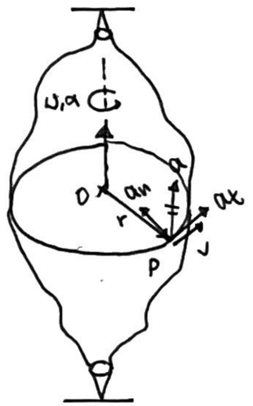
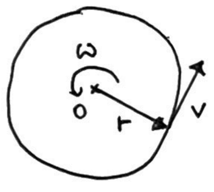
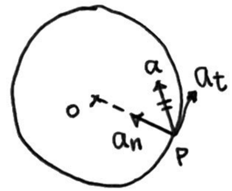
1. Relative Motion







$\underline{v} = \underline{\omega} \times \underline{r}$ ... (1)

$\underline{a} = \frac{d\underline{v}}{dt} = \underbrace{\frac{d\underline{\omega}}{dt}}_{\underline{\alpha}} \times r_p + \underline{\omega} \times \underbrace{\frac{dr_p}{dt}}_{\underline{v} = \underline{\omega} \times r_p}$

$\underline{a} = \underline{\alpha} \times \underline{r_p} + \underline{\omega} \times (\underline{\omega} \times \underline{r_p})$ ... (2)

$\underline{a} = \underline{a_t} + \underline{a_n} = \underline{\alpha} \times \underline{r} - \omega^2 \underline{r}$ ... (3)

As a scalar form,

$v = \omega r$ ... (4)

$\left.\begin{array}{l} a_t = \alpha r \\ a_n = \omega^2 r \end{array}\right\} a = \sqrt{a_t^2 + a_n^2}$ ... (5) ... (6)

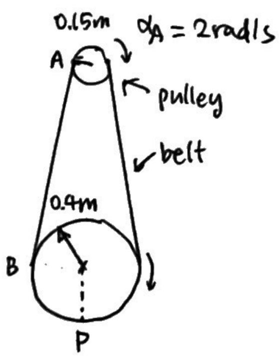
Angular motion :

$\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$, $\alpha d\theta = \omega d\omega$ ... (7)

Given constant acceleration :

$\left\{\begin{array}{l} \omega = \omega_0 + \alpha t \\ \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2 \\ \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0) \end{array}\right.$ ... (8)

Example 1 (Pulley)



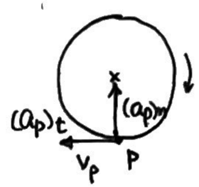
belt same speed and tangential acceleration :

$\left.\begin{array}{l} v = \omega_A r_A = \omega_B r_B \rightarrow 7.090(0.15) = \omega_B(0.4) \rightarrow \omega_B = 2.659 \text{ rad/s} \\ a_t = \alpha_A r_A = \alpha_B r_B \rightarrow 2(0.15) = \alpha_B(0.4) \rightarrow \alpha_B = 0.75 \text{ rad/s}^2 \end{array}\right\}$ From A to B

$\omega_A^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0) = 0^2 + 2(2)(4\pi - 0) = 16\pi \rightarrow \omega_A = 7.090$ rad/s ← at pulley

$v_P, a_P = ?$ after 2 rev at pulley

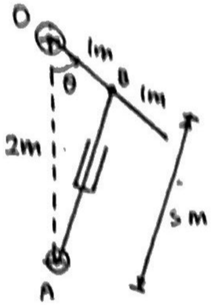
* $\theta_A = 2\text{rev} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 4\pi$ rad



$v_P = \omega_B r_B = 2.659(0.4) = 1.06$ m/s

$\left.\begin{array}{l} (a_p)_t = \alpha_B r_B = (0.75)(0.4) = 0.3 \text{ m/s}^2 \\ (a_p)_n = \omega_B^2 r_B = 2.659^2(0.4) = 2.828 \text{ m/s}^2 \end{array}\right\} a = \sqrt{a_n^2 + a_t^2} = \sqrt{0.3^2 + 2.828^2} = \underline{\underline{2.84 \text{ m/s}^2}}$

Example 2



→ cylinder extend at constant rate of 0.5m/s

determine angular velocity and angular acceleration of window.

Position coordinate: $s^2 = 1^2 + 2^2 - 2(1)(2)\cos\theta = 5 - 4\cos\theta$ ... (1)

$$s\big|_{\theta=30^\circ} = \sqrt{5-4\cos 30^\circ} = 1.239$$

Take time derivative: $s^2 = 5 - 4\cos\theta \rightarrow 2s\underbrace{\frac{ds}{dt}}_{v_s} = 4\sin\theta\underbrace{\frac{d\theta}{dt}}_{\omega}$

note that $\frac{ds}{dt} = v_s = 0.5\,m/s$, at $\theta = 30^\circ$, $\rightarrow$ $s v_s = 2\sin\theta\,\omega$

$$2(1.239)(0.5) = 4\sin 30^\circ\,\omega \rightarrow \omega = 0.6197\ rad/s$$

Take time derivative of (2):

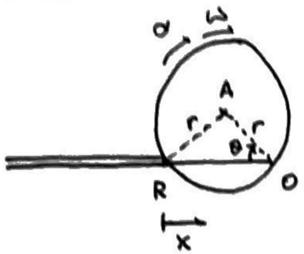
$$\underbrace{\frac{ds}{dt}}_{v_s}\cdot v_s + s\cdot\underbrace{\frac{dv_s}{dt}}_{a_s} = 2\sin\theta\underbrace{\frac{d\omega}{dt}}_{\alpha} + 2\cos\theta\underbrace{\frac{d\theta}{dt}}_{\omega}\cdot\omega$$

$$\rightarrow\quad v_s^2 + s\cancel{a_s} = 2\sin\theta\cdot\alpha + 2\cos\theta\,\omega^2$$

$$\rightarrow\quad (0.5)^2 = 2\sin 30^\circ\cdot\alpha + 2\cos 30^\circ\cdot(0.6197)^2$$

$$\alpha = -0.415\ rad/s \qquad \square$$

Example 3



determine the velocity and acceleration of rod R.

Position Coordinate: (relate x and θ)

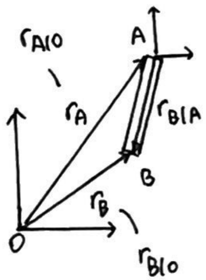
$$x = 2r\cos\theta$$

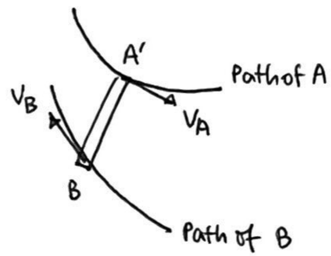
$$\frac{dx}{dt} = 2\cancel{\frac{dr}{dt}}\cos\theta + 2r(-\sin\theta)\frac{d\theta}{dt} \rightarrow v = -2r\sin\theta\,\omega$$

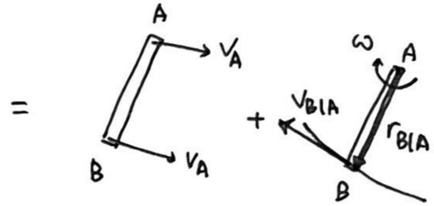
$$\frac{dv}{dt} = -2r\cos\theta\underbrace{\frac{d\theta}{dt}}_{\omega}\cdot\omega - 2r\sin\theta\frac{d\omega}{dt}$$

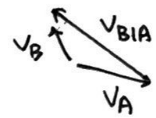
$$= -2r(\omega^2\cos\theta + \alpha\sin\theta) \qquad \square$$

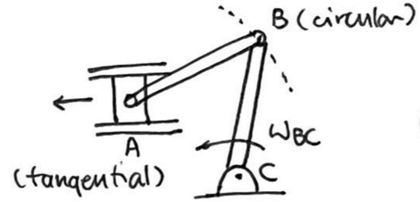
Relative motion

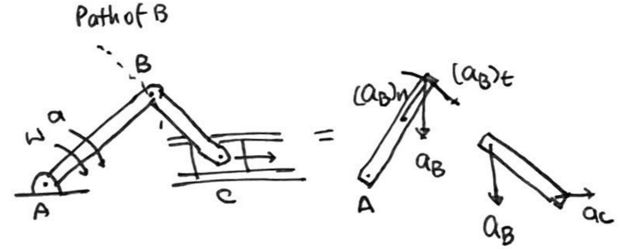












Position:

$$r_B = r_A + r_{B/A} \quad \cdots (1)$$

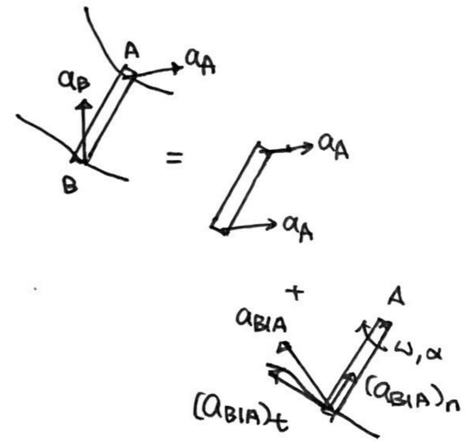
↓

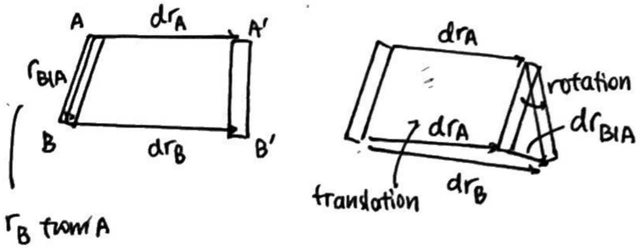
$$dr_B = dr_A + dr_{B/A}$$

translation and rotation; translation of A; rotation about A

$$\rightarrow \underbrace{\frac{dr_B}{dt}}_{V_B} = \underbrace{\frac{dr_A}{dt}}_{V_A} + \underbrace{\frac{dr_{B/A}}{dt}}_{V_{B/A}} \rightarrow V_B = V_A + \omega \times r_{B/A} \quad \cdots (2)$$

$V_{B/A}$: velocity with respect to A. of B





↓

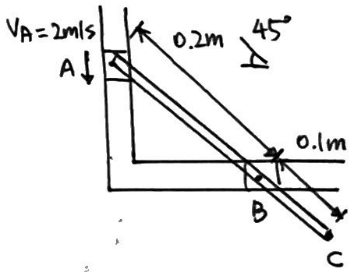
$$\underbrace{\frac{dV_B}{dt}}_{a_B} = \underbrace{\frac{dV_A}{dt}}_{a_A} + \underbrace{\frac{dV_{B/A}}{dt}}_{}$$

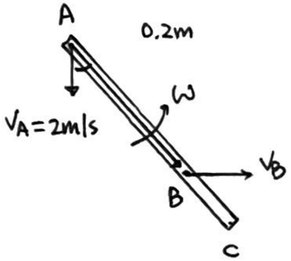
$$a_{B/A} = (a_{B/A})_t + (a_{B/A})_n$$

↓

$$a_B = a_A + \underbrace{\alpha \times r_{B/A}}_{\text{tangential}} - \underbrace{\omega^2 r_{B/A}}_{\text{normal}} \quad \cdots (3)$$

Example 4





Find the velocity of B when $\theta = 45^\circ$

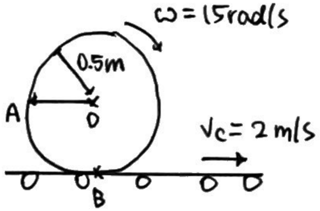
$$\underline{V}_B = \underline{V}_A + \underline{\omega} \times \underline{r}_{B|A}$$

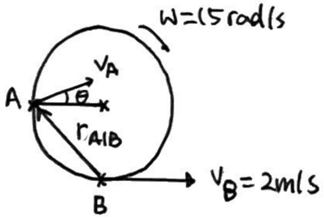
$$V_B\hat{i} = -2\hat{j} + (\omega\hat{k} \times (0.2\cos 45^\circ\,\hat{i} - 0.2\sin 45^\circ\,\hat{j}))$$

$$V_B\hat{i} = -2\hat{j} + 0.2\omega\sin 45^\circ\,\hat{j} + 0.2\omega\cos 45^\circ\,\hat{i}$$

Comparing coefficient: $\left.\begin{array}{l} V_B = 0.2\omega\cos 45^\circ \\ 2 = 0.2\omega\sin 45^\circ \end{array}\right\}$ $\begin{array}{l} \omega = 14.1 \text{ rad/s} \circlearrowleft \\ V_B = 2 \text{ m/s} \rightarrow \end{array}$

Example 5





determine velocity of A

* rolls without slipping

$$V_A = V_B + \omega \times r_{A|B}$$

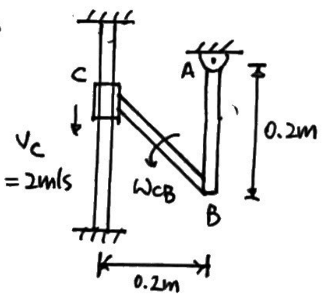
$$(V_A)_x\hat{i} + (V_A)_y\hat{j} = 2\hat{i} + (-15\hat{k}) \times (-0.5\hat{i} + 0.5\hat{j})$$

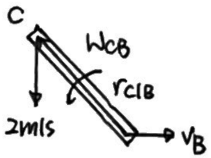
$$= 2\hat{i} + 7.50\hat{j} + 7.50\hat{i} = 9.50\hat{i} + 7.50\hat{j}$$

$\rightarrow$ $\left.\begin{array}{l} (V_A)_x = 9.5 \\ (V_A)_y = 7.5 \end{array}\right\}$ $V_A = \sqrt{9.5^2 + 7.5^2} = 12.1 \text{ m/s}$

$$\theta = \tan^{-1}\left(\frac{7.5}{9.5}\right) = 38.3^\circ$$

Example 6





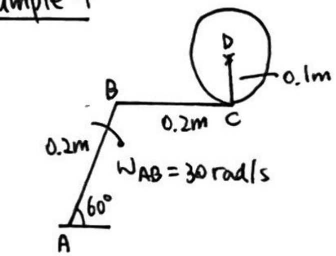
$$V_C = V_B + \omega_{CB} \times r_{C|B}$$

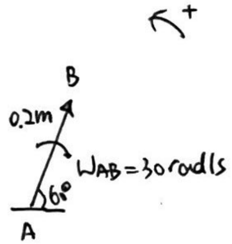
$$-2\hat{j} = V_B\hat{i} + \omega_{CB}\hat{k} \times (-0.2\hat{i} + 0.2\hat{j})$$

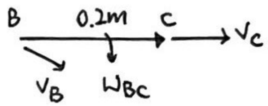
$$= V_B\hat{i} + (-0.2\omega_{CB}\hat{i} - 0.2\omega_{CB}\hat{j})$$

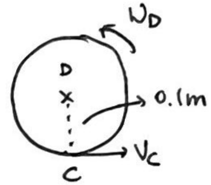
$\rightarrow$ $\begin{array}{l} 2 = 0.2\omega_{CB} \\ V_B = 0.2\omega_{CB} \end{array}$ $\rightarrow$ $\begin{array}{l} \omega_{CB} = 10 \text{ rad/s} \\ V_B = 2 \text{ m/s} \end{array}$ □

Example 7









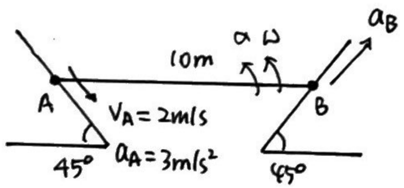
determine angular velocity of BC and the wheel at this instant.

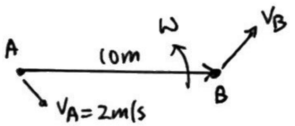
$$v_B = \omega_{AB} \times r_{B|A} + \cancel{r_A}$$
$$= -30\hat{k} \times (0.2\cos 60^\circ\,\hat{i} + 0.2\sin 60^\circ\,\hat{j})$$
$$= 5.2\hat{i} - 3.0\hat{j} \text{ m/s}$$

$$v_C = v_B + \omega_{BC} \times r_{C|B}$$
$$v_C\hat{i} = 5.2\hat{i} - 3.0\hat{j} + (-\omega_{BC}\hat{k}) \times (0.2\hat{i})$$
$$v_C\hat{i} = 5.2\hat{i} + (-0.2\omega_{BC} - 3.0)\hat{j}$$
$$\rightarrow v_C = 5.2 \text{ m/s}$$
$$-0.2\omega_{BC} - 3.0 = 0 \rightarrow \omega_{BC} = 15 \text{ rad/s}$$

$$v_C = \omega_D \times r_{C|D} + \cancel{v_D}$$
$$5.2\hat{i} = \omega_D\hat{k} \times (-0.1\hat{j})$$
$$\omega_D = 52.0 \text{ rad/s}$$

Example 8



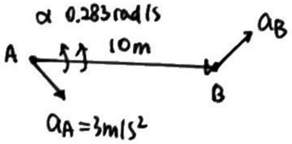


Find ω: $v_B = v_A + \omega \times r_{B|A}$

$$v_B\cos 45^\circ\,\hat{i} + v_B\sin 45^\circ\,\hat{j} = 2\cos 45^\circ\,\hat{i} - 2\sin 45^\circ\,\hat{j} + \overbrace{\omega\hat{k} \times (10\hat{i})}^{10\omega\hat{j}}$$

$$v_B\cos 45^\circ = 2\cos 45^\circ$$
$$v_B\sin 45^\circ = -2\sin 45^\circ + 10\omega$$
$$\rightarrow v_B = 2 \text{ m/s}, \quad \omega = \frac{4\sin 45^\circ}{10} = 0.283 \text{ rad/s}$$

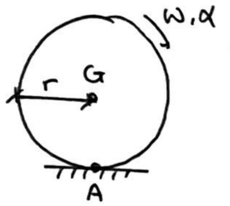
Find α: $a_B = a_A + \overbrace{\alpha \times r_{B|A}}^{(a_{B|A})_t} - \omega^2 r_{B|A}$



$$3\cos 45^\circ\,\hat{i} - 3\sin 45^\circ\,\hat{j} + \alpha\hat{k} \times (10\hat{i}) - (0.283)^2(10\hat{j}) = a_B\cos 45^\circ\hat{i} + a_B\sin 45^\circ\,\hat{j}$$

$$\begin{cases} a_B\cos 45^\circ = 3\cos 45^\circ - (0.283)^2(10) \\ a_B\sin 45^\circ = -3\sin 45^\circ + \alpha \cdot 10 \end{cases} \rightarrow \begin{matrix} a_B = 1.87 \text{ m/s}^2 \ \angle 45^\circ \\ \alpha = 0.344 \text{ rad/s}^2 \end{matrix}$$

Example 9



determine v_G, a_G and a_A if rolls without slipping.

rolls without slipping : $v_A = 0$

$$v_G = \cancel{v_A} + \omega \times r_{G/A}$$
$$= \omega(-\hat{k}) \times r\hat{j} = r\omega\,\hat{i}$$

