Fault Analysis – Balance and Unbalance

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Fault and What Causes Fault?

- Insulation Failure creates a "Short Circuit"
    - An easier path for the current → What about "series high impedance fault"?
- A large fault current flows through short circuit → Can the fault have a smaller current than load?
- This fault current must be interrupted quickly because: → Is there any minimum / maximum limit to fault clearance?
    - Equipment Damage and Human Danger
    - Sustained Interruption (due to fault repair)
    - Momentary Interruption (due to equipment reset)
    - Voltage Dip and Load Rejection
    - System Perturbation – Instability and Blackout
    - Reduced Reliability – SAIFI vs SAIDI
- What are the possible causes of fault?
    - Lightning – Direct Strike, Back Flashover, Induced Voltage
    - Intruders – Contact between bare conductors e.g. branches, animals (e.g. squirrels, snakes, large birds)
    - Movement of conductors due to wind, sagging due to high current
    - Defects in insulating material – Ageing of solid insulation, contamination of oil, pollution
    - Damage to solid insulator – Rats, ants, vandalism, construction (i.e., digging up the street), water tree
    - Pollution on the insulators due to fire hill (e.g. LCE – YUE 2: 20240331)

Types of Fault

Transient Fault

- Fault disappears once the fault current has been interrupted (e.g. Lightning Fault)
- Component can be put back in service very quickly (e.g. by DAR)

Permanent Fault

- Insulation is permanently damaged
- Component must be repaired or replaced before being put back in service

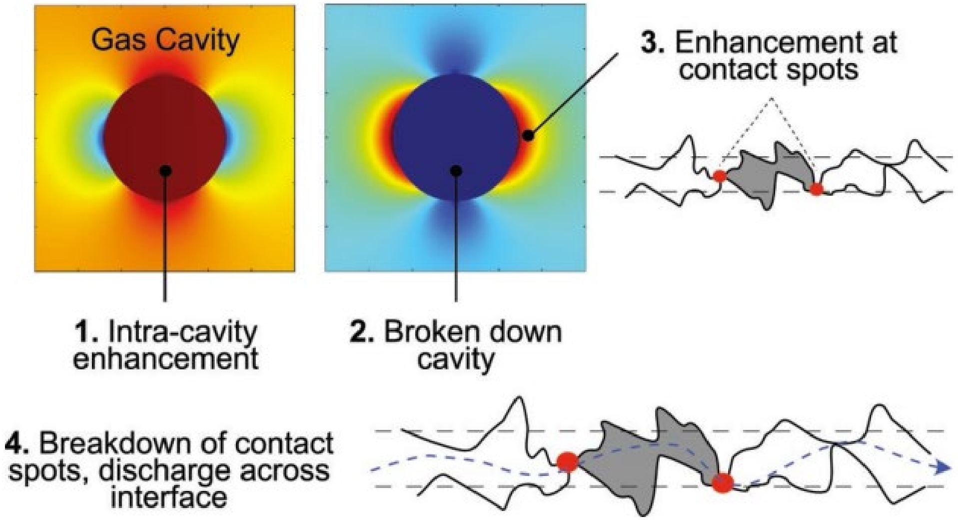
Balanced faults (a.k.a. symmetrical faults)

- All three phases affected in the same way
- Can be studied using single phase model

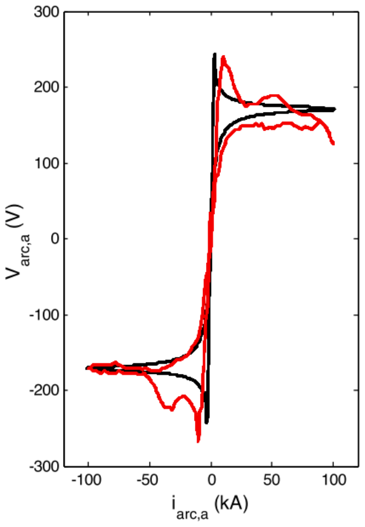
Unbalanced faults

- Fault Current vs Voltage Shift (e.g. pole factor)
- Not all phases are affected in the same way
- Behaviour in all three phases is no longer symmetrical
- Sequence Network Analysis

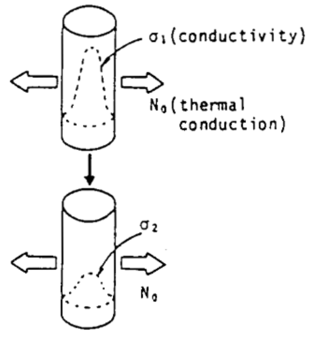
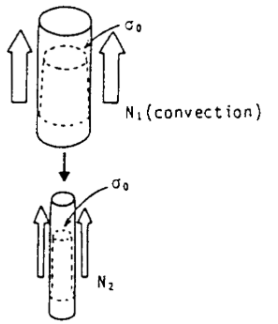
Evolving fault – Fault is a dynamic behavior



Which type of fault is more severe?



- Fault Level intrinsically describe the maximum fault current available at a point at the voltage level.
  - Maximum Fault Level = maximum fault current available at any point at the voltage level.
  - Determine in first day when designing the system for breaker procurement (breaking capacity).
  - System to be operated within the fault level.
  - Minimum Fault Level = minimum of the maximum fault current at some point at the voltage level.
  - To ensure enough fault current such that protective device has enough fault current to trigger.
- A more severe fault is one with a larger fault current
  - SLG fault > LLL fault due to $Z_0 < Z_1$
  - Balanced three phase faults are usually the most severe in general
- Resistance Fault
  - Bolted Fault / Solid Fault: $R_F = 0$
  - Arc Fault: $R_F \neq 0$ (Arc dynamics)
  - DC offset dependent on Inception Angle and X/R ratio

Model name	Mayr	Cassie
Arc conductance	By change of ionization degree	By change of arc diameter
	$G \propto \exp\left(\frac{Q}{Q_0}\right)$	$G \propto Q$
Heat loss	By thermal conduction	By thermal convection
	$N = N_0$ (Constant)	$N \propto Q$
Dynamic characteristic expression	$\frac{1}{G}\cdot\frac{dG}{dt} = \frac{1}{\theta}\left(\frac{EI}{N_0} - 1\right)$	$\frac{1}{G}\cdot\frac{dG}{dt} = \frac{1}{\theta}\left(\frac{E^2}{E_0^2} - 1\right)$
	Small current region	Large current region
Conceptual diagram		

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Fault Handling

Relay's Work

1. Condition Monitoring with Prognostic Analysis

   Fault are evolving, and it should be detected with arranged maintenance outage.

2. Fault Detection (including Fault Phase Identification)

   Must be done quickly before it becomes a bigger problem: 10ms – 100ms

   Cleared by unit protection (a sensitive detection with no time delay)

3. Fault Clearance

   Interrupt the fault current using a circuit breaker or a fuse: CB mechanism (30ms – 60ms) / Fuse Dynamics

4. Fault Location (in % or km if Feeder Fault)

   Determine the fault location with fault location theory
   (e.g. impedance based, high frequency incremental, travelling wave or AI approach)

   Identify the faulted equipment upon arrival of site staff (maloperation or real fault)

5. Fault Repair

   Only if fault is not a transient fault
   Take weeks or months

6. Energization (or Auto-Reclose in case of OHL fault)

Recall Per Unit Calculation

- Given $S_B^{3\varphi}$ and $V_B^{3\varphi}$, base current and base impedance are:

$$Z_B = \frac{V_{B,L-L}^2}{S_B^{3\varphi}} = \frac{\left(\sqrt{3}V_{B,L-N}\right)^2}{3S_B^{1\varphi}} = \frac{V_{B,L-N}^2}{S_B^{1\varphi}}$$

$$I_B^{3\varphi} = \frac{S_B^{3\varphi}}{\sqrt{3}V_{B,L-L}} = \frac{3S_B^{1\varphi}}{\sqrt{3}\left(\sqrt{3}V_{B,L-N}\right)} = \frac{S_B^{1\varphi}}{V_{B,L-N}} = I_B^{1\varphi}$$

- Base Change Formula: $Z_2 = Z_1\left(\frac{V_1}{V_2}\right)^2\left(\frac{S_2}{S_1}\right) \xrightarrow[V_1=V_2]{} Z_2 = Z_1\left(\frac{S_2}{S_1}\right)$

- Fault Current:

$$I_F[\mathrm{pu}] = \frac{\mathrm{V[pu]}}{\mathrm{Z_{eq}[pu]}} = \frac{1}{\mathrm{Z_{eq}[pu]}} \longrightarrow I_F[\mathrm{A}] = I_F[\mathrm{pu}] \times I_B[A] = I_F[\mathrm{pu}] \times \frac{S_B^{3\varphi}}{\sqrt{3}V_{B,L-L}}$$

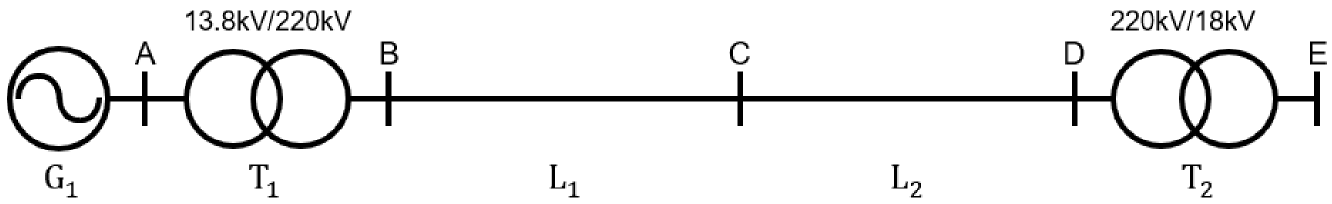
$$\|$$

$$S_F[\mathrm{pu}] = \frac{(\mathrm{V[pu]})^2}{\mathrm{Z_{eq}[pu]}} = \frac{1}{\mathrm{Z_{eq}[pu]}} \longrightarrow S_F[\mathrm{A}] = S_F[\mathrm{pu}] \times S_B[A] = S_F[\mathrm{pu}] \times S_B^{3\varphi}$$

$$S_F[\mathrm{A}] = I_F[A] \times \sqrt{3}V_{B,L-L}$$

- Major Problem – Determine equivalent impedance $\mathrm{Z_{eq}[pu]}$
- The magnitude of fault level [MVA] does not mean it can generate such an output, as the actual voltage is not same as the base voltage anyway. Yet, it is a measure for comparing the fault severity at different voltage level.

Example 9.1



Given that

- All impedances are in per unit on a 50 MVA basis
- Resistances and shunt admittances are neglected
- No loads on the system
- All voltages are at nominal value

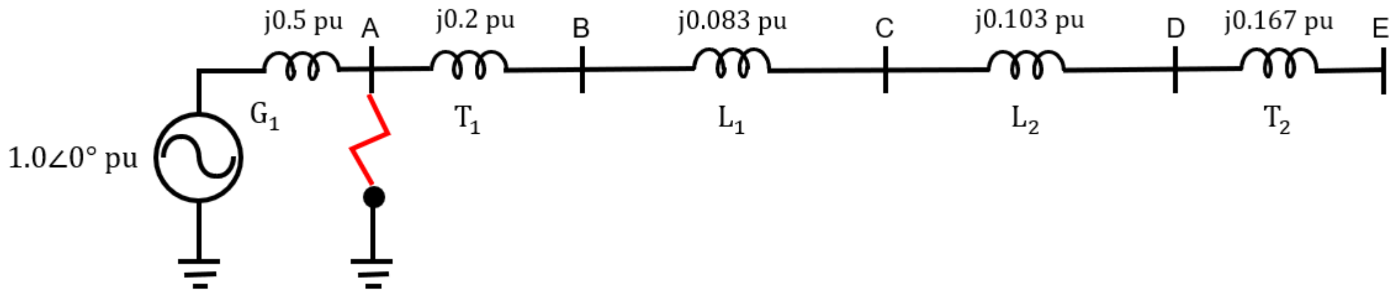
Component	Reactance (pu)
G_1	0.5
T_1	0.2
L_1	0.083
L_2	0.103
T_2	0.167

Compute the fault level at each level (A, B, C, D and E).

Solution

Consider a fault occurred at A.

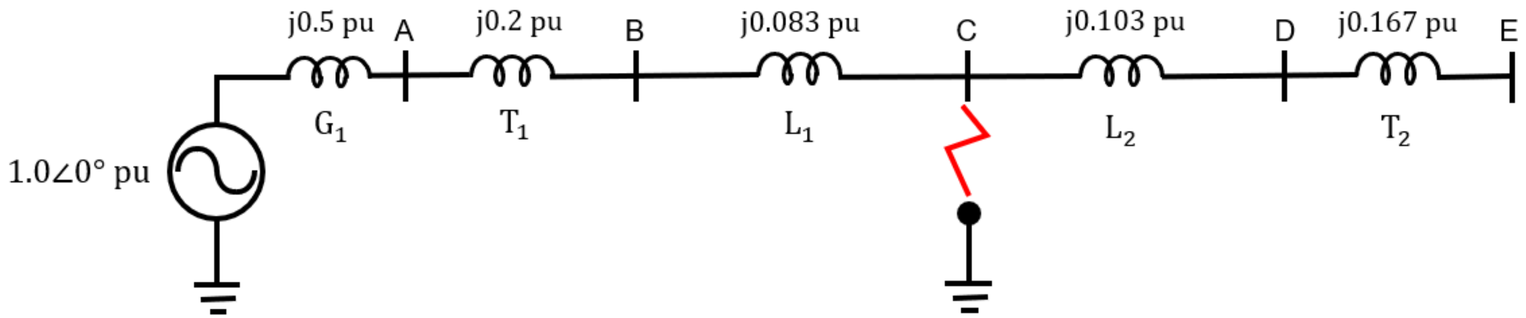
$$\bar{I}_A^F = \frac{1.0\angle 0^\circ}{j0.5} = 2.0\angle -90^\circ \text{pu}$$



Fault with Resistance:

$$\bar{I}_A^F = \frac{1.0\angle 0^\circ}{R_f + j0.5}$$

Example 9.1 (cont')



- Consider a fault at C.

$$\overline{I_C^F} = \frac{1.0\angle 0°}{j0.5 + j0.2 + j0.083} = 1.28\angle - 90°pu$$

- Similarly,

Bus	A	B	C	D	E
$\lvert I_F \rvert$ (pu)	2.0	1.43	1.28	1.13	0.95
V_B (kV)	13.8	220	220	220	18
I_B (A)	2,092	131.2	131.2	131.2	1,604
$\lvert I_F \rvert$ (A)	4,184	188	168	148	1524
FL (MVA)	100	71.5	64	56.5	47.5

$$I_F[\text{pu}] = 1/Z_{eq}[\text{pu}]$$

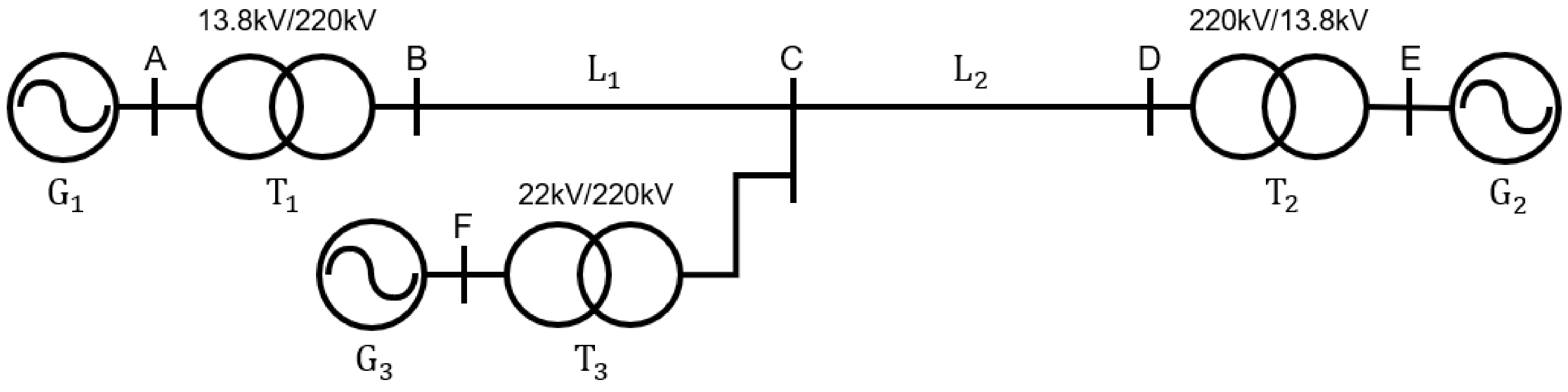
$$I_B[\text{A}] = S_B[\text{MVA}]/\sqrt{3}V_B^2\ [\text{kV}]$$

$$I_F[\text{A}] = I_F[\text{pu}] \times I_B[\text{A}]$$

$$\text{FL}[\text{MVA}] = \sqrt{3}\text{V}_\text{B}[\text{kV}] \times \text{I}_\text{F}[\text{kA}]$$

- Fault current [A] depends on the nominal voltage [kV] at fault location.
- Fault Level [MVA] is the convenient way of comparing the severity of faults at different voltage levels.

Example 9.2: System with Multiple Source



Assumptions:

- No loads in the system
- No currents flowing before the fault
- All voltages (including internal emfs) at nominal value (1.0 pu)

Determine

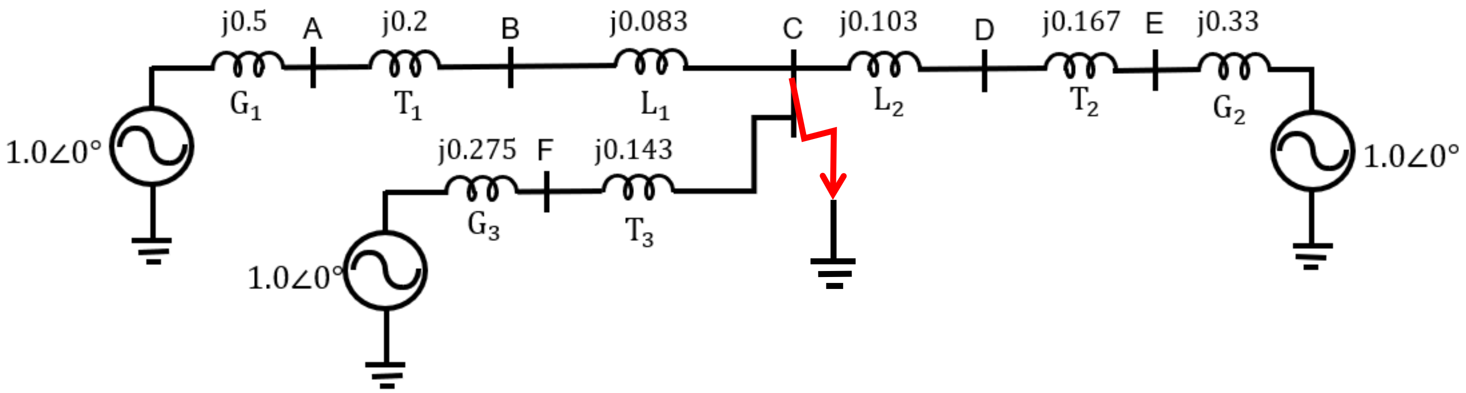
(a) Equivalent source impedance for a bolted fault at C.

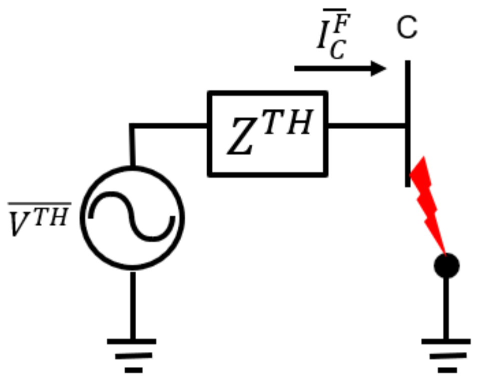
(b) Maximum fault current at C.

(c) Maximum fault current at B.

