25}$$

Substitute Equation (21) into Equation (25):

$$I_s = (1 + n_t) \cdot I_2 + Y_m \cdot \left(\frac{1}{1 + n_t} \cdot V_L + \frac{Z_t}{1 + n_t} \cdot I_2\right)$$

$$I_s = \frac{Y_m}{1 + n_t} \cdot V_L + \left(\frac{Y_m \cdot Z_t}{1 + n_t} + 1 + n_t\right) \cdot I_2$$

Two-Winding Autotransformer

$$I_s = c \cdot V_L + d \cdot I_2 \tag{26}$$

$$c = \frac{Y_m}{1 + n_t} \qquad d = \frac{Y_m \cdot Z_t}{1 + n_t} + 1 + n_t \qquad (27)\ (28)$$

In sum,

$$\boxed{\begin{gathered} V_s = a \cdot V_L + b \cdot I_2 \quad (22) \\ I_s = c \cdot V_L + d \cdot I_2 \quad (26) \\ a = \frac{1}{1 + n_t} \qquad b = \frac{Z_t}{1 + n_t} \qquad c = \frac{Y_m}{1 + n_t} \qquad d = \frac{Y_m \cdot Z_t}{1 + n_t} + 1 + n_t \end{gathered}}$$

Equations (23), (24), (27), and (28) define the generalized constants relating the source voltage and current as a function of the output voltage and current for the "step-up" autotransformer.

Two-Winding Autotransformer

The two-winding transformer can also be connected in the "step-down" connection by reversing the connection between the shunt and series winding as shown in Fig.4.

Generalized constants can be developed for the "step-down" connection following the same procedure as that for the step-up connection.

Applying KVL in the secondary circuit, $E_1 - E_2 = V_L + Z_t \cdot I_2$ (29)

Using the "ideal" transformer relationship of Equation (5),

$$I_1 = \frac{N_2}{N_1} \cdot I_2 = n_t \cdot I_2 \quad (5)$$

$$E_1 - n_t \cdot E_1 = V_L + Z_t \cdot I_2 \quad (30)$$

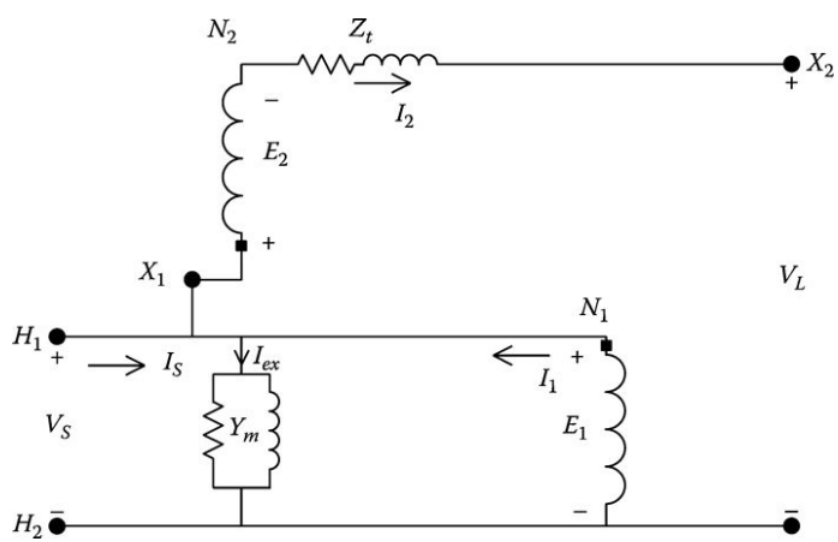


Fig.4 Step-down autotransformer

Two-Winding Autotransformer

Since the source voltage VS is equal to E1 and I2 is equal to IL, Equation (30) can be modified to

$$V_s = \frac{1}{1-n_t} \cdot V_L + \frac{Z_t}{1-n_t} \cdot I_2 \quad (31)$$

$$V_s = a \cdot V_L + b \cdot I_2 \quad (32)$$

$$a = \frac{1}{1-n_t} \qquad b = \frac{Z_t}{1-n_t} \quad (33)\ (34)$$

It is observed at this point that the only difference between the a and b constants of Equations (23) and (24) for the step-up connection and Equations (33) and (44) for the step-down connection is the sign in front of the turns ratio (n_t).

$$a = \frac{1}{1+n_t} \qquad b = \frac{Z_t}{1+n_t} \quad (23)\ (24)$$

This will also be the case for the c and d constants. Therefore, for the step-down connection, the c and d constants are defined by

$$c = \frac{Y_m}{1-n_t} \qquad d = \frac{Y_m \cdot Z_t}{1-n_t} + 1 - n_t \quad (35)\ (36)$$

The only difference between the definitions of the generalized constants is the sign of the turn ratio n_t. In general, then, the generalized constants can be defined by

$$a = \frac{1}{1 \pm n_t} \qquad b = \frac{Z_t}{1 \pm n_t} \qquad c = \frac{Y_m}{1 \pm n_t} \qquad d = \frac{Y_m \cdot Z_t}{1 \pm n_t} + 1 \pm n_t \quad (37)\ (38)\ (39)\ (40)$$

In Equations (37) through (40), the sign of nt will be positive for the step-up connection and negative for the step-down connection.

Two-Winding Autotransformer

$$V_s = a \cdot V_L + b \cdot I_2 \qquad (32)$$

As with the two-winding transformer, it is sometimes necessary to relate the output voltage as a function of the source voltage and the output current. Solving Equation (32) for the output voltage:

$$V_L = \frac{1}{a} \cdot V_s - \frac{b}{a} \cdot Z_t \cdot I_2 \qquad (41)$$

$$V_L = AV_s - BI_2 \qquad (42)$$

$$A = \frac{1}{a} = 1 \pm n_t \qquad B = -\frac{b}{a} Z_t \qquad (43)\ (44)$$

The generalized equations for the step-up and step-down autotransformers have been developed. They are of the same form as was derived for the two-winding transformer and for the line segment. For the single-phase autotransformer, the generalized constants are single values but will be expanded later to 3 × 3 matrices for three-phase autotransformers.

Autotransformer Rating

The kVA rating of the autotransformer is the product of the rated input voltage V_S times the rated input current I_S or the rated load voltage V_L times the rated load current I_L. Define the rated kVA and rated voltages of the two-winding transformer and autotransformer as follows:

- kVA_{xfm} represents the kVA rating of the two-winding transformer.
- kVA_{auto} represents the kVA rating of the autotransformer.
- $V_{rated\_1} = E_1$ represents rated source voltage of the two-winding transformer.
- $V_{rated\_2} = E_2$ represents rated load voltage of the two-winding transformer.
- V_{auto_S} represents rated source voltage of the autotransformer.
- V_{auto_L} represents rated load voltage of the autotransformer.

For the following derivation, neglect the voltage drop through the series winding impedance:

$$V_{auto\_L} = E_1 \pm E_2 = (1 \pm n_t) \cdot E_1 \tag{45}$$

The rated output kVA is then

$$kVA_{auto} = V_{auto\_L} \cdot I_2 = (1 \pm n_t) \cdot E_1 \cdot I_2 \tag{46}$$

$$I_2 = \frac{I_1}{n_t} \rightarrow \quad kVA_{auto} = \frac{(1 \pm n_t)}{n_t} \cdot E_1 \cdot I_1 \tag{47}$$

$$E_1 \cdot I_1 = kVA_{xfm}$$

Therefore,

$$kVA_{auto} = \frac{(1 \pm n_t)}{n_t} \cdot kVA_{xfm} \tag{48}$$

Equation (48) gives the kVA rating of a two-winding transformer when connected as an auto-transformer.

Example 2

A two-winding transformer is connected as a "step-up" autotransformer, the turns ratio is $n_t = 0.1$, the KVA rating is 750. Determine the kVA and voltage ratings of the autotransformer.

The rated kVA of the autotransformer using Equation 47 is given by

$$\text{kVA}_{\text{auto}} = \frac{(1 + 0.1)}{0.1} \cdot 750 = 825\ \text{kVA}$$

The voltage ratings are

$$V_{auto\_S} = V_{rated\_1} = 2400\ V$$

$$V_{auto\_L} = V_{rated\_1} + V_{rated\_1} = 2400 + 240 = 2640\ V$$

Therefore, the autotransformer would be rated as 825 kVA, 2400–2640 V.

