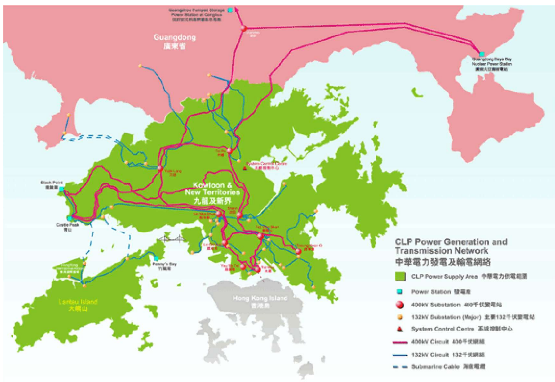
Chapter 2 Transmission Line Fundamental

Nearly all power system analysis, protection and control requires understanding to line parameters, no matter if cable or overhead line is used. Line resistance, inductance and capacitance have been derived in this chapter. Skin effect and proximity effect have been described. Based on the line parameters, classification of power line has been done followed by modelling of different parts of power system. Two-port model, which is the commonly used transmission line model, is presented for further load flow analysis. Transmission parameters, characteristics impedance, image impedance, surge impedance loading, etc. are also presented.

2.1 Classification of Lines

Based on the line length and hence intrinsic capacitance effect, power transmission line can be divided into:

Category	Length
1. Short Transmission Line	< 50 km
2. Middle Transmission Line	50 – 150 km
3. Long Transmission Line	> 150 km



Short Line Model requires only consideration in line resistance and inductance, with negligible capacitance effect. **Middle Line Model** starts to have effect in its intrinsic capacitance. Yet, a lumped pi-model is enough for accuracy. **Long Line Model** has a large capacitive effect, even larger than the inductive effect. Hence, it is often the line resistance and a lossless line model with distributed nature of capacitance and inductance is employed.

It is noted that the overhead lines (OHL) in Hong Kong is shorter than 60km. Hence, a short line model is enough.

2.2 Line Parameters – Resistance

Resistance of a material where current is uniformly distributed can be determined from the following formula:

$$\boxed{R = \rho \frac{l}{A}} \tag{2.1}$$

where l is the length of the conducting material, A is the cross-sectional area of the conducting material, ρ is the resistivity of the conducting material.

It is often line resistance with (2.1) is used for simulation. Yet, the actual resistance is often larger given the non-uniformity in current distribution and resistivity due to proximity effect and skin effect, which is affected by line geometry or configuration. The following table is often given by conductor manufacturer assuming standard configuration.

	Cross-Section Area				Diameter			Approx. Current-Carrying Capacity	Resistance (mΩ/km)					60 Hz Reactances (Dm = 1 m)	
		Aluminum								AC (60 Hz)					
Code	Total (mm²)	(kcmil)	(mm²)	Stranding Al/Steel	Conductor (mm)	Core (mm)	Layers	(Amperes)	DC 25°C	25°C	50°C	75°C	GMR (mm)	X_1 (Ω/km)	X_0 (MΩ/km)
–	1521	2 776	1407	84/19	50.80	13.87	4		21.0	24.5	26.2	28.1	20.33	0.294	0.175
Joree	1344	2 515	1274	76/19	47.75	10.80	4		22.7	26.0	28.0	30.0	18.93	0.299	0.178
Thrasher	1235	2 312	1171	76/19	45.77	10.34	4		24.7	27.7	30.0	32.2	18.14	0.302	0.180
Kiwi	1146	2 167	1098	72/7	44.07	8.81	4		26.4	29.4	31.9	34.2	17.37	0.306	0.182
Bluebird	1181	2 156	1092	84/19	44.75	12.19	4		26.5	29.0	31.4	33.8	17.92	0.303	0.181
Chukar	976	1 781	902	84/19	40.69	11.10	4		32.1	34.1	37.2	40.1	16.28	0.311	0.186
Falcon	908	1 590	806	54/19	39.24	13.08	3	1 380	35.9	37.4	40.8	44.3	15.91	0.312	0.187
Lapwing	862	1 590	806	45/7	38.20	9.95	3	1 370	36.7	38.7	42.1	45.6	15.15	0.316	0.189
Parrot	862	1 510	765	54/19	38.23	12.75	3	1 340	37.8	39.2	42.8	46.5	15.48	0.314	0.189
Nuthatch	818	1 510	765	45/7	37.21	9.30	3	1 340	38.7	40.5	44.2	47.9	14.78	0.318	0.190
Plover	817	1 431	725	54/19	37.21	12.42	3	1 300	39.9	41.2	45.1	48.9	15.06	0.316	0.190

Temperature Effect: Metal resistivity ρ varies essentially linearly with the temperature. i.e. $\rho(T) = \rho_0(1 + \alpha(T - T_0))$
Hence, the resistance depending on temperature can be estimated as

$$\frac{R_2}{R_1} = \frac{T_0 - T_2}{T_0 - T_1} \tag{2.2}$$

Skin Effect: Current is made of free electrons, which are negatively charged, and all free electrons will try to repulse each other. Electron density becomes high at periphery and low at centre. This results in high resistance at centre. This is known as skin effect. It increases effective resistance and hence overall loss.

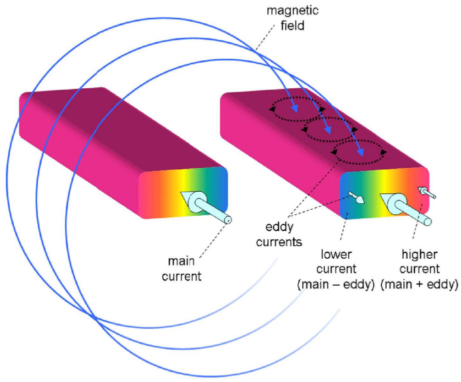
In AC, other than non-uniform resistive effect, there is an effect of non-uniform flux linkage. Internal flux linkage is high at the centre and minimum at the periphery. Thus, overall impedance is high at centre and minimum at periphery. Thus, skin effect is more in AC than in DC.

As the AC current density J can be modelled as $J = J_S e^{-(1+j)d/\delta}$, skin depth is thus defined as the depth below the surface of the conductor at which the current density has fallen to 37% of J_S. Skin depth can be determined with $\delta = \sqrt{\pi f \mu_r \mu_0 \sigma}$, where f is electrical frequency, μ_r is relative permeability and σ is electrical conductivity.

The effective AC resistance with consideration of skin effect can be derived with Al-Asadi (1988):

$$R_{ac} = \frac{1}{\pi\sigma\delta\left(1 - e^{-\frac{r}{\delta}}\right)\left(2r - \delta\left(1 - e^{-\frac{r}{\delta}}\right)\right)} \tag{2.3}$$

Proximity Effect: When two or more conductors are close each other, the AC magnetic field generated by each one affects the neighboring conductors, thus inducing eddy currents in such conductors. Hence, the electric current distribution within the conductors will change when compared with that of an isolated conductor. This phenomenon is known as proximity effect. The result is that the current is concentrated in the areas of the conductor farthest away from nearby conductors carrying current in the same direction.



Proximity effect with Skin effect forms the **ratio between AC Resistance and DC Resistance.** The following empirical formula is given by IEC60287-1-1.

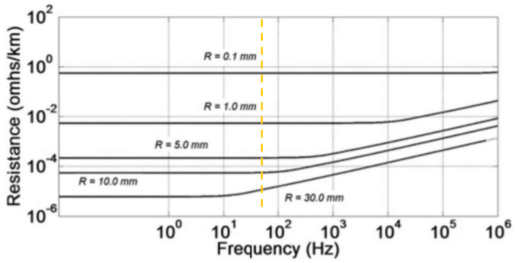
$$\frac{R_{AC}}{R_{DC}} = 1 + F_s + F_p \tag{2.4}$$

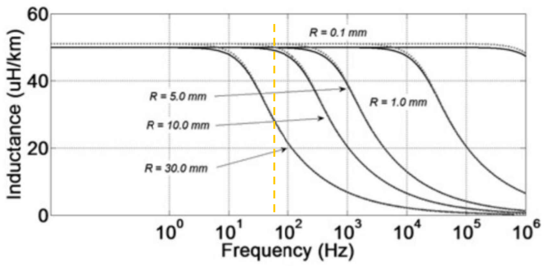
where F_s is skin effect factor and F_p is proximity effect factor, given by

$$x^4 = \left(\frac{8\pi f}{R_{dc}10^7}\right)^2 \qquad F_s = \frac{x^4}{192 + 0.8x^4} \qquad F_p = \frac{2.9x^4}{192 + 0.8x^4}\left(\frac{d_c}{s}\right)^2$$

where d_c = diameter of conductors, and s = geometric mean of lengths between conductors.

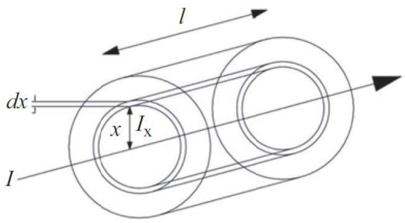
The following shows the effect of conductor thickness and frequency to line AC resistance and inductance.





