Multidisciplinary Design Optimization (MDO): Single-Level and Multi-Level Methods

AE 6310: Optimization for the Design of Engineered Systems

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Single-level MDO Approaches

Single-level approaches use only <u>one</u> optimizer– specifically at the system level.

These approaches distribute the analysis to partitioned subsystems but design/optimization is kept centralized at the system-level optimizer.

Two of the most common single-level approaches are:

- Multidisciplinary Feasible (MDF)
- Individual Disciplinary Feasible (IDF)

MDO Terminology

Shared design variables (x_s) – design variables used by more than one subsystem.

Local design variables (x_i) – design variables used by only one subsystem

Let $X = \{x, x_s\}$ and $X_i = \{x_i, x_{si}\}$, where *i* refers to some subsystem.

Coupling variables (y) – e.g. y_{21} is the output of subsystem 2 that is a required input for subsystem 1

Multidisciplinary Feasible (MDF)

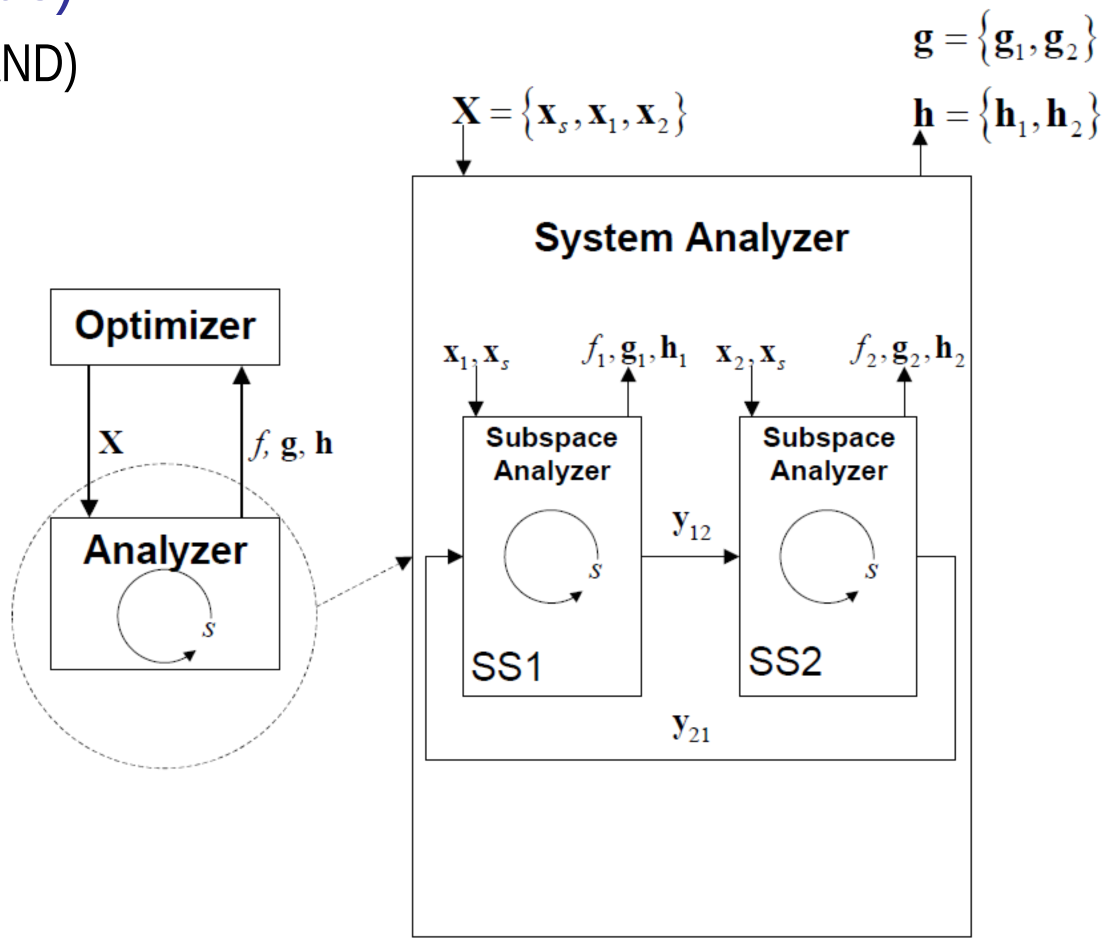
Synonyms (or similar methods):

- Nested Analysis and Design (NAND)
- Single-NAND-NAND (SNN)
- All-in-One
- One-at-a-Time
- All-at-Once (AAO)

Multidisciplinary feasibility

- Solution is *consistent* across all disciplines (subsystems) at each function call of optimizer.
- Solution may still be **infeasible** *in terms of constraints*.

