Aircraft Flight Dynamics, Stability, and Control

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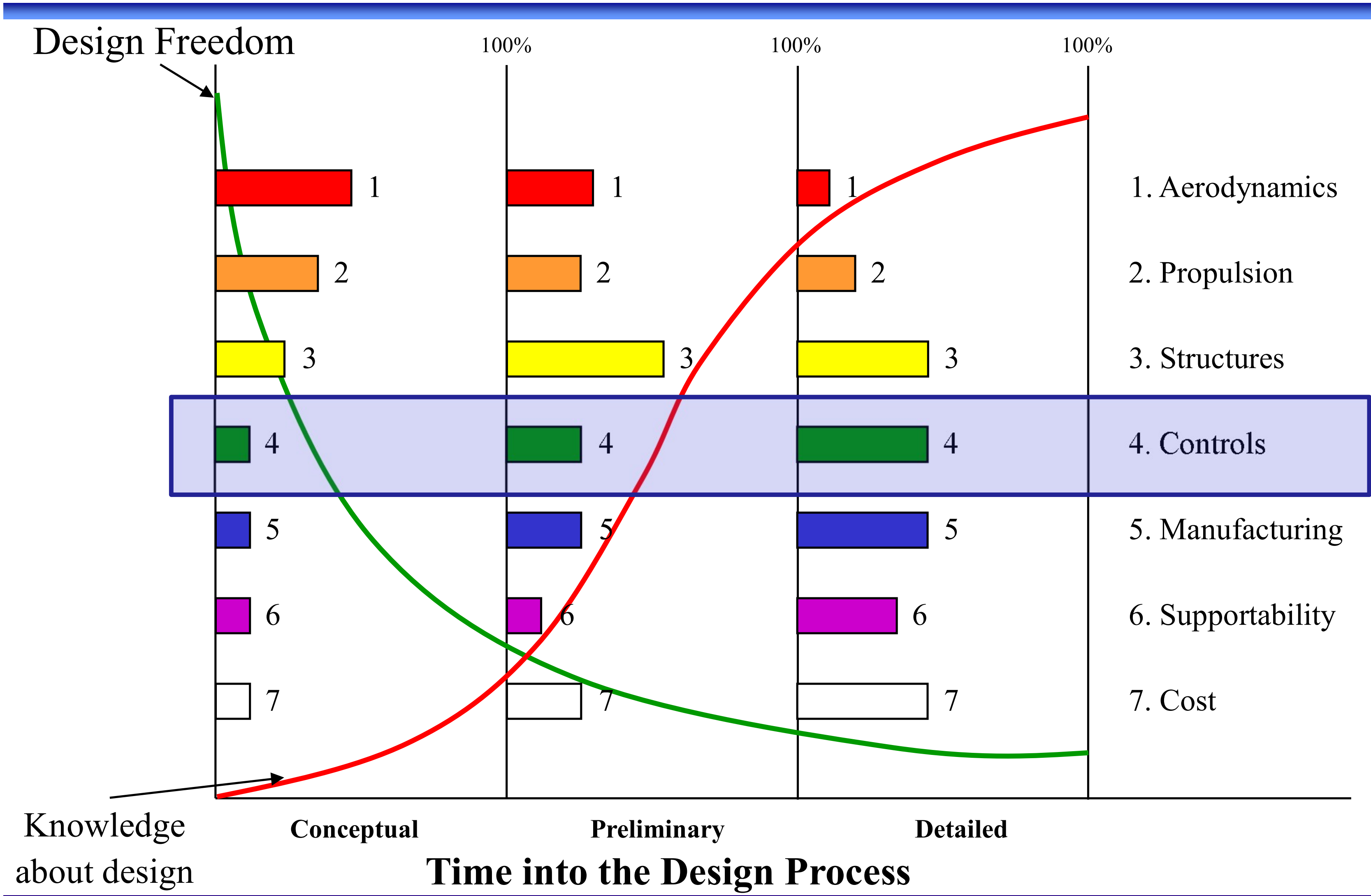
Uneven Distribution of Knowledge Effects



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Atmospheric Flight Mechanics

Atmospheric Flight Mechanics
encompasses three major disciplines

Performance

- Determination of performance characteristics
- Range, endurance, rate of climb, takeoff & landing distances, fuel burn, flight path optimization, etc.
- Point mass assumption, with focus on L, D, T, W forces

Flight Dynamics

- Analysis of motion due to internal or external disturbances
- Determination of stability and control capabilities
- Requires complete equations of motion (6-DoF for rigid body)

Aeroelasticity

- Analysis of phenomena associated with interactions between inertial, elastic, and aerodynamic forces
- Deals with both static and dynamic aeroelastic phenomena

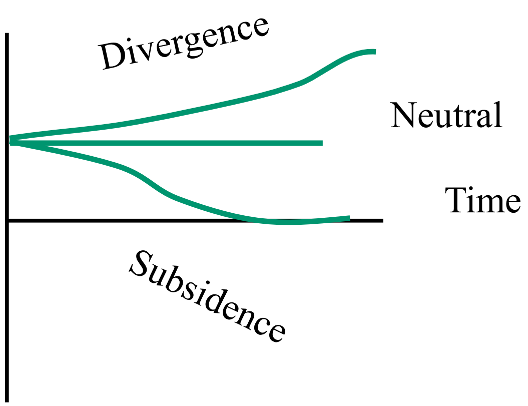
Nelson, R.C., "Flight Stability and Automatic Control", 2nd Ed., McGraw Hill, 1997

Stability & Control (or Versus?)

- Stability is defined as the aircraft's tendency to return to its equilibrium position after it has been disturbed either by pilot action or atmospheric phenomena

- **Static stability** is the initial tendency to return to original equilibrium state
  - Statically stable, statically unstable, neutrally stable

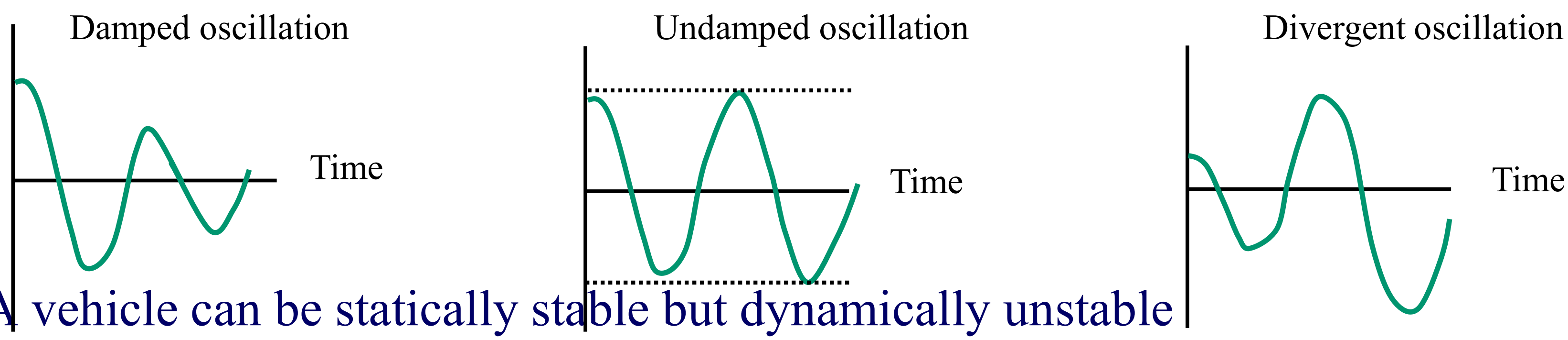
- **Dynamic stability** deals with the time history of the motion of the vehicle after it is disturbed from the equilibrium point
  - Non-oscillatory motion: subsidence, divergence, neutral



Nelson, R.C., "Flight Stability and Automatic Control", 2nd Ed., McGraw Hill, 1997

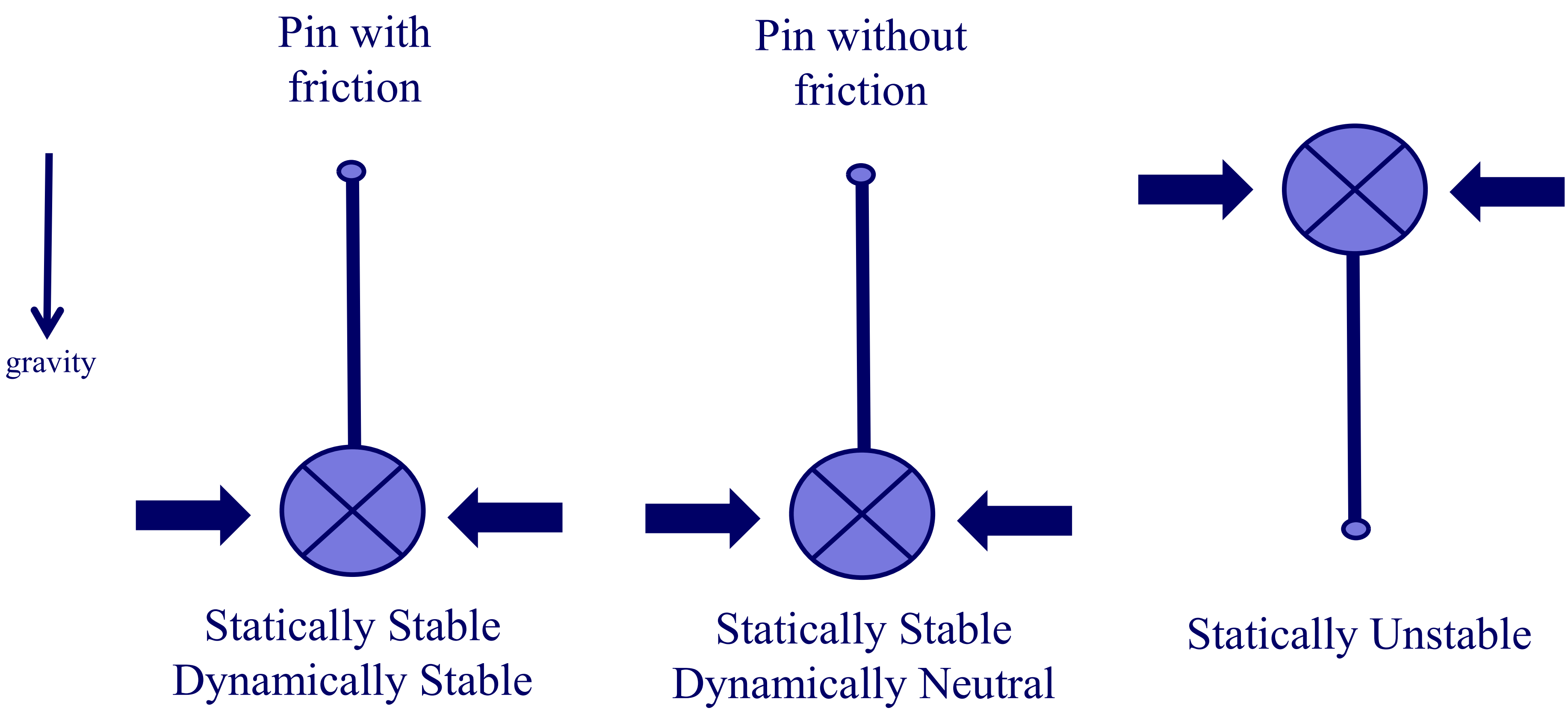
Stability & Control (or Versus?)

- Dynamic stability often manifests as oscillatory motion
  - Damped, undamped, divergent

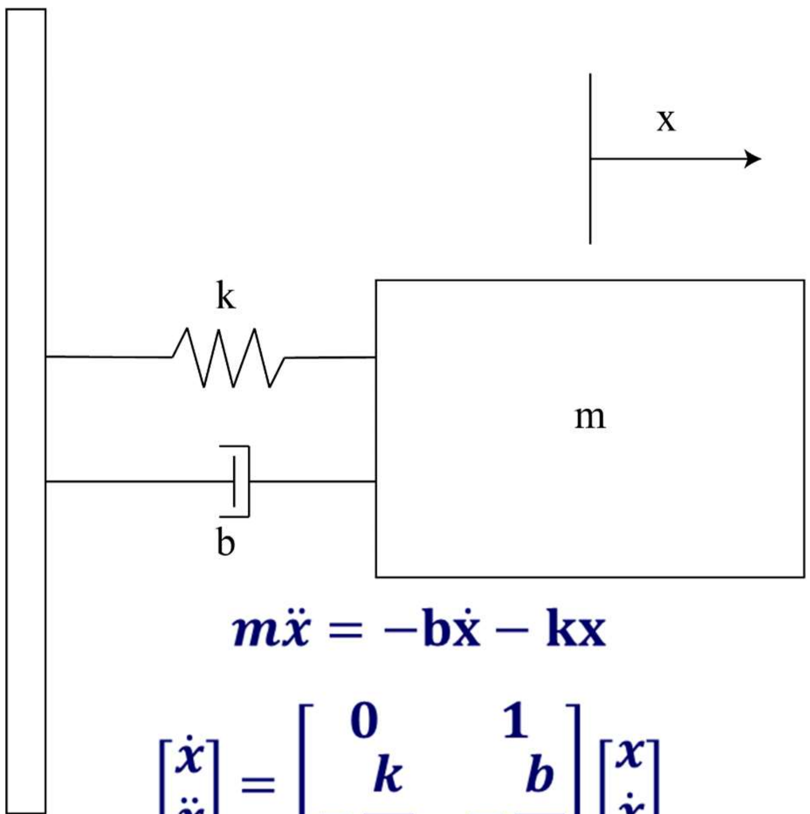


- A vehicle can be statically stable but dynamically unstable
  - Static stability does not guarantee dynamic stability
- However, for a vehicle to be dynamically stable, it must also be statically stable
- **Control** is defined as the ability of the aircraft to be maneuvered as desired from one flight condition to another

Stability: Pendulum Example



Stability: Mass-Spring-Damper



$$m\ddot{x} = -\mathbf{b}\dot{\mathbf{x}} - \mathbf{kx}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\dot{X} = AX$$

- Behavior can be simply described with Newton's 2nd law
  - 2nd order ODE
- System is **autonomous**, **linear**, and **time-invariant**
- Is this system statically and dynamically stable?
  - How does the stability change as the system parameters change?

Eigenvalues and Eigenvectors

- Consider the eigenproblem for the system
$$\dot{\boldsymbol{X}} = A\,\boldsymbol{X}$$
- The ***eigenvalues*** $\lambda = \{\lambda_1, \dots, \lambda_n\}$ are the roots of the characteristic equation
$$\mathbf{det}(\boldsymbol{\lambda}\,\boldsymbol{I} - \boldsymbol{A}) = \mathbf{0}$$
- The ***eigenvector*** $\boldsymbol{v_i}$ for each eigenvalue $\boldsymbol{\lambda_i}$ is found by solving the equation
$$\boldsymbol{A v_i} = \boldsymbol{\lambda_i v_i}$$
- Eigenvectors are typically normalized to a magnitude of unity
- Eigenvalues and eigenvectors can be real or complex, with implications on dynamic stability

An Example Eigenproblem

Consider a simple 2 x 2 system

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Find eigenvalues of this system

$$(A - \lambda I) = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (2-\lambda)^2 - 1 = 0$$

$$\lambda_1 = 1, \lambda_2 = 3$$

Find eigenvector for $\boldsymbol{\lambda_1 = 1}$

$$Av_1 = \lambda_1 v_1 \rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = 1 \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix}$$

$$a_1 + b_1 = 0 \rightarrow v_1 = \begin{Bmatrix} 0.707 \\ -0.707 \end{Bmatrix}$$

Find eigenvector for $\boldsymbol{\lambda_2 = 3}$

$$Av_2 = \lambda_2 v_2 \rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} a_2 \\ b_2 \end{Bmatrix} = 3 \begin{Bmatrix} a_2 \\ b_2 \end{Bmatrix}$$

$$a_2 - b_2 = 0 \rightarrow v_2 = \begin{Bmatrix} 0.707 \\ 0.707 \end{Bmatrix}$$

Eigenvalues of the MSD System

- Consider the eigenproblem for the system

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

- Construct the characteristic equation:

$$\det\left(\begin{bmatrix} \lambda & -1 \\ \frac{k}{m} & \lambda - \frac{b}{m} \end{bmatrix}\right) = 0 \rightarrow \lambda^2 + \frac{b}{m}\lambda + \frac{k}{m} = 0$$

$$\lambda = \frac{-\left(\frac{b}{m}\right) \pm \sqrt{\left(\frac{b}{m}\right)^2 - 4\left(\frac{k}{m}\right)}}{2}$$

- In general the values of λ will be complex numbers of the form

$$\boldsymbol{\lambda = -\sigma \pm j\omega_d}$$

Interpretation of Eigenvalues

- Consider a general complex conjugate eigenvalue pair:
  $\boldsymbol{\lambda = -\sigma \pm j\, \omega_d}$ $\boldsymbol{\sigma}$**:** damping rate $\boldsymbol{\omega_d}$**:** damped natural frequency

- Purely real eigenvalue ($\omega_d = 0$):
  - non-oscillatory mode, damping = σ
  - $\boldsymbol{Re(\lambda) = 0} \rightarrow \sigma = 0$: rigid-body mode, describing rigid-body displacement
  - $\boldsymbol{Re(\lambda) < 0} \rightarrow \sigma > 0$: convergent mode, exponential return to equilibrium
  - $\boldsymbol{Re(\lambda) > 0} \rightarrow \sigma < 0$: divergent mode, exponential deviation from equilibrium

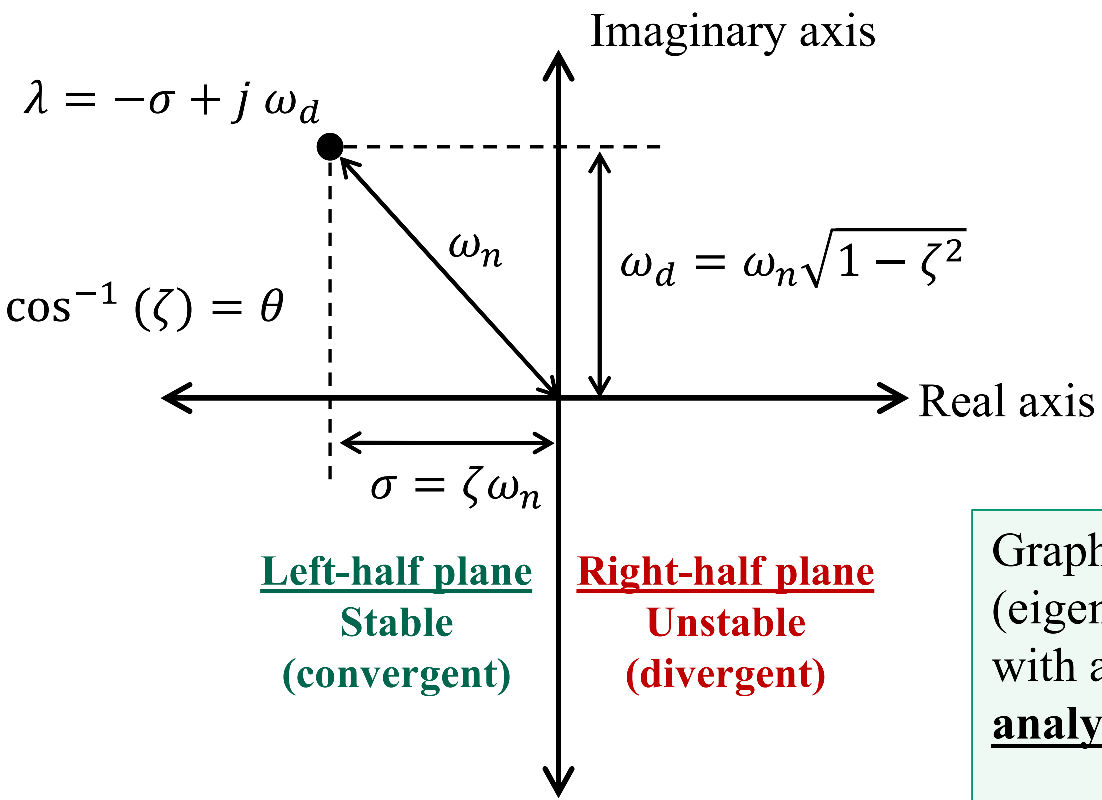
Interpretation of Eigenvalues

- Consider a general complex conjugate eigenvalue pair:
  $\boldsymbol{\lambda = -\sigma \pm j\,\omega_d}$ $\boldsymbol{\sigma}$**:** damping rate $\boldsymbol{\omega_d}$**:** damped natural frequency

- Complex pair of eigenvalues ($\omega_d \neq 0$):
  - oscillatory mode, damping = σ, damped natural frequency = ω_d
  - $\boldsymbol{Re(\lambda) = 0} \rightarrow \sigma = 0$: undamped oscillatory mode
  - $\boldsymbol{Re(\lambda) < 0} \rightarrow \sigma > 0$: damped oscillatory mode
  - $\boldsymbol{Re(\lambda) > 0} \rightarrow \sigma < 0$: divergent oscillatory mode

Interpretation of Eigenvalues

$\boldsymbol{\lambda = -\sigma \pm j\,\omega_d}$ $\boldsymbol{\sigma}$: damping rate $\boldsymbol{\omega_d}$: damped natural frequency



Undamped natural frequency

$$\omega_n = \sqrt{\sigma^2 + \omega_d^2}$$

Damping ratio

$$\zeta = \frac{\sigma}{\sqrt{\sigma^2 + \omega_d^2}}$$

Graphical analysis of how the roots (eigenvalues) of a dynamical system change with a change in gain is called **root locus analysis**.

This type of graphical representation is called a **root locus plot**.

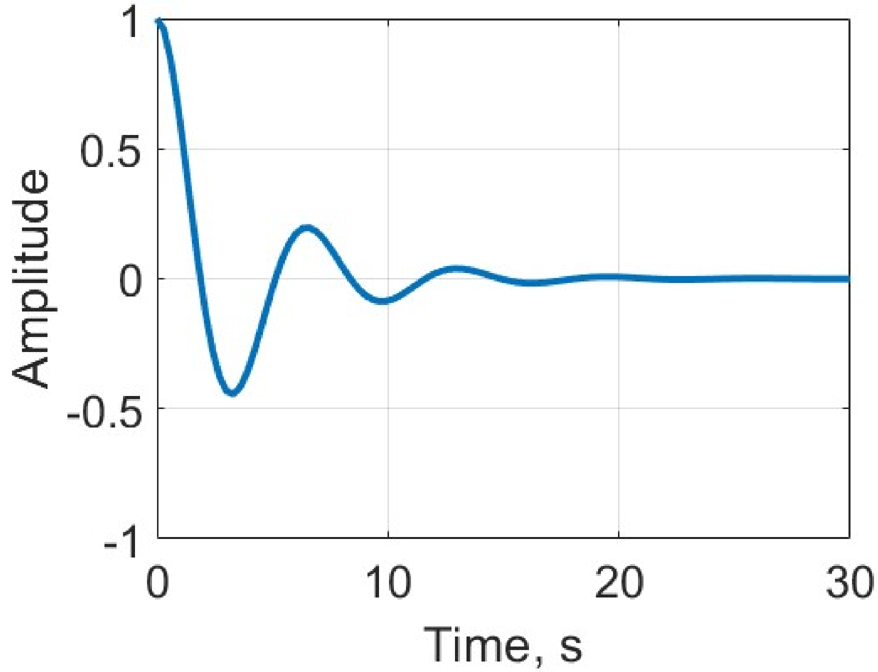
Stability of the MSD

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\boldsymbol{\lambda} = \frac{-\left(\frac{b}{m}\right) \pm \sqrt{\left(\frac{b}{m}\right)^2 - 4\left(\frac{k}{m}\right)}}{2}$$

$$\boldsymbol{\lambda = -\sigma \pm j\omega_d}$$

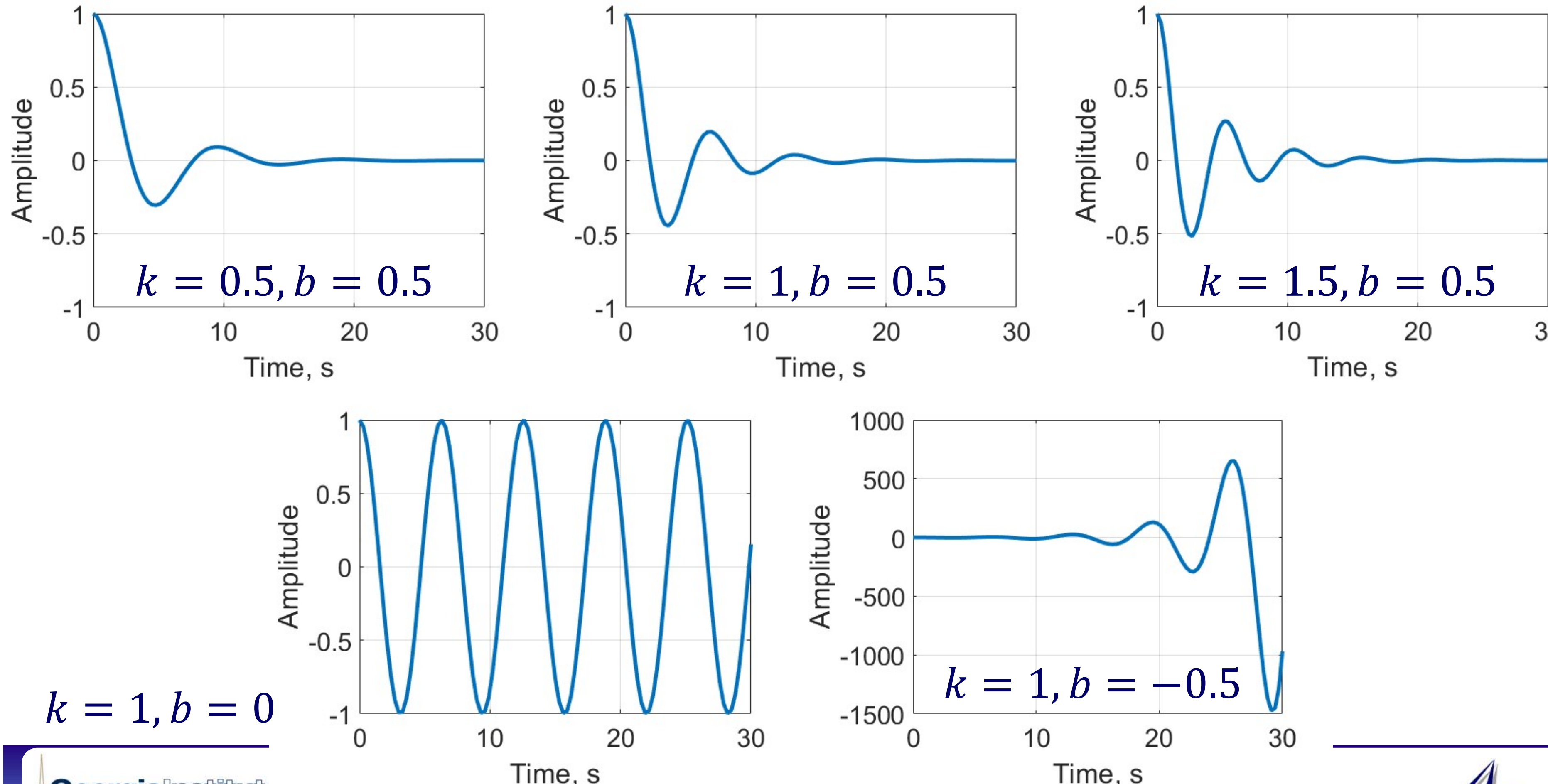
- In general for positive values of k and b of similar magnitude, the system is statically stable and dynamically stable with damped oscillations



$m = 1, k = 1, b = 0.5$

Stability of the MSD

- The behavior of the system is highly dependent upon the value of the **system parameters**



Reference Frames

- A reference frame is an orthonormal set of axes attached to an origin, which may be stationary or moving

- Positions, translation and angular velocities, forces, moments, etc. are vector quantities. A reference frame is needed to represent them with numbers
- An inertial frame of reference is one which is stationary with respect to the distant stars. For such frames, Newton's 2nd Law ($\bar{F} = m\,\bar{a}_c$) and Euler's Law ($\bar{M}_c = \dot{\bar{H}}_c$) can be applied in these stated forms

- In many cases in aircraft dynamics, it is more convenient (or necessary) to use reference frames whose origins (and axes) are fixed relative to the vehicle. These reference frames are not inertial
- As a result, the equations of motion developed using such reference frames have additional terms that account for the movement of the reference frame

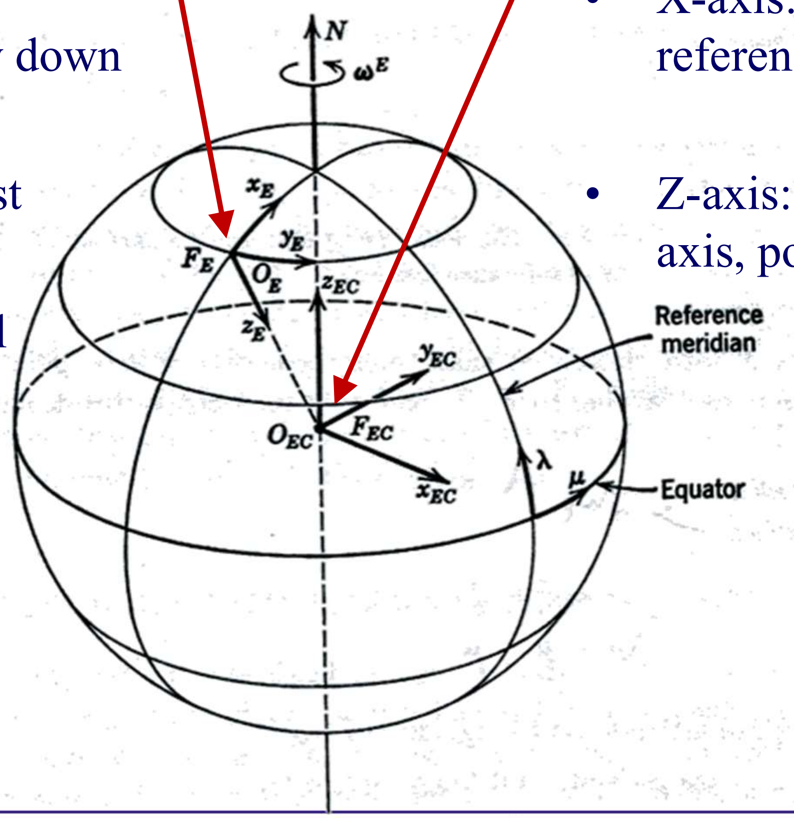
Earth-Fixed Reference Frames

Earth-Surface Earth-Fixed (ESEF)

- Origin: Point on Earth's surface, preferably near vehicle
- Z-axis: Directed vertically down
- X-axis: North, Y-axis: East
- XY plane: local horizontal plane

Earth-Center Earth-Fixed (ECEF)

- Origin: Center of Earth
- X-axis: direction fixed by reference point on equator
- Z-axis: along Earth's rotation axis, pointing North



Etkin, B., "Dynamics of Atmospheric Flight", John Wiley & Sons Inc., 1972

Wind Axes and Body-Fixed Axes

Wind Axes

- Origin: Usually C.G.
- X-axis: along velocity vector relative to atmosphere
- Z-axis: lies on plane of symmetry (if present)

Body-fixed Axes

- Origin: Usually C.G.
- X-axis: longitudinal direction (multiple possibilities)
- Y-axis: rightward

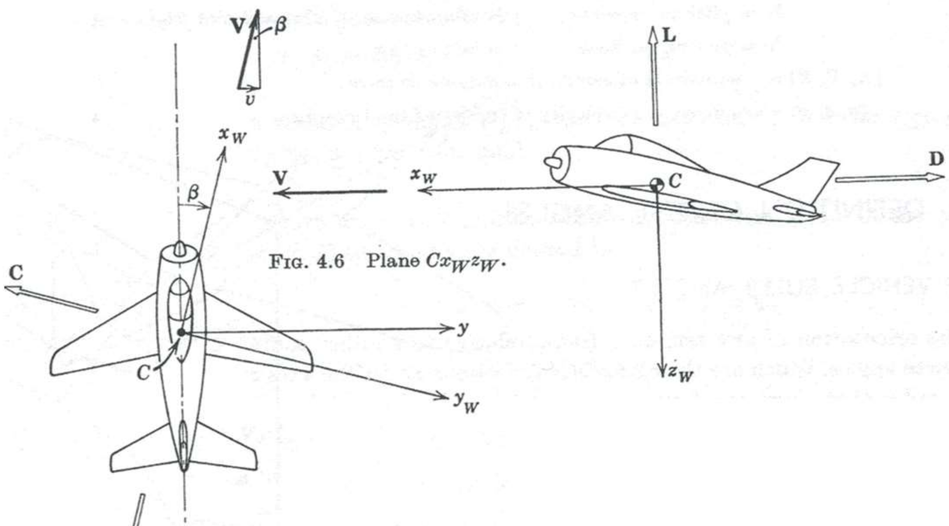


FIG. 4.6 Plane Cx_Wz_W.

