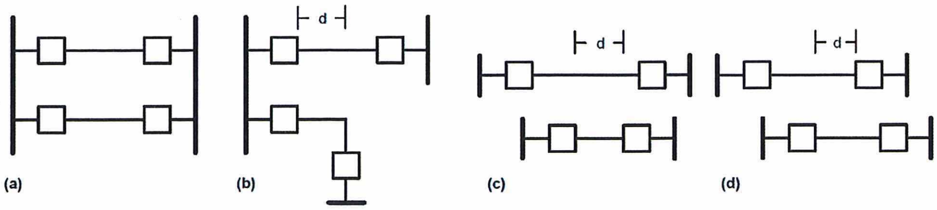
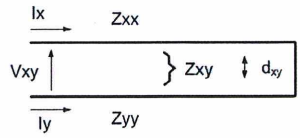
B1 Modelling of Mutually Coupled Lines for Distance Protection

When two or more lines are running parallel to each other, mutual impedances between the lines modify the voltage and current profile measured in the protective relays. It is only an ideal conditions that a transmission line can be fully represented by decoupled zero- and positive-sequence impedance when mutual coupling effect is concerned. Here we discuss the modelling of mutually coupled lines, application of ground distance elements and mutual compensation for distance elements.



B1.1 Modelling of Mutually Coupled Lines

To perform analysis on unbalanced network, interactions between different phases and mutually coupled lines, such as parallel lines in Figure (a), parallel lines with one circuit switched off and isolated, and parallel lines with one circuit isolated and earthed, or other topologies with part of the lines running in parallel, coupled in one side or earthed in one side, phase network is considered.

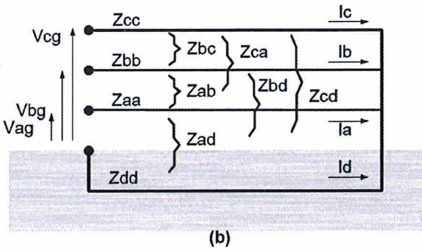


(a)

Consider the self- and mutual- impedance with modified Carson's Equation:

$$\begin{cases} Z_{xx} = r_x + j\,0.07537\left(\dfrac{f}{60}\right)\ln\left(\dfrac{1}{GMR_x}\right) \; [\Omega/\text{km}] \\ Z_{xy} = j\,0.07357\left(\dfrac{f}{60}\right)\ln\left(\dfrac{1}{d_{xy}}\right) \; [\Omega/\text{km}] \end{cases} \tag{B1.1}$$

where f= system frequency, r_x = line resistance, GMR_x = geometric mean radius and d_{xy} = distance between lines respectively.



(b)

In a three-phase power system, the ground return could be modelled with an equivalent ground conductor as shown in Figure (b). The result is applicable to N-lines with current returning through ground.

In the analysis of three-conductor system, we take phase 'a' as the 'reference' conductor and write the equation for Vag.

$$V_{ag} = Z_{aa}I_a + Z_{ab}I_b + Z_{ac}I_c - (Z_{dd}I_d + Z_{ad}I_a + Z_{bd}I_b + Z_{cd}I_c) \tag{B1.2}$$

It simplifies to

$$V_{ag} = (Z_{aa} - Z_{ad})I_a + (Z_{ab} - Z_{bd})I_b + (Z_{ca} - Z_{cd})I_c + (Z_{ad} - Z_{dd})I_d \tag{B1.3}$$

With KCL: $I_d = -(I_a + I_b + I_c)$, the equation is further simplified to:

$$V_{ag} = (Z_{aa} - 2Z_{ad} + Z_{dd})I_a + (Z_{ab} - Z_{bd} - Z_{ad} + Z_{dd})I_b + (Z_{ca} - Z_{cd} - Z_{ad} + Z_{dd})I_c \tag{B1.4}$$

In short, it can be written as

$$V_{ag} = Z_{AA}I_a + Z_{AB}I_b + Z_{AC}I_c \tag{B1.5}$$

where Z_{AA} is the self-impedance of the 'a' conductor, Z_{AB} and Z_{AC} impedances are the mutual impedance to the 'b' and 'c' conductor respectively.

Putting (B1.1) to find the expressions for Z_{AA}, Z_{AB}, Z_{AC} and further mathematical simplification, the impedances are defined as

$$\begin{cases} Z_{AA} = r_a + r_d + j\,0.07537\left(\dfrac{f}{60}\right)\left[\ln\left(\dfrac{1}{GMR_a}\right) + \ln\left(\dfrac{d_{ad}^2}{GMR_d}\right)\right] \; [\Omega/\text{km}] \\ Z_{AB} = r_d + j\,0.07357\left(\dfrac{f}{60}\right)\left[\ln\left(\dfrac{1}{d_{ab}}\right) + \ln\left(\dfrac{d_{ad}d_{bd}}{GMR_d}\right)\right] \; [\Omega/\text{km}] \end{cases} \tag{B1.6}$$

The above development effectively removed the ground return from the self- and mutual- impedance of the three conductors. Yet, the unfortunate part is that the 'equivalent' ground conductor is not physical nor is its GMR or the distance to other conductors known.

In 1926, John Carson published a classic paper deriving equations for EM waves propagating in electrical conductors and returning through grounds. Yet, Carson provided an approximation to the self and mutual impedance for any 'i' and 'j' conductors above ground with typical earth resistivity of σ_e = 100 ohms/m³.

$$\begin{cases} Z_{ii} = r_i + 0.05919\left(\dfrac{f}{60}\right) + j\,0.07537\left(\dfrac{f}{60}\right)\left[\ln\left(\dfrac{1}{GMR_i}\right) + 6.7458\right] \; [\Omega/\text{km}] \\ Z_{ij} = 0.05919\left(\dfrac{f}{60}\right) + j0.07537\left(\dfrac{f}{60}\right)\left[\ln\left(\dfrac{1}{d_{ij}}\right) + 6.7458\right] \; [\Omega/\text{km}] \end{cases} \tag{B1.7}$$

For any arrangement of conductors (a, b, c) of a single circuit transmission line, the matrix Z_{abc} describes the line impedance using (B1.7)

$$Z_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \tag{B1.8}$$

For any arrangement of conductors of a double-circuit line or two parallel single circuit lines (a, b, c, a', b', c'), the matrix Zabc'' describes the line impedance.

