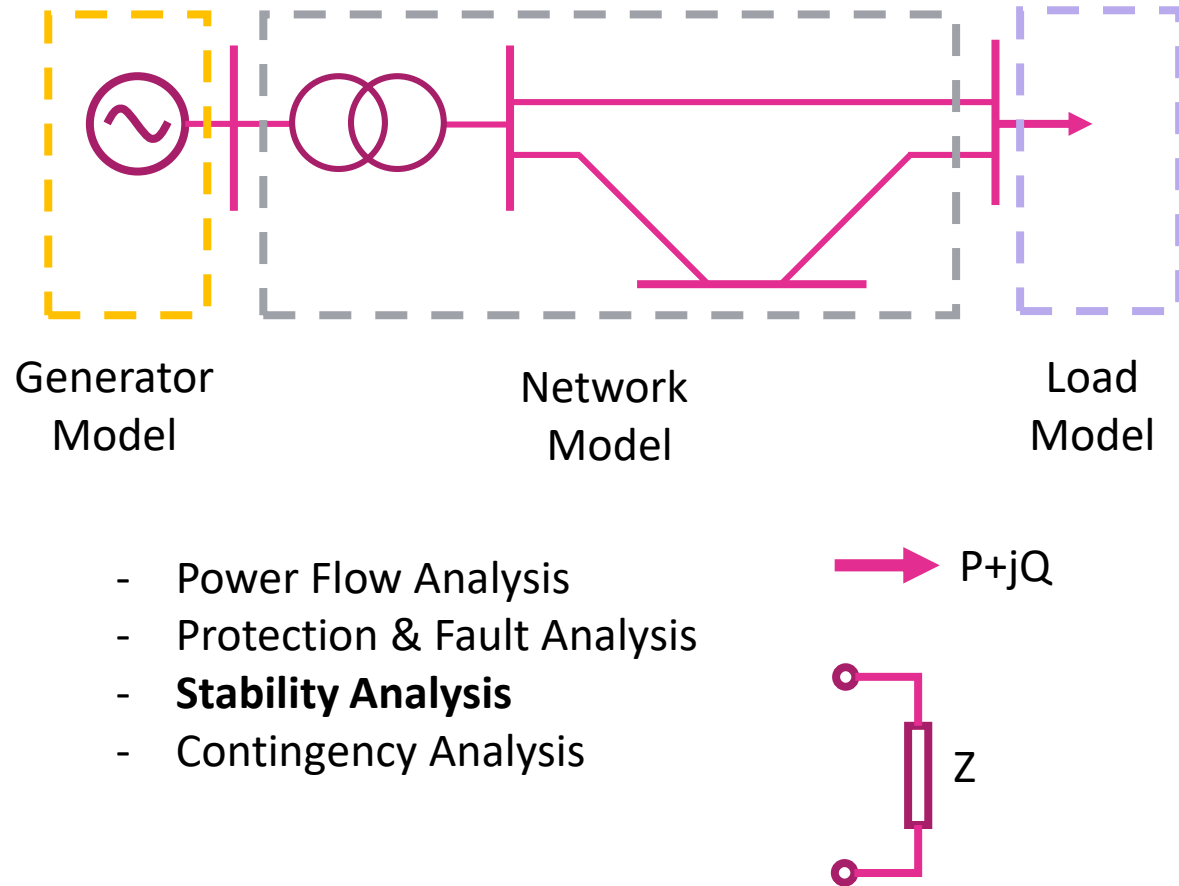


# Conservative Voltage Reduction (CVR) – Load Model Study

**Karl M.H. LAI**

# Load Model

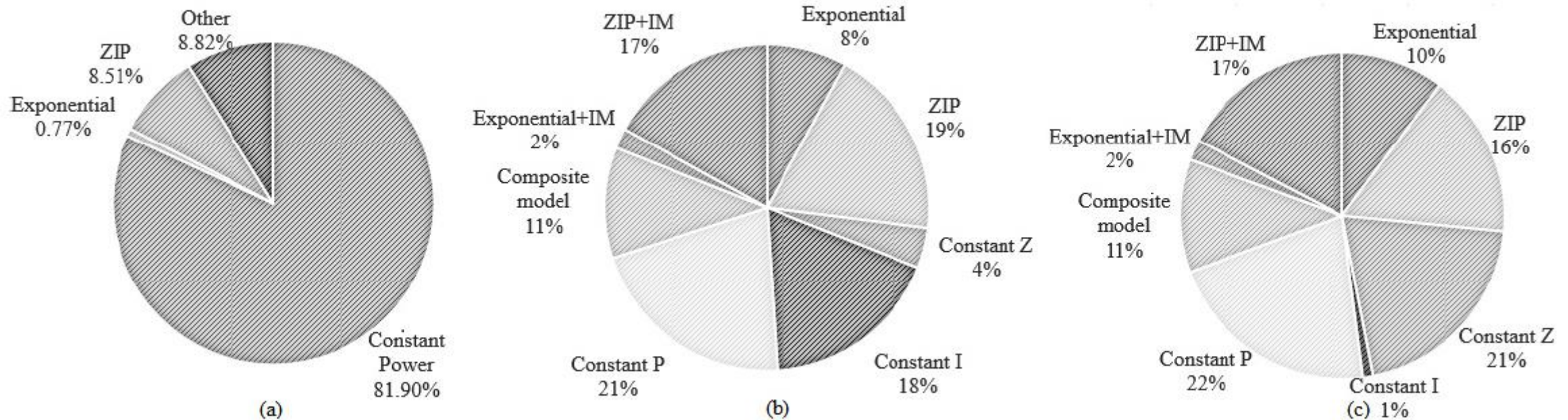


Why Load Model is important?

- **Meter may not be accurate; insufficient meters have not enough data**
- **State estimation:** data for operation & control
- **Voltage control and stability:** voltage depends heavily on load
- **Energy Management:** Load energy and loss energy are dependent on voltage (and frequency), lower voltage (within limit) can reduce loss
- **Protection:** Distance Relay setting varies with load composition of the system

# Load Model – A survey

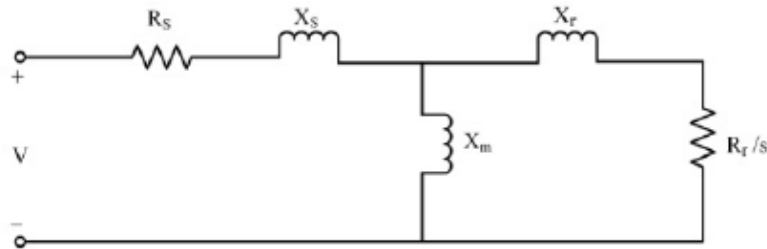
Model to be used for (a) steady and transient studies; (b) real power modelling; (c) reactive power modelling.



Most are constant power, others are ZIP or exponential based. Why? → easy to model (but does it reflect the truth?)  
To know the load model, it is either by components-based (The answer is given), or measurement-based (ill-conditioned)

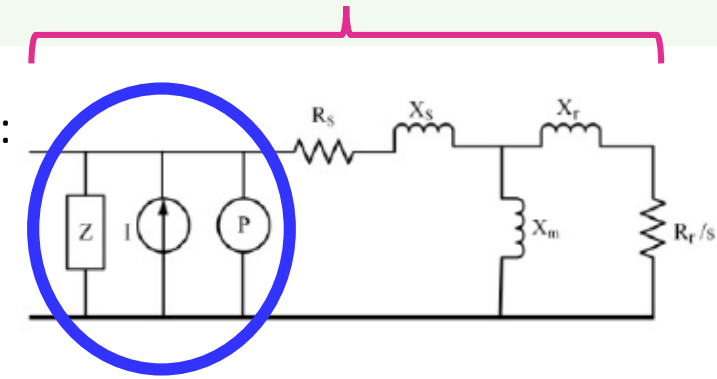
# Load Model – A summary

IM load:



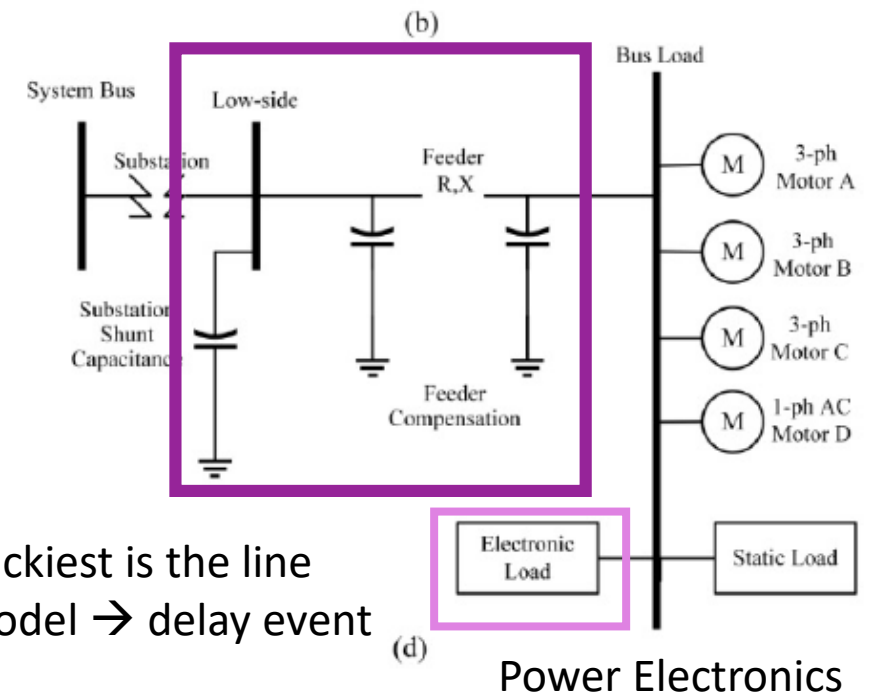
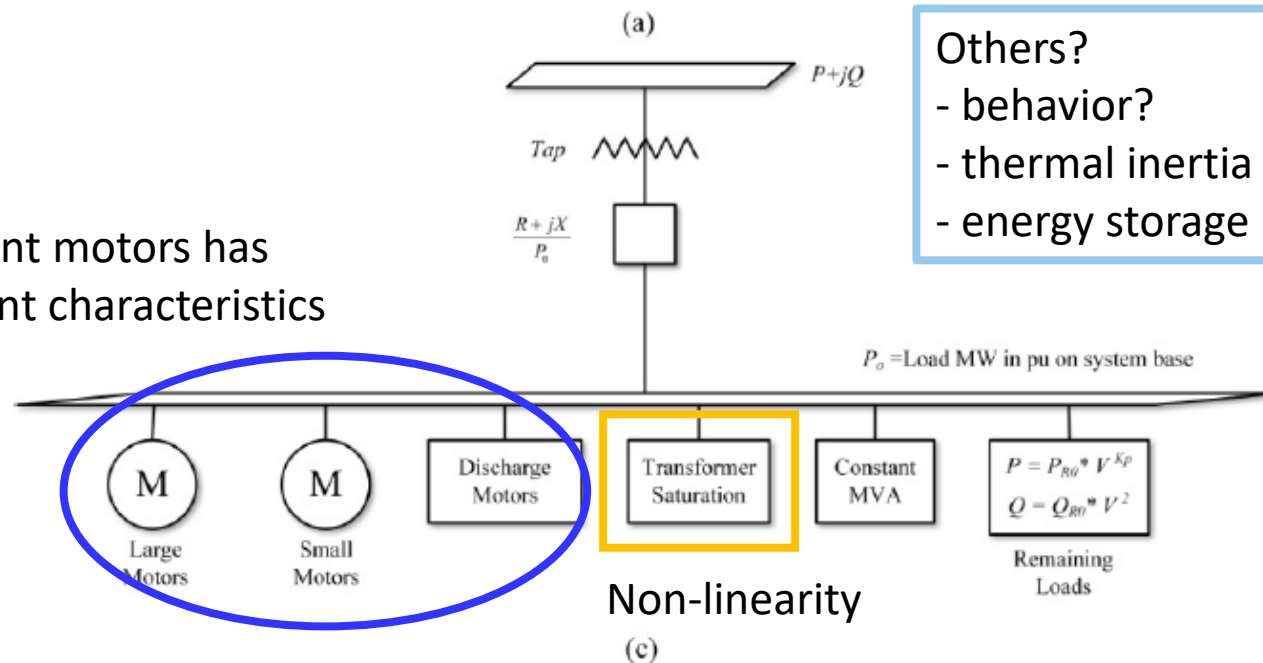
\* Similar to model  
a load with voltage  
source

ZIP load:  
 $P, Q (V, f)$

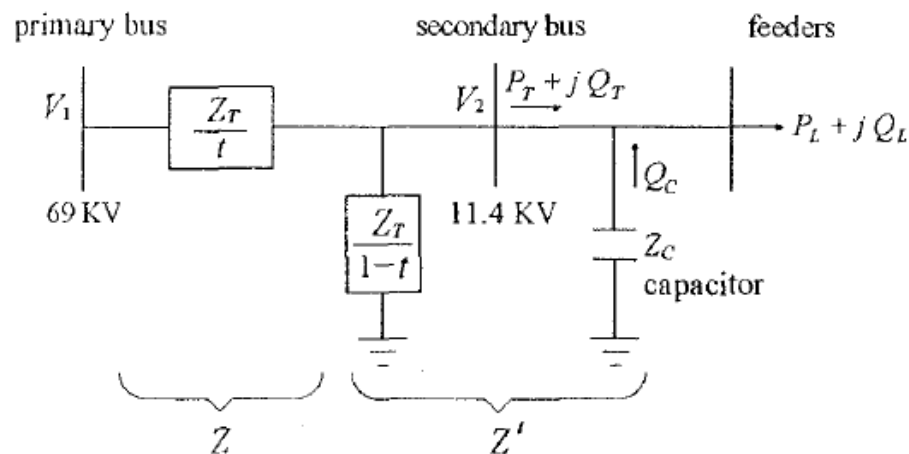
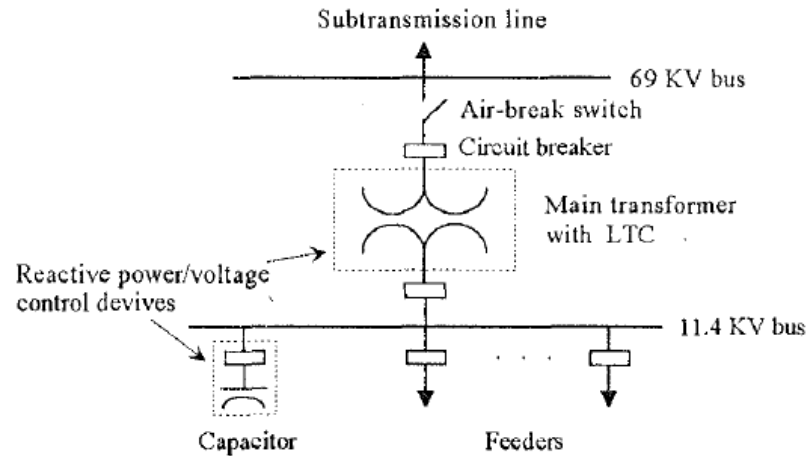


Composite load

Different motors has  
different characteristics



# Static Loads: ZIP and Q/V Control



Consider a feeder with LTC and capacitor bank to control Q and V.

ZIP load is to assume the load is constant Z, constant I and constant P in nature, with

$$\begin{cases} P_L = P_D(aV^2 + bV + c)(1 + k_1f) \\ Q_L = Q_D(dV^2 + eV + f)(1 + k_2f) \end{cases}$$

To control the position of load tap changer (LTC), P and Q can be written as

$$\begin{cases} P_L = \frac{|V_1||V_2|}{Z} \sin \angle(V_1 - V_2) = \frac{|V_1||V_2|}{Z} \sin \theta_{12} \\ Q_L = \left( -\frac{|V_2|^2}{Z} + \frac{|V_1||V_2|}{Z} \cos \theta_{12} \right) - \frac{|V_2|^2}{Z'} \end{cases}$$

Combining,

$$\left( Q_L + \frac{|V_2|^2}{Z''} \right)^2 = \frac{|V_1|^2 |V_2|^2}{Z^2} - P_L^2$$

where  $Z' = \frac{Z_T}{1-t} \parallel Z_C$  and  $Z'' = Z \parallel Z'$

# Static Loads: ZIP and Q/V Control

Let  $|Z''| = K |Z_T|$ .

The equation can be written in form of  $A|V_2|^4 + B|V_2|^3 + [C - (Kt)^2|V_1|^2]|V_2|^2 + D|V_2| + E = 0$

The transformer ratio  $t$  can be calculated.

In practice, there are rules to switch these cap bank and transformer LTC.

1. The main transformer PF should keep *as high as possible*, and larger than 0.8.
2. The secondary bus voltage deviation should be *as small as possible*, and smaller than  $\pm 5\%$
3. The switching operation of cap bank and LTC daily should be smaller than 30 and 6, *as less as possible*.
4.  $\Delta V(\text{kV}) = 0.23(Q_{\text{bank}}(\text{MVar}))/6$

Fuzzy logic is employed to provide these priors into the calculation.

The operation of LTC and CB in  $I = 1-24$  hours can be represented with the following objective function.

