Distance Protection
Impedance Based Fault Location

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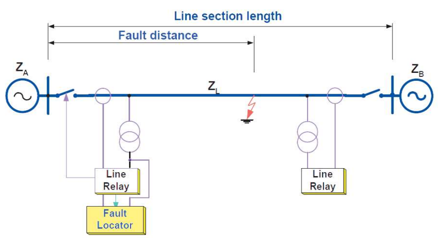
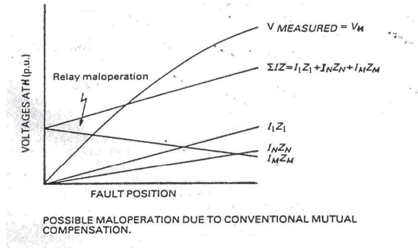
Introduction

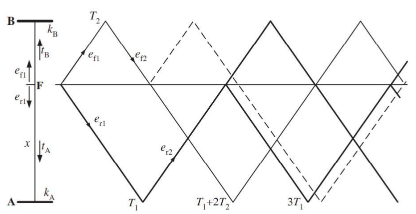
- Fault Location is a process aimed at locating the distance of occurred fault (in km or in percentage of line) in a transmission line with the highest possible accuracy.
- Fault Locators vs Protective Relays –
  - Accuracy of Fault Location (may require compensation factors, e.g. mutual coupling, which is not often used in protection due to its de-stabilizing effect)
  - Speed of Determining Fault Position (loss of information after fault clearance)
  - Speed of Transmission of Data from Remote Site (comparison of pre-fault information of all terminals helps improve the accuracy, which is not required in distance protection)
  - Used Data Window (Windowing affects resolution. Low time resolution with small time window can provide high frequency resolution, which shows any discontinuities and reflection)
  - Digital Filtering of Input Signals and Complexity of Calculation (Filtering creates delay, in which may not be preferred for fault detection and clearance purpose)

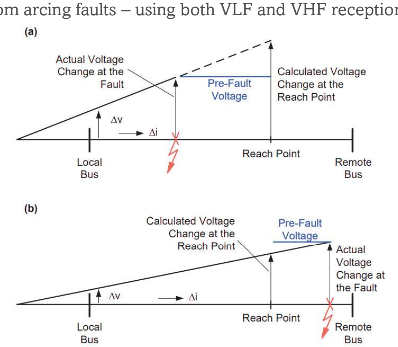




Introduction

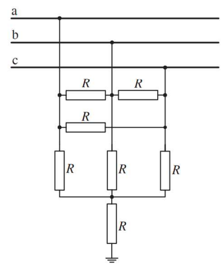
- Fault Location Technologies
  - Techniques based on Fundamental Frequency Current and Voltage – Mainly on Impedance-baed Measurement
  - Techniques based on Travelling Wave Phenomenon
  - Techniques based on High Frequency Components of Current and Voltage Generated by Faults
  - Knowledge-based Approaches
  - Unconventional Techniques (e.g. Fault Indicators – installed to indicate if the fault gone through the section; Monitoring transients of induced radiation from arcing faults – using both VLF and VHF reception)

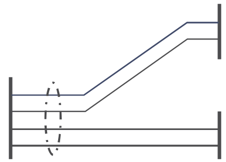




Factors Influencing Accuracy in Fault Location

1. Insufficient Information (Only have one side waveform for Tee circuit)
2. Inaccurate Compensation for Reactance Effect using One-End Measurement (Due to incorrect input of local source strength and that of other terminals)
3. Inaccurate Fault-Type (Faulted Phase) Identification (phase-to-earth or phase-to-phase loop does not cover all fault types)
4. Inaccurate Line Parameter (Line Length could be known with some error) (Difficulties in obtaining accurate zero-sequence impedance, which varies over the years according to meteorological condition)
5. Inaccurate Compensation for Mutual Effects on Zero Sequence Component (Line may not going all in parallel and Current may not be crossed for input)
6. Insufficient Accuracy of Line Model (Line capacitance effect and transposition effect may not be included)
7. Presence of Compensation Devices or Voltage Protection Device (Shunt reactors, Series reactors, Phase Shifting Transformers, MOV)
8. Load Flow Unbalance
9. Combined Effect of Fault Resistance and Load
10. Error of Current and Voltage Transformers and Unfaithful Reproduction of Primary Signals due to Limited Bandwidth
11. System Non-homogeneity with different System Infeed Condition (Remote / Third terminal infeed, tapped load with zero-sequence sources)





Difficulties in Fault Location – Reactance Effect

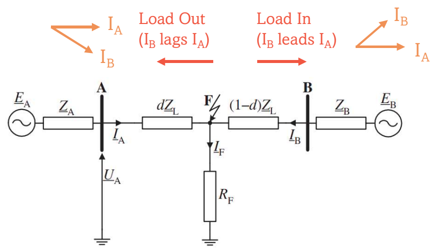
Consider a line with two sources A and B.

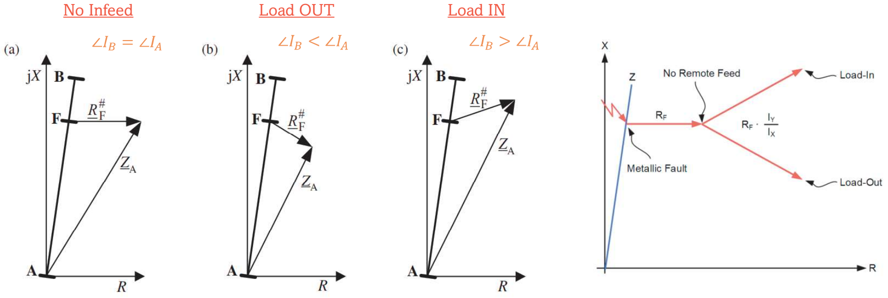
$$U_A = I_A\, dZ_L + (I_A + I_B)R_F$$

$$\frac{U_A}{I_A} = dZ_L + \left(1 + \frac{I_B}{I_A}\right)R_F$$

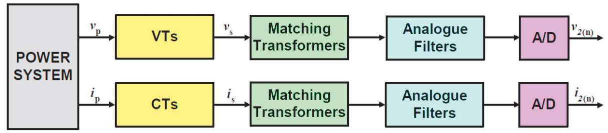
The measured impedance depends on the fault resistance R_F and the current at both ends I_A and I_B.

Reactance Effect is the impedance seen in R-X plane is no longer purely resistive with pre-fault load flow.

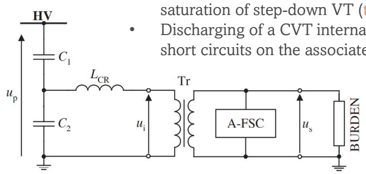


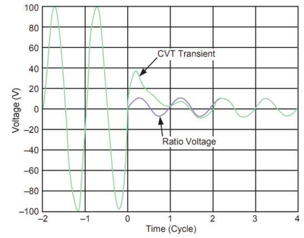


Voltage and Current Measurement Chain

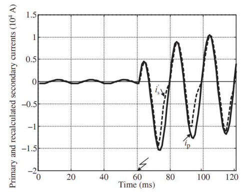


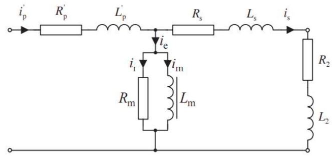
- CVT Measurement
  - Non-linear Oscillations under magnetic core saturation of step-down VT (transient overreach)
  - Discharging of a CVT internal energy during short circuits on the associated transmission line.