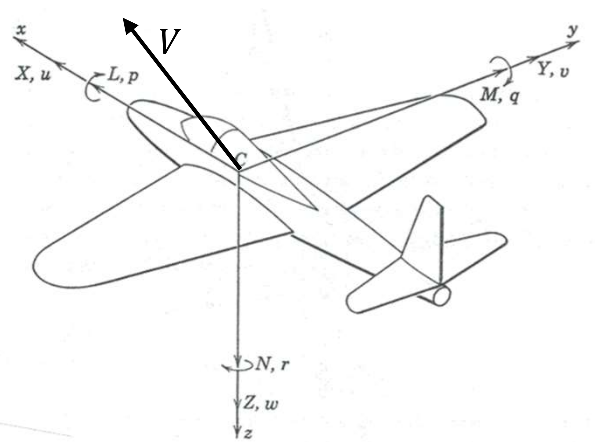


FIG. 4.8 Notation for body axes.

L = rolling moment $\quad$ p = rate of roll
M = pitching moment $\quad$ q = rate of pitch
N = yawing moment $\quad$ r = rate of yaw
$[X, Y, Z]$ = components of resultant aerodynamic force
$[u, v, w]$ = components of velocity of C relative to atmosphere

The angle of attack and angle of sideslip rotate the wind axes into the body-fixed axes or vice versa

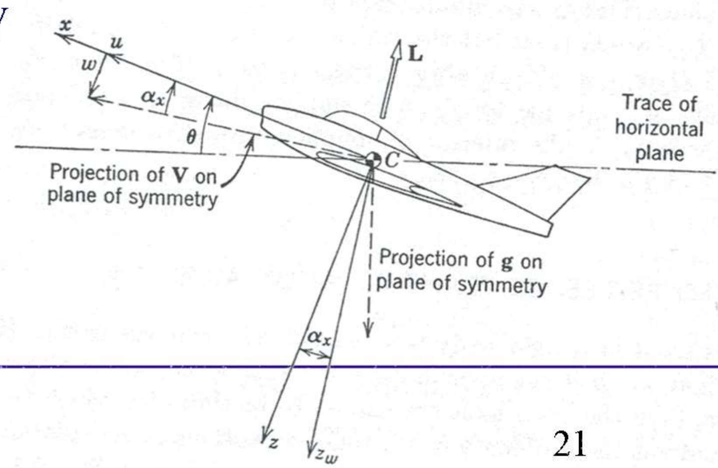
$$V_\infty = \sqrt{u^2 + v^2 + w^2}, \qquad \alpha = \tan^{-1}\left(\frac{w}{u}\right), \qquad \beta = \sin^{-1}\left(\frac{v}{V_\infty}\right)$$

Etkin, B., “Dynamics of Atmospheric Flight”, John Wiley & Sons Inc., 1972

Stability Axes and Principal Axes

Stability Axes

- Special case of body-fixed axes used to study disturbances from reference condition
- For symmetric flight condition, coincides with wind axes in reference condition, departs from it and moves with the body during disturbances
- For asymmetric flight condition, x-axis lies on projection of V_∞ on plane of symmetry, and z-axis also lies on plane of symmetry



Principal Axes

- Body-fixed axes have the advantage that components of the inertia tensor do not change with change in orientation (attitude)
- Principal axes have a further advantage that the inertia tensor is a diagonal matrix
  - Only moments of inertia (diagonal terms) are present
  - All products of inertia (off-diagonal) terms are zero

$$I_c^* = \begin{bmatrix} I_{xx}^* & 0 & 0 \\ 0 & I_{yy}^* & 0 \\ 0 & 0 & I_{zz}^* \end{bmatrix}$$

Rotation Between Reference Frames

- Commonly convenient to work in multiple reference frames
  - e.g: Wind axis convenient for aerodynamic analysis while body axis convenient for kinematic analysis
- Can express vectors generated in one reference frame in a different frame through rotation matrix

$$L_1(X_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos X_1 & \sin X_1 \\ 0 & -\sin X_1 & \cos X_1 \end{bmatrix}$$

$$L_2(X_2) = \begin{bmatrix} \cos X_2 & 0 & -\sin X_2 \\ 0 & 1 & 0 \\ \sin X_2 & 0 & \cos X_2 \end{bmatrix}$$

$$L_3(X_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos X_1 & \sin X_1 \\ 0 & -\sin X_1 & \cos X_1 \end{bmatrix}$$

<u>From vehicle carried frame to body fixed frame</u>

$$x_B = L_{BV} x_V$$

$$L_{BV} = L_1(\phi) \cdot L_2(\theta) \cdot L_3(\psi)$$

$$L_{BV} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix}$$

<u>From wind axes to body-fixed axes</u>

$$x_B = L_{BW} x_W$$

$$L_{BW} = L_2(\alpha_x) \cdot L_3(\beta)$$

$$L_{BW} = \begin{bmatrix} \cos\alpha_x\cos\beta & -\cos\alpha_x\sin\beta & -\sin\alpha_x \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha_x\cos\beta & -\sin\alpha_x\sin\beta & \cos\alpha_x \end{bmatrix}$$

Etkin, B., "Dynamics of Atmospheric Flight", John Wiley & Sons Inc., 1972

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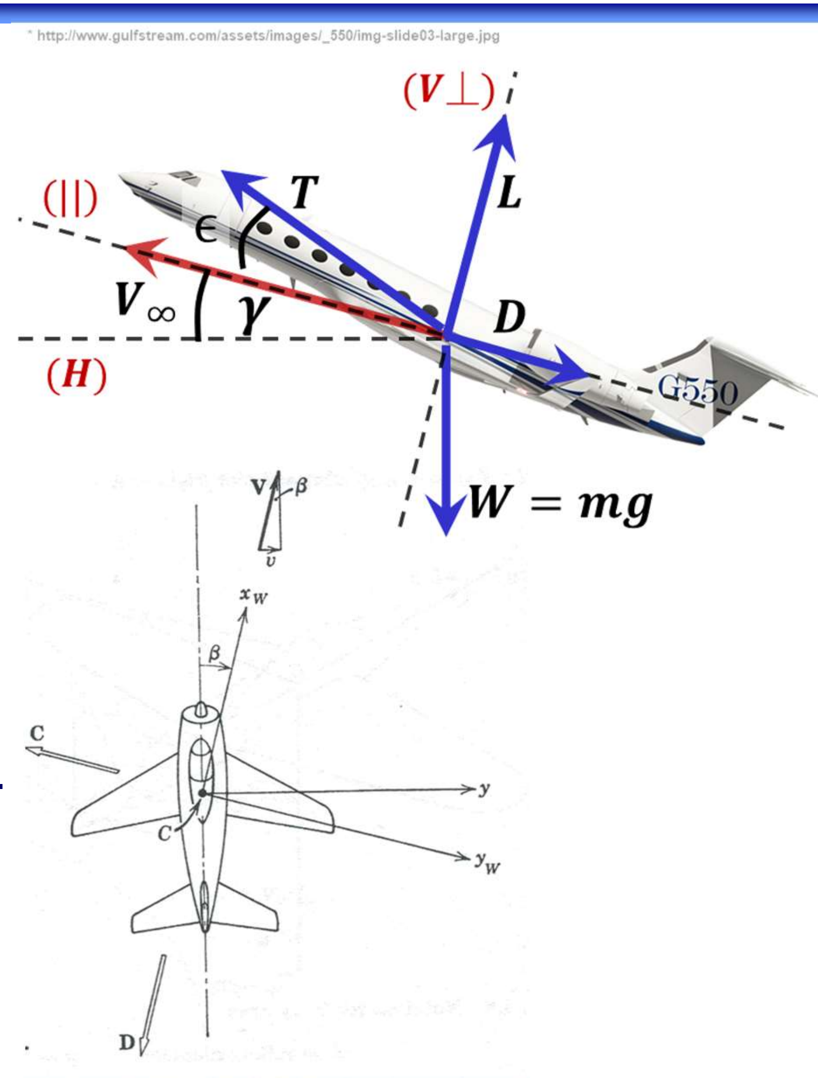
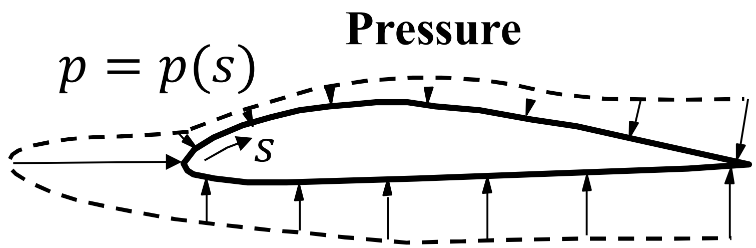
Transitioning to Rigid Body Motion

- Early performance analysis can typically be conducted using a point-mass assumption
  - Total forces are assumed to act at some reference point
- S&C analysis typically involves distributed forces and moments, which prevents the use of the point-mass assumption
- Will also consider the full set of rigid-body degrees of freedom
  - Results in more vehicle states and corresponding equations

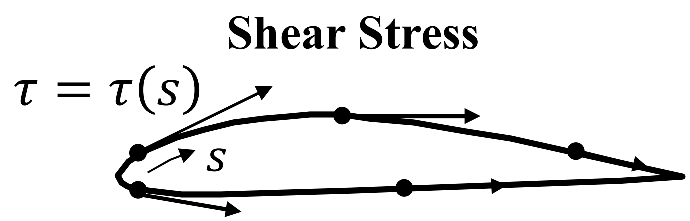


Aerodynamic Forces on a Body

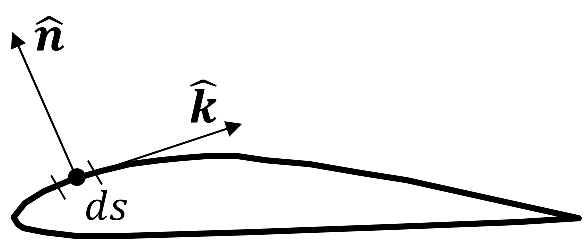
The resultant aerodynamic reaction $\bar{R}$ is the integrated effect of the pressure and shear forces acting on the body moving within the fluid



acts perpendicular to the surface

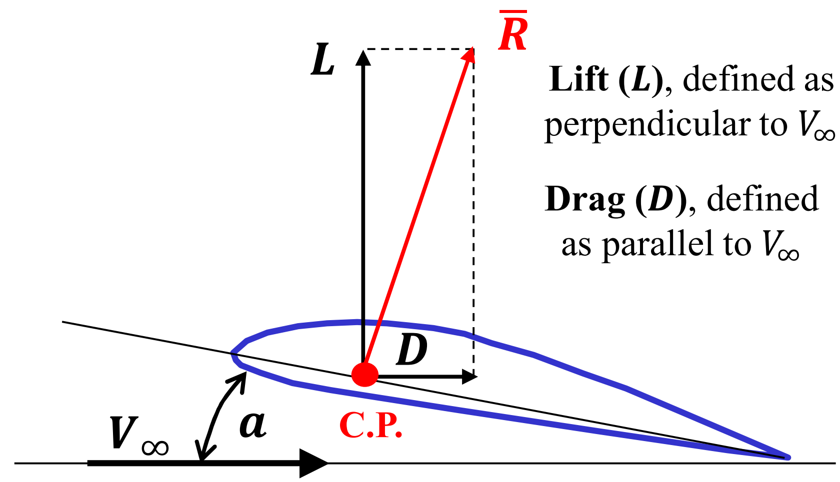


acts parallel to the surface



Integrate $p(s)$ and $\tau(s)$ around the surface of the body to get the total force:

$$\bar{R} = \int\int p(s)\,\hat{n}\,ds + \int\int \tau(s)\,\hat{k}\,ds$$

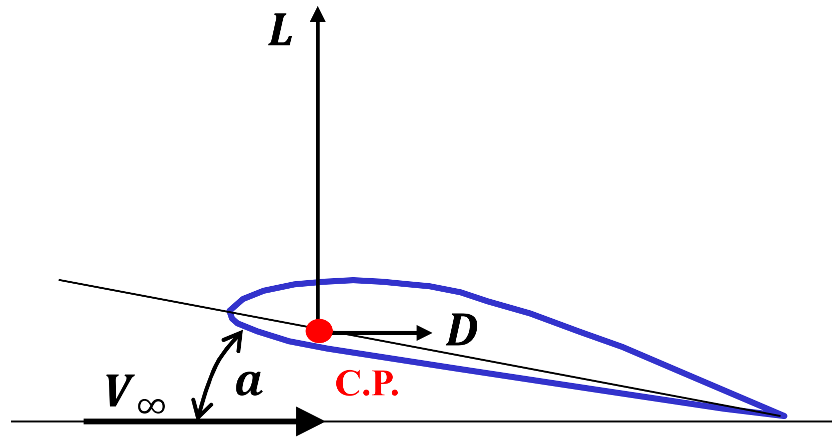


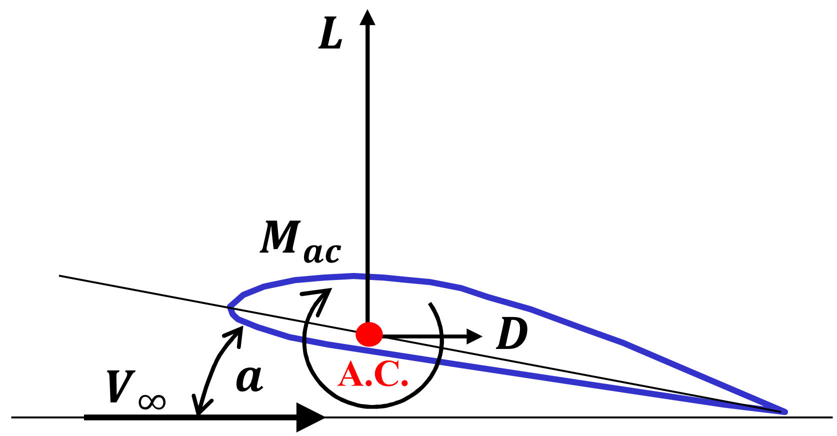
Lift (*L*), defined as perpendicular to V_∞

Drag (*D*), defined as parallel to V_∞

These aerodynamic forces act at the **center of pressure** (the centroid of the distributed load)

Center of Pressure and Aerodynamic Center





The aerodynamic forces act at the **center of pressure** (the centroid of the distributed load)

The net aerodynamic moment taken about this point is zero. However, the point moves with change in angle of attack α

This makes it inconvenient to use the C.P. in flight dynamics analyses, since its movement would have to be tracked

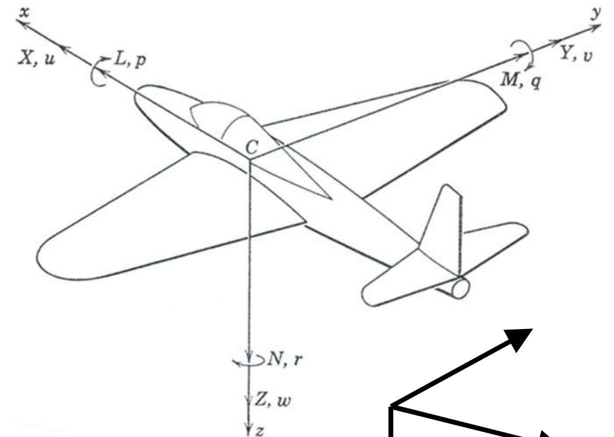
L and D can be transferred to a different point, with a moment added to maintain equivalency

A point called the **aerodynamic center** (A.C.) exists on the airfoil such that the moment M_{ac} about it is invariant with angle of attack α

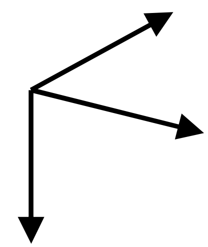
The location of the A.C. can be found experimentally. It is near 25%-chord for subsonic and near 45%-chord for supersonic flow

Kinematics for 6-DoF Rigid Body Aircraft Motion

- The <u>translational velocity</u> can be expressed either in body-fixed axes as $\bar{V}_b = \{u, v, w\}$ or in wind axes as $\bar{V}_w = \{V_\infty, 0{,}0\}$
- The <u>angular velocity</u> can also be expressed in either of these two axes: $\bar{\omega} = \{p, q, r\}$ or $\bar{\omega}_w = \{p_w, q_w, r_w\}$



- The <u>position</u> of the aircraft (x_f, y_f, z_f) is expressed with respect to an axis system fixed to the ground
- The <u>attitude</u> of the aircraft is expressed through the Euler angles (ϕ, θ, ψ) or $(\phi_w, \theta_w, \psi_w)$ that rotate the earth-fixed axes to the body or wind axes



6 x 1st order ODEs

$$\begin{Bmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{z}_f \end{Bmatrix} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$

- Kinematic relationships allow the velocities and rates to be converted to the time rates of position and attitude variables

$$\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & s_\phi s_\theta / c_\theta & c_\phi s_\theta / c_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix} \begin{Bmatrix} p \\ q \\ r \end{Bmatrix}$$

- Integrating these allows the aircraft motion to be "tracked"

The Force Equations

- Newton's 2nd Law, written in vector form for an accelerating frame of reference:

$$\bar{F}_{aero} + \bar{F}_{prop} + \bar{F}_{grav} = m\left(\dot{\bar{V}} + \tilde{\omega}\bar{V}\right)$$

- The gravitational force can be expressed in terms of the Euler angles ϕ, θ, ψ (if wind axes are used, the same relationships are valid with ϕ_w, θ_w, ψ_w

$$\bar{F}_{grav} = mg\,\{-\sin\theta\,, \qquad \sin\phi\cos\theta\,, \qquad \cos\phi\cos\theta\}$$

The Force Equations

- If body-fixed axes are chosen, then $\bar{V} = \{u, v, w\}$, $\bar{F}_{aero} = \{X, Y, Z\}$, $\bar{F}_{prop} = \{T_{x,b}, T_{y,b}, T_{z,b}\}$

$$\begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{Bmatrix} = \frac{1}{m} \begin{Bmatrix} X + T_{x,b} - mg\sin\theta + rv - qw \\ Y + T_{y,b} + mg\sin\phi\cos\theta + pw - ru \\ Z + T_{z,b} + mg\cos\phi\cos\theta + qu - pv \end{Bmatrix}$$

3 x 1st order ODEs

- If wind axes are chosen, then $\bar{V} = \{V_\infty, 0, 0\}$, $\bar{F}_{aero} = -\{D, C, L\}$, $\bar{F}_{prop} = \{T_{x,w}, T_{y,w}, T_{z,w}\}$

$$\begin{aligned} m\dot{V} &= T_{x,w} - D - mg\sin\theta_w \\ mVr_w &= T_{y,w} - C + mg\cos\theta_w\sin\phi_w \\ -mVq_w &= T_{z,w} - L + mg\cos\theta_w\cos\phi_w \end{aligned}$$

The Moment Equations

- The sum of externally applied moments equals the rate of change of angular momentum
  - Angular momentum of the vehicle, expressed in body-fixed axes

$$\bar{H}_C = I_C\bar{\omega} + \sum \bar{h}_r$$

  - $\sum \bar{h}_r$ is the net angular momentum contribution of rotating internal systems, e.g. propellers, jet engines, etc.

- Taking a time derivative, and remembering that the body-fixed axes are not inertial, the moment equations are obtained as

Net moments due to both aero and thrust

$$\begin{Bmatrix} M_{x,b} \\ M_{y,b} \\ M_{z,b} \end{Bmatrix} = I_C\dot{\bar{\omega}} + \dot{I}_C\bar{\omega} + \tilde{\omega} I_C \bar{\omega} + \sum \dot{\bar{h}}_r + \tilde{\omega}\sum \bar{h}_r$$

The Moment Equations

$$\begin{Bmatrix} M_{x,b} \\ M_{y,b} \\ M_{z,b} \end{Bmatrix} = I_C \dot{\bar{\omega}} + \dot{I}_C \bar{\omega} + \tilde{\omega} I_C \bar{\omega} + \sum \dot{\bar{h}}_r + \tilde{\omega} \sum \bar{h}_r$$

- For the restricted case of a rigid body ($\dot{I}_C = 0$) with no rotating internal systems ($\sum \bar{h}_r = \bar{0}$), and with an x-z plane of symmetry ($I_{xy} = I_{yz} = 0$), the moment equations simplify to:

$$\begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{Bmatrix} = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix}^{-1} \begin{Bmatrix} M_{x,b} + (I_{yy} - I_{zz})qr + I_{xz}\, pq \\ M_{y,b} + (I_{zz} - I_{xx})pr + I_{xz}(r^2 - p^2) \\ M_{z,b} + (I_{xx} - I_{yy})pq - I_{xz}\, qr \end{Bmatrix}$$

3 x 1st order ODEs

The Rigid Body 6-DoF Equations of Motion

- The system of 12, coupled, nonlinear 1st order differential equations does not (in general) have analytical solutions, but can be integrated numerically to yield time histories of:

Translational velocity

$$\begin{Bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{Bmatrix} = \frac{1}{m}\begin{Bmatrix} X + T_{x,b} - mg\sin\theta + rv - qw \\ Y + T_{y,b} + mg\sin\phi\cos\theta + pw - ru \\ Z + T_{z,b} + mg\cos\phi\cos\theta + qu - pv\end{Bmatrix}$$

Rotational velocity

$$\begin{Bmatrix}\dot{p}\\ \dot{q}\\ \dot{r}\end{Bmatrix} = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz}\end{bmatrix}^{-1}\begin{Bmatrix} M_{x,b} + \left(I_{yy} - I_{zz}\right)qr + I_{xz}\,pq \\ M_{y,b} + (I_{zz} - I_{xx})pr + I_{xz}(r^2 - p^2) \\ M_{z,b} + \left(I_{xx} - I_{yy}\right)pq - I_{xz}\,qr\end{Bmatrix}$$

Position vector

$$\begin{Bmatrix}\dot{x}_f\\ \dot{y}_f\\ \dot{z}_f\end{Bmatrix} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta\end{bmatrix}\begin{Bmatrix}u\\ v\\ w\end{Bmatrix}$$

Euler angles (attitude)

$$\begin{Bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{Bmatrix} = \begin{bmatrix} 1 & s_\phi s_\theta/c_\theta & c_\phi s_\theta/c_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi/c_\theta & c_\phi/c_\theta\end{bmatrix}\begin{Bmatrix}p\\ q\\ r\end{Bmatrix}$$

The dependency of aircraft dynamics on design and operating parameters is not easily discernable from numerical solutions.

To obtain some closed-form analytical solutions, these equations have to be linearized

Linearization – The Small Disturbance Theory

- Linearization using small disturbance theory requires that the aircraft motion be confined to small deviations about some steady equilibrium condition
- Each variable in the E.o.M. is written as its equilibrium reference state value plus a small disturbance from that state: $X = X_0 + \Delta X$
  - This applies to (1) kinematic variables, (2) forces, and (3) moments
- The forces and moments, which are functions of kinematic variables and control deflections are approximated using **partial derivatives evaluated at the reference flight condition**

$$\Delta M_{x,b} = \begin{bmatrix} \frac{\partial M_{x,b}}{\partial u} & \frac{\partial M_{x,b}}{\partial v} & \frac{\partial M_{x,b}}{\partial w} \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta v \\ \Delta w \end{Bmatrix} + \begin{bmatrix} \frac{\partial M_{x,b}}{\partial p} & \frac{\partial M_{x,b}}{\partial q} & \frac{\partial M_{x,b}}{\partial r} \end{bmatrix} \begin{Bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{Bmatrix}$$

$$+ \begin{bmatrix} \frac{\partial M_{x,b}}{\partial \dot{u}} & \frac{\partial M_{x,b}}{\partial \dot{v}} & \frac{\partial M_{x,b}}{\partial \dot{w}} \end{bmatrix} \begin{Bmatrix} \Delta \dot{u} \\ \Delta \dot{v} \\ \Delta \dot{w} \end{Bmatrix} + \begin{bmatrix} \frac{\partial M_{x,b}}{\partial \delta_a} & \frac{\partial M_{x,b}}{\partial \delta_e} & \frac{\partial M_{x,b}}{\partial \delta_r} \end{bmatrix} \begin{Bmatrix} \Delta \delta_a \\ \Delta \delta_e \\ \Delta \delta_r \end{Bmatrix}$$

Small Disturbance Theory Example

- Start with one of the nonlinear equations

$$\dot{u} = \frac{1}{m}(X + T_{x,b} - mg\sin\theta + rv \; - qw)$$

- Apply perturbated state relationships

$$(\dot{u}_0 + \Delta\dot{u}) = \frac{1}{m}\begin{bmatrix} X + T_{x,b} - mg\sin(\theta_0 + \Delta\theta) \\ +(r_0 + \Delta r)(v_0 + \Delta v) - (q_0 + \Delta q)(w_0 + \Delta w) \end{bmatrix}$$

- Recall that these are small deviations about some steady equilibrium condition
  - Typically choose symmetric rectilinear flight, but this is not a restrictive decision

Small Disturbance Theory Example

- At the reference condition we see

$$\dot{u}_0 = 0 = \frac{1}{m}\left[X_0 + T_{x,b,0} - mg\sin(\theta_0)\right] = \frac{1}{m}\left[F_{x,b,0} - mg\sin\theta_0\right]$$

- The small perturbation case also can be simplified

$$\Delta\dot{u} = \frac{1}{m}\left[F_{x,b} - mg\sin(\theta_0 + \Delta\theta) + \Delta r\Delta v - \Delta q\Delta w\right]$$

- Now we can simplify this further considering the small angle approximation and that the product of small values is negligible

$$\Delta\dot{u} = \frac{1}{m}\left[F_{x,b} - mg\sin\theta_0 + mg\cos\theta_0\,\Delta\theta\right]$$

Small Disturbance Theory Example

- Next a first-order expansion of the force

$$F_{x,b} = F_{x,b,0} + \frac{\partial F_{x,b}}{\partial \dot{u}}\Delta\dot{u} + \frac{\partial F_{x,b}}{\partial \dot{w}}\Delta\dot{w} + \frac{\partial F_{x,b}}{\partial u}\Delta u + \frac{\partial F_{x,b}}{\partial w}\Delta w + \frac{\partial F_{x,b}}{\partial q}\Delta \mathrm{q} + \frac{\partial F_{x,b}}{\partial \delta_e}\Delta\delta_e$$

- Substituting in to the equation of motion and rearranging

$$\left(\mathrm{m} - \frac{\partial F_{x,b}}{\partial \dot{u}}\right)\Delta\dot{u} - \frac{\partial F_{x,b}}{\partial \dot{w}}\Delta\dot{w}$$
$$= \left(F_{x,b,0} - mg\sin\theta_0\right) + \frac{\partial F_{x,b}}{\partial u}\Delta u + \frac{\partial F_{x,b}}{\partial w}\Delta w + \frac{\partial F_{x,b}}{\partial q}\Delta \mathrm{q}$$
$$+ mg\cos\theta_0\,\Delta\theta + \frac{\partial F_{x,b}}{\partial \delta_e}\Delta\delta_e$$

- This is now linear equation that we can rewrite in matrix format

$$\begin{bmatrix} \frac{\mathrm{W}}{\mathrm{g}} - \frac{\partial F_{x,b}}{\partial \dot{u}} & -\frac{\partial F_{x,b}}{\partial \dot{w}} \end{bmatrix} \begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_{x,b}}{\partial u} & \frac{\partial F_{x,b}}{\partial w} & \frac{\partial F_{x,b}}{\partial q} & \mathrm{W}\cos\theta_0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \frac{\partial F_{x,b}}{\partial \delta_e}\Delta\delta_e$$

Linearized Longitudinal and Lateral Equations

- Introducing the derivatives into the E.o.M., applying small angle approximations, and neglecting second order terms, we get
  - The linearized longitudinal equations

$$\begin{bmatrix} W/g - F_{x,b_{\dot{u}}} & -F_{x,b_{\dot{w}}} & 0 & 0 & 0 & 0 \\ -F_{z,b_{\dot{u}}} & W/g - F_{z,b_{\dot{w}}} & 0 & 0 & 0 & 0 \\ -M_{y,b_{\dot{u}}} & -M_{y,b_{\dot{w}}} & I_{yy} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{x}_f \\ \Delta\dot{z}_f \\ \Delta\dot{\theta} \end{Bmatrix} = \begin{bmatrix} F_{x,b_u} & F_{x,b_w} & F_{x,b_q} & 0 & 0 & -W\cos\theta_0 \\ F_{z,b_u} & F_{z,b_w} & F_{z,b_q} + V_0 W/g & 0 & 0 & -W\sin\theta_0 \\ M_{y,b_u} & M_{y,b_w} & M_{y,b_q} & 0 & 0 & 0 \\ \cos\theta_0 & \sin\theta_0 & 0 & 0 & 0 & -V_0\sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 & 0 & 0 & 0 & -V_0\cos\theta_0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta x_f \\ \Delta z_f \\ \Delta\theta \end{Bmatrix} + \begin{Bmatrix} F_{x,b_{\delta_e}} \\ F_{z,b_{\delta_e}} \\ M_{y,b_{\delta_e}} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Delta\delta_e$$

  - The linearized lateral equations

$$\begin{bmatrix} W/g & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{xx} & -I_{xz} & 0 & 0 & 0 \\ 0 & -I_{xz} & I_{zz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{y}_f \\ \Delta\dot{\phi} \\ \Delta\dot{\psi} \end{Bmatrix} = \begin{bmatrix} F_{y,b_v} & F_{y,b_p} & F_{y,b_r} - V_0 W/g & 0 & W\cos\theta_0 & 0 \\ M_{x,b_v} & M_{x,b_p} & M_{x,b_r} & 0 & 0 & 0 \\ M_{z,b_v} & M_{z,b_p} & M_{z,b_r} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & V_0\cos\theta_0 \\ 0 & 1 & \tan\theta_0 & 0 & 0 & 0 \\ 0 & 0 & \sec\theta_0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta y_f \\ \Delta\phi \\ \Delta\psi \end{Bmatrix} + \begin{bmatrix} F_{y,b_{\delta_a}} & F_{y,b_{\delta_r}} \\ M_{x,b_{\delta_a}} & M_{x,b_{\delta_r}} \\ M_{z,b_{\delta_a}} & M_{z,b_{\delta_r}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{Bmatrix}$$

Linearized Equations of Motion

$$\begin{bmatrix} W/g - F_{x,b_{\dot{u}}} & -F_{x,b_{\dot{w}}} & 0 & 0 & 0 & 0 \\ -F_{z,b_{\dot{u}}} & W/g - F_{z,b_{\dot{w}}} & 0 & 0 & 0 & 0 \\ -M_{y,b_{\dot{u}}} & -M_{y,b_{\dot{w}}} & I_{yy} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{x}_f \\ \Delta\dot{z}_f \\ \Delta\dot{\theta} \end{Bmatrix} = \begin{bmatrix} F_{x,b_u} & F_{x,b_w} & F_{x,b_q} & 0 & 0 & -W\cos\theta_0 \\ F_{z,b_u} & F_{z,b_w} & F_{z,b_q} + V_0 W/g & 0 & 0 & -W\sin\theta_0 \\ M_{y,b_u} & M_{y,b_w} & M_{y,b_q} & 0 & 0 & 0 \\ \cos\theta_0 & \sin\theta_0 & 0 & 0 & 0 & -V_0\sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 & 0 & 0 & 0 & -V_0\cos\theta_0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta x_f \\ \Delta z_f \\ \Delta\theta \end{Bmatrix} + \begin{Bmatrix} F_{x,b_{\delta_e}} \\ F_{z,b_{\delta_e}} \\ M_{y,b_{\delta_e}} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Delta\delta_e$$

$$\begin{bmatrix} W/g & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{xx} & -I_{xz} & 0 & 0 & 0 \\ 0 & -I_{xz} & I_{zz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\dot{v} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{y}_f \\ \Delta\dot{\phi} \\ \Delta\dot{\psi} \end{Bmatrix} = \begin{bmatrix} F_{y,b_v} & F_{y,b_p} & F_{y,b_r} - V_0 W/g & 0 & W\cos\theta_0 & 0 \\ M_{x,b_v} & M_{x,b_p} & M_{x,b_r} & 0 & 0 & 0 \\ M_{z,b_v} & M_{z,b_p} & M_{z,b_r} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & V_0\cos\theta_0 \\ 0 & 1 & \tan\theta_0 & 0 & 0 & 0 \\ 0 & 0 & \sec\theta_0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta y_f \\ \Delta\phi \\ \Delta\psi \end{Bmatrix} + \begin{bmatrix} F_{y,b_{\delta_a}} & F_{y,b_{\delta_r}} \\ M_{x,b_{\delta_a}} & M_{x,b_{\delta_r}} \\ M_{z,b_{\delta_a}} & M_{z,b_{\delta_r}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{Bmatrix}$$

- The linearized 6-DoF equations are considerably simpler than the nonlinear 6-DoF ones
  - Valid iff aircraft motion is confined to small deviations about equilibrium condition
  - Valid for most commonly encountered flying conditions
  - Not applicable to spinning motion, stall recovery, or large-amplitude rapid maneuvers

Stability & Control Derivatives

- Traditional flight dynamics discussions will reference a vehicle's ***stability derivatives***

- Stability (and control) derivatives are the partial derivatives evaluated about a reference condition
  – Often expressed with respect to *nondimensional* form of forces or moments

$$\mathrm{M}_{\mathrm{X},\mathrm{u}} = \frac{\partial \mathrm{M}_{\mathrm{X}}}{\partial u} \qquad C_{M_u} = \frac{\partial C_M}{\partial u}$$

Some Key Stability & Control Derivatives

	Lift Force (C_L)	Side Force (C_Y)	Roll Mom. (C_l)	Pitch Mom. (C_m)	Yaw Mom. (C_n)
AOA (α)	$\boldsymbol{C_{L_\alpha}}$			$\boldsymbol{C_{m_\alpha}}$	
AOA rate ($\hat{\dot{\alpha}}$)	$\boldsymbol{C_{L_{\hat{\dot{\alpha}}}}}$			$\boldsymbol{C_{m_{\hat{\dot{\alpha}}}}}$	
Sideslip (β)		$\boldsymbol{C_{Y_\beta}}$	$\boldsymbol{C_{l_\beta}}$		$\boldsymbol{C_{n_\beta}}$
Roll rate ($\hat{p}$)		$\boldsymbol{C_{Y_{\hat{p}}}}$	$\boldsymbol{C_{l_{\hat{p}}}}$		$\boldsymbol{C_{n_{\hat{p}}}}$
Pitch rate ($\hat{q}$)	$\boldsymbol{C_{L_{\hat{q}}}}$			$\boldsymbol{C_{m_{\hat{q}}}}$	
Yaw rate ($\hat{r}$)		$\boldsymbol{C_{Y_{\hat{r}}}}$	$\boldsymbol{C_{l_{\hat{r}}}}$		$\boldsymbol{C_{n_{\hat{r}}}}$
Aileron (δ_a)			$\boldsymbol{C_{l_{\delta_a}}}$		$\boldsymbol{C_{n_{\delta_a}}}$
Elevator (δ_e)	$\boldsymbol{C_{L_{\delta_e}}}$			$\boldsymbol{C_{m_{\delta_e}}}$	
Rudder (δ_r)		$\boldsymbol{C_{Y_{\delta_r}}}$	$\boldsymbol{C_{l_{\delta_r}}}$		$\boldsymbol{C_{n_{\delta_r}}}$

1st or 2nd order "stiffness"

Rate damping derivatives

Control powers

Roll/yaw axis coupling

State-Space Representation of Aircraft Dynamics

- Note that the linearized equations are of the form:

$$\Delta\dot{\boldsymbol{X}} = A\,\Delta\boldsymbol{X} + B\,\Delta\boldsymbol{U}$$

- This is called the state-space representation of a dynamical system, in which
  - $\boldsymbol{X} \in \boldsymbol{R}^{\boldsymbol{n}}$: state vector
  - $\boldsymbol{U} \in \boldsymbol{R}^{\boldsymbol{p}}$: control or input vector
  - $\boldsymbol{A} \in \boldsymbol{R}^{\boldsymbol{n}\,\mathrm{x}\,\boldsymbol{n}}$: state or system matrix
  - $\boldsymbol{B} \in \boldsymbol{R}^{\boldsymbol{n}\,\mathrm{x}\,\boldsymbol{p}}$: control or input matrix

$$\Delta\dot{X}_{\text{long}} = \mathrm{A}_{\text{long}}\Delta X_{\text{long}} + \mathrm{B}_{\text{long}}\Delta U_{long}$$

$$\Delta\dot{X}_{\text{lat}} = \mathrm{A}_{\text{lat}}\Delta X_{\text{lat}} + \mathrm{B}_{\text{lat}}\Delta U_{lat}$$

$$\Delta X_{\text{long}} = \begin{Bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta x_f \\ \Delta z_f \\ \Delta\theta \end{Bmatrix} \qquad \Delta X_{\text{lat}} = \begin{Bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta y_f \\ \Delta\phi \\ \Delta\psi \end{Bmatrix}$$

- To assess the dynamic stability of the uncontrolled (or controls-fixed) system, we set $\Delta\boldsymbol{U} = \overline{\boldsymbol{0}}$, and look at the stability of the system $\Delta\dot{\boldsymbol{X}} = A\,\Delta\boldsymbol{X}$
- This can be done by solving the ***eigenproblem***, which yields the system's eigenvalues and corresponding eigenvectors

$$\Delta U_{long} = \Delta\delta_e$$

$$\Delta U_{lat} = \begin{Bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{Bmatrix}$$

Contents

- Fundamental Concepts and Reference Frames
- Aircraft Equations of Motion
- **Longitudinal Stability**
  - Flying Wing, Wing-Tail, and Canard-Wing Configurations
  - Additional factors affecting longitudinal stability
- Longitudinal Control and Maneuverability
- Lateral/Directional Stability, Control, and Maneuverability
- Stability in Steady Flight
  - Longitudinal Modes
    - Phugoid and Short Period Modes
    - Open Loop Response to Control Inputs
  - Lateral Modes
- Additional Topics

Trim

- When the controls are set so that the resultant forces and the moments about the center of gravity are all zero, the aircraft is said to be in trim, which simply means static equilibrium.

