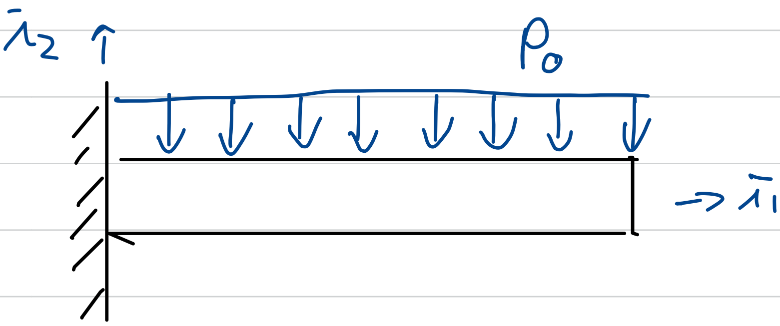
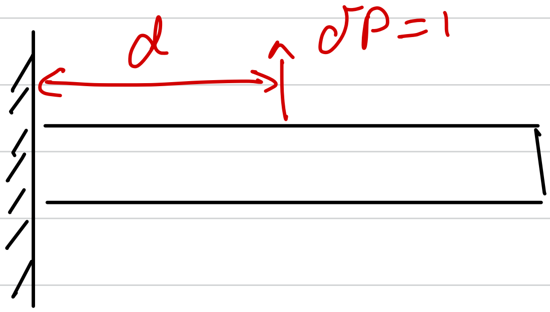
Ex #1:

H^c_{33} is known

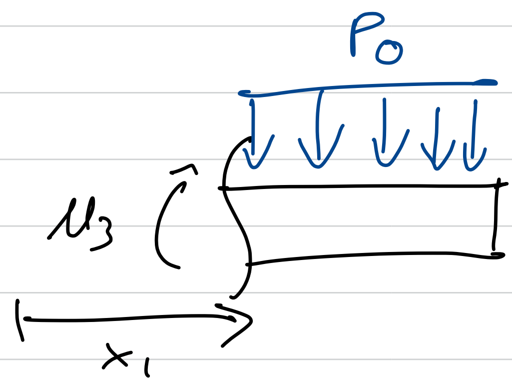
$H^c_{23} = 0$



Find $u_2(x_1)$



Find $u_3(x_1)$



$$u_3 = -\frac{P_0 (l - x_1)^2}{2}$$

Find $\hat{u}_3(x_1)$

$$\hat{u}_3 = \begin{cases} \delta P (d - x_1) & x_1 \leq d \\ 0 & x_1 \geq d \end{cases}$$

$$\Delta \cdot \delta P = \int_0^L \frac{u_3 \hat{u}_3}{H_{33}^c} \, dx_1$$

$$\Delta \cdot \delta P = \int_0^d -\delta P (d - x_1) \frac{P_0 (L - x_1)^2}{2 H_{33}^c} \, dx_1 + \int_d^L -(0) \frac{P_0 (L - x_1)^2}{2 H_{33}^c} \, dx_1$$

