Introduction to Power System - [Lecture Hour = 2 hours]

Per Phase System, Three Phase Transformer & Per Unit Calculation

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Three Phase Systems

High-power equipment such as generators, transformers, and transmission lines are built as three phase equipment. However, at the distribution level, depending on the voltage/power rating, a mixture of single phase and three-phase systems is used.

The three-phase system has many advantages over the single-phase system.

1. Three-phase systems produce a rotating magnetic field inside the alternating current (ac) motors and, therefore, cause the motors to rotate without the need for extra controls.
2. Three-phase generators produce more power than single-phase generators of equivalent volume.
3. Three-phase transmission lines transmit three times the power of single-phase lines.
4. Three-phase systems are more reliable; when one phase is lost, the other two phases can still deliver some power to the loads

Line Parameter and Phase Parameter

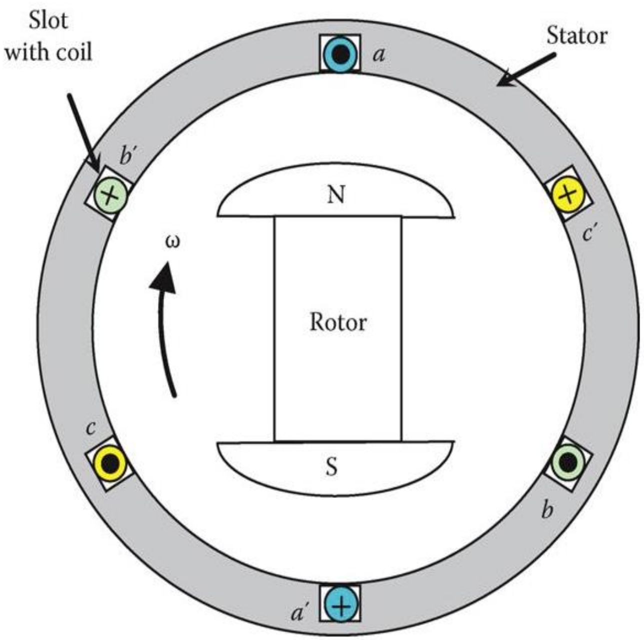
- Power System can be divided into three distinct parts: source, transmission and load.
- The currents in the transmission line conductors are called line currents. The voltages of the transmission line are classified by two variables: (1) line-to-neutral voltage and (2) line-to-line voltage
- The currents flowing through the source coil (generator or transformer) is phase current. The line-to-ground voltage is also called phase voltage. Phase voltage is the voltage between any line and the ground (or neutral).

Generation of Three-Phase Voltages

- Faraday's law: When conductor cuts magnetic field lines, a voltage is induced across the conductor.
- The generator consists of an outer frame called stator and a rotating magnet (permanent magnet or DC field coil) called rotor.
- When the magnet is spinning clockwise inside the machine by an external prime mover, the magnetic field then cuts all coils and, therefore, induces a voltage across each of them. (Φ → V)
- If each coil is connected to a load impedance, a current would flow into the load, and the generator produces electric energy that is consumed by the load. (V → I)
- The voltage across any of the three coils can be expressed by Faraday's law:

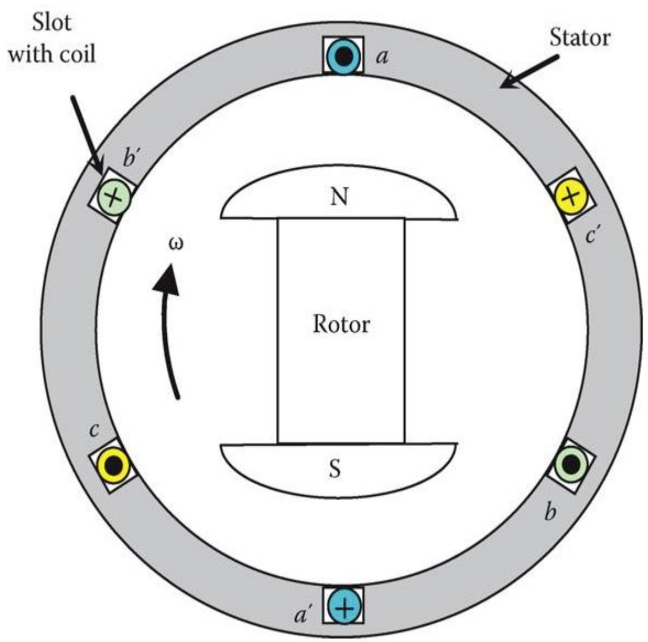
$$\boxed{e = nBl\omega}$$

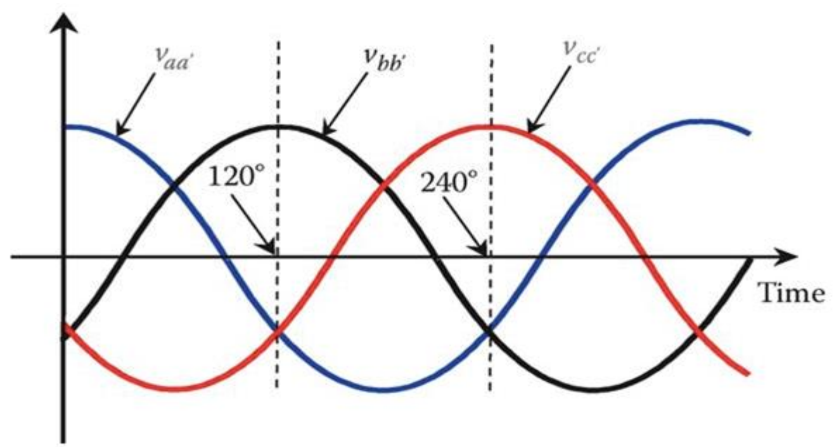
e is the voltage induced across the coil
n is the number of turns in the coil
B is the flux density of the magnetic field
I is the length of the slot
ω is the angular speed



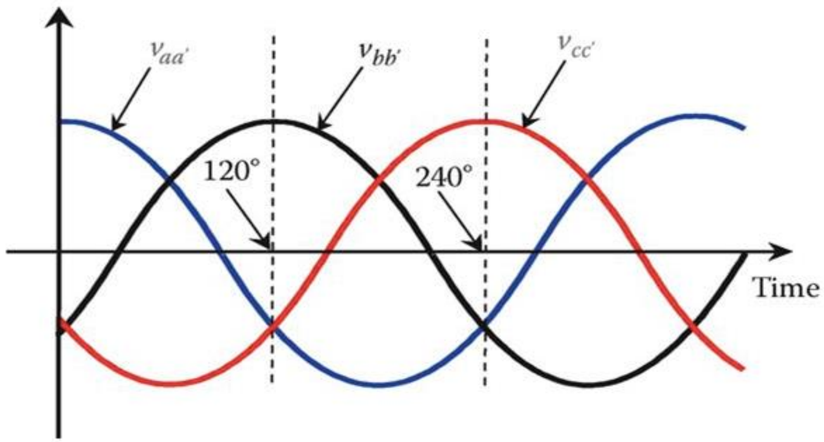
Generation of Three-Phase Voltages

- Voltage induced is proportional to perpendicular component of the magnetic field w.r.t. conductor.
- At the rotor position in Figure, coil a–a' has the maximum perpendicular field and, hence, it has the <u>maximum induced voltage</u>. Coil b–b' will have its maximum voltage when the rotor moves clockwise by 120°, and coil c–c' will have its maximum voltage when the rotor is at 240° position.
- If the rotation is continuous, the voltage across each coil is sinusoidal, all coils have the same magnitude of the maximum voltage, and the induced voltages are shifted by 120°. → Three Phase System





Generation of Three-Phase Voltages: Mathematical Expression



<u>Time domain Representation</u>

$$v_{aa'} = V_{max} \cos \omega t$$

$$v_{bb'} = V_{max} \cos(\omega t - 120°)$$

$$v_{cc'} = V_{max} \cos(\omega t - 240°)$$

<u>Phasor domain Representation</u>

$$\bar{V}_{aa'} = \frac{V_{\max}}{\sqrt{2}} \angle 0° = V \angle 0°$$

$$\bar{V}_{bb'} = \frac{V_{\max}}{\sqrt{2}} \angle -120° = V \angle -120°$$

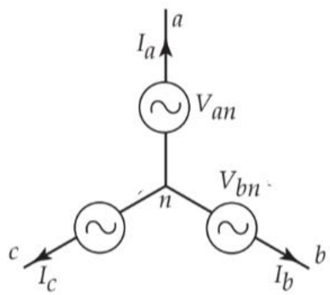
$$\bar{V}_{cc'} = \frac{V_{\max}}{\sqrt{2}} \angle 120° = V \angle 120°$$

where,
$\bar{V}$ is the phasor voltage in complex number form
V is the magnitude of the rms voltage

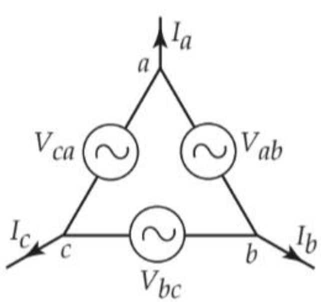
Main Components

A balanced 3-phase system has:

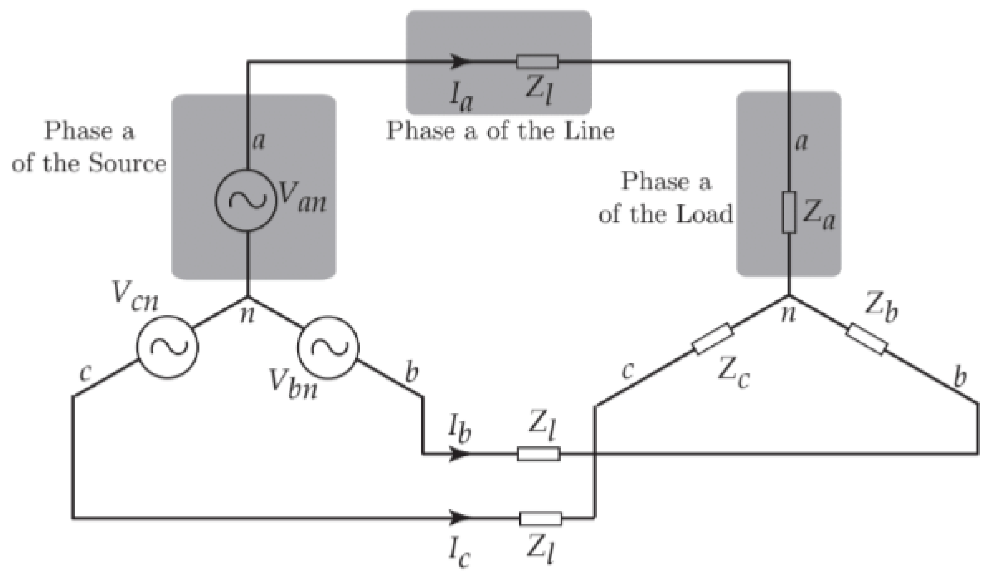
- Three sinusoidal voltage sources with equal magnitude, but with a phase shift of $120°$.
- equal loads on each phase.
- equal line impedance connecting the sources to the loads



Y-Connected



Δ-Connected

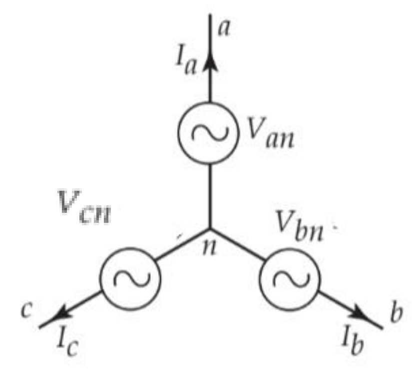


Y- Connected Voltage Source

$$v_{an} = V_{max}\cos(\omega t + \alpha)$$
$$v_{bn} = V_{max}\cos(\omega t + \alpha - 120^o)$$
$$v_{cn} = V_{max}\cos(\omega t + \alpha - 240^o)$$

$\longrightarrow$

$$V_{an} = V\angle\alpha$$
$$V_{bn} = V\angle\alpha - 120°$$
$$V_{cn} = V\angle\alpha - 240°$$



Y-Connected

In every balanced three-phase system, the sum of abc variables is ALWAYS zero:

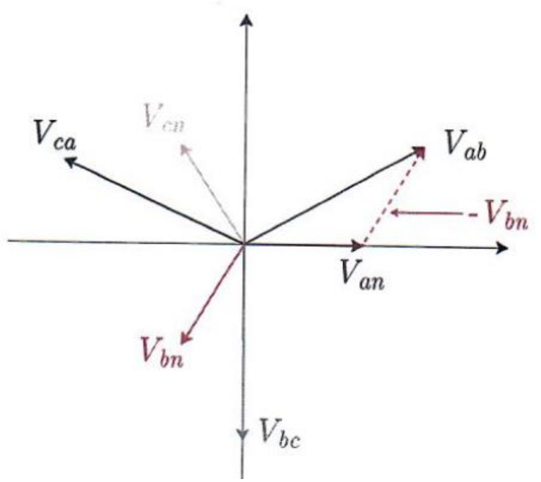
$$\boxed{V_{an} + V_{bn} + V_{cn} = 0}$$

Line-to-Line Voltage (with $\sqrt{3}$) as convention in power system.

$$V_{ab} = V_{an} - V_{bn} = V\,(1\angle\alpha - 1\angle(\alpha - 120°))$$
$$= \sqrt{3}V\angle\alpha + 30°$$

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V\angle\alpha - 90°$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V\angle\alpha - 210°$$

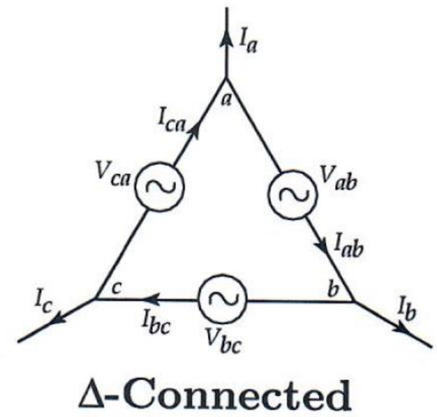


Δ - Connected Voltage Source

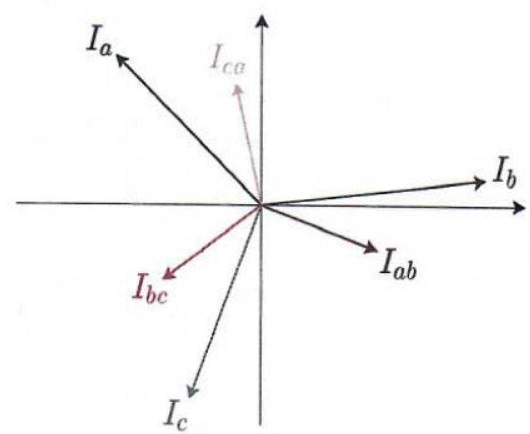
For the **Δ** connection, Phase voltages equal Line voltages.

