1 Introduction

Buckling

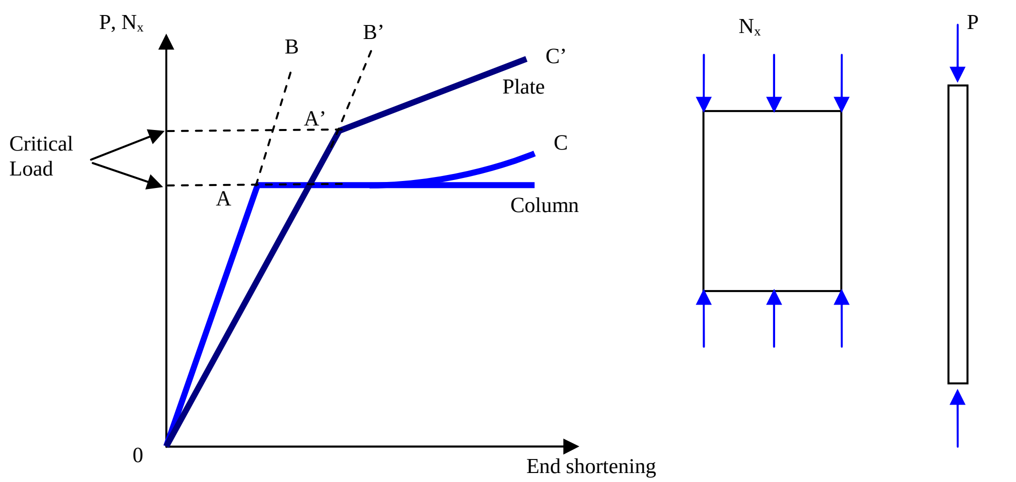
- such kind of structure behavior under load that with a little increase of the load, the deformation is very large
- most aero structures are very thin structures, thus if they will buckle is a big concern

Load Behavior

The critical condition is affected by load behavior. It is not just a function of geometry and material properties.

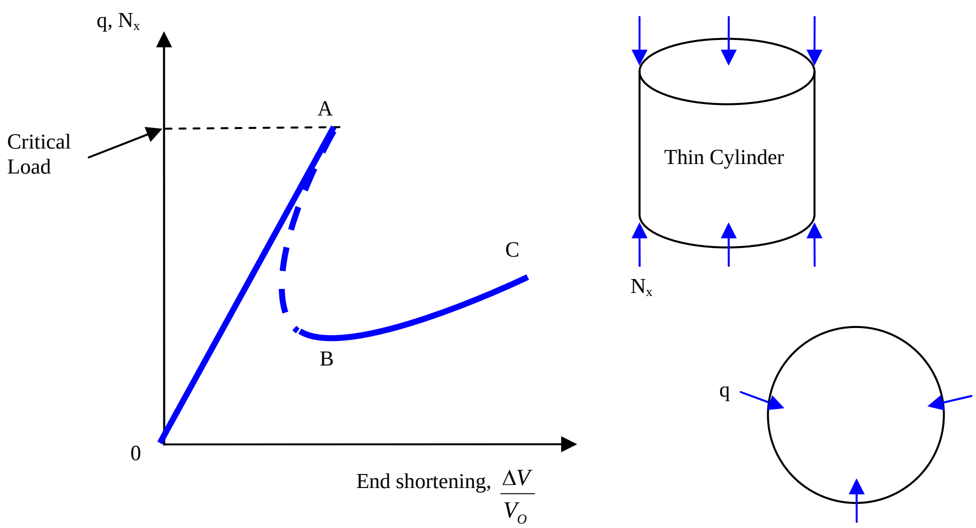
Types of Buckling

- **Classical or Bifurcation Buckling**
  As the load passes through its critical stage, the structure passes from its unbuckled equilibrium configuration to an infinitesimally close buckled equilibrium configuration.



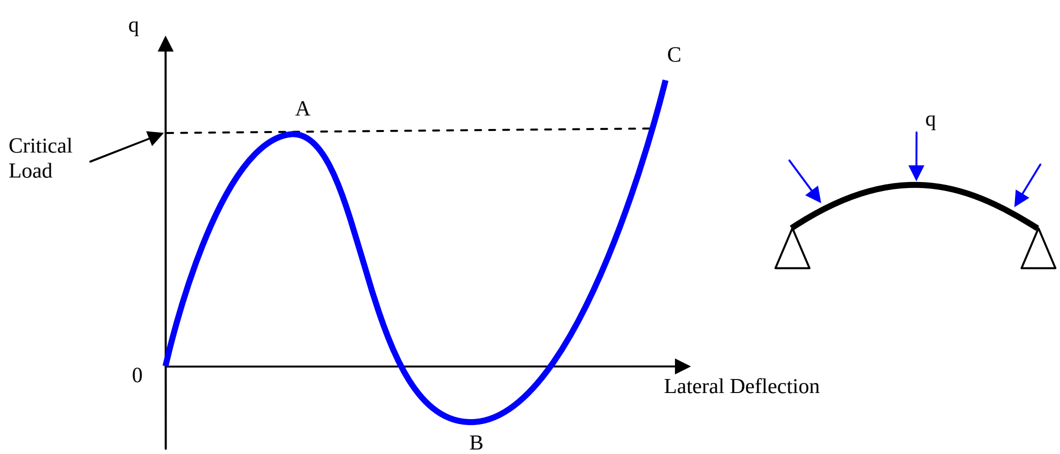
  - **Bifurcation**
  - **Stable**

- **Finite-Disturbance Buckling**
  The loss of stiffness after buckling is so great that the buckled equilibrium configuration can only be maintained by returning to an earlier level of loading.



