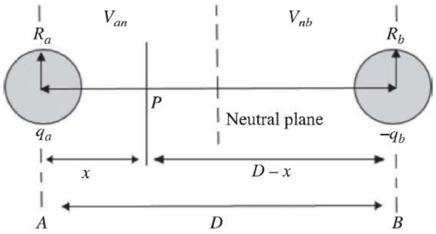
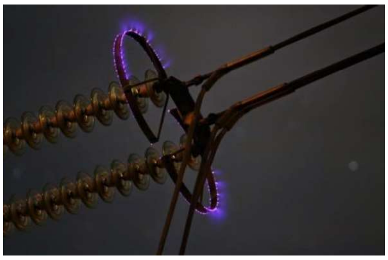
Chapter 4 Clearance, Corona and Surges

Insulation Coordination is the "selection of insulation strength consistent with the expected overvoltage to obtain an acceptable risk of failure with the characteristics of surge protective devices". Overhead line is prone of lightning surges and hence equipment connected are also prone of surges. It is noted that lightning surges are in order of microseconds (us), and protective devices are not fast enough to operate and clear the fault (by shorting to ground?). This chapter aims to discuss corona and clearance, insulation strength, switching overvoltage in transmission lines, shielding, travelling waves, and arrester.

4.1 Corona

Corona effects on conductors are generated by the disruption of air dielectrics around the conductor, when the electrical field reaches the critical surface gradient. The critical gradient for smooth round cylinders with 10 mm radius is about 27 kV_{rms}/cm, failing to 21kV_{rms}/cm for stranded conductors. As the corona discharges are not permanent but occur in form of sparks around the conductor, electromagnetic radiation are emitted from the conductor, causing different types of undesirable effects. The main nuisances of a too high conductor surface gradient are the emission of radio interference (RIV) at the AM band and the production of audible noise (AN) in the vicinity of the line, besides the generation of corona losses. Hence, the maximum conductor surface gradient has to be kept below certain limits in order to avoid undue impacts of electric origin. In practice, it is recommended that the conductor surface gradient of overhead conductors should be limited to around 17 kV_{rms}/cm.





Consider two conductors of a transmission line as shown. Distance between two conductors is D. Consider a point P in between the two conductors. Electric field at point P can be written as

$$G_P = G_a + G_b = \frac{1}{2\pi\varepsilon}\frac{q}{x} + \frac{1}{2\pi\varepsilon}\frac{q}{D-x} \tag{4.1}$$

This equation shows that the nearer to the conductor, electric stress will be more. Now if it is assumed that point P is located nearer to the conductor *a,* then D will be much in compared to x, i.e. 1/(D – x) ~ 0. The electric stress will be maximum near the surface, i.e. x = r.

$$G_P = \frac{1}{2\pi\varepsilon}\frac{q}{x} + \frac{1}{2\pi\varepsilon}\frac{q}{D-x} \approx \frac{1}{2\pi\varepsilon}\frac{q}{x}\bigg|_{x=r} = \frac{1}{2\pi\varepsilon}\frac{q}{r} \tag{4.2}$$

From voltage equation, we can write

$$V_{ab} = V_{an} + V_{bn} = \frac{q}{\pi\varepsilon}\ln\frac{D}{r} \rightarrow \boxed{G_{max} = \frac{1}{2\pi\varepsilon}\frac{q}{r} = \frac{V_{ab}}{2r\ln\frac{D}{r}} = \frac{V_{an}}{r\ln\frac{D}{r}}} \tag{4.3}$$

Phase to neutral voltage may be expressed in terms of maximum electric stress as follows.

$$V = V_{an} = G_{max}\, r\ln\frac{D}{r} \tag{4.4}$$

Voltage at which ionization starts is known as corona voltage (V_C) which can be expressed with G_0, the electric stress when corona starts.

$$V_C = G_0 r\ln\frac{D}{R} \tag{4.5}$$

At air pressure of 76 cm mercury and 23°C temperature, electric stress or voltage gradient required to ionize air or break the air is G_0 = 30 kV/cm (max). Electric stress depends on the density of air and this relation is proportional. Mathematically stress can be written as

$$G = G_0\delta = G_0\frac{3.92b}{273+t} \tag{4.6}$$

where δ is known as air density factor, b is barometric air pressure and t is temperature. Hence, considering air density factor,

$$V_C = G_0\delta r\ln\frac{D}{R} = \boxed{G_0\frac{3.92b}{273+t}r\ln\frac{D}{R}} \tag{4.7}$$

Minimum phase-to-neutral voltage at which visual effect or glow is observed due to ionization is known as visual corona voltage. Mathematically, visual corona voltage is expressed as

$$V_v = m_0 G_0\delta\left(1+\frac{0.3}{\sqrt{\delta r}}\right)\ln\frac{D}{R} \tag{4.8}$$

Peak's formula:

$$P_{loss} = \frac{21\times10^6}{\ln^2\frac{D}{r}} fV^2F \;\; [\mathrm{kW}] \qquad \frac{V}{V_C} < 1.8 \tag{4.9}$$

Peterson's formula:

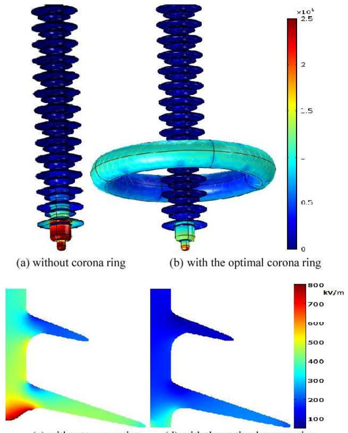
$$P_{loss} = \frac{244\times10^3}{\delta}(f+23)\sqrt{\frac{r}{D}}(V-V_C)^2 \;\; [\mathrm{kW}] \qquad \frac{V}{V_C} > 1.8 \tag{4.10}$$

(4.9) and (4.10) shows that corona loss increases with the square of voltage indicating that it will be high in EHVAC lines. Also, it shows that corona has two parts: (i) frequency – independent part, indicating that corona loss will be present in DC line also with comparatively less magnitude and (ii) frequency – dependent part, indicating that corona loss in AC is more than that of DC lines.

To reduce corona discharge, increase of distance, increase of line conductor radius, reduction of line voltage, use of HVDC, utilization of bundled conductor and corona ring can help.

Corona ring is widely used in OHL with transmission voltage 220kV or above to reduce corona discharge. It is made of conducting materials of ring shape of a larger diameter. It is electrically connected with line conductor and kept at conductor potential. Corona discharge depends on gradient of electric field which is high near the surface of the conductor. If there is any sharp end or sharp surface, electric field gradient increases. If potential gradient exceeds dielectric strength of air (30kV/cm) or critical disruptive voltage of air, plasma of ionized air forms and corona discharge takes place.

Corona discharge leads to corona losses at EHV system. It also increases line capacitance and hence charging and discharging current of the line. Triple-frequency current flows in grounded AC system and hence triple frequency in voltage waveform.


(a) without corona ring (b) with the optimal corona ring
(c) without corona ring (d) with the optimal corona ring

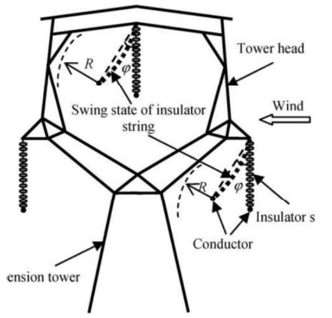
4.2 Clearance

To determine the clearance requirement, both fast-front or slow-front overvoltage or power-frequency voltage exhibit at line-to-ground voltage, i.e. to any grounded objects, or line-to-line voltage should be considered.

The required withstand voltage of an air gap is determined by considering an x % probability of being exceeded taking the critical voltage $U_{50\%}$ into account, so that

$$U_{rw} = U_x = U_{50\%} - n_x \sigma_u \quad (4.11)$$

where $U_{50\%}$ is the 50% withstand voltage of the air gap, σ_u standard deviation and n_x the number of standard deviations.



For transient stresses (fast-front and slow-front overvoltages), the required withstand voltage should be the **90%** withstand voltage of air gap. It is determined as a function of the 50% withstand voltage by the following relation

$$U_{rw} = U_{(x=90\%)} = U_{50\%} - 1.3\,\sigma_u$$

As far as the power frequency voltage is concerned, the required withstand voltage U_{rw} is considered as a deterministic parameter.

$$U_{rw} = U_{(x=100\%)} = U_{50\%} - 3\,\sigma_u$$

The required design withstand voltage can be determined by employing the deviation factor K_z and coefficient of variation v_u through:

$$U_{rw} = U_{50\%} - n_x\sigma_u = (1 - v_u n) U_{50\%} = K_z U_{50\%}$$

Overvoltage Type	K_Z
Lightning Impulse	0.961
Switching Surge	0.922
Power Frequency	0.910

In general, the **type of air gap** affects its dielectric withstand strength. The 50% withstand voltage $U_{50\%}$ of an air gap of any arrangement can be equated as a function of the voltage of rod-to-plane ($U_{50\%rp}$) air gap through

$$U_{50\%} = K_g U_{50\%rp}$$

where K_g designated the gap factor. For every type of voltage stress, the respective gap factor can be related to the gap factor for switching overvoltage as follows.

Overvoltage Type	Formula
Slow-Front OV	$K_{g\_sf} = K_g$
Fast-Front OV	$K_{g\_ff} = 0.74 + 0.26K_g$
Power Frequency	$K_{g\_pf} = 1.35\,K_g - 0.35K_g^2$

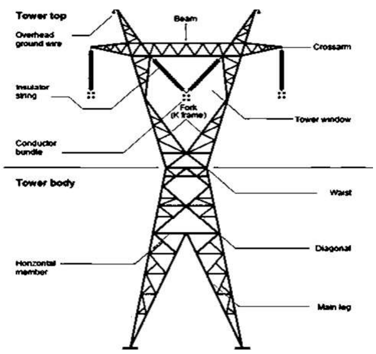
(4.12)

K_g	SF_OV	FF_OV	PFV
Rod-to-Plane	1.00	1.00	1.00
Conductor to obstacle	1.30	1.08	1.16
Conductor to Plane	1.15	1.04	1.09
Conductor to Tower Window	1.25	1.07	1.14
Conductor to Tower	1.45	1.12	1.22
Conductor to Guy	1.40	1.10	1.20
Conductor to Conductor	1.60	1.16	1.26

Hence, the required design withstand voltage resulted is

$$U_{rw} = K_z K_g U_{50\%rp} \quad (4.13)$$

The values of air gap factors, applicable to slow-front overvoltages, depend on the air gap arrangement. Four types arrangements considered include conductor-to-obstacles (**external clearance**), conductor-to-conductor in tower window, i.e. I-string or V-string in the tower window (**internal clearance**), conductor-to-tower from free swinging insulator (internal clearance) and conductor-to-conductor.



The relation between withstand voltage of an air gap and the clearance distance d can be expressed as

$$U_{rw} = K_z K_g f(d) \quad (4.14)$$

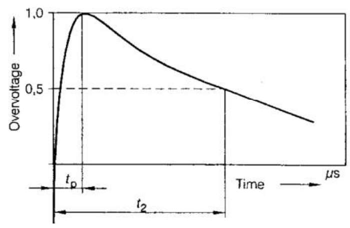
The withstand voltage of any self-restoring insulation regarding slow-front overvoltages with front time 250us and time to half value 2500us is significantly lower than the corresponding voltage for fast-front overvoltages of the same polarity. Practically, the withstand voltage of a rod-to-plane air gap up to 25 m distance, for positive polarity and the standardized **slow-front** wave shape, can be determined by

$$U_{50\%rp\_sf} = 1080\ln(0.46d + 1) \quad (4.15)$$

where d is the clearance distance in m and $U_{50\%rp\_sf}$ represents the peak value of impulse in kV.

For **standard lightning impulses** with a front time of 1.2 us and time to half value of 50 us, and for rod to plane air gaps up to 10 m distance, the critical polarity is determined by

$$U_{50\%rp\_ff} = 530d \quad (4.16)$$



The 50% withstand voltage $U_{50\%rp}$ of a rod-to-plane air gap for power frequency voltage is determined by

$$U_{50\%rp\_pf} = 750\sqrt{2}\ln(1 + 0.55d^{1.2}) \quad (4.17)$$

From the above calculation, phase-to-earth clearance D_{el} and phase-to-phase clearance D_{pp} for lightning surge (fast-front, ff) and switching surge (slow-front, sf) can be calculated. Power frequency clearance is also given.

Lightning Surge Withstand Voltage (kV)	D_{el_ff} (m)	D_{pp_ff} (m)
400	0.77	0.85
600	1.14	1.26
800	1.50	1.68
1000	1.88	2.08
1200	2.23	2.50
1400	2.61	2.92
1600	2.98	3.33

Switching Surge Withstand Voltage (kV)	D_{el_sf} (m)	D_{pp_sf} (m)
400	0.88	1.02
600	1.44	1.67
800	2.07	2.45
1000	2.84	3.41
1200	3.71	4.57
1400	4.77	5.97
1600	6.02	7.66

Power Frequency of Equipment (kV)	D_{pf_pe} (m)	D_{pf_pp} (m)
145	0.27	0.42
245	0.43	0.69
420	0.70	1.17
525	0.86	1.47

To determine the overvoltage (kV) to be withstood and corresponding clearance, the following formula can be used.

Overvoltage Type	Rod-to-Plane Voltage Formula	Clearance Formula
Fast-Front OV	$U_{rp} = U_{90\%ff\_ins} = 530K_{z\_ff}K_{g\_ff\_ins}d_{ins}$	$D_{el\_ff} = U_{90\%ff_{ins}}/(530K_aK_{z_{ff}}K_{g_{ff}})$ $D_{el\_ff} = 1.2U_{90\%ff_{ins}}/(530K_aK_{z_{ff}}K_{g_{ff}})$
Slow-Front OV	Phase to Earth: $U_{rp} = K_{cs}U_{2\%sf}$ Phase to Phase: $U_{rp} = 1.4K_{cs}U_{2\%sf}$	$D_{el\_sf} = 2.17\ \exp\left(\frac{K_{cs}U_{2\%sf}}{1080K_aK_{z_{sf}}K_{g_{sf}}} - 1\right)$ $D_{pp\_sf} = 2.17\ \exp\left(\frac{1.4K_{cs}U_{2\%sf}}{1080K_aK_{z_{sf}}K_{g_{sf}}} - 1\right)$
Power Frequency	Phase to Earth: $U_{rp} = \sqrt{2}U_s/\sqrt{3}$ Phase to Phase: $U_{rp} = \sqrt{2}U_s$	$D_{pf\_pe} = 1.64\ \exp\left(\frac{U_s}{750\sqrt{3}K_aK_{zpf}K_{gpf}} - 1\right)$ $D_{pf\_pp} = 1.64\ \exp\left(\frac{U_s}{750K_aK_{zpf}K_{gpf}} - 1\right)$

(4.18)

where K_{z_ff} is deviation factor (K_{z_ff} = 0.961), $K_{g\_ff\_ins}$ is lightning impulse gap factor for insulator set and d_{ins} the flashover distance of insulator set, $U_{2\%sf}$ is the overvoltage with 2% probability to be exceed and K_{cs} is the risk of failure (1.05, corresponding to flashover risk of 0.001), and U_S is highest system voltage.

Example

Given a line located at altitude of 500m above sea level and highest power frequency voltage is 420kV. The lightning withstand voltage $U_{90\%ff\_ins}$ is 1780kV and statistical switching overvoltage $U_{2\%\_sf}$ is 1050kV. Taking K_{cs} = 1.05,

- Phase-to-Earth: U_{rp} = 1.05 (1050) = 1103 kV, Phase to Phase above 1100kV: 1544kV

The altitude factor K_a = 0.992 for overvoltage, K_a = 0.975 for phase-to-earth and K_a = 0.982 for phase-to-phase. The deviation factor from (4.12) are K_{z_ff} = 0.961, K_{z_sf} = 0.922, K_{z_pf} = 0.910, the resulting clearances for fast-front overvoltage from (4.18):

Fast Front Overvoltage:

Phase to Tower:

$$D_{el\_ff} = \frac{U_{90\%ff_{ins}}}{530K_aK_{z_{ff}}K_{g_{ff}}} = \frac{1780\text{ kV}}{530\ \times 0.992 \times 0.961 \times 1.12} \boxed{= 3.15\text{ m}}$$

Phase to Phase:

$$D_{el\_ff} = \frac{1.2U_{90\%ff_{ins}}}{530K_aK_{z_{ff}}K_{g_{ff}}} = 1.2 \times 3.15 \boxed{= 3.64\text{ m}}$$

Slow Front Overvoltage:

Phase to Tower:

$$D_{el\_sf} = 2.17\ \exp\left(\frac{K_{cs}U_{2\%sf}}{1080K_aK_{z_{sf}}K_{g_{sf}}} - 1\right) = 2.17\left(\frac{1103}{1080 \times 0.992 \times 0.992 \times 1.45} - 1\right) = 2.52\text{ m}$$

Phase to Phase:

$$D_{pp\_sf} = 2.17\ \exp\left(\frac{1.4 \times 1103}{1080 \times 0.992 \times 0.992 \times 1.45} - 1\right) = 3.60\text{ m}$$

Power Frequency:

$$D_{pf\_pe} = 1.64\ \exp\left(\frac{420}{750\sqrt{3} \times 0.975 \times 0.922 \times 1.22} - 1\right)^{0.833} = 0.61\text{ m}$$

For swing action, probability of wind velocity should be considered. Gumbel Distribution for wind velocity V_τ corresponding to a given return period τ (in years) can be determined from

$$V_\tau = \bar{V} - \sigma_V\left[0,45 + \ln(-\ln(1 - \frac{1}{\tau})) \times \frac{\sqrt{6}}{\pi}\right] \tag{4.19}$$

(4.19) can be used to establish wind velocity having a high probability of occurrence during one year, e.g. wind velocity with a return period of two year and to derive the yearly time distribution of wind velocity, Weibull distribution is used.

$$P(V \le V_\tau) = 1 - \exp\left[-\left(\frac{V_\tau}{V_\eta}\right)^\beta\right] \tag{4.20}$$

where β for extreme statistics is 2.0.

Swing angle of an **insulator set** may be related to wind velocity by

$$\bar{\phi}_{ins} = \tan^{-1}\left(\frac{\frac{\rho}{2}C_C V_R^2 G_L D a_W + \frac{Q_{Wins}}{2}}{W_C + \frac{W_{ins}}{2}}\right) \tag{4.21}$$

given that ρ = air density depending on temp, RH and altitude above sea level, C_C = drag factor (1.0) for stranded conductors, V_R = reference wind velocity, $G_{L=}$ = span factor taking into account the effect of span length, D = conductor diameter, a_W= wind span, Q_{Wins} = wind load on insulator set, W_C = effective conductor weight with the differences in level to conductor attachment, W_{ins} = dead weight of insulator set.