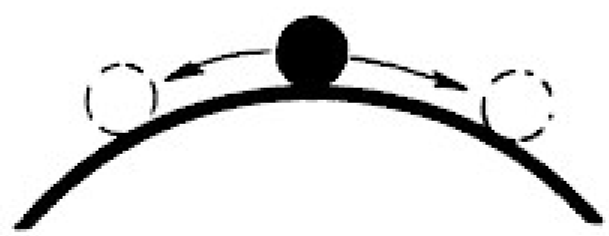
$$\sum F_x = \sum F_y = \sum F_z = 0, \qquad \sum L_{cg} = \sum M_{cg} = \sum N_{cg} = 0$$

- The ease with which the vehicle can be trimmed is an important aspect of the aircraft characteristics known as handling qualities
  - A difficult-to-trim airplane will be tiresome or even dangerous to fly
  - Such an aircraft will never be popular among pilots

Trim

- For an aircraft to be ***statically stable*** in rotation, disturbances in roll, pitch, or yaw axes must result in generation of restoring moments tending to return the aircraft to the original equilibrium (trimmed) attitude

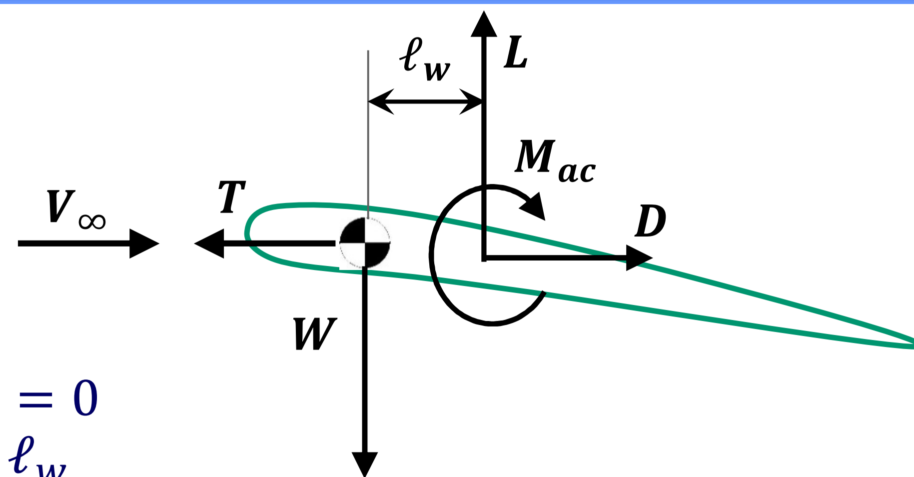
- Note: Simply being able to trim at a flight condition does not imply that the vehicle is dynamically (or even statically) stable



Pitch Trim & Stability of a Flying Wing

- Equilibrium of forces: $T = D, L = W$

- Summation of pitching moments about CG must also be zero:



$$M = M_{ac} - \ell_w L = 0$$
$$\rightarrow C_m = C_{m_{ac}} - \frac{\ell_w}{\bar{c}} C_L = 0$$

- For a given speed, altitude, and weight, $C_L = \frac{W}{\frac{1}{2}\rho V^2 S} > 0$

- Solving for the moment arm, $\ell_w = C_{m_{ac}} \frac{\bar{c}}{C_L}$

- The above is the requirement for ***static equilibrium*** **-** we also need ***static stability***

Pitch Trim & Stability of a Flying Wing

- For static stability, a restoring pitching moment must develop in response to a disturbance in the pitch axis, i.e., an angle of attack disturbance $\Delta\alpha$
  - Restoring means, if $\Delta\alpha > 0$, then $\Delta C_m < 0$. And if $\Delta\alpha < 0$, then $\Delta C_m > 0$
  - Thus, the general pitch static stability criterion for any aircraft: $C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} < 0$

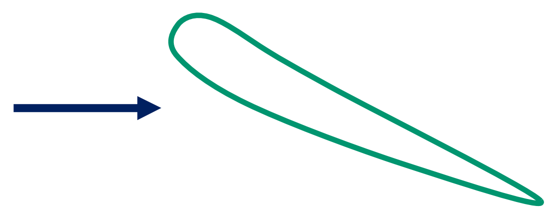
- Recall: $C_m = C_{m_{ac}} - \frac{\ell_w}{\bar{c}} C_L$. Differentiating this w.r.t. α and imposing $\frac{\partial C_m}{\partial \alpha} < 0$

$$\frac{\partial C_m}{\partial \alpha} = \frac{\partial C_{m_{ac}}}{\partial \alpha} - \frac{\ell_w}{\bar{c}} \frac{\partial C_L}{\partial \alpha} < 0$$

- From the definition of aerodynamic center, $\frac{\partial C_{m_{ac}}}{\partial \alpha} = 0$. And recall, $\ell_w = C_{m_{ac}} \frac{\bar{c}}{C_L}$
- Using these, the pitch static stability criterion simplifies to the requirement that:

$$-\frac{C_{m_{ac}}}{C_L} \frac{\partial C_L}{\partial \alpha} < 0 \rightarrow C_{m_{ac}} > 0$$

Pitch Trim & Stability of a Flying Wing

Criterion →	Static equilibrium	Static stability	
Requirement →	$\ell_w = C_{m_{ac}} \frac{\bar{c}}{C_L}$	$C_{m_{ac}} > 0$	
Symmetric **($C_{m_{ac}} = 0$)**	✓ $\ell_w = 0$ (place CG at AC)	✗	
Positive camber **($C_{m_{ac}} < 0$)**	✓ $\ell_w < 0$ (place CG behind AC)	✗	
Negative camber **($C_{m_{ac}} > 0$)**	✓ $\ell_w > 0$ (place CG ahead of AC)	✓	

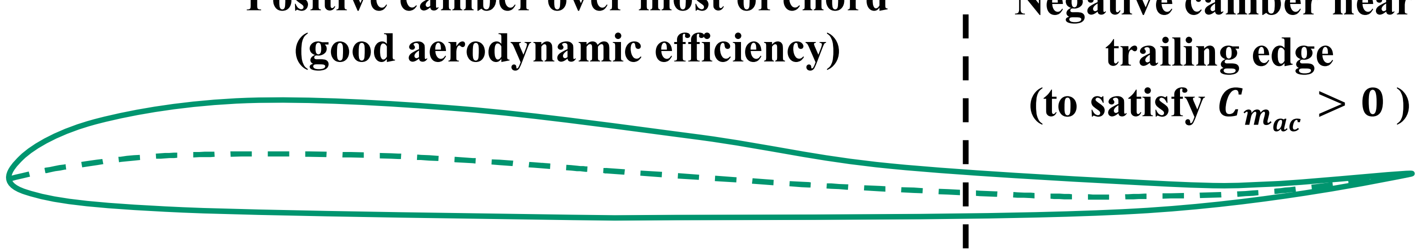
- Negative camber will get the job done, but is quite inefficient
  - Low $C_{L,max}$ and lift-to-drag ratio

Pitch Trim & Stability of a Flying Wing

Criterion →	**Static equilibrium**	**Static stability**
Requirement →	$\ell_w = C_{m_{ac}} \dfrac{\bar{c}}{C_L}$	$C_{m_{ac}} > 0$
Reflexed Trailing Edge ($C_{m_{ac}} > 0$)	✓ $\ell_w > 0$ (place CG ahead of AC)	✓

Positive camber over most of chord (good aerodynamic efficiency)

Negative camber near trailing edge (to satisfy $C_{m_{ac}} > 0$)



While static stability is achievable, pure flying wings usually require some form of stability augmentation to obtain acceptable handling qualities

Pitch Stability of Combined Lifting Surfaces

- The majority of fixed-wing aircraft use multiple lifting surfaces
  - The primary (main) lifting surface is commonly called the wing

- Additional lifting surface(s) contribute to ***pitch stability*** and also allow ***pitch control***
  - A tail, or tail-plane (a.k.a. stabilizer), located aft of the main wing
  - A canard, or fore-plane, located ahead of the main wing
- Thus, wing-tail, canard-wing, and canard-wing-tail configurations are possible

F-16 Fighting Falcon (wing-tail)

https://c1.staticflickr.com/3/2906/13873859334_7106cff425_b.jpg

JAS 39 Gripen (canard-wing)

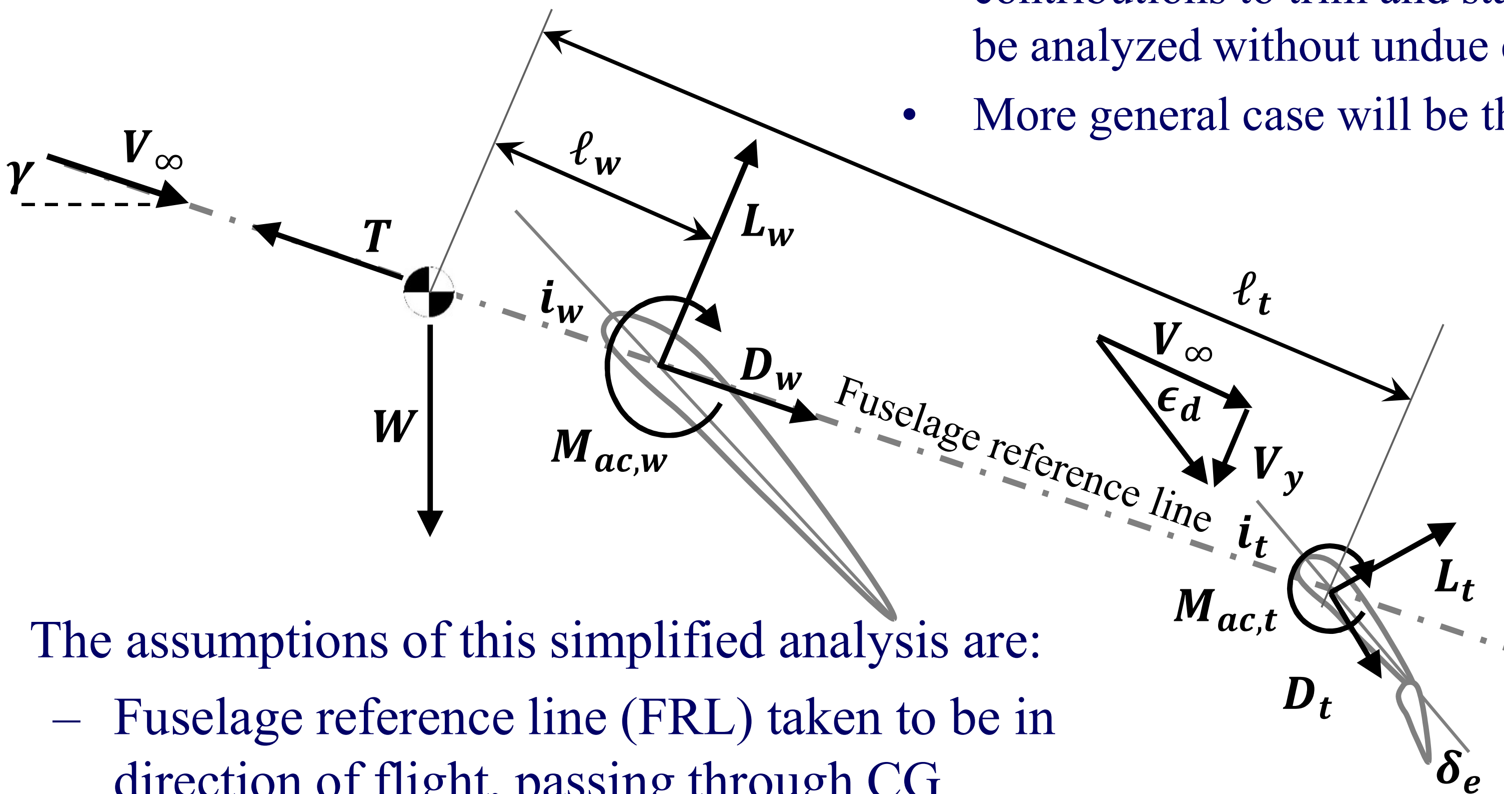
https://en.wikipedia.org/wiki/Canard_(aeronautics)

Sukhoi Su-30 MKI (canard-wing-tail)

https://s-media-cache-ak0.pinimg.com/564x/93/d6/01/93d6012be76c0d0723e6116ad9a42f86.jpg

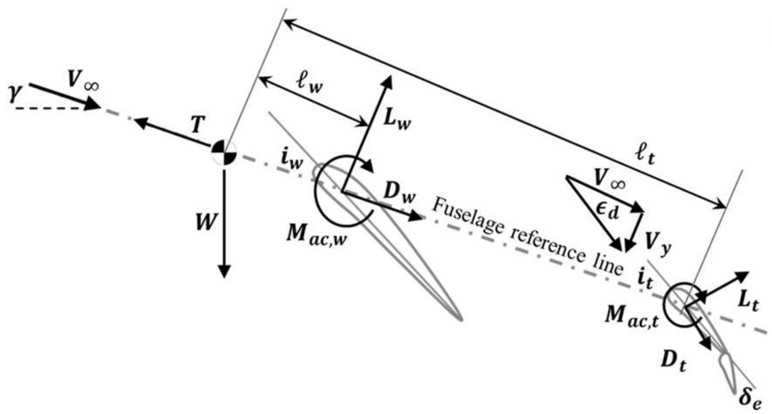
Wing & Aft Tail Combination

- A simplified analysis will allow the major contributions to trim and static stability to be analyzed without undue complexity
- More general case will be then be shown



- The assumptions of this simplified analysis are:
  - Fuselage reference line (FRL) taken to be in direction of flight, passing through CG
  - The thrust line and the aerodynamic centers (AC) of both wing and tail lie on FRL

Simplified Pitch Equilibrium Analysis



$$T = D_w + D_t \cos \epsilon_d + L_t \sin \epsilon_d + W \sin \gamma$$

$$L_w + L_t \cos \epsilon_d - D_t \sin \epsilon_d = W \cos \gamma$$

$$M_{cg} = M_{ac,w} + M_{ac,t} - \ell_w L_w - \ell_t L_t \cos \epsilon_d + \ell_t D_t \sin \epsilon_d = 0$$

- If the downwash angle is small, then $\cos \epsilon_d \approx 1, D_t \sin \epsilon_d \ll L_t$

$$\frac{L_w}{\frac{1}{2}\rho_\infty V_\infty^2 S_w} + \left(\frac{\frac{1}{2}\rho_\infty V_t^2}{\frac{1}{2}\rho_\infty V_\infty^2}\right)\left(\frac{S_t}{S_w}\right)\frac{L_t}{\frac{1}{2}\rho_\infty V_t^2 S_t} = \frac{W \cos \gamma}{\frac{1}{2}\rho_\infty V_\infty^2 S_w} \Rightarrow \boxed{C_{L,w} + \eta_t \left(\frac{S_t}{S_w}\right) C_{L,t} = C_W \cos \gamma}$$

Tail efficiency, $\eta_t \in [0.8, 1.2]$

$$\frac{M_{cg}}{\frac{1}{2}\rho_\infty V_\infty^2 S_w \bar{c}_w} = \frac{M_{ac,w}}{\frac{1}{2}\rho_\infty V_\infty^2 S_w \bar{c}_w} + \eta_t \left(\frac{S_t\ \bar{c}_t}{S_w\ \bar{c}_w}\right)\frac{M_{ac,t}}{\frac{1}{2}\rho_\infty V_t^2 S_t \bar{c}_t} - \frac{\ell_w}{\bar{c}_w}\frac{L_w}{\frac{1}{2}\rho_\infty V_\infty^2 S_w} - \left(\frac{S_t\ \ell_t}{S_w\ \bar{c}_w}\right)\eta_t \frac{L_t}{\frac{1}{2}\rho_\infty V_t^2 S_t} = 0$$

$$\boxed{C_{m,cg} = C_{m,ac,w} + \eta_t \left(\frac{S_t\ \bar{c}_t}{S_w\ \bar{c}_w}\right) C_{m,ac,t} - \frac{\ell_w}{\bar{c}_w}\ C_{L,w} - \left(\frac{S_t\ \ell_t}{S_w\ \bar{c}_w}\right)\eta_t\ C_{L,t} = 0}$$

Tail Lift: Positive, Negative, or Zero?

- Solving the previous two relationships for wing and tail lift yields:

$$C_{L,w} = C_W \cos\gamma - \frac{S_t}{S_w}\eta_t C_{L,t} \quad C_{L,t} = \frac{S_w \bar{c}_w}{S_t (\ell_t - \ell_w)\eta_t}\left[C_{m,ac,w} + \frac{S_t \bar{c}_t}{S_w \bar{c}_w}\eta_t C_{m,ac,t} - \frac{\ell_w}{\bar{c}_w} C_{L,W}\right]$$

- If tail carries negative lift ($C_{L,t} < 0$), the wing lift has to exceed the weight of the aircraft ($C_{L,w} > C_W$), which is clearly inefficient

- A tail carrying positive lift ($C_{L,t} > 0$) is more efficient, but still not as efficient as if the same amount of lift was carried by the wing
  - Wing AR > tail AR, which means tail sees more induced drag than wing for same lift

Tail Lift: Positive, Negative, or Zero?

- So for aerodynamic efficiency alone (and not trim or stability), a good preliminary design choice is to trim with no elevator deflection with zero tail load ($C_{L,t} = 0$) at a design weight and flight condition (i.e. $C_{L,w} = C_W$)

- This is achieved by positioning the wing A.C. relative to the C.G. such that

$$C_{m,ac,w} + \frac{S_t \bar{c}_t}{S_w \bar{c}_w} \eta_t \, C_{m,ac,t} - \frac{\ell_w}{\bar{c}_w} C_W = 0$$

$$\boxed{\ell_w = \frac{\bar{c}_w}{C_W}\left[C_{m,w} + \frac{S_t \, \bar{c}_t}{S_w \, \bar{c}_w} \eta_t \, C_{m,ac,t}\right]}$$

Wing and Tail Mounting Angles

Wing

- At trim, under the assumed conditions, wing lift: $C_{L,w} = C_W = W/\left(\frac{1}{2}\rho_\infty V_\infty^2 S_w\right)$

- Referring to the wing's lift curve, $C_{L,w} = C_{L_{\alpha,w}}(\alpha + i_w - \alpha_{L0,w})$

- The angle of attack α is measured relative to the fuselage reference line (F.R.L.), and we may want this to be aligned with the freestream ($\alpha = 0$) at the design condition
- This allows us to determine the wing mounting angle as

$$i_w = \alpha_{L0,w} + W/\left(\frac{1}{2}\rho_\infty V_\infty^2 S_w C_{L_{\alpha,w}}\right)$$

Tail

- The downwash seen by the tail is expressed as $\epsilon = \epsilon_0 + \left(\frac{\partial\epsilon}{\partial\alpha}\right)\alpha$

- The tail AOA is given by:

$$\alpha_t = \alpha + i_t - \epsilon = \alpha\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) + i_t - \epsilon_0$$

- If the tail is symmetric, then for $C_{L,t} = 0$, we require that $\alpha_t = 0$

- Solving the above, the tail mounting angle is then numerically equal to the downwash at zero angle of attack of the F.R.L.

$$i_t = \epsilon_0$$

Elevator Angle to Trim

- For flight conditions other than the design condition, the airplane will have to be trimmed with elevator deflection $\delta_e \neq 0$
- In this case, the tail lift (assuming a symmetrical airfoil) is given by

$$C_{L,t} = C_{L_\alpha,t}\left[\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\alpha + i_t - \epsilon_0 + \epsilon_e\,\delta_e\right]$$

  The new quantity ϵ_e is called the ***elevator effectiveness***. Can be thought of as "change of effective tail AOA per unit elevator deflection"

- Further, elevator deflection causes a moment about the tail A.C., which for small elevator deflections is proportional to the deflection: $C_{m,ac,t} = C_{m,ac,t_{\delta_e}}\,\delta_e$
- These relationships can be substituted into the original lift and pitching moment relationships, which can then be solved for the following two quantities
  - Angle of attack at trim, α_{trim}
  - Elevator deflection to achieve trim, $\delta_{e,trim}$

Simplified Analysis of Pitch Stability

- After a lot of algebra and grouping of terms, the following matrix equations are obtained:

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_e}} \\ C_{m_\alpha} & C_{m_{\delta_e}} \end{bmatrix} \begin{Bmatrix} \alpha_{trim} \\ \delta_{e,trim} \end{Bmatrix} = \begin{Bmatrix} C_L - C_{L_0} \\ -C_{m_0} \end{Bmatrix}$$

$$C_{L_\alpha} = C_{L_{\alpha,w}} + \frac{S_t}{S_w} \eta_t \, C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$ Effective lift curve slope of wing + tail

$$C_{L_{\delta_e}} = \frac{S_t}{S_w} \eta_t \, C_{L_{\alpha,t}} \, \epsilon_e$$ Elevator lift effectiveness

$$C_{m_\alpha} = -\frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}} - \frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$ Pitch stiffness

$$C_{m_{\delta_e}} = \frac{S_t \bar{c}_t}{S_w \bar{c}_w} \eta_t C_{m,ac,t_{\delta_e}} - \frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} \epsilon_e$$ Elevator control power

$$C_{L_0} = C_{L_{\alpha,w}} \left(i_w - \alpha_{L0,w}\right) + \frac{S_t}{S_w} \eta_t \, C_{L_{\alpha,t}} (i_t - \epsilon_0)$$ Total lift (wing + tail) at zero AOA

$$C_{m_0} = C_{m,ac,w} - \frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}} \left(i_w - \alpha_{L0,w}\right) - \frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} (i_t - \epsilon_0)$$ Total pitching moment (wing + tail) at zero AOA

Lift coefficient in trimmed, steady-level flight

$$C_L = \frac{W}{\frac{1}{2} \rho_\infty V_\infty^2 \, S_w}$$

Static Stability in Pitch

- Recall the general pitch static stability criterion: $\frac{\partial C_m}{\partial \alpha} < 0$

- Recall also the equation for pitching moment, expressed in terms of wing and (symmetric) tail lift curve slopes, incidence angles, and tail downwash:

$$C_m = C_{m,ac,w} + \frac{S_t \bar{c}_t}{S_w \bar{c}_w} \eta_t C_{m,t_{\delta_e}} \delta_e - \frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}} \left(\alpha + i_w - \alpha_{L0,w} \right)$$
$$- \frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} \left[\left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \alpha + i_t - \epsilon_{d,0} + \epsilon_e \delta_e \right]$$

- Taking the derivative with respect to AOA gives the static stability criterion:

$$\frac{\partial C_m}{\partial \alpha} = -\frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}} - \frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) < 0$$

- Now, we can analyze how to ensure static stability of the wing and aft tail combination by studying the contributions of both wing and aft tail

Static Stability in Pitch

- The static stability criterion for the wing and aft tail configuration:

$$\mathrm{C}_{\mathrm{m}_\alpha} = \frac{\partial C_m}{\partial \alpha} = \underbrace{-\frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}}}_{\textbf{Wing contribution}} - \underbrace{\frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)}_{\textbf{Aft tail contribution}} < 0$$

- The downwash gradient $\left(\frac{\partial \epsilon}{\partial \alpha}\right) < 1$, and so the aft tail is always stabilizing. The stabilizing influence of the tail is proportional to tail volume $S_t \ell_t$
  - Often expressed as horizontal tail volume ratio $V_h = \frac{S_t \ell_t}{S_w \bar{c}_w}$

Static Stability in Pitch

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} = -\frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}} - \frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) < 0$$

- The wing's contribution depends on the sign of the wing's moment arm ℓ_w
  - If $\ell_w > 0$ (CG forward of wing AC), the wing is stabilizing. However, the tail will carry negative lift for all operating conditions
  - If $\ell_w < 0$ (CG aft of wing AC), the wing is de-stabilizing. In that case, the tail volume ratio V_h has to be large enough to ensure that $C_{m_\alpha} < 0$

- Important: ℓ_w, ℓ_t, V_h are all affected by ***CG location***. CG location has a ***dramatic*** impact on pitch stability

Stick-Fixed Neutral Point & Static Margin

- Just like the pitching moment about the aerodynamic center of an airfoil is invariant to AOA, a similar point exists for the entire airplane. It is called stick-fixed neutral point
  - If the CG is located at this point, the airplane is neutrally stable ($C_{m_\alpha} = 0$)

- We can find this point for the wing-tail combination by solving for the CG location that gives $C_{m_\alpha} = 0$

Numerator: Pitch stiffness of wing + tail combination, C_{m_α}

$$\frac{\ell_{np}}{\bar{c}_w} = -\frac{\frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}} + \frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)}{C_{L_{\alpha,w}} + \frac{S_t}{S_w} \eta_t \, C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)}$$

Denominator: Lift curve slope of wing + tail combination, C_{L_α}

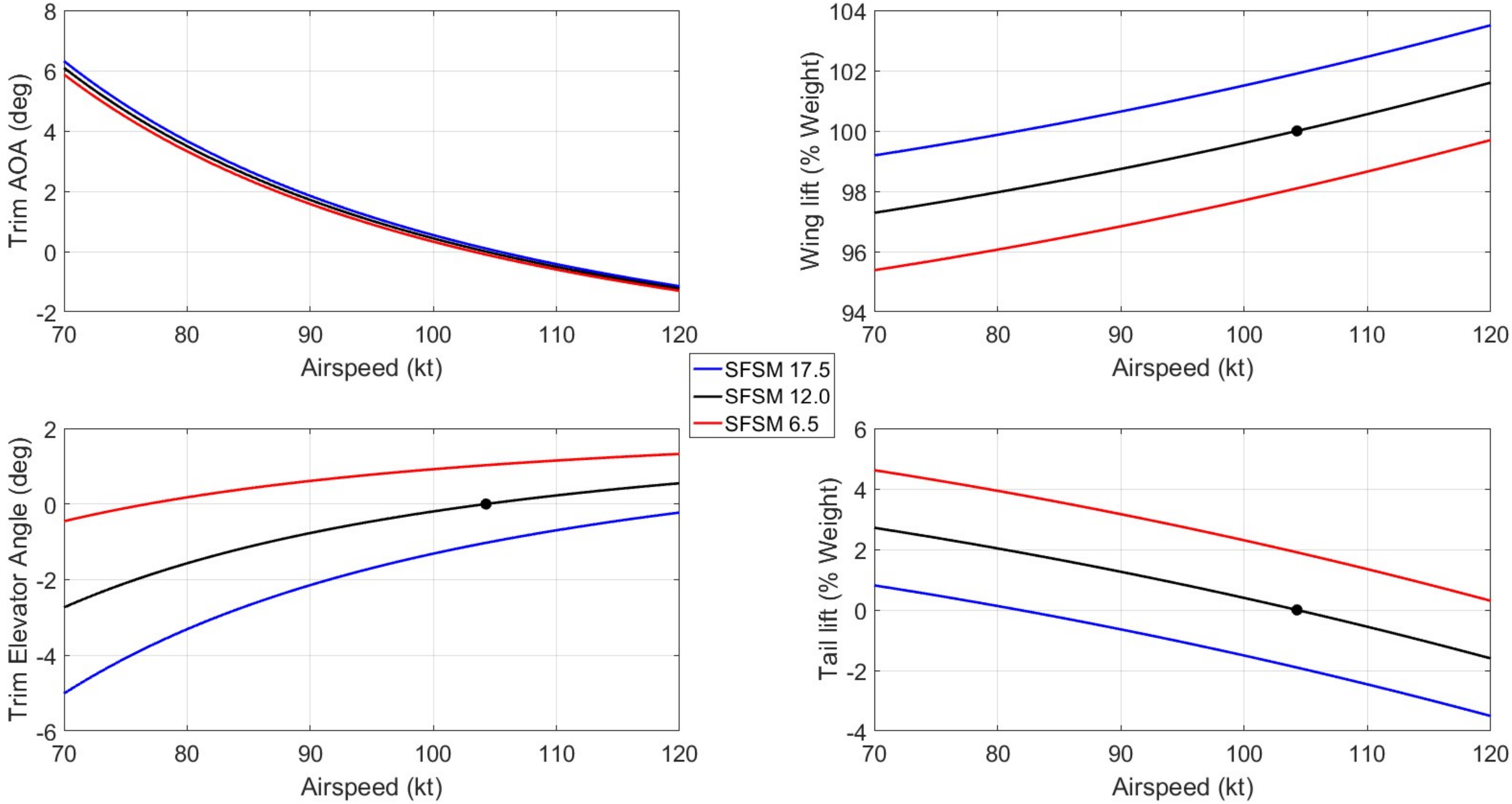
Stick-Fixed Neutral Point & Static Margin

$$\frac{\ell_{np}}{\bar{c}_w} = -\frac{\frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}} + \frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)}{C_{L_{\alpha,w}} + \frac{S_t}{S_w} \eta_t C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)}$$