- **Bifurcation**
- **Unstable**

- **Snap-Through**
  A visible and sudden jump from one equilibrium configuration to another equilibrium configuration for which displacements are larger than in the first nonadjacent equilibrium states



- **No Bifurcation**
- **Unstable**

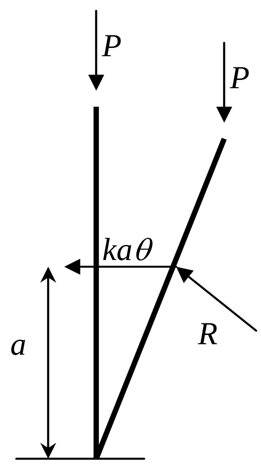
2 Basic Approaches

1. Classical Equilibrium
   - Based on equilibrium equation

- Critical load (point) is major concern
- Eigenvalue problem
- Not always successful with non-conservative

2. Energy or potential –conservative
   - System must be conservative

3. Dynamic (Applicable to most cases)
   - Equation of motion as function of loads
   - Employs periodic function
   - Searching diverging point (value) of w



Classical:

Linearized (θ is small)

Moment about o is zero.

$-Pl\theta + (ka\theta)a = 0$

$(Pl - ka^2)\theta = 0$

$\theta = 0 \quad or \quad Pl = ka^2$

Note: linearization does not give information about the second path.

Nonlinear (θ is small)

$$-Pl\sin\theta + R\frac{a}{\cos\theta} = 0$$

$$-Pl\sin\theta + \frac{ka^2\sin\theta}{\cos^3\theta} = 0$$

$$\theta = 0,\ Pl = \frac{ka^2}{\cos^3\theta}$$

Energy:

Linearized (θ is small)

$$U_T = \frac{1}{2}k(a\theta)^2 - pl(1-\cos\theta)$$

$$\frac{dU_T}{d\theta} = ka^2\theta - pl\sin\theta = (ka^2 - pl)\theta = 0$$

$$\frac{d^2U_T}{d\theta^2} = ka^2 - pl\cos\theta$$

$$\left.\frac{d^2U_T}{d\theta^2}\right|_{\theta=0} = ka^2 - pl$$

$$p < \frac{ka^2}{l} \quad : \quad \left.\frac{d^2U_T}{d\theta^2}\right|_{\theta=0} > 0 \quad \Rightarrow \quad \textit{Stable}$$

$$p > \frac{ka^2}{l} \quad : \quad \left.\frac{d^2U_T}{d\theta^2}\right|_{\theta=0} < 0 \quad \Rightarrow \quad \textit{Unstable}$$

Nonlinear (θ is small)

$$U_T = \frac{1}{2}k(a\tan\theta)^2 - pl(1-\cos\theta)$$

$$\frac{dU_T}{d\theta} = (-pl + \frac{ka^2}{\cos^3\theta})\sin\theta = 0,$$

$$\frac{d^2U_T}{d\theta^2} = (-pl + \frac{ka^2}{\cos^3\theta})\cos\theta + 3ka^2\frac{\sin^2\theta}{\cos^4\theta}$$

For secondary path $P_{eq}l = \dfrac{ka^2}{\cos^3\theta}$ into $\dfrac{d^2U_T}{d\theta^2}$

$$\frac{d^2U_T}{d\theta^2} = 0 + 3ka^2 \frac{\sin^2\theta}{\cos^4\theta} \geq 0; STABLE$$

Dynamic: $I\ddot{\theta} + M = 0$

$$M(\theta) = -pl\sin\theta + \frac{ka^2\sin\theta}{\cos^3\theta} \quad \theta_0; equilibrium, \theta = \theta_0 + \phi$$

$$M(\theta) = M(\theta_0 + \phi) = M(\theta_0) + \phi(\frac{dM}{d\theta})_{\theta=\theta_0} + \frac{1}{2!}\phi^2(\frac{d^2M}{d\theta^2})_{\theta=\theta_0}$$

at equilibrium $M(\theta_0) = 0$

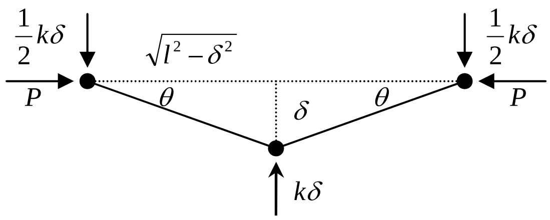
$$\therefore M(\theta) = \phi\left[(-pl + \frac{ka^2}{\cos^3\theta_0})\cos\theta_0 + 3ka^2\frac{\sin^2\theta_0}{\cos^4\theta_0}\right]$$