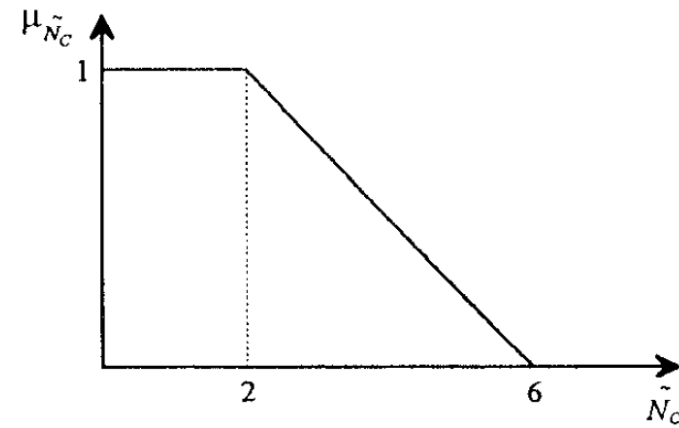
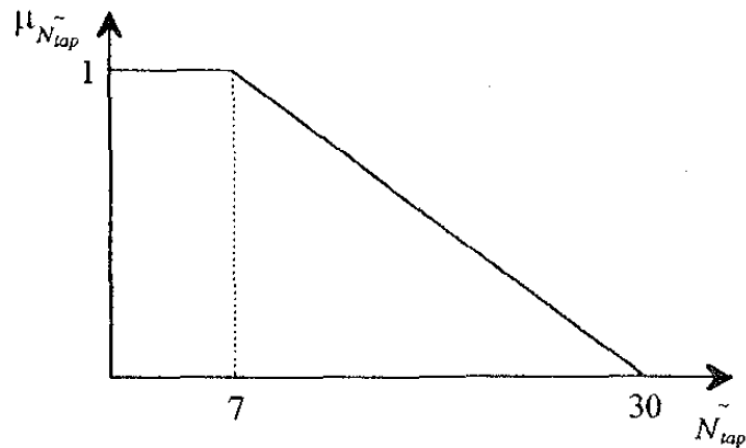
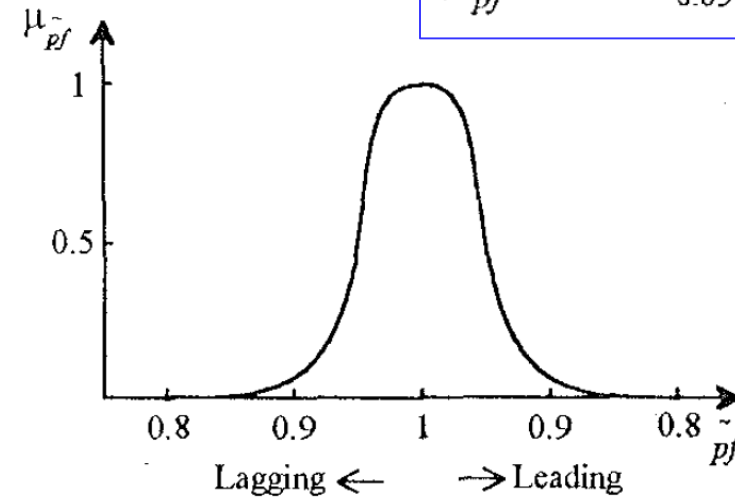
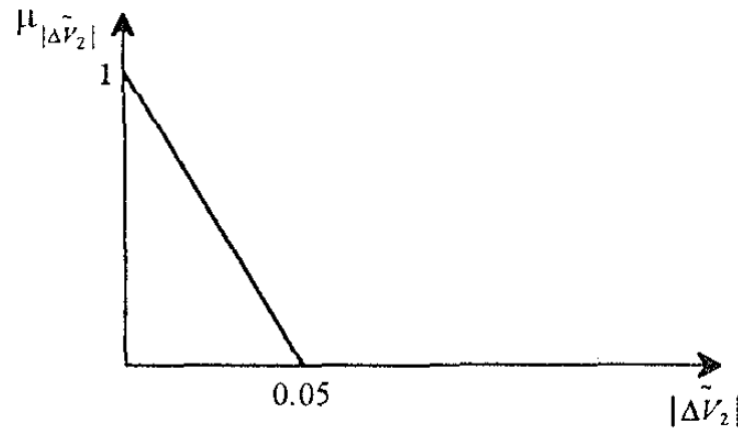
$$J = \sum_{i=1}^{24} \mu_{\Delta V_{2i}} + \sum_{i=1}^{24} \mu_{pfi} + \mu_{N_{tap}} + \mu_{N_C} \quad \text{subject to} \quad \begin{aligned} N_{tap} &= \sum (TAP_i - TAP_{i-1}) \leq 30, N_C \leq 6, \\ V_{min} &\leq V \leq V_{max}, pf_{min} < pf \end{aligned}$$

where  $\mu_{\Delta V_{2i}}, \mu_{pfi}, \mu_{N_{tap}}, \mu_{N_C}$  are the membership function of the requirements, and  $TAP_i$  is the tap position at hour  $i$ .

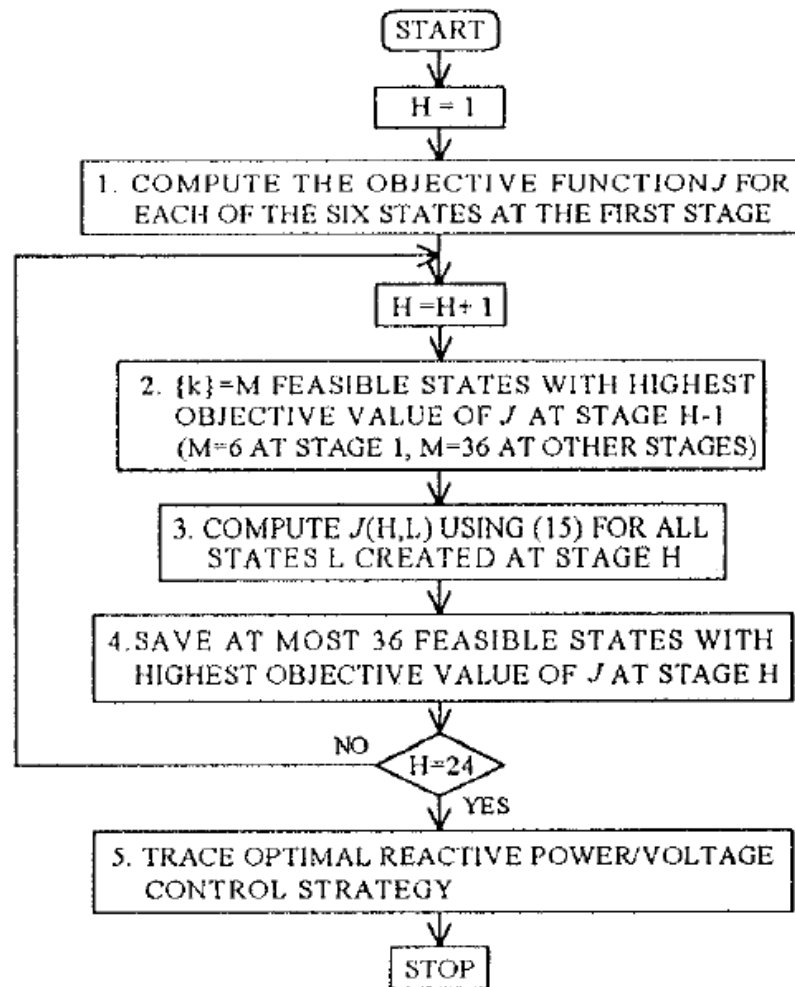
# Static Loads: ZIP and Q/V Control

## Membership Functions

$$\mu_{\tilde{p}f} = [1 + (\frac{\tilde{p}f - 1}{0.05})^4]^{-1} \quad \tilde{p}f \geq 0$$



# Static Loads: ZIP and Q/V Control

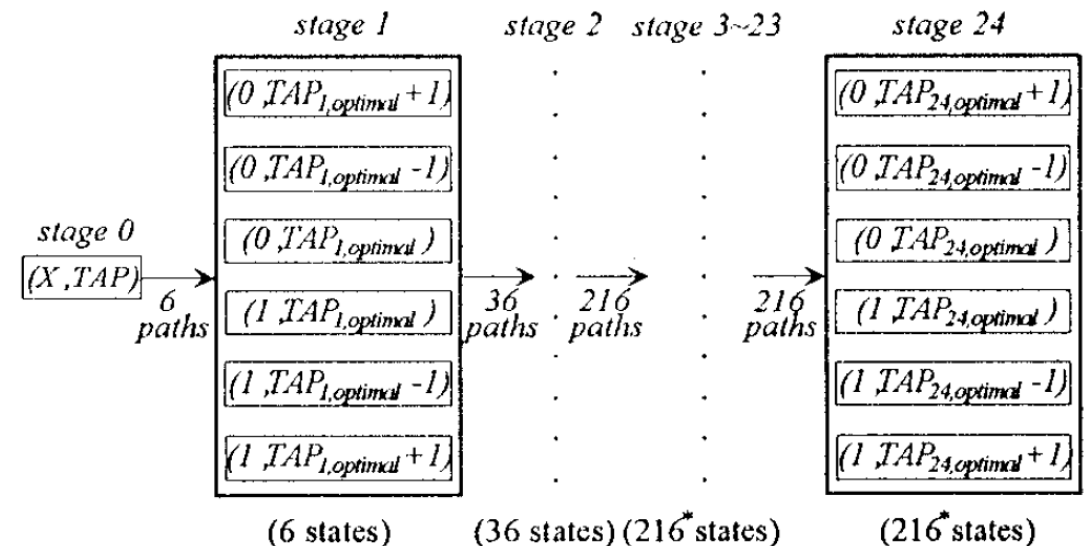


To obtain a Q/V dispatching schedule with maximum  $J$  for hour  $H$  and stage  $L$ ,

$$J(H, L) = \max(J(H-1, K) + J_H(H, L))$$

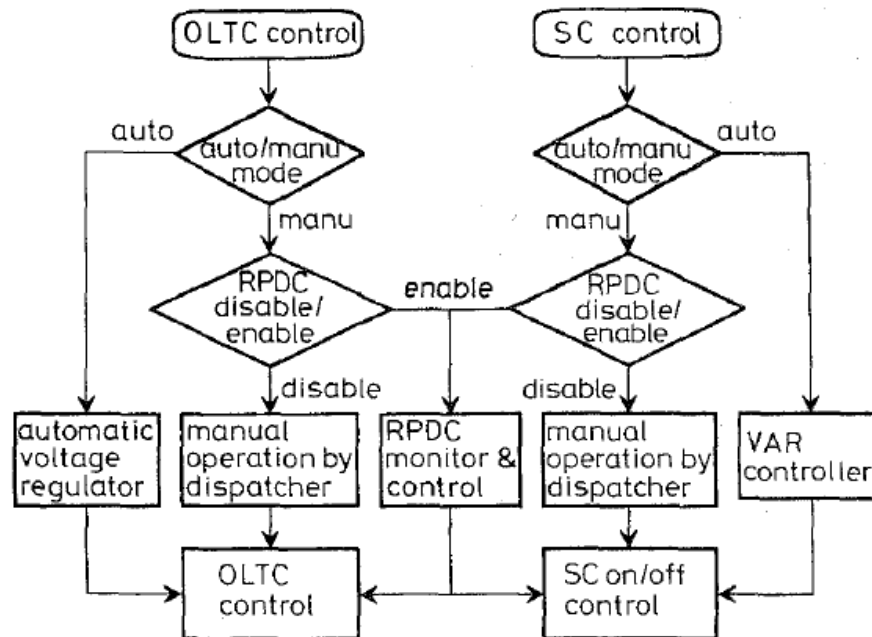
$$\text{where } J_H = \begin{cases} \mu_{\Delta V_{2i}} + \mu_{pfi} & i = 1 - 23 \\ \mu_{\Delta V_{2i}} + \mu_{pfi} + \mu_{N_{tap}} + \mu_{N_C} & i = 24 \end{cases}$$

Optimal  $\pm 1$  are used to reduce computation complexity





# Static Loads: ZIP and Q/V Control



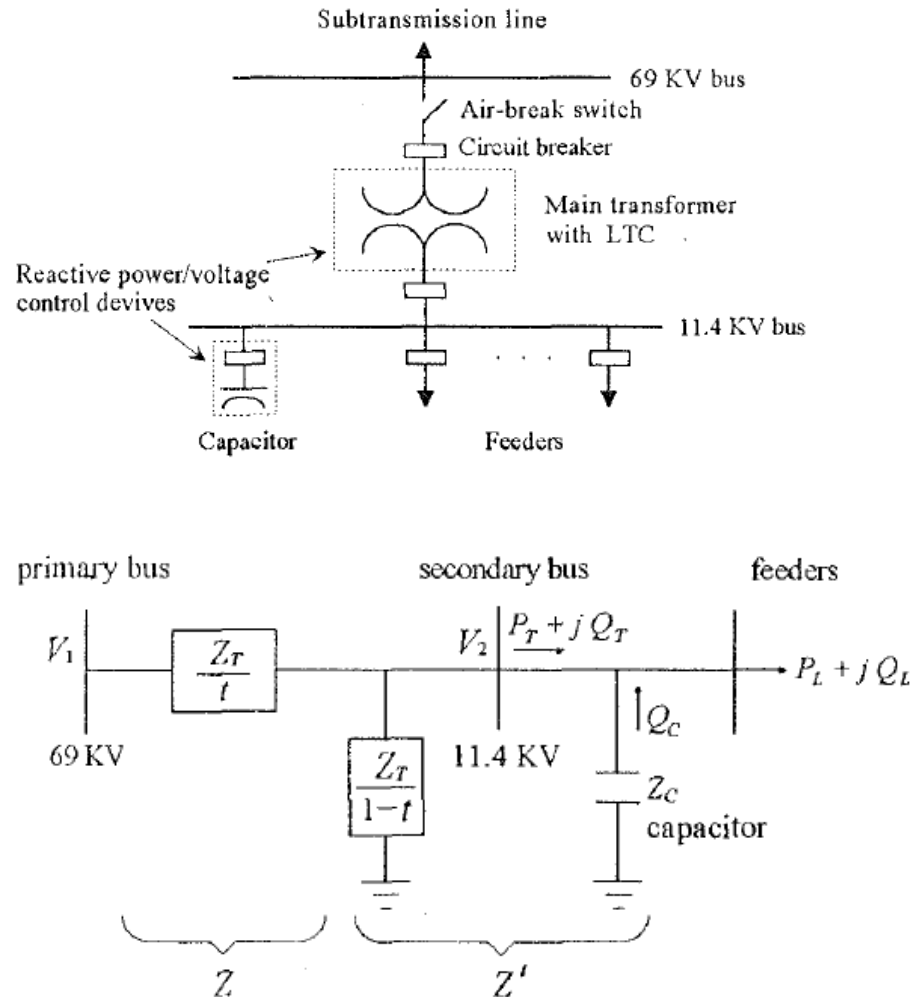
The current Q/V control is shown in the flowchart.

Yet there are difficulties as shown:

1. VR and controllers function independently, hence coordination between OLTC and SC on/off control is impossible
2. Aged VR with delayed response to voltage deviations causes OLTC failure to act at normal position when SC is switched ON/OFF or when there is severity load variation.
3. Unnecessary frequent operation of SC by controller may be experienced with temporary changes in Q flow over Tx
4. The preset values of controllers must be reset manually following the change of SC in different seasons. To coordinate VR and controllers properly, and to optimize the use of OLTC and SC, the system is unable to do so.

Hence, a Q/V control schedule is generated with load forecasted.

# Static Loads: ZIP and Q/V Control



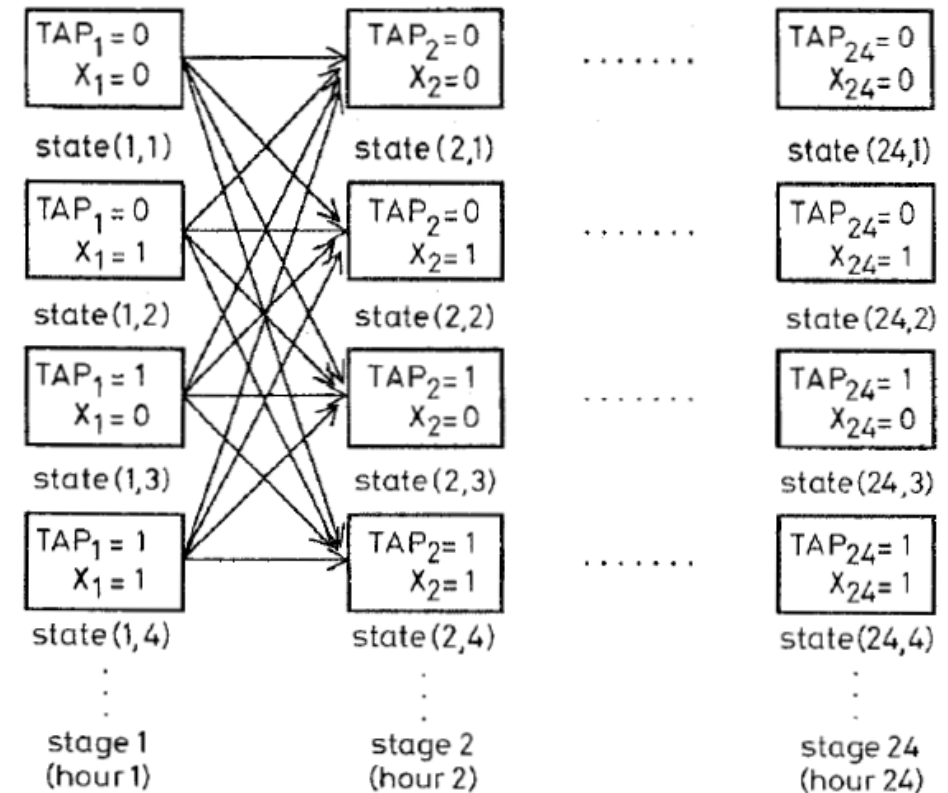
With similar requirement in the fuzzy logic control, a dynamic programming approach is proposed, the goal is to minimize objective value along path.

$$X_i = \begin{cases} 1 & SC \text{ on} \\ 0 & SC \text{ off} \end{cases}$$

$$TAP_i = -8, -7, \dots, 7, 8$$

(at i-th hour)

