AE6114 - Fundamentals of Solid Mechanics

Fall 2020

Homework 3: Balance Laws

Due at the indicated time on Canvas, on Tuesday, Oct 13^{th} 2020

Problem 1

As shown in class, the traction $\mathbf{t}$ on an internal surface of the body $\mathcal{B}$ with normal $\mathbf{n}$ is given by Cauchy's relation:

$$t_i = \sigma_{ij} n_j$$

Show that σ_{ij} are the components of a second order tensor.

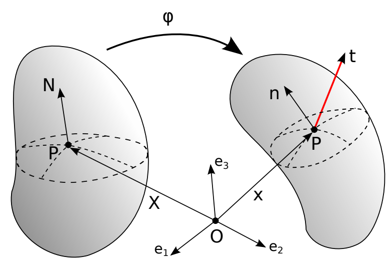

Figure 1:

Problem 2

A slab at rest occupies the following region in the deformed configuration:

$$-a \leq x_1 \leq a, \quad -a \leq x_2 \leq a, \quad -h \leq x_3 \leq h$$

It has the following stress distribution:

$$\sigma_{11} = -\frac{p(x_1^2 - x_2^2)}{a^2}, \quad \sigma_{22} = \frac{p(x_1^2 - x_2^2)}{a^2}, \quad \sigma_{12} = \sigma_{21} = \frac{2px_1x_2}{a^2},$$

with the rest of the Cauchy stress components being zero.

1. Examine whether there are any body forces within the slab.
2. Calculate the tractions acting on the faces $x_1 = \pm a$, and the faces $x_3 = \pm h$.

Problem 3

The Cauchy stress tensor at a point in a solid is given by

$$\sigma_{11} = -3, \quad \sigma_{12} = 1, \quad \sigma_{13} = 2, \quad \sigma_{22} = 0, \quad \sigma_{23} = T, \quad \sigma_{33} = 0$$

where T is a constant. Find all values of T that would result in a traction-free plane through the point and, for each T, determine the orientation (or normal vector) of that plane.

Problem 4

The components of the Cauchy stress tensor at a point in a solid are given, in the basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, by:

$$[\sigma] = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

1. Calculate the stress vector (traction) $\mathbf{t}$ on a surface element with normal $[0, -1, 1]^T$.
2. Give the normal and shear components of this stress vector (indicating both magnitude and direction of the components).
3. Find the principal stresses and principal directions of the stress tensor.
4. Write the matrix representation of the stress tensor σ with respect to the eigenbasis (basis that consists of principal directions).

Problem 5

The components of the Cauchy stress tensor in a solid *at rest* are given, in the basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, by:

$$[\sigma] = \frac{1}{4}\rho\omega^2 \begin{bmatrix} x_1^2 & 2x_1x_2 & 0 \\ 2x_1x_2 & x_2^2 & 0 \\ 0 & 0 & 2(x_1^2 + x_2^2) \end{bmatrix}$$

where ρ is the density (constant) and ω is constant. Find the body force $\mathbf{b}$ that must be acting on this body.

Problem 6

Consider a general stress state at a point given by the Cauchy stress tensor σ. Let $\sigma_1 \geq \sigma_2 \geq \sigma_3$ be the eigenvalues of σ and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ the corresponding eigenvectors. Show that the largest shear stress is $\frac{1}{2}(\sigma_1 - \sigma_3)$ and it corresponds to directions $\frac{1}{\sqrt{2}}(\mathbf{v}_1 + \mathbf{v}_3)$ and $\frac{1}{\sqrt{2}}(\mathbf{v}_1 - \mathbf{v}_3)$