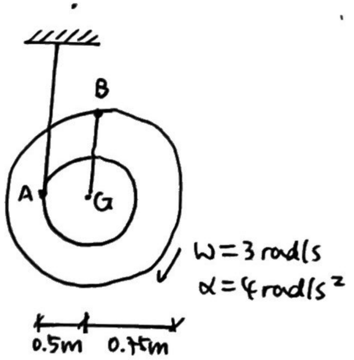
$$\frac{dv_G}{dt} = a_G = \cancel{\frac{dr}{dt}}\,\omega + \underbrace{\frac{d\omega}{dt}}_{\alpha}\, r = r\alpha\,\hat{i}$$

$$a_G = a_A + \alpha \times r_{G/A} - \omega^2 r_{G/A}$$

$$+ r\alpha\,\hat{i} = (a_A)_x\,\hat{i} + (a_A)_Y\,\hat{j} + \underbrace{\alpha(-\hat{k}) \times r\,\hat{j}}_{\alpha r\,\hat{i}} - \omega^2 \cdot r\,\hat{j}$$

$$\begin{cases} (a_A)_x = 0 \\ (a_A)_Y = r\omega^2 \end{cases} \rightarrow a_A = r\omega^2$$

Example 10



Find a_B

$$a_G = r\alpha = 0.5 \cdot (4) = 2\,m/s^2$$

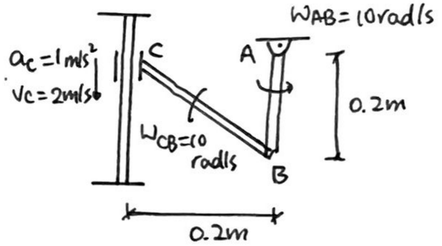
$$a_B = a_G + \alpha \times r_{B/G} - \omega^2 r_{B/G}$$

$$(a_B)_x\,\hat{i} + (a_B)_y\,\hat{j} = \underbrace{4(-\hat{k}) \cdot (0.75\,\hat{j})}_{3\,\hat{i}} - (3)^2(0.75\,\hat{j}) + (-2\,\hat{j})$$

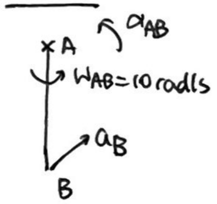
$(a_B)_x = 3\ m/s^2 \rightarrow$

$(a_B)_Y = -3^2 \times 0.75 - 2 = -8.75\,m/s^2 \downarrow \qquad a_B = \sqrt{3^2 + 8.75^2} = 9.25\,m/s^2$

Example 11

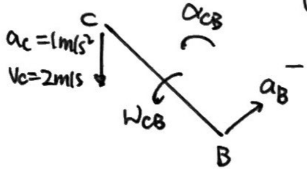


Link AB



$$a_B = \cancel{a_A} + \alpha_{AB} \times r_{B/A} - \omega^2 r_{B/A}$$
$$= \alpha_{AB}\hat{k} \times (-0.2\,\hat{j}) - 10^2(-0.2\,\hat{j})$$
$$= 0.2\alpha_{AB}\,\hat{i} + 20\,\hat{j}$$

Link BC



$$a_C = a_B + \alpha_{CB} \times r_{C/B} - \omega^2 r_{C/B}$$

$$-1\,\hat{j} = 0.2\alpha_{AB}\,\hat{i} + 20\,\hat{j} + \alpha_{CB}\hat{k} \times (-0.2\,\hat{i} + 0.2\,\hat{j}) - 10^2(-0.2\,\hat{i} + 0.2\,\hat{j})$$

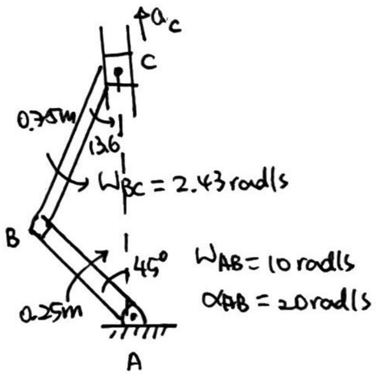
$$\begin{cases} 0.2\alpha_{AB} = 0.2\alpha_{BC} - 20 \\ 20 = -1 + 0.2\alpha_{CB} + 20 \end{cases}$$

$\alpha_{CB} = 5\,rad/s^2$ ↺

$\alpha_{AB} = -95\,rad/s^2$ ↺

□

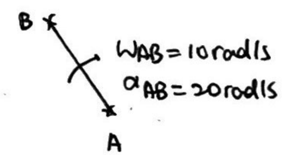
Example 12



$$r_{B/A} = -0.25\cos 45^\circ\,\hat{i} + 0.25\cos 45^\circ\,\hat{j} = -0.177\,\hat{i} + 0.177\,\hat{j}$$

$$r_{C/B} = 0.75\sin 13.6^\circ\,\hat{i} + 0.75\cos 13.6^\circ\,\hat{j} = 0.176\,\hat{i} + 0.729\,\hat{j}$$

Consider link AB

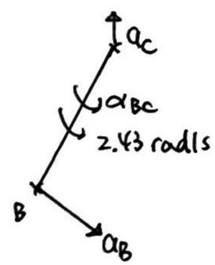


$$a_B = \cancel{a_A} + \alpha \times r_{B/A} - \omega^2 r_{B/A}$$

$$= -20\,\hat{k} \times (-0.177\,\hat{i} + 0.177\,\hat{j}) - 10^2(-0.177\,\hat{i} + 0.177\,\hat{j})$$

$$= 21.21\,\hat{i} - 14.14\,\hat{j}\ \text{m/s}^2 \quad (25.49\ \text{m/s}^2)$$

Consider link BC



$$a_C = a_B + \alpha_{BC} \times r_{C/B} - \omega^2 r_{C/B}$$

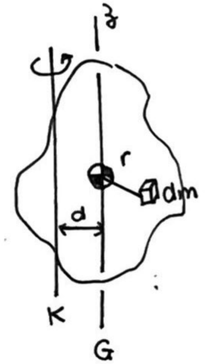
$$a_C\,\hat{j} = 21.21\,\hat{i} - 14.14\,\hat{j} + \underbrace{\alpha_{BC}\,\hat{k} \times (0.176\,\hat{i} + 0.729\,\hat{j})}_{-0.729\,\alpha_{BC}\,\hat{i} + 0.177\,\alpha_{BC}\,\hat{j}} - 2.43^2(0.176\,\hat{i} + 0.729\,\hat{j})$$

$$\begin{cases} 0 = 20.17 - 0.729\,\alpha_{BC} \\ a_C = 0.177\,\alpha_{BC} - 18.45 \end{cases}$$

$$\rightarrow \begin{cases} \alpha_{BC} = 27.7\ \text{rad/s}^2 \circlearrowleft \\ a_C = -13.5\ \text{m/s}^2 \uparrow \end{cases}$$

□

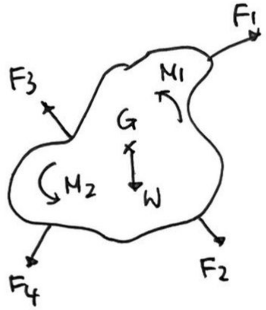
2. KINETICS : FORCE AND ACCELERATION



$$I = \int_m r^2\,dm = \int_V r^2 \rho\, dV$$

Rotating the object about K:

$$I_K = I_G + md^2$$



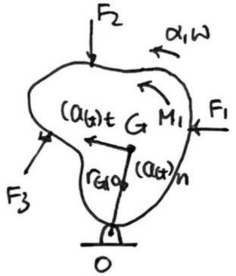
General Motion

$$\Sigma F_x = m(a_G)_x$$

$$\Sigma F_y = m(a_G)_y$$

$$\Sigma M_G = I_G \cdot \alpha$$

Motion - Rotation about a fixed axis



$$\Sigma F_n = m(a_G)_n = m(\omega^2 r_G)$$

$$\Sigma F_t = m(a_G)_t = m(\alpha r_G)$$

$$\Sigma M_G = I_G \cdot \alpha$$

Consider centre of gravity at G and pinned at O,

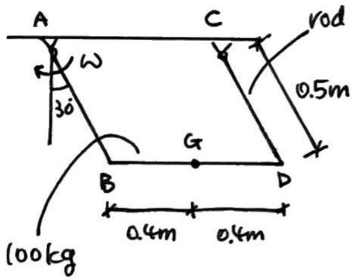
$$\Sigma M_G = I_O \cdot \alpha \leftarrow I_O = I_G + md^2$$

$$= r \cdot m(a_G)_t + I_G \alpha$$

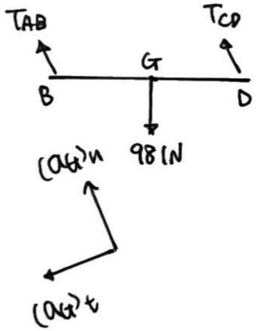
$$= mr^2 \alpha + I_G \alpha$$

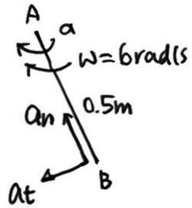
$$= \underline{(mr^2 + I_G)\alpha}$$

Example 13



force at each rod.





$(a_G)_n = \omega^2 r = 6^2(0.5) = 18\,m/s^2$

$\nwarrow \Sigma F_n = m(a_G)_n$: $T_{AB} + T_{CD} - 981\cos 30^\circ = (100)(18)$ ... (1)

$\swarrow \Sigma F_t = m(a_G)_t$: $981 \sin 30^\circ = 100 \cdot (a_G)_t$ ... (2)

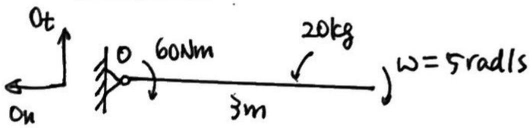
$\Sigma M_G = 0$: $-T_{AB}\cos 30^\circ \cdot 0.4 + T_{CD}\cos 30^\circ \cdot 0.4 = 0$ ... (3)

$\rightarrow$ (3): $T_{AB} = T_{CD}$

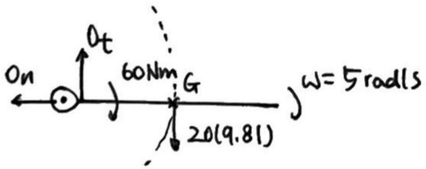
(1): $T_{AB} = T_{CD} = 1325\,N$

(2): $(a_G)_t = 4.905\,m/s^2$

Example 14



determine angular acceleration and O_t, O_n



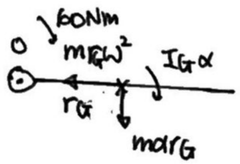
distance of O from G

About G: $\Sigma F_n = m\omega^2 r_G$: $O_n = (20)(5)^2(1.5) = 750N$

$\Sigma F_t = m\alpha r_G$: $O_t - 20(9.81) = (20)(\alpha)(1.5)$

$\curvearrowright \Sigma M_G = I_G \alpha$: $O_t(1.5) + 60 = (\frac{1}{12}(20)(3)^2)\alpha$

$$\rightarrow \begin{cases} O_t = 19.05N \\ \alpha = 5.90\,rad/s \end{cases}$$

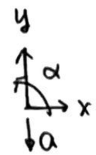


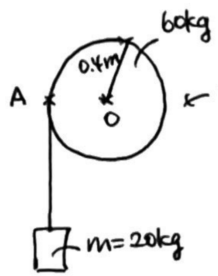
About O: $\Sigma M_O = I_O \alpha$:

$60 + 20(9.81)(1.5) = (\frac{1}{12}(20)(3)^2 + 20 \times 1.5^2)\alpha$

$\alpha = 5.90\,rad/s$

Example 15

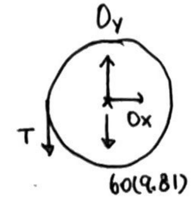


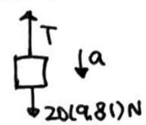


← radius of gyration:

$k_O = 0.25m$

determine the drum's angular acceleration





$I_O = mk_O^2 = 60(0.25)^2 = 3.75\ kg\cdot m^2$

For the mass block: $20(9.81) - T = 20\cdot a = 20\cdot(0.4)\alpha$

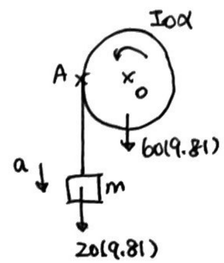
For the drum: $\Sigma M_O = I_O\alpha$: $T(0.4) = 3.75\cdot\alpha$

$\left.\right\}$ $T = 106N$

$a = 4.52 m/s^2$

$\rightarrow \alpha = 4.52/0.4 = 11.3\ rad/s$ ↺

Consider the drum as single system

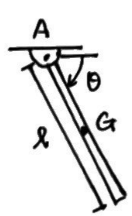


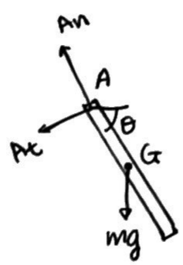
force due to m only, the drum's force is cancelled

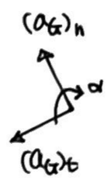
Take moment about O: $\underbrace{20(9.81)}_{mg}\underbrace{(0.4)}_{r_{A/O}} = \left(3.75 + 20(0.4)^2\right)\alpha$

$\alpha = 11.3\ rad/s$

Example 16







Determine An and At at $\theta = 90°$

motion

↖ $\Sigma F_n = m\omega^2 r_G$: $A_n - mg\sin\theta = m\omega^2\frac{l}{2}$

↙ $\Sigma F_t = m\alpha r_G$: $A_t - mg\cos\theta = m\alpha\cdot\frac{l}{2}$

note: $I_A = \frac{1}{3}ml^2$

↻ $\Sigma M_A = I_A r_G$: $mg\cos\theta\cdot\frac{l}{2} = (\frac{1}{3}ml^2)\cdot\alpha$ ... (3)

kinematic

From (3): $\alpha = \frac{mg\cos\theta\frac{l}{2}}{\frac{1}{3}ml^2} = \frac{3g\cos\theta}{2l}$

$\boxed{\omega d\omega = \alpha d\theta} \rightarrow \int_0^{\omega}\omega d\omega = \int_{0°}^{90°}\left(\frac{3g}{2l}\right)\cos\theta d\theta \rightarrow \frac{\omega^2}{2} = \frac{3g}{2l}\cdot(-\sin\theta)\Big|_{0°}^{90°}$

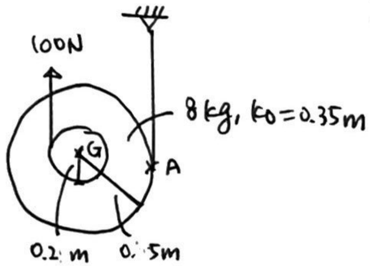
$\rightarrow \omega^2 = \frac{3g}{l}$

$\therefore$ $A_n = mg\underbrace{\sin\theta}_{\theta=90°} + m\cdot\frac{3g}{l}\cdot\frac{l}{2} = \frac{5}{2}mg$

$A_t = mg\cancel{\cos}\theta + m\cdot\cancel{\alpha}\cdot\frac{l}{2} = 0$

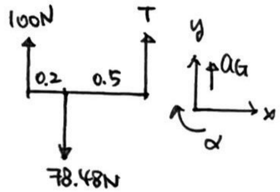
$\frac{3g\cos\theta}{2l}$

Example 17



determine the angular acceleration of spool

Solution I



$I_G = m k_G^2 = 8(0.35)^2 = 0.980\ kg\cdot m^2$

$\uparrow \Sigma F_y = m(a_G)_y$: $T + 100 - 78.48 = 8(a_G)_y = 8(0.5)\alpha$

$\circlearrowleft \Sigma M_G = I_G \cdot \alpha$: $100(0.2) - T(0.5) = 0.980 \cdot \alpha$

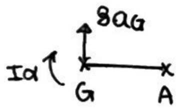
assume the spool rolls without slipping at A, i.e. $a_A = 0$

$a_G = \cancel{a_A} + \alpha \times r_{G/A} - \omega^2 r_{G/A}$

$= \alpha(0.5)$ $\qquad \omega = 0$

$\rightarrow \begin{cases} \alpha = 10.3\ rad/s \\ a_G = 5.16\ m/s \\ T = 19.8\ N \end{cases}$

Solution II



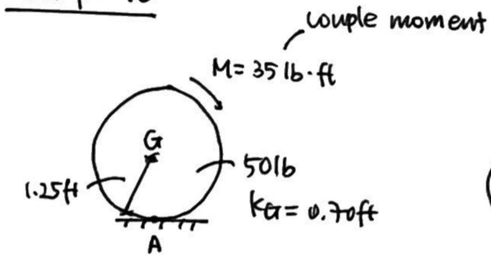
$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A$: $\underbrace{100(0.7) - 78.48(0.5)}_{\text{external force about A}} = \underbrace{I\alpha + m a_G \cdot r_{AG}}_{\text{due to net force \& net moment (rotation)}}$

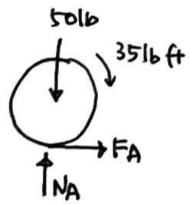
$= 0.980\alpha + 8 \cdot (0.5)^2 \cdot \alpha \rightarrow \alpha = 10.3\ rad/s$

Solution III

$\Sigma M_A = I_A \cdot \alpha$: $\overbrace{100(0.7) - 78.48(0.5)}^{M_{net}} = (I_G + m \cdot r^2)\alpha = (0.980 + 0.8 \cdot 0.5^2)\alpha \rightarrow \alpha = 10.3\ rad/s$

Example 18





determine the acceleration of its mass centre G

* $\mu_s = 0.3$ $\quad \mu_k = 0.25$

$I_G = m k_G^2 = \dfrac{50lb}{32.2ft/s^2}(0.70ft)^2 = 0.7609\ slug\cdot ft^2$

$\Sigma F_x = m(a_G)_x$: $F_A = \dfrac{50lb}{32.2ft/s^2} \cdot a_G \rightarrow a_G = r \cdot \alpha = 1.25ft \cdot \alpha$

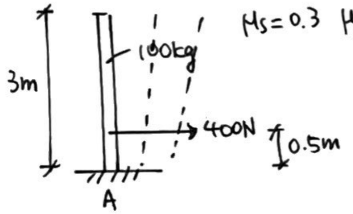
$\Sigma F_y = m(a_G)_y$: $N_A - 50lb = 0 \rightarrow N_A = 50lb$

$\circlearrowleft \Sigma M_A = I_G \alpha$: $35lb\cdot ft - F_A \cdot 1.25ft = 0.7609\ slug\cdot ft^2 \cdot \alpha$

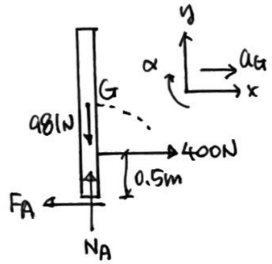
$\begin{cases} F_A = 21.3\ lb \\ \alpha = 11.0\ rad/s^2 \\ a_G = 13.7\ ft/s^2 \end{cases}$

Example 21

Example 19



$\mu_s = 0.3 \quad \mu_k = 0.25$ determine the pole's angular acceleration



$\xrightarrow{+} \Sigma F_x = m(a_G)_x$: $400 - \underset{\mu_k N_A}{F_A} = (100) a_G$

$+\uparrow \Sigma F_y = m(a_G)_y$: $981 - N_A = 0$

$+\circlearrowleft \Sigma M_G = I_G \cdot \alpha$: $F_A(1.5) - 400(1.0) = \underbrace{\left(\frac{1}{12}(100)(3)^2\right)}_{I_G \text{ at } G} \cdot \alpha$

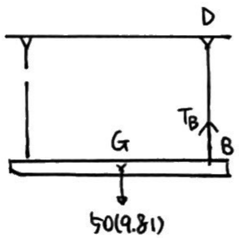
$a_G = r \cdot \alpha$

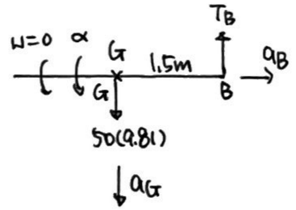
$\left.\begin{array}{l} F_A = 300N \\ N_A = 981N \\ a_G = 1 \text{ m/s} \\ \alpha = 0.67 \text{ rad/s} \end{array}\right\}$

$\frac{F_A}{N_A} = \frac{300}{981} = 0.306 > 0.3$

$\rightarrow$ slip.