- This yields a very important relationship: $C_{m_\alpha} = -\left(\frac{\ell_{np}}{\bar{c}_w}\right) C_{L_\alpha}$
  - The quantity $\left(\frac{\ell_{np}}{\bar{c}_w}\right)$ is called the stick-fixed static margin: the distance between CG and stick-fixed neutral point, expressed as fraction of MAC of main wing

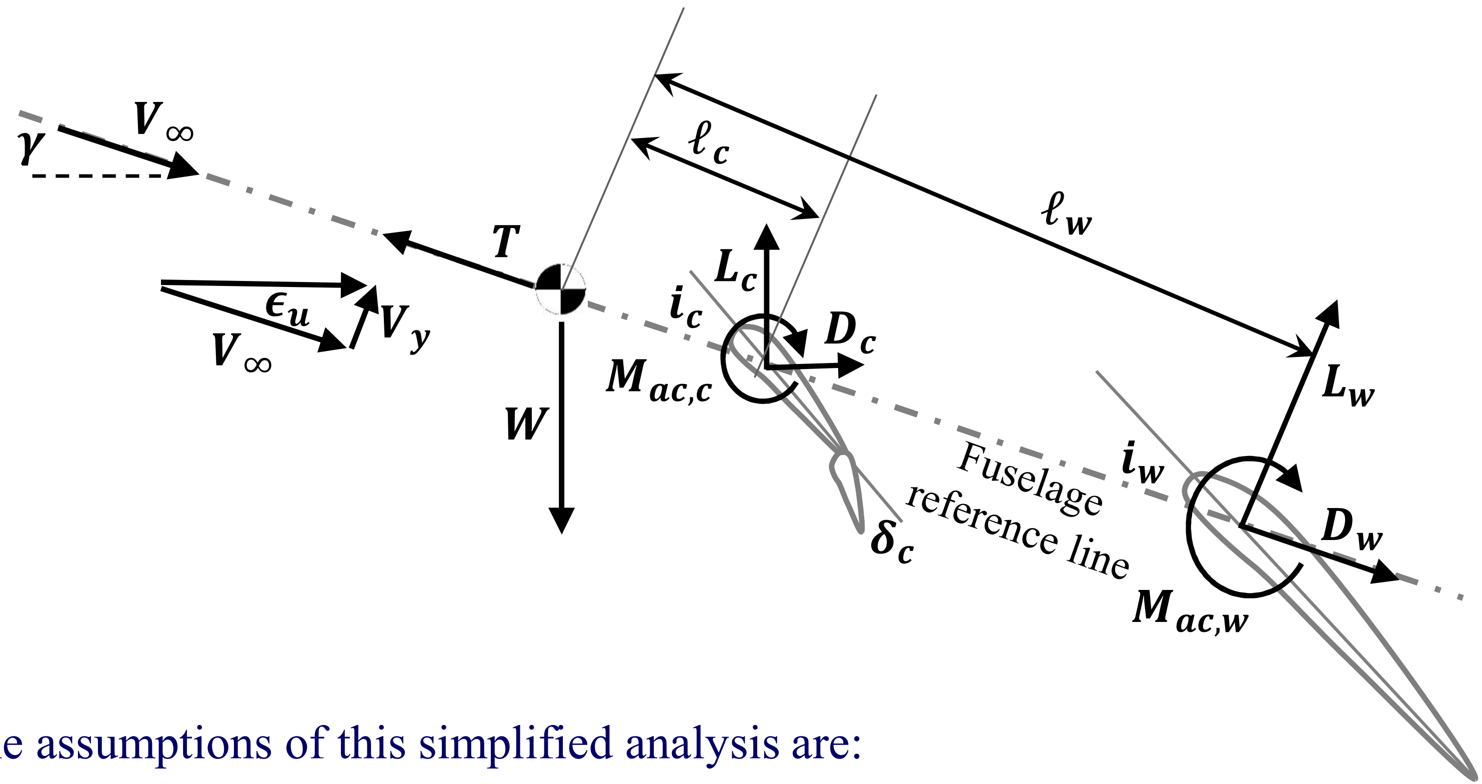
- Static stability requires a positive stick-fixed static margin, which requires that $\ell_{np} > 0$, i.e., CG must be located forward of stick-fixed neutral point

Effect of Airspeed and CG Location on Trim



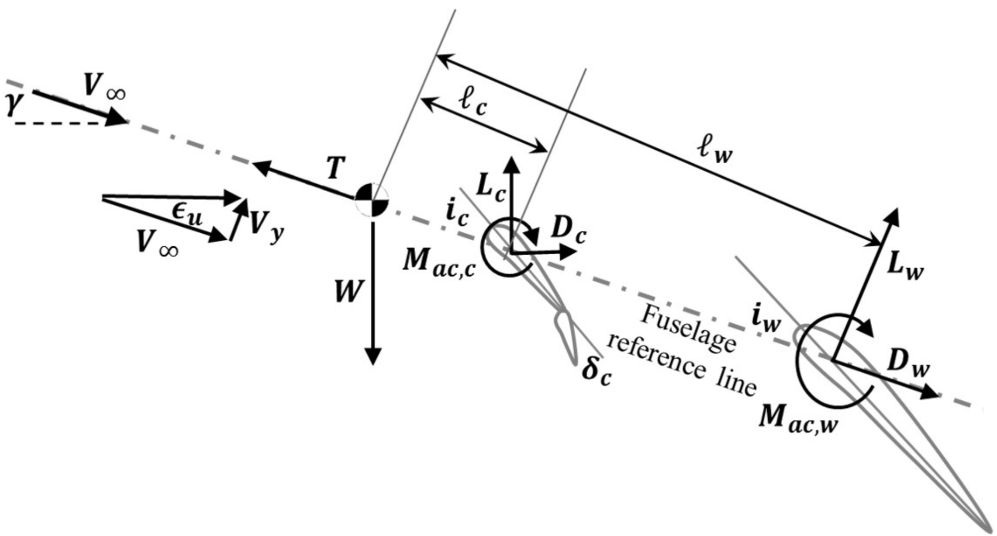
$W = 2700\ lb, S_w = 180\ ft^2, S_t = 36\ ft^2, \bar{c}_w = 5.45\ ft, \bar{c}_t = 3\ ft, C_{L_{\alpha,w}} = 4.44, C_{L_{\alpha,t}} = 3.97, C_{m,ac,t,\delta_e} = -0.55, \eta_t = 1.0, \frac{\partial\epsilon}{\partial\alpha} = 0.44,$
$\epsilon_{d,0} = 0.04041, \epsilon_e = 0.60, \alpha_{L0,w} = -2.2°, C_{m,ac,w} = -0.053$

Wing & Canard Combination



- The assumptions of this simplified analysis are:
  - Fuselage reference line (FRL) taken to be in direction of flight, passing through CG
  - The thrust line and the aerodynamic centers (AC) of both wing and tail lie on FRL

Simplified Pitch Equilibrium (Trim) Analysis



Support weight through lift:

$$C_{L_w} + \frac{S_c}{S_w} C_{L_c} = \frac{W \cos\gamma}{\frac{1}{2}\rho_\infty V_\infty^2 S_w} = C_W \cos\gamma$$

Balance out (trim) pitching moments:

$$C_m = C_{m,ac,w} + \frac{S_c \bar{c}_c}{S_w \bar{c}_w} C_{m,ac,c} - \frac{\ell_w}{\bar{c}_w} C_{L,w} - \frac{S_c \ell_c}{S_w \bar{c}_w} C_{L,c} = 0$$

$$C_{L,w} = \underbrace{\left(\frac{\bar{c}_w}{\ell_w - \ell_c}\right)}_{>\,0} \left[\underbrace{C_{m,ac,w}}_{\leq\,0} + \underbrace{\frac{S_c\, \bar{c}_c}{S_w\, \bar{c}_w} C_{m,ac,c}}_{\leq\,0} - \frac{\ell_c}{\bar{c}_w} \underbrace{C_W \cos\gamma}_{>\,0}\right]$$

$$C_{L,c} = \underbrace{\left(\frac{\bar{c}_c}{\ell_w - \ell_c}\right)}_{>\,0} \left[\underbrace{\frac{S_w \ell_w}{S_c \bar{c}_c} C_W \cos\gamma}_{>\,0} - \underbrace{\frac{S_w \bar{c}_w}{S_c\, \bar{c}_c} C_{m,ac,w}}_{\leq\,0} - \underbrace{C_{m,ac,c}}_{\leq\,0}\right]$$

For the wing to carry positive lift ($C_{L,w} > 0$), the canard must be located <u>ahead</u> of the CG ($\ell_c < 0$)

Unlike aft-tail configurations, in a canard-wing configuration, the canard <u>always</u> carries <u>positive lift</u> ($C_{L,c} > 0$)
- For aero efficiency, canards are designed with high aspect ratio

Simplified Pitch Stability Analysis

- Recall the pitching moment of the canard-wing configuration

$$C_m = C_{m,ac,w} + \frac{S_c \bar{c}_c}{S_w \bar{c}_w} C_{m,ac,c} - \frac{\ell_w}{\bar{c}_w} C_{L,w} - \frac{S_c \ell_c}{S_w \bar{c}_w} C_{L,c} \Leftarrow C_{L,c} = C_{L_{\alpha,c}}(\alpha + i_c - \alpha_{L0,c} + \epsilon_u + \epsilon_c \delta_c)$$

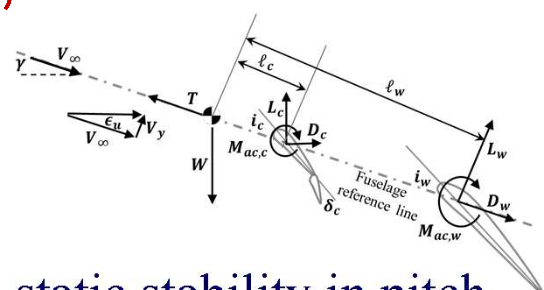
$$C_{L,w} = C_{L_{\alpha,w}}(\alpha + i_w - \alpha_{L0,w})$$

- Differentiating w.r.t. α and enforcing the static pitch stability criterion $C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} < 0$ yields the condition for static stability in pitch for the canard-wing configuration:

$$\frac{\partial C_m}{\partial \alpha} = \boxed{-\frac{\ell_w}{\bar{c}_w} C_{L_{\alpha,w}}} - \boxed{\frac{S_c \ell_c}{S_w \bar{c}_w} C_{L_{\alpha,c}} \left(1 + \frac{\partial \epsilon_u}{\partial \alpha}\right)} < 0$$

Since CG is <u>ahead</u> of AC of wing ($\ell_w > 0$), the <u>wing</u> is <u>always stabilizing</u>

Since CG is aft of AC of canard ($\ell_c < 0$), the <u>canard</u> is <u>always de-stabilizing</u>

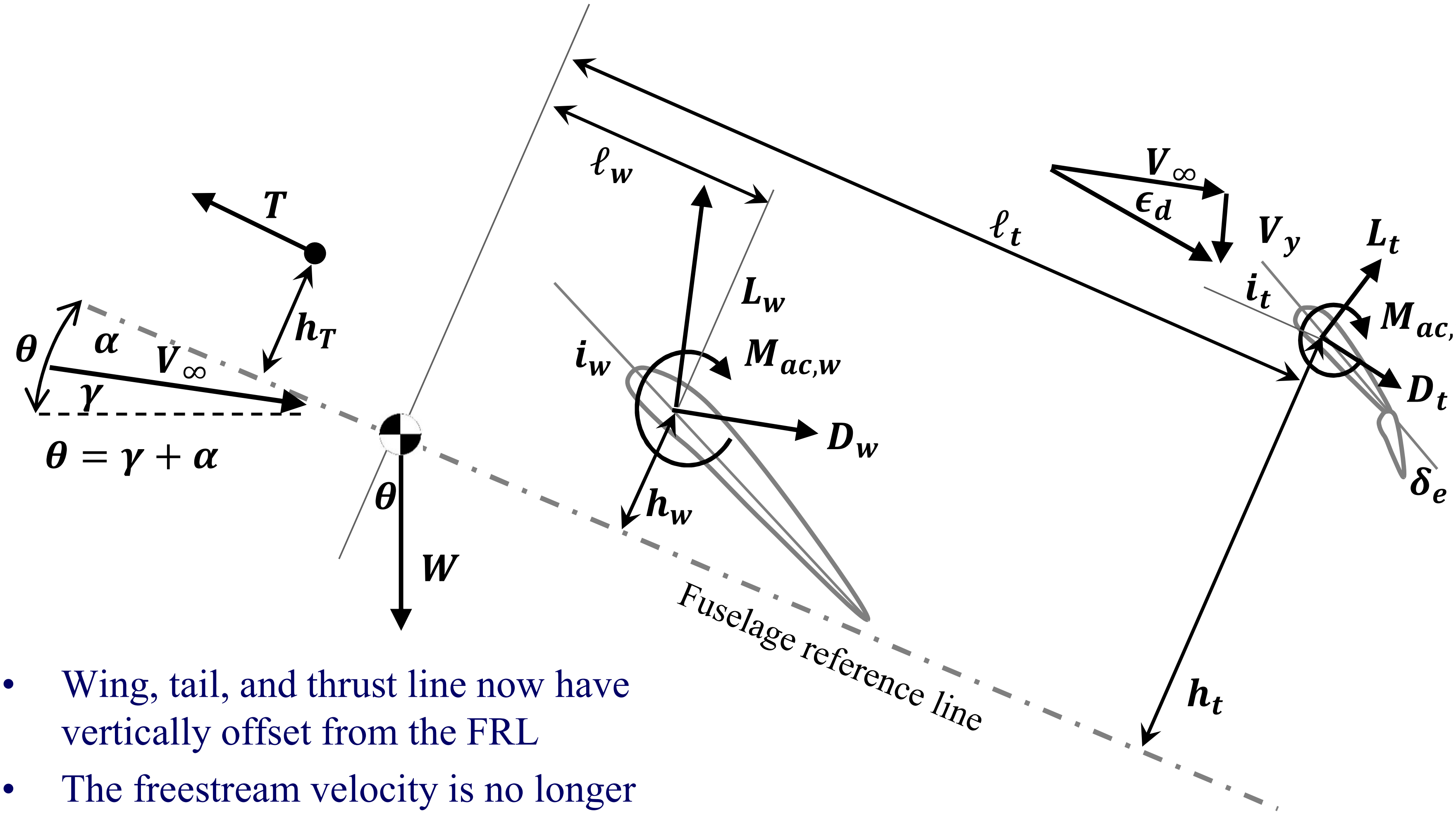


- Thus, it is the main wing (and not the canard) that provides static stability in pitch for a canard-wing configuration

Understanding how a Canard is Different

- Similar to an aft tail, it is possible to define a canard volume ratio, $V_c = \frac{S_c(-\ell_c)}{S_w \bar{c}_w}$
- The trim requirement expressed in terms of V_c: $V_c = \frac{\ell_w}{\bar{c}_w}\frac{C_{L_w}}{C_{L_c}} - \frac{C_{m,ac,w}}{C_{L_c}} - \frac{S_c\,\bar{c}_c}{S_w\,\bar{c}_w}\frac{C_{m,ac,c}}{C_{L_c}}$
- The $C_{m_\alpha} < 0$ requirement in terms of V_c: $V_c < \frac{\ell_w}{\bar{c}_w}\frac{C_{L\alpha,w}}{C_{L\alpha,c}(1+\partial\epsilon_u/\partial\alpha)}$
- Canard sizing is a tradeoff between pitch control and pitch stability
- Combining the above relationships: $\frac{C_{L,w}}{C_{L,c}} < \underbrace{\frac{C_{L\alpha,w}}{C_{L\alpha,c}(1+\partial\epsilon_u/\partial\alpha)}}_{\textbf{typically} < 1} + \underbrace{\frac{\bar{c}_w}{\ell_w}\frac{C_{m,ac,w}}{C_{L,c}} + \frac{S_c\bar{c}_c}{S_w\ell_w}\frac{C_{m,ac,c}}{C_{L,c}}}_{\textbf{typically} < 0}$
- Therefore, $C_{L_c} > C_{L,w}$, which means that the canard will stall before the main wing
  - It is very difficult to stall the main wing through a gradual maneuver

Vertical Offset of Wing, Tail, and Thrust



- Wing, tail, and thrust line now have vertically offset from the FRL
- The freestream velocity is no longer parallel to the FRL

Vertical Offset of Wing, Tail, and Thrust

Force balance in the x and z directions

$$C_T = C_W \sin\theta + C_{D,w}\cos\alpha - C_{L,w}\sin\alpha + \frac{S_t}{S_w}\eta_t\left[C_{D,t}\cos(\alpha-\epsilon_d) - C_{L,t}\sin(\alpha-\epsilon_d)\right]$$

$$C_W \cos\theta = C_{L,w}\cos\alpha + C_{D,w}\sin\alpha + \frac{S_t}{S_w}\eta_t\left[C_{L,t}\cos(\alpha-\epsilon_d) + C_{D,t}\sin(\alpha-\epsilon_d)\right]$$

Pitching moment balance.

Note 1: there is a moment due to the thrust axis offset

Note 2: $C_{m_\alpha} = \partial C_m/\partial\alpha$ will now be a function of α

$$\begin{aligned} C_m = C_{m,ac,w} &- \left(\frac{\ell_w}{\bar{c}_w}\cos\alpha + \frac{h_w}{\bar{c}_w}\sin\alpha\right)C_{L,w} + \left(\frac{h_w}{\bar{c}_w}\cos\alpha - \frac{\ell_w}{\bar{c}_w}\sin\alpha\right)C_{D,w} \\ &+ \frac{S_t\bar{c}_t}{S_w\bar{c}_w}\eta_t C_{m,ac,t} - \left[\frac{S_t\ell_t}{S_w\bar{c}_w}\cos(\alpha-\epsilon_d) + \frac{S_t h_t}{S_w\bar{c}_w}\sin(\alpha-\epsilon_d)\right]\eta_t C_{L,t} \\ &+ \left[\frac{S_t h_t}{S_w\bar{c}_w}\cos(\alpha-\epsilon_d) - \frac{S_t\ell_t}{S_w\bar{c}_w}\sin(\alpha-\epsilon_d)\right]\eta_t C_{D,h} - \frac{h_T}{\bar{c}_w}C_T = 0 \end{aligned}$$

In addition to lift and pitching moment about AC, we also have to build up the drag on the wing and tail

$$C_{L,w} = C_{L_\alpha,w}(\alpha + i_w - \alpha_{L0,w}) \qquad C_{L,t} = C_{L_{\alpha,t}}(\alpha + i_t - \alpha_{L0,t} - \epsilon_d + \epsilon_e\delta_e)$$

$$C_{D,w} = C_{D_{0,w}} + K_{1,w}\,C_{L,w} + K_{2,w}\,C_{L,w}^2 \qquad C_{D,t} = C_{D_{0,t}} + K_{1,t}\,C_{L,t} + K_{2,t}\,C_{L,t}^2$$

$$C_{m,ac,t} = C_{m,ac,t_{\delta_e}}\delta_e$$

This set of nonlinear equations has to be solved numerically to determine the trim point, and also to determine the static stability at the trim point (which may be different at different trim points)

Contribution of Fuselage and Nacelles

- Contribution of the fuselage and nacelles to static stability is typically destabilizing (often, the effect can be quite significant)

- Aero forces and moments on the fuselage are complex and affected by interactions with wing and tail. Computer simulations and/or wind tunnel tests are typically required

- However, for a first approximation in early design, the effect of the fuselage and nacelles on airplane lift is usually neglected. The effect of moment may be assessed using an experimental correlation (Hoak , 1960):

$$(\Delta C_m)_{fus} = -2\frac{S_f \ell_f}{S_w \bar{c}_w}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{\frac{3}{2}}\right]\alpha_f$$

- S_f: max fuselage cross-sectional area, c_f: length of fuselage, d_f: diameter of circle of area S_f, ℓ_f: distance that fuselage center of pressure is aft of aircraft CG (typically, $\ell_f < 0$), α_f: AOA of fuselage minimum drag axis

Hoak, D.E., "USAF Stability and Control Datcom", AFWAL-TR-83-3048, 1960

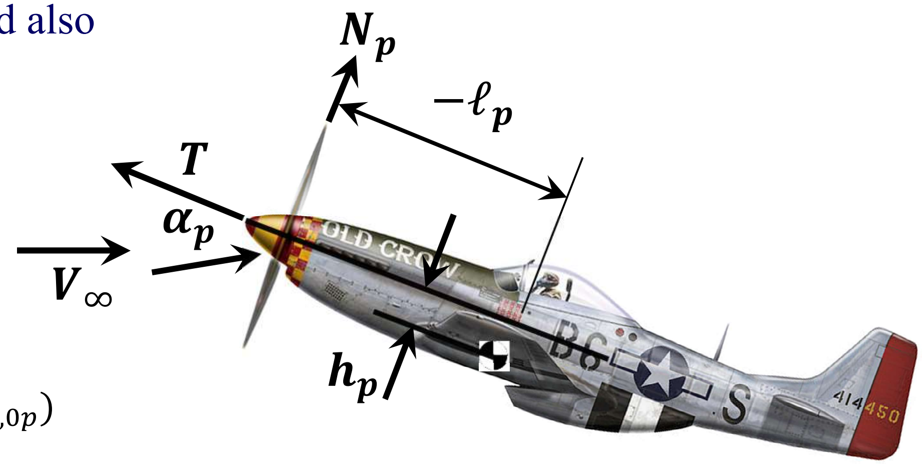
Contribution of Running Propellers

- Affect both trim and static stability due to (1) direct effects: due to forces and moments on prop and (2) Indirect effects: due to complex prop slipstream interactions
- Considering the direct effects only:

$$(\Delta C_m)_p = -\frac{h_p}{\bar{c}_w} C_T - \frac{2\, d_p^2 \ell_p^2}{S_w \bar{c}_w}\left(\frac{C_{N,p_\alpha}}{J^2}\right)\alpha_p$$

- Prop incidence angle depends on downwash $\epsilon_{d,p}$ and also thrust axis inclination α_{0p}: $\alpha_p = \alpha - \epsilon_{d,p} + \alpha_{0p}$
- The net impact can be represented as

$$(\Delta C_m)_p = (\Delta C_{m_0})_p + (\Delta C_{m_\alpha})_p\,\alpha$$



- The first term affects trim:

$$(\Delta C_{m_0})_p = -\frac{h_p}{\bar{c}_w} C_T - \frac{2\, d_p^2 \ell_p}{S_w \bar{c}_w}\left(\frac{C_{N,p_\alpha}}{J^2}\right)(\alpha_{0p} - \epsilon_{d,0p})$$

- The second term is the prop's effect on static stability

$$(\Delta C_{m_\alpha})_p = -\frac{2\, d_p^2 \ell_p}{S_w \bar{c}_w}\left(\frac{C_{N,p_\alpha}}{J^2}\right)(1 - \partial\epsilon_{d,p}/\partial\alpha)$$

$$C_T = \frac{T}{\frac{1}{2}\rho_\infty V_\infty^2 S_w}$$

- Tractor prop: destabilizing $[\ell_p < 0 \rightarrow (\Delta C_{m_\alpha})_p > 0]$
- Pusher prop: stabilizing $[\ell_p > 0 \rightarrow (\Delta C_{m_\alpha})_p < 0]$

$\left(\frac{C_{N,p_\alpha}}{J^2}\right)$ has to be read off a chart

d_p: prop diameter

Contribution of Running Jet Engines

- Similar to a propeller, running jet engines also affect both trim and static stability in pitch through direct and indirect effects.
  - Direct effects: Due to moments from thrust offset and normal force
  - Indirect effects: Due to flow entrainment by the jet, interaction of jet exhaust with other airplane surfaces (neglected in early design)

- In a very similar manner to a propeller, the net pitching moment impact due to a jet can be represented as

$$(\Delta C_m)_j = \left(\Delta C_{m_0}\right)_j + \left(\Delta C_{m_\alpha}\right)_j \alpha$$

$$\left(\Delta C_{m_0}\right)_j = -C_T \left[\frac{h_j}{\bar{c}_w} + \frac{\eta_{p,i}}{2(1-\eta_{p,i})}\frac{\ell_j}{\bar{c}_w}\left(\alpha_{0j} - \epsilon_{d,0j}\right)\right]$$

$$\left(\Delta C_{m_\alpha}\right)_p = -C_T \frac{\eta_{p,i}}{2\left(1-\eta_{p,i}\right)}\frac{\ell_j}{\bar{c}_w}\left(1 - \partial\epsilon_{d,j}/\partial\alpha\right)$$

$\boldsymbol{\eta_{p,i}}$**:** ideal propulsive efficiency of jet engine

Geometric parameters are identical to those shown for the propeller airplane, with subscript ‘j’ for jet, instead of ‘p’ for propeller

Phillips, W.F., “Mechanics of Flight”, John Wiley & Sons Inc., 2004

Contents

- Fundamental Concepts and Reference Frames
- Aircraft Equations of Motion
- Longitudinal Stability
  - Flying Wing, Wing-Tail, and Canard-Wing Configurations
  - Additional factors affecting longitudinal stability
- **Longitudinal Control and Maneuverability**
- Lateral/Directional Stability, Control, and Maneuverability
- Stability in Steady Flight
  - Longitudinal Modes
    - Phugoid and Short Period Modes
    - Open Loop Response to Control Inputs
  - Lateral Modes
- Additional Topics

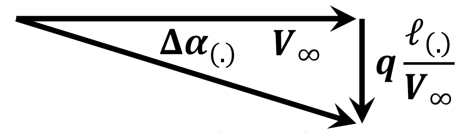
Some Key Stability & Control Derivatives

	Lift Force (C_L)	Side Force (C_Y)	Roll Mom. (C_l)	Pitch Mom. (C_m)	Yaw Mom. (C_n)
AOA (α)	C_{L_α}			C_{m_α}	
AOA rate ($\hat{\dot{\alpha}}$)	$C_{L_{\hat{\dot{\alpha}}}}$			$C_{m_{\hat{\dot{\alpha}}}}$	
Sideslip (β)		C_{Y_β}	C_{l_β}		C_{n_β}
Roll rate ($\hat{p}$)		$C_{Y_{\hat{p}}}$	$C_{l_{\hat{p}}}$		$C_{n_{\hat{p}}}$
Pitch rate ($\hat{q}$)	$C_{L_{\hat{q}}}$			$C_{m_{\hat{q}}}$	
Yaw rate ($\hat{r}$)		$C_{Y_{\hat{r}}}$	$C_{l_{\hat{r}}}$		$C_{n_{\hat{r}}}$
Aileron (δ_a)			$C_{l_{\delta_a}}$		$C_{n_{\delta_a}}$
Elevator (δ_e)	$C_{L_{\delta_e}}$			$C_{m_{\delta_e}}$	
Rudder (δ_r)		$C_{Y_{\delta_r}}$	$C_{l_{\delta_r}}$		$C_{n_{\delta_r}}$

1st or 2nd order "stiffness" — **Rate damping derivatives**

Control powers — **Roll/yaw axis coupling**

Effect of Pitch Rate on Lift and Pitching Moment



$$\Delta\boldsymbol{\alpha}_{(.)} = \tan^{-1}\left(\boldsymbol{q}\frac{\ell_{(.)}}{\boldsymbol{V}_\infty}\right) \approx \boldsymbol{q}\frac{\ell_{(.)}}{\boldsymbol{V}_\infty}$$

- Consider an airplane in a pull-up maneuver.
  The pitch rate q is non-dimensionalized as $\hat{q} = q\,\dfrac{\bar{c}_w}{2V_\infty}$
- The pitch rate changes the wing and tail incidence angles
  - For wing: $\Delta\alpha_w = \dfrac{q\ell_w}{V_\infty} = \dfrac{2\ell_w}{\bar{c}_w}\hat{q}$ and for tail: $\Delta\alpha_t = \dfrac{q\ell_t}{V_\infty} = \dfrac{2\ell_t}{\bar{c}_w}\hat{q}$

- Change in airplane C_L due to the pitch rate: $(\Delta C_L)_q = C_{L_{\alpha,w}}\Delta\alpha_w + \dfrac{S_t}{S_w}\eta_t\,\mathrm{C}_{\mathrm{L}_{\alpha,t}}\Delta\alpha_t$

$$(\Delta C_L)_q = \left(\frac{2\ell_w}{\bar{c}_w}C_{L_{\alpha,w}} + 2\,\frac{S_t\ell_t}{S_w\bar{c}_w}\eta_t\mathrm{C}_{\mathrm{L}_{\alpha,t}}\right)\hat{q} = \boldsymbol{C}_{\boldsymbol{L}_{\hat{\boldsymbol{q}}}}\,\hat{q}$$

- Change in airplane C_m due to pitch rate: $(\Delta C_m)_q = -\dfrac{\ell_w}{\bar{c}_w}C_{L_{\alpha,w}}\Delta\alpha_w -$
  $\dfrac{\ell_t}{\bar{c}_w}\dfrac{S_t}{S_w}\eta_t\,\mathrm{C}_{\mathrm{L}_{\alpha,t}}\Delta\alpha_t$

$$(\Delta C_m)_q = \left(-\frac{2\ell_w^2}{\bar{c}_w^2}C_{L_{\alpha,w}} - 2\,\frac{S_t\ell_t^2}{S_w\bar{c}_w^2}\eta_t\mathrm{C}_{\mathrm{L}_{\alpha,t}}\right)\hat{q} = \boldsymbol{C}_{\boldsymbol{m}_{\hat{\boldsymbol{q}}}}\hat{q}$$

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Effect of Pitch Rate on Lift and Pitching Moment

$$(\Delta C_L)_q = \left(\frac{2\ell_w}{\bar{c}_w} C_{L_{\alpha,w}} + 2\frac{S_t \ell_t}{S_w \bar{c}_w} \eta_t \mathrm{C}_{\mathrm{L}_{\alpha,t}}\right)\hat{q} = \boldsymbol{C_{L_{\hat{q}}}}\,\hat{q}$$

$$(\Delta C_m)_q = \left(-\frac{2\ell_w^2}{\bar{c}_w^2} C_{L_{\alpha,w}} - 2\frac{S_t \ell_t^2}{S_w \bar{c}_w^2} \eta_t \mathrm{C}_{\mathrm{L}_{\alpha,t}}\right)\hat{q} = \boldsymbol{C_{m_{\hat{q}}}}\hat{q}$$

- The coefficient $\boldsymbol{C_{L_{\hat{q}}}}$ represents change in lift due to pitch rate
- The coefficient $\boldsymbol{C_{m_{\hat{q}}}}$ represents change in pitching moment due to pitch rate and is called the pitch damping coefficient (derivative).

- They have very important effects on aircraft dynamic response in pitch

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Elevator Angle Per g

Lift and pitching moment equations:

$$\boldsymbol{C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\hat{q}}}\hat{q} + C_{L_{\delta_e}}\delta_e}$$

$$\boldsymbol{C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\hat{q}}}\hat{q} + C_{m_{\delta_e}}\delta_e}$$

- Recall that for the pull-up maneuver:
  - Pitch rate: $q = \frac{(n-1)g}{V_\infty} \rightarrow \hat{q} = (n-1)\frac{g\bar{c}_W}{2\,V_\infty^2}$
  - Also: $L = n\,W \rightarrow C_L = n\,C_W$
- Using these to solve the lift and pitching moment equations for the elevator deflection:

$$\delta_e = -\frac{(nC_W - C_{L,0})C_{m_\alpha} + C_{L_\alpha}C_{m_0} + \left(C_{L_\alpha}C_{m_{\hat{q}}} - C_{L_{\hat{q}}}C_{m_\alpha}\right)(n-1)\left(\frac{g\bar{c}_W}{2\,V_\infty^2}\right)}{C_{L_\alpha}C_{m_{\delta_e}} - C_{L_{\delta_e}}C_{m_\alpha}}$$

- Differentiating this w.r.t. load factor gives elevator angle per g:

$$\frac{\partial\delta_e}{\partial n} = -\frac{C_W C_{m_\alpha} + (C_{L_\alpha}C_{m_{\hat{q}}} - C_{L_{\hat{q}}}C_{m_\alpha})\left(\frac{g\bar{c}_W}{2\,V_\infty^2}\right)}{C_{L_\alpha}C_{m_{\delta_e}} - C_{L_{\delta_e}}C_{m_\alpha}}$$

Elevator Angle Per g

- Elevator angle per g:

$$\frac{\partial \delta_e}{\partial n} = -\frac{C_W C_{m_\alpha} + (C_{L_\alpha} C_{m_{\hat{q}}} - C_{L_{\hat{q}}} C_{m_\alpha})\left(\frac{g \bar{c}_w}{2\, V_\infty^2}\right)}{C_{L_\alpha} C_{m_{\delta_e}} - C_{L_{\delta_e}} C_{m_\alpha}}$$

- This provides a measure of airplane maneuverability
  – Smaller magnitude implies more maneuverable aircraft (less elevator needed per g)
  – But if it is too small, the airplane will become difficult to fly

What is the impact of:

- Airspeed?
- Weight?
- CG position?
- pitch damping?
- elevator control power?