For line current, it is a sum of current from the connected two sources.

$$
\begin{aligned}
I_a &= I_{ca} - I_{ab} = \sqrt{3} I_{ca} \angle 30^\circ \\
I_b &= I_{ab} - I_{bc} = \sqrt{3} I_{ab} \angle 30^\circ \\
I_c &= I_{bc} - I_{ca} = \sqrt{3} I_{bc} \angle 30^\circ
\end{aligned}
$$



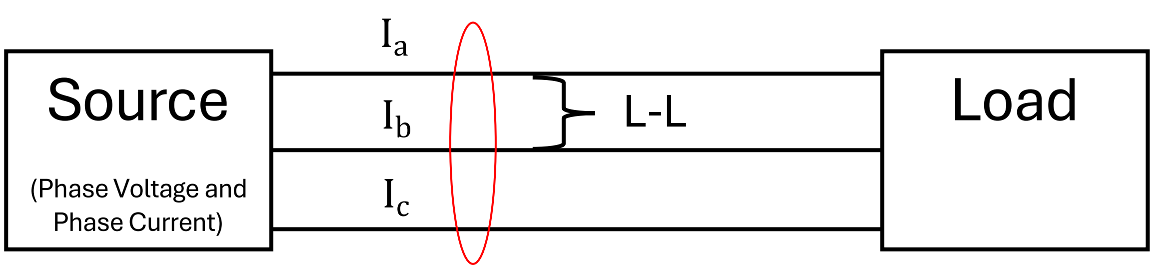
Line Currents are also balanced.



Measurement from outside, the currents are; I_a/ I_b/ I_c

Measurement from outside, the voltages are; V_{ab}/ V_{bc}/ V_{ca}

Note: Line voltage = Phase Voltage in Delta Connection



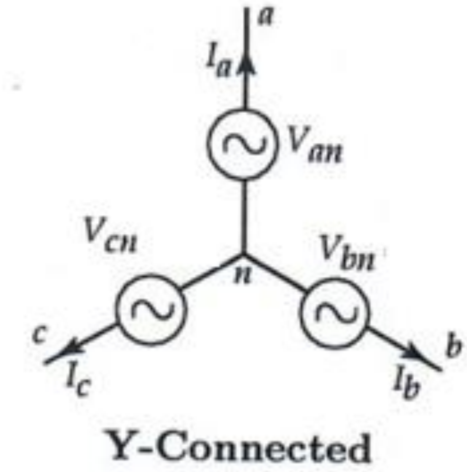
Measurement from Outside

Y- Connected Voltage Source

Summary

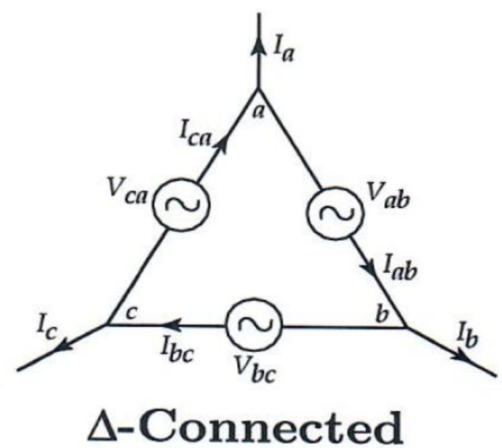
For Y-connected source:

$$V_{line} = \sqrt{3}V_{phase}\angle 30°$$
$$I_{line} = I_{phase}$$
$$S_{3-\emptyset} = 3V_{phase}\, I^* = V_{an}\,.\,I_a^* + V_{bn}\,.\,I_b^* + V_{cn}\,.\,I_c^*$$



For **Δ**-Connection source:

$$V_{line} = V_{phase}$$
$$I_{line} = \sqrt{3}I_{phase}\angle 30°$$
$$S_{3\phi} = 3V_{phase}\, I_{phase}^* = V_{ab}I_{ab}^* + V_{bc}I_{bc}^* + V_{ca}I_{ca}^*$$

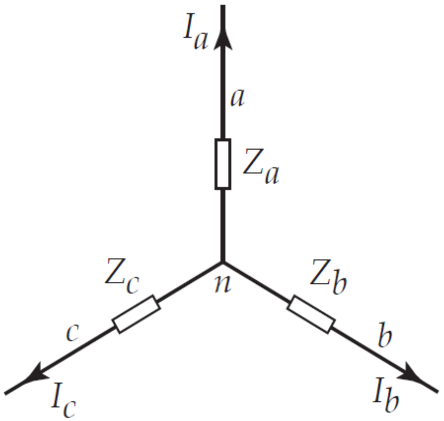


Y-Connected Load and Δ -Connected Load

For wye-connected load:

Phase & Line Currents are the same.

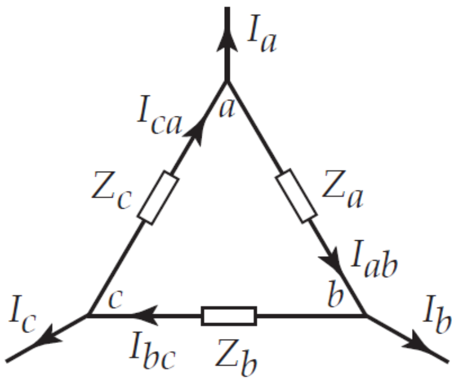
$$V_{ab} = V_{an} - V_{bn} = \sqrt{3}V_{an} \cdot e^{j30^\circ}$$
$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_{bn} \cdot e^{j30^\circ}$$
$$V_{ca} = V_{cn} - V_{an} = \sqrt{3}V_{cn} \cdot e^{j30^\circ}$$



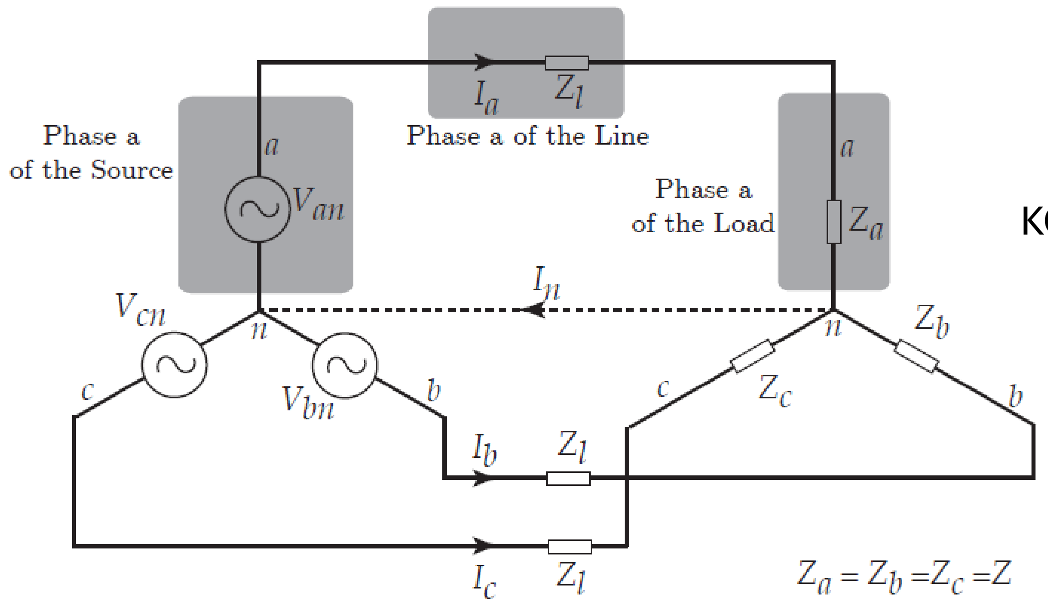
For delta-connected load:

Phase & Line Voltages are the same.

$$I_a = I_{ca} - I_{ab} = \sqrt{3}\, I_{ca}\, e^{j30^o}$$
$$I_b = I_{ab} - I_{bc} = \sqrt{3}\, I_{ab}\, e^{j30^o}$$
$$I_c = I_{bc} - I_{ca} = \sqrt{3}\, I_{bc}\, e^{j30^o}$$



Neutral Current and Unbalance Condition



KCL:

$$I_n = I_a + I_b + I_c$$

With KCL: $I_n = I_a + I_b + I_c$

$$I_n = \frac{V_{an}}{Z_l + Z_a} + \frac{V_{b_n}}{Z_l + Z_b} + \frac{V_{cn}}{Z_l + Z_c} = \frac{1}{Z_l + Z_a}(V_{an} + V_{bn} + V_{cn})$$

$$I_n = \frac{1}{Z_l + Z_a}(0) = 0$$

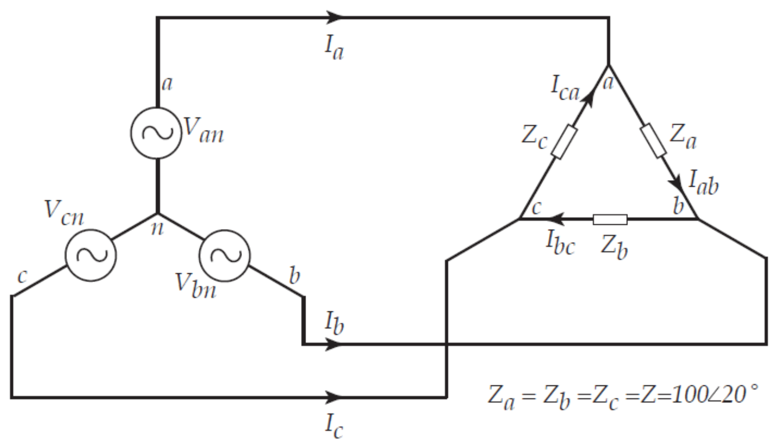
If there is any unbalance, in terms of source, load or lines, there is neutral voltage shift and neutral current flow. It is important in sizing neutral conductor and design earthing system.

Example 1 - Delta Load with Wye Source

Assume a Δ-connected load is supplied from a 3-phase 13.8kV (L-L) source with $Z = 100\angle 20°\Omega$.

(a) Find I_a, I_b, and I_c.

(b) Determine the load apparent power of the source.



Solution

$$V_{ab} = 13.8\angle 0°\text{kV}$$

$$V_{bc} = 13.8\angle -120°\text{kV}$$

$$V_{ca} = 13.8\angle -240°\text{kV}$$

$$I_{ab} = \frac{13.8\text{k}\angle 0°}{100\angle 20°} = 138\angle -20°\ \text{A}$$

$$I_{bc} = \frac{13.8\text{k}\angle -120°}{100\angle 20°} = 138\angle -140°\ \text{A}$$

$$I_{ca} = \frac{13.8\text{k}\angle -240°}{100\angle 20°} = 138\angle -260°\text{A}$$

Example 1 (Delta Load with Wye Source) cont'

From phase component to line component:

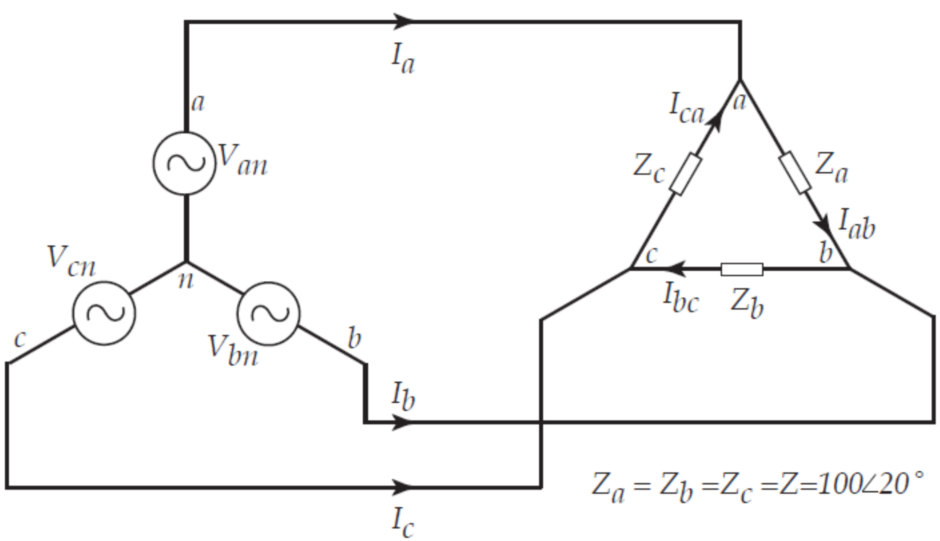
$$I_a = I_{ab} - I_{ca} = (138\angle -20°) - (138\angle -260°)$$

$$I_a = 239\angle -50° A$$

$$I_b = 239\angle -170° A$$

$$I_c = 239\angle -290° A$$

$$S = 3 \times V_{ab} I_{ab}^{*} = 3 \times 13.8\angle 0° kV \times 138\angle 20° = 5.7\angle 20° MVA = 5.37 + j1.95 MVA$$



Δ- Y Transformation

To simplify analysis of balanced 3 Phase systems:

- **Δ**-connected sources can be replaced by an equivalent Y-connected sources with

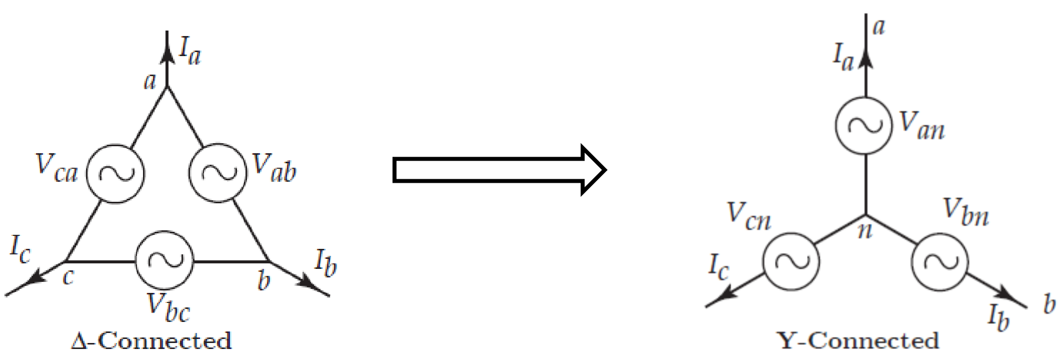
$$\boxed{V_{phase,Y} = \frac{V_{line,\Delta}}{\sqrt{3}\angle 30^\circ}}$$

- **Δ**-connected loads can be replaced by an equivalent Y-connected loads with

$$\boxed{Z_Y = \frac{Z_\Delta}{3}}$$

For the source,

$$V_{ab,Y} = V_{an,Y} - V_{bn,Y} = \sqrt{3}V_{an,Y}\angle 30^{o}$$
$$V_{ab,\Delta} = \sqrt{3}V_{an,Y}\angle 30^{o}$$
$$V_{an,Y} = \frac{V_{ab,\Delta}}{\sqrt{3}}\angle -30^{o}$$



Three Phase Transformers Combinations

Three phase transformers are of **Δ - Δ**, Y - Y, **Δ** - Y, and Y - **Δ** combinations.

Δ - Δ

- Common in Transmission Systems
- Reliability
- 3rd harmonics propagation
- Cannot be grounded
- Cannot supply very unbalanced system
- cannot handle a large amount of single-phase load.