Let $\delta P = 1$

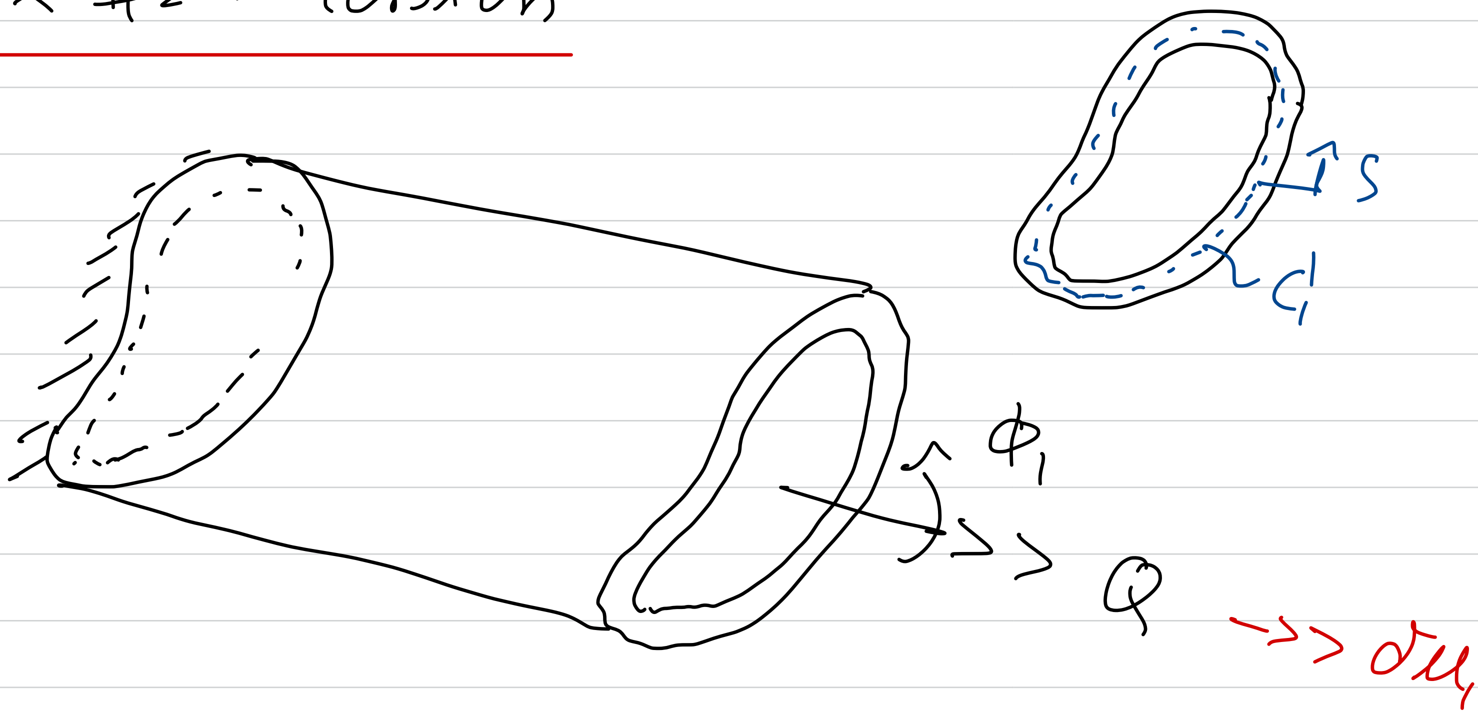
$$\Delta = \frac{-P_0}{2 H_{33}^c} \int_0^d (d - x_1)(L - x_1)^2 \, dx_1$$

$$\Delta = \frac{-P_0}{2 H_{33}^c} \left(\frac{L^2 d^2}{2} - \frac{L d^3}{3} + \frac{d^4}{12} \right)$$

* Vary d to find u_2 at any x_1

$$\boxed{u_2(x_1) = -\frac{P_0}{2 H_{33}^c} \left(\frac{L^2 x_1^2}{2} - \frac{L x_1^3}{3} + \frac{x_1^4}{12} \right)}$$

Ex #2 : Torsion



$$\phi_1 \, \delta M_1 = -\delta W_I' = \int_V \gamma_s \, \delta \tau_s \, dV$$

Recall that τ_s is a traction at s in the plane

$$\phi_1 \, \delta M_1 = \int_0^L \int_{C_1'} \gamma_s \cdot \delta \tau_s \, t \, ds \, dx_1$$

$$\gamma_s = \frac{\tau_s}{G}, \quad \tau_s = \frac{M_1}{2A_c t} \rightarrow \gamma_s = \frac{M_1}{2A_c t G}$$

$$\gamma_s = \frac{Q}{2A_c t G}, \quad \hat{\tau}_s = \frac{\delta M_1}{2A_c t}$$

$$\phi_1 \,\delta\mathcal{M}_1 = \int_0^L \int_{C_1'} \frac{Q}{2A_c t G} \cdot \frac{\delta\mathcal{M}_1}{2A_c t} \cdot t\, ds\, dx_1$$