Example 20





Motion

$\Sigma F_y = m(a_G)_y$: $T_B - 50(9.81) = 50(a_G)_y$

$\Sigma M_G = I\alpha$: $T_B \cdot (1.5m) = \frac{1}{12}(50)(3)^2 \cdot \alpha$

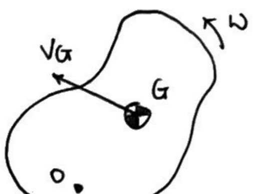
Kinematics

$a_G = a_B + \alpha \times r_{G/B} - \cancel{\omega^2} r_{G/B}$ $\qquad (a_B)_x = 0$

$-(a_G)_y \hat{j} = (a_B)_x \hat{i} + \alpha \cdot \hat{k} \times (-1.5\hat{i}) \rightarrow (a_G)_y = 1.5\alpha$

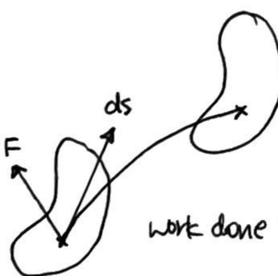
$\left.\begin{array}{l} \alpha = 4.905 \text{ rad/s} \\ T_B = 123N \\ (a_G)_y = 7.36 \text{ m/s}^2 \end{array}\right\}$

3. WORK AND ENERGY



$KE = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 \quad \cdots (1)$

if $v_G = r\omega \rightarrow KE = \frac{1}{2}(I_G + mr^2)\omega^2 = \frac{1}{2} I_O \omega^2 \quad \cdots (2)$

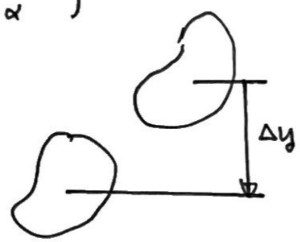


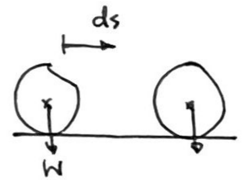
work done by force:

$U_F = \int F \cdot dr = \int_s F\cos\theta \cdot ds \quad \cdots (3)$

constant force: $U_{F,const} = F\cos\theta \cdot s$

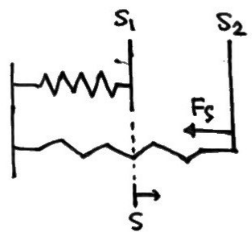
weight (potential energy): $PE = m \cdot g \cdot \Delta y \quad \cdots (4)$





$\int W \cdot ds = 0$

as $\cos\theta |_{\theta = 90^\circ} = 0$



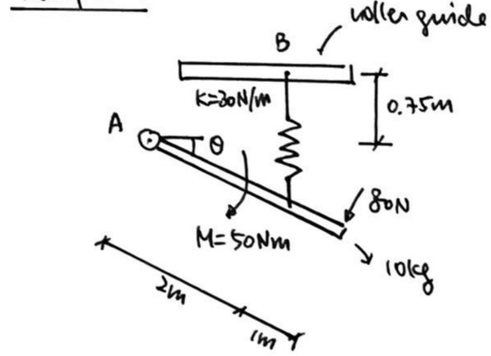
Work of spring:

$F_s = ks$

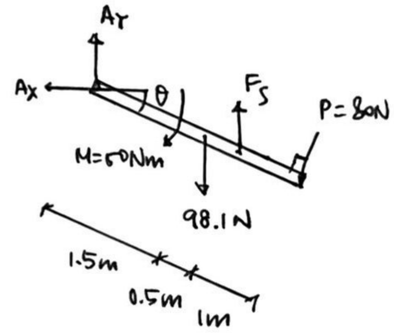
$\rightarrow U_s = -\left(\frac{1}{2} k s_2^2 - \frac{1}{2} k s_1^2\right)$

negative as displacement oppose the force

Example 21



determine work done by all forces ~~with~~ from $\theta = 0^\circ$ to $\theta = 90^\circ$



Weight W: $PE = mg\Delta y = 98.1\,(\overset{90^\circ}{1.5} - \overset{0^\circ}{0}) = 147.2\,J$

Coupled moment M: $U_M = M\Delta\theta = 50\left(\frac{\pi}{2} - 0\right) = 78.5\,J$

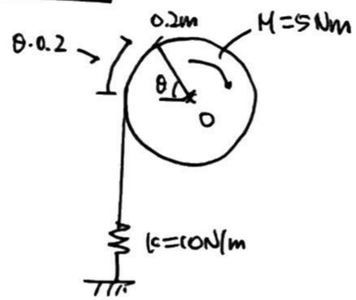
Spring force F_S: $U_S = -\left(\frac{1}{2}(30)(2+0.75) - \frac{1}{2}(30)(0.75-0.5)\right) = -75.0\,J$

Force P: $U_P = 80\left(\frac{\pi}{2} \cdot 3\right) = 377\,J$

Pin Reaction A_x, A_y: $U_{Ax} = U_{Ay} = 0$ (no displacement)

Total Work: $U = 147.2 + 78.5 - 75.0 + 377 = 528\,J$

Example 22



determine the angle it must rotate to attain angular velocity $\omega = 2$ rad/s starting from rest.

KE: $KE_1 = 0$

$KE_2 = \frac{1}{2} I_O \omega_2^2 = \frac{1}{2}\left(\frac{1}{2}(30)(0.2m)^2\right)(2\,rad/s)^2 = 1.2\,J$

Balance of work and energy:

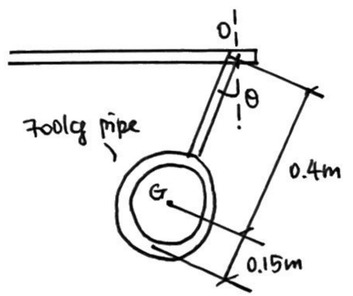
$$\{\cancel{KE_1}\} + \{\Sigma U_{1-2}\} = \{KE_2\}$$

$$\{M\Delta\theta - \Delta(\tfrac{1}{2}ks^2)\} = \{KE_2\}$$

$$\{5Nm \cdot (\theta - 0) - \tfrac{1}{2}(10N/m)(\theta \cdot 0.2)^2\} = 1.2$$

$$5\theta - 0.2\theta^2 = 1.2 \rightarrow \theta = 0.2423\ rad.$$

Example 23

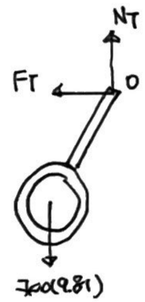


the pipe undergoes a swinging motion with $\theta = 30^\circ$ if is momentarily at rest. find the normal and frictional force it requires to support the pipe at $\theta = 0^\circ$

$KE|_{\theta=30^\circ} = 0$

$I_G = mr^2$ (thin ring)

$KE|_{\theta=0^\circ} = \frac{1}{2} I_O \omega_2^2 = \frac{1}{2}(I_G + mr^2)\omega_2^2 = \frac{1}{2}(700kg \times (0.15m)^2 + 700kg \times (0.4m)^2)\omega_2^2 = 63.875\omega_2^2$



* displacement $\Delta h = 0.4 - 0.4\cos 30^\circ = 0.05359m$

Principle of work and energy: $\{\cancel{KE_1}\} + \{U_{1-2}\} = \{KE_2\}$

$700kg \cdot 9.81 m/s^2 \cdot 0.05359 = 63.875\omega_2^2 \rightarrow \omega_2 = 2.4$ rad/s

$\underline{\text{at } \theta = 0^\circ} \rightarrow \omega_2$

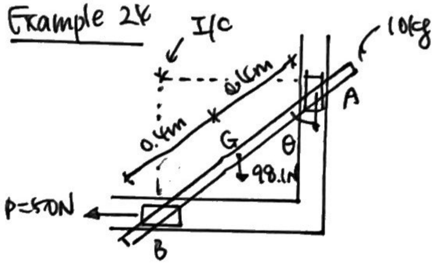
Equation of motion:

$\overset{+}{\leftarrow} \Sigma F_t = ma_t$: $F_T = 700\overbrace{(a_G)_t}^{r\alpha}$ $\rightarrow F_t = 0$

$+\uparrow \Sigma F_n = ma_n$: $N_T - 700(9.81) = 700\underbrace{(0.4)(2.4)^2}_{r\omega^2 = a_n}$ $\rightarrow N_T = 8480N$

$\Sigma M_O = I_O\alpha$: $0 = [(700)(0.15)^2 + (700)(0.4)^2]\alpha$ $\rightarrow \alpha = 0$

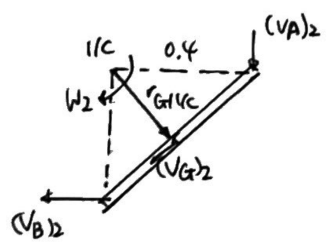
Example 24



initially at rest = $\theta = 0^\circ$

slider block at B with P = 50N

determine angular velocity at $\theta = 45^\circ$



$\{KE_1\} = 0$

G is varying → take an instantaneous centre

$\{KE_2\} = \frac{1}{2}mV_{G_2}^2 + \frac{1}{2}I_G\omega_2^2 = \frac{1}{2}(10)(V_G^2) + \frac{1}{2}(\frac{1}{12}(10)(0.8)^2)\omega_2^2$

$= 5V_{G_2}^2 + 0.2667\omega_2^2$

$(V_G)_2 = r_{G/IC}\omega_2 = 0.4\tan 45^\circ \omega_2 = 0.4\omega_2 \rightarrow \{KE_2\} = 1.0667\omega_2^2$

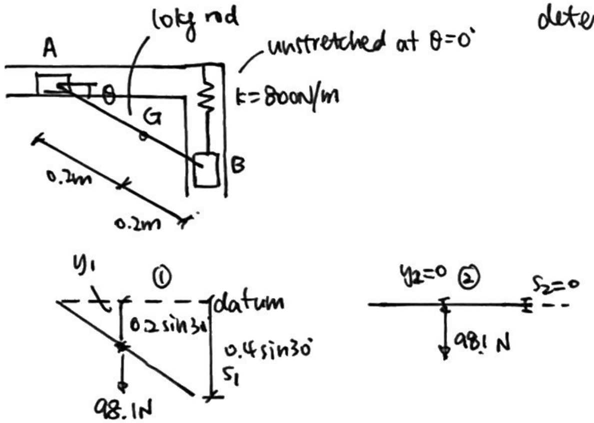
Energy Balance: $\{KE_1\} + \{U_{1-2}\} = \{KE_2\}$

$0 + \{W\Delta y + P\Delta s\} = 1.0667\omega_2^2$

$0 + 98.1(0.4 - 0.4\cos 45^\circ) + (50)(0.4\sin 45^\circ) = 1.0667\omega_2^2$

$\omega_2 = 6.11$ rad/s

Example 25



determine ω_{AB} at $\theta=0°$ if the rod release from rest when $\theta=30°$

$\{KE_1\} = 0$

$\{KE_2\} = \frac{1}{2} I_A \omega_{AB}^2 = \frac{1}{2}(I_0 + m.d^2)\,\omega_{AB}^2 = \frac{1}{2}\left(\frac{1}{12}(0.4)^2(10) + 10(0.2)^2\right)\cdot \omega_{AB}^2$ *

$= 0.2667\,\omega_{AB}^2$

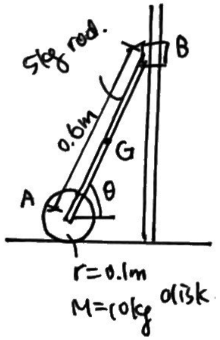
$\{U_{1-2}\} = \{W\Delta y + U_s\}$

$= \{98.1(0.2\sin 30° - 0) - \frac{1}{2}(800)(0.4\sin 30°)^2\} = 6.19\text{J}$

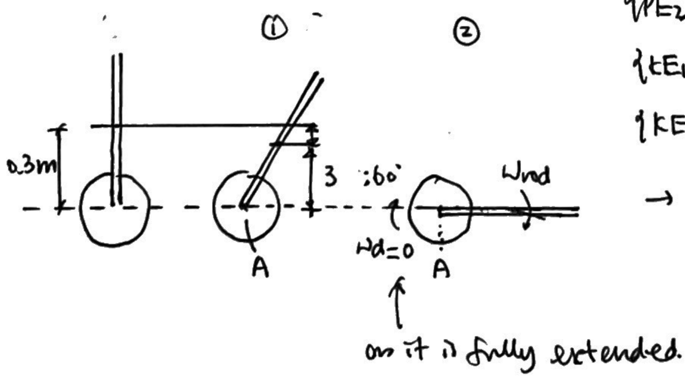
$\rightarrow \{\cancel{KE_1}\} + \{U_{1-2}\} = \{KE_2\}$

$6.19 = 0.2667\,\omega_{AB}^2 \rightarrow \omega_{AB} = 4.818$ rad/s ↶

Example 26



release from rest at $\theta = 60°$, find angular velocity of the rod when $\theta = 0°$



$\{PE_1\} = 5(9.81)(0.3\sin 60°) = 12.743\text{J}$

$\{PE_2\} = 0$

$\{KE_1\} = 0$

$\{KE_2\} = \frac{1}{2} I_A \omega_{rod}^2 = \frac{1}{2}\left(\frac{1}{3}(5)(0.6)^2\right)\omega_{rod}^2 = 0.3\,\omega_{rod}^2$ J

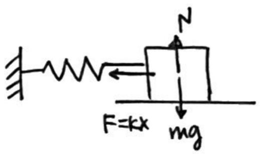
w.r.t. A

$\rightarrow \{KE_1\} + \{PE_1\} = \{KE_2\} + \{PE_2\}$

$12.743 = 0.3\,\omega_{rod}^2$

$\omega_{rod} = 6.517$ rad/s

4. VIBRATION



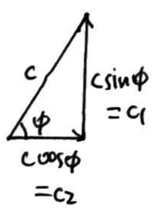
undamped oscillation

$\xrightarrow{+} \Sigma F_x = m\ddot{x}: \quad -kx = m\ddot{x}$

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2} x = 0$$

$$\rightarrow x = c_1 \cos\Big(\underbrace{\sqrt{\tfrac{k}{m}}}_{\text{natural frequency}} x\Big) + c_2 \sin\Big(\sqrt{\tfrac{k}{m}}\, x\Big)$$



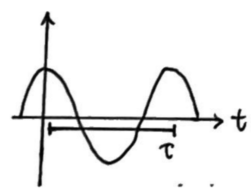
$$x = c\sin\phi \cos\sqrt{\tfrac{k}{m}}\,x + c\cos\phi \sin\sqrt{\tfrac{k}{m}}\,x$$

$$= c\sin(\underbrace{\phi}_{\text{phase angle}} + \sqrt{\tfrac{k}{m}}\,x)$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{1}{f} \text{ (period)}$$

$$= 2\pi\sqrt{\frac{m}{k}}$$

↑ frequency



For energy method:

$$T + V = \text{constant}$$

$$\left(\tfrac{1}{2}m\dot{x}^2\right) + \tfrac{1}{2}kx^2 = \text{constant}$$

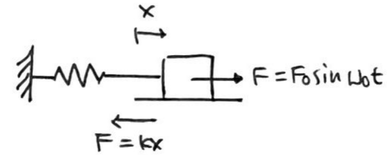
$$\downarrow \frac{d}{dt}$$

$$m\dot{x}\ddot{x} + kx\dot{x} = 0$$

$$\dot{x}(m\ddot{x} + kx) = 0 \rightarrow \dot{x} \neq 0$$

$$m\ddot{x} + kx = 0$$

$$\ddot{x} + \frac{k}{m}x = 0 \rightarrow \omega_n = \sqrt{\frac{k}{m}}$$



$\xrightarrow{+} \Sigma F_x = m\ddot{x}: \quad F_0 \sin\omega_0 t - kx = m\ddot{x}$

$$\ddot{x} - \frac{k}{m}x = \frac{F_0}{m}\sin\omega_0 t$$

$$x = \underbrace{x_c}_{\text{complementary soln}} + x_p \rightarrow \text{particular soln due to forced oscillation}$$

$$x_c = C\sin(\phi + \sqrt{\tfrac{k}{m}}\,t) \qquad x_p = X\sin\omega_p t$$

$$\ddot{x} + \frac{k}{m}x = \frac{F_0}{m}\sin\omega_0 t \rightarrow -X\omega_p^2 \cancel{\sin\omega_p t} + \frac{k}{m}X\cancel{\sin\omega_p t} = \frac{F_0}{m}\cancel{\sin\omega_p t}$$

$$\rightarrow -X\left(\omega_p^2 - \frac{k}{m}\right) = \frac{F_0}{m}$$

$$\rightarrow X = \frac{F_0/m}{k/m - \omega_p^2}$$

note that $k/m = \omega_n$:

$$\boxed{X = \frac{F_0/k}{1 - (\omega_p/\omega_n)^2}}$$

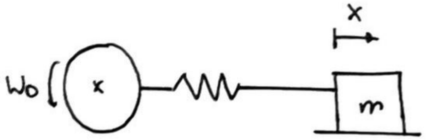
$$\rightarrow x = x_c + x_p = \underbrace{C\sin(\omega_n t + \phi)}_{\substack{\text{free vibration}\\ \text{damped down}\\ \text{by friction}\\ \rightarrow \text{transient}}} + \underbrace{\frac{F_0/k}{1 - (\omega_p/\omega_n)^2}\sin\omega_p t}_{\substack{\text{forced vibration}\\ \rightarrow \text{steady state}}}$$

$$\text{magnification factor } MF = \frac{X}{F_0/k}$$

X ← amplitude of steady state

F_0/k ← static deflection

Periodic Support Displacement



$\delta = \delta_0 \sin \omega_0 t$

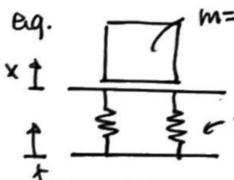
not force

due to support displacement

$$-k(x - \delta_0 \sin \omega_0 t) = m\ddot{x}$$

$$\rightarrow \quad m\ddot{x} + kx = k\delta_0 \sin \omega_0 t$$

eg.