$$\theta_0 = 0; primary \Rightarrow I\ddot{\phi} + \phi(ka^2 - pl) = 0$$

post-buckled: $I\ddot{\phi} + \phi(3ka^2\frac{\sin^2\theta_0}{\cos^4\theta_0}) = 0$

at critical point: $P_{cr} = \frac{ka^2}{l}, \theta_0 = 0$

$$I\ddot{\phi} + \frac{3}{2}ka^2\phi^3 = 0$$



Energy Equation

$$U_T = \frac{1}{2}k\delta^2 - P(2l - 2\sqrt{l^2-\delta^2})$$

$$\frac{dU_T}{d\delta} = k\delta - P\frac{2\delta}{\sqrt{l^2-\delta^2}} = 0, \ for\ P_{cr} \Rightarrow \delta^2 << l^2,$$

$$\frac{d^2U_T}{d\delta^2} = k - \frac{2P}{\sqrt{l^2-\delta^2}} - \frac{2P\delta^2}{(l^2-\delta^2)^{3/2}}$$

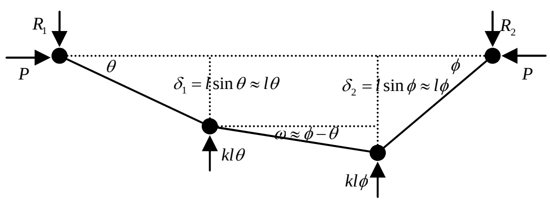
Lagrangian equation of motion

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right)-\frac{\partial K}{\partial q}=-\frac{\partial U}{\partial q}$$

$$K=2\frac{1}{2}I\dot{\theta}^2,$$

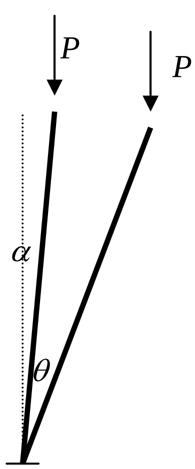
$$U=\frac{1}{2}k\delta^2-P\left[2l-2l\cos\theta\right]$$

$$\approx\frac{1}{2}k(l\theta)^2-P\left[2l-2l(1-\frac{\theta^2}{2})\right]$$



Energy Method

$$U_T=\frac{1}{2}k(l\theta)^2+\frac{1}{2}(kl\phi)^2$$
$$-P(3l-l\cos\theta-l\cos\phi-l\cos(\phi-\theta)\quad \frac{\partial U_T}{\partial\theta}=\frac{\partial U_T}{\partial\phi}=0,\ P=\begin{Bmatrix} kl/3 \\ kl \end{Bmatrix}$$
$$=\frac{1}{2}k(l\theta)^2+\frac{1}{2}(kl\phi)^2-Pl(\theta^2+\phi^2-\phi\theta)$$



$$U_T = \frac{1}{2}C(\theta-\alpha)^2 - Pl(\cos\alpha - \cos\theta)$$

$$\frac{dU_T}{d\theta} = C(\theta-\alpha) - Pl\sin\theta = 0$$

$$\frac{d^2U_T}{d\theta^2} = C - Pl\cos\theta$$

for post-buckled path

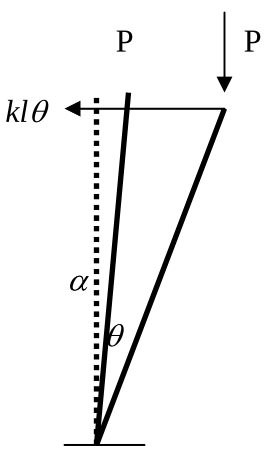
$$\frac{d^2U_T}{d\theta^2} = C\left(1 - \frac{\theta-\alpha}{\tan\theta}\right)$$

set $\frac{d^2U_T}{d\theta^2} = 0$ for P_{cr}

$$\theta_{cr} - \alpha = \tan\theta_{cr} \approx \theta_{cr} + \frac{\theta^3}{3} \Rightarrow \therefore \theta_{cr} \approx (-3\alpha)^{1/3}$$

$$P_{cr} = \frac{C}{l}\frac{(\theta_{cr}-\alpha)}{\sin\theta_{cr}} = \frac{C}{l}\frac{1}{\cos\theta_{cr}} \approx \frac{C}{l}\frac{1}{1-\frac{\theta_{cr}^{\ 2}}{2}}$$

$$\approx \frac{C}{l}\left(1 + \frac{\theta_{cr}^{\ 2}}{2}\right) \approx \frac{C}{l}\left(1 + \frac{3^{2/3}}{2}\alpha^{\frac{2}{3}}\right)$$



$$U_T = \frac{1}{2}k(l\sin\theta - l\sin\alpha)^2 - Pl(\cos\alpha - \cos\theta)$$

$$\frac{dU_T}{d\theta} = kl^2(\sin\theta - \sin\alpha)\cos\theta - pl\sin\theta = 0$$

$$\frac{d^2U_T}{d\theta^2} = kl^2(\cos 2\theta + \sin\alpha\sin\theta) - Pl\cos\theta$$