Suppose now that the autotransformer is supplying rated kVA at rated voltage with a power factor of 0.9 lagging, determine the source voltage and current:

$$V_L = V_{auto\_L} = 2640\angle 0\ V$$

$$I_L = \frac{kVA_{auto} \cdot 1000}{V_{auto\_L}} = \frac{825{,}000}{2{,}640}\angle - \cos^{-1}(0.9) = 312.5\angle - 25.84\ A$$

Determine the generalized constants

$$a = \frac{1}{1 + 0.1} = 0.9091 \qquad d = \frac{(1.92 - j8.52) \cdot 10^{-4} \cdot (0.0122 + j0.0235\,)}{1 + 0.1} + 0.1 + 1 = 1.1002 - j0.000005$$

$$b = \frac{0.0122 + j0.0235}{1 + 0.1} = 0.0111 + j0.0214 \qquad c = \frac{(1.92 - j8.52) \cdot 10^{-4}}{1 + 0.1} = (1.7364 - j7.7455) \cdot 10^{-4}$$

Example 2

Applying the generalized constants,

$$V_s = a \cdot 2640\angle 0 + b \cdot 312.5\angle - 25.84 = 2406.0\angle 0.1\ V$$

$$I_s = c \cdot 2640\angle 0 + d \cdot 312.5\angle - 25.84 = 345.06\angle - 26.11\ A$$

When the load-side voltage is determined knowing the source voltage and load current, the A and B parameters are needed:

$$A = \frac{1}{n_t} = 1.1 \qquad B = Z_t = 0.0111 + j0.0235$$

The load voltage is then

$$V_L = A \cdot 2406.04\angle 0.107 - B \cdot 312.5\angle - 25.84 = 2640\angle 0\ V$$

Rework this example by setting the transformer impedances and shunt admittance to zero.

When this is done the generalized matrices are

$$a = \frac{1}{1+n_t} = 0.9091 \qquad b = \frac{Z_t}{1+n_t} = 0 \qquad c = \frac{Y_m}{1+n_t} = 0 \qquad d = \frac{Y_m \cdot Z_t}{1+n_t} + 1 + n_t = 1.1$$

Using these matrices the source voltage and currents are

$$V_s = a \cdot V_L + b \cdot I_L = 2400\angle 0 \qquad I_s = c \cdot V_L + d \cdot I_L = 343.75\angle - 25.8$$

The "errors" for the source voltages and currents by ignoring the impedances and shunt admittance are

$$Error_V = \left(\frac{2406 - 2400}{2406}\right) \cdot 100 = 0.25\% \qquad Error_1 = \left(\frac{345.07 - 343.75}{345.07}\right) \cdot 100 = 0.38\%$$

Step-Voltage Regulator

A step-voltage regulator consists of an autotransformer and a LTC mechanism. The voltage change is obtained by changing the taps of the series winding of the autotransformer. The position of the tap is determined by a control circuit (line drop compensator).

Standard step-regulators contain a reversing switch enabling a ±10% regulator range, usually in 32 steps. This amounts to a 5/8% change per step or 0.75 V change per step on a 120 V base.

Step-regulators can be connected in a “Type A” or “Type B” connection according to the ANSI/IEEE C57.15-1986 standard [2]. The more common Type B connection is shown in Fig.5.

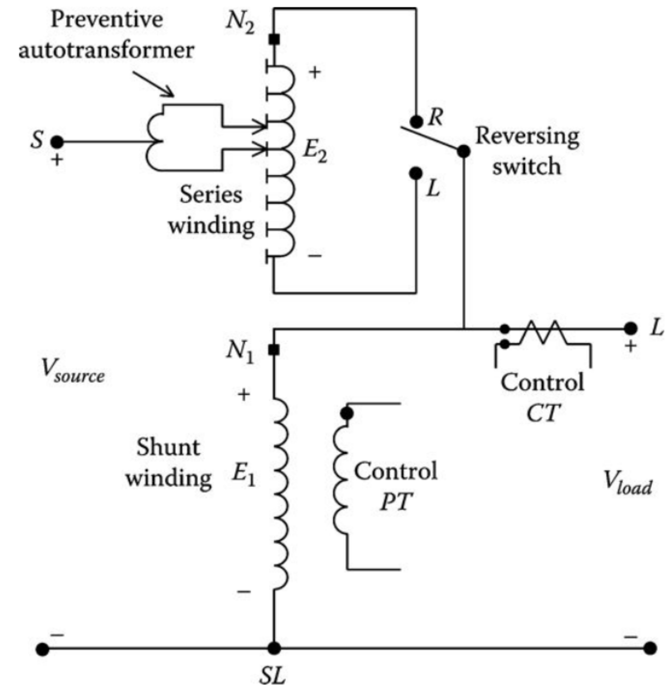


Fig.5 Type “B” step-voltage regulator

The step-voltage regulator control circuit requires the following settings:

- ***Voltage level***: The desired voltage (on 120 V base) to be held at the “load center.” The load center may be the output terminal of the regulator or a remote node on the feeder.
- ***Bandwidth***: The allowed variance of the load center voltage from the set voltage level. The voltage held at the load center will be ±1/2 of the bandwidth. For example, if the voltage level is set to 122 V and the bandwidth set to 2 V, the regulator will change taps until the load center voltage lies between 121 and 123 V.

[2] *IEEE Standard Requirements, Terminology, and Test Code for Step-Voltage and Induction-Voltage Regulators*, ANSI/IEEE C57.15-1986, Institute of Electrical and Electronic Engineers, New York, 1988

Step-Voltage Regulator

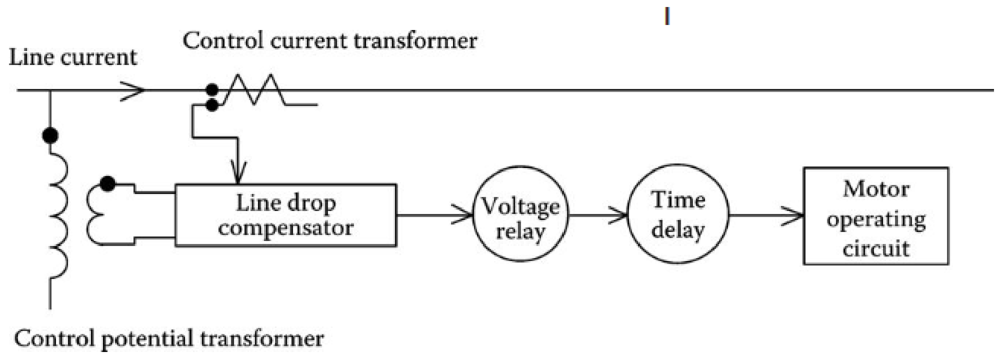


Fig.6 Step-voltage regulator control circuit

- ***Time delay:*** Length of time that a raise or lower operation is called for before the actual execution of the command. This prevents taps changing during a transient or short time change in current.

- ***Line drop compensator:*** Set to compensate for the voltage drop (line drop) between the regulator and the load center. The settings consist of R and X settings in volts corresponding to the equivalent impedance between the regulator and the load center. This setting may be zero if the regulator output terminals are the "load center."

The required rating of a step-regulator is based upon the kVA transformed and not the kVA rating of the line. In general, this will be 10% of the line rating since rated current flows through the series winding, which represents the ±10% voltage change. The kVA rating of the step-voltage regulator is determined in the same manner as that of the previously discussed autotransformer.

Type A Step-Voltage Regulator

- The detailed equivalent circuit and abbreviated equivalent circuit of a Type A step-voltage regulator in the "raise" position are shown in Fig.7.
- As shown in Fig.7, the primary circuit of the system is connected directly to the shunt winding of the Type A regulator. The series winding is connected to the shunt winding and, in turn, via taps, to the regulated circuit. In this connection, the core excitation varies because the shunt winding is connected directly across primary circuit.
- When the Type A connection is in the "lower" position, the reversing switch is connected to the "L" terminal. The effect of this reversal is to reverse the direction of the currents in the series and shunt windings. Fig.8 shows the equivalent circuit and abbreviated circuit of the Type A regulator in the lower position.

