AE6114 - Fundamentals of Solid Mechanics - Fall 2020

Final Exam - Tuesday, Dec 1^{st} 2020

PLEASE READ CAREFULLY BEFORE STARTING THE EXAM

1. **Do not read the exam until you are ready to take it.**
2. Once you have read any part of the exam, it is considered that you started taking it, so all rules described below apply.
3. This is a take-home exam.
4. Even though there is no time limit, the exam must be taken *in one continuous session* so plan your schedule carefully.
5. During the exam, you are allowed to use any handwritten notes produced by you *before you started the exam*. These might include your own summaries, class notes and homework assignments.
6. No collaboration is allowed, and no textbooks or any other resource other than your handwritten notes can be used during the exam.
7. Solve the exam on your own sheets of paper.
8. Write your name on the top-right corner of this page and in the same place for every solution page.
9. Submission deadline for the exam is **Thursday, Dec 3 (11:59 pm EST) for Section A** students and **Saturday, Dec 5 (11:59 pm EST) for Section Q** students.

Problem 1

A strain gage rosette (see Figure 1) with 3 gages is placed at a point P on a body. One gage (B) is aligned in the x_2 direction with the other two (A and C) aligned 60 degrees on either side of the x_2 direction. The body is then stressed and the strains from the 3 gages are measured to be ϵ_A, ϵ_B, and ϵ_C.

1. Given that a strain gage gives the normal strain in the direction of its orientation determine all components of the infinitesimal strain tensor, assuming that the components ϵ_{13}, ϵ_{23} and ϵ_{33} are zero (i.e. plane strain).

2. Assuming the body was not necessarily under plane strain, and you were allowed to attach a strain gage in any orientation, what is the smallest number of strain gages you would need to determine the entire strain tensor? Describe how you would place the strain gages.

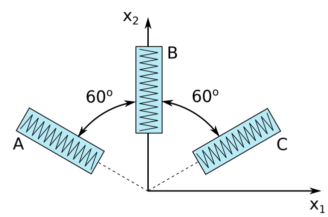

Figure 1: Strain gage rosette for problem 1.

Problem 2

An engineer conducts the following two tests to find the constitutive response of a thin sheet of material of thickness t.

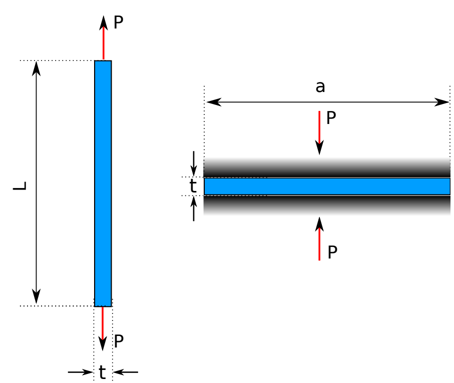

Figure 2: Schematics for problem 2.

1. In the first test, a thin strip of material (thickness t, length L, and width w, with $L \gg w, t$) is subject to tension along its length.

2. In the second test, the material is placed between two rigid square plates of side a ($a \gg t$) and is subject to uniform compression in the thickness direction while remaining laterally constrained, i.e., there are no strains in the in-plane directions.

In each of the tests, the load and the displacement are measured. The engineer computes the elastic stress-strain response of the material from each of these tests and finds them to be different. Please help the engineer derive the stress-strain response of these two tests and investigate if any relation exists between these two tests. State all your assumptions clearly. Assume the material is linearly elastic and isotropic, with Young's modulus E, and Poisson's ratio ν. How would you compute the elastic constants from the results of these two tests?

Problem 3

Consider the plate ($0 < x_1 < a$; $-b < x_2 < b$) shown in Figure 3 and subject to the following loads:

- The surfaces $x_2 = \pm b$ are stress-free
- The surface $x_1 = a$ is subject to a force per unit length of $\tau(x_2)$ pointed in the x_2 direction, giving a total force equal to F, i.e.

$$F = \int_{-b}^{b} \tau(x_2) dx_2$$

- The surface $x_1 = 0$ is subject to suitable loads to keep the plate in equilibrium.
- Zero body force.