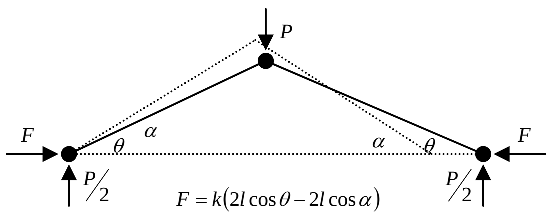
for post-buckled path

$$\frac{d^2U_T}{d\theta^2} = \frac{kl^2}{\sin\theta}\left(\sin\alpha - \sin^3\theta\right)$$

for P_{cr}, $Set\ \frac{d^2U_T}{d\theta^2} = 0 \Rightarrow \therefore \sin\theta_{cr} = \left(\sin\alpha\right)^{\frac{1}{3}}$

$$P_{cr} = kl\left(1 - \left(\sin\alpha\right)^{2/3}\right)^{3/2} \approx kl\left(1 - \left(\alpha\right)^{2/3}\right)^{3/2} \approx kl\left(1 - \frac{3}{2}\left(\alpha\right)^{2/3}\right)$$

Snap-Through



$$U_T = \frac{1}{2}k\left(2l\cos\theta - 2l\cos\alpha\right)^2$$
$$- Pl\left(\sin\alpha - \sin\theta\right)$$

$$\frac{dU_T}{d\theta} = 0 \Rightarrow \frac{P}{4kl} = \sin\theta - \cos\alpha\tan\theta$$

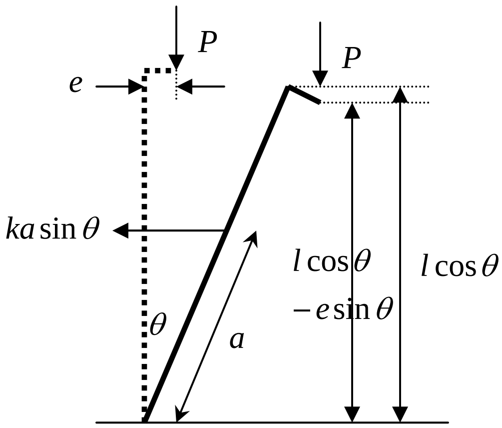
$$\frac{d^2U_T}{d\theta^2} = 4kl^2(-\cos^2\theta + \cos\alpha\cos\theta)$$
$$+ 4kl^2\sin^2\theta - pl\sin\theta$$

for post-buckled path

$$\frac{d^2U_T}{d\theta^2} = 4kl^2\left(\frac{\cos\alpha}{\cos\theta} - \cos^2\theta\right)$$

for critical $\frac{d^2U_T}{d\theta^2} = 0 \Rightarrow \cos^3\theta_{cr} = \cos\alpha$

$$\frac{P_{cr}}{4kl} = \sin\theta_{cr} - \cos\alpha\tan\theta_{cr}$$



$$U_T = \frac{1}{2}k(a\sin\theta)^2 - P[l - (l\cos\theta - e\sin\theta)]$$

$$\frac{\partial U_T}{\partial\theta} = ka^2\sin\theta\cos\theta - Pl\left(\sin\theta + \frac{l}{e}\cos\theta\right) = 0$$

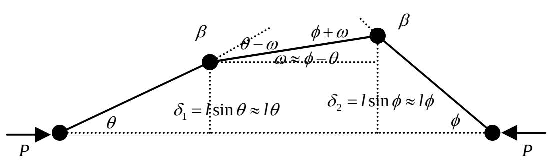
$$\Rightarrow P_{eql} = \frac{ka^2}{l}\frac{\sin\theta}{\tan\theta + e/l}$$

$$\frac{\partial^2 U_T}{\partial\theta^2} = ka^2\frac{\cos^2\theta}{\tan\theta + e/l}\left[-\tan^3\theta + \frac{e}{l}\right]$$

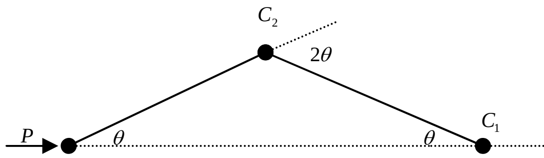
for critical load

$$\frac{\partial^2 U_T}{\partial\theta^2} = 0 \Rightarrow \tan\theta_{cr} = \left(\frac{e}{l}\right)^{\frac{1}{3}}$$

$$P_{cr} = \frac{ka^2}{l}\frac{\sin\theta_{cr}}{\tan\theta_{cr} + e/l} = \frac{ka^2}{l}\left[1 + \left(\frac{e}{l}\right)^{2/3}\right]^{-3/2}$$



$$U_T = \frac{1}{2}\beta(\theta-\omega)^2 + \frac{1}{2}\beta(\phi+\omega)^2$$
$$- Pl[3 - (\cos\theta + \cos\omega + \cos\phi)]$$