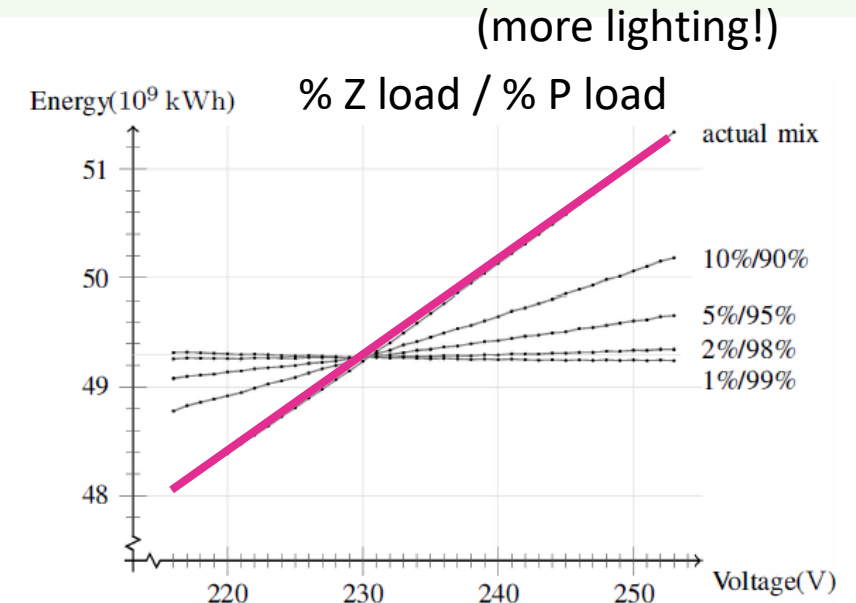
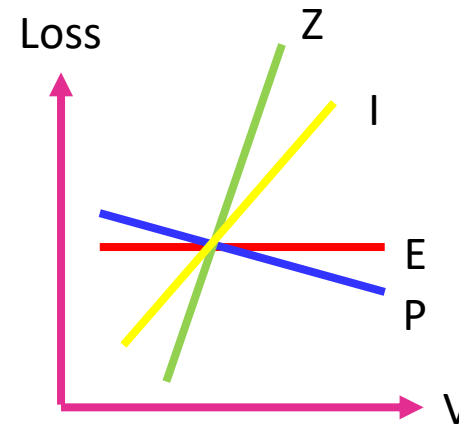
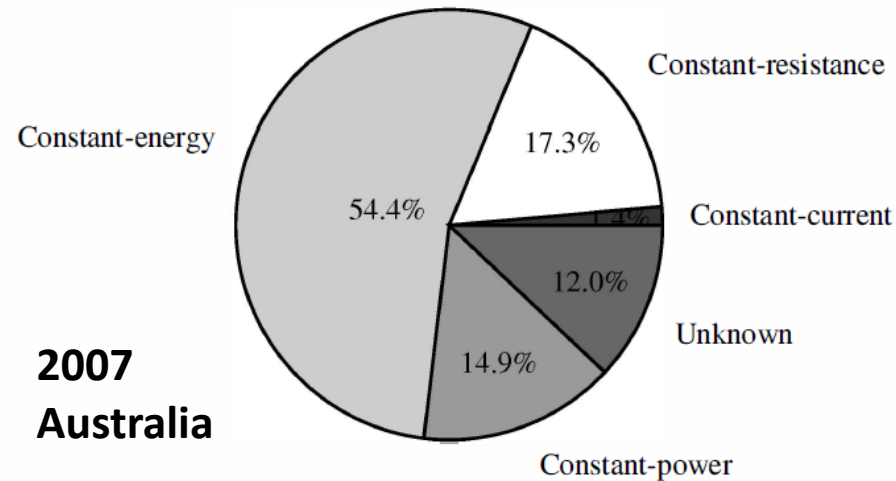
$(2 \times 17)^{24}$  paths  
 ->  $TAP_{\text{optimal}}$  searches first. (with prior)



# Energy Saving for ZIP Load

Divided into:

- Constant Z load: lower V  $\rightarrow$  lower  $P_{\text{loss}}$
- Constant I load: lower V  $\rightarrow$  lower  $P_{\text{loss}}$
- Constant P load: lower V  $\rightarrow$  higher I,  $P_{\text{loss}}$
- Constant E load: lower V  $\rightarrow$  longer time



$$\% \text{ saving} = 2 \times \% \text{ voltage reduction} \times \% \text{ Z load}$$

e.g. for 2.5% voltage reduction and 20% Z load, there is 1% energy reduction

Sensitivity Analysis: 1. devices used more often, 2. Incorrect power data, 3. devices required more energy, 4. change in load type

# Static Loads: Load Mix Based

Other than traditional static loads, such as ZIP, or exponential as shown, load composition based on classes (e.g. industrial) and classes (e.g. lighting) are used.

**ZIP Model:**

$$P_{\text{total}} = P_0 \left[ Z_p \left( \frac{V}{V_0} \right)^2 + I_p \left( \frac{V}{V_0} \right) + P_p \right]$$

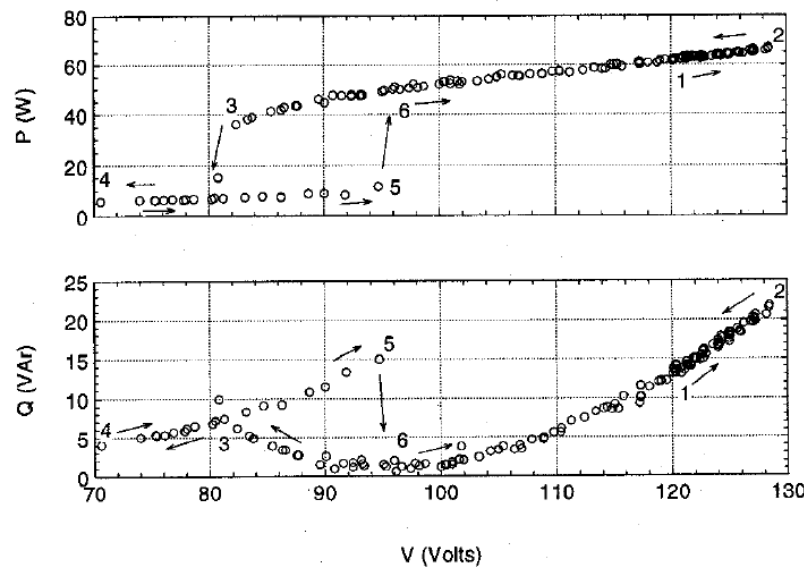
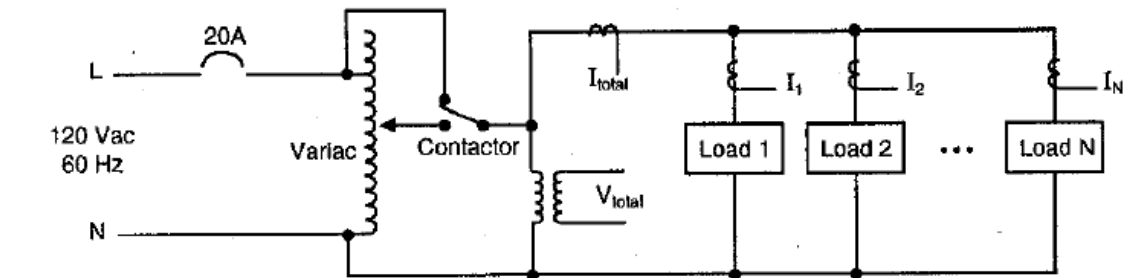
$$Q_{\text{total}} = Q_0 \left[ Z_q \left( \frac{V}{V_0} \right)^2 + I_q \left( \frac{V}{V_0} \right) + P_q \right]$$

**Exponential Model:**

$$P_{\text{total}} = S_0 * k_p * \left( \frac{V}{V_0} \right)^{np}$$

$$Q_{\text{total}} = S_0 * k_q * \left( \frac{V}{V_0} \right)^{nq}$$

A load test is performed to see the ZIP coefficient.



(steady state P(V),  
Q(V) measurement)

# Static Loads: Load Mix Based

(Exponential Model)

No	Load	S0 (VA)	pf	Vmin (pu)	Active Power			Reactive Power			Exponent	
					Z	I	P	Z	I	P	$\partial P/\partial V$	$\partial Q/\partial V$
1a	dryer heater	5400	1.00	0.50	0.96	0.05	-0.01	0.00	0.00	0.00	1.96	0.00
1b	dryer motor	515	0.45	0.50	1.91	-2.23	1.33	2.51	-2.34	0.83	1.58	2.68
2	advanced washing machine	654	0.61	0.50	0.05	0.31	0.63	-0.56	2.20	-0.65	0.42	1.09
3a	adjustable frequency drive 1	1800	0.79	0.75	0.43	0.61	-0.05	-1.21	3.47	-1.26	1.48	1.05
3b	adjustable frequency drive 2	1780	0.79	0.75	3.19	-3.84	1.65	1.09	-0.18	0.09	2.54	2.00
4a	heatpump 1 (split design)	1160	0.93	0.75	0.72	-0.98	1.25	14.78	-23.72	9.93	0.47	5.85
4b	heatpump 2 blower	575	0.74	0.87	5.46	-14.21	9.74	-14.85	31.59	-15.74	-3.28	1.90
4c	heatpump 2 compressor	5700	0.90	0.87	0.85	-1.40	1.56	22.92	-40.39	18.47	0.29	5.45
5	refrigerator/freezer	1030	0.84	0.76	1.19	-0.26	0.07	0.59	0.65	-0.24	2.12	1.84
6	battery charger	6430	0.76	0.61	3.51	-3.94	1.43	5.80	-7.26	2.46	3.08	4.34
7a	electronic compact fluorescent 1	60	0.99	0.50	0.14	0.77	0.09	-0.06	-0.34	-0.60	1.05	-0.46
7b	electronic compact fluorescent 2	53	0.97	0.50	0.16	0.79	0.05	0.18	-0.83	-0.35	1.12	-0.47
7c	conventional magnetic compact fluorescent	151	0.49	0.83	0.34	1.31	-0.65	3.03	-2.89	0.86	1.99	3.17
8a	electronically ballasted fluorescent 1	1335	1.00	0.65	-2.48	5.46	-1.97	0.00	0.00	0.00	0.49	0.00
8b	electronically ballasted fluorescent 2	1172	0.98	0.54	-1.60	3.58	-0.98	0.79	-0.16	0.36	0.38	1.43
9a	electronic dimming ballast	62	1.00	0.63	-0.16	1.77	-0.62	0.00	0.00	0.00	1.46	0.00
9b	external fluorescent dimmer	157	0.94	0.78	-0.48	1.89	-0.41	12.21	-18.38	7.16	0.93	6.05
10	high pressure sodium lamps	341	0.99	0.73	0.98	-0.03	0.06	29.84	-45.26	14.41	1.92	14.43
11a	office equipment 1	1020	1.00	0.67	0.34	-0.32	0.98	0.00	0.00	0.00	0.36	0.00
11b	office equipment 2	800	1.00	0.67	0.08	0.07	0.85	0.00	0.00	0.00	0.24	0.00
12	microwave oven	1361	1.00	0.83	-2.78	6.06	-2.28	0.00	0.00	0.00	0.50	0.00
13a	industrial heater/blower	1350	0.99	0.50	0.98	0.02	0.00	0.69	0.25	0.06	1.98	1.63
13b	baseboard heater	1200	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00

- Notes:
1. Reactive power ZIP coefficients adding to -1 indicate capacitive reactive power load characteristics
  2. Vmin represents the minimum voltage (in pu) for which the load model is valid (dropout voltage)
  3. S0 is the measured apparent power (VA) at rated voltage (not nameplate)
  4. pf is the load power factor
  5.  $\partial P/\partial V$  and  $\partial Q/\partial V$  are the measured slopes at rated voltage

(ZIP Model)

Load	Load Factor	Vmin (pu)	kp	np	kq	nq	Notes
1a	1.00	0.50	1	1.95	0	0	II T
1b	1.00	0.50	0.45	0.77	0.892	2.13	II T
2	0.68	0.50	0.567	0.34	0.744	1.51	II
3a	0.78	0.75	0.795	1.47	0.627	1.34	II 1
3b	0.77	0.75	0.786	2.12	0.625	1.98	II 2
4a	1.00	0.75	0.94	0.34	0.376	4.12	II T
4b	0.81	0.87	0.915	-3.34	0.831	0.86	II T
4c	0.91	0.87	0.729	0.33	0.35	5.74	II T
5	1.00	0.76	0.853	2.11	0.558	1.89	I T
6	0.97	0.61	0.761	2.59	0.650	4.06	II
7a	1.00	0.50	0.988	0.95	-0.156	0.31	I C
7b	1.00	0.50	0.963	1.03	-0.247	0.46	I C
7c	1.25	0.83	0.495	2.07	0.892	3.21	I C, M
8a	0.87	0.65	N/A	N/A	0	0	I C
8b	0.89	0.54	0.976	0.89	0.179	1.21	I C
9a	0.78	0.63	0.992	1.64	N/A	N/A	I C
9b	0.68	0.78	0.947	1.0	0.333	5.84	I C, M
10	1.08	0.73	0.981	1.90	-0.055	-4.25	I C
11a	0.16	0.67	0.95	0.24	0	0	I C
11b	0.16	0.67	1	0.20	0	0	I C
12	1.00	0.83	0.999	0.83	0.045	24.17	I
13a	0.94	0.50	1	1.98	0.016	1.42	I T
13b	1.00	0.00	1	2	0	0	I T

- Notes:
- C. Composite of several devices totalling 15 A
  - T. Thermostatic load
  - M. Conventional magnetic technology
  - I. Single-phase load, Figure I
  - II. Two-phase load, Figure II
  - 1. Constant torque; represents compressors - most motors
  - 2. Constant speed; represents fans/pumps - fraction of all motors
  - Vmin. Minimum voltage (pu) for which load model is valid
  - Load Factor. Ratio of S0 (Table II) to nameplate VA

# Static Loads: Load Mixed Based

Considerations:

1. It gives more information on the **actual load** (e.g. lighting, air conditioning, heater)
2. It can be used to build a model with **demand response** and **tariff**. Yet the difficulty is to know the real time information on what load is connected. → **AMI** required.

System load meeting capability (LMC) and boundary condition can be known.

Figure IV: Effect of Load Increase on Voltage of 115 kV Bus

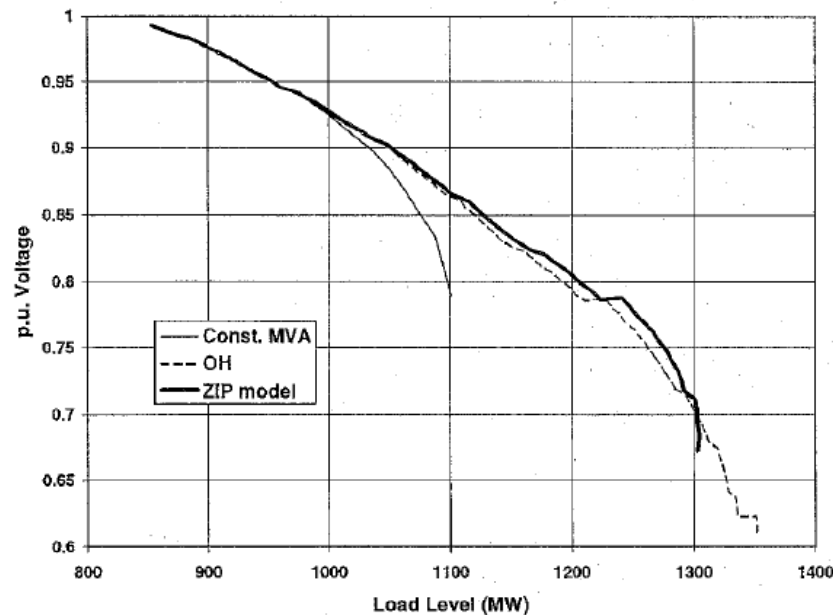
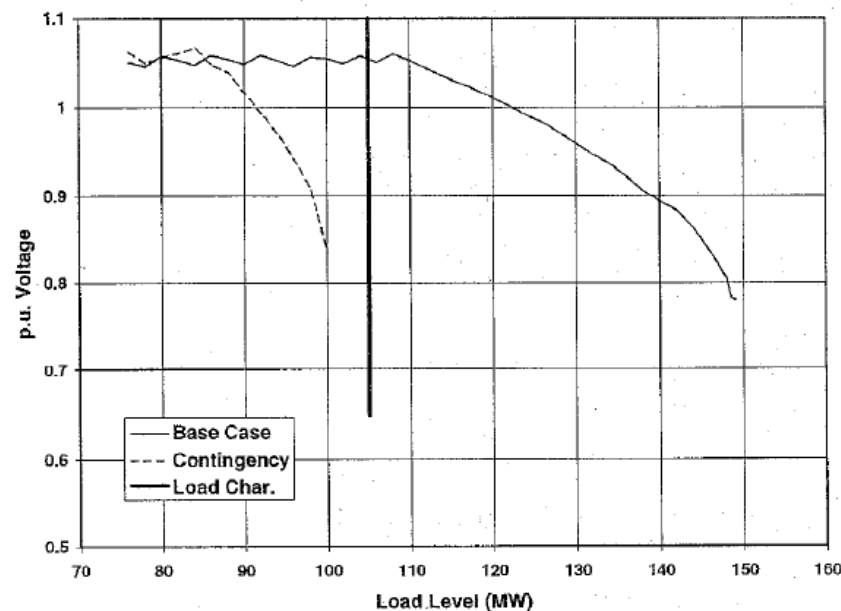


Figure V: Voltage of Load Bus for Constant MVA Load Model



Constant MVA

= constant power load

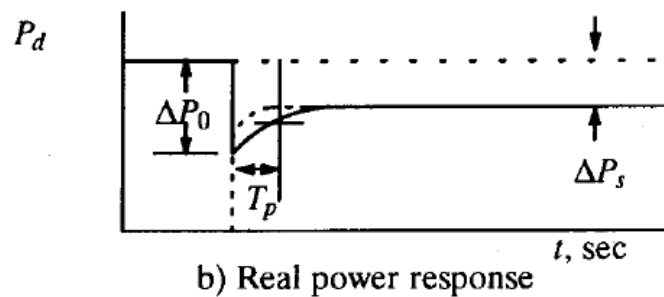
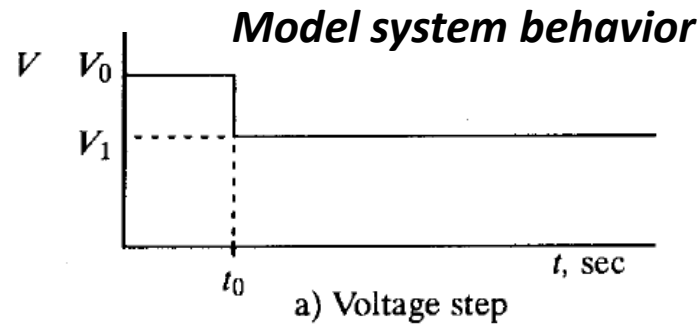
OH =  $P(\text{const } I, Z)$ ,  $Q(\text{const } Z)$

ZIP = load mixed based.

3. Some load **may not restart** after islanding or fault. Actual voltage stability/ behavior depends more on actual load behavior.



# Dynamic Loads: Exponential Recovery Model



Observation:

- Sudden jump with  $V$
- Exponential rise to reach steady state

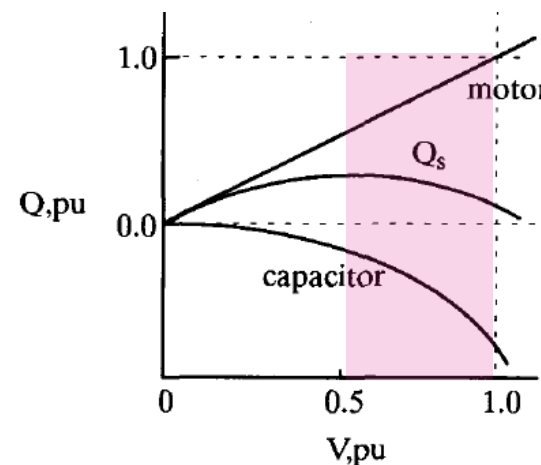
General Load Model (1<sup>st</sup> order dynamic):  $f(\dot{P}_d, P_d, \dot{V}, V) = 0$

Exponential Recovery Model:  $T_p \dot{P}_d + P_d = P_s(V) + \frac{d}{dt} K_p(V)$

\* At steady state:  $P_d = P_s(V) = C_s V^{a_p} \rightarrow$  static load is a function of  $V$ .

