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HW #7

* Graded assignments were not given back (that I know of) so these are just my answers

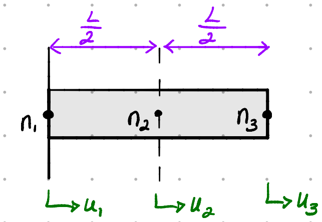
↳ If you have corrected answers or find any mistakes, let me know! :)

Problem 3

Solve the same problem as before using a piece-wise linear approximation for the displacement field. That is, divide the bar in n segments, assume the displacement field is linear within each segment, writing the linear function in terms of the (unknown) displacements at the end points of each segment. We call these points nodes, and the corresponding unknown values nodal displacements. More precisely:

1. Solve in particular the cases for $n = 2, 4, 8,$ and 16. You may use your preferred math program or programming language for this step.

$$n=2 \begin{cases} n_1 \text{ at } x_1 = 0 \\ n_2 \text{ at } x_1 = \frac{L}{2} \\ n_3 \text{ at } x_1 = L \end{cases}$$



using $\hat{u}(x_1) = a x_1 + b$

then
$$\hat{u} = \begin{cases} \frac{u_2 - u_1}{L/2} x_1 + u_1 , & 0 \le x_1 \le \frac{L}{2} \\ \frac{u_3 - u_2}{L/2}\left(x_1 - \frac{L}{2}\right) + u_2 , & \frac{L}{2} \le x_1 \le L \end{cases}$$

$\hat{u}$ = slope(x) + intercept

from displacement BCs : $\hat{u}(0) = 0 \Rightarrow u_1 = 0$

for Matlab:

$$\hat{u}_1 = \int_0^{L/2} \frac{u_2}{L/2} x_1$$

$$\hat{u}_2 = \int_{L/2}^{L} \frac{u_3 - u_2}{L/2}\left(x_1 - \frac{L}{2}\right) + u_2$$

$$\hat{u}_2(L) = u_3$$

Putting everything in Matlab gives

$$n=2 \begin{cases} u_2 = \dfrac{L(4P+3Lq)}{8E} = \dfrac{3qL^2}{8} + \dfrac{PL}{E} & \text{at } x_1 = \dfrac{L}{2} \\ u_3 = \dfrac{L(2P+Lq)}{2E} = \dfrac{qL^2}{2E} + \dfrac{PL}{E} & \text{at } x_1 = L \end{cases}$$

$n=4 \Rightarrow$

n_1 at $x_1 = 0$

n_2 at $x_1 = \frac{L}{4}$

n_3 at $x_1 = \frac{L}{2}$

n_4 at $x_1 = \frac{3L}{4}$

n_5 at $x_1 = L$

using $\hat{u}(x_1) = ax_1 + b$

$0 \to \frac{L}{4}: \quad \dfrac{u_2 - u_1}{L/4} x_1 + u_1 \qquad (u_1 = 0)$

$\frac{L}{4} \to \frac{L}{2}: \quad \dfrac{u_3 - u_2}{L/4}\left(x_1 - \frac{L}{4}\right) + u_2$

$\frac{L}{2} \to \frac{3L}{4}: \quad \dfrac{u_4 - u_3}{L/4}\left(x_1 - \frac{L}{2}\right) + u_3$

$\frac{3L}{4} \to L: \quad \dfrac{u_5 - u_4}{L/4}\left(x_1 - \frac{3L}{4}\right) + u_4$

$0 \to \frac{L}{4}: \quad \dfrac{4u_2}{4} \qquad \frac{L}{4} \to \frac{L}{2}: \quad \dfrac{4(u_3 - u_2)}{L} x_1 - u_3 + u_2 + u_2$

$\frac{L}{2} \to \frac{3L}{4}: \quad \dfrac{4(u_4 - u_3)}{L} x_1 - 2u_4 + 2u_3 + u_3$

$\frac{3L}{4} \to L: \quad \dfrac{4(u_5 - u_4)}{L} - 3u_5 + 3u_4 + u_4$

Putting everything in Matlab gives:

$$n=4 \begin{cases} u_2 = \dfrac{L(8P+7Lq)}{32E} = \dfrac{7qL^2}{32E} + \dfrac{PL}{4E} & \text{at } x_1 = \dfrac{L}{4} \\ u_3 = \dfrac{L(4P+3Lq)}{8E} = \dfrac{3qL^2}{8E} + \dfrac{PL}{2E} & \text{at } x_1 = \dfrac{L}{2} \\ u_4 = \dfrac{3L(8P+5Lq)}{32E} = \dfrac{15qL^2}{32E} + \dfrac{3PL}{4E} & \text{at } x_1 = \dfrac{3L}{4} \\ u_5 = \dfrac{L(2P+Lq)}{2E} = \dfrac{qL^2}{2E} + \dfrac{PL}{E} & \text{at } x_1 = L \end{cases}$$

for $n=8$:

$0 \to \frac{L}{8}, \quad \frac{L}{8} \to \frac{L}{4}, \quad \frac{L}{4} \to \frac{3L}{8}, \quad \frac{3L}{8} \to \frac{L}{2}, \quad \frac{L}{2} \to \frac{5L}{8}, \quad \frac{5L}{8} \to \frac{3L}{4}, \quad \frac{3L}{4} \to \frac{7L}{8}, \quad \frac{7L}{8} \to L$

putting everything in matlab gives:

$u_2 = \dfrac{15qL^2}{128} + \dfrac{PL}{8E} \quad \text{at } x_1 = \dfrac{L}{8}$

$u_3 = \dfrac{7qL^2}{32E} + \dfrac{PL}{4E} \quad \text{at } x_1 = \dfrac{L}{4}$

$$u_4 = \frac{39qL^2}{128} + \frac{3PL}{8E} \quad \text{at } x_1 = \frac{3L}{8}$$

$$u_5 = \frac{3qL^2}{8} + \frac{PL}{2E} \quad \text{at } x_1 = \frac{L}{2}$$

$$u_6 = \frac{55qL^2}{128E} + \frac{5PL}{8E} \quad \text{at } x_1 = \frac{5L}{8}$$

$$u_7 = \frac{15qL^2}{32E} + \frac{3PL}{4E} \quad \text{at } x_1 = \frac{3L}{4}$$

$$u_8 = \frac{63qL^2}{128E} + \frac{7PL}{8E} \quad \text{at } x_1 = \frac{7L}{8}$$

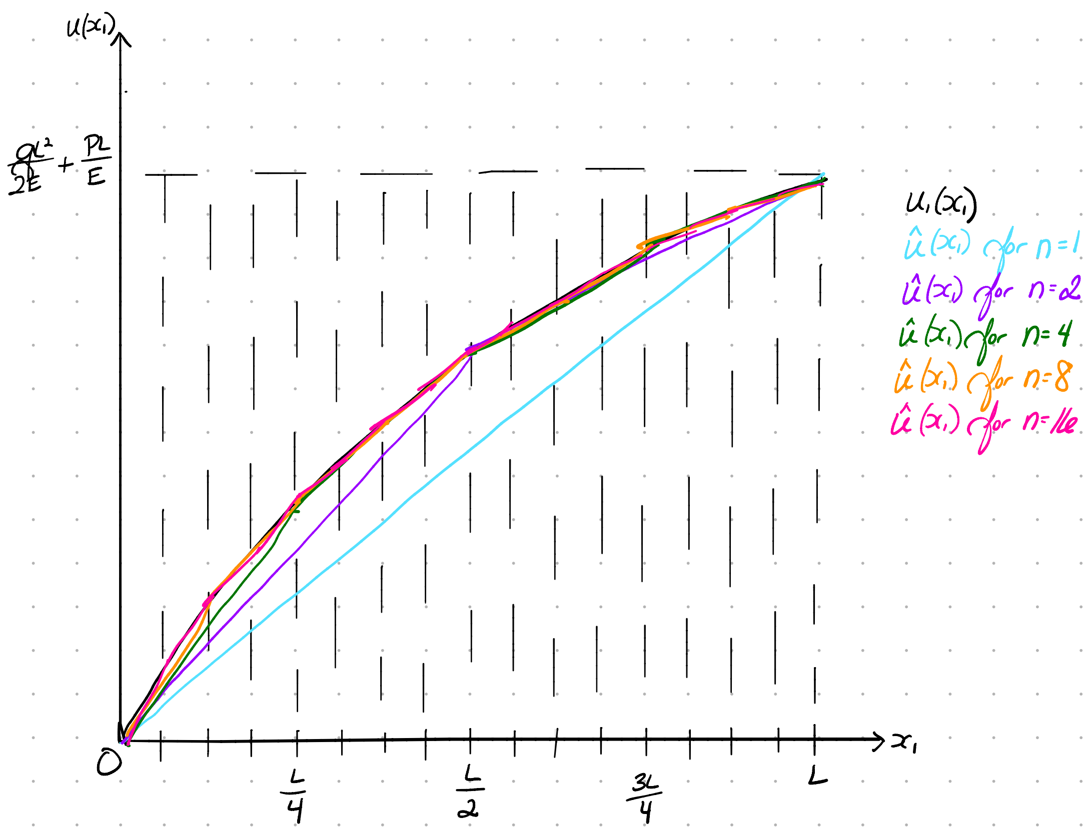
$$u_9 = \frac{qL^2}{2E} + \frac{PL}{E} \quad \text{at } x_1 = L$$

See code for $n=16$

I'm the self-proclaimed World's Worst Coder, so if you have a code that works, please let me know

I couldn't figure out how to plot everything in Matlab because I needed symbolic points (I think) so I did it by hand:
(for plot)

2. Plot the solutions for all cases and compare with the exact solution obtained in Problem 1.



3. Make a plot of the value of the potential Π for each case vs the number of elements. Add to the plot the horizontal line corresponding to the exact value of Π known from Problem 1.

I have no clue how to code this ...