$$U_T = \frac{1}{2}C_1\theta^2 + \frac{1}{2}C_2\theta^2 - P(2l - 2l\cos\theta)$$

$$\frac{dU_T}{d\theta} = C_1\theta + 4C_2\theta - 2Pl\sin\theta = 0$$

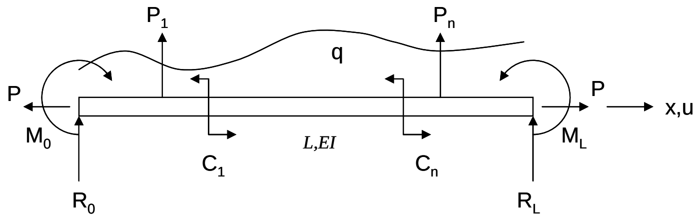
$$\Rightarrow P_{eql} = \frac{C_1 + 4C_2}{2l}\frac{\theta}{\sin\theta}$$

$$P_{cr} = \lim_{\theta\to 0}\frac{C_1 + 4C_2}{2l}\frac{\theta}{\sin\theta} = \frac{C_1 + 4C_2}{2l}$$

$$\frac{d^2U_T}{d\theta^2} = C_1 + 4C_2 - 2Pl\cos\theta = 0$$

for post-buckled path

$$\frac{d^2U_T}{d\theta^2} = C_1 + 4C_2(1 - \frac{\theta}{\tan\theta}) > 0$$



D.E

$P_x = 0 \rightarrow P = const$

$$\left(EIw_{xx}\right)_{xx} - Pw_{xx} = q + \sum_{i=1}^{n} p_i \delta\left(x - x_i\right) - \sum_{j=1}^{m} C_j \eta\left(x - x_j\right)$$

B.C

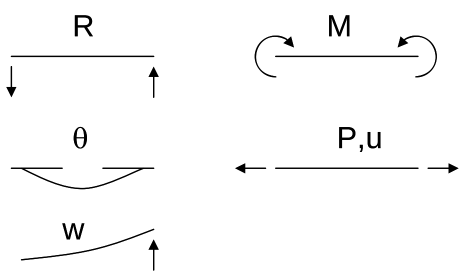
Reaction $\quad Pw_x - \left(EIw_{xx}\right)_x = R$

Moment $\quad EIw_{xx} = M$

Deflection $\quad w =$

Elongation $\quad u =$

Angle $\quad w_x =$



Solution

$w = A_1 \sin kx + A_2 \cos kx + A_3 x + A_4 + w_p$

S-S: $P_{cr} = \dfrac{\pi^2 EI}{L^2}$, Fix-free: $P_{cr} = \dfrac{\pi^2 EI}{4L^2}$

C-C: $P_{cr} = \dfrac{4\pi^2 EI}{L^2}$,

C-S: $P_{cr} = \dfrac{4.49^2 EI}{L^2}$, $\tan 2u - 2u = 0$

Critical load is not depend on the transverse shear load.

Two End Couple

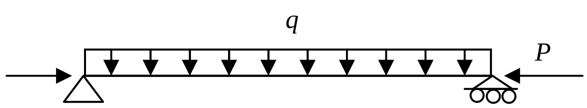
$$\theta_A = \frac{M_A L}{3EI}\Psi(u) + \frac{M_B L}{6EI}\Phi(u)$$

$$\theta_B = \frac{M_B L}{3EI}\Psi(u) + \frac{M_A L}{6EI}\Phi(u)$$

$$\Psi(u) = \frac{3}{2u}\left(\frac{1}{2u} - \frac{1}{\tan 2u}\right)$$

$$\Phi(u) = \frac{3}{u}\left(\frac{1}{\sin 2u} - \frac{1}{2u}\right) \text{ where } u = \frac{kL}{2}$$

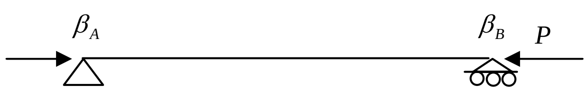
$$P = 0 \Rightarrow u = 0 \Rightarrow \Phi(0), \Psi(0) = 1$$



$$\theta_A = \theta_B = \frac{9L^3}{24EI}\chi(u), \chi(u) = \frac{3(\tan u - u)}{u^3}$$

2nd order differential equation

$$EIW_{,xx} + PW = -R_0 x + M_0$$



Symmetric Buckling ($\beta_A = \beta_B$)

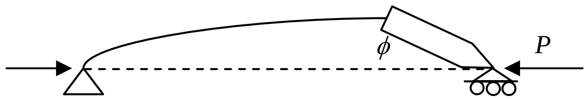
$$\frac{\tan u}{u} = -\frac{2EI}{\beta L}$$

Anti-symmetric Buckling ($\beta_A = \beta_B$)

$$\frac{3}{u}\left(\frac{1}{u} - \frac{1}{\tan u}\right) = -\frac{6EI}{\beta L}$$