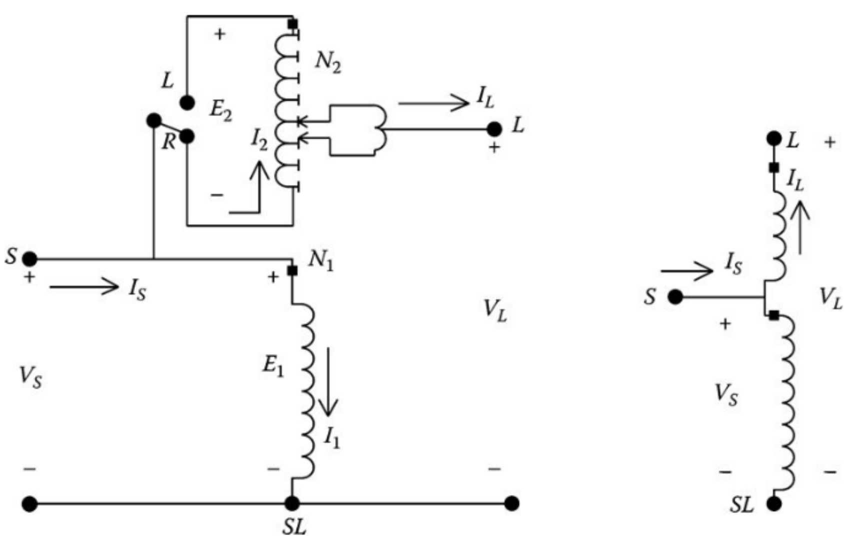


Fig.7 Type A step-voltage regulator in the raise position

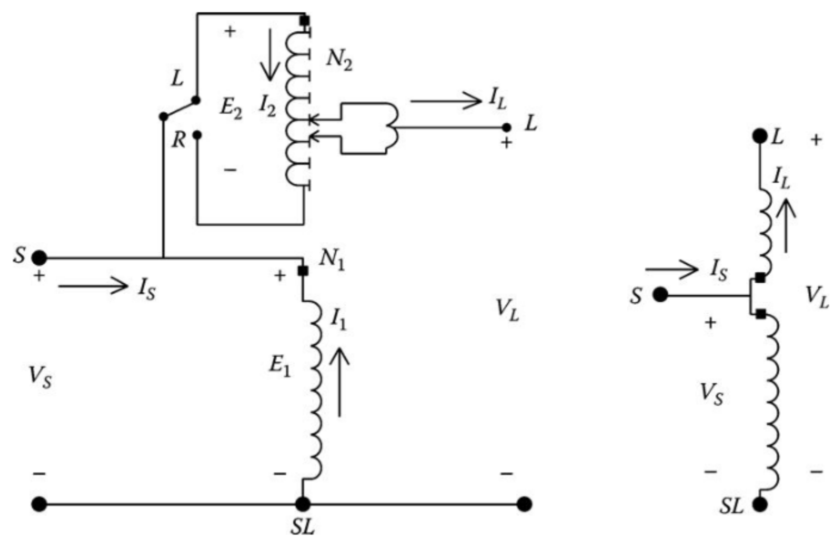


Fig.8 Type A step-voltage regulator in the lower position

Type B Step-Voltage Regulator

- The more common connection for step-voltage regulators is the Type B. Since this is the more common connection, the defining voltage and current equations for the voltage regulator will be developed only for the Type B connection.
- The detailed and abbreviated equivalent circuits of a Type B step-voltage regulator in the "raise" position are shown in Fig.9.

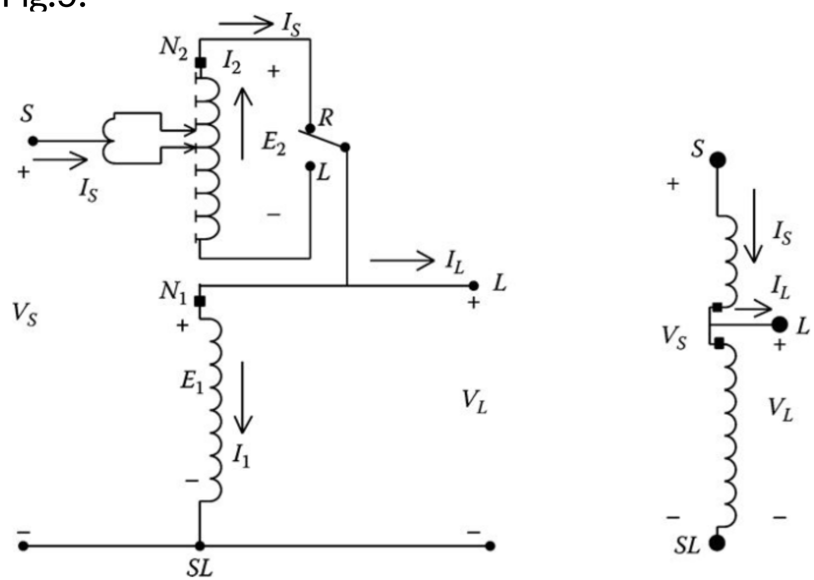


Fig.9 Type B step-voltage regulator in the raise position

- The primary circuit of the system is connected, via taps, to the series winding of the regulator in the Type B connection. The series winding is connected to the shunt winding, which is connected directly to the regulated circuit. In a Type B regulator, the core excitation is constant because the shunt winding is connected across the regulated circuit.

Type B Step-Voltage Regulator

The defining voltage and current equations for the regulator in the raise position are as follows:

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} \qquad N_1 \cdot I_1 = N_2 \cdot I_2 \tag{61}$$

$$V_s = E_1 - E_2 \qquad I_L = I_s - I_1 \tag{62}$$

$$V_L = E_1 \qquad I_2 = I_s \tag{63}$$

$$E_2 = \frac{N_2}{N_1} \cdot E_1 = \frac{N_2}{N_1} \cdot V_L \qquad I_1 = \frac{N_2}{N_1} \cdot I_1 = \frac{N_2}{N_1} \cdot I_s \tag{64}$$

$$V_s = (1 - \frac{N_2}{N_1}) \cdot V_L \qquad I_L = (1 - \frac{N_2}{N_1}) \cdot I_s \tag{65}$$

$$V_s = a_R \cdot V_L \qquad I_L = a_R \cdot I_s \tag{66}$$

$$a_R = (1 - \frac{N_2}{N_1}) \tag{67}$$

Equations (66) and (67) are the necessary defining equations for modeling a Type B regulator in the raise position.

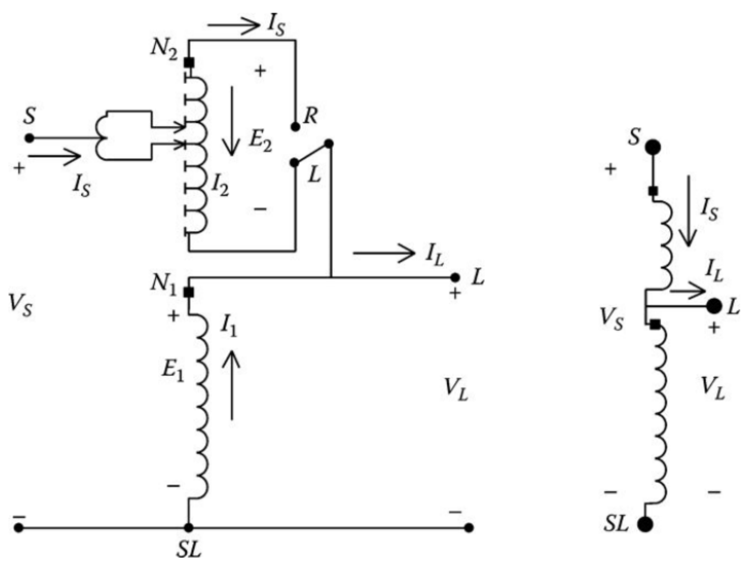


Fig.10 Type B step-voltage regulator in the lower position

Type B Step-Voltage Regulator

The defining voltage and current equations for the Type B step-voltage regulator in the lower position are as follows:

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} \qquad N_1 \cdot I_1 = N_2 \cdot I_2 \qquad (68)$$

$$V_s = E_1 + E_2 \qquad I_L = I_s + I_1 \qquad (69)$$

$$V_L = E_1 \qquad I_2 = I_s \qquad (70)$$

$$E_2 = \frac{N_2}{N_1} \cdot E_1 = \frac{N_2}{N_1} \cdot V_L \qquad I_1 = \frac{N_2}{N_1} \cdot I_1 = \frac{N_2}{N_1} \cdot (I_s) \qquad (64)$$

$$a_R = 1 - \frac{N_2}{N_1} \qquad (67)$$

$$V_s = (1 + \frac{N_2}{N_1}) \cdot V_L \qquad I_L = (1 + \frac{N_2}{N_1}) \cdot I_s \qquad (72)$$

$$V_s = a_R \cdot V_L \qquad I_L = a_R \cdot I_s \qquad (73)$$

$$a_R = 1 + \frac{N_2}{N_1} \qquad (74)$$

$$a_R = 1 + \frac{N_2}{N_1} \qquad (74)$$

Equations (67) and (74) give the value of the effective regulator ratio as a function of the ratio of the number of turns on the series winding (N_2) to the number of turns on the shunt winding (N_1).