- CT Measurement
  - Phase-Shifting Effect on CT Saturation, which acquired increased excitation current leading to missing current output

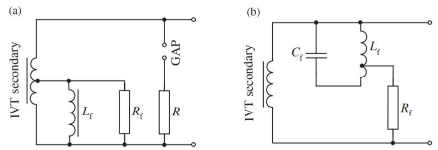


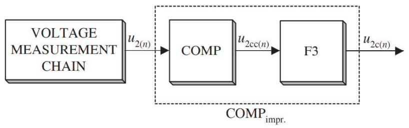


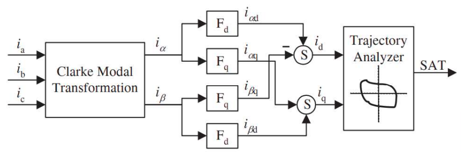
Solution to Inaccurate Voltage and Current Measurement

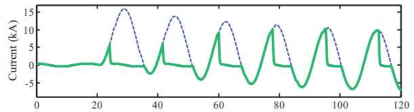
For CVT Ferro-resonance,

1. Addition of Suppression Circuit
2. Dynamic Compensation
   (Inversion of CVT transfer function)









For CT Saturation, (Require CT Saturation Detection)

1. Use of Voltage Signal alone for fault location
2. Use of Voltage and Current Signal, but excluding current from saturated CTs
3. Reconstruction of CT primary Current by Inrush or Fault Waveform
4. Allowing current from saturated CT to be used without intervals of linear transformation

Differential Equation Based Relay Measurement

- Consider a line in short-line model, i.e. R+jX with lumped parameters.

$$u(t) = Ri(t) + L\frac{di(t)}{dt}$$

- Discretizing with sampling frequency T_s

$$u[k] = Ri[k] + L\frac{i[k] - i[k-1]}{T_s}$$

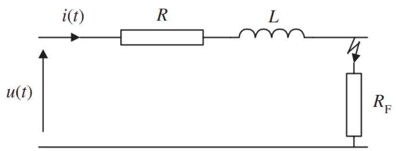
$$u[k+1] = Ri[k+1] + L\frac{i[k+1] - i[k]}{T_s}$$

- Least Square Estimation

Quadrilateral ways to estimate the (R+jX) measured

$$\underbrace{\begin{pmatrix} u[k] \\ u[k+1] \\ \vdots \\ u[k+m] \end{pmatrix}}_{\triangleq \mathbf{u}} = \underbrace{\begin{pmatrix} i[k] & \frac{i[k]-i[k-1]}{T_s} \\ i[k+1] & \frac{i[k+1]-i[k]}{T_s} \\ \vdots & \vdots \\ i[k+m] & \frac{i[k+m]-i[k+m-1]}{T_s} \end{pmatrix}}_{\triangleq \mathbf{I}} \underbrace{\boxed{\begin{pmatrix} R \\ L \end{pmatrix}}}_{\triangleq \mathbf{z}}$$

$$\mathbf{u} = \mathbf{I}\mathbf{z}$$
$$\mathbf{I}^T\mathbf{u} = \mathbf{I}^T\mathbf{I}\mathbf{z}$$
$$\mathbf{z} = (\mathbf{I}^T\mathbf{I})^{-1}\mathbf{I}^T\mathbf{u}$$



Fault Analysis – Fault Loop and Coefficients

Aim –

- Improve fault location accuracy by introducing compensation for shunt capacitance
- Limit influence of uncertain parameters on fault location accuracy
- Obtain simple formulae by applying generalized fault loop model and fault model

Recall **symmetric component** –

$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix}, \qquad a = 1\angle 120^o$$

Generalized Fault Loop Model

$$U_A - d\, Z_{1L} I_A - R_F(a_{F1}I_{F1} + a_{F2}I_{F2} + a_{F0}I_{F0}) = 0$$

where

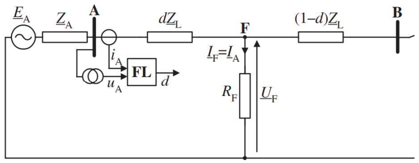
d, R_F – unknown distance to fault, fault resistance

U_A, I_A – fault loop voltage and current (dependent on fault type)

Z_{1L} – positive sequence line impedance

I_{F1}, I_{F2}, I_{F0} – symmetrical components of total fault current

a_{F1}, a_{F2}, a_{F2} – weighting factors (dependent on fault type)



Fault Analysis – Fault Loop and Coefficients

Example – Determine the weighting factors for A-G Fault.

$$\begin{pmatrix} I_{F0} \\ I_{F1} \\ I_{F2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} I_{Fa} \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} I_{Fa} \\ I_{Fa} \\ I_{Fa} \end{pmatrix}$$

which results in $I_{F1} = I_{F2} = I_{F0} = 0.33 I_{Fa}$. It means that we can represent the fault current as follows.

- $I_F = I_{F1} + I_{F2} + I_{F0}$
- $I_F = 3I_{F1}$
- $I_F = 3I_{F2}$
- $I_F = 3I_{F0}$
- $I_F = 1.5I_{F2} + 1.5I_{F0}$

Different priority with respect to using a particular symmetrical component can be applied.

- Use of zero-sequence is generally **avoided**, since zero-sequence impedance for OHL is often considered as an uncertain parameter.
- Use of negative-sequence, instead of positive-sequence, is **preferred**, as line shunt capacitance charging is more extensive for positive sequence
- Use of both positive and negative sequence, excluding the zero-sequence (e.g. $I_F = 1.5I_{F1} + 1.5I_{F2}$) is much preferred, as it allows calculation to be made simpler.

Fault Analysis – Fault Loop and Coefficients

Consider fault current I_F to be flexibly decomposed, or mapped, to certain portion of sequence currents, according to the fault nature. $I_F = a_{F0}I_{F0} + a_{F1}I_{F1} + a_{F2}I_{F2}$

Given that the use of zero-sequence should be eliminated, and priority of using negative sequence should be larger than positive sequence,

Fault Type	Total Fault Current	a_{F1}	a_{F2}	a_{F0}
A-G	I_{Fa}	0	3	0
B-G	I_{Fb}	0	$-1.5 + j\,1.5\sqrt{3}$	0
C-G	I_{Fc}	0	$-1.5 - j\,1.5\sqrt{3}$	0
A-B	$I_{Fa} - I_{Fb}$	0	$1.5 - j\,0.5\sqrt{3}$	0
B-C	$I_{Fb} - I_{Fc}$	0	$j\,\sqrt{3}$	0
C-A	$I_{Fc} - I_{Fa}$	0	$-1.5 - j\,0.5\sqrt{3}$	0
A-B-G	$I_{Fa} - I_{Fb}$	$1.5 + j\,0.5\sqrt{3}$	$-1.5 - j\,0.5\sqrt{3}$	0
B-C-G	$I_{Fb} - I_{Fc}$	$-j\,\sqrt{3}$	$j\,\sqrt{3}$	0
C-A-G	$I_{Fc} - I_{Fa}$	$1.5 - j\,0.5\sqrt{3}$	$1.5 + j\,0.5\sqrt{3}$	0
A-B-C (A-B-C-G)*	$I_{Fa} - I_{Fb}$	$1.5 + j\,0.5\sqrt{3}$	$1.5 - j\,0.5\sqrt{3}$**	0

Can be eliminated. i.e. $I_F = a_{F1}I_{F1} + a_{F2}I_{F2}$

Note –
* inter-phase fault loop *a-b* is considered. Other fault loops can be taken as well.
** the coefficient is different from zero, but I_2 is not present in the signal.