Swing angle for a **conductor** alone follows

$$\bar{\phi}_C = \tan^{-1}\left(\frac{\frac{\rho}{2}C_C V_R^2 G_L D a}{m_C g a}\right) \tag{4.22}$$

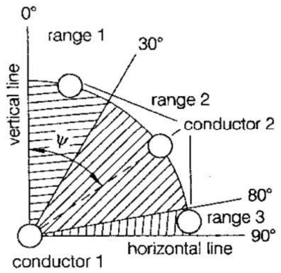
given that a = span length and m_C = mass of conductor per unit length.

Using the 5-minute mean wind velocity is a conservative assumption, as confirmed by observation. It can be assumed that the actual swing angles for a given mean wind velocity follow a Normal distribution function with a mean value of $\bar{\phi}$ resulting from (4.21) and (4.22), whereas the standard deviation is given by the reference wind velocity V_R with

$$\sigma_\phi = 2.25\left(1 - \exp\left(-\frac{V_R^2}{230}\right)\right) \tag{4.23}$$

As a result, the most unfavorable conductor position assuming swing angles can be calculated with $\phi_C = \bar{\phi}_C \pm 2\sigma_\phi$.

Range of Swing Angle ϕ_C	Relative Position between Conductor 2 and 1		
k_C	$\psi < 30^o$	$30^o < \psi < 80^o$	$\psi > 80^o$
> 65°	0.95	0.75	0.70
55° – 65°	0.85	0.70	0.65
40° – 55°	0.75	0.65	0.62
< 40°	0.70	0.62	0.60



Minimum clearance c_{min} of conductors at midspan in still air should be at least

$$c_{min} = k_C\sqrt{f_C + l_k} + 0.75D_{pp} \;\; [\mathrm{m}] \tag{4.24}$$

However, this should not less than k_C in [m] in case of phase-to-phase clearance. Between conductor and earth wire, it applies:

$$c_{min} = k_C\sqrt{f_C + l_k} + 0.75D_{el} \;\; [\mathrm{m}] \tag{4.24}$$

where f_C = sag of the conductor at temp of 40°C [m], l_k = length of that part of the insulator set swinging orthogonally to the line direction [m], k_C = coefficient in table, D_{pp}, D_{el} = minimum clearance (phase-to-phase, phase-to-earth) in m.

Example

A 500kV OHL is located at an altitude of 500m above sea level and will be equipped with 22 cap and pin insulators U160BL. The highest power frequency voltage is 550kV. The lightning withstand voltage $U_{90\%ff\_ins}$ is 1725kV. The statistical switching voltage $U_{2\%sf}$ is 1175kV. The slow-front overvoltage to be considered follow (2.18) taking K_{CS} = 1.05 into account:

Phase-to-Earth: $U_{rp} = K_{cs}\,U_2\%sf = 1.05\,(1175) = 1234\text{ kV}$; Phase-to-Phase at voltage above 1100kV: $U_{rp} = 1.4\,(1.05)(1175) = 1727\text{kV}$

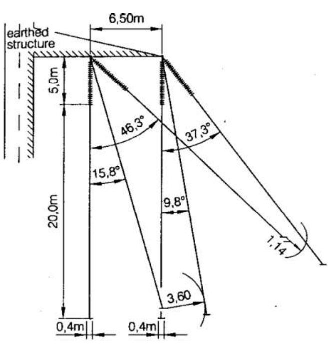
The resulting clearance for fast-front OV, slow front OV and power-frequency can be determined with (4.18).

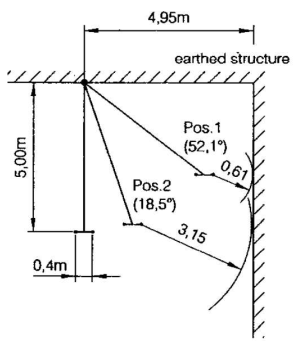
(FF-OV): D_{el} = 3.05 m, D_{pp} = 3.53 m; (SF-OV): D_{el} = 2.97 m, D_{pp} = 4.30 m; (PF-V): D_{pf_pe} = 0.89 m, D_{pf_pp} = 1.53 m.

For design of the tower top geometry, swing angle having a probability of occurrence of 1% or more during a year should be combined with the clearance distance necessary to withstand switching or lightning surges. To take care of the scattering of swing angles under a given wind velocity, (4.23) can be used. For V_R = 32.0 m/s with a standard deviation of 2.25°, the extreme angle from $\bar{\phi}_C \pm 2\sigma_\phi$ = 47.6° + 4.5° = 52.1° for insulator and 41.8° + 4.5° = 46.3° for conductors. [Note: 47.6° is calculated for the **400kV tower** insulator with (4.23) given that G_L = 0.94 for 400m span for 50 years return period]. With an insulator set 5 m long and a conductor sag of 20 m, the case of fast-front overvoltage is pre-dominant and extreme swing angle combined with the clearance is considered. Given the electrical clearance in [m] for the 400kV tower as

Overvoltage Type	Phase-to-Phase	Phase-to-Earth
FF-OV	D_{pp} = 3.64	D_{el} = 3.04
SF-OV	D_{pp} = 3.60	D_{el} = 2.19
PF-V	D_{pf} = 1.14	D_{pf} = 0.60

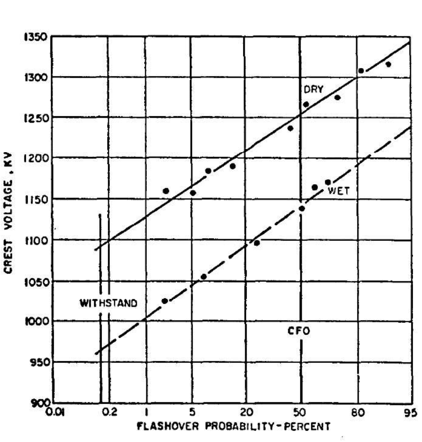
Phase-to-Tower Window	Phase-to-Tower Body	Phase-to-Obstacles
D_{el} = 3.29	D_{el} = 3.15	D_{el} = 3.26
D_{el} = 3.13	D_{el} = 2.52	D_{el} = 2.95
D_{pf} = 0.66	D_{pf} = 0.61	D_{pf} = 0.61

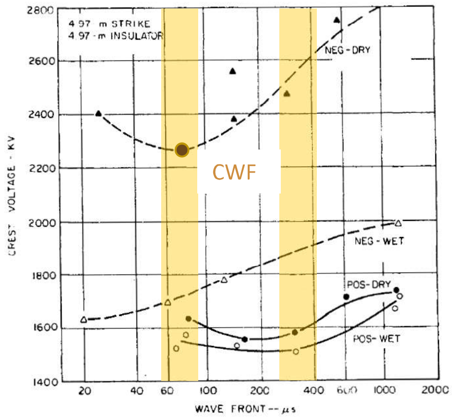


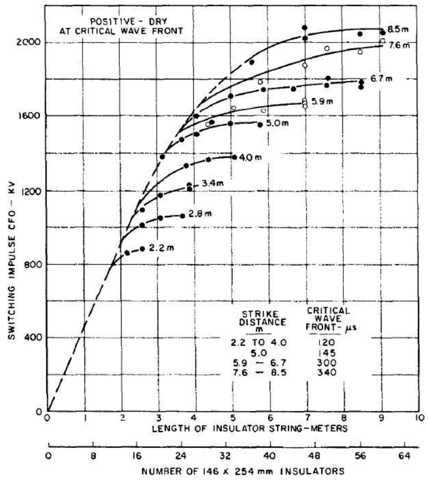


4.3 Insulation Strength Characteristics

As discussed in previous section, the insulation strength is described by the electrical dielectric strength to lightning impulses, switching impulses, temporary overvoltage and power frequency voltages. This section aims to discuss the characteristics of air-porcelain insulation subjected to lightning and switching impulses.





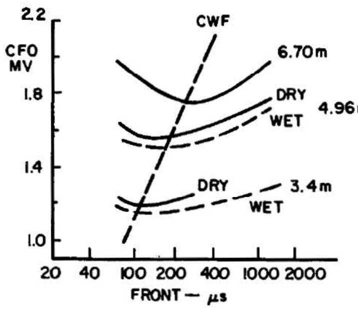


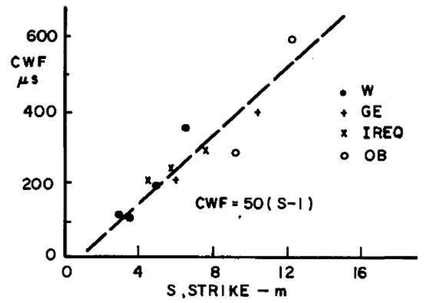
Flashover Probability and crest voltage [kV] is plotted on Gaussian probability paper and shows that the insulation strength characteristics can be approximated by a Gaussian Distribution with mean (CFO, crest flashover) and standard deviation σ_f. Usually the S.D. is given in per unit or percentage of the CFO.

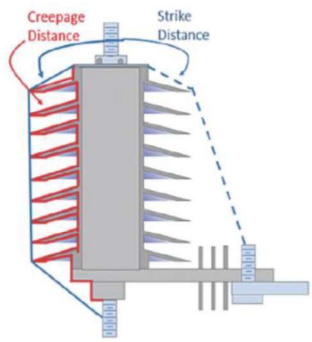
$$\boxed{V_3 = CFO - 3\sigma_f = CFO\left(1 - 3\,\frac{\sigma_f}{CFO}\right)} \tag{4.25}$$

$$p = F(V) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{Z} e^{-\frac{z^2}{2}}\,dZ, \qquad Z = \frac{V - CFO}{\sigma_f} \tag{4.26}$$

The effect of wave front for wet and dry conditions and positive and negative polarity is as shown. The U-shaped curves shows that there exists a wave front that produces minimum insulation strength. This is called the critical wave front (CWF). Wet condition decreases the CFO, more for negative polarity and less for positive polarity. **Positive-Wet conditions** are the most severe. When the insulator length increases, the CFO increases until the insulator length is equal to the strike distance. [If insulator strength < strike distance, flashover will occur across the insulators, and thus the insulator string limits the tower strength. If strike distance is less than the insulator length, flashovers will occur across the air strike distance, and the strike distance is limiting]







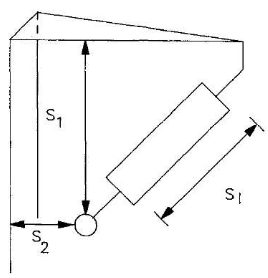
Strike distance is the shortest distance through the surrounding medium between terminal electrode. It is evident that the critical wave front increases with strike distance and approximately, with CWF+ and CWF- for positive and negative polarity in microseconds, and S is the strike distance.

$$CWF^{+} = 50(S-1) \approx 50S, \qquad CWF^{-} \approx 10S$$

Consider the maximum CFO of a tower window, the relationship of CFO and strike distance can be approximated by

$$CFO = k_g\frac{3400}{1+\frac{8}{S}}, \qquad k_g = 1.25 + 0.005\left(\frac{h}{S} - 6\right) + 0.25(e^{-\frac{8W}{S}} - 0.20) \tag{4.27}$$

where k_g is the gap factor, h is the conductor height and W is the tower width.



For other conditions,

- **Wet conditions** decrease the CFOs by 4%, i.e., multiply (4.27) by 0.96.
- **Outside phase** has an 8% higher CFOs, i.e., multiply (4.27) or (4.28) by 1.08.
- The CFOs and V_3 should be increased by 10% for **wave fronts of 1000ps or longer**, i.e., multiply (4.27) by 1.10.
- The **insulator string length** should be a minimum of 1.05 times the strike distance.
- For **I-string insulators**, the CFOs may be estimated by (4.27) multiplied by 1.08.
- S is the **minimum** of the three distances (1) the strike distance to the tower side, (2) the strike distance to the upper truss, and (3) the insulator string length divided by 1.05.

The usual line design assumes thunderstorms or wet conditions under average altitude. Therefore, the CFO under these conditions, CFO_A, may be obtained by

$$CFO_A = \delta^m CFO_S = 0.96 k_g \delta^m \frac{3400}{1+\frac{8}{S}} \tag{4.28}$$

or if the strike distance is desired, then

$$\boxed{S=\frac{8}{\frac{3400(0.96)k_g\delta^m}{CFO_A}-1}}, \qquad m=1.25G_0(G_0-0.2),\ G_0=\frac{CFO_S}{500S},\delta=e^{-A/8.6} \tag{4.29}$$

where A is altitude in [km].

Example

Determine the center phase strike distance and number of standard insulators for a 500kV (550kV max) line to be constructed at an altitude of 1000 m. The maximum switching surge is 2.0 pu (1 pu = 450kV) and W = 1.5 m and h = 15 m. Assume that all surges have a front equal to the critical wave front. Design for wet conditions and let σ_f/CFO = 5%.

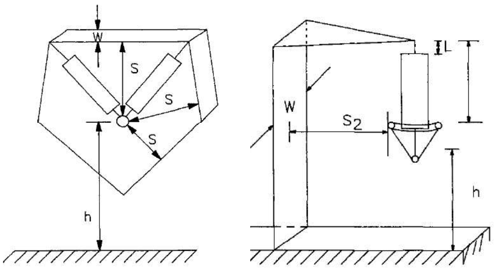
The CFO required at 1000 m is obtained from (4.25),

$$CFO_A=\frac{E_m}{1-3\left(\frac{\sigma_F}{CFO}\right)}=\frac{900}{0.85}=1059\text{ kV}$$

Also with the relative air density = 0.890. As gap factor k_g and G_0 are both functions of strike distance, strike distance cannot be obtained directly. To calculate S, (4.29) is used, and with a first guess k_g = 1.2, m = 0.5 and S = 3.2. Iteratively on S,

S	k_g	CFO_S	G_0	m	S
3.2	1.199	1118	0.699	0.436	3.18
3.18	1.199	1113	0.700	0.438	3.18

Therefore, the strike distance is 3.18 m and the minimum insulator length is 5% greater (3.34 m), which translates to **23 insulators**.



For Conductor Window Gap - Centre Phase:

$$k_g=1.25+0.005\left(\frac{h}{S}-6\right)+0.25\left(e^{-\frac{8W}{S}}-0.2\right) \tag{4.30}$$

For Conductor Crossarm – Outside Phase:

$$k_g=1.45+0.015\left(\frac{h}{S}-6\right)+0.35\left(e^{-\frac{8W}{S}}-0.2\right)+0.135\left(\frac{S_2}{S_1}-1.5\right) \tag{4.31}$$

Exercise

Determine the tower strike distance and insulator length for a single circuit 500kV (550kV max(transmission line under the following conditions. Note: 1 per unit = 450 kV.

Towers have V-strings on all phases.
The fronts of all switching surges are equal to the critical wave front.
Line length = 200 km with three towers per km.
Line altitude = 1000 m.
Maximum switching surge = 2.052 per unit.
σ_f/CFO = 5%.
Wet conditions decreases the dry CFO by 4%
Tower width = 1.8 m, conductor height = 20 m.

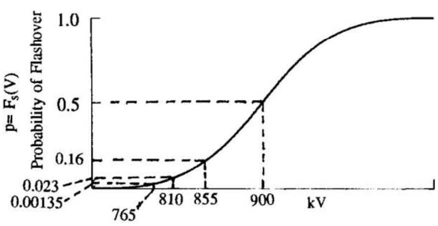
Also, calculate the probability of at least one flashover on the line for a switching surge of 2.052 per unit and also for a switching surge of 900 kV.

4.4 Switching Overvoltage in Transmission Lines

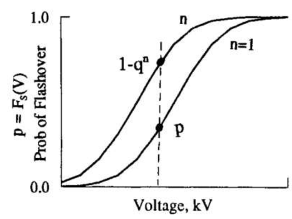
Strength distribution of one tower is modelled as Gaussian Distribution as discussed in previous sections. With a line consisting of several towers, the generated SOVs are applied to all these towers simultaneously and the probability of flashover given a SOV changes to

$$P(FO|V)=1-q^n=1-(1-p)^n \tag{4.32}$$

where p is the probability of flashover for a single tower insulation, q is the probability of no flashover and n is the number of towers.

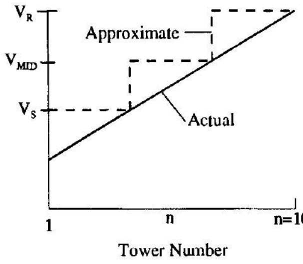


		$n=1$ tower		$n=100$ towers	
V, pu/kV	$P(V)$	P[FO|V]	P[FO]	P[FO|V]	P[FO]
1.9/855	0.01	.1587	.00159	1.00000	0.01000
1.8/810	0.05	.02275	.00114	.8999	0.04499
1.7/765	0.10	.001350	.00014	.12636	0.01264
1.6/720	0.21	.00003267	.00001	.00797	0.00326
1.5/675	0.27	.00000028	.00000	.000028	0.00001
		Total	0.00288		0.07090
		SSFOR	0.288/100		7.09/100



The calculation of switching surge flashover rate (SSFOR) proceeds as before and is shown for 100-tower line. As expected, the SSFOR increases to 7.09/100, but again, only the upper tail of the SOV distribution is important.

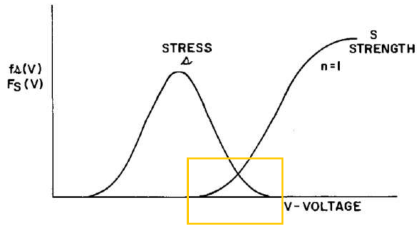
With **different voltage along the line** (smaller at breaker end and large at remote end), it is assumed that V_S / V_R = 0.9 and V_{mid} is the midpoint voltage. Assume there are 33 towers with each voltage (V_S, V_{mid}, V_R). Given that the probability of no-flashover | V_R is q_R^n, the probability of no flashover | V_{mid} (0.95 x 1.9 = 1.805 pu) is q_{mid}^n and the probability of no flashover | V_S (0.9 x 1.9 = 1.71 pu) is q_S^n. The probability of flashover given all these value is then

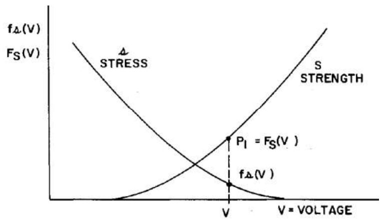


$$P(FO|V) = 1 - q_R^n q_{mid}^n q_S^n \quad (4.32)$$

V, pu/kV	P(V)	q_R	q_{mid}	q_s	$P[FO \mid V]$ $1 - q_R^n q_{mid}^n q_S^n$	P[FO]
1.9/855	0.01	.84134	.97441	.99813	.99866	.00999
1.8/810	0.05	.97725	.99813	.99993	.56111	.02806
1.7/765	0.10	.99865	.99994	1.00000	.04549	.00455
1.6/720	0.21	.99997	1.00000	1.00000	.00108	.00023
					Total	.04283
					SSFOR	4.28/100

The SSFOR decreases from 7.09% to 4.28%.





