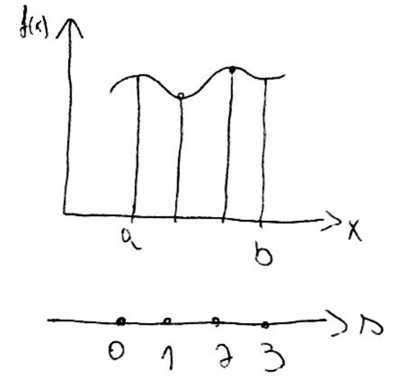
KARLOS - GRAU 3 FECHADA



$$I = \int_a^b f(x)\,dx \simeq h \cdot \int_0^3 g(s)\,ds$$

$$g(s) = \sum_{k=0}^{3} \binom{s}{k} \Delta^k f_0 \qquad x(s) = a + s \cdot h$$

$$\frac{dx}{ds} = h$$

$k=0$: f_0

$k=1$: $s \cdot (f_1 - f_0)$

$k=2$: $\frac{1}{2} \cdot (s^2 - s) \cdot (f_2 - 2f_1 + f_0)$

$k=3$: $\frac{1}{6} \cdot (s^3 - 3s^2 + 2s) \cdot (f_3 - 3f_2 + 3f_1 - f_0)$

$$g(s) = f_0 + s \cdot (f_1 - f_0) + \frac{1}{2} \cdot (s^2 - s) \cdot (f_2 - 2f_1 + f_0) + \frac{1}{6} \cdot (s^3 - 3s^2 + 2s) \cdot (f_3 - 3f_2 + 3f_1 - f_0)$$

$$I \simeq h \cdot \int_0^3 g(s)\,ds = h \cdot \Big\{ f_0 \int_0^3 ds + (f_1 - f_0) \cdot \int_0^3 s\,ds + \frac{1}{2} \cdot (f_2 - 2f_1 + f_0) \cdot \Big[\int_0^3 s^2\,ds - \int_0^3 s\,ds \Big] +$$
$$+ \frac{1}{6} \cdot (f_3 - 3f_2 + 3f_1 - f_0) \cdot \Big[\int_0^3 s^3\,ds - 3\int_0^3 s^2\,ds + 2\int_0^3 s\,ds \Big] \Big\} =$$

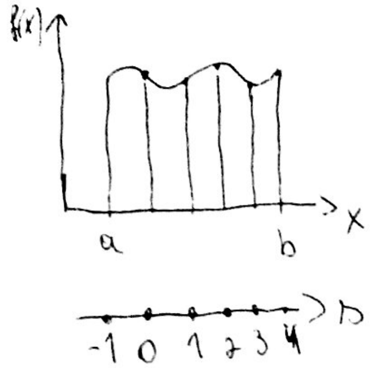
$$= h \cdot \Big\{ f_0 \cdot s\Big|_0^3 + (f_1 - f_0) \cdot \frac{s^2}{2}\Big|_0^3 + \frac{1}{2} \cdot (f_2 - 2f_1 + f_0) \cdot \Big[\frac{s^3}{3}\Big|_0^3 - \frac{s^2}{2}\Big|_0^3 \Big] +$$
$$+ \frac{1}{6} \cdot (f_3 - 3f_2 + 3f_1 - f_0) \cdot \Big[\frac{s^4}{4}\Big|_0^3 - 3 \cdot \frac{s^3}{3}\Big|_0^3 + 2 \cdot \frac{s^2}{2}\Big|_0^3 \Big] \Big\} =$$

$$= h \cdot \Big\{ 3f_0 + (f_1 - f_0) \cdot \frac{9}{2} + \frac{1}{2} \cdot (f_2 - 2f_1 + f_0) \cdot \frac{9}{2} + \frac{1}{6} \cdot (f_3 - 3f_2 + 3f_1 - f_0) \cdot \frac{9}{4} \Big\} =$$

$$= h \Big\{ f_0 \cdot \Big(3 - \frac{9}{2} + \frac{9}{4} - \frac{3}{8}\Big) + f_1 \cdot \Big(\frac{9}{2} - \frac{9}{2} + \frac{9}{8}\Big) + f_2 \cdot \Big(\frac{9}{4} - \frac{9}{8}\Big) + f_3 \cdot \Big(\frac{9}{24}\Big) \Big\} =$$

$$= h \cdot \Big\{ \frac{3}{8} \cdot f_0 + \frac{9}{8} \cdot f_1 + \frac{9}{8} \cdot f_2 + \frac{3}{8} \cdot f_3 \Big\} = \boxed{\frac{3}{8} \cdot h \cdot \{ f_0 + 3f_1 + 3f_2 + f_3 \}}$$