\* At transient state, the dynamics are determined  $P_s(V)$  and  $K_p = \int_0^V k_p(\sigma) d\sigma$

Solution:  $P_d(t) = P_s(v_1) + \left[ P_s(v_0) - \frac{1}{T_p} K_p(V_0) - P_s(V_1) + \frac{1}{T_p} K_p(V_1) \right] e^{-\frac{t-t_0}{T_p}}$



Voltage stability:

1. If  $\frac{dQ_s}{dV} < 0$ , the system is unstable
2. A (real/reactive) load overshoot will occur.

# Dynamic Loads: Static Voltage Stability

$$T_p \dot{Q}_d + Q_d = Q_s(V) + \frac{d}{dt} K_q(V)$$

$$Q_d = Q_l(V) \text{ (steady state } Q = \text{ reactive power flow in statics)}$$

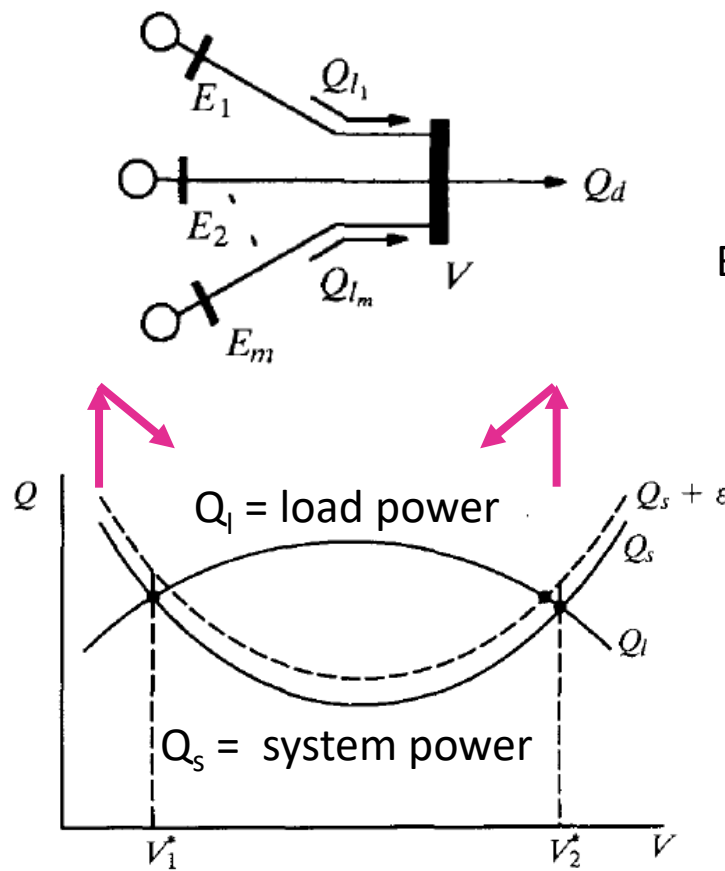
By definition, the system is stable if a decrease in load results in an increase of  $V$

$$\text{i.e. } \frac{dV}{dQ_d} < 0$$

An increase of system power  $\varepsilon$  will shift the system.

For  $V_2$ , the system power increases. To restore the balance between load and system, the voltage decreases.  $\rightarrow V_2$  is a stable operating point

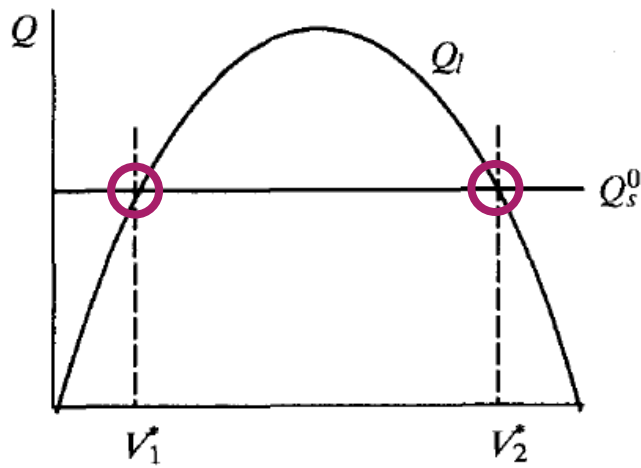
For  $V_1$ , the system power increases, the voltage shifts right (i.e. increases)  $\rightarrow V_1$  is an unstable point.





# Dynamic Loads: Dynamic Voltage Stability

$V_2$  is stable, in statics  
or in dynamics;



$V_1$  is steady-state unstable;  
dynamic unstable when  $T_q$  is small  
enough

From  $T_p \dot{Q}_d + Q_d = Q_s(V) + \frac{d}{dt} K_q(V)$   $Q_d = Q_l(V) \rightarrow \frac{dQ_d}{dt} = \frac{dQ_l}{dV} \dot{V}$   
 $C_d(V) \dot{V} = Q_s(V) - Q_l(V)$ , where  $C_d(V) = \frac{d}{dV} (T_q Q_l - K_q)$  (dynamic coeff.)

1. If  $C_d(V) = \frac{d}{dV} (T_q Q_l - K_q) < 0$  for  $V > 0$ ,

Then  $V_1^*$  and  $V_2^*$  are unstable and stable respectively.

2. The linearized version of differential equations is:  $C_d(V^*) \Delta \dot{V} = J_s(V^*) \Delta V$

where  $\Delta V = V - V^*$ . The stability is guaranteed by:  $\frac{J_s(V^*)}{C_d(V^*)} < 0$

3. If  $J_s(V^*) > 0$ , i.e. steady state stable,  $C_d(V_2^*) < 0 \rightarrow \frac{k_q(V_2^*)}{T_q} > \frac{dQ}{dV} \big|_{V_2}$

which is physically impossible to be unstable in dynamics.

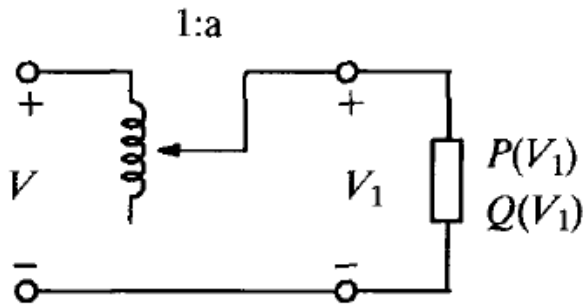
# Dynamic Loads: Exponential Recovery Model

1. Static Loads:  $P_d = P_s(V) = a_1 V^2 + a_2 V + a_3$

2. Induction Motor Load:  $T_p \dot{P}_d + P_d = P_s(V) + \frac{d}{dt} K_p(V) \rightarrow \dot{P}_d + F_p P_d = F_s V + k_p \dot{V}$  \* Based on experiment

Other Formulations:  $\begin{cases} P_d = P_0 + K_{pv}(V + T_{pv}\dot{V}) \\ Q_d = Q_0 + K_{qv}V \end{cases} \quad \begin{cases} P_d = P_s(V) + k_p \dot{V} \\ Q_d = Q_s(V) \end{cases}$

3. Load Tap Changer (LTC)



$T\dot{a} = V^0 - aV_1$ ,  $V^0$  is voltage set point and  $T$  is speed of change in LTC

When  $V_0$  goes to  $V_1$  at  $t_0$ , the tap cannot change suddenly,  $P(a_0 V_0) \rightarrow P(a_1 V_0)$

$$a(0) = a_0 \quad a(\infty) = \frac{V^0}{V_1} \quad P_d(\infty) = P(a(\infty)V_1) = P(V_0)$$

Power can restore back to initial value

$$T_p \dot{P}_d + P_d = P_s(V) + \frac{d}{dt} K_p(V) \rightarrow P_s(V) = P(V^0), K_p = T_p P(a_0 V), T_p = \frac{T}{V_1}$$

4. Heating Load:

$$T_p \dot{P}_d + P_d = P_s(V) + \frac{d}{dt} K_p(V) \rightarrow \dot{P}_d + \frac{r}{T} \frac{P_d^2}{V} (P_d - P_L) = \frac{2P_d}{V} \dot{V}$$

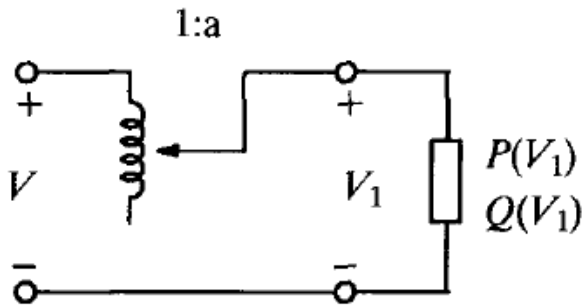
# Constant Power Load response to LTC change

Consider a constant power load with load tap change.

$$Q_d = (aV^2)B = Q_1(V)$$

Linearize,

$$(2a^2VB + J_s)\Delta V = -2aV^2B\Delta a$$



From the load tap change model,  $T\dot{a} = V^0 - aV_1$

$$T\Delta a = -V\Delta a - a\Delta V$$

Combining,

$$T(1 + a^2BX)\Delta a = -XJ_s\Delta a$$

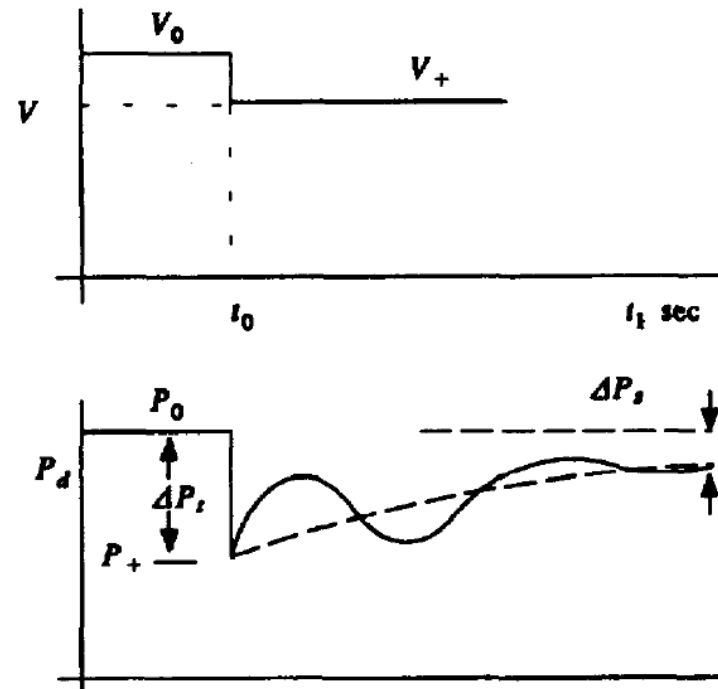
For stability,

$$J_s = \frac{dQ}{dV} > 0 \quad (\text{steady state criterion})$$

The dynamic equations can be converted with:

$$T_p\dot{Q}_d + Q_d = Q_s(V) + \frac{d}{dt}K_q(V) \quad T_q = \frac{T}{2V}, \quad Q_s = \sqrt{Q_dBV^0}, \quad k_q = \frac{TQ_d}{V^2}$$

# General Dynamic Loads: Extension of ERM



Generalized Model:

$$f\left(P_d^{(n)}, P_d^{(n-1)}, \dots, \dot{P}_d, P_d, V^{(m)}, V^{(m-1)}, \dots, \dot{V}, V\right) = 0$$

The transfer function could be expressed as:

$$G(s) = \frac{(b_m s^m + b_{m-1} s^{m-1} + \dots + b_0)}{s^n + a_{n-1} s^{n-1} + \dots + a_0}$$

It can be transformed as:

$$\begin{cases} \dot{x}_p = F x_p + G \omega \\ P_r = H^T x_p \end{cases}$$

where  $x_p$  is an  $n$ -dimension state vector

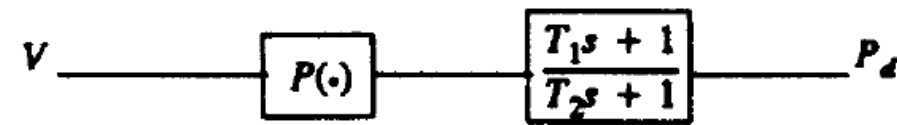
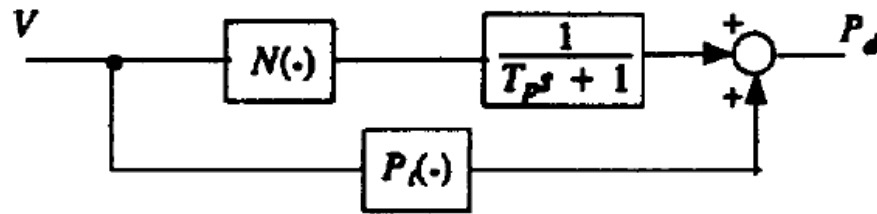
$$\text{and } \begin{cases} \omega = N_1(V) \\ P_l = N_2(V) \\ P_d = P_r + P_t \end{cases}$$

In general, the load model can be represented by non-linear system

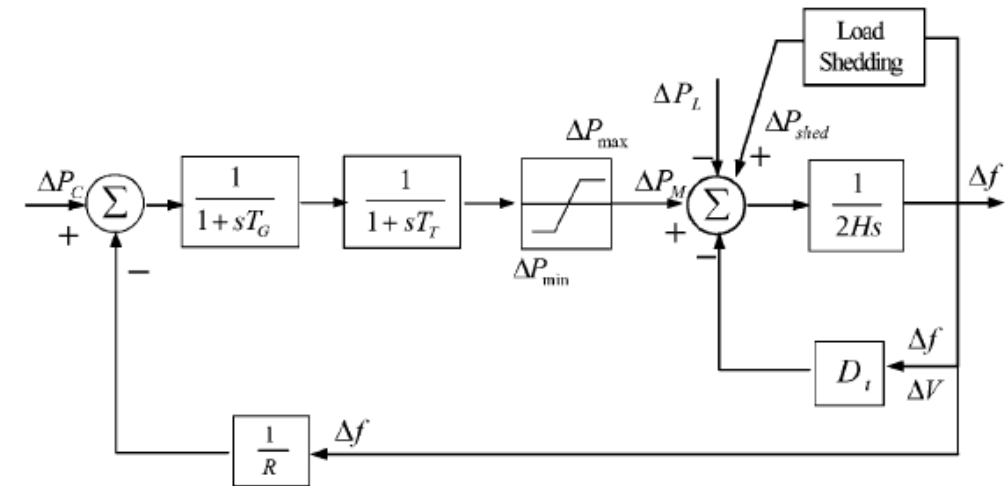
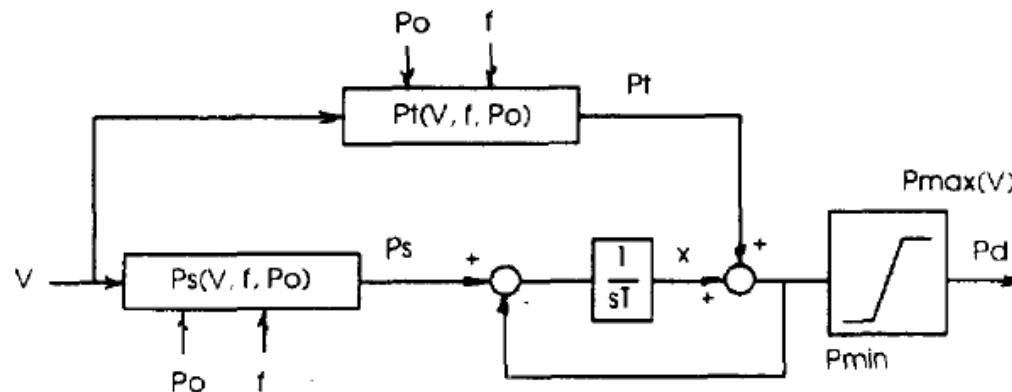
$$\dot{x}_p = f(x, u; \theta) \text{ and } y = g(x, u; \theta)$$

# Dynamic Loads: Exponential Recovery Model

For the first order load model, the following transfer function can be applied.