The probability that a voltage V occurs is $f_s(V)dV$ and the probability of a flashover given that V occurs is $P(FO|V) = F_S(V) = p$. The incremental probability of a flashover for a voltage V is denoted as dP and is therefore the multiplication of these values

$$dP = p \cdot f_s(V)dV = F_S(V) f_s(V)dV \quad (4.32)$$

The total probability of flashover considering all SOVs is the sum of (4.32) for all SOVs or

$$SSFOR = P(F) = \frac{1}{2}\int_{E_1}^{E_M} F_S(V) f_s(V)dV \quad (4.33)$$

where SSFOR is the switching surge flashover rate. The integration is taken from E_1, the minimum voltage (1.0 pu), to E_M, the maximum SOV. The integral is multiplied by ½ as the distribution of SOV is composed of all positive and negative. As the insulation strength of negative polarity is significantly larger than that of positive polarity, hence the negative polarity SOVs are neglected. (4.33) can be further expanded to

$$SSFOR = \int_{E_1}^{E_M} f_s(V)\left[\int_{-\infty}^{V} f_S(V)dV\right]dV \quad (4.34)$$

Assume both distribution of stress and strength are Gaussian. Then in short notation form with N signifying a Normal Distribution. $f_s(V) = N(\mu_0, \sigma_0)$ and $f_S(V) = N(\mu_s = CFO, \sigma_f)$

where S denotes the insulation strength and s denotes the stress or the SOVs.

The probability of a flashover or a failure, P(FO) is defined, as before, where the strength is less than the stress or

$$P(FO) = P(S < s) = P[(S - s) < 0] \quad (4.35)$$

Let Z = S – s, or more formally, a random variable Z equals the random variable S minus the random variable s. Since both distribution are normal, the distribution Z will be normal given that

$$\mu_Z = CFO - \mu_0, \qquad \sigma_Z = \sqrt{\sigma_f^2 + \sigma_0^2} \quad (4.36)$$

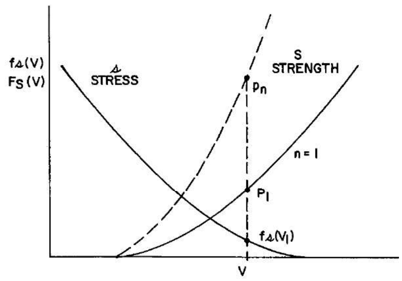
The resultant density function f(Z) has its area of interest below 0, i.e. S – s < 0. Therefore,

$$P(FO) = \int_{-\infty}^{0} f(Z)dZ, SSFOR = P(FO) = 1 - F\left[\frac{CFO - \mu_0}{\sqrt{\sigma_f^2 + \sigma_0^2}}\right] \quad (4.37)$$

Assume a CFO of 900kV, a σ_f of 45kV, μ_0 of 675kV and a σ_0 of 90kV. Then

$$SSFOR = \frac{1}{2}P(FO) = \frac{1}{2}[1 - F(2.236)] = 0.0064$$

It means 0.64 flashover per 100 breaker closing operations.



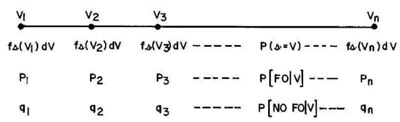
Returning to the general case depicted by (4.32),

$$P(FO|V) = 1 - q^n = 1 - (1 - p)^n \quad (4.32)$$

where p is the probability of flashover of one tower.

Then (4.33) must be modified by replacing p,

$$SSFOR = P(F) = \frac{1}{2}\int_{E_1}^{E_M} (1 - (1 - p)^n)\, f_s(V)dV \quad (4.38)$$



With the line parameter, given the remote end voltage V_n, the sectionalized voltage must be V_1, V_2, … Hence, the probability of switching overvoltage (SOV) is equal, i.e.

$$f_s(V_1)dV_1 = f_s(V_2)dV_2 = \cdots = f_s(V_n)dV_n \quad (4.39)$$

To simplify, we will use $f_s(V)dV$ and noted that $f_s(V)$ is the probability density function at the opened end of the line.

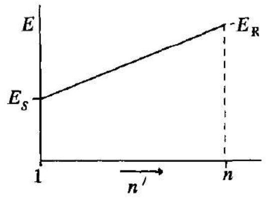
The probability of no flashover on tower 1 is q_1, the probability of no flashover on tower is q_2, … Therefore, the probability of at least one flashover on the line for a single switching operation is

$$1 - q_1 q_2 \cdots q_n \tag{4.40}$$

Considering all switching operations, the SSFOR is

$$SSFOR = P(F) = \frac{1}{2}\int_{E_1}^{E_M}\left(1 - \prod_{i=1}^{n} q_i\right) f_s(V)dV \tag{4.41}$$

Denote the 2% SOV as E_2.

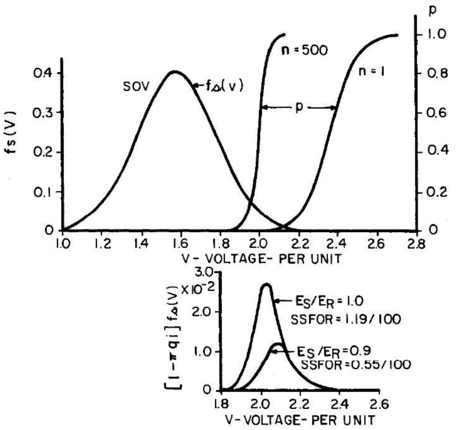


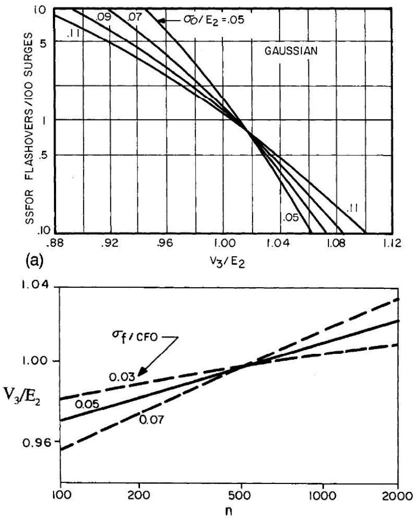
$$E_2 = \mu_0 + 2.054\sigma_0 \;\rightarrow\; \mu_0 = E_2\left(1 - 2.054\frac{\sigma_0}{E_2}\right) \tag{4.42}$$

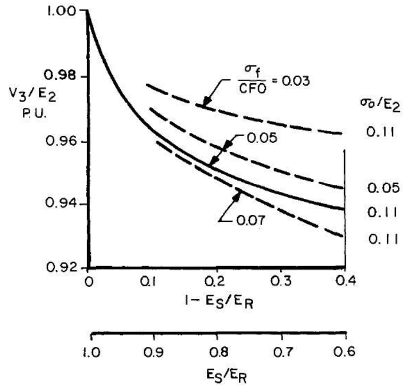
Represent also the voltage profile along the line as

$$E(x) = E_S + (E_R - E_S)\frac{x}{L} \;\rightarrow\; \frac{E}{E_R} = \gamma + (1-\gamma)\frac{x}{L} \qquad \gamma = \frac{E_S}{E_R} \tag{4.43}$$

Denote also the strength-stress ratio V_3 / E_2 (V_3 = strength at $3\sigma_f$ and E_2 is 2% SOV).

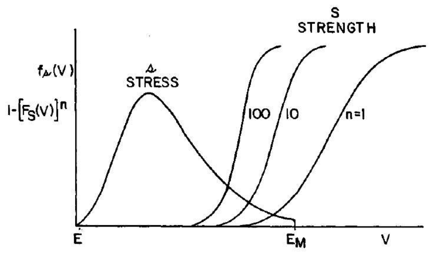






Brown's Method for estimating SSFOR

Assume for every switching operation, the SOVs are identical at each of the towers, i.e. E_S / E_R = 1.0. As the number of tower increases, the strength characteristics becomes steeper, or the S.D. becomes smaller. For any specific voltage, the probability of flashover increases from p to $(1 - q^n)$. If the strength can be represented by a single-valued function located at a voltage equal to CFO_n, the probability of flashover or the SSFOR is simply

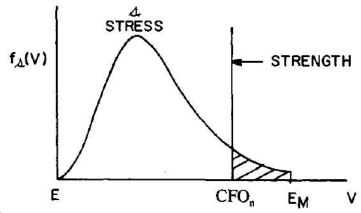


$$SSFOR = \frac{1}{2}\int_{CFO_n}^{E_M} f_s(V)dV \tag{4.44}$$

which can be easily evaluated for any SOV distribution, i.e.

$$\boxed{SSFOR = \frac{1}{2}\left[F\left(\frac{E_m - \mu_0}{\sigma_0}\right) - F\left(\frac{CFO_n - \mu_0}{\sigma_0}\right)\right]} \tag{4.45}$$

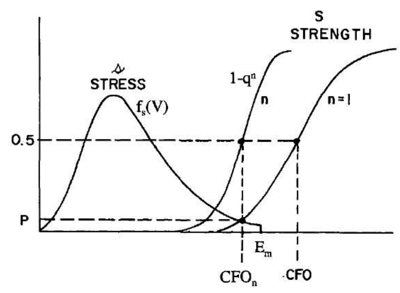
where the first term is approximately 1.0.



The CFO_n is the CFO for n towers and can be obtained from a knowledge of the strength characteristics for a single tower as illustrated in figure on the right. As for the CFO of a single tower, the CFO_n for n tower is defined at a probability of 0.5. Therefore,

$$0.5 = 1 - (1-p)^n \;\rightarrow p = 1 - \sqrt[n]{0.5} \tag{4.46}$$

From the value of p, by the formula for the reduced variate,



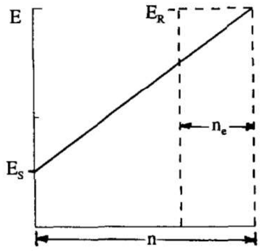
$$Z_f = \frac{CFO_n - CFO}{\sigma_f} \;\rightarrow CFO_n = CFO\left(1 + Z_f\frac{\sigma_f}{CFO}\right) \tag{4.47}$$

Example

For n = 200 tower and p = 0.003460, F(Z) of 1 – p (0.996540), Z = 2.70 and therefore Z_f is approximately = - 2.70. Therefore, CFO_n = 0.865. Assume σ_f/ CFO = 5%. If the CFO = 1000kV and SOV distribution is Gaussian with E_2 = 900kV, E_m = 999kV (one S.D above E_2) and σ_0 / E_2 = 0.11, μ_0 = 696.67 kV, the SSFOR with (4.45) is

$$SSFOR = \frac{1}{2}\left[F\left(\frac{999 - 696.67}{99}\right) - F\left(\frac{865 - 696.67}{99}\right)\right] = \frac{1}{2}[F(3.05) - F(1.70)] = 2.23\%$$

To consider the SOV profile, and equivalent number of towers n_e is calculated and then used in (4.46). The value of n_e, is the number of towers having an E_s/E_v = 1.00, which gives the same SSFOR as the actual number of towers with the specified E_s/E_v, as illustrated in Fig. 22. The equivalent number of towers may be estimated from the equation



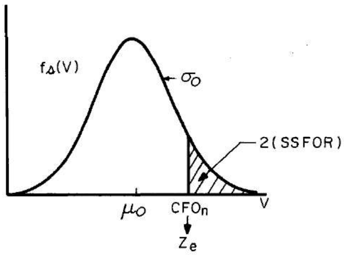
$$n_e = \min(n, \frac{k_n}{1-\gamma}\frac{\sigma_f}{CFO}n), \qquad \gamma = \frac{E_S}{E_R} \tag{4.48}$$

Over the practical range of 30 to 500 towers, an average value of k_n of 0.4 may be used.

As an example, consider the previous example for the Gaussian distribution with 200 towers but now assume an E_S/E_R of 0.9. Thus n_e = 40. Continuing, the CFO_n is 895 kV, and therefore the SSFOR reduces to 1.15/100.

An overwhelming advantage of the method proposed by Brown is that it can be used to obtain directly an estimate of the required value of V_3/E_2 given a value of SSFOR, which is the usual design problem. To develop this method, note that (4.47) may be placed in terms of V_3, i.e.,

$$CFO_n = \frac{V_3}{K_F}, \qquad K_F = \frac{1-3\left(\frac{\sigma_f}{CFO}\right)}{1+Z_f\left(\frac{\sigma_f}{CFO}\right)} \tag{4.49}$$



Neglecting E_m, the SSFOR is

$$SSFOR = \frac{1}{2}\int_{CFO_m}^{\infty} f_s(V)dV = \frac{1}{2}[1-F(Z_e)] \ \rightarrow \ 2(SSFOR) = 1 - F(Z_e) \tag{4.50}$$

where

$$Z_e = \frac{CFO_n - \mu_0}{\sigma_0} \tag{4.51}$$

To obtain Z_e, as obtained by (4.51), Normal distribution table can be used. Using the relationship of (4.49) for CFO_n, and the relationship between μ_0 and E_2, it produces

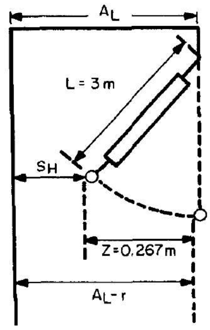
$$\frac{V_3}{E_2} = K_f K_G \ \rightarrow K_G = 1 - (2.054 - Z_e)\frac{\sigma_0}{E_2} \tag{4.52}$$

Example

Assume the ratio of V_3/E_2 desired for an SSFOR of 1.5/100. n = 200, σ_0/E_2 = 0.08 and σ_f / CFO = 0.05, then p = 0.00346, Z_f = - 2.70, K_f = 0.9827. For a SSFOR of 1.5/100, the value is obtained from $F(Z_e)$ = 1 – 2(0.015) = 0.97. From Table, Z_e = 1.88 and hence K_G = 0.9862. V_3/E_2 must be 0.9691. Consider E_S/E_R = 0.90 and and σ_f / CFO = 0.05, then n_e = 40. Proceeding the previous result, K_f = 0.9507 and V_3/E_2 = 0.9376.

Consider a Weibull distribution for wind speed

$$F(v) = 1 - e^{-4.6\left(\frac{v}{v_{100}}\right)^{\beta}} \tag{4.53}$$



where v_{100} is the 100-hour wind speed.
The swing angle can alternatively represented by

$$\tan\alpha_s = k_1 v^{1.6}, \qquad k_1 = 1.138 \times 10^{-4}\ \frac{D}{W}/\frac{V}{H} \tag{4.54}$$

Consider v_{100} = 50 km/h, (D/W)/(V/H) = 1.5, β = 1.9. Then for v = v_{100}, F(v) = 0.99 and α_s = 5.1°. Therefore $P(\alpha_s > 5.1°)$ = 0.01. Consider the system in the figure, where arm length A_L is

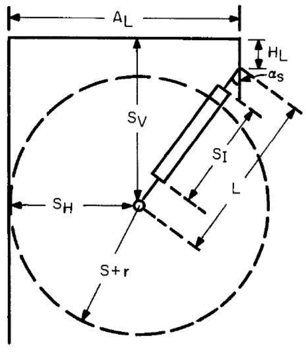
$$A_L = S_H + r + L\sin\alpha_s \tag{4.55}$$

Hence,

$$P((A_L - S_H - r) \geq 0.267\ \text{m}) = 0.01$$

As a general procedure, v_d, the designed velocity is used.

$$v_d = 0.6\ v_{100} \tag{4.56}$$



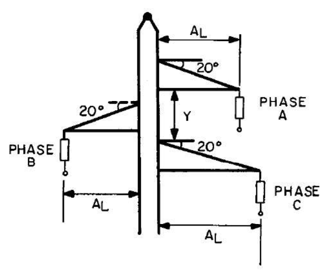
To clarify the procedure, the steps of the calculations are

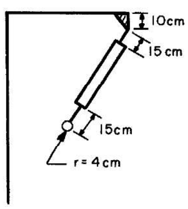
1. Proceed in exactly the same manner as for the outside phase V-string design, that is, for a given SSFOR, find V_3/E_2 and strike distance S.
2. Use a design wind speed of 60% of 100-hour wind speed, calculate the swing angle.
3. Knowing the insulator strength S_I, the hanger length H_L, and lower connection length to the conductor, the total connection length L is known. All steel member earthed must be outside the clearance circle, denoted by

$$\begin{aligned} A_L &\geq L\sin\alpha_S + S_H + r \\ H_L &\geq S_V + r - L\cos\alpha_S \\ S_I &\geq 1.05S \end{aligned} \tag{4.57}$$

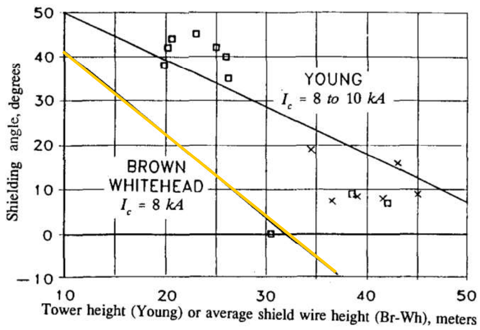
Exercise

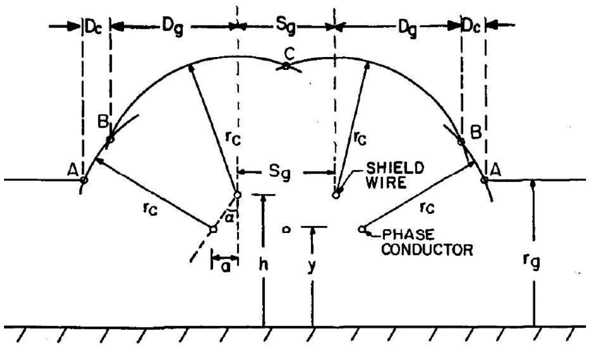
Determine the crossarm lengths A_l and the crossarm separation distance Y for 345 kV, 362 kV max tower. Design the line for a SSFOR of 1 flashover per 100 breaker operations. The arm length for phase C must be 1.0m longer than the arm length for phase A. The insulator string must contain a minimum of 18 insulators (146 x 254mm). Use the following data:





Gaussian SOV with E_2 of 2.8 pu, σ_0/E_2 of 0.07, E_S/E_R = 0.9.
Altitude = 2000m, tower width = 1.3 m, I-string insulator on all phases,
σ_f/ CFO = 0.05, 100 hour wind speed = 60km/h, β = 1.9, D/W = 1.3, V/H = 1.0,
Conductor height = 15 m, number of tower = 500





Brown and Whitehead developed the Geometric Model and a stroke angle distribution was added (Young assumed vertical strokes). Their recommendation on shielding angle is plotted against the *average* shielding wire height. It is noted that Young's recommendation uses the height at the tower. Geometric Model is to examine the sensitivity of shielding angle with tower or line height.