← four spring, k = 800N/m

$\delta = 10 \sin 8t$ mm

$$-4k(x - \delta_0 \sin \omega_0 t) = m\ddot{x}$$

$$\rightarrow \quad \ddot{x} + \frac{4k}{m} x = \frac{4k\delta_0}{m} \sin \omega_0 t$$

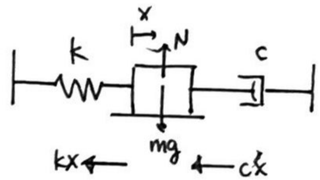
→ natural frequency $= \sqrt{\frac{4k}{m}}$

$$\omega_n = \sqrt{4k/m} = 12.65 \text{ rad/s}$$

$$X = \frac{\delta_0}{1 - (\omega_0/\omega_n)^2} = \frac{10}{1 - (8/12.65)^2} = 16.7\text{mm}$$

→ resonant occurs at $\omega_0 = \omega_n = 12.65$ rad/s

Viscous Damped Oscillation



$$-kx - c\dot{x} = m\ddot{x}$$

$$\rightarrow \quad \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m} x = 0$$

Let $x = e^{\lambda x}$

$$\lambda^2 e^{\lambda x} + \frac{c}{m}\lambda e^{\lambda x} + \frac{k}{m} e^{\lambda x} = 0$$

$$\rightarrow \quad \lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0$$

$$\lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

Critical damping at $\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0$

$$\rightarrow \quad c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

Over-damped: $\Delta = \left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} > 0 \rightarrow c > c_c$

$\lambda_1, \lambda_2 \in \mathbb{R}$

$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$ (non-vibrating)

Critically damped: $x = (A + Bt)e^{-\omega_n t}$ $\quad \lambda_1 = \lambda_2 = -\frac{c_c}{2m}$

Underdamped: $x = D\left(e^{-\frac{c}{2m}t} \sin(\omega_d t + \phi)\right)$

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \omega_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2}$$

damping factor $= c/c_c$

Viscous Forced Vibration

$= k\delta_0$ if displacement

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega_0 t$$

$$x_p = X' \sin(\omega_0 t - \phi')$$

$\rightarrow \quad -mX'\omega_0^2 \sin(\omega_0 t - \phi') + cX'\omega_0 \cos(\omega_0 t - \phi')$

$\quad + kX' \sin(\omega_0 t - \phi') = F_0 \sin \omega_0 t \quad \cdots (1)$

consider $\omega_0 t - \phi' = 0 \quad \sin(\cdot) = 0 \quad \cos(\cdot) = 1$

(1) becomes: $\quad cX'\omega_0 = F_0 \sin \phi' \quad \cdots (2)$

consider $\omega_0 t - \phi' = \frac{\pi}{2} \quad \sin(\cdot) = 1 \quad \cos(\cdot) = 0$

$$-mX'\omega_0^2 + kX' = F_0 \cos \phi' \quad \cdots (3)$$

$(2)^2 + (3)^2$: $\quad c^2 X'^2 \omega_0^2 + (mX'\omega_0^2 - kX')^2 = F_0^2$

$$X'^2\left((c\omega_0)^2 + (m\omega_0^2 - k)^2\right) = F_0^2$$

$$X' = \frac{F_0}{\sqrt{(c\omega_0)^2 + (m\omega_0^2 - k)^2}}$$

$\frac{(2)}{(3)}$: $\quad \tan \phi' = \frac{c\omega_0}{k - m\omega_0^2}$

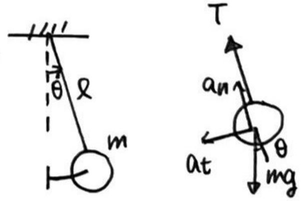
with $\omega_n = \sqrt{\frac{k}{m}}$ and $c_c = 2m\sqrt{\frac{k}{m}}$

$$X' = \frac{F_0/k}{\sqrt{(1 - (\omega_0/\omega_n)^2)^2 + (2(c/c_c)(\omega_0/\omega_n))^2}}$$

$$\tan \phi' = \frac{2(c/c_c)(\omega_0/\omega_n)}{1 - (\omega_0/\omega_n)^2}$$

$$MF = \frac{1}{\sqrt{(1 - (\omega_0/\omega_n)^2)^2 + (2(c/c_c)(\omega_0/\omega_n))^2}}$$

Example 27



$\overset{+}{\swarrow} \Sigma F_t = ma_t:$ $\quad mg\sin\theta = ma_t \quad \tau(\ell\ddot{\theta}) = \ddot{s}$

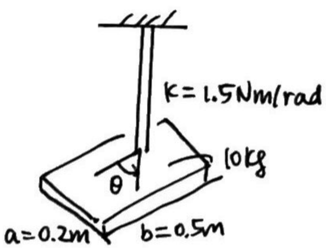
$mg\sin\theta = m\cdot\ell\ddot{\theta}$

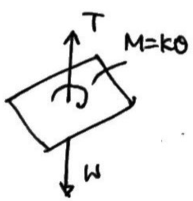
$\rightarrow \ddot{\theta} + \frac{g}{\ell}\theta = 0 \quad (\sin\theta \approx \theta$

$\rightarrow \omega_n = \sqrt{g/\ell}$

$\tau = 2\pi/\omega_n = 2\pi\sqrt{\ell/g}$

Example 28





$\Sigma M_O = I_O\alpha: \quad -k\theta = I_O\ddot{\theta}$

$\ddot{\theta} + \frac{k}{I_O}\theta = 0$

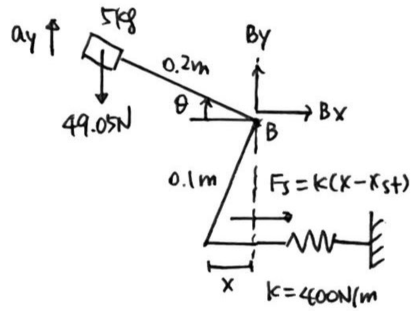
$\omega_n = \sqrt{k/I_O}$

$I_O = \frac{1}{12}(10)(0.2^2 + 0.5^2) = 0.1083\ kg\cdot m^2$

$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{I_O/k} = 2\pi\sqrt{0.1083/1.5}$

$= 1.69s$

Example 29



at $\theta=0$: $49.05(0.2) = 0.1(400)(x_{st})$

$\circlearrowleft + \Sigma M_B = \Sigma(M_k)_B$

$49.05(0.2) + 0.1\underbrace{(400)}_{k}(x - x_{st}) = -5kg\cdot a_y\ (0.2m)$

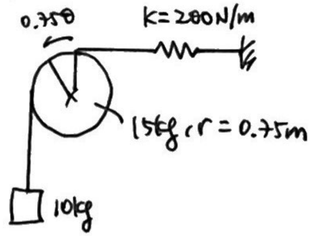
$kx(0.1) = -5a_y(0.2)$

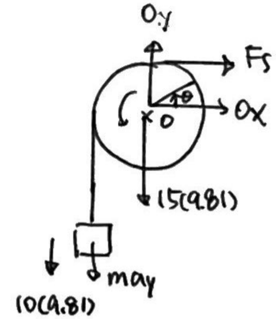
$x = 0.1\theta \qquad a_y = 0.2\ddot{\theta}$

$\ddot{\theta} + 20\theta = 0 \quad \omega_n^2 = 20 \quad \omega_n = 4.47\ rad/s$

$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{4.47} = 1.40s$

Example 30





as a particle with net force $10a_y$ and distance 0.75 from O

$10(9.81)(0.75) - F_s(0.75) = \left(\frac{1}{2}Mr^2\right)\ddot{\theta} + (10a_y)(0.75)$

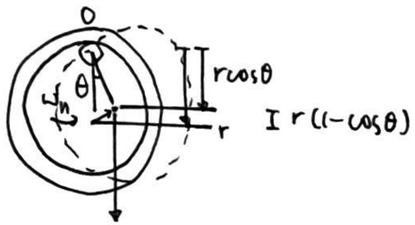
$F_s = -ks = -(200)(0.75\theta) + 200(0.75\theta_0) \qquad s = 0.75\theta, \quad \ddot{s} = a_y = 0.75\ddot{\theta}$

where $10(9.81)(0.75) = 200(0.75)\theta_0$ at the steady state

$\rightarrow -200(0.75\theta)(0.75) = (\frac{1}{2}(15)(0.75)^2 + \overbrace{10(0.75)(0.75)}^{md^2})\ddot{\theta}$

$9.84\ddot{\theta} + 112.5\theta = 0 \qquad \omega_n = \sqrt{112.5/9.84} = 3.38\ rad/s$

Example 31



$\{KE_1\} = 0$

$\{KE_2\} = \frac{1}{2} I_O \omega_n^2 = \frac{1}{2}(mr^2 + mr^2)\dot{\theta}^2 = mr^2\dot{\theta}^2$

$\{PE_1\} = 0$

$\{PE_2\} = -mg(r\cos\theta)$

$\rightarrow \quad mr^2\dot{\theta}^2 = -mg(r\cos\theta)$

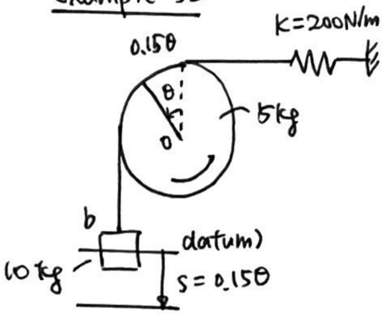
$\downarrow \frac{d}{dt}$

$2mr^2 \cdot \dot{\theta}\ddot{\theta} = +mgr\sin\theta\,\dot{\theta} \approx mgr\theta\dot{\theta}$

$\ddot{\theta} + \frac{g}{2r}\theta = 0$

$\omega_n = \sqrt{\frac{g}{2r}} \rightarrow \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2r}{g}}$

Example 32



$\{KE_1\} = 0$

$\{KE_2\} = \frac{1}{2} m_b v_b^2 + \frac{1}{2} I_O \omega_d^2$

$= \frac{1}{2}(10)(0.15\dot{\theta})^2 + \frac{1}{2}(\frac{1}{2}(5)(0.15)^2)\dot{\theta}^2$

$= 0.1406\,\dot{\theta}^2$

$\{PE_1\} = 0$

$\{PE_2\} = 10(9.81)(0.15\theta) - \frac{1}{2}(200)(0.15\theta + s_{st})^2$

$\{KE_2\} + \{PE_2\} = 0.1406\dot{\theta}^2 + 100(s_{st} + 0.15\theta)^2 - 14.715\theta$

$\downarrow \frac{d}{dt}$

$s_{st} = W/k = 98.1/200$ → taking the elongation at start

$0.28125\,\dot{\theta}\ddot{\theta} + 200(s_{st} + 0.15\theta)(0.15\dot{\theta}) - 14.72\dot{\theta} = 0$

$\downarrow$

$\ddot{\theta} + 16\theta = 0$

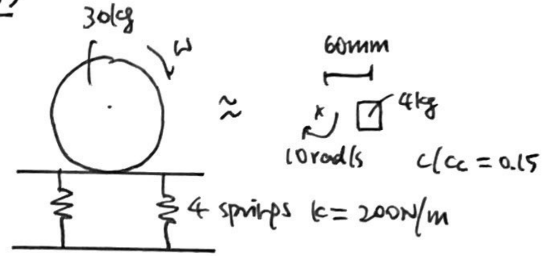
$\omega_n = \sqrt{16} = 4\,\text{rad/s}$

$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{4} = 1.57s$

Example 33

F_0? ω_n?

x'?



* $F_0 = ma_n^2 = mr\omega_0^2 = 4kg \cdot 0.06m \cdot (10rad/s)^2 = 24N$

* $\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(200)}{30}} = 5.164\,\text{rad/s}$

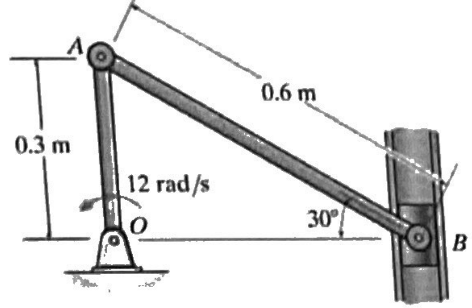
$x' = \frac{F_0/k}{\sqrt{(1-(\omega_0/\omega_n)^2)^2 + (2(c/c_n)(\omega_0/\omega_n))^2}}$

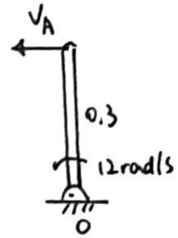
$= \frac{24/800}{\sqrt{(1-(10/5.164)^2)^2 + 2(0.15)(10/5.164)^2}}$

$= 0.0107\,m = 10.7\,mm.$

1. Relative Motion

F16–10. If crank OA rotates with an angular velocity of $\omega = 12$ rad/s, determine the velocity of piston B and the angular velocity of rod AB at the instant shown.



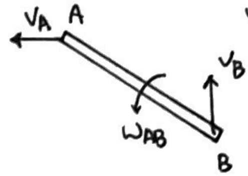


$v_A = \cancel{v_O}^{\,*} + \omega_{OA} \times r_{A/O}$

$= 12\hat{k} \times (0.3\hat{j}) = -3.6\hat{i}$

$v_B = v_A + \omega_{AB} \times r_{B/A}$

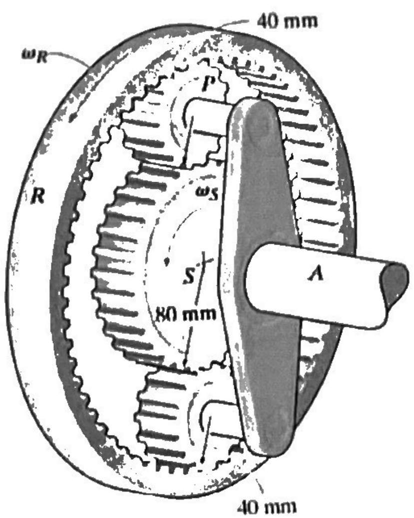
$= -3.6\hat{i} + \omega_{AB}\hat{k} \times (0.6\cos 30\,\hat{i} \;\; -0.6\sin 30\,\hat{j})$



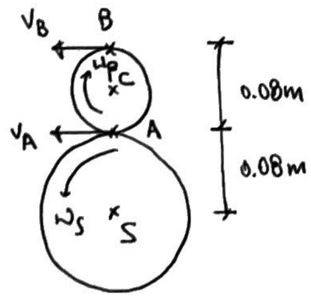
$v_B\hat{j} = (-3.6 + 0.6\sin 30^\circ\,\omega_{AB})\hat{i} \;+ \omega_{AB}\cdot 0.6\cos 30^\circ\,\hat{j}$

$\begin{cases} 3.6 = 0.6\sin 30^\circ\,\omega_{AB} \\ v_B = \omega_{AB}\,0.6\cos 30^\circ \end{cases} \rightarrow \begin{cases} \omega_{AB} = 12 \text{ rad/s} \circlearrowleft \\ v_B = 6.235 \text{ m/s} \uparrow \end{cases}$

***16–64.** The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.



$\omega_R = 0 \rightarrow v_B = \cancel{\omega}^{\,*} \times r_{B/S} = 0$



this requires $v_c = 0$

$v_A = v_A$

$\omega_S \times r_{A/S} = \omega_P \times r_{A/C}$

$\omega_P = \omega_S \times \dfrac{r_{A/S}}{r_{A/C}} = 5 \times \dfrac{0.08}{0.04}$? $= 10$ rad/s ?

$v_A = \cancel{v_B}^{\,*} + \omega_c \times r_{A/B} = \omega_c(-\hat{k}) \times (-0.08\hat{j})$

$5(0.08) = -0.08\,\omega_c$

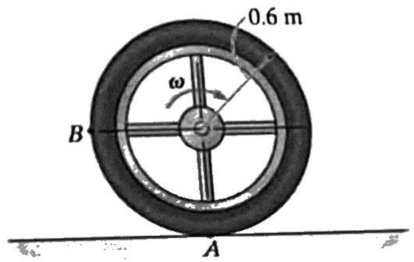
due to s

$\omega_c = -5$ rad/s

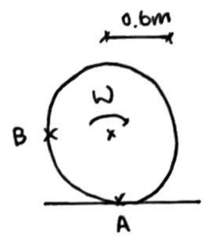
$v_c = \cancel{v_B}^{\,*} + \omega \times r_{c/B} \rightarrow v_c = (-5\hat{k}) \times (\overset{0.04}{\cancel{-40}}\,\hat{j}) = 0.2\,\hat{i}$

shaft. $\omega_A = \dfrac{v_c}{r_{c/s}} = \dfrac{0.2}{0.12} = 1.67$ rad/s

F16–8. The wheel rolls without slipping with an angular velocity of $\omega = 10$ rad/s. Determine the magnitude of the velocity of point B at the instant shown.



F16–8

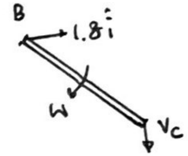


$v_A = 0$ without slipping

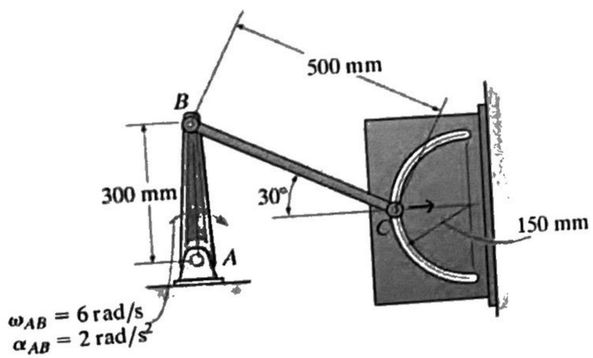
$v_B = \cancel{v_A} + \omega_{AB} \times r_{B/A}$

$= -10\hat{k} \times (-0.6\hat{i} + 0.6\hat{j}) = 6\hat{i} + 6\hat{j}$

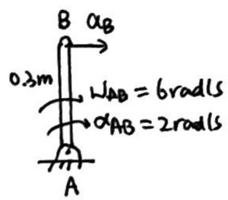
$\rightarrow |v_B| = \sqrt{6^2+6^2} = 8.485 m/s$



•16–121. Crank AB rotates with an angular velocity of $\omega_{AB} = 6$ rad/s and an angular acceleration of $\alpha_{AB} = 2$ rad/s^2. Determine the acceleration of C and the angular acceleration of BC at the instant shown.