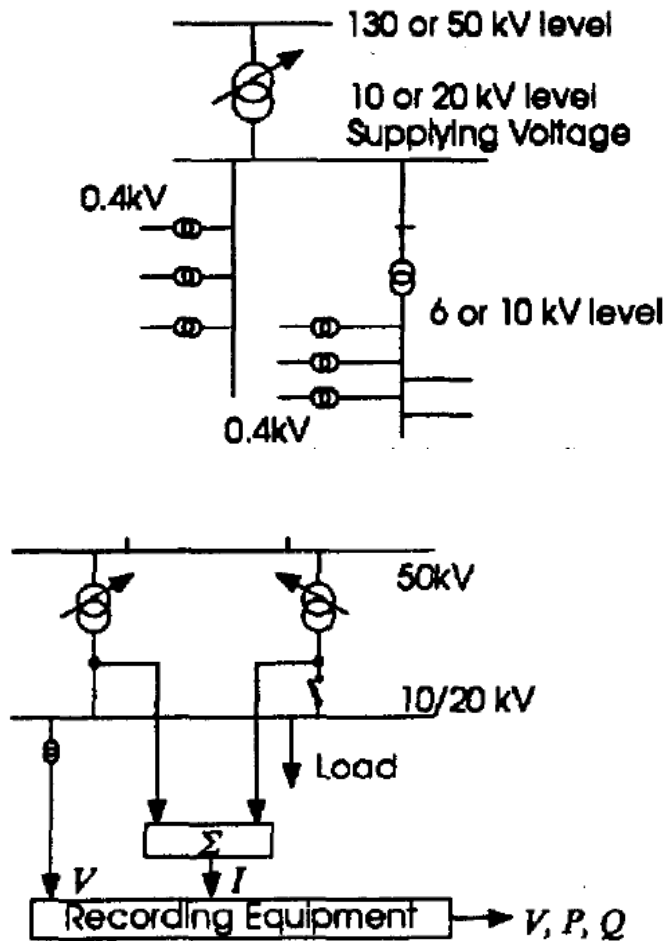


In a generalized case, with state-space representation to high order ODEs, the transfer function can be expressed as:

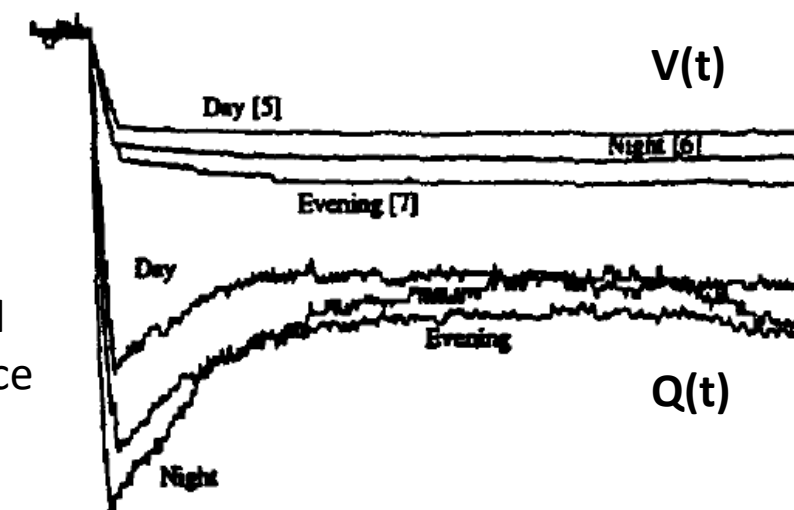
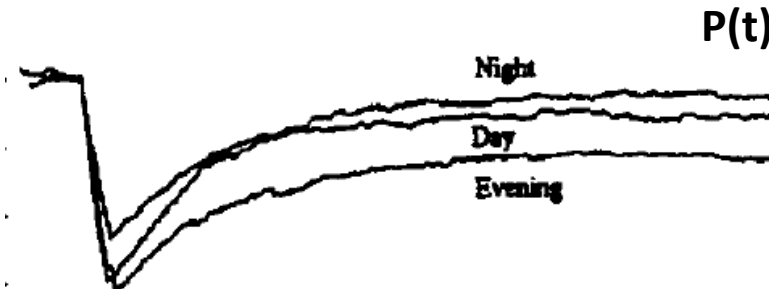
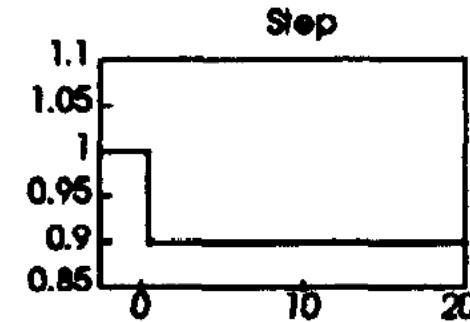
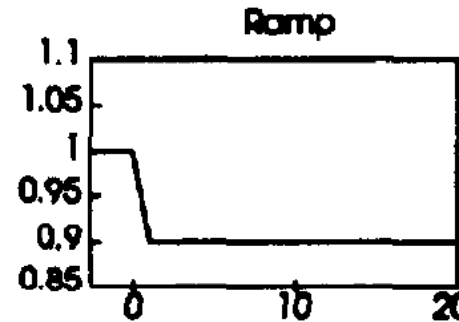


# Dynamic Loads: Field Test

Within 5 mins, half of initial active power reduction is restored. Initially, the composite load is almost equal to a constant Z load, finally the P curve stabilizes between constant I and constant P load.



A ramp and a step were introduced to the system.



- initially, composite load  $\approx$  const. Z load
- Finally, const. I and const. P load balance
- P recovers at low V with high I, hence Q increases a bit after V drops.

# Dynamic Loads: Field Test

Load Model for parameter identification:

$$T_p \frac{dP_r}{dt} + P_r = N_p(V); \quad N_p(V) = \underbrace{P_0 \left( \frac{V}{V_0} \right)^{\alpha_s}}_{\text{= steady part}} - \underbrace{P_0 \left( \frac{V}{V_0} \right)^{\alpha_t}}_{\text{= transient part}}$$

$$P_d = P_r + P_0 \left( \frac{V}{V_0} \right)^{\alpha_t}$$

$$T_q \frac{dQ_r}{dt} + Q_r = N_q(V); \quad N_q(V) = Q_0 \left( \frac{V}{V_0} \right)^{\beta_s} - Q_0 \left( \frac{V}{V_0} \right)^{\beta_t}$$

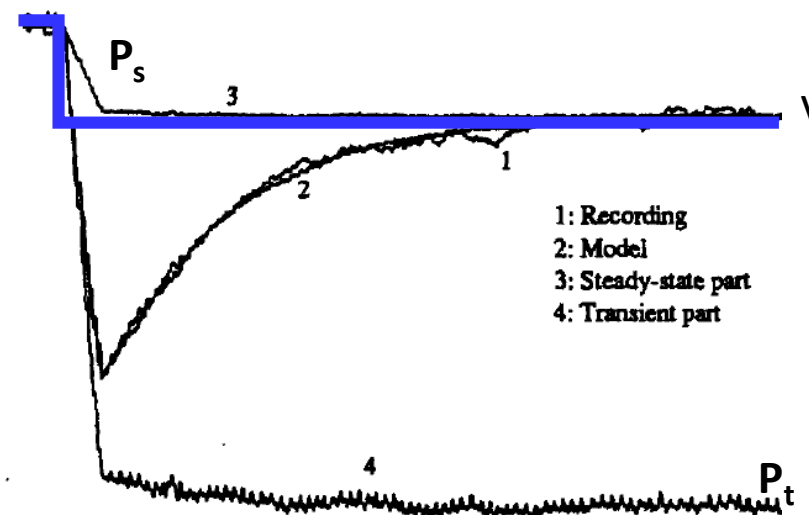
$$Q_d = Q_r + Q_0 \left( \frac{V}{V_0} \right)^{\beta_t}$$

\* This is an **exponential load**.

Curve fitting for least square method (LSM) is used.  
 $V_0, P_0, Q_0$  are chosen as the state before ramp/step.

$$\alpha_t = \log \left( \frac{P_+}{P_0} \right) / \log \left( \frac{V_+}{V_0} \right) \quad \beta_t = \log \left( \frac{Q_+}{Q_0} \right) / \log \left( \frac{V_+}{V_0} \right)$$

$\alpha_t$  and  $T_p$  ( $\beta_t$  and  $T_q$ ) are obtained by using non-linear least square method (NLSM)



# Dynamic Loads: Measurement-Based

To perform experiment/ online monitoring of load:

- Step 1) Obtain a set of input–output data derived from a set of measurement.
- Step 2) Select a load model structure.
- Step 3) Estimate its parameters using a suitable method and estimation criterion.
- Step 4) Validate the derived model with the parameters obtained in Step 3.
- Step 5) If validation criterion is not met, take remedy actions; for example, try another estimation method, or try another model structure and go to Step 3.

Error function to minimize:

$$\min_{p \in Z} \varepsilon(p) = \min_{p \in Z} \frac{1}{2} \sum_{k=1}^N (y(k) - \hat{y}(k))^2$$

Linearized Dynamic Load Model (GNLD)

$$\Delta \dot{x}_p(t) = -\frac{1}{T_P} \Delta x_p(t) + \frac{P_0}{T_P} \left( \frac{N_{ps} - N_{pt}}{V_0} \right) \Delta V(t)$$

$$\Delta P_d(t) = \Delta x_p(t) + \frac{P_0 N_{pt}}{V_0} \Delta V(t)$$

$$\Delta \dot{x}_q(t) = -\frac{1}{T_q} \Delta x_q(t) + \frac{Q_0}{T_q} \left( \frac{N_{qs} - N_{qt}}{V_0} \right) \Delta V(t)$$

$$\Delta Q_d(t) = \Delta x_q(t) + \frac{Q_0 N_{qt}}{V_0} \Delta V(t).$$

- Without solving DAE, and with discrete input, the linearized model can run faster
- better result compared to induction motor model, ZIP and exponential (in P), better result compared to adaptive model and exponential recovery (in Q)



# Dynamic Loads: Linearized Induction Motor

First-order Induction Motor Model:

$$\Delta P + T_{pp} \frac{d\Delta P}{dt} = K_{pv} \left( \Delta V + T_{pv} \frac{d\Delta V}{dt} \right)$$

$$\Delta Q = K_{qv} \Delta V + K_{qp} \Delta P$$

\* For load modelling purpose, 1<sup>st</sup> order is enough,  
But for motor modelling purpose, 3<sup>rd</sup> or 5<sup>th</sup> order  
can be used.

A frequency dependent IM linearized model:

$$\Delta P = K_{pf} \Delta f + K_{pv} \left( \Delta V + T_{pv} \frac{d\Delta V}{dt} \right)$$

$$\Delta Q = K_{qf} \Delta f + K_{qv} \Delta V + K_{qp} \Delta P$$

Remarks:

The window can be Gaussian/Laplacian and it may be adaptive as event triggered.

Composite Load: Induction Motor + ZIP

$$P_{IM} + P_{ZIP} = P_{Total}$$

$$Q_{IM} + Q_{ZIP} = Q_{Total}$$

Parameter Estimation:

$$\Delta \hat{P}_n = A \cdot \Delta \hat{P}_{n-1} + B \cdot \Delta V_n + C \cdot (\Delta V_n - \Delta V_{n-1})$$

$$\Delta \hat{Q}_n = K_{qv} \Delta V_n + K_{qp} \Delta \hat{P}_n$$

$$\Delta \hat{P}_n = \hat{P}_n - P_0, \quad \Delta \hat{Q}_n = \hat{Q}_n - Q_0, \quad \Delta V_n = V_n - V_0$$

$$A = \frac{1}{1 + \frac{h}{T_{pp}}}, \quad B = \frac{h \cdot K_{pv} / T_{pp}}{1 + \frac{h}{T_{pp}}}, \quad C = \frac{K_{pv} \cdot T_{pv} / T_{pp}}{1 + \frac{h}{T_{pp}}}$$

\* T<sub>pp</sub>, K<sub>pv</sub>, T<sub>pv</sub>, K<sub>pq</sub> are parameters to be estimated;  
h = sampling time (NOT necessary to be constant)

# Application: Voltage Stability under fault

5 state (D-Q) model:

$$v_{qs} = r_s i_{qs} + \frac{\omega}{\omega_b} \psi_{ds} + \frac{p}{\omega_b} \psi_{qs}$$

$$v_{ds} = r_s i_{ds} - \frac{\omega}{\omega_b} \psi_{qs} + \frac{p}{\omega_b} \psi_{ds}$$

$$v'_{qr} = r'_r i'_{qr} + \left( \frac{\omega - \omega_r}{\omega_b} \right) \psi'_{dr} + \frac{p}{\omega_b} \psi'_{qr}$$

$$v'_{dr} = r'_r i'_{dr} - \left( \frac{\omega - \omega_r}{\omega_b} \right) \psi'_{qr} + \frac{p}{\omega_b} \psi'_{dr}$$

The static model (or linearized model) cannot show the quick drop of power and fail to predict the trajectory under a fault.

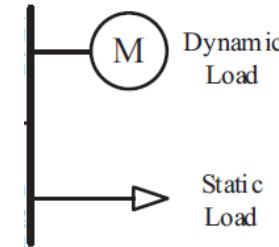
Static Model:

$$T_e = \left( \frac{3}{2} \right) \cdot \left( \frac{poles}{2} \right) \cdot \left( \frac{1}{\omega_b} \right) \cdot (\psi'_{qr} i'_{dr} - \psi'_{dr} i'_{qr})$$

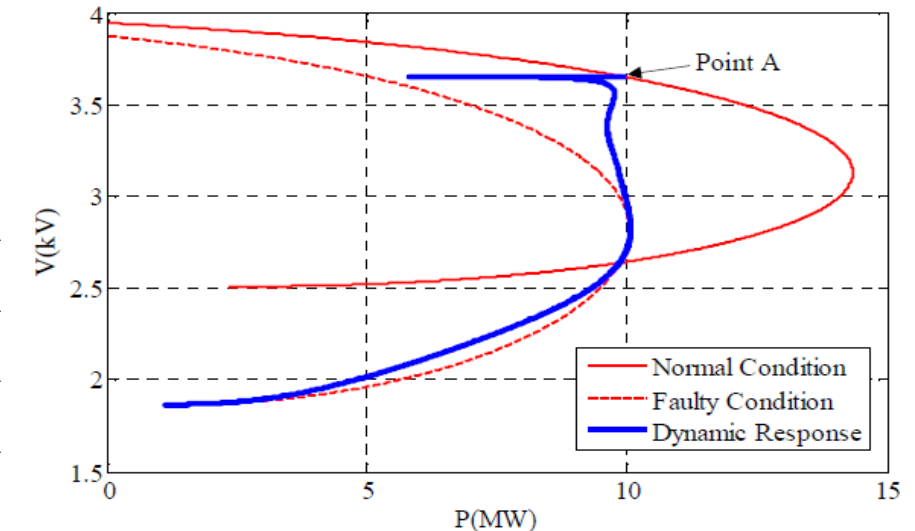
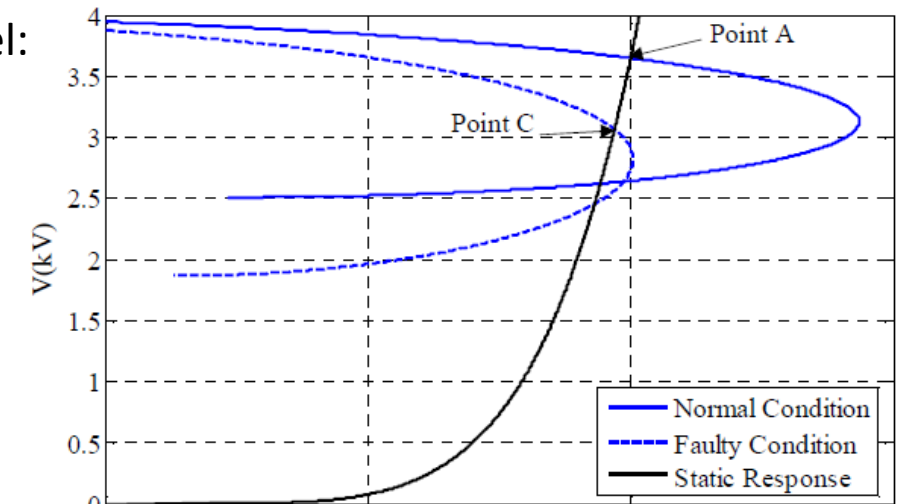
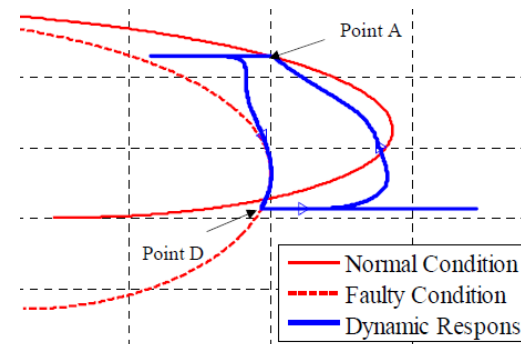
$$T_e = 2H \cdot p \left( \frac{\omega_r}{\omega_b} \right) + T_L$$

$$P = \frac{3}{2} (v_{ds} i_{ds} + v_{qs} i_{qs})$$

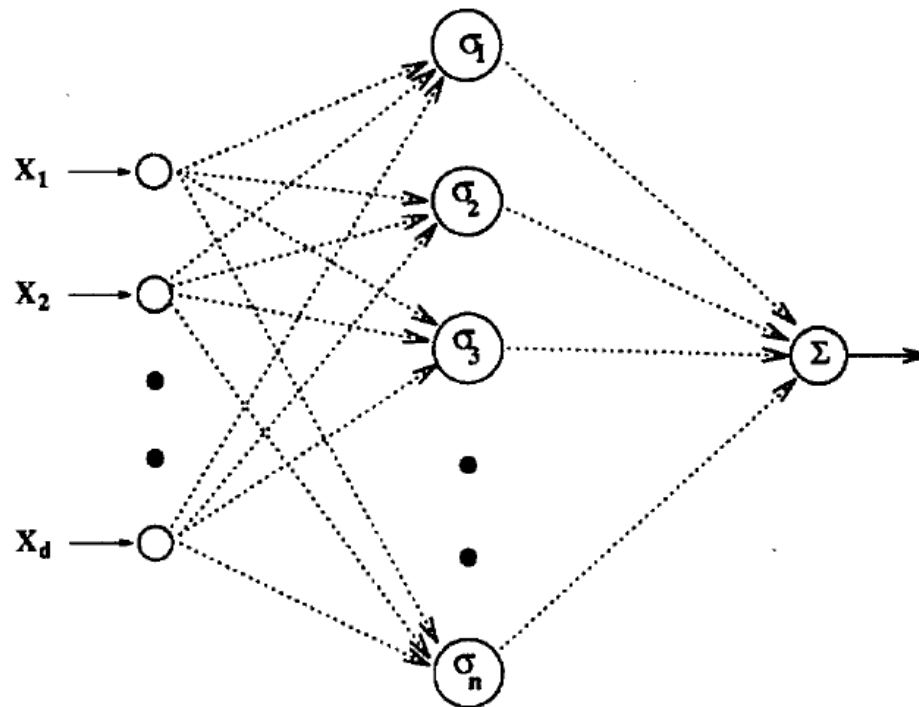
$$Q = \frac{3}{2} (v_{qs} i_{ds} - v_{ds} i_{qs})$$



Dynamic Model:



# Dynamic Loads: Artificial Neural Network (ANN)



Input

Hidden Layer

Output

Feedforward ANN Method

ANN is adaptive and trainable, and it is highly parallel. It can tackle static and dynamics situation.

The feedforward network can be represented as

$$y = \sum_i \sigma_i \left( \sum_j w_{ij} x_j + b_i \right)$$

The activation function is sigmoid:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

As the actual load model is complex, with

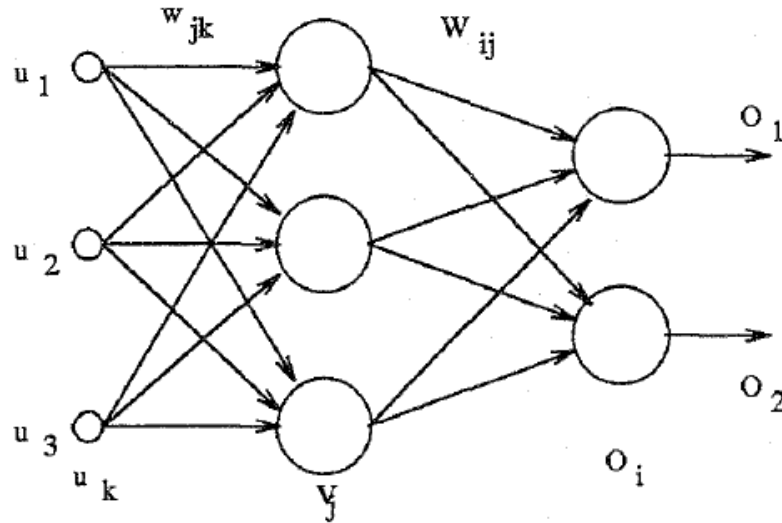
$$\left( \frac{P}{P_0} \right)_{K+1} = f_p \left( \left( \frac{P}{P_0} \right)_K, V_{K+1}, V_K \right), \quad Q_{K+1} = f_q(Q_K, V_{K+1}, V_K)$$

Quadratic Loss & Cross Entropy can be used for back propagation.