2.3 Line Parameters – Inductance

Inductance of a transmission line conductor forms due to the flux linkage which occurs both internally and externally. Thus, the total inductance of a conductor can be mathematically expressed as $L_T = L_{in} + L_{ex}$, where L_{in} is the inductance due to internal flux linkage, and L_{ex} is the inductance due to external flux linkage.



Consider the differential element of a conductor.
Given Ampere's Circuital Law:

$$\oint H \cdot dl = \sum I \quad \rightarrow \quad 2\pi x H_x = I_x = \frac{\pi x^2}{\pi r^2} I = \frac{x^2}{r^2} I \tag{2.5}$$

Hence, the magnetic flux density becomes

$$B_x = \mu H_x = \frac{\mu x}{2\pi r^2} I \tag{2.6}$$

Consider an elementary ring at radius x of width dx and also consider an elementary area dA formed by the elementary distance dix in the radial direction with length 1m. Flux distribution $d\phi$ and hence the flux linkage $d\psi$ created by $d\phi$ are given by

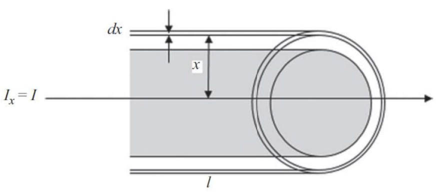
$$d\phi = B_x dA = \frac{\mu x}{2\pi r^2} I dx \tag{2.7}$$

$$d\psi = \frac{\pi x^2}{\pi r^2} d\phi = \frac{x^2}{r^2} \frac{\mu x}{2\pi r^2} I dx = \frac{\mu x^3}{2\pi r^4} I dx \tag{2.8}$$

Hence, the total flux linkage and hence line inductance due to internal flux linkage can be written as

$$\psi_{in} = \int_0^r \frac{\mu x^3}{2\pi r^4} I dx = \frac{\mu I}{8\pi} \tag{2.9}$$

$$L_{in} = \frac{\psi_{in}}{I} = \frac{\mu}{8\pi} = \frac{10^{-7}}{2} \text{ H/m} \tag{2.10}$$



For external flux, the flux linkage can be shown as

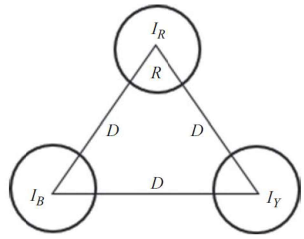
$$\psi_{ex} = \int_r^D \frac{\mu}{2\pi x} I dx = \frac{\mu I}{2\pi} \ln\frac{D}{r} \tag{2.11}$$

$$L_{ex} = \frac{\psi_{ex}}{I} = \frac{\mu}{2\pi} \ln\frac{D}{r} = 2 \times 10^{-7} \ln\frac{D}{r} \text{ H/m} \tag{2.12}$$

Total Inductance of a conductor is

$$L_T = L_{in} + L_{ex} = \frac{1}{2} \times 10^{-7} + 2 \times 10^{-7} \ln\frac{D}{r} = 2 \times 10^{-7} \ln\frac{D}{re^{-\frac{1}{4}}} = 2 \times 10^{-7} \ln\frac{D}{r'} \qquad (2.13)$$

where r' = 0.78 r.



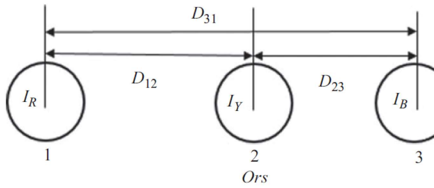
For the inductance of transmission line having three-phase conductors put in symmetry, let the current in R, Y and B phase be I_R, I_Y, I_B respectively. Flux linkage in R phase will be created by I_R (self) and by I_Y and I_B (mutual).

Flux linkage in R phase:

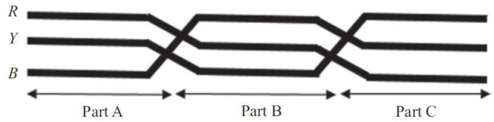
$$\psi_R = 2 \times 10^{-7} \left\{ I_R \ln\frac{1}{R} + I_Y \ln\frac{1}{D} + I_B \ln\frac{1}{D} \right\} \qquad (2.14)$$

If the system is *balanced*, $I_R + I_Y + I_B = 0$

$$\psi_R = 2 \times 10^{-7} \left\{ I_R \ln\frac{1}{R} - I_R \ln\frac{1}{D} \right\} = 2 \times 10^{-7} I_R \ln\frac{D}{R} \qquad (2.15)$$



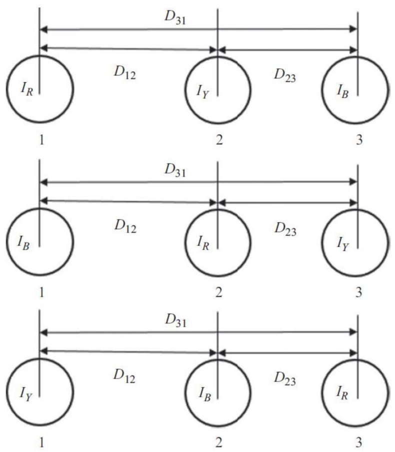
If the conductors are put unsymmetrically, with unequal inter-distances, the flux linkage to each phase will be different. Therefore, the output voltage is unbalanced.
i.e. $\psi_R \neq \psi_B \neq \psi_Y \;\rightarrow\; L_R \neq L_B \neq L_Y \;\rightarrow\; X_R \neq X_B \neq X_Y \;\rightarrow\; IX_R \neq IX_B \neq IX_Y$



Transposition is alternate change of position of phase conductors. A transposed three-phase three-wire transmission line is shown in the figure. It helps equalize the average flux linkage and hence the impedance in each phase.

Consider flux linkage in R phase for transposed unsymmetrical lines.

$$\begin{aligned}
\psi_{R1} &= 2 \times 10^{-7} \left\{ I_R \ln\frac{1}{R_1} + I_Y \ln\frac{1}{D_{12}} + I_B \ln\frac{1}{D_{31}} \right\} \\
\psi_{R2} &= 2 \times 10^{-7} \left\{ I_R \ln\frac{1}{R_1} + I_Y \ln\frac{1}{D_{23}} + I_B \ln\frac{1}{D_{12}} \right\} \qquad (2.16) \\
\psi_{R3} &= 2 \times 10^{-7} \left\{ I_R \ln\frac{1}{R_1} + I_Y \ln\frac{1}{D_{31}} + I_B \ln\frac{1}{D_{23}} \right\}
\end{aligned}$$



Average Flux linkage in phase R will be

$$\bar{\psi}_R = \frac{\psi_{R1} + \psi_{R2} + \psi_{R3}}{3} \qquad (2.17)$$

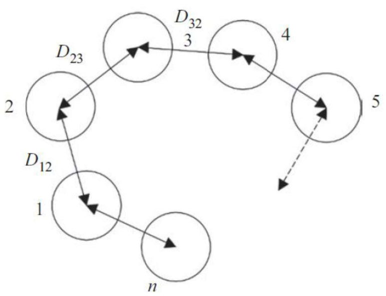
$$\begin{aligned}
\bar{\psi}_R &= \frac{1}{3} \times 2 \times 10^{-7} \left(3I_R \ln\frac{1}{R_1} + I_Y \ln\frac{1}{D_{12}D_{23}D_{31}} + I_B \ln\frac{1}{D_{12}D_{23}D_{31}} \right) \\
&= 2 \times 10^{-7} \left(I_R \ln\frac{1}{R_1} + I_Y \ln\frac{1}{\sqrt[3]{D_{12}D_{23}D_{31}}} + I_B \ln\frac{1}{\sqrt[3]{D_{12}D_{23}D_{31}}} \right) \qquad (2.18)
\end{aligned}$$

If the system is balanced, $I_R + I_Y + I_B = 0$

$$\bar{\psi}_R = 2 \times 10^{-7} I_R \ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{R_1} \qquad (2.19)$$

Let $D = \sqrt[3]{D_{12}D_{23}D_{31}}$,

$$\bar{\psi}_R = 2 \times 10^{-7} I_R \ln\frac{D}{R_1} \;\rightarrow\; L_R = 2 \times 10^{-7} \ln\frac{D}{R_1} \qquad (2.20)$$



Denote Geometric Mean Distance (GMD) for n-conductors as

$$D^* = \sqrt[n]{D_{12}D_{23} \dots D_{(n-1)n}D_{n1}} \qquad (2.21)$$

and Geometric Mean Radius (GMR) as

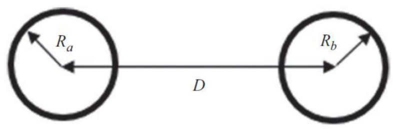
$$R^* = \sqrt[n]{R_{12}R_{23} \dots R_{(n-1)n}R_{n1}} \qquad (2.22)$$

The inductance of a line can be written as

$$\boxed{L_R = 2 \times 10^{-7} \ln\frac{D^*}{R^*}} \qquad (2.23)$$

2.4 Line Parameters – Capacitance

Consider a single phase two-wire transmission line with distance between two conductor D, their radii be R_a and R_b and equal and opposite charge $q_a = -q_b$.



