AE6114 - Fundamentals of Solid Mechanics - Fall 2020

Test 1 - Thurs, Sep 24^{th} 2020

Notes: (i) Solve the exam on your own sheets of paper; (ii) write your name on the top-right corner of this page and in the same place for every solution page; and (iii) attach the formula sheet used during the exam.

Problem 1 [50 Points]

1. Let $\underline{a}$, $\underline{b}$, and $\underline{c}$ be rank one tensors and let $\cdot$ and $\times$ represent the dot and cross product operators respectively. Show that:
   (a) $\underline{a} \cdot (\underline{b} \times \underline{c}) = \underline{b} \cdot (\underline{c} \times \underline{a}) = \underline{c} \cdot (\underline{a} \times \underline{b})$
   (b) $\underline{a} \times (\underline{b} \times \underline{c}) = \underline{b}(\underline{a} \cdot \underline{c}) - \underline{c}(\underline{a} \cdot \underline{b})$
2. Show that the change in length of an infinitesimal fiber in a given direction is frame indifferent, that is, that the operation $\underline{\mathbf{N}} \cdot \underline{\underline{\mathbf{C}}} \cdot \underline{\mathbf{N}}$ gives the same result independently of the frame adopted to represent the Cauchy-Green strain tensor $\underline{\underline{\mathbf{C}}}$ and the fiber direction $\underline{\mathbf{N}}$.

Problem 2 [50 Points]

Let the material and spatial coordinates be measured in the same rectangular Cartesian coordinate system with basis vectors $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$. Consider the following deformation:

$$\begin{aligned} x_1 &= X_1 \\ x_2 &= X_2 \\ x_3 &= X_3 + \alpha(X_1 + X_2) \end{aligned}$$

1. Find the matrices of the deformation gradient tensor $\mathbf{F}$ and the Lagrangian strain tensor $\mathbf{E}$. Does the deformation gradient change with position? Does this deformation change volume?
2. Without any further assumptions, find:
   (a) *Stretch* of the element that is in the direction $\mathbf{e}_2$ in the reference configuration
   (b) *Change of angle* between the elements that are in the directions $\mathbf{e}_2$ and $\mathbf{e}_3$ in the reference configuration.
3. Find the displacement field and components of the infinitesimal strain tensor $\boldsymbol{\epsilon}$. For this specific case, under what conditions is $\boldsymbol{\epsilon}$ a good description of the strain? Show that the Lagrangian strain tensor $\mathbf{E}$ reduces to $\boldsymbol{\epsilon}$ under those conditions.
4. For the infinitesimal strain tensor $\boldsymbol{\epsilon}$ find:
   (a) The maximum value of extensional strain and its direction.
   (b) The maximum value of shear strain and its directions.