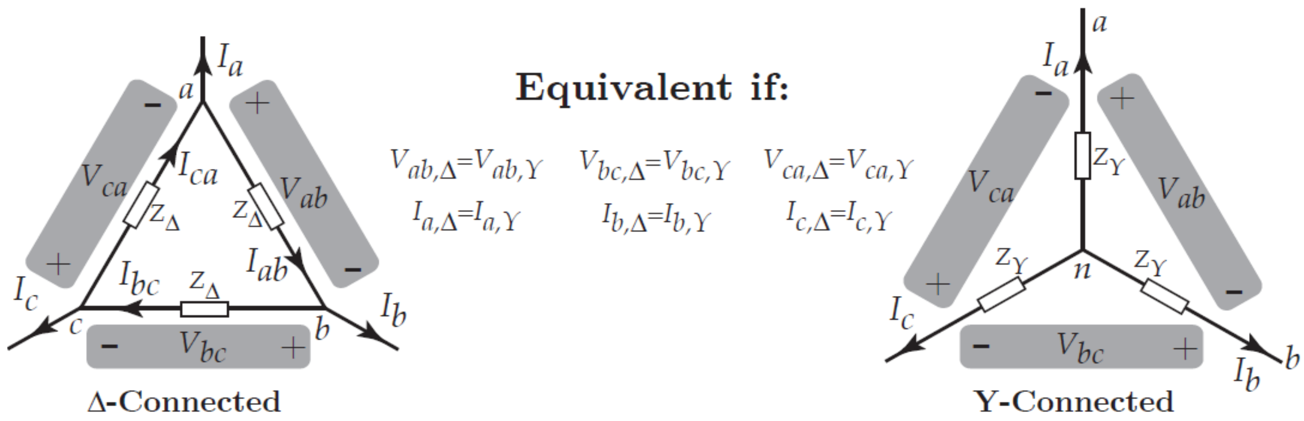
Y – Y

- Can be grounded on each side.
- No phase shift
- No 3rd harmonics propagation.

Δ – Y

- Most commonly used connection scheme (Step-down transformer between Transmission and Distribution.
- Presence of Neutral/ Can be grounded.
- 3rd harmonics propagation.
- Reliable
- Phase shift

Δ- Y Transformation - Loads



From Δ-side: we have,

$$I_a = I_{ca} - I_{ab} = \frac{V_{ca}}{z_\Delta} - \frac{V_{ab}}{z_\Delta} = \frac{V_{ca} - V_{ab}}{z_\Delta}$$

From Y-side: we have,

$$V_{ab} = V_{an} - V_{bn} = I_b \cdot z_Y - I_a \cdot z_Y. \qquad V_{ca} = V_{cn} - V_{an} = I_a \cdot z_Y - I_c \cdot z_Y$$

$$V_{ca} - V_{ab} = Z_Y(2I_a - I_b - I_c)$$

$$I_a + I_b + I_c = 0 \Rightarrow I_a = -I_b - I_c \cdot \frac{Z_0}{3}$$

$$V_{ca} - V_{ab} = z_Y \cdot 3I_a$$

$$\boxed{z_Y = \frac{z_\Delta}{3}}$$

Per Phase Analysis

Per phase analysis allows analysis of balanced 3-phase systems with the same effort as for a single-phase system. Per phase analysis is applicable to balanced 3-phase systems with all loads and sources Y-connected and no mutual inductance between phases.

In such systems:

- All neutrals are at the same potential.
- All phases are COMPLETELY *decoupled*.
- All system values are the same sequence as sources. The sequence order we have been using (phase b lags phase a and phase c lags phase b) is known as positive sequence.
- Convert all Δ load/sources to equivalent Y's.
- Keep in mind that all naturals are at the same potential.

Procedures -

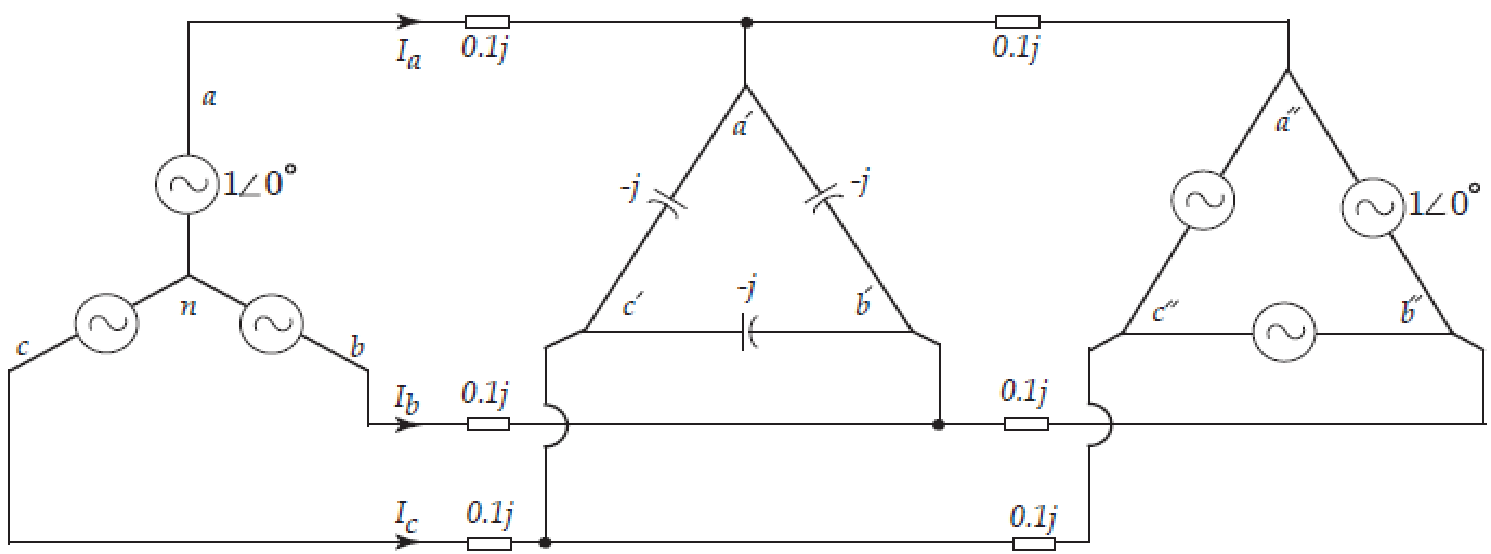
1. Solve phase "a" independent of the other phases.
2. Total system power is then $S = 3V_a I_a^*$.
3. If desired, phase "b" and "c" values can be determined by inspection (i.e., $\pm 120^\circ$ degree phase shifts).
4. If necessary, go back to original circuit to determine line-line values or internal Δ values.

Example 2 - Per Phase Analysis

Assume a 3 phase, Y-connected generator with $V_{an} = 1\angle 0°$ volts supplies a Δ-connected load with $Z_\Delta = -j$ through a transmission line with impedance of $0.1j$ per phase. The load is also connected to a Δ connected generator with $V_{ab} = 1\angle 0°$ through a second transmission line which also has an impedance of $j0.1$ per phase. Find:

(a) load voltage $V_{a'b'}$

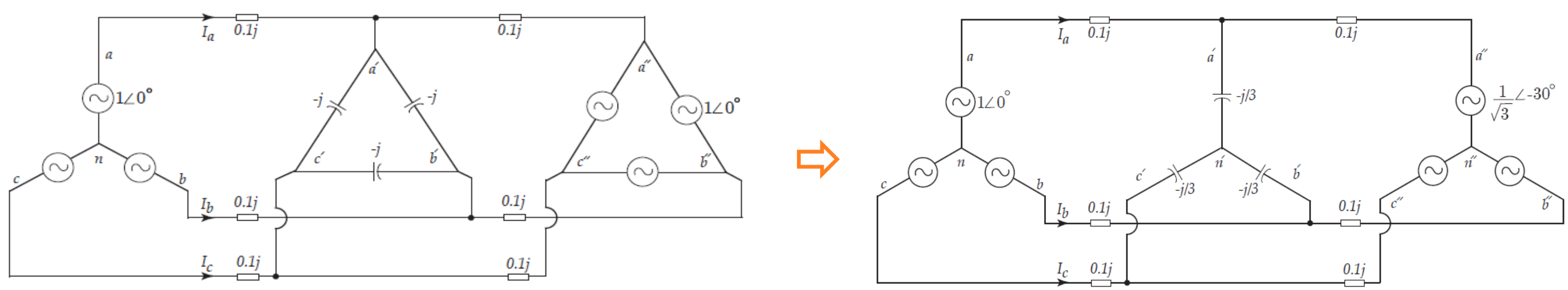
(b) The total power supplied by each generator, S_Y and S_Δ.



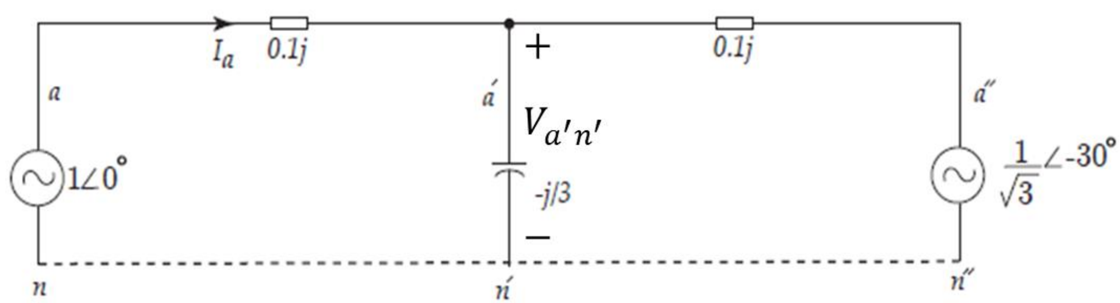
Example 2 - Per Phase Analysis

Solution

1. Convert all delta source and load into wye.



2. Since it is a balanced network, put it in per phase analysis.



KCL at a’: $$(V_{a'n'} - 1\angle 0)(-10j) + V_{a'n'}(3j) + \left(V_{a'n'} - \frac{1}{\sqrt{3}}\angle -30^\circ\right)(-10j) = 0$$

Example 2 - Per Phase Analysis

Therefore,

$$\left(10j + \frac{10}{\sqrt{3}}\angle 60^\circ\right) = V_{a'n'}(10j - 3j + 10j) \rightarrow V_{a'n'} = 0.9\angle -10.9^\circ \mathrm{V}$$

3. Obtain other two phase voltages with from the reference phase by shifting 120°.

$$V_{a'n'} = 0.9\angle -10.9^\circ \text{ V}$$

$$V_{b'n'} = 0.9\angle -130.9^\circ \text{ V} \quad V_{c'n'} = 0.9\angle -250.9^\circ \text{ V}$$

4. Put it in phase voltage with the conversion factor.

$$V_{a'b'} = V_{a'n'} - V_{b'n'} = 1.56\,\angle 19.1^\circ \text{ V}$$

5. Calculate the source power [VA] and load power [VA]

$$\mathrm{S_{Y,Gen}} = 3\mathrm{V_{an}}\mathrm{I_a^*} = 3\mathrm{V_{an}}\left(\frac{\mathrm{V_{an}} - \mathrm{V_{a'n'}}}{\mathrm{j0.1}}\right)^* = 5.1 + \mathrm{j}3.5\mathrm{VA}$$

$$\mathrm{S_{\Delta,Gen}} = 3\mathrm{V_{a''n''}}\mathrm{I_{a''}^*} = 3\mathrm{V_{a''n''}}\left(\frac{\mathrm{V_{a''n''}} - \mathrm{V_{a'n'}}}{\mathrm{j0.1}}\right)^* = -5.1 - \mathrm{j}4.7\mathrm{VA}$$

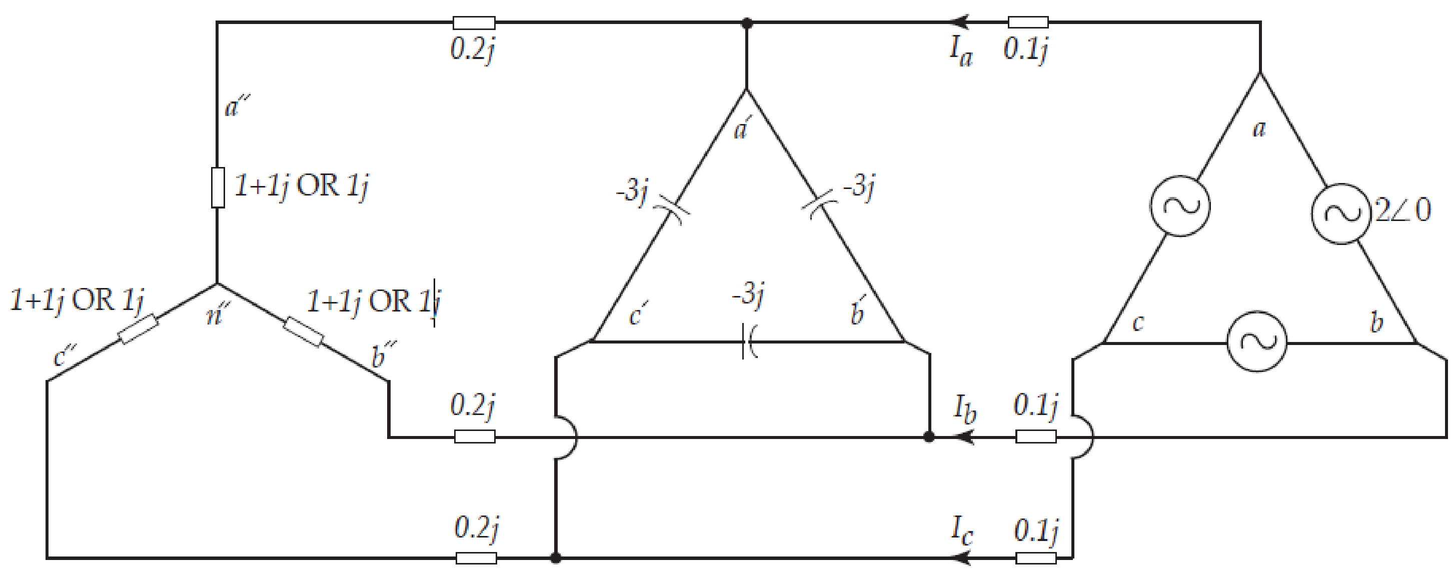
Note: We calculate the complex power in the transformed circuit. Since Δ source and its transformed Y source are equivalent, their complex power is the same.

Example 3 - Per Phase Analysis

Assume a 3-phase, Δ-connected generator with $V_{ab} = 2\angle 0°$ volts supplies a Δ-connected load with $Z_{\Delta} = -3j\Omega$ through a transmission line with impedance of $0.1j\Omega$ per phase. The load is also connected to another Y-connected load with impedance of Z_{load} through a second transmission line which has an impedance of $0.2j\ \Omega$ per phase.

