

# Seat apportionment in the European Parliament and Brexit

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# 1 Introduction

On 23rd June 2016, citizens of the United Kingdom decided in the European Union membership referendum to leave the EU (so-called Brexit).<sup>[1]</sup> Since then, there have been discussions about how to leave the EU, but as of December 2019, the UK is still a member. On 12th December 2019, Boris Johnson, a politician who is strongly for Brexit, won general election in the UK, resulting in 365 seats won out of the total of 650 seats<sup>[2]</sup>, and became the prime minister. With such a majority in the House of Commons, things might finally start to move, and the UK could leave the EU as of 31st January, 2020.<sup>[1]</sup>

One of the aspects of leaving the EU is that the seats of the leaving country in several EU institutions need to be handled. One of those institutions is the European Parliament, often abbreviated as EP.

It is already decided how the seats belonging to the UK will be redistributed among the remaining members of the EU. Once the UK actually leaves, out of the current 73 British seats, 27 are planned to be reassigned, while the remaining 46 will be empty, effectively shrinking the size of the Parliament from 751 seats down to 705.<sup>[3]</sup> There are already some questions: Why not just reassign all the seats? How come that the shrink has the size of 46? But before the answers, one needs to understand how the apportionment in the EP actually works.

This paper elaborates on the seat apportionment in the European Parliament and tries to answer those questions.

## 2 Seat Apportionment in the EP

The seat apportionment method used for the European Parliament is defined in the Treaty on European Union, Art. 14, par. 2:<sup>[4]</sup>

*The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats.*

To be more expressive, building a mathematical model should help. Let  $S$  be the tuple of EU member states.

**Definition 2.1.** *Apportionment method*<sup>[5, p. 55]</sup>

$$A : \{S\} \times \mathbb{N}^{|S|} \times \mathbb{N} \rightarrow \mathbb{N}^{|S|}$$

takes the tuple of member states, their populations and the parliament size and yields the number of seats allocated for each member state, let it be  $(a_1, a_2, \dots, a_{|S|})$ . Then,  $\sum(a_1, a_2, \dots, a_{|S|})$  is equal to the parliament size (the whole parliament is distributed).

Let  $P \in \mathbb{N}$  be the size of the European Parliament,  $A_{EP}$  be the apportionment method used to calculate the seat allocation in the EP, and  $p \in \mathbb{N}^{|S|}$  the populations used in that calculation. Then, from the EU Treaty paragraph shown above, the following could be derived (degressive proportionality will be discussed in a moment):

$$P \leq 751 \tag{1}$$

$$\min A_{EP}(S, p, P) \geq 6 \tag{2}$$

$$\max A_{EP}(S, p, P) \leq 96 \tag{3}$$

The Treaty of Lisbon from December 2009<sup>[6]</sup> grants the right to decide the actual apportionment to the European Council.<sup>[7]</sup>

These rules do not just look ambiguous – they really are. That is probably one of many reasons why Cambridge Compromise took place in 2011. It refines the definition of degressive proportionality so that it can be used for the need of seat apportionment in the EP, and also, it suggests an apportionment method.<sup>[8]</sup> As per the Compromise, to be *degressively proportional* means the following: <sup>[8, p. 11]</sup>

*The principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State **before rounding to whole numbers** must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State.*

The bold part is bold also in the original text and it is really important. For more details, see [8, p. 10].

Translated into the language of mathematics:

**Definition 2.2.** Let  $A$  be an apportionment method. *Apportionment method without rounding*

$$A^* : \{S\} \times \mathbb{N}^{|S|} \times \mathbb{N} \rightarrow \mathbb{R}^{|S|}$$

takes the same parameters as  $A$  and yields an apportionment without rounding. If rounding was applied in  $A^*$ , then  $A^* = A$ .

Let  $A_{EP}^*$  be the apportionment method used to calculate the seat allocation in the EP, but without rounding. Then (note that  $S_i$  stands for  $i$ -th item of tuple  $S$ ):

$$\forall i, j \in 1, \dots, |S| : p_i \geq p_j \implies A_{EP}^*(S, p, P)_i \geq A_{EP}^*(S, p, P)_j \quad (4)$$

$$\forall i, j \in 1, \dots, |S| : p_i \geq p_j \implies \frac{p_i}{A_{EP}^*(S, p, P)_i} \geq \frac{p_j}{A_{EP}^*(S, p, P)_j} \quad (5)$$

The apportionment method suggested by Cambridge Compromise is *base+prop* method: each member is apportioned a *base* number of seats, and then the remaining seats are distributed proportionally by the population of the members. More specifically:<sup>[8, p. 12]</sup>

- assign to each member a *base* number of seats, denoted as  $b$ ,
- take a divisor  $d$  and assign to each member  $i$  additional  $p_i/d$  seats, where  $p_i$  is the population of member  $i$ ,
- perform a rounding so that each member has an integral number of seats; if this number exceeds the maximum, use the maximum instead,
- if the sum of all assigned seats is not equal to the Parliament size, adjust  $d$ .