Prob. 16–121



$a_B = \cancel{a_A} + \alpha \times r_{B/A} - \omega^2 r_{B/A}$

$= -2\hat{k} \times 0.3\hat{j} - 6^2 \cdot 0.3\hat{j}$

$= 0.6\hat{i} - 10.8\hat{j}$

$v_B = \cancel{v_A} + \omega \times r_{B/A}$

$= -6\hat{k} \times 0.3\hat{j} = 1.8\hat{i}$

$v_C = v_B + \omega_{BC} \times r_{C/B}$

$-v_C\hat{j} = 1.8\hat{i} + (-\omega_{BC}\hat{k}) \times (0.5\cos 30°\hat{i} - 0.5\sin 30°\hat{j})$

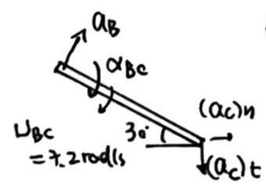
$-v_C\hat{j} = (1.8 - 0.25\omega_{BC})\hat{i} - 0.433\omega_{BC}\hat{j}$

$\rightarrow$ $1.8 - 0.25\omega_{BC} = 0$

$-v_C = -0.433\omega_{BC}$

$\omega_{BC} = 7.2$ rad/s ✓

$v_C = 3.12$ m/s ✓



$a_C = a_B + \alpha_{BC} \times r_{C/B} - \omega^2 r_{C/B}$

$= 0.6\hat{i} - 10.8\hat{j}$

$+ (-\alpha_{BC}\hat{k}) \times (0.433\hat{i} - 0.25\hat{j})$

$- 7.2^2 (0.433\hat{i} - 0.25\hat{j})$

$64.9\hat{i} - (a_C)_t\hat{j} = (0.6 - \alpha_{BC} \cdot 0.25 - 22.45)\hat{i}$

$-(10.8 + 0.433\alpha_{BC} - 12.96)\hat{j}$

$* \; (a_C)_n = \frac{v_C^2}{r}$

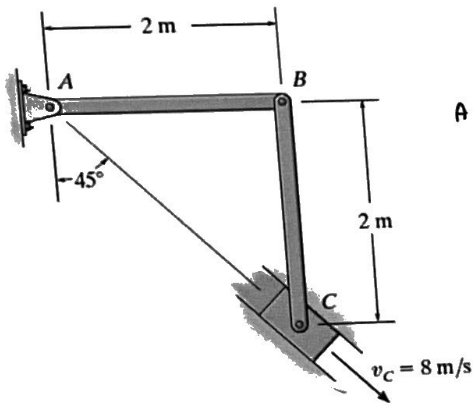
$= \frac{3.12^2}{0.15} = 64.9$

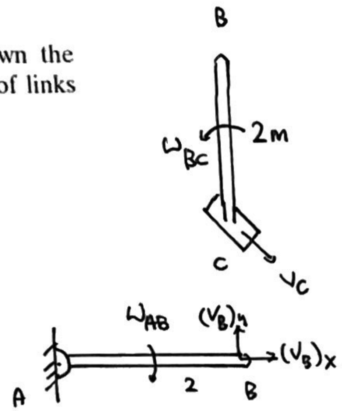
$\alpha_{BC} = 347$ rad/s^2

$(a_C)_t = 2.16 - 0.433\alpha_{BC} = -148$

$a = \sqrt{64.9^2 + 148.1^2} = 161.69$ m/s^2

*16–60. The slider block C moves at 8 m/s down the inclined groove. Determine the angular velocities of links AB and BC, at the instant shown.





link AB: $(v_B)_x\hat{i} + (v_B)_y\hat{j} = \cancel{v_A} + \omega_{AB}(-\hat{k}) \times r_{B/A}$

$= \omega_{AB}(-\hat{k}) \times (2\hat{i})$ $\quad (v_B)_x = 0$

$= -2\omega_{AB}\hat{j} \quad \rightarrow \quad (v_B)_y = -2\omega_{AB}$

link BC: $v_C = v_B + \omega_{BC} \times r_{C/B}$

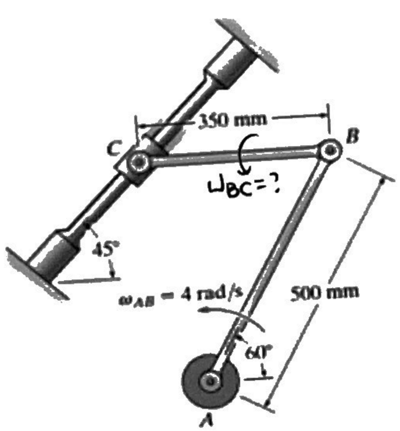
$\frac{8}{\sqrt{2}}\hat{i} - \frac{8}{\sqrt{2}}\hat{j} = (v_B)_y\hat{j} + \omega_{BC}\hat{k} \times (-2\hat{j})$

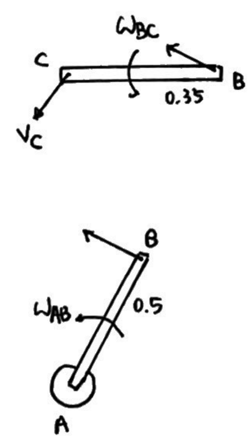
$= \omega_{BC}(2)\hat{i} + (-2\omega_{AB})\hat{j}$

$\rightarrow \frac{8}{\sqrt{2}} = \omega_{BC}(2) \rightarrow \omega_{BC} = 2\sqrt{2}$ rad/s

$-\frac{8}{\sqrt{2}} = -2\omega_{AB}$ $\quad \omega_{AB} = 2\sqrt{2}$ rad/s

*16–68. Knowing that angular velocity of link AB is $\omega_{AB} = 4$ rad/s, determine the velocity of the collar at C and the angular velocity of link CB at the instant shown. Link CB is horizontal at this instant.





link AB: $v_B = \cancel{v_A} + \omega_{AB} \times r_{B/A}$

$= 4\hat{k} \times (0.5\cos 60°\,\hat{i} + 0.5\sin 60°\,\hat{j})$

$= 1\hat{j} - 1.73\hat{i}$

link BC: $v_C = v_B + \omega_{BC} \times r_{C/B}$

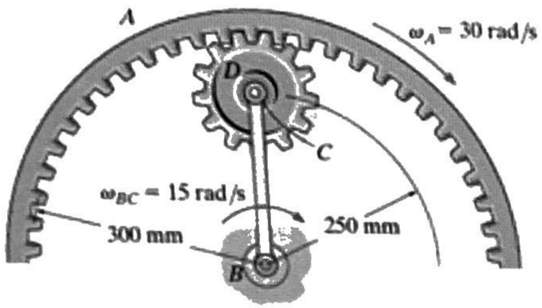
$-\frac{v_C}{\sqrt{2}}\hat{i} - \frac{v_C}{\sqrt{2}}\hat{j} = -1.73\hat{i} + 1\hat{j} + \omega_{BC}\hat{k} \times (-0.35\hat{i})$

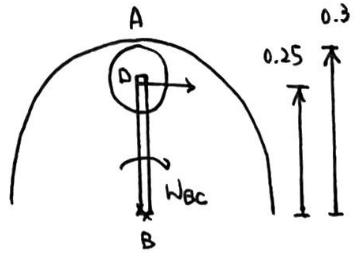
$= -1.73\hat{i} + (1 - 0.35\,\omega_{BC})\hat{j}$

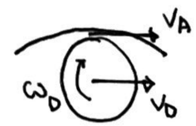
$\rightarrow -\frac{v_C}{\sqrt{2}} = -1.73 \quad \rightarrow v_C = 2.45$ m/s

$-\frac{v_C}{\sqrt{2}} = 1 - 0.35\,\omega_{BC} \quad \omega_{BC} = \frac{\left(-\frac{2.45}{\sqrt{2}} - 1\right)}{(-0.35)} = 7.81$ rad/s

16–78. If the ring gear A rotates clockwise with an angular velocity of $\omega_A = 30$ rad/s, while link BC rotates clockwise with an angular velocity of $\omega_{BC} = 15$ rad/s, determine the angular velocity of gear D.







$v_A = \omega_A \times r_{A/B} = 30(0.3) = 9$ m/s

$v_D = \omega_{BC} \times r_{C/B} = 15(0.25) = 3.75$ m/s

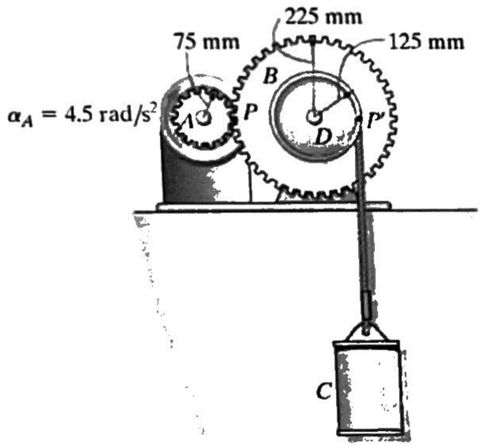
$v_A = v_D + \omega_D \times r_{A/D}$

$9\hat{i} = 3.75\hat{i} + (-\omega_D)\hat{k} \times 0.05\hat{j}$

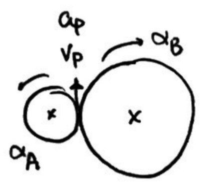
$= 3.75\hat{i} + 0.05\omega_D\hat{i}$

$\rightarrow 9 = 3.75 + 0.05\omega_D \rightarrow \omega_D = \underline{105 \text{ rad/s}}$

F16–6. For a short period of time, the motor turns gear A with a constant angular acceleration of $\alpha_A = 4.5$ rad/s^2, starting from rest. Determine the velocity of the cylinder and the distance it travels in three seconds. The cord is wrapped around pulley D which is rigidly attached to gear B.



F16–6



$a_p|_{\text{due to } A} = a_p|_{\text{due to } B} \rightarrow \alpha_A r_A = \alpha_B r_B$

$\rightarrow \alpha_B = \alpha_A \frac{r_A}{r_B} = 4.5 \cdot \frac{75}{225} = 1.5 \text{ rad/s}^2$

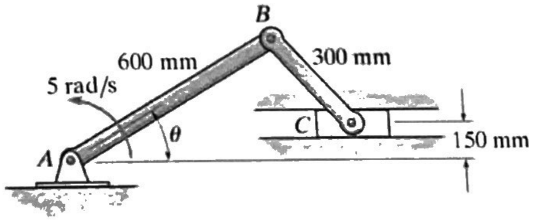
$\omega_B = \cancel{\omega_B}|_0 + \alpha_B t = 1.5 \text{ rad/s}^2 \cdot 3\text{s} = 4.5 \text{ rad/s}$

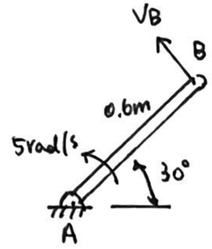
$\theta_B = \cancel{(\theta_B)}|_0 + \cancel{\omega_B}|_0 \cdot t + \frac{1}{2}\alpha_B t^2 = \frac{1}{2}(1.5)(3)^2 = 6.75 \text{ rad.}$

$v_C = r_C \cdot \omega_B = 4.5 \text{ rad/s} \cdot 0.125 = 0.5625 \text{ m/s}$

$s_C = r_C \cdot \theta_B = 0.125 \cdot 6.75 = \underline{0.844 \text{ m.}}$

•16–41. Crank AB rotates with a constant angular velocity of 5 rad/s. Determine the velocity of block C and the angular velocity of link BC at the instant $\theta = 30^\circ$.

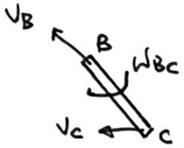


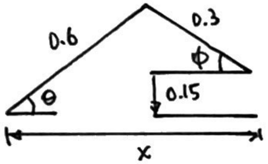


$v_B = \cancel{v_A} + \omega_{AB} \times r_{B/A}$

$= 5\hat{k} \times (0.6\cos 30^\circ\,\hat{i} + 0.6\sin 30^\circ\,\hat{j})$

$= 2.598\,\hat{j} - 1.5\,\hat{i}$





$x = 0.6\cos\theta + 0.3\cos\phi$ $\quad \theta = 30^\circ \to \phi = 30^\circ$

$0.6\sin\theta = 0.3\sin\phi + 0.15$

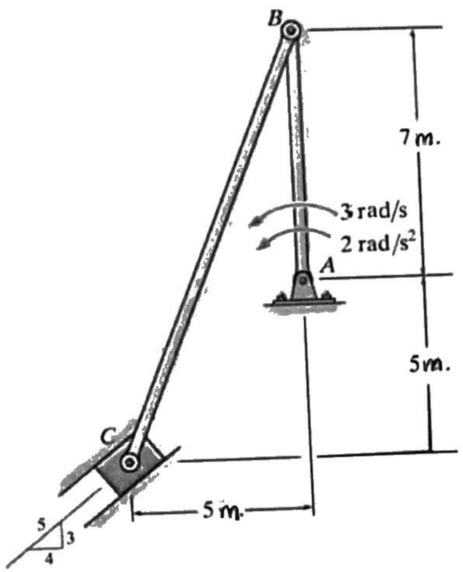
$v_C = v_B + \omega_{BC} \times r_{C/B}$

$-v_C\,\hat{i} = -1.5\,\hat{i} + 2.598\,\hat{j} + (-\omega_{BC}\hat{k}) \times (0.3\cos 30^\circ\,\hat{i} - 0.3\sin 30^\circ\,\hat{j})$

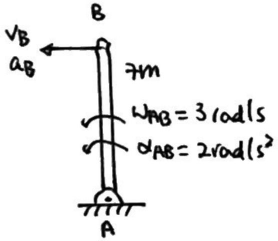
$= -1.5\,\hat{i} + 2.598\,\hat{j} - 0.26\,\omega_{BC}\,\hat{j} - 0.15\,\omega_{BC}\,\hat{i}$

$\to \begin{cases} -v_C = -1.5 - 0.15\,\omega_{BC} \\ 0 = 2.598 - 0.26\,\omega_{BC} \end{cases} \to \begin{cases} \omega_{BC} = 10 \text{ rad/s} \\ \underline{\underline{v_C = 3 \text{ m/s}}} \end{cases}$

***16–116.** At the given instant member AB has the angular motions shown. Determine the velocity and acceleration of the slider block C at this instant.



Prob. 16–116

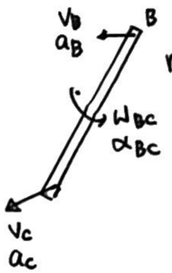


$v_B = -3 \times 7 = -21\,\hat{i}$

$a_B = \cancel{a_A} + \alpha_{AB} \times r_{B/A} - \omega_{AB}^2\, r_{B/A}$

$= 2\hat{k} \times (7\hat{j}) - 9(7\hat{j}) = -14\,\hat{i} - 63\,\hat{j}$

$v_C = v_B + \omega_{BC} \times r_{C/B}$



$r_{C/B} = -5\hat{i} - 12\hat{j}$

$-0.8\,v_C\,\hat{i} - 0.6\,v_C\,\hat{j} = -21\,\hat{i} + \omega_{BC}\,\hat{k} \times (-5\hat{i} - 12\hat{j})$

$= (-21 + 12\,\omega_{BC})\,\hat{i} + (-5\,\omega_{BC})\,\hat{j}$

$\to \begin{cases} -0.8\,v_C = -21 + 12\,\omega_{BC} \\ -0.6\,v_C = -5\,\omega_{BC} \end{cases}$ $\quad v_C = 9.375$ m/s, $\omega_{BC} = 1.125$ rad/s

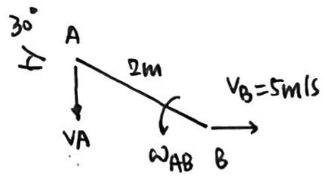
$a_C = a_B + \alpha_{BC} \times r_{C/B} - \omega_{BC}^2\, r_{C/B}$

$-0.8\,a_C\,\hat{i} - 0.6\,a_C\,\hat{j} = -14\,\hat{i} - 63\,\hat{j} + \alpha_{BC}\,\hat{k} \times (-5\hat{i} - 12\hat{j}) - 1.125^2(-5\hat{i} - 12\hat{j})$

$= -14\,\hat{i} - 63\,\hat{j} + 12\,\alpha_{BC}\,\hat{i} - 5\,\alpha_{BC}\,\hat{j} + 6.33\,\hat{i} + 13.5\,\hat{j}$

$\begin{cases} -0.8\,a_C = -14 + 12\,\alpha_{BC} + 6.33 \\ -0.6\,a_C = -63 - 5\,\alpha_{BC} + 13.5 \end{cases} \to \begin{cases} a_C = 56.46 \text{ m/s}^2 \\ \alpha_{BC} = -3.12 \text{ rad/s}^2 \end{cases}$

*16–120. The slider block moves with a velocity of $v_B = 5$ ft/s and an acceleration of $a_B = 3$ ft/s². Determine the acceleration of A at the instant shown.

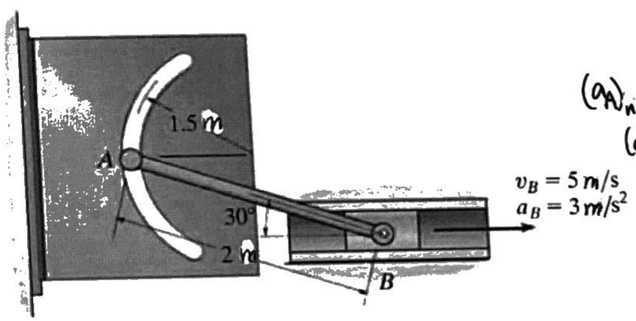


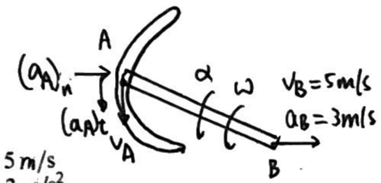
$V_A = V_B + \omega_{AB} \times r_{A/B}$

$-V_A \hat{j} = 5\hat{i} + \omega_{AB}\hat{k} \times (-2\cos 30^\circ \hat{i} + 2\sin 30^\circ \hat{j})$

$-V_A \hat{j} = 5\hat{i} + -2\sin 30^\circ \omega_{AB}\hat{i} - 2\cos 30^\circ \omega_{AB}\hat{j}$

$\begin{cases} 0 = 5 - 2\sin 30^\circ \omega_{AB} \\ -V_A = -2\cos 30^\circ \omega_{AB} \end{cases} \rightarrow \begin{cases} \omega_{AB} = 5 \text{ rad/s} \\ V_A = 8.66 \text{ m/s} \end{cases} \rightarrow (a_A)_n = \frac{v_A^2}{r} = \frac{8.66^2}{1.5} = 50 \text{ m/s}^2$





$a_A = a_B + \alpha_{AB} \times r_{A/B} - \omega^2 r_{A/B}$

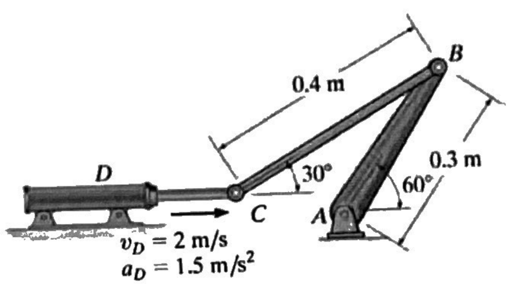
$50\hat{i} - (a_A)_t\hat{j} = +\hat{i} + \alpha_{AB}\hat{k} \times (-2\cos 30^\circ \hat{i} + 2\sin 30^\circ \hat{j})$
$- 5^2(-2\cos 30^\circ \hat{i} + 2\sin 30^\circ \hat{j})$

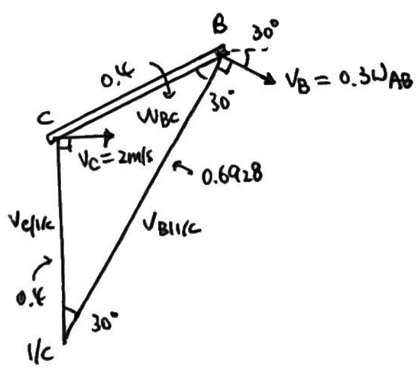
$50\hat{i} - (a_A)_t\hat{j} = 3\hat{i} - \alpha_{AB}\hat{i} + 25\sqrt{3}\hat{i} - \sqrt{3}\alpha_{AB}\hat{j} - 25\hat{j}$

$\begin{cases} 47 = -\alpha_{AB} + 25\sqrt{3} \\ -(a_A)_t = -\sqrt{3}\alpha_{AB} - 25 \end{cases} \rightarrow \begin{cases} \alpha_{AB} = [illegible] \text{ rad/s}^2 \\ (a_A)_t = 18.59 \text{ m/s}^2 \end{cases}$

$\rightarrow a_A = \sqrt{18.59^2 + 50^2} = 53.3 \text{ m/s}^2$

•16–125. The hydraulic cylinder is extending with the velocity and acceleration shown. Determine the angular acceleration of crank AB and link BC at the instant shown.