Example 9.2: System with Multiple Source (cont')

Solution





(a) Thevenin Voltage $V^{TH} = 1.0\angle 0°$ pu (determine voltage for open circuit at C)

Thevenin Impedance

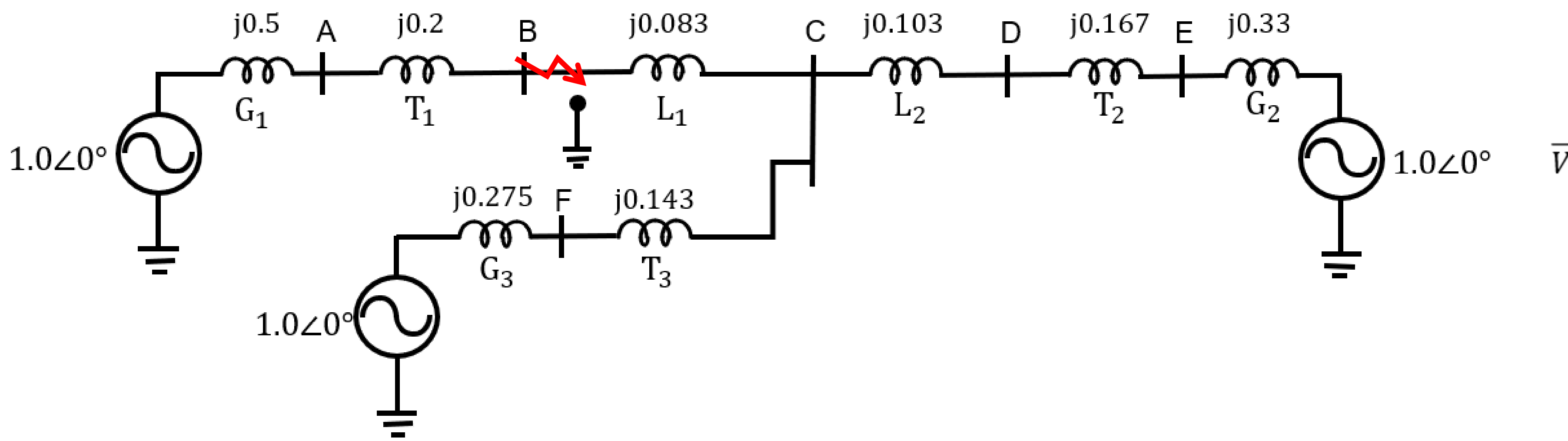
$$Z_C^{TH} = j(0.33 + 0.167 + 0.103) \parallel j(0.5 + 0.2 + 0.083) \parallel j(0.275 + 0.143) = j0.187 \text{ pu}$$

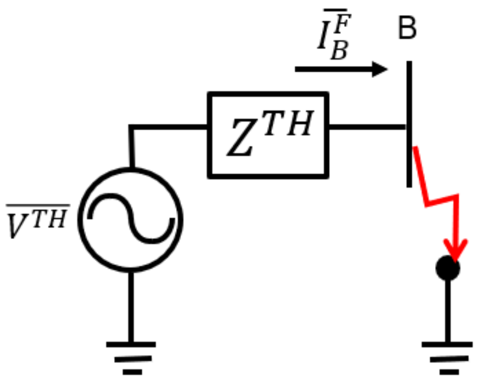
(b) Maximum Fault Current with Bolted Fault

$$\overline{V_C^{TH}} = 1.0\angle 0° \text{ pu} \qquad Z_C^{TH} = j0.187 \text{ pu}$$

$$\overline{I_C^F} = \frac{\overline{V_C^{TH}}}{Z_C^{TH}} = \frac{1.0\angle 0°}{j0.187} = 5.348\angle -90° \text{ pu}$$

Example 9.2: System with Multiple Source (cont')





(c) Fault Current at B

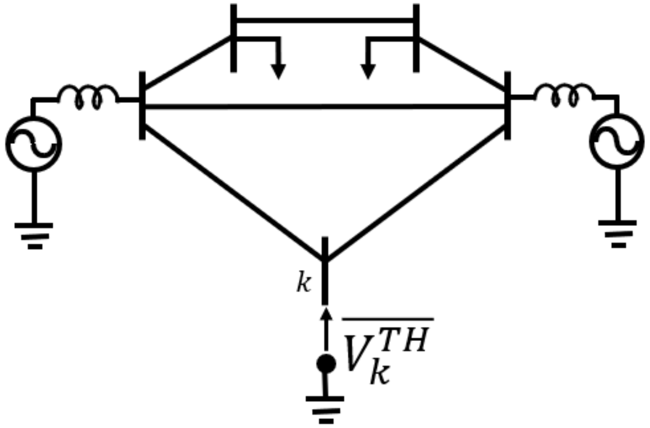
$$\overline{V_B^{TH}} = 1.0\angle 0° p.u.$$

$$Z_B^{TH} = j(0.5 + 0.2) \parallel \{j0.083 + [j(0.275 + 0.143) \parallel j(0.33 + 0.167 + 0.103)]\} = j0.224\, pu$$

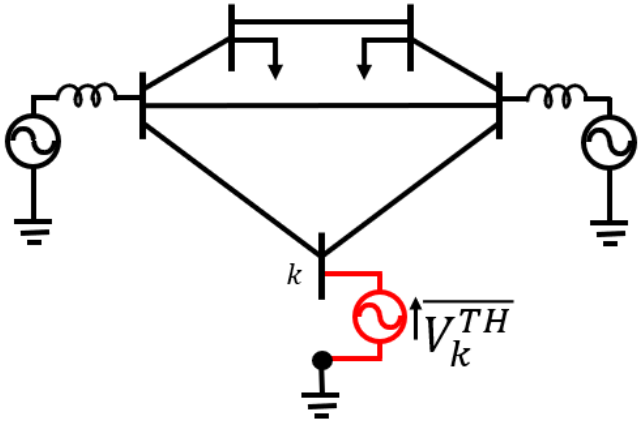
$$\overline{I_B^F} = \frac{\overline{V_B^{TH}}}{Z_B^{TH}} = \frac{1.0\angle 0°}{j0.224} = 4.465\angle -90° p.u.$$

Balanced fault calculations in large systems

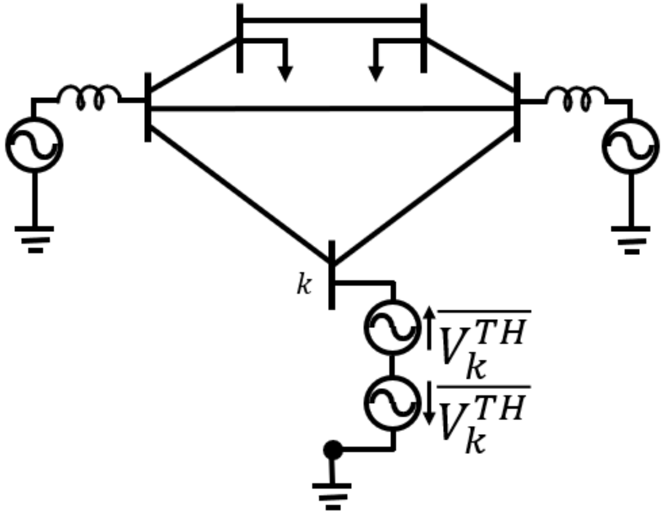
- Goals:
  - Develop a systematic and scalable technique for computing fault currents
  - Remove simplifying assumptions: No loads in the system / All voltages are at nominal value
- Approach:
  - Thevenin equivalent
  - Superposition theorem



Run Pre-Fault Load Flow
to Determine Thevenin Voltage



Imagine adding a voltage source with
same voltage as the Thevenin Voltage



Adding a reverse voltage source to
make bus k as zero voltage

Superposition Theorem

From circuit theory:

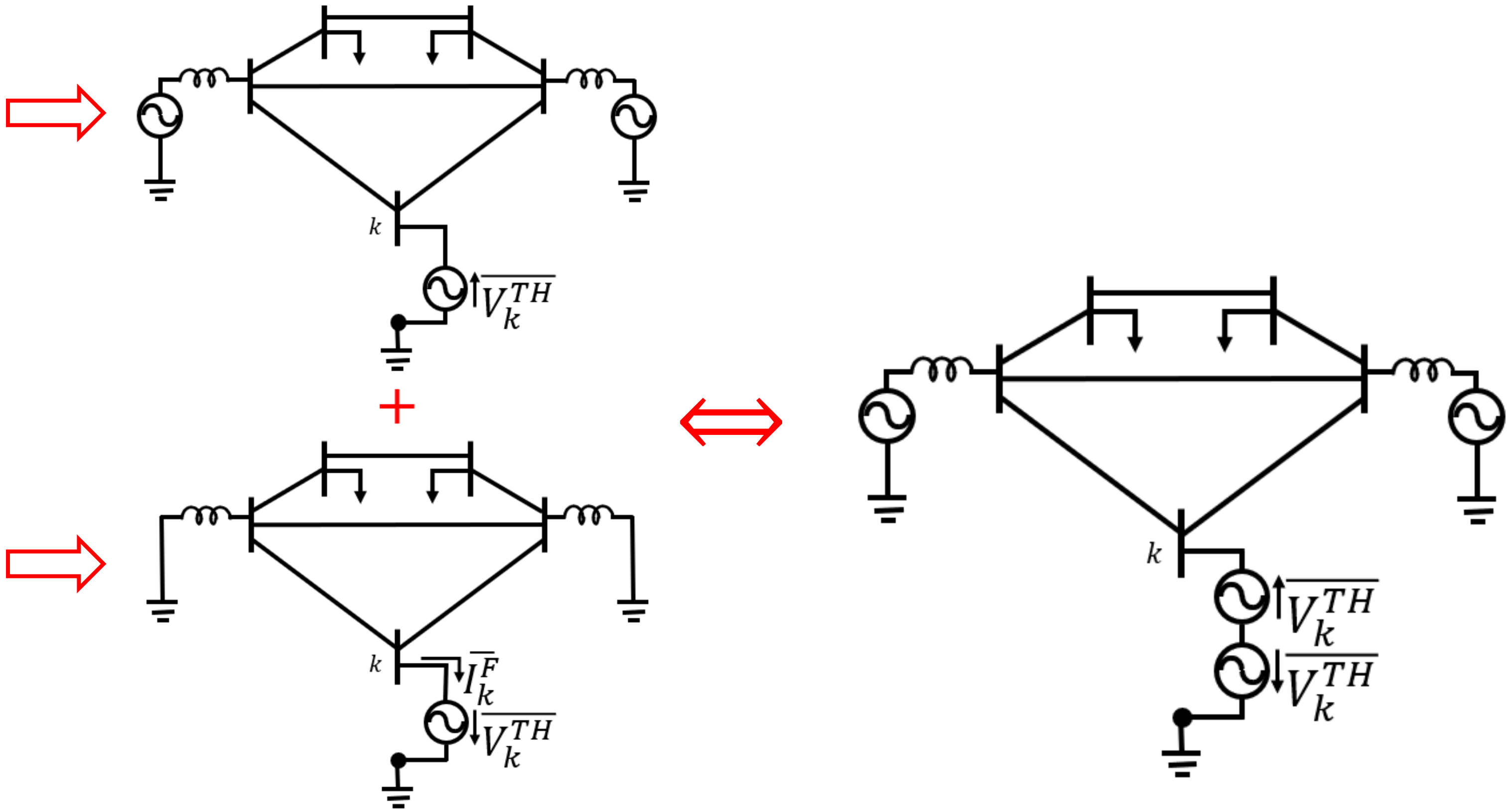
- A linear circuit with multiple sources can be analyzed by calculating the voltages and currents resulting from each source acting separately
- The combined effects of all the sources can then be calculated by adding the voltages and currents produced by each sources taken separately

Pre-fault conditions:

- No current flows through the additional voltage source $\overline{V_k^{TH}}$.
- Normal load currents in the rest of the network.

Fault conditions:

- Fault current $\overline{I_k^F}$ flows driven by voltage source $-\overline{V_k^{TH}}$ at bus k.



Fault Current Calculation with Nodal Analysis

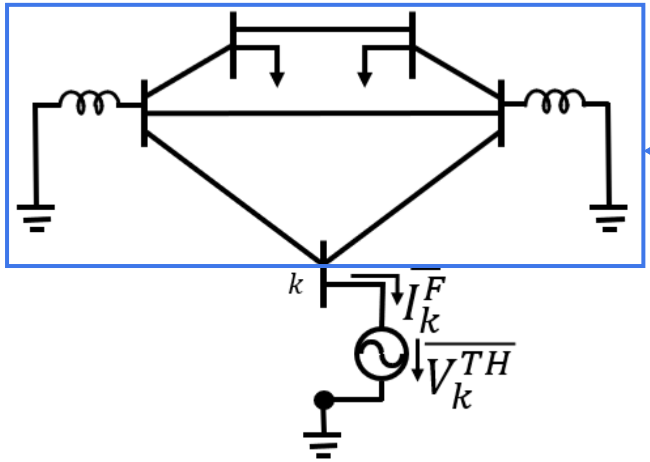
- Calculate the fault current with nodal analysis

$$\boxed{YV = I}$$

Y: Admittance matrix

V: Vector of nodal voltages

I: Vector of injected currents



Replace by an equivalent (i.e., Thevenin) impedance

⇩

Need a technique that scales up to any size network

⇩

Nodal analysis

$$Y\begin{pmatrix} \overline{\Delta V_1} \\ \vdots \\ -\overline{V_k^{TH}} \\ \vdots \\ \overline{\Delta V_n} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ -\overline{I_k^F} \\ \vdots \\ 0 \end{pmatrix} \Leftarrow$$

$-\overline{I_k^F}$: is the only injected current

$\overline{\Delta V_j}$: change in voltage at bus j due to fault current

Taking inverse on admittance matrix -

$$Z\begin{pmatrix} 0 \\ \vdots \\ -\overline{I_k^F} \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} \overline{\Delta V_1} \\ \vdots \\ -\overline{V_k^{TH}} \\ \vdots \\ \overline{\Delta V_n} \end{pmatrix}$$

Z: impedance matrix of the circuit

Fault Current Calculation with Nodal Analysis

- Consider row k of this equation:

Z_{kk}: diagonal element (k, k) of Z

Z_{ik}: diagonal element (i, k) of Z

$$Z\begin{pmatrix}0\\ \vdots\\ -\overline{I_k^F}\\ \vdots\\ 0\end{pmatrix}=\begin{pmatrix}\overline{\Delta V_1}\\ \vdots\\ -\overline{V_k^{TH}}\\ \vdots\\ \overline{\Delta V_n}\end{pmatrix}$$

$$Z_{kk}\left(-\overline{I_k^F}\right)=-\overline{V_k^{TH}}$$

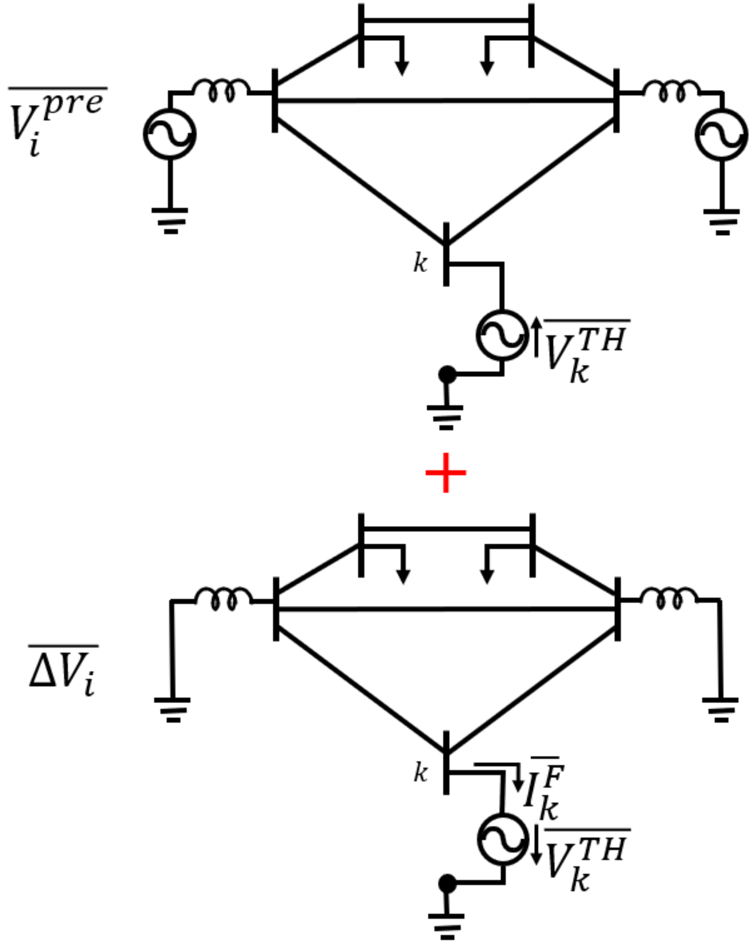
$$\boxed{\overline{I_k^F}=\frac{\overline{V_k^{TH}}}{Z_{kk}}}$$

- Voltage at bus i –

  Consider other rows of impedance matrix,

$$\boxed{\overline{\Delta V_i}=-Z_{ik}\overline{I_k^F}=-\frac{Z_{ik}}{Z_{kk}}\overline{V_k^{TH}}}\qquad i=1,\cdots n\quad i\neq k$$

$$\overline{V_i^F}=\overline{V_i^{pre}}+\overline{\Delta V_i}=\overline{V_i^{pre}}-\frac{Z_{ik}}{Z_{kk}}\overline{V_k^{TH}}\qquad i=1,\cdots n$$

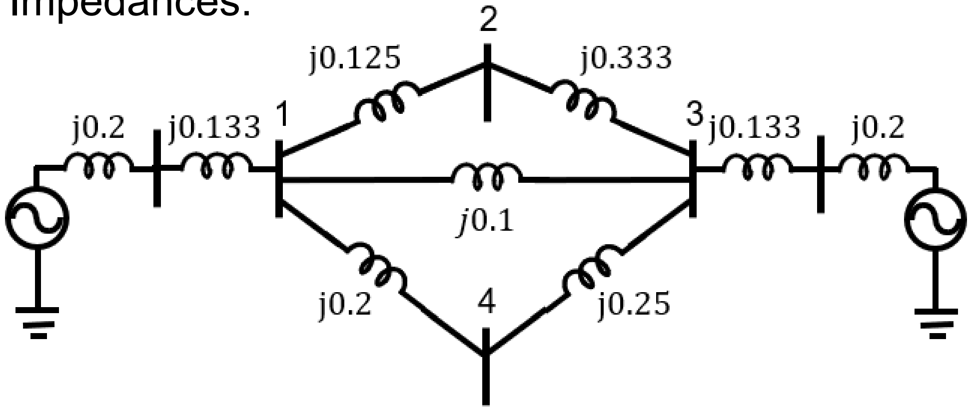


Example 9.3: Fault calculation using the impedance matrix

Determine fault current for a bolted, three-phase fault at bus 2.

Neglect loads and assume nominal voltage at all buses.

