AE6310: Optimization for the Design of Engineered Systems

Quiz 2
March 7th, 2019

Briefly answer the following questions on the paper provided. Organize your work and be careful to properly answer all parts of each question.
This quiz is closed book. The length of the quiz is 40 minutes.

It may be helpful to recall the following formula:

$$||p^*(\lambda)||^2 = \sum_{i=1}^{n} \frac{(g^T q_i)^2}{(\lambda + \lambda_i)^2}$$

1. (30 points) Sketch the following cases graphically. Be careful to draw the contour lines and constraints correctly. In all cases indicate the exact trust region step, the unconstrained model minimizer, if any, and the Cauchy point. Make these points distinct, whenever possible, so that the Cauchy point is not the exact trust region step, etc.

   (a) A positive definite quadratic model with an inactive trust region constraint

   (b) A positive definite quadratic model with an active trust region constraint

   (c) An indefinite quadratic model

2. (35 points) The following question is based on a quadratic model function $m(x+p) = \frac{1}{2}p^T Bp + g^T p$

$$B = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \qquad g = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

   (a) If $\Delta = 1/2$, find the Cauchy step for this problem

   (b) What is the maximum trust radius size Δ where the trust region constraint is still active?

   (c) If $\Delta = 1/3$, find the multiplier λ and the step p^* to the exact model minimizer

3. (35 points) This question is based on the following constrained optimization problem.

$$\begin{aligned} \min \quad & f(x) = x_1^2 - 2x_1 + x_2^2 + x_1 x_2 \\ \text{s.t.} \quad & 2 - x_1 - x_2 \leq 0 \end{aligned}$$

   (a) State the Karush–Kuhn–Tucker (KKT) conditions for an objective function $f(x) : \mathbb{R}^n \to \mathbb{R}$ and constraint functions $c(x) : \mathbb{R}^n \to \mathbb{R}^m$, such that $c(x) \leq 0$.

   (b) Solve the KKT conditions for the problem stated above.