Calculate the complex power of the second load for two scenarios:

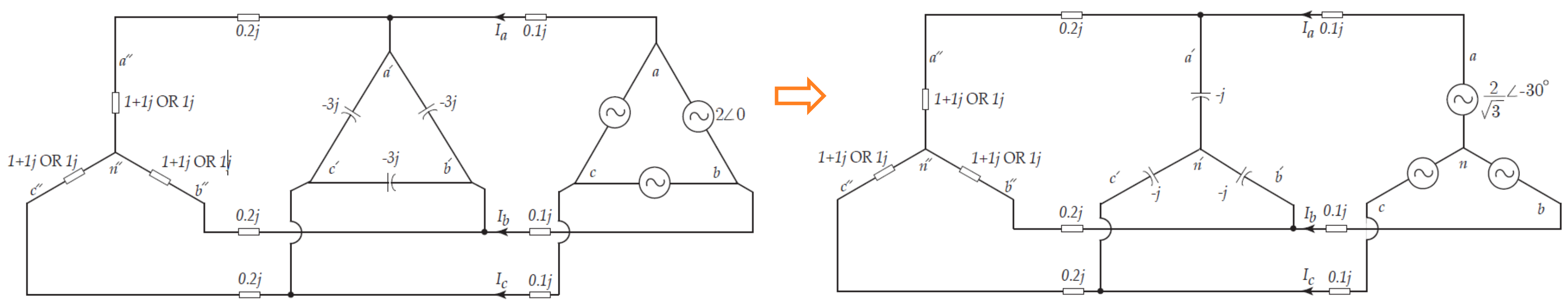
i) $Z_{\text{load}} = j1\Omega$ and ii) $Z_{\text{load}} = 1 + j1\Omega$



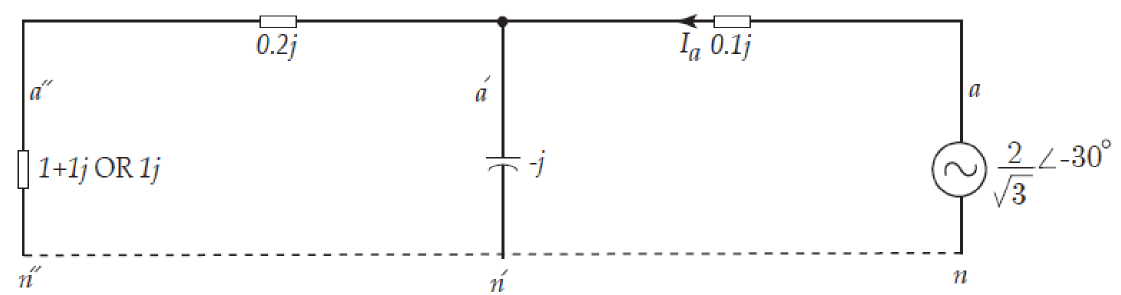
Example 3 - Per Phase Analysis

Solution

1. Convert all delta into wye.



2. Put it in per phase analysis.



By KCL:

$$\frac{V_{a'n'}}{0.2j+1+1j}+\frac{V_{a'n'}}{-j}+\frac{V_{a'n'}-\dfrac{2}{\sqrt{3}}\angle-30^\circ}{0.1j}=0$$

Example 3 - Per Phase Analysis

Therefore,

$$\frac{-20}{\sqrt{3}}\angle 60^\circ = V_{a'n'}\left(\frac{1}{1+1.2j} + j - 10j\right) \rightarrow V_{a'n'} = 1.21\angle -32.47^\circ \text{V}$$

$$I_{load\ 1,a} = \frac{V_{a'n'}}{1+1j+0.2j} = 0.78\angle -82.66^\circ \text{A}$$

$$V_{a''n''} = I_{load1,a}\,(1+1j) = 1.1\angle -37.68^\circ \text{ V}$$

$$S_{load\ 1} = 3V_{a''n''}I^*_{load1,a} = 1.81 + j1.81 \text{ VA}$$

For another load: 1 + j1Ω, with KCL:

$$\frac{V_{a'n'}}{0.2j+1j} + \frac{V_{a'n'}}{-j} + \frac{V_{a'n'} - \frac{2}{\sqrt{3}}\angle -30^\circ}{0.1j} = 0$$

$$\frac{-20}{\sqrt{3}}\angle 60^\circ = V_{a'n'}\left(\frac{1}{1.2j} + j - 10j\right) \rightarrow V_{a'n'} = 1.17\angle -30^\circ \text{ V}$$

$$I_{load\ 2,a} = \frac{V_{a'n'}}{1j+0.2j} = 0.98\angle -120^\circ \text{A}$$

$$V_{a''n''} = I_{\text{load }2,a}(1j) = 0.98\angle -30^\circ \text{ V}$$

$$S_{load\ 2} = 3V_{a''n''}I^*_{load\ 2,a} = 0 + j2.87 \text{ VA}$$

Key Messages

- Three-phase systems are usually used at high power/voltage levels.
- Three-phase systems are more economical for transferring energy as they need less amount of wire to transmit the same amount of energy compared to single or two-phase systems.
- To analyze complicated three-phase BALANCED systems, $\Delta - \mathrm{Y}$ transformation is the key element.
- In three-phase BALANCED systems, when all Δ loads and sources are transformed to Y connections, neutral points are at the same potential. Hence, per phase analysis can be applied.

Power in Single Phase Circuits

- The instantaneous power delivered to a load is,

$$\begin{aligned} p(t) &= \mathrm{v(t)i(t)} \\ &= \sqrt{2}\,\mathrm{V}\sin\omega t \sqrt{2} I \sin(\omega t - \phi) \\ &= \mathrm{VI}\cos\phi - VI\cos(2\omega t - \phi) \end{aligned}$$

- This power clearly has an average value, $\mathrm{P} = \mathrm{VI}\cos\phi$ (W). This is referred to as the "real power." The second, double frequency, term has no time averaged value, and therefore does no work.

- $\cos\phi$ is the "power factor"

- "Reactive Power," $\mathrm{Q} = \mathrm{VI}\sin\phi$ (VAR)

- $\mathrm{Q} > 0 \Rightarrow \phi > 0 \Rightarrow \mathrm{R} - \mathrm{L}$ load (motor, wires). $\mathrm{Q} < 0 \Rightarrow \phi < 0 \Rightarrow R - C$ load (battery).

- Complex Power, $S = \mathrm{P} + j\mathrm{Q} = \mathrm{VI}^*$ (* is complex conjugate)

Power in Three Phase Circuits

- The instantaneous power is:

$$p(t) = p_a(t) + p_b(t) + p_c(t)$$

$$p_a(t) = \sqrt{2}V_{an}\cos(\omega t)\sqrt{2}I_a\cos(\omega t - \phi)$$

$$p_b(t) = \sqrt{2}V_{an}\cos(\omega t - 2\pi/3)\sqrt{2}I_a\cos(\omega t - 2\pi/3 - \phi)$$

$$p_c(t) = \sqrt{2}V_{an}\cos(\omega t + 2\pi/3)\sqrt{2}I_a\cos(\omega t + 2\pi/3 - \phi)$$

$$p(t) = 2V_{an}I_a\left[\cos(\omega t)\sqrt{2}\cos(\omega t - \phi) + \cos\left(\omega t - \frac{2\pi}{3}\right)\cos\left(\omega t - \frac{2\pi}{3} - \phi\right) + \cos\left(\omega t + \frac{2\pi}{3}\right)\cos(\omega t)\right]$$

$$= V_{an}I_a\left[\cos(2\omega t - \phi)\cos(\phi) + \cos\left(2\omega t - \frac{4\pi}{3} - \phi\right)\cos(\phi) + \cos\left(2\omega t + \frac{4\pi}{3} - \phi\right)\cos(\phi)\right]$$

$$= 3V_{an}I_a\cos(\phi) = \mathrm{P}$$

3 x power with one more wire compared to single phase

Ideal Transformers

First, we review the voltage/current relationships for an ideal transformer assuming

- no real power losses,
- magnetic core has infinite permeability,
- no leakage flux.

Primary is usually the side with the higher voltage, but may be the low voltage side on a generator step-up transformer.

Assume we have flux ϕ_m in magnetic material. Then:

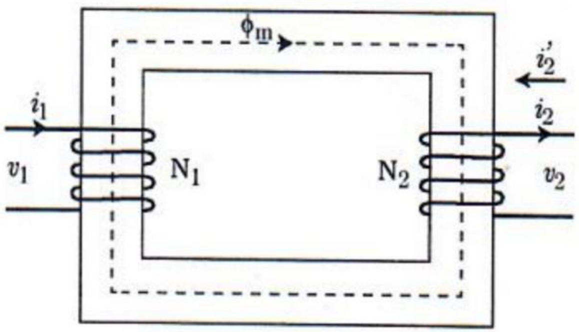
$$\lambda_1 = N_1\phi_m \text{ and } \lambda_2 = N_2\phi_m$$

$$v_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\phi_m}{dt} \text{ and } v_2 = \frac{d\lambda_2}{dt} = N_2\frac{d\phi_m}{dt}$$

$$\frac{d\phi_m}{dt} = \frac{v_1}{N_1} = \frac{v_2}{N_2} \rightarrow \frac{v_1}{v_2} = \frac{N_1}{N_2} = a = \text{turn ratio}$$

Faraday's Law: $$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

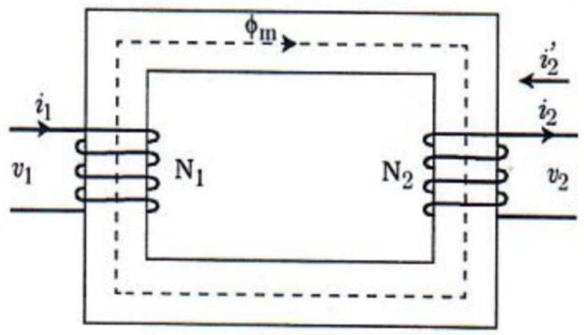
Flux Balance: $$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$



Current/Voltage Relationships

To get the current relationships, use ampere's law:

$$\oint H.dl = N_1 i_1 + N_2 i_2' = N_1 i_1 - N_2 i_2$$



where, $i_2' = -i_2$

$$\frac{BL}{\mu} = HL = N_1\ i_1 + N_2 i_2'$$

Assuming uniform flux density in the core,

$$\frac{\phi_m \times \text{ length}}{\mu \times \text{ area}} = N_1 i_1 + N_2 i_2'$$

If μ is infinite, then $\quad 0 = N_1 i_1 + N_2 i_2'$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

Matrix Form:

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

Polarity

- Current owing into a dot on the primary winding will induce a current owing out of the dot on the corresponding secondary winding.
- Depending on how the windings are connected to the bushings, the polarities can be additive or subtractive.

Impedance Transformation

Calculate the primary voltage and current for an impedance load on the secondary.

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ \frac{v_2}{Z} \end{bmatrix}$$

$$v_1 = av_2 \text{ and } i_1 = \frac{v_2}{aZ}$$

$$Z_{eq} = \frac{v_1}{i_1} = \frac{av_2}{\frac{v_2}{aZ}} = a^2 Z$$

$$\boxed{Z_1 = a^2 Z_2}$$

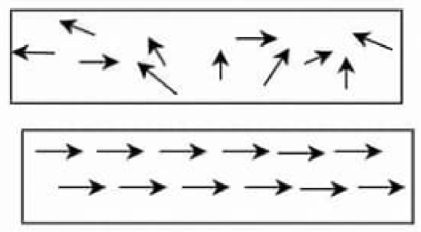
This is the equivalent impedance in primary side.

Reflecting the impedance from LV side to HV side, the impedance is amplified.

Real Transformer and Transformer Losses

Real Transformers

- have losses,
- have leakage flux,
- have finite permeability of magnetic core.

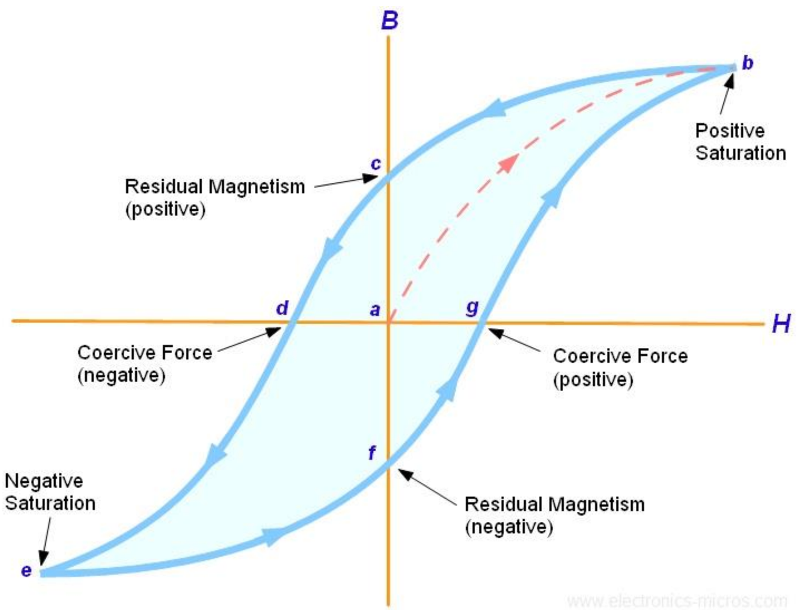


Transformer Losses

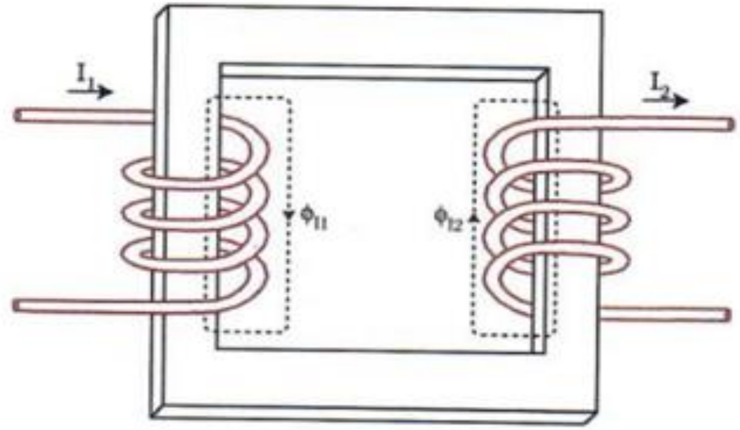
Real power losses are due to winding losses (i^2R) and core losses due to eddy currents and hysteresis.

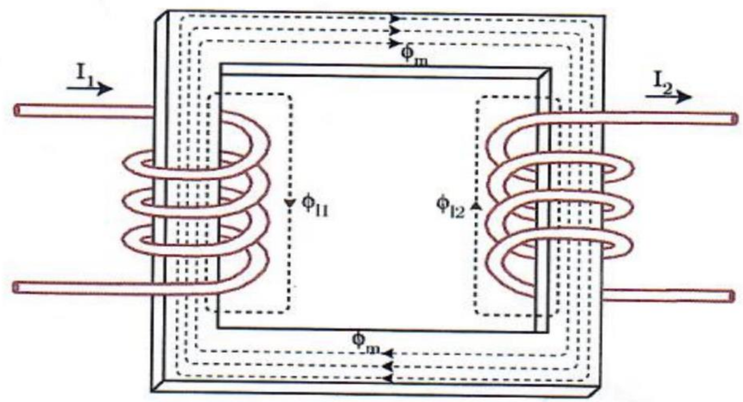
Eddy currents arise because of changing flux in core. Eddy currents are reduced by laminating the core.

Hysteresis losses are proportional to area of BH curve and the frequency. These losses are reduced by using material with a thin BH curve.