$$Z''_{abc} = \begin{bmatrix} Z_{abc} & Z_{aa'} \\ Z_{aa'} & Z_{abc'} \end{bmatrix} = \left[\begin{array}{ccc|ccc} Z_{aa} & Z_{ab} & Z_{ac} & Z_{a'a} & Z_{a'b} & Z_{a'c} \\ Z_{ab} & Z_{bb} & Z_{bc} & Z_{a'b} & Z_{b'b} & Z_{b'c} \\ Z_{ac} & Z_{bc} & Z_{cc} & Z_{a'c} & Z_{b'c} & Z_{c'c} \\ \hline Z_{a'a} & Z_{a'b} & Z_{a'c} & Z_{a'a'} & Z_{a'b'} & Z_{a'c'} \\ Z_{a'b} & Z_{b'b} & Z_{b'c} & Z_{a'b'} & Z_{b'b'} & Z_{b'c'} \\ Z_{a'c} & Z_{b'c} & Z_{c'c} & Z_{a'c'} & Z_{b'c'} & Z_{c'c'} \end{array}\right] \tag{B1.9}$$

(B1.8) and (B1.9) are often referred to as the phase impedance matrix. When ground wires are presented, they can be included in the above matrices with Kron's reduction techniques. Phase impedances are useful to for steady-state unbalanced flow analysis. Yet, the symmetrical component impedances are of interest to the protection engineers because the positive- and negative- sequence impedance are of concerns in transmission line protection. Using the phase impedance matrix in (B1.9), we have

$$[\Delta V''_{abc}] = [Z''_{abc}][I''_{abc}] \tag{B1.10}$$

Transferring the 012-domain to abc-domain with T-matrix,

$$[T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \longrightarrow \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \end{bmatrix} [\Delta V''_{012}] = [Z''_{abc}] \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \end{bmatrix} [I''_{012}] \tag{B1.11}$$

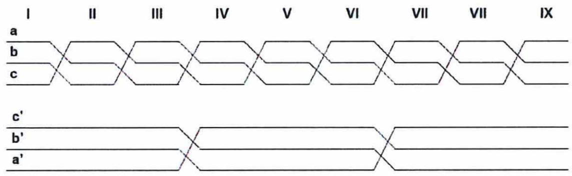
We have the sequence impedance as follows,

$$[Z''_{012}] = \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \end{bmatrix}^{-1} [Z''_{abc}] \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \end{bmatrix} = [T']^{-1}[Z''_{abc}][T'] \tag{B1.12}$$

The main diagonal impedances of the 6 x 6 matrix are the zero-, positive- and negative- sequence self impedance of conductor arrangement. The off-diagonal ones are the mutual impedance.

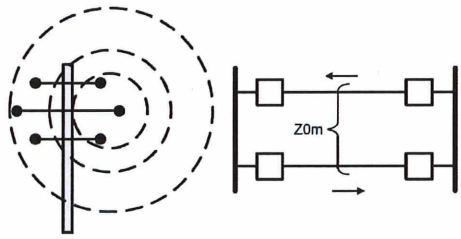
When the position of the phases is re-arranged in the same physical setup, it is said that the line is transposed, to balance the effect of mutual impedance in each phase and equate to length of three lines. If the Z_{012}'' impedance matrix is calculated, it would be the sum of the symmetric components, i.e.

$$Z''_{012} = \frac{1}{N}\sum_{i=1}^{N} Z_{012_i}'' \tag{B1.13}$$



Flux linkages depends on the total current flowing in one circuit and link this circuit to the other circuit. Hence, for positive- and negative- sequence current, the effective current in one transmission line is zero, and the flux linkage to the parallel line is negligible. However, the zero-sequence flux linking the other transmission line is significant as it does not add to zero.

In conclusion, Z_{0m} (zero-sequence mutual) is always present, regardless of phasing arrangement of the conductors or the number of transposition in the way. Its magnitude will vary according to geometry, phasing and transposition, and Z_{0m} is always measurable. In some instant, it will be comparable to the positive-sequence impedance.



To model the zero-sequence mutual flow in one transmission line, the mutual effect can be visualized as an induced voltage in parallel line proportional to Z_{0m} I_0.

B1.2 Distance Protection and Fault Location

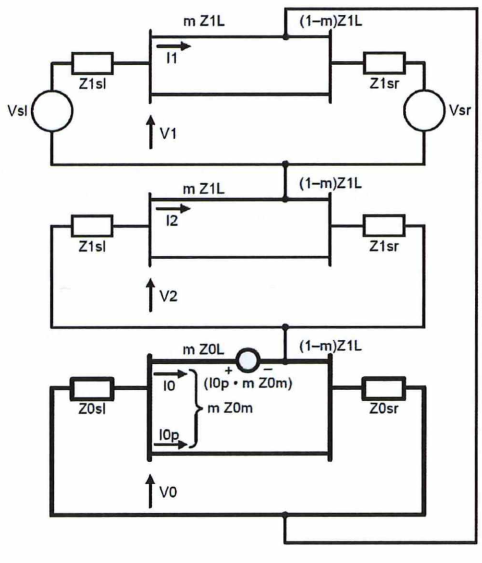
Consider a phase-to-ground fault in one of the parallel lines with zero-sequence mutual coupling.

At the fault location, with no fault resistance considered:

$$(V_1 - mZ_{1L}I_1) + (V_2 - mZ_{1L}I_2) + (V_0 - mZ_{0L}I_0 - mZ_{0m}I_{0p}) = 0 \tag{B1.14}$$

$$V_1 + V_2 + V_0 = mZ_{1L}I_1 + mZ_{1L}I_2 + mZ_{0L}I_0 + mZ_{0m}I_{0p} \tag{B1.15}$$

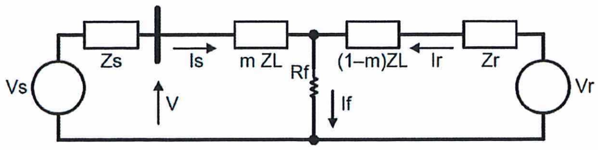
$$\boxed{V_A = mZ_{1L}\left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}}3I_0 + \frac{Z_{0m}}{3Z_{1L}}3I_{0p}\right)} \tag{B1.16}$$



(B1.16) implies that the zero-sequence mutual impedance (Z_{0m}) is needed to accurately measure the apparent impedance to the fault. It also depends the relay voltage and current to be used in distance relay, i.e. I_A, I_N and I_{0p}.

It is noted that the following affects the accuracy of fault location.

1. fault resistance R_F
2. infeed and outfeed current (I_r / I_s)
3. Inaccuracy of Line impedance calculation
4. Errors in CT and VT.