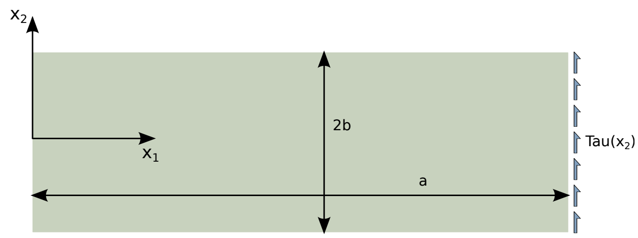

Figure 3: Schematics for problem 3.

Which of the following is a solution of the elasticity problem (for suitable constants A, B, C, and D)? Explain why and give the correct values of A, B, C, and D.

1. $\sigma_{11} = Ax_2 + Bx_1x_2 + D\cos\left(\frac{\pi x_1}{2a}\right)$
   $\sigma_{22} = B\sin\left(\frac{\pi x_2}{b}\right)$
   $\sigma_{12} = -\frac{1}{2}Bx_2^2 - C$
2. $\sigma_{11} = Ax_2 + Bx_1x_2$
   $\sigma_{22} = D$
   $\sigma_{12} = -\frac{1}{2}Bx_2^2 - C$
3. $\sigma_{11} = Ax_2 + Bx_1x_2 + \frac{B}{2}(x_1 - a)^2$
   $\sigma_{22} = \frac{1}{2}Bx_2^2 + D$
   $\sigma_{12} = -\frac{1}{2}Bx_2^2 - B(x_1 - a)x_2 + C$

Problem 4

Determine whether the following statements are true or false. Explain your answer briefly (in one to three sentences):

1. When a body is rigidly displaced, the resulting infinitesimal strain tensor is zero.

2. For any motion, the straight line segments in the reference configuration are transformed into straight line segments in the current configuration.

3. The potential energy of a system satisfying static equilibrium is minimized.

4. When the Poisson's ratio $\nu = 0$, the solid is incompressible (its volume remains constant no matter what stresses are applied)

5. In a *homogeneous*, isotropic material, the hydrostatic pressure may depend on the deviatoric strain

6. The axes of principal strain always coincides with the axes of principal stress.

Problem 5

Consider a dam represented as a wedge with two infinitely long sides (i.e., we do not consider in this problem how the dam is supported by the ground). The dam also extends infinitely in the x_3 direction. The vertical side is subjected to the pressure $\rho_w g x_2$ of water. The inclined side is traction-free. The two sides make angle β as shown. The dam is also subject to its own weight, i.e., the body force $\rho_d g$ is acting on the dam (ρ_d is the density of the dam).

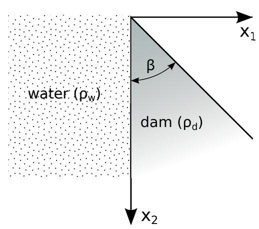

Figure 4: Schematics for problem 5.

1. This problem has a body force. Accounting for that fact, write down the relations between the Airy stress function and stresses in this problem and the equation that the Airy stress function must satisfy.

2. Using the Airy stress function $\phi = A_1 x_1^3 + A_2 x_1^2 x_2 + A_3 x_1 x_2^2 + A_4 x_2^3$, find the stress distribution inside the dam.

Bonus Problem

An Euler-Bernoulli beam is a beam that satisfy the following kinematic assumptions: 1) straight lines perpendicular to the neutral axis remain straight after deformation, 2) straight lines perpendicular to the neutral axis keep the same length after deformation ($\epsilon_{22} = 0$), and 3) straight lines perpendicular to the neutral axis remain perpendicular after deformation ($\epsilon_{12} = 0$).

For the Euler-Bernoulli beam depicted in Figure 5, derive the strong formulation of the problem (differential equation and traction boundary conditions) by using the principle of minimum potential energy. As in the last homework assignment, you will need to use rules of variation and integration by parts (perhaps multiple times).

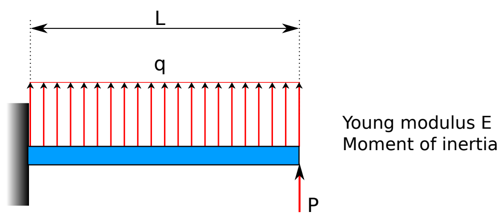

Figure 5: Beam configuration for problem 6 (bonus problem).