CS 310
Assignment 131

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Problem 1 — Use the definitions to prove or disprove the statement $n^4 - 2n^2 + 5 \in \Omega(n^3)$, and illustrate this graphically.

Answer: The definition requires us to find c and n_0 so that

$$n^4 - 2n^2 + 5 \geq cn^3 \text{ when } n \geq n_0$$

If this is true, then we can rearrange the inequality so that

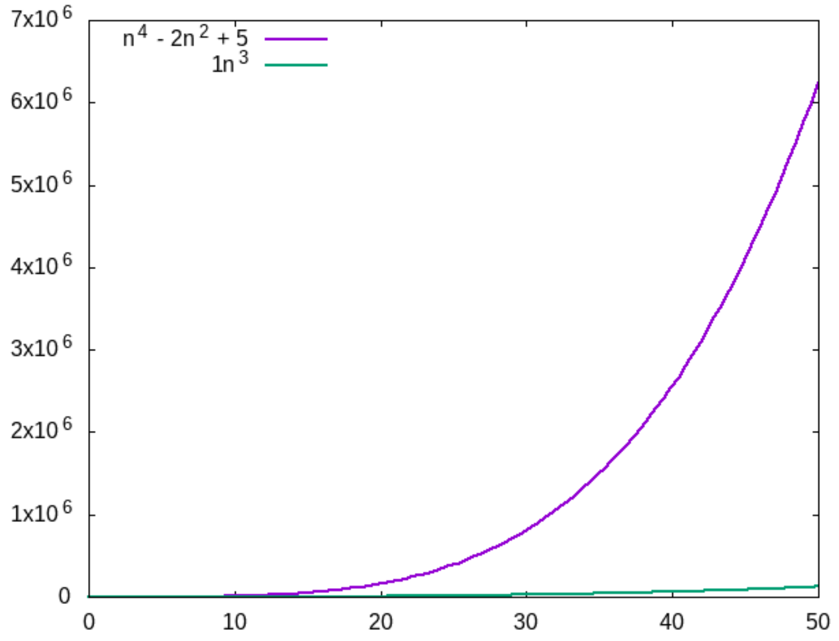
$$\frac{n^4 - 2n^2 + 5}{n^3} \geq c$$
$$n - \frac{2}{n} + \frac{5}{n^3} \geq c$$

We can then take the derivative of the left and find it's minimum by setting it equal to 0.

$$\frac{d}{dn}(n - \frac{2}{n} + \frac{5}{n^3}) = 0$$
$$1 + \frac{2}{n^2} - \frac{15}{n^4} = 0$$
$$n^4 + 2n^2 - 15 = 0$$
$$n = \sqrt{3}, -\sqrt{3}$$

$\sqrt{3} \approx 1.731$ so we will choose 2 for n_0, and $c \leq \frac{13}{8}$. We will choose 1 for c.

This is graphically illustrated by the following plot that shows $n^4 - 2n^2 + 5$ along with the standard function n^3 scaled by the constant coefficient $c = 1$.



Problem 2 — Either prove the following assertion using the definitions or disprove it with a specific counterexample:

$$\text{if } T(n) \in O(f(n)) \text{ then } f(n) \in \Omega(T(n))$$

Answer: If we assume $T(n) \in O(f(n))$ them we can assume there is a $c > 0$ and $n > n_0$ such that $T(n) \leq c * f(n)$.

If we rearrange this inequality, we find that $f(n) \geq \frac{1}{c} * T(n)$.

Since $c > 0$ we can assume $\frac{1}{c} > 0$ and can rename it to c_2.

So we write $f(n) \geq c_2 {*} T(n)$. This satisfies the definition of $\Omega(n)$, which means $f(n) \in \Omega(T(n))$.

Problem 3 — For the following algorithm, explain what it computes, state what the input size for analysis is, state what basic operations should be counted for analyzing it, state exactly how many operations are executed as a function of the input size, and state the efficiency class to which it belongs.

```
void foo(vector<unsigned>& array)
{
  for (size_t pass_indx = array.size() - 1; pass_indx > 0; pass_indx--)
  {
    for (size_t compare_indx = 0; compare_indx < pass_indx; compare_indx++)
    {
      if (array.at(compare_indx) > array.at(compare_indx + 1))
      {
        swap(array.at(compare_indx), array.at(compare_indx + 1));
      }
    }
  }
}
```

Answer: This algorithm is a simple bubble sort. It takes an array an performs swaps when it sees elements out of order. The input size, n, for this algorithm will simply be the size of the vector.