Fault Analysis – Fault Loop and Coefficients

Determine the coefficients to transfer zero-sequence current back to sum of positive and negative-sequence current for *b-c-g* fault. i.e. $I_{F0} = b_{F1}I_{F1} + b_{F2}I_{F2}$

$$\begin{pmatrix} I_{F0} \\ I_{F1} \\ I_{F2} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}\begin{pmatrix} 0 \\ I_{Fb} \\ I_{Fc} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} I_{Fb} + I_{Fc} \\ aI_{Fb} + a^2 I_{Fc} \\ a^2 I_{Fb} + aI_{Fc} \end{pmatrix}$$

The sum of positive and negative- sequence current equals

$$I_{F1} + I_{F2} = \frac{1}{3}\left((a + a^2)I_{Fb} + (a^2 + a)I_{Fc}\right) = -\frac{1}{3}(I_{Fb} + I_{Fc}) = -I_{F0}$$

Hence, $I_{F0} = -1\, I_{F1} - 1\, I_{F2}$, i.e. b_1 = -1 and b_2 = -1

Given the fault characteristics, the zero-sequence current can be obtained by a linear sum of other sequence current.

$$I_{F0} = b_{F1}I_{F1} + b_{F2}I_{F2}$$

Fault Type	b_{F1}	b_{F2}
A-G	0	1
B-G	0	$-0.5 + j\,0.5\sqrt{3}$
C-G	0	$-0.5 - j\,0.5\sqrt{3}$
A-B-G	$0.5 - j\,0.5\sqrt{3}$	$0.5 + j\,0.5\sqrt{3}$
B-C-G	-1	-1
C-A-G	$0.5 + j\,0.5\sqrt{3}$	$0.5 - j\,0.5\sqrt{3}$

Fault Analysis – Fault Loop and Coefficients

- Define Conversion Factor k_F such that

$$I_{F1}=\frac{\Delta I_{A1}}{k_{F1}}=\frac{I_{A1}-I_{A1}^{pre}}{k_{F1}},\qquad I_{F2}=\frac{I_{A2}}{k_{F2}},\qquad I_{F0}=\frac{I_{A0}}{k_{F0}}$$

- Conversion Factor, or distribution factor, according to the impedance network.

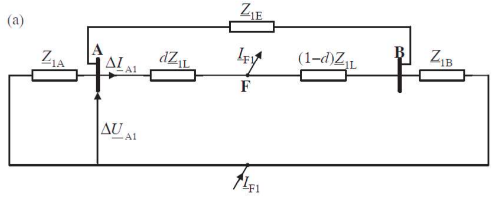
$$k_{F1}=k_{F2}=\frac{K_1 d+L_1}{M_1},\qquad k_{F0}=\frac{K_0 d+L_0}{M_0}$$

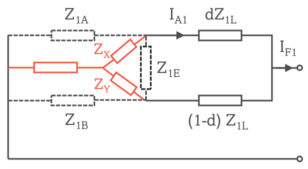
- From Delta-Wye Transformation,

$$Z_X=\frac{Z_{1A}Z_{1E}}{Z_{1A}+Z_{1B}+Z_{1E}},\qquad Z_Y=\frac{Z_{1B}Z_{1E}}{Z_{1A}+Z_{1B}+Z_{1E}}$$

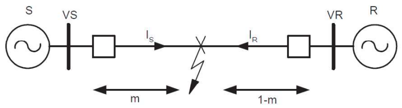
$$I_{A1}=I_F\frac{Z_Y+(1-d)Z_{1L}}{Z_X+Z_Y+Z_{1L}}=I_F\frac{Z_{1B}Z_{1E}+Z_{1L}Z_\Sigma+(-Z_{1L}Z_\Sigma)d}{(Z_{1A}+Z_{1B})Z_{1E}+Z_{1L}Z_\Sigma}$$

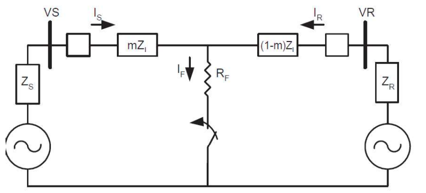
Extra Link	Coefficient
$Z_{1E}\neq\infty$	$K_1=-Z_{1L}Z_{1E}-(Z_{1A}+Z_{1B})Z_{1L}$ $L_1=Z_{1L}(Z_{1A}+Z_{1B})+Z_{1E}(Z_{1L}+Z_{1B})$ $M_1=(Z_{1A}+Z_{1B})(Z_{1E}+Z_{1L})+Z_{1L}Z_{1E}$
$Z_{1E}\to\infty$	$K_1=-Z_{1L}\quad L_1=Z_{1L}+Z_{1B}$ $M_1=Z_{1A}+Z_{1B}+Z_{1L}$





Revisit on Old Techniques – Simple Reactance Method





Voltage Drop from Source S - $V_s=m\,Z_{1L}\,I_s+R_F\,I_F$

For A-G Fault, $V_S=V_{AG},\ I_S=I_A+K_N I_N$

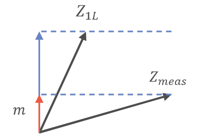
The goal is to minimize the effect of $R_F\,I_F$ term.

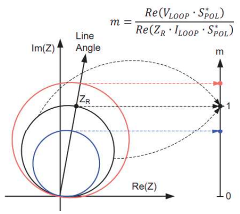
$$\frac{V_S}{I_S}=m\,Z_{1L}+R_F\frac{I_F}{I_S}\ \rightarrow\ \mathrm{Im}\left(\frac{V_S}{I_S}\right)=\mathrm{Im}(m\,Z_{1L})+\mathrm{Im}(R_F\frac{I_F}{I_S})$$

If $R_F=0$ or $\angle I_S=\angle I_F$, the latter term is zero.

Hence, the fault location (percentage of line) is

$$\boxed{m=\frac{\mathrm{Im}\left(\frac{V_S}{I_S}\right)}{\mathrm{Im}(Z_{1L})}}$$





One End Fault Location Optimization – Takagi Method

Superimposed circuit is a current divider of fault current.

$$\Delta I_A=\frac{(1-d)Z_L+Z_B}{Z_A+Z_L+Z_B}I_F$$

where $\Delta I_A=I_A-I_A^{pre}$ is the incremental current determined from the moment of fault inception occurrence.

Hence, the total fault current is

$$I_F=\frac{\Delta I_A}{k_F},\qquad k_F=|k_F|\angle k_F=\frac{(1-d)Z_L+Z_B}{Z_A+Z_L+Z_B}$$

where k_F is fault current distribution factor.

From KVL -

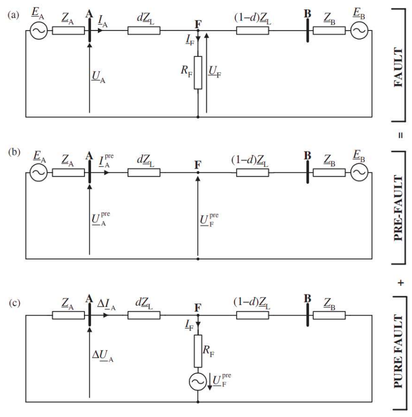
$$U_A-dZ_LI_A-R_FI_F=0\ \rightarrow\ U_A-dZ_LI_A-R_F\frac{\Delta I_A}{|k_F|\angle k_F}=0$$

Multiplying with $\Delta I_A^*\angle k_F$ and taking the imaginary part,

$$U_A\Delta I_A^*\angle k_F-dZ_LI_A\Delta I_A^*\angle k_F\boxed{-R_F\frac{\Delta I_A\Delta I_A^*}{|k_F|}}=0$$

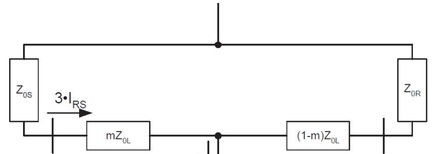
Real Component

$$\boxed{d=\frac{\mathrm{Im}(U_A\cdot\Delta I_A^*\angle k_F)}{\mathrm{Im}(Z_LI_A\cdot\Delta I_A^*\angle k_F)}}$$

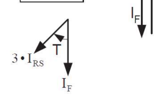


This is coined as **Takagi Method**

One End Fault Location Optimization – Modified Takagi Method



Takagi Method - $d=\frac{\mathrm{Im}(U_A\cdot\Delta I_A^*\angle k_F)}{\mathrm{Im}(Z_LI_A\cdot\Delta I_A^*\angle k_F)}$



It requires data storing for pre-fault condition.

Let's consider projecting the equation to neutral current $I_N=3I_0$.