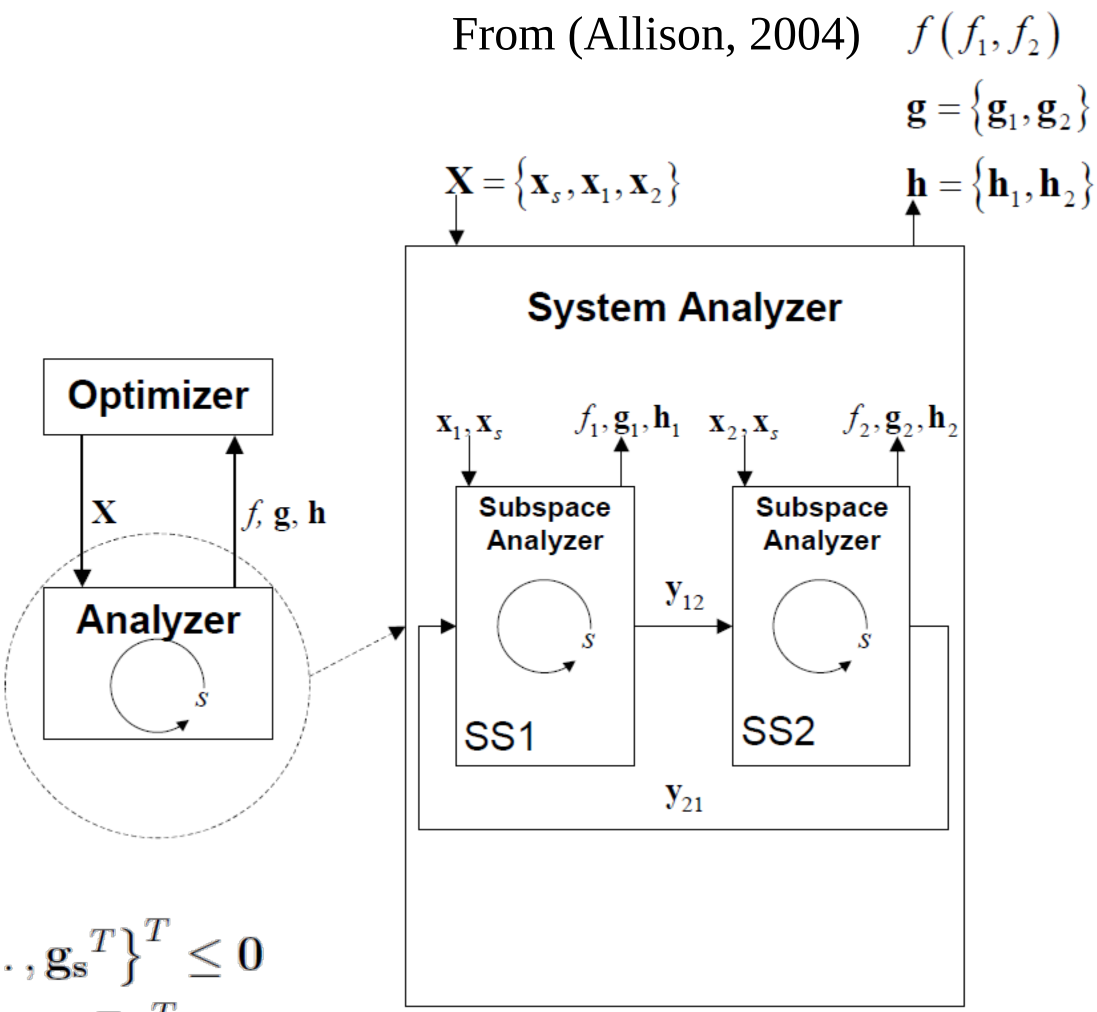
From (Allison, 2004)



Multidisciplinary Feasible (MDF)

System solver coordinates all subsystems and returns a consistent, converged solution.

- Optimizer "sees" a "normal" function with only original design variables to control.

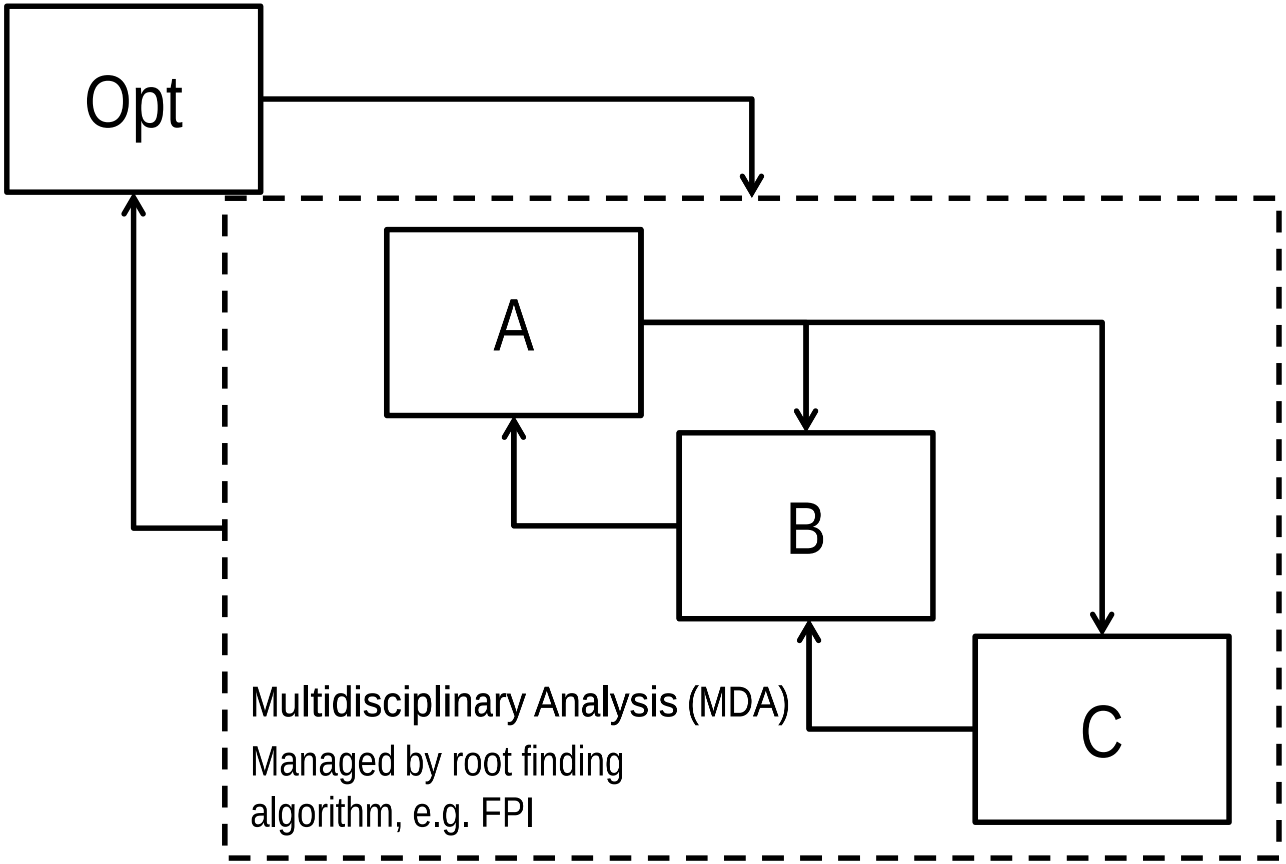


$$\min_{\mathbf{X}=\{\mathbf{x},\mathbf{x_s}\}} \quad f(\mathbf{X})$$

$$\text{subject to} \quad \mathbf{g}(\mathbf{X}) = \{\mathbf{g_1}^T, \mathbf{g_2}^T, \ldots, \mathbf{g_s}^T\}^T \leq \mathbf{0}$$