# Dynamic Loads: Artificial Neural Network (ANN)

Suppose the targeted output is  $t_i$ , for  $i = 1, 2, \dots, n$

Let  $W_{ij}$  = weight from the hidden unit to the output unit, and  $w_{jk}$  = weight from the input to the hidden unit.



$$\Delta W_{ij} = -\eta \frac{\partial E}{\partial W_{ij}} = \eta \delta_i V_j$$

$$\delta_i = g'(c_i)[t_i - O_i]$$

$$\Delta w_{jk} = -\eta \frac{\partial E}{\partial w_{jk}} = -\eta \sum \frac{\partial E}{\partial V_j} \frac{\partial V_j}{\partial w_{jk}} = \eta \delta_j u_k$$

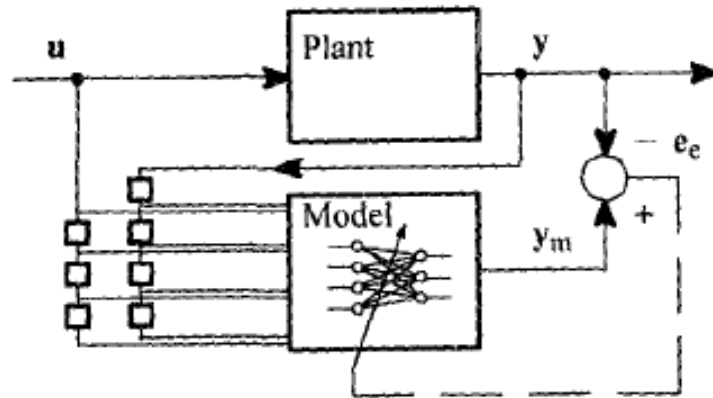
$$\delta_j = g'(h_j) \sum_i W_{ij} \delta_i$$

$$w_{ij}^{\text{new}} = w_{ij}^{\text{old}} + \Delta w_{ij}$$

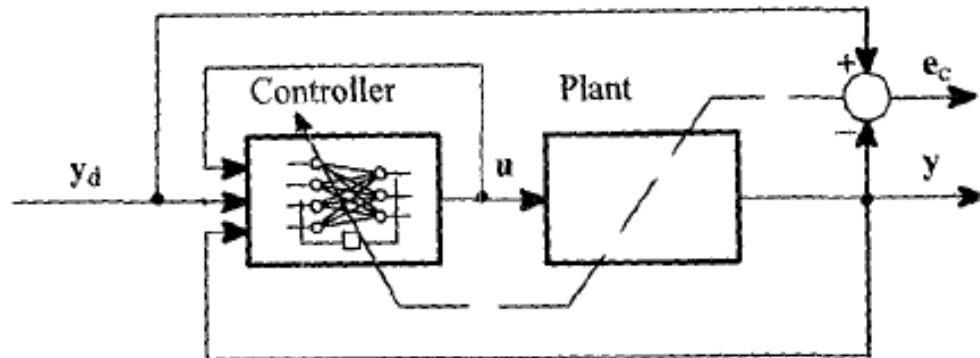
where  $g$  is activation function,  $V$  is the summation,  $O_i$  is the output.

# Dynamic Loads: ANN with control

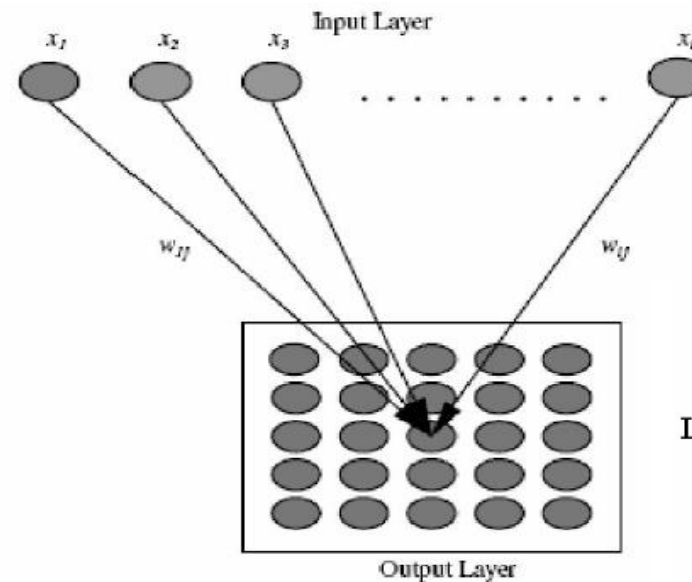
1. A model is trained to obtain plant behavior.



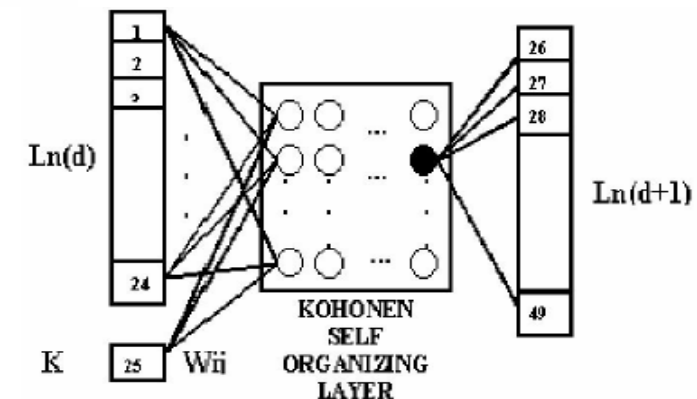
2. Inverse Dynamic Controller controls the plant with error back-propagation through plant.



3. In a long term, load forecasting (in different time frame) is needed. A self-organizing maps stored different cost function is applied. The weight of cost function with minimum value is amended. Neighborhood radius and learning rate are updated. This is coined as Kohonen network.

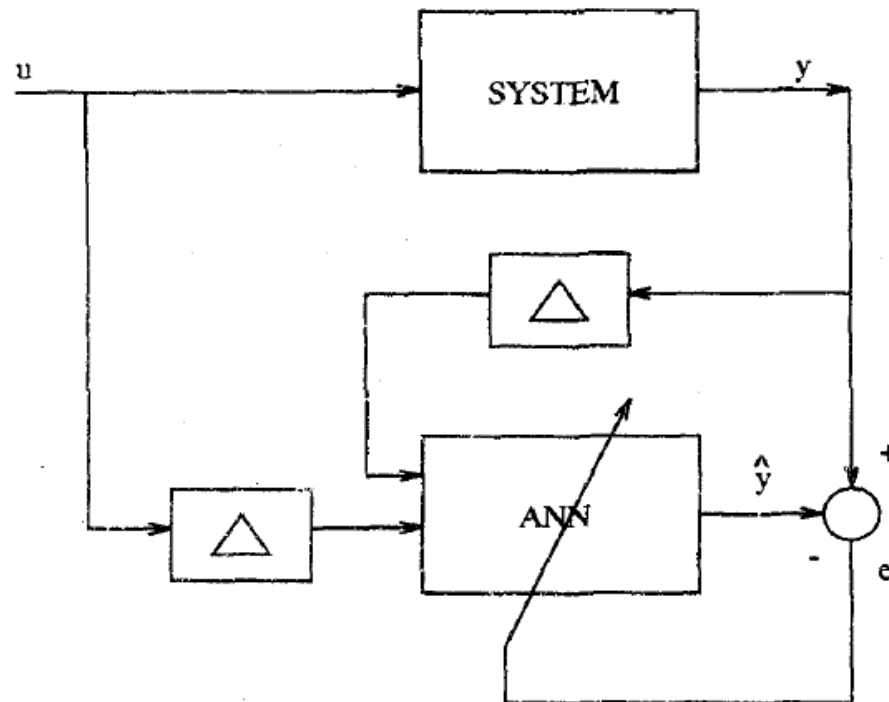


Information like population, peak day, maintenance are imposed to the network.



# Dynamic Loads: Artificial Neural Network (ANN)

A non-linear, autoregressive, moving average with exogenous inputs (NARMAX) model with n-th order ODE can be used to describe the load dynamics.



Identification of Power System Load Dynamics with NN

This techniques allow NN works independently with power system load → it can be used for forecasting.

A third order induction motor model can be used for training.

$$T_o' \frac{dE_q'}{dt} = -\frac{X}{X'} E_q' + \frac{X - X'}{X'} V + T_o' \omega_s (\nu - 1) E_d'$$

$$T_o' \frac{dE_d'}{dt} = -\frac{X}{X'} E_d' - T_o' \omega_s (\nu - 1) E_q'$$

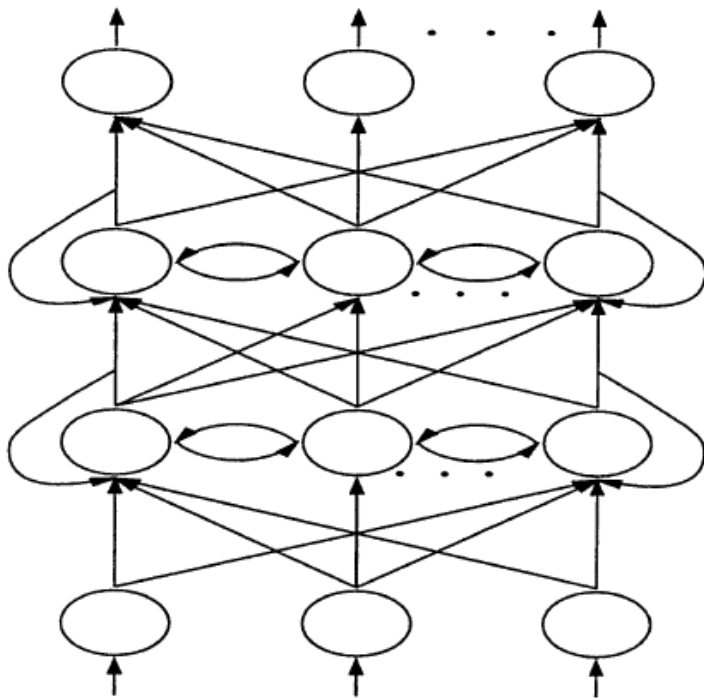
$$2H \frac{d\nu}{dt} = \frac{VE_d'}{X'} - T_m$$

$$Q = \frac{V^2}{X'} - \frac{VE_q'}{X'}$$

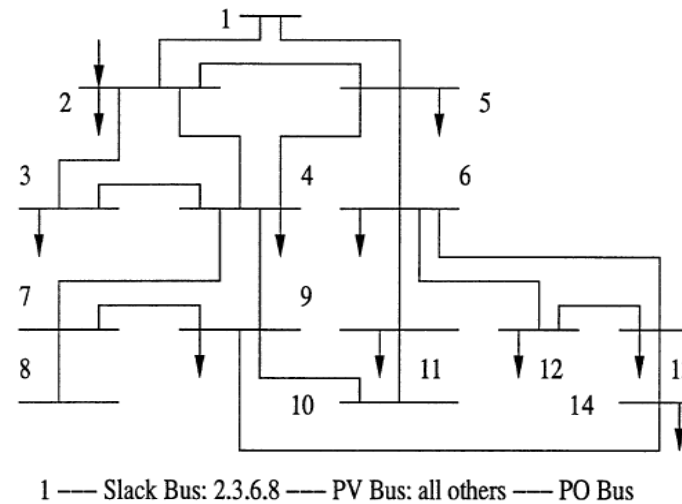
$$P = \frac{VE_d'}{X'}$$

# Dynamic Loads: Recurrent Neural Network

## Recurrent Neural Network

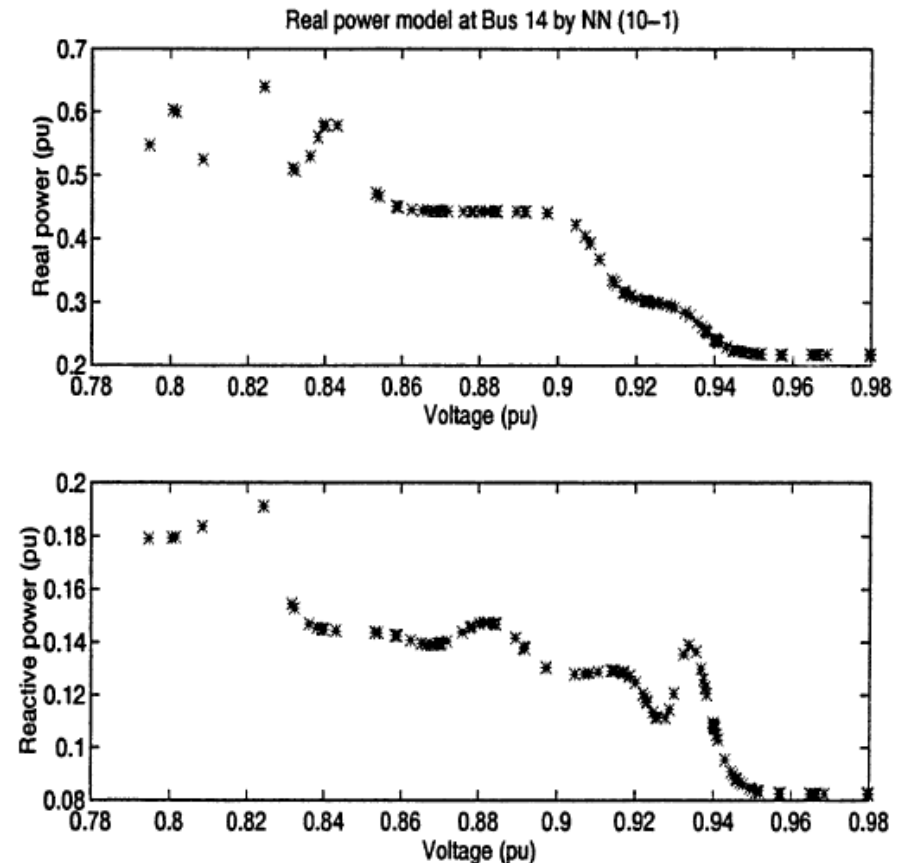


For a 14 buses system,



$$\begin{aligned}x(k+1) &= f(x(k), u(k)) \\z(k+1) &= g(z(k), x(k)) \\y(k) &= h(z(k))\end{aligned}$$

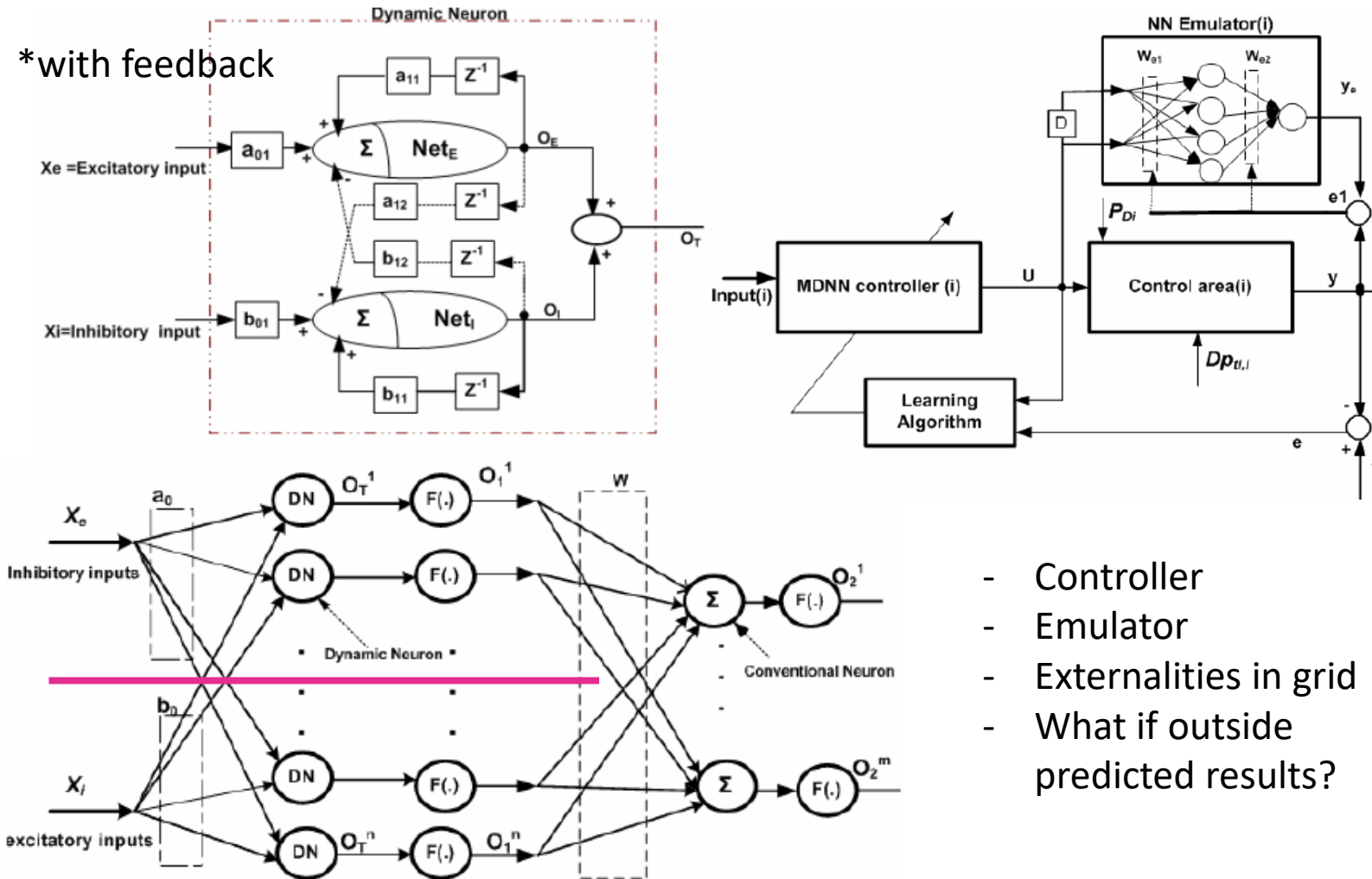
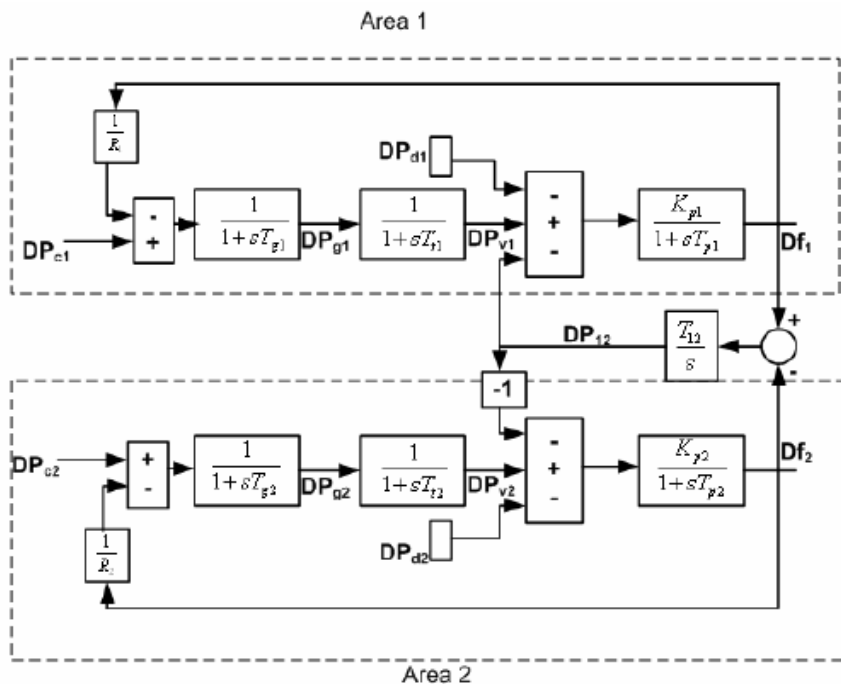
Recurrent NN is used to include aggregate effect of load and input change.  
Inner layer is to provide flexibility and improve efficiency.





