An Energy Based Approach to Constraint Analysis – Example

Presented by
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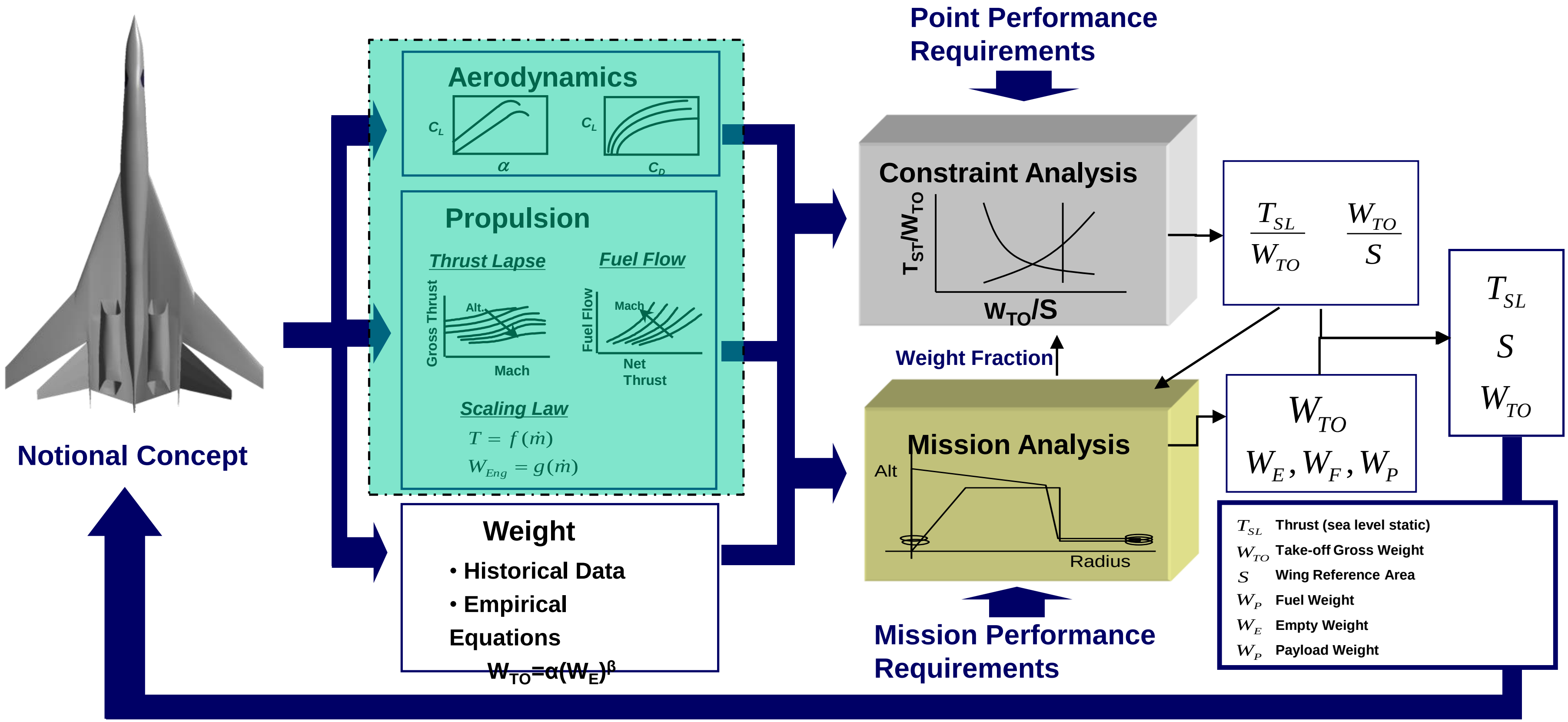
Example: Air-to-Air Fighter Specifications (Table 2)

