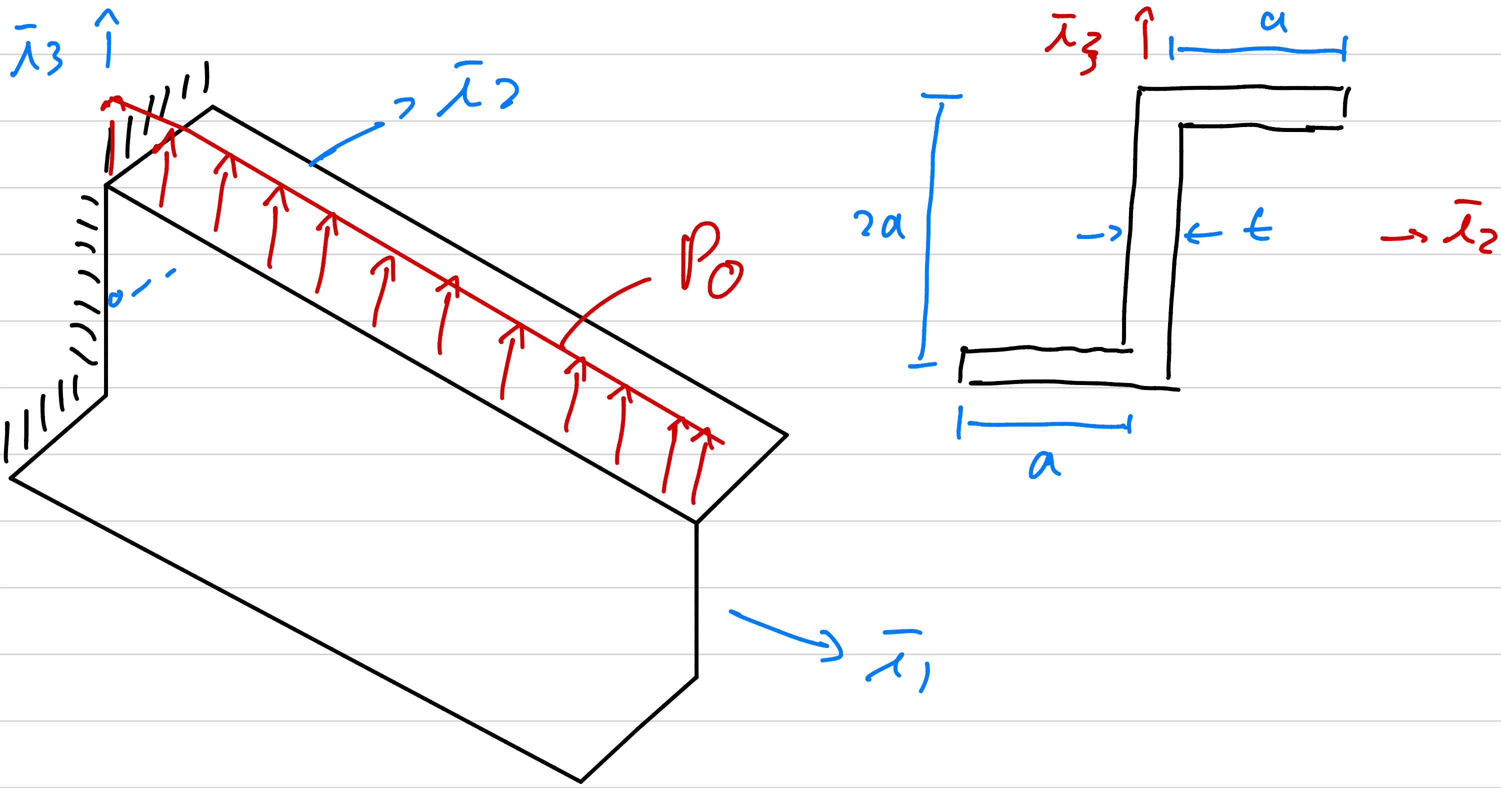
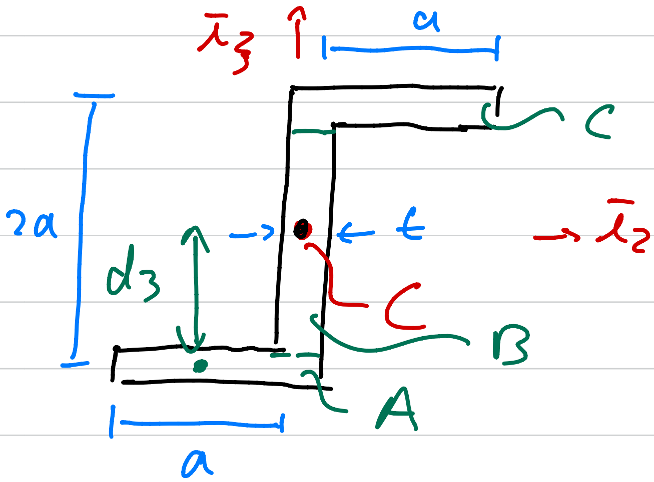
BENDING OF A "Z" SECTION