$\omega_{BC} = \frac{V_C}{r_{C/IC}} = \frac{2}{0.4} = 5 \text{ rad/s}$

$V_B = r_{B/IC} \cdot \omega_{BC} = 0.6928 \cdot 5$
$= 3.464 \text{ m/s}$
$\omega_{AB} = 3.464 / 0.3 = 11.547 \text{ rad/s}$

$a_B = \cancel{a_A} + \alpha_{AB} \times r_{B/A} - \omega_{AB}^2 r_{B/A}$ $\quad r_{B/A} = 0.15\hat{i} + 0.15\sqrt{3}\hat{j}$

$= -\alpha_{AB}\hat{k} \times (0.15\hat{i} + 0.15\sqrt{3}\hat{j}) - 133.3(0.15\hat{i} + 0.15\sqrt{3}\hat{j})$

$= (0.15\sqrt{3}\alpha_{AB} - 20)\hat{i} + (-0.15\alpha_{AB} - 34.63)\hat{j}$

$a_B = a_C + \alpha_{BC} \times r_{B/C} - \omega_{BC}^2 r_{B/C}$ $\quad r_{B/C} = 0.2\sqrt{3}\hat{i} + 0.2\hat{j}$

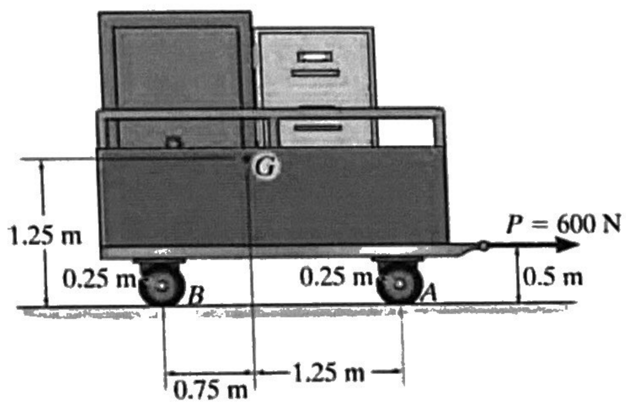
$= 1.5\hat{i} + \alpha_{BC}\hat{k} \times (0.2\sqrt{3}\hat{i} + 0.2\hat{j}) - 25(0.2\sqrt{3}\hat{i} + 0.2\hat{j})$

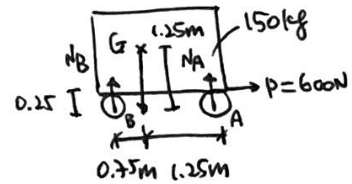
$= (1.5 + -0.2\alpha_{BC} - 5\sqrt{3})\hat{i} + (0.2\sqrt{3}\alpha_{BC} - 5)\hat{j}$

$\rightarrow \begin{cases} 0.15\sqrt{3}\alpha_{AB} - 20 = -0.2\alpha_{BC} + 7.160 \\ -(0.15\alpha_{AB} + 34.63) = 0.2\sqrt{3}\alpha_{BC} - 5 \end{cases} \rightarrow \begin{cases} \alpha_{AB} = -160.4 \text{ rad/s} \\ \alpha_{BC} = 172.93 \text{ rad/s} \end{cases}$

2. Planar Kinetics

17–51. The trailer with its load has a mass of 150 kg and a center of mass at G. If it is subjected to a horizontal force of $P = 600$ N, determine the trailer's acceleration and the normal force on the pair of wheels at A and at B. The wheels are free to roll and have negligible mass.





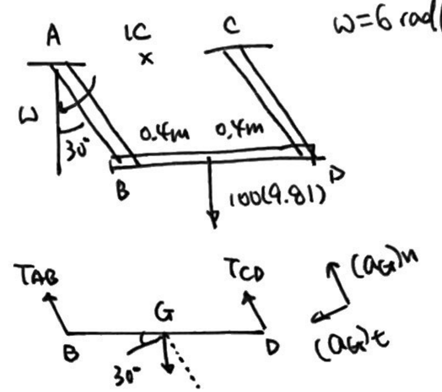
$\xrightarrow{+} \sum F_x = ma_x : 600 = 150a_x \rightarrow a_x = 4\ m/s^2$

$ma_{net}\ r_{G/A}$

$\circlearrowleft+ \sum M_A = \sum(M_k)_A : -N_B(2\ ) + 150(9.81)(1.25) - 600(0.25) = -150(4)(1.25)$

$N_B = 1.219\ kN$

$+\uparrow \sum F_y = ma_y : N_B + N_A = 150(9.81) \rightarrow N_A = \underline{\underline{252.5\ N}}$



$\omega = 6\ rad/s$

$\sum F_t = ma_t : \quad 981 \sin 30° = 100\, a_t$

$= 100\,\alpha\,(0.5) \quad r_{G/IC} \quad \rightarrow \alpha = 9.81\ rad/s^2$

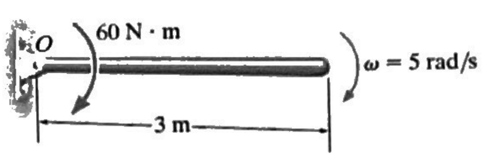
$\sum F_n = ma_n : \quad T_{AB} + T_{CD} - 981 \cos 30° = 100(6)^2(0.5)$

$T_{AB} + T_{CD} = 2649.57\ N$

$\sum M_G = 0 : \quad T_{AB} \cos 30° \cdot 0.4 = T_{CD} \cos 30° \cdot 0.4 \rightarrow T_{BC} = T_{CD}$

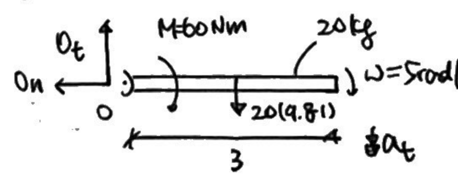
no rotation about only translation $\rightarrow T_{AB} = T_{CD} = \underline{\underline{1.325\ kN}}$

At the instant shown in Fig. 17–15*a*, the 20-kg slender rod has an angular velocity of $\omega = 5$ rad/s. Determine the angular acceleration and the horizontal and vertical components of reaction of the pin on the rod at this instant.



* $I_G = \frac{1}{12} ml^2$

$I_O = \frac{1}{3} ml^2$



Consider rotation at O:

$\sum F_t = ma_t = m\alpha r : \quad -O_t + 20(9.81) = 20\alpha(1.5)$

$\sum F_n = ma_n = m\omega^2 r : \quad O_n = 20(5)^2(1.5) = 750\ N$

$\sum M_O = I_O \alpha : \quad 60 + 20(9.81)(1.5) = \underbrace{\frac{1}{3}(20)(3)^2}_{I_O} \cdot \alpha$

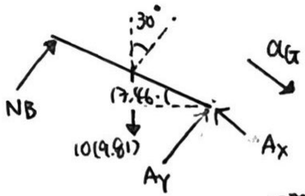
$\alpha = 5.905\ rad/s^2$

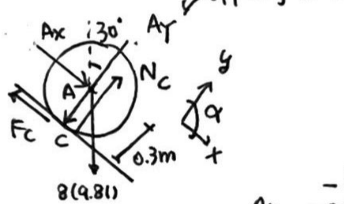
$O_t = -[20(5.905)(1.5) - 20(9.81)] = \underline{\underline{19.05\ N}}$

112

***17–112.** The assembly consists of an 8-kg disk and a 10-kg bar which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s = 0.6$ and $\mu_k = 0.4$, respectively. Neglect friction at B.

•17–113. Solve Prob. 17–112 if the bar is removed. The coefficients of static and kinetic friction between the disk and inclined plane are $\mu_s = 0.15$ and $\mu_k = 0.1$, respectively.





At disk: $\overset{+}{\searrow} \Sigma F_x = ma_G$: $A_x - \overset{F_c}{\cancel{0.6N_c}} + 8(9.81)\sin 30° = 8a_G = 8(0.3)\alpha$

$+\nearrow \Sigma F_y = 0$: $A_y = N_c - 8(9.81)\cos 30°$

$+\circlearrowleft \Sigma M_A = I_A\alpha$: $F_c(0.3) = \frac{1}{2}(8)(0.3)^2 \cdot \alpha$

At bar: $\overset{+}{\searrow} \Sigma F_x = ma_G$: $10(9.81)\sin 30° - A_x = 10a_G = 10(0.3)\alpha$

$+\nearrow \Sigma F_y = 0$: $N_B + A_y = 10(9.81)\cos 30°$

$\circlearrowleft \Sigma M_G = 0$: $N_B(0.5\cos 17.46) - A_x(0.5\sin 17.46) - A_y(0.5\cos 17.46) = 0$

$A_x - \overset{-F_c}{\cancel{0.6N_c}} - 2.4\alpha = -39.24 \rightarrow A_x = 3.6\alpha - 39.24 \rightarrow A_x = 8.928\,N$

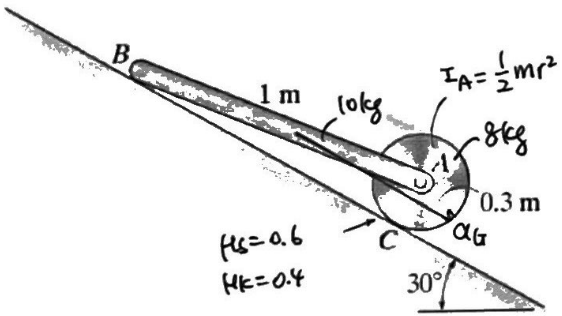
$A_y - N_c = -67.97 \qquad \rightarrow N_c = 109.04\,N.$

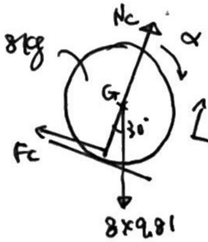
$F_c = 1.2\alpha \qquad \rightarrow F_c = 16.05\,N \qquad \frac{F_c}{N_c} = \frac{16.05}{109.04} = 0.147$ → not fully used μ_s

$A_x + 3\alpha = 49.05 \rightarrow \alpha = 13.38\ rad/s^2$

$A_y + N_B = 84.95$

$N_B - 0.314A_x - A_y = 0 \rightarrow N_B - A_y = 2.803$ $\Big\} \rightarrow A_y = 41.07\,N$, $N_B = 43.87\,N$





$\Sigma F_x = ma_x$: $-F_c + 8(9.81)(\sin 30°) = 8a_G = 8(0.3)\alpha \rightarrow F_c + 2.4\alpha = 39.24$

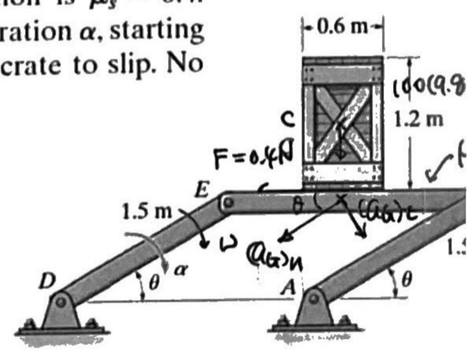
$\Sigma F_y = ma_y$: $N_c = 8(9.81)(\cos 30°) \qquad \underbrace{0.3\alpha = a_G}_{\rightarrow \text{no slip } X}$ $\qquad F_c = 13.08\ N$

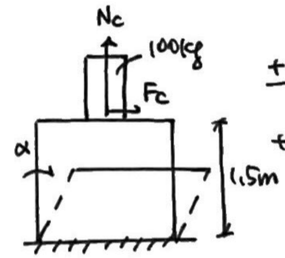
$\Sigma M_G = I_G\alpha$: $F_c \cdot 0.3 = \frac{1}{2}(8)(0.3)^2\alpha \rightarrow F_c = 1.2\alpha \qquad \alpha = 10.9\ rad/s^2$

$N_c = 67.97\,N$

Put $F_c = 0.1N_c \rightarrow a_G = 4.06\ m/s^2 \quad \alpha = 5.66\ rad/s^2 \qquad \frac{F_c}{N_c} = 0.19 > 0.15.$

17–55. The 100-kg uniform crate C rests on the elevator floor where the coefficient of static friction is $\mu_s = 0.4$. Determine the largest initial angular acceleration α, starting from rest at $\theta = 90°$, without causing the crate to slip. No tipping occurs.

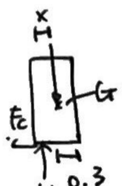




$\overset{+}{\rightarrow} \Sigma F_x = ma_x$: $F_c = 100(1.5)\alpha = 150\alpha = \mu_s N_c$

$+\uparrow \Sigma F_y = ma_y$: $N_c - 100(9.81) = 100(1.5)\omega^2$

$N_c = 981$



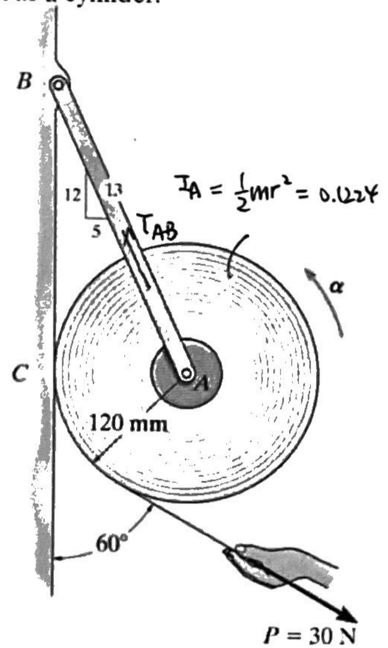
$\Sigma M_G = 0$: $\qquad F_c(0.6) = N_c(x) = 981x$

at static friction: $150\alpha = 0.4(981) \rightarrow \alpha = 2.616\ rad/s$

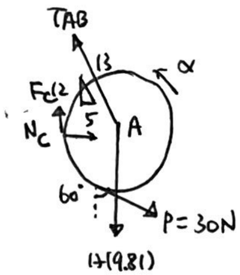
$\rightarrow 0.4(981)(0.6) = 981x \rightarrow x = 0.24m$

as $x = 0.24 < 0.3 \rightarrow$ it intends to slide before it tips

•17–89. A 17-kg roll of paper, originally at rest, is supported by bracket AB. If the roll rests against a wall where the coefficient of kinetic friction is $\mu_C = 0.3$, and a constant force of 30 N is applied to the end of the sheet, determine the tension in the bracket as the paper unwraps, and the angular acceleration of the roll. For the calculation, treat the roll as a cylinder.



Prob. 17–89



$\Sigma F_x = m(a_G)_x$: $30\sin 60° - T_{AB}\frac{5}{13} + N_C = 0$ kinetic friction

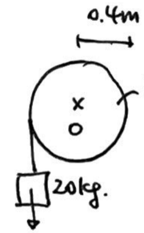
$\Sigma F_y = m(a_G)_y$: $T_{AB}\cdot\frac{12}{13} - 17(9.81) - 30\,\cancel{\sin}\cos 60° + 0.3N_C = 0$

$\Sigma M_A = I\alpha$: $30(0.12) - 0.3N_C(0.12) = 0.1224\,\alpha$

$N_C - \frac{5}{13}T_{AB} = -25.98$; $N_C = 44.23$ N

$0.3N_C + \frac{12}{13}T_{AB} = 181.77$; $T_{AB} = 182.5$ N

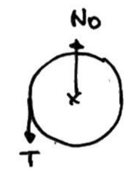
$\alpha = 16.4$ rad/s



$I_o = mk_o = 60\times 0.25$
$= 15.$

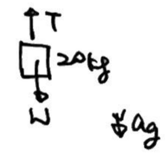
$\Sigma M_o = I_o\alpha$: $20(9.81)(0.4) = (15 + 20\times 0.4^2)\cdot\alpha$

$\alpha = 4.31$ rad/s



Block: $20(9.81) - T = 20a_g = 20(0.4)\alpha$

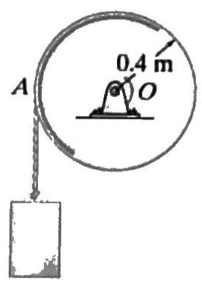
drum: $\Sigma M_o = I_o\alpha$: $T(0.4) = (15)\alpha$



$20(9.81) - \frac{15\alpha}{0.4} = 20(0.4)\alpha$

$20(9.81)(0.4) = (20(0.4)^2 + 15)\alpha$

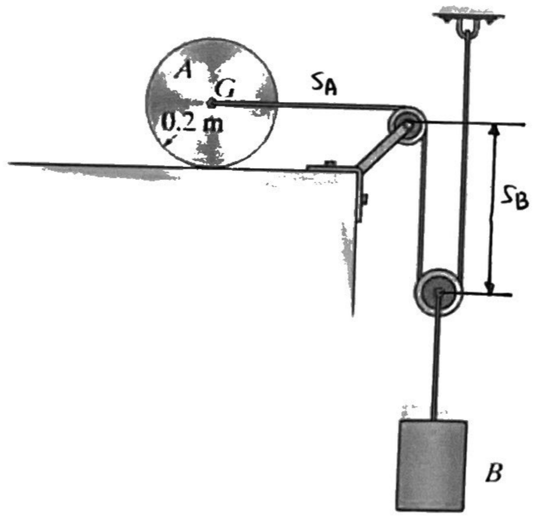
$\alpha = 4.31$ rad/s



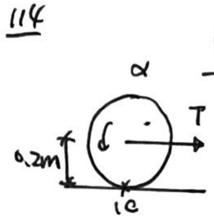
The drum shown in Fig. 17–16a has a mass of 60 kg and a radius of gyration $k_O = 0.25$ m. A cord of negligible mass is wrapped around the periphery of the drum and attached to a block having a mass of 20 kg. If the block is released, determine the drum's angular acceleration.

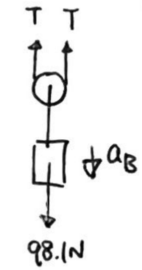
17–114. The 20-kg disk A is attached to the 10-kg block B using the cable and pulley system shown. If the disk rolls without slipping, determine its angular acceleration and the acceleration of the block when they are released. Also, what is the tension in the cable? Neglect the mass of the pulleys.

17–115. Determine the minimum coefficient of static friction between the disk and the surface in Prob. 17–114 so that the disk will roll without slipping. Neglect the mass of the pulleys.