In the final analysis, the only difference between the voltage and current equations for the Type B regulator in the raise and lower positions is the sign of the turns ratio (N_2/N_1). The actual turns ratio of the windings is not known. However, the particular tap position will be known. Equations (67) and (74) can be modified to give the effective regulator ratio as a function of the tap position. Each tap changes the voltage by 5/8% or 0.00625 per unit. Therefore, the effective regulator ratio can be given by

$$a_R = 1 \pm 0.00625 \cdot \text{Tap} \qquad (75)$$

Generalized Constants

In previous chapters and sections of this chapter, generalized a, b, c, and d constants have been developed for various devices. It can now be shown that the generalized a, b, c, and d constants can also be applied to the step-voltage regulator. For both Type A and Type B regulators, the relationship between the source voltage and current to the load voltage and current is of the form:

Type A $$V_s = \frac{1}{a_R} \cdot V_L \qquad I_s = a_R \cdot I_L \tag{76}$$

Type B $$V_s = a_R \cdot V_L \qquad I_L = \frac{1}{a_R} \cdot I_s \tag{77}$$

Therefore, the generalized constants for a single-phase step-voltage regulator become

Type A $$\mathrm{a} = \frac{1}{a_R} \qquad b = 0 \qquad \mathrm{c} = 0 \qquad \mathrm{d} = a_R \qquad A = a_R \qquad \mathrm{B} = 0 \tag{78}$$

Type B $$\mathrm{a} = a_R \qquad b = 0 \qquad \mathrm{c} = 0 \qquad \mathrm{d} = \frac{1}{a_R} \qquad A = \frac{1}{a_R} \qquad \mathrm{B} = 0 \tag{79}$$

$$a_R = 1 \pm 0.00625 \cdot \mathrm{Tap} \tag{75}$$

The a_R is given by Equation (75) and the sign convention is given in Tab.1.

	Type A	Type B
Raise	+	-
Lower	-	+

Line Drop Compensator (LDC)

The changing of taps on a regulator is controlled by the "line drop compensator."

Fig.11 shows an analog circuit of the compensator circuit and how it is connected to the distribution line through a potential transformer and a current transformer.

Older regulators are controlled by an analog compensator circuit. Modern regulators are controlled by a digital compensator. The digital compensators require the same settings as the analog, because it is easy to visualize, the analog circuit will be used in this section. However, understand that the modern digital compensators perform the same function for changing the taps on the regulators.

The compensator input voltage is typically 120 V, which requires the potential transformer in Fig.11 to reduce the rated voltage down to 120 V. For a regulator connected line to ground, the rated voltage is the nominal line-to-neutral voltage, while for a regulator connected line to line, the rated voltage is the line-to-line voltage.

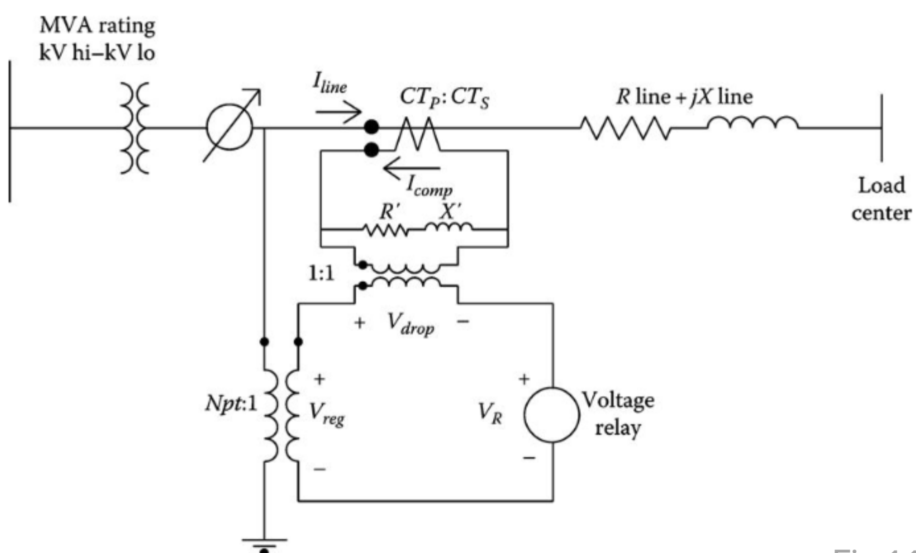


Fig.11 Line drop compensator circuit

Line Drop Compensator (LDC)

The current transformer turns ratio is specified as CT_P: CT_S where the primary rating (CT_P) will typically be the rated current of the feeder.

The setting that is most critical is that of *R*' and *X*' calibrated in volts. These values must represent the equivalent impedance from the regulator to the load center.

The basic requirement is to force the per-unit line impedance to be equal to the per-unit compensator impedance.

It is essential that a consistent set of base values be developed wherein the per-unit voltage and currents in the line and in the compensator are equal.

The consistent set of base values is determined by selecting a base voltage and current for the line circuit and then computing the base voltage and current in the compensator by dividing the system base values by the potential transformer ratio and current transformer ratio, respectively. For regulators connected line to ground, the base system voltage is selected as the rated line-to-neutral voltage (V_{LN}), and the base system current is selected as the rating of the primary winding of the current transformer (CT_P).

Tab.2 gives “table of base values” and employs these rules for a regulator connected line to ground.

Base	Line Circuit	Compensator Circuit
Voltage	V_{LN}	$\frac{V_{LN}}{N_{PT}}$
Current	CT_P	CT_S
Impedance	$\mathrm{Zbase}_{\mathrm{line}} = \frac{V_{LN}}{CT_P}$	$\mathrm{Zbase}_{\mathrm{comp}} = \frac{V_{LN}}{N_{PT} \cdot CT_S}$

Line Drop Compensator (LDC)

With the table of base values developed, the compensator R and X settings in Ohms can be computed by first computing the per-unit line impedance:

$$R_{pu} + jX_{pu} = \frac{Rline_{\Omega} + jXline_{\Omega}}{Zbase_{line}} \qquad R_{pu} + jX_{pu} = (Rline_{\Omega} + jXline_{\Omega}) \cdot \frac{CT_P}{V_{LN}} \tag{80}$$

The per-unit impedance of Equation (80) must be the same in the line and in the compensator. The compensator impedance in Ohms is computed by multiplying the per-unit impedance by the base compensator impedance:

$$Rcomp_{\Omega} + jXcomp_{\Omega} = \left(R_{pu} + jX_{pu}\right) \cdot Zbase_{comp} = (Rline_{\Omega} + jXline_{\Omega}) \frac{CT_P}{V_{LN}} \cdot \frac{V_{LN}}{N_{PT} \cdot CT_s} \tag{81}$$

$$= (Rline_{\Omega} + jXline_{\Omega}) \frac{CT_P}{N_{PT} \cdot CT_s} \Omega$$

Equation (81) gives the value of the compensator *R* and *X* settings in Ohms. The compensator *R* and *X* settings in volts are determined by multiplying the compensator *R* and *X* in Ohms times the rated secondary current (CT_S) of the current transformer:

$$R' + jX' = (Rline_{\Omega} + jXline_{\Omega}) \cdot CT_s = (Rline_{\Omega} + jXline_{\Omega}) \frac{CT_P}{N_{PT} \cdot CT_s} \cdot CT_s \tag{82}$$

$$= (Rline_{\Omega} + jXline_{\Omega}) \frac{CT_P}{N_{PT}} \text{ V}$$

Knowing the equivalent impedance in Ohms from the regulator to the load center, the required value for the compensator settings in volts is determined by using Equation (82). This is demonstrated in Example 7.4.