Consider the network as shown.

$$V = I_s\, mZ_L + I_f R_f \tag{B1.17}$$

where I_s is the fault current measured locally and $I_f = I_{fs} + I_{fr}$ is the fault current contributed from Vs and Vr.
Consider the change of measured current ΔI_s

$$\Delta I_s = I_{fs} - I_{spf} \tag{B1.18}$$

where I_{spf} is the pre-fault Vs side current. The largest source of error in the equations comes from the unknown fault resistance R_f.
Multiply both sides with the conjugate of ΔI_{fs}.

$$V\Delta I_{fs}^* = mZ_L I_s \Delta I_{fs}^* + R_f I_f I_{fs}^* \tag{B1.19}$$

It is noted that ΔI_f and I_{fs} have the same phase for a homogeneous system (similar source impedance angle in both side). It is noted that $R_f I_f I_s^*$ is a real number. Hence, Takagi Algorithm suggests

$$\boxed{m = \frac{Im\{V\Delta I_{fs}^*\}}{Im\{Z_L I_s \Delta I_{fs}^*\}}} \tag{B1.20}$$

A smarter choice for the reference projection base is the negative sequence current with pre-fault $I_2 = 0$.

$$\boxed{m = \frac{Im\{VI_2^*\}}{Im\{Z_L I_s I_2^*\}}} \tag{B1.21}$$

Consider (B1.21) with (B1.16):

$$V_A = mZ_{1L}\left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}} 3I_0 + \frac{Z_{0m}}{3Z_{1L}} 3I_{0p}\right) \tag{B1.16}$$

$$m = \frac{Im\{V_A I_{2A}^*\}}{Im\left\{Z_{1L}\left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}} 3I_0 + \frac{Z_{0m}}{3Z_{1L}} 3I_{0p}\right) I_{2A}^*\right\}} \tag{B1.22}$$

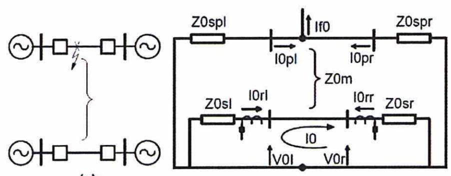
For more than 2 circuits,

$$m = \frac{Im\{V_A I_{2A}^*\}}{Im\left\{Z_{1L}\left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}} 3I_0 + \frac{Z_{0m}}{3Z_{1L}} 3I_{0p1} + \frac{Z_{0m}}{3Z_{1L}} 3I_{0p2} + \cdots\right) I_{2A}^*\right\}} \tag{B1.23}$$

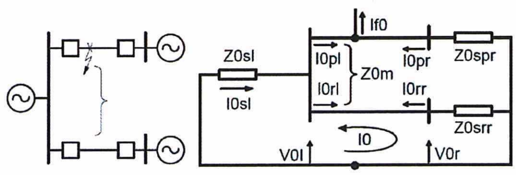
(B1.22) and (B1.23) can be optimized further with the non-homogeneity in the negative –sequence network to make the fault location more accurate.

Zero-sequence mutual impedance between two parallel lines can affect directional comparison systems using ground directional elements polarized with zero-sequence quantities. These quantities are zero-sequence voltage U_0, zero-sequence current I_0 and zero-sequence current from the transformer neutral.

Consider the zero-sequence ground directional that determines its direction based on the sign of V_0/I_0 ratio. For a forward fault, $V_0/I_0 = -Z_{s0}$ (negative impedance). For a reverse fault, the impedance is positive. Traditional ground directional elements use V_0 as the polarizing voltage, and I_0 as operating current.

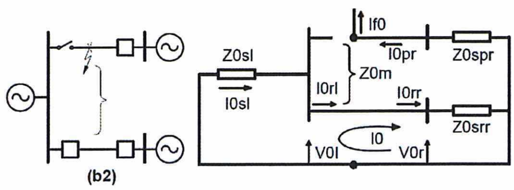


(a)



(b1)

It illustrates the fact that zero-sequence mutual can create problems with directional comparison system that use ground direction elements for ground fault detection, if by virtue of CB switching or configuration (line proximity), two isolated zero-sequence networks are formed.



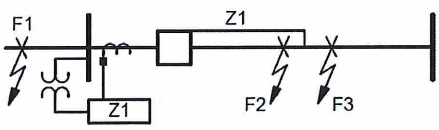
(b2)

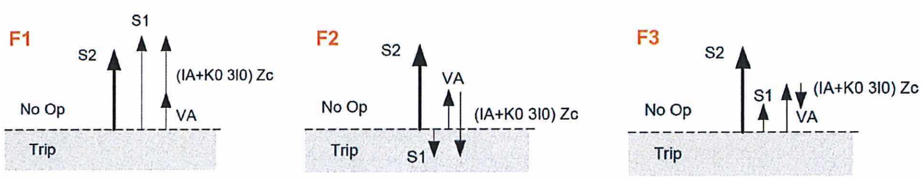
Negative sequence ground directional elements are often considered as a reliable choice for parallel line directional application.

Consider the operating and polarizing quantities of a distance element.

$$\text{Operating -} \quad S_1 = V_A - \left(I_A + \frac{Z_{L0} - Z_{L1}}{3Z_{L1}} 3I_0\right) Z_c \tag{B1.23}$$

$$\text{Polarizing -} \quad S_2 = V_1$$





(a) (b) (c)

The choice of S_2 provides the distance element with certain beneficial characteristics required in line distance protection, such as expansion, directionality, single-pole trip suitability and adaptivity to load flow.

Using (B1.16) as the basis to eliminate the effect of the zero-sequence mutual impedance, the proper operating quantity S1 should be

$$V_A = mZ_{1L}\left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}}3I_0 + \frac{Z_{0m}}{3Z_{1L}}3I_{0p}\right) \quad \text{(B1.16)}$$

$$S_1 = V_A - \left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}}3I_0 + \frac{Z_{0m}}{3Z_{1L}}3I_{0p}\right)Z_c \quad \text{(B1.24)}$$

Voltage V_A is the restraining quantity, while the sum of the current (I_A, I_0 and I_{0p} with their respective zero-sequence compensating factors) and the reach setting (Z_c) determines the operation of the unit. If the voltage is larger than the current (S1 > 0), the unit does not operate. If the currents are larger than the voltage (S1 < 0), the unit operates.