Impedances:



Solution

Admittance Matrix:

$$Y = \begin{pmatrix} -j26 & j8 & j10 & j5 \\ j8 & -j11 & j3 & 0 \\ j10 & j3 & -j20 & j4 \\ j5 & 0 & j4 & -j9 \end{pmatrix}$$

Impedance Matrix:

$$Z = Y^{-1} = j\begin{pmatrix} 0.182 & 0.174 & 0.151 & 0.168 \\ 0.174 & 0.261 & 0.160 & 0.167 \\ 0.151 & 0.160 & 0.182 & 0.165 \\ 0.168 & 0.167 & 0.165 & 0.278 \end{pmatrix}$$

Bolted LLL Fault Current:

$$\overline{I_2^F} = \frac{\overline{V_2^{TH}}}{Z_{22}} = \frac{1.0\angle 0°}{j0.261} = 3.83\angle -90° p.u.$$

Voltage at other bus:

$$\overline{V_i^F} = \overline{V_i^{pre}} + \overline{\Delta V_i} = \overline{V_i^{pre}} - \frac{Z_{ik}}{Z_{kk}}\overline{V_k^{TH}} \qquad \overline{V_i^{pre}} = 1.0\angle 0° \text{pu} \quad i = 1, \cdots 4$$

$$\overline{V_1^F} = 1.0\angle 0° - \frac{j0.174}{j0.261} 1.0\angle 0° = 0.334\angle 0° \text{ pu.} \qquad \overline{V_4^F} = 1.0\angle 0° - \frac{j0.167}{j0.261} 1.0\angle 0° = 0.358\angle 0° \text{ pu}$$

$$\overline{V_3^F} = 1.0\angle 0° - \frac{j0.160}{j0.261} 1.0\angle 0° = 0.388\angle 0° \text{ pu.}$$

Generator Modeling for Fault Current Calculation

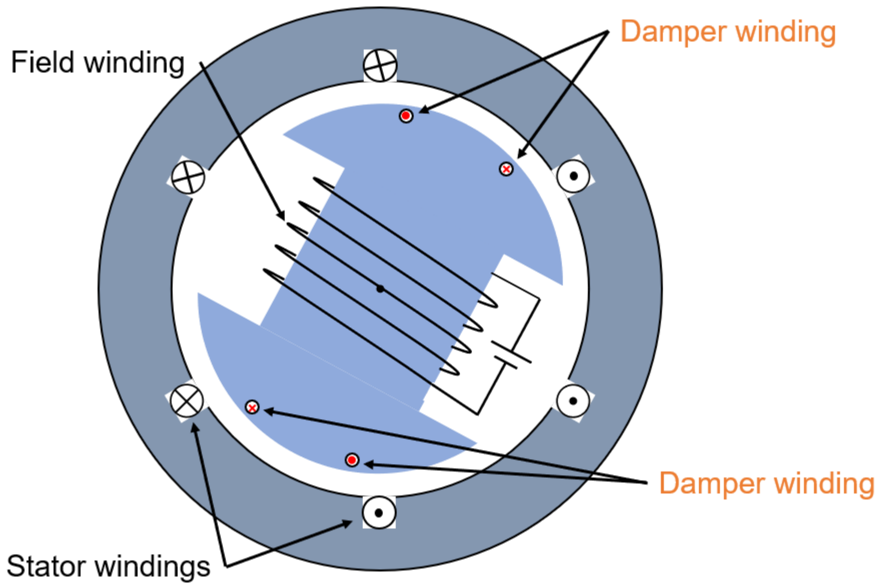
- It was assumed that generators behaved like ideal voltage sources
- Need to enhance our generator model to reflect what happens when the stator current changes rapidly due to a fault
- Model what happens to the magnetic flux inside the generator

Damper Winding

- Short circuit windings in the pole face
- No effect in the steady state
- Designed to dampen mechanical oscillations of the rotor

Magnetic Coupling

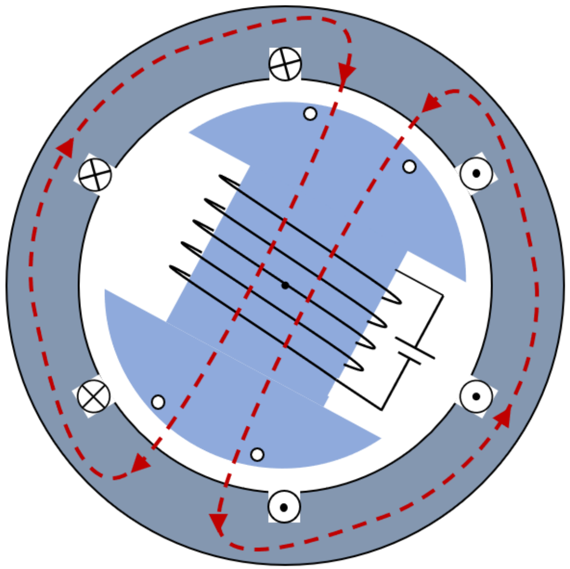
- Stator, field, and damper windings are magnetically coupled
- Current flowing in each of these windings affects the magnetic flux in the other windings



Generator Modeling for Fault Current Calculation

Steady State Condition

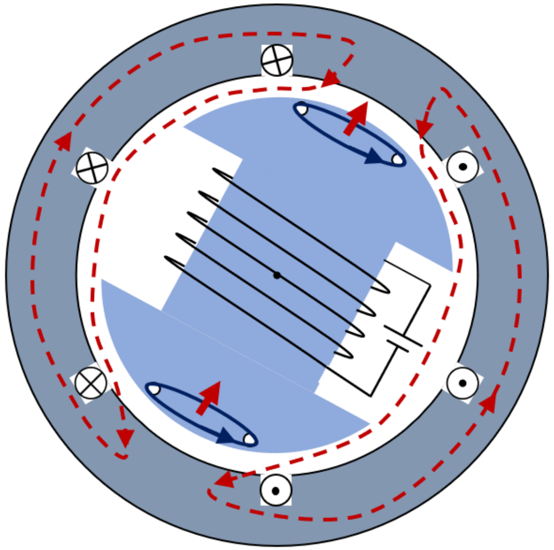
- Field current creates rotor flux
- Three-phase stator current creates stator flux
- Stator flux rotates in synchronism with the rotor
- Stator flux linking the field and damper windings is constant
- Stator flux path is mostly through iron → low reluctance path
- Self inductance of stator winding L_S and synchronous reactance $X_S = \omega L_S$ are large



Sudden Increase in Stator Current

- Increase in stator current → increase in stator flux
- Lenz's law: a change in flux induces current that opposes the change in flux
- Current induced in damper and field windings prevents larger stator flux from entering the rotor
- Stator flux pushed into the airgap, which is a higher reluctance path

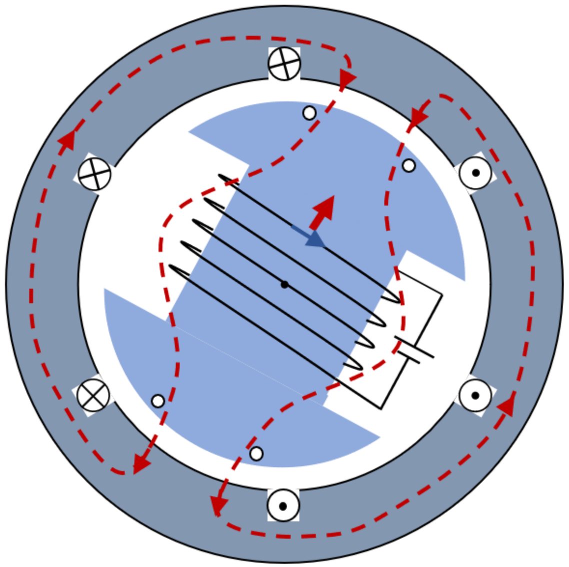
  → Lower reactance



Generator Modeling for Fault Current Calculation

Decay of Induced Damper Current

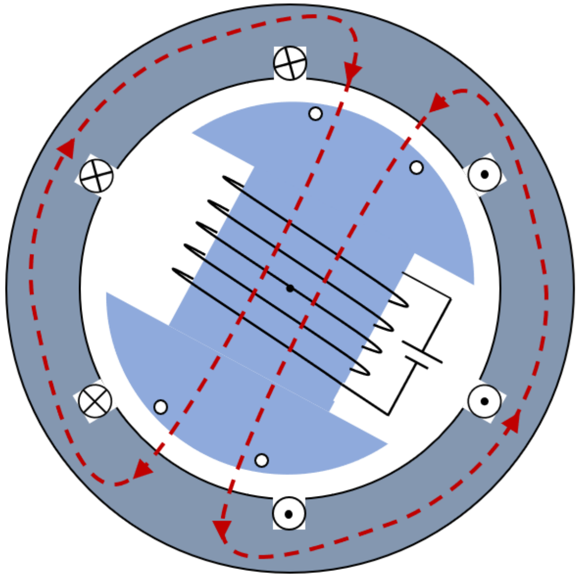
- Currents induced in the damper windings decay with time constant (L / R) of damper windings
- Increased stator flux partially penetrates the rotor
- Induced current prevents this increased stator flux from penetrating the field winding
- Flux path partially through iron
- Lower reluctance → Higher reactance



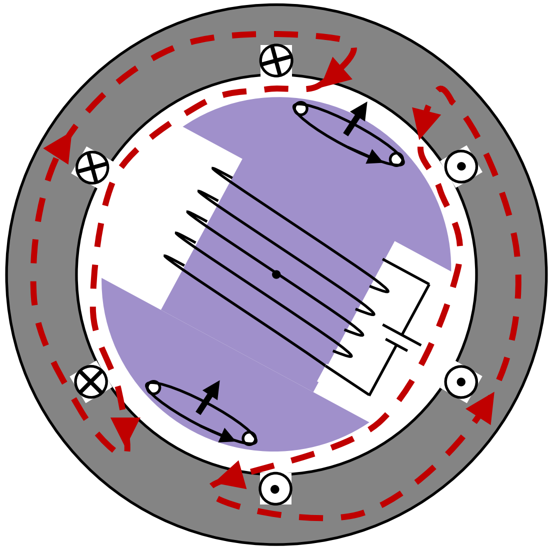
Back to Steady State Condition

- Currents induced in the field winding decays with time constant (L / R) of these windings
- Lower reluctance

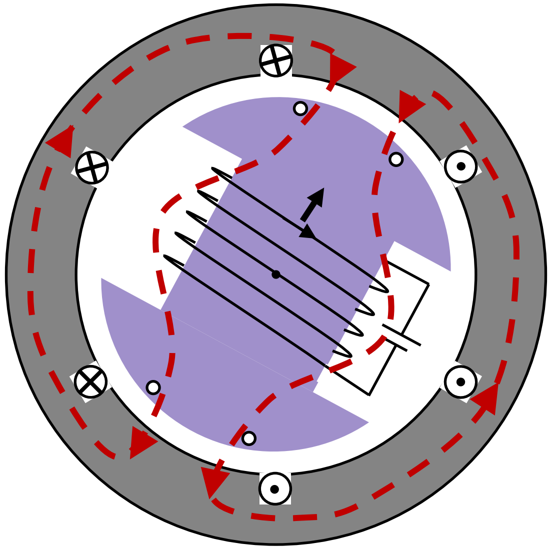
  → Reactance returns to its higher steady state value



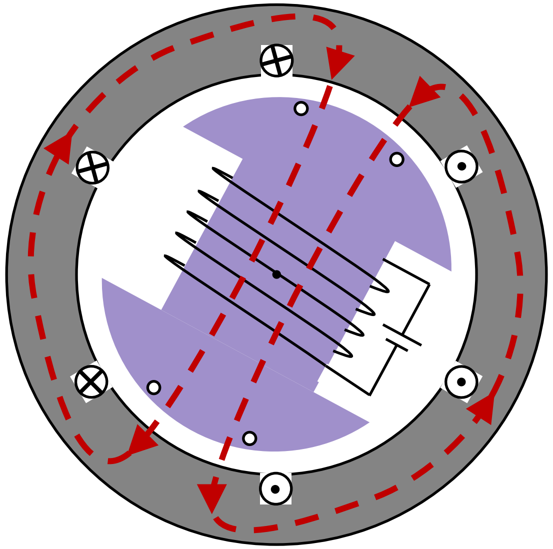
Generator Modeling for Fault Current Calculation



Immediately after
the fault



After the damper currents
have decayed



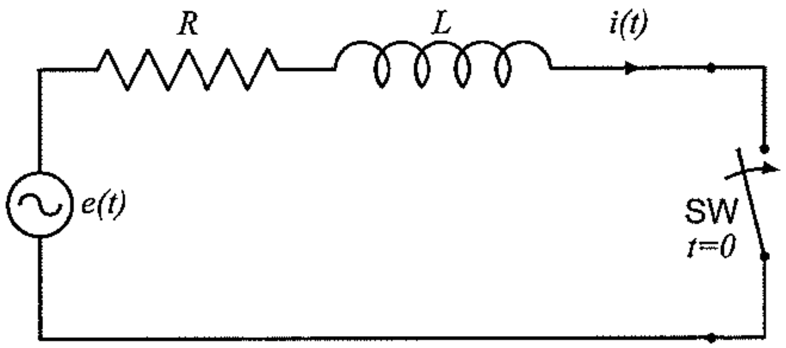
After the field current
has decayed

Continuous evolution driven by time constants (L / R)

Subtransient state Valid for a few 10 ms	Transient state Valid for a few 100 ms	Steady state
Sub-transient reactance X''	Transient reactance X'	Synchronous reactance X_S

$$\boxed{X'' < X' < X_S}$$

Series RL Circuit Transient



- R, L represents fault impedance, including source impedance.
- SW closed at t = 0 to represent a solid fault.
- If $$e(t) = \sqrt{2}E\sin(\omega t + \alpha)$$
  then $$L\frac{di(t)}{dt} + Ri(t) = \sqrt{2}E\sin(\omega t + \alpha)$$
  where α determines the magnitude of voltage when the circuit is closed, i.e. inception angle.
- The solution of the differential equation is:

$$i(t) = \underbrace{\frac{\sqrt{2}E}{Z}\sin(\omega t + \alpha - \theta)}_{\text{Symmetrical Short Circuit Current}} + \underbrace{\frac{\sqrt{2}E}{Z}\sin(\theta - \alpha)\, e^{-\frac{t}{T}}}_{\text{DC Offset Current}}$$

$$= i_{ac}(t) + i_{dc}(t)$$

where

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X^2} \qquad T = \frac{L}{R}$$

$$\theta = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{X}{R}$$

Worst Case: $\sin(.) = 1, e^{-\frac{t}{T}} = 1$

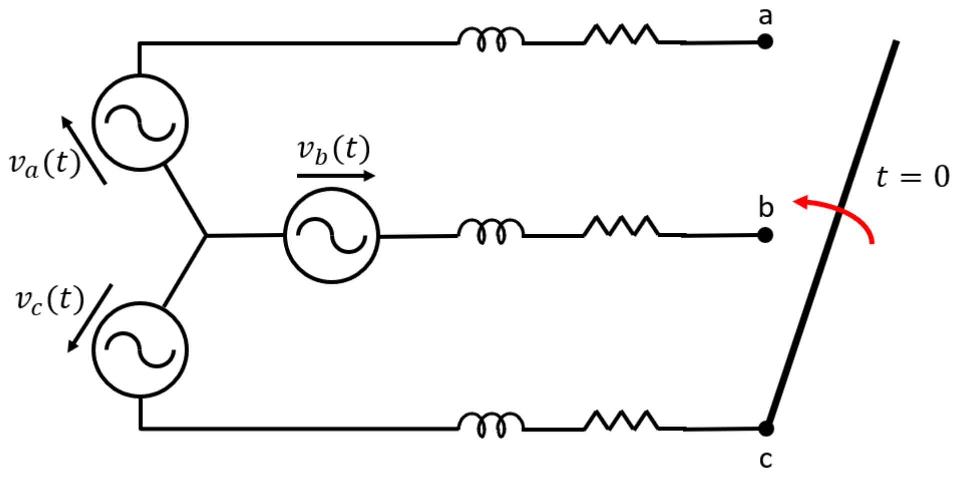
Maximum Possible Current:

$$i_{mm} = 2\frac{\sqrt{2}E}{Z}$$

(Doubling Effect)

Fault in Three Phase RL Circuit

- Inception Angle is different for each phase
  → DC offset is different in each phase
- Possibly leading to missing zero-crossing at each phase
  - Current chopping without natural damping
  - Phase Detector Circuit may not behave

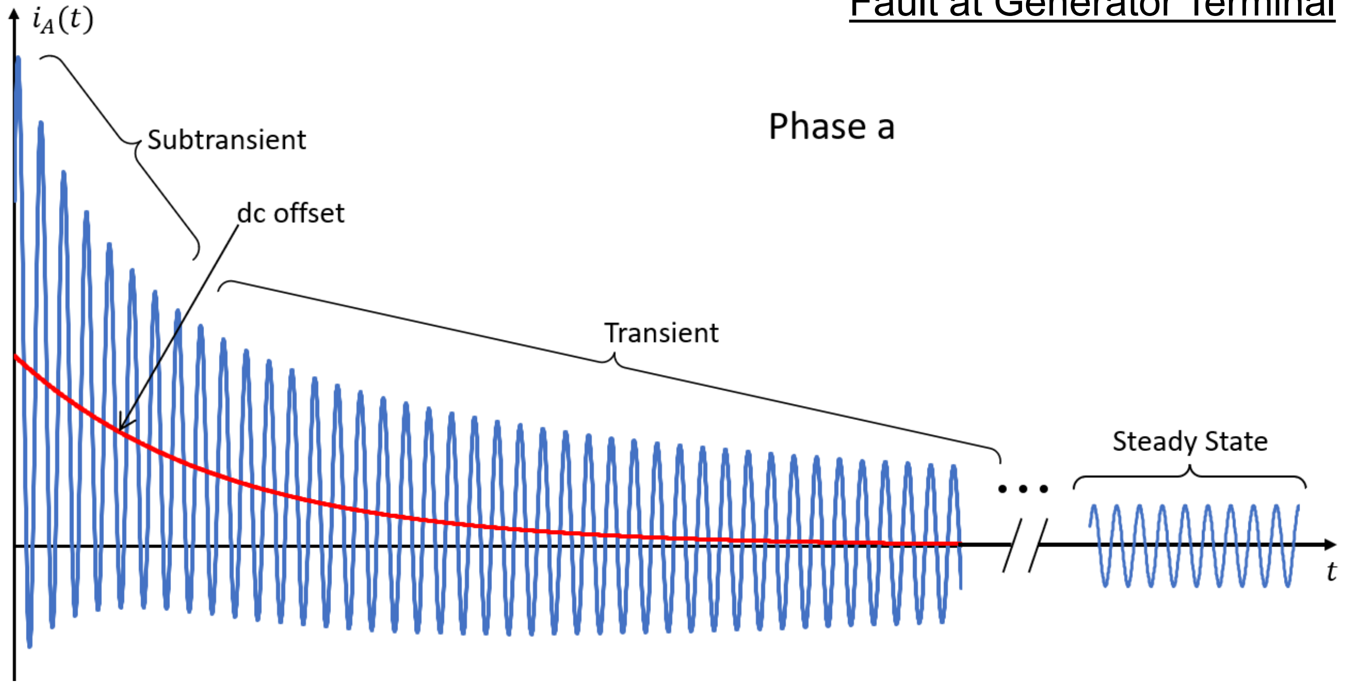


$$v_a(t) = \sqrt{2}V\,\sin(\omega t + \varphi)$$

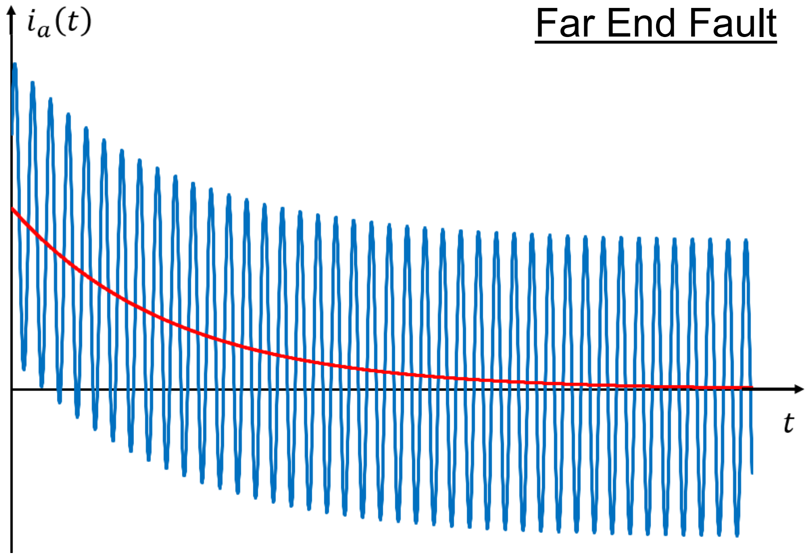
$$v_b(t) = \sqrt{2}V\,\sin(\omega t + \varphi - 120°)$$

$$v_c(t) = \sqrt{2}V\,\sin(\omega t + \varphi + 120°)$$

Fault at Generator Terminal

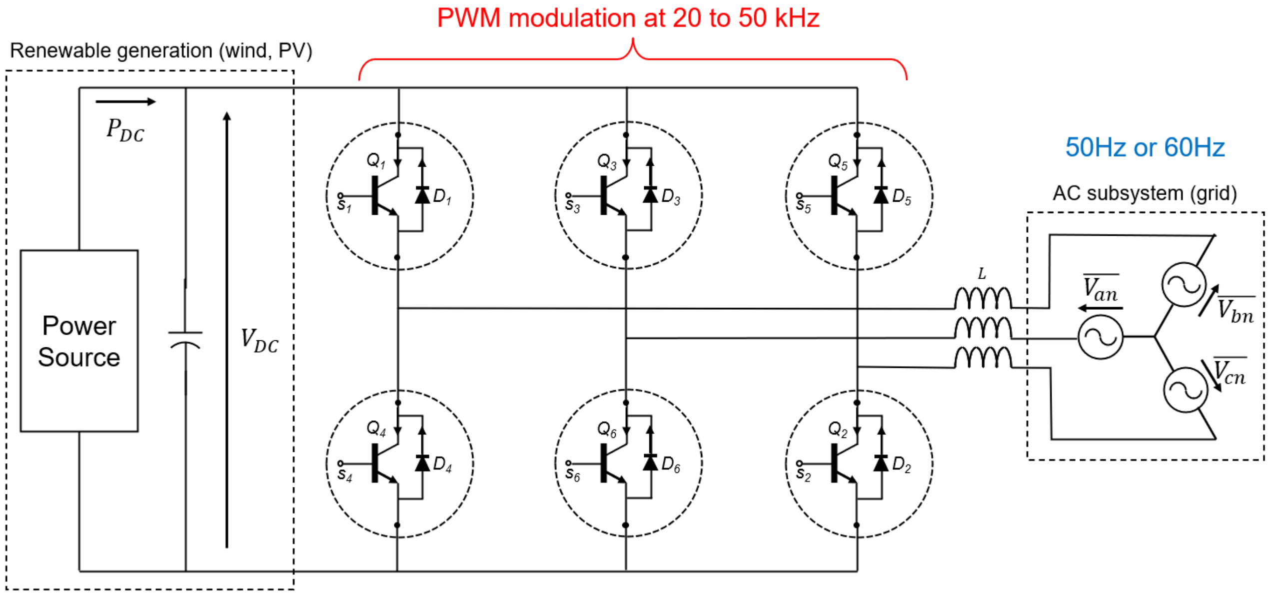


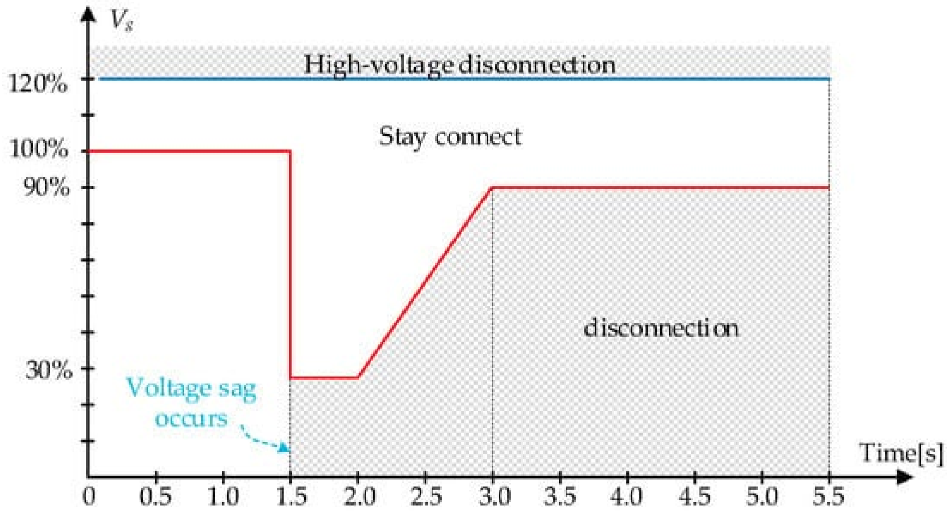
Far End Fault



Fault and IBR Plant

- Inverter shutdown or low voltage output may be resulted to limit possible fault current due to UV detection.
- With high switching and sampling frequency, inverter can be controlled fast to limit fault current to avoid damaging the IGBT / MOSFET.
- Inverter output is limited by control.
- It creates difficulties for fault detection especially for the one with OCEF.
- Low Voltage Ride-Through (LVRT) may be triggered to ensure the IBR plant connected in the grid to avoid large generation-load imbalance.