Example 4

The substation transformer is rated 5000 kVA, 115/4.16kV Dyn11, and the equivalent line impedance from the regulator to the load center is 0.3 + j0.9 Ω.

1. Determine the potential transformer and current transformer ratings for the compensator circuit.

The rated line-to-ground voltage of the substation transformer is

$$V_s = \frac{4160}{\sqrt{3}} = 2401.8$$

To provide approximately 120 V to the compensator, the potential transformer ratio is

$$N_{PT} = \frac{2400}{120} = 20$$

The rated current of the substation transformer is $I_{rated} = \dfrac{5000}{\sqrt{3} \cdot 4.16} = 693.9$

The primary rating of the CT is selected as 700 A, and if the compensator current is reduced to 5 A, the CT ratio is

$$CT = \frac{CT_P}{CT_S} = \frac{700}{5} = 140$$

2. Determine the R and X setting of the compensator in Ohms and volts

Applying Equation (82) to determine the setting in volts

$$R' + jX' = (0.3 + j0.9)\frac{700}{20} = 10.5 + j31.5\ V$$

Example 4

The *R* and *X* settings in Ohms are determined by dividing the settings in volts by the rated secondary current of the current transformer:

$$Rline_{ohms} + jXline_{ohms} = \frac{10.5 + j31.5}{5} = 2.1 + j6.3\ \Omega$$

Understand that the *R* and *X* settings on the compensator control board are calibrated in volts.

The substation transformer is supplying 2500 kVA at 4.16 kV and 0.9 PF lag. The regulator has been set so that $R' + jX' = 10.5 + j31.5\ V$

c. Determine the tap position of the regulator that will hold the load center voltage at the desired voltage level and within the bandwidth. This means that the tap on the regulator needs to be set so that the voltage at the load center lies between 119 and 121 V.

The first step is to calculate the actual line current:

$$I_{line} = \frac{2500}{\sqrt{3} \cdot 4.16} \angle - a\,cos(0.9) = 346.97\angle - 25.84\ A$$

The current in the compensator is then

$$I_{comp} = \frac{346.97\angle - 25.84}{140} = 2.4783\angle - 25.84\ A$$

The input voltage to the compensator is

$$V_{reg} = \frac{2401.8\angle 0}{20} = 120.09\angle 0\ V$$

Example 4

The voltage drop in the compensator circuit is equal to the compensator current times the compensator *R* and *X* values in Ohms:

$$V_{drop} = (2.1 + j6.3) \cdot 2.4783\angle -25.84 = 16.458\angle 45.7\ V$$

The voltage across the voltage relay is

$$V_R = V_{reg} - V_{drop} = 120.09\angle 0 - 16.458\angle 45.7 = 109.24\angle -6.19\ V$$

The voltage across the voltage relay represents the voltage at the load center. Since this is well below the minimum voltage level of 119, the voltage regulator will have to change taps in the raise position to bring the load center voltage up to the required level. Recall that on a 120 V base, one step change on the regulator changes the voltage 0.75 V. The number of required tap changes can then be approximated by

$$\text{Tap} = \frac{119 - 109.24}{0.75} = 13.02$$

This shows that the final tap position of the regulator will be "raise 13." With the tap set at +13, the effective regulator ratio assuming a Type B regulator is

$$a_R = 1 - 0.00625 \cdot 13 = 0.9188$$

The generalized constants for modeling the regulator for this operating condition are

$$a = a_R = 0.9188 \qquad b = 0 \qquad c = 0 \qquad d = \frac{1}{0.9188} = 1.0884$$

Example 4

e. calculate the actual voltage at the load center with the tap set at +13 assuming the 2500 kVA at 4.16 kV measured at the substation transformer's low-voltage terminals.

The actual line-to-ground voltage and line current at the load-side terminals of the regulator are

$$V_L = \frac{V_s}{a} = \frac{2401.8\angle 0}{0.9188} = 2614.2\angle 0\ V$$

$$I_L = \frac{I_s}{d} = \frac{346.97\angle -25.84}{1.0884} = 318.77\angle -25.84\ A$$

The actual line-to-ground voltage at the load center is

$$V_{LC} = V_L - Z_{line} \cdot I_L = 2614.2\angle 0 - (0.3 + j0.9) \cdot 318.77\angle -25.84$$
$$= 2412.8\angle -5.15\ V$$

On a 120 V base, the load center voltage is

$$VLC_{120} = \frac{V_{LC}}{N_{pt}} = \frac{2412.8\angle -5.15}{20} = 120.6\angle -5.15\ V$$

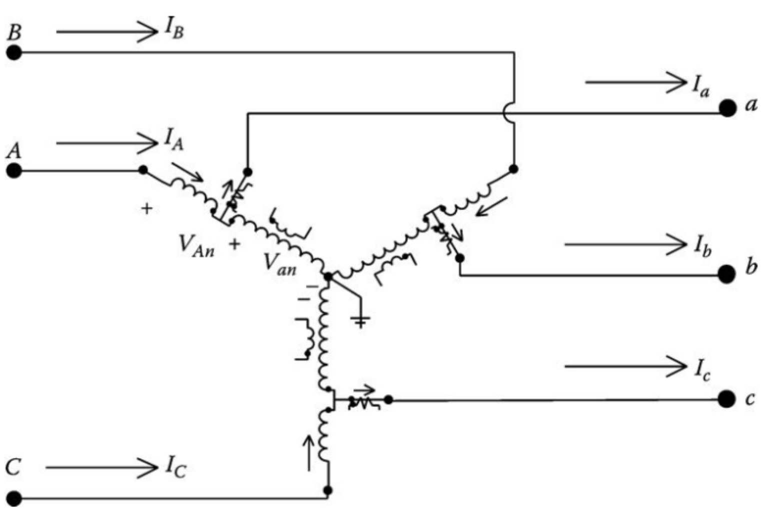
The +13 tap is an approximation and has resulted in a load center voltage within the bandwidth. However, since the regulator started in the neutral position, the taps will be changed one at a time until the load center voltage is inside the 119 lower bandwidth. Remember that each step changes the voltage by 0.75 V. Since the load center voltage has been computed to be 120.6 V, it would appear that the regulator went one step more than necessary.

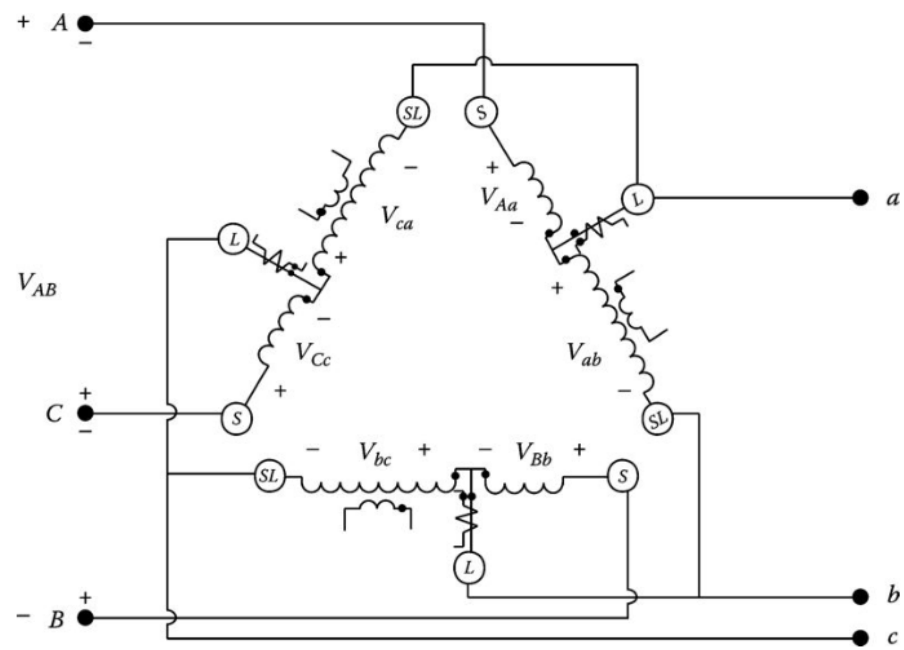
Tap	Voltage
0	109.2
1	110.1
2	110.9
3	111.7
…	…
10	117.8
11	118.8
12	119.7