When a distance relay is NOT measuring the parallel line zero-sequence current, the extra term is the error in the measurement due to the zero-sequence mutual impedance. It can be concluded that the error of not measuring the parallel line zero-sequence current shows as an additional terms adding to the restraining voltage, i.e.

$$S_1 = \left(V_A + \frac{Z_{0m}}{3Z_{1L}}3I_{0p}Z_c\right) - \left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}}3I_0\right)Z_c, \qquad V_{res} = V_A + \frac{Z_{0m}}{3Z_{1L}}3I_{0p}Z_c \quad \text{(B1.25)}$$

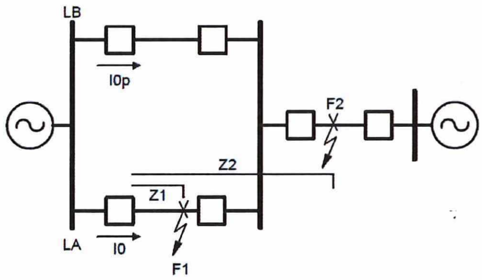
Overreaching zones, e.g. Zone 2, should be compensated to measure the zero-sequence mutual does not reduce the desired reach. Overreaching zones are used in directional comparison scheme (POTT for example), and the relay setting should make sure that the overreaching zones reach past the remote terminal or more. For F2 fault in the figure, both parallel lines carry the same direction and magnitude of zero-sequence current ($I_0 = I_{0p}$), therefore the ground distance calculation (B1.24) becomes

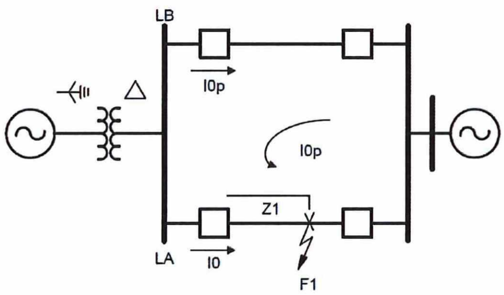
Parallel Line Application:
$$\boxed{S_1 = V_A - \left(I_A + \frac{Z_{0L} - Z_{1L} + Z_{0m}}{3Z_{1L}}3I_0\right)Z_c} \quad \text{(B1.26)}$$

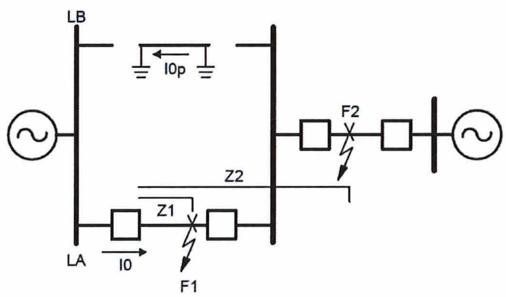
For a parallel line application, one could set

$$K_{01} = \frac{Z_{L0} - Z_{L1}}{3Z_{L1}}, \qquad Z_{0F} = \frac{Z_{L0} - Z_{L1} + Z_{0m}}{3Z_{L1}} \quad \text{(B1.27)}$$

The K01 factor is the setting for the underreaching zone (Zone 1), while K0F factor is the setting for the overreaching zone (Zone 2). With this choice of K0 factors, it is acknowledged that Zone 1 may underreach (but never overreach), and Zone 2 may overreach (to the remote terminal).







As an interesting case for parallel lines, where only one terminal has a ground source, the lower line F1 fault would make the $3I_0$ current flows in the opposite directions, but their magnitude will be the same. Because of the direction of I_0, the effect is to make zone 1 ground distance overreach.

Using (B1.16) with measured $3I_0$ equals to $-3I_{0p}$, an appropriate K0 factor can be found.

Single Point Earth:
$$\boxed{S_1 = V_A - \left(I_A + \frac{Z_{0L} - Z_{1L} - Z_{0m}}{3Z_{1L}}3I_0\right)Z_c,} \qquad K_{01} = \frac{Z_{L0} - Z_{L1} - Z_{0m}}{3Z_{1L}} \quad \text{(B1.28)}$$

Another consideration occurs when the overhead line is under maintenance, such that it is isolated and earthed. A ground fault in the operating line will produce a zero-sequence current in the earthed loop. The I_{0p} current is in the opposite direction as I_0 per the flux linkage law.

$$I_{0p} = \left(\frac{Z_{0m}}{Z_{L0}}\right)I_0 \quad \text{(B1.29)}$$

Substitute (B1.29) into (B1.16),

$$S_1 = V_A - \left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}}3I_0 + \frac{Z_{0m}}{3Z_{1L}}(-3I_{0p})\right)Z_c \quad \text{(B1.30)}$$

$$S_1 = V_A - \left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}}3I_0 + \frac{Z_{0m}}{3Z_{1L}}\left(\frac{Z_{0m}}{Z_{L0}}(-3I_0)\right)\right)Z_c \quad \text{(B1.31)}$$

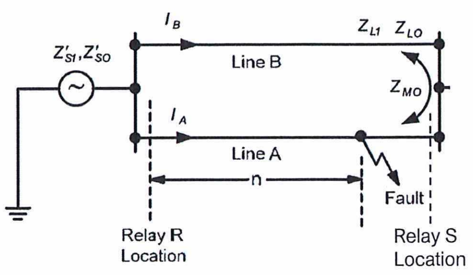
Isolated and Earth:
$$\boxed{S_1 = V_A - \left(I_A + \frac{Z_{0L} - Z_{1L} - \left(\frac{Z_{0m}^2}{Z_{L0}}\right)}{3Z_{1L}}3I_0\right)Z_c} \rightarrow K_{01} = \frac{Z_{0L} - Z_{1L} - \left(\frac{Z_{0m}^2}{Z_{L0}}\right)}{3Z_{1L}} \quad \text{(B1.32)}$$

Recall the compensation factor –

$$V_A = I_A nZ_{L1} + I_N Z_N = I_A nZ_{L1} + I_N(K_N nZ_{L1}) = (I_A + K_N I_N)nZ_{L1} \quad \text{(B1.33)}$$

$$V_A = nZ_{1L}\left(I_A + \frac{Z_{0L}-Z_{1L}}{3Z_{1L}} I_N\right) = nZ_{1L}(I_A + K_N I_N) \quad \text{(B1.16)}$$

- For OHL application, $\angle Z_{L0} \approx \angle Z_{L1},\ \angle K_N \approx 0$ → Scalar Compensation
- For Cable application, $\angle Z_{L0} \neq \angle Z_{L1},\ \angle K_N \neq 0$ → Vector Compensation

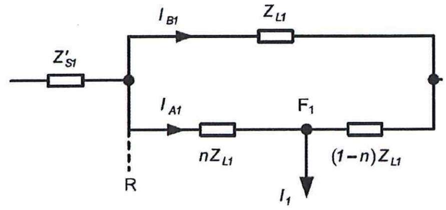


Consider a phase-to-ground fault on Line A.
Faulted phase voltage as seen by relay R:

$$V_A = V_{A1} + V_{A2} + V_{A0} \quad \text{(B1.34)}$$

$$= I_{A1}\ nZ_{L1} + I_{A2}\ nZ_{L1} + I_{A0}\ nZ_{L0} + I_{B0}\ nZ_{0m}$$

$$= nZ_{L1}\left(I_{A1} + I_{A2} + I_{A0}\frac{Z_{L0}}{Z_{L1}} + I_{B0}\frac{Z_{0m}}{Z_{L1}}\right)$$