Cambridge Compromise suggests values  $b = 5$  and rounding upwards<sup>[8, p. 12]</sup>, which guarantees that each member gets at least 6 seats.<sup>1</sup>

In 2013, the European Council released a decision which established the composition of the European Parliament, being effective from the parliamentary term 2014 – 2019 until Brexit actually happens.<sup>[9]</sup> It implements rules (4) and (5) and strengthens rules (2) and (3) by saying that  $\min A_{EP}(S, p, P) = 6$  and  $\max A_{EP}(S, p, P) = 96$  to “*reflect as closely as possible the sizes of the respective populations of Member States*”<sup>[9, Art. 1]</sup>. From the proposed apportionment<sup>[9, Art. 3]</sup>, by summing up all the seats, it reveals that  $P = 751$ , thus strengthening also rule (1). The Decision also mentions that the population data shall be taken from Eurostat<sup>[9, Art. 2]</sup>. Therefore, the newest population data available at the time of releasing the Decision is as of 1st January 2013<sup>[10]</sup>.

<sup>1</sup>Provided it has a non-zero population.

However, the thing which is not followed in the Decision is the proposed apportionment method. Thus, the following can be said:

**Claim 2.3.**  $A_{EP}$ , as defined by the Treaty on European Union<sup>[4]</sup> and the Decision of the European Council<sup>[9]</sup> (that is, by rules from (1) to (5)), is ambiguous.

*Proof.* Let  $A_{B+P}$  be base+prop apportionment method with base 5 and upwards rounding. We show that  $A_{B+P}$  also obeys the rules from (1) to (5) and  $A_{B+P} \neq A_{EP}$ . Compare those two apportionments in a table:

State	Population	A(BP)	A(BP)*	Pop/Seat*	A(EP)	A(EP) - A(BP)
Malta	422,509	6	5.50	76,783	6	0
Luxembourg	537,039	6	5.63	95,239	6	0
Cyprus	865,878	7	6.03	143,594	6	-1
Estonia	1,320,174	7	6.57	200,926	6	-1
Latvia	2,023,825	8	7.40	273,214	8	0
Slovenia	2,058,821	8	7.44	276,385	8	0
Lithuania	2,971,905	9	8.53	348,191	11	2
Croatia	4,262,140	11	10.07	423,247	11	0
Ireland	4,609,779	11	10.48	439,712	11	0
Slovakia	5,410,836	12	11.43	473,118	13	1
Finland	5,426,674	12	11.45	473,723	13	1
Denmark	5,602,628	12	11.66	480,307	13	1
Bulgaria	7,284,552	14	13.66	533,064	17	3
Austria	8,451,860	16	15.05	561,435	18	2
Sweden	9,555,893	17	16.36	583,839	20	3
Hungary	9,908,798	17	16.78	590,261	21	4
Portugal	10,487,289	18	17.47	600,121	21	3
Czechia	10,516,125	18	17.50	600,592	21	3
Greece	11,003,615	19	18.08	608,287	21	2
Belgium	11,137,974	19	18.24	610,322	21	2
Netherlands	16,779,575	25	24.96	672,248	26	1
Romania	20,020,074	29	28.81	694,776	32	3
Poland	38,062,535	51	50.27	757,044	51	0
Spain	46,727,890	61	60.58	771,268	54	-7
Italy	59,685,227	76	75.99	785,338	73	-3
United Kingdom	63,905,342	82	81.01	788,765	73	-9
France	65,600,350	84	83.03	790,025	74	-10
Germany	80,523,746	96	96.00	838,789	96	0
<b>Total</b>	<b>505,163,053</b>	<b>751</b>			<b>751</b>	<b>0</b>

Population as of 1<sup>st</sup> Jan 2013 by Eurostat  
A(BP) = base+prop method with base 5 and upwards rounding  
A(BP)\* = base+prop method with base 5 without rounding  
Pop/Seat\* = Population needed for one seat in A(BP)\*  
A(EP) = Apportionment by the Decision  
A(EP) - A(BP) = difference between the two apportionments

Figure 1: Comparison of  $A_{B+P}$  and  $A_{EP}$ . (Source: own calculation)