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

The Stick-Fixed Maneuver Point

- Using the results $C_{m_\alpha} = -\left(\frac{\ell_{np}}{\bar{c}_w}\right) C_{L_\alpha}$ and $\ell_{np} = x_{np} - x_{cg}$, in the $\partial\delta_e/\partial n$ relationship:

$$\frac{\partial\delta_e}{\partial n} = -\frac{C_W - C_{L_{\hat{q}}}\left(\frac{g\bar{c}_w}{2V_\infty^2}\right)}{C_{m_{\delta_e}} + C_{L_{\delta_e}}\left[\frac{x_{np} - x_{cg}}{\bar{c}_w}\right]}\left[\frac{C_{m_{\hat{q}}}\left(\frac{g\bar{c}_w}{2V_\infty^2}\right)}{C_W - C_{L_{\hat{q}}}\left(\frac{g\bar{c}_w}{2V_\infty^2}\right)} - \frac{x_{np}}{\bar{c}_w} - \frac{x_{cg}}{\bar{c}_w}\right]$$

- There is a CG location for which the red term in the above becomes zero, giving $\frac{\partial\delta_e}{\partial n} = 0$

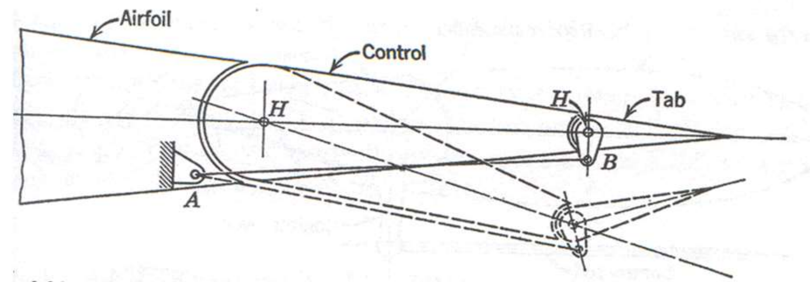
- The CG location that gives $\frac{\partial\delta_e}{\partial n} = 0$ is called the stick-fixed maneuver point
- The distance between CG and SFMP is called the stick-fixed maneuver margin

The Stick-Fixed Maneuver Point

$$\frac{\partial \delta_e}{\partial n} = -\frac{C_W - C_{L_{\hat{q}}}\left(\frac{g\bar{c}_w}{2V_\infty^2}\right)}{C_{m_{\delta_e}} + C_{L_{\delta_e}}\left[\frac{x_{np} - x_{cg}}{\bar{c}_w}\right]}\left[\frac{C_{m_{\hat{q}}}\left(\frac{g\bar{c}_w}{2V_\infty^2}\right)}{C_W - C_{L_{\hat{q}}}\left(\frac{g\bar{c}_w}{2V_\infty^2}\right)} - \frac{x_{np}}{\bar{c}_w} - \frac{x_{cg}}{\bar{c}_w}\right]$$

- The stick-fixed MP $\left(\frac{\partial \delta_e}{\partial n} = 0\right)$ is always aft of the stick-fixed neutral point $\left(C_{m_\alpha} = 0\right)$
  - Stick-fixed maneuver margin is always greater than stick-fixed static margin
- Note: A statically unstable airplane (stick-fixed SM < 0) can be reliably flown by a human pilot
  - Requires a Stability Augmentation System (SAS), i.e., computer control
- Warning! Even a computerized SAS cannot control an airplane with negative stick-fixed maneuver margin
  - The CG must always be kept forward of the SFMP in all airplanes

Control Surface Hinge Moments and Trim Tabs



6.26 Geometry of geared tab.

- Consider the case of the elevator (but the concept applies to all hinged controls)
- To trim the aircraft at a given flight condition requires a certain elevator deflection
  - At all but one deflection, airflow will cause a hinge moment about the hinge-line
  - In a reversible flight control system, the pilot will have to apply a force to the control column to offset this hinge moment. This will be quite… fatiguing
- A trim tab is used to offset (zero out) the hinge moment, and "trim" the control. Thus, the elevator "trims" the airplane. The elevator tab "trims" out the elevator c
  - All of this can be studied using hinge moment coefficients

$$C_{H,e} = C_{H,e_0} + C_{H,e_{\alpha_t}}\alpha_t + C_{H,e_{\delta_e}}\delta_e + C_{H,e_{\delta_{tt}}}\delta_{tt}$$

- "Letting go" of the yoke/stick ("stick-free") results in the elevator "floating" to a position $\delta_{e,free}$ where $C_{H,e} = 0$
- "Trimming out" the elevator control force means setting δ_{tt} such that $C_{H,e} = 0$, *at the elevator angle that you need for trim.* The trim tab allows $\delta_{e,trim} = \delta_{e,free} \rightarrow C_{H,e} = 0$

How Heavy Are the Controls? Control Force Gradient

- Consider a simplified relationship between hinge moment and control force: $F_p = \frac{H_e}{\ell_g}$
  - ℓ_g represents the effective gearing of the elevator control system

- The control force (that the pilot must apply) can be expressed as (skipping derivation)

$$F_p = \eta_t \frac{S_e \bar{c}_e}{S_w \ell_g} \left[C_1 \left(n - \frac{V_\infty^2}{V_{trim}^2} \right) W + C_2 (n-1) \frac{\rho_\infty g \, S_w \bar{c}_w}{4} \right]$$

- Note that at trimmed steady-level flight, $V_\infty = V_{trim}, n = 1 \rightarrow F_p = 0$
- Differentiating F_p w.r.t. V_∞ with $V = V_{trim}, n =$ constant gives the control force gradient

$$\left(\frac{\partial F_p}{\partial V_\infty} \right)_{\substack{V = V_{trim} \\ n = \text{const}}} = -2 \, \eta_t \frac{S_e \, \bar{c}_e}{S_w \ell_g} \; C_1 \frac{W}{V_{trim}}$$

Phillips, W.F., “Mechanics of Flight”, John Wiley & Sons Inc., 2004

How Heavy Are the Controls? Control Force Gradient

$$\left(\frac{\partial F_p}{\partial V_\infty}\right)_{\substack{V=V_{trim} \\ n=\text{ const}}} = -2\,\eta_t \frac{S_e\,\bar{c}_e}{S_w \ell_g}\; C_1\, \frac{W}{V_{trim}}$$

- Note from the above that controls become "heavier" at higher gross weight W and lower trim speeds $V_\infty = V_{trim}$
  - They also increase with $\bar{c}_e$, which is $\propto$ airplane size

- There is a certain CG location at which the control force gradient $\left(\frac{\partial F_p}{\partial V_\infty}\right) = \mathbf{0}$. This is called the **stick-free neutral point**
  - It is the neutral point if no control force were exerted on a hypothetical frictionless, massless flight control system

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

How Heavy Are the Controls? Control Force Per g

- The control force (that the pilot must apply) can be expressed as (skipping derivation)

$$F_p = \eta_t \frac{S_e \bar{c}_e}{S_w \ell_g} \left[C_1 \left(n - \frac{V_\infty^2}{V_{trim}^2} \right) W + C_2 (n-1) \frac{\rho_\infty g\, S_w \bar{c}_w}{4} \right]$$

- Taking a derivative w.r.t. load factor at constant airspeed gives the longitudinal control force per g:

$$\left(\frac{\partial F_p}{\partial n} \right)_{V=const} = \eta_t \frac{S_e \bar{c}_e}{S_w \ell_g} \left(C_1\, W + C_2 \frac{\rho_\infty g\, S_w \bar{c}_w}{4} \right)$$

- Note that it increases in magnitude with increasing gross weight and decreasing altitude
- There is a CG location where the control force per g becomes zero, $\boldsymbol{\left(\frac{\partial F_p}{\partial n} \right) = 0}$. This is called the **stick-free maneuver point**

Constants C_1, C_2 depend on aircraft stability and control derivatives as well as hinge moment derivatives

$$C_1 = \frac{C_{m_{\delta_e}} C_{H,e_{\alpha_t}} \left(1 - \frac{\partial \epsilon_d}{\partial \alpha} \right) - C_{m_\alpha} C_{H,e_{\delta_e}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{L_{\delta_e}} C_{m_\alpha}}$$

$$C_2 = \frac{2 \ell_h C_{H,e_{\alpha_t}}}{\bar{c}_w} + \frac{\left(C_{L_{\delta_e}} C_{m_{\hat{q}}} - C_{L_{\hat{q}}} C_{m_{\delta_e}} \right) C_{H,e_{\alpha_t}} \left(1 - \frac{\partial \epsilon_d}{\partial \alpha} \right) - \left(C_{L_\alpha} C_{m_{\hat{q}}} - C_{L_{\hat{q}}} C_{m_\alpha} \right) C_{H,e_{\delta_e}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{L_{\delta_e}} C_{m_\alpha}}$$

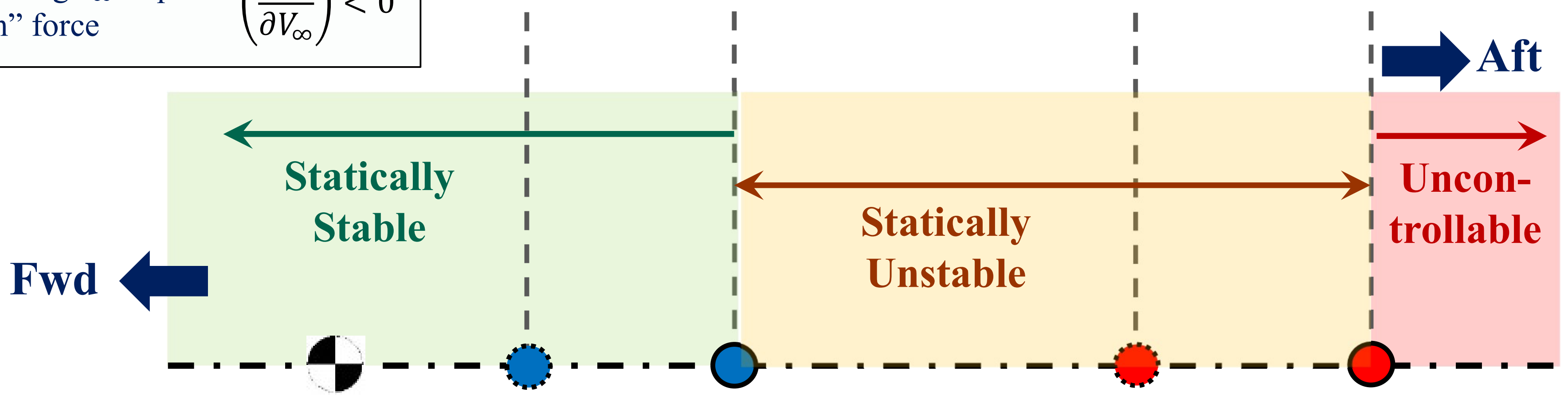
Summary – Neutral & Maneuver Points

Sign convention: "Pull" force and "down elevator" are positive

If CG lies ahead of all neutral and maneuver points:

- Increasing n requires "up" elevator $\left(\frac{\partial \delta_e}{\partial n}\right) < 0$
- Increasing n requires "pull" force $\left(\frac{\partial F_p}{\partial n}\right) > 0$
- Increasing V_∞ requires "down" elevator $\left(\frac{\partial \delta_e}{\partial V_\infty}\right) > 0$
- Increasing V_∞ requires "push" force $\left(\frac{\partial F_p}{\partial V_\infty}\right) < 0$

Stick-free Neutral Point	**Stick-fixed Neutral Point**	**Stick-free Maneuver Point**	**Stick-fixed Maneuver Point**
(Zero control force gradient)	(Zero pitch stiffness)	(Zero control force per g)	(Zero elevator angle per g)
$\left(\frac{\partial F_p}{\partial V_\infty}\right) = 0$	$C_{m_\alpha} = 0$ $\left(\frac{\partial \delta_e}{\partial V_\infty}\right) = 0$	$\left(\frac{\partial F_p}{\partial n}\right) = 0$	$\left(\frac{\partial \delta_e}{\partial n}\right) = 0$



Ground Effect

- Flow around the aircraft is significantly modified when it flies in close proximity to the ground (typically within one wingspan) – this is referred to as "ground effect"
- The proximity to the ground ***reduces the downwash*** for wing and other components
  - Increases lift curve slope, reduces induced drag
- Ground effect typically produces a substantial ***rearward shift in the neutral point*** for a conventional airplane
  - Increase is wing and tail lift curve slopes tend to move the N.P. forward
  - However, the dominant effect is from reduction in aft tail downwash, which increases the stabilizing effect of an aft tail, moving the N.P. aft
- Thus, ground effect increases the amount of up-elevator needed to trim at low airspeeds
  - Also increases the control force gradient, making reversible controls feel heavier
- The combination of (1) low airspeed, (2) forward C.G., and (3) ground effect under approach/landing conditions can be a critical constraint on elevator sizing

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Effect of Deployed Flaps and Landing Gear

- Deploying landing gear generates nose-down pitching moment, as it is below the CG
- Deploying flaps (1) increases max lift coefficient, (2) reduces zero-lift AOA, (3) causes additional nose-down pitching moment about wing AC, and (4) increases wing drag

- Recall the solution for trim angle of attack α_{trim} and elevator angle $\delta_{e,trim}$ for flight at weight coefficient C_W. The changes in these quantities due to flaps + gear deployment:

$$\alpha_{trim} = \frac{(C_W - C_{L_0})C_{m_{\delta_e}} + C_{L_{\delta_e}} C_{m_0}}{C_{L_\alpha}\left[C_{m_{\delta_e}} + C_{L_{\delta_e}}\left(l_{np}/\bar{c}_w\right)\right]} \Rightarrow (\Delta\alpha_{trim})_{fg} = \frac{-C_{m_{\delta_e}}\left(\Delta C_{L_0}\right)_{fg} + C_{L_{\delta_e}}\left(\Delta C_{m_0}\right)_{fg}}{C_{L_\alpha}\left[C_{m_{\delta_e}} + C_{L_{\delta_e}}\left(l_{np}/\bar{c}_w\right)\right]}$$

$$\delta_{e,trim} = \frac{(C_W - C_{L_0})(l_{np}/\bar{c}_w) - C_{m_0}}{\left[C_{m_{\delta_e}} + C_{L_{\delta_e}}\left(l_{np}/\bar{c}_w\right)\right]} \Rightarrow \left(\delta_{e,trim}\right)_{fg} = \frac{-\left(\frac{l_{np}}{\bar{c}_w}\right)\left(\Delta C_{L_0}\right)_{fg} - \left(\Delta C_{m_0}\right)_{fg}}{\left[C_{m_{\delta_e}} + C_{L_{\delta_e}}\left(l_{np}/\bar{c}_w\right)\right]}$$

Note: denominators are negative

$C_{m_{\delta_e}} < 0, C_{L_{\delta_e}} > 0$

$|C_{m_{\delta_e}}| > |C_{L_{\delta_e}}|$

- When flaps + gear are deployed, $\left(\Delta C_{L_0}\right)_{fg} > 0$ and $\left(\Delta C_{m_0}\right)_{fg} < 0$. The net effect:
  - Angle of attack is significantly reduced in landing configuration (good: better visibility)
  - For low static margin, more up-elevator. For high static margin, some down-elevator

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Elevator Sizing

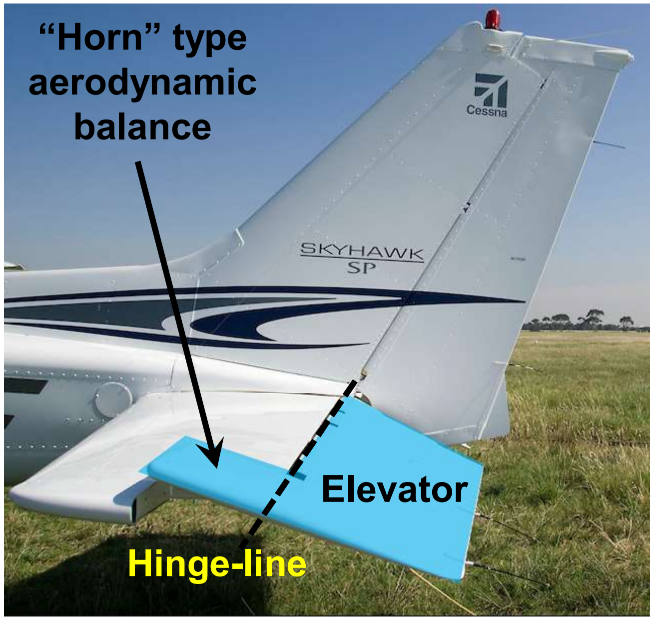
- The elevator must be sized to have sufficient authority to trim the airplane to achieve $C_{L_{max}}$ at forward CG and in ground effect with gear extended
  - For both flaps extended and flaps retracted (either may occur during landing)
  - For a sized elevator, this constraint may be used to specify forward CG limit
  - If forward CG limit has been otherwise established, this constraint can be used to size the elevator
  - Elevators are typically a constant fraction of the stabilizer chord, extending up to 95% or to the tip of the stabilizer (Raymer)
  - Flutter tendencies should be minimized by using mass balancing and aerodynamic balancing (the latter also reduces required control forces)
- For transonic and supersonic airplanes, the ***entire horizontal stabilizer rotates***
  - If this is done to aid in trim, it is called a trimmable horizontal stabilizer
  - If this is done to aid in control as well, it is often called a "stabilator"

Aircraft	Elevator C_e/C
Fighter/attack	0.30[a]
Jet transport	0.25[b]
Jet trainer	0.35
Biz jet	0.32[b]
GA single	0.45
GA twin	0.36
Sailplane	0.43

a) Supersonic usually all-moving only.
b) Often all-moving plus elevator.

Raymer, D., "Aircraft Design: A Conceptual Approach", 4th Ed. AIAA Education Series

Let’s Check Out Some Empennages



GA aircraft elevator with notch or “horn”-type aerodynamic balance. Located forward of the hinge-line, the horn deflects into the flow in the opposite direction of the elevator, and cancels out some of the hinge moment

Trimmable Horizontal Stabilizer on commercial jet transport. This allows the aircraft to be trimmed with the elevator faired (neutral). The elevators provide primary pitch control

F-16 aircraft with stabilators. There is no separate elevator. The stabilator provides pitch stability and pitch control

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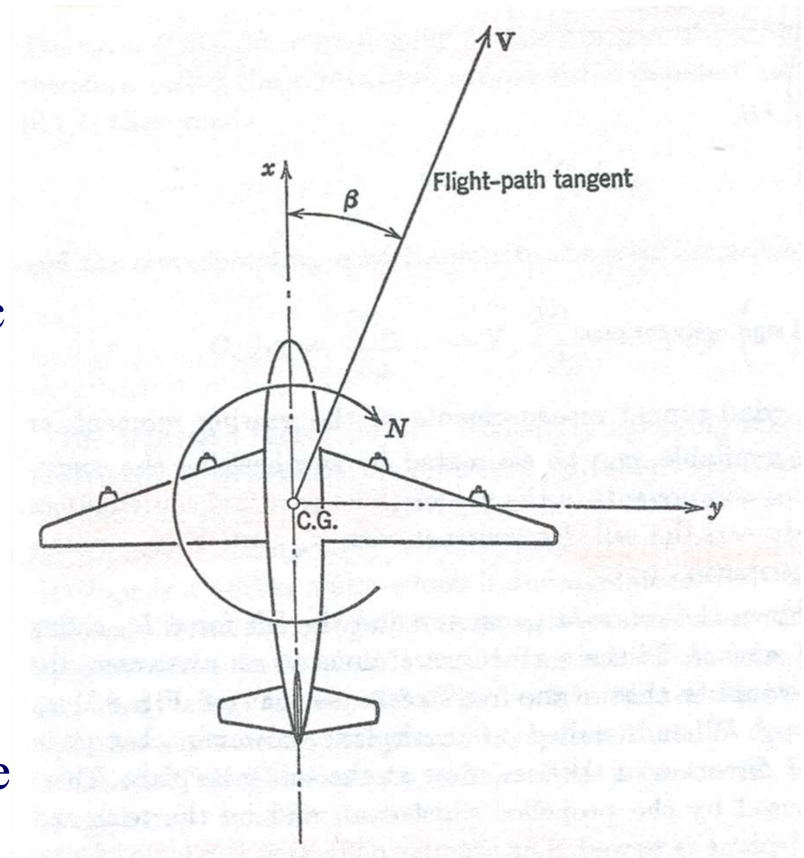
https://i.stack.imgur.com/9RQY4.jpg
https://en.wikipedia.org/wiki/Stabilator

Contents

Yaw Stiffness (Directional / Weathervane Stability)

- Analogous to pitch stiffness, a directionally stable aircraft should develop a moment that tends to reduce any sideslip disturbance.
- The criterion for positive yaw stiffness (static stability in yaw) is:
  $C_{n_\beta} = \frac{\partial C_n}{\partial \beta} > 0$
- The primary stabilizing contribution is from an aft tail
- Contributions from fuselage and propeller are typically destabilizing
- Wing contribution is not significant unless the wing is highly swept



Effect of Vertical Tail on Side Force

- Some immediate analogies with the longitudinal case:
  - Vertical tail volume ratio: $V_v = \frac{S_v \ell_v}{S_w b_w}$, sidewash σ and sidewash gradient $\frac{\partial \sigma}{\partial \beta}$, vertical tail efficiency η_v
- Contribution of vertical tail to side force:

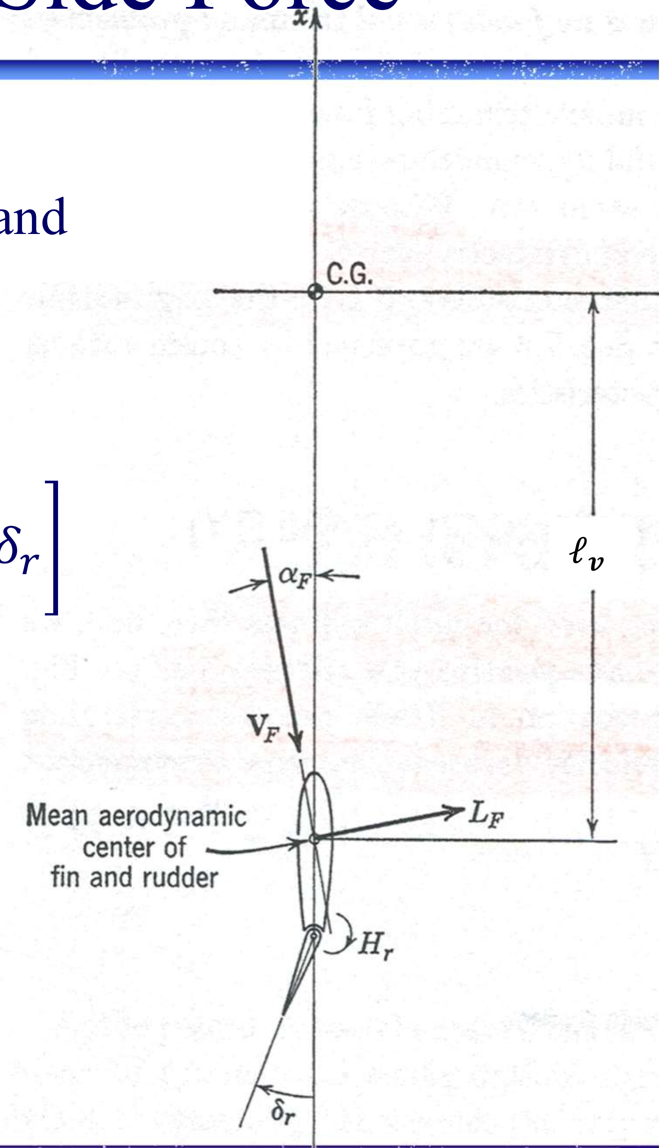
$$(\Delta C_Y)_v = -\eta_v \frac{S_v}{S_w} C_{L_{\alpha,v}} \left[\left(1 - \frac{\partial \sigma}{\partial \beta} \right) \beta - \epsilon_{s,0} + i_v - \epsilon_r \, \delta_r \right]$$

- Taking a partial derivative w.r.t. sideslip β

$$\left(\boldsymbol{C}_{\boldsymbol{Y}_{\boldsymbol{\beta}}} \right)_v = -\eta_v \frac{S_v}{S_w} C_{L_{\alpha,v}} \left(1 - \frac{\partial \sigma}{\partial \beta} \right)$$

- Taking a partial derivative w.r.t. rudder angle δ_r

$$\boldsymbol{C}_{\boldsymbol{Y}_{\boldsymbol{\delta_r}}} = \eta_v \frac{S_v}{S_w} C_{L_{\alpha,v}} \epsilon_r$$



Effect of Vertical Tail on Yawing Moment

- Considering the vertical tail moment arm ℓ_v, the yawing moment contribution can be summed up as:
$$(\Delta C_n)_v = \eta_v \frac{S_v \ell_v}{S_w b_w}\left[C_{L_{\alpha,v}}\left(1 - \frac{\partial\sigma}{\partial\beta}\right)\beta - C_{L_{\alpha,v}}(\epsilon_{s,0} + i_v) - \left(\epsilon_r C_{L_{\alpha,v}} - \frac{\bar{c}_v}{\ell_v} C_{m,ac,v_{\delta_r}}\right)\delta_r\right]$$

- Taking a partial derivative w.r.t. sideslip β
$$\left(\boldsymbol{C_{n_\beta}}\right)_v = \eta_v \frac{S_v \ell_v}{S_w b_w} C_{L_{\alpha,v}}\left(1 - \frac{\partial\sigma}{\partial\beta}\right)$$

- Taking a partial derivative w.r.t. rudder angle δ_r
$$\boldsymbol{C_{n_{\delta_r}}} = -\eta_v \frac{S_v \ell_v}{S_w b_w}\left(C_{L_{\alpha,v}}\epsilon_r - \frac{\bar{c}_v}{\ell_v} C_{m,ac,v_{\delta_r}}\right)$$

Effect of Fuselage + Nacelles & Direct Effect of Propellers

- Fuselage + nacelles: Analogous to the preliminary estimate for contribution to pitch stability, except with sideslip angle β replacing angle of attack α

$$(\Delta C_n)_{fu} = 2\frac{S_f \ell_f}{S_w b_w}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{\frac{3}{2}}\right]\beta_f \quad \Rightarrow \quad \left(C_{n_\beta}\right)_{fus} = 2\frac{S_f \ell_f}{S_w b_w}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{\frac{3}{2}}\right]$$

- Direct effect of propellers: Also analogous to the approach for pitch stability

$$(\Delta C_n)_p = \left(\Delta C_{n_0}\right)_p + \left(\Delta C_{n_\alpha}\right)_p \alpha + \left(\Delta C_{n_\beta}\right)_p \beta$$

$$\left(\Delta C_{n_0}\right)_p = -\frac{y_{bp}}{b_w} C_{T,p} + \frac{2\,d_p^3}{S_w b_w}\left(\frac{C_{n,p_\alpha}}{J^2}\right)(\alpha_{0p} - \epsilon_{d,0p})$$

$$\left(\Delta C_{n_\alpha}\right)_p = \frac{2\,d_p^3}{S_w \bar{c}_w}\left(\frac{C_{n,p_\alpha}}{J^2}\right)(1 - \partial\epsilon_{d,p}/\partial\alpha)$$

$C_{n,p_\alpha} < 0$ for clockwise-turning propellers. Thus these create a yawing moment to the left at positive AOA. This is called "p-factor"

The interaction of the rotating propeller slipstream with vertical tail in particular also generates yawing moments (leftward, for clockwise prop)

$$\left(\Delta C_{n_\beta}\right)_p = \frac{2\,d_p^2 \ell_p}{S_w \bar{c}_w}\left(\frac{C_{N,p_\alpha}}{J^2}\right)(1 - \partial\epsilon_{s,p}/\partial\beta)$$

As before, tractor propellers ($\ell_p < 0$) are destabilizing in yaw, while pusher propellers ($\ell_p > 0$) are stabilizing

Hoak, D.E., "USAF Stability and Control Datcom", AFWAL-TR-83-3048, 1960

Turning Tendencies due to Rotating Propeller

- This discussion applies to propellers turning clockwise when viewed from behind. The effects will reverse from counter-clockwise turning propellers

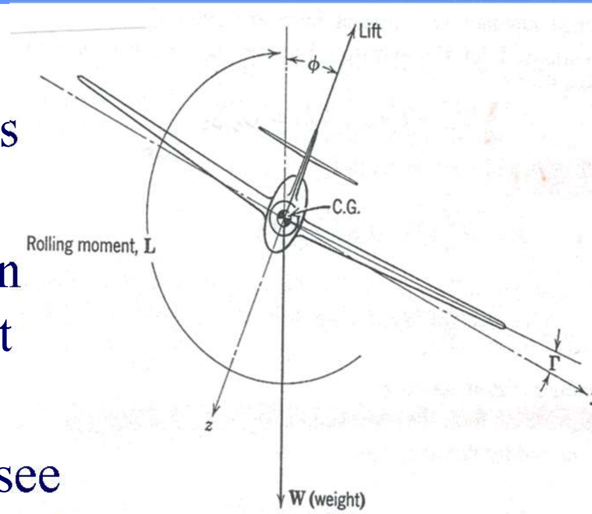
- Overall, a clockwise propeller has a left turning tendency, due to 4 causes:
  - Torque
  - P-Factor:
  - Gyroscopic moment:
  - Slipstream interaction:

Rudder Sizing and Control Power ($C_{n_{\delta_r}}$)

- The rudder has to be sized to permit aircraft control for a variety of constraining conditions:
  - **Coordinated turns**: against adverse yaw (most significant when flying at low airspeed)
  - **Crosswind landings**: rudder must be powerful enough to overcome the natural yaw stiffness of the aircraft and allow alignment with runway
  - **Asymmetric power conditions**: Must permit positive directional control after failure of the critical engine at minimum controllable airspeed
  - **Spin recovery**: The rudder must be powerful enough to oppose and stop the yaw rotation in a spin and permit recovery from the spin

$$\boldsymbol{C_{n_{\delta_r}}} = -\eta_v \frac{S_v \ell_v}{S_w b_w}\left(C_{L_{\alpha,v}} \epsilon_r - \frac{\bar{c}_v}{\ell_v} C_{m,ac,v_{\delta_r}}\right) = -\eta_v V_v \left(C_{L_{\alpha,v}} \epsilon_r - \frac{\bar{c}_v}{\ell_v} C_{m,ac,v_{\delta_r}}\right)$$

Roll Stiffness (Dihedral Effect, $C_{l\beta}$)



- Unlike pitch and yaw, ***there is no 1st order roll stiffness***
- Airplanes still have an inherent tendency to fly with wings level due to the presence of wing dihedral
- When there is a roll disturbance, a component of weight in the direction of the roll causes a sideslip to develop in that direction (i.e., if $\phi > 0$, then $\beta > 0$)
- The presence of dihedral causes the "windward" wing to see a higher angle of attack. This develops a restoring moment that reduces the roll ($\phi > 0 \rightarrow \beta > 0 \rightarrow C_l < 0$)

$$\left(C_{l_\beta}\right)_{\Gamma w} = -\frac{2\Gamma}{3\pi}\kappa_l \, C_{L_{\alpha},w}$$

Γ: dihedral (rad)

κ_l: chart look-up

- The derivative $C_{l_\beta} < 0$ is used as a measure of the dihedral effect. The mounting position of the wing relative to fuselage (low, mid, or high) also has an impact, with high wing having the strongest roll stiffness and low wing the least
- Excessive roll stiffness may be dealt with using some negative dihedral, called anhedral

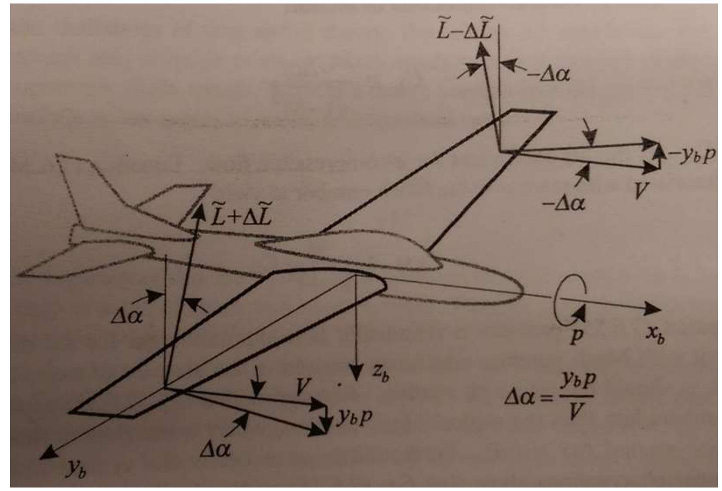
Roll Damping ($C_{l_{\hat{p}}}$)

- For most airplanes, the rolling motion is heavily damped. The primary influence is from the wing

- As the aircraft rolls, the roll rate causes the down-moving wing to see a higher angle of attack, and thus more lift.

- This creates an opposing rolling moment proportional to roll rate (i.e., a damping moment)

- This is captured through the roll damping derivative, $\mathbf{C_{l_{\hat{p}}}}$
- Note that at very high roll rates, the down-moving wing may stall. What then?



Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Aileron Sizing and Control Power $(C_{l_{\delta_a}})$

- The aileron control power $\left(C_{l_{\delta_a}}\right)$ denotes the rolling moment authority of the ailerons. Ailerons are typically sized based on a steady-state roll rate criterion $\left(\frac{p_{ss}b_w}{2V_0}\right)$

- An undesirable side-effect of aileron deflection is **adverse yaw**, caused by the down-deflected aileron generating more drag
- This causes a yawing moment in the opposite direction as the rolling moment
  - Captured using the derivative $\mathbf{C_{n_{\delta_a}}}$

Balance between rolling moments due to maximum aileron input and roll damping

$$C_l = C_{l_{\delta_a}} \Delta\delta_a + C_{l_{\hat{p}}} \hat{p}_{ss} = 0$$

$$C_{l_{\delta_a}} \Delta\delta_a + C_{l_{\hat{p}}} \frac{p_{ss} b_w}{2V_\infty} = 0$$

$$\left(\frac{p_{ss} b_w}{2V_\infty}\right) = -\frac{C_{l_{\delta_a}} \Delta\delta_a}{C_{l_{\hat{p}}}}$$

Low to medium maneuverability aircraft, e.g., cargo & transport airplanes:

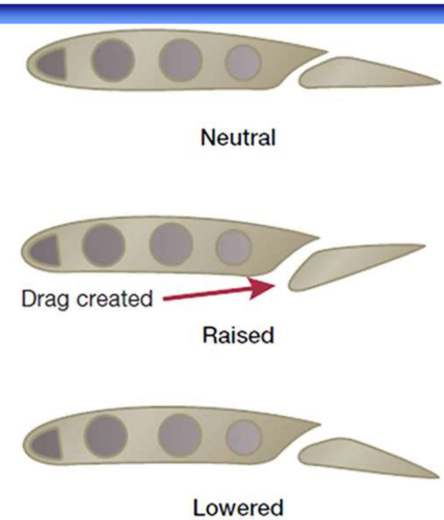
$$\left|\left(\frac{p_{ss} b_w}{2V_\infty}\right)\right| \geq 0.07$$

High maneuverability aircraft, e.g., fighters and aerobatic airplanes

$$\left|\left(\frac{p_{ss} b_w}{2V_\infty}\right)\right| \geq 0.09$$

Aileron Sizing and Control Power ($C_{l_{\delta_a}}$)

- Adverse yaw can be reduced or eliminated using:
  - **Differential aileron deflection**: Deflecting the up-deflected aileron more
  - **Frise ailerons**: where the leading edge of the up-deflected aileron projects into the flow to create drag
  - **Differential spoiler deflections**: deploying spoilers only on the down-moving wing. This creates rolling and yawing moments in the same direction



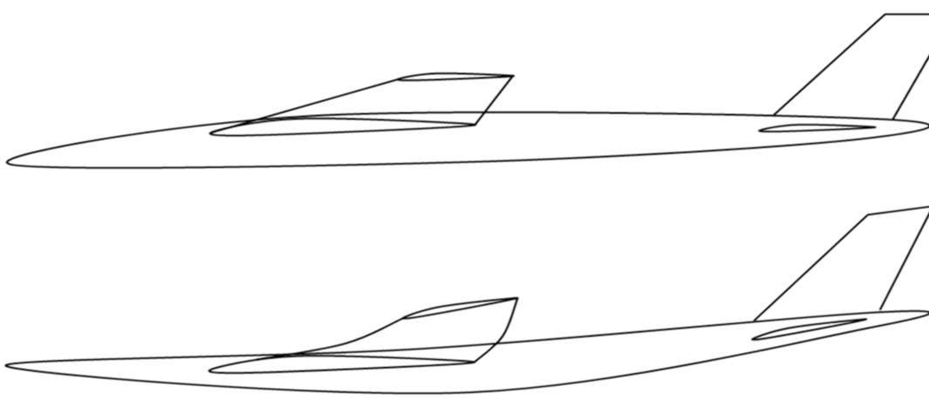
https://www.aviationsafetymagazine.com/features/two-aileron-types/

https://www.airlineratings.com/did-you-know/what-are-spoilers-and-what-are-they-used-for/

A Look at Lateral Control Surfaces

Aeroelastic Effects

- Common flexing occurs in the fuselage bending longitudinally and spanwise, as well as torsional bending of the wing
- Flexing of a swept wing will cause the lift-curve slope to be reduced
  - Negatively effects stability
  - Typical swept-wing transport at high subsonic speed can see a wing lift-curve slope reduction of about 20%, tail pitching moment reduction of 30%, 50% less elevator effectiveness, and the wing aerodynamic center moved 10% forward
- Aileron effectiveness can be **reduced to over 100%** due to ailerons twisting the wing in the opposite direction of aileron travel (known as ***aileron reversal***)



Aircraft Performing a Steady Turn

- Build up the longitudinal and lateral forces and moments
- <u>Assumption:</u> Excess power equation is satisfied implicitly

Longitudinal

Lift $C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\hat{q}}}\hat{q} + C_{L_{\delta_e}}\delta_e$

Pitching moment $C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_\beta}\beta + C_{m_{\hat{q}}}\hat{q} + C_{m_{\delta_e}}\delta_e$

Lateral

Side force $C_Y = C_{Y_\beta}\beta + C_{Y_{\hat{p}}}\hat{p} + C_{Y_{\hat{r}}}\hat{r} + C_{Y_{\delta_r}}\delta_r$

Rolling moment $C_l = C_{l_\beta}\beta + C_{l_{\hat{p}}}\hat{p} + C_{l_{\hat{r}}}\hat{r} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r$

Yawing moment $C_n = C_{n_\alpha}\alpha + C_{n_\beta}\beta + C_{n_{\hat{p}}}\hat{p} + C_{n_{\hat{r}}}\hat{r} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r$

- Collect relationships into matrix equation
- We want to solve for $\boldsymbol{X}$

$$\underbrace{\begin{bmatrix} C_{L_\alpha} & 0 & 0 & C_{L_{\delta_e}} & 0 \\ 0 & C_{Y_\beta} & 0 & 0 & C_{Y_{\delta_r}} \\ 0 & C_{l_\beta} & C_{l_{\delta_a}} & 0 & C_{l_{\delta_r}} \\ C_{m_\alpha} & C_{m_\beta} & 0 & C_{m_{\delta_e}} & 0 \\ C_{n_\alpha} & C_{n_\beta} & C_{n_{\delta_a}} & 0 & C_{n_{\delta_r}} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{Bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\delta_a} \\ \boldsymbol{\delta_e} \\ \boldsymbol{\delta_r} \end{Bmatrix}}_{\boldsymbol{X}} = \underbrace{\begin{Bmatrix} C_L - C_{L_0} \\ C_Y \\ C_l \\ C_m - C_{m_0} \\ C_n \end{Bmatrix}}_{\boldsymbol{B}} - \underbrace{\begin{bmatrix} 0 & C_{L_{\hat{q}}} & 0 \\ C_{Y_{\hat{p}}} & 0 & C_{Y_{\hat{r}}} \\ C_{l_{\hat{p}}} & 0 & C_{l_{\hat{r}}} \\ 0 & C_{m_{\hat{q}}} & 0 \\ C_{n_{\hat{p}}} & 0 & C_{n_{\hat{r}}} \end{bmatrix}}_{\boldsymbol{C}} \underbrace{\begin{Bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{Bmatrix}}_{\boldsymbol{D}}$$

$$\boxed{A\boldsymbol{X} = B - CD \rightarrow \quad \boldsymbol{X} = A^{-1}B + A^{-1}CD}$$

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Aircraft Performing a Steady Turn

- For aircraft in steady, climbing, coordinated turn: $C_L = n\, C_W \cos\gamma = \; C_W \cos\gamma \,/\, \cos\phi$
- For a coordinated turn, the side force should be zero: $C_Y = 0$
- There are no unbalanced rolling, pitching, or yawing moments.
  Therefore, $C_l = C_m = C_n = 0$
- The turn rate is related to bank angle and speed as: $\Omega = g \tan\phi / V_\infty$
  - If this is resolved in the body-fixed axes, we get the body-fixed angular velocities

    $$p = -\Omega \sin\gamma, \qquad q = \Omega\cos\gamma\sin\phi\,, \qquad r = \Omega\cos\gamma\cos\phi$$
  - Remember, these need to be converted to non-dimensional angular rates

    $$\hat{p} = \frac{p b_w}{2V_\infty}, \qquad \hat{q} = \frac{q \bar{c}_w}{2V_\infty}, \qquad \hat{r} = \frac{r b_w}{2V_\infty}$$

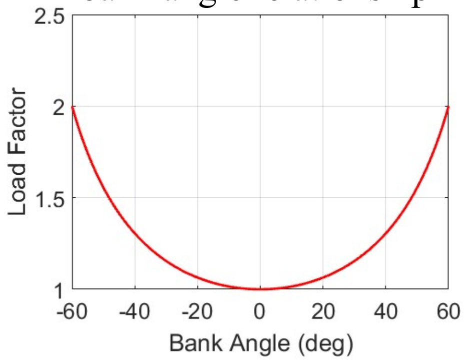
- We can now solve for $X = \{\alpha, \beta, \delta_a, \delta_e, \delta_r\}$ over a range of bank angles $\phi \in [-60^\circ, +60^\circ]$ to study the steady turn performance. For simplicity, we shall do this for $\gamma = 0$

$$\begin{bmatrix} C_{L_\alpha} & 0 & 0 & C_{L_{\delta_e}} & 0 \\ 0 & C_{Y_\beta} & 0 & 0 & C_{Y_{\delta_r}} \\ 0 & C_{l_\beta} & C_{l_{\delta_a}} & 0 & C_{l_{\delta_r}} \\ C_{m_\alpha} & C_{m_\beta} & 0 & C_{m_{\delta_e}} & 0 \\ C_{n_\alpha} & C_{n_\beta} & C_{n_{\delta_a}} & 0 & C_{n_{\delta_r}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\delta_a} \\ \boldsymbol{\delta_e} \\ \boldsymbol{\delta_r} \end{Bmatrix} = \begin{Bmatrix} C_W \dfrac{\cos\gamma}{\cos\phi} - C_{L_0} \\ 0 \\ 0 \\ -C_{m_0} \\ 0 \end{Bmatrix} - \begin{bmatrix} 0 & C_{L_{\hat{q}}} & 0 \\ C_{Y_{\hat{p}}} & 0 & C_{Y_{\hat{r}}} \\ C_{l_{\hat{p}}} & 0 & C_{l_{\hat{r}}} \\ 0 & C_{m_{\hat{q}}} & 0 \\ C_{n_{\hat{p}}} & 0 & C_{n_{\hat{r}}} \end{bmatrix} \begin{Bmatrix} -b_w \sin\gamma \\ \bar{c}_w \cos\gamma \sin\phi \\ b_w \cos\gamma\cos\phi \end{Bmatrix} \left(\frac{g\tan\phi}{2V_\infty^2} \right)$$

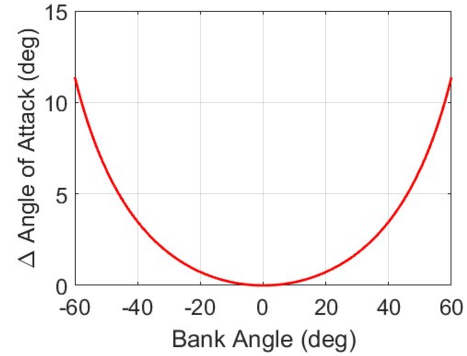
Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Aircraft Performing a Steady Turn

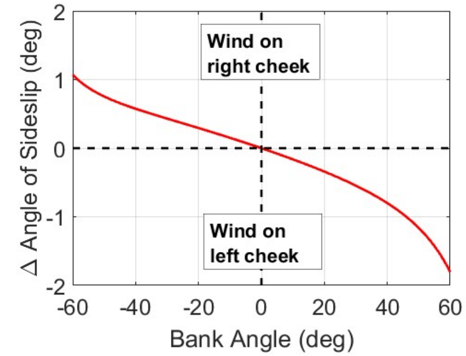
We are already familiar with the load factor vs. bank angle relationship



AOA must increase with bank angle to generate the additional lift needed



A coordinated turn requires $C_Y = 0$, not zero sideslip



$C_{l_\beta} < 0$

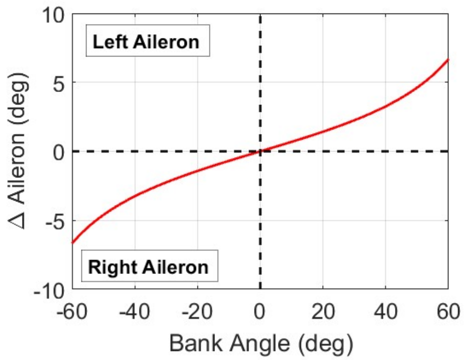
$C_{l_{\hat{p}}} < 0$

$C_{l_p} < 0$

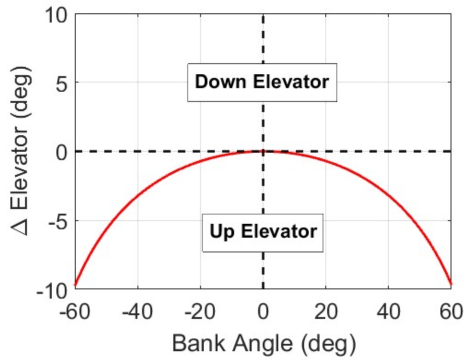
$C_{l_{\delta_a}} < 0$

$C_{l_{\hat{r}}} > 0$

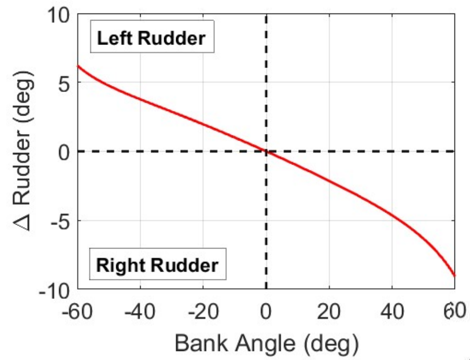
$C_{l_{\delta_r}} > 0$



"Hold bank with aileron": input direction depends on rolling moment balance



"Bank-and-yank": higher bank angles need more up-elevator ("yank") to achieve the higher AOA



"Keep the ball centered": This ensures $C_Y = 0$, and requires rudder input "into" the turn

$$\delta_a = -\frac{1}{C_{l_{\delta_a}}}\left(C_{l_\beta}\beta + C_{l_{\hat{p}}}\hat{p} + C_{l_{\hat{r}}}\hat{r} + C_{l_{\delta_r}}\delta_r\right)$$

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004 (Example 6.7.1)

Contents

- Fundamental Concepts and Reference Frames
- Aircraft Equations of Motion
- Longitudinal Stability
  - Flying Wing, Wing-Tail, and Canard-Wing Configurations
  - Additional factors affecting longitudinal stability
- Longitudinal Control and Maneuverability
- Lateral/Directional Stability, Control, and Maneuverability
- **Stability in Steady Flight**
  - Longitudinal Modes
    - Phugoid and Short Period Modes
    - Open Loop Response to Control Inputs
  - Lateral Modes
- Additional Topics

State-Space Representation of Aircraft Dynamics

- Note that the linearized equations are of the form:

$$\Delta\dot{\boldsymbol{X}} = A\,\Delta\boldsymbol{X} + B\,\Delta\boldsymbol{U}$$

- This is called the state-space representation of a dynamical system, in which
  - $\boldsymbol{X} \in \boldsymbol{R}^{\boldsymbol{n}}$: state vector
  - $\boldsymbol{U} \in \boldsymbol{R}^{\boldsymbol{p}}$: control or input vector
  - $\boldsymbol{A} \in \boldsymbol{R}^{\boldsymbol{n}\,\mathrm{x}\,\boldsymbol{n}}$: state or system matrix
  - $\boldsymbol{B} \in \boldsymbol{R}^{\boldsymbol{n}\,\mathrm{x}\,\boldsymbol{p}}$: control or input matrix

$$\Delta\dot{X}_{\mathrm{long}} = \mathrm{A}_{\mathrm{long}}\Delta X_{\mathrm{long}} + \mathrm{B}_{\mathrm{long}}\Delta U_{long}$$

$$\Delta\dot{X}_{\mathrm{lat}} = \mathrm{A}_{\mathrm{lat}}\Delta X_{\mathrm{lat}} + \mathrm{B}_{\mathrm{lat}}\Delta U_{lat}$$

$$\Delta X_{\mathrm{long}} = \begin{Bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta x_f \\ \Delta z_f \\ \Delta\theta \end{Bmatrix} \qquad \Delta X_{\mathrm{lat}} = \begin{Bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta y_f \\ \Delta\phi \\ \Delta\psi \end{Bmatrix}$$

- To assess the dynamic stability of the uncontrolled (or controls-fixed) system, we set $\Delta\boldsymbol{U} = \overline{\boldsymbol{0}}$, and look at the stability of the system $\Delta\dot{\boldsymbol{X}} = A\,\Delta\boldsymbol{X}$
- This can be done by solving the ***eigenproblem***, which yields the system's eigenvalues and corresponding eigenvectors

$$\Delta U_{long} = \Delta\delta_e$$

$$\Delta U_{lat} = \begin{Bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{Bmatrix}$$

Eigenvalues and Eigenvectors

- Consider the eigenproblem for the system
$$\Delta\dot{\boldsymbol{X}} = A\,\Delta\boldsymbol{X}$$
- The eigenvalues $\lambda = \{\lambda_1, \dots, \lambda_n\}$ are the roots of the characteristic equation:
$$\mathbf{det}(\boldsymbol{A} - \boldsymbol{\lambda}\,\boldsymbol{I}) = \mathbf{0}$$
- The eigenvector $\boldsymbol{v_i}$ for each eigenvalue $\boldsymbol{\lambda_i}$ is found by solving the equation
$$\boldsymbol{Av_i} = \boldsymbol{\lambda_i v}$$
- Eigenvectors are typically normalized to a magnitude of unity
- Eigenvalues and eigenvectors can be real or complex, with implications on dynamic stability

Consider a simple 2 x 2 system

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Find eigenvalues of this system

$$(A - \lambda I) = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (2-\lambda)^2 - 1 = 0$$

$$\lambda_1 = 1, \lambda_2 = 3$$

Find eigenvector for $\boldsymbol{\lambda_1} = \mathbf{1}$

$$Av_1 = \lambda_1 v_1 \rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = 1 \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix}$$

$$a_1 + b_1 = 0 \rightarrow v_1 = \begin{Bmatrix} 0.707 \\ -0.707 \end{Bmatrix}$$

Find eigenvector for $\boldsymbol{\lambda_2} = \mathbf{3}$

$$Av_2 = \lambda_2 v_2 \rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} a_2 \\ b_2 \end{Bmatrix} = 3 \begin{Bmatrix} a_2 \\ b_2 \end{Bmatrix}$$

$$a_2 - b_2 = 0 \rightarrow v_2 = \begin{Bmatrix} 0.707 \\ 0.707 \end{Bmatrix}$$

Interpretation of Eigenvalues

- Consider a general complex conjugate eigenvalue pair:

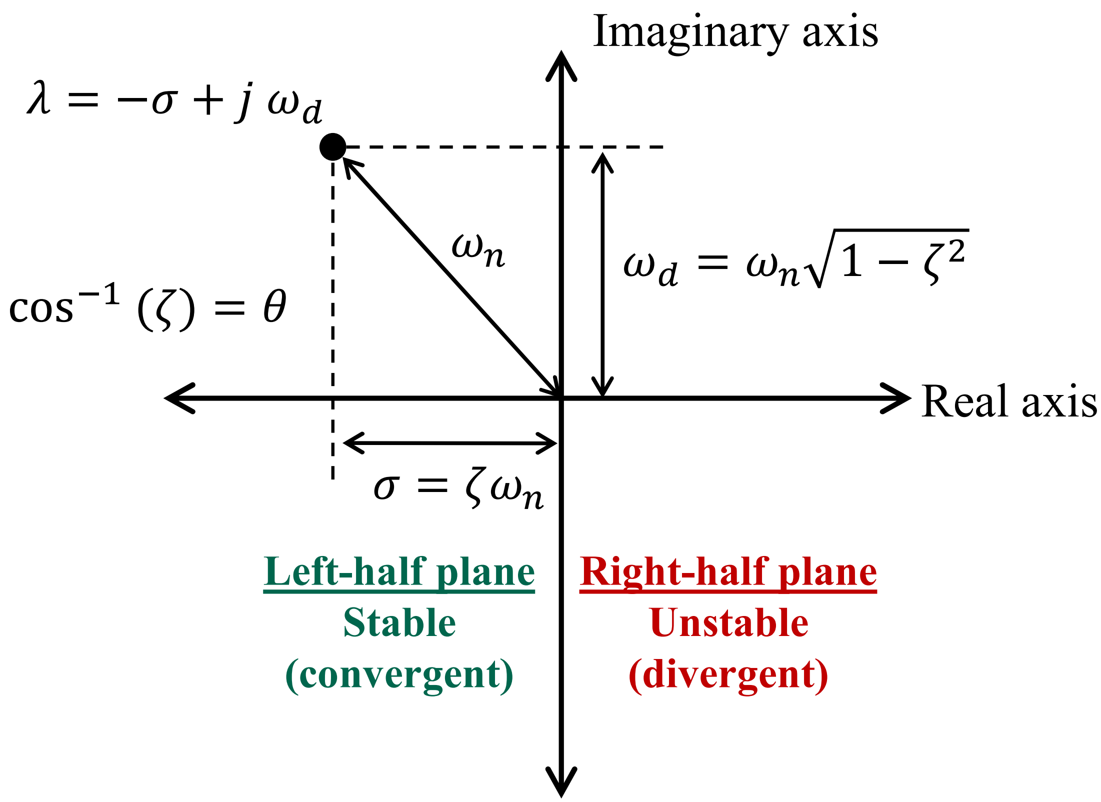
  $\boldsymbol{\lambda = -\sigma \pm j\, \omega_d}$ $\boldsymbol{\sigma}$**:** damping rate $\boldsymbol{\omega_d}$**:** damped natural frequency

- Purely real eigenvalue ($\omega_d = 0$):
  - **non-oscillatory mode**, damping = σ
  - $\boldsymbol{Re(\lambda) = 0} \rightarrow \sigma = 0$: rigid-body mode, describing rigid-body displacement
  - $\boldsymbol{Re(\lambda) < 0} \rightarrow \sigma > 0$: convergent mode, exponential return to equilibrium
  - $\boldsymbol{Re(\lambda) > 0} \rightarrow \sigma < 0$: divergent mode, exponential deviation from equilibrium

- Complex pair of eigenvalues ($\omega_d \neq 0$):
  - **oscillatory mode**, damping = σ, damped natural frequency = ω_d
  - $\boldsymbol{Re(\lambda) = 0} \rightarrow \sigma = 0$: undamped oscillatory mode
  - $\boldsymbol{Re(\lambda) < 0} \rightarrow \sigma > 0$: damped oscillatory mode
  - $\boldsymbol{Re(\lambda) > 0} \rightarrow \sigma < 0$: divergent oscillatory mode

Interpretation of Eigenvalues

$\boldsymbol{\lambda = -\sigma \pm j\,\omega_d}$ $\boldsymbol{\sigma}$**:** damping rate $\boldsymbol{\omega_d}$**:** damped natural frequency



Undamped natural frequency

$$\omega_n = \sqrt{\sigma^2 + \omega_d^2}$$

Damping ratio

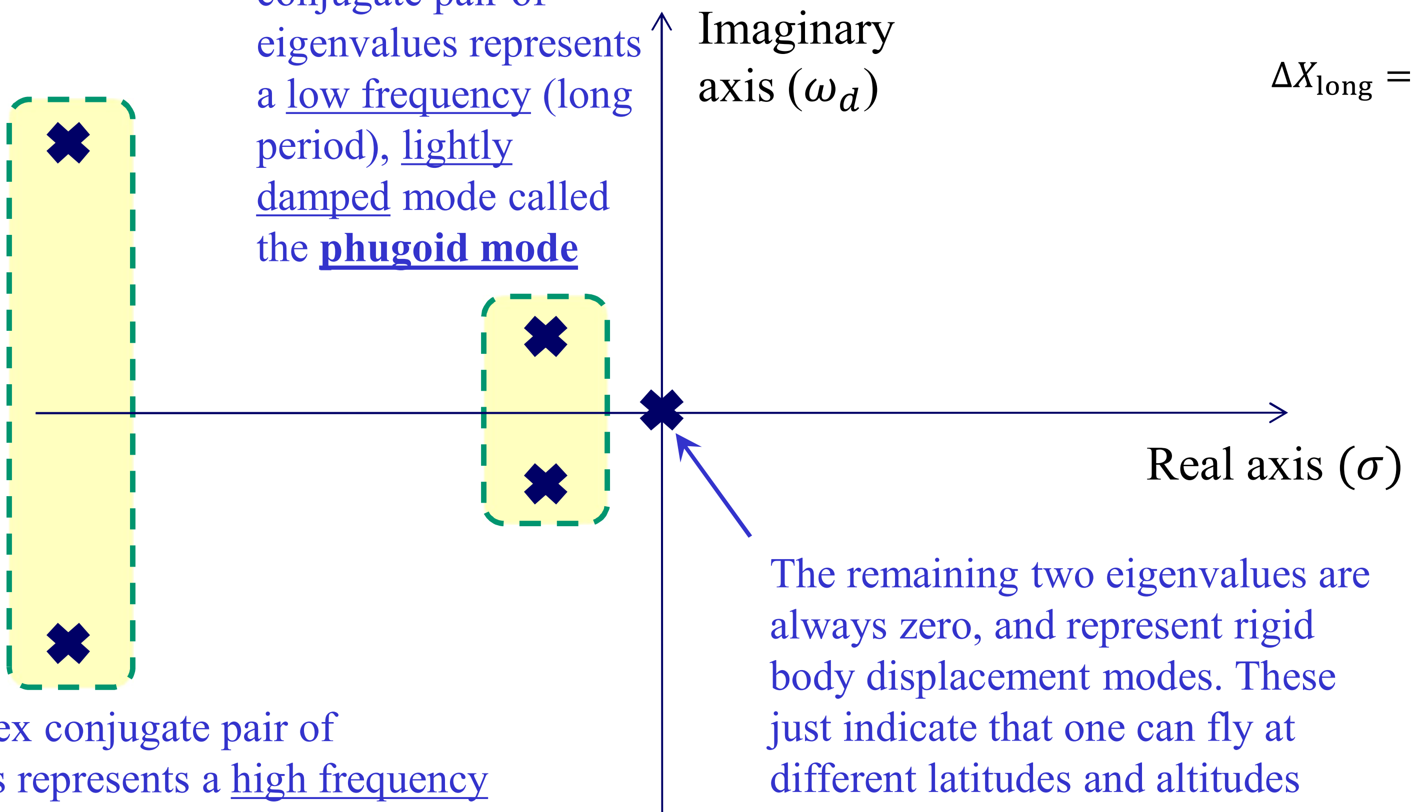
$$\zeta = \frac{\sigma}{\sqrt{\sigma^2 + \omega_d^2}}$$

Longitudinal Modes – Eigenproblem Solution

6 states, therefore 6 eigenvalues

$$\Delta X_{\text{long}} = \begin{Bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta x_f \\ \Delta z_f \\ \Delta \theta \end{Bmatrix}$$

Another complex conjugate pair of eigenvalues represents a low frequency (long period), lightly damped mode called the **phugoid mode**



The remaining two eigenvalues are always zero, and represent rigid body displacement modes. These just indicate that one can fly at different latitudes and altitudes

One complex conjugate pair of eigenvalues represents a high frequency (short period), highly damped mode called the **short period mode**

(not drawn to scale)

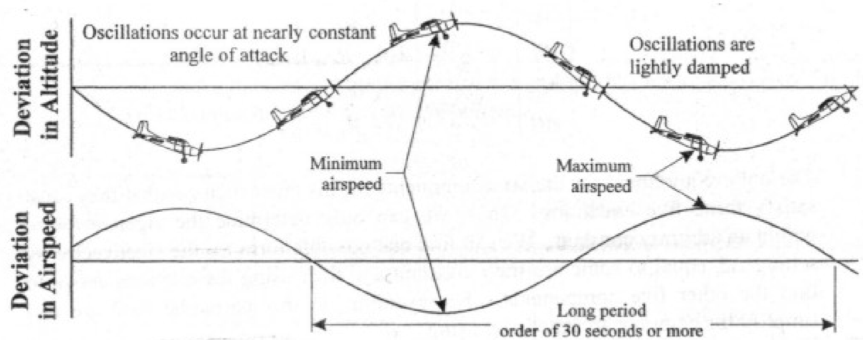
Longitudinal Modes: Phugoid & Short Period

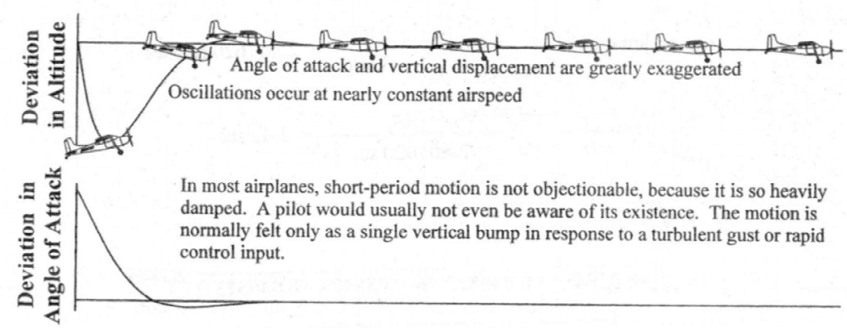
Phugoid (Long Period) Mode

- Aircraft exchanges translational KE & PE
- Speed leads the pitch angle (or FPA) by about 90 deg in phase
- Very little AoA variation
- Long period, lightly damped. Damping reduces with increasing L/D. Damping can even be negative at certain airspeeds
- Light phugoid damping objectionable in IFR flight, but usually not in VFR flight

Short Period Mode

- Aircraft exchanges rotational KE & PE
- There is negligible speed variation
- Rapid AoA and pitch variations which lead variation in vertical displacement (altitude) by about 90 deg
- If high frequency and heavily damped, airplane responds rapidly to elevator input without undesirable overshoot





Short Period Mode Approximation

- Damped natural frequency ($\omega_{d,sp}$):

$$\omega_{d,sp} = \frac{2V_0}{\bar{c}_w}\sqrt{\left(R_{z_\alpha}R_{m_{\hat{q}}} - R_{m_\alpha}\right) - \left(\frac{R_{z_\alpha} + R_{m_q} + R_{m_{\hat{\alpha}}}}{2}\right)^2}$$

$$R_{m_\alpha} = \frac{\rho S_w \bar{c}_w^3}{8\, I_{yy}} C_{m_\alpha}$$

$$R_{m_{\hat{q}}} = \frac{\rho S_w \bar{c}_w^3}{8\, I_{yy}} C_{m_{\hat{q}}}$$

- Damping rate (σ_{sp}):

$$\sigma_{sp} = -\frac{V_0}{\bar{c}_w}\left(R_{z_\alpha} + R_{m_{\hat{q}}} + R_{m_{\hat{\alpha}}}\right)$$

$$R_{z_\alpha} = \frac{\rho S_w \bar{c}_w}{4\frac{W}{g}}\left(-C_{L_\alpha} - C_D\right)$$

$$R_{m_{\hat{\alpha}}} = \frac{\rho S_w \bar{c}_w^3}{8\, I_{yy}} C_{m_{\hat{\alpha}}}$$

- Increasing the pitch damping derivative $C_{m_{\hat{q}}}$ and AOA-rate damping derivative $C_{m_{\hat{\alpha}}}$ will increase short period damping

- Increasing pitch stiffness $|C_{m_\alpha}|$ (static stability) will increase short period frequency

- Note that altitude effects enter some of the coefficients through density (ρ). Damping is higher in the denser lower atmosphere. The frequency does not vary much with altitude

Phugoid Mode Approximation

- Approximate phugoid damped natural frequency ($\omega_{d,p}$):

$$\omega_{d,p} = \frac{2V_0}{\bar{c}_w}\left[\frac{R_{z_\mu}}{2}\sqrt{-\frac{4R_{gx}}{R_{z_\mu}}R_s - \left(\frac{R_{x_\mu}}{R_{z_\mu}} + R_d\right)^2}\right]$$

- Approximate phugoid damping rate (σ_p):

$$\sigma_p = -\frac{2V_0}{\bar{c}_w}\left[\frac{R_{z_\mu}}{2}\left(\frac{R_{x_\mu}}{R_{z_\mu}} + R_d - R_p\right)\right]$$

- Phugoid damping can be increased by *reducing* the lift-to-drag ratio, but this is clearly inefficient. An alternative (if phugoid) damping is unacceptable is to use a phugoid suppression system (example of SAS)

- Damping and frequency are higher at denser, lower altitudes and lower at higher altitudes

$$R_s = \frac{R_{m_\alpha}}{R_{m_\alpha} - R_{z_\alpha}R_{m_{\hat{q}}}}$$

$$R_d = \frac{R_{x_\alpha}R_{m_{\hat{q}}}}{R_{m_\alpha} - R_{z_\alpha}R_{m_{\hat{q}}}}$$

$$R_d = \frac{R_{x_\alpha}R_{m_{\hat{q}}}}{R_{m_\alpha} - R_{z_\alpha}R_{m_{\hat{q}}}}$$

$$R_p = R_{gx}R_s\left(\frac{R_{z_\alpha} + R_{m_{\hat{q}}}}{R_{m_\alpha} - R_{z_\alpha}R_{m_{\hat{q}}}}\right)$$