The outer `for` loop will have $2(n-1)$ instructions.

The inner `for` loop will run $2(n-1)$ instructions the first time and $2(n-1)$ the next, and so on for n times. As a formal series this is $\sum_{i=1}^{n-1}(2(n-i))$ instructions. Since $\sum_{i=1}^{n-1}(2(n-i)) = \frac{n(n-1)}{2} = \frac{1}{2}(n^2-n)$, this will be our highest order term at n^2.

The inner `if` statement will run an extra 2 or 4 instructions every time the inner `for` loop runs. This doesn't effect the complexity.

The best case (already sorted) formula is

$$2n + \sum_{i=1}^{n-1}(2(n-i)+2)$$

The worst case (reverse order) formula is

$$2n + \sum_{i=1}^{n-1}(2(n-i)+4)$$

The efficiency of this algorithm is $T(n) \in \Theta(n^2)$. Since this algorithm will run all the way through every time both the best and worst case have the same complexity.

Problem 4 — Write a C++ program that implements the algorithm in problem 3, counts the number of basic operations, and outputs the input size and the count of basic operations to the cerr stream. Run this program many times with many different inputs and capture the results.

Answer: See the submitted file `rambergk131.cpp`.

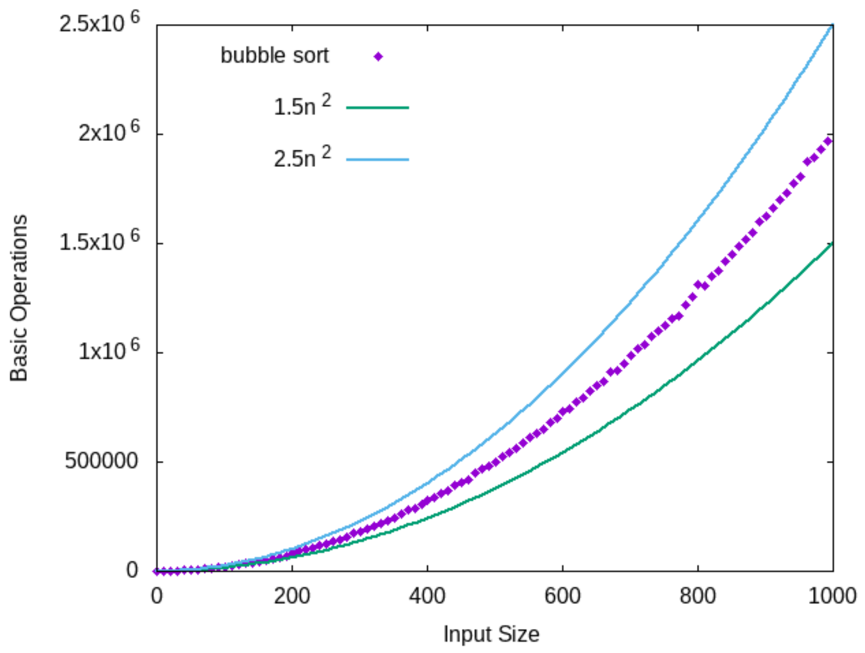
Problem 5 — Using the output of the program in Problem 4, create a plot of input size vs. basic operations, along with one or more standard functions properly scaled, to illustrate your analysis in Problem 3.

Answer: When the program is run with the command

```
for n in `seq 1 10 1000`;
do
  ./rambergk131 $n;
done 1> /dev/null 2> results.dat
```

and the resulting data file is plotted with gnuplot, the following is produced. Also plotted on the same axes is the standard function n^2 scaled by 1.5 and 2.5 which illustrates that

$$T(n) \in \Theta(n^2)$$


2.5x10 6
2x10 6
1.5x10 6
1x10 6
Basic Operations
Input Size
bubble sort
1.5n 2
2.5n 2