$$\mathbf{h}(\mathbf{X}) = \{\mathbf{h_1}^T, \mathbf{h_2}^T, \ldots, \mathbf{h_s}^T\}^T = \mathbf{0}$$

Multidisciplinary Feasible (MDF)



Multidisciplinary Feasible (MDF)

MDF is non-hierarchic.

Effective if nested iteration converges quickly (typically true when coupling is weak) and analyses are computationally inexpensive.

Allows the use of legacy computational analysis tools without modification. May need to standardize data format for communication.

Multidisciplinary Feasible (MDF)

Solution may be sub-optimal if more than one exists.

Lack of ability to easily parallelize implies that the method may be very inefficient.

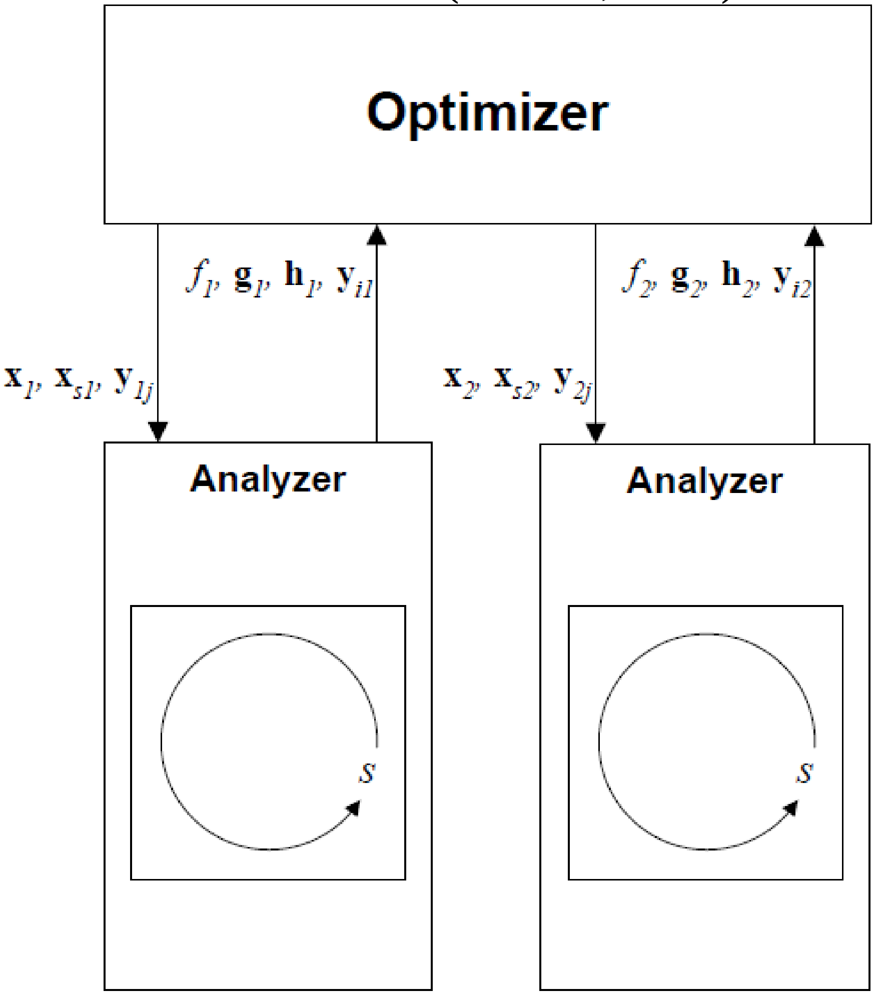
If paired with FPI to converge the system, limitations of FPI are retained.

Individual Discipline Feasible (IDF)

Synonyms (or similar methods):

- Simultaneous Analysis and Design (SAND)
- Single-SAND-NAND (SSN)

From (Allison, 2004)



Individual discipline feasibility

- Each discipline (subsystem) satisfies its governing equations at each iteration.
- Solution is not MDF until optimizer converges.

Individual Discipline Feasible (IDF)

Optimizer coordinates subsystems by controlling design *and* coupling variables

$$\min_{\mathbf{X}=\{\mathbf{x},\mathbf{x_s}\},\mathbf{y}} \quad f(\mathbf{X},\mathbf{y})$$