114





Disk: $T(0.2) = (\underbrace{\tfrac{1}{2}(20)(0.2)^2}_{\frac{1}{2}mr^2} + \underbrace{20(0.2)^2}_{md^2})\alpha$ $\quad T = 6\alpha$

Block: $98.1 - 2T = 10a_B$

no slip → $a_A = 0.2\alpha$

→ $a_B = -0.1\alpha$

$\begin{cases} T = 6\alpha \\ 98.1 - 2T = -\alpha \end{cases} \rightarrow \begin{cases} \alpha = 7.546 \text{ rad/s} \\ T = 1.258\text{ N.} \end{cases} \rightarrow \begin{cases} a_A = 1.51 \text{ m/s}^2 \\ a_B = -0.755 \text{ m/s}^2 \end{cases}$

$S_A + 2S_B = \ell$

$\downarrow \frac{d^2}{dt^2}$

$a_A = -2a_B$

115 minimum coeff. disk:

$\Sigma F_x = ma_x: \; T - F_c = 20a_A$

$\Sigma F_y = ma_y: \; 20(9.81) = N_c = 196.2\text{N}$

$\Sigma M_{IC} = I_{IC}\alpha: \; T(0.2) = -(\tfrac{1}{2}(20)(0.2)^2 + 20(0.2)^2)\alpha$

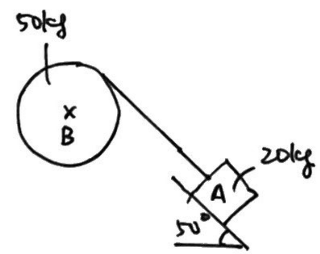
$\rightarrow T = 6\alpha$

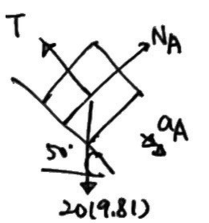
Block: $98.1 - 2T = 10a_B$

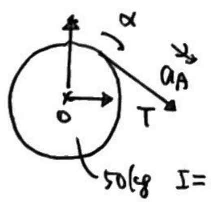
→ $a_B = 0.755 \text{ m/s}^2$, $\alpha = -7.55 \text{ rad/s}^2$, $T = 45.3\text{N}$, $a_A = 1.51\text{ m/s}^2$

$T - F_c = 20a_A \rightarrow F_c = 15.09\text{N}$

→ $\frac{F_c}{N_c} = \frac{15.09}{196.2} = 0.0769 = \mu_{min}$







$I = \frac{1}{2}(50)(0.5)^2 = 6.25$

17-47 A rope is wrapped around the uniform 50-kg pulley B and attached to the 20-kg block A. If the system is released from rest, find (a) the initial acceleration of A; and (b) the velocity of A after it has moved 1 m down the incline. Neglect friction.

Block: $\Sigma F_t = ma_A: 20(9.81)\sin 50° - T = 20a_A = 20(0.5)\alpha = 10\alpha$

$\Sigma F_N = 0: 20(9.81)\cos 50° = N_A$

$* \; 150 = T + 10\alpha$

Pulley: $\Sigma M_O = I_O\alpha$:

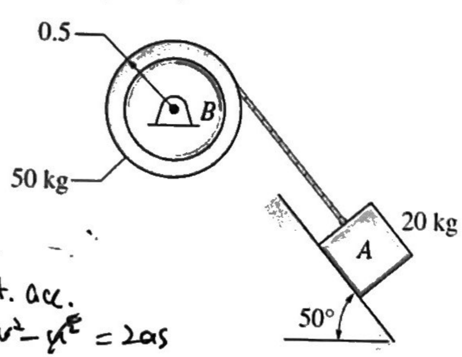
$T(0.5) = 6.25\alpha$

$T = 12.5\alpha$

$\alpha = 6.67$ rad/s

$T = 83.3$N

$a_A = 6.67 \times 0.5 = 3.33 \text{ m/s}^2$



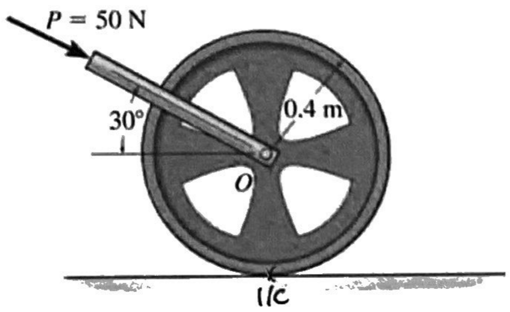
const. acc.

$v^2 - \cancel{v_0^2} = 2as$

$v = \sqrt{2(3.33)(1)} = \underline{2.58 \text{ m/s.}}$

3. Work and Energy

F18–4. The 50-kg wheel is subjected to a force of 50 N. If the wheel starts from rest and rolls without slipping, determine its angular velocity after it has rotated 10 revolutions. The radius of gyration of the wheel about its mass center O is $k_O = 0.3$ m.



$I_o = mk_o = 50(0.3)^2 = 4.5.$

$\{\cancel{KE_1}\}^{0} + \{U_{1-2}\} = \{KE_2\}$

from rest

$\{KE_2\} = \frac{1}{2}mv_o^2 + \frac{1}{2}I_o\omega^2 \quad (\text{or } \frac{1}{2}I_{IC}\omega^2)$

$= \frac{1}{2}(50 \times 0.4^2 + 4.5)\omega^2$

$= 625\omega^2$

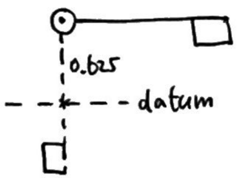
$\{U_{1-2}\} = P\cos\theta \cdot s \xrightarrow{} r\theta$

$= 50\cos 30° \cdot 0.4 \cdot \overbrace{10 \cdot 2\pi}^{\theta} = 1088.3\text{ J}$

$\rightarrow 1088.3 = 625\omega^2$

$\omega = 1.319$ rad/s

•18–5. The pendulum of the Charpy impact machine has a mass of 50 kg and a radius of gyration of $k_A = 1.75$ m. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.



$\{KE_1\} = 0$

$\{PE_1\} = 50(9.81)\overset{1.25}{\cancel{(0.625)}}$

$= \cancel{306.5625}$ 613.125

$\{KE_2\} = \frac{1}{2}I_A\omega^2 = \frac{50(1.75)^2}{2}\omega^2$

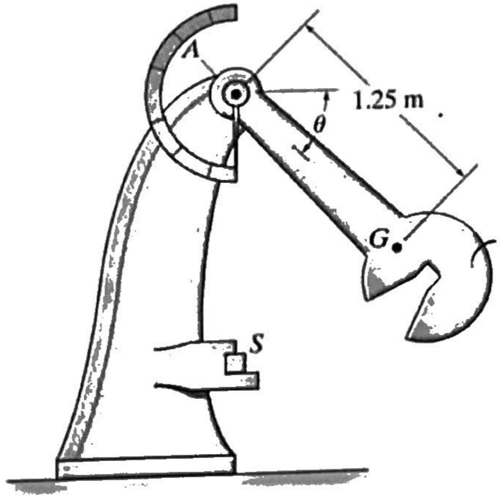
$= 76.5625\omega^2$

$\{PE_2\} = 0$

$\{KE_1\} + \{PE_1\} = \{KE_2\} + \{PE_2\}$

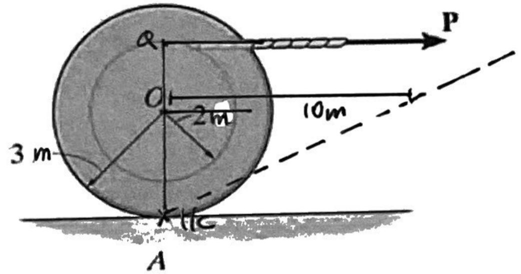
$\underset{613.125}{\cancel{306.5625}} = 76.5625\omega^2$

~~ω = 2 rad/s~~ $\omega = 2.83$ rad/s



50kg, $k_A = 1.75$ m.

•18–9. The spool has a weight of 150 kg and a radius of gyration $k_O = 2.25$ m. If a cord is wrapped around its inner core and the end is pulled with a horizontal force of $P = 40$ N, determine the angular velocity of the spool after the center O has moved 10 m to the right. The spool starts from rest and does not slip at A as it rolls. Neglect the mass of the cord.



$I_O = mk_O^2 = 150\text{kg}\cdot(2.25\text{m})^2 = 759.4$

$\{\cancel{KE_1}\} + \{U_{1-2}\} = \{KE_2\}$

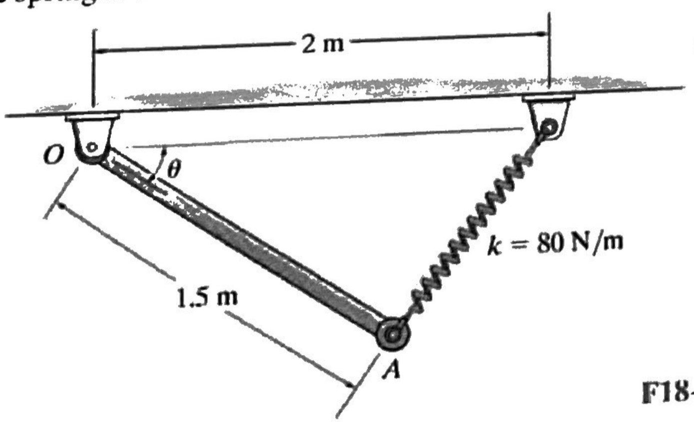
$P\Delta s = \frac{1}{2} I_O \omega^2$

$40(16.67) = \frac{1}{2}(759.4)\omega^2$

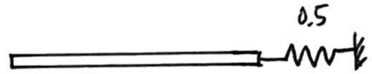
$\omega = 0.75$ rad/s

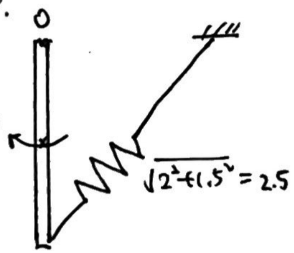
$* \ \Delta s|_{\text{w.r.t. } Q} = \frac{10}{3} \times 5 = 16.67$ m

F18–10. The 30-kg rod is released from rest when $\theta = 0°$. Determine the angular velocity of the rod when $\theta = 90°$. The spring is unstretched when $\theta = 0°$.



F18–10





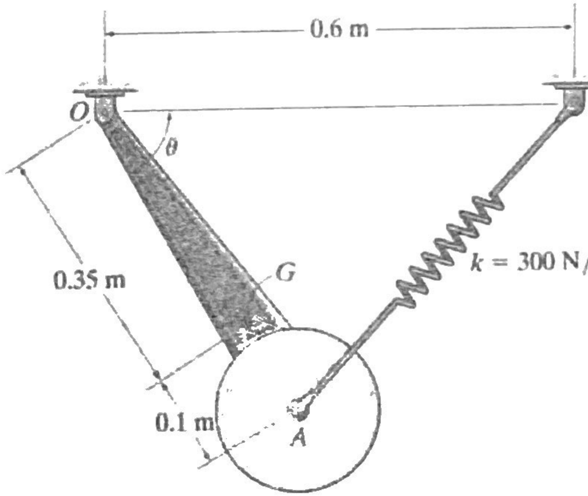
$\{\cancel{KE_1}\} + \{U_{1-2}\} = \{KE_2\}$

$-\frac{1}{2}(80)(\underset{-0.5}{2.5})^2 + (30)(9.81)(1.5/2) = \frac{1}{2}\overbrace{(\frac{1}{3}(30)(1.5)^2)}^{11.25}\omega^2$

$60.725 = 11.25\omega^2$

$\omega = 2.32$ rad/s

18–51. The 30 kg pendulum has its mass center at G and a radius of gyration about point G of $k_G = 300$ mm. If it is released from rest when $\theta = 0°$, determine its angular velocity at the instant $\theta = 90°$. Spring AB has a stiffness of $k = 300$ N/m and is unstretched when $\theta = 0°$.

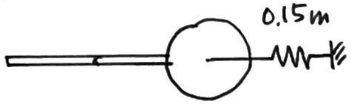


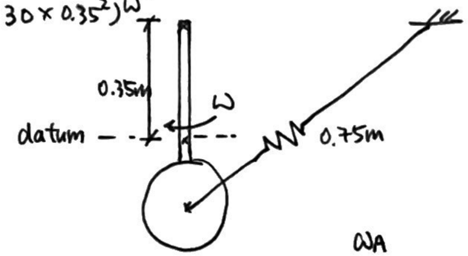
$\{\cancel{KE_1}\} + \{U_{1-2}\} = \{KE_2\}$

$30(9.81)(0.35) - \frac{1}{2}(300)(0.75-0.15)^2 = \frac{1}{2}(30\times 0.3^2 + 30\times 0.35^2)\omega^2$

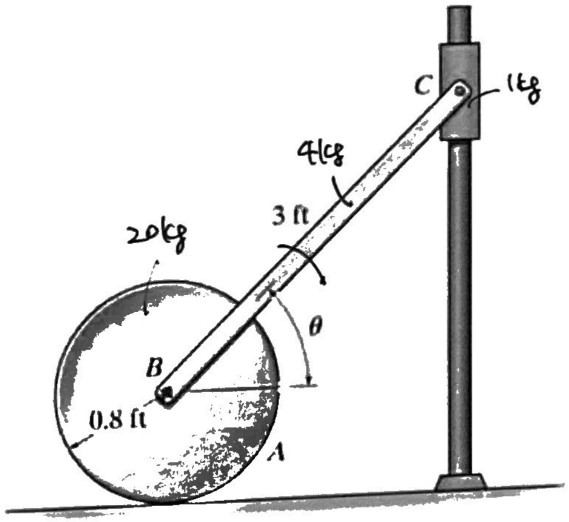
$49.005 = 3.1875\,\omega^2$

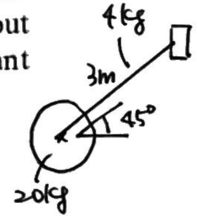
$\omega = 3.92$ rad/s

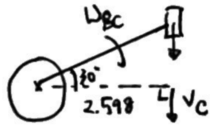




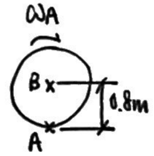
18–46. The system consists of a 20-kg disk A, 4-kg slender rod BC, and a 1-kg smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 30°$. The system is released from rest when $\theta = 45°$.





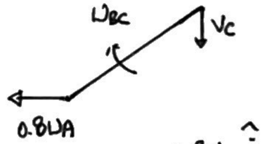


$v_C = 2.598\,\omega_{BC}$



$v_B = 0.8\,\omega_A$

$\omega_{BC} = v_C/2.598 = 0.385\,v_C$



$-0.8\omega_A\,\hat{i} = -v_C\,\hat{j} + (-\omega_{BC}\,\hat{k})\times(-3\cos30°\,\hat{i} + 3\sin30°\,\hat{j})$

$= -v_C\,\hat{j} + 1.5\omega_{BC}\,\hat{i} + 1.5\sqrt{2}\,\omega_{BC}\,\hat{i}$

$\rightarrow \begin{cases} -0.8\omega_A = 1.5\omega_{BC} \\ v_C = 1.5\sqrt{2}\,\omega_{BC} \end{cases} \rightarrow \begin{array}{l} v_C = \\ \omega_A = 1.875\,\omega_{BC} \\ v_A = 1.5\,\omega_{BC} \end{array}$

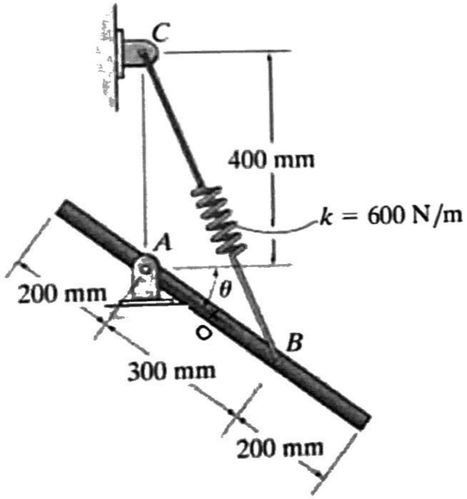
$\overset{1.5}{4}(\sin45° - \sin30°)(9.81) + 1(\underset{-3}{(\sin45° - \sin30°)}(9.81) = \frac{1}{2}\underbrace{\left(\frac{1}{2}(20)(0.8m)^2\right)}_{I_O = \frac{1}{2}mr^2 \text{ disk}}\underbrace{(1.875\,\omega_{BC})^2}_{\omega_A^2}$

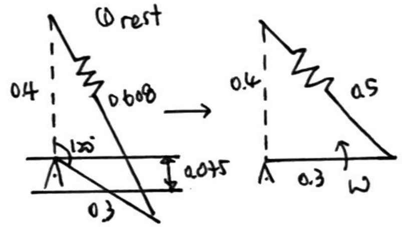
$+ \underbrace{\frac{1}{2}(20)(1.5\,\omega_{BC})^2}_{\text{disk } \frac{1}{2}mv^2} + \underbrace{\frac{1}{2}\left(\frac{1}{3}(4)(\cancel{3})^2\right)\omega_{BC}^2}_{I_B = \frac{1}{3}ml^2 \text{ rod}}$

$+ \underbrace{\frac{1}{2}(1)(2.598\,\omega_{BC})^2}_{\frac{1}{2}mv_c^2 \text{ collar.}}$

$\omega_{BC} = 1.536$ rad/s

18–54. If the 6-kg rod is released from rest at $\theta = 30°$, determine the angular velocity of the rod at the instant $\theta = 0°$. The attached spring has a stiffness of $k = 600$ N/m, with an unstretched length of 300 mm.





$I_O = \frac{1}{12}(6)(0.7)^2$

$= 0.245$

$I_A = 0.245 + (6)(0.15)^2$

$= 0.38$

28.510 − 4.4145

$\{\cancel{KE_1}\} + \{U_{1-2}\} = \{KE_2\}$

16.459

$-0.075(6)(9.81) - \frac{1}{2}(600)((0.5-0.3)^2 - (0.608-0.3)^2)$

4.4145 28.4592

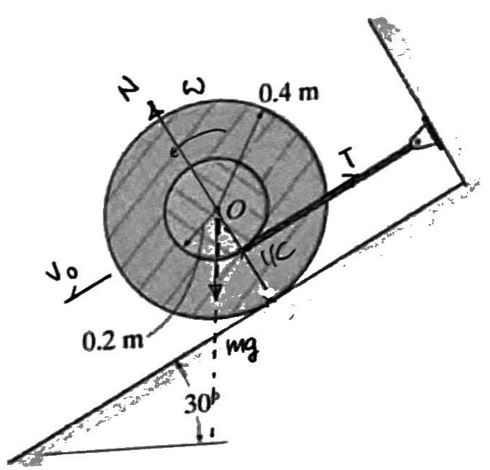
$= \frac{1}{2}(0.38)(\omega)^2$

612

$\omega =$ rad/s

7.98

F18–8. The 50-kg reel has a radius of gyration about its center O of $k_O = 300$ mm. If it is released from rest, determine its angular velocity when its center O has traveled 6 m down the smooth inclined plane.



$v_O = \omega r_{O/IC} = 0.2\omega$

$I_{IC} = (50)(0.3)^2 + (50)(0.2)^2 = 6.5$

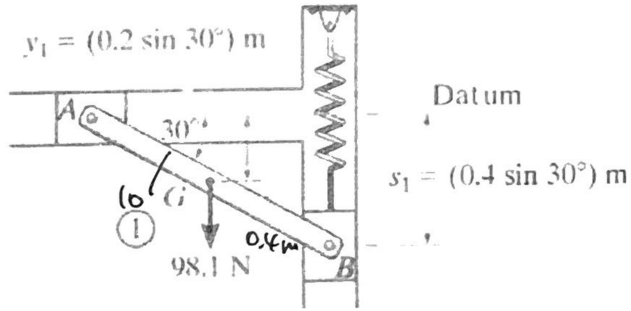
rest $\{\cancel{KE_1}\} + \{PE_1\} = \{KE_2\} + \{\cancel{PE_2}\}$

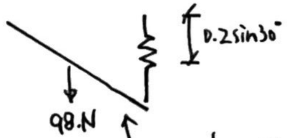
$(50)(9.81)(6\sin 30°) = \frac{1}{2}(6.5)\omega_0^2 + 0$

$1471.5 = \frac{1}{2}(6.5)v_0^2$

$\omega_0 = 21.278$ rad/s

The 10-kg rod AB shown in Fig. 18–18a is confined so that its ends move in the horizontal and vertical slots. The spring has a stiffness of $k = 800$ N/m and is unstretched when $\theta = 0°$. Determine the angular velocity of AB when $\theta = 0°$, if the rod is released from rest when $\theta = 30°$. Neglect the mass of the slider blocks.