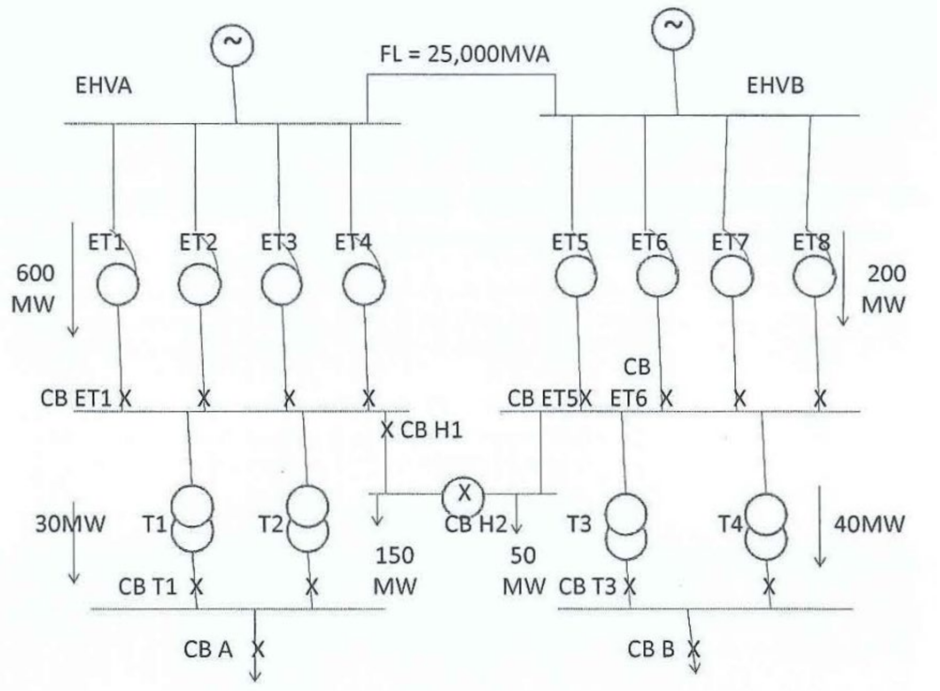
Exercise 9

1. A system has 3 generators G1, G2 and G3 connected respectively to bus A, B and C. Bus A and B are connected by a reactor circuit R1 , while Bus B and C are connected by another reactor circuit R2. The ratings are given below:

G1: 10MVA, 11kV, 10% reactance	R1: 10MVA, 11kV, 5% reactance
G2: 20MVA, 11kV, 15% reactance	R2: 8MVA, 11kV, 4% reactance
G3: 20MVA, 11kV, 15% reactance	

Estimate the fault level at Bus A.

2. In the network shown below, if the fault level at substation EHVA and EHVB is 25,000MVA, total power flow on ET1 to ET4 is 600MW and that on ET5 to ET8 is 200MW. If the rating of each ET transformer is 240MVA, 400/132kV and a reactance of 9%. Determine the subsequent fault level at CB H2 after this Circuit Breaker is closed.

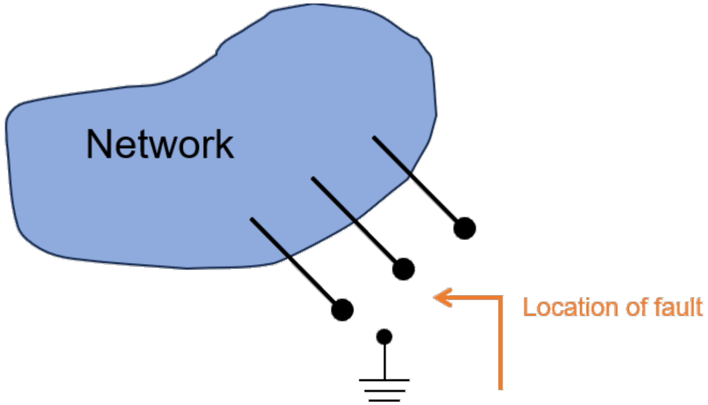


Fault Current Calculation and Sequence Network

- Pre-fault Condition: Currents stay in the intact network
- Fault Condition: Currents flow from the network into the ground or into another phase of the network
- To calculate these currents, a model of the network as seen from the fault is needed.

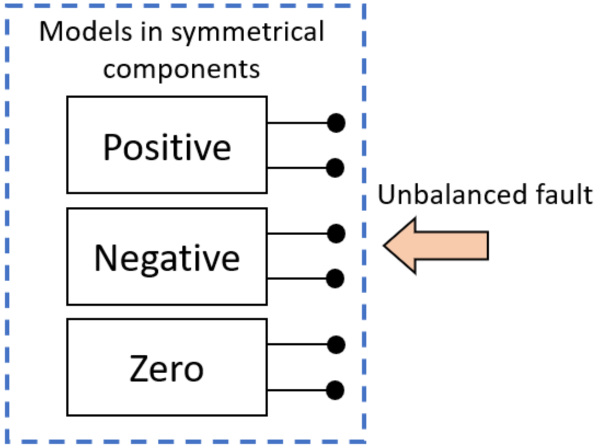
<u>For Unbalance Fault</u>

- Unbalanced faults break the three-phase symmetry of the system
- Unbalanced faults can only be analyzed using a three-phase model
- For a n-bus system, the model would be 3n x 3n with cross coupling elements.



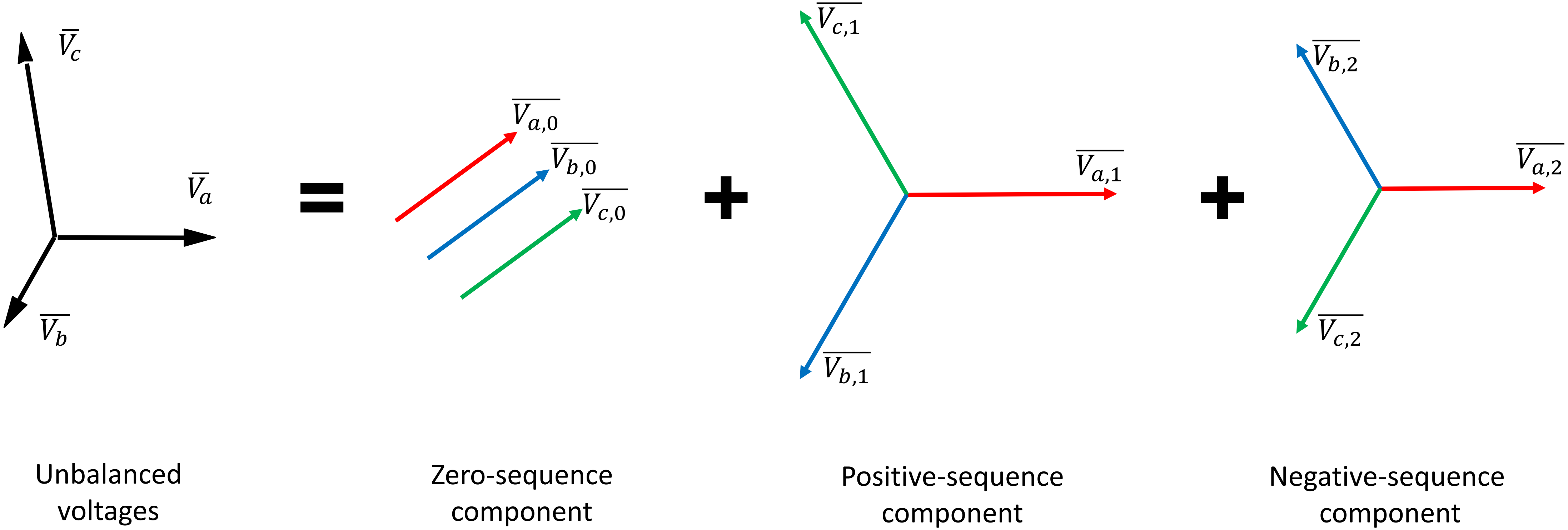
<u>Sequence Network</u>

- Transform variables from three-phase to symmetrical components
- Decouples three-phase network model into three decoupled models
- Each model represents a symmetrical component
- Using these models simplifies analysis of unbalanced faults



Symmetrical Components

- Any unbalanced 3φ voltage or current can be decomposed into the sum of three balanced three-phase voltages or currents called sequence components:



Symmetrical Components

- Phase Variables can be transformed from symmetrical components (one-to-one mapping)

$$\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix} \qquad \boxed{\begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix}} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \boxed{\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix}}$$

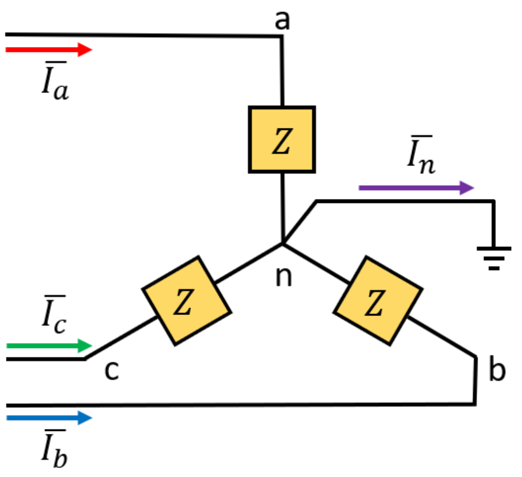
$$\boldsymbol{V_P} = \boldsymbol{A V_S} \qquad \boldsymbol{V_S} = \boldsymbol{A^{-1} V_P}$$

- Similarly, phase current can be transformed to sequence current with the transformation matrix A

$$\begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix} \qquad \boxed{\begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix}} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \boxed{\begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix}}$$

$$\boldsymbol{I_P} = \boldsymbol{A I_S} \qquad \boldsymbol{I_S} = \boldsymbol{A^{-1} I_P}$$

- For neutral current, $\overline{I_n} = \overline{I_a} + \overline{I_b} + \overline{I_c}$.
  From the matrix equation, it is shown that $\overline{I_n} = 3\overline{I_0} = \overline{I_a} + \overline{I_b} + \overline{I_c}$.
- Hence, zero-sequence current can be measured with neutral CT, or calculated with phase currents.



Sequence Networks Representation of a Load

- Unbalance voltage and current in network makes the analysis complex, as the three phase are coupled, i.e. affecting each others. Symmetrical components can replace the 3φ network by decoupled sequence network.
- KVL around loop a-n-g: $\bar{V_a} = \bar{I_a} Z_Y + \bar{I_n} Z_n$

$$\bar{I_n} = \bar{I_a} + \bar{I_b} + \bar{I_c}$$

$$\bar{V_a} = \bar{I_a}(Z_Y + Z_n) + \bar{I_b} Z_n + \bar{I_c} Z_n$$

- Similarly for loop b-n-g and c-n-g: $\bar{V_b} = \bar{I_a} Z_n + \bar{I_b}(Z_Y + Z_n) + \bar{I_c} Z_n$

$$\bar{V_c} = \bar{I_a} Z_n + \bar{I_b} Z_n + \bar{I_c}(Z_Y + Z_n)$$

- Phase Representation for balance load in unbalance input:

$$\begin{pmatrix} \bar{V_a} \\ \bar{V_b} \\ \bar{V_c} \end{pmatrix} = \begin{pmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{pmatrix} \begin{pmatrix} \bar{I_a} \\ \bar{I_b} \\ \bar{I_c} \end{pmatrix}$$

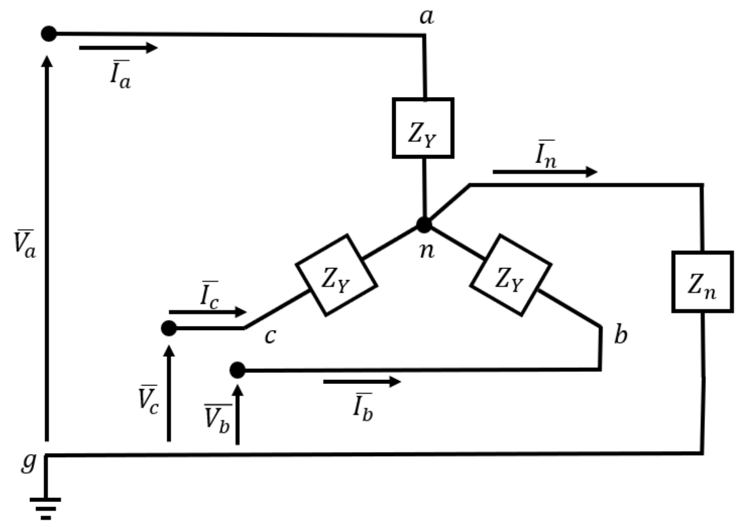
- Given that

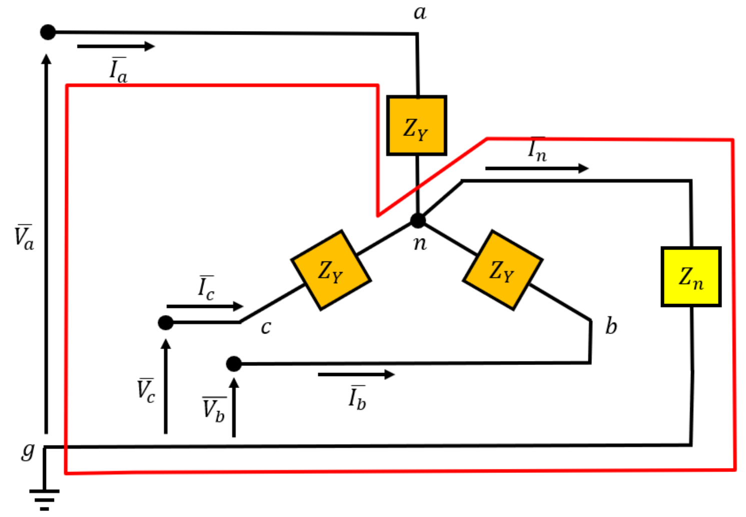
$$\boldsymbol{V_P} = \boldsymbol{Z_P I_P} \quad \text{(Phase Representation)}$$

$$\boxed{\boldsymbol{A^{-1} V_P}} = \boldsymbol{A^{-1} Z_P} \boxed{\boldsymbol{I_P}} \quad \text{(Multiply A}^{-1}\text{ at both sides)}$$

$$\boxed{\boldsymbol{V_S}} = \boldsymbol{A^{-1} Z_P} \boxed{\boldsymbol{A\, I_S}}$$

$$\boldsymbol{V_S} = \boldsymbol{Z_S\, I_S}$$





Sequence Networks Representation of a Load

- Sequence Impedance transformed from phase impedance

$$\boldsymbol{Z_S} = \boldsymbol{A}^{-1}\boldsymbol{Z_P}\boldsymbol{A} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

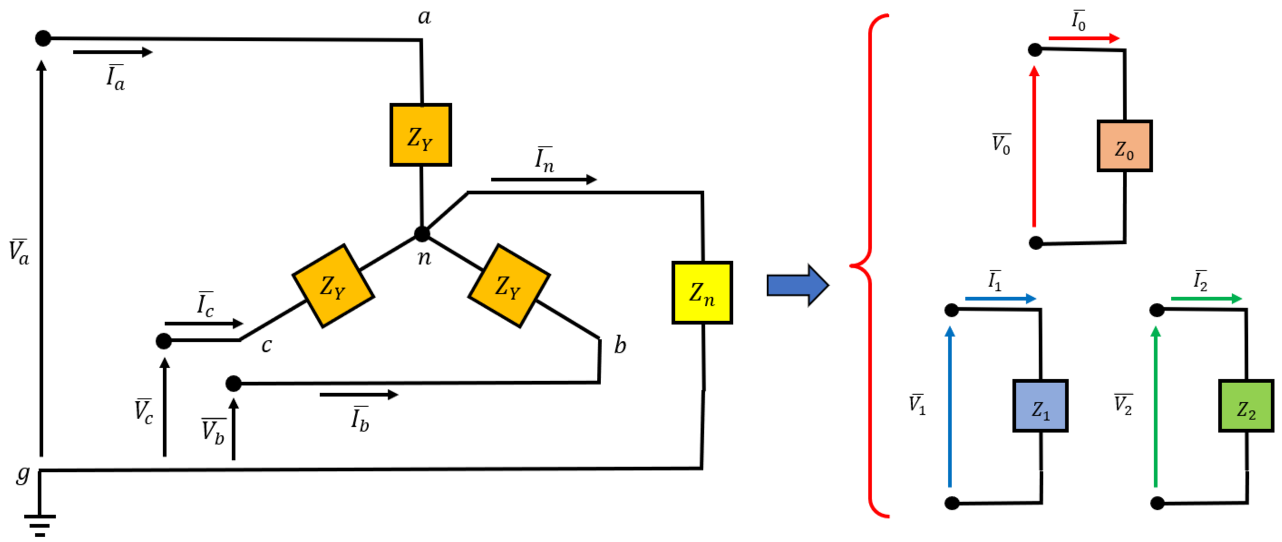
$$\boxed{\boldsymbol{Z_S} = \begin{pmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{pmatrix}}$$

Decoupled without Cross Terms

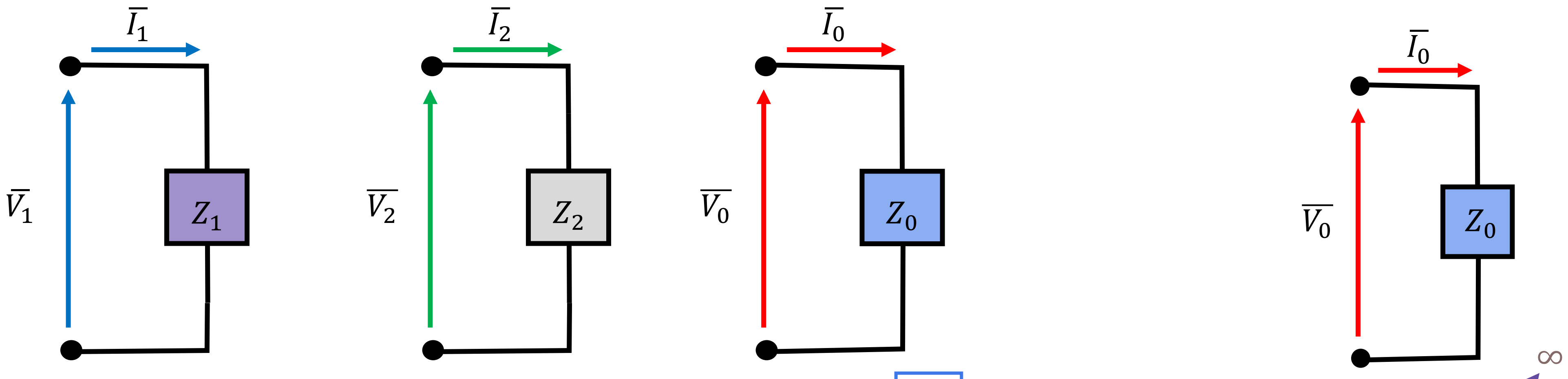
- Sequence Representation for balance load in unbalance network.

$$\boldsymbol{V_S} = \boldsymbol{Z_S}\,\boldsymbol{I_S} \quad \begin{cases} \overline{V_0} = (Z_Y + 3Z_n)\overline{I_0} = Z_0\overline{I_0} \\ \overline{V_1} = Z_Y\,\overline{I_1} = Z_1\overline{I_1} \\ \overline{V_2} = Z_Y\,\overline{I_2} = Z_2\overline{I_2} \end{cases}$$

- Converting to sequence representation <u>decouples</u> the equations such that I_1 is only affected by V_1 ; I_2 is only affected by V_2 ; and I_0 is only affected by V_0.