Consider the zero-sequence network.

$$\frac{I_F}{I_N}=\frac{(Z_{0S}+Z_{0L}+Z_{0R})}{(1-m)Z_{0L}+Z_{0R}}=A\angle T\ \rightarrow I_F=I_N\,A\angle T$$

Recall the fault equation $U_A-mZ_LI_A-R_FI_F=0$ and project the equation onto $I_N\angle T$

$$U_A\,(I_N\angle T)^*-mZ_LI_A(I_N\angle T)^*-R_F\,I_NA\angle T\,(I_N\angle T)^*=0$$

$$\mathrm{Im}(U_A\,I_N^*\angle-T)-m\,\mathrm{Im}(Z_LI_A\,I_N^*\angle-T)-\mathrm{Im}(AR_FI_NI_N^*)=0$$

$$\boxed{m=\frac{\mathrm{Im}(U_A\,I_N^*\angle-T)}{\mathrm{Im}(Z_LI_A\,I_N^*\angle-T)}}$$

This is coined as **Modified Takagi Method**

Consider KVL in fault loop A.

$$U_A - dZ_L I_A - R_F(a_{F1} I_{F1} + a_{F2} I_{F2}) = 0$$

Divide the equation with I_A to obtain seen impedance.

$$Z_A = \boxed{\frac{U_A}{I_A}} - dZ_L - R_F\left(a_{F1}\frac{\Delta I_{A1}}{|k_F|\angle k_F}\frac{1}{I_A} + a_{F2}\frac{\Delta I_{A2}}{|k_F|\angle k_F}\frac{1}{I_A}\right) = 0$$

Put the equation in R + jX coordinate,

$$R_A + jX_A - d(R_L + jX_L)$$

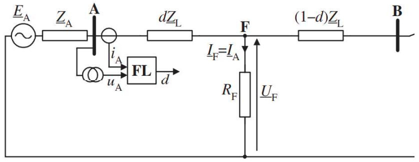
$$-R_F\left(\underbrace{\mathrm{Re}\left\{\frac{a_{F1}\Delta I_{A1} + a_{F2}\Delta I_{A2}}{I_A|k_F|\angle k_F}\right\}}_{\triangleq\, a} + \underbrace{\mathrm{Im}\left\{\frac{a_{F1}\Delta I_{A1} + a_{F2}\Delta I_{A2}}{I_A|k_F|\angle k_F}\right\}}_{\triangleq\, b}\right) = 0$$

Comparing the Coefficient,

$$\begin{cases} R_A = d\, R_L + R_F\, a \\ X_A = d\, X_L + R_F\, b \end{cases} \rightarrow \begin{pmatrix} R_A \\ X_A \end{pmatrix} = \begin{pmatrix} R_L & a \\ X_L & b \end{pmatrix}\begin{pmatrix} d \\ R_F \end{pmatrix}$$

The fault location (percentage of line length) can be represented by –

$$\boxed{d = \frac{X_A}{X_L} - \frac{\frac{R_A}{X_L}\tan\angle Z_L - \frac{X_A}{X_L}}{\frac{a}{b}\tan\angle Z_L - 1}}$$



This is coined as
Wisz Method

- Remote source infeed often leads to error when considering the fault resistance part.
- To eliminate such error, a complete network model which considers the actual distribution of fault current is employed. Compensation for fault resistance voltage drop requires pre-fault current and corresponding source impedance.
- Consider the Generalized Fault Model – $Z_A - dZ_L - R_F\left(\frac{a_{F1}\Delta I_{A1} + a_{F2}\Delta I_{A2}}{I_A|k_F|\angle k_F}\right) = 0$
- Consider the Conversion Factor – $k_F = \frac{\Delta I_A}{I_F} = |k_F|\angle k_F = \frac{K_1 d + L_1}{M_1}$
- Combining the two to form a quadratic equation - $A_2 d^2 + A_1 d + A_0 + A_{00} R_F = 0$

$$Z_A - dZ_L - R_F\left(\frac{M_1}{K_1 d + L_1}\right)\left(\frac{a_{F1}\Delta I_{A1} + a_{F2}\Delta I_{A2}}{I_A}\right) = 0$$

$$K_1 Z_L\, d^2 + (Z_{1L} L_1 - Z_A K_1)\, d - L_1 Z_A + \frac{1}{I_A} M_1 (a_{F1}\Delta I_{A1} + a_{F2}\Delta I_{A2}) R_F = 0$$

- This equation can be written down separately for the Real and Imaginary parts. Combine them in a way such that fault resistance is eliminated yields the quadratic formula for a sought distance to fault.

$$A_2 d^2 + A_1 d + A_0 + A_{00} R_F = 0 \rightarrow \; B_2 d^2 + B_1 d + B_0 = 0$$

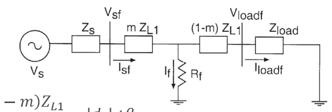
where:

$B_2 = \mathrm{Re}(A_2)\mathrm{Im}(A_{00}) - \mathrm{Im}(A_2)\mathrm{Re}(A_{00})$, $B_1 = \mathrm{Re}(A_1)\mathrm{Im}(A_{00}) - \mathrm{Im}(A_1)\mathrm{Re}(A_{00})$

$B_0 = \mathrm{Re}(A_0)\mathrm{Im}(A_{00}) - \mathrm{Im}(A_0)\mathrm{Re}(A_{00})$

This is coined as
Eriksson's Method

Measure the pre-fault load impedance by $Z_{Load} = \frac{V_{PS}}{I_{PS}} - Z_{L1} = R + jX$



Determine current distribution coefficient

$$d_s = \frac{\Delta I_S}{I_F} = \frac{I_{SF} - I_{PS}}{I_F} = \frac{Z_{Load} + (1-m)Z_{L1}}{Z_S + Z_{Load} + Z_{L1}} = |d_s|\angle\beta_s$$

Change in Current is given by $\Delta I_S = I_{SF} - I_{PS} = |\Delta I_S|\angle\lambda_S$

From Fault Loop Analysis,

$$\frac{V_{SF}}{I_{SF}} = mZ_{L1} + R_F\left(\frac{I_F}{I_{SF}}\right) \rightarrow \; V_{SF} = mZ_{L1}I_{SF} + R_F\left(\frac{\Delta I_S}{d_s}\right) = mZ_{L1}I_{SF} + R_F\frac{|\Delta I_S|}{|d_s|}\angle(\lambda_s - \beta_s)$$

Define $\angle(\lambda_s - \beta_s) = \angle\Phi$ and $R_F|\Delta I_S|/|d_s| = \mathrm{D}$, $\quad V_{SF} = mZ_{L1}I_{SF} + D\angle\Phi$

This equation has two unknowns – D and m. To deduce the solution for m, the percentage line length, we can

1. Solve the linear equation (by comparing coefficient).

$$m = \frac{\mathrm{Re}(V_{SF})\mathrm{Im}(\Phi) - \mathrm{Im}(V_{SF})\mathrm{Re}(\Phi)}{R_{L1}M - X_{L1}N}$$

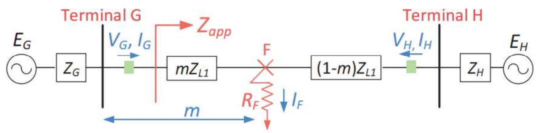
$$M = \mathrm{Re}(I_{SF})\mathrm{Im}(\Phi) - \mathrm{Im}(I_{SF})\mathrm{Re}(\Phi), \qquad N = \mathrm{Re}(I_{SF})\mathrm{Re}(\Phi) - \mathrm{Im}(I_{SF})\mathrm{Im}(\Phi)$$

This is coined as
Novosel's Method
(1998)

2. Projecting the equation onto $\angle\Phi$ and obtain m by neglecting the real part.

$$m = \frac{\mathrm{Im}(V_{SF}\angle - \Phi)}{\mathrm{Im}(Z_{L1}I_{SF}\angle - \Phi)}$$

Note – this solution requires current memory I_{PS} and pre-fault load information Z_{Load}.