$A_{B+P}$  clearly obeys rules (1), (2) and (3): Malta being the least populous country gets 6 seats, Germany being the most gets 96 seats, and all 751 seats are distributed. Rule (4) is also obeyed, as the population and the  $A_{B+P}^*$  columns strictly increase and so does the  $Pop/Seat^*$  column, which is what rule (5) says. So  $A_{B+P}$  follows all the requirements of  $A_{EP}$ , but clearly  $A_{B+P} \neq A_{EP}$ .  $\square$

While for example the law of Czech Republic in conjunction with Czech Statistical Office precisely define the method for apportionment in the Czech parliament<sup>[11]</sup> (d'Hondt method<sup>[12]</sup>), the European one seems not to be the case.<sup>2</sup>

Figure 1 shows that  $A_{EP}$  tends to assign more seats to mid-size states while snatching seats from big states. The table below shows comparison of  $A_{EP}$  with other used apportionment methods:

<sup>2</sup>On 24 Dec 2019, I personally sent a query using the official contact form to ask the details about  $A_{EP}$ . No answer so far.

State	Population	A(H)	A(J)	A(dH)	A(S)	Average	A(EP)	A(EP) - Average
Malta	422,509	6	6	6	6	6	6	0
Luxembourg	537,039	6	6	6	7	6.25	6	-0.25
Cyprus	865,878	6	7	7	7	6.75	6	-0.75
Estonia	1,320,174	6	7	7	8	7	6	-1
Latvia	2,023,825	6	8	8	8	7.5	8	0.5
Slovenia	2,058,821	6	8	8	8	7.5	8	0.5
Lithuania	2,971,905	6	9	9	9	8.25	11	2.75
Croatia	4,262,140	6	11	11	11	9.75	11	1.25
Ireland	4,609,779	7	11	11	11	10	11	1
Slovakia	5,410,836	8	12	12	12	11	13	2
Finland	5,426,674	8	12	12	12	11	13	2
Denmark	5,602,628	8	12	12	13	11.25	13	1.75
Bulgaria	7,284,552	11	14	14	14	13.25	17	3.75
Austria	8,451,860	12	16	16	16	15	18	3
Sweden	9,555,893	14	17	17	17	16.25	20	3.75
Hungary	9,908,798	15	17	17	18	16.75	21	4.25
Portugal	10,487,289	16	18	18	18	17.5	21	3.5
Czechia	10,516,125	16	18	18	18	17.5	21	3.5
Greece	11,003,615	16	19	19	19	18.25	21	2.75
Belgium	11,137,974	16	19	19	19	18.25	21	2.75
Netherlands	16,779,575	25	25	25	26	25.25	26	0.75
Romania	20,020,074	30	29	29	29	29.25	32	2.75
Poland	38,062,535	56	51	51	50	52	51	-1
Spain	46,727,890	69	61	61	60	62.75	54	-8.75
Italy	59,685,227	89	76	76	76	79.25	73	-6.25
United Kingdom	63,905,342	95	82	82	81	85	73	-12
France	65,600,350	96	84	84	82	86.5	74	-12.5
Germany	80,523,746	96	96	96	96	96	96	0
<b>Total</b>	<b>505,163,053</b>	<b>751</b>	<b>751</b>	<b>751</b>	<b>751</b>	<b>751</b>	<b>751</b>	<b>0</b>

Population as of 1<sup>st</sup> Jan 2013 by Eurostat  
A(H) = Hamilton's method, leftover seats are distributed by round-robin  
A(J) = Jefferson's method (upwards, downwards, arithmetic, geometric and harmonic rounding happen to produce the same result)  
A(dH) = d'Hondt method  
A(S) = Spline method, mentioned as an alternative to base+prop method in the Compromise  
Average = Average of the previous four columns  
A(EP) = Apportionment by the Decision  
A(EP) - Average = Difference between the real number of seats and the average of used methods

Figure 2: Comparison of apportionment methods. (Source: own calculation; for more details, check the implementation at [13].)

It reveals that all of the conventional methods seem to prefer more populous member states. Maybe that is one of the reasons a new apportionment method was invented. Unfortunately, its details are not published, and it does not quite resemble any of the other methods.

### 3 Redistribution of the British seats

Basically, there are three options what to do with British seats: redistribute them, leave them empty, or a combination of both. The last option is the one which the European Council opted for: taking place – 27 will be reassigned and 46 will remain empty<sup>[14]</sup>. Again, there are no details of the apportionment method. The table in Figure 3 compares the new apportionment with other apportionment methods. Let  $A_{BR}$  be the apportionment method used by the European Council for Brexit purposes.