Karlos - Grau 3 Aberta



$$I = \int_a^b f(x)\,dx \simeq h \int_{-1}^{4} g(s)\,ds$$

○ Mantemos $g(s)$ que nem o da Grau 3 Fechada

$$x(s) = a + h + s.h \qquad \frac{dx}{ds} = h$$

$$I \simeq h \cdot \int_{-1}^{4} g(s)\,ds = h \cdot \left\{ f_0 \cdot s \Big|_{-1}^{4} + (f_1 - f_0) \cdot \frac{s^2}{2}\Big|_{-1}^{4} + \frac{1}{2} \cdot (f_2 - 2f_1 + f_0) \cdot \left[\frac{s^3}{3}\Big|_{-1}^{4} - \frac{s^2}{2}\Big|_{-1}^{4} \right] + \right.$$

$$\left. + \frac{1}{6} \cdot (f_3 - 3f_2 + 3f_1 - f_0) \cdot \left[\frac{s^4}{4}\Big|_{-1}^{4} - 3 \cdot \frac{s^3}{3}\Big|_{-1}^{4} + 2 \cdot \frac{s^2}{2}\Big|_{-1}^{4} \right] \right\} =$$

$$= h \cdot \left\{ f_0 \cdot 5 + (f_1 - f_0) \cdot \frac{15}{2} + \frac{1}{2} \cdot (f_2 - 2f_1 + f_0) \cdot \frac{85}{6} + \frac{1}{6} \cdot (f_3 - 3f_2 + 3f_1 - f_0) \cdot \frac{55}{4} \right\} =$$

$$= h \cdot \left\{ f_0 \cdot \left(5 - \frac{15}{2} + \frac{85}{12} - \frac{55}{24}\right) + f_1 \cdot \left(\frac{15}{2} - \frac{85}{6} + \frac{55}{8}\right) + f_2 \cdot \left(\frac{85}{12} - \frac{55}{8}\right) + f_3 \cdot \left(\frac{55}{24}\right) \right\} =$$

$$= h \cdot \left\{ f_0 \cdot \frac{55}{24} + f_1 \cdot \frac{5}{24} + f_2 \cdot \frac{5}{24} + f_3 \cdot \frac{55}{24} \right\} =$$

$$= \boxed{\frac{5}{24} \cdot h \cdot (11 f_0 + f_1 + f_2 + 11 f_3)}$$

$$s\Big|_{-1}^{4} = 5$$

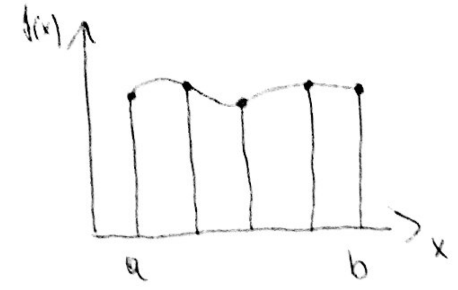
$$\frac{s^2}{2}\Big|_{-1}^{4} = \frac{15}{2}$$

$$\frac{s^3}{3}\Big|_{-1}^{4} = \frac{65}{3}$$

$$\frac{s^4}{4}\Big|_{-1}^{4} = \frac{255}{4}$$

$$\frac{s^5}{5}\Big|_{-1}^{4} = 205$$

KANLOS - GRAU 4 FECHADA



$$I = \int_a^b f(x)\,dx \simeq \int_0^4 g(s)\,ds$$

$$x(s) = a + sh \quad \frac{dx}{ds} = h$$

$$g(s) = \sum_{k=0}^{4} \binom{s}{k} \Delta^k f_0$$

$k=0 : f_0$

$k=1 : s\,(f_1 - f_0)$

$k=2 : \frac{1}{2}\cdot(s^2 - s)\cdot(f_2 - 2f_1 + f_0)$

$k=3 : \frac{1}{6}\cdot(s^3 - 3s^2 + 2s)\cdot(f_3 - 3f_2 + 3f_1 - f_0)$

$k=4 : \frac{1}{24}\cdot(s^4 - 6s^3 + 11s^2 - 6s)\cdot(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$

$$g(s) = f_0 + s\cdot(f_1 - f_0) + \frac{1}{2}\cdot(s^2 - s)\cdot(f_2 - 2f_1 + f_0) + \frac{1}{6}\cdot(s^3 - 3s^2 + 2s)\cdot(f_3 - 3f_2 + 3f_1 - f_0) + \frac{1}{24}\cdot(s^4 - 6s^3 + 11s^2 - 6s)\cdot(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$

$$I = h\int_0^4 g(s)\,ds = h\cdot\Big\{f_0\cdot s\Big|_0^4 + (f_1 - f_0)\cdot\frac{s^2}{2}\Big|_0^4 + \frac{1}{2}\cdot(f_2 - 2f_1 + f_0)\cdot\left[\frac{s^3}{3}\Big|_0^4 - \frac{s^2}{2}\Big|_0^4\right] +$$