Consider unsynchronized measurement with synchronizing operation $e^{j\delta}$. Fault voltage can be represented by

$$V_{Fi} = (V_{Gi} - mZ_{L1}I_{Gi})e^{j\delta}, \qquad V_{Fi} = V_{Hi} - (1-m)Z_{Li}I_{Hi}$$

Combining the two equations, we have $\quad e^{j\delta} = \frac{(V_{Hi} - Z_{Li}I_{Hi}) - mZ_{Li}I_{Hi}}{(V_{Gi} - mZ_{L1}I_{Gi})}$

To eliminate the synchronizing operator, we take the absolute value

$$|e^{j\delta}| = \left|\frac{(V_{Hi} - Z_{Li}I_{Hi}) - mZ_{Li}I_{Hi}}{V_{Gi} - mZ_{L1}I_{Gi}}\right| = 1$$

This can be expanded to a quadratic equation.

$$\boxed{(|Z_{Li}I_{Gi}|^2 - |Z_{Li}I_{Hi}|^2)m^2 + (-2\,\mathrm{Re}(V_{Gi}(Z_{Li}I_{Gi})^* + (V_{Hi} - Z_{Li}I_{Hi})(Z_{Li}I_{Hi})^*)m + (|V_{Gi}|^2 - |V_{Hi} - Z_{Li}I_{Hi}|^2) = 0}$$

It is noted that $0 < m < 1$ and

use i = 2 (negative sequence component) for unbalance fault;

use i = 1 (positive sequence component) for balanced fault.

This is coined as
Zivanovic's Method
(2008)

Two End Fault Location Optimization – Tziouvaras Method

Relay S -

$$V_{2F} = -I_{2S}(Z_{2S} + mZ_{2L})$$

Relay R –

$$V_{2F} = -I_{2R}(Z_{2R} + (1-m)Z_{2L})$$

Eliminating V_{2F} from the equations –

$$I_{2R} = I_{2S}\,\frac{Z_{2S} + m\,Z_{2L}}{Z_{2R} + (1-m)Z_{2L}}$$

To avoid alignment of data set, take the magnitude of the equation at both sides.

$$|I_{2R}| = \left| I_{2S}\,\frac{Z_{2S} + m\,Z_{2L}}{Z_{2R} + (1-m)Z_{2L}} \right|$$

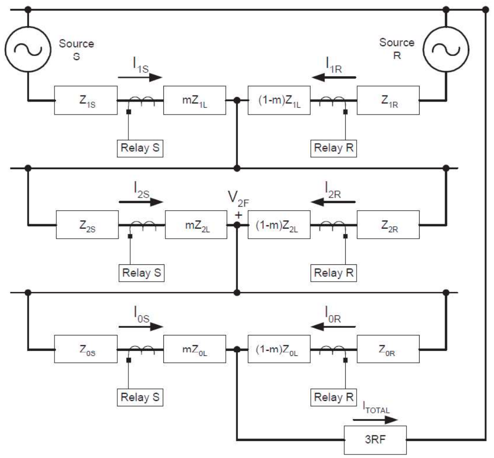
It can be written as

$$|I_{2R}|^2 = \left| \frac{I_{2S}Z_{2S} + mI_{2S}\,Z_{2L}}{Z_{2R} + Z_{2L} - mZ_{2L}} \right|^2$$

This can be written in form of a quadratic equation.

$$Am^2 + Bm + C = 0$$

It requires negative sequence impedance of line and sources at both ends.



Parallel Lines Fault Location – Izykowski's Method

- Consider the voltage drop along Route 1 and Route 2

$$dZ_{L1}I_{A1} = (Z_{\|} + (1-d)Z_{L1})I_{\|} \;\rightarrow (1-d)Z_{L1}I_{\|} + Z_{\|}I_{\|} = I_{A1}Z_{L1} - (1-d)I_{A1}Z_{L1}$$

$$(1-d)Z_{L1}(I_{\|} + I_{A1}) = I_{A1}Z_{L1} - Z_{\|}I_{\|} \;\rightarrow (1-d)Z_{L1}I_{F1} = I_{A1}Z_{L1} - -Z_{\|}I_{\|}$$

- Hence, fault current (positive sequence and negative sequence) can be represented by

$$I_{F1} = \frac{I_{A1} - \frac{Z_{\|1}}{Z_{L1}}I_{\|1}}{1-d}, \qquad I_{F2} = \frac{I_{A2} - \frac{Z_{\|2}}{Z_{L2}}I_{\|2}}{1-d}$$

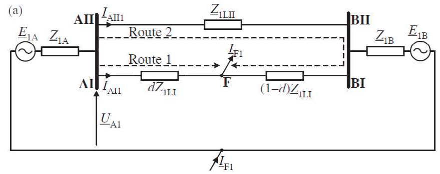
- With KVL, $U_A - dZ_{L1}I_{A1} - R_F I_F = 0$

$$U_A - dZ_{L1}I_{A1} - \frac{R_F}{1-d}\overbrace{\left[a_{F1}\left(I_{A1} - \frac{Z_{\|1}}{Z_{L1}}I_{\|1}\right) + a_{F2}\left(I_{A2} - \frac{Z_{\|2}}{Z_{L2}}I_{\|2}\right)\right]}^{\triangleq N_{12}} = 0$$

- Comparing the Coefficient and Eliminating the term $R_F / (1-d)$,

$$d = \frac{\mathrm{Im}(U_A)\mathrm{Re}(N_{12}) - \mathrm{Re}(U_A)\mathrm{Im}(N_{12})}{\mathrm{Im}(Z_{L1}I_A)\mathrm{Re}(N_{12}) - \mathrm{Re}(Z_{L1}I_A)\mathrm{Im}(N_{12})} \;\rightarrow\; \boxed{d = \frac{\mathrm{Im}(U_A\,N_{12}^*)}{\mathrm{Im}(Z_{L1}I_A\,N_{12}^*)}}$$

This is coined as **Izykowski's Method** (2004)



Distance Protection –

$$Z_A^{DP} = \frac{U_A}{I_A}$$

Fault Location –

$$Z_A^{FL} = \frac{\mathrm{Im}(U_A\,N_{12}^*)}{\mathrm{Im}(Z_{L1}I_A\,N_{12}^*)}Z_{1L}$$

Parallel Lines Fault Location

Comparison of fault-loop impedance measurements

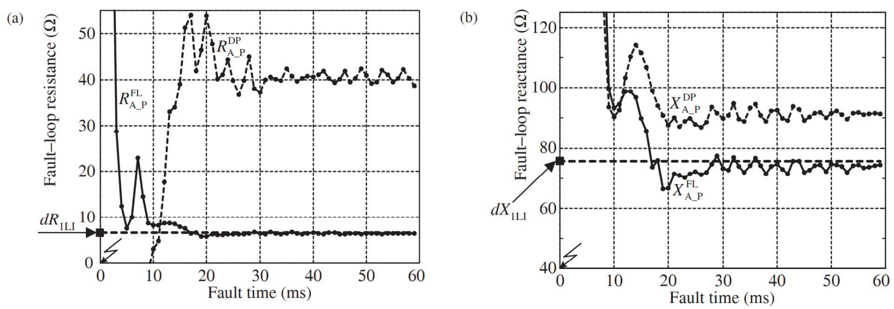
Distance Protection –

$$Z_A^{DP} = \frac{U_A}{I_A}$$

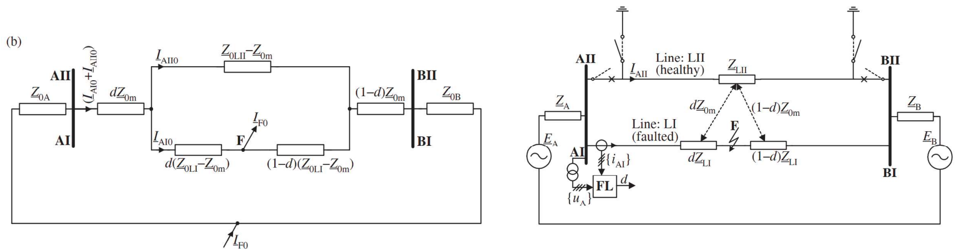
Fault Location –

$$Z_A^{FL} = \frac{\mathrm{Im}(U_A\,N_{12}^*)}{\mathrm{Im}(Z_{L1}I_A\,N_{12}^*)}Z_{1L}$$

For solid faults ($R_F = 0$), the apparent impedance is equal to the portion of positive sequence line impedance, i.e. $Z_A = d\,Z_{1L}$. Other faults, due to the "reactance effect", the apparent impedance Z_A^{DP} is not a strict measure of distance to fault. The measurement is affected by reactance effect and presence of pre-fault power flow.