Mission Phases & Segments		Conditions
1-2	Warm-up & take-off A – Warm-up B – Acceleration C - Rotation	2000 ftPA, 100°F 60s mil power k_{TO}=1.2, μ_{TO}=0.05 max power $M_{TO}.t_R$ = 3s, max power
2-3	Accelerate & climb D – Acceleration E – Climb/acceleration	Min. time-to-climb path M_{TO} =>M_{CL}/2000ft PA, 100°F, mil power M_{CL}/2000ft PA, 100°F => BCM/BCA, mil power
3-4	Subsonic cruise climb	BCM/BCA, $\Delta s_{23} + \Delta s_{34}$ = 150 nmi.
4-5	Descend	BCM/BCA => M_{CAP}/30k ft..
5-6	Combat air patrol	30k ft.., 20 min
6-7	Supersonic penetration F – Acceleration G – Penetration	30,000 ft.. M_{CAP} => 1.5M/30 kft, max power 1.5 M, $\Delta s_F + \Delta s_G$ = 100 n. mi., no after-burn (AB)
7-8	Combat H – Turn 1 I – Turn 2 J – Acceleration	30,000 ft.. 1.6 M, one 360° 5g sustained turn, with AB 0.9 M, two 360° 5g sustained turn, with AB 0.8 => 1.6 M, Δt <50s, max power
	Deliver expendables	1.6 M /30k ft.., 1309 lb.
8-9	Escape dash	1.5 M/30k ft.., Δs_{89} = 25 n. mi., no AB
9-10	Min. time climb	1.5 M/30k ft.. => BCM/BCA
10-11	Subsonic cruise climb	BCM/BCA, Δs_{89} = 150 n. mi.
11-12	Descend	BCM/BCA => M_{loiter}/10k ft..
12-13	Loiter	M_{loiter}/10k ft.., 20 min
13-14	Descend & land	M_{loiter}/10k ft.. => 2000 ft.. PA, 100°F
	Maximum Mach Number	2.0M/40k, max power

Traditional Aircraft Sizing Process



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Preliminary Estimates for AAF

In order to proceed, you need:

- C_{Lmax}
- Lift drag polar
- Engine data, including lapse rates

These are obtained from the figures and the equations to come…

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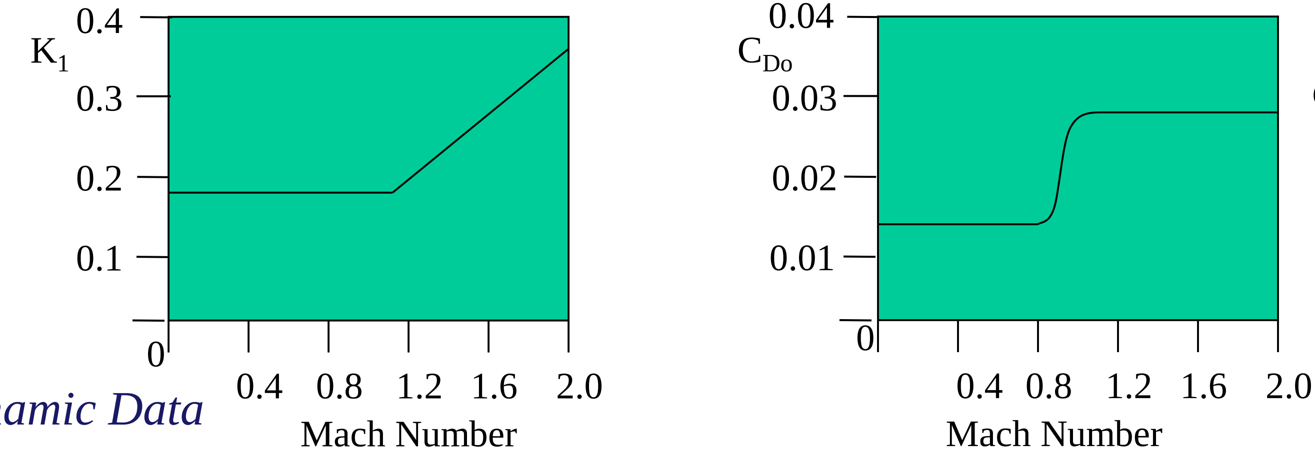
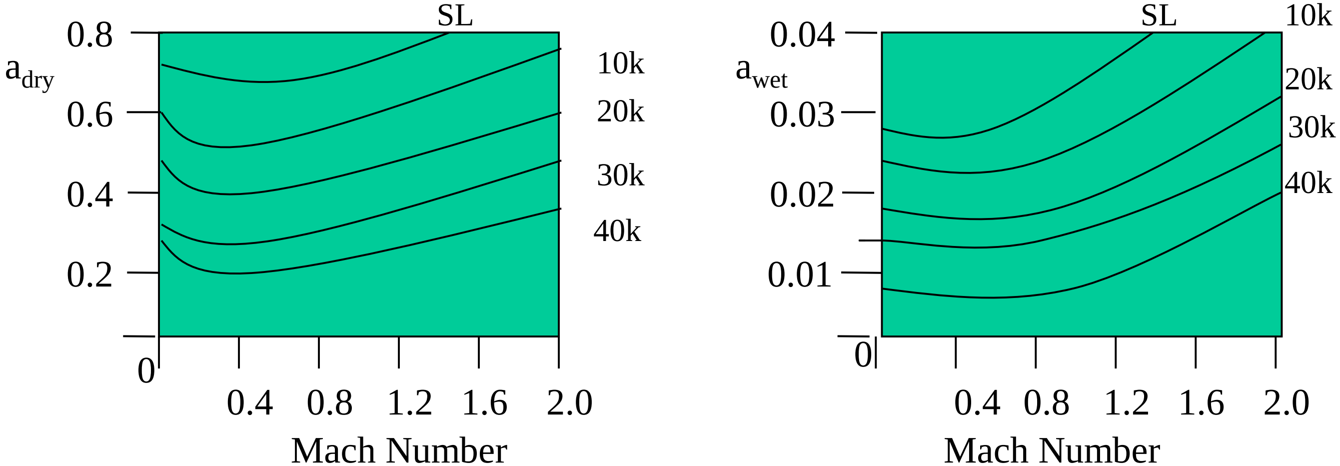
Preliminary Estimates for AAF