Sequence Networks Representation of a Load



$Z_1 = Z_Y$ $\qquad$ $Z_2 = Z_Y$ $\qquad$ $Z_0 = Z_Y + \boxed{3Z_n}$ $\qquad$ $Z_0 = Z_Y + \cancel{3Z_n}$

$\underbrace{\qquad\qquad\qquad}$ Z_n does not appear in Z_1 or Z_2

Because $\overline{I_n} = 3\overline{I_0}$

Z_0 is infinity (i.e. zero seq network opened) with ungrounded Y connection of transformer.

- Zero-Sequence network heavily dependent on transformer configuration (grounded or ungrounded, wye or delta, with or without zigzag).
- Balance Voltage will be reflected at positive sequence network = per phase representation. (~ 3 phase fault)
- If the loads are <u>unbalanced</u>, the sequence network is <u>coupled</u>.

Example 10.6: Unbalanced Voltage with Balanced Load (wye)

Each branch of a Y-connected load has an impedance of $9 + j3\Omega$. The neutral point of this load is connected to ground through an impedance of $j2\Omega$. The following set of unbalanced line-to-neutral voltages is applied to this load:

$$\overline{V_a} = 100\angle 0° \, V \quad \overline{V_b} = 90\angle 240° \, V \quad \overline{V_c} = 110\angle 120° \, V$$

Calculate the current in each phase and in the neutral connection.

Solution

$$Z_0 = Z_Y + 3Z_n = 9 + j3 + 3 \times j2 = 9 + j9\Omega \qquad Z_1 = Z_Y = 9 + j3\Omega \qquad Z_2 = Z_Y = 9 + j3\Omega$$

Sequence Voltage can be found with transformation matrix A.

$$\overline{V_0} = 5.77\angle 90° \, V \quad \overline{V_1} = 100\angle 0° \, V \quad \overline{V_2} = 5.77\angle -90° \, V$$

Sequence Current is evaluated with decoupled impedance in sequence network.

$$\overline{I_0} = \frac{\overline{V_0}}{Z_0} = \frac{5.77\angle 90°}{9 + j9} = 0.453\angle 45° \text{ A} \qquad \overline{I_1} = \frac{\overline{V_1}}{Z_1} = \frac{100\angle 0°}{9 + j3} = 10.54\angle -18.43° \, A$$

$$\overline{I_2} = \frac{\overline{V_2}}{Z_3} = \frac{5.77\angle -90°}{9 + j3} = 0.608\angle -108.43° \text{ A}$$

Sequence Current can be found with transformation matrix A.

$$\overline{I_a} = 10.75\angle -19.51° \, A \quad \overline{I_b} = 9.57\angle -136.77° \, A \quad \overline{I_c} = 11.32\angle +101.20° \, A$$

$$\overline{I_n} = 3\overline{I_0} = 1.359\angle 45° \, A$$

Example 10.7: Unbalanced Voltage with Balanced Load (delta)

Repeat the calculations of Example 10.5 assuming that the branch impedances are connected in delta. Replace the delta-connected load by its Y-connected equivalent.

Solution

With delta-wye transformation, and delta without grounding path,

$$Z_Y = \frac{Z_\Delta}{3} = \frac{9 + j3}{3} = 3 + j1\Omega \text{ and } Z_n = \infty$$

$$Z_0 = \infty \qquad Z_1 = 3 + j1\Omega \qquad Z_2 = 3 + j1\Omega$$

Sequence Current to be evaluated with decoupled sequence impedance

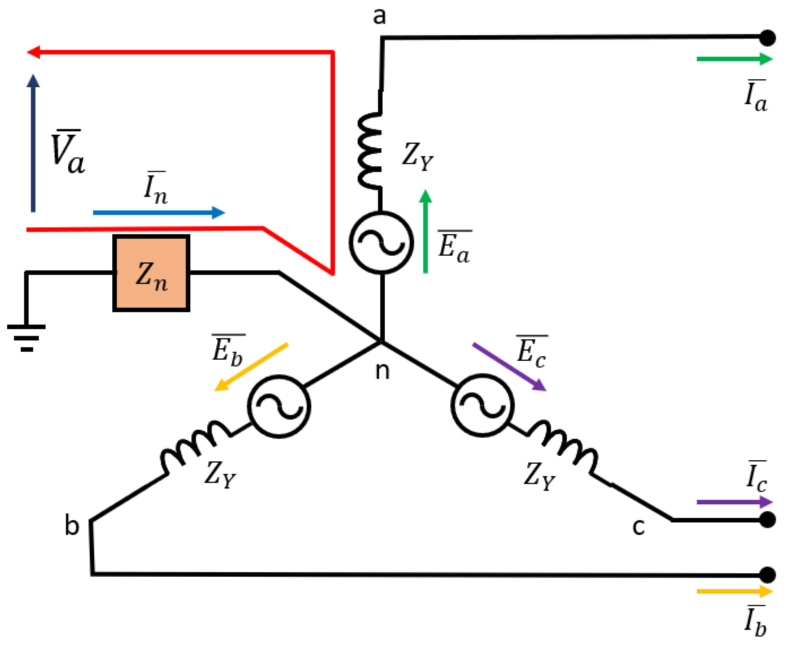
$$\bar{I}_0 = \frac{\bar{V}_0}{Z_0} = 0 \qquad \bar{I}_1 = \frac{\bar{V}_1}{Z_1} = \frac{100\angle 0°}{3 + j1} = 31.62\angle - 18.43°\ A$$

$$\bar{I}_2 = \frac{\bar{V}_2}{Z_2} = \frac{5.77\angle - 90°}{3 + j1} = 1.824\angle - 108.43°\ A$$

Phase Current can be found with transformation matrix A.

$$\bar{I}_a = 31.67\angle - 21.73°\ A \qquad \bar{I}_b = 30.05\angle - 136.69°\ A \qquad \bar{I}_c = 33.21\angle + 103.14°\ A$$

Sequence Network Representation of a Generator



- KVL applied to the path a-n-g:

$$\bar{V}_a = \bar{E}_a - \bar{I}_a Z_Y - \bar{I}_n Z_n$$

$$\bar{V}_a = \bar{E}_a - \bar{I}_a Z_Y - (\bar{I}_a + \bar{I}_b + \bar{I}_c) Z_n$$

- Similarly for phase b and c:

$$\bar{V}_a = \bar{E}_a - \bar{I}_a (Z_Y + Z_n) - \bar{I}_b Z_n - \bar{I}_c Z_n$$

$$\bar{V}_b = \bar{E}_b - \bar{I}_b (Z_Y + Z_n) - \bar{I}_a Z_n - \bar{I}_c Z_n$$

$$\bar{V}_c = \bar{E}_c - \bar{I}_c (Z_Y + Z_n) - \bar{I}_a Z_n - \bar{I}_c Z_n$$

- Phase Representation in Matrix Form:

$$\begin{pmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{pmatrix} = \begin{pmatrix} \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \end{pmatrix} - \begin{pmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{pmatrix} \begin{pmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{pmatrix}$$

$$\mathbf{A^{-1}E_P} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \end{pmatrix} = \begin{pmatrix} 0 \\ \bar{E}_1 \\ 0 \end{pmatrix} = \mathbf{E_S}$$

- Transform the quantities into sequence form:

$$\mathbf{V_P} = \mathbf{E_P} - \mathbf{Z_P I_P}$$

$$\mathbf{A^{-1}V_P} = \mathbf{A^{-1}E_P} - \mathbf{A^{-1}Z_P I_P}$$

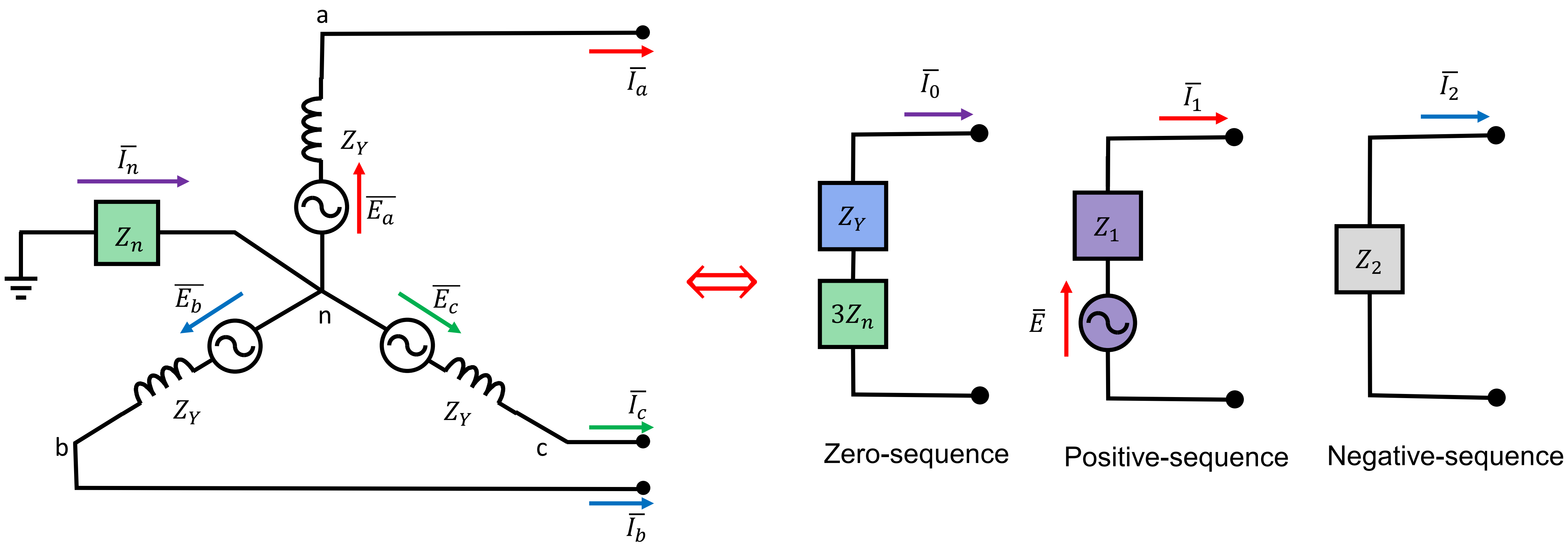
$$\mathbf{V_S} = \mathbf{A^{-1}E_P} - \mathbf{A^{-1}Z_P A I_S}$$

$$\boxed{\mathbf{V_S} = \mathbf{E_S} - \mathbf{Z_S I_S}}$$

$$\mathbf{A^{-1}Z_P A} = \mathbf{Z_S} = \begin{pmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{pmatrix}$$

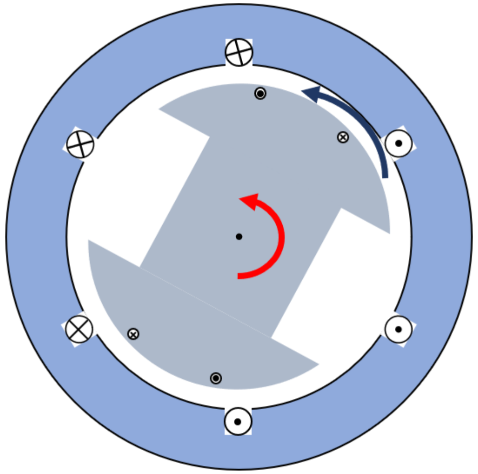
$$\boxed{\begin{pmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \bar{E}_1 \\ 0 \end{pmatrix} - \begin{pmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{pmatrix} \begin{pmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{pmatrix}}$$

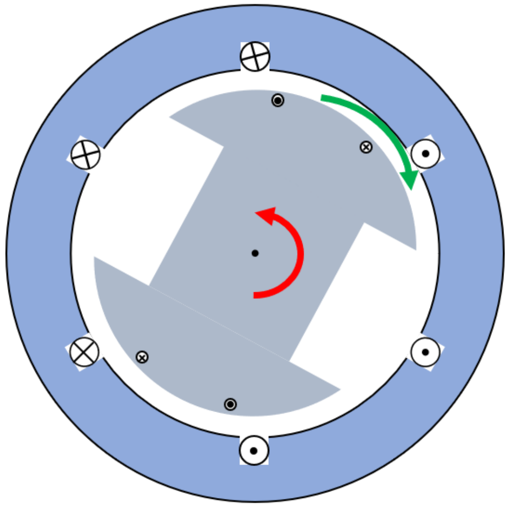
Sequence Network Representation of a Generator

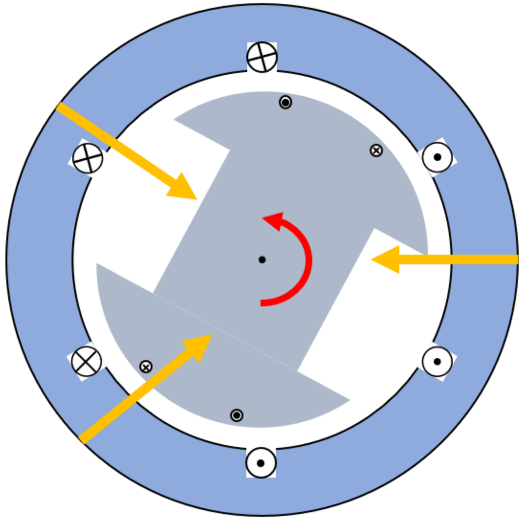


- From the previous generator analysis, depending on the time frame considered, Z_1 is the synchronous (X_d), transient (X_d'), or sub-transient impedance (X_d").
- $Z_0 = Z_{g0} + 3Z_n$ → zero sequence current can flow through a generator only if it is Y-connected and the neutral is connected to ground

Sequence Network Representation of a Generator







Effect of Positive Sequence Currents

- Create a flux wave that rotates in the <u>same</u> direction as the rotor
- Flux wave is in synchronism with the rotor
- Path of positive sequence magnetic flux is mostly through iron
- Low reluctance, high impedance Z_1

Effect of Negative Sequence Currents

- Create a flux wave that rotates in the <u>opposite</u> direction as the rotor
- Flux cannot penetrate the rotor
- Path of negative sequence magnetic path is mostly through the air
- High reluctance, low impedance
- $Z_2 < Z_1$

Effect of Zero Sequence Currents

- Flux created by all three phases are <u>in phase</u>
- Resulting flux cannot penetrate the rotor
- Zero sequence magnetic flux path is mostly through the air
- High reluctance, low impedance
- $Z_{g0} < Z_2 < Z_1$

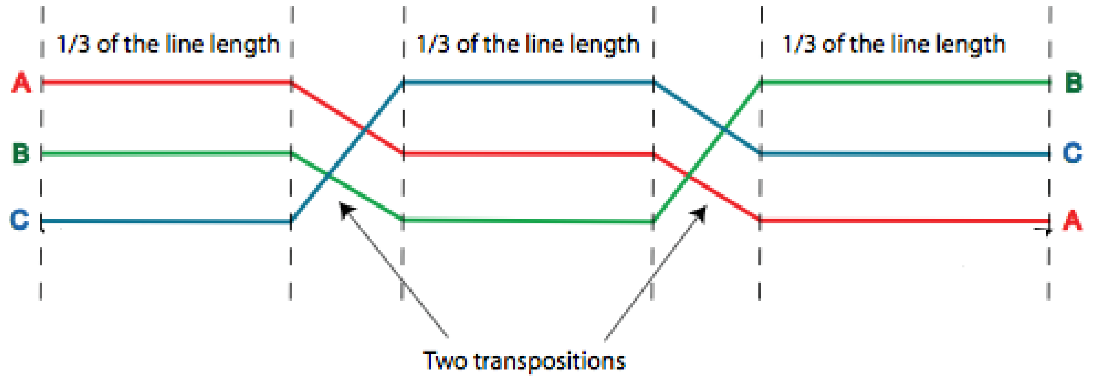
Sequence Impedance in Generator in theory should not be the same.

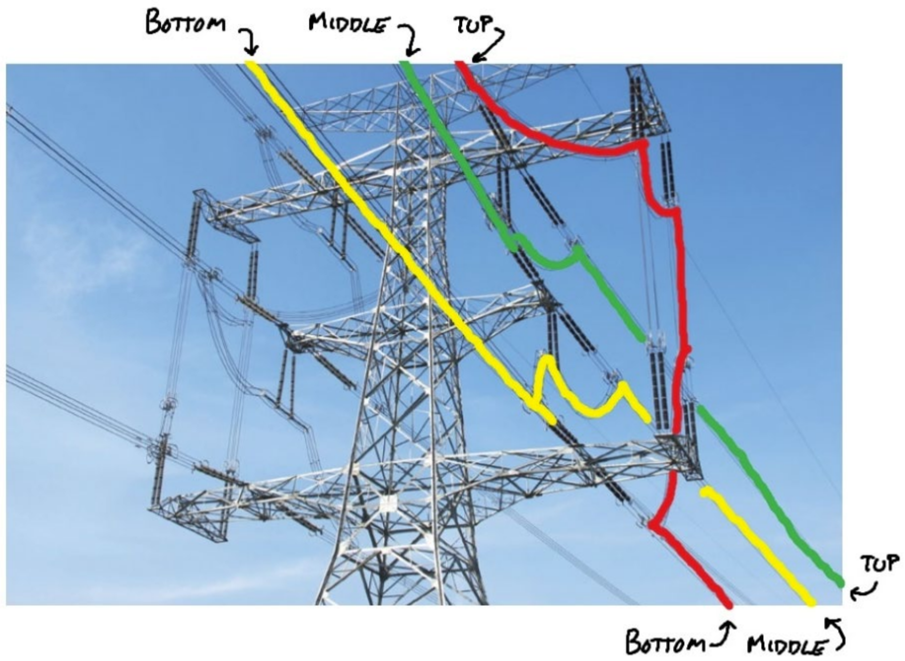
Sequence Network Representations of Lines and Cables

- Reactance of lines and cables reflects their magnetic field
- Depends on the geometry of conductors and their spacing (recall transmission line modelling in Power Transmission Course)
- If the phases are arranged symmetrically → $Z_1 = Z_2$
- Unequal distance between the phases

  → Unequal positive and negative sequence magnetic fields

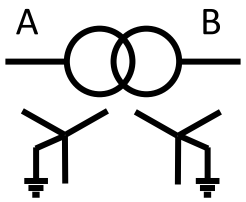
  → $Z_1 \neq Z_2$
- Need regular transposition to restore balance
- Problem – How does transposition imply possible faults type?

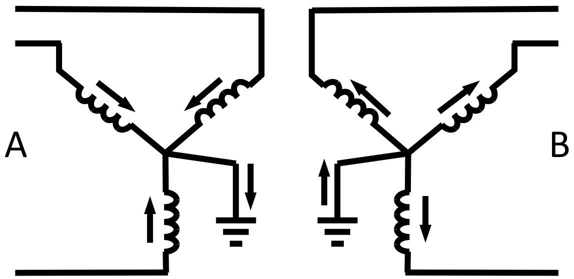


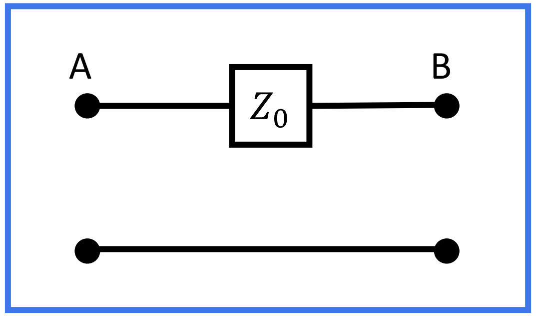


Sequence Network Representations of Transformers

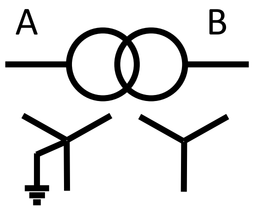
- Ideal transformer implies $I_P / I_S = N_S / N_P$ with flux balancing. Primary and secondary current must be in phase, and secondary current must balance primary current.
- Depending on the connection of the primary and secondary windings, this may or may not be possible for the zero-sequence.
- For Grounded Y – Grounded y connection, primary zero-sequence current can be balanced by a secondary zero sequence current. There is thus a path for zero-sequence currents through this transformer.