By Gauss's Law

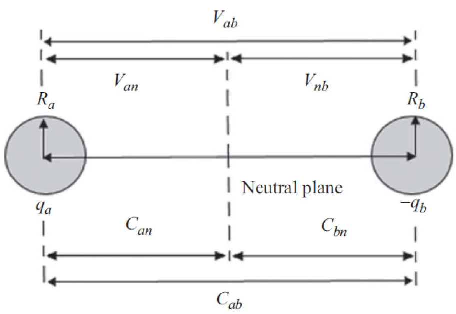
$$\oint E \cdot dA = \frac{q}{\varepsilon_0} \rightarrow E(x) = \frac{q}{2\pi x\, \varepsilon_0} \tag{2.24}$$

Voltage develop across a and b due to charge a becomes

$$V_{ab-q_a} = \int_{R_a}^{D} E\, dx = \frac{q_a}{2\pi\varepsilon_0} \ln\frac{D}{R_a} \tag{2.25}$$

Similarly, voltage across a and b due to charge b becomes

$$V_{ab-q_b} = \frac{q_b}{2\pi\varepsilon_0} \ln\frac{R_b}{D} \tag{2.26}$$



Hence, the net voltage becomes

$$V_{ab} = V_{ab-q_a} + V_{ab-q_b} = \frac{q_a}{2\pi\varepsilon_0}\ln\frac{D}{R_a} + \frac{q_b}{2\pi\varepsilon_0}\ln\frac{R_b}{D} = \frac{q}{\pi\varepsilon}\ln\frac{D}{\sqrt{R_a R_b}} \tag{2.27}$$

Therefore, capacitance across two conductor becomes

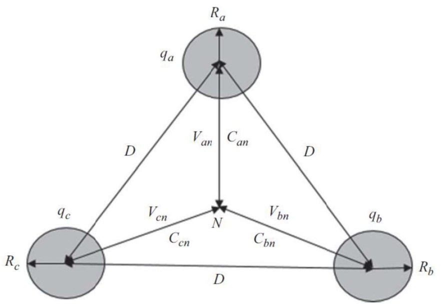
$$C_{ab} = \frac{q}{V_{ab}} = \frac{\pi\varepsilon}{\ln\frac{D}{R}} \tag{2.28}$$

Consider the neutral plane n between two lines.
As the net capacitance can be written as

$$C_{ab} = \frac{C_{an}}{2} = \frac{C_{bn}}{2} \tag{2.29}$$

Therefore,

$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{R}} \tag{2.30}$$



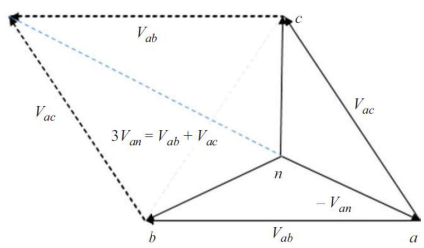
Consider the capacitance of symmetrically placed three-phase conductor

$$V_{ab} = V_{ab-q_a} + V_{ab-q_b} + V_{ab-q_c}$$

$$V_{ab} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D}{R_a} + \frac{q_b}{2\pi\varepsilon}\ln\frac{D}{R_b} + \frac{q_c}{2\pi\varepsilon}\ln\frac{D}{D} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{R_a} + q_b \ln\frac{R_b}{D}\right) \tag{2.31}$$

Similarly

$$V_{ac} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{R_a} + q_c \ln\frac{R_c}{D}\right) \tag{2.32}$$



Let the radius of each conductor be R and for balance system $q_a + q_b + q_c = 0$.
Therefore

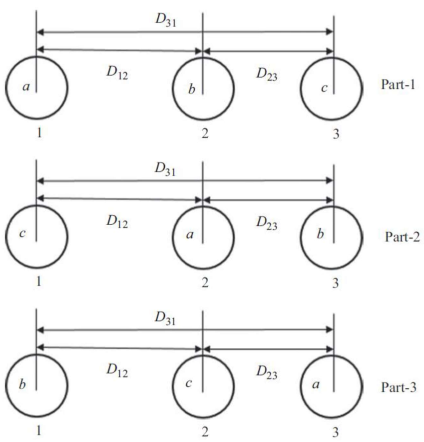
$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln\frac{D}{R} - q_a \ln\frac{R}{D}\right) = \frac{1}{2\pi\varepsilon} 3q_a \ln\frac{D}{R} \tag{2.33}$$

Consider the phasor diagram, $V_{ab} + V_{bc} = 3V_{an}$

$$V_{an} = \frac{V_{ab} + V_{ac}}{3} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D}{R} = V_{bn} = V_{cn} \tag{2.34}$$

Capacitance across neutral to a phase can be written as

$$C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{R}\right)} \tag{2.35}$$



Consider again the transposed unsymmetrical lines.

$$\begin{aligned}
V_{ab1} &= V_{ab-q_a} + V_{ab-q_b} + V_{ab-q_c} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D_{12}}{R_a} + \frac{q_b}{2\pi\varepsilon}\ln\frac{R_b}{D_{12}} + \frac{q_c}{2\pi\varepsilon}\ln\frac{D_{23}}{D_{31}} \\
V_{ab2} &= V_{ab-q_a} + V_{ab-q_b} + V_{ab-q_c} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D_{23}}{R_a} + \frac{q_b}{2\pi\varepsilon}\ln\frac{R_b}{D_{23}} + \frac{q_c}{2\pi\varepsilon}\ln\frac{D_{31}}{D_{12}} \\
V_{ab3} &= V_{ab-q_a} + V_{ab-q_b} + V_{ab-q_c} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D_{31}}{R_a} + \frac{q_b}{2\pi\varepsilon}\ln\frac{R_b}{D_{31}} + \frac{q_c}{2\pi\varepsilon}\ln\frac{D_{12}}{D_{23}}
\end{aligned} \tag{2.36}$$

The average voltage across a and b will be

$$\bar{V}_{ab} = \frac{V_{ab1} + V_{ab2} + V_{ab3}}{3} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D_{12}D_{23}D_{31}}{R_a^3} + \frac{q_b}{2\pi\varepsilon}\ln\frac{R_b^3}{D_{12}D_{23}D_{31}} + \frac{q_c}{2\pi\varepsilon}\ln\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}$$

$$\bar{V}_{ab} = \frac{3}{2\pi\varepsilon}\left(q_a \ln\frac{D^*}{R_a} + q_b \ln\frac{R_b}{D^*}\right) \tag{2.37}$$

Similarly,

$$\bar{V}_{ac} = \frac{3}{2\pi\varepsilon}\left(q_a \ln\frac{D^*}{R_a} + q_c \ln\frac{R_c}{D^*}\right) \tag{2.37}$$

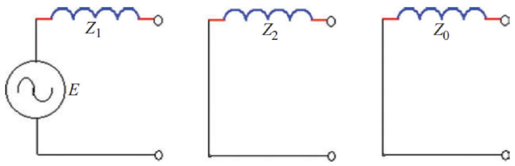
Let the radius of each conductor be R and for balance system $q_a + q_b + q_c = 0$.

$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln\frac{D}{R} - q_a \ln\frac{R}{D}\right) = \frac{1}{2\pi\varepsilon} 3q_a \ln\frac{D}{R} \tag{2.38}$$

Again from the phasor diagram, $3V_{an} = V_{ab} + V_{ac}$
The capacitance across neutral to phase a is

$$\boxed{C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{R}}} \tag{2.39}$$

2.5 Line Parameters – Sequence Impedance



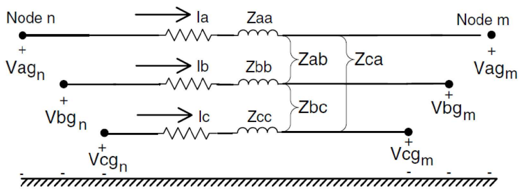
Positive Sequence Impedance Z_1, Negative Sequence Impedance Z_2 and Zero Sequence Impedance Z_0 are the impedance the component as seen in source when the source is in positive rotation, negative rotation and in-phase rotation respectively. Among these networks, only positive sequence network has voltage source (is it always true?), as generators rotates in positive sequence.

For linear, symmetrical and static system, positive and negative sequence are equal in magnitude, and they are equal to the impedance seen at balanced condition, provided the supply is balanced. Yet, in rotating machines, positive and negative sequence are usually different in magnitude due to machines construction. Zero-sequence impedance is usually different from other two sequence impedance as zero sequence path is anywhere the in-phase current can go, such as grounding, circulation and charging current.

An unbalance system can be decoupled into three symmetrical components which are balanced. In an unbalanced system, zero-sequence current and voltage may or may not exist in the system. Some special case shall be considered.

- In balanced system, zero-sequence component is absent. (Is it always true?)
- In unbalanced star-connected four-wire system, zero-sequence component flows though neutral wire. It is equal to the sum of the three-line currents.
- In unbalanced ungrounded star-connected three-wire system, there is no zero-sequence current as there is no return or neutral path for zero-sequence current.
- In unbalanced delta connected system, zero-sequence current circulates through the mesh. In line current, it is absent.