$C_D = K_1 C_L^2 + C_{Do}$

$C_{Lmax} = 2.0$

(a) Aerodynamic Data



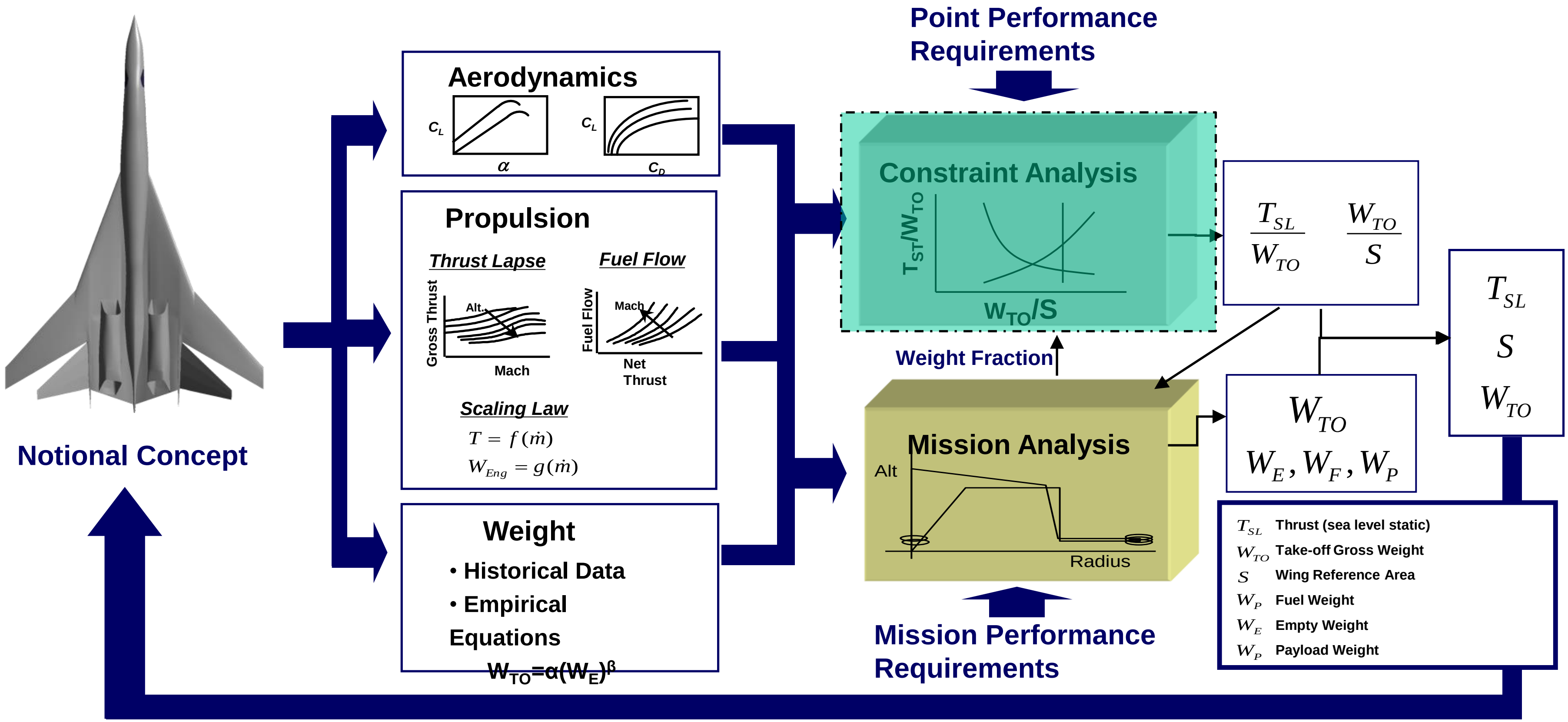
(b) Installed Thrust Lapse

$$\frac{T_{dry}}{T_{SL}} = a_{dry} = 0.72\{0.88 + 0.245(|M - 0.6|)^{1.4}\} * \sigma^{0.7}$$