Assume with only vertical stroke, distance D_C and D_g are defined in the figure and are the exposure distance for phase conductor and shield wires, respectively. Therefore the specific value of current for which the arcs of figure are drawn, the number of strokes terminates on the phase conductor, or shielding failure rate SFR, is the area formed by D_C and length of the line times the ground flash density, i.e.

$$SFR|I = 2N_g L D_C \tag{4.58}$$

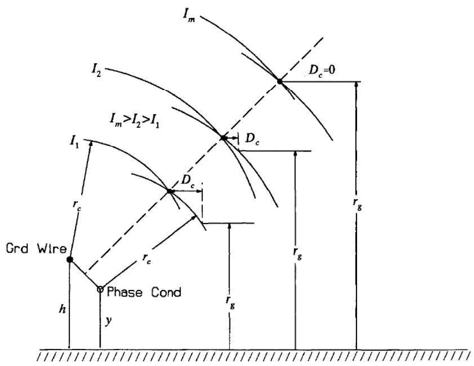
The probability of occurrence of this current is f(I) dI, so that the incremental failure d(SFR) is

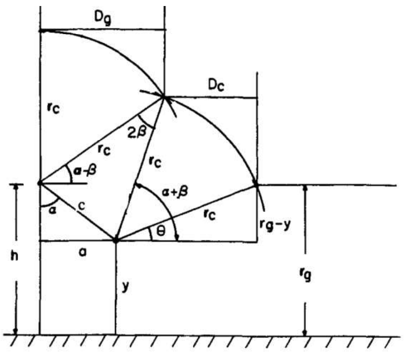
$$d(SFR) = 2N_g L D_c f(I) dI \quad \rightarrow SFR = 2N_g L \int_3^{I_m} D_c f(I) dI \tag{4.59}$$

As noted, the integration limits are 3kA and I_m, where I_m is the maximum current at and above which no strokes will terminate on the phase conductor. As the current increases, D_C, decreases until a point is reached at which all the three striking distances meet and D_C becomes zero. This point is defined by the current I_M. Since the lowest value of current in CIGRE data is 3kA, it is selected as the minimum value. Yet, other investigators believes that the values of 1kA or 2kA is more reasonable.

Above I_m, the exposure distance for the shield wires becomes D_g' as defined. The number of strokes or flashes to the shield wire N(G) is

$$N(G) = N_g L \left[\int_3^{I_m} (2D_g + S_g) f(I) dI + \int_{I_m}^{\infty} (2D_g' + S_g) f(I) dI \right] \tag{4.60}$$





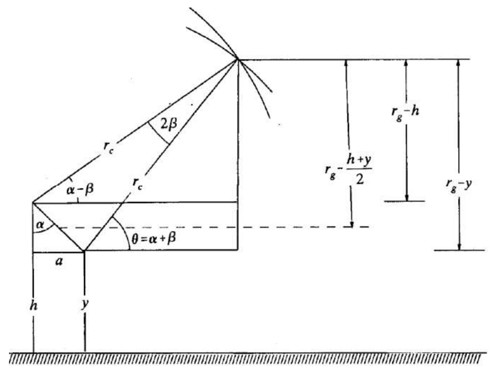


Figure in the middle shows one side of the shield wire-phase conductor diagram. The angle between two radii r_C is defined as β and is

$$2\beta = \sin^{-1}\frac{c}{2r_c} = \sin^{-1}\frac{\sqrt{a^2 + (h-y)^2}}{2r_c} = \sin^{-1}\frac{(h-y)\sqrt{1+\tan^2\alpha}}{2r_c} \tag{4.61}$$

The angles θ and α are

$$\theta = \sin^{-1}\frac{r_g - y}{r_c}, \qquad \alpha = \tan^{-1}\frac{a}{h-y} \tag{4.62}$$

From the figure,

$$D_c = r_c[\cos\theta - \cos(\alpha+\beta)]$$
$$D_g = r_c \cos(\alpha - \beta) \tag{4.63}$$

If r_g is less than or equal to y, set θ to zero in (4.63).

Consider figure on the right where all striking distances coincide at a single point, where I_m is defined. From this diagram, the value of r_g at I_m or r_{gm} is found by first finding the value of a

$$a = \sqrt{r_c^2 - (r_g - h)^2} - \sqrt{r_c^2 - (r_g - y)^2} \tag{4.64}$$

and thus

$$r_{gm} = \frac{h+y}{2k_0}\left[1 + \sqrt{1 - k_0\left(1 + \left(\frac{a}{h+y}\right)^2\right)}\right], \qquad k_0 = 1 - \gamma^2 \sin^2\alpha, \qquad \gamma = \frac{r_c}{r_g} \tag{4.66}$$

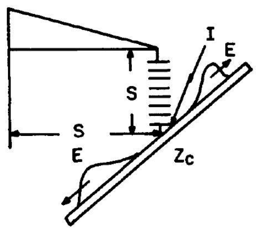
Also from the figure,

$$\sin\alpha = \frac{r_{gm} - \frac{h+y}{2}}{\sqrt{r_{cm}^2 - \frac{c^2}{4}}}, \qquad r_{cm}^2 \gg \frac{c^2}{4} \quad \rightarrow \boxed{r_{gm} = \frac{\frac{h+y}{2}}{1-\gamma\sin\alpha}} \tag{4.67}$$

r_{rgm} and I_m are related by

$$r_g = AI^b \;\rightarrow\; I_m = \left(\frac{r_{gm}}{A}\right)^{\frac{1}{b}} \tag{4.68}$$

Shielding Failure Rate (SFR) is the number of strokes that terminate on the phase conductor. Not all of these results in flashover. However, if the voltage produced by a stroke to the conductor exceeds the CFO, flashover occurs. Thus, the SFR includes both the stroke that causes flashover and those that do not. To determine the flashover rate, the voltage on the conductor and across line insulation E is



$$E = I\left(\frac{Z_c}{2}\right) \tag{4.68}$$

where Z_C is the surge impedance of the phase conductor.

If the voltage E is set to the CFO, negative polarity, then the critical current, at and above which flashover occurs, is

$$I_c = \frac{2(CFO)}{Z_c} \tag{4.69}$$

Revising (4.59) for the SFR to obtain the SFFOR,

$$SFFOR = 2N_gL\int_{I_C}^{I_m} D_c f(I)dI \tag{4.70}$$

Define "perfect" shielding as a SFFOR of zero. It is noted from (4.70) that this can be achieve by setting I_C to I_m. Hence,

$$\begin{cases} \alpha_p - \beta = \sin^{-1}\frac{r_g - h}{r_C} \\ \alpha_p + \beta = \sin^{-1}\frac{r_g - y}{r_C} \end{cases} \rightarrow \;\alpha_p = \frac{1}{2}\left[\sin^{-1}\frac{r_g - h}{r_c} + \sin^{-1}\frac{r_g - y}{r_c}\right] \tag{4.71}$$

Another way is to determine first the horizontal distance a_p for perfect shielding

$$a_p = \sqrt{r_c^2 - (r_g - h)^2} - \sqrt{r_c^2 - (r_g - y)^2} \;\rightarrow \boxed{\alpha_p = \tan^{-1}\frac{a_p}{h-y}} \tag{4.72}$$

Also, it can also be represented by

$$\alpha_p = \sin^{-1}\frac{r_g - \frac{h+y}{2}}{\sqrt{r_c^2 - \left(\frac{c^2}{4}\right)}}, \qquad \frac{c}{2} \ll r_C \;\rightarrow\; \alpha_p = \sin^{-1}\frac{r_g - \frac{h+y}{2}}{r_C} \tag{4.72}$$

Before attempting an analysis of the alternate striking distance, the stroke angle should be considered. In Young's original derivation of the geometric model, only vertical strokes were considered. That is, the downward leader was assumed to be perpendicular to the line, and previous developed equations apply for vertical strokes. Later, Whitehead and his associates developed the concept that the downward leader could approach the line from any direction with probability density function

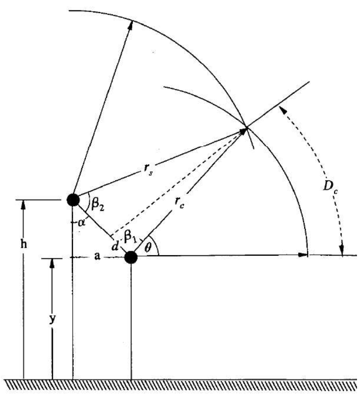
$$f(\psi) = k\cos^2\psi \tag{4.73}$$

where ψ is the angle to the vertical axis and varies between -90° and 90°. This assumption adds a considerable degree of complexity to the calculation of SFFOR and only increases the SFFOR by about 10 – 29%. Note that if $r_g < (h-y)/2$, the shielding angle is negative with (4.71). Consider the distribution, a near-horizontal stroke can occur.

Eriksson's modified geometric model includes two striking distance r_S to shield wire and r_C to the phase conductor, with equation

$$r_s = 0.67\,I^{0.74}h^{0.6}, \qquad r_c = 0.67\,I^{0.74}y^{0.6} \tag{4.74}$$

As shown, the striking distance to ground does not exist. As a default condition, any downward leader that does not meet the arc described by r_C will terminate to ground. Thus, all stroke angles are considered, and all are considered equally likely. Yet, the downward leader is not permitted to travel below the height of the phase conductor and then travel upward to the phase conductor. The exposure of the phase conductor is specified by the arc D_C and therefore



$$D_c = r_c\theta \tag{4.75}$$

Therefore, the SFR and SFFOR are

$$SFR = 2N_gL\int_3^{I_m} D_c f(I)dI\,, \qquad SFFOR = 2N_gL\int_{I_C}^{I_m} D_c f(I)dI \tag{4.76}$$

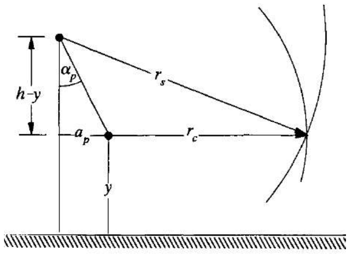
From the figure, D_C can be found with

$$d = \frac{r_c^2 - r_s^2 + c^2}{2c}, \qquad \beta_1 = \cos^{-1}\frac{d}{r_c}, \beta_2 = \cos^{-1}\frac{c-d}{r_s}, \theta = \alpha - \beta_1 + \frac{\pi}{2} \tag{4.77}$$

or to be complete,

$$D_C = \left(\alpha - \beta_1 + \frac{\pi}{2}\right)r_c \tag{4.78}$$

Again, the perfect horizontal distance a_p and the perfect shielding angle α_p can be obtained by



$$\alpha_p = \sqrt{r_s^2 - (h-y)^2} - r_c \qquad \alpha_p = \tan^{-1}\frac{a_p}{h-y} \tag{4.79}$$

For a specific value of a, the maximum current I_m can also be obtained from the figure.

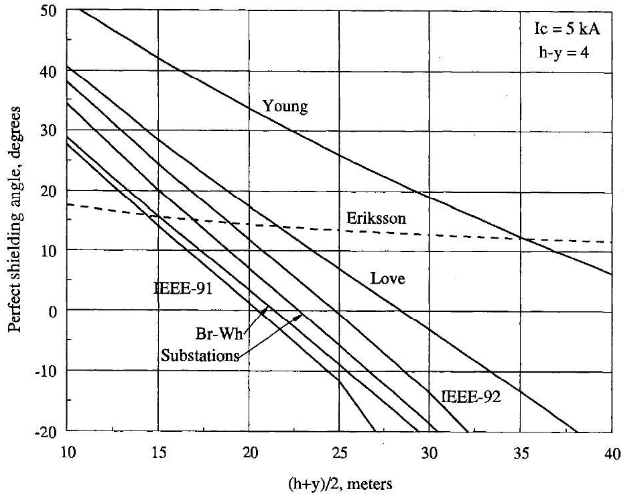
The maximum value of r_c, r_{cm} is

$$r_{cm} = \frac{1}{\gamma^2-1}\left(a + \sqrt{a^2 + c^2(\gamma^2-1)}\right), \qquad \gamma = \frac{r_s}{r_c} = \left(\frac{h}{y}\right)^{0.6} \tag{4.80}$$

and thus,

$$I_m = \left(\frac{r_{cm}}{0.67y^{0.6}}\right)^{\frac{1}{0.74}} \tag{4.81}$$

The equation developed must be solved by numerical integration. An approximation calculating the SFFOR was suggested by Anderson. Observing D_C when $I = I_M$ is zero, Anderson suggested that the average value of D_C over the interval from I_C to I_M is half of the value of D_C at $I = I_C$. More formally, let D_{CC} equal the value of D_C at I_C. Since D_{CC} is constant, it can be taken out of the integral.



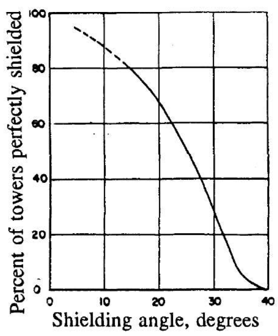
$$\begin{aligned} SFFOR &= 2N_gL\frac{D_{CC}}{2}\int_{I_c}^{I_m} f(I)dI \\ &= 2N_gL\frac{D_{CC}}{2}\left[Q(I_c) - Q(I_m)\right] \end{aligned} \tag{4.82}$$

where Q(I) = 1 – F(I), and can be approximated with

$$Q = 1 - 0.31e^{-\frac{Z^2}{1,6}}, \qquad Z = \ln(\frac{I}{61.1})/1.33 \tag{4.83}$$

Example

Two shield wires are located at an average height of 30 m, and the conductors are at 26 m. The shielding angle is 25° and the critical current is 10 kA. Also the ground flash density is 4.



With (4.67) and (4.68), r_{gm} = 52.72, I_m = 16.63 kA.

At I_C, with (4.61) – (4.63), r_g = 36.0, r_c = 39.9, β = 3.17°, θ = 14.5°, D_c = 3.459 m.

$$Z_c = \ln(\frac{10}{61.1})/\,1.33 = -1.361 \;\rightarrow Q_c = 0.9026$$

At I_M,

$$Z_M = \ln(\frac{16.63}{61.1})/\,1.33 = -0.9782 \;\rightarrow Q_m = 0.8295$$

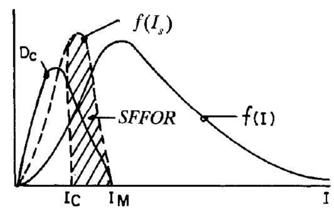
Therefore, SFFOR = 2 (4) (100) (3.459/2) (1/1000) (0.0731) = 0.10 / 100 km – years.

With a shielding angle of 35°, r_{gm} = 77.1, I_m = 27.6, D_c = 7.41, Z_c = 1.361, Z_m = -0.3103, Q_c = 0.9026, Q_m = 0.6095, SFFOR = 0.87 / 100 km – years.

Let the shielding failure flashover current equal I_F.

$$F(I_f) = \frac{2N_gL}{SFFOR}\int_3^{I_f} D_cf(I)dI \qquad I_c \le I_f \le I_m \tag{4.84}$$

Let the shielding current equals I_S.



$$F(I_S) = \frac{2N_gL}{SFR}\int_3^{I_S} D_cf(I)dI \qquad 3 \le I_s \le I_m \tag{4.84}$$

Let the current to the shield wires equals I_G. For current between 3 and I_m

$$F(I_G) = \frac{N_gL}{N(G)}\left[2\int_3^{I_G} D_gf(I)dI + S_g\int_3^{I_G} f(I)dI\right] \tag{4.85}$$

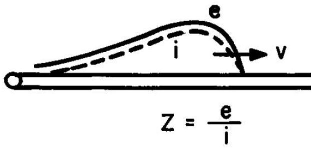
For current greater than I_m,

$$\begin{aligned} F(I_G) &= \frac{N_gL}{N(G)}\left[2\int_3^{I_G} D_gf(I)dI + \int_{I_G}^{\infty} D_g{'}f(I)dI + S_g\int_3^{I_G} f(I)dI\right] \\ f(I_G) &= \frac{N_gL}{N(G)}\left[2D_g'f(I) + S_gf(I)\right] \end{aligned} \tag{4.86}$$

The equation for density of $f(I_g)$ is illustrated in figure. The crosshatched area is the SFFOR.

4.6 A Review of Travelling Waves

Recall the surge impedance of a line (Z) and travelling velocity (v) as



$$Z = \sqrt{\frac{L}{C}} \qquad v = \frac{1}{\sqrt{LC}} \tag{4.87}$$

For Overhead lines (single conductor having a radius r located at height h above ground with earth of zero resistivity),

$$L = 0.2\ln\frac{2h}{r}\ [\mu\mathrm{H/m}], \qquad C = \frac{10^{-3}}{18\ln\frac{2h}{r}}\ [\mu\mathrm{F/m}] \tag{4.88}$$

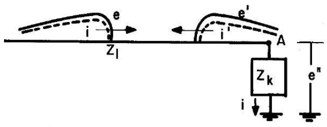
$$Z = 60\ln\frac{2h}{r}\ [\Omega], \qquad v = 30\ \mathrm{m}/\mu s$$

For cables, the velocity is a special case of the general phenomenon that the velocity of propagation varies inversely as the square root of the permittivity of the medium. For cable, the permittivity k varies from 2.4 to 4.0, the surge impedance and velocity of propagation are

$$Z = \frac{60}{\sqrt{k}}\ln\frac{r_2}{r_1}, \qquad v = \frac{300}{\sqrt{k}}\ \mathrm{m}/\mu\mathrm{s} \quad = 150\mathrm{m/us} \tag{4.89}$$

The surge impedance of a cable varies from about 30 – 60 ohms and the velocity of propagation is about 1/3 to ½ of the speed of light.