Effect of Leakage Flux





Not all flux is within the transformer core.

$$\lambda_1 = \lambda_{l1} + N_1\phi_m$$

$$\lambda_2 = \lambda_{l2} + N_2\phi_m$$

Assuming a linear magnetic medium, we get:

$$\lambda_{l1} \triangleq L_{l1} i_1 \text{ and } \lambda_{l2} \triangleq L_{l2} i_2'$$

Thus,

$$v_1 = r_1 i_1 + L_{l1}\frac{di_1}{dt} + N_1\frac{d\phi_m}{dt}$$

$$v_2 = \boxed{r_2 i_2'} + \boxed{L_{l2}\frac{di_2'}{dt}} + \boxed{N_2\frac{d\phi_m}{dt}}$$

Winding Resistance

Voltage induced by flux leakage

Voltage induced by ϕ_m

Effect of Finite Core Permeability

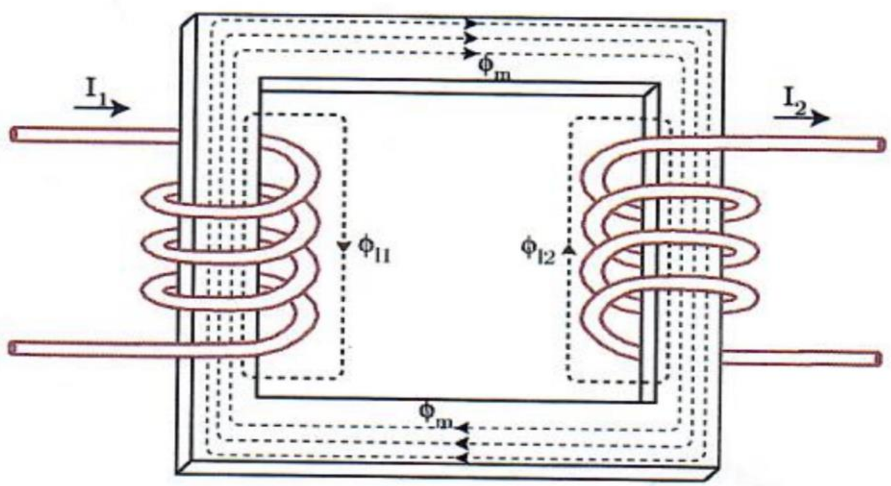
Finite core permeability means a non-zero mmf is required to maintain ϕ_m in the core:

$$N_1 i_1 - N_2 i_2 = \frac{\text{length}}{\mu \times \text{ area}} \phi_m = \boxed{\mathcal{R}\phi_m}$$

This value is usually modelled as magnetizing current.

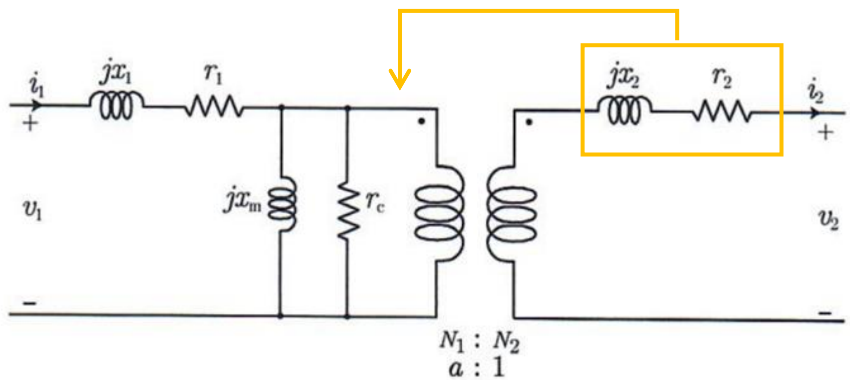
$$i_1 = \frac{\mathcal{R}\phi_m}{N_1} + \frac{N_2}{N_1} i_2 = i_m + \frac{N_2}{N_1} i_2$$

where $i_m = \mathcal{R}\phi_m / N_1$ is the magnetizing current.



Transformer Equivalent Circuit

- Using the previous relationships, we can derive an equivalent circuit model for the real transformer.

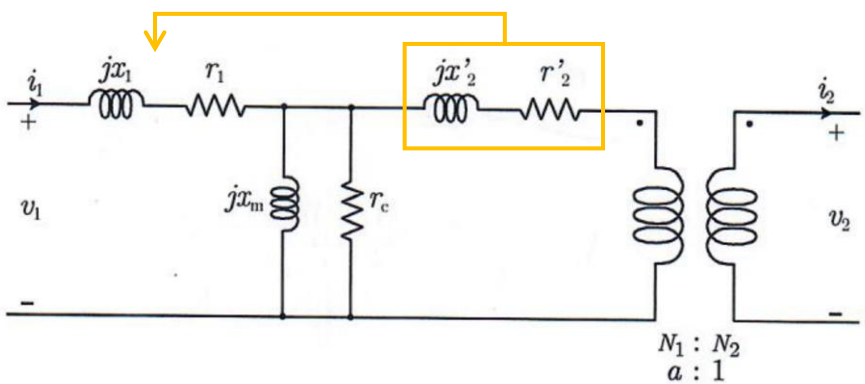


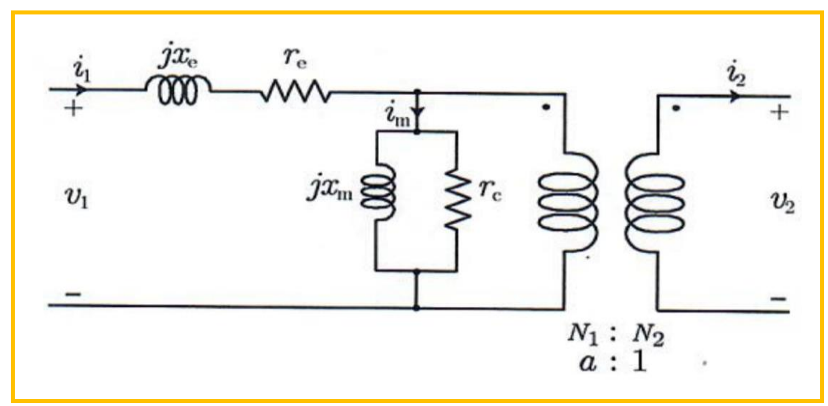
This model is further simplified by referring all impedances to the primary side:

$$r_2' = a^2 r_2 \qquad x_2' = a^2 x_2$$

Moreover, since the impedance of the parallel branch is so high, approximate the equivalent circuit:

$$\boxed{r_e = r_1 + r_2' \qquad x_e = x_1 + x_2'}$$

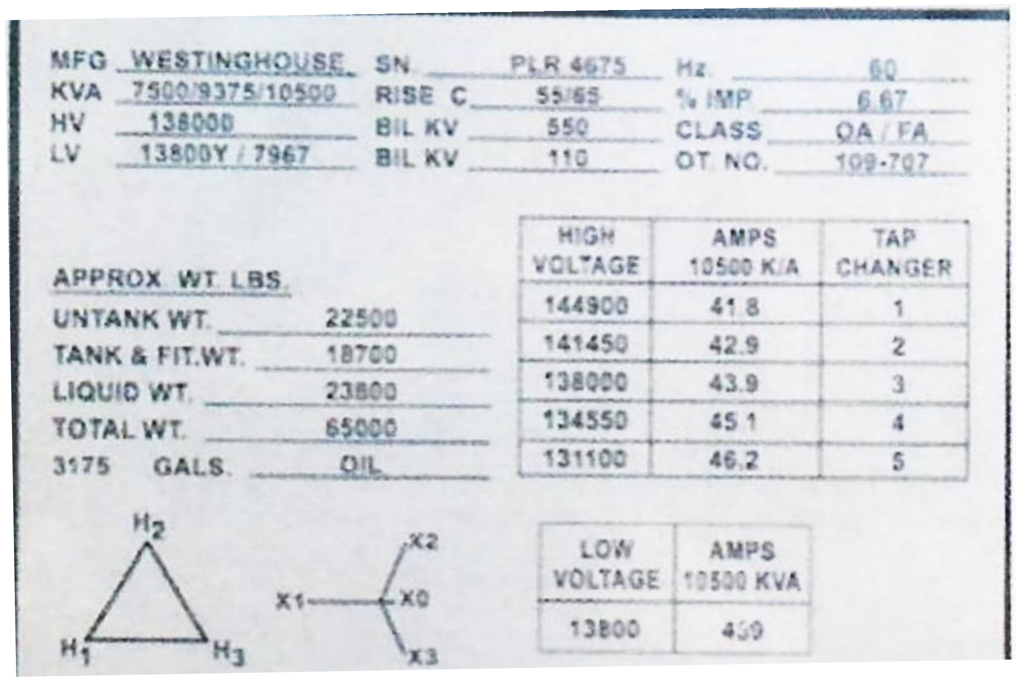




Calculation of Model Parameters

The parameters of the model are determined based upon:

- **Nameplate data**: Given rated voltages and power.
- **Open circuit test**: Rated voltage is applied to primary with secondary open; measure the primary current and losses (the test may also be done applying the voltage to the secondary, calculating the values, then referring the values back to the primary side).
- **Short circuit test**: With secondary shorted, apply voltage to primary to get rated current to flow; measure voltage and losses.



MFG	WESTINGHOUSE	SN	PLR 4675	Hz	60
KVA	7500/9375/10500	RISE C	55/65	% IMP	6.67
HV	138000	BIL KV	550	CLASS	OA / FA
LV	13800Y / 7967	BIL KV	110	OT NO.	109-707

APPROX WT LBS

UNTANK WT.	22500
TANK & FIT.WT.	18700
LIQUID WT.	23800
TOTAL WT.	65000
3175 GALS.	OIL

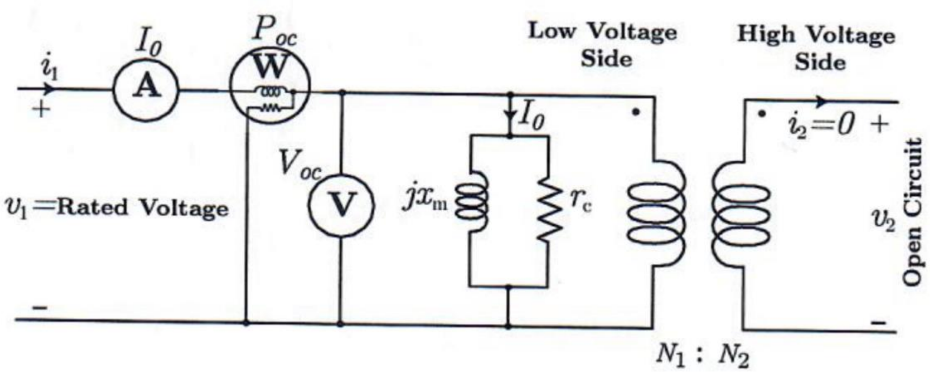
HIGH VOLTAGE	AMPS 10500 KVA	TAP CHANGER
144900	41.8	1
141450	42.9	2
138000	43.9	3
134550	45.1	4
131100	46.2	5

LOW VOLTAGE	AMPS 10500 KVA
13800	439

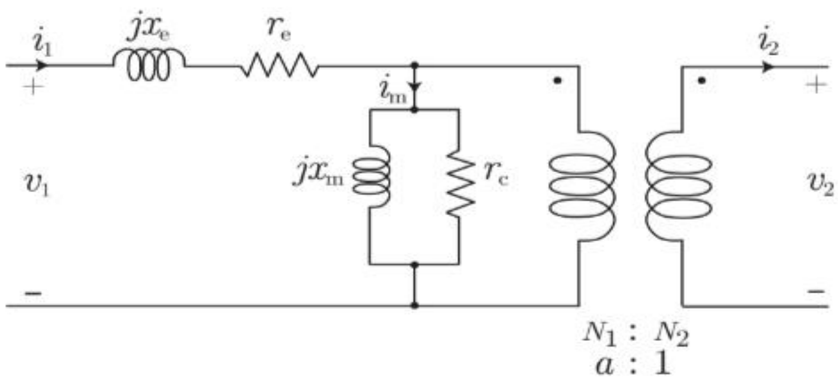
Open Circuit Test

- **Open circuit test** is conducted to determine the parameters of the magnetizing branch: x_m, r_c.
- In this test, **HV side** is **open-circuited**, and rated voltage is applied to LV side.
- Current, voltage, and power at LV side are measured.
- In such a case, the voltage drop in the leakage inductance and winding resistance is negligible due to very low primary current. Therefore, the transformer equivalent circuit is as shown below.

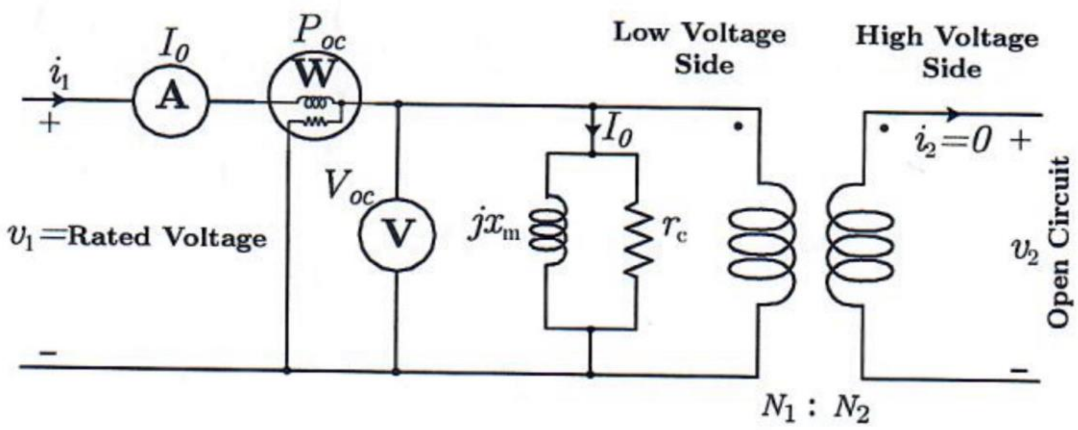
Measurement Circuit:



Simplified Equivalent Circuit:



Open Circuit Test



The watt-meter shows the core loss. Therefore,

$$\boxed{r_c = \frac{V_{oc}^2}{P_{oc}}}$$

Moreover, the magnitude of the admittance of the branch is:

$$\left\| \frac{1}{r_c} + \frac{1}{jx_m} \right\| = \sqrt{\frac{1}{r_c^2} + \frac{1}{x_m^2}} = \frac{I_0}{V_{oc}} \rightarrow \boxed{x_m = \frac{1}{\sqrt{\left(\frac{I_0}{V_{oc}}\right)^2 - \frac{1}{r_c^2}}}}$$

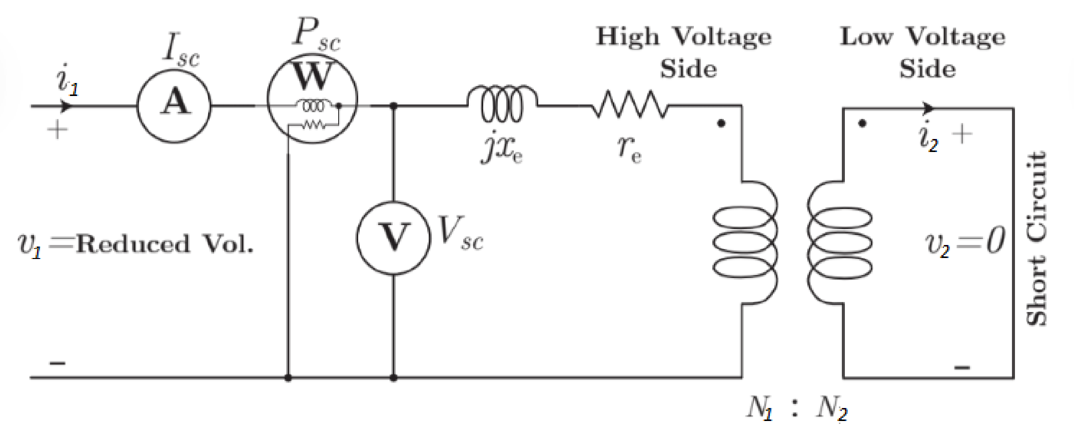
Short Circuit Test

- **Short circuit test** is to determine the parameters of the windings x_e, r_e.
- **LV side** is **short-circuited**. A reduced voltage is applied to HV side to reach rated current flows.
- Current, voltage, and power at HV side are measured.
- In such a case, the magnetizing branch can be neglected since the magnetizing current is negligible compared to the rated current.
- The watt-meter shows the winding loss.

$$\boxed{r_e = \frac{P_{sc}}{I_{sc}^2}}$$

- The magnitude of the winding impedance is

$$\|r_e + jx_e\| = \frac{V_{sc}}{I_{sc}} \rightarrow \boxed{x_e = \sqrt{\left(\frac{V_{sc}}{I_{sc}}\right)^2 - r_e^2}}$$



Example 4 – Transformer Parameter Calculation

These data were obtained when open-circuit and short circuit tests were conducted on a single-phase transformer with the power rating of 50kVA with the rated frequency of $60\ \text{Hz}$. The rated voltages of the high-voltage and low-voltage sides are $2400\ \text{V}$ and $240\ \text{V}$, respectively.

