Potential for Internal and External forces

$$W = -\Delta\phi$$

* Both internal and external forces, it conservative, can be derived from a potential

* In an elastic body the internal forces are the stresses

$$W_I = -\Delta A$$ Strain Energy (Deformation Energy, Internal Energy).

$$W_E = -\Delta\Phi$$

Total Work

$$W = W_E + W_I = -\Delta A - \Delta\phi = -\Delta\Pi$$

$$A = \hat{A}(\epsilon)$$

$$W_I = -\left(\hat{A}(\epsilon) - \underbrace{A(\epsilon=0)}_{=0}\right)$$

$$W_I = -\hat{A}(\epsilon)$$

Principle of Stationary Potential Energy

Aside $\quad W_I = -A(\epsilon_x)$

$$dW_I = -\frac{\partial A}{\partial \epsilon_x}\cdot d\epsilon_x$$

$$\delta W_I = -\frac{\partial A}{\partial \epsilon_x}\cdot \delta\epsilon_x$$

The virtual quantities behave like the variational operator "d"

From P.V.W

$$\delta W_I + \delta W_E = 0$$

$$\delta (W_I + W_E) = 0$$

$$\delta \Pi = 0$$

* A system is in equilibrium if the first variation of the total potential is zero for all virtual displacements.

Let the system have N generalized coordinates q_i

$$\delta \Pi = \sum_{i=1}^{N} \underbrace{\frac{\partial \Pi}{\partial q_i}}_{=0} \delta q_i = 0$$

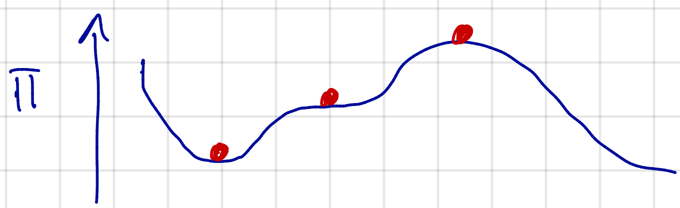
ARBITRARY

$$\Rightarrow \frac{\partial \Pi}{\partial q_i} = 0$$

$$d\Pi = \sum_{i=1}^{N} \underbrace{\frac{\partial \Pi}{\partial q_i}}_{=0} \underline{dq_i} = 0$$

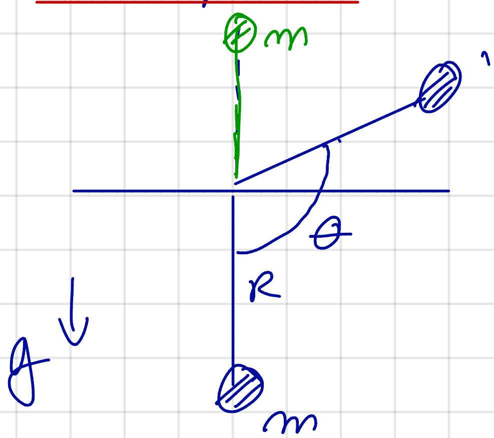
$$\Rightarrow \boxed{d\Pi = 0}$$

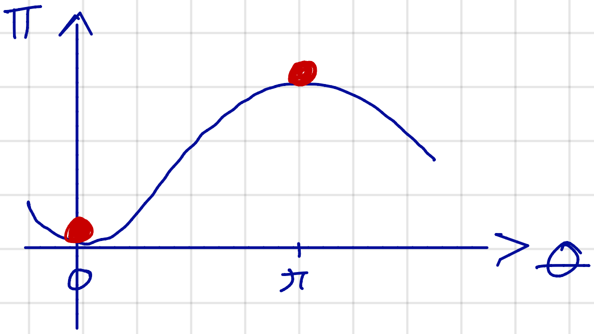
Total differential is zero. This occurs at critical points.



* If a structural system is in equilibrium, the total potential energy has a stationary value.

Example:





$$\Pi = mg\,(R - R\cos(\theta))$$

* For a stable equilibrium Π is a relative minimum

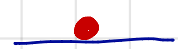
Consider a system w/ one generalized coordinate x

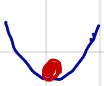
$\Pi(x)$ $\quad$ $x \rightarrow$ an equilibrium point

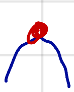
$$\Pi(x+\Delta x) = \Pi(x) + \underbrace{\left.\frac{d\Pi}{dx}\right|_x}_{=0} \Delta x + \frac{1}{2}\left.\frac{d^2\Pi}{dx^2}\right|_x (\Delta x)^2 + \dots$$

$$\Delta\Pi = \Pi(x+\Delta x) - \Pi(x)$$

$$\Delta\Pi = \frac{1}{2}\underbrace{\left.\frac{d^2\Pi}{dx^2}\right|_x}_{\text{Determines the sign}} \underbrace{(\Delta x)^2}_{>0}$$



 $\dfrac{d^2\Pi}{dx^2} > 0$ $\quad$ Potential is a minimum
System is stable

 $\dfrac{d^2\Pi}{dx^2} < 0$ $\quad$ Potential is a maximum
System is unstable

$\dfrac{d^2\Pi}{dx^2} = 0$

- Saddle Point
- Neutrally Stable

Principle of minimum total potential energy

* A conservative system is in stable equilibrium if and only if the total potential energy is a minimum w.r.t. changes in the generalized coordinates.