General Case ($\beta_A \neq \beta_B$)

$$\left(\frac{1}{\beta_A}+\frac{L}{3EI}\Psi(u)\right)\left(\frac{1}{\beta_B}+\frac{L}{3EI}\Psi(u)\right)-\left(\frac{L}{6EI}\Phi(u)\right)^2=0$$

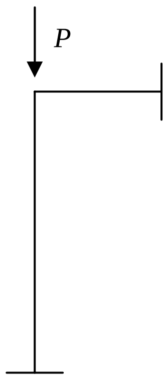


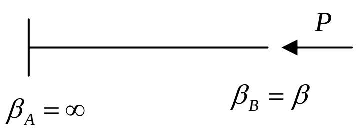
$$\phi \approx \sin\phi = \frac{W(L)}{a} \Rightarrow W_{,x}(L) = -\frac{W(L)}{a}$$

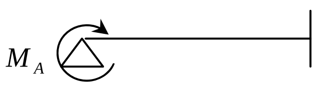
$$M(L)+PW(L)-V(L)a\cos\phi = 0$$

$$EIW_{,xx}(L)+PW(L)+EIW_{,xxx}(L)a+PaW_{,x}(L)=0$$

$$\Rightarrow W_{,xx}(L)+aW_{,xxx}(L)=0$$





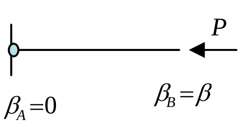


$$\left(\frac{L}{3EI}\Psi(u)\right)\left(\frac{1}{\beta}+\frac{L}{3EI}\Psi(u)\right)-\left(\frac{L}{6EI}\Phi(u)\right)^2=0$$

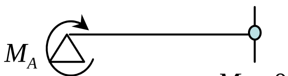
$$\theta_A=\frac{M_A b}{3EI_1}+\frac{M_B b}{6EI_1}$$

$$\theta_B=\frac{M_B b}{3EI_1}+\frac{M_A b}{6EI_1}=0 \Rightarrow M_B=-\frac{1}{2}M_A$$

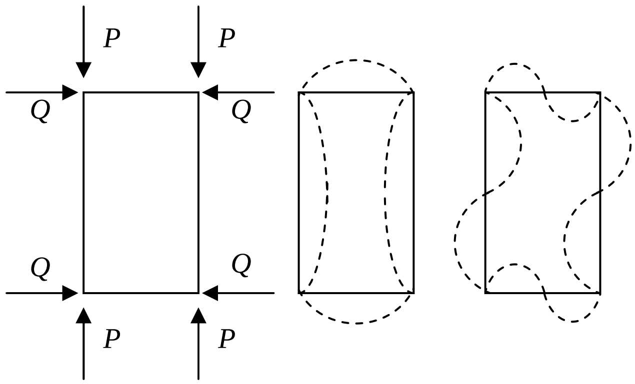
$$\theta_A=\frac{M_A b}{4EI_1} \Rightarrow \beta=\frac{4EI_1}{b}$$



$$\frac{1}{\beta}+\frac{L}{3EI}\Psi(u)=0$$



$$\theta_A=\frac{M_A b}{3EI_1} \Rightarrow \beta=\frac{3EI_1}{b}$$



1) Symmetric (No Q)

$$\frac{\tan u}{u} = -\frac{2EI}{\beta L} \text{ and } \theta_A = \frac{M_o b}{3EI_1} + \frac{M_0 b}{6EI_1} \Rightarrow \beta = \frac{2EI_1}{b}$$

if $EI = EI_1$, b=L, $\dfrac{\tan u}{u} = -1$

2) Anti-symmetric (No Q)

$$\frac{3}{u}\left(\frac{1}{u} - \frac{1}{\tan u}\right) = -\frac{6EI}{\beta L} \text{ and}$$

$$\theta_A = \frac{M_o b}{3EI_1} + \frac{-M_0 b}{6EI_1} \Rightarrow \beta = \frac{6EI_1}{b}$$

3) Symmetric (With Q)

$$\frac{\tan u}{u} = -\frac{2EI}{\beta L} \text{ and}$$

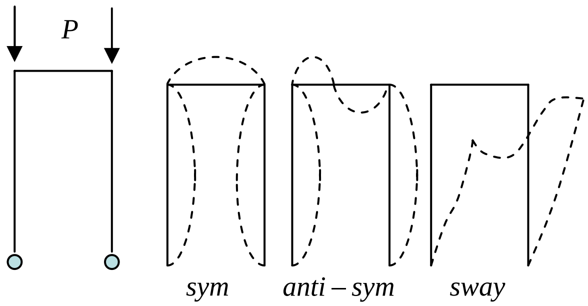
$$\theta_A = \frac{M_O b}{3EI_1}\Psi(u_1) + \frac{M_O b}{6EI_1}\Phi(u_1) = \frac{M_O b}{2EI_1}\frac{\tan u_1}{u_1}$$

$$\beta = \frac{2EI_1}{b}\frac{u_1}{\tan u_1}$$

where $u_1 = \dfrac{k_1 b}{2}, k_1^{\;2} = \dfrac{Q}{EI}$

char. Eqn: $\dfrac{\tan u}{u} = -\dfrac{EIb}{EI_1 L}\dfrac{\tan u_1}{u_1}$

Portal Frame



1)sym

$$\frac{1}{\beta}+\frac{L}{3EI}\Psi(u)=0 \text{ and } \theta_A=\frac{M_o b}{3EI_1}+\frac{M_0 b}{6EI_1}\Rightarrow \beta=\frac{2EI_1}{b}$$