Let $\delta\mathcal{M}_1 = 1$

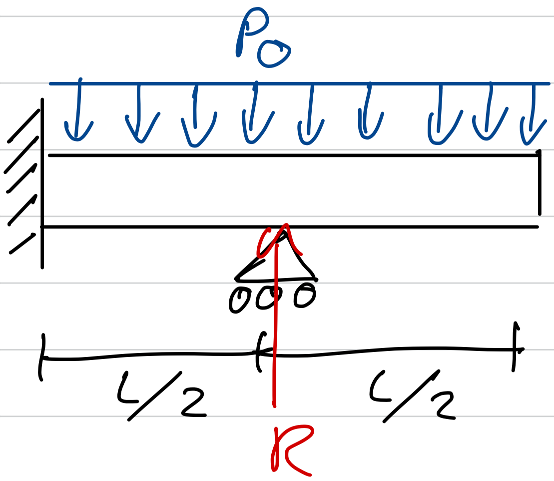
$$\phi_1 = \frac{Q}{4A_c^2} \int_0^L \left[\int_{C_1'} \frac{ds}{Gt} \right] dx_1$$

$$= \frac{Q}{4A_c^2} \cdot L \int_{C_1'} \frac{ds}{Gt}$$

$$K_1 = \frac{\phi_1}{L} = \frac{Q}{4A_c^2} \int_{C_1'} \frac{ds}{Gt}$$

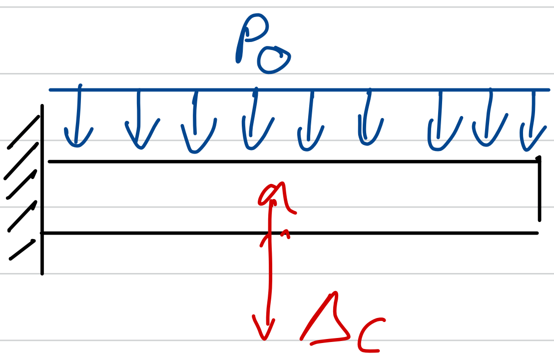
$$\boxed{K_1 = \frac{\mathcal{M}_1}{4A_c^2} \int_{C_1'} \frac{ds}{Gt}}$$

Statically Indeterminate Problems

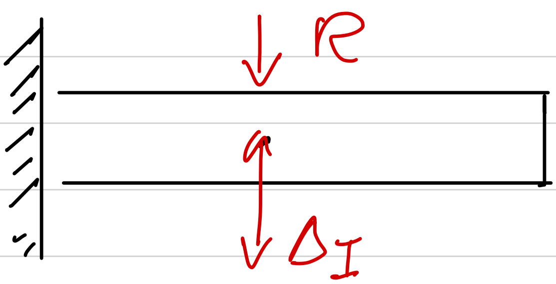


Need to Find R

Release the extra constraint!