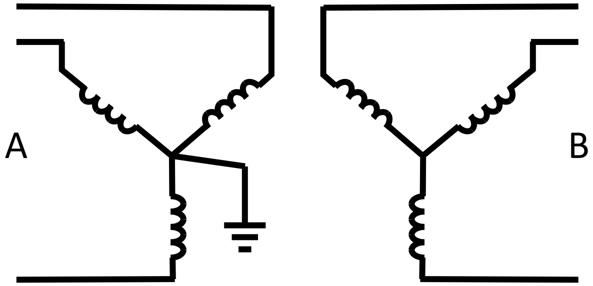


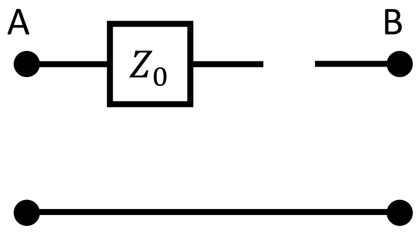




- For Grounded Y – Ungrounded y connection, primary zero-sequence current cannot be balanced by a secondary zero sequence current. Therefore, no zero-sequence current can flow in or out of this transformer.

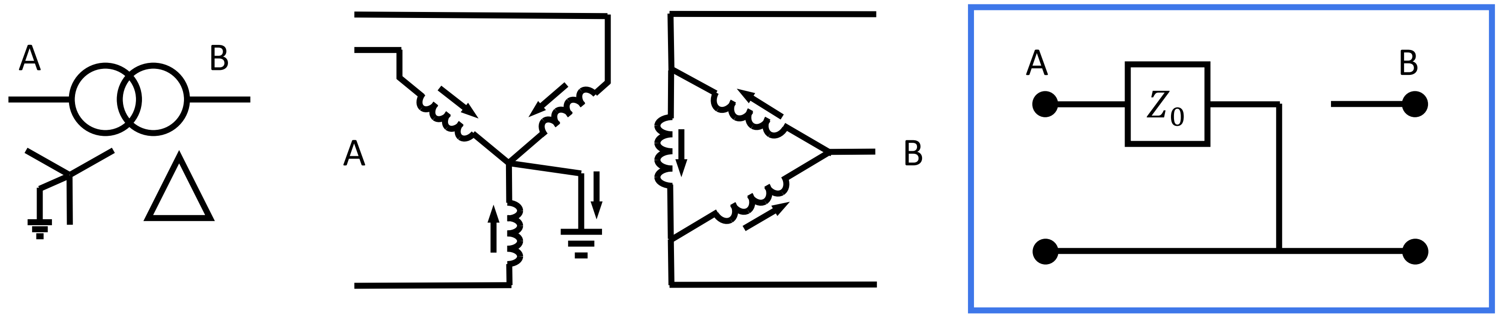




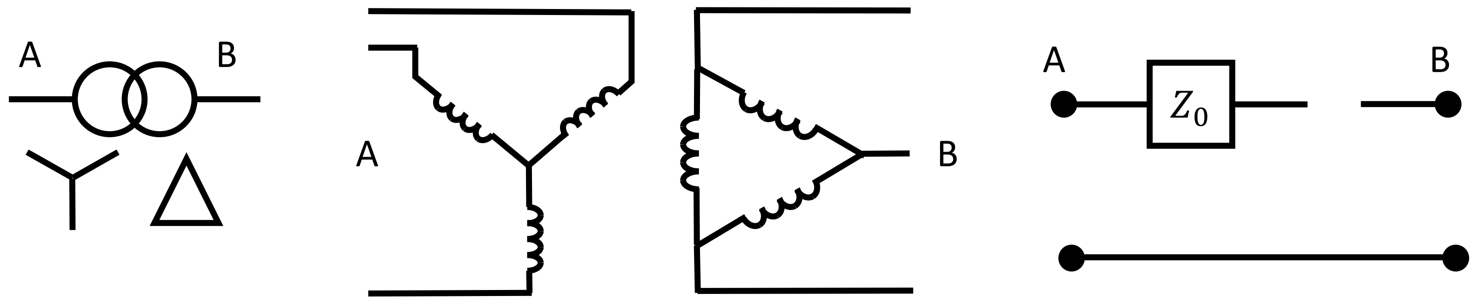


Sequence Network Representations of Transformers

- For Grounded Y – delta connection, primary zero-sequence current can be balanced by a circulating zero sequence in the secondary. There is thus a path for zero sequence currents on the grounded-Y side, but not on the delta side.



- For Ungrounded Y – delta connection, there is no path for a zero-sequence current on the primary side. No zero-sequence current can therefore flow in or out of this transformer.



Sequence Impedances

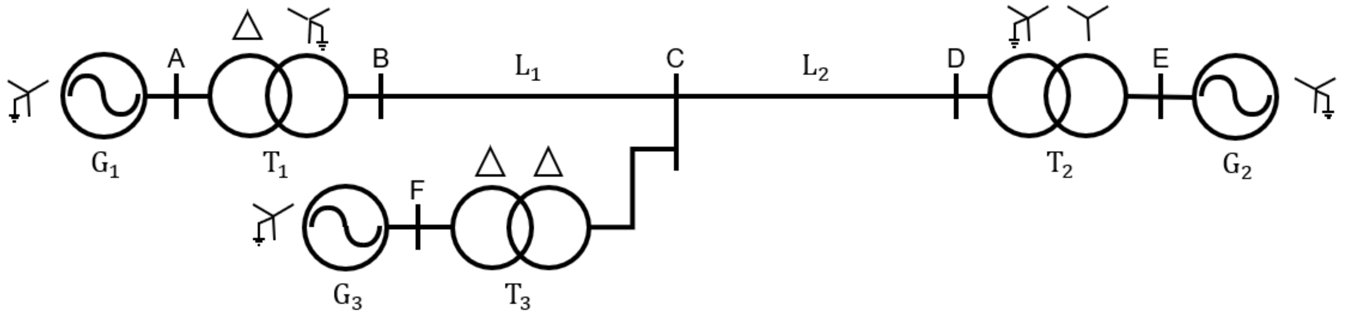
- Positive-sequence impedance Z_1
  - Same as impedance of single-phase representation of three-phase circuits

- Negative-sequence impedance Z_2
  - Equal to positive-sequence impedance for components that do not differentiate between the phase sequences
  - $Z_2 = Z_1$ for passive loads, transformers and transposed lines
  - $Z_2 \neq Z_1$ for synchronous and induction machines, and un-transposed lines

- Zero-sequence impedance Z_0
  - Mainly represents the impedance on earthing path
  - Usually different from Z_1 and Z_2
  - Because $\overline{I_n} = 3\overline{I_0}$, Z_0 is infinite if there is no connection between neutral and ground
  - Because primary and secondary currents must be balanced, the connection of a transformer windings affects its zero-sequence model → What is the effect of using delta-wye transformer in 11kV/380V?
  - Some transformer connections block zero sequence currents

Example 10.8: Determine Sequence Network

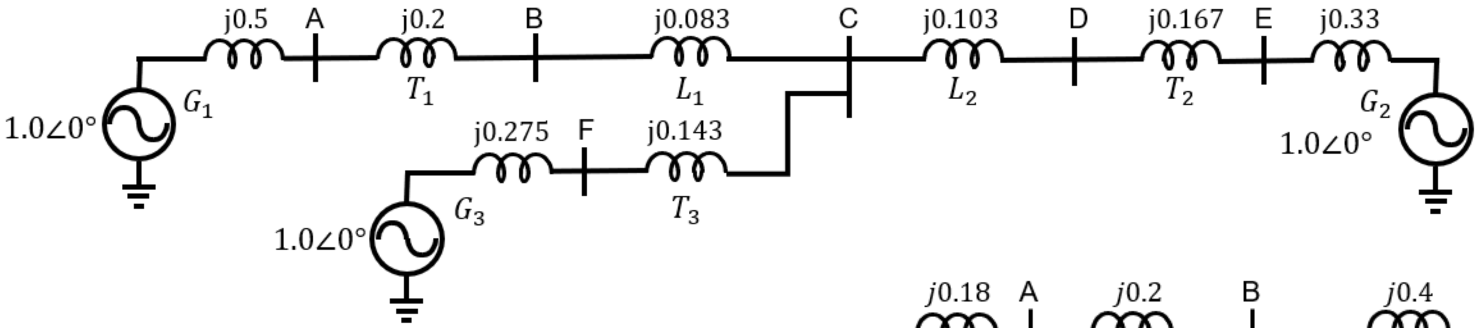
Determine the positive, negative and zero-sequence network for the following network.



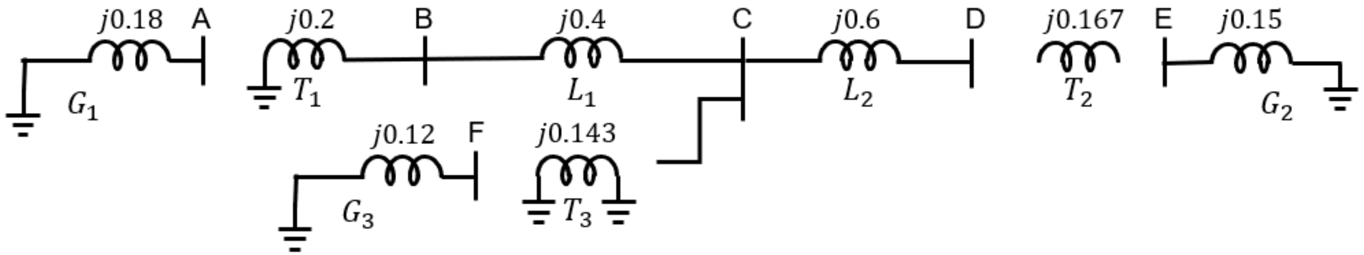
	X_1(p.u.)	X_2(p.u.)	X_0 (p.u.)
G_1	0.5	0.5	0.18
G_2	0.33	0.33	0.15
G_3	0.275	0.275	0.12
T_1	0.2	0.2	0.2
T_2	0.167	0.167	0.167
T_3	0.143	0.143	0.143
L_1	0.083	0.083	0.4
L_2	0.103	0.103	0.6

Solution

<u>Positive and Negative Sequence Network </u>(with or without voltage source)



<u>Zero Sequence Network</u>

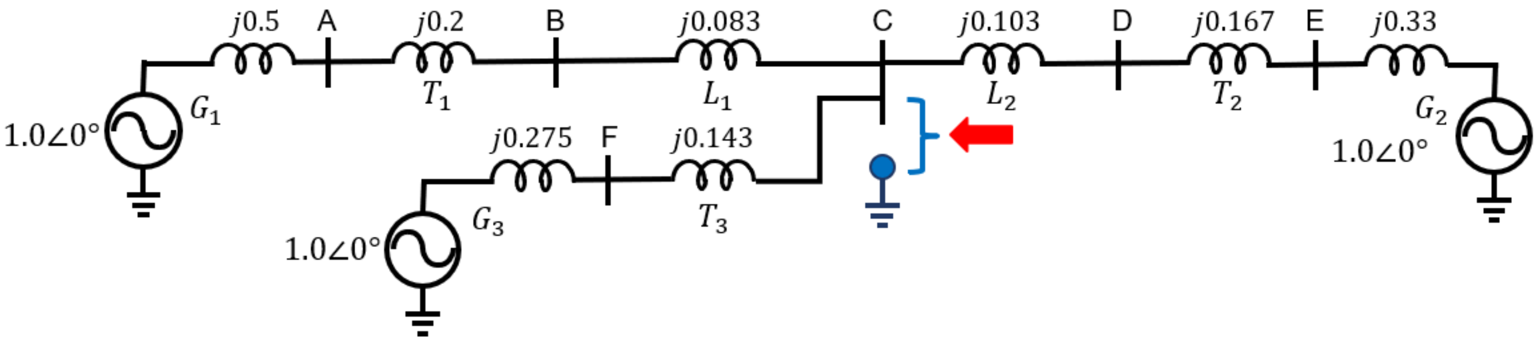


Example 10.9: Determine Sequence Impedance

Determine the sequence impedance at **node C** in Example 10.8 by using Thevenin network.

Solution

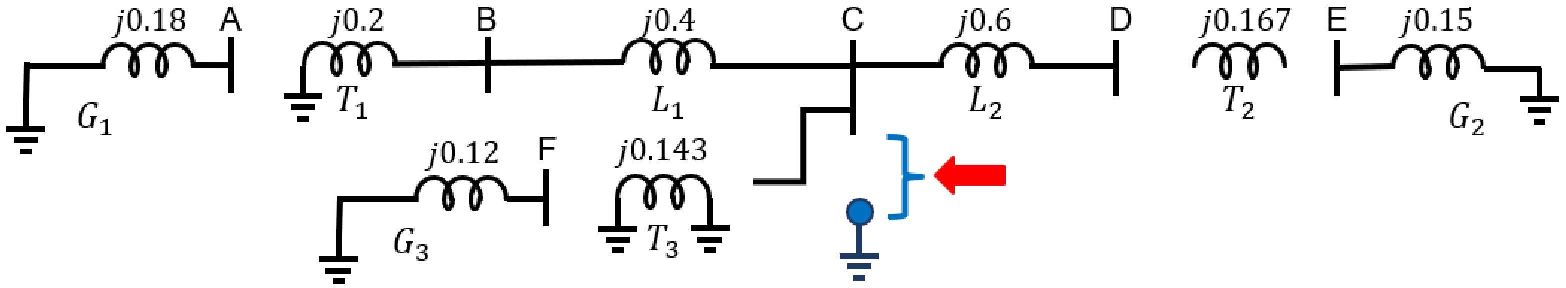
Positive Sequence Impedance:



$$\overline{V_C^{TH,1}} = 1.0\ \mathrm{p.u.}$$

$$Z_C^{TH,1} = j(0.33 + 0.167 + 0.103) \parallel j(0.5 + 0.2 + 0.083) \parallel j(0.275 + 0.143) = j0.187\mathrm{p.u.}$$

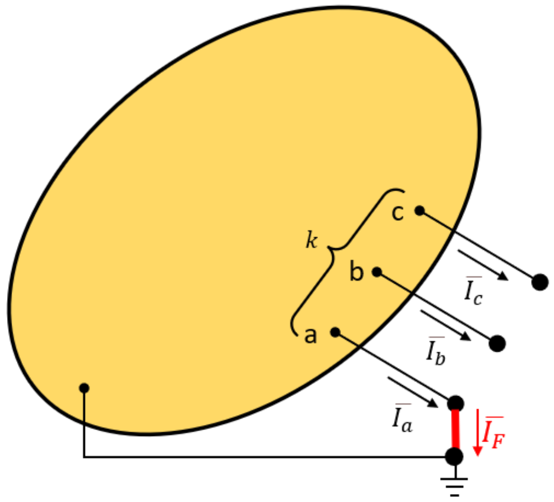
Zero Sequence Impedance:



$$\overline{V_C^{TH,0}} = 0$$

$$Z_C^{TH,0} = j(0.2 + 0.4) = j0.6\ p.u.$$

Fault Analysis with Symmetrical Component (SLG Fault)

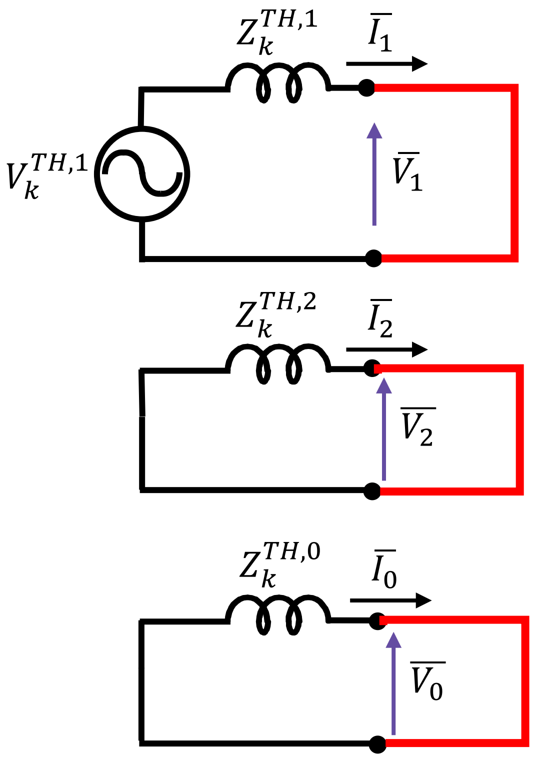


Consider a balance 3φ fault at node k.

$$\overline{V_a} = \overline{V_b} = \overline{V_c} = 0$$

$$\begin{pmatrix} \overline{V_0} \\ \overline{V_1} \\ \overline{V_2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} \overline{V_a} \\ \overline{V_b} \\ \overline{V_c} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}\begin{pmatrix} 0 \\ \overline{I_1} \\ 0 \end{pmatrix} = \begin{pmatrix} \overline{I_1} \\ a^2\overline{I_1} \\ a\overline{I_1} \end{pmatrix}$$



$$\overline{I_1} = \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1}}$$

$$\overline{I_2} = 0$$

$$\overline{I_0} = 0$$

Balance Fault creates balanced current, represented with the only current flow at positive sequence network.

For a single-line-to-ground (SLG) fault,

- Pre-fault: $\overline{I_a} = \overline{I_b} = \overline{I_c} = 0$
- Without loss of generality, assume that the fault is on phase a.
- $\overline{I_a} = \overline{I_F} \neq 0,\ \ \overline{I_b} = \overline{I_c} = 0$

Fault Analysis with Symmetrical Component

For a single-line-to-ground (SLG) fault, $\overline{I_a} = \overline{I_F} \neq 0 \quad \overline{I_b} = \overline{I_c} = 0$

$$\begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} \overline{I_F} \\ 0 \\ 0 \end{pmatrix} \longrightarrow \overline{I_0} = \overline{I_1} = \overline{I_2} = \frac{1}{3}\overline{I_F}$$

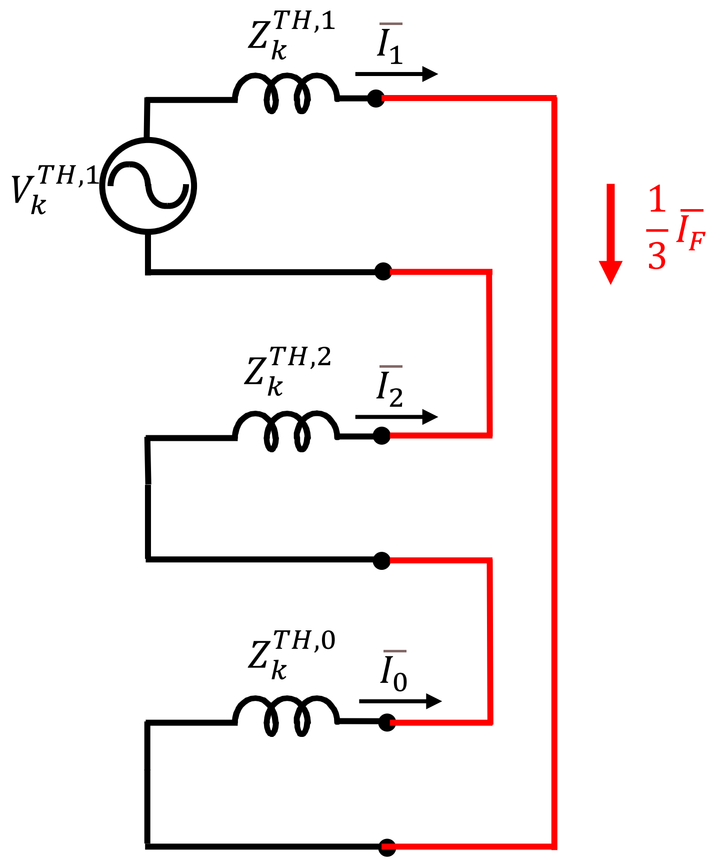
The equality in sequence current implies the three networks are connected in series.

$$\overline{I_0} = \overline{I_1} = \overline{I_2} = \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2} + Z_k^{TH,0}}$$

$$\boxed{\overline{I_a} = \overline{I_F} = \frac{3\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2} + Z_k^{TH,0}}}$$

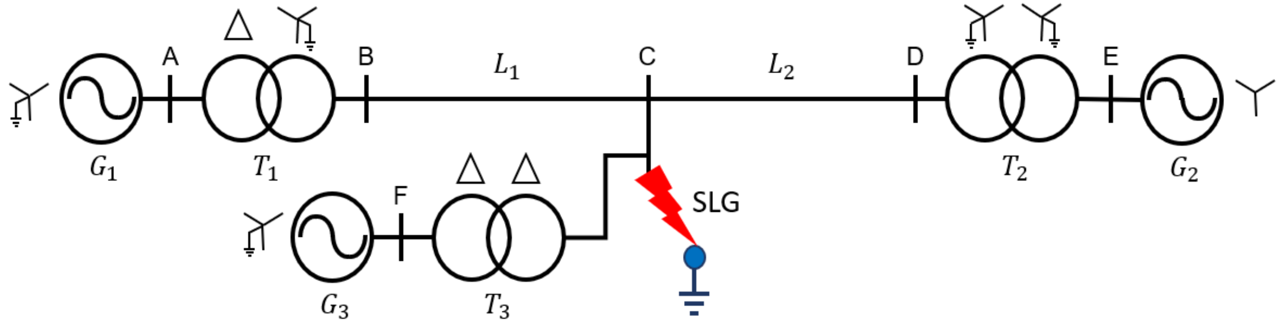
If $Z_k^{TH,1} = Z_k^{TH,2} = Z_k^{TH,0}$, $\quad \overline{I_a} = \frac{3\overline{V_k^{TH,1}}}{3Z_k^{TH,1}} = \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1}}$ (three phase fault current)

It implies that SLG fault current > LLL fault current when $Z_k^{TH,1} > Z_k^{TH,0}$.