$A_{BR}$  makes sure nobody loses their seats. Most of the seats are reassigned to the biggest states, which are, though, still quite underrepresented, compared to other apportionment methods. It could be said that they benefit the most, but at the price of still having not as many seats as they could have. Thus, the goal of the cut was probably not to hurt or let anyone benefit.

There is a much more likely scenario: currently, the Parliament is full, and thus nobody can join the European Union at the moment, as they would not get their at least 6 EP seats. According to Eurostat, the population of the EU on 1st January 2019 is 513 481 690, therefore, one seat out of 751 represents roughly 680 000 citizens. By freeing 46 seats, there is suddenly room for one or more states with the total population up to about 31 000 000 citizens to immediately join the EU – without the cut, they would need to wait for the next parliamentary term. That is not enough for Ukraine, but almost all states at the Balkans could potentially fit.

State	Population	A(EP)	A(BP)	A(J)	A(dH)	A(S)	Avg	A(BR)	A(BR) - A(EP)	A(BR) - Avg
Malta	475,701	6	6	6	6	6	6.00	6	0	0.00
Luxembourg	602,005	6	6	6	6	7	6.33	6	0	-0.33
Cyprus	864,236	6	7	6	7	7	6.67	6	0	-0.67
Estonia	1,319,133	6	7	7	7	8	7.33	7	1	-0.33
Latvia	1,934,379	8	8	8	8	8	8.00	8	0	0.00
Slovenia	2,066,880	8	8	8	8	8	8.00	8	0	0.00
Lithuania	2,808,901	11	9	9	9	9	9.00	11	0	2.00
Croatia	4,105,493	11	11	10	11	11	10.67	12	1	1.33
Ireland	4,830,392	11	12	11	12	12	11.67	13	2	1.33
Slovakia	5,443,120	13	12	12	12	13	12.33	14	1	1.67
Finland	5,513,130	13	13	12	13	13	12.67	14	1	1.33
Denmark	5,781,190	13	13	13	13	13	13.00	14	1	1.00
Bulgaria	7,050,034	17	15	14	15	15	14.67	17	0	2.33
Austria	8,822,267	18	17	17	17	17	17.00	19	1	2.00
Hungary	9,778,371	20	18	18	18	18	18.00	21	1	3.00
Sweden	10,120,242	21	18	18	18	19	18.33	21	0	2.67
Portugal	10,291,027	21	19	19	19	19	19.00	21	0	2.00
Czechia	10,610,055	21	19	19	19	19	19.00	21	0	2.00
Greece	10,741,165	21	19	19	19	19	19.00	21	0	2.00
Belgium	11,398,589	21	20	20	20	20	20.00	21	0	1.00
Netherlands	17,181,084	26	28	28	28	27	27.67	29	3	1.33
Romania	19,530,631	32	31	31	31	30	30.67	33	1	2.33
Poland	37,976,687	51	54	55	54	54	54.33	52	1	-2.33
Spain	46,658,447	54	65	66	65	65	65.33	59	5	-6.33
Italy	60,483,973	73	83	84	83	82	83.00	76	3	-7.00
United Kingdom	66,273,576	73	0	0	0	0	0.00	0	-73	0.00
France	66,926,166	74	91	93	91	90	91.33	79	5	-12.33
Germany	82,792,351	96	96	96	96	96	96.00	96	0	0.00
<b>Total</b>	<b>512,379,225</b>	<b>751</b>	<b>705</b>	<b>705</b>	<b>705</b>	<b>705</b>	<b>705</b>	<b>-46</b>	<b>0</b>	<b>0</b>

Population as of 1<sup>st</sup> Jan 2018 by Eurostat  
A(EP) = Apportionment by the Decision 2013  
Hamilton's method is not applicable here, as it assigns 7 seats to Malta  
A(J) = Jefferson's method with standard rounding (among all roundings, it resembles A(BR) most)  
A(dH) = d'Hondt method  
A(S) = Spline method, mentioned as an alternative to base+prop method in the Compromise  
Avg = Average of the previous four columns  
A(BR) = Apportionment by the Decision 2018 (including Brexit)  
A(BR) - A(EP) = Gain/loss of the seats after Brexit  
A(BR) - Avg = Difference between A(BR) and Avg

Figure 3: Comparison of apportionment methods after Brexit. (Source: own calculation; for more details, check the implementation at [13].)

## 4 Conclusion

While looking for the reasoning for the proposed redistribution of the British seats in the European Parliament, a much more serious problem arose: the apportionment method used in the EP is ambiguous and seem to produce quite different results from other, more common apportionment methods – it reveals to be disadvantageous to more populous states. However, that is pretty much all we know. It would be definitely worth defining it properly!