# Application: Load frequency Control with NN

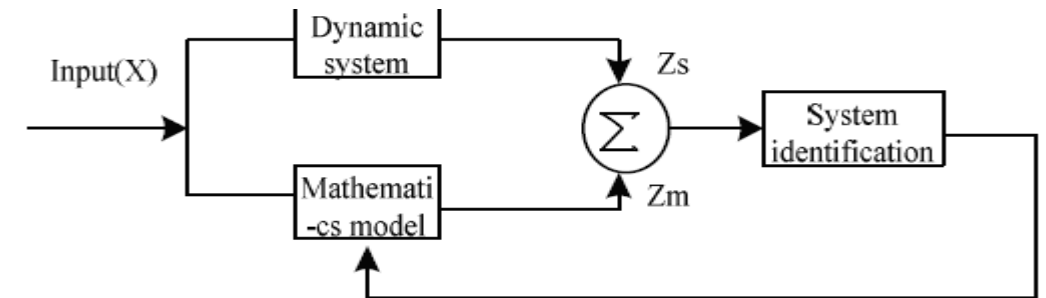
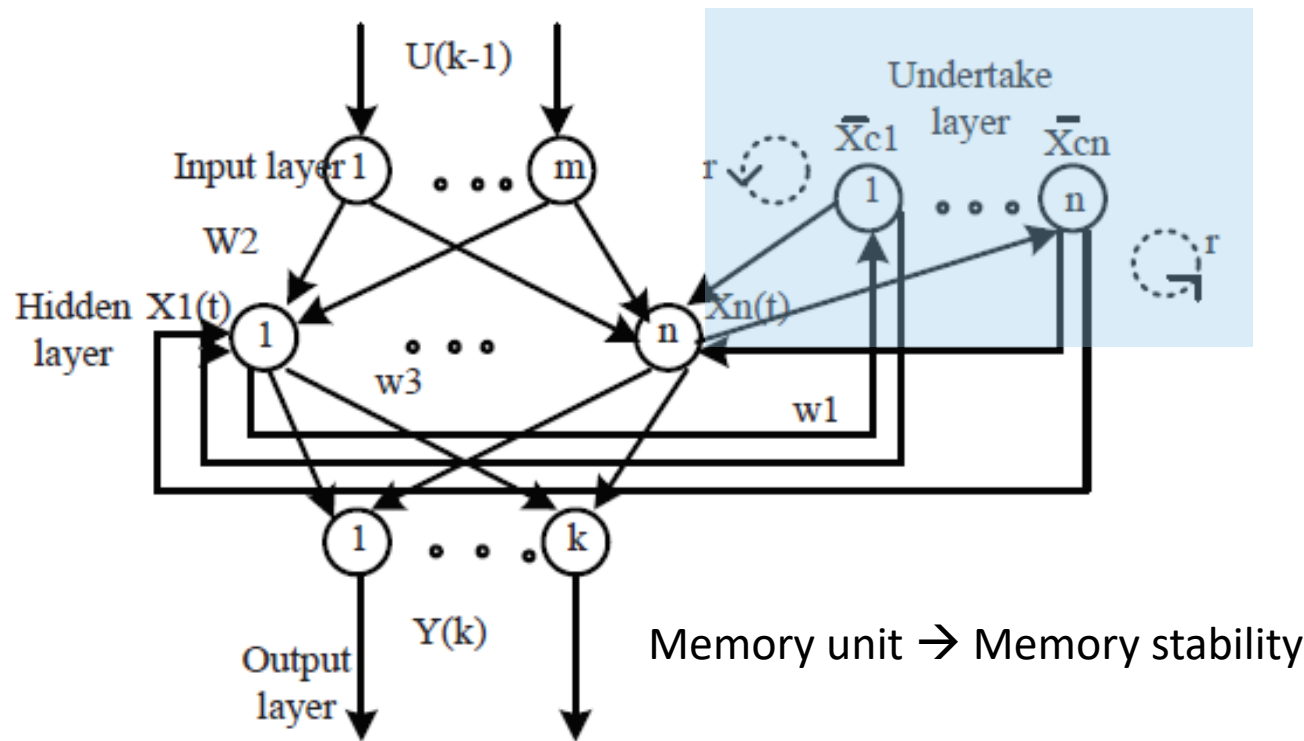
For a 2 area load frequency problem, a modified dynamic neuron model is used.



- Controller
- Emulator
- Externalities in grid
- What if outside predicted results?



# Application: Elman Network for composite load



Error calculation:

$$e_j^k = \left[ \sum_{t=1}^r d_t \cdot v_{jt} \right] b_j (1 - b_j) \quad t = 1, 2, \dots, r$$

Update the weight:

$$w_{ij}(N+1) = w_{ij}(N) + \beta e_j^k a_i^k$$

$$\theta_{ij}(N+1) = \theta_{ij}(N) + \beta e_j^k$$

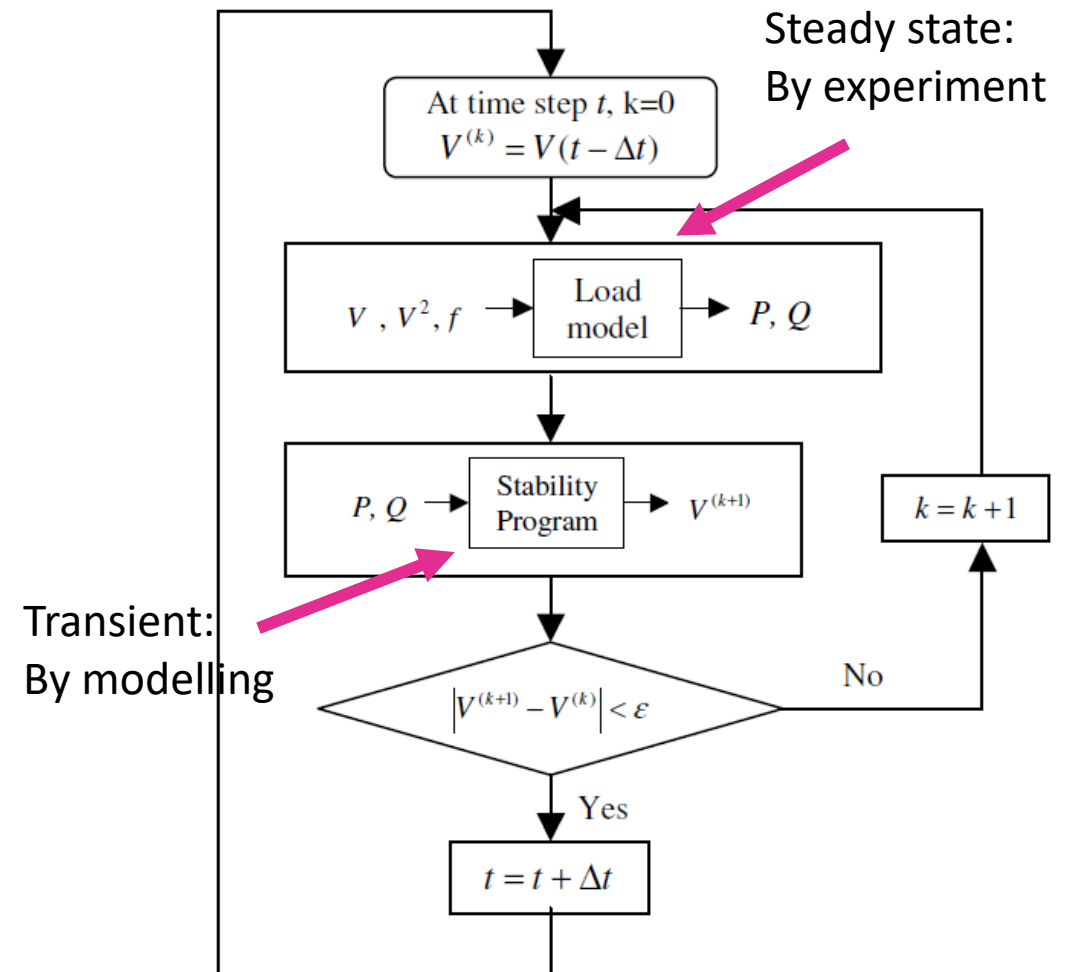
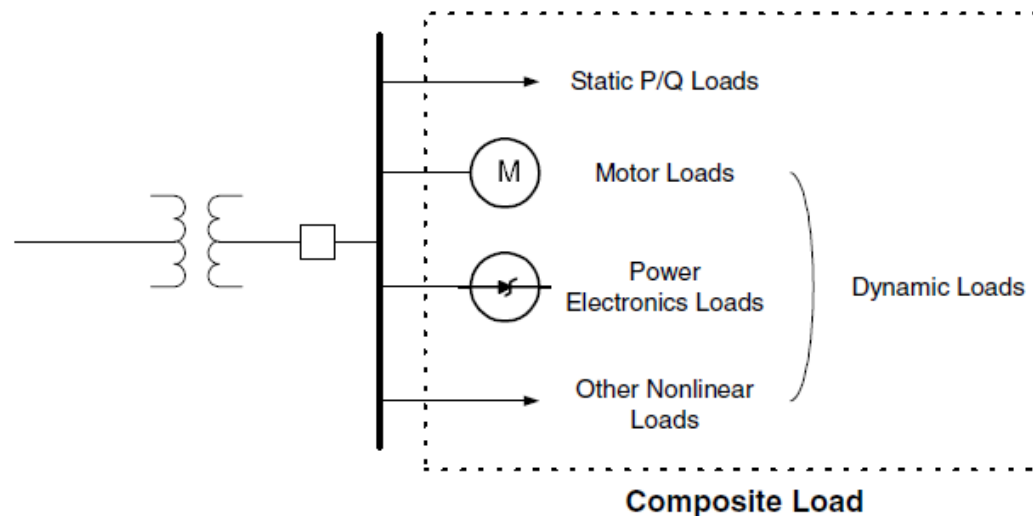
$$i = 1, 2, \dots, m, \quad j = 1, 2, \dots, c, \quad 0 < \beta < 1$$

# Composite Load = Dynamic + Static Load

Different loads have different behaviors.  
e.g. static ZIP load, power electronics and drives,  
motors, energy storage

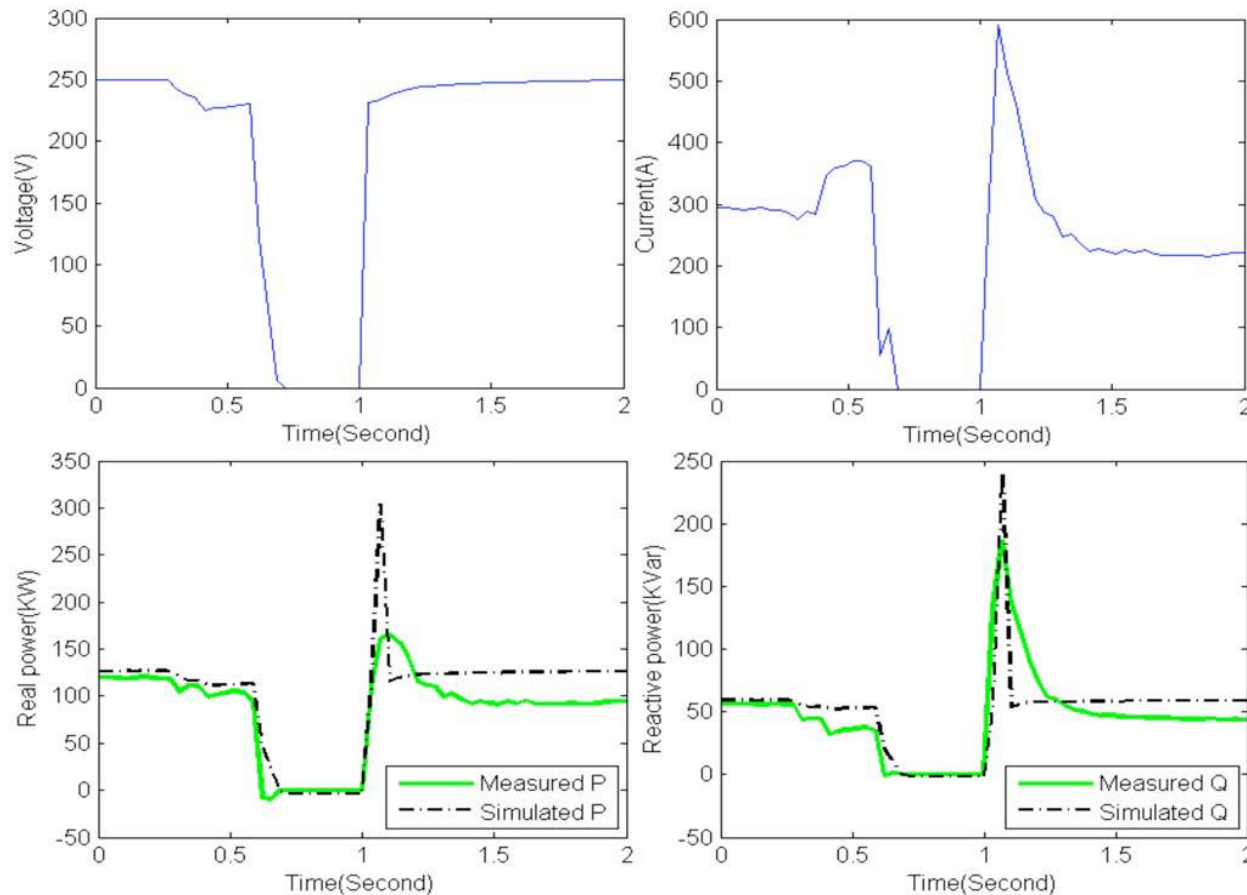
Question:

*Can assumed model capture  $P, f, V$  phenomena?*



# Composite Load = IM + ZIP

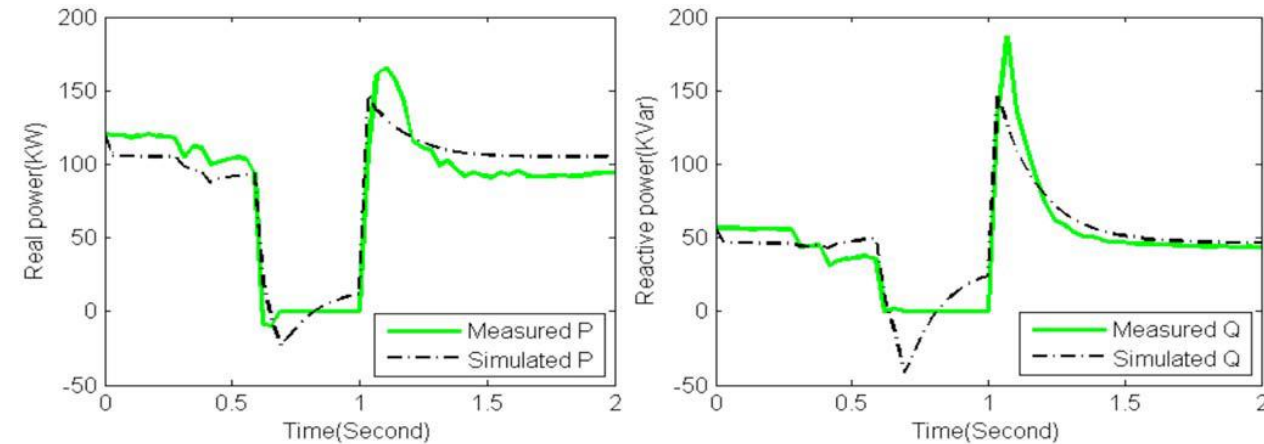
## Pure ZIP Load Modelling with Voltage Event



1. ZIP tends to have higher overshoot as compared to Composite and experiment; but Composite overestimates fall of real and reactive power.
2. Dynamic Load consists of exponential part. It underestimates the overshoot of power with voltage.