Example 10.10: Single Line to Ground Fault at Phase C

Consider a SLG fault at node C.

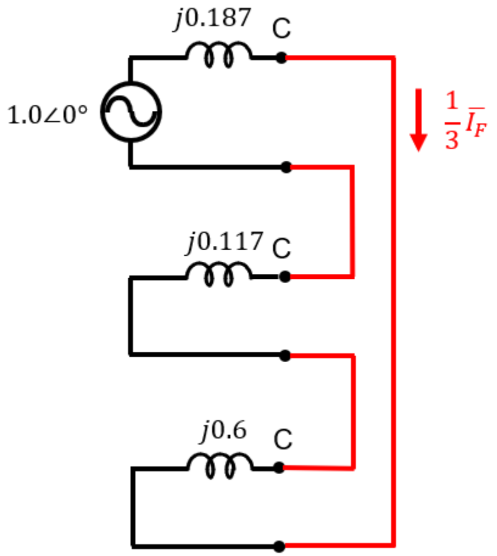


	X_1(p. u.)	X_2(p. u.)	X_0 (p. u.)
G_1	0.5	0.2	0.18
G_2	0.33	0.15	0.15
G_3	0.275	0.1	0.12
T_1	0.2	0.2	0.2
T_2	0.167	0.167	0.167
T_3	0.143	0.143	0.143
L_1	0.083	0.083	0.4
L_2	0.103	0.103	0.6

It is given that the positive seq., negative seq., and zero seq. Thevenin impedance are j0.6, j0.187 and j0.117 respectively.

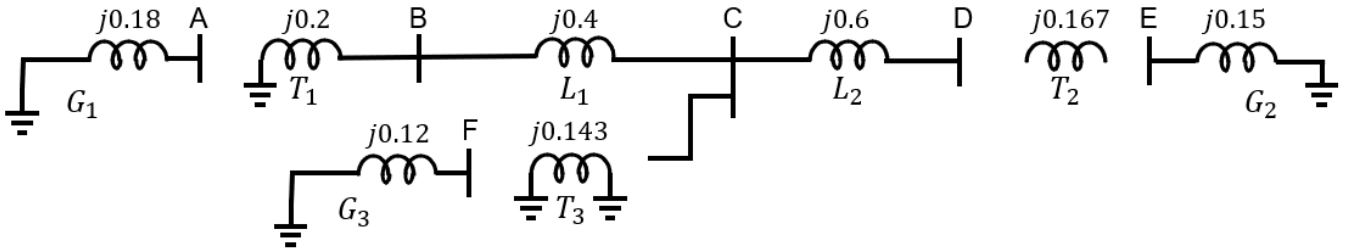
Solution

$$\bar{I_F} = \frac{3 \times 1.0\angle 0°}{j(0.6 + 0.187 + 0.117)} = 3.32\angle -90° p.u.$$

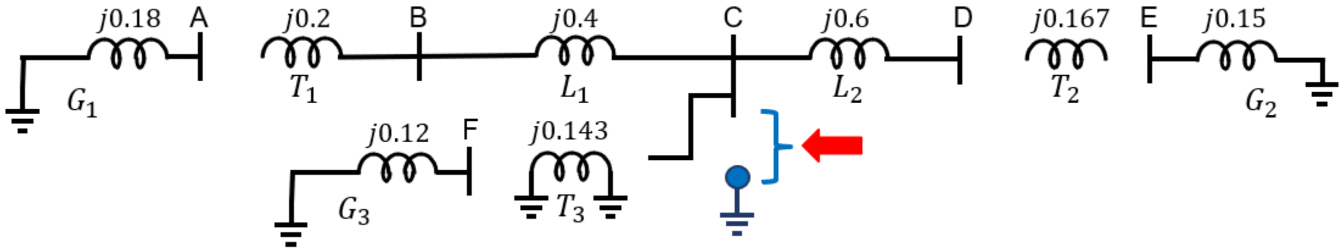


Importance of Grounding Connection

Zero sequence circuit with
T_1 delta-grounded Y:



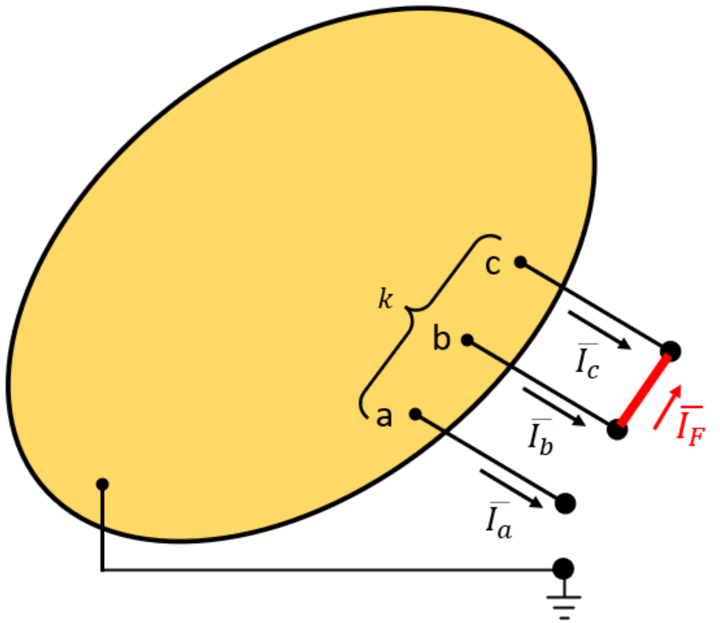
Zero sequence circuit with
T_1 delta-Y:



$$Z_C^{TH,0} = \infty \quad \Longrightarrow \bar{I}_F = \frac{3\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2} + Z_k^{TH,0}} = 0$$

- Is having a zero fault current a good thing or a bad thing?
  - For the protection system to operate, there must be a large enough fault current.
    (Note: there are other methods to detect faults in an ungrounded system.)
  - Undetected faults represent a safety hazard because a normal action can then have unexpected consequences
    (There is no monitoring on the zero-sequence network. It is possible that the transformer becomes ungrounded as the earthing bar of the transformer is stolen.)

Fault Analysis with Symmetrical Component (LL Fault)



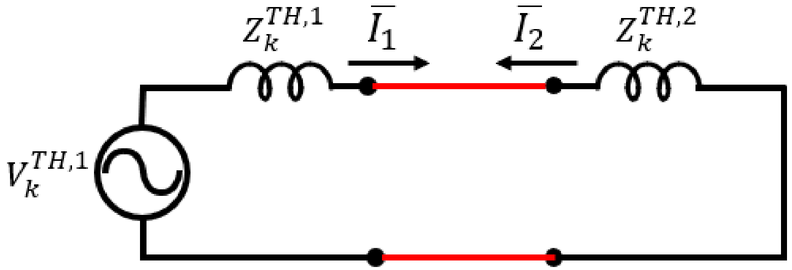
Consider a Line-to-Line Fault at node k:

- Pre-fault: $\overline{I_a} = \overline{I_b} = \overline{I_c} = 0$
- Without loss of generality, assume that the fault is between phases b and c
- Fault Condition: $\overline{I_a} = 0, \quad \overline{I_b} = -\overline{I_c} = \overline{I_F}$

$$\begin{pmatrix} \overline{I_0} \\ \overline{I_1} \\ \overline{I_2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} 0 \\ \overline{I_F} \\ -\overline{I_F} \end{pmatrix}$$

$$\begin{cases} \overline{I_0} = \frac{1}{3}(\overline{I_F} - \overline{I_F}) = 0 \\ \overline{I_1} = \frac{1}{3}(a - a^2)\overline{I_F} \\ \overline{I_2} = \frac{1}{3}(a^2 - a)\overline{I_F} = -\overline{I_1} \end{cases} \longrightarrow$$

$\overline{I_2} = -\overline{I_1}$ implies the positive and negative sequence network is connected <u>in parallel</u>, without connected to the zero-sequence network

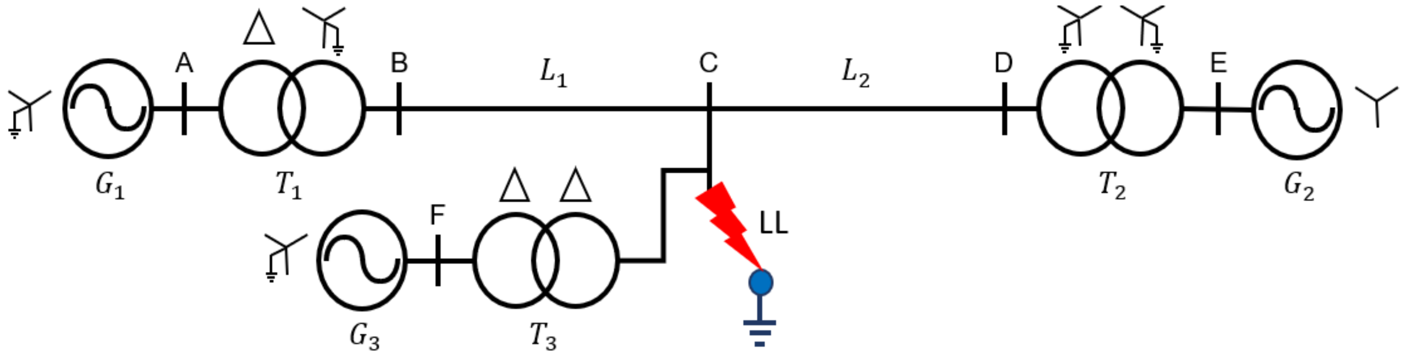


$$\overline{I_1} = -\overline{I_2} = \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2}}$$

$$\overline{I_F} = \frac{3}{j\sqrt{3}}\overline{I_1} = -j\sqrt{3} \times \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2}}$$

Example 10.11: Line to Line Fault at Phase B-C

Consider a SLG fault at node C.

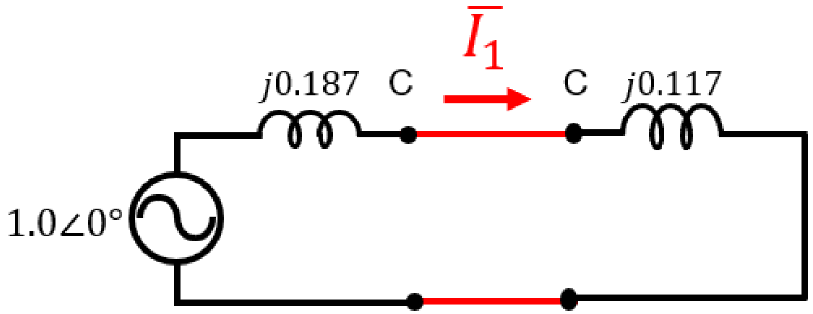


It is given that the positive seq., negative seq., and zero seq. Thevenin impedance are j0.6, j0.187 and j0.117 respectively.

Solution

$$\overline{I_F} = \frac{3}{j\sqrt{3}}\overline{I_1} = -j\sqrt{3} \times \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + Z_k^{TH,2}}$$

$$\overline{I_F} = -j\sqrt{3} \times \frac{1.0\angle 0°}{j(0.187 + 0.117)} = -5.70\angle 0°\, p.u.$$



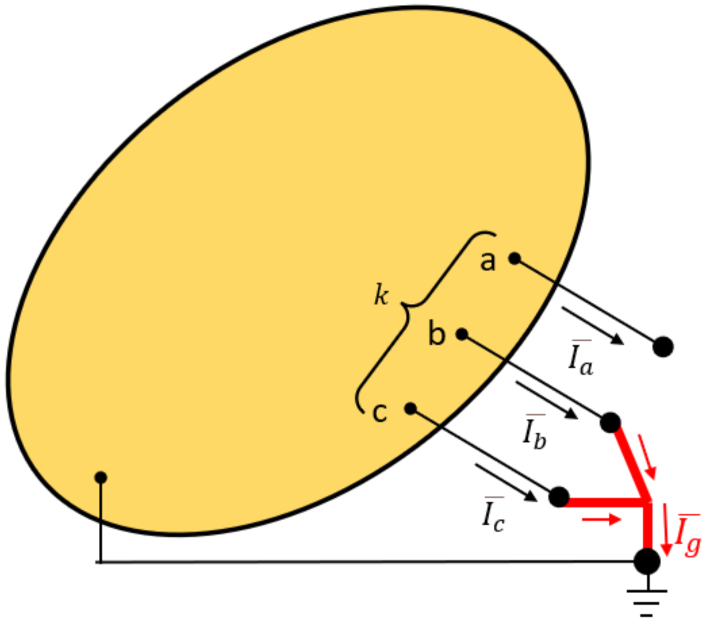
Fault Analysis with Symmetrical Component (LLG Fault)

Consider a double line to ground fault at node k (b-c-g fault).

Pre-fault: $\bar{I}_a = \bar{I}_b = \bar{I}_c = 0$

Fault Condition: $\bar{I}_a = 0 \quad \bar{I}_g = \bar{I}_b + \bar{I}_c \longrightarrow \begin{pmatrix} 0 \\ \bar{I}_b \\ \bar{I}_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{pmatrix}$

$$\bar{V}_a \neq 0 \quad \bar{V}_b = \bar{V}_c = 0 \longrightarrow \begin{pmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} \bar{V}_a \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} \bar{V}_a \\ \bar{V}_a \\ \bar{V}_a \end{pmatrix}$$

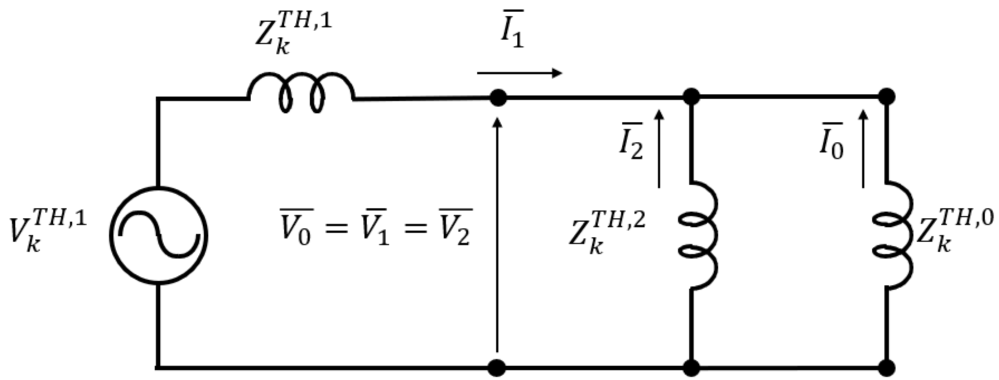


From the analysis, $\bar{V}_0 = \bar{V}_1 = \bar{V}_2 = \frac{1}{3}\bar{V}_a, \qquad \bar{I}_0 + \bar{I}_1 + \bar{I}_2 = 0$

It implies that the sequence networks are <u>connected in parallel</u>.

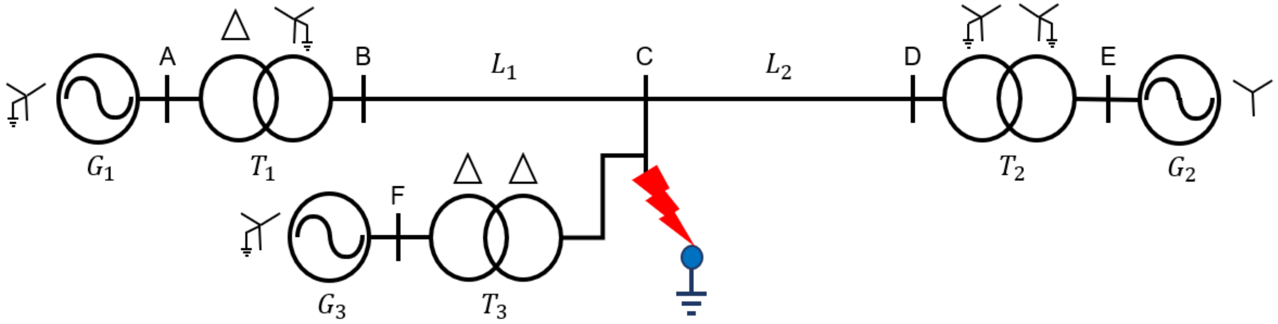
$$\bar{I}_1 = \frac{\overline{V_k^{TH,1}}}{Z_k^{TH,1} + \dfrac{Z_k^{TH,2} Z_k^{TH,0}}{Z_k^{TH,2} + Z_k^{TH,0}}}, \qquad \bar{V}_0 = \bar{V}_1 = \bar{V}_2 = \overline{V_k^{TH,1}} - Z_k^{TH,1} \bar{I}_1$$

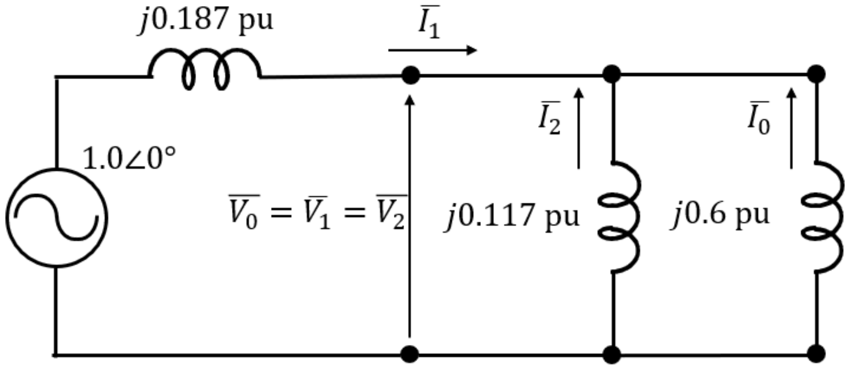
$$\bar{I}_0 = -\frac{\bar{V}_0}{Z_k^{TH,0}}, \qquad \bar{I}_2 = -\frac{\bar{V}_2}{Z_k^{TH,2}}$$



Example 10.10: Double Line to Ground Fault at Phase C

Consider a LLG fault at node C.