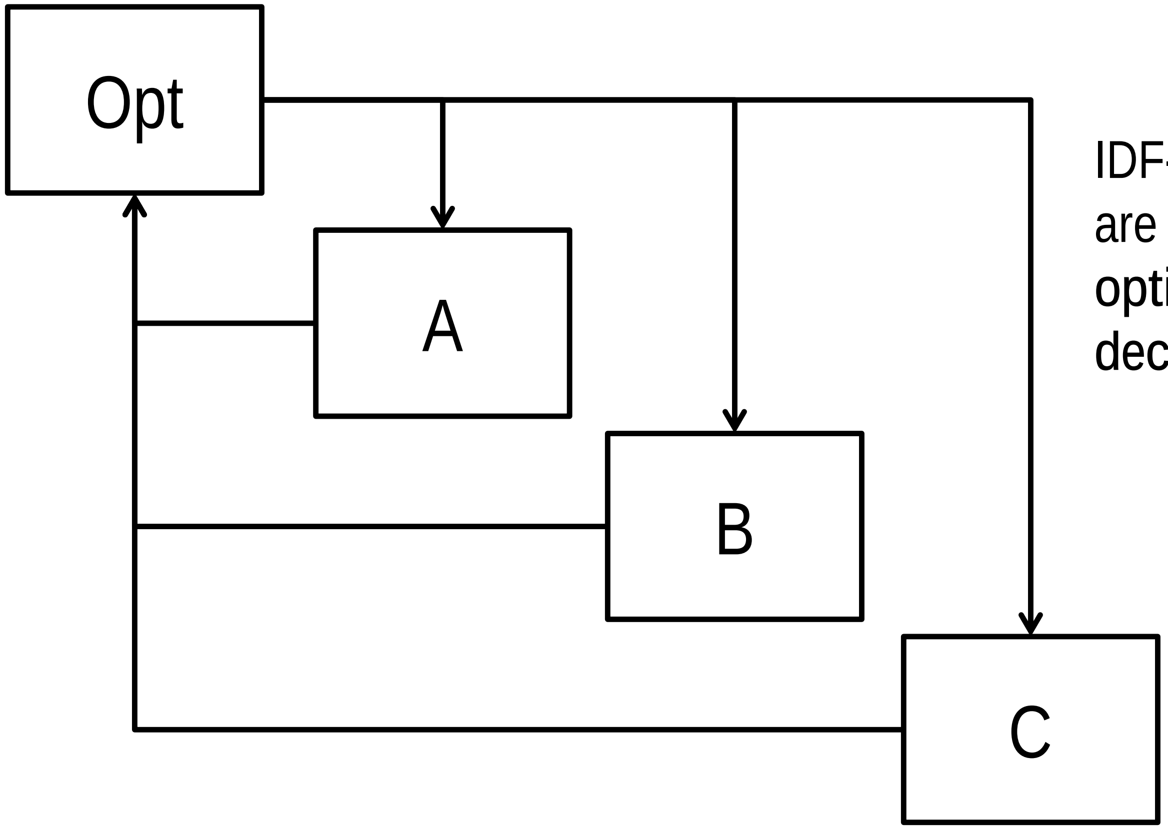
$$\text{subject to} \quad \mathbf{g}(\mathbf{X},\mathbf{y}) = \{\mathbf{g_1}^T, \mathbf{g_2}^T, \ldots, \mathbf{g_s}^T\}^T \leq 0$$

$$\mathbf{h}(\mathbf{X},\mathbf{y}) = \{\mathbf{h_1}^T, \mathbf{h_2}^T, \ldots, \mathbf{h_s}^T\}^T = 0.$$

$$\boxed{\mathbf{h_{aux}}(\mathbf{X},\mathbf{y}) = \mathbf{y}(\mathbf{X},\mathbf{y}) - \mathbf{y} = 0.}$$

Auxiliary constraints (compatibility constraints) in the optimizer enforce consistency between computed and “guessed” coupling variables.

Individual Discipline Feasible (IDF)



IDF-like methods are sometimes called optimizer-based decomposition (OBD)

The optimizer ensures consistency by enforcing compatibility constraints. In other words, the root-finding problem of convergence is handled directly by the optimizer, not FPI or Newton's Method.

Individual Discipline Feasible (IDF)

IDF is hierarchical

More parallelizable (each disciplinary tool could be run on a separate processor)

Improved convergence properties.

More likely to find optimal solution if multiple consistent solutions exist.

More taxing for optimizer due to extra variables to control.

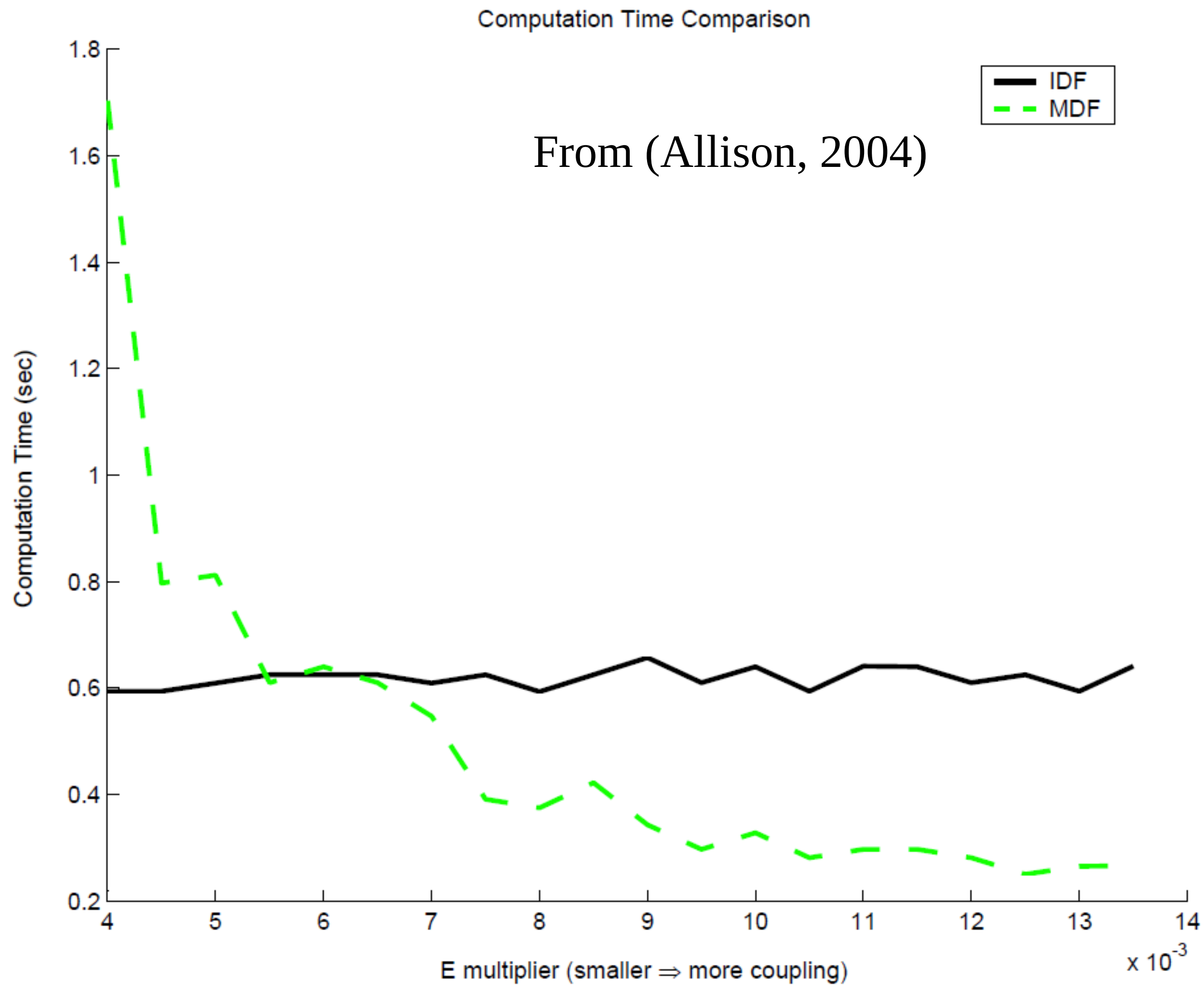
Produces inconsistent solution if optimizer fails to converge.

