Potential field solution when the potential at the surface of the cylinder is V_0, the radius is a, and the field strength is E_0:

$$\Phi(r,\theta) = V_0 + E_0 cos\theta \left(\frac{a^2}{r} - r \right)$$

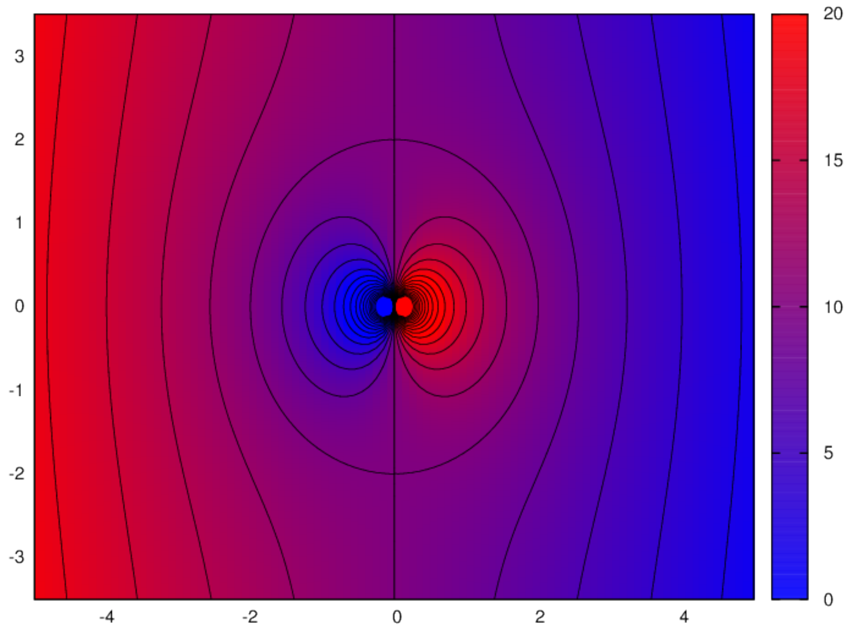

Figure 1: Shape of potential (scalar) field when $a = 2$, $E_0 = 2$, and $V_0 = 10$. Electric field lines are perpendicular to the drawn contour lines.

Electric field (set $E_0 = a = 1$):

$$E(x,y) = -\nabla\Phi(x,y) = \left(\frac{2x^2}{(x^2+y^2)^2} - \frac{1}{x^2+y^2} + 1 \right) \underline{i} + \frac{2xy}{(x^2+y^2)^2} \underline{j}$$

Problem: take care of singularity inside conductor (potential field should be constant inside conductor). Define it piecewise?

Question: is V_0 a function of E_0 since we know the cylinder is a perfect conductor?