1) Find the displacement field.
$\bar{u}_1, u_2, u_3$

2) Find σ_1

3) Find the neutral axis.



E = CONSTANT

COMPUTE STIFFNESSES

$$S = \int_A E\,dA = E\,(at + 2at + at)$$

$$S = 4Eat$$

Parallel Axis Theorem

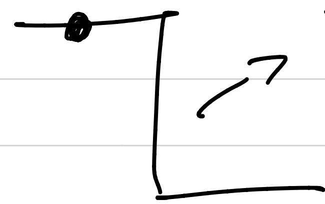
$$H_{22} = H_{22}^{L} + S\,d_3^2 \qquad H_{23} = H_{23}^{L} + S\,d_2 d_3$$

$$H_{33} = H_{33}^{L} + S\,d_2^2$$

$$H_{22}^{C} = H_{22}^{L} + S\,d_3^2$$

$$= E\left[\underbrace{\frac{t\,(2a)^3}{12}}_{B} + \underbrace{\left(\frac{a\,t^3}{12} + (at)\,a^2\right)2}_{A,\,C}\right]$$

$$H_{22}^{C} \approx \frac{8}{3} a^3 t \cdot E$$

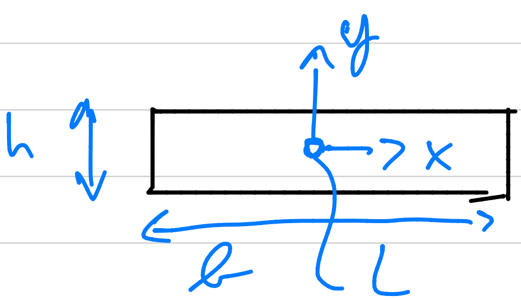


$H_{23}^{C} = -a^3 t E$

$$H_{33}^{C} = E\left(\underbrace{\frac{(2a)t^3}{12}}_{B} + \underbrace{\left(\frac{t a^3}{12} + (at)\left(\frac{a}{2}\right)^2 \right) 2}_{A,\,C} \right)$$

$$H_{33}^{C} \approx \frac{2}{3} a^3 t E$$

$$H_{23}^{L} = \int_A E\, x_2\, x_3\, dA$$



$$H_{23}^{L} = E\left[\frac{x^2}{2}\right]_{-\frac{b}{2}}^{\frac{b}{2}} \left[\frac{y^2}{2}\right]_{-\frac{h}{2}}^{\frac{h}{2}}$$

$$H_{23}^{L} = 0$$

$$H_{23}^{C} = E\Big[\underbrace{0}_{B} + \underbrace{(at)\overset{d_2}{\left(-\frac{a}{2}\right)}\overset{d_3}{(-a)}}_{A}$$

$$+ \underbrace{(at)\left(\frac{a}{2}\right)(a)}_{C} \Big] = a^3 t E$$

* H_{23}^{C} CAN BE NEGATIVE

$$S \frac{d^2 \bar{u}_1}{dx_1^2} = 0$$

$$H_{23}^c \frac{d^4 u_3}{dx_1^4} + H_{33}^c \frac{d^4 u_2}{dx_1^4} = 0$$

$$H_{22}^c \frac{d^4 u_3}{dx_1^2} + H_{23}^c \frac{d^4 u_2}{dx_1^4} = P_0$$

BOUNDARY CONDITIONS

@ $x_1 = 0$ $\bar{u}_1 = u_2 = u_3 = 0$

$$\frac{du_2}{dx_1} = \frac{du_3}{dx_1} = 0$$

@ $x_1 = L$ $N_1 = 0$, $M_2 = M_3 = 0$

$V_2 = V_3 = 0$

$$N_1 = S \left. \frac{d\bar{u}_1}{dx_1} \right|_L = 0 \rightarrow \frac{d\bar{u}_1}{dx_1} = 0$$