The cut of size 46 a little big compensates the underrepresentation of big members and is decent to fit any single country in Europe and not (yet) in the EU except Ukraine and Russia immediately. Time will tell whether the cut was really needed.

## References

- [1] Wikipedia contributors. *Brexit*. <https://en.wikipedia.org/w/index.php?title=Brexit&oldid=935613116>. 13 Jan 2020. [Online; cit. 30 Jan 2020]
- [2] Elise Uberoi, Carl Baker, Richard Cracknell. *General Election 2019: full results and analysis*. <https://researchbriefings.parliament.uk/ResearchBriefing/Summary/CBP-8749>. 19 Dec 2019. [Online; cit. 30 Jan 2020]
- [3] EU Press. *General Election 2019: full results and analysis*. <https://www.europarl.europa.eu/news/en/headlines/eu-affairs/20180126ST094114/eu-elections-how-many-meps-will-each-country-get-in-2019>. 7 Feb 2018. [Online; cit. 30 Jan 2020]
- [4] Official Journal of the European Union. *Consolidated Version of the Treaty on European Union*. <https://eur-lex.europa.eu/resource.html?uri=cellar:>

- 2bf140bf-a3f8-4ab2-b506-fd71826e6da6.0023.02/DOC\_1&format=PDF. 26 Oct 2012. [Online; cit. 30 Jan 2020]
- [5] Philip Grech. *Mathematics in Politics and Law*. [https://moodle-app2.let.ethz.ch/pluginfile.php/763650/mod\\_resource/content/16/MATHEMATICS%20IN%20POLITICS%20AND%20LAW.pdf](https://moodle-app2.let.ethz.ch/pluginfile.php/763650/mod_resource/content/16/MATHEMATICS%20IN%20POLITICS%20AND%20LAW.pdf). 2019. [Online; cit. 30 Jan 2020]
  - [6] Eeva Pavy. *Fact Sheet on the European Union: The Treaty of Lisbon*. <http://www.europarl.europa.eu/factsheets/en/sheet/5/the-treaty-of-lisbon>. Nov 2019. [Online; cit. 30 Jan 2020]
  - [7] Wikipedia contributors. *Treaty of Lisbon*. [https://en.wikipedia.org/w/index.php?title=Treaty\\_of\\_Lisbon&oldid=937179628](https://en.wikipedia.org/w/index.php?title=Treaty_of_Lisbon&oldid=937179628). 23 Jan 2020. [Online; cit. 30 Jan 2020]
  - [8] Geoffrey Grimmet and collaborators. *The allocation between the EU Member States of the seats in the European Parliament*. [https://www.europarl.europa.eu/RegData/etudes/note/join/2011/432760/IPOL-AFCO\\_NT\(2011\)432760\\_EN.pdf](https://www.europarl.europa.eu/RegData/etudes/note/join/2011/432760/IPOL-AFCO_NT(2011)432760_EN.pdf). 29 Jan 2011. [Online; cit. 30 Jan 2020]
  - [9] European Council. *EUROPEAN COUNCIL DECISION establishing the composition of the European Parliament*. <https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32013D0312&from=EN>. 28 Jun 2013. [Online; cit. 30 Jan 2020]
  - [10] Eurostat. *Population on 1 January*. <https://ec.europa.eu/eurostat/databrowser/view/tps00001/default/table?lang=en>. 6 Nov 2019. [Online; cit. 30 Jan 2020]
  - [11] Pavel Kuklík and Josef Baxa. *Metody pro přepočet hlasů na mandáty*. [https://www.czso.cz/csu/czso/metody\\_pro\\_prepecet\\_hlasu\\_na\\_mandaty](https://www.czso.cz/csu/czso/metody_pro_prepecet_hlasu_na_mandaty). 3 Oct 2012. [Online; Czech; cit. 30 Jan 2020]
  - [12] Wikipedia contributors. *D'Hondt method*. [https://en.wikipedia.org/w/index.php?title=D%27Hondt\\_method&oldid=933523280](https://en.wikipedia.org/w/index.php?title=D%27Hondt_method&oldid=933523280). 23 Jan 2020. [Online; cit. 30 Jan 2020]
  - [13] Karel Jílek. *ep-apportionment*. <https://github.com/karlosss/ep-apportionment/tree/e387d085a49755da8f14e14ff4b734b791c6656d>. 29 Jan 2020. [Online; GitHub repository; cit. 30 Jan 2020]
  - [14] European Council. *EUROPEAN COUNCIL DECISION establishing the composition of the European Parliament*. <https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32018D0937&from=EN>. 19 Jun 2018. [Online; cit. 30 Jan 2020]