Consider Three-Phase Line Segment Model:



$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_n = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m + \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{2.40}$$

or in simple matrix form,

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}][I_{abc}] \tag{2.41}$$

The line-to-ground phase voltage can be written as a linear function of sequence voltage,

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}}_{[A_S]} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \tag{2.42}$$

Similarly, current can be written as

$$[I_{abc}]_n = [A_S][I_{012}] \tag{2.43}$$

(2.41) can be represented in form after transformation.

$$[VLG_{012}]_n = [A_S]^{-1}[VLG_{abc}]_n$$
$$[VLG_{012}]_n = [A_S]^{-1}[VLG_{abc}]_m + [A_S]^{-1}[Z_{abc}][A_S][I_{012}] \tag{2.44}$$
$$[VLG_{012}]_n = [VLG_{012}]_m + [Z_{012}][I_{012}]$$

In expanded form,

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}_n = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}_m + \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{2.45}$$

where Z_{00}, Z_{11}, Z_{22} are zero sequence, positive sequence and negative sequence. The off-diagonal terms are mutual coupling between sequence, which is equal to zero in ideal case.

Note: Assume the line is transposed, the self and mutual inductance can be written as:

$$z_s = \frac{z_{aa} + z_{bb} + z_{cc}}{3} \ \Omega/\text{km}$$

$$z_m = \frac{z_{ab} + z_{bc} + z_{ca}}{3} \ \Omega/\text{km}$$

$$[Z_{abc}] = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix} \Omega/\text{km}$$

$$z_{00} = z_s + 2z_m \ \ \Omega/\text{km}$$
$$z_{11} = z_{22} = z_s - z_m \ \ \Omega/\text{km}$$

Is it possible to have a component with $z_{00} < z_{11}$?
What is the ratio of z_{00} to z_{11}?

Carson (1926) has proposed a generalized model, i.e. no spacing, conductor size, transposition and balanced input requirement , for self- and mutual impedance. The equation is applicable for both cable and OHL. Yet, the calculation is rather tedious without computer.

Carson Equations (Simplified Version):

Self Impedance of Conductor i:

$$\hat{z}_{ii} = r_i + 4\omega P_{ii} G + j\left(X_i + 2\omega G \cdot \ln\frac{S_{ii}}{RD_i} + 4\omega Q_{ii} G\right) \ \Omega/\text{mile}$$

Mutual Impedance between Conductor i and j:

$$\hat{z}_{ij} = 4\omega P_{ij} G + j\left(2\omega G \cdot \ln\frac{S_{ij}}{D_{ij}} + 4\omega Q_{ij} G\right) \ \Omega/\text{mile}$$

where

$\hat{z}_{ii}$ = self impedance of conductor i in Ω/mile
$\hat{z}_{ij}$ = mutual impedance between conductors i and j in Ω/mile
r_i = resistance of conductor i in Ω/mile
$\omega = 2\pi f$ = system angular frequency in radians per second
$G = 0.1609344 \times 10^{-3}$ Ω/mile
RD_i = radius of conductor i in feet
GMR_i = Geometric Mean Radius of conductor i in feet
f = system frequency in Hertz
ρ = resistivity of earth in Ω-meters

Interim Parameter (Simplified Version):

$$P_{ij} = \frac{\pi}{8} \qquad Q_{ij} = -0.03860 + \frac{1}{2}\ln\frac{2}{k_{ij}} \qquad k_{ij} = 8.565 \times 10^{-4} \cdot S_{ij} \cdot \sqrt{\frac{f}{\rho}}$$

Assume earth resistivity = 100Ω/m and f = 60 Hz,

$$\hat{z}_{ii} = r_i + 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{GMR_i}\right) + 7.93402\right] \ \Omega/\text{mile}$$

$$\hat{z}_{nn} = r_n + 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{GMR_n}\right) + 7.93402\right] \ \Omega/\text{mile}$$

$$\hat{z}_{ij} = 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{D_{ij}}\right) + 7.93402\right] \ \Omega/\text{mile}$$

$$\hat{z}_{in} = 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{D_{in}}\right) + 7.93402\right] \ \Omega/\text{mile}$$

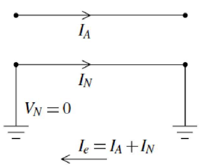
Transforming to sequence impedance,

$$z_{00} = \hat{z}_{ii} + 2 \cdot \hat{z}_{ij} - 3 \cdot \left(\frac{\hat{z}_{in}^2}{\hat{z}_{nn}}\right) \ \Omega/\text{mile}$$

$$z_{11} = z_{22} = \hat{z}_{ii} - \hat{z}_{ij}$$

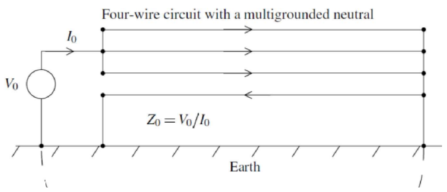
$$z_{11} = z_{22} = r_i + j0.12134 \cdot \ln\left(\frac{D_{ij}}{GMR_i}\right) \ \Omega/\text{mile}$$

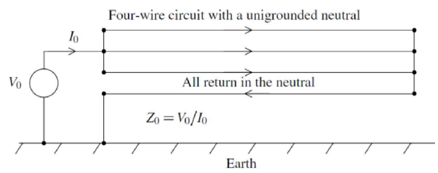
Yet, the actual modelling may be involved by design and operation topology.



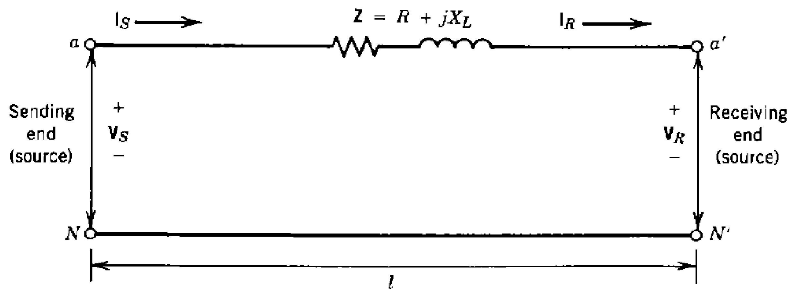
$V_N = 0 = Z_{AN} I_A + Z_{NN} I_N$

So, $I_N = -\dfrac{Z_{AN}}{Z_{NN}} I_A$





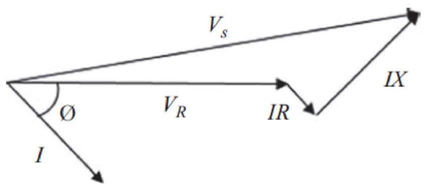
2.6 Line Model – Short Line Model



It is often line resistance with (2.1) is used for simulation. Yet, the actual resistance is often larger given the non-uniformity in current distribution and resistivity due to proximity effect and skin effect, which is affected by line geometry or configuration. The following table is often given by conductor manufacturer assuming standard configuration.

Voltage Regulation, which is to avoid voltage dropping too much through a transmission line, must be considered in power system operation. For short lines,

$$\text{Regulation} = \frac{(\text{sending end voltage at full load} - \text{receiving end voltage at full load})}{\text{receiving end voltage at full load}} \tag{2.46}$$



In Phasor Calculation, the sending-end voltage can be written as

$$V_S^2 = (V_S \cos\phi + IR)^2 + (V_R \sin\phi + IX)^2 \tag{2.47}$$

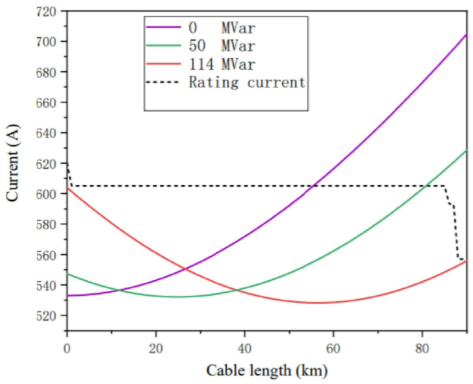
$$V_S = \sqrt{(V_S \cos\phi + IR)^2 + (V_R \sin\phi + IX)^2}$$

$$= \sqrt{V_S^2 + 2V_R IR\cos\phi + 2V_R IX\sin\phi + (IR)^2(IX)^2}$$

$$= \sqrt{V_S^2 + 2V_R IR\cos\phi + 2V_R IX\sin\phi} \tag{2.48}$$

$$V_S = V_R\left(1 + \frac{2IR\,cos\,\phi + 2V_R IX\,sin\,\phi}{V_R}\right)^{\frac{1}{2}} \tag{2.49}$$

$$V_S = V_R\left(1 + \frac{IR\cos\phi + V_R IX\sin\phi}{V_R}\right) \quad (2.50)$$



Hence regulation (%) can be represented as

$$\boxed{\varepsilon(\%) = \frac{V_S - V_R}{V_S} = \frac{IR\cos\phi + IX\sin\phi}{V_S}} \quad (2.51)$$

It is often the voltage drop across a short line, e.g. distribution line, are represented by either one of these.

$$\begin{aligned} \Delta V &= IR\cos\phi + IX\sin\phi \\ &= \frac{PR + QX}{V_S} \\ &= \frac{P}{\xi S_k} + \frac{Q}{S_k} \\ \Delta V_{pu} &= P_{pu}R_{pu} + Q_{pu}X_{pu} \end{aligned} \quad (2.52)$$

where ξ = X / R and S_k = short circuit capacity.