It is given that the positive seq., negative seq., and zero seq. Thevenin impedance are j0.6, j0.187 and j0.117 respectively.

$$\overline{I_1} = \frac{1.0\angle 0°}{j\left(0.187 + \frac{0.117 \times 0.6}{0.117 + 0.6}\right)} = 3.51\angle -90° \text{ pu}$$

$$\overline{V_0} = \overline{V_1} = \overline{V_2} = 1.0\angle 0° - j\,0.187 \times 3.51\angle -90° = 0.343\angle 0° \text{ pu}$$

$$\overline{I_2} = -\frac{0.343\angle 0°}{j0.117} = 2.93\angle 90^{o} \text{ pu}$$

$$\overline{I_0} = -\frac{0.343\angle 0°}{j0.6} = 0.572\angle 90° \text{ pu}$$

$$\begin{pmatrix} \overline{I_a} \\ \overline{I_b} \\ \overline{I_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} 0.572\angle 90° \\ 3.51\angle -90° \\ 2.93\angle 90° \end{pmatrix} = \begin{pmatrix} 0 \\ 5.64\angle 171.21° \\ 5.64\angle 8.79° \end{pmatrix}$$

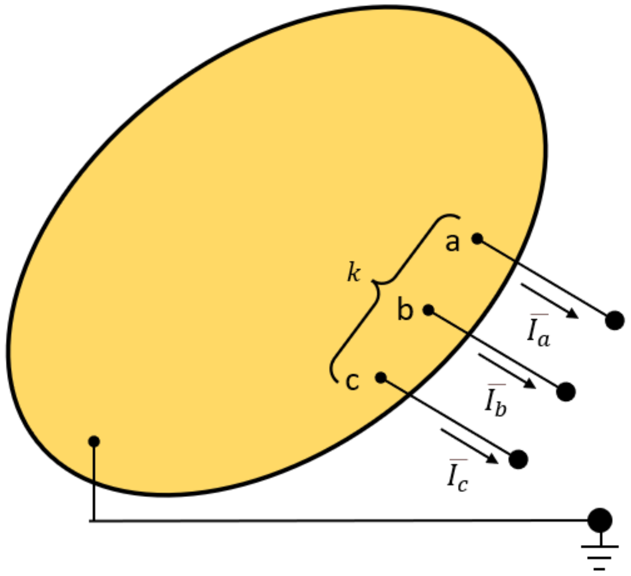
$$\overline{I_g} = 5.64\angle 171.21° + 5.64\angle 8.79° = 1.72\angle 90° \text{pu}$$

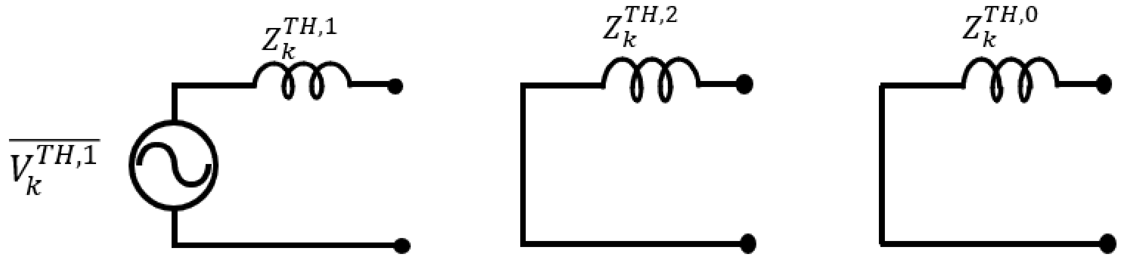
Unbalanced Faults in Large Network

Small networks → calculate Thevenin equivalents by hand

Large networks → need systematic scalable technique

1. Build the admittance matrices of the decoupled, pre-fault positive-, negative, and zero-sequence networks
2. Invert these matrices to obtain the corresponding impedance matrices
3. The <u>diagonal elements</u> of these impedance matrices are the Thevenin equivalent impedances of the sequence networks
4. Connect these Thevenin equivalent impedances as before to calculate the fault currents



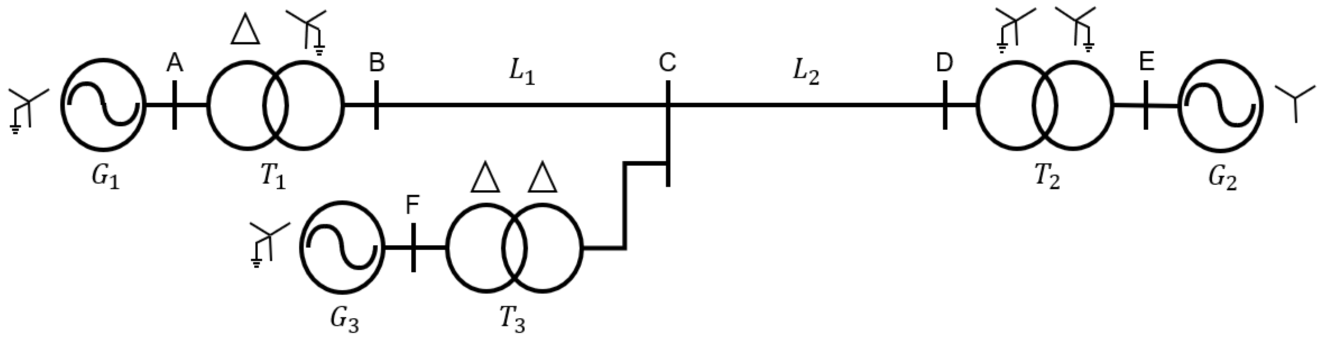


$$Z_k^{TH,1} = Z_1\,(k,k)$$

$$Z_k^{TH,2} = Z_2\,(k,k)$$

$$Z_k^{TH,0} = Z_0\,(k,k)$$

Example 10.13: SLG Fault at Large Network

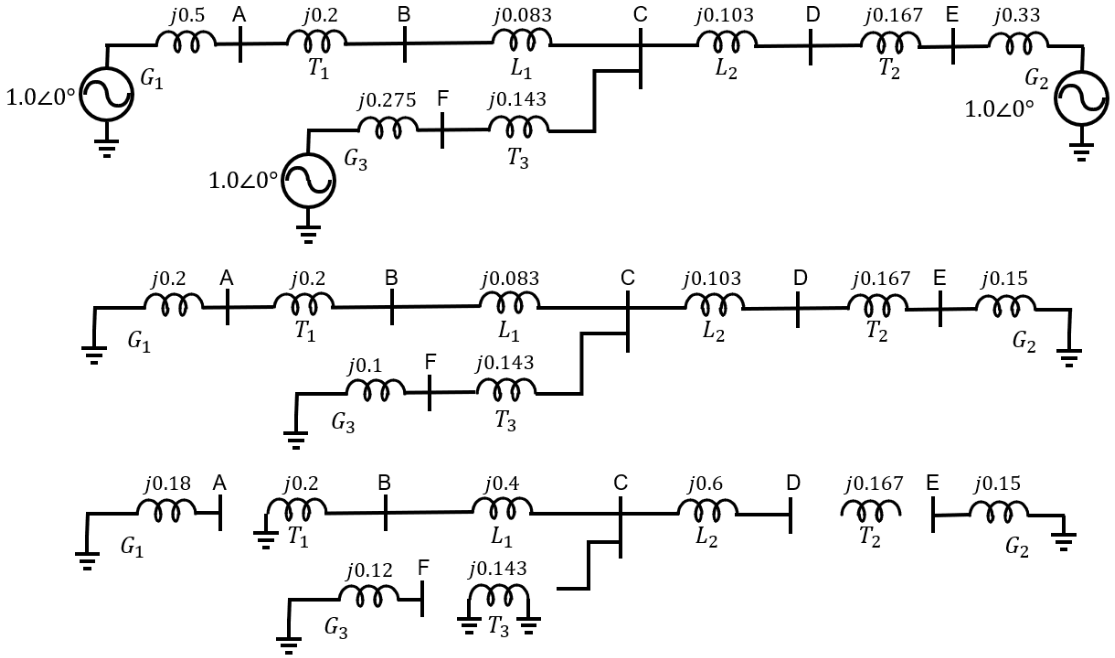


Determine the SLG fault at all node (A, B, C, D, E and F) at the network.

Solution

Positive Sequence Admittance Matrix:

$$Y_1 = j\begin{pmatrix} -7 & 5 & 0 & 0 & 0 & 0 \\ 5 & -17 & 12 & 0 & 0 & 0 \\ 0 & 12 & -28.71 & 9.71 & 0 & 7 \\ 0 & 0 & 9.71 & -15.71 & 6 & 0 \\ 0 & 0 & 0 & 6 & -9 & 0 \\ 0 & 0 & 7 & 0 & 0 & -10.64 \end{pmatrix}$$



Example 10.13: SLG Fault at Large Network

Negative Sequence Admittance Matrix:

$$Y_2 = j\begin{pmatrix} -10 & 5 & 0 & 0 & 0 & 0 \\ 5 & -17 & 12 & 0 & 0 & 0 \\ 0 & 12 & -28.75 & 9.71 & 0 & 7 \\ 0 & 0 & 9.71 & -15.71 & 6 & 0 \\ 0 & 0 & 0 & 6 & -12.66 & 0 \\ 0 & 0 & 7 & 0 & 0 & -17 \end{pmatrix}$$

Zero Sequence Admittance Matrix:

$$Y_0 = j\begin{pmatrix} -5.55 & 0 & 0 & 0 & 0 & 0 \\ 0 & -7.5 & 2.5 & 0 & 0 & 0 \\ 0 & 2.5 & -4.16 & 1.66 & 0 & 0 \\ 0 & 0 & 1.66 & -1.66 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & -8.33 \end{pmatrix}$$

Sequence Impedance:

$$Z_1 = j\begin{pmatrix} 0.257 & 0.160 & 0.120 & 0.099 & 0.066 & 0.079 \\ 0.160 & 0.224 & 0.168 & 0.139 & 0.093 & 0.110 \\ 0.120 & 0.168 & 0.187 & 0.156 & 0.104 & 0.123 \\ 0.099 & 0.139 & 0.156 & 0.214 & 0.143 & 0.102 \\ 0.066 & 0.093 & 0.104 & 0.143 & 0.206 & 0.068 \\ 0.079 & 0.110 & 0.123 & 0.102 & 0.068 & 0.175 \end{pmatrix}$$

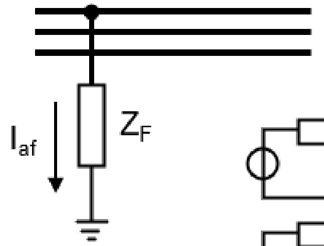
$$Z_2 = j\begin{pmatrix} 0.137 & 0.074 & 0.048 & 0.036 & 0.017 & 0.020 \\ 0.074 & 0.149 & 0.096 & 0.073 & 0.034 & 0.040 \\ 0.048 & 0.096 & 0.116 & 0.088 & 0.042 & 0.048 \\ 0.036 & 0.073 & 0.088 & 0.144 & 0.068 & 0.036 \\ 0.017 & 0.034 & 0.042 & 0.068 & 0.111 & 0.017 \\ 0.020 & 0.040 & 0.048 & 0.036 & 0.017 & 0.079 \end{pmatrix}$$

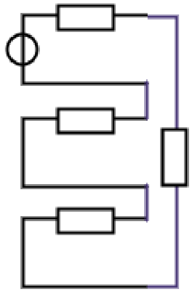
$$Z_0 = j\begin{pmatrix} 0.180 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.200 & 0.200 & 0.200 & 0.000 & 0.000 \\ 0.000 & 0.200 & 0.600 & 0.600 & 0.000 & 0.000 \\ 0.000 & 0.200 & 0.600 & 1.202 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.500 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.120 \end{pmatrix}$$

$$\boxed{\overline{I_F} = \frac{3\overline{V_k^{TH,1}}}{Z_1(k,k) + Z_2(k,k) + Z_0(k,k)}}$$

Node	$Z_0(k,k)$	$Z_1(k,k)$	$Z_2(k,k)$	SLG	LLL
A	0.18	0.257	0.137	5.226	3.891
B	0.2	0.224	0.149	5.236	4.464
C	0.6	0.187	0.116	3.322	5.348
D	1.202	0.214	0.144	1.923	4.673
E	1.500	0.206	0.111	1.651	4.854
F	0.120	0.175	0.079	8.021	5.714

Unbalance Fault: Summary

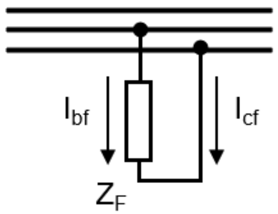


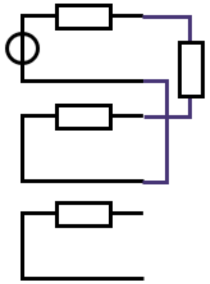


Single Line-to-Ground Fault:

$$\begin{pmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{pmatrix}\begin{pmatrix} I_{af} \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{pmatrix} = \frac{I_f}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \rightarrow I_f = 3I_a^+ = \frac{3V_f}{Z^0 + Z^+ + Z^- + 3Z_F}$$

$$\begin{pmatrix} V_{af} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{pmatrix}\boxed{\begin{pmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{pmatrix}} \rightarrow V_{af} = V^+ + V^- + V^0 \rightarrow V_{af} = Z_F I_{af} = Z_F(3I_a^0)$$





Line-to-Line Fault:

$$\begin{pmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{pmatrix}\boxed{\begin{pmatrix} 0 \\ I_{bf} \\ -I_{bf} \end{pmatrix}} \rightarrow I_a^0 = 0, I_a^+ = -I_a^- = (\alpha - \alpha^2)I_{bf} \rightarrow I_f = \frac{I_a^+}{\alpha - \alpha^2}$$

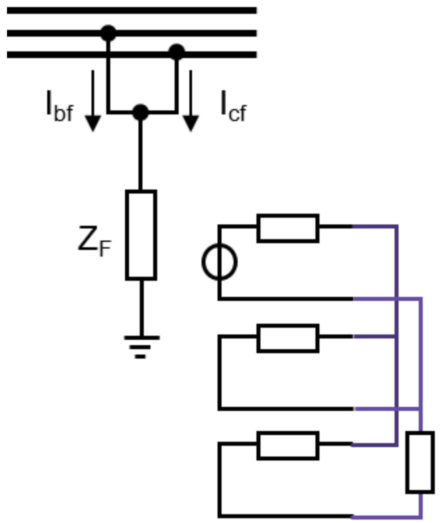
$$\begin{cases} V_{bf} - V_{cf} = I_{bf}Z_F \\ I_a^+ = -I_a^- \end{cases} \rightarrow \begin{cases} (\alpha^2 - \alpha)(V_a^+ - V_a^-) = (\alpha^2 I_a^+ + \alpha I_a^-)Z_F \\ V_a^+ = V_f - Z^+ I_a^+ \end{cases} \rightarrow I_a^+ = -I_a^- = \frac{V_f}{Z^+ + Z^- + Z_F}$$

Note that $V_{af}^+ = V_{af}^- = V_{am}^+ - Z^+ I_a^+ = V_{am}^- - Z^- I_a^-$

Distance:

$$\begin{cases} V_b = V^0 + \alpha^2 V^+ + \alpha V^- \\ V_c = V^0 + \alpha V^+ + \alpha^2 V^- \end{cases} \rightarrow \begin{cases} V_b - V_c = (\alpha^2 - \alpha)(V^+ - V^-) \\ I_b - I_c = (\alpha^2 - \alpha)(I^+ - I^-) \end{cases} \rightarrow \boxed{\frac{V_b - V_c}{I_b - I_c} = Z_{1F}}$$

Unbalance Fault: Summary



Double-Line-to-Ground Fault: $I_{af} = 0, \qquad V_{bf} = V_{cf} = (I_{bf} + I_{cf})Z_F$

$$\begin{pmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{pmatrix}\begin{pmatrix} 0 \\ I_{bf} \\ I_{cf} \end{pmatrix} \rightarrow \boxed{I_a^0 = \frac{(I_{bf} + I_{cf})}{3} = \frac{I_f}{3}} \quad \rightarrow V_{bf} = V_{cf} = (I_{bf} + I_{cf})Z_F$$

$$= 3Z_F I_a^0$$

$$\begin{pmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{pmatrix}\begin{pmatrix} V_{af} \\ V_{bf} \\ V_{cf} \end{pmatrix} \rightarrow 3V_{af}^0 = V_{af} + 2V_{bf} \qquad \rightarrow V_a^+ = V_a^- = V_a^0 - 3Z_F I_a^0$$

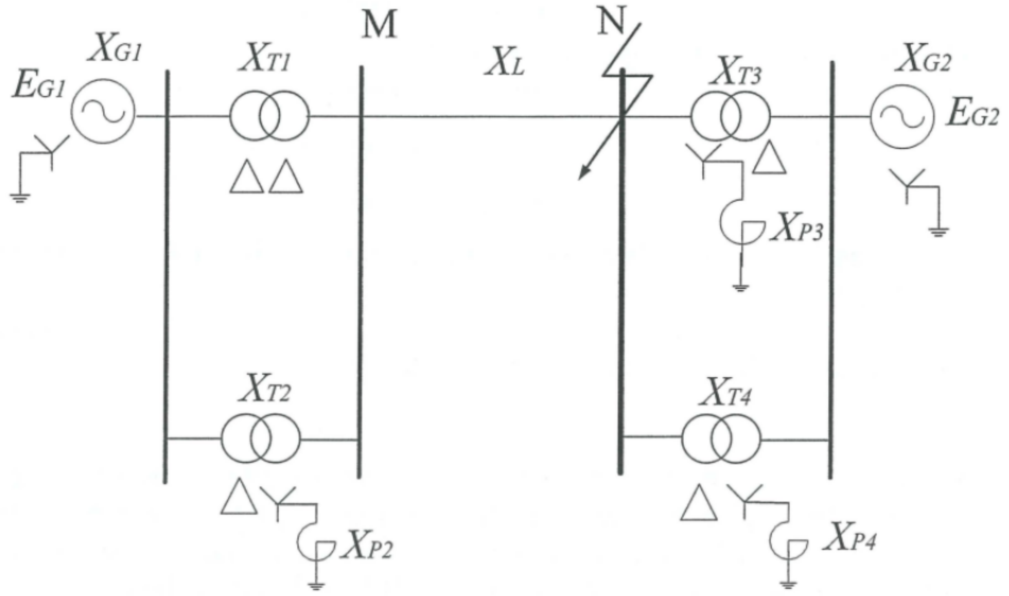
$$V_a^+ = V_a^-$$

- To evaluate unbalance faults, we apply sequence network to simplify the coupled analysis of phase network (3n x 3n) to decoupled 3 (n x n) network.
- Fault type implies the coupling (connection) of the 3 x sequence network. By evaluate pre-fault impedance matrix in network, one can compute the corresponding fault current with the connection.
- It is also possible to apply sequence components for fault detection, e.g.
  - I_0 for earth fault detection, but I_0 is susceptible to mutual coupling and zero-sequence network in secondary circuit.
  - I_2 for unbalance fault detection, but I_2 could be too small to measure, and I_2 network may not be accurate due to asymmetry in Z_1 and Z_2 for machines and converter-based sources.
  - I_0 and I_2 for phase comparison.

Exercise 10.1

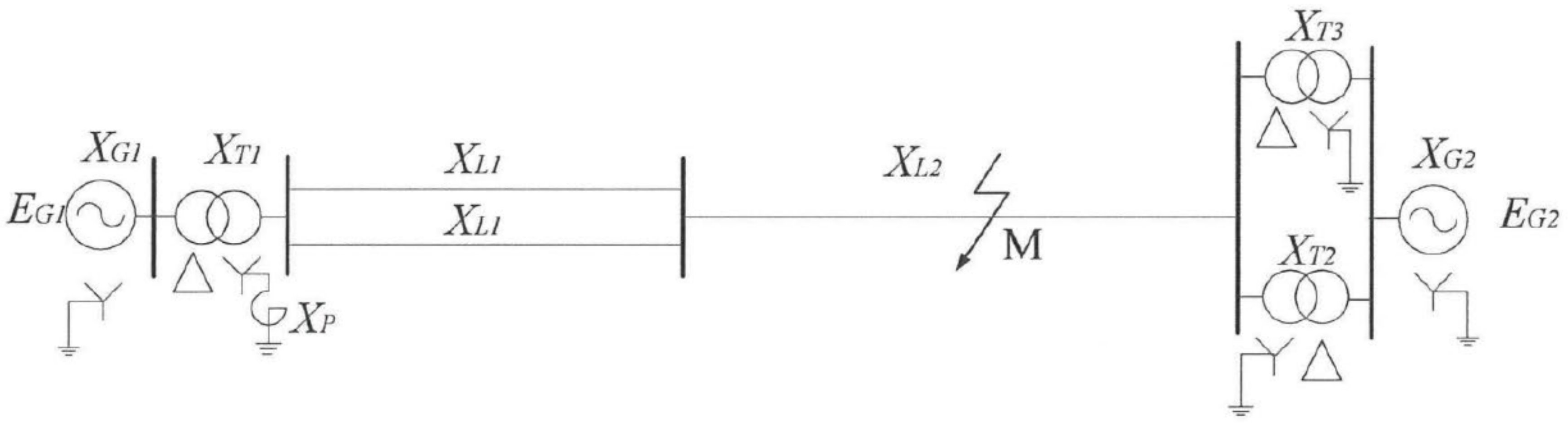
a) For the following system, please draw the complete positive-sequence, negative-sequence and zero-sequence network connection (phase A) for the double line-to-ground fault (phases B and C to ground fault) at bus N.

b) Prefault voltage: $E_{G1} = 1\angle 0^o, E_{G2} = 1\angle 0^o$(p.u.)
Parameters (p.u.):
Generators: $X_{G1}^+ = X_{G1}^- = X_{G1}^0 = 0.2, X_{G2}^+ = X_{G2}^- = X_{G2}^0 = 0.2$
Line: $X_L^+ = X_L^- = X_L^0 = 0.2$
Transformers: $X_{T1}^+ = X_{T1}^- = X_{T1}^0 = 0.4,\ \ X_{T2}^+ = X_{T2}^- = X_{T2}^0 = 0.4$
$X_{T3}^+ = X_{T3}^- = X_{T3}^0 = 0.4,\ \ X_{T4}^+ = X_{T4}^- = X_{T4}^0 = 0.4$
$X_{P2} = X_{P3} = X_{P4} = 0.2$
For a single line-ground fault (phase A) at point N, please find the fault current at phase A.



Exercise 10.2

a) For the following system, please draw the complete positive-sequence, negative-sequence and zero-sequence network connection(phase A) for the double line-to-ground faults (phases B and C to ground faults) at bus M (M is the middle of X_{L2}).



b) Prefault voltage: $E_{G1} = 1\angle 0^o, E_{G2} = 1\angle 0^o$(p.u.)

Parameters (p.u.):

Generators: $X_{G1}^{+} = X_{G1}^{-} = X_{G1}^{0} = 0.2, X_{G2}^{+} = X_{G2}^{-} = X_{G2}^{0} = 0.2$

Line: $X_{L1}^{+} = X_{L1}^{-} = X_{L1}^{0} = 0.4,\ X_{L2}^{+} = X_{L2}^{-} = X_{L2}^{0} = 0.4$

Transformers: $X_{T1}^{+} = X_{T1}^{-} = X_{T1}^{0} = 0.6,\ \ X_{T2}^{+} = X_{T2}^{-} = X_{T2}^{0} = 0.4$

$X_{T3}^{+} = X_{T3}^{-} = X_{T3}^{0} = 0.4$

$X_P = 0.8$

For a single line-ground fault (phase A) at point N, please find the fault current at phase A.