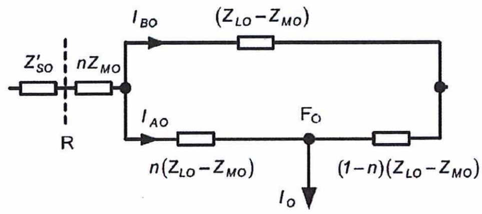
Positive Sequence Network

Faulted phase current as seen by relay R:

$$I_A = I_{A1} + I_{A2} + I_{A0} + K_N I_{A0}, \qquad K_N = (Z_{L0} - Z_{L1})/Z_{L1} \quad \text{(B1.35)}$$

$$= I_{A1} + I_{A2} + I_{A0}\left(\frac{Z_{L0}}{Z_{L1}}\right) \qquad I_{A1} = I_{A2} = I_{A0}$$

$$= I_{A0}\left(2 + \frac{Z_{L0}}{Z_{L1}}\right) = I_{A0}\frac{2Z_{L1} + Z_0}{Z_{L1}}$$



Zero Sequence Network

Impedance as seen by relay R:

$$\frac{V_A}{I_A} = Z_A = \frac{nZ_{L1}\left(I_{A1} + I_{A2} + I_{A0}\frac{Z_{L0}}{Z_{L1}} + I_{B0}\frac{Z_{0m}}{Z_{L1}}\right)}{I_{A0}\frac{2Z_{L1} + Z_0}{Z_{L1}}} \quad \text{(B1.36)}$$

$$= nZ_{L1}\left(1 + \boxed{\frac{I_{B0}}{I_{A0}}\frac{Z_{0m}}{2Z_{L1} + Z_{L0}}}\right)$$

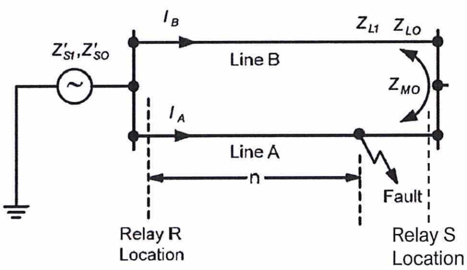
1. $Z_{0m} = 0$, $Z_A = nZ_{L1}$
2. Z_{0m} causes the relay to underreach by a factor of $\frac{I_{B0}}{I_{A0}}\frac{Z_{0m}}{2Z_{L1} + Z_{L0}}$

Faulted Phase Voltage as seen by relay S:

$$V_{S,A} = V_{S,A1} + V_{S,A2} + V_{S,A0} \quad \text{(B1.37)}$$

$$= I_{S,A1}\ (1-n)Z_{L1} + I_{S,A2}\ (1-n)Z_{L1} + I_{S,A0}\ (1-n)Z_{L0} + (-I_{S,A0})\ nZ_{0m}$$

$$= (1-n)Z_{L1}\left(I_{S,A1} + I_{S,A2} + I_{S,A0}\frac{Z_{L0}}{Z_{L1}} - I_{S,A0}\frac{Z_{0m}}{Z_{L1}}\right)$$



Faulted Phase Current as seen by relay S:

$$I_{S,A} = I_{S,A1} + I_{S,A2} + I_{S,A0} + K_N I_{S,A0}, \qquad K_N = (Z_{L0} - Z_{L1})/Z_{L1} \quad \text{(B1.38)}$$

$$= I_{S,A1} + I_{S,A2} + I_{S,A0}\left(\frac{Z_{L0}}{Z_{L1}}\right) \qquad I_{S,A1} = I_{S,A2} = I_{S,A0}$$

$$= I_{S,A0}\left(2 + \frac{Z_{L0}}{Z_{L1}}\right) = I_{S,A0}\frac{2Z_{L1} + Z_0}{Z_{L1}}$$

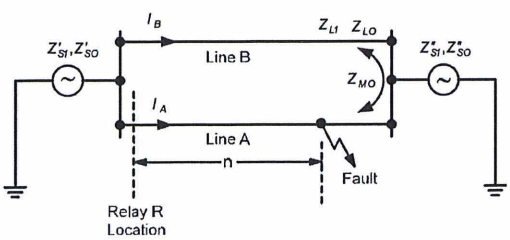
Impedance seen by relay S:

$$\frac{V_{S,A}}{I_{S,A}} = Z_{S,A} = \frac{(1-n)Z_{L1}\left(I_{S,A1} + I_{S,A2} + I_{S,A0}\frac{Z_{L0}}{Z_{L1}} - I_{S,A0}\frac{Z_{0m}}{Z_{L1}}\right)}{I_{S,A0}\frac{2Z_{L1} + Z_0}{Z_{L1}}} \quad \text{(B1.39)}$$

$$= (1-n)Z_{L1}\left(1 - \boxed{\frac{Z_{0m}}{2Z_{L1} + Z_{L0}}}\right)$$

1. $Z_{0m} = 0$, $Z_A = nZ_{L1}$
2. Z_{0m} causes the relay to overreach by a factor of $\frac{Z_{0m}}{2Z_{L1} + Z_{L0}}$

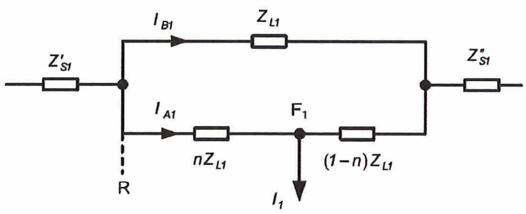
Consider the mutual coupling effect with double source infeed.
Impedance seen by relay R:



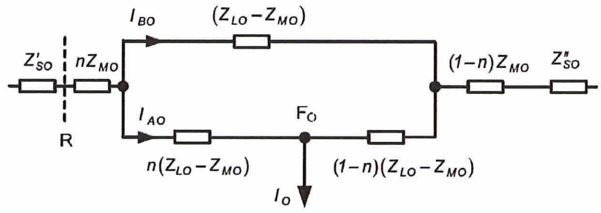
$$Z_A = nZ_{L1}\left[1 + \frac{\left(\frac{I_{B0}}{I_{A0}}\right)M}{2\left(\frac{I_{A1}}{I_{A0}}\right) + K}\right], \qquad M = \frac{Z_{M0}}{Z_{L1}}, K = \frac{Z_{L0} - Z_{L1}}{Z_{L1}} \quad \text{(B1.40)}$$

$$\frac{I_{B0}}{I_{A0}} = \frac{nZ''_{S0} - (1-n)Z'_{S0}}{(2-n)Z''_{S0} + (1-n)(Z'_{S0} + Z_{L0} + Z_{M0})}$$