*Questions: When to use which?*

## Composite Load Modelling with Voltage Event



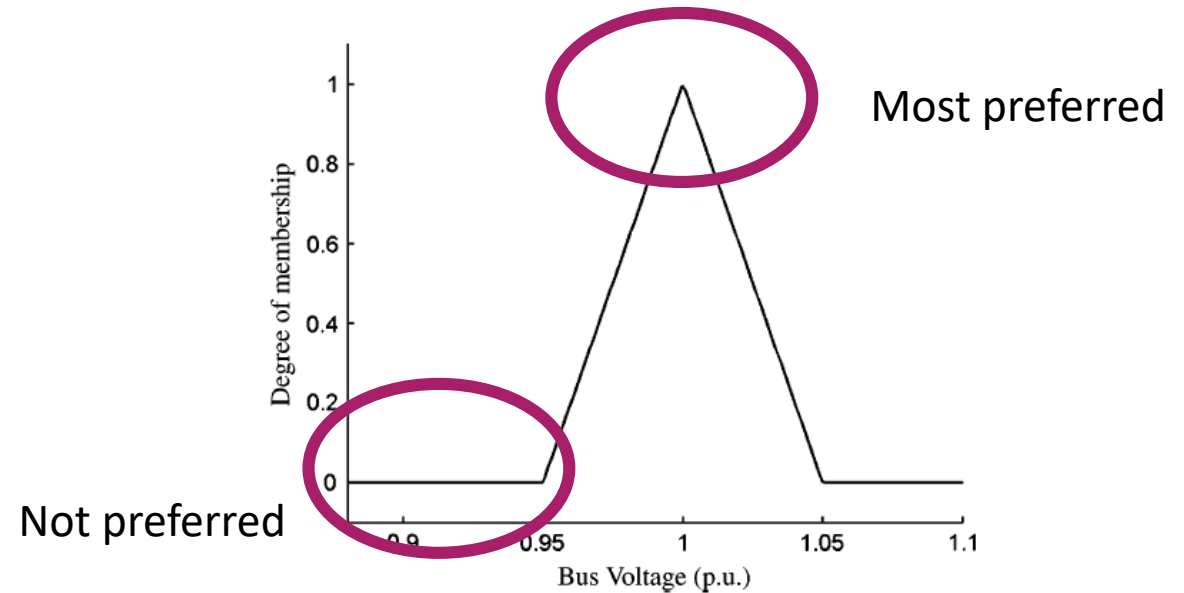
# Application: Cap Bank/VR Placement for CVR

Questions:

1. What form of voltage regulators are used?  
(closed delta, as it can still regulate with open delta.)
2. How much fixed/switched cap is installed? Where should it be installed?
3. What to be used to solve this type of questions?  
(Fuzzy + SPEA2) as fuzzy can input prior knowledge.
4. What are the objective function?  
  - maintain voltage within limit for all buses
  - power flow balance (and power within limit)
  - Cap bank should minimize energy loss

$$f_1 = \left( \sum_{i=1}^{nl} K_{Ei} \Delta E_i \right) + K_C(n_f, n_{sw}, C_{inst}) \quad \text{(energy loss cost, installation cost)}$$

$$f_2 = \frac{1}{nl} \sum_{p=1}^{nl} \left( \frac{1}{nb} \sum_{i=1}^{nb} (1 - \mu(v_i)) \right) \quad \text{(voltage of all buses within limit)}$$



Part I Capacitors														Part II Regulators						
Bus				Units of capacitors on load:										Bus Address ( <i>n'</i> )		tap on load:				
				<i>p</i>				<i>i</i>				<i>l</i>				<i>p</i>	<i>i</i>	<i>l</i>		
27	51	13	45	1	1	2	0	0	1	1	3	3	1	2	3	2	7	5	3	
1°				2°				3°				4°		1°	2°	3°	4°			

# Application: CVR based on AMI

Observation:  $\text{CVR}_f = \% E / \% V = 0.92\%$   
i.e. 0.92% energy reduction with 1% voltage drop

Why not primary sensing? → don't know voltage at PCC  
(all based on modelling and state estimation)

Why not customer metering? → intrusive, expensive  
\* Reported case: overloaded distribution transformer  
excessive voltage drop in conductor

Consideration:

1. Frequency of data sensing  
e.g. daily peak report, 15 min read, exceptional reporting  
load dependent frequency sensing
2. Frequency of switching in hardware
3. Setting of DMS and CVR control  
→ what to do? who to decide in hierarchy?

Improvement may required?

- addition of transformer → split load/ shorten secondary
- reconductoring → solve voltage drop problem

Consideration and Results:

- Modelled primary voltage is not a reliable predictor of metered voltage
- lowest voltage spreads across circuit (ring/radial) and in fact they are more close to substations
- load relief and reconductoring to solve overload?
- Capacitive effect of underground cable shows less overloading and better voltage of related feeders.
- Operational Benefits: resolve issue prior to detection of customers  
e.g. low voltage, open phase, loose connection, wrong tap on distributed Tx, errors in residential meters

# Application: Economic Analysis for CVR

With load flow simulation over a period of time, the following could be found.

- Distribution kW and kWh system losses
- Annual energy kWh consumption
- Peak kW Demand
- System Voltage Range and Averages
- Tap Changing Equipment Operation
- Reactive kVAr demand and kVArh components

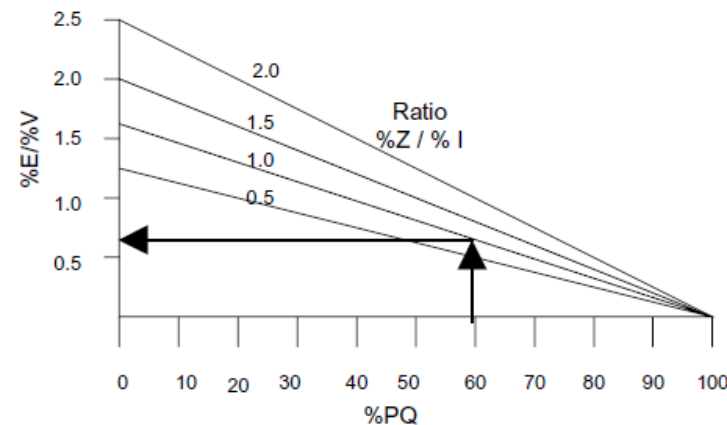
The following could be included for the analysis.

- Optimal switching configurations
- Phase balancing improvements
- Optimal capacitor placement
- Regulator line drop compensation settings
- Time range load flow

The following are modifications for base data model.

- Add, delete, relocate line regulators
- Add, delete, relocate line switched shunt capacitors
- Add, delete, relocate line fixed shunt capacitors
- Add conductors (new construction)
- Reconductor existing lines
- Re-phase existing lines
- Revise capacitor switch controls
- Revise regulator line drop compensation controls

A diagram could be plotted for CVR factor:

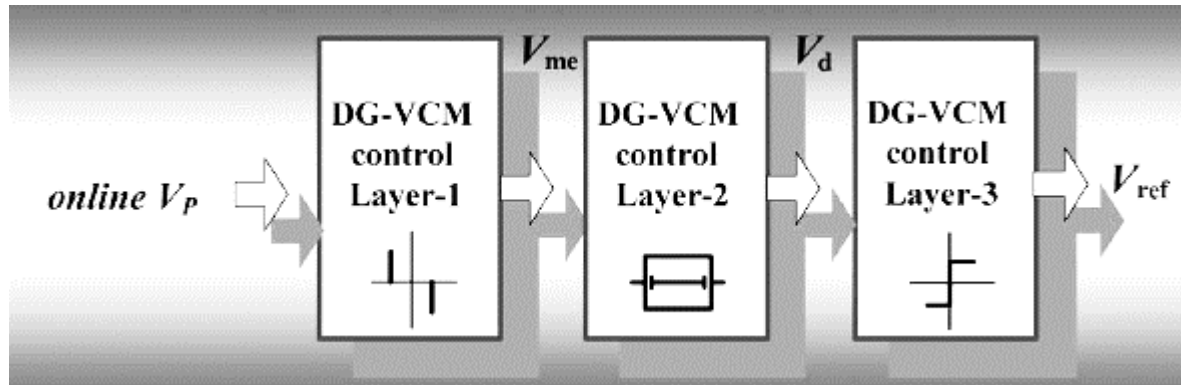


# Application: Coordination in DG grid

Consideration:

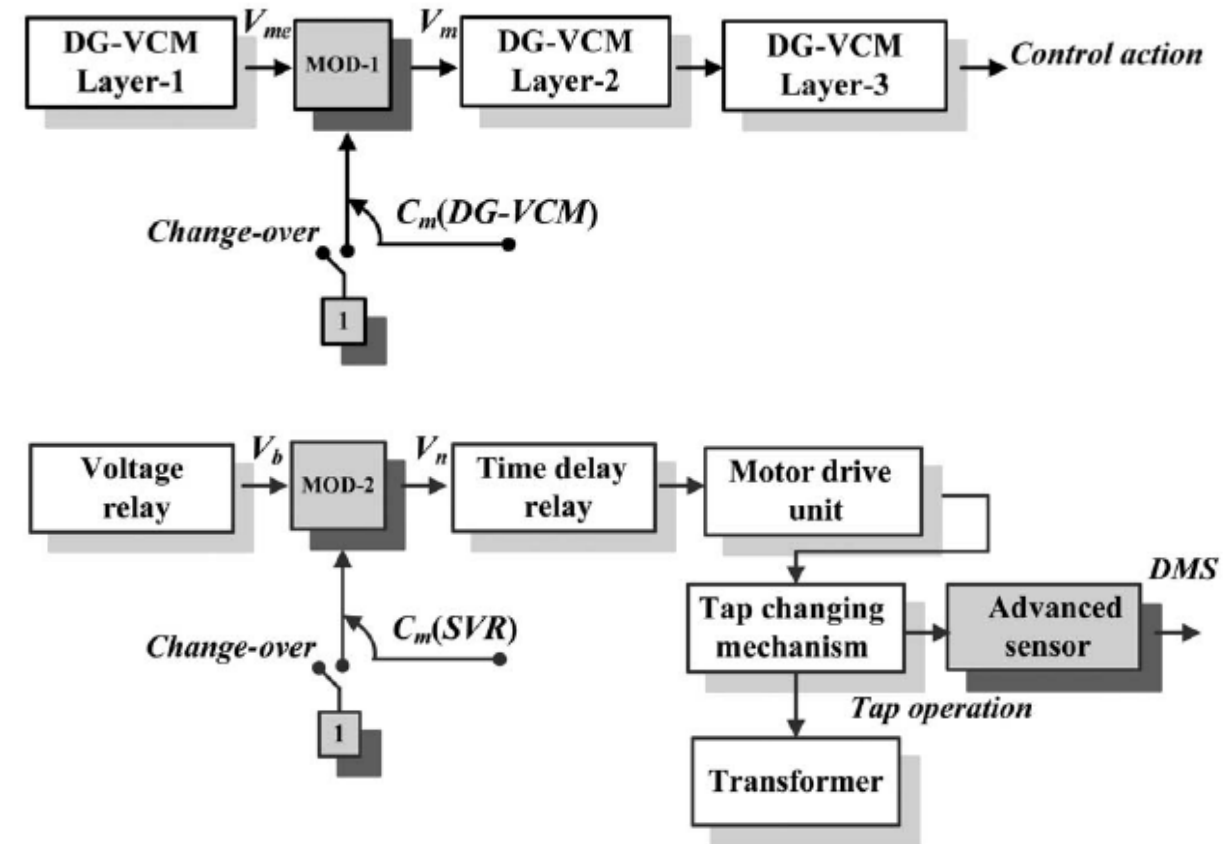
1. maximize voltage support by DG through VCM
2. Real time control of all DG converter and VR.

1. What is the voltage control module (VCM) of DG?  
deadband, limiter, hysteresis



2. Coordination between DGs and VR requires edge detection control layer.

Switching control, zone control, \*delay





# Application: Non-intrusive Load Monitoring

To monitor **load mix** non-intrusively (i.e. not installing any sensors to the client side), *power magnitude* or *V-I characteristics* are often obtained.

But how to detect the load mix?

## 1. Kernel Method

Suppose  $n$  switches are switching over time.  
The system is  $n$  RLC coupled differential equation.  
Putting it into Hilbert Space and Apply Kernel.

$$F(\omega, \mathbf{i}_j) = \sum_{k=1}^N w_k K(\mathbf{i}_j, \mathbf{i}_k) + w_0.$$

$$K(\mathbf{u}, \mathbf{v}) = e^{-\frac{\|\mathbf{u}-\mathbf{v}\|^2}{2p^2}}$$

This can successfully reduce the dimension by using dot product.

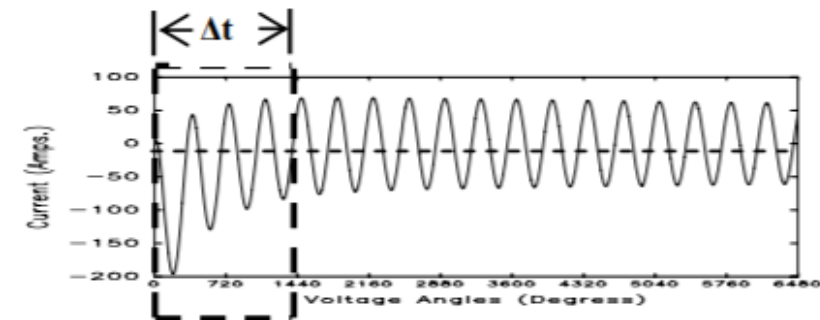
## 2. Principle Component Analysis (PCA)

Eigen-decomposition can provide a lower dimension separation with less noise and large inter-class variance.

$$\frac{1}{C} \sum_{i=1}^C \frac{1}{K} \sum_{k=1}^K [Y(i, k) - \mu_{full}] [Y(i, k) - \mu_{full}]^T.$$

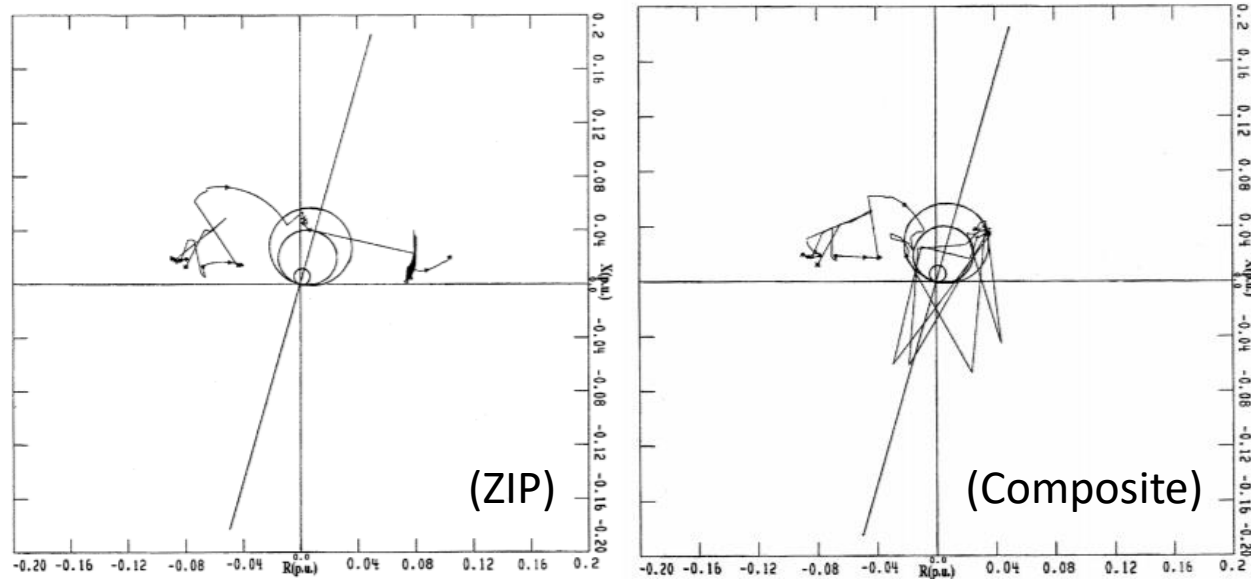
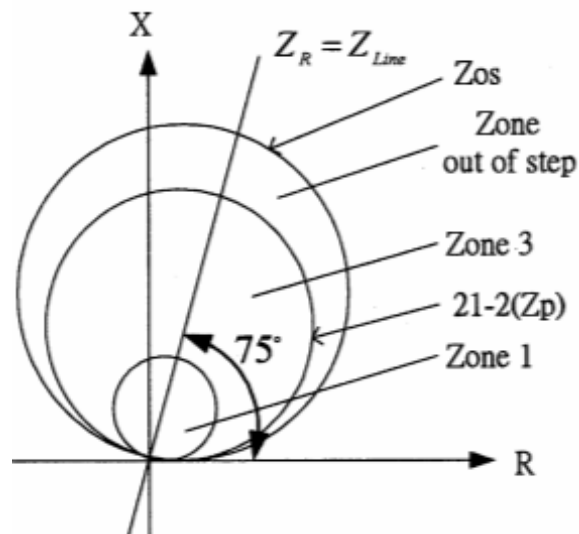
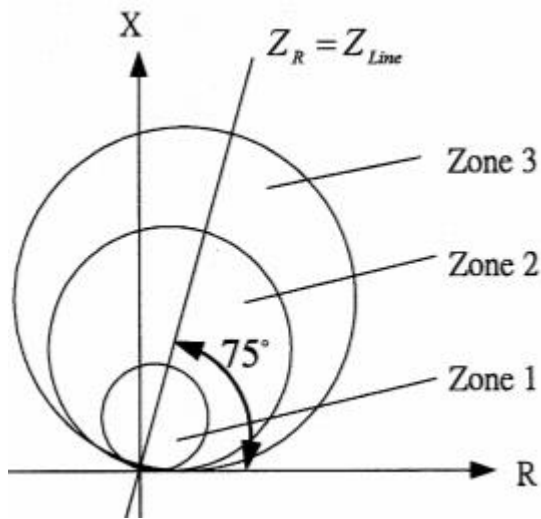
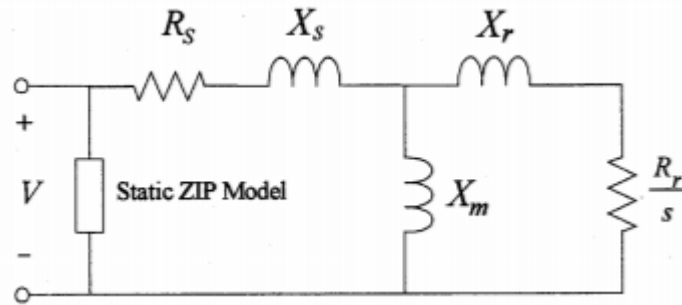
## 3. Frequency domain

To recognize the load, the frequency spectrum can be analyzed by HARR or wavelet transform. Neural Network can also help.





# Application: Distance Relay with Load



1. Load Models: motor, composite, exponential experienced significant swings than others with impedance trajectories.
2. Blocking time: composite model results the worse blocking time.
3. Suggested load model for relay setting is PTI IEEE.