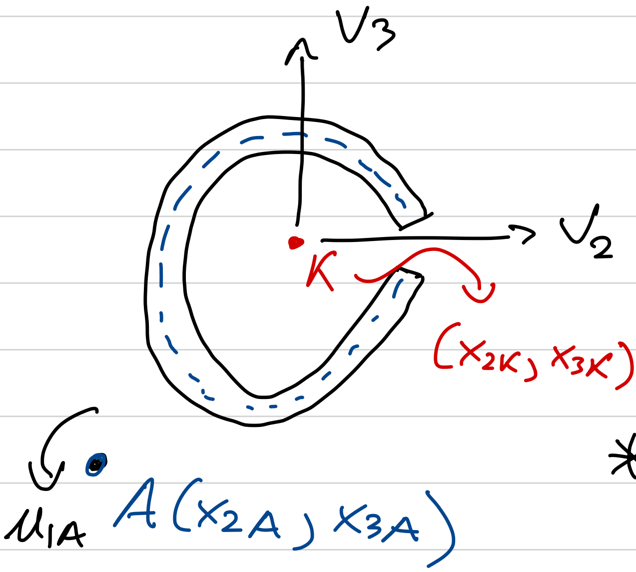
Shear Center



* Assume the beam is subjected to shear forces V_2 and V_3 only ($M_1 = 0$)

* The resulting shear flow should generate no torque.

* The problem above is not well defined, the lines of action for the shear forces are not given.

* The torque generated by the shear flow must vanish when computed w.r.t. the shear center

$$M_{1K} = \int_{C_1'} f \cdot r_k \, ds = 0$$

* A beam bends without twisting, if and only if, the transverse loads are applied about the shear center.

1) Find the location at K by solving

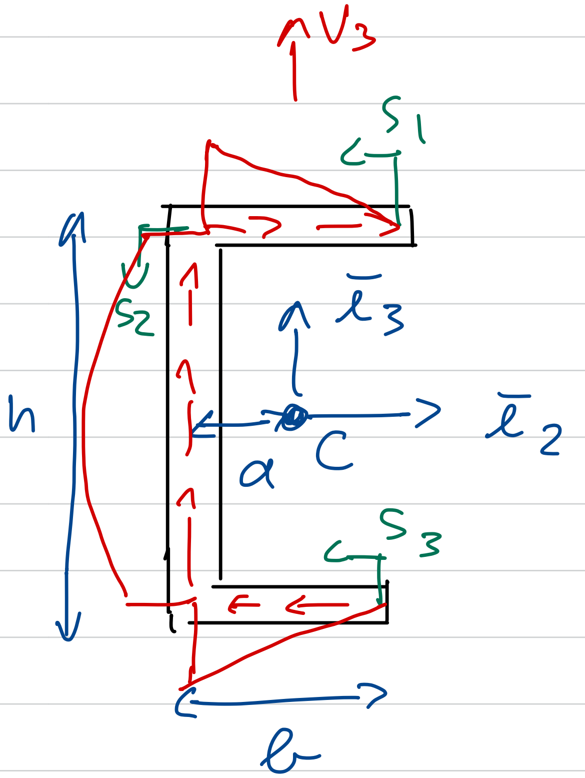
$$\boxed{\mathcal{M}_{1K} = \int_{C_1'} t \, r_t = 0}$$

$\Sigma \mathcal{M}$: $\mathcal{M}_{1K} = \mathcal{M}_{1A} - (x_{2K} - x_{2A}) \cdot V_3$