With a surge of voltage *e* and current *i*, the transmitted voltage e" and current i" and reflected voltage e' and current i' are found by



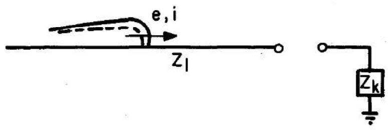
$$\boxed{\begin{aligned} e'' &= \frac{2Z_k}{Z+Z_k}e, & i'' &= \frac{e''}{Z_k} = \frac{2Z}{Z+Z_k}i \\ e' = e'' - e &= \frac{Z_k - Z}{Z+Z_k}e, & i' &= \frac{e'}{Z_k} = \frac{Z_k - Z}{Z+Z_k}i \end{aligned}} \tag{4.90}$$

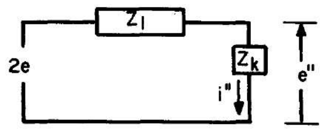
For short circuit (Z_k = 0), e" = 0, i" = 2i; e' = - e, i' = -i.

For open circuit (Z_k = inf), e" = 2e, i" = 0; e' = e, i' = i. (4.91)

That is, the voltage doubles at an open circuit, current doubles at a short circuit.

Thevenin's theorem can be applied to the circuit to obtain the voltage across the impedance Z_k. First open the circuit at the point of discontinuity and calculate the opened circuit voltage. From (4.91), this open circuit voltage is equal to 2e. Next, find the impedance of the circuit by "standing" at the open circuit point and looking backward. Then we find the impedance as simply Z. Thevenin's equivalent circuit is as shown, which can be used to calculate the voltage e" and the current i". The voltage e' and the current i' can be found by noting that e' = e" - e and i' = i – i".



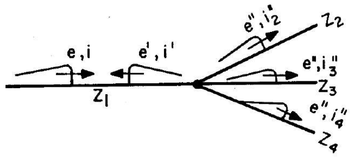


For a **capacitor load**, assume Z_k is a capacitor C_k and e is a unit step function with magnitude E. Then

$$e'' = \frac{\frac{2}{C_k s}}{Z + \frac{1}{C_k s}}\frac{E}{s} = \frac{2E}{ZC_k}\frac{1}{s\left(s + \frac{1}{ZC_k}\right)} \tag{4.92}$$

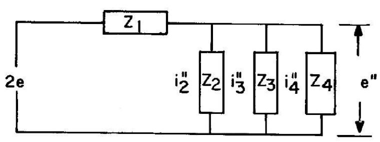
Then

$$e'' = 2E\left(1 - e^{-\frac{t}{ZC_k}}\right), \qquad e' = E\left(1 - 2e^{-\frac{t}{ZC_k}}\right) \tag{4.93}$$



For an **n-Line station**, with one incoming line and (n – 1) outgoing lines. Let the lines have surge impedance Z_1, Z_2, Z_3, etc. Thus assuming four lines,

$$Z_k = \frac{Z_2 Z_3 Z_4}{Z_2 Z_3 + Z_2 Z_4 + Z_3 Z_4} \tag{4.94}$$



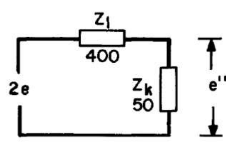
Apply Thevenin circuit and the transmitted voltages on each of the outgoing lines will be equal to e''. The voltage on the lines are the same, but the current on each line will differ with different surge impedance.

$$e'' = \frac{2Z_k}{Z_1 + Z_k}e \ \rightarrow\ i_2'' = \frac{e''}{Z_2}, i_3'' = \frac{e''}{Z_3}, i_4'' = \frac{e''}{Z_4} \tag{4.95}$$

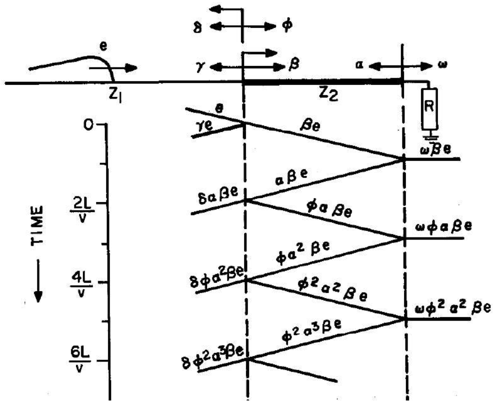
Assume all lines connected are in equal surge impedance,

$$e'' = \frac{2\left[\frac{Z}{n-1}\right]}{Z + \left[\frac{Z}{n-1}\right]} = \frac{2e}{n} \tag{4.96}$$

For a **line-cable junction**, when a surge initiated from overhead line with surge impedance 400 ohms to cable of 50 ohms, the voltage at line cable junction e" = 2/9 e and reflected surge voltage is e' = e" – e = - 7/9 e. Low impedance of cable reduces incoming surge.



Lattice Diagram is often used in hand to determine voltage at junctions after several reflections. For a finite length cable, surges are reflected from the end of the cable and arrive back at the cable-line junction and usually tend to increase the voltage at this point. Now consider that the cable is of finite length as shown. And to add general interest, also assume that the end of the cable is terminated in a resistor R.



The first task is to calculate the reflection and transmission coefficients and place them on the "sign" posts. That is, for example, for a surge e traveling on the cable toward the line, the signpost tells us that the voltage transmitted onto the line is δe and that the surge reflected can be determined using (4.90). Thus,

$$\beta = \frac{2Z_2}{Z_1+Z_2}, \qquad \gamma = \frac{Z_2-Z_1}{Z_1+Z_2}, \qquad \omega = \frac{2R}{R+Z_2}$$
$$\alpha = \frac{R-Z_2}{R+Z_2}, \qquad \delta = \frac{2Z_1}{Z_1+Z_2}, \qquad \phi = \frac{Z_1-Z_2}{Z_1+Z_2} \tag{4.97}$$

To clarify, the voltage e arriving at the line-cable junction transmits a surge of βe and reflects a surge of γe . The transmitted surge travels to the end of the cable and reflects a surge of $\alpha\beta e$. The voltage produced at the end of the cable is $\omega\beta e$. Now this reflected surge travels back to the line-cable junction and transmits a voltage and $\delta\alpha\beta e$ reflects a voltage $\phi\alpha\beta e$, etc.

The voltage at any location or point on the cable can be calculated by adding those voltages that arrive at the selected location. However, they must be added w.r.t their time of arrival at this location. For example, the voltage at the line-cable junction e_T is

$$e_T = e[\beta + \delta\alpha\beta(t-2T) + \delta\phi\alpha^2\beta(t-4T) + \delta\phi^2\alpha^3\beta(t-6T) + \cdots \tag{4.98}$$

where T is the time required for a surge to travel one length of the cable L, i.e.

$$T = L/v \tag{4.99}$$

Note that the first term of (4.98), βe have been taken as $(1+\gamma)e$, and the second term $\delta\alpha\beta e$ is taken as $(\alpha\beta + \phi\alpha\beta)e$, depending on which side of the dotted line is traversed. With $\beta = 1+\gamma,\ \delta = 1+\phi$, (4.98) can be simplified to

$$e_T = \beta e[1 + \delta\alpha(t-2T) + \delta\phi\alpha^2(t-4T) + \delta\phi^2\alpha^3(t-6T) + \cdots \tag{4.100}$$

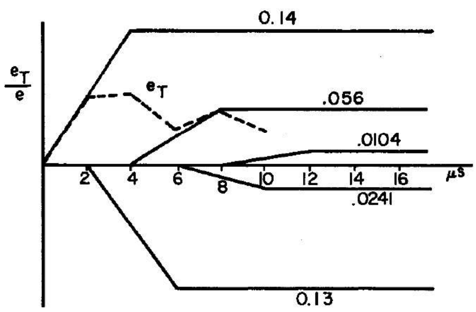
Example

Let $Z_1 = 400$, $Z_2 = 30$, $R = 10$,

$$\alpha = -0.5, \beta = 0.14, \delta = 1.86, \omega = 0.5, \gamma = -0.86, \phi = 0.86$$

Let e be defined by a linear front and an infinite tail. Let also the time to crest of this surge equal 4 us and the travel time T be 1us. Then

$$e_T = e[0.14 - 0.13(t-2) + 0.056(t-4) - 0.0241(t-6) + \cdots]$$

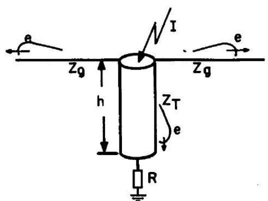


and

$$e_T(t=\infty) = \beta e[1 + \delta\alpha(1 + \phi\alpha + \phi^2\alpha^2 + \cdots] = \beta e\left(1 + \frac{\delta\alpha}{1-\phi\alpha}\right) \tag{4.101}$$

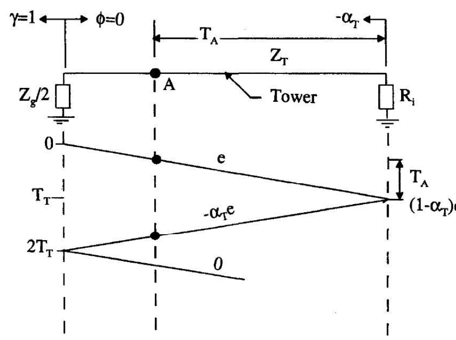
$$= \frac{2R}{R+Z_1}e = 0.049e$$

Consider first stroke of a flash terminates at the **tower top** as illustrated in the figure. A voltage e is produced at the top of the tower, creating a traveling wave that travels down the tower and out on the overhead ground wires. The voltage e is the product of the stroke current I and the combined impedance of the tower and the ground wires, i.e.



$$e = \frac{Z_T\left(\frac{Z_g}{2}\right)}{Z_T + \frac{Z_g}{2}}I = \frac{Z_T Z_g}{Z_g + 2Z_T}I \tag{4.102}$$

where Z_g is the ground wire surge impedance and Z_T is the surge impedance of the tower. As a good approximation, $Z_T = Z_g/2$, hence (4.102) becomes $e = Z_T/2\ I = Z_g/4\ I$.

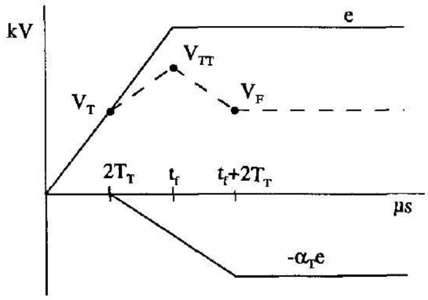


Let the tower travel time be T_T and assume that the waveshape of the voltage e is defined as a linear rising front and an infinite or constant tail. Let the time to crest equal t_f. To calculate the voltage at the top of the tower, or at any point along the tower, lattice diagram is used. The coefficient are calculated as

$$\alpha_T = \frac{Z_T - R_i}{Z_T + R_i} \approx \frac{Z_g - 2R_i}{Z_g + 2R_i}, \qquad \gamma = \frac{2Z_g}{Z_g + 2Z_T}, \qquad \phi = \frac{Z_g - 2Z_T}{Z_g + 2Z_T} \tag{4.103}$$

Note that the reflection coefficient α_T in (4.103) must be used as a negative value in the lattice diagram.

Three voltage magnitudes are of interest, V_{TT} the crest voltage, V_T the voltage at the tower top prior to any reflections from the footing resistance, and V_v, the final voltage.



$$V_T = \frac{2T_T}{t_f} e = Z_T \frac{T_T}{t_f} I = L_T \frac{I}{t_f} \tag{4.104}$$

where L_T is the total inductance of the tower and $L_T = Z_T\, T_T$. The factor I / t_f is the rate of rise or steepness of the front and is frequently denoted as S_i or more simply dI/dt. Thus V_T is very simply the voltage drop caused by the tower, L dI/dt.

$$V_{TT} = e - \alpha_T e \frac{t_f - 2T_T}{t_f} = (1 - \alpha_T)e + \alpha_T e \frac{2T_T}{t_f} \tag{4.105}$$

$$= \left[\frac{R_i Z_g}{Z_g + 2T_i} + \alpha_T Z_T \frac{T_T}{t_f}\right] I = \left[R_e + \alpha_T Z_T \frac{T_T}{t_f}\right] I = K_{TT} I$$

$$V_F = (1 - \alpha_T)e = R_e I \tag{4.106}$$

The voltage across the footing resistance, V_R, and current through this resistance I_R are

$$V_R = \frac{R_i Z_T}{Z_T + R_i}, \qquad I_R = \frac{Z_T}{Z_T + R_i} I = \frac{R_e}{R_i} I \tag{4.107}$$

For $R_i << Z_T$, which is the normal case, $\alpha_T < 1$ and V_{TT} is the initial tower component of voltage multiplied by α_T plus the final voltage V_F. The final voltage in per unit of the stroke current is the footing resistance in parallel with half the ground wire surge impedance. And if $Z_g >> R_i$, then the final voltage is simply IR_i.

Example

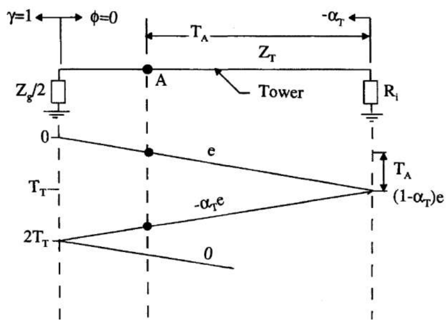
Z_g = 350 ohms, Z_T = 200 ohms, R_i = 20 ohms, h = 30m and t_f = 2 us. Then R_e = 17.95 ohm,

$$\frac{V_T}{I} = 10 \text{ ohms}, \qquad \frac{V_{TT}}{I} = K_{TT} = 17.95 + 8.13 = 26.13 \text{ ohms}$$

$$\frac{V_F}{I} = 17.95 \text{ ohms}, \qquad \frac{V_R}{I} = 17.95 \text{ ohms}, \qquad \frac{I_R}{I} = 0.898$$

First note that even for a low tower footing resistance of 20 ohms, the footing resistance component is dominant, about 78% of V_{TT}. However, the tower component s 22% of V_{TT}. Also note that the current through the footing resistance is about 90% of the stroke current; little current travels out the ground wires.

It is noted that voltage at point A can be represented by

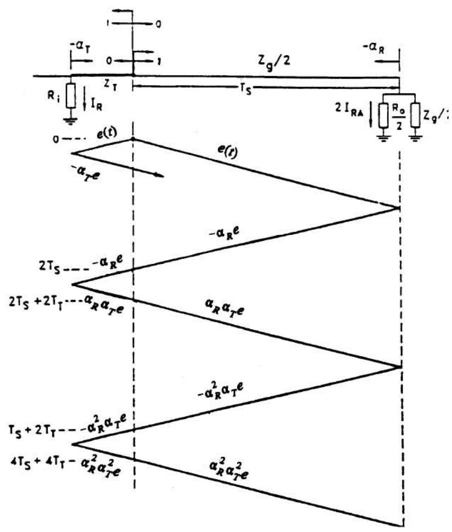


$$V_{TA} = \left(R_e + \alpha_T Z_T \frac{T_A}{t_f}\right) I = K_{TA} I \tag{4.108}$$

Consider the **effect of reflection from adjacent towers**. The large current flowing through the footing resistance of the struck tower results in a decreased resistance. However, as will be shown, the current flowing through the footing resistance of the adjacent towers is only a few percent of that of the struck tower, and therefore this footing resistance will remain at approximately the measured or low current value. As before, to simplify, let $Z_T = Z_g/2$ so that

$$\alpha_T = \frac{Z_T - R_i}{Z_T + R_i} \approx \frac{Z_g - 2R_i}{Z_g + 2R_i}, \qquad \alpha_R = \frac{Z_g}{Z_g + 2R_0} \tag{4.109}$$

Define T_T as the tower travel time and T_S as the span travel time, from the lattice diagram, the voltage at struck tower top, e_{TT} is



$$e_{TT} = (1 - \alpha_T(t - 2T_T)e(1 - \alpha_R(t - 2T_s) + \alpha_R \alpha_T\big(t - (2T_s + 2T_T)\big) \tag{4.110}$$
$$-\alpha_R^2 \alpha_T\big(t - (4T_s + 2T_T)\big) + (\alpha_R \alpha_T)^2\big(1 - (4T_s + 4T_T)\big) - \cdots$$

Neglecting the travel time of the struck tower for reflections,

$$e_{TT} = (1 - \alpha_T(t - 2T_T))e(1 - \alpha_R(1 - \alpha_T)((t - 2T_s) + \alpha_R \alpha_T(t - 4T_s) \tag{4.111}$$
$$+(\alpha_R \alpha_T)^2(t - 6T_s) - \cdots)$$

The quantity $\big(1 - \alpha_T(t - 2T_T)\big)e$ is the value of e_{TT} developed previously when reflections from adjacent towers were neglected. Therefore this quantity can be replaced by K_{TT}.

For purposes of simplification and approximation, assume that the voltage described by ($K_{TT}I$) has a linearly rising front. Then the crest voltage becomes

$$V_{TT} = K_{SP} K_{TT} I \tag{4.112}$$

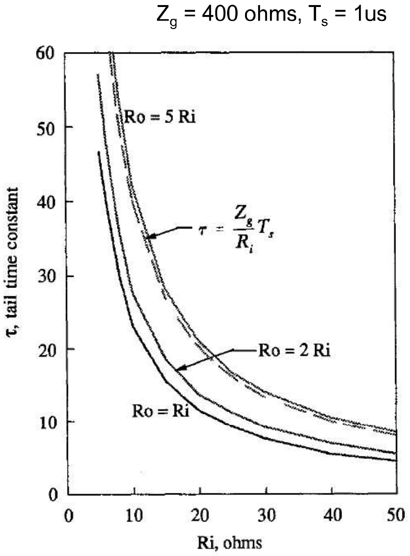
$$K_{SP} = 1 - \alpha_R(1 - \alpha_T)\left[\left(1 - \frac{2T_s}{t_f}\right) + \alpha_R \alpha_T \left(1 - \frac{4T_s}{t_f}\right) + (\alpha_R \alpha_T)^2 \left(1 - \frac{6T_s}{t_f}\right) + \cdots\right]$$

Reflections from other towers can further reduce the crest voltage, providing they arrive before crest voltage is attained at the struck tower.

