

$$\begin{array}{c}
 \begin{array}{cc}
 & \mathcal{C}\left(\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}\right) \cup \mathcal{C}\left(\begin{pmatrix} V_2 \end{pmatrix}\right) \\
 \begin{array}{c} V_1 \\ V_2 \end{array} \downarrow \begin{array}{|cc|cc|} \hline & 1 & 1 & -1 & & 0 \\ & -1 & 1 & -1 & & \\ & 1 & & 1 & & \\ \hline & 0 & & & 1 & \\ & & & & -1 & 1 \\ \hline \end{array} \\
 & (D_6^{(n)})_w
 \end{array}
 \end{array}
 \left. \vphantom{\begin{array}{c} \begin{array}{cc} & \mathcal{C}\left(\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}\right) \cup \mathcal{C}\left(\begin{pmatrix} V_2 \end{pmatrix}\right) \\ \begin{array}{c} V_1 \\ V_2 \end{array} \downarrow \begin{array}{|cc|cc|} \hline & 1 & 1 & -1 & & 0 \\ & -1 & 1 & -1 & & \\ & 1 & & 1 & & \\ \hline & 0 & & & 1 & \\ & & & & -1 & 1 \\ \hline \end{array} \\ & (D_6^{(n)})_w \end{array} \right\} \sum_{x \in V_1} \widehat{\text{raid}}_x = 0$$

$\Rightarrow \widehat{\text{raid}}_x \text{ is in } \mathbb{Z}$
 $\Rightarrow \det = 0$