For the two-port line model, it is a linear, bilateral and passive network with two point. Voltage and current of sending end of two port network can be represented in terms of voltage and current of receiving end.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (2.53)$$

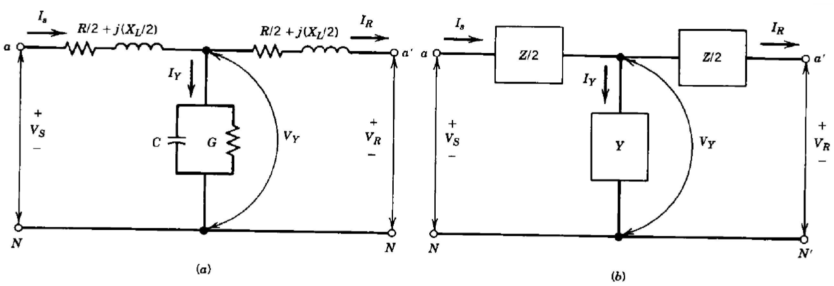
where A is reverse voltage gain, B is transfer impedance, C is transfer admittance and D is reverse current gain, as

$$A = \left.\frac{V_S}{V_R}\right|_{I_R=0}, \quad B = \left.\frac{V_S}{I_R}\right|_{V_R=0}, \quad C = \left.\frac{I_S}{V_R}\right|_{I_R=0}, \quad D = \left.\frac{I_S}{I_R}\right|_{V_S=0} \quad (2.54)$$

For short lines, with KCL and KVL,

$$\begin{cases} V_S = V_R + ZI_R \\ I_S = I_R \end{cases} \rightarrow \begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} \rightarrow \boxed{[ABCD]_{SL} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}} \quad (2.55)$$

2.6 Line Model – Middle Line Model



The capacitance starts to take place in middle line model. An equivalent T-network or Pi network is employed.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (2.53)$$

With KVL,

$$V_S = I_S \times \frac{1}{2}Z + I_R \times \frac{1}{2}Z + V_R = \left[I_R + \left(V_R + I_R \times \frac{1}{2}Z\right)Y\right]\frac{1}{2}Z + V_R + I_R \times \frac{1}{2}Z$$

or

$$V_S = \left(1 + \frac{1}{2}ZY\right)V_R + \left(Z + \frac{1}{4}YZ^2\right)I_R \quad (2.56)$$

and KCL,

$$I_S = I_R + \left(V_R + I_R \times \frac{1}{2}Z\right)Y$$

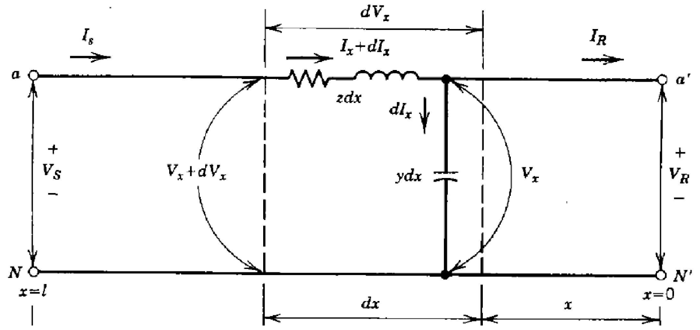
Or

$$I_S = Y \times V_R + \left(1 + \frac{1}{2}ZY\right)I_R \quad (2.57)$$

Hence, the ABCD matrix becomes:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}YZ & Z + \frac{1}{4}YZ^2 \\ Y & 1 + \frac{1}{2}YZ \end{bmatrix}\begin{bmatrix} V_R \\ I_R \end{bmatrix} \rightarrow \boxed{[ABCD]_{ML} = \begin{bmatrix} 1 + \frac{1}{2}YZ & Z(1 + \frac{1}{4}YZ) \\ Y & 1 + \frac{1}{2}YZ \end{bmatrix}} \quad (2.58)$$

2.6 Line Model – Long Line Model



$$\frac{d^2\mathbf{V}_x}{dx^2} = \mathbf{yzV}_x \tag{2.231}$$

The lumped model starts to become inaccurate in long line. A distributed line model is used instead.
The voltage drop in the differential element is

$$dV_x = (V + dV_x) - V_x = (I_x + dI_x)zdx \approx I_x z\, dx \tag{2.59}$$

Similarly, the incremental charging current in the differential element is

$$dI_x = V_x y\, dx \tag{2.60}$$

It becomes the differential equations

$$\frac{dV_x}{dx} = zI_x \quad \text{and} \quad \frac{dI_x}{dx} = yV_x \tag{2.61}$$

and hence,

$$\frac{d^2V_x}{dx^2} = yzV_x \tag{2.62}$$

At x = 0, $V_X = V_R$ and $I_X = I_R$. Therefore, the solution for the second order ODE gives

$$V(x) = (\cosh\sqrt{yz}x)V_R + \left(\sqrt{\frac{z}{y}}\sinh\sqrt{yz}x\right)I_R$$

$$I(x) = \left(\sqrt{\frac{y}{z}}\sinh\sqrt{yz}x\right)V_R + (\cosh\sqrt{yz}x)I_R \tag{2.63}$$

Let γ = propagation constant per unit length = $\sqrt{yz}$

Z_c = characteristics (surge / natural) impedance of line per unit length = $\sqrt{z/y}$

(2.63) can be written as

$$V(x) = (\cosh\gamma x)V_R + (Z_c \sinh\gamma x)I_R$$

$$I(x) = \left(\frac{1}{Z_c}\sinh\gamma x\right)V_R + (\cosh\gamma x)I_R \tag{2.64}$$

Further, $\gamma = \alpha + j\beta$

where α = attenuation constant (decrement in voltage and current per unit length in direction of travel)

β = phase constant (or phase change between two voltage, or current per unit length apart).

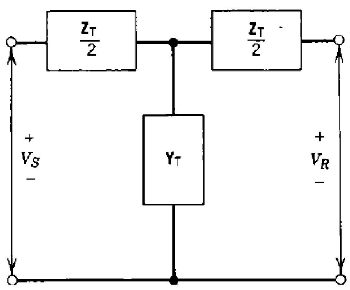
Putting into two-port model,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{2.53}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh\gamma x & Z_c\sinh\gamma x \\ \frac{1}{Z_c}\sinh\gamma x & \cosh\gamma x \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \rightarrow \boxed{[ABCD]_{LL} = \begin{bmatrix} \cosh\gamma x & Z_c\sinh\gamma x \\ \frac{1}{Z_c}\sinh\gamma x & \cosh\gamma x \end{bmatrix}} \tag{2.65}$$

Lumping the differential element to a lumped T-model similar to Middle Line Model, and recall (2.58)

$$[ABCD]_T = \begin{bmatrix} 1 + \frac{1}{2}Y_T Z_T & Z_T(1 + \frac{1}{4}Y_T Z_T) \\ Y_T & 1 + \frac{1}{2}Y_T Z_T \end{bmatrix} \tag{2.58}$$



Solving Y_T and $Z_T / 2$,

$$\frac{Z_T}{2} = \frac{A-1}{C} = \frac{\cosh\theta - 1}{Y_c\sinh\theta} \rightarrow Z_T = 2Z_c\tanh\gamma l \rightarrow \boxed{\frac{Z_T}{2} = \frac{\frac{Z}{2}\tanh\left(\frac{1}{2}\sqrt{YZ}\right)}{\frac{1}{2}\sqrt{YZ}}} \tag{2.66}$$

$$Y_T = C = Y_c\sinh\theta = \frac{\sinh\gamma l}{Z_c} \rightarrow \boxed{Y_T = \frac{Y\sinh\sqrt{YZ}}{\sqrt{YZ}}} \tag{2.67}$$

where $Y = yl$ and $Z = zl$.

2.7 Wave Propagation in Lossless Line

Characteristics impedance refers to a parameter which carries characteristic information of a power line. It is derived from maximum impedance and minimum impedance either from sending end or from receiving end. It is given as the square root of open-circuit impedance (maximum impedance) and short circuit impedance (minimum impedance) of any side, i.e.

$$Z_{OC}=\frac{A}{C}=\frac{\cosh\gamma x}{\frac{1}{Z_c}\sinh\gamma x}=\frac{Z_C\cosh\gamma x}{\sinh\gamma x} \tag{2.68}$$

$$Z_{SC}=\frac{B}{D}=\frac{Z_c\sinh\gamma x}{\cosh\gamma x} \tag{2.69}$$

Therefore, Characteristics impedance can be written as

$$\sqrt{Z_{OC}Z_{SC}}=\sqrt{\frac{AB}{CD}}=\sqrt{\frac{Z_C\cosh\gamma x}{\sinh\gamma x}\times\frac{Z_c\sinh\gamma x}{\cosh\gamma x}}=Z_C \tag{2.70}$$

For Long Line Model, $\boxed{Z_C=\sqrt{Z/Y}.}$

Propagation of electromagnetic wave depends on the strength of electric field and magnetic field governed by the voltage and the current of the line. Behavior of propagation wave is analyzed by velocity, propagation constant and characteristic impedance of the line. For lossless line, real part of impedance (r) and real part of admittance (g) are zero. Mathematically, r = 0, g = 0. Propagation Constant can be written as

$$\gamma=j\omega\sqrt{LC}=j\beta \tag{2.71}$$

Velocity of Wave Propagation is

$$v=\frac{\omega}{\beta}=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{\frac{\mu_0}{2\pi}\ln\frac{D}{r}\times\frac{2\pi\varepsilon_0}{\ln\frac{D}{r}}}}\boxed{=\frac{1}{\sqrt{\mu_0\varepsilon_0}}} \tag{2.72}$$