For example, the first reflection from the second tower, arriving at $4T_s$, is equal to

$$\Delta e_{TT} = -\alpha_R(1-\alpha_T)(1-\alpha_R)^2 \tag{4.113}$$

For the stroke to the tower, the tail of the stroke current was assumed infinite, i.e., the crest current was held constant, and therefore the tails of the tower voltages were also infinite. Even though reflections from adjacent towers do not decrease the crest voltages at the struck tower, they will **decrease the tail** or **time to half value.** To assess the magnitude of this decrease, the surge impedance and length of the shield wire is replaced by its equivalent inductance, the tower is neglected, and additional inductive-resistive pi-sections are added to represent the entire line. For an infinite line, the final voltage approaches zero, and, as may be noted from this network, the method of achieving zero voltage is through time constants consisting of the inductance and various combinations of R_o and R_i.



However, the tail or voltage e_R for times equal to or greater than $t_f + 2T_T$ can be approximated by a single time constant τ such that

$$e_R(t) = V_F e^{-\left(t-\frac{(t_F+2T_T)}{\tau}\right)} \tag{4.114}$$

The time constant can be conservatively estimated by the following equation, which is simply an L/R time constant with inductance L as the inductance of a span, i.e.

$$\tau = \frac{Z_g}{R_i}T_s, \qquad L = \frac{Z_g}{c}(\text{span length}) = Z_g T_S \tag{4.115}$$

The voltage at the adjacent tower is

$$e_A = (1-\alpha_T)(1-\alpha_R)e\left[1+\alpha_R\alpha_T\left(1-\frac{2T_s}{t_f}\right)+(\alpha_R\alpha_T)^2\left(1-\frac{4T_s}{t_f}\right)+\cdots\right] \tag{4.116}$$

Where the votlage s is (I_L Z_g/2) and I_L, the current flowing out on the shield wires is

$$I_L = \frac{2R_i}{Z_g+2R_i}I \tag{4.117}$$

Combining these equations and remembering that the result is twice this current, and using the equations for the current through the footing resistance of struck tower, the current through the footing of the adjacent tower I_{RA} is

$$I_{RA} = \frac{8R_i^2}{(Z_g+2R_0)(Z_g+2R_i)}I_R\left(1+\alpha_R\alpha_T\left(1-\frac{2T_s}{t_f}\right)+(\alpha_R\alpha_T)^2\left(1-\frac{4T_s}{t_f}\right)+\cdots\right) \tag{4.118}$$

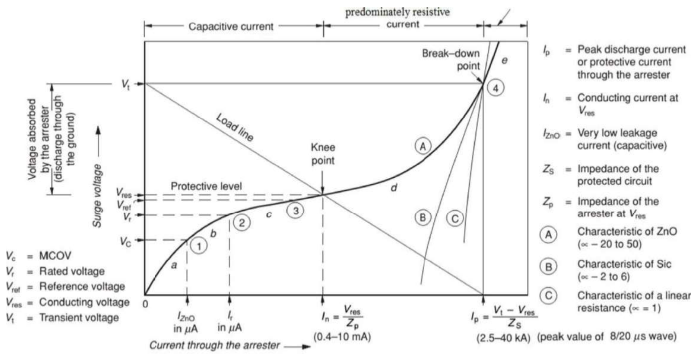
As a numerical result, I_{RA} = 0.065 I_R.

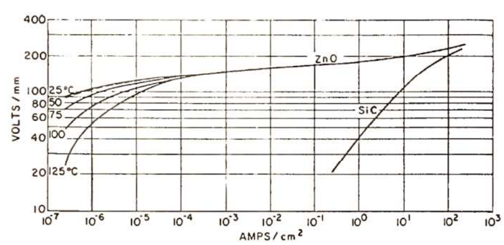
4.7 Arresters and Corona Effect to Waves

Different voltage surge limiting devices (e.g. spark gap, arc horn, surge arrester, lightning arresters) are installed at different locations along the overhead lines to perform insulation coordination and limit the voltage before reaching the substation equipment and causing damages. The major problem is where to install, and which phase to install, i.e. the optimal placement of arresters.

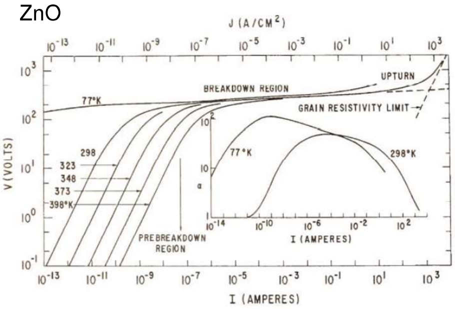
Table 1 - Voltage Surge Limiting Devices

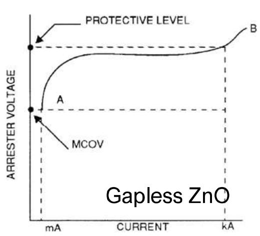
Device Type	Relative Strengths	Limitations
Carbon Block	Very inexpensive.	Low impedance; characteristics vary with time and temperature; rarely used except in telephone service.
Selenium	Relatively good energy absorption rating.	Large; costly; limited temperature rise; significant leakage current.
Silicon Carbide	Inexpensive; can have large energy absorption rating.	High leakage current; limited resistance change with voltage; less stable than ZnO.
Spark gap	Can handle large currents; low leakage current.	Low impedance when arcing; difficult to interrupt; costly in small sizes; breakdown voltage sensitive to environment.
Zener diode	Very sharply defined onset of limiting; low dynamic resistance; stable characteristics.	Limited power dissipation; relatively expensive.
Zinc Oxide	Inexpensive; can have large energy absorption rating; large design range; very good α; temperature insensitive.	Leakage is temperature dependent; some change of characteristics under heavy surges.

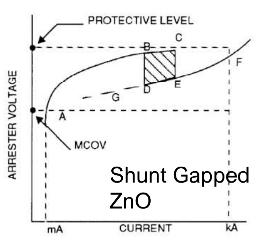


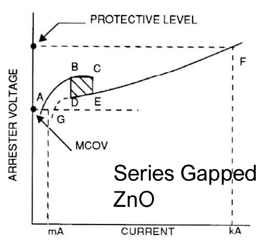


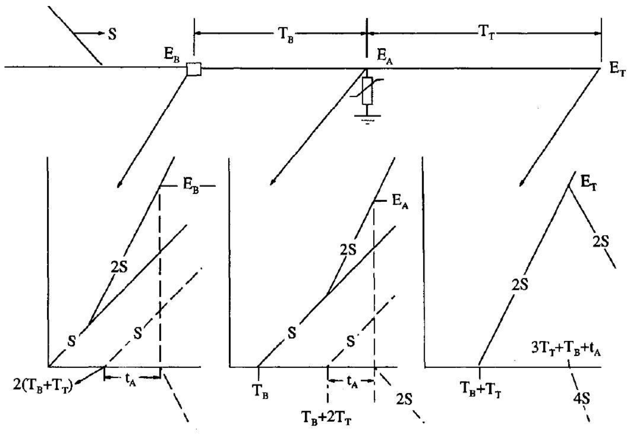
ZnO

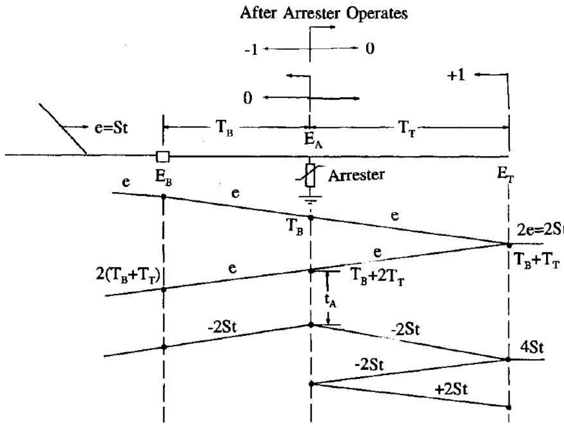










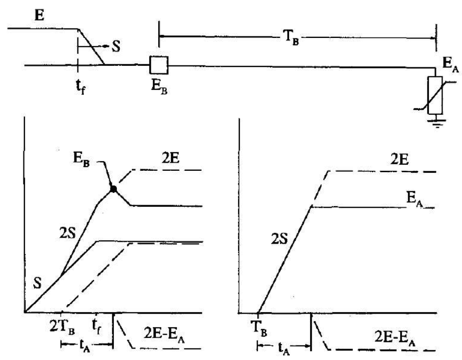


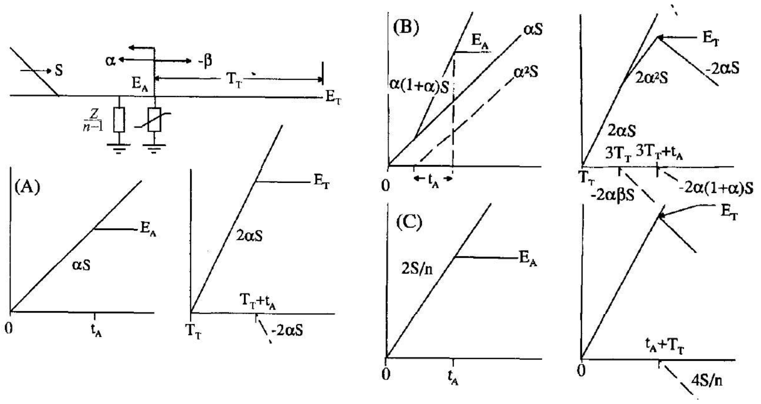
To demonstrate further the use of traveling wave theory, consider the effect of arresters in limiting the surge voltage at locations remote from the arrester. Arrester is considered as an "ideal" or constant-voltage arrester. That is, the assumption is that the arrester maintains a constant voltage E_A that is independent of the current being discharged by the arrester. Further assumed is that the arrester appears as an **opened circuit** until this voltage is reached. At that time, the arrester appears as a **short circuit**, since the voltage is held constant. This assumption does produce some useful equations.

With an arrester located in front of a transformer, which is here represented by an opened circuit and located behind a breaker. Note that the transformer should not be modeled as an opened circuit but as a **capacitance to ground** (2nF). The effect of the capacitance is to increase the voltage at the transformer and increase the current through the arrester.

Assume that a surge having a **front steepness** of S and an unlimited crest voltage travels in toward the breaker-arrester-transformer. The travel time between the arrester and the breaker is defined as T_B, and the travel time between the arrester and the transformer is defined as T_T. The surge arrives at the breaker at time zero, at the arresters at time T_B, and at the transformer at time $T_B + T_T$. Because of the opened circuit, the steepness at the transformer doubles, and a reflected surge having a steepness S travels back toward the arrester and breaker. Upon arrival at these other two locations, the steepness doubles. This situation continues with no further reflections until the arrester operates. Assume that the arrester operates at time t_A after the reflection from the transformer. Therefore the voltage at the arrester can be described by the equation

$$\begin{cases} E_A = 2ST_T + 2St_A \\ E_T = E_A + 2ST_T \\ E_B = E_A + 2ST_B \end{cases} \rightarrow \begin{cases} E_T = E_A + 2ST_T \\ E_B = E_A + 2ST_B \end{cases} \tag{4.119}$$





As noted by the equations, at both locations the voltage is increased by twice the steepness multiplied by the travel time between the arrester and the equipment. The maximum voltage at the transformer is $2E_A$, which occurs at $ST_T / E_A = 0.5$.

The maximum voltage at the breaker is the crest magnitude of the incoming surge plus half of the arrester voltage, where t_f is the time to crest of the incoming surge. The maximum voltage is

$$E_A = 2St_A, E_B = E + St_A = E + \frac{E_A}{2} \quad \text{for } (T_A + 2T_B) \geq t_f \tag{4.120}$$

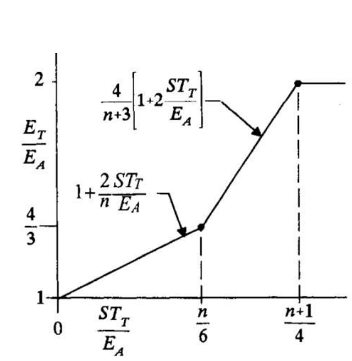
Now consider an **n-line station** and determine the voltage at the opened circuit, the transformer. The surge voltage with steepness S approaches the arrester and is reduced to as after passing the arrester. The transmitted and reflected coefficients for a surge approaching the arrester from the right are shown as follows. Moreover, to obtain the maximum voltage at the arrester, assume that the arrester operation occurs before a reflection returns from the opened end of the line, as depicted in figure (a). The arrester voltage and the voltage at the opened end are

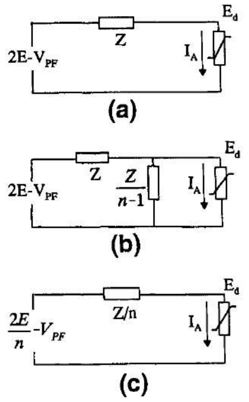
$$\alpha = \frac{2}{n+1}, \quad \beta = \frac{n-1}{n+1}, \quad \alpha = 1 - \beta \quad \rightarrow \begin{cases} E_A = \alpha St_A \\ E_T = 2\alpha St_A = 2E_A \end{cases} \tag{4.121}$$

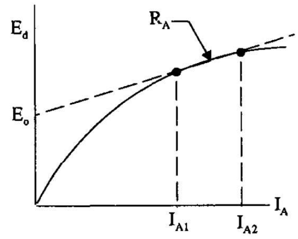
And the maximum voltage at the transformer is twice the arrester voltage.

Next, consider that the arrester operates after the first reflection from the transformer as shown in figure (b) where reflections are shown by the dotted lines. The voltages at the arrester and transformer are

$$\begin{cases} E_A = 2\alpha ST_T + \alpha(1+\alpha)St_A \\ E_T = 4\alpha ST_T + 2\alpha^2 St_A \end{cases} \rightarrow E_T = \frac{4}{n+3}(E_A + 2ST_T) \tag{4.122}$$







Comparing (4.121) and (4.122), the maximum voltage occurs at

$$\frac{ST_T}{E_A}=\frac{n+1}{4} \tag{4.123}$$

for small values of T_T, as shown in figure (c), the voltage steepness at the arrester and the transformer are equal at 2S/n. The arrester operates at time t_A, and the voltage at the transformer occurs at $t_A + T_T$, resulting in the equations

$$E_A=\frac{2S}{n}t_A\,,\qquad E_T=\frac{2S}{n}(t_A+T_T)=E_A+\frac{2S}{n}T_T \tag{4.124}$$

Equating (4.122) and (4.124) shows that the two equations intersect at $ST_T / E_A = n / 6$, at which point $E_T / E_A = 4 / 3$.

The arrester current I_A is therefore

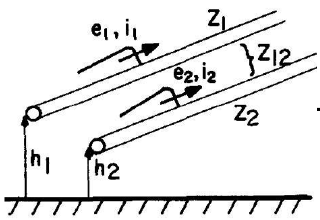
$$I_A=\frac{2E-E_d-V_{PF}}{Z}=\frac{2E-E_A}{Z} \tag{4.125}$$

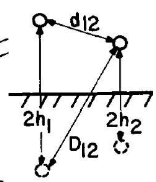
For an n-line station, the circuit of figure (b) applies, which using Thevenin's theorem can be reduced to that of figure (c). Thus, in general

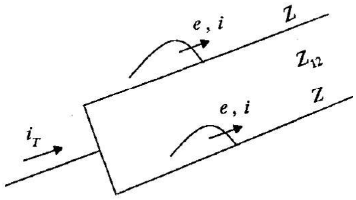
$$I_A=\frac{2E/n-E_d-V_{PF}}{Z/N}=\frac{2E/n-E_A}{Z/n} \tag{4.126}$$

The arrester discharge voltage E_d is a function of the arrester discharge current Z_A, and therefore these values are not independent. With the discharge voltage-current characteristic shown, and two currents Z_{A1} and Z_{A2}, the characteristic can be approximated as a straight line per the dotted line. Therefore the arrester discharge voltage is $E_d = E_0 + I_A R_A$

Arrester current, kA	R_A, ohms	E_0, kV
3		
→	5.0⟶	198
5		
→	4.0⟶	203
10		
→	3.4⟶	209
15		
→	2.2⟶	227
20		







With this modification, the arrester current becomes

$$I_A=\frac{2E/n-E_0-V_{PF}}{Z/n+R_A} \tag{4.127}$$

To illustrate by example, assume a single line, n = 1, and that Z = 400 ohms, E = 1500 kV, and V_m = 90 kV. Assume that the arrester characteristics for an 84 kV MCOV station class arrester are given per Table. The values of E_n and R_A are also given. Note first that the maximum current is 2E/Z or 3000/400 = 7.5 kA, and therefore as a first step assume that the current is below 7.5 kA, or is between 5 and 10 kA. Using these characteristics from Table 1, the current is I_A = (3000-203-90)/404 = 6.70kA. The arrester discharge voltage is E_d = 203 + 6.7(4) = 230 kV.

Consider two conductor separated by a distance d_{12} with unequal heights above ground. The radii of the conductors may be different, r_1 and r_2 for conductor 1 and 2. Z_1 and Z_2 are the self-surge impedance of each conductor, while Z_{12} is the mutual-surge impedance between conductors.

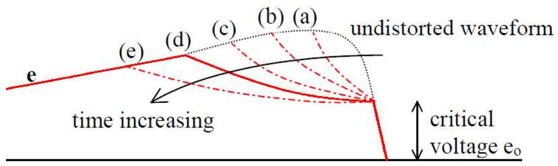
$$Z_1=60\ln\frac{2h_1}{r_1},\ Z_2=60\ln\frac{2h_2}{r_2},\qquad Z_{12}=60\ln\frac{D_{12}}{d_{12}} \tag{4.128}$$

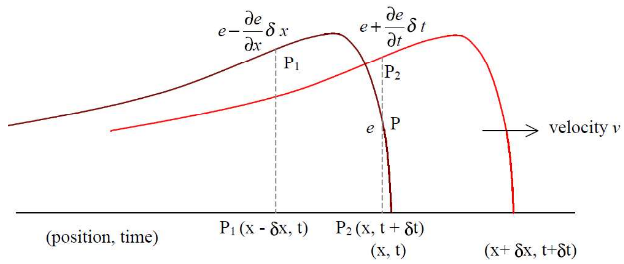
The travelling wave equations are similar to those for a single conductor except a voltage is induced by a current in the other conductor. Thus

$$\begin{cases} e_1=i_1Z_1+i_2Z_{12} \\ e_2=i_1Z_{12}+i_2Z_2 \end{cases} \to \begin{cases} i_1=\dfrac{Z_2e_1-Z_{12}e_2}{D} \\ i_2=\dfrac{Z_1e_2-Z_{12}e_1}{D} \end{cases} \quad D=Z_1Z_2-Z_{12}^2 \tag{4.129}$$

Consider a combined surge impedance of two conductor Z_e with total current i_T.

$$Z_e=\frac{e}{i_T}=\frac{e}{i_1+i_2},\qquad i=i_1=i_2=\frac{Z-Z_{12}}{D}e\ \to\ Z_e=\frac{e}{2i}=\frac{Z^2-Z_{12}^2}{2(Z-Z_{12})}=\frac{Z+Z_{12}}{2} \tag{4.130}$$





In general, for n conductors, if $Z = Z_1 = Z_2 = Z_3$ etc, and $Z_m = Z_{12} = Z_{13} = Z_{23}$ etc, then

$$Z_n = \frac{Z + (n-1)Z_m}{n} \tag{4.131}$$

As the voltage on a conductor is increased, a threshold voltage is reached, above which streamers emanate from the conductor, thus increasing the radius of the conductor. The streamer formation could be viewed as an increase in conductor radius, which therefore increases the **capacitance to ground**. Since the inductance of the conductor remains constant, this increase in capacitance results in a **decrease in the velocity of propagation** and a **decrease in conductor surge impedance**. This decrease in velocity results in a attenuation and distortion of the surge voltage. That is, the wave front is *pushed back* so that the steepness of the surge is deceased, and depending on the tail of the initial surge, the crest voltage is also decreased.