Three-Phase Step-Voltage Regulator

Three single-phase step-voltage regulators can be connected externally to form a three-phase regulator. When three single-phase regulators are connected, each regulator has its own compensator circuit, and, therefore, the taps on each regulator are changed separately. Typical connections for single-phase step-regulators are

1. Single phase
2. Three regulators connected in grounded wye
3. Three regulators connected in closed delta
4. Two regulators connected in open delta
5. Two regulators connected in “open wye” (sometimes referred to as “V” phase)





Three-Phase Step-Voltage Regulator

A three-phase regulator has the connections between the single-phase windings internal to the regulator housing. The three-phase regulator is “gang” operated so that the taps on all windings change the same, and, as a result, only one compensator circuit is required. For this case, it is up to the engineer to determine which phase current and voltage will be sampled by the compensator circuit. Three-phase regulators will only be connected in a three-phase wye or closed delta.

Many times the substation transformer will have LTC windings on the secondary. The LTC will be controlled in the same way as a gang-operated three-phase regulator.

In the regulator models to be developed in the next sections, the phasing on the source side of the regulator will use capital letters A, B, and C. The load-side phasing will use lower case letters a, b, and c.

Wye-Connected Regulators

In Fig.12 the polarities of the windings are shown in the "raise" position. When the regulator is in the "lower" position, a reversing switch will have reconnected the series winding so that the polarity on the series winding is now at the output terminal. Regardless of whether the regulator is raising or lowering the voltage, the following equations apply:

Voltage equations:

$$\begin{bmatrix} V_{An} \\ V_{Bn} \\ V_{Cn} \end{bmatrix} = \begin{bmatrix} a_{R\_a} & 0 & 0 \\ 0 & a_{R\_b} & 0 \\ 0 & 0 & a_{R\_c} \end{bmatrix} \cdot \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \quad (84)$$

$$[VLN_{ABC}] = [\mathrm{a}] \cdot [VLN_{abc}] + [\mathrm{b}] \cdot [I_{abc}] \quad (85)$$

where a_{R_a}, a_{R_b}, and a_{R_c} represent the effective turns ratios for the three single-phase regulators.

Current equation:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} \frac{1}{a_{R\_a}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_b}} & 0 \\ 0 & 0 & \frac{1}{a_{R\_c}} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (86)$$

$$[I_{ABC}] = [\mathrm{c}] \cdot [VLG_{abc}] + [\mathrm{d}] \cdot [I_{abc}] \quad (87)$$

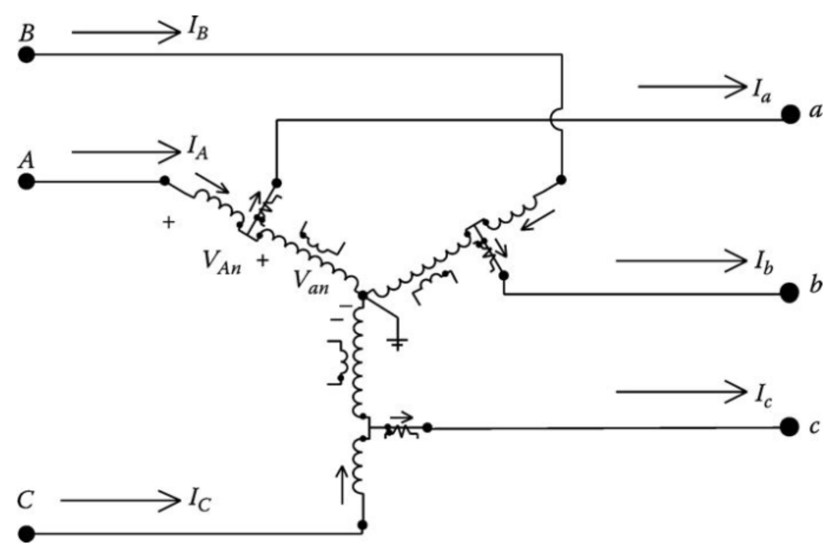


Fig.12 Wye-connected Type B regulators

Wye-Connected Regulators

$$[VLN_{ABC}] = [\mathrm{a}] \cdot [VLN_{abc}] + [\mathrm{b}] \cdot [I_{abc}] \quad (85)$$

$$[I_{ABC}] = [\mathrm{c}] \cdot [VLG_{abc}] + [\mathrm{d}] \cdot [I_{abc}] \quad (87)$$

- Equations (85) and (87) are of the same form as the generalized equations that were developed for the three-phase line segment. For a three-phase wye-connected step-voltage regulator, neglecting the series impedance and shunt admittance, the forward and backward sweep matrices are therefore defined as

$$[a] = \begin{bmatrix} a_{R\_a} & 0 & 0 \\ 0 & a_{R\_b} & 0 \\ 0 & 0 & a_{R\_c} \end{bmatrix} \quad (88)$$

$$[b] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (89)$$

$$[c] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (90)$$

$$[d] = \begin{bmatrix} \frac{1}{a_{R\_a}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_b}} & 0 \\ 0 & 0 & \frac{1}{a_{R\_c}} \end{bmatrix} \quad (91)$$

$$[A] = \begin{bmatrix} \frac{1}{a_{R\_a}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_b}} & 0 \\ 0 & 0 & \frac{1}{a_{R\_c}} \end{bmatrix} \quad (92)$$

$$[B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (93)$$

- In Equations (88), (91), and (93), the effective turns ratio for each regulator must satisfy $0.9 \le a_{R\_abc} \le 1.1$ in 32 steps of 0.625%/step (0.75 V/step on 120 V base).

Closed Delta-Connected Regulators

Three single-phase Type B regulators can be connected in a closed delta as shown in Fig.13. In the figure, the regulators are shown in the "raise" position.

The closed delta connection is typically used in three-wire delta feeders. Note that VT for this connection are monitoring the load-side line-to-line voltages and CT are not monitoring the load-side line currents.

The relationships between the source side and currents and the voltages are needed. Equations (64) through (67) define the relationships between the series and shunt winding voltages and currents for a step-voltage regulator that must be satisfied no matter how the regulators are connected.

KVL is first applied around a closed loop starting with the line-to-line voltage between phases A and C on the source side. It defines the various voltages:

$$V_{AB} = V_{Aa} + V_{ab} - V_{Bb} \quad (94)$$

$$V_{Bb} = -\frac{N_2}{N_1} \cdot V_{bc} \quad (95)$$

$$V_{Aa} = -\frac{N_2}{N_1} \cdot V_{ab} \quad (96)$$

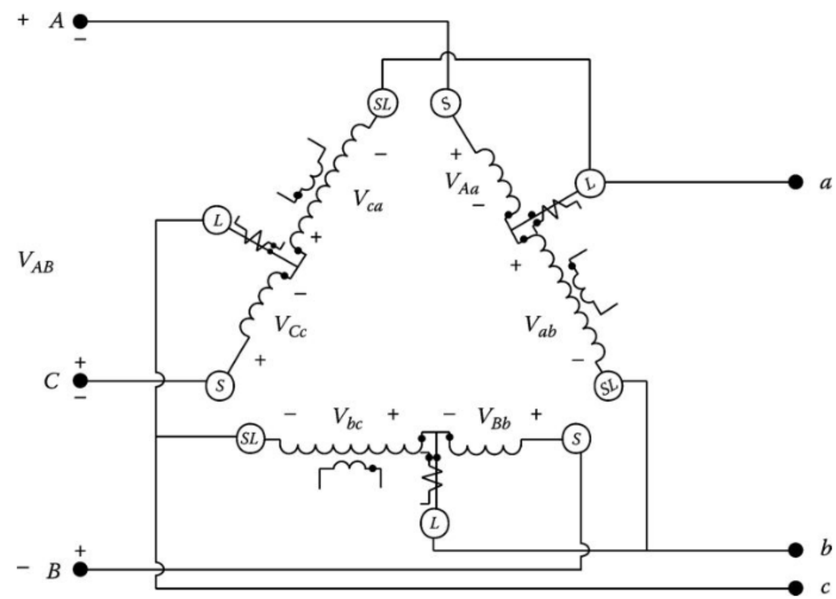


Fig.13 Wye-connected Type B regulators

Closed Delta-Connected Regulators

Substitute Equations (95) and (96) into Equation (94) and simplify:

$$V_{AB} = \left(1 - \frac{N_2}{N_1}\right) \cdot V_{bc} + \frac{N_2}{N_1} \cdot V_{bc} \;\; = a_{R\_ab} \cdot V_{ab} + \left(1 - a_{R\_bc}\right) \cdot V_{bc} \tag{97}$$