2) anti-sym

$$\frac{1}{\beta}+\frac{L}{3EI}\Psi(u)=0 \text{ and}$$

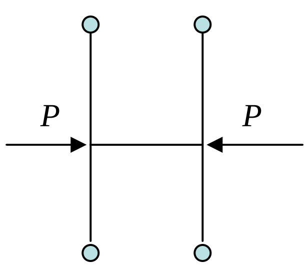
$$\theta_A=\frac{M_o b}{3EI_1}+\frac{-M_0 b}{6EI_1}\Rightarrow \beta=\frac{6EI_1}{b}$$

3) sway

$$-\left(\frac{2u}{L}\right)^6\sin(2u)+\frac{\beta}{EI}\left(\frac{2u}{L}\right)^5\cos(2u)=0 \text{ and } \theta_A=\frac{M_o b}{3EI_1}+\frac{-M_0 b}{6EI_1}\Rightarrow \beta=\frac{6EI_1}{b}$$

4) small modification (pin ·clamped)

$$\tan(2u)=-\frac{2u}{L}\frac{EI}{\beta} \text{ and } \theta_A=\frac{M_o b}{3EI_1}+\frac{-M_0 b}{6EI_1}\Rightarrow \beta=\frac{6EI_1}{b}$$



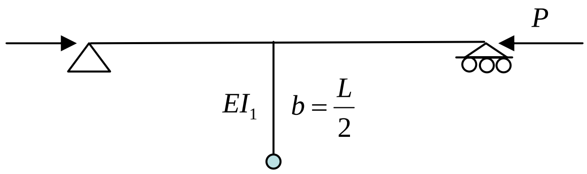
1) sym

$$\frac{\tan u}{u}=-\frac{2EI}{\beta L} \text{ and}$$

$$\theta_B=\frac{\frac{M}{2}L}{3EI}=\frac{ML}{6EI}\Rightarrow \beta=\frac{M}{\theta}=\frac{6EI}{L}$$

2) anti-sym

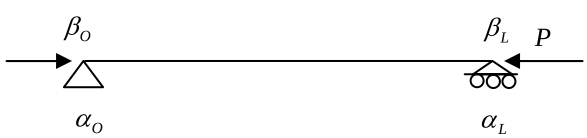
$$\frac{3}{u}\left(\frac{1}{u}-\frac{1}{\tan u}\right)=-\frac{6EI}{\beta L}$$ and same β



$\frac{1}{\beta}+\frac{L}{3EI}\Psi(u)=0$ and

$$\theta_A=\frac{Mb}{3EI_1}\Rightarrow\beta=\frac{M/2}{\theta_A}=\frac{3EI}{2b}=\frac{3EI}{L}$$

char eqn: $1+\Psi(u)=0$



$$\bar{\alpha}_0=\frac{\alpha_0}{EI},\bar{\alpha}_L=\frac{\alpha_L}{EI},\bar{\beta}_0=\frac{\beta_0}{EI},\bar{\beta}_L=\frac{\beta_L}{EI}$$

char eqn.

$$\left[-(\bar{\alpha}_0+\bar{\alpha}_L)\left(\frac{u}{L}\right)^6+\{\bar{\beta}_0\bar{\beta}_L(\bar{\alpha}_0+\bar{\alpha}_L)+\bar{\alpha}_0\bar{\alpha}_L L\}\left(\frac{u}{L}\right)^4+\bar{\alpha}_0\bar{\alpha}_L(\bar{\beta}_0+\bar{\beta}_L-\bar{\beta}_0\bar{\beta}_L L)\left(\frac{u}{L}\right)^2\right]\sin u$$
$$+\left[(\bar{\alpha}_0+\bar{\alpha}_L)(\bar{\beta}_0+\bar{\beta}_L)\left(\frac{u}{L}\right)^5-\bar{\alpha}_0\bar{\alpha}_L L(\bar{\beta}_0+\bar{\beta}_L)\left(\frac{u}{L}\right)^3-2\bar{\alpha}_0\bar{\alpha}_L\bar{\beta}_0\bar{\beta}_L\frac{u}{L}\right]\cos u+2\bar{\alpha}_0\bar{\alpha}_L\bar{\beta}_0\bar{\beta}_L\frac{u}{L}=0$$

$$P_{cr}=\frac{{u_{cr}}^2 EI}{L^2},\ u=KL$$

c-c: $\alpha_0=\alpha_L=\beta_0=\beta_L=\infty$

divide by $\bar{\alpha}_0\bar{\alpha}_L\bar{\beta}_0\bar{\beta}_L$

$$-L\left(\frac{u}{L}\right)^2\sin u-2\left(\frac{u}{L}\right)\cos u+2\left(\frac{u}{L}\right)=0$$