Consider the energy associated with a surge waveform

$$Q = \frac{1}{2}Ce^2 + \frac{1}{2}Li^2, \qquad i = \frac{e}{Z_0} = e\sqrt{\frac{C}{L}} \;\rightarrow\; Q = Ce^2 \tag{4.132}$$

As illustrated in the figure. Let the voltage at a point P at the position x be e at time t, i.e. e = e(x, t). The voltage at position P_1 just behind P would be

$$e - \frac{\partial e}{\partial x} v dt \tag{4.133}$$

If the voltage is above corona inception, it would not remain at this value, but attain a value $e - \frac{\partial e}{\partial x} dt$ at P at time t + dt, when the surge at P_1 moves toward P_2. Thus, corona causes a depression in the voltage and hence loss

$$e - v\frac{\partial e}{\partial x}dt \;\rightarrow\; e + \frac{\partial e}{\partial t}dt; \quad Q \;\rightarrow\; C\left[\left(e - v\frac{\partial e}{\partial x}dt\right)^2 - \left(e + \frac{\partial e}{\partial t}dt\right)^2\right] = -2Ce\left(v\frac{\partial e}{\partial x} + \frac{\partial e}{\partial t}\,dt\right) \tag{4.134}$$

The energy to create a corona field is proportional to the square of excess voltage, i.e. $k(e - e_0)^2$.
Thus the energy required to change the voltage is given by

$$e \rightarrow \left(e + \frac{\partial e}{\partial t}dt\right): \;\; Q_{cor} = k\left[\left(e + \frac{\partial e}{\partial t}dt - e_0\right)^2 - (e - e_0)^2\right] = 2k(e - e_0)\frac{\partial e}{\partial t}dt \tag{4.135}$$

The loss of energy causing distortion must be equal to the change in energy required. Thus,

$$-2Ce\left(v\frac{\partial e}{\partial x} + \frac{\partial e}{\partial t}\right)dt = \; 2k(e - e_0)\frac{\partial e}{\partial t}dt \tag{4.136}$$

Rearranging and simplifying gives the equation

$$v\frac{\partial e}{\partial x} = -\left(1 + \frac{k}{C}\frac{e - e_0}{e}\right)\frac{\partial e}{\partial t} \;\xrightarrow[\text{ideal}]{}\; v\frac{\partial e}{\partial x} = -\frac{\partial e}{\partial t} \tag{4.137}$$

Thus we see that the wave velocity has decreased below the normal propagation velocity, and that the wave velocity of an increment of voltage at e has a magnitude given by

$$v_e = \frac{v}{1 + \frac{k}{c}\frac{e - e_0}{e}} \tag{4.138}$$

Time of travel for an element at e when it travels a distance x is given by

$$t = \frac{x}{v_e} = \frac{x}{v}\left(1 + \frac{k}{c}\frac{e - e_0}{e}\right) \rightarrow \; \Delta t = \left(\frac{x}{v_e} - \frac{x}{v}\right) = \frac{x}{v}\frac{k}{c}\frac{e - e_0}{e} \;\rightarrow\; \frac{\Delta t}{x} = \frac{k}{vc}\left(1 - \frac{e_0}{e}\right) \tag{4.139}$$

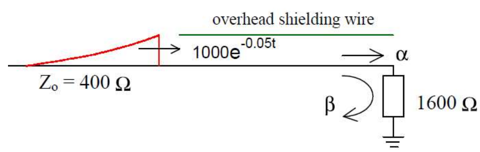
Example

A transformer has an impulse insulation level of 1050 kV and is to be operated with an insulation margin of 15% under lightning impulse conditions. The transformer has a surge impedance of 1600S and is connected to a transmission line having a surge impedance of 400 . A short length of overhead earth wire is to be used for shielding the line near the transformer from direct strikes. Beyond the shielded length, direct strokes on the phase conductor can give rise to voltage waves of the form 1000 $e^{-0.05t}$ kV (where t is expressed in s). If the corona distortion in the line is represented by the expression

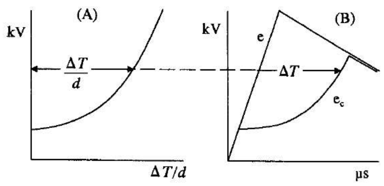
$$\frac{\Delta t}{x} = \frac{1}{B}\left(1 - \frac{e_0}{e}\right) \quad \mu s/\mathrm{m}$$

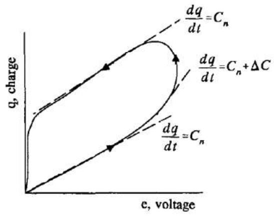
where B = 110m/ s and e_0 = 200 kV, determine the minimum length of shielding wire necessary in order that the transformer insulation will not fail due to lightning surges.

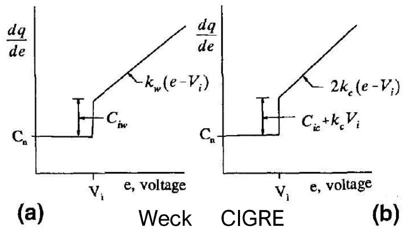
Transmission coefficient: α = 2(1600)/(1600+400) = 1.6
For BIL of 1050 kV and insulation margin of 15%.
Maximum permissible voltage = 1050 x 85% = 892.5 kV.



Since the voltage increased by the transmission coefficient 1.6 at terminal equipment, the maximum permissible incident voltage must be decreased by this factor.







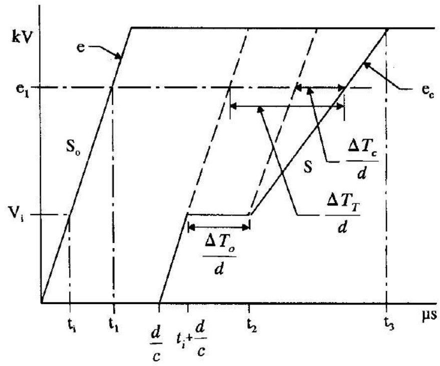
Thus maximum permissible incident surge = 892.5/1.6 = 557.8 kV

For transformer insulation to be protected by the shielding wire, the distortion caused must reduce the surge to a magnitude of 557.8kV.

Therefore, 1000 $e^{-0.05\,t}$ = 557.8. This gives the delay time t = 11.6 us.

Substitute in the equation gives 11.67 / x = 1/ 100 (1 – 200 / 557.8). It gives x = 2002 m = 2.0 km.

Hence, the minimum length of shielding wire required is 2 km.



Following tests on transmission lines, tests on conductors and oscillograms is obtained on charge - voltage (a q-e curve) as illustrated in the figure. Since the capacitance is dq/de, to be noted is that as the voltage increases, the capacitance increases until the crest voltage is attained. The dotted line indicates the natural capacitance of the conductor, C_n, and an additional dotted line represents an increased capacitance, C_n + ΔC. After the crest of the voltage, the q-e curve indicates that the capacitance returns to the natural capacitance of the conductor.

Voltage	CIGRE Capacitance
$e < E_i$	C_n
$e = E_i$	$C_n + C_{ic} + k_C V_i$
$e > E_i$	$C_n + C_{ic} + k_C (2e - V_i)$

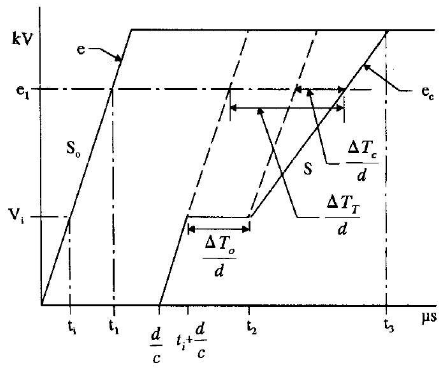
To derive the equations, the CIGRE equation for capacitance is used. For e = V_i, using the capacitance in the table, the value of $\Delta T_0/d$ is

$$\frac{\Delta T_0}{d}=\frac{1}{v_0}-\frac{1}{c}=\sqrt{L(C_n+C_{in}+k_cV_i)}-\sqrt{LC_n}=\sqrt{LC_n}(\sqrt{1+\frac{C_{ic}+k_cV_i}{C_n}}-1) \quad (4.140)$$

As an approximation,

$$\frac{\Delta T_0}{d}\approx\sqrt{LC_n}\left(1+\frac{C_{ic}+k_cV_i}{C_n}-1\right)\approx\frac{1}{2}Z_0(C_{ic}+k_cV_i) \quad (4.141)$$

Above Corona Start Voltage



Again using the CIGRE formulation and referring to the figure, at a voltage e_1,

$$\frac{\Delta T_c}{d}=\sqrt{L(C_n+C_{ic}+k_C(2e_1-V_i)}-\sqrt{L(C_n+C_{ic}+k_CV_i)} \quad (4.142)$$

$$=\sqrt{LC_n}\left[\sqrt{1+\left(C_n+\frac{k_C(2e_1-V_i)}{C_n}\right)}-\sqrt{1+\frac{C_{ic}+k_cV_i}{C_n}}\right]$$

Again approximating,

$$\frac{\Delta T_c}{d}\approx\sqrt{LC_n}\left[1+\frac{1}{2}\left(C_n+\frac{k_C(2e_1-V_i)}{C_n}\right)-\left(1+\frac{1}{2}\frac{C_{ic}+k_cV_i}{C_n}\right)\right] \quad (4.143)$$

$$=Z_0k_c(e_1-V_i)$$

To find the steepness S of the surge note first the steepness of e, denoted as S_0, is

$$S_0=\frac{e_1-V_i}{t_1-t_i} \quad (4.144)$$

$$S=\frac{e_1-V_i}{t_3-t_2}=\frac{e_1-V_i}{t_1-t_i+\Delta t_c}=\frac{S_0}{1+\Delta T\frac{S_0}{e_1}-V_i}=\frac{S_0}{1+k_cZ_0S_0d} \quad (4.145)$$

Setting

$$K_c=\frac{1}{k_cZ_0} \quad (4.146)$$

Then

$$S=\frac{S_0}{1+\frac{S_0d}{K_c}}\ \rightarrow\ \frac{1}{S}-\frac{1}{S_0}=\frac{d}{K_c}\ \xrightarrow[S_0=\infty]{}\ S=\frac{K_c}{d} \quad (4.147)$$

Conductor diameter inches/mm	E_i, kV	$\Delta t_0/d$, μs/km	K_c, kV-km/μs	C_{iw}, pF/m	$k_w = 2k_c$ pF/kV-m	C_{ic} pF/m
0.927/23.5	270	0.472	1132	1.92	3.6×10^{-3}	1.43
1.65/41.9	570	0.379	1531	1.66	2.9×10^{-3}	0.85
2.00/50.8	420	0.492	2432	2.21	1.9×10^{-3}	1.82

Corona Inception Voltage

The corona inception voltage for a single conductor can be estimated from the equation

$$V_i = \frac{Z_0 r E_0}{60} \tag{4.148}$$

where E_0 is the critical gradient usually in kV/m, r is the conductor radius in cm, and Z_0 is the natural or non-corona surge impedance. The critical gradient in CIGRE is

$$E_0 = 23\left(1 + \frac{1.22}{d^{0.37}}\right)\ \mathrm{kV/m} \tag{4.149}$$

For bundled conductors, an equation for the equivalent radius r_{eq} is provide.

$$r_{eq} = \frac{nr}{1 + 2(n-1)\sin\dfrac{\pi}{n}\dfrac{r}{A}} \approx \frac{nr}{1 + 2(n-1)\dfrac{\pi}{n}\dfrac{r}{A}} \approx nr \tag{4.150}$$

where A is the subconductor spacing in cm and n is the number of subconductors. Approximately the equivalent radius is equal to n times the conductor radius. The surge impedance Z_0 becomes the surge impedance of the bundle conductor.

$$Z_0 = \left(rA\left(\sin\frac{\pi}{n}\right)^{-1}\prod_{i=1}^{n-1}\sin\frac{i\pi}{n}\right)^{\frac{1}{n}} \tag{4.151}$$

Since the capacitance increases above the corona inception voltage, it would be expected that the surge impedance would decrease. Let Z_c be the surge impedance above corona, then

$$Z_c = \sqrt{\frac{L}{C_n + \Delta C}} = Z_0\sqrt{\frac{C_n}{C_n + \Delta C}} \tag{4.152}$$

Recall

$$C_n + \Delta C = \frac{10^{-3}}{18\ln\dfrac{2h}{R_c}} \tag{4.153}$$

$$Z_c = \sqrt{0.2\ln\frac{2h}{r}18\ln\frac{2h}{R_c}10^3} = \sqrt{60\ln\frac{2h}{r}\ 60\ln\frac{2h}{R_c}} = \sqrt{Z_0 Z_c{'}} \tag{4.154}$$

To obtain a corona radius,

$$Z_c' = \frac{Z_C^2}{Z_0} \rightarrow \ln R_c = \ln 2h - \frac{Z_c^2}{60 Z_0} \tag{4.155}$$

For the completeness, ZnO arrester, spark gap and arc modelling is provided.

ZnO Arrester

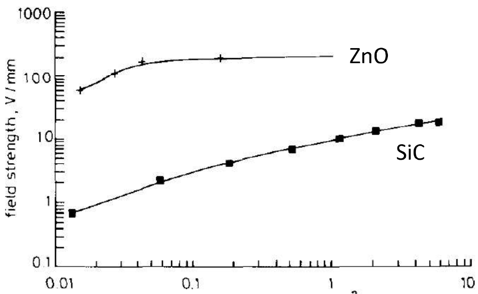
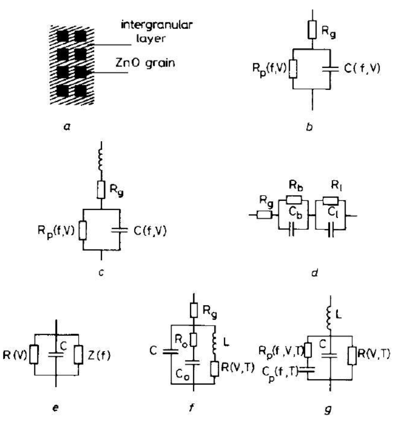
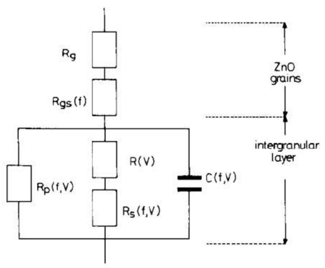


Fig. 1 *E/J characteristics for ZnO and SiC materials*



a Idealised model of microstructure [19]
b Voltage and frequency-dependent parallel circuit. Series R_g represents ZnO grain resistance [3, 20]
c Addition of inductance for high frequency and impulse responses [8, 14]
d Combined equivalent circuit for barrier and intergranular layer [10]
e Separation of voltage and frequency-dependent resistance elements [21]
f R_0, C_0 are polarisation elements, R is DC resistance [22]

In view of the above, the proposed equivalent circuit embraces the following observations:
(i) the increase of current and power consumption with increasing frequency in the prebreakdown region.
(ii) the decrease of current and power consumption with increasing frequency in the breakdown region
(iii) the upturn region, where the voltage is limited by the ZnO grain resistance, which will produce smaller current for faster surges at a given applied voltage.



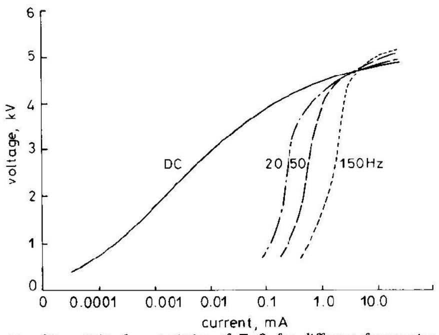
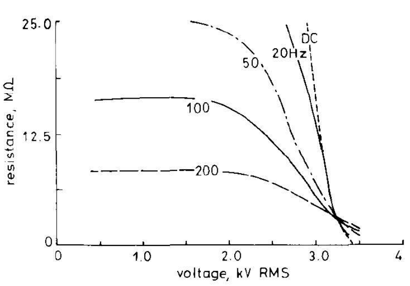
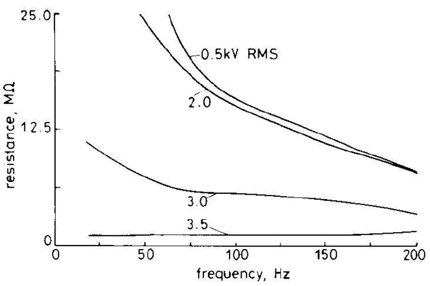
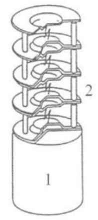


Fig. 9A *V/I characteristics of ZnO for different frequencies (peak values)*

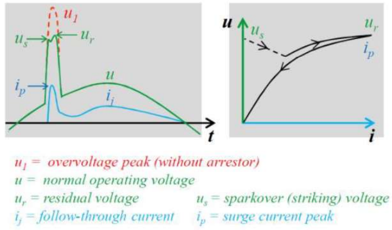




Ideal Condition:
- When voltage exceeds peak operating voltage, the arrester becomes **conductive** (weak resistor) allowing the surge energy to be discharged without increasing voltage over the protected device.
- Immediately after excess energy is discharged, the arrestor regains its insulating state



Reality:
- Limited energy discharge capacity (only applicable to relatively short duration overvoltage)
- Discharge of overvoltage is NOT immediate
- Leakage current is present even in insulating mode



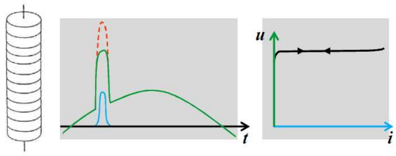
Disk spark gap (2) in series with SiC resistor (1) encased in a porcelain shell

Dividing the spark gap into sections decreases breakdown voltage scatter and flattens the steep transient resulting from flashover. The nonlinear resistor limits the earth fault current so that arcing is extinguished by itself (high current with low resistance, low current with high resistance.)