Compute Δ_C
using ULM



Compute Δ_I

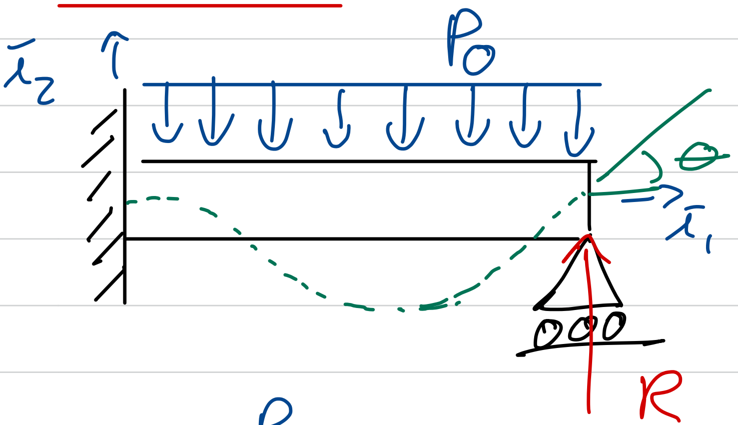
using ULM

What is the value of R
such that

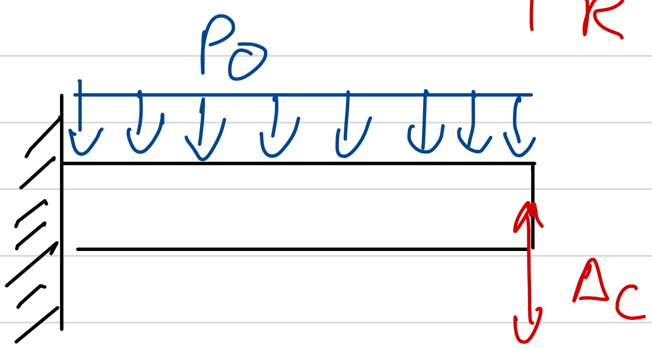
$$\Delta_I + \Delta_C = 0$$

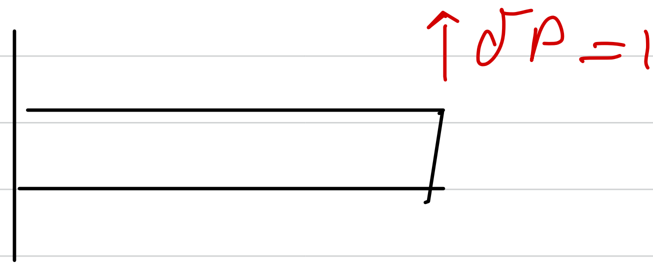
Ex #3

$H_{23}^c = 0$



Find the tip rotation θ

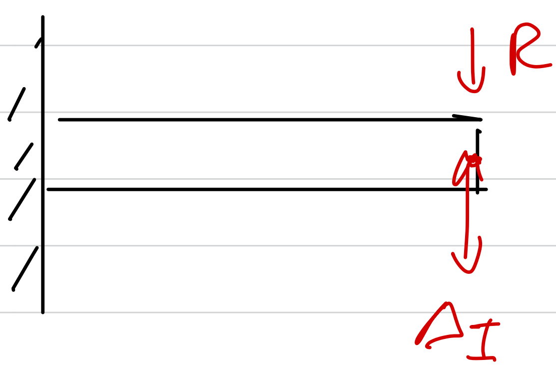




$$\Delta_C = \int_0^L \frac{\mathcal{M}_3 \hat{\mathcal{M}}_3}{H_{33}^c}\, dx_1 = \frac{1}{H_{33}^c}\int_0^L -(L-x_1)\, p_0 \frac{(L-x_1)^2}{2}\, dx_1$$

$$= \frac{-p_0}{2H_{33}^c}\int_0^L (L-x_1)^3\, dx_1 = -\frac{p_0}{2H_{33}^c}\cdot\frac{L^4}{4}$$

$$\Delta_C = \frac{-p_0 L^4}{8H_{33}^c}$$



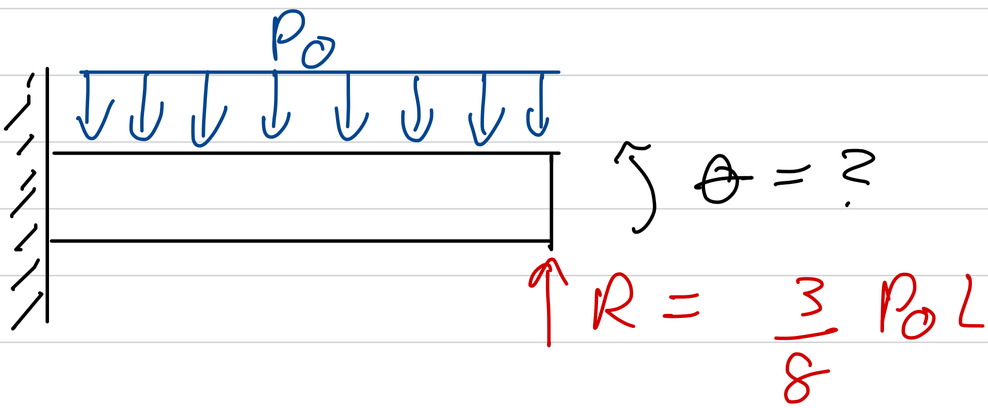
$$\Delta_I = \frac{1}{H_{33}^c}\int_0^L -(L-x_1)\, R\, (L-x_1)\, dx_1$$