$$+ \frac{1}{6}\cdot(f_3 - 3f_2 + 3f_1 - f_0)\cdot\left[\frac{s^4}{4}\Big|_0^4 - 3\cdot\frac{s^3}{3}\Big|_0^4 + 2\cdot\frac{s^2}{2}\Big|_0^4\right] + \frac{1}{24}\cdot(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)\cdot\left[\frac{s^5}{5}\Big|_0^4 - 6\cdot\frac{s^4}{4}\Big|_0^4 + 11\cdot\frac{s^3}{3}\Big|_0^4 - 6\cdot\frac{s^2}{2}\Big|_0^4\right]$$

$$= h\cdot\Big\{f_0\cdot 4 + (f_1 - f_0)\cdot 8 + (f_2 - 2f_1 + f_0)\cdot\frac{40}{6} + (f_3 - 3f_2 + 3f_1 - f_0)\cdot\frac{16}{6} +$$

$$+ (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)\cdot\left(\frac{112}{360}\right)\Big\} =$$

$$= h\cdot\Big\{f_0\cdot\left(4 - 8 + \frac{40}{6} - \frac{16}{6} + \frac{112}{360}\right) + f_1\cdot\left(8 - \frac{80}{6} + \frac{48}{6} - \frac{56}{45}\right) + f_2\cdot\left(\frac{40}{6} - \frac{48}{6} + \frac{84}{45}\right) +$$

$$+ f_3\cdot\left(\frac{16}{6} - \frac{64}{45}\right) + f_4\cdot\left(\frac{14}{45}\right)\Big\} = \boxed{\frac{2}{45}\cdot h\cdot(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4)}$$

KARCOS - GRAU 4 ABERTA

$$I = \int_a^b f(x)\,dx \cong h \cdot \int_{-1}^{5} g(s)\,ds$$

$$x(s) = a + h + s \cdot h \qquad \frac{dx}{ds} = h$$

o MANTEMOS $g(s)$ QUE VEM NA GRAU 4 FECHADA

$$g(s) = f_0 + s \cdot (f_1 - f_0) + \frac{1}{2} \cdot (s^2 - s) \cdot (f_2 - 2f_1 + f_0) + \frac{1}{6} \cdot (s^3 - 3s^2 + 2s) \cdot (f_3 - 3f_2 + 3f_1 - f_0)$$
$$+ \frac{1}{24} \cdot (s^4 - 6s^3 + 11s^2 - 6s) \cdot (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$

$$I \cong h \int_{-1}^{5} g(s)\,ds = h \cdot \Big\{ f_0 \cdot s\Big|_{-1}^{5} + (f_1 - f_0) \cdot \frac{s^2}{2}\Big|_{-1}^{5} + \frac{1}{2} \cdot (f_2 - 2f_1 + f_0) \cdot \left(\frac{s^3}{3}\Big|_{-1}^{5} - \frac{s^2}{2}\Big|_{-1}^{5}\right) +$$
$$+ \frac{1}{6} \cdot (f_3 - 3f_2 + 3f_1 - f_0) \cdot \left[\frac{s^4}{4}\Big|_{-1}^{5} - 3 \cdot \frac{s^3}{3}\Big|_{-1}^{5} + 2 \cdot \frac{s^2}{2}\Big|_{-1}^{5}\right]$$
$$+ \frac{1}{24} \cdot (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0) \cdot \left[\frac{s^5}{5}\Big|_{-1}^{5} - 6 \cdot \frac{s^4}{4}\Big|_{-1}^{5} + 11 \cdot \frac{s^3}{3}\Big|_{-1}^{5} - 6 \cdot \frac{s^2}{2}\Big|_{-1}^{5}\right]\Big\} =$$

$$= h \cdot \Big\{ f_0 \cdot 6 + (f_1 - f_0) \cdot 12 + (f_2 - 2f_1 + f_0) \cdot 15 + (f_3 - 3f_2 + 3f_1 - f_0) \cdot 9 + (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0) \cdot \frac{33}{10} \Big\} =$$

$$= h \cdot \Big\{ f_0 \left(6 - 12 + 15 - 9 + \frac{33}{10}\right) + f_1 \cdot \left(12 - 30 + 27 - \frac{132}{10}\right) + f_2 \left(15 - 27 + \frac{198}{10}\right) + f_3 \left(9 - \frac{132}{10}\right)$$
$$+ f_4 \cdot \left(\frac{33}{10}\right) \Big\} =$$

$$= h \cdot \Big\{ f_0 \cdot \frac{33}{10} - f_1 \cdot \frac{42}{10} + f_2 \cdot \frac{78}{10} - f_3 \cdot \frac{42}{10} + f_4 \cdot \frac{33}{10} \Big\} =$$

$$= \boxed{\frac{3}{10} \cdot h \cdot \{ 11f_0 - 14f_1 + 26f_2 - 14f_3 + 11f_4 \}}$$

$$s\Big|_{-1}^{5} = 6$$

$$\frac{s^2}{2}\Big|_{-1}^{5} = 12$$

$$\frac{s^3}{3}\Big|_{-1}^{5} = 42$$

$$\frac{s^4}{4}\Big|_{-1}^{5} = 156$$

$$\frac{s^5}{5}\Big|_{-1}^{5} = \frac{3126}{5}$$