	Voltage (V)	Current (A)	Power (W)
HV Winding O/C	240	4.85	180
LV Winding S/C	50	20.8	600

Calculate the equivalent circuit of the transformer referred to the low voltage side.

Solution

Using the open-circuit test data, r_c and x_m can be calculated:

$$r_c = \frac{V_{oc}^2}{P_{oc}} = \frac{240^2}{180} = 320\Omega$$

$$x_m = \frac{1}{\sqrt{\left(\frac{I_0}{V_{oc}}\right)^2 - \frac{1}{r_c^2}}} = \frac{1}{\sqrt{\frac{4.85}{240} - \frac{1}{320^2}}} = 50.08\Omega$$

Example 4 – Transformer Parameter Calculation

Using the short-circuit test data, r_e and x_m can be calculated:

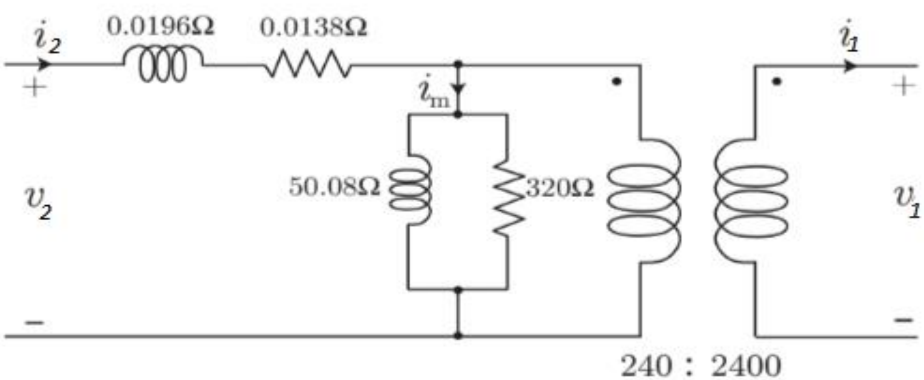
$$r_e = \frac{P_{sc}}{I_{sc}^2} = \frac{600}{20.8^2} = 1.38\Omega$$

$$x_e = \sqrt{\left(\frac{V_{sc}}{I_{sc}}\right)^2 - r_e^2} = \sqrt{\left(\frac{50}{20.8}\right)^2 - 1.38^2} = 1.96\Omega$$

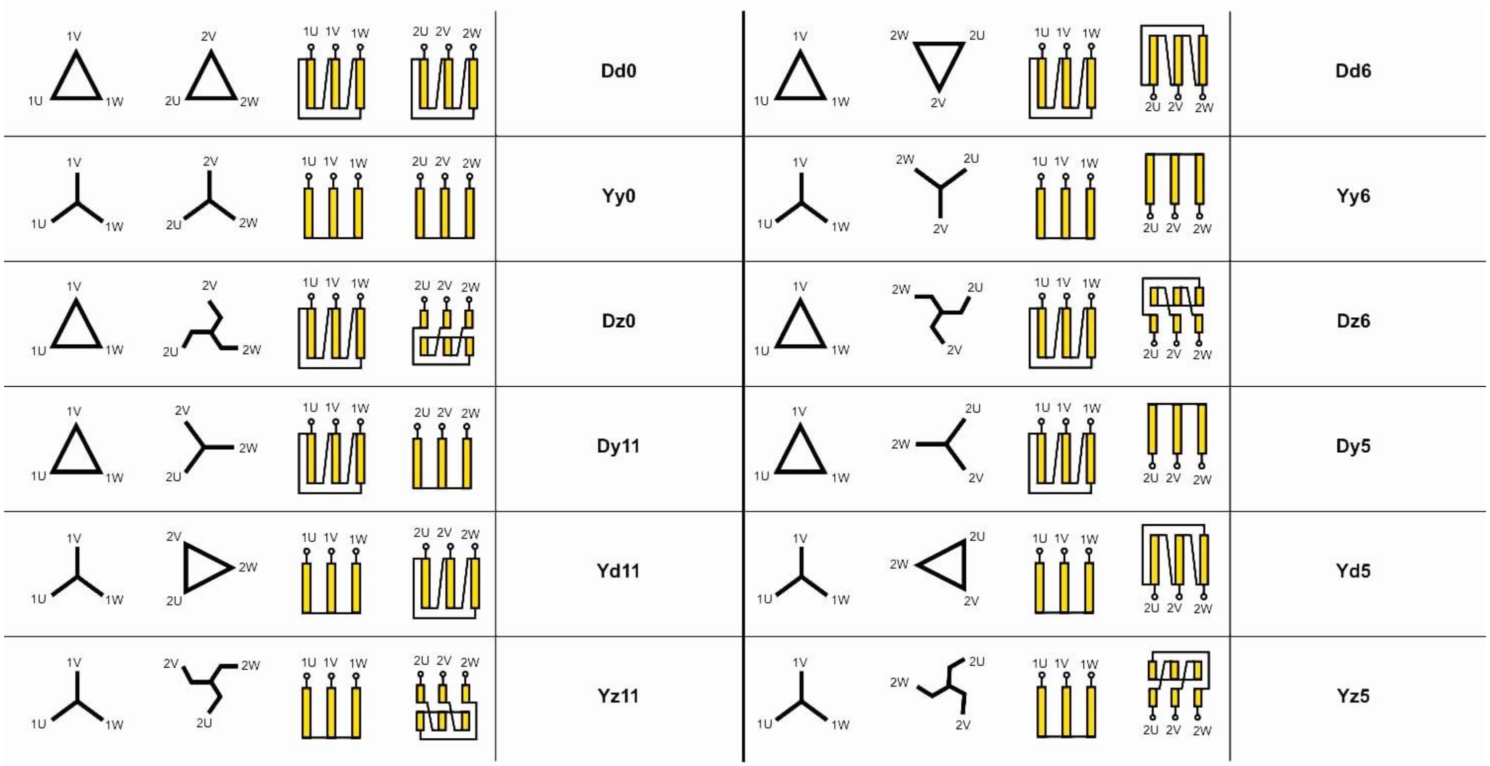
It must be noted that x_e and r_e are at HV side. Since the equivalent model at LV side is asked, x_e and r_e must be referred to the low voltage side.

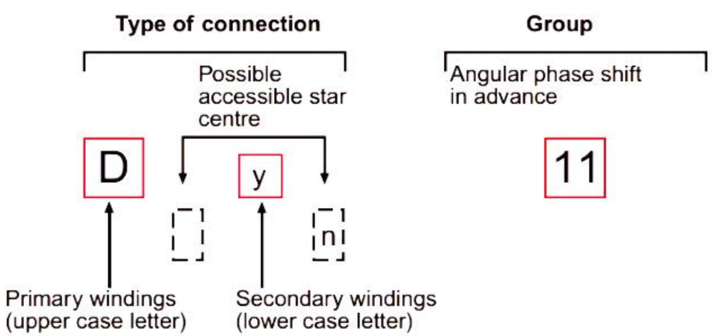
$$d = \frac{N_2}{N_1} = \frac{V_{\text{low}}}{V_{\text{high}}} = \frac{240}{2400} = 0.1$$

$$r_e' = d^2 \times r_e = 0.1^2 \times r_e = 0.0138\Omega \qquad x_e' = d^2 \times x_e = 0.1^2 \times x_e = 0.0196\Omega$$

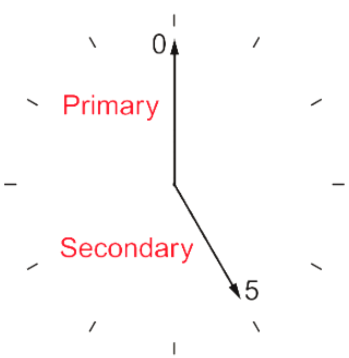


Vector Group Notation

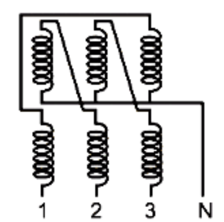


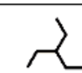


- **Y** – star connection
- **D** – delta connection
- **Z** – zigzag connection



Zigzag





Z or z

Used on the secondary side of distribution transformers

Y – Y Connection

The turns ratio of the transformer is N_{I}/N_2, while the line-to-line voltage ratio of the transformer is V_{ab}/V_{AB}.

$$\frac{N_1}{N_2} = \frac{V_{an}}{V_{AN}} = \frac{\sqrt{3}V_{an}}{\sqrt{3}V_{AN}} = \frac{V_{ab}}{V_{AB}}$$

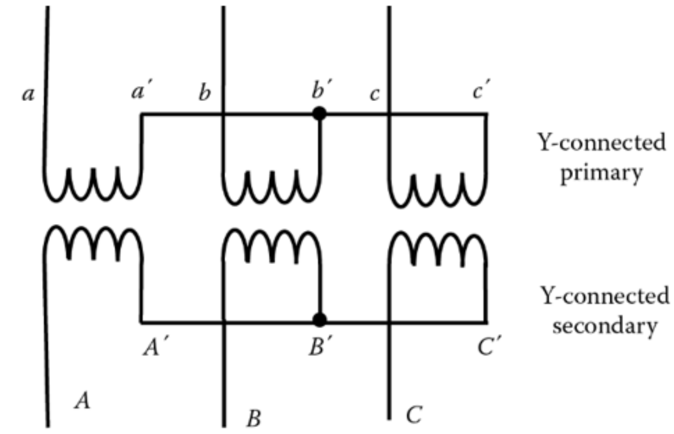
The current ratio is the inverse of the turn ratio.

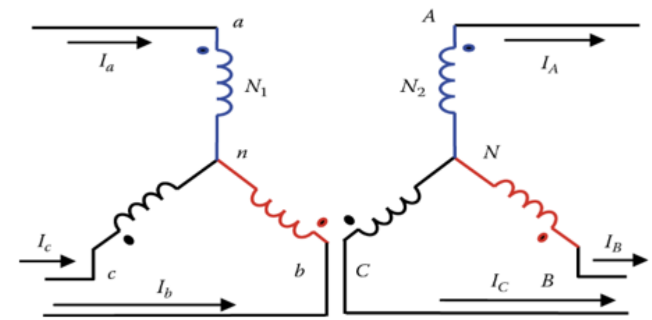
$$\frac{N_1}{N_2} = \frac{I_A}{I_a}$$

Y-Y connections are common in transmission systems.

<u>Advantages</u>

- ability to ground each side
- no phase shift introduced





Delta-Delta Connection

- Currents in transformer windings are phase current
- Currents in lines feeding the transformers are line current.

Turns ratio of transformer is N_{I}/N_2

Voltage ratio of transformer is ratio of voltages V_{ab}/V_{AB}, which are the same as the ratio of the line-to-line voltages in the delta configuration.

Keep in mind that the voltage per turn is constant in any winding. Hence,

$$V_T = \frac{V_{ab}}{N_1} = \frac{V_{AB}}{N_2}$$

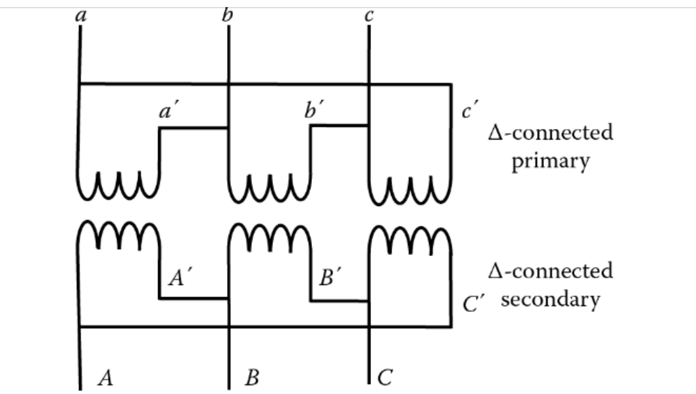
$$\frac{N_1}{N_2} = \frac{V_{ab}}{V_{AB}}$$

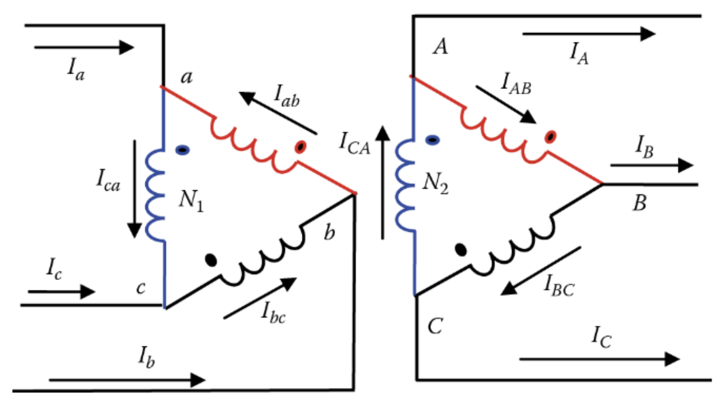
Using the magnetomotive force equation, we can compute the current ratio

$$\mathfrak{I} = I_{ab}N_1 = I_{AB}N_2$$

$$\frac{N_1}{N_2} = \frac{I_{AB}}{I_{ab}}$$

- Disadvantage: Cannot be grounded.



