Exam:

- HW1 / HW2
- All material until last class
- email at 9:25am
- Open notes & HWs + summaries prepared for studying.
- same level of complexity as homework
- Ok to use mathematica / MATLAB, but be sure to submit.

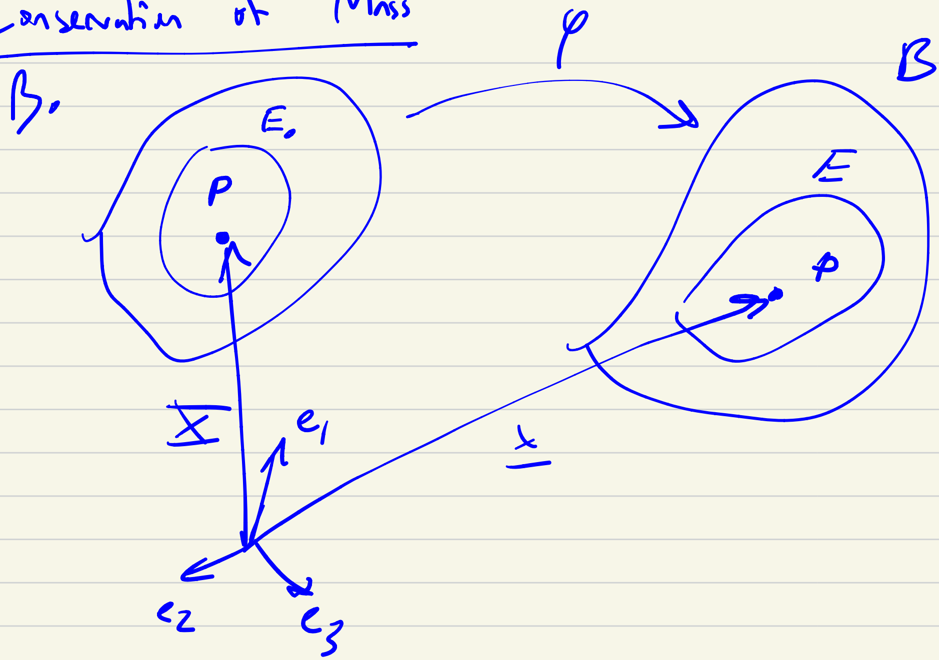
Prof. Rimoli's # (626) 429 -2241

So far, no physics.

Balance Laws

- Previously we derived kinematic quantities to describe the possible deformed configurations of a continuum medium
- These quantities on their own cannot predict the configuration a body will adopt as a result of a given applied loading
- To do so requires a generalization of the laws of mechanics, originally developed for collections of particles, to a continuum medium.

Conservation of Mass



(E_0, E arbitrary subset of the bodies)

$\mathcal{B}_0 \equiv$ body in reference configuration

$\mathcal{B} \equiv \varphi(\mathcal{B}_0) \equiv$ body in deformed configuration

$E_0 \equiv$ subbody in ref config; $E_0 \subset \mathcal{B}_0$ (↑ subset)

$E \equiv \varphi(E_0) \equiv$ sub body in the deformed config; $E \subset \mathcal{B}$

The mass of any subbody E_0 must remained unchanged by the deformation:

$$m_0(E_0) = m(E) \quad \forall E_0 \subset \mathcal{B}_0$$

$$\Longrightarrow \int_{E_0} dm_0 = \int_E dm$$

We define mass as : $\rho_0 = dm_0/dV$;

$\rho = dm/dv$

$$\Rightarrow \int_{E_0} \rho_0 \, dV = \int_E \rho \, dv \; ; \quad \forall E_0 \subset B_0$$

(ρ/V don't have to stay the same, but mass stays the same)

We know from kinematics that

$$dv = J \, dV \; ; \qquad J = \det(\underline{\underline{F}})$$

$$\int_{E_0} \rho_0 \, dV = \int_{E_0} \rho J \, dV \; , \; \forall E_0 \subset B_0$$

(change limit of integration)

$$\Rightarrow \int_{E_0} (\rho J - \rho_0) \, dV = 0 \, , \quad \forall E_0 \subset B_0$$

(This integral has to be zero for any E_0)

In order for the previous equation to be satisfied for all E_0, it must be satisfied pointwise:

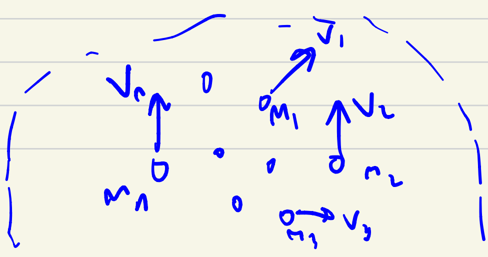
$$\boxed{J_\rho = \rho_0} \quad (\text{local expression})$$

(The only way for any E_0 is for the function itself to be zero.

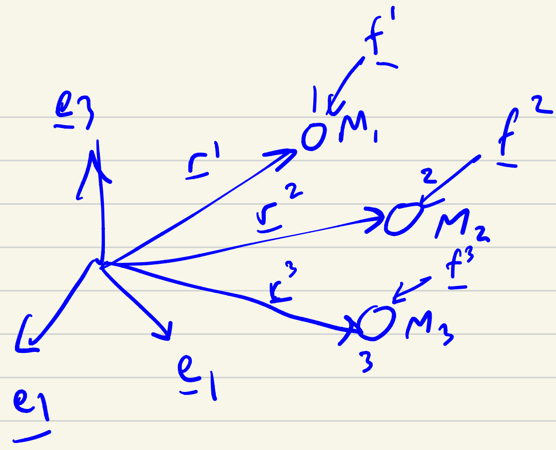
Balance of linear momentum

Newton's second law for a system of particles establishes:

$$\frac{D}{Dt}(\underline{L}) = \underline{F}^{ext}; \quad \underline{L} = \sum_{\alpha=1}^{N} m^\alpha \dot{\underline{r}}^\alpha; \quad \underline{F}_{ext} = \sum_{\alpha=1}^{N} \underline{f}^\alpha$$



velocity ($d\underline{r}/dt$)



$\underline{L} \equiv$ Linear momentum of the system

$M^\alpha \equiv$ mass of particle α

$\dot{\underline{r}}^\alpha \equiv$ velocity of particle α

$\underline{f}^\alpha \equiv$ force acting on particle α

how to go from sys of particles to continuum?

For a continuum system, we consider a continuous distribution of matter divided into infinitesimal elements of mass dm, or volume elements dv with mass

$$dm = \rho \, dv$$

The linear momentum of a single differential element is $d\underline{L} = \dot{\underline{x}}\, dm$

Integrating over the body gives total momentum of B

$$\boxed{\underline{L}(B) = \int_B dL = \int_B \dot{x}\, dm = \int_B \dot{\underline{x}}\, \rho\, dv}$$

The balance of linear momentum is then:

$$\boxed{\frac{D}{Dt} \int_B \dot{x}\, \rho\, dv = \underline{F}^{ext}(B)} \quad (1)$$