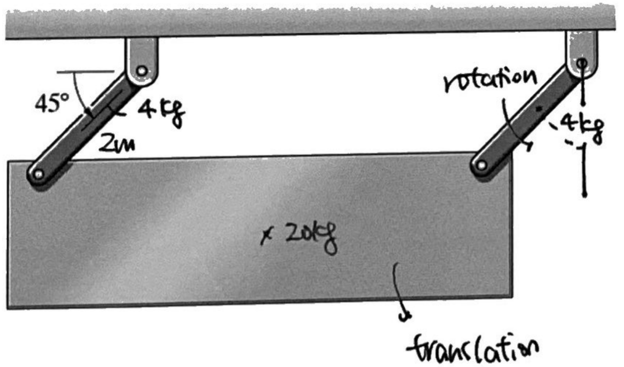
$I_A = \frac{1}{3}(10)(0.4)^2$

$\{KE_1\} + \{U_{1-2}\} = \{KE_2\}$

$$\underbrace{-(10)(9.81)(0.2\sin 30°)}_{-6.19} - \overbrace{\tfrac{1}{2}(800)(0^2-(0.4\sin 30°)^2)}^{+16} = \overbrace{\tfrac{1}{2}(\underbrace{\tfrac{1}{3}(10)(0.4)^2}_{I_A})}^{0.2667}\omega^2$$

$\omega = 6.06$ rad/s

19.33 The 2-ft slender bars each weigh 4 lb, and the rectangular plate weighs 20 lb. If the system is released from rest in the position shown, what is the velocity of the plate when the bars are vertical?



$2 - 2\cos 45°$

$\{KE_1\} = 0$

$$\{KE_2\} = \underbrace{\tfrac{1}{2}(\tfrac{1}{3}(4)(2)^2)\omega^2 \times 2}_{\text{bar}} + \underbrace{\tfrac{1}{2}(20)(\overbrace{2\omega}^{v})^2}_{\text{plate}} = 45.33\omega^2$$

$\{PE_1\} = (4+4+20)(9.81)(2-2\cos 45°) = 160.9$

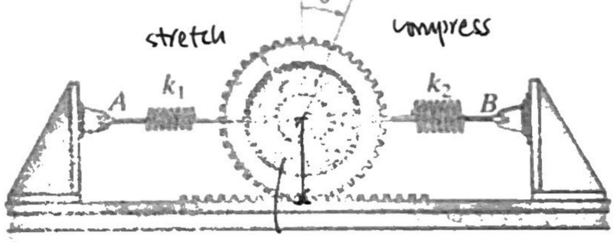
$\{PE_2\} = 0$

$45.33\omega^2 = 160.9$

$\omega = 1.88$ rad/s

4. Oscillations and Vibrations

***22–40.** The gear of mass m has a radius of gyration about its center of mass O of k_O. The springs have stiffnesses of k_1 and k_2, respectively, and both springs are unstretched when the gear is in an equilibrium position. If the gear is given a small angular displacement of θ and released, determine its natural period of oscillation.



elastic energy: $U_s = \frac{1}{2}k_1(r\theta)^2 + \frac{1}{2}k_2(r\theta)^2 = \frac{1}{2}(k_1+k_2)r^2\theta^2$

kinetic energy: $KE = \frac{1}{2}I_{IC}\omega^2 = \frac{1}{2}m(k_O^2+r^2)\dot{\theta}^2$

$I_O = mk_O^2$

$I_{IC} = mk_O^2 + mr^2$

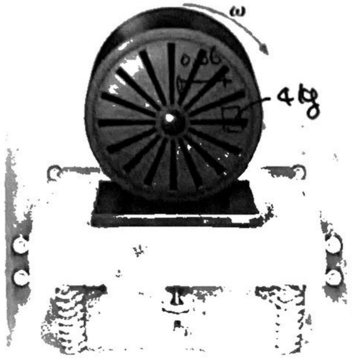
$KE+PE = \frac{1}{2}(k_1+k_2)r^2\theta^2 + \frac{1}{2}m(k_O^2+r^2)\dot{\theta}^2$

$\frac{d}{dt}$

$(k_1+k_2)r^2\theta\cancel{\dot{\theta}} + m(k_O^2+r^2)\cancel{\dot{\theta}}\ddot{\theta} = 0 \rightarrow \ddot{\theta} + \frac{(k_1+k_2)r^2}{m(k_O^2+r^2)}\theta = 0 \rightarrow \omega_n = \frac{r\sqrt{k_1+k_2}}{\sqrt{m(k_O^2+r^2)}}$

$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi\sqrt{m(k_O^2+r^2)}}{r\sqrt{k_1+k_2}}$

The 30-kg electric motor shown in Fig. 22–18 is supported by *four* springs, each spring having a stiffness of 200 N/m. If the rotor is unbalanced such that its effect is equivalent to a 4-kg mass located 60 mm from the axis of rotation, determine the amplitude of vibration when the rotor is turning at $\omega_0 = 10$ rad/s. The damping factor is $c/c_c = 0.15$.



$k = 200\,N/m \times 4$

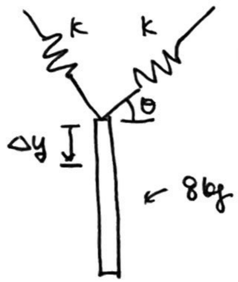
* $F_0 = ma_n = mr\omega^2 = (4)(0.06)(10)^2 = 24$

* $\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{200(4)}{30}} = 5.16$

* $x' = \frac{F_0/k}{\sqrt{(1-(\omega_0/\omega_n)^2)^2 + (2(c/c_c)(\omega_0/\omega_n))^2}} = \frac{24/4\times200}{\sqrt{(1-10/5.16)^2 + (2(0.15)(10/5.16))^2}}$

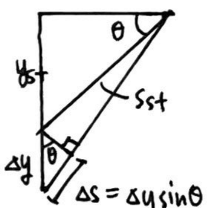
$= 0.0107 = 10.7\,mm$

spring → energy.



$k = 40N/m$

$S_{st} =$



Force Balance :

$mg = 2k \cdot S_{st} \cdot \sin\theta$

$\downarrow \quad S_{st} = \frac{mg}{2k\sin\theta}$

$$E = 2\left(\tfrac{1}{2}k(S_{st} + y\sin\theta)^2\right) - mgy + \tfrac{1}{2}m\dot{y}^2$$

$$\frac{dE}{dt} = 0 \rightarrow 2k(S_{st} + y\sin\theta)\cdot\sin\theta\,\dot{y} - mg\dot{y} + \tfrac{1}{2}m(2\dot{y})\ddot{y} = 0$$

$$2k\left(\frac{mg}{2k\sin\theta} + y\sin\theta\right)\cdot\sin\theta - mg + \tfrac{1}{2}m\ddot{y} = 0$$

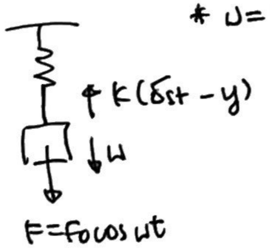
$$\tfrac{1}{2}m\ddot{y} + (\cancel{mg} + 2ky\sin^2\theta) - \cancel{mg} = 0$$

$$\ddot{y} + \frac{4k\sin^2\theta}{m}y = 0$$

$$\omega_n = \sqrt{\frac{4k\sin^2\theta}{m}}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi\sqrt{m}}{\sqrt{4k\sin^2\theta}}$$

* $\omega = k\delta_{st}$ at stable



$$\rightarrow F_0\cos\omega t - ky = m\ddot{y}$$

$$\ddot{y} + \frac{k}{m}y = \frac{F_0}{m}\cos\omega t$$

$$y_c = A\sin\sqrt{\tfrac{k}{m}}t + B\cos\sqrt{\tfrac{k}{m}}y$$

$$y_p = c\cos\omega t$$

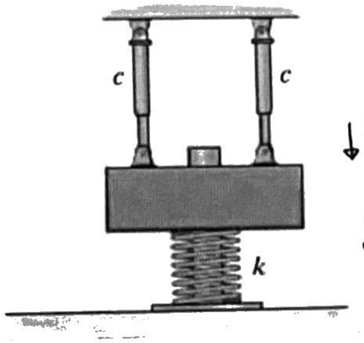
$$-c\omega^2\cancel{\cos\omega t} + \frac{k}{m}c\cancel{\cos\omega t} = \frac{F_0}{m}\cancel{\cos\omega t}$$

$$c\left(\frac{k}{m} - \omega^2\right) = \frac{F_0}{m}$$

$$c = \frac{F_0/m}{\frac{k}{m} - \omega_0^2} = \frac{F_0/k}{1 - (\frac{\omega_0}{\omega_n})^2}$$

$$\rightarrow y = A\sin\sqrt{\tfrac{k}{m}}t + B\cos\sqrt{\tfrac{k}{m}}t + \frac{F_0/k}{1 - (\omega_0/\omega_n)^2}\cos\omega t.$$

•**22–57.** Two identical dashpots are arranged parallel to each other, as shown. Show that if the damping coefficient $c < \sqrt{mk}$, then the block of mass m will vibrate as an underdamped system.



$\downarrow 2c\dot{y}$

$\downarrow ky$

$$-ky - 2c\dot{y} = m\ddot{y}$$

$$\ddot{y} + \frac{2c}{m}\dot{y} + \frac{k}{m}y = 0$$

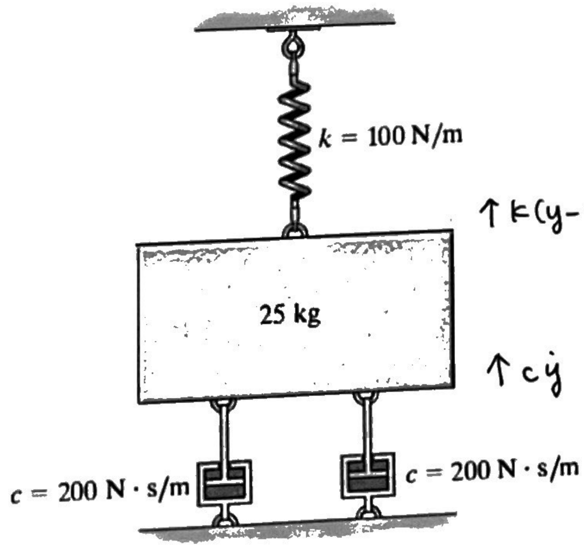
underdamp coefficient requires $\Delta < 0$

$$\left(\frac{2c}{m}\right)^2 - 4(1)\left(\frac{k}{m}\right) < 0$$

$$\frac{c^2}{m^2} - \frac{k}{m} < 0 \rightarrow c^2 < km$$

$$c < \sqrt{km}$$

*****22–68.** Determine the differential equation of motion for the damped vibratory system shown. What type of motion occurs?



$\uparrow k(y - y_{st})$

$\downarrow W = ky_{st}$

$\uparrow c\dot{y}$

$$\Sigma F_y = m\ddot{y} : \quad -W - k(y - y_{st}) - 2c\dot{y} = m\ddot{y}$$

$$\ddot{y} + \frac{2c}{m}\dot{y} + \frac{k}{m}y = 0$$

$m = 25$

$k = 100$

$c = 200$

$$\ddot{y} + \frac{2(200)}{25}\dot{y} + \frac{(100)}{25}y = 0$$

$$\ddot{y} + 16\dot{y} + 4y = 0$$

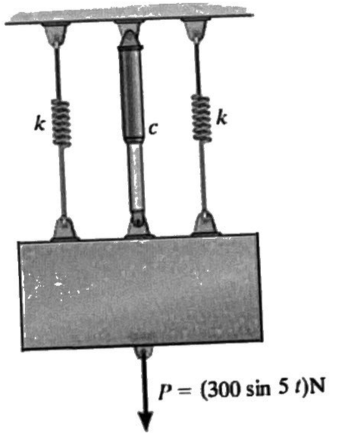
~~$\Delta = 0 \rightarrow$ critically~~

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{4} = 2$$

$$c_c = 2m\omega_n = 2(25)(2) = 100$$

$c = 200 \rightarrow c > c_c \rightarrow$ overdamped.

22–62. If the 30-kg block is subjected to a periodic force of $P = (300 \sin 5t)$ N, $k = 1500$ N/m, and $c = 300$ N·s/m, determine the equation that describes the steady-state vibration as a function of time.



$$\omega_n = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2(1500)}{30}} = 10$$

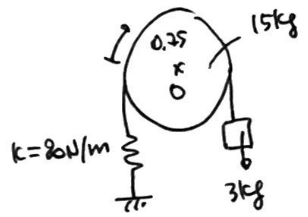
$$c_c = 2m\omega_n = 2(30)(10) = 600$$

$c < c_c \rightarrow$ underdamped

$$Y = \frac{F_0/k_{eq}}{\sqrt{(1-\omega_0/\omega_n)^2 + (2\,c/c_n \cdot \omega_0/\omega_n)^2}} = \frac{300/2(1500)}{\sqrt{(1-10/5)^2 (2 \cdot 300/600 \cdot 10/5)^2}} = 0.1109.$$

$$\phi' = \tan^{-1}\left(\frac{2\frac{c}{c_n}\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right) = \tan^{-1}\frac{2 \cdot \frac{300}{600} \cdot \frac{10}{5}}{1-\left(\frac{10}{5}\right)^2} = 0.588\text{ rad } (= 105.86°)$$

$\rightarrow y = 0.1109 \sin(5t + 105.86°)$



$s = 0.75\theta \rightarrow \dot{s} = 0.75\dot{\theta}$

$$KE + PE = \frac{1}{2}I_0\omega^2 + \frac{1}{2}ks^2 + mgs$$

$$= \frac{1}{2}\left(\frac{1}{2}(15)(0.75)^2\right)\omega^2 + \frac{1}{2}(80)\left((0.75)\theta - \delta_{st}\right) + 3(9.81)(0.75)\theta$$

$~~= 2.109\omega^2 + 300\theta + 22.07\theta~~$

$~~= 1.186\dot{\theta}^2 + 52.07\theta~~$

$k\delta_{st} = mg$

$$\delta_{st} = \frac{3(9.81)}{80} = 0.3678$$

$$\theta_{st} = \frac{0.3678}{0.75} = 0.49$$

$$30(\theta - 0.49)^2 + 22.07\theta$$

$$KE + PE = 1.582\dot{\theta}^2 + \cancel{4}$$

$$\frac{d}{dt}$$

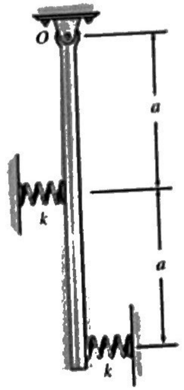
$$0 = 3.164\dot{\theta}\ddot{\theta} + 60(\theta - 0.49)\dot{\theta} + 2\cancel{2}.07\dot{\theta}$$

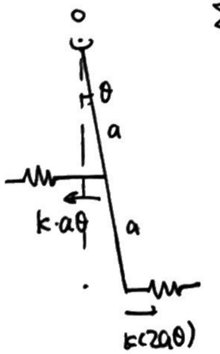
$$3.164\ddot{\theta} + \overset{60}{\cancel{150}}\theta = 0$$

$$\ddot{\theta} + 18.96\theta = 0$$

$$\omega_n = \sqrt{18.96} = 4.35 \text{ rad/s}$$

***22–36.** The slender rod has a mass m and is pinned at its end O. When it is vertical, the springs are unstretched. Determine the natural period of vibration.



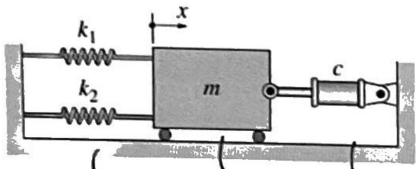


$$\Sigma M_O = I_O\alpha : \; k(a\theta) - k(2a\theta) = \frac{1}{3}m(2a)^2 \cdot \ddot{\theta}$$

$$0 = \frac{4ma^2}{3}\ddot{\theta} + 2ka\theta$$

$$0 = \ddot{\theta} + \frac{3k}{2ma}\theta$$

$$\omega_n = \sqrt{\frac{3k}{2ma}}$$



50 0.2 ?

The system shown in the figure is underdamped with the damping factor $\zeta = 0.25$. If the initial condition on the motion of the block are $x = 0$ and $x' = 4\ m/s$.

1. Determine the displacement of the block at $t = 0.1\ s$. Use the data $m = 0.2\ kg$, $k_1 = 20\ N/m$, and $k_2 = 30\ N/m$
2. How many circles does the system in last example requires to drop the amplitude to less than 5% of the original one.

$x = De^{-\frac{c}{2m}t} \sin(\omega t + \varphi)$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{0.2}} = 15.8\ rad/s$

$\frac{c}{2m} = \frac{1.58}{2(0.2)} = 3.95$

$c_c = 2m\omega_n = 2(0.2)(15.8) = 6.32$

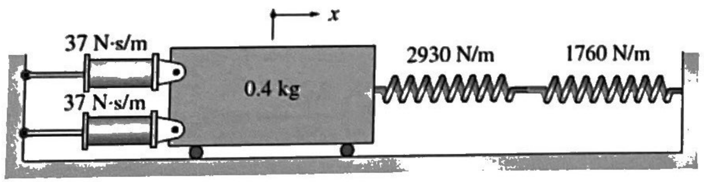
$\zeta = \frac{c}{c_c} = 0.25 \rightarrow c = 6.32 \times 0.25 = 1.58.$

$\omega_d = \sqrt{\frac{k}{m}}\sqrt{1-\zeta^2} = 15.8\sqrt{1-0.25^2} = 15.298\ rad/s$

$x = De^{-3.95t} \sin(15.309t + \varphi)$

$e^{-3.95t} = 0.05 \rightarrow t = \frac{\ln 0.05}{-3.95}$

20.23 The system is released from rest at $x = 25$ mm, where x is measured from the position where the springs are unstretched. (a) Determine if the system is underdamped, critically damped, or overdamped. (b) Derive the expression for $x(t)$.



2×37

$c = 74$

ζ

$k_{eq} = \frac{2930 \times 1760}{2930 + 1760} = 1100$

$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{1100}{0.4}} = 52.4\ rad/s$

$c_c = 2m\omega_n = 2(0.4)(52.4) = 41.92.$

$\because c_{eq} > c_c \rightarrow$ overdamped.

Consider $m\ddot{x} + c\dot{x} + kx = 0$

$0.4\ddot{x} + 74\dot{x} + 1100x = 0$

$0.4\lambda^2 + 74\lambda + 1100 = 0$

$\lambda^2 + 185\lambda + 2750 = 0$

$\lambda = \frac{-185 \pm \sqrt{185^2 - 4(2750)}}{2} = -92.5 \pm 76.19$

$= -16.3, -168.7$

$\therefore x = Ae^{-16.3t} + Be^{-168.7t}$

$x(0) = 0.025 \quad \dot{x}(0) = 0$