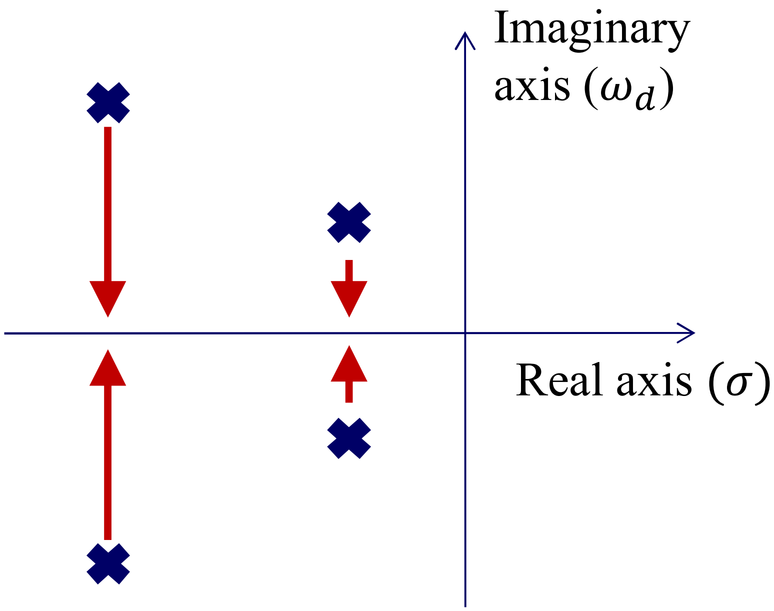
$$R_{gx} = \frac{g\bar{c}_w}{2\,V_0^2}$$

$$R_{x_\mu} = \frac{\rho S_w \bar{c}_w}{4W/g}\left[-2C_D - M\,C_{D_M} + \frac{T_V\cos(\alpha_{T0})}{\frac{1}{2}\rho V_0 S_w}\right]$$

$$R_{z_\mu} = \frac{\rho S_w \bar{c}_w}{4W/g}\left[-2C_L - M\,C_{L_M} - \frac{T_V\sin(\alpha_{T0})}{\frac{1}{2}\rho V_0 S_w}\right]$$

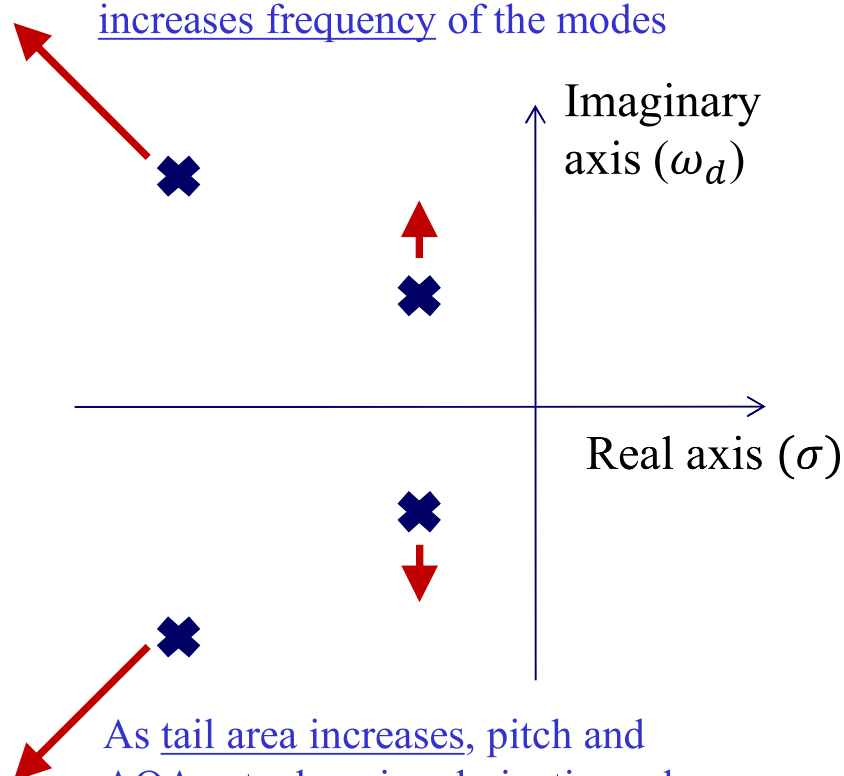
Effect of CG Position and Tail Area on Longitudinal Modes

As the CG moves aft, the frequency of both longitudinal modes reduces (longer period)



As the CG moves further rearward, the modes become aperiodic, and ultimately unstable

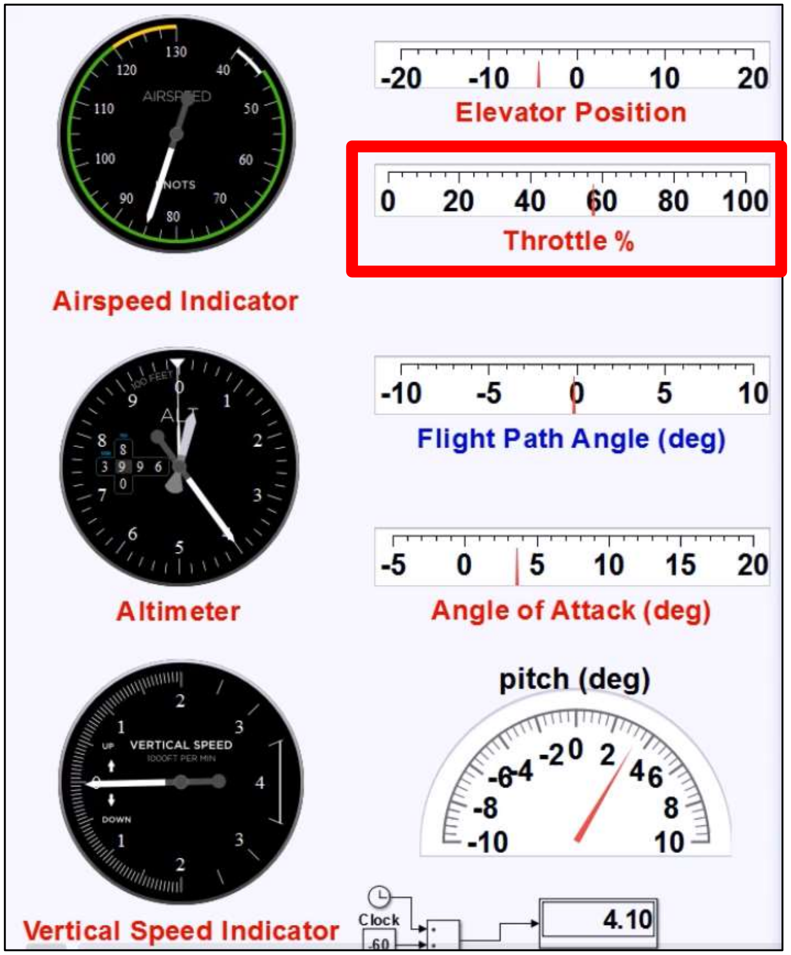
As tail area increases, static stiffness increases, which increases frequency of the modes

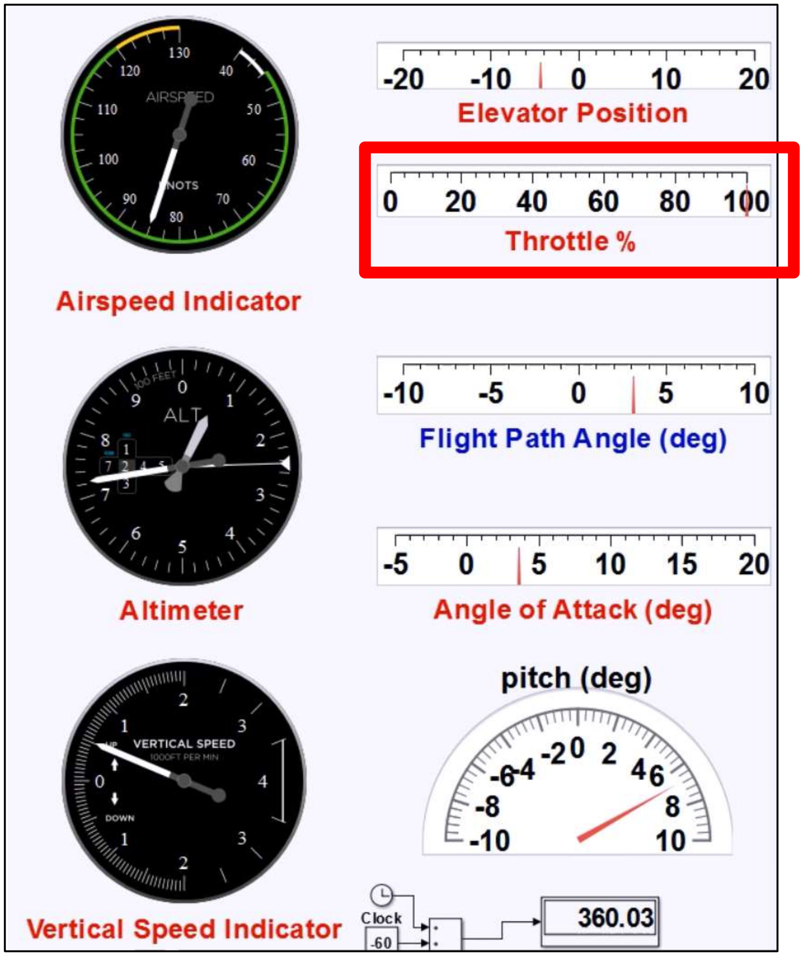


As tail area increases, pitch and AOA-rate damping derivatives also increase in magnitude, which increases short period mode damping

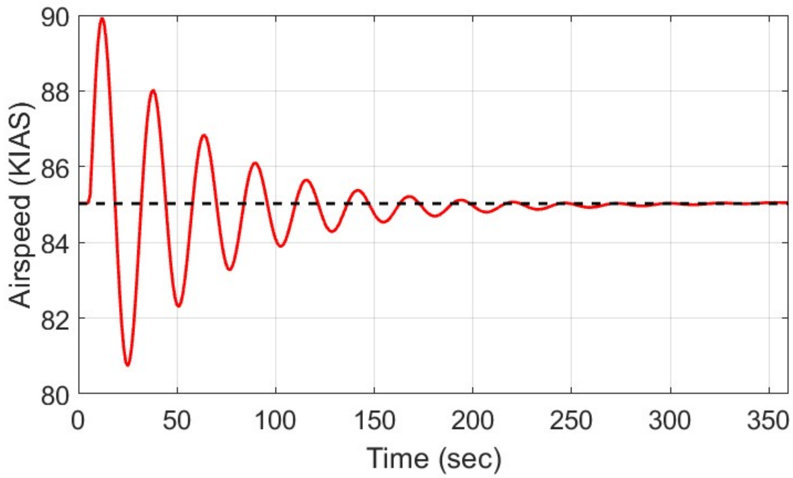
Effect of Throttle Step Input (Elevator Fixed)

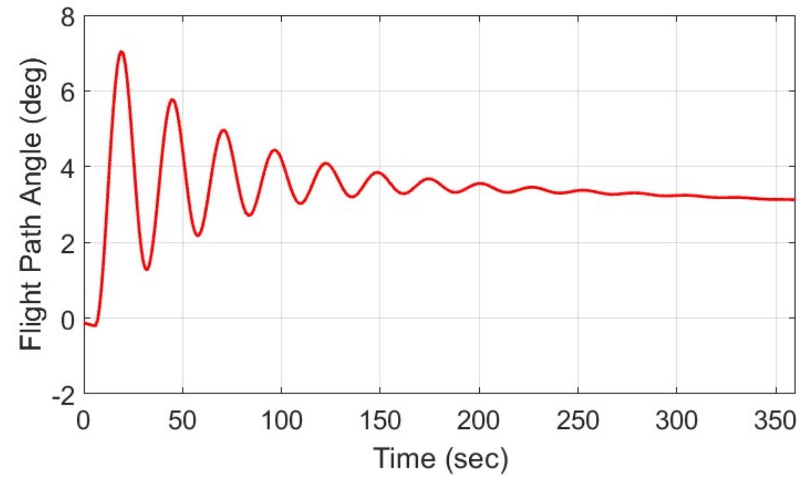
- Full throttle applied at t = 5 sec. What's the net effect after t = 6 min?

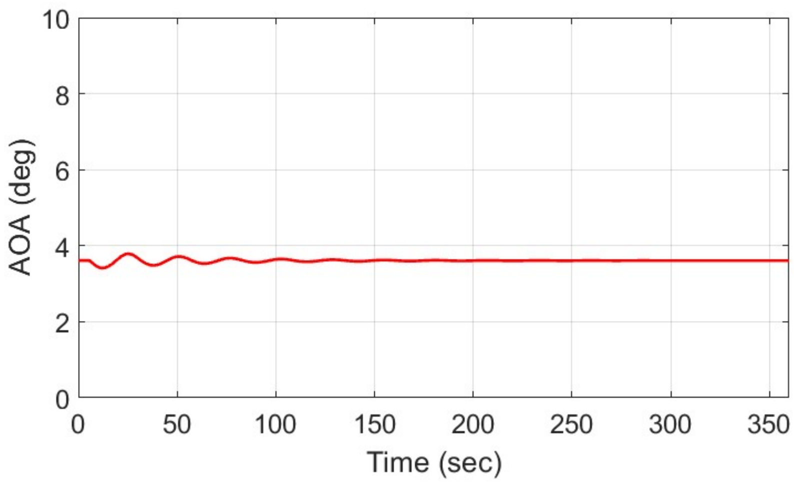


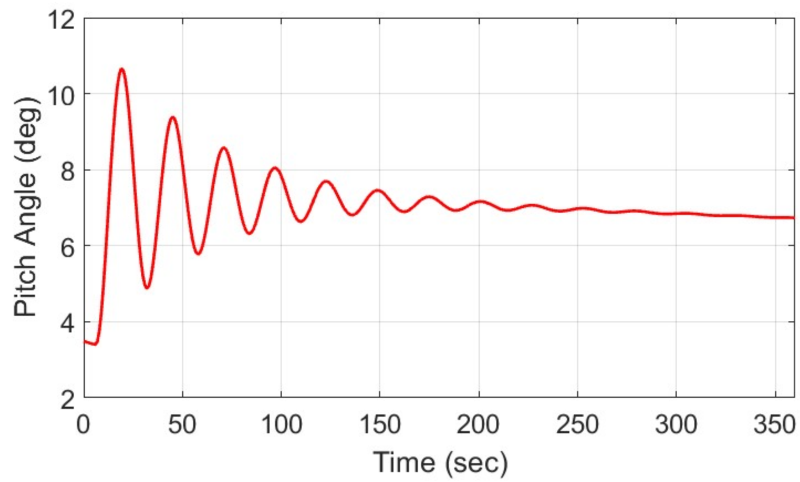


Effect of Throttle Step Input (Elevator Fixed)



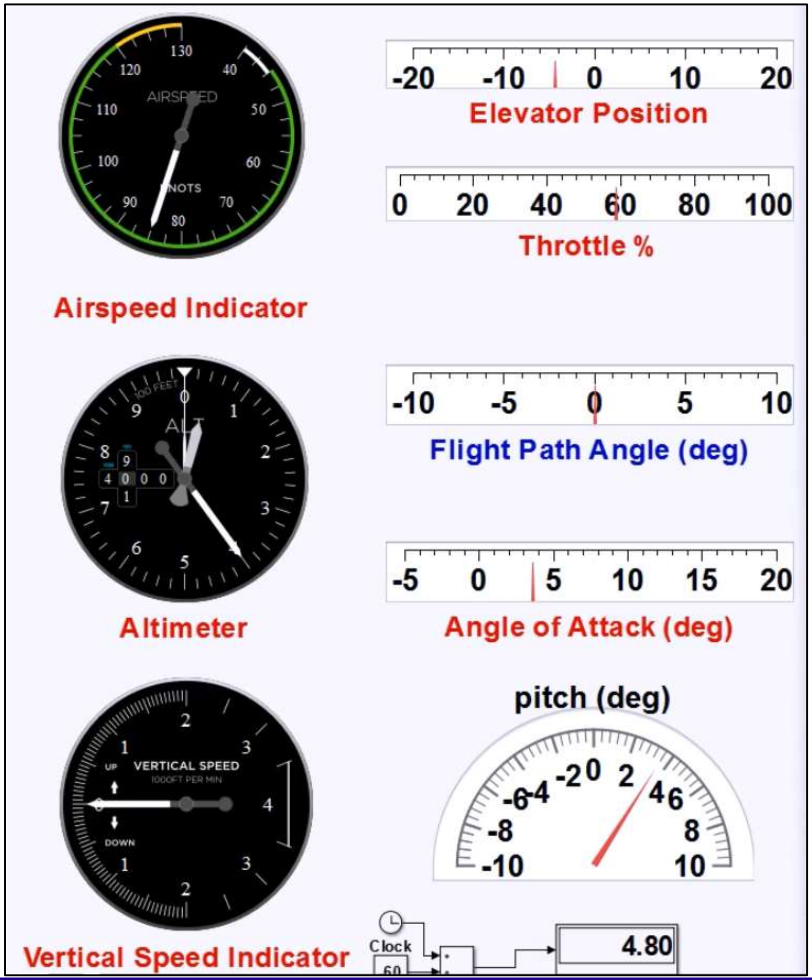


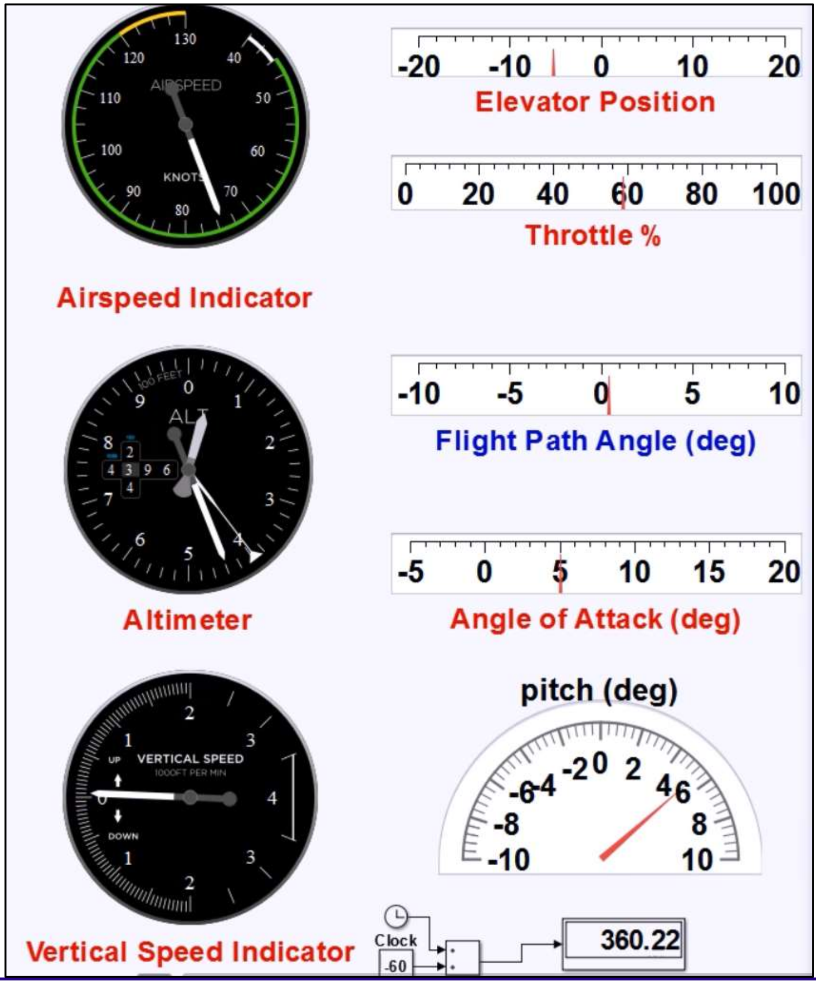




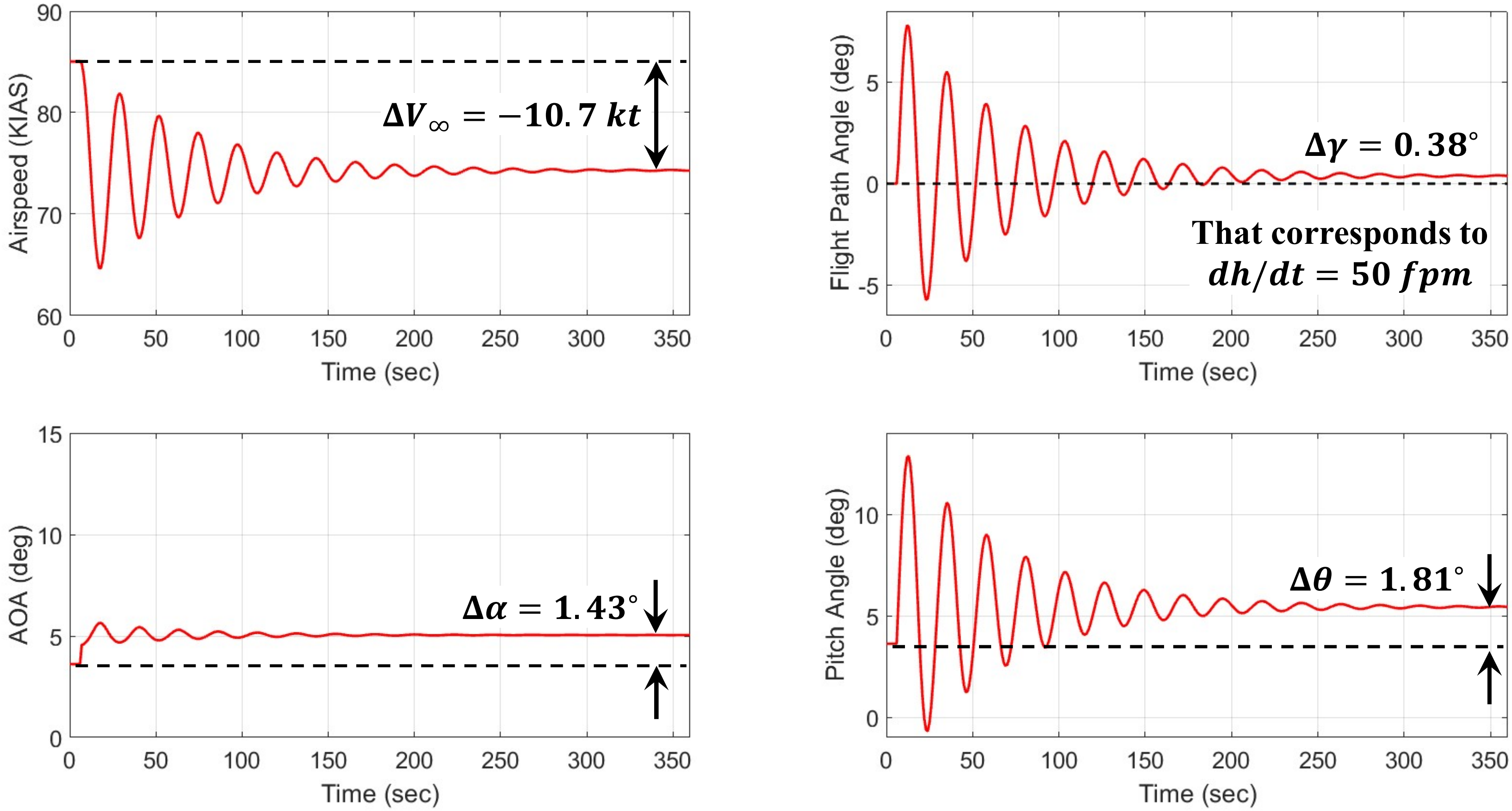
Effect of Elevator Step Input (Throttle Fixed)

- −1° elevator step input at t = 5 sec. What's the net effect after t = 6 min?

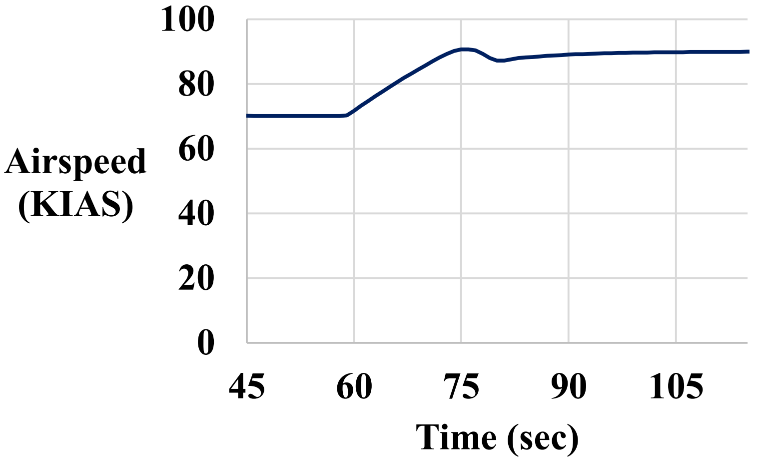


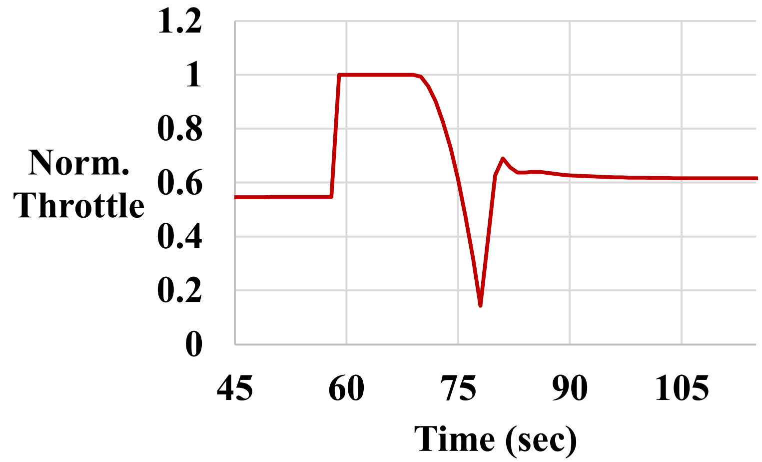


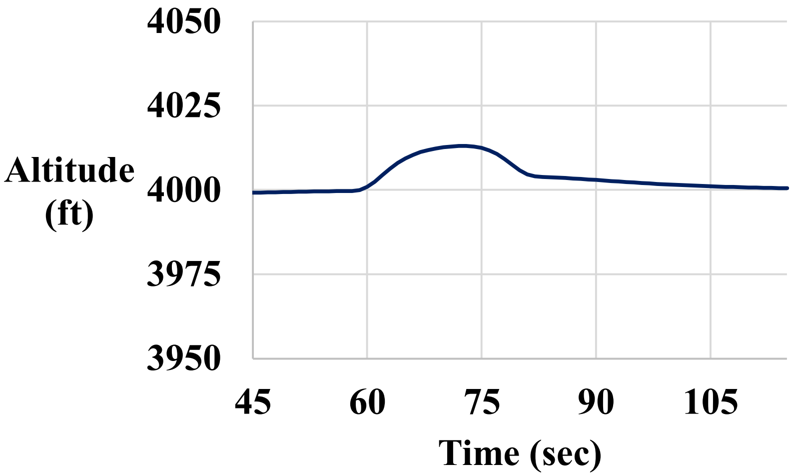
Effect of Elevator Step Input (Throttle Fixed)

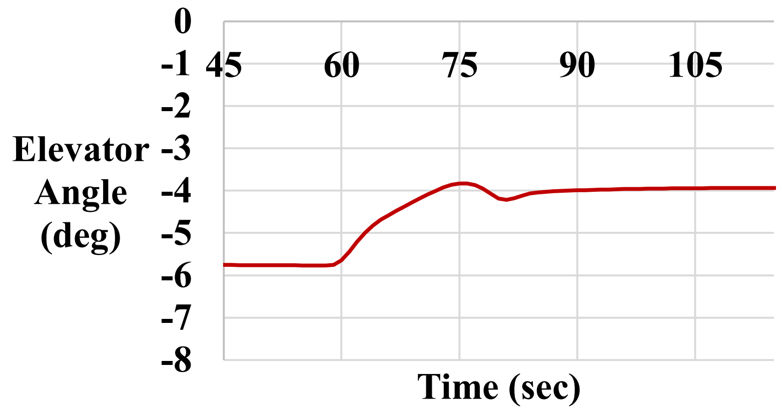


Acceleration Requires Both Throttle & Elevator Inputs





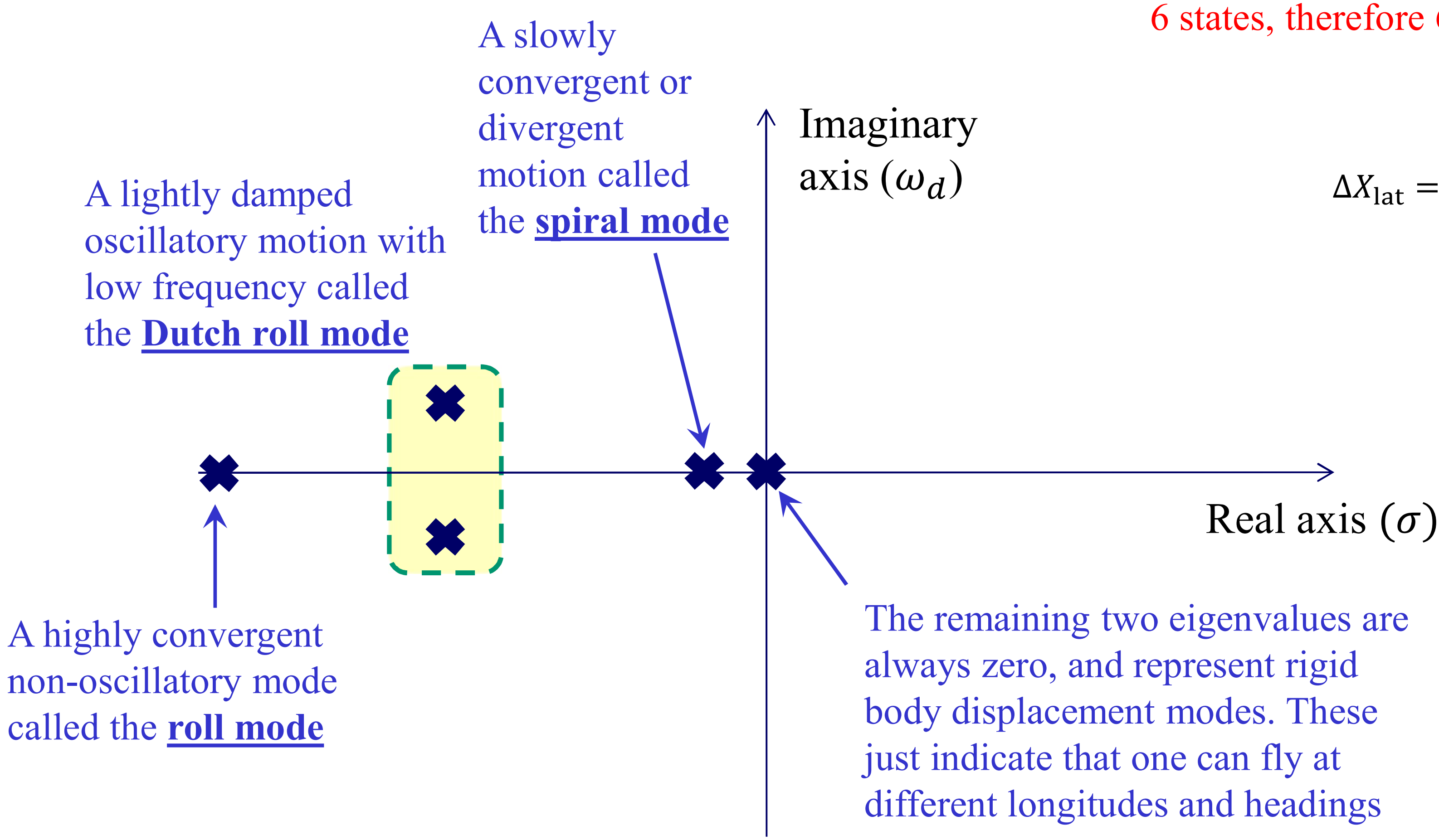




Lateral Modes – Eigenproblem Solution

6 states, therefore 6 eigenvalues

$$\Delta X_{\text{lat}} = \begin{Bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta y_f \\ \Delta \phi \\ \Delta \psi \end{Bmatrix}$$



(not drawn to scale)

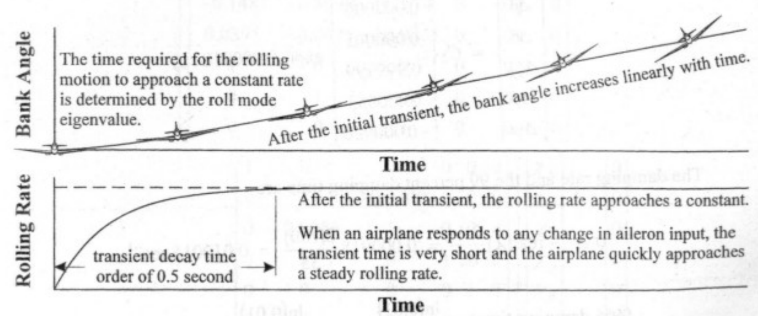
The Roll Mode

- The roll mode is very heavily damped and is of no concern to pilots/passengers

- Because it is overdamped and highly convergent, transients in rolling motion are quickly damped out and a steady rolling rate is quickly approached
  - As a result, pilots tend to associate aileron input with the steady state roll rate (i.e., larger aileron input yields higher steady state roll rate)

- If roll damping is insufficient, pilots will feel that they are commanding roll acceleration instead of roll rate, and will give a poor handling quality rating

- Approximate roll mode damping rate, $\sigma_r = -\frac{\rho S_w b_w^2 V_0}{4\, I_{xx}}\, C_{l_{\hat{p}}}$
  - Clearly, it is driven by the magnitude of the roll damping derivative
  - Roll damping reduces with reducing speed



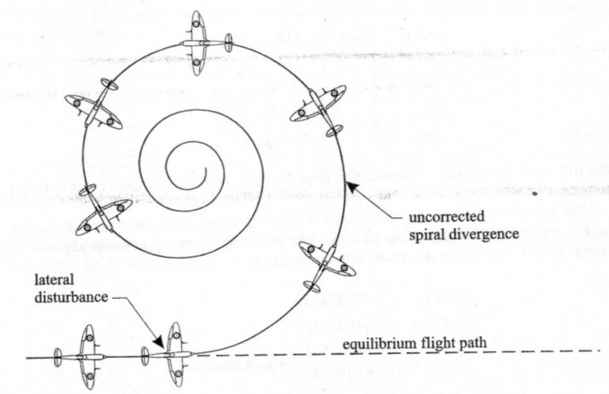
The Spiral Mode

- The spiral mode is usually convergent (damping reduces at lower speeds). If divergent, it is so slowly divergent that the pilot usually corrects for it without any problem

- If an airplane with an unstable spiral mode is disturbed laterally, it will begin a slow spiral in the direction of the disturbance.
  - As the spiral deviation becomes large, longitudinal motion will also be induced and an ever-tightening high-speed spiral dive may result
  - This can be a problem if inadvertently entering clouds while VFR or suffering failure of attitude indicating instruments

- Approximate spiral mode damping rate: $\sigma_s = \frac{g}{V_0}\left(\frac{C_{l_\beta} C_{n_{\hat{r}}} - C_{l_{\hat{r}}} C_{n_\beta}}{C_{l_\beta} C_{n_{\hat{p}}} - C_{l_{\hat{p}}} C_{n_\beta}}\right)$

- Root locus analysis reveals that increasing the dihedral effect (C_{l_β} more negative) makes the spiral mode more stable. Increasing directional stability (C_{n_β} more positive) makes the spiral mode less stable.