$$+ (x_{3K} - x_{3A}) V_2 = 0$$

$$\boxed{\mathcal{M}_{1A} = (x_{2K} - x_{2A}) V_3 - (x_{3K} - x_{3A}) V_2}$$

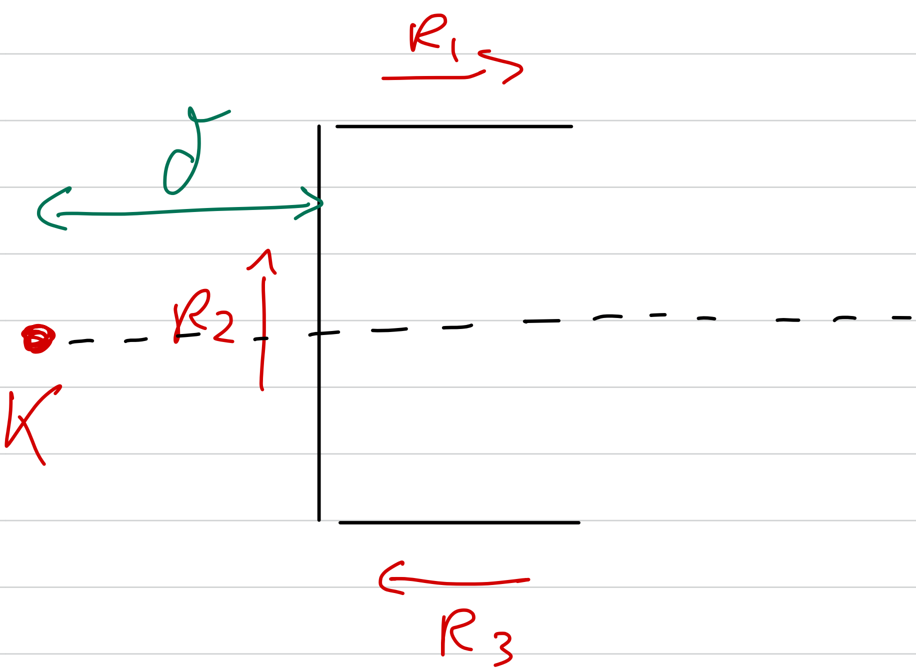
Shear Center in a "C"-channel



$$f(s_1) = -\frac{E t h s_1}{2} \frac{V_3}{H_{22}^c}$$

$$f(s_2) = E t \left(-\frac{b h}{2} - \frac{s_2 h}{2} + \frac{s_2^2}{2} \right) \frac{V_3}{H_{22}^c}$$

$$f(s_3) = \frac{E t h s_3}{2} \frac{V_3}{H_{22}^c}$$



Procedure

1) If there is a symmetry plane the shear center must be on it.

2) Apply a load V_3 only ($V_2 = 0$) solve for x_{2K}

3) Apply a load V_2 only ($V_3 = 0$) solve for x_{3K}.

$$\mathcal{M}_{1K} = \int_0^{b} f(s_1) \cdot \frac{h}{2} \, ds - \int_0^{h} f(s_2) \cdot d \, ds - \int_0^{b} f(s_3) \frac{h}{2} \, ds = 0$$

$$\mathcal{M}_{1K} = -R_1 \frac{h}{2} + R_2 d - R_3 \frac{h}{2} = 0$$

$$d = \frac{\frac{h}{2}(R_1 + R_3)}{R_2}$$

$$R_1 = -\int_0^{b} -\frac{E t h S_1}{2} \frac{V_3}{H_{22}^c}\, ds$$

$$R_1 = \frac{E t h b^2}{4} \frac{V_3}{H_{22}^c}$$

$$R_2 = -\int_0^{h} E t \left(-\frac{b h}{2} - \frac{S_2 h}{2} + \frac{S_2^2}{2} \right) \frac{V_3}{H_{22}^c}\, ds$$

$$R_2 = E t \left(\frac{b h^2}{2} + \frac{h^3}{12} \right) \frac{V_3}{H_{22}^c}$$

$$H_{22}^c = E t \left(\frac{h^3}{12} + \frac{b h^2}{2} \right)$$

$$\boxed{R_2 = V_3}$$

$$R_3 = \int_0^{b} \frac{E t h S_3}{2} \frac{V_3}{H_{22}^c}\, ds$$

$$= \frac{E t h b^2}{4} \frac{V_3}{H_{22}^c}$$

$$\Sigma F_{\bar{\imath}_2} : \quad R_1 - R_3 = 0 \qquad R_1 = R_3$$

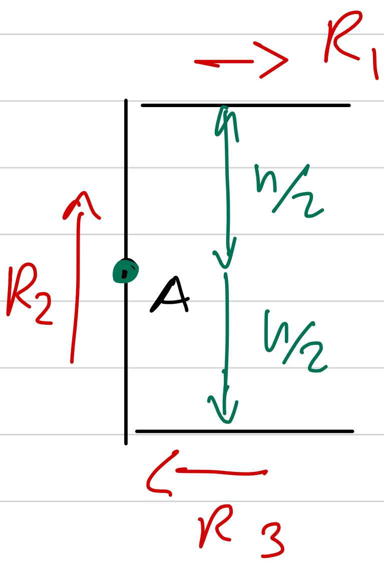
$$\delta = \frac{h}{2}\,\frac{(R_1 + R_3)}{R_2}$$

$$= \frac{h}{2}\,\frac{1}{2}\,E\,t\,h\,\ell^2\,\frac{V^3}{H^c_{22}}\cdot\frac{1}{V_3}$$

$$= \frac{1}{4}\,\frac{E\,t\,h^2\ell^2}{H^c_{22}}$$

$$\boxed{\delta = \frac{1}{4}\,\frac{h^2\ell^2}{\left(\frac{h^3}{12} + \frac{\ell h^2}{2}\right)}} = \frac{3\,\ell}{h/\ell + 6}$$

Approch #2:



$$M_{1A} = -\frac{h}{2}R_1 - \frac{h}{2}R_3$$

$$-\frac{h}{2}(R_1 + R_2) = \underbrace{(x_{2K} - x_{2A})}_{\delta}V_3$$

$$\delta = -\frac{h}{2}\,\frac{(R_1 + R_2)}{V_3}$$