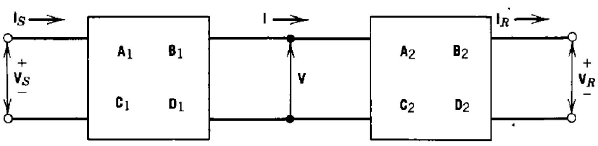
Wavelength of propagation wave in lossless line is

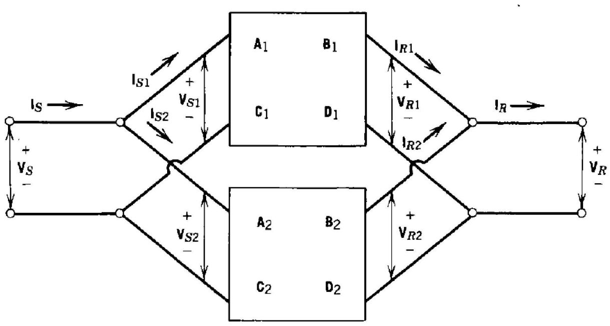
$$\lambda=\frac{v}{f}=\boxed{\frac{1}{f\sqrt{LC}}} \tag{2.73}$$

The sending end voltage and current can also be represented by the sum of incident wave and reflected wave.

$$\boxed{\begin{aligned}V_S&=\frac{1}{2}(V_R+I_RZ_C)e^{\alpha l}e^{j\beta l}+\frac{1}{2}(V_R-I_RZ_C)e^{-\alpha l}e^{-j\beta l}\\ I_S&=\frac{1}{2}\left(\frac{V_R}{Z_c}+I_R\right)e^{\alpha l}e^{j\beta l}-\frac{1}{2}\left(\frac{V_R}{Z_c}-I_R\right)e^{-\alpha l}e^{-j\beta l}\end{aligned}} \tag{2.74}$$

2.8 Series and Parallel Network





Network are often put in series, e.g. a cable connected with an OHL, or parallel, e.g. parallel feeders.

Series Network

For the network models,

$$\begin{bmatrix}V_s\\I_s\end{bmatrix}=\begin{bmatrix}A_1&B_1\\C_1&D_1\end{bmatrix}\begin{bmatrix}V\\I\end{bmatrix}\qquad\begin{bmatrix}V\\I\end{bmatrix}=\begin{bmatrix}A_2&B_2\\C_2&D_2\end{bmatrix}\begin{bmatrix}V_R\\I_R\end{bmatrix} \tag{2.74}$$

Substituting,

$$\begin{bmatrix}V_s\\I_s\end{bmatrix}=\begin{bmatrix}A_1&B_1\\C_1&D_1\end{bmatrix}\begin{bmatrix}A_2&B_2\\C_2&D_2\end{bmatrix}\begin{bmatrix}V_R\\I_R\end{bmatrix} \tag{2.75}$$

$$\boxed{\begin{bmatrix}V_s\\I_s\end{bmatrix}=\begin{bmatrix}A_1A_2+B_1C_2&A_1B_2+B_1D_2\\C_1A_2+D_1C_2&C_1B_2+D_1D_2\end{bmatrix}\begin{bmatrix}V_R\\I_R\end{bmatrix}} \tag{2.76}$$

Parallel Network

It is given that

$$\begin{bmatrix}V_{S1}\\I_{S1}\end{bmatrix}=\begin{bmatrix}A_1&B_1\\C_1&D_1\end{bmatrix}\begin{bmatrix}V_{R1}\\I_{R1}\end{bmatrix}\qquad\begin{bmatrix}V_{S2}\\I_{S2}\end{bmatrix}=\begin{bmatrix}A_2&B_2\\C_2&D_2\end{bmatrix}\begin{bmatrix}V_{R2}\\I_{R2}\end{bmatrix} \tag{2.77}$$

and

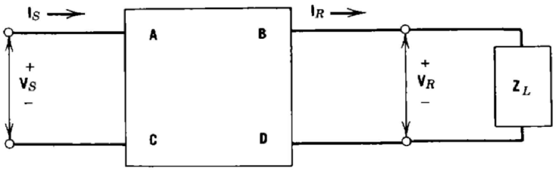
$$\begin{cases}V_S=V_{S1}=V_{S2}\\V_R=V_{R1}=V_{R2}\end{cases}\text{and}\quad\begin{cases}I_s=I_{S1}+I_{S2}\\I_R=I_{R1}+I_{R2}\end{cases} \tag{2.78}$$

Substituting,

$$\boxed{\begin{bmatrix}V_s\\I_s\end{bmatrix}=\begin{bmatrix}\frac{A_1B_2+B_1A_2}{B_1+B_2}&\frac{B_1B_2}{B_1+B_2}\\C_1+C_2+\frac{(A_1-A_2)(D_2-D_1)}{B_1+B_2}&\frac{B_1D_2+D_1B_2}{B_1+B_2}\end{bmatrix}\begin{bmatrix}V_R\\I_R\end{bmatrix}} \tag{2.79}$$

2.9 Terminated Lines

All lines are terminated with loads, which is often a function of receiving end voltage P = P(V_R) or a constant, i.e. ohmic.



Given a two-port model in (2.53),

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{2.53}$$

or

$$V_S = AV_R + BI_R \quad I_S = CV_R + DI_R \tag{2.80}$$

Given $V_R = Z_L I_R$, the impedance becomes

$$Z_{in} = \frac{V_S}{I_S} = \frac{AV_R + BI_R}{CV_R + DI_R} = \frac{AZ_L + B}{CZ_L + D} \tag{2.81}$$

Assume a symmetrical transposed long transmission line with

$$A = \cosh\sqrt{YZ} = \cosh\theta\,, \qquad B = \sqrt{\frac{Z}{Y}}\sinh\sqrt{YZ} = Z_c\sinh\theta\,, \qquad C = \frac{1}{Z_c}\sinh\theta\,, \qquad D = A = \cosh\theta \tag{2.82}$$

the input impedance becomes,

$$\boxed{Z_{in} = \frac{Z_L\cosh\theta + Z_c\sinh\theta}{\frac{Z_L}{Z_c}\sinh\theta + \cosh\theta}} \tag{2.83}$$

If the load impedance is chosen to be equal to the characteristic impedance, that is, $Z_L = Z_c$, the input impedance become $Z_{in} = Z_c$, which is independent of θ and line length. The value of the voltage is constant all along the line.

2.10 Lossless Lines

For a lossless line $\gamma = j\beta\ (\alpha = 0)$ and the hyperbolic function $\cosh\gamma x = \cosh j\beta x = \cos\beta x$ and $\sinh\gamma x = \sinh j\beta x = j\sin\beta x$, the equations for the rms voltage and current along the line, is given by

$$V(x) = \cos\beta x\, V_R + jZ_c\sin\beta x\, I_R \qquad I(x) = j\frac{1}{Z_c}\sin\beta x\, V_R + \cos\beta x\, I_R \tag{2.84}$$

At sending end $x = l$

$$V_S = \cos\beta l\, V_R + jZ_c\sin\beta l\, I_R \qquad I_S = j\frac{1}{Z_c}\sin\beta l\, V_R + \cos\beta l\, I_R \tag{2.85}$$

For hand calculation, it is easier to use (2.84) and (2.85). The termination condition can be easily obtained from the above equation. For example, the open-circuit line $I_R = 0$. The no-load receiving voltage is

$$\boxed{V_{R(nl)} = \frac{V_S}{\cos\beta l}} \tag{2.86}$$

At no-load, the line current is entirely capacitive charging current, and the receiving end voltage is higher than the sending end voltage. It is coined as Ferranti Effect. For a solid short circuit at the receiving end, $V_R = 0$ and

$$V_S = jZ_c\sin\beta l\, I_R\,, \qquad I_S = I_R\cos\beta l \tag{2.87}$$

2.11 Transmission Limit

The power handling ability of a line is limited by the thermal rating and stability limit. The increase in conductor temperature due to I^2R loss stretches the conductor and it increases the sag between transmission tower. For cables, it can even lead to insulation failure or conductor melting. Thermal limit is specified by the current carrying capacity of the conductor in manufacturer data. Thermal loading limit of the line is

$$S_{th} = 3V_\phi I_{th} \tag{2.88}$$

The thermal limit is calculated by the heat transfer equation, i.e.

$$I_{th}^2 R_{ac} = q_{rad} + q_{conv} - q_s \tag{2.89}$$

The convective heat loss q_{conv} is estimated by the empirical formula

$$q_{cond} = \max\left(\left(1.01 + 0.371\left(\frac{\rho VD}{\mu}\right)^{0.52}\right)K_f(T_c - T_a), 0.1695\left(\frac{\rho VD}{\mu}\right)K_f(T_c - T_a)^{0.6}\right) \tag{2.90}$$

where $\rho, V, D, \mu, K_f, T_c$ and T_a are air density, wind velocity, conductor diameter (in), air viscosity, thermal conductivity of air, conductor temperature and surrounding temperature respectively.