MDF/IDF Comparison

Comparison of computation times vs. coupling strength

Recall our example of wing rigidity…

Here, E refers to the elastic modulus.



Multi-Level MDO Approaches

Multi-level approaches use an optimizer at the system level as well as multiple optimizers at the subsystem level.

Useful when problem scale is too large for one optimizer to handle

These approaches distribute both analysis to partitioned subsystems and design to subsystem-level optimizers.

Two popular multi-level approaches are:

- Collaborative Optimization (CO)
- Analytical Target Cascading (ATC)

Collaborative Optimization (CO)

Motivated by organizational implications of the MDA; historical ties to the aerospace design process

Bi-level – CO is intended for early design phases where partitioning by discipline is common, and all disciplines are assumed to be on the same level.

Collaborative Optimization (CO)

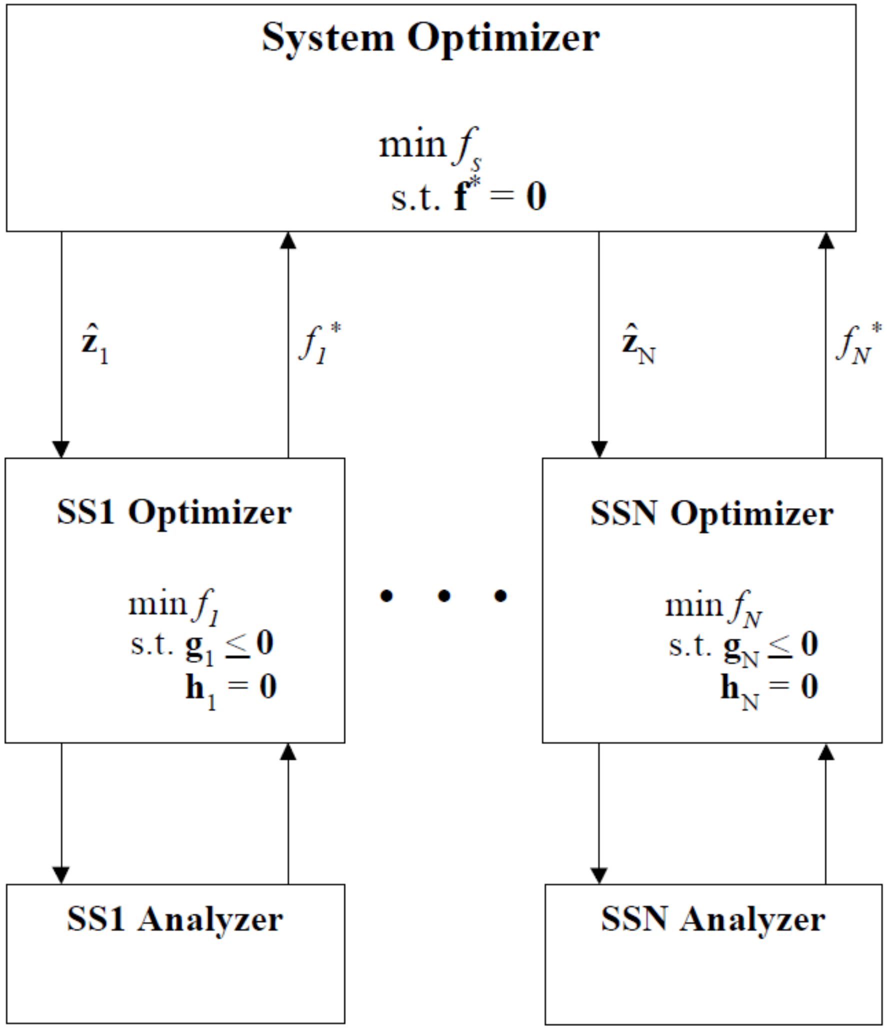
System optimizer selects target vector ($\hat{\mathbf{z}}_i$) for subsystems where,

$$\hat{\mathbf{z}}_i = \left\{ \hat{\mathbf{x}}_{si}, \hat{\mathbf{y}}_{ij}, \hat{\mathbf{y}}_{ji} \right\}$$