Positive Sequence Network

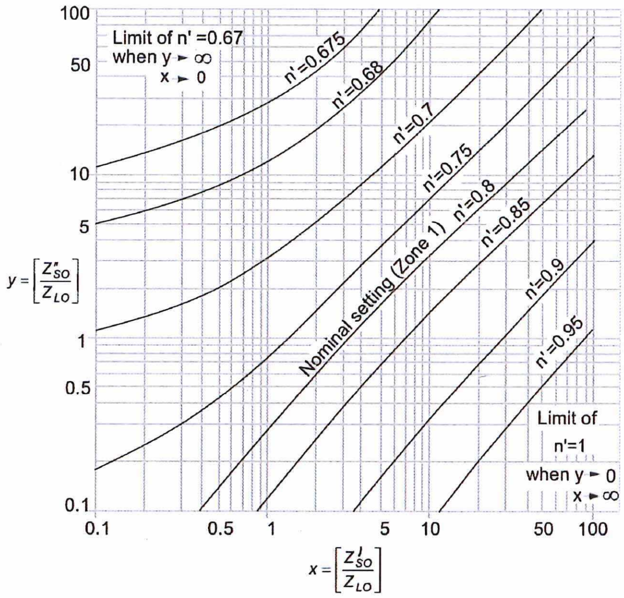


Zero Sequence Network



$$I_{A1} = \frac{(2-n)Z''_{S1} + (1-n)(Z'_{S1}+Z_{L1})}{2(Z'_{S1}+Z''_{S1})+Z_{L1}} I_1$$

$$I_{A0} = \frac{(2-n)Z''_{S0} + (1-n)(Z'_{S0}+Z_{L0}+Z_{M0})}{2(Z'_{S0}+Z''_{S0})+Z_{L0}+Z_{M0}} I_1 \qquad \text{(B1.40)}$$



Using the above formulae, families of reach curve may be constructed, with n' as the effective per unit reach of a relay set to protect 80% of the line. It has been assumed that an infinite busbar is located at each line end, that is Z_{S1}' and Z_{S1}" are both zero.

It is shown that

- R can be underreach or overreach, according to the relative values of zero-sequence source to line impedance ratio. The range is from 0.67 to 1.
- Relay R tends to underreach, whereas the relay at the opposite line end tends to overreach. Hence, the Zone 1 characteristics of the relays at both ends of the feeder overlap for an earth fault anywhere is the feeder.

Consider the mutual coupling effect with single circuit isolated and earthed.

Fault phase voltage seen by R: $I_{G1} = I_{G2} = I_{G0}, \quad I_{H0} = I_{G0}\left(\frac{Z_{M0}}{Z_{L0}}\right)$

$$V_R = V_{R1} + V_{R2} + V_{R0} = I_{G1}Z_{L1} + I_{G2}Z_{L2} + I_{G0}Z_{L0} - I_{H0}Z_{M0}$$

$$= I_{G1}\left(Z_{L1} + Z_{L2} + Z_{L0} - \frac{Z_{M0}^2}{Z_{L0}}\right) = I_{G1}\left(2Z_{L1} + Z_{L0} - \frac{Z_{M0}^2}{Z_{L0}}\right) \qquad \text{(B1.41)}$$

Faulted current as seen by relay R:

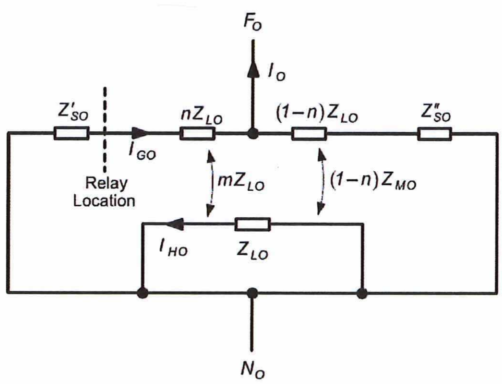
$$I_R = I_{G1} + I_{G2} + I_{G0}\frac{Z_{L0}}{Z_{L1}}$$

$$= I_{G1}\frac{2Z_{L1}+Z_{L0}}{Z_{L1}} \qquad \text{(B1.41)}$$

Impedance as seen by relay R:

$$\frac{V_A}{I_A} = Z_A = \frac{I_{G1}\left(2Z_{L1} + Z_{L0} - \frac{Z_{M0}^2}{Z_{L0}}\right)}{I_{G1}\frac{2Z_{L1}+Z_{L0}}{Z_{L1}}} \qquad \text{(B1.42)}$$

$$= Z_{L1}\left(1 \boxed{- \frac{Z_{M0}^2}{Z_{L0}(2Z_{L1}+Z_{L0})}}\right)$$



As a numerical example on how different operating conditions affects the accuracy of distance element, overreaching or underreaching,

Double Circuit In Service	**Adjacent Circuit Switched Out**	**Adjacent Circuit Earthed**
Z_1 = 0.709 + j 8.314 m p.u.	Z_1 = 0.714 + j 8.652 m p.u.	Z_1 = 0.717 + j 8.622 m p.u.
Z_0 = 10.75 + j 36.43 m p.u.	Z_0 = 5.675 + j 24.522 m p.u.	Z_0 = 2.191 + j 18.90 m p.u.
K_{F1} = **(Z_0/ Z_1 – 1)/3** = 1.1925	K_{F2} = 0.6384	K_{F3} = 0.4003

If Normal Setting = Double Line In Service

$K_R = K_{F1}$ = 1.1925 **$Z_F = Z_R$ (1+ K_R) / (1 + K_F)**

(i) When adjacent circuit is switched out:
$Z_F = Z_R$ (1+ 1.1925) / (1 + 0.6384) = 1.34 Z_R
Relay **overreaches** by 34%
Z1 = 80% x 1.34 = 107% of Line
Z2 = 120% x 1.34 = 161% of Line
Z3 = 160% x 1.34 = 214% of Line

(ii) When adjacent circuit is earthed:
$Z_F = Z_R$ (1+1.1925) / (1+ 0.4003) = 1.57 Z_R
Relay **overreaches** by 56%
Z1 = 80% x 1.57 = 126% of Line
Z2 = 120% x 1.57 = 188% of Line
Z3 = 160% x 1.57 = 251% of Line