- However, the Dutch roll mode behaves in the exact opposite manner!

The Dutch Roll Mode

Dutch Roll Simulation
(courtesy Prof. Eric Johnson)

- Damped oscillatory motion with combination of sideslip, roll, and yaw.
  - When amplitude is large, longitudinal motion is also induced
  - Damping reduces as lower speeds

- Period for this motion is of the order of a few seconds. Thus, it can be very annoying or uncomfortable for pilots and passengers if damping is insufficient

- Approximate damped natural frequency and damping rate for Dutch roll mode:

$$\omega_{d,dr} = \frac{2V_0}{b_w}\sqrt{(1 - R_{y_{\hat{r}}})R_{n_\beta} + R_{y_\beta}R_{n_{\hat{r}}} + R_{D_s} - \left(\frac{R_{y_\beta} + R_{n_{\hat{r}}}}{2}\right)^2}$$

$$\sigma_{dr} = -\frac{V_0}{b_w}\left(R_{y_\beta} + R_{n_{\hat{r}}} - R_{D_c} + R_{D_p}\right)$$

- Dutch roll mode frequency increases if directional stability (C_{n_β}) is increased.

- Increasing yaw damping ($|C_{n_{\hat{r}}}|$) will result in better Dutch roll damping. On the other hand, increasing dihedral effect ($|C_{l_\beta}|$) makes the Dutch roll less stable. These are opposite to what is observed for the spiral mode

Nelson, R.C., "Flight Stability and Automatic Control", 2nd Ed., McGraw Hill, 1997

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Spiral Mode vs. Dutch Roll Mode - Tradeoff

- Designing good Dutch roll characteristics while maintaining acceptable roll, yaw, and spiral stability is challenging
- The only way to increase both Dutch roll and spiral damping is to increase yaw damping (higher $|C_{n_{\hat{r}}}|$)
  - This is difficult to achieve using geometric design changes alone

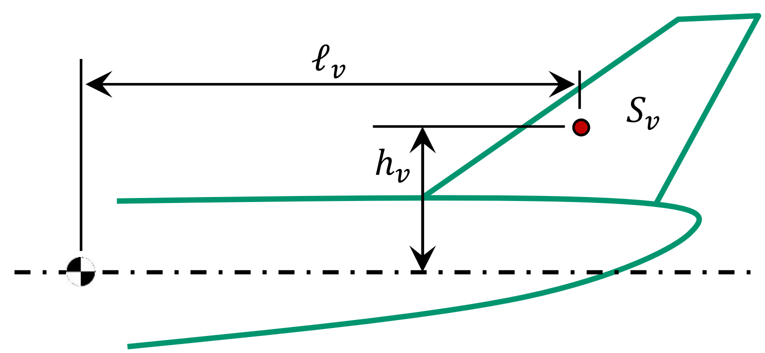
With increasing magnitudes of these derivatives…		Dutch Roll Mode Damping	Spiral Mode Damping
Roll stability	$\lvert C_{l_\beta}\rvert$	Decreases	Increases
Roll damping	$\lvert C_{l_{\hat{p}}}\rvert$	Increases	Decreases
Yaw stability	$\lvert C_{n_\beta}\rvert$	Increases	Decreases
Yaw damping	$\lvert C_{n_{\hat{r}}}\rvert$	Increases	Increases

$$\left(\boldsymbol{C_{n_\beta}}\right)_v = \eta_v \frac{S_v \ell_v}{S_w b_w} C_{L_{\alpha,v}} \left(1 - \frac{\partial \sigma}{\partial \beta}\right)$$

$$\left(\boldsymbol{C_{l_\beta}}\right)_v = -\eta_v \frac{S_v h_v}{S_w b_w} C_{L_{\alpha,v}} \left(1 - \frac{\partial \sigma}{\partial \beta}\right)$$

$$\left(\boldsymbol{C_{n_{\hat{r}}}}\right)_v = -\eta_v \frac{2\, S_v \ell_v^2}{S_w b_w^2} C_{L_{\alpha,v}} \left(1 - \frac{\partial \sigma}{\partial \beta}\right)$$

<u>**Note:**</u> Attempting to change the vertical tail area to increase yaw damping ($|C_{n_{\hat{r}}}|$) will also result in changes to the other derivatives



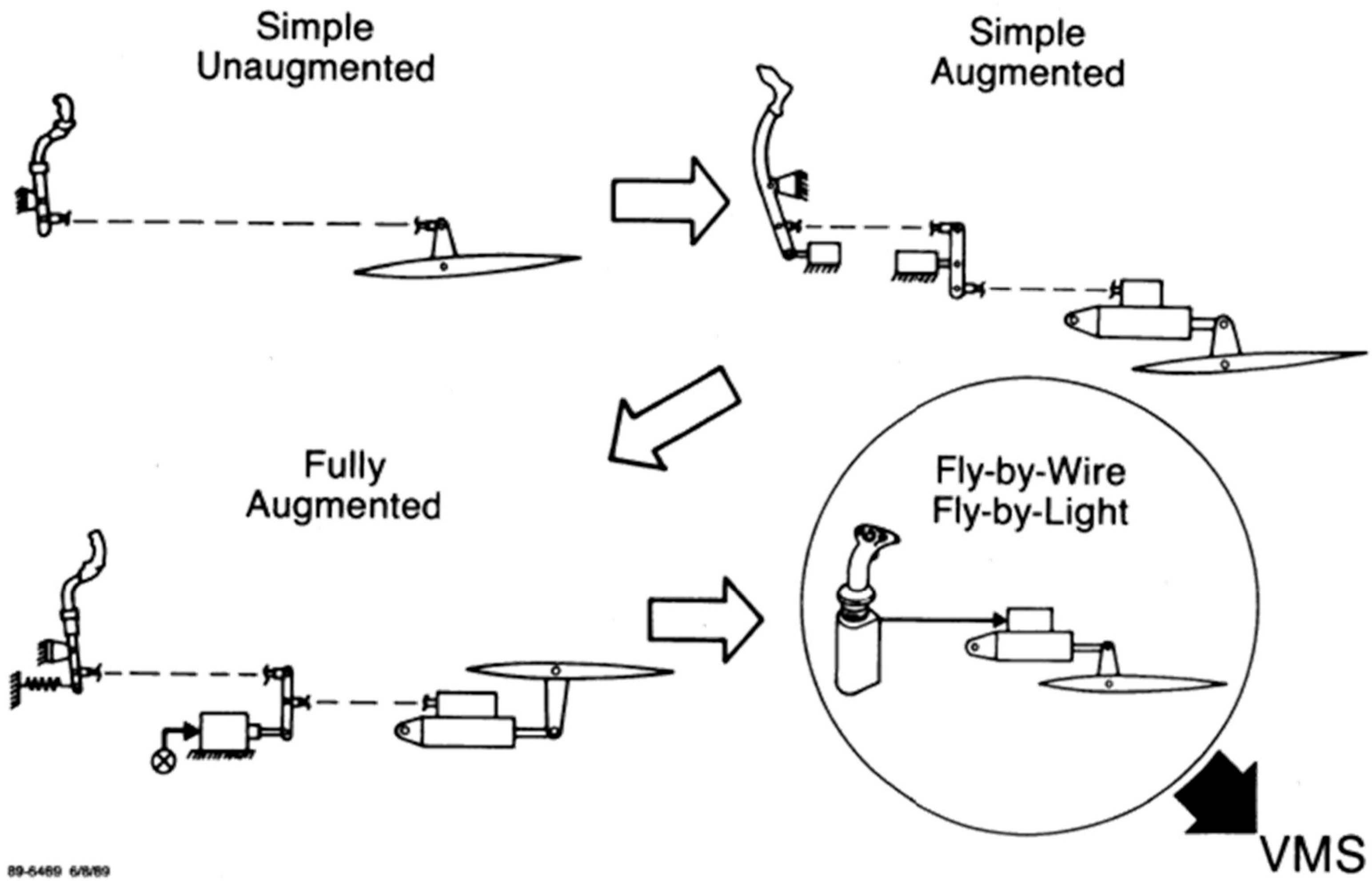
Contents

Beyond the Basics

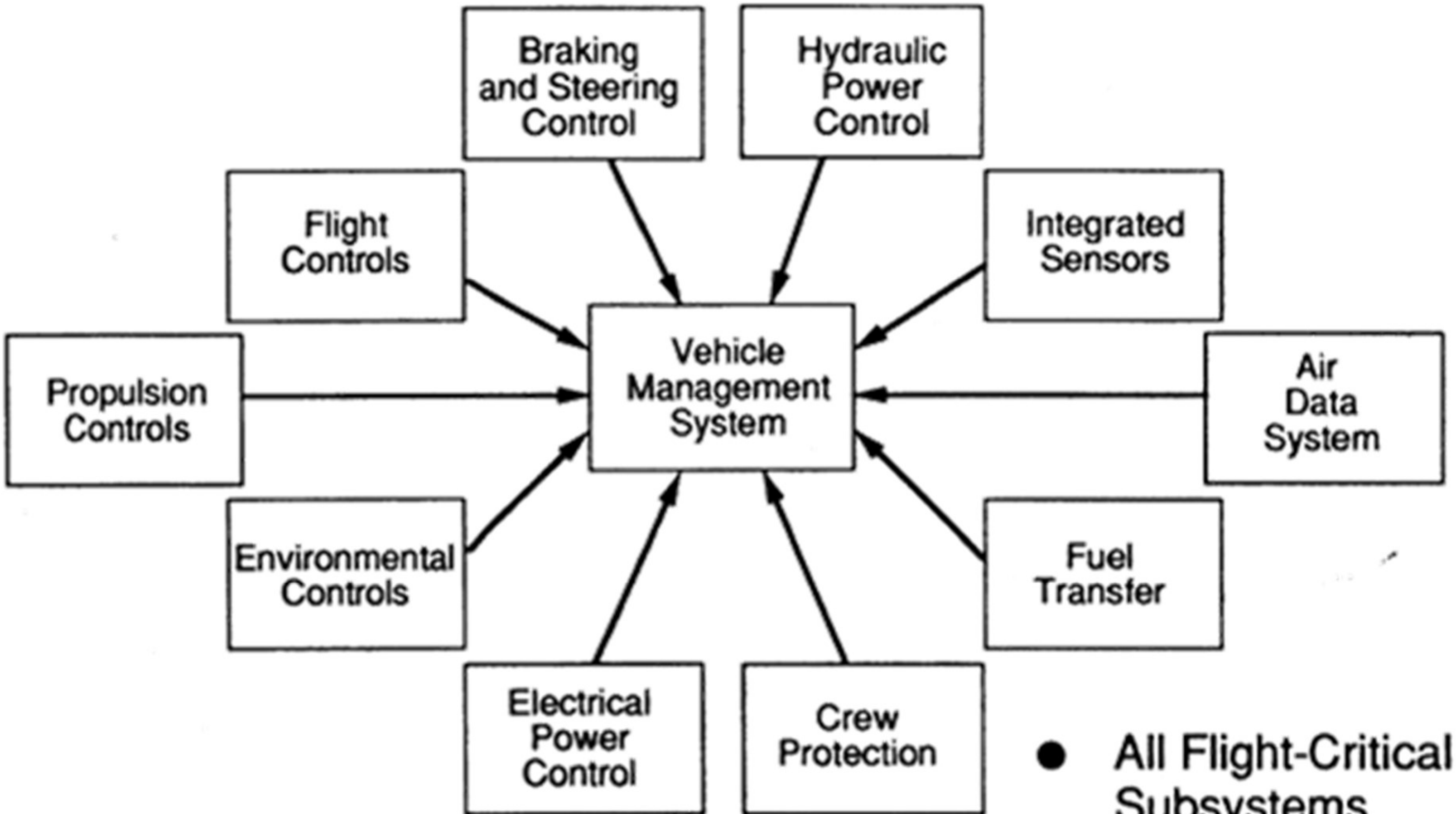
- With an understanding of the basic notions of aircraft stability & control one may begin to entertain more advanced considerations
  - Design and integration of physical control systems
  - Vehicle Management
  - Vehicle Handling Qualities
  - Stability Augmentation Systems
  - Relaxed Stability
  - Control-Configured Vehicle Designs

- Most of these topics are natural extension of the fundamental dynamics we have covered already
  - Generally requires supporting analysis to discern underlying vehicle dynamics
  - Vehicle dynamic behavior can then be exploited or corrected

Control System Development

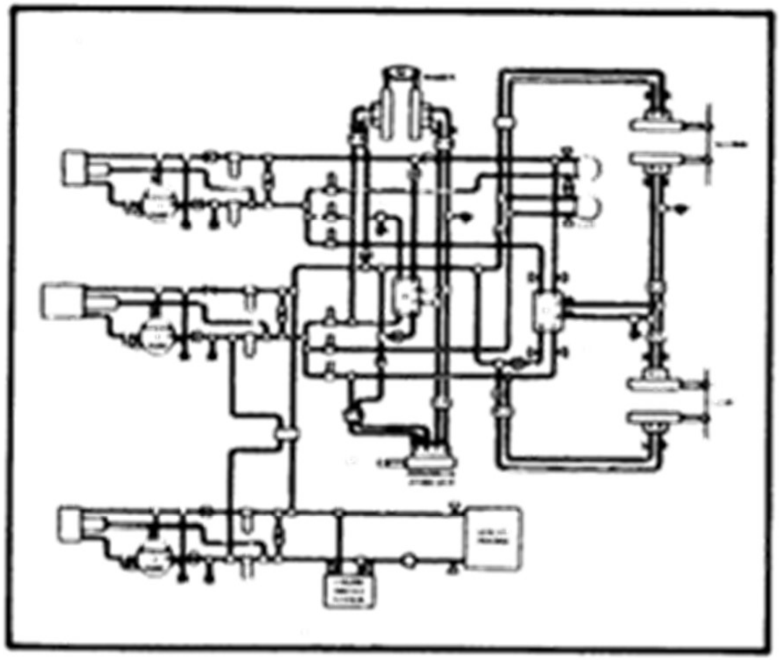


The Vehicle Management System (VMS)



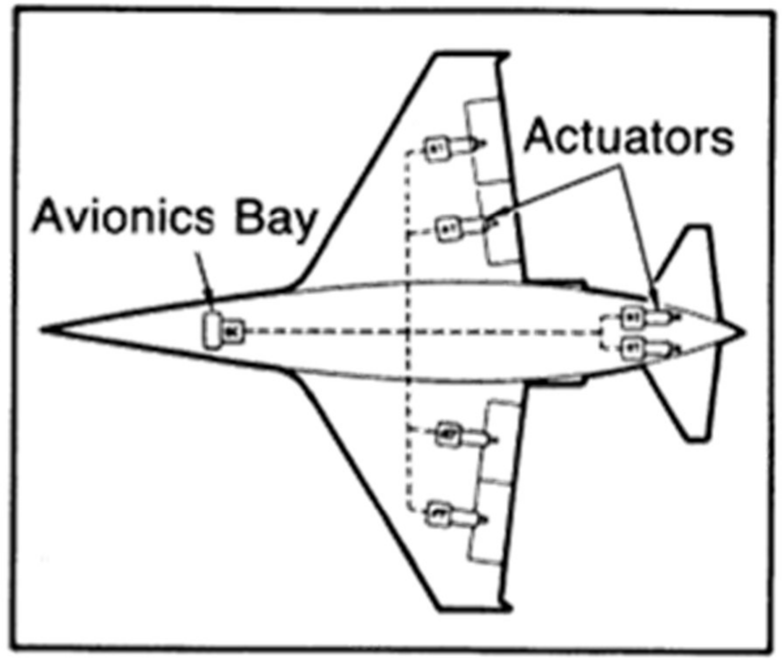
- All Flight-Critical Subsystems
- Integrated Controls

VMS Actuation



Centralized Hydraulics

Versus

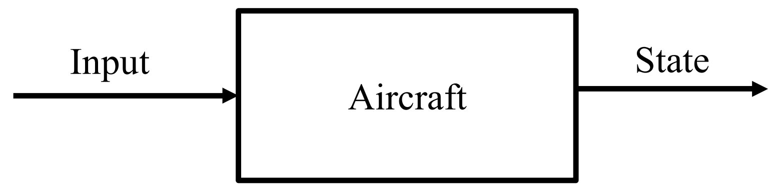


Distributed Hydraulics (EHA)

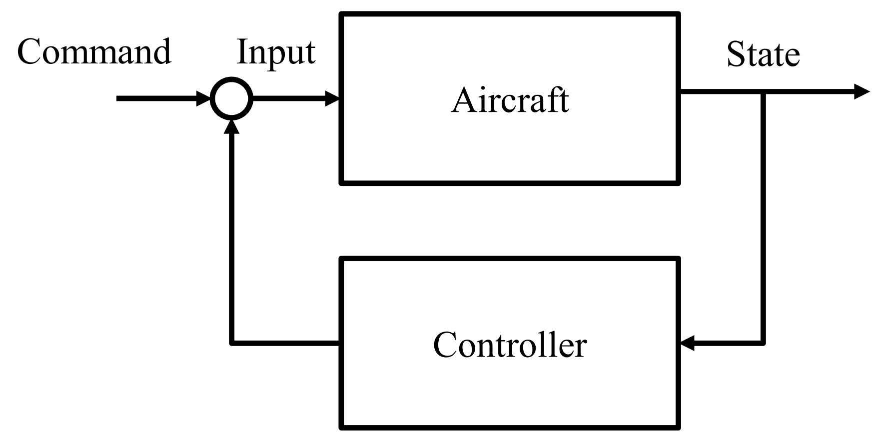
Open-Loop vs. Closed-Loop

- Earlier we discussed the longitudinal and lateral dynamic properties of aircraft
  - "How does the aircraft respond to various inputs?"
- The "feel" of an aircraft (i.e. it's open-loop characteristics) are typically assessed using **Handling Qualities Ratings**
- When the state of the vehicle is used to help determine the input (**feedback control**) we "close the loop" on our system
- Closed-loop system allows for:
  - Steady-state error reduction
  - Improve open-loop characteristics
  - Augment pilot capabilities
  - Provide automatic control (e.g. autopilot)

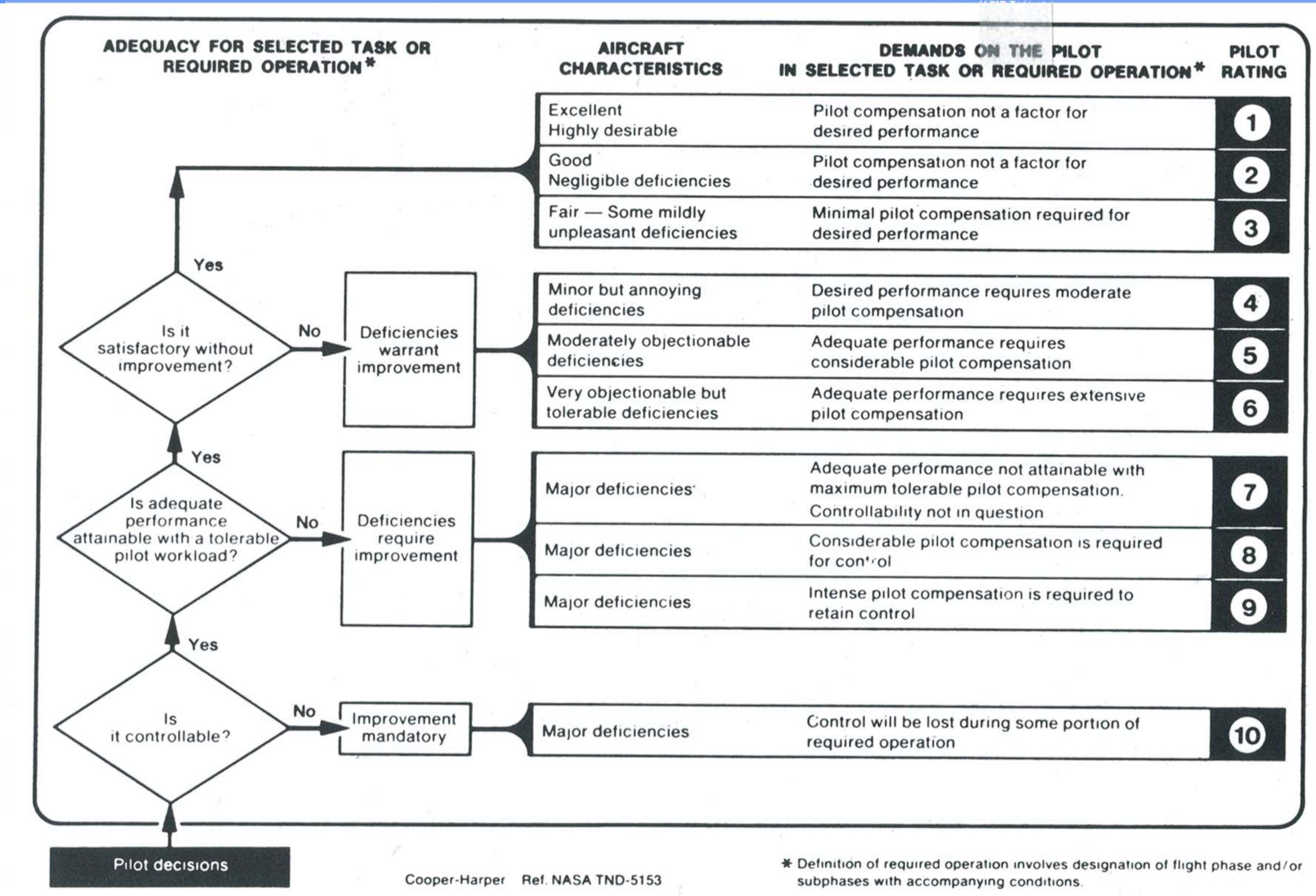
Open-Loop System



Closed-Loop System



Cooper Harper Handling Quality Ratings



Aircraft and Flight Phase Classification

Aircraft Classification	Aircraft Size and Type
Class I	Small, light aircraft such as: Light utility Primary trainer Light observation
Class II	Medium-weight, low- to medium-maneuverability aircraft such as: Heavy utility/search and rescue Light or medium transport/cargo/tanker Early warning/electronic countermeasures/airborne command, control or communications relay Antisubmarine Assault transport Reconnaissance Tactical bomber Heavy attack Trainer for Class II
Class III	Large, heavy, low- to medium-maneuverability aircraft such as: Heavy transport/cargo/tanker Heavy bomber Patrol/early warning/electronic countermeasures/airborne command, control or communications relay Trainer for Class III
Class IV	High-maneuverability aircraft such as: Fighter-interceptor Attack Tactical reconnaissance Observation Trainer for Class IV

Table 10.2.3. Aircraft classification, from the U.S. Military Specification on flying qualities.

Flight Phase	Piloting Task Classification
Category A	Nonterminal flight phases that require rapid maneuvering, precision tracking, or precise flight-path control. Included in the category are air-to-air combat, ground attack, weapon delivery or launch, aerial recovery, reconnaissance, in-flight refueling (receiver), terrain following, antisubmarine search, and close-formation flying.
Category B	Nonterminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight-path control may be required. Included in the category are climb, cruise, loiter, in-flight refueling (tanker), descent, emergency descent, emergency deceleration, and aerial delivery.
Category C	Terminal flight phases that are normally accomplished using gradual maneuvers and usually require accurate flight-path control. Included in this category are takeoff, catapult takeoff, approach, wave-off/go-around and landing.

Longitudinal Mode Requirements from HQ Perspective

Short Period Mode

Level	Category A & C Flight Phases		Category B Flight Phases	
	Minimum ζ	Maximum ζ	Minimum ζ	Maximum ζ
1	0.35	1.30	0.30	2.00
2	0.25	2.00	0.20	2.00
3	0.15	–	0.15	–

Table 10.3.1. Short-period damping ratio requirements for all aircraft classes.

Phugoid (Long Period Mode)

Level	All Flight Phases for all Aircraft Categories Minimum Phugoid Damping
1	Damping ratio should be greater than 0.04
2	Damping ratio should be greater than 0.00
3	Time-to-double-amplitude should be greater than 55 sec

Table 10.3.2. Phugoid damping requirements for all aircraft classes.

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Lateral Mode Requirements from HQ Perspective

Roll Mode

Flight Phase Category	Aircraft Class	Handling Quality Level 1	Level 2	Level 3
A & C	I, IV	1.0	1.4	10
	II, III	1.4	3.0	10
B	ALL	1.4	3.0	10

Table 10.3.3. Roll mode damping requirements, maximum time constant ($1/\sigma$) in seconds.

Spiral Mode

Flight Phase Category	Aircraft Class	Handling Quality Level 1	Level 2	Level 3
A	I, IV	12	12	4
	II, III	20	12	4
B & C	ALL	20	12	4

Table 10.3.4. Spiral mode damping requirements, minimum time-to-double-amplitude in seconds.

Dutch Roll Mode

Level	Flight Phase Category	Aircraft Class	Minimum[5] ζ	Minimum[5] $\zeta\omega_n$ (rad/sec)	Minimum[6] ω_n (rad/sec)
1	A	IV-CO[1]&GA[2]	0.4	0.4	1.0
		I, IV-other	0.19	0.35	1.0
		II, III	0.19	0.35	0.4
	B	ALL	0.08	0.15	0.4
	C	I, II-C[3], IV	0.08	0.15	1.0
		II-L[4], III	0.08	0.10	0.4
2	ALL	ALL	0.02	0.05	0.4
3	ALL	ALL	0.00	—	0.4

[1]CO: combat, [2]GA: ground attack, [3]C: carrier-based, [4]L: land-based,
[5]The minimum damping requirement is that yielding the greatest damping ratio,
[6]The minimum frequency requirement is always based on the undamped natural frequency.

Table 10.3.6. The Dutch roll frequency and damping requirements.

Long Period Mode

Level	All Flight Phases for all Aircraft Categories Minimum Damping for the Oscillatory Roll-Spiral Mode
1	The $\zeta\omega_n$ product should be greater than 0.50 rad/sec
2	The $\zeta\omega_n$ product should be greater than 0.30 rad/sec
3	The $\zeta\omega_n$ product should be greater than 0.15 rad/sec

Table 10.3.5. Total damping requirements for the seldom-encountered low-frequency mode that is sometimes called the lateral phugoid.

For some modern aircraft with low wing area, the roll damping may be quite low. Then, it is occasionally possible for the usual roll and spiral modes to combine to form a complex pair. In this case, the aircraft will exhibit a long-period lateral oscillation, called the lateral phugoid or the roll-spiral mode. It is not desirable, but not objectionable either if there is sufficient damping

Phillips, W.F., "Mechanics of Flight", John Wiley & Sons Inc., 2004

Stability Augmentation Systems (SAS)

- Recall the state-space representation of the aircraft dynamical system:

$$\dot{\boldsymbol{X}} = A\,\boldsymbol{X} + B\,\boldsymbol{U}$$

- Suppose the modes (eigenvalues) of the uncontrolled system ($\boldsymbol{A}$) are either unstable or undesirable (e.g. insufficient damping)
- Split the control $\boldsymbol{U}$ into a parts supplied by pilot $\boldsymbol{U_p}$ and the SAS $\boldsymbol{U_{sas}}$

$$\boldsymbol{U} = \boldsymbol{U_p} + \boldsymbol{U_{sas}}$$

- Design $\boldsymbol{U_{sas}}$ to be a function of the feedback of aircraft states: $\boldsymbol{U_{sas}} = -\boldsymbol{KX}$
- The augmented system is now given by

$$\dot{\boldsymbol{X}} = (A - BK)\boldsymbol{X} + B\boldsymbol{U_p}$$

- By tuning the elements (called gains) within matrix $\boldsymbol{K}$, it is possible to
  - Stabilize an otherwise unstable aircraft
  - Modify an unacceptable mode to have acceptable characteristics
  - Change the entire “feel” of the vehicle

Relaxed Stability

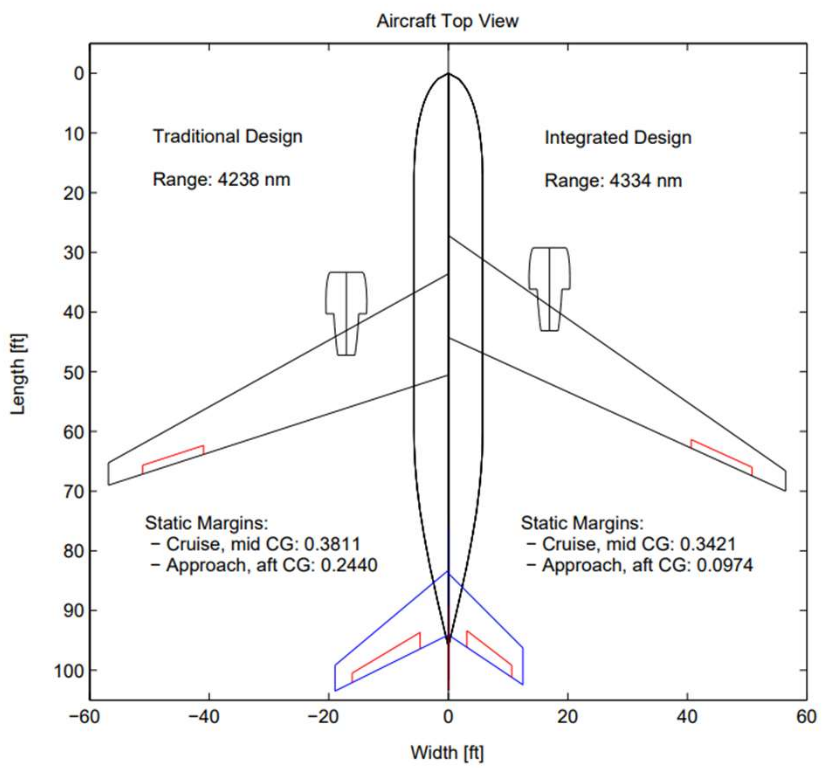
- Most aircraft are designed to be statically stable while maintaining adequate performance

- There are some classes of aircraft where reduced static stability, or even instability, is accepted in order to gain other system benefits

Potential Benefits of Relaxed Stability

- Increased maneuverability is the most common motivation for relaxed stability
  - As static margin is reduced the vehicle's pitch stiffness is also reduced
  - Reduced stiffness allows for more responsive vehicle maneuverability

- For more traditional vehicles without large maneuverability demands, relaxed stability can still be of interest
  - Reduce the size of control surfaces
  - Reduction of total drag at trim conditions



Perez, Ruben E., Hugh HT Liu, and Kamran Behdinan. "Relaxed static stability aircraft design via longitudinal control-configured multi-disciplinary design optimization methodology." *Canadian Aeronautics and Space Journal* 52.1 (2006): 1-14.

Control-Configured Vehicle Design

- Traditionally a vehicle's controllers are designed in the preliminary and detailed design stages
  - Controllers are designed to augment the stability or performance of a fixed design

- This traditional approach carries some inherent risk, particularly for vehicles that require SAS to overcome relaxed stability

Control-Configured Vehicle Design

- Alternative methods to integrate control design considerations alongside vehicle design are generally known as Control-Configured Vehicle (CCV) Design

- General approaches balance vehicle characteristics with stability augmentation to meet desired vehicle handling or maneuverability requirements
  - Requires greater level of detail for design knowledge (aerodynamics, mass properties, etc.) in order to estimate S&C properties

- Some control considerations pertaining to vehicle details (e.g. actuator dynamics) will be needed to refine controllers in later design stages