Parallel Lines Fault Location – Eriksson's Method



Recall Generalized Fault Model with Distributed Factors –

$$U_A - dZ_{L1}I_A - R_F\left(\frac{a_{F1}\Delta I_{A1}}{k_{F1}} + \frac{a_{F2}\Delta I_{A2}}{k_{F2}} + \frac{a_{F2}\Delta I_{A0}}{k_{F0}}\right) = 0$$

where measured current I_A is a sum of several factors.

$$I_A = I_A^{SL} + a_0\frac{Z_{0m}}{Z_{L1}}I_{\|} = \left(a_1I_{A1} + a_2I_{A2} + a_0\frac{Z_{0L}}{Z_{1L}}I_{A0}\right) + \cdots$$

It is also noted that zero-sequence current can be represented numerically by positive sequence and negative sequence current.

$$I_{F0} = b_{F1}I_{F1} + b_{F2}I_{F2}$$

Recall for parallel overhead line, we have,

- Phase-Shifting Effect on CT Saturation, which acquired increased excitation current leading to missing current output

$$I_{F1} = \frac{\Delta I_{A1} - \frac{Z_{\|}}{Z_{L1}}\Delta I_{\|}}{1-d} \;\rightarrow\; I_{F0} = \frac{\Delta I_{A0} - P_0 I_{\|}}{1-d}$$

where

$$P_0\Big|_{\text{parallel run}} = \frac{Z_{\|0} - Z_{0m}}{Z_{0L} - Z_{0m}}, \qquad P_0\Big|_{\text{adjacent earthed}} = -\frac{Z_{\|0}}{Z_{0m}}$$

Parallel Lines Fault Location – Eriksson's Method

Combining the two equation, the parallel (healthy) line current can be found.

$$\left.\begin{aligned} I_{F0} &= b_{F1}I_{F1} + b_{F2}I_{F2} \\ I_{F0} &= \frac{\Delta I_{A0} - P_0 I_{\parallel}}{1-d} \end{aligned}\right\} \quad \frac{\Delta I_{A0} - P_0 I_{\parallel}}{1-d} = b_{F1}I_{F1} + b_{F2}I_{F2} \;\rightarrow\; I_{\parallel} = \frac{1}{P_0}\left[\Delta I_{A0} - (1-d)(b_{F1}\Delta I_{A1} + b_{F2}\Delta I_{A2})\left(\frac{M_1}{K_1 d + L_1}\right)\right]$$

Putting back the healthy line current into I_A, and with the fault loop model,

$$U_A - dZ_{L1}I_A - \frac{R_F}{k_F}(a_{F1}\Delta I_{A1} + a_{F2}\Delta I_{A2}) = 0 \qquad I_A = I_A^{SL} + a_0\frac{Z_{0m}}{Z_{L1}}I_{\parallel}$$

$$U_A - dZ_{L1}\left\{I_A^{SL} + a_0\frac{Z_{0m}}{Z_{L1}}\frac{1}{P_0}\left[\Delta I_{A0} - (1-d)(b_{F1}\Delta I_{A1} + b_{F2}\Delta I_{A2})\left(\frac{M_1}{K_1 d + L_1}\right)\right]\right\} - \frac{R_F}{k_F}(a_{F1}\Delta I_{A1} + a_{F2}\Delta I_{A2}) = 0$$

which can be put to quadratic form $\boxed{A_2 d^2 + A_1 d + A_0 + A_{00}R_F = 0}$

Same as **Eriksson's Method**

where

$$A_2 = -Z_{L1}K_1 I_A^{SL} - \frac{Z_{0m}}{P_0}(K_1 I_{A0} - (b_{F1}\Delta I_{A1} + b_{F2}\Delta I_{A2})M_1$$

$$A_1 = K_1 U_A - Z_{L1}L_1 I_A^{SL} - \frac{Z_{0m}}{P_0}L_1 I_{A0}$$

$$A_0 = L_1 U_A$$

$$A_0 = -M_1(a_{F1}\Delta I_{A1} + a_{F2}\Delta I_{A2})$$

Parallel Line Fault Location – Djuric's Method

Given the Conversion Factors and Zero-Sequence Current Conversion.

$$I_{F1} = \frac{\Delta I_{A1}}{k_{F1}}, \qquad I_{F2} = \frac{\Delta I_{A2}}{k_{F2}}, \qquad I_{F0} = \frac{\Delta I_{A0}}{k_{F0}} \qquad k_{F1} = k_{F2} = \frac{K_1 d + L_1}{M_1}, \qquad k_{F0} = \frac{K_0 d + L_0}{M_0}$$

$I_{F0} = b_{F1}I_{F1} + b_{F2}I_{F2}$

Combining, we have

$$\Delta I_{A0}\left(\frac{M_0}{K_0 d + L_0}\right) = (b_{F1}\Delta I_{A1} + b_{F2}\Delta I_{A2})\left(\frac{M_1}{K_1 d + L_1}\right)$$

Put d as the main term,

$$\boxed{d = -\frac{L_1[M_0 I_{A0} - M_1(b_{F1}\Delta I_{A1} + b_{F2}\Delta I_{A2})]}{K_1[M_0 I_{A0} - M_1(b_{F1}\Delta I_{A1} + b_{F2}I_{A2})]}}$$

This is coined as **Djuric's Method** (1998)

It requires

- Correct formulation of network (page 12) for parameter K_1, L_1, M_1 and K_0, L_0, M_0.
- Correct detection of fault type for parameter b_{F1} and b_{F2}. (Note – still need correct current measurement)
- Correct current measurement (without CT saturation) and memory.

To reduce the effect of CT saturation, voltage signal is replaced for the distance calculation.

$$\Delta I_{A1} = -\frac{\Delta U_{A1}}{Z_{1A}}, \qquad \Delta I_{A2} = -\frac{\Delta U_{A2}}{Z_{1A}}, \qquad \Delta I_{A0} = -\frac{\Delta U_{A0}}{Z_{0A}}$$

Hence, the distance (percentage length) could be represented as -

$$d = \boxed{-\frac{Z_{1A}L_1 M_0 U_{A0} - Z_{0A}L_0 M_1(b_{F1}\Delta I_{A1} + b_{F2}\Delta I_{A2})}{Z_{1A}K_1 M_0 U_{A0} - Z_{0A}K_0 M_1(b_{F1}\Delta I_{A1} + b_{F2}\Delta I_{A2})}}$$

This is coined as **Izykowski's Method** (2008)

Fault Location with Arc-Voltage Estimation

Assume arc voltage waveform is a square wave in phase with arc current. Hence,

$$u_F(t) = U_{arc}\,\mathrm{sgn}(i_A(t)) + R_F i_F(t) + \xi(t) \qquad \xi(t) = \text{noise}$$