$$\left(\frac{u}{2}\cos\frac{u}{2}-\sin\frac{u}{2}\right)\sin\frac{u}{2}=0$$

$$\therefore\frac{u}{2}=\pi\Rightarrow P_{cr}=\frac{4\pi^2 EI}{L^2}$$

c-free: $\alpha_0 = \infty, \alpha_L = 0, \beta_0 = \infty, \beta_L = 0$

divide by $\overline{\alpha}_0 \overline{\beta}_0$

$$\left(\frac{u}{L}\right)^5 \cos u = 0$$

$$\therefore u = \frac{\pi}{2} \Rightarrow P_{cr} = \frac{\pi^2 EI}{4L^2}$$

c-pin: $\alpha_0 = \alpha_L = \infty, \beta_0 = \infty, \beta_L = 0$

divide by $\overline{\alpha}_0 \overline{\alpha}_L \overline{\beta}_0$

$$\tan u = u$$

$$\therefore u = 4.49 \Rightarrow P_{cr} = \frac{4.49^2 EI}{L^2}$$

Transverse shear

: decreases critical load

$$P_{cr} = \frac{P_E}{1 + \frac{nP_E}{AG}}, \qquad P_E = \frac{\pi^2 EI}{L^2}$$

Engesser formula

$$P_{cr} = \frac{P_E}{1 + \frac{\pi^2 E}{L^2} \frac{wh^3}{12} \frac{1}{whG}} = \frac{P_E}{1 + \frac{\pi^2 h^2 E}{12 L^2 G}}$$

$\frac{E}{G} \uparrow$ → Transverse shear $\uparrow$

$\frac{h}{L} \uparrow$ → Transverse shear $\uparrow$

Part A

1. The use of equilibrium approach can only establish the existence of bifurcation points.
2. The dynamics approach can only be used is the system is conservative.
3. The critical load an imperfect system which is not sensitive to imperfections is not a function of the imperfection amplitude.
4. The critical load for a system which exhibits snap-through buckling can be found by making small displacement approximations, i.e. by neglecting geometric nonlinearities.
5. A system which, when perfect, exhibits unstable bifurcation, is expected to be sensitive to imperfections.
6. The energy approach can only be used if the system is conservative.
7. The dynamic approach can only be sued if the system is conservative.
8. If a system undergoes bifurcational buckling, it is possible to find the critical load by making small displacement approximation.

Part B

1. The effect of transverse shear on column buckling is to increase the critical load.
2. The Southwell plot is used to obtained experimentally the critical load of the perfect system by doing an experiment on an imperfect column.
3. The Superposition Principle on column analysis can be used if the two superposed cases have different end fixities provided they still have the same axial load.
4. The internal members of a frame can be considered as beams with both ends clamped.
5. The critical load of a frame is determined only by the properties of the frame member which is loaded in compression (and no by the rest of the frame).
6. The effect of transverse shear depends on the ratio of extensional to shear modulus, E/G, but it is independent of the thickness over length ratio, i.e. only the material (and not the geometry) plays a role.
7. The critical load of a simply-supported column loaded by an axial load P and a concentrated transverse load, Q, in the middle, is less than the critical load of exactly the same column loaded only by the axial load P.
8. The trial displacement functions in the Rayleigh and Timoshenko quotients should be kinematically admissible.
9. If a frame can buckle in a symmetric mode, this would always be the dominant one and, therefore, there is no need to examine other modes such as antisymmetric.

Part A: 1) F 2) F 3) F 4) F 5) T 6) T 7) F 8) T

Part B: 1) F 2) T 3) F 4) F 5) F 6) F 7) F 8) T 9)F

3 Math

$$\int (a+bx)^n\,dx = \frac{(a+bx)^{n+1}}{b(n+1)}$$

$$\int \frac{1}{a+bx}\,dx = \frac{1}{b}\ln(a+bx)$$

$$\int x^n \ln x\,dx = x^{n+1}\left[\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right]$$

$$\int x e^{ax}\,dx = \frac{e^{ax}(ax-1)}{a^2}$$

$$\int x^2 e^{ax}\,dx = \frac{e^{ax}(a^2x^2-2ax+2)}{a^3}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x \tan y}$$

$$\sin 3x = 3\cos^2 x \sin x - \sin^3 x$$

$$\cos 3x = \cos^3 x - 3\sin^2 x \cos x$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\csc x \big/ \sec x = \cot x = \sin x$$

$$1 \big/ \csc x = \cos x \big/ \cot x = \sin x$$

$$1+\tan^2 x = \sec^2 x,\ \tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}, \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

$$\cos x \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\sin x = \frac{\tan x}{(1+\tan^2 x)^{1/2}}$$

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\cdots$$

$$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\cdots$$

$$\tan x \approx x + \frac{x^3}{3} + \frac{2x^5}{15}\cdots$$

$$\frac{1}{1-x} \approx 1 + x + x^2 + \cdots$$