The same procedure can be followed to determine the relationships between the other line-to-line voltages. The final three-phase equation is

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = \begin{bmatrix} a_{R\_ab} & 1 - a_{R\_bc} & 0 \\ 0 & a_{R\_bc} & 1 - a_{R\_ca} \\ 1 - a_{R\_ab} & 0 & a_{R\_ca} \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \tag{98}$$

$$[VLL_{ABC}] = [\mathrm{a}] \cdot [VLL_{abc}] + [\mathrm{b}] \cdot [I_{abc}] \tag{99}$$

The relationship between source and load line currents starts with applying KCL at the load-side terminal *a*.

$$I_a = I'_a + I_{ca} = I_A - I_{ab} + I_{ca} \tag{100}$$

$$I_{ab} = \frac{N_2}{N_1} \cdot I_A \qquad I_{ca} = \frac{N_2}{N_1} \cdot I_c \qquad (101) \quad (102)$$

Substitute Equations (101) and (102) into Equation (100) and simplify:

$$I_a = \left(1 - \frac{N_2}{N_1}\right) \cdot I_A + \frac{N_2}{N_1} \cdot I_c = a_{R\_ab} \cdot I_A + \left(1 - a_{R\_ca}\right) \cdot I_c \tag{103}$$

Closed Delta-Connected Regulators

The same procedure can be followed at the other two load-side terminals. The resulting three-phase equation is

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} a_{R\_ab} & 0 & 1-a_{R\_ca} \\ 1-a_{R\_ab} & a_{R\_bc} & 0 \\ 0 & 1-a_{R\_bc} & a_{R\_ca} \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad (104)$$

$$[I_{abc}] = [\mathrm{D}] \cdot [I_{ABC}] \quad (105)$$

The general form needed for the standard model is

$$[I_{ABC}] = [\mathrm{c}] \cdot [VLL_{ABC}] + [\mathrm{d}] \cdot [I_{abc}] \quad (106)$$

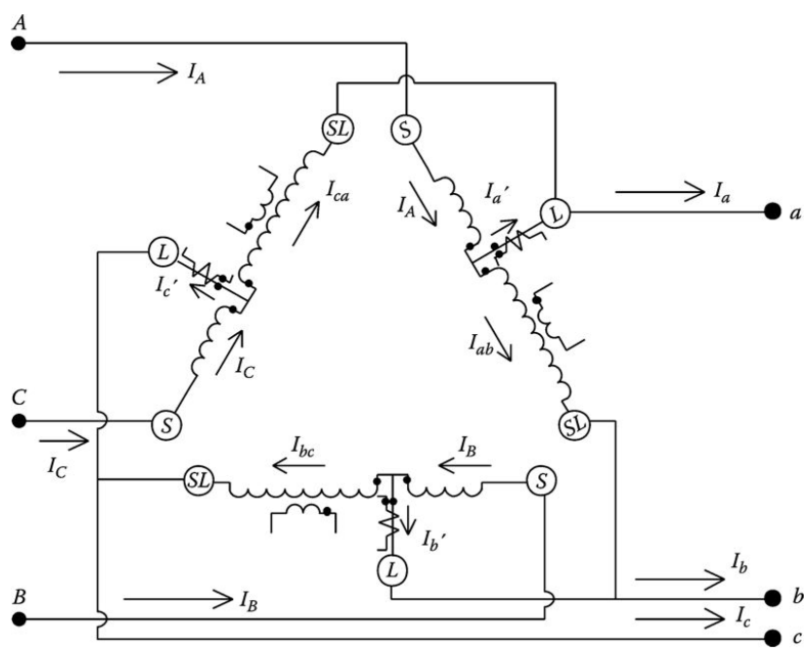


Fig.14 Closed Delta-connected regulators with currents

Open Delta-Connected Regulators

Two Type B single-phase regulators can be connected in the "open" delta connection. Shown in Fig.15 is an open delta connection where two single-phase regulators have been connected between phases *AB* and *CB*.

Two additional open connections can be made by connecting the single-phase regulators between phases *BC* and *AC* and between phases *CA* and *BA*.

The open delta connection is typically applied to three-wire delta feeders. Note that VT monitor the line-to-line voltages and CT monitor the line currents. Basic voltage and current relations of the individual regulators are used to determine the relationships between the source-side and load-side voltages and currents.

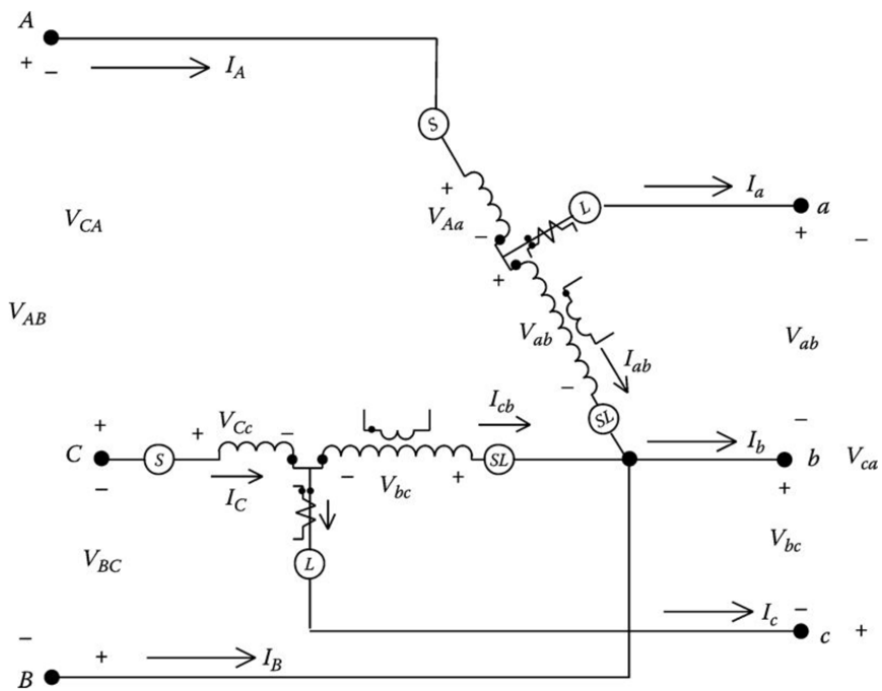


Fig.15 Open delta connection

Open Delta-Connected Regulators

The voltage VAB across the first regulator consists of the voltage across the series winding plus the voltage across the shunt winding:

$$V_{AB} = V_{Aa} + V_{ab} \tag{107}$$

Paying attention to the polarity marks on the series and shunt windings, the voltage across the series winding is

$$V_{Ab} = -\frac{N_2}{N_1} \cdot V_{ab} \tag{108}$$

Substituting Equation (108) into Equation (107) yields

$$V_{AB} = \left(1 - \frac{N_2}{N_1}\right) \cdot V_{ab} = a_{R\_ab} \cdot V_{ab} \tag{109}$$

Following the same procedure for the regulator connected across VBC, the voltage equation is

$$V_{BC} = \left(1 - \frac{N_2}{N_1}\right) \cdot V_{bc} = a_{R\_cb} \cdot V_{bc} \tag{110}$$

KVL must be satisfied so that

$$V_{CA} = -(V_{AB} + V_{BC}) = -a_{R\_ab} \cdot V_{ab} - a_{R\_cb} \cdot V_{bc} \tag{111}$$

$$V_{AB} = V_{Aa} + V_{ab} \tag{107}$$

$$V_{AB} = \left(1 - \frac{N_2}{N_1}\right) \cdot V_{ab} = a_{R\_ab} \cdot V_{ab} \tag{109}$$

Open Delta-Connected Regulators

Equations (107) through (109) can be put into matrix form:

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = \begin{bmatrix} a_{R\_ab} & 0 & 0 \\ 0 & a_{R\_cb} & 0 \\ -a_{R\_ab} & -a_{R\_cb} & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \tag{112}$$

Equation (112) in generalized form is

$$[VLL_{ABC}] = [a_{LL}] \cdot [VLL_{abc}] + [b_{LL}] \cdot [I_{abc}] \tag{113}$$

$$[a_{LL}] = \begin{bmatrix} a_{R\_ab} & 0 & 0 \\ 0 & a_{R\_cb} & 0 \\ -a_{R\_ab} & -a_{R\_cb} & 0 \end{bmatrix} \tag{114}$$

The effective turns ratio of each regulator is given by Equation (75). Again, as long as the series impedance and shunt admittance of the regulators are neglected, $[b_{LL}]$ is zero.