$$-M_2 = \left[H_{22}^{c} \frac{d^2 u_3}{dx_1^2} + H_{23}^{c} \frac{d^2 u_2}{dx_1^2} \right]_L = 0$$

$$M_3 = \left[H_{33}^{c} \frac{d^2 u_2}{dx_1^2} + H_{23}^{c} \frac{d^2 u_3}{dx_1^2} \right]_L = 0$$

$$\frac{d M_3}{dx_1} + V_2 = 0$$

$$\left[H_{22}^{c} \frac{d^3 u_3}{dx_1^3} + H_{23}^{c} \frac{d^3 u_2}{dx_1^3} \right]_L = 0$$

$$\left[H_{33}^{c} \frac{d^3 u_2}{dx_1^3} + H_{23}^{c} \frac{d^3 u_3}{dx_1^3} \right]_L = 0$$

$$S \frac{d\bar{u}_1}{dx_1} = \cancel{A}^{\,0} \qquad @ x_1 = L, \ \frac{d\bar{u}_1}{dx_1} = 0$$

$$\rightarrow A = 0$$

$$S \bar{u}_1 = B \qquad @ x_1 = 0, \ \bar{u}_1 = 0$$

$$\rightarrow B = 0$$

$$\boxed{\bar{u}_1 = 0}$$

$$H^c_{23} A + H^c_{33} B = 0$$

$$H^c_{22} A + H^c_{23} B = P_0$$

$$A = -\frac{H^c_{33}}{H^c_{23}} B$$

$$H^c_{22}\left(-\frac{H^c_{33}}{H^c_{23}} B\right) + H^c_{23} B = P_0$$

$$B\left(\frac{H^c_{23} H^c_{23} - H^c_{22} H^c_{33}}{H^c_{23}}\right) = P_0$$

$$B = \frac{d^4 u_2}{dx_1^4} = P_0 \cdot \frac{H^c_{23}}{H^c_{23} H^c_{23} - H^c_{22} H^c_{33}} = -\frac{d}{7a^3tE} P_0$$

$$A = \frac{d^4 u_3}{dx_1^4} = -P_0 \cdot \frac{H^c_{33}}{H^c_{23} H^c_{23} - H^c_{22} H^c_{33}} = \frac{6}{7a^3tE} P_0$$

BOUNDARY CONDITIONS

$$\frac{d^2 u_2}{dx_1^2} = 0 \qquad \frac{d^2 u_3}{dx_1^2} = 0 \qquad @\ x_1 = L$$

$$\frac{d^3 u_2}{dx_1^3} = 0\ , \qquad \frac{d^3 u_2}{dx_1^3} = 0 \qquad @\ x_1 = L$$

SOLVE

$$\frac{d^3 u_2}{dx_1^3} = C x_1 + A_1 \quad ; \ C = \frac{-d}{7a^3tE} P_0$$

$$@\ x_1 = L \qquad 0 = CL + A_1 \rightarrow A_1 = -CL$$

$$\frac{d^3 u_2}{dx_1^3} = C x_1 - CL$$

$$\frac{d^2 u_2}{dx_1^2} = \frac{C x_1^2}{2} - CLx + A_2$$

$$@\ x_1 = L \qquad 0 = \frac{CL^2}{2} - CL^2 + A_2 \rightarrow A_2 = \frac{CL^2}{2}$$

$$\frac{d^2 u_2}{dx_1^2} = \frac{C x_1^2}{2} - CLx_1 + \frac{CL^2}{2}$$

$$\frac{du_2}{dx_1} = C\frac{x_1^3}{6} - CL\frac{x_1^2}{2} + \frac{CL^2}{2}x_1 + A_3$$

@ $x_1 = 0$ $\qquad 0 = A_3 \quad \rightarrow \quad A_3 = 0$

$$u_2 = C\left[\frac{x_1^4}{18} - \frac{Lx_1^3}{6} + L^2\frac{x_1^2}{4}\right] + A_4$$