Subsystem optimizer minimizes

$$f_i = \left\| \mathbf{z}_i - \hat{\mathbf{z}}_i \right\|_2^2$$

subject to any disciplinary constraints by selecting local ($\mathbf{x}_i$) and shared ($\mathbf{x}_{si}$) design variables

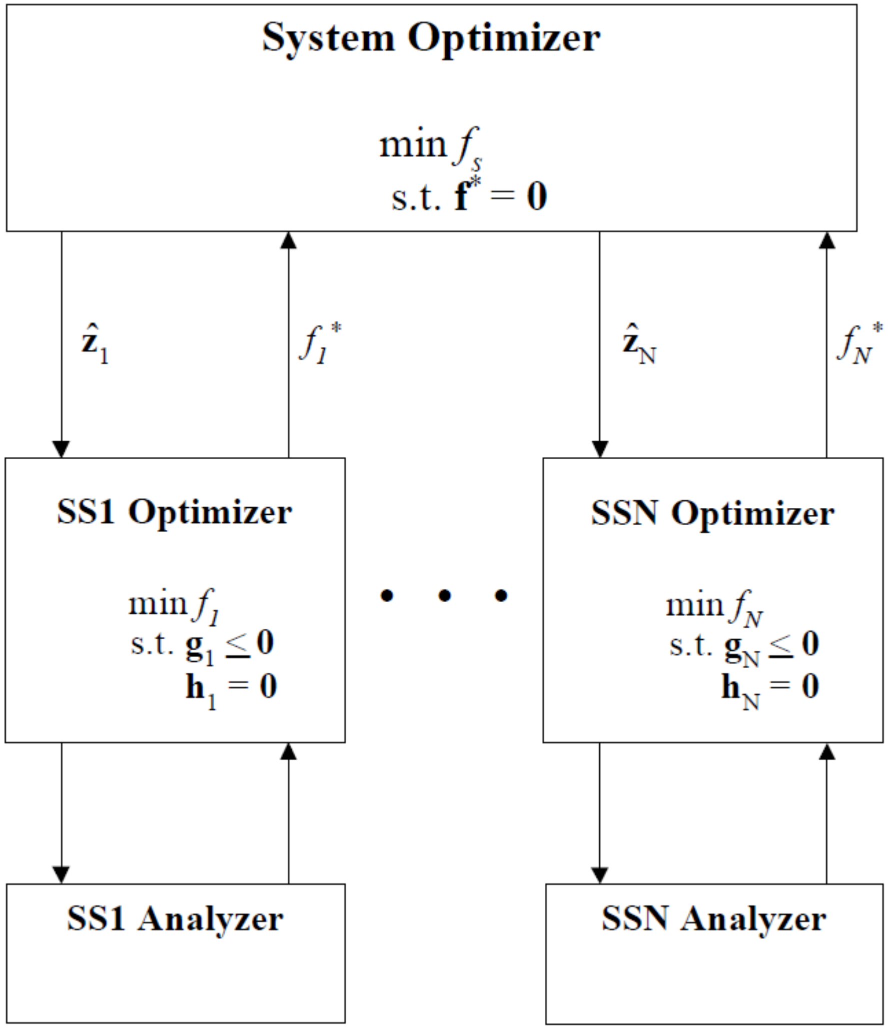


From (Allison, 2004)

Collaborative Optimization (CO)

System optimizer's task is to minimize the system-level objective function ($f_s(\hat{\mathbf{z}})$) while driving the design towards consistency via the equality constraint.

$$f^* = \mathbf{z} - \hat{\mathbf{z}}$$



From (Allison, 2004)

Analytical Target Cascading (ATC)

Originates from product development needs in the automotive industry.

Multi-level (perhaps more than 2 levels) – ATC supports the multi-level hierarchical structure common to product development industries (often due to partitioning by physical boundaries).

MDO Terminology

$\mathbf{x}$ *Local design variables*: Design variables that are each inputs to only one subspace.

$\mathbf{x}_s$ *Shared design variables*: Design variables that are each inputs to more than one subspace.

$\mathbf{y}$ *Coupling variables*: Quantities that are passed from one subspace to another that are not original design variables, but rather artifacts of decomposition.

From (Allison, 2004)

ATC Terminology

$\mathbf{y}^{\dagger}$ *Linking variables*: Quantities that are input to more than one subspace. These could be either shared variables (original design variables) or coupling variables (not original design variables).

$\mathbf{x}^{\dagger}$ *Local decision variables*: Variables that a particular subspace determines the value of. May or may not be original design variables.

R *Responses*: Values generated by subspaces required as inputs to respective parent subspaces. May or may not be coupling variables.

T *Targets*: Values set by parent subspaces to be matched by the corresponding quantities from child subspaces. Targets may exist for either responses or linking variables.

From (Kim, 2003)

ATC Example

$$\text{Minimize} \quad f = x_1^2 + x_2^2$$

$$x_3, x_4, \ldots, x_{14}$$

where

$$R_1 = x_1 = r_1(x_3, x_4, x_5) = (x_3^2 + x_4^{-2} + x_5^2)^{1/2}$$

$$R_2 = x_2 = r_2(x_5, x_6, x_7) = (x_5^2 + x_6^2 + x_7^2)^{1/2}$$

Assign to system-level optimizer

$$R_3 = x_3 = r_3(x_8, x_9, x_{10}, x_{11}) = (x_8^2 + x_9^{-2} + x_{10}^{-2} + \boxed{x_{11}^2})^{1/2}$$

$$R_4 = x_6 = r_4(x_{11}, x_{12}, x_{13}, x_{14}) = (\boxed{x_{11}^2} + x_{12}^2 + x_{13}^2 + x_{14}^2)^{1/2}$$