If Normal Setting = Adjacent Line Switched Out

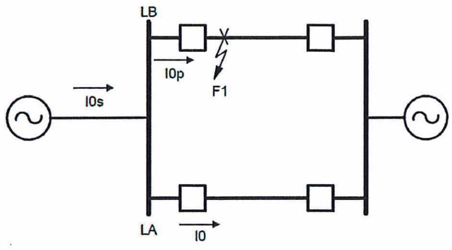
$K_R = K_{F1}$ = 0.63839

(i) When Double Circuit is in Service:
$Z_F = Z_R$ (1+0.6389)/(1+1.1925) = 0.75 Z_R
Relay **underreaches** by 25%
Z1 = 80% x 0.75 = 60% of Line (only 20% is covered)
Z2 = 120% x 0.75 = 90% of Line (Now set **Z2 = Z3**)
Z3 = 160% x 0.75 = 120% of Line

(ii) When adjacent circuit is earthed:
$Z_F = Z_R$ (1+ 0.6384) / (1+ 0.4003) = 1.17 Z_R
Relay **overreaches** by 17%
Z1 = 80% x 1.17 = 94% of Line
Z2 = 120% x 1.17 = 140% of Line
Z3 = 160% x 1.17 = 187% of Line

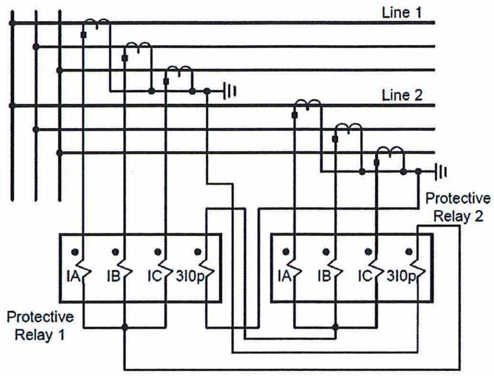
Consider a strong infeed with large zero-sequence current I_{0s} to the ground fault in the parallel line. Recall the compensated Z_{1L} value

$$Z_{1L} = \frac{V_A}{I_A + \frac{(Z_{0L} - Z_{1L})}{3Z_{1L}}3I_0 + \frac{Z_{0m}}{3Z_{1L}}3I_{0p}} \approx \frac{V_A}{\left(\frac{Z_{0m}}{3Z_{1L}}\right)3I_{0p}} \quad \text{(B1.43)}$$



The unfaulted ground distance relay receives the very large ground fault zero-sequence measurement from the parallel line relay. If $3I_{0p}$ is very large, the ground distance unit of the healthy line will measure a small impedance.

Line protection relays can accommodate an additional current input to measure the parallel line zero-sequence current as illustrated in the circuitry.

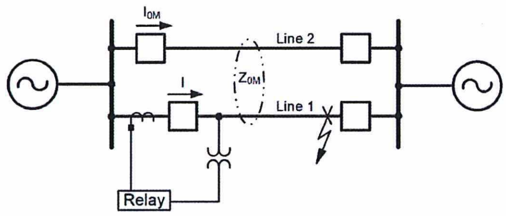


Yet, it also depends on:
1. Local availability of another line current
2. No current information in the parallel line isolated and earthed
3. Compensation factor may increase the error in healthy line
4. Incorrect installation (e.g. with four parallel lines in one tower)

Consider the apparent impedance Z_{APP} measured by the AG ground distance element on Line 1.

$$Z_{APP} = \frac{V_a}{I_a + k_0 I_r} = mZ_{1L} + \boxed{mZ_{0m}\frac{I_{0m}}{I_a + k_0 I_r}} \quad \text{(B1.44)}$$

Z_{APP} includes the error term due to I_{0m}, in which is compensated by the simple factor $K_N = (Z_0 - Z_1)/3Z_1$. It shows that the error term is
- Positive, when current I_{0M} and I_0 flows in the same direction (underreach)
- Negative, when current I_{0M} and I_0 flows in opposite direction (overreach)



The relative directions of currents in mutually coupled lines (which determine ground distance element overreaching or underreaching) depends on
- Existing mutual coupling
- System topology
- Fault location
- Line Taps

Note: System topology changes during fault clearance due to sequential CB tripping.

Given the apparent impedance for AG element (with voltage V_a and current $I_a + k_0 I_r$) in (B1.44)

$$Z_{APP} = \frac{V_a}{I_a + k_0 I_r} = mZ_{1L} + mZ_{0m}\frac{I_{0m}}{I_a + k_0 I_r} \quad \text{(B1.45)}$$

The measured impedance for different switching status of the coupled lines are as follows.

Status of Coupled Line	In service	Switched Out	Isolation and Earth
Measured Impedance	$Z_{APP} = Z_{1L} + \frac{Z_{OM}}{3(1+k_0)}$	$Z_{APP} = Z_{1L}$	$Z_{APP} = Z_{1L} - \frac{Z_{0M}^2}{3Z_{0L}(1+k_0)}$

assumed that
- The phase current and residual current of the protected line are equal ($I_a = I_r$)
- The coupled line residual current is equal to the residual current of the protected lines ($I_r = 3I_{OM}$)

Detecting the status of coupled line can help perform mutual compensation on the apparent impedance to find Z_{1L}.

Another way is to use the zero-sequence current from the coupled line with

$$V_a = mZ_{1L}\left(I_a + k_0 I_r + \boxed{\frac{Z_{OM}}{Z_{1L}}I_{OM}}\right) \quad \text{(B1.46)}$$

The terms in paratheses in (B1.46) is the current required to eliminate the impedance measurement error. It includes an additional compensation term that contains I_{OM} and it causes the aforementioned problem.

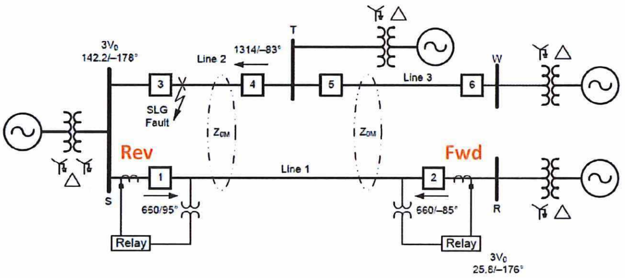
A simpler way is to apply k_0 setting that consider mutual coupling effect, which is introduced throughout this appendix.