NOTE: T_{SL} = Std.sea level max power static thrust

$$\frac{T_{wet}}{T_{SL}} = a_{wet} = \{0.94 + 0.38(M - 0.4)^2\} * \sigma^{0.7}$$

Traditional Aircraft Sizing Process



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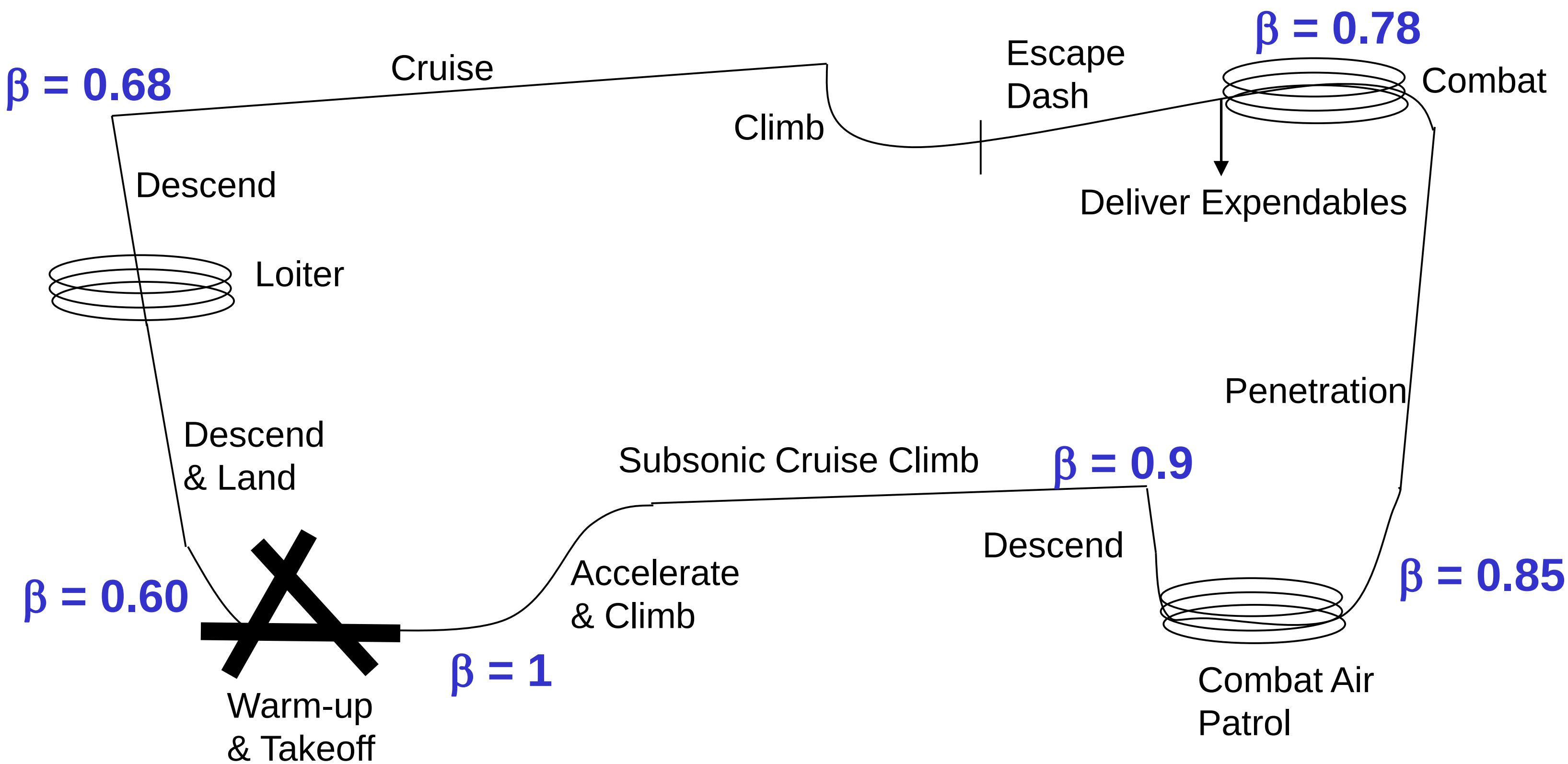
Initial Weight Fraction Guess

Instantaneous weight fraction, β, for a Typical Fighter Aircraft

(Figure 4)



Constraint Analysis

- Creating constraint boundaries on a takeoff thrust loading (T_{SL}/W_{TO}) vs wing loading (W_{TO}/S) diagram
- Created by re-arranging terms so as to create an equation in the following form:

$$f\{(T_{SL}/W_{TO}),(W_{TO}/S)\}=0$$

- Proceed with an initial estimate for β for each segment for a typical fighter

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Phase 1-