Assign to subsystem optimizers

subject to

$$g_1: \frac{x_3^{-2} + x_4^2}{x_5^2} \leqslant 1 \qquad g_2: \frac{x_5^2 + x_6^{-2}}{x_7^2} \leqslant 1 \qquad g_3: \frac{x_8^2 + x_9^2}{x_{11}^2} \leqslant 1$$

$$g_4: \frac{x_8^{-+2} + x_{10}^2}{x_{11}^2} \leqslant 1 \qquad g_5: \frac{x_{11}^2 + x_{12}^{-2}}{x_{13}^2} \leqslant 1 \qquad g_6: \frac{x_{11}^2 + x_{12}^2}{x_{14}^2} \leqslant 1$$

$$x_3, x_4, \ldots, x_{14} \geqslant 0$$

From (Kim, 2003)

$$\mathbf{P}_s: \quad \underset{x_3, x_4, \ldots, x_7, x_{11}, \varepsilon_1, \varepsilon_2, \varepsilon_3}{\text{Minimize}} \quad x_1^2 + x_2^2 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

subject to

$$\left(x_{11} - x_{11ss1}^L\right)^2 + \left(x_{11} - x_{11ss2}^L\right)^2 \le \varepsilon_1$$

$$\left(x_3 - x_3^L\right)^2 \le \varepsilon_2, \left(x_6 - x_6^L\right)^2 \le \varepsilon_3,$$

$$g_1: \ \frac{x_3^{-2} + x_4^2}{x_5^2} \le 1, \ g_2: \ \frac{x_5^2 + x_6^{-2}}{x_7^2} \le 1, x_3, x_4, \ldots, x_7, x_{11} \ge 0$$

From (Kim, 2003)

$$x_1 = r_1(x_3, x_4, x_5) = \left(x_3^2 + x_4^{-2} + x_5^2\right)^{1/2}$$

$$x_2 = r_2(x_5, x_6, x_7) = \left(x_5^2 + x_6^2 + x_7^2\right)^{1/2}$$

$$\mathbf{P}_{ss1}: \quad \underset{x_8, x_9, x_{10}, x_{11}}{\text{Minimize}} \left(x_3 - x_3^U\right)^2 + \left(x_{11} - x_{11}^U\right)^2$$

subject to

$$g_3: \ \frac{x_8^2 + x_9^2}{x_{11}^2} \le 1, g_4: \ \frac{x_8^{-2} + x_{10}^2}{x_{11}^2} \le 1$$

$$x_8, x_9, x_{10}, x_{11} \ge 0$$

x_{11}

$$\mathbf{P}_{ss2}: \quad \underset{x_{11}, x_{12}, x_{13}, x_{14}}{\text{Minimize}} \left(x_6 - x_6^U\right)^2 + \left(x_{11} - x_{11}^U\right)^2$$

subject to

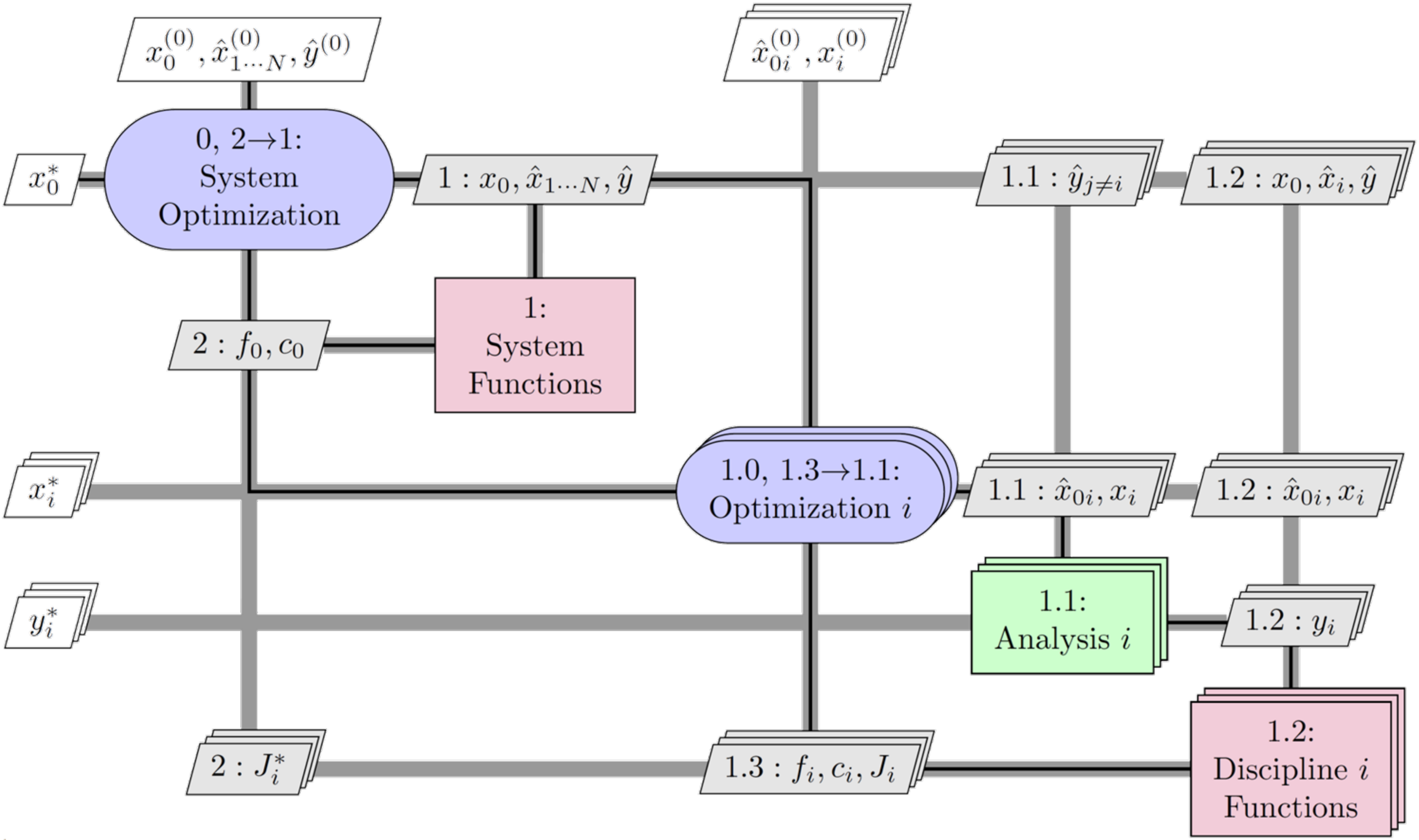
$$g_5: \ \frac{x_{11}^2 + x_{12}^{-2}}{x_{13}^2} \le 1, g_6: \ \frac{x_{11}^2 + x_{12}^2}{x_{14}^2} \le 1$$