Equation (114) gives the line-to-line voltages on the source side as a function of the line-to-line voltages on the load side of the open delta using the generalized matrices. Up to this point, the relationships between the voltages have been in terms of line-to-neutral voltages.

Open Delta-Connected Regulators

In next chapter, the [W] matrix is derived. This matrix is used to convert line-to-line voltages to equivalent line-to-neutral voltages.

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] \tag{115}$$

$$[W] = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

The line-to-neutral voltages are converted to line-to-line voltages by

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] \tag{116}$$

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[VLL_{ABC}] = [a_{LL}] \cdot [VLL_{abc}] + [b_{LL}] \cdot [I_{abc}] \tag{113}$$

Convert Equation (113) to line-to-neutral form:

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] = [W] \cdot [a_{LL}] \cdot [D] \cdot [VLN_{abc}]$$

$$[VLN_{ABC}] = [a_{reg}] \cdot [VLN_{abc}]$$

$$[a_{reg}] = [W] \cdot [a_{LL}] \cdot [D] \tag{117.a}$$

Open Delta-Connected Regulators

When the load-side line-to-line voltages are needed as function of the source-side line-to-line voltages, the necessary equation is

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} \frac{1}{a_{R\_ab}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_cb}} & 0 \\ -\frac{1}{a_{R\_ab}} & -\frac{1}{a_{R\_cb}} & 0 \end{bmatrix} \begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} \quad (117.b)$$

$$[VLL_{abc}] = [A_{LL}] \cdot [VLL_{ABC}] \quad (118)$$

$$[A_{LL}] = \begin{bmatrix} \frac{1}{a_{R\_ab}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_cb}} & 0 \\ -\frac{1}{a_{R\_ab}} & -\frac{1}{a_{R\_cb}} & 0 \end{bmatrix} \quad (119)$$

Equation (118) is converted to line-to-neutral form by

$$\begin{aligned} [VLN_{abc}] &= [A_{reg}] \cdot [VLN_{ABC}] \\ [A_{reg}] &= [W] \cdot [A_{LL}] \cdot [D] \end{aligned} \quad (120)$$

There is no general equation for each of the elements of $[A_{reg}]$. The matrix $[A_{reg}]$ must be computed according to Equation (120).

Open Delta-Connected Regulators

Referring to Fig.15, the current equations are derived by applying KCL at the *L* node of each regulator:

$$I_A = I_a + I_{ab} \tag{121}$$

$$I_{ab} = \frac{N_2}{N_1} \cdot I_A \rightarrow \left(1 - \frac{N_2}{N_1}\right) \cdot I_A = I_a \tag{122}$$

Hence,

$$I_A = \frac{1}{a_{R\_ab}} \cdot I_a \tag{123}$$

Similarly,

$$I_C = \frac{1}{a_{R\_cb}} \cdot I_c \tag{124}$$

In matrix form,

$$\begin{bmatrix} I_A \\ I_B \\ I_c \end{bmatrix} = \begin{bmatrix} \frac{1}{a_{R\_ab}} & 0 & 0 \\ -\frac{1}{a_{R\_ab}} & 0 & -\frac{1}{a_{R\_cb}} \\ 0 & 0 & \frac{1}{a_{R\_cb}} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{126}$$

$$[I_{ABC}] = [c_{reg}] \cdot [VLN_{ABC}] + [d_{reg}] \cdot [I_{abc}] \tag{127}$$

When the series impedances and shunt admittances are neglected, the constant matrix $[c_{reg}]$ will be zero.

Open Delta-Connected Regulators

The load-side line currents as a function of the source line currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} a_{R\_ab} & 0 & 0 \\ -a_{R\_ab} & 0 & -a_{R\_cb} \\ 0 & 0 & a_{R\_cb} \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{129}$$

$$[I_{abc}] = [D_{reg}] \cdot [I_{ABC}] \tag{130}$$

$$[D_{reg}] = \begin{bmatrix} a_{R\_ab} & 0 & 0 \\ -a_{R\_ab} & 0 & -a_{R\_cb} \\ 0 & 0 & a_{R\_cb} \end{bmatrix} \tag{131}$$

The determination of the *R* and *X* compensator settings for the open delta follows the same procedure as that of the wye-connected regulators.

However, care must be taken to recognize that in the open delta connection the voltages applied to the compensator are line to line and the currents are line currents. The open delta–connected regulators will maintain only two of the line-to-line voltages at the load center within defined limits. The third line-to-line voltage will be dictated by the other two (KVL). Therefore, it is possible that the third voltage may *not be within the defined limits*.

Open Delta-Connected Regulators

With reference to Fig.16, an equivalent impedance between the regulators and the load center must be computed. Since each regulator is sampling line-to-line voltages and a line current, the equivalent impedance is computed by taking the appropriate line-to-line voltage drop and dividing by the sampled line current.

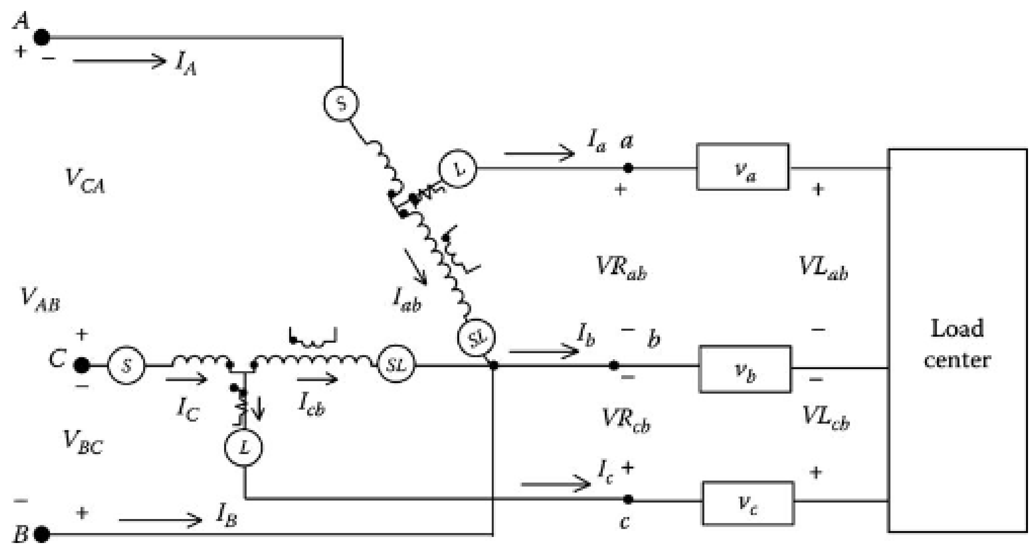


Fig.16 Open delta connected to a load center

For the open delta connection shown in Figure 7.16, the equivalent impedances are computed as

$$Zeq_a = \frac{VR_{ab} - VL_{ab}}{I_a} \qquad Zeq_c = \frac{VR_{cb} - VL_{cb}}{I_c} \qquad (132)\ (133)$$

The units of these impedances will be in system Ohms. They must be converted to compensator volts. For the open delta connection, the potential transformer will transform the system line-to-line rated voltage down to 120 V.