Consider the line model with $R_i + jX_i$

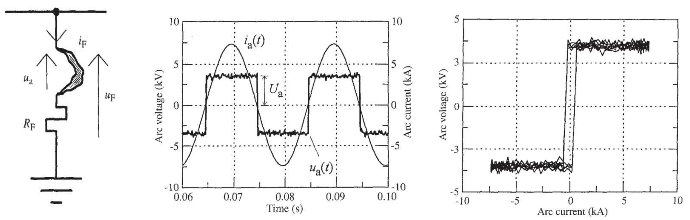
$$\begin{cases} u_1 = Ri_1 + L\,di_1/dt + u_{F1} \\ u_2 = Ri_2 + L\,di_2/dt + u_{F2} \\ u_0 = R_0 i_0 + L_0\,di_0/dt + u_{F0} \end{cases}$$

Combining the three equations,

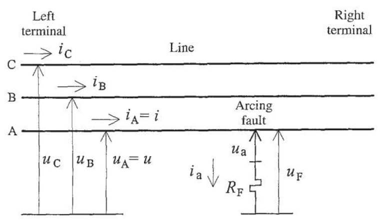
$$U_A = R(I_A - I_0) + R_0 I_0 + L\left(\frac{dI_A}{dt} + K_L\frac{dI_0}{dt}\right) + u_F$$

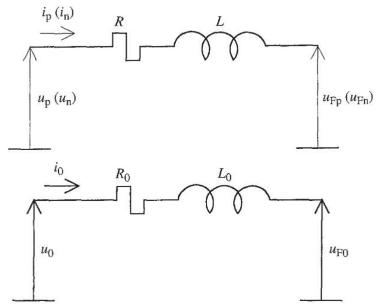
Let $i_F = k_a i_0$

$$U_A = R(I_A - I_0) + L\left(\frac{dI_A}{dt} + K_L\frac{dI_0}{dt}\right) + U_{arc}\,\mathrm{sgn}(I_A) + (R_0 + k_a R_F)I_0 + \varepsilon$$



Fault Location with Arc-Voltage Estimation





Discretizing,

$$U_{A[k]} = \left\{r\left(I_{A[k]} - I_{0[k]}\right) + \frac{x}{\omega}\left(\frac{I_{A[k+1]} - I_{A[k-1]}}{2T} + K_L\frac{I_{0[k+1]} - I_{0[k-1]}}{2T} + \right)\right\} l + \mathrm{sgn}\left(I_{A[k]}\right) U_{arc} + I_{0[k]}R_e + \varepsilon$$

Hence, it can be written as

$$u_k = (a_{k1} \;\; a_{k2} \;\; a_{k3})\begin{pmatrix} l \\ U_{arc} \\ R_e \end{pmatrix} + \varepsilon \;\rightarrow\; \overbrace{\begin{pmatrix} u_k \\ u_{k-1} \\ \dots \\ u_{k-m} \end{pmatrix}}^{\triangleq \mathbf{u}} = \overbrace{\begin{pmatrix} a_{k1} & a_{k2} & a_{k3} \\ a_{(k-1)1} & a_{(k-1)2} & a_{(k-1)3} \\ \vdots & \vdots & \vdots \\ a_{(k-m)1} & a_{(k-m)2} & a_{(k-m)3} \end{pmatrix}}^{\triangleq \mathbf{A}} \overbrace{\begin{pmatrix} l \\ U_{arc} \\ R_e \end{pmatrix}}^{\triangleq \mathbf{x}} + \overbrace{\begin{pmatrix} \varepsilon_k \\ \varepsilon_{k-1} \\ \dots \\ \varepsilon_{k-m} \end{pmatrix}}^{\triangleq \boldsymbol{\varepsilon}}$$

Djuric (1998)

Stacking all time steps and perform a least-square for minimum square error,

$$\boxed{l = (1\;0\;0)\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{u}}$$

Accuracy Improvement by Application of Distributed Line Model

Distributed-Parameter Model works similar to Lumped-Parameter Model, except the correction factor for shunt (sh) and series (th) impedance effect. The determination of fault location is performed iteratively with $d_{[n-1]}$ as the previous iteration.

$$U_A - dZ_L\left(a_1 I_{A1} + a_2 I_{A2} + a_0 \frac{Z_{0L}}{Z_{1L}} I_{A0}\right) - R_F\left(a_{F1} I_{F1} + a_{F2} I_{F2} + a_{F0} I_{F0}\right) = 0$$

Introduce the distribution parameter,

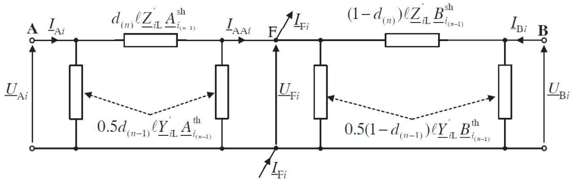
$$U_A - d_{[n]} Z_L\left(a_1 A_{1[n-1]}^{\text{sh}} I_{A1}^{\text{comp}} + a_2 A_{2[n-1]}^{\text{sh}} I_{A2}^{\text{comp}} + a_0 A_{0[n-1]}^{\text{sh}} \frac{Z_{0L}}{Z_{1L}} I_{A0}^{\text{comp}}\right) - R_F\left(a_{F1} I_{F1} + a_{F2} I_{F2} + a_{F0} I_{F0}\right) = 0$$

where

$$A_{i[n-1]}^{\text{sh}} = \frac{\sinh\left(\gamma_i d_{[n-1]} l\right)}{\gamma_i d_{[n-1]} l}, A_{i[n-1]}^{\text{th}} = \frac{\tanh\left(0.5\gamma_i d_{[n-1]} l\right)}{0.5\gamma_i d_{[n-1]} l} \quad i = 0, 1, 2$$

and the sequence current after deducing shunt capacitance current is

$$I_{Ai}^{\text{comp}} = I_{Ai} - \frac{Y_{iL}}{2} d_{[n-1]} l A_{i[n-1]}^{\text{th}} U_{Ai} \quad i = 0, 1, 2$$



Comparison between Impedance Based Fault Location Techniques

Input Data	Simple Reactance	Takagi	Modified Takagi	Eriksson	Novosel et al.	Synchronized Two-ended	Unsynchronized Two-ended
Fault Event Data							
Fault Type	✓	✓	✓	✓	✓		
Fault Voltage[1] Current Phasor (Local End)	✓	✓	✓	✓	✓	✓	✓
Fault Voltage Phasor[1] (Remote End)						✓	✓
Fault Current Phasor (Remote End)						✓	✓
Synchronized Data						✓	
Prefault Current Phasor		✓		✓	✓		
Prefault Voltage Phasor					✓		
Transmission Line Parameters							
Line Length	✓	✓	✓	✓	✓	✓	✓
Positive-sequence Line Impedance	✓	✓	✓	✓	✓	✓	✓
Zero-sequence Line Impedance	✓	✓	✓	✓	✓		
Source Impedance Parameters							
Positive-sequence Source Impedance (Local End)				Optional	Optional		
Positive-sequence Source Impedance (Remote End)				✓			
Negative-sequence Source Impedance (Local End)							
Negative-sequence Source Impedance (Remote End)							
Zero-sequence Source[2] Impedance (Local End)			Optional				
Zero-sequence Source Impedance (Remote End)			✓				

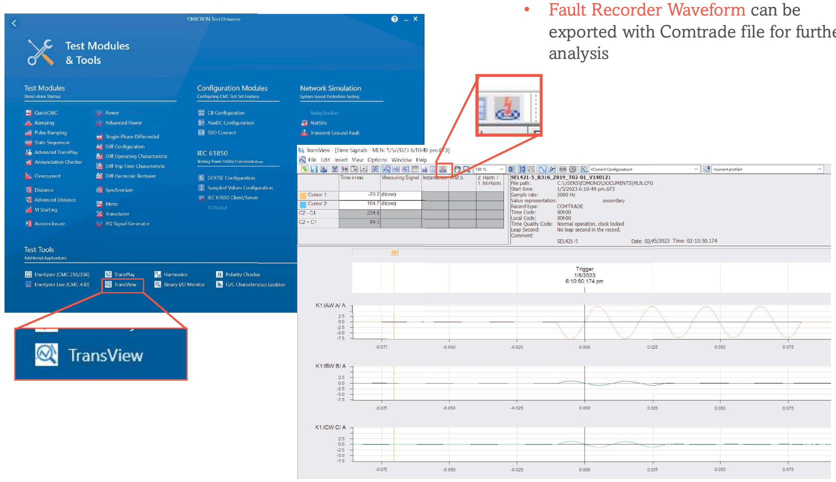
Comparison between Impedance Based Fault Location Techniques