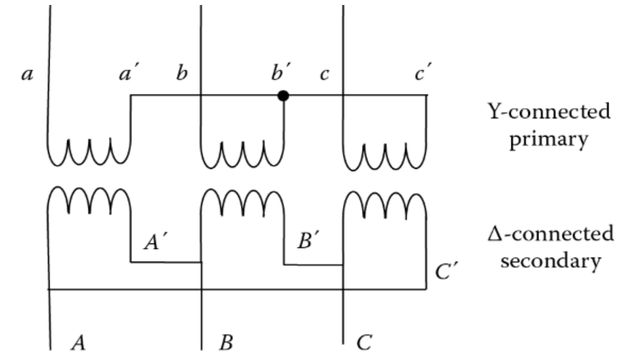


Delta-Wye Connection

Voltage across N_1 is the phase voltage of the primary circuit V_{an} and voltage across N_2 is line-to-line voltage of secondary circuit V_{AB}. Since voltage turns ratio as

$$V_T = \frac{V_{an}}{N_1} = \frac{V_{AB}}{N_2}$$
$$\frac{N_1}{N_2} = \frac{V_{an}}{V_{AB}}$$

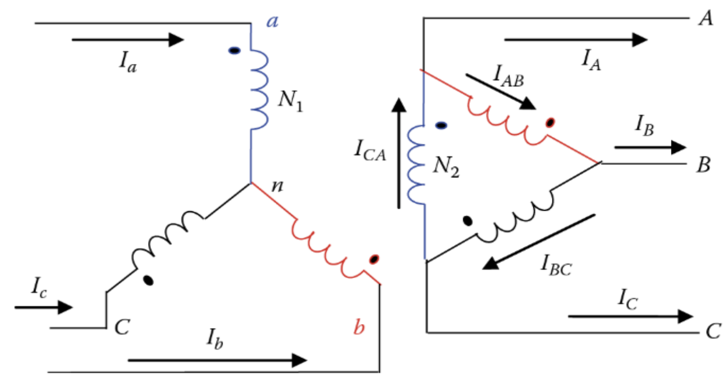
$$\frac{N_1}{N_2} = \frac{I_{AB}}{I_a}$$



Similarly, current through N_1 is line current of primary circuit I_a and current through N_2 is phase current of the secondary winding I_{AB}.

Hence, we can compute the current ratio using the ampere-turn equation.

$$\Im = I_a N_1 = I_{AB} N_2$$
$$\frac{N_1}{N_2} = \frac{I_{AB}}{I_a}$$



Δ - Y Connection: V/I Relationships

Voltage Relationship

$$V_{AB} = \sqrt{3}V_{An}\angle 30^\circ$$

$$V_{AB} = a \cdot V_{an}$$

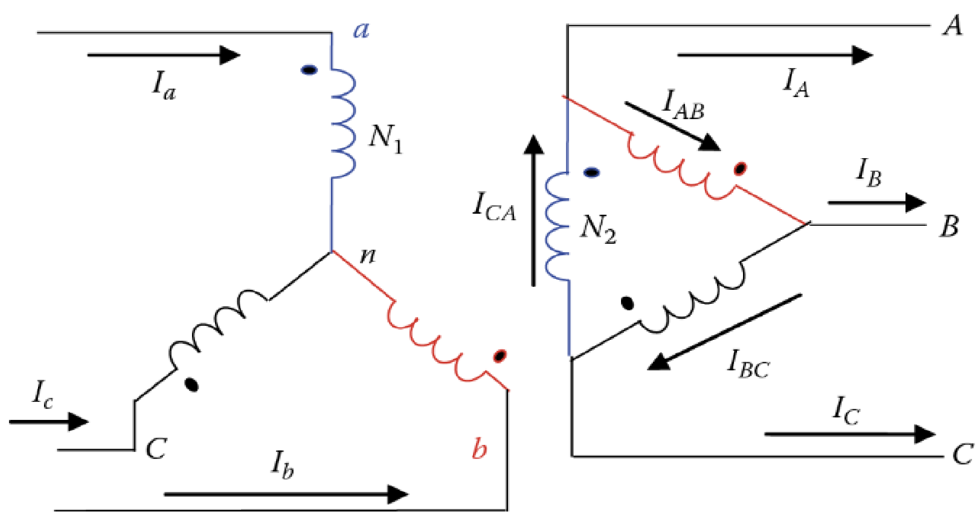
$$V_{an} = \frac{\sqrt{3}V_{An} \cdot \angle 30^\circ}{a}$$

Current Relationship

$$\frac{I_{AB}}{I_a} = \frac{1}{a} \rightarrow I_a = aI_{AB}$$

$$I_A = \sqrt{3}I_{AB}\angle -30^\circ \rightarrow I_{AB} = \frac{1}{\sqrt{3}}I_A\angle 30^\circ$$

$$I_a = a\frac{1}{\sqrt{3}}I_A\angle 30^\circ$$



Example 5 – Parameters in Delta-Wye Transformer

A 25kVA transformer has a voltage ratio of $20\text{kV}(\mathbf{Y})/10\text{kV}(\mathbf{\Delta})$. Compute the following:

1. Winding voltages
2. Turns ratio
3. Line currents in the primary and secondary circuits
4. Phase currents of the transformer

Solution

1. Winding Voltage

The primary windings of the transformer are connected in wye. Hence, the voltage across the winding is

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} = \frac{20}{\sqrt{3}} = 11.55\text{kV}$$

The secondary windings are connected in delta. Hence, the winding voltage is equal to the line-to-line voltage

$$V_{AB} = 10\text{kV}$$

Example 5 – Parameters in Delta-Wye Transformer

2. Turn Ratio

The turns ratio is the ratio of the voltages of the windings

$$\frac{N_1}{N_2} = \frac{11.55}{10} = 1.155$$

3. Line Current in HV and LV sides

$$I_a = \frac{S}{\sqrt{3}V_{ab}} = \frac{25}{\sqrt{3} \times 20} = 0.7217 \text{ A}$$

$$I_A = \frac{S}{\sqrt{3}V_{AB}} = \frac{25}{\sqrt{3} \times 10} = 1.4434 \text{ A}$$

4. Phase currents of the transformer

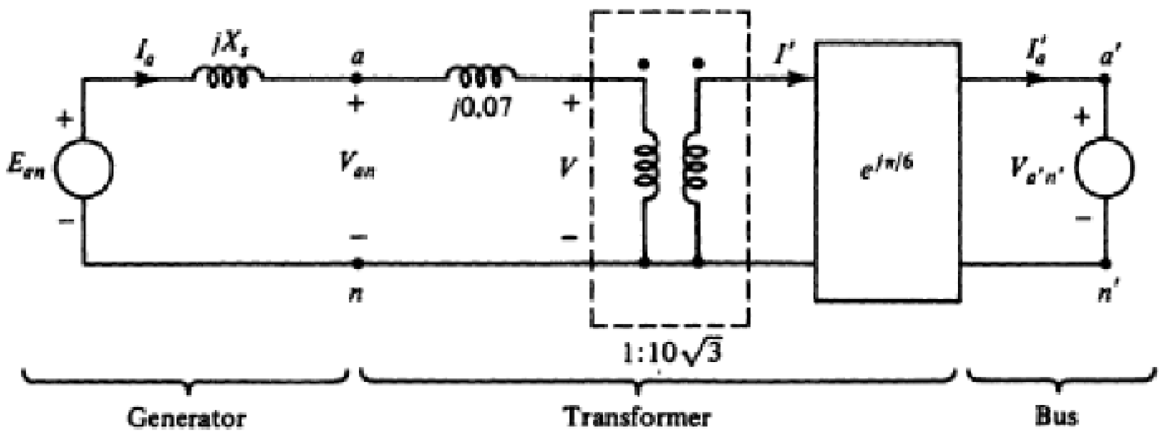
Since the primary windings are connected in wye, the phase current of the primary winding is the same as the line current of the primary circuit.

The secondary windings are connected in delta, hence the phase current of the secondary winding is

$$I_{AB} = \frac{I_A}{\sqrt{3}} = \frac{1.4434}{\sqrt{3}} = 0.8333 \text{ A}$$

Example 6 – Parameters in Delta-Wye Transformer

Generator with Δ-Y step up transformer. Transformer is made of identical 1ϕ transformers with $X_l = 0.21\Omega,\ n = N_1/N_2 = 10$. Each generator phase is modeled as a Thevenin equivalent circuit. The transformer delivers 100MW at 0.9 lagging power factor at 230kV to the bus.



Find the primary current, primary voltage LL, and 3ϕ complex power supplied by the generator.

Solution

$$V_{a'n'} = 230/\sqrt{3}\angle 0^\circ\text{kV} = 132.8\angle 0^\circ\text{kV}$$

$$S' = \frac{100 \times 10^6}{0.9 \times 3}\angle 25.84^\circ = 37.04\angle 25.84^\circ\text{MVA}$$

$$I_a' = \left(\frac{S'}{V_{a'n'}}\right)^* = \left(\frac{37.04\angle 25.84^\circ}{132.8\angle 0^\circ}\right)^* = 278.9\angle -25.84^\circ$$

Example 6 – Parameters in Delta-Wye Transformer

By current conversion factor,

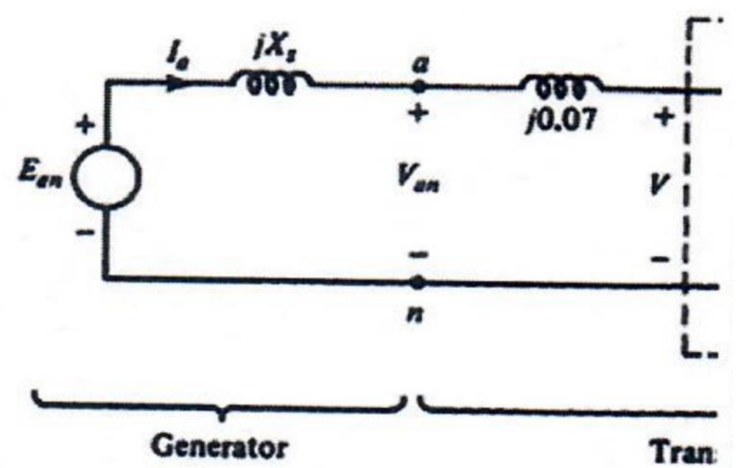
$$I_a = 10\sqrt{3}\angle -30^{\text{o}}\ I_a' = 4830.6\angle -55.84^{\circ}$$

$$\begin{aligned}
\mathrm{V_{an}} &= V + j0.071 I_a \\
&= \frac{1}{10\sqrt{3}} 132.8 \times 10^3 \angle -30^{\circ} + j0.07 \times 4830.6\angle 55.84^{\text{o}} \\
&= 7667.2 \angle -30^{\text{o}} + 338.1 \angle 34.16^{\text{o}} \\
&= 7820.5\angle -27.77^{\circ}\mathrm{V}
\end{aligned}$$

$$\mathrm{V_{ab}} = \sqrt{3}\angle -30^{\text{o}} V_{an} = 13.55\angle -57.77^{\circ}\mathrm{kV}$$

$$\mathrm{S}_{1\phi} = V_{an} I_a^* = 37.77 \angle 28.07^{\circ}\mathrm{MVA}$$

$$S_{3\phi} = 3S = 113.33 \angle 28.07^{\circ}\mathrm{MVA}$$



Transformer Efficiency and Voltage Regulation

Transformer Efficiency is given by

$$\eta = \frac{P_O}{P_{in}} = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output power}}{\text{Output power} + \text{Losses}}$$

$$= \frac{V_2' I_2' \cos\theta_2}{V_2' I_2' \cos\theta_2 + P_{cu} + P_{Fe}} \times 100\%.$$

The losses of the transformer are due to R_1, R_2, and R_o. The losses in R_1 and R_2 are copper losses P_{cu} (the windings are made of copper material). The loss in Ro is known as iron loss P_{Fe} (the core is made of iron).

Voltage regulation of transformer indicates the voltage reduction due to various parameters of the transformer.

$$\varepsilon = \frac{|V_{\text{no load}}| - |V_{\text{full load}}|}{|V_{\text{full load}}|}$$

$|V_{no\ load}|$ is magnitude of open circuit voltage measured at the load terminals

$\left|V_{full\ load}\right|$ is magnitude of voltage at load terminals when rated current is delivered to the load

At no load, (open circuit) $I_2 = 0, V_{no\ load} = V_1, V_{full\ load} = V_2'$

$$\varepsilon = \frac{V_1 - aV_2}{aV_2} * 100\%\ , where\ a = \frac{N_1}{N_2}$$

Per Unit Calculation

Motivation
It would be difficult to continuously refer impedance to the different sides of transformer.

Mathematically,

$$\text{Quantity in Per Unit} = \frac{\text{Actual Quantity}}{\text{Base value of the Quantity}}$$

For Single Phase System,

Pick base power S_B [MVA] and voltage base V_B [kV].

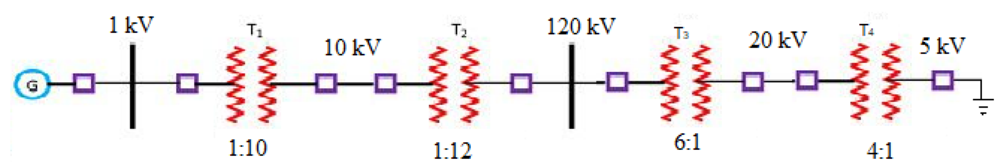
Calculate current base I_B and Impedance base Z_B.

$$S_B = V_B \cdot I_B^*$$

$$I_B = \left(\frac{S_B}{V_B}\right)^* = \frac{S_B}{V_B}$$

$$Z_B = \frac{(V_B)^2}{S_B}$$

[Note: Base values are real numbers]



S_B is for the entire system.

V_B is for each different voltage level.

Base voltages are related by transformer turns ratio.