@ $x_1 = 0$ $\qquad 0 = A_4 \quad \rightarrow \quad A_4 = 0$

$u_2 = 0$

$$\rightarrow u_2 = -\frac{9}{7}\frac{p_0}{a^3tE}\left(\frac{x_1^4}{18} - L\frac{x_1^3}{6} + L^2\frac{x_1^2}{4}\right)$$

$$\eta = x_1/L$$

$$\rightarrow u_2 = -\frac{3}{56}\frac{L^4}{a^3t}\frac{p_0}{E}\left(\eta^4 - 4\eta^3 + 6\eta^2\right)$$

$$\rightarrow u_3 = \frac{1}{28}\frac{L^4p_0}{a^3tE}\left(\eta^4 - 4\eta^3 + 6\eta^2\right)$$

STRESS

$$\sigma_1 = E\left(\bar{\epsilon}_1 + x_3 K_2 - x_2 K_3\right)$$

$$K_2 = -\frac{d^2 u_3}{dx_1^2}, \quad K_3 = \frac{d^2 u_2}{dx_1^2}; \quad \bar{\epsilon} = \frac{d\bar{u}_1}{dx_1}$$

$$\sigma_1 = E\frac{P_0 L^4}{a^3 t E}\left[-x_3 \frac{1}{28}\left(\frac{12\eta^2}{L^2} - \frac{24\eta}{L^2} + \frac{12}{L^2}\right)\right.$$

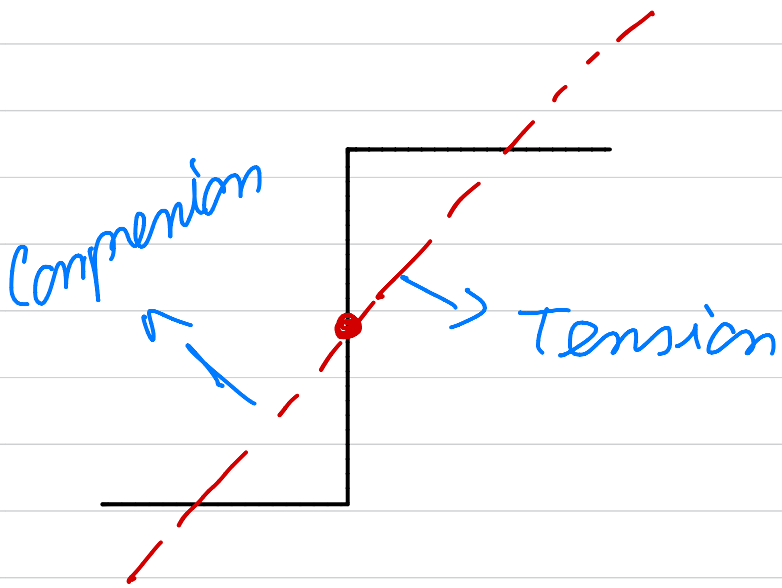
$$\left. + x_2 \frac{3}{56}\underbrace{\left(\frac{12\eta^2}{L^2} - \frac{24\eta}{L^2} + \frac{12}{L^2}\right)}_{\frac{12}{L^2}(1-\eta)^2}\right]$$

$$\sigma_1 = \frac{3}{14}\frac{P_0 L^4}{a^3 t E}\underbrace{(3x_2 - 2x_3)}_{\text{Dependence on } x_2,\, x_3}\underbrace{(1-\eta)^2}_{\text{Dependence on } x_1}$$

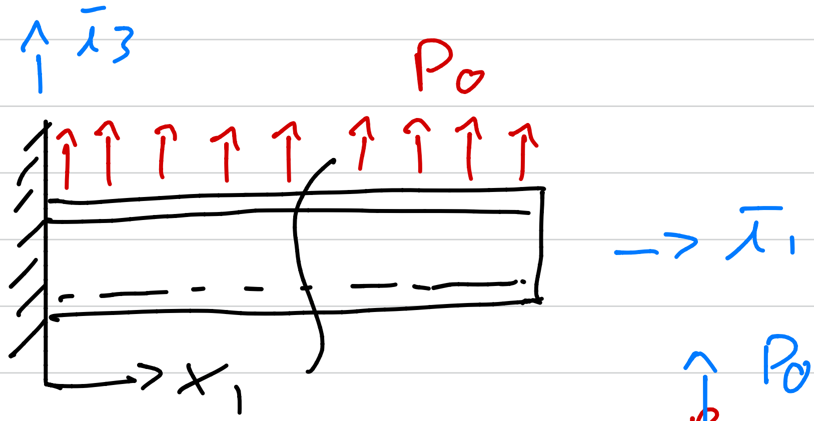
→ Stress is a max at $\eta = 0$, the root.