Input Data	Simple Reactance	Takagi	Modified Takagi	Eriksson	Novosel et al.	Synchronized Two-ended	Unsynchronized Two-ended	Unsynchronized Current-only Two-ended
Instrument Transformer Challenges								
Loss of Potential	✓	✓	✓	✓	✓	✓	✓	
CT Saturation	✓	✓	✓	✓	✓	✓	✓	✓
Delta-connected[1] Potential Transformers	✓	✓	✓	✓	✓	✓	✓	
Power System Challenges								
System Load	✓							
Non-homogeneous System	✓	✓						
Parallel Lines[2] (Mutual Coupling)	✓	✓	✓	✓	✓			✓
Fault Related Challenges								
Fault Resistance	✓	✓						
Fault Incidence Angle (DC Offset)	✓	✓	✓	✓	✓	✓	✓	✓
Application Related Challenges								
Two-terminal Lines	✓	✓			N/A			
Soil Type, Weather[3] Temperature, Season (Earth Resistivity)	✓	✓	✓	✓	✓			
Untransposed Lines	✓	✓	✓	✓	✓	✓	✓	✓
Tapped Lines[4]	✓	✓	✓	✓	✓	✓	✓	✓
Three-terminal Lines[4]	✓	✓	✓	✓	✓	✓	✓	✓

[1] Delta-connected PTs pose a problem in locating single line-to-ground faults only. If the zero-sequence impedance of the local source is available, estimate the corresponding line-to-ground voltages.
[2] Mutual coupling affects the accuracy of one-ended algorithms in locating single line-to-ground faults only. If transmission lines are parallel for the entire line length, then the residual current from the parallel line can improve the accuracy of one-ended methods. The unsynchronized current-only two-ended method, however, requires the negative-sequence currents at both ends of the parallel line for all fault types.
[3] Earth resistivity affects the accuracy of locating single line-to-ground faults only.
[4] It is possible to modify two-ended methods for application to tapped lines and three-terminal lines.

Fault Location in Practice – Omicron TransView

A circuit YUE RMU 2 – MLN No.2 was under fault in 2023-05-02.
Backup Distance (typed SEL421) has been activated, but the fault locator was failed to locate the fault.
Omicron Test Universe – TransView was employed to perform such fault location.



- Fault Recorder Waveform can be exported with Comtrade file for further analysis

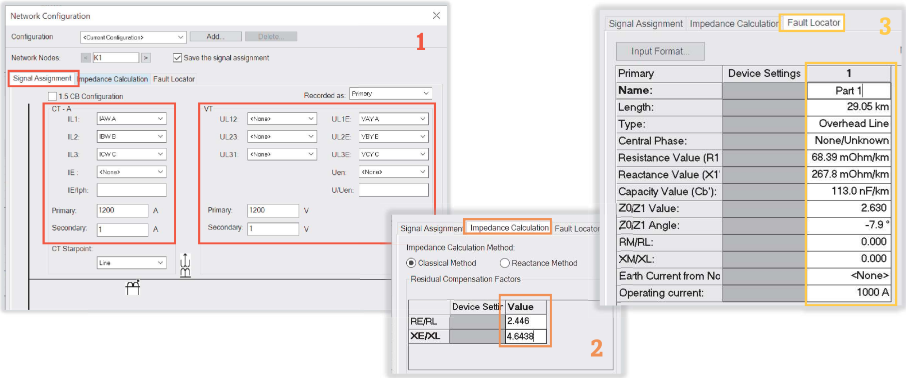
Fault Location in Practice – Omicron TransView

- Given CT Ratio = 1200/1 and VT Ratio = 132000/1 = 1200/1
- Given the circuit parameter

Circuit Total:

Type	Z0/Z1	Length(m)	R1(PU)	X1(PU)	Ro(PU)	Xo(PU)	Xc(PU)
Double Circuit in Service (OHL) + Single Circuit Only (OHL) + Cables	2.63	29054.6	1.137e-02	4.453e-02	5.280e-02	1.085e-01	-4.645e+00

- After clicking the "Fault Locator" function, input the information as follows.



Fault Location in Practice – Omicron TransView

For YUE RMU 2 – MLN 2

Circuit Total:

Type	Z0/Z1	Length(m)	R1(PU)	X1(PU)	Ro(PU)	Xo(PU)	Xc(PU)
Double Circuit in Service (OHL) + Single Circuit Only (OHL) + Cables	2.63	29054.6	1.137e-02	4.453e-02	5.280e-02	1.085e-01	-4.645e+00

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(132\text{kV})^2}{100\text{MVA}} = 174.74$$

$$R_1 = \frac{R_1[pu] \times Z_{base}}{L[km]} = \frac{(1.137 \times 10^{-2}) \times 174.74}{29.05} = 68.4\text{m}\Omega/\text{km}$$

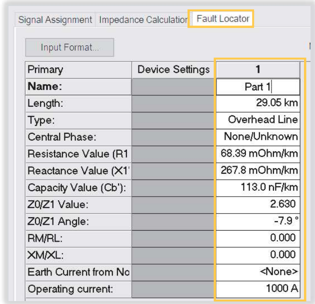
$$X_1 = \frac{X_1[pu] \times Z_{base}}{L[km]} = \frac{(4.453 \times 10^{-2}) \times 174.74}{29.05} = 267.8\text{m}\Omega/\text{km}$$

$$X_C = X_C[pu] \times Z_{base} \times L[km] = \frac{1}{j\omega C}$$

$$\rightarrow Cb' = \frac{1}{2\pi 50\,(4.645 \times 174.74 \times 29.05)} = 135\text{nF/km}$$

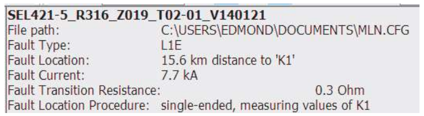
$$\frac{Z_0}{Z_1} = \frac{R_0 + jX_0}{R_1 + jX_1} = \frac{5.280 \times 10^{-2} + j1.085 \times 10^{-1}}{1.137 \times 10^{-2} + j4.453 \times 10^{-2}} = 2.63\angle -11.6^o$$

$$\frac{R_E}{R_L} = \frac{5.280 \times 10^{-2}}{1.137 \times 10^{-2}} = 4.643, \quad \frac{X_E}{X_L} = \frac{1.085 \times 10^{-1}}{4.453 \times 10^{-2}} = 2.661$$

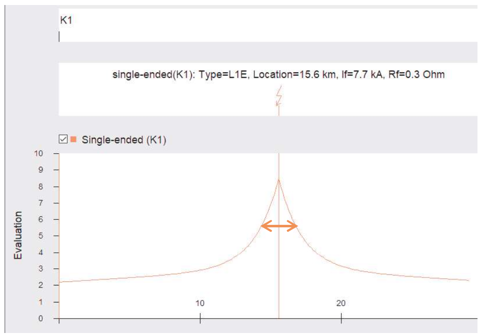


Fault Location in Practice – Omicron TransView

- The system fault was located as follows.
  "15.6km from Node K1" (given that only Node K1 information is available)



SEL421-5_R316_Z019_T02-01_V140121
File path: C:\USERS\EDMOND\DOCUMENTS\MLN.CFG
Fault Type: L1E
Fault Location: 15.6 km distance to 'K1'
Fault Current: 7.7 kA
Fault Transition Resistance: 0.3 Ohm
Fault Location Procedure: single-ended, measuring values of K1



- It was noted that fault location may not converge or may provide faulty location, due to the possible error in Page 4.