Per Unit Calculation

$$V_{p\cdot u} = \frac{V}{V_B} \qquad P_{p\cdot u} = \frac{P}{S_B}$$

$$S_{p\cdot u} = \frac{S}{S_B} \qquad Q_{p\cdot u} = \frac{Q}{S_B}$$

$$I_{p\cdot u} = \frac{I}{I_B} \qquad R_{p.u} = \frac{R}{Z_B}$$

$$Z_{p\cdot u} = \frac{Z}{Z_B}. \qquad X_{p\cdot u} = \frac{X}{Z_B}.$$

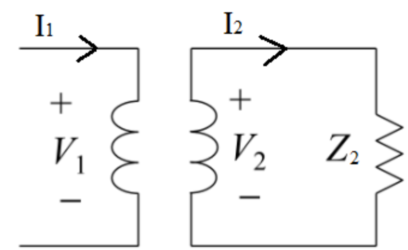
- Per Unit Values of Primary and Secondary Side of a transformer are **EQUAL.**

$$a = \frac{N_1}{N_2} \qquad n = \frac{N_2}{N_1}$$

Transformation Ratio

$$V_2 = n\,.V_1 = \frac{V_1}{a} \qquad I_2 = \frac{I_1}{n} = a.I_1$$

Real Values



$N_1 : N_2$

<u>Choose Base Values</u>

S_B

V_{1B} & $V_{2B} = \frac{V_{1B}}{a} = nV_{1B}$

Per Unit Voltage:

$$V_{2,pu} = \frac{V_2}{V_{2B}} = \frac{n\cdot V_1}{n\cdot V_{1B}} = \frac{V_1}{V_{1B}} = V_{1,pu}$$

Per Unit Current:

$$I_2, pu = \frac{I_2}{I_{2B}} = \frac{I_2}{\frac{S_B}{V_{2B}}} = \frac{V_{2B}\cdot I_2}{S_B} = \frac{n\cdot V_{1B}\cdot I_2}{S_B} = \frac{n\cdot V_{1B}\cdot a\cdot I_1}{S_B} = \frac{V_{1B}\cdot I_1}{S_B.} = \frac{I_1}{\frac{S_B}{V_{1B}}} = \frac{I_1}{I_{1B}} = I_{1,PU}.$$

Per Unit Calculation

- Per Unit Values of Primary and Secondary Side of a transformer are **EQUAL.**

Voltage:

$$V_{2,pu} = \frac{V_2}{V_{2B}} = \frac{n \cdot V_1}{n \cdot V_{1B}} = \frac{V_1}{V_{1B}} = V_{1,pu}$$

Current:

$$I_2, pu = \frac{I_2}{I_{2B}} = \frac{I_2}{\frac{S_B}{V_{2B}}} = \frac{V_{2B} \cdot I_2}{S_B} = \frac{n \cdot V_{1B} \cdot I_2}{S_B} = \frac{n \cdot V_{1B} \cdot a \cdot I_1}{S_B} = \frac{V_{1B} \cdot I_1}{S_B.} = \frac{I_1}{\frac{S_B}{V_{1B}}} = \frac{I_1}{I_{1B}} = I_{1,PU}.$$

Impedance:

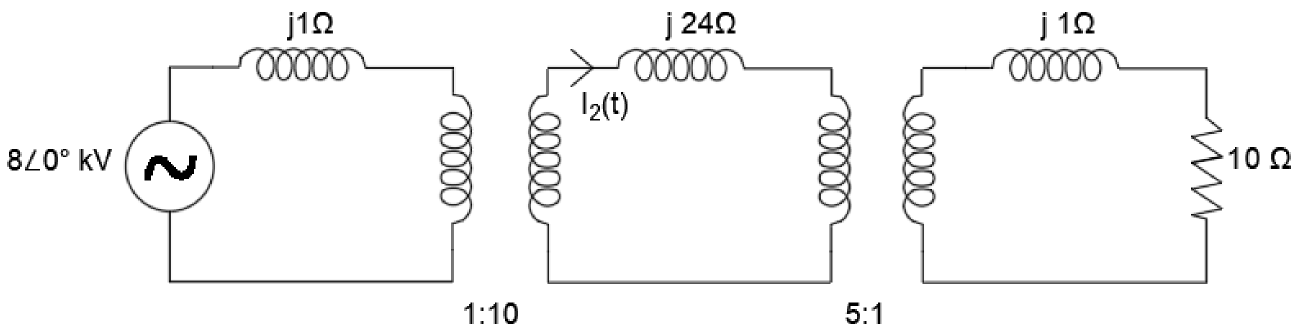
$$Z_{2B} = \frac{V_{2B}^2}{S_B} = \frac{n^2 \cdot V_{1B}^2}{S_B} = n^2 \cdot Z_{1B}$$

$$Z_{2;\, p.u} = \frac{Z_2}{Z_{2B}} = \frac{n^2 \cdot z_e}{n^2 \cdot z_{1B}} = \frac{z_e}{z_{1B}} = z_{e;\, p.u}.$$

Under per unit calculation, the primary and secondary quantities through the transformers are the same, hence we don’t need to perform any conversion between voltage level.

Example 7: Per Unit Value

Solve for the current, load voltage & load power in the following circuit using P.U. analysis with an $S_B = 100\text{MVA}$ and base voltage of $V_{B1} = 8\text{kV}, V_{B2} = 80\text{kV}, \text{and } V_{B3} = 16\text{kV}.$



Solution

1. Choose Voltage base [kV] and Power base [MVA]

$$V_{B1} = 8\text{kV};\ \ V_{B2} = 80\text{kV};\ \ V_{B3} = 16\text{kV}$$

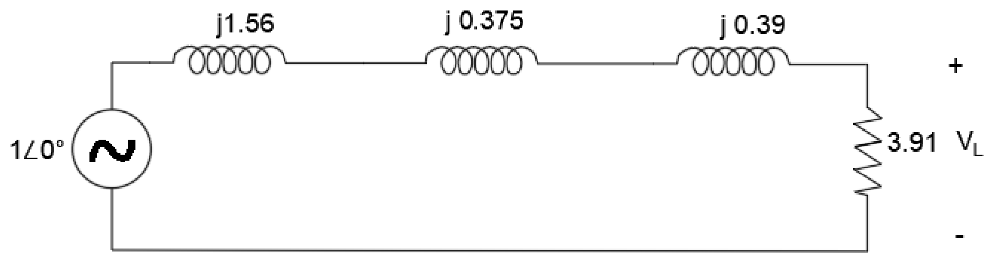
2. Calculate the impedance base [Ω]

$$Z_{B1} = \frac{(8\ \text{k}V)^2}{100\text{MVA}} = 0.64\Omega.$$

$$Z_{B_2} = \frac{(80\ \text{k}V)^2}{100MVA} = 64\Omega.$$

$$Z_{B3} = \frac{(16\ \text{k}V)^2}{100\text{MVA}} = 2.56\Omega.$$

Example 7: Per Unit Value



3. Calculate the variables in per unit circuit.

$$I = \frac{1\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ p.u$$

$$V_L = 1\angle 0^\circ - 0.22\angle(-30.8^\circ) \times j2.327 = 0.859\angle -30.8^\circ\ p.u.$$

$$S_L = V_L \cdot I_L^* = 0.189\ p.u.$$

$$S_G = V_G \cdot I_G^* = 1\angle 0^\circ * 0.22\ \angle 30.8^\circ = 0.22\angle 30.8^\circ\ p.u$$

4. Base Conversion back to Actual Quantities

$$I_{2B} = \frac{100MVA}{80kV} = 1250\ A$$

$$V_L^{actual} = 0.859\angle(-30.8^\circ) \times 16kV = 13.7\angle(-30.8^\circ\ )\ \mathrm{kV}$$

$$S_L^{actual} = 0.189\angle 0^\circ \times 100MVA = 18.9\angle 0^\circ \mathrm{MVA}$$

$$S_G^{actual} = 0.22\ \angle 30.8^\circ \times 100MVA = 22\angle 30.8^\circ \mathrm{MVA}$$

$$I_2^{actual} = 0.22\angle -30.8^\circ \times 1250\ A = 275\angle -30.8^\circ \mathrm{A}$$

Per Unit Calculation in Three - Phase

- Step 1: Pick a Three-phase base power for the entire system $S_B^{3\phi}$
- Step 2: Pick a base voltage for each different voltage level, $V_{B,\,LL}$
  - L-L Base $V_{B,LL}$
  - L-N Base $V_{B,LL/\sqrt{3}}$
- Step 3: Calculate Base Impedance

$$Z_B = \frac{\left(\frac{V_{B,LL}}{\sqrt{3}}\right)^2}{\left(\frac{S_B^{3\phi}}{3}\right)} = \frac{V_{B,LL}^2}{S_B^{3\phi}}$$

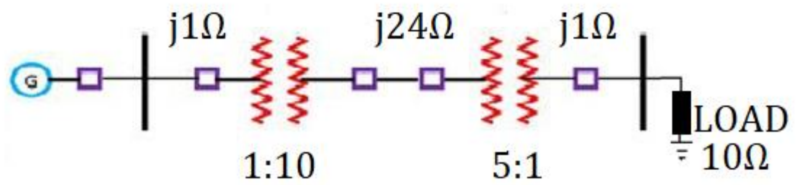
- Step 4: Calculate Base current

$$I_B^{3\phi} = \frac{\frac{S_B^{3\phi}}{3}}{\frac{V_{B,LL}}{\sqrt{3}}} = \frac{S_B^{3\phi}}{\sqrt{3}V_{B,LL}}$$

- Step 5: Convert actual values to p.u.

Example 8 – Per Unit Calculation in 3 Phase

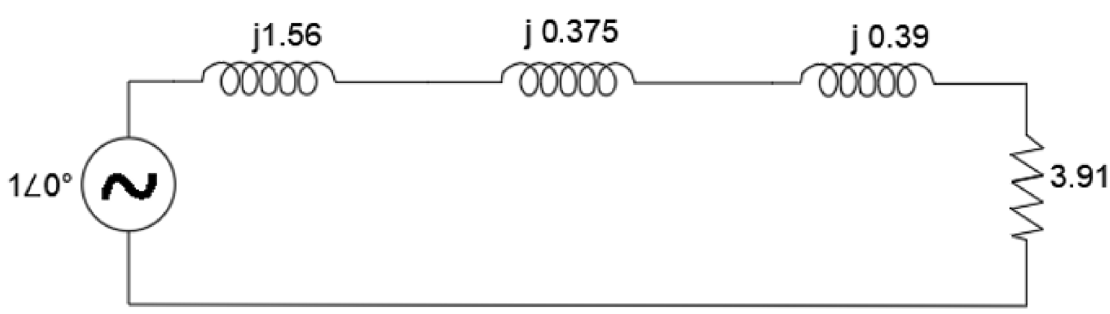
Solve for the current, load voltage and load power in the previous circuit, assuming a 3ϕ base power of 300 MVA, and line to line base voltages of 13.8 kV, 138 kV and 27.6 kV (square root of 3 larger than the 1ϕ example voltages). Also assume the generator is Y-connected so its line-to-line voltage is 13.8 kV.



Solution

Base Voltage: $V_{B1} = 13.8kV, V_{B2} = 138kV$ and $V_{B3} = 27.6\ kV$

Convert the per phase actual circuit into per unit circuit.



Example 8 – Per Unit Calculation in 3 Phase

- Calculate the variables in per unit circuit.

$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle - 30.8^\circ \text{ p.u.}$$

$$V_L = 1.0\angle 0^\circ - 0.22\angle - 30.8^\circ \times 2.327\angle 90^\circ = 0.859\angle - 30.8^\circ \text{ p.u.}$$

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

- Convert back to actual value.

$$I_B^{Middle} = \frac{300MVA}{\sqrt{3}\,(138kV)} = 1250 \text{ A}$$

$$V_L^{actual} = 0.859\angle(-30.8^\circ) \times 27.6kV = 23.8\angle(-30.8^\circ\,)\,kV$$

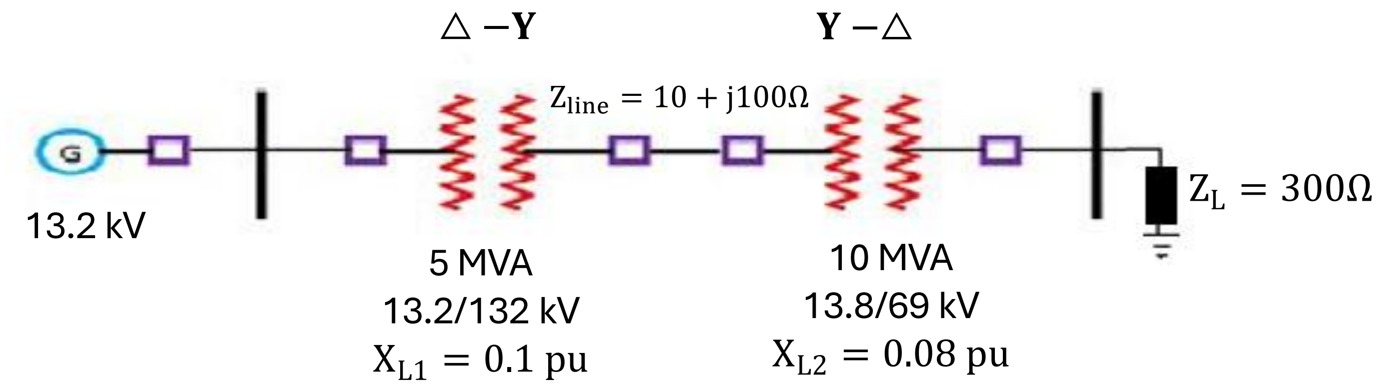
$$S_L^{actual} = 0.189\angle 0^\circ \times 300MVA = 0.567\angle 0^\circ MVA$$

$$S_G^{actual} = 0.22\,\angle 30.8^\circ \times 300MVA = 0.66\angle 30.8^\circ MVA$$

$$I_{Middle}^{actual} = 0.22\angle - 30.8^\circ \times 1250\,A = 275\angle - 30.8^\circ A$$

Example 9 – Base Change

Find the equivalent per unit value for each component.



Solution

1. Select the power base and voltage base.

$$S_B^{3\phi} = 10\text{ MVA}$$

$$V_{3B}^{\mathrm{L-L}} = 69\text{ kV} \qquad V_{2B}^{\mathrm{L-L}} = 138\text{ kV} \qquad V_{1B}^{\mathrm{L-L}} = 138\text{ kV} \times \frac{13.2kV}{132kV} = 13.8kV$$

2. Calculate the impedance base.

$$Z_{3B} = \frac{\left(V_{3B}^{\mathrm{L-L}}\right)^2}{S_B^{3\phi}} = \frac{69^2}{10} = 476\Omega \qquad Z_{2B} = \frac{\left(V_{2B}^{\mathrm{L-L}}\right)^2}{S_B^{3\phi}} = \frac{138^2}{10} = 1904\Omega$$

$$Z_{1B} = \frac{\left(V_{1B}^{\mathrm{L-L}}\right)^2}{S_B^{3\phi}} = \frac{13.8^2}{10} = 19.04\Omega$$

Example 9 – Base Change

2. Calculate the impedance base.

$$\mathrm{Z}_{\text{load}} = \frac{350\Omega}{476\Omega} = 0.63\ \text{p.u.}$$

$$\mathrm{Z}_{\text{line}} = \frac{10 + \mathrm{j}100\Omega}{1904\Omega} = 0.005 + \mathrm{j}0.05\ \text{p.u.}$$

Calculate also source voltage due to unmatched voltage base.

$$E_s = \frac{13.2\mathrm{k}V}{13.8\ \mathrm{k}V} = 0.96\ \text{p.u.}$$

3. Calculate transformer impedance with base change formula.

$$X_{\mathrm{L2}} = \frac{0.08 \times \frac{(V_{3B}^{L-L})^2}{S_B^{3\phi}}}{Z_{3B}} = \frac{0.08 \times \frac{(V_{3B}^{L-L})^2}{S_B^{3\phi}}}{\frac{(V_{3B}^{L-L})^2}{S_B^{3\phi}}} = 0.08$$

$$X_{\mathrm{L1}} = \frac{0.1 \times \frac{(13.2\mathrm{kV})^2}{5\ \mathrm{MVA}}}{Z_{1B}} = \frac{0.1 \times \frac{(13.2)^2}{5}}{\frac{(13.8)^2}{10}} = 0.183\ \text{p.u}$$

Power System Analysis and Protection

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