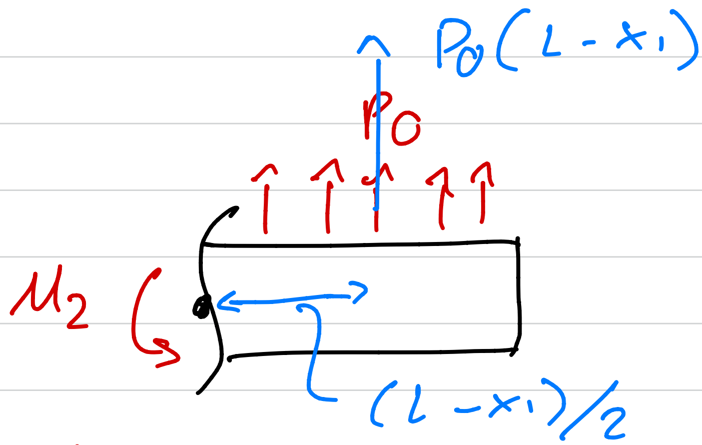
Neutral axis is defined by $\sigma_1 = 0$

$$3x_2 - 2x_3 = 0 \rightarrow x_3 = \frac{3}{2} x_2$$



SOLVE USING SECTIONAL CONSTITUTIVE EQUATIONS





$$M_2 + P_0 (L - x_1) \frac{(L-x_1)}{2} = 0$$

$$\begin{cases} M_2 = -\dfrac{P_0 (L-x_1)^2}{2} \\ M_3 = 0 \end{cases}$$

B.Cs @ $x_1 = 0$ $\quad u_2 = u_3 = 0$

$$\frac{du_2}{dx_1} = \frac{du_3}{dx_1} = 0$$

$$K_2 = \frac{H_{33}^{c}}{\Delta H} M_2 = -\frac{H_{33}^{c}}{\Delta H}\frac{P_0}{2}(L-x_1)^2$$

$$\frac{d^2 u_3}{dx_1^2} = \frac{H_{33}^{c}}{\Delta H}\frac{P_0}{2}(L-x_1)^2 = \frac{H_{33}^{c}}{\Delta H}\frac{P_0}{2}\left(L^2-2Lx_1+x_1^2\right)$$

$$\frac{du_3}{dx_1^2} = \frac{H_{33}^{c}}{\Delta H}\frac{P_0}{2}\left(L^2x_1 - Lx_1^2 + \frac{x_1^3}{3}\right) + \cancelto{0}{A_1}$$

$$u_3 = \underbrace{\frac{H_{33}^{c}}{\Delta H}}\frac{P_0}{2}\left(L^2\frac{x_1^2}{2} - L\frac{x_1^3}{3} + \frac{x_1^4}{12}\right)$$

$$u_3 = \frac{6}{7}\frac{1}{a^3tE}\frac{P_0}{2}\left(L^2\frac{x_1^2}{2} - L\frac{x_1^3}{3} + \frac{x_1^4}{12}\right)$$

$$\eta = x_1/L$$

$$* \; u_3 = \frac{1}{28}\frac{P_0L^4}{a^3tE}\left(6\eta^2 - 4\eta^3 + \eta^4\right)$$

STRESS

$$\sigma_1 = E(\bar{\epsilon}_1 + x_3 K_2 - x_2 K_3)$$

$$\sigma_1 = E\left[\cancel{\frac{N_1}{S}} + x_3 \frac{H_{33}^c M_2 + H_{23}^c \cancel{M_3}}{\Delta H} - x_2 \frac{H_{23}^c M_2 + H_{22}^c \cancel{M_3}}{\Delta H}\right]$$

$$\sigma_1 = E \frac{1}{\Delta H}\left(x_3 H_{33}^c - x_2 H_{23}^c\right) - \frac{P_0}{2}(L - x_1)^2$$

$$H_{33}^c = \frac{2}{3} a^3 t E, \quad H_{23}^c = a^3 t E$$

$$\sigma_1 = \frac{E}{3} \frac{a^3 t E}{\Delta H}\left(3x_2 - 2x_1\right) P_0 (L - x_1)^2$$