The radiative heat loss q_{rad} (W/ft) is estimated by

$$q_{rad} = 0.138\, D\varepsilon\left[\left(\frac{T_c + 273}{100}\right)^4 - \left(\frac{T_a + 273}{100}\right)^4\right] \tag{2.91}$$

where D = conductor diameter (in) and ε = emissivity (0.23 – 0.91 for bare conductor).

The solar radiation heat gain q_s is

$$q_s = 3.96D\sin\theta \tag{2.92}$$

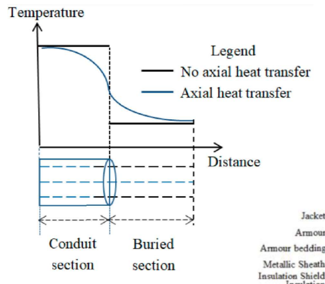
where θ = effective incident angle of sun ray.

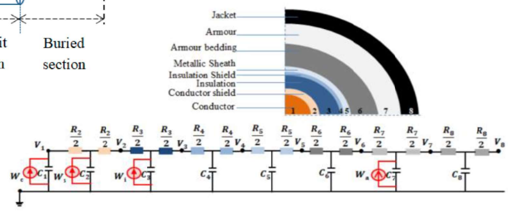
For a thermal insulated cable, i.e. buried in ground, the current rating is calculated with the thermal equation

$$I_R = \sqrt{(\Delta T - \frac{W_d[0.5R_{t1} + n(R_{t2} + R_{t3} + R_{t4}))}{R_e[R_{t1} + n(1+\lambda_1)R_{t2} + n(1+\lambda_1+\lambda_2)(R_{t3}+R_{t4})]}} \tag{2.93}$$

where

ΔT: difference between the maximum allowable conductor temperature and the ambient temperature;
W_d: dielectric losses for the insulation surrounding the conductor;
R_{t1}: thermal resistance between one conductor and the sheath;
R_{t2}: thermal resistance between the sheath and the armour;
R_{t3}: thermal resistance of the external serving of the cable;
R_{t4}: thermal resistance between the cable surface and the surrounding medium;
λ_1: ratio of losses in the metal sheath to total losses in all conductors;
λ_2: ratio of losses in the armouring to total losses in all conductors;
R_e: electric resistance of the conductor evaluated at the maximum allowable conductor temperature;
n: number of cable conductors.





Real power transfer over the line for a lossless line is given by

$$P = \frac{V_R V_S}{X}\sin\delta \tag{2.94}$$

The theoretical limit is when $\delta = 90^o$. The practical load angle alone is limited to no more than 30 to 45°. This is because of the generator and transformer impedance added to the line will result in a larger δ for a given load. For planning and other purpose, it is very useful to express the power transfer formula in terms of surge impedance loading (SIL), and construct the line loadability curve. For a lossless line $X' = Z_c \sin\beta l$, (2.94) can be written as

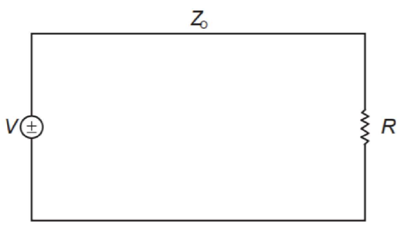
$$P_{3\phi} = \left(\frac{|V_{S(L-L)}|}{V_{rated}}\right)\left(\left|\frac{V_{R(L-L)}}{V_{rated}}\right|\right)\left(\frac{V_{rated}^2}{Z_c}\right)\frac{\sin\delta}{\sin\beta l} \tag{2.94}$$

The first two terms are per unit voltage denoted by $V_{s,pu}$ and $V_{R,\ pu}$ and the third term is recognized as SIL. (2.94) can be further written as

$$P_{3\phi} = \frac{|V_{s,pu}||V_{R,pu}|SIL}{\sin\beta l}\sin\delta = \frac{|V_{s,pu}||V_{R,pu}|SIL}{\sin\frac{2\pi l}{\lambda}}\sin\delta \tag{2.95}$$

Heavy load appreciably larger than SIL will produce large dip in voltage, hence compensation is needed.

2.11 Reflection and Refraction of Waves in Transmission Lines



Consider a lossless transmission line which has a surge impedance of Z_0 terminated through a resistance R as shown. When the wave travels along the line and absorbs any change (line end, change of series or shunt impedance), then it is partly or totally reflected.

The expression for reflected current is

$$i'' = -\frac{v''}{Z_0} \tag{2.96}$$

where v'' and i'' are the reflected voltage and current waves, respectively.

Let v and i be the transmitted voltage and current waves, and v' and i' be the incident waves.

Incident current: $i' = \frac{v'}{Z_0}$ Transmitted current: $i = \frac{v}{Z_0}$ (2.97)

Since $i = i' + i''$ and $v = v' + v''$, the transmitted voltage is

$$\frac{v}{R} = \frac{v'}{Z_0} - \frac{v''}{Z_0} \rightarrow \boxed{v = \frac{2R}{Z_0 + R}v'} \tag{2.98}$$

and, transmitted current

$$i = \frac{v}{R} = \frac{2}{Z_0 + R}v' = \frac{v'}{Z_0}\frac{2Z_0}{Z_0 + R} \rightarrow \boxed{i = i' \times \frac{2Z_0}{Z_0 + R}} \tag{2.99}$$

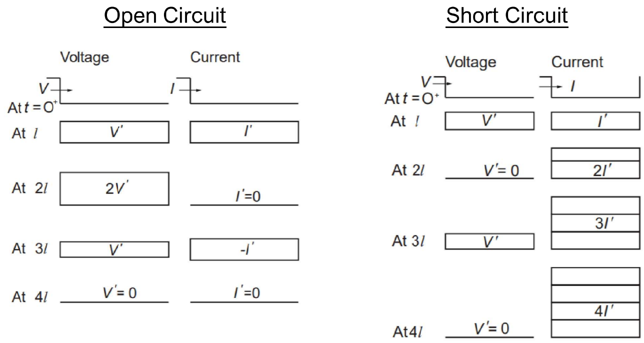
Substitute for v in terms of v' + v''

$$\frac{v' + v''}{R} = \frac{v'}{Z_0} - \frac{v''}{Z_0} \rightarrow \boxed{v'' = v' \times \frac{R - Z_0}{R + Z_0}} \tag{2.100}$$

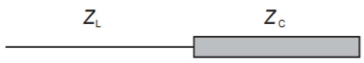
$$i'' = -\frac{v''}{Z_0} \rightarrow \boxed{i'' = i' \times \frac{R - Z_0}{R + Z_0}} \tag{2.101}$$

Open circuit $(R \to \infty)$:
Transmitted coefficient for voltage wave = 2; Transmitted coefficient for current wave = 0
Reflection coefficient for voltage wave = 1; Reflection coefficient for current wave = -1

Short circuit $(R \to 0)$:
Transmitted coefficient for voltage wave = 0; Transmitted coefficient for current wave = 2
Reflection coefficient for voltage wave = -1; Reflection coefficient for current wave = 1



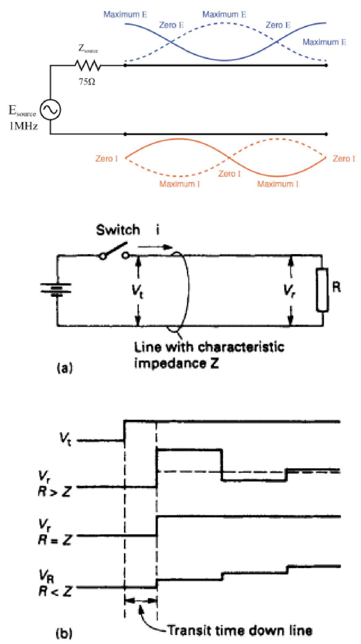
Consider a feeder with mix of overhead line Z_L and Z_c.

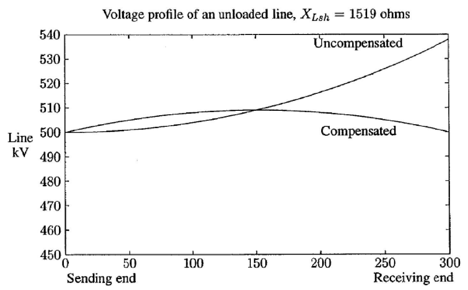


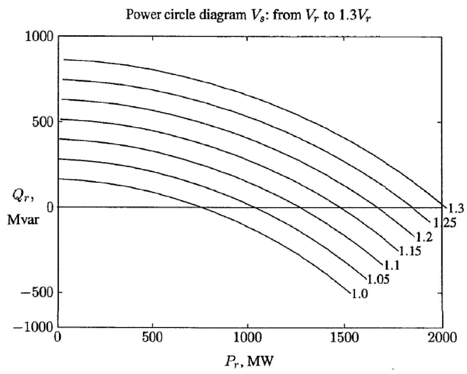
	Transmitted	Reflected	
Voltage	$v = v' \times \dfrac{2Z_C}{Z_C + Z_L}$	$v'' = v' \times \dfrac{Z_C - Z_L}{Z_C + Z_L}$	(2.102)
Current	$i = -i' \times \dfrac{2Z_C}{Z_C + Z_L}$	$i'' = -i' \times \dfrac{Z_C - Z_L}{Z_C + Z_L}$	

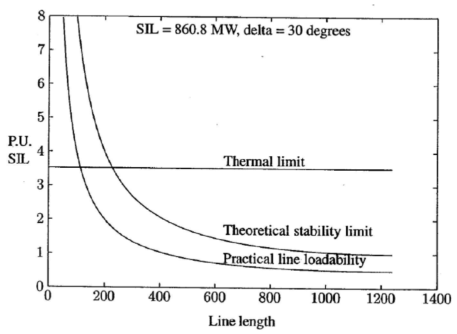
As a numerical example, an OHL with 1.3 mH/km and 0.09 uF/km respectively connected in series with an underground cable of 0.2 mH/km and 0.3 uF/km. From this, the surge impedance of OHL and cable are $Z_L = 120.18\Omega, Z_C = 25.82\Omega$ respectively. If a voltage surge of 100kV is initiated from OHL side, V_T = 35.27 kV, V_R = - 64.63 kV, I_T = 1.37kA, I_R = - 537.74A. If a voltage surge of 100kV is initiated from cable side, V_T = 164.63kV, V_R = 64. 63 kV, I_T = 1.37kA, I_R = -2.503 kA. To fully understand the possible voltage and current surge, Lattice diagram is often used to record surge voltage and current after several incidents.