$$x_{11}, x_{12}, x_{13}, x_{14} \ge 0$$

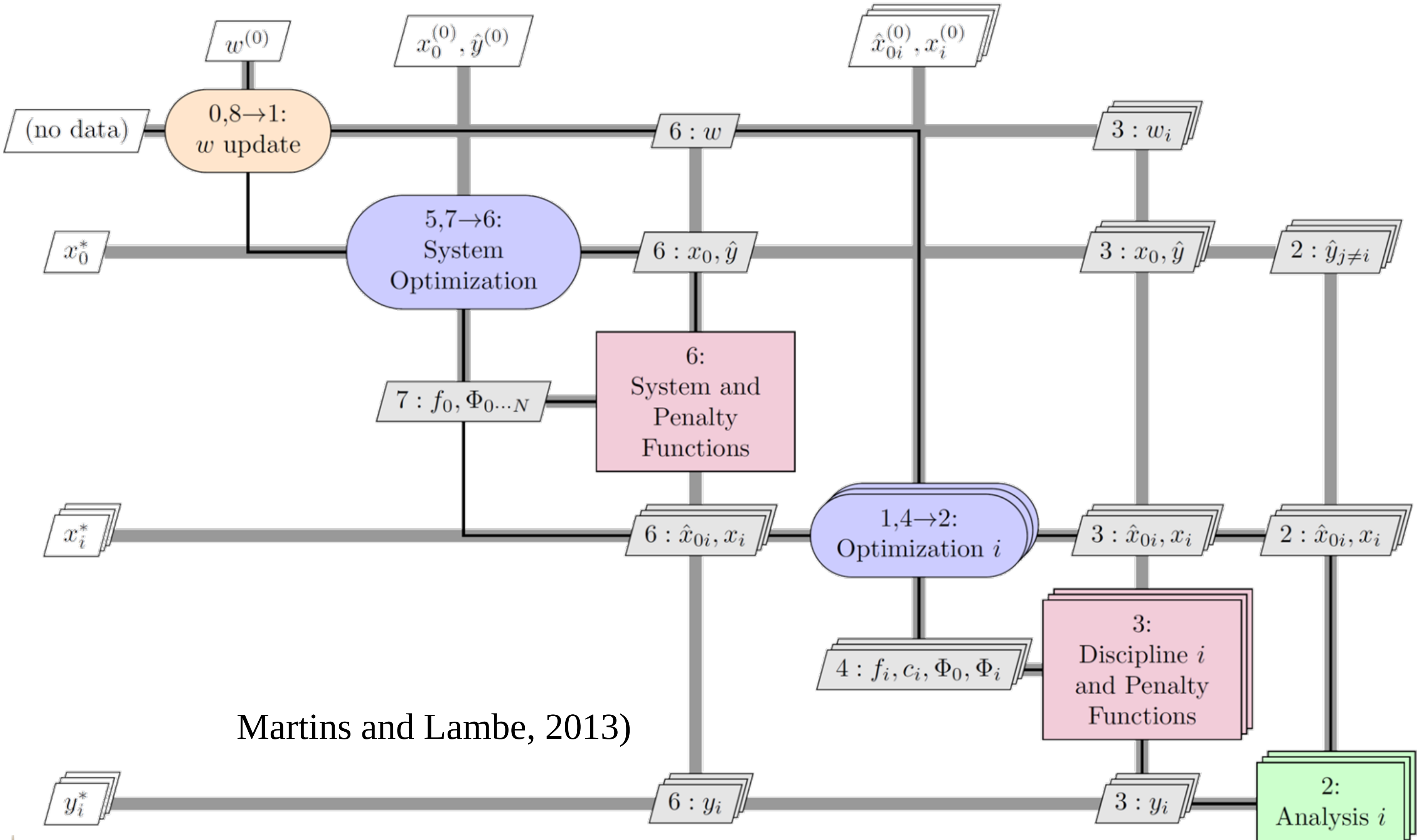
$$x_3 = r_3(x_8, x_9, x_{10}, x_{11}) = \left(x_8^2 + x_9^{-2} + x_{10}^{-2} + x_{11}^2\right)^{1/2}$$

$$x_6 = r_4(x_{11}, x_{12}, x_{13}, x_{14}) = \left(x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2\right)^{1/2}$$

XDSM for CO


$x_0^{(0)}, \hat{x}_{1\cdots N}^{(0)}, \hat{y}^{(0)}$
$\hat{x}_{0i}^{(0)}, x_i^{(0)}$
x_0^*
0, 2→1: System Optimization
$1 : x_0, \hat{x}_{1\cdots N}, \hat{y}$
$1.1 : \hat{y}_{j \neq i}$
$1.2 : x_0, \hat{x}_i, \hat{y}$
$2 : f_0, c_0$
1: System Functions
x_i^*
1.0, 1.3→1.1: Optimization i
$1.1 : \hat{x}_{0i}, x_i$
$1.2 : \hat{x}_{0i}, x_i$
y_i^*
1.1: Analysis i
$1.2 : y_i$
$2 : J_i^*$
$1.3 : f_i, c_i, J_i$
1.2: Discipline i Functions

XDSM for ATC



Martins and Lambe, 2013)