- As voltage over the arrestor exceeds sparkover (striking) voltage u_s, the spark gap is ignited.
- Surge current i_p grows to a value determined by the overvoltage magnitude.
- Residual voltage u_r (maximum voltage over arrestor during operation) is determined by the discharge current and nonlinear resistor magnitude
- After the overvoltage has passed, the arrestor remains conductive and follow-through current i_j (fed by the power frequency voltage) is present until the spark gap is extinguished (voltage becomes zero)

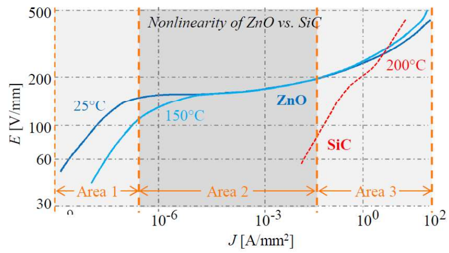
Metal Oxide Varistor (MOV)

With no rapid voltage changes, no breakdown voltage scatter and hence there is insignificant back current.



Area 1: ZnO penetrating current decreases radically under voltage threshold value (high resistivity). Poorly conductive surface layer determines magnitude of current.
- At small currents the resistance of the ZnO element decreases as temperature increases (negative thermal coefficient).
- Sufficient cooling needed to assure that the arrestor does not become unstable (thermal run-away) and break.

Area 2: Tunnel effect – more current penetrates through surface layer into ZnO core.
Area 3: Tunnel effect throughout entire material. Magnitude of current determined by core. Resistivity of material is very small.



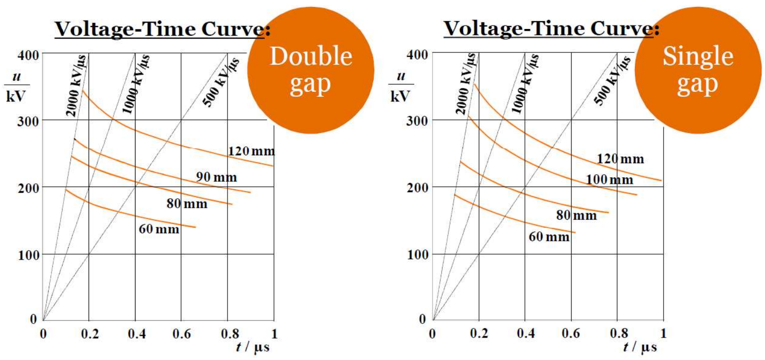
Spark Gap

Simple device consisting of two electrodes – one connected to the conductor to be protected and the other to ground.

Spark gaps form a weak point enabling overvoltage to flow to earth instead of to the protected device.

Breakdown voltage can be adjusted with gap distance.

It is a cheaper and simpler solution for protecting smaller pole transformers is to use a spark gap



500 kV/µs: Direct lightning stroke to conductor
1000 – 2000 kV/µs: Back flashover (rare)

Arc Model

In Mayr's Arc Model, assuming constant arc diameter, constant arc power loss, Saha's expression of arc conductivity, etc. the following expression is deduced.:

$$\frac{1}{G}\frac{dG}{dt} = \frac{1}{\theta}\left(\frac{EI}{N_0} - 1\right) \qquad (4.156)$$

where
G = arc conductivity, θ = arc time constant,
E = arc voltage, I = arc current, N_0 = arc loss constant

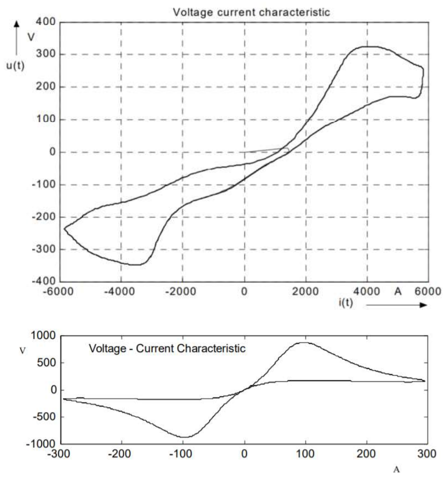
Introduce Laplace operator,

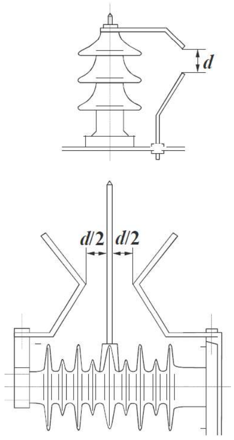
$$G_0 = \frac{I^2}{N_0} \rightarrow G = \frac{G_0}{1 + s\theta} \qquad (4.157)$$

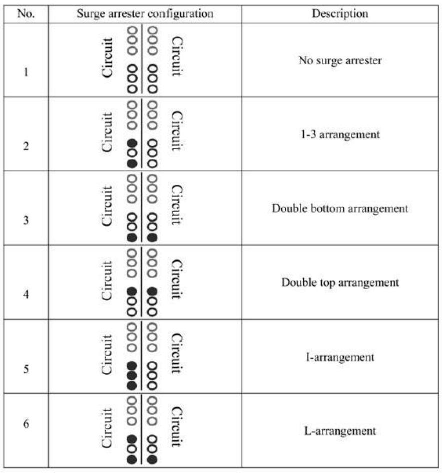
Cassie arc model assumes heat loss depends on arc flow (convective loss), and heat loss, stored heat and conductance depend on cross section area.
Hence,

$$\frac{1}{G}\frac{dG}{dt} = \frac{1}{\theta}\left(\frac{E^2}{E_0^2} - 1\right) \qquad (4.158)$$

$$G_0 = G^2 = \left(\frac{I}{E}\right)^2 \rightarrow G_0 = \frac{I^2}{E_0^2}\frac{1}{1 + s\theta} \qquad (4.159)$$







No.	Surge arrester configuration	Description
1	Circuit / Circuit	No surge arrester
2	Circuit / Circuit	1-3 arrangement
3	Circuit / Circuit	Double bottom arrangement
4	Circuit / Circuit	Double top arrangement
5	Circuit / Circuit	I-arrangement
6	Circuit / Circuit	L-arrangement

○275kV circuit without SA ○132kV circuit without SA ●132kV circuit with SA

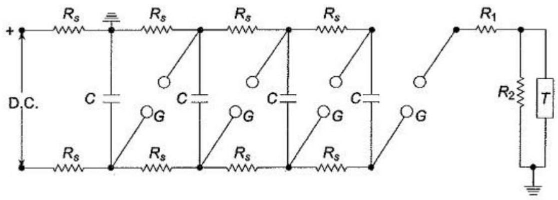
Configuration	Lightning current																							
	122 kA						150 kA						170 kA						200 kA					
	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2
no arrester	X	X					X	X				X	X	X	X	X		X	X	X	X	X	X	X
1–3 arrangement		X						X				X		X	X	X		X		X	X	X		X
double bottom		X						X	X	X				X	X	X	X			X	X	X	X	
double bottom	X						X					X	X		X	X		X	X		X	X		X
I-arrangement		X						X				X		X		X		X		X		X		X
I-arrangement								X						X	X	X				X	X	X		

Configuration	Footing resistance																							
	10 Ω						20 Ω						30 Ω						50 Ω					
	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2
no arrester	X	X					X	X				X	X	X	X	X		X	X	X	X	X	X	X
1–3 arrangement		X						X				X		X	X	X		X		X	X	X		X
double bottom		X						X						X	X	X				X	X	X	X	
double bottom	X						X					X	X		X	X		X	X		X	X		X
I-arrangement		X						X				X		X		X		X		X				X
I-arrangement								X						X	X	X				X	X	X		

Configuration	Phase angle																							
	0°						70°						180°						300°					
	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2	R1	R2	B1	B2	Y1	Y2
no arrester	X	X							X	X					X	X		X	X					X
1 3 arrangement		X							X	X								X						X
double bottom		X							X	X					X	X								
double bottom	X								X	X								X	X					X
I-arrangement		X								X								X						X
I-arrangement									X	X														

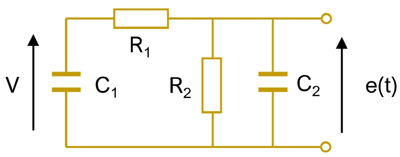
4.8 Marx Generator

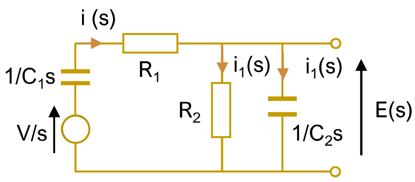
To generate the lightning impulse for HV testing, Marx Generator is often used as the impulse generator with double exponential output, i.e.

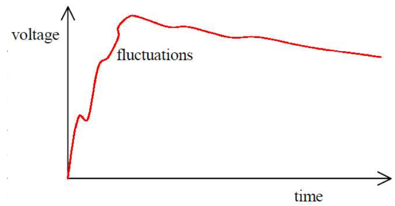


$$e(t) = \frac{V}{C_2 R_1}\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t}) \tag{4.159}$$

It is given by the following circuit.







The circuit analysis is as follows.

$$\frac{V}{s} = i(s)\frac{1}{C_1 s} + i(s)R_1 + i_1(s)R_2 \qquad E(s) = i_1(s)R_2 = i_2(s)\frac{1}{C_2 s} \qquad i(s) = i_1(s) + i_2(s) \tag{4.160}$$

$$i_1(s)\left(R_1 + R_2 + \frac{1}{C_1 s}\right) + i_2(s)\left(R_1 + \frac{1}{C_1 s}\right) = \frac{V}{s} \qquad i_2(s) = C_2 R_2 s\, i_1(s) \tag{4.161}$$

$$i_1(s)\left(R_2 + R_1 + \frac{1}{C_1 s} + R_1 C_2 R_2 s + \frac{R_2 C_2}{C_1}\right) = \frac{V}{s} \rightarrow \quad E(s) = i_1(s)R_2 = \frac{V C_1 R_2}{R_1 R_2 C_1 C_2 s^2 + (C_1 R_2 + C_1 R_1 + C_2 R_2)s + 1} \tag{4.162}$$

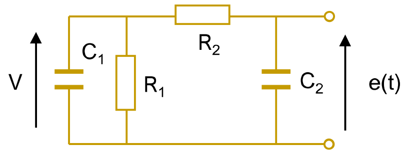
If α and β are the solutions of the equation

$$R_1 R_2 C_1 C_2 s^2 + (C_1 R_2 + C_1 R_1 + C_2 R_2)s + 1 = 0 \tag{4.163}$$

Then E(s) can be expressed as

$$E(s) = \frac{V}{R_1 C_2}\frac{1}{(s+\alpha)(s+\beta)} = \frac{V}{R_1 C_2}\frac{1}{\beta - \alpha}\left(\frac{1}{s+\alpha} - \frac{1}{s+\beta}\right) \rightarrow e(t) = \frac{V}{C_2 R_1}\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t}) \tag{4.164}$$

An alternative form of the circuit is as shown in the figure.



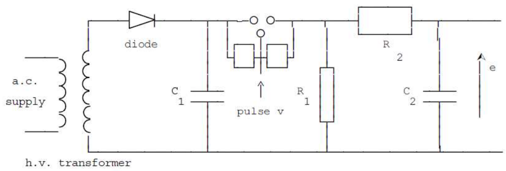
Consider again for (4.164). The peak value of this voltage occurs when

$$\frac{de(t)}{dt} = 0 \rightarrow \alpha e^{-\alpha t} = \beta e^{-\beta t} \tag{4.165}$$

With $\beta \gg \alpha$,

$$e^{(\beta-\alpha)t} = \frac{\beta}{\alpha} = e^{\beta t} \rightarrow E_{max} = \frac{V}{C_2 R_1}\frac{1}{\beta - \alpha}\left(1 - \alpha - \frac{\alpha}{\beta}\right) \approx \frac{V}{\beta C_2 R_1} \tag{4.166}$$

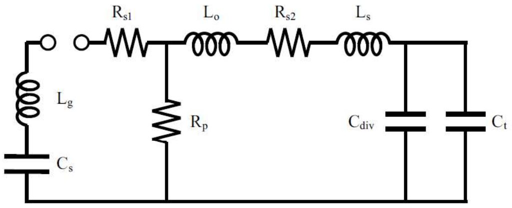
After reaching the peak, the voltage falls to half maximum in time t_2 given by

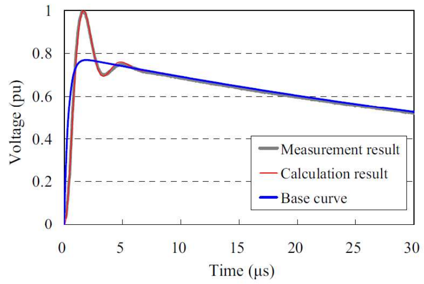


$$e^{-\beta t_2} \ll e^{-\alpha t_2} \rightarrow \frac{V}{2\beta R_1 C_2} \approx \frac{V}{\beta C_2 R_1} e^{-\alpha t_2} \rightarrow e^{-\alpha t_2} \approx \frac{1}{2} \tag{4.166}$$

Question:
How to size R_1, R_2, C_1, C_2 with the requirement on α, β

It is noted with intrinsic wiring inductance, it can lead to overshoot and hence the failure of tested object with increased testing voltage.





Lightning Protection	Feasibility Check	Economic Viability
Add or extend Shielding wire(s)	• Lines are generally unshielded due to sensitive reasons • Strongly depends on tower design • Not effective for high footing resistance	• High material & labor costs • Power interruption is required • **Non-economical solution**
Increase BIL (insulator replacement)	• Strongly depends on tower design and system clearances • Leads to travelling / propagating waves on the line for **high footing resistances**	• High material & labor costs • Power interruption is required • **Non-economical solution**
Improved tower Footing Resistances	• Additional copper counterpoise might be completely inefficient with high soil resistance • Only efficient for shielded lines • Eliminates only back-flashovers and doesn't influence shielding failures.	• Moderate installation costs • Improvement & Cost-efficiency is not guaranteed
Install Line Surge Arresters	• Versatile & Large feasibility • Highest protective effectiveness even for high footing resistances in all terrains • Eliminate all types of lightning failures.	• Low material & labor costs • Live installation possible. • Cost-efficient solution.

Reference

[1] Hileman A.R. (1999), Insulation Coordination for Power System
[2] Chattopadhyay S. & Das A. (2021), Overhead Electric Power Lines: Theory and practice
[3] Giraudet F. (2017) Line Surge Arresters: Applications, Designs, Trends, Monitoring and Recommendation
[4] Halim S. A. et al (2016), Lightning back flashover tripping patterns on a 275/132 kV quadruple circuit transmission line in Malaysia

Summary

4.1 Corona

Max. Electric Stress: $$G_{max} = \frac{1}{2\pi\varepsilon}\frac{q}{r} = \frac{V_{ab}}{2r\ln\frac{D}{r}} = \frac{V_{an}}{r\ln\frac{D}{r}}$$

Corona Voltage: $$V_C = G_0\delta r\ln\frac{D}{R} = G_0\frac{3.92b}{273+t}r\ln\frac{D}{R}$$

4.2 Clearance

Overvoltage Type	Rod-to-Plane Voltage Formula	Clearance Formula
Fast-Front OV	$U_{rp} = U_{90\%ff\_ins} = 530K_{z\_ff}K_{g\_ff\_ins}d_{ins}$	$D_{el\_ff} = U_{90\%ff_{ins}}/(530K_aK_{z_{ff}}K_{g_{ff}})$ $D_{el\_ff} = 1.2U_{90\%ff_{ins}}/(530K_aK_{z_{ff}}K_{g_{ff}})$
Slow-Front OV	Phase to Earth: $U_{rp} = K_{cs}U_{2\%_{sf}}$ Phase to Phase: $U_{rp} = 1.4K_{cs}U_{2\%_{sf}}$	$D_{el\_sf} = 2.17\exp\left(\frac{K_{cs}U_{2\%sf}}{1080K_aK_{z_{sf}}K_{g_{sf}}} - 1\right)$ $D_{pp\_sf} = 2.17\exp\left(\frac{1.4K_{cs}U_{2\%sf}}{1080K_aK_{z_{sf}}K_{g_{sf}}} - 1\right)$
Power Frequency	Phase to Earth: $U_{rp} = \sqrt{2}U_s/\sqrt{3}$ Phase to Phase: $U_{rp} = \sqrt{2}U_s$	$D_{pf\_pe} = 1.64\exp\left(\frac{U_s}{750\sqrt{3}K_aK_{zpf}K_{gpf}} - 1\right)$ $D_{pf\_pp} = 1.64\exp\left(\frac{U_s}{750K_aK_{zpf}K_{gpf}} - 1\right)$

4.3 Insulation Strength Characteristics

CFO: $$V_3 = CFO - 3\sigma_f = CFO\left(1 - 3\frac{\sigma_f}{CFO}\right)$$

Strike Distance: $$S = \frac{8}{\frac{3400(0.96)k_g\delta^m}{CFO_A} - 1}$$

4.4 Switching Overvoltage in Transmission Lines

$$SSFOR = \frac{1}{2}\left[F\left(\frac{E_m - \mu_0}{\sigma_0}\right) - F\left(\frac{CFO_n - \mu_0}{\sigma_0}\right)\right]$$

4.5 Shielding of Transmission Lines

Strike Radius to Ground: $$r_{gm} = \frac{\frac{h+y}{2}}{1 - \gamma\sin\alpha}$$

Shield Distance & Shielding Angle: $$a_p = \sqrt{r_c^2 - (r_g - h)^2} - \sqrt{r_c^2 - (r_g - y)^2} \rightarrow \alpha_p = \tan^{-1}\frac{a_p}{h - y}$$

4.6 A Review of Travelling Waves

$$e'' = \frac{2Z_k}{Z + Z_k}e, \quad i'' = \frac{e''}{Z_k} = \frac{2Z}{Z + Z_k}i$$

$$e' = e'' - e = \frac{Z_k - Z}{Z + Z_k}e, \quad i' = \frac{e'}{Z_k} = \frac{Z_k - Z}{Z + Z_k}i$$

THE END