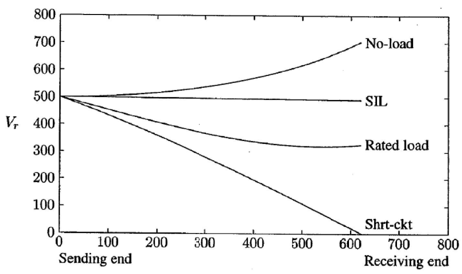
Additional Information:









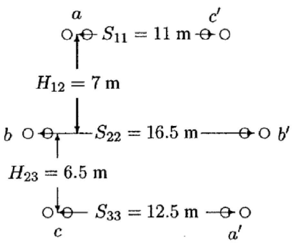


Any network will be based on cable, and at the high speeds used there are aspects of transmission line theory that need to be considered. Consider simple circuit (a). At the instance that the switch closes, the source voltage does not know the value of the load at the far end of the line. The initial current step, i, is therefore determined not by the load, but by the characteristics of the cable (dependent on the inductance and capacitance per unit length). A line therefore has a characteristic impedance. The initial current step will therefore be V/Z where Z is the characteristic impedance.

THE END

Example 1: The line feeds two balanced three-phase loads that are connected in parallel. The first load is Y- connected and has an impedance of 30 + j 40 Ω per phase. The second load is Δ – connected and has an impedance of 60 – j 45 Ω. The line is energized at the sending end from a three phase balanced supply of line voltage 207.85 V.
Taking the phase voltage V_a as reference, determine:
(a) The current, real power, and reactive power drawn from the supply.
(b) The line voltage at the combined loads.
(c) The current per phase in each load.
(d) The total real and reactive powers in each load and the line.
(e) The size of compensation device needed at load side.

Example 2: A 345-kV double circuit three phase transposed line is composed of two ACSR per phase with vertical conductor configuration as shown below. The conductors have a diameter of 0.036m and a GMR of 0.014 m. The bundled spacing is 0.457 m. With the following configuration, find the impedance per phase per kilometer of the line.



Example 3: A three phase, 132-kV Transmission Line is connected to a 50MW load at 0.85 lagging power factor. The line constants of 54km long line are $Z = 95\angle 78^o \Omega$ and Y $= 0.001\angle 90^o\ S$. Using the nominal T-circuit representation, calculate:
(a) The ABCD matrix
(b) Sending end voltage and current
(c) Sending end power factor
(d) Efficiency of transmission

Example 4: A single circuit, 60-Hz, three phase transmission line is 150mi long. The line is connected to a lad of 50MVA at a lagging power factor of 0.85 at 138kV. The line constants are given as R = 0.1858 Ω/mi, L= 2.60 mH/mi, C = 0.012 uF/mi. Calculate without considering wave reflection,
(a) The ABCD matrix
(b) Sending end Voltage and Current
(c) Sending end power and power factor
(d) Power Loss in line and hence transmission efficiency
(e) Percentage Voltage Regulation
(f) Sending end Charging Current at No Load.

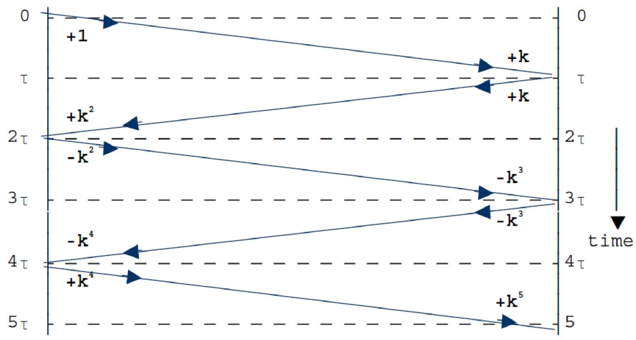
Calculate with wave reflection,
(a) Attenuation constant and phase change constant per km of line.
(b) Wavelength and Velocity of Propagation.
(c) Incident and Reflected Voltage at receiving end of line.
(d) Line Voltage at Receiving end of Line
(e) Incident and Reflected Voltage at sending end of Line
(f) Line Voltage at Sending end

Example 5: For a voltage surge with magnitude 1 per unit, prove that the voltage at open end after n[th] reflection is given by

$$V_R = 2k\frac{1-(-k^2)^n}{1+k^2}$$

Also, prove that the maximum voltage and steady state voltage is

$$V_{R(Max)} = 2k \qquad\qquad V_{R(\infty)} = \frac{2k}{1+k^2}$$



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Chapter 2 – Summary

Line Parameter: $R = \rho\frac{l}{A}$ $\qquad L_R = 2\times 10^{-7}\ln\frac{D^*}{R^*}$ $\qquad C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{R}}$

Voltage Regulation: $\varepsilon(\%) = \frac{V_S - V_R}{V_S} = \frac{IR\cos\phi + IX\sin\phi}{V_S}$

Line Model: $[ABCD]_{SL} = \begin{bmatrix}1 & Z\\ 0 & 1\end{bmatrix}$ $\qquad [ABCD]_{ML} = \begin{bmatrix}1+\frac{1}{2}YZ & Z(1+\frac{1}{4}YZ)\\ Y & 1+\frac{1}{2}YZ\end{bmatrix}$ $\qquad [ABCD]_{LL} = \begin{bmatrix}\cosh\gamma x & Z_c\sinh\gamma x\\ \frac{1}{Z_c}\sinh\gamma x & \cosh\gamma x\end{bmatrix}$

Conversion from Long Line Model to T-circuit: $Y_T = \frac{Y\sinh\sqrt{YZ}}{\sqrt{YZ}}$ $\qquad \frac{Z_T}{2} = \frac{\frac{Z}{2}\tanh\left(\frac{1}{2}\sqrt{YZ}\right)}{\frac{1}{2}\sqrt{YZ}}$

γ = Propagation Constant per unit length = $\sqrt{yz} = \alpha + j\beta$

Z_c = Characteristics Impedance of line per unit length = $\sqrt{z/y}$

Propagation Wave Velocity: $v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\qquad$ Propagation Wave Wavelength: $\lambda = \frac{v}{f} = \frac{1}{f\sqrt{LC}}$

Sending End Voltage and Current with reflection:

$$V_S = \frac{1}{2}(V_R + I_R Z_C)e^{\alpha l}e^{j\beta l} + \frac{1}{2}(V_R - I_R Z_C)e^{-\alpha l}e^{-j\beta l}$$

$$I_S = \frac{1}{2}\left(\frac{V_R}{Z_c} + I_R\right)e^{\alpha l}e^{j\beta l} - \frac{1}{2}\left(\frac{V_R}{Z_c} - I_R\right)e^{-\alpha l}e^{-j\beta l}$$

Series Network:

$$\begin{bmatrix}V_s\\ I_s\end{bmatrix} = \begin{bmatrix}A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2\\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2\end{bmatrix}\begin{bmatrix}V_R\\ I_R\end{bmatrix}$$

Parallel Network:

$$\begin{bmatrix}V_s\\ I_s\end{bmatrix} = \begin{bmatrix}\frac{A_1B_2 + B_1A_2}{B_1 + B_2} & \frac{B_1B_2}{B_1 + B_2}\\ C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} & \frac{B_1D_2 + D_1B_2}{B_1 + B_2}\end{bmatrix}\begin{bmatrix}V_R\\ I_R\end{bmatrix}$$

Input Impedance in terminated lines: $Z_{in} = \frac{Z_L\cosh\theta + Z_c\sinh\theta}{\frac{Z_L}{Z_c}\sinh\theta + \cosh\theta}$

No load Receiving Voltage: $V_{R(nl)} = \frac{V_S}{\cos\beta l}$

Reflected and Transmitted Voltage and Current: $V_T = \frac{2R}{Z_0 + R}V_{in}$ $\qquad I_T = \frac{2Z_0}{Z_0 + R}I_{in}$ $\qquad V_R = \frac{R - Z_0}{R + Z_0}V_{in}$ $\qquad I_R = \frac{R - Z_0}{R + Z_0}I_{in}$