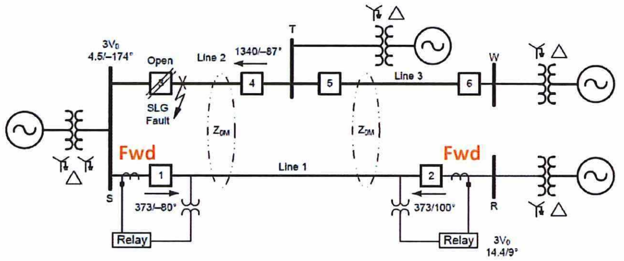
$$V_a = Z_{1L}\left(I_a + \boxed{\left(k_0 + \frac{Z_{OM}}{3Z_{1L}}\right)} I_r\right), \qquad I_r = 3I_{OM} \quad \text{(B1.47)}$$

Status of Coupled Line	In service	Switched Out	Isolation and Earth
K_0 value	$K_0' = \frac{Z_{OL} - Z_{1L} + Z_{OM}}{3Z_{1L}}$	$K_0 = \frac{Z_{OL} - Z_{1L}}{3Z_{1L}}$	$K_0'' = \frac{Z_{OL} - Z_{1L} - \frac{Z_{0M}^2}{Z_{OL}}}{3Z_{1L}}$

The suggestion is to apply the value of K_0 to Zone 1 and K_0' to Zone 2 when the coupled line is in service. To put one line isolated and earthed, the user can apply K_0'' to Zone 1 and keep K_0' value to Zone 2.

Fault Location is performed with the information AFTER the distance element / distance protection decided to trip. The typical time period starting from fault inception is around 1 cycle decision due to the 1-cycle filter, and the latter 30ms – 80ms CB opening time, depending on the CB driving mechanism.



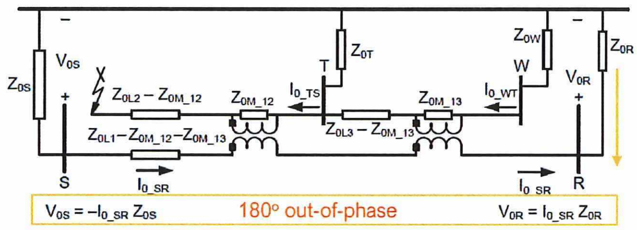


Voltage Reversal Problem in Directional Guards

Zero sequence polarizing quantity (current/ voltage) reversal occurs when mutual coupling is stronger than electrical coupling, i.e.

- Two lines running in parallel in isolated network
- Opening of CB and connected in one side for parallel network
- Lines in different voltage running in parallel
- Coupled lines in looped system

The healthy line will have zero-sequence current flowing towards one neutral and voltage reversal occurs at one bus such that it is indicated as an in-zone fault and tripped the line.



In conclusion,

1. Transposition and appropriate phase arrangement allows smaller unbalance in transmission lines, but it does not eliminate the zero-sequence mutual impedance between parallel lines.
2. Zero-sequence mutual impedance that cannot be eliminated even with transposition can be comparable to the positive-sequence impedance of the line.
3. Zero-sequence mutual compensation (measurement of parallel line current) benefits single-ended fault location algorithm.
4. Ground directional element polarized with zero-sequence quantities and used in pilot relaying scheme can determine a forward direction at both terminal when the zero-sequence sources are independent. Yet, zero-sequence based directional element is not reliable with mutual coupling, non-single point earthing in secondary circuit or sequential tripping with current flip. Negative-sequence based directional element is much preferred.
5. Ground distance protection should ensure that
   - Zone 1 does not overreach due to zero-sequence mutual.
   - Overreaching zone such as Zone 2, should not underreach the remote terminal
   - With proper selection of K0 factor, the above goal can be achieved.
6. Measurement of $3I_{0p}$ for ground distance relaying is not practicable due to the extra wiring and additional logic in the line protection.

Reference:

[1] ALSTOM (2011), Chapter 13 – Protection of Complex Transmission Circuits, Network Protection and Automation Guide
[2] Calero F. (2015), Mutual Impedance in Parallel Lines – Protective Relaying and Fault Location Considerations
[3] Tziouvaras D. A. et al (2014), Protecting Mutually Coupled Transmission Lines: Challenges and Solutions
[4] OHMEGA406 Instruction Manual
[5] CIGRE WG B5.15 (2008), Modern Distance Protection – Functions and Applications

Formula:

Carson's Formula

$$\begin{cases} Z_{ii} = r_i + 0.05919\left(\frac{f}{60}\right) + j\,0.07537\left(\frac{f}{60}\right)\left[\ln\left(\frac{1}{GMR_i}\right) + 6.7458\right] \quad [\Omega/\text{km}] \\ Z_{ij} = 0.05919\left(\frac{f}{60}\right) + j0.07537\left(\frac{f}{60}\right)\left[\ln\left(\frac{1}{d_{ij}}\right) + 6.7458\right] \quad [\Omega/\text{km}] \end{cases} \tag{B1.7}$$

Transformation from phase impedance to sequence impedance

$$[Z''_{012}] = \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \end{bmatrix}^{-1} [Z''_{abc}] \begin{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \end{bmatrix} = [T']^{-1}[Z''_{abc}][T'] \tag{B1.12}$$

Compensation Factor

$$V_A = mZ_{1L}\left(I_A + \frac{Z_{0L} - Z_{1L}}{3Z_{1L}} 3I_0 + \frac{Z_{0m}}{3Z_{1L}} 3I_{0p}\right) \tag{B1.16}$$

Takagi Algorithm

$$m = \frac{Im\{VI_2^*\}}{Im\{Z_L I_s I_2^*\}} \tag{B1.21}$$

SLG Fault Underreaching

$$\frac{V_A}{I_A} = Z_A = \frac{nZ_{L1}\left(I_{A1} + I_{A2} + I_{A0}\frac{Z_{L0}}{Z_{L1}} + I_{B0}\frac{Z_{0m}}{Z_{L1}}\right)}{I_{A0}\frac{2Z_{L1} + Z_0}{Z_{L1}}} = nZ_{L1}\left(1 + \frac{I_{B0}}{I_{A0}}\frac{Z_{0m}}{2Z_{L1} + Z_{L0}}\right) \tag{B1.36}$$

Status of Coupled Line	In service	Switched Out	Isolation and Earth
Measured Impedance	$Z_{APP} = Z_{1L} + \frac{Z_{0M}}{3(1+k_0)}$	$Z_{APP} = Z_{1L}$	$Z_{APP} = Z_{1L} - \frac{Z_{0M}^2}{3Z_{0L}(1+k_0)}$
K_0 value	$K_0' = \frac{Z_{0L} - Z_{1L} + Z_{0M}}{3Z_{1L}}$	$K_0 = \frac{Z_{0L} - Z_{1L}}{3Z_{1L}}$	$K_0'' = \frac{Z_{0L} - Z_{1L} - \frac{Z_{0M}^2}{Z_{0L}}}{3Z_{1L}}$