$$\Delta_I = -\frac{R}{H_{33}^c}\,\frac{L^3}{3}$$

Want $\Delta_I + \Delta_C = 0$

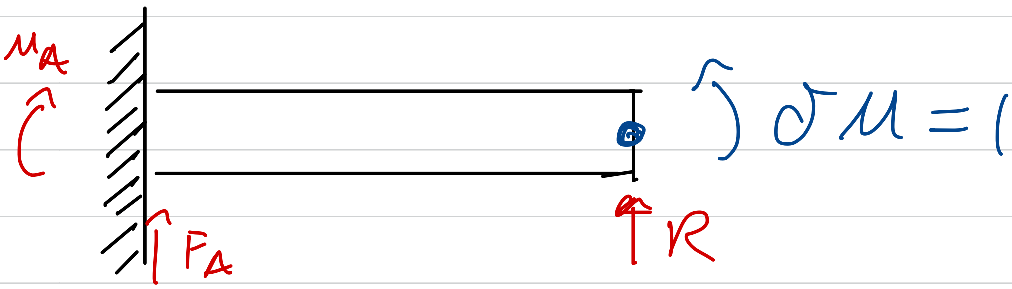
$$-\frac{P_0 L^4}{8 H_{33}^c} - \frac{R L^3}{3 H_{33}^c} = 0$$

$$\boxed{R = -\frac{3}{8} P_0 L}$$



$$\theta \cdot \delta M = \int_0^L \frac{\hat{M}_3 M_3}{H_{33}^c} dx_1$$

$$M_3 = R(L - x_1) - \frac{P_0 (L - x_1)^2}{2}$$



$\hat{M}_3 \rightarrow$ <u>Any</u> Statically admissible moment in eq. w/ the applied δM.

1) Let $R = 0$

$$\hat{M}_3 = \delta M$$

2) Let $M_A = 0$

$$RL = \delta M \qquad R = \delta M / L$$

$$\hat{M}_3 = -R(L - x_1) + \delta M$$

$$= -\delta M_3/L\,(L - x_1) + \delta M$$

$$\hat{M}_3 = \frac{x_1}{L}\delta M$$

$$\theta = \frac{1}{H_{33}^{C}} \int_0^L \Bigg[\overbrace{\frac{3}{8} P_0 L (L - x_1) - \frac{P_0 (L - x_1)^2}{2}}^{u_3} \Bigg] \cdot \underbrace{(1)}_{\hat{u}_3} dx_1$$

$$\theta = \frac{1}{H_{33}^{C}} \left(\frac{3}{8} P_0 L \frac{L^2}{2} - \frac{P_0}{2} \frac{L^3}{3} \right)$$

$$\boxed{\theta = \frac{P_0}{H_{33}^{C}} \frac{L^3}{48}}$$

2) $\hat{u}_3 = \frac{x_1}{L} \delta u = \frac{x_1}{L}$

$$\theta = \frac{1}{H_{33}^{C}} \int_0^L \left[\frac{3}{8} P_0 L (L - x_1) - \frac{P_0}{2} (L - x_1)^2 \right] \cdot \left(\frac{x_1}{L} \right) dx_1$$

$$\theta = \frac{1}{H_{33}^{C}} \int_0^L \left[\frac{3}{8} P_0 (L x_1 - x_1^2) - \frac{P_0}{2L} (L^2 x_1 - 2L x_1^2 + x_1^3) \right]$$

$$\theta = \frac{1}{H_{33}^{C}} \left[\frac{3}{8} P_0 \left[L \frac{x_1^2}{2} - \frac{x_1^3}{3} \right]_0^L - \frac{P_0}{2L} \left[L^2 \frac{x_1^2}{2} - 2L \frac{x_1^3}{3} + \frac{x_1^4}{4} \right]_0^L \right)$$

$$\theta = \frac{P_0 L^3}{H_{33}^c}\left(\frac{3}{8}\cdot\frac{1}{6} - \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right)\right)$$

$$\boxed{\theta = \frac{P_0 L^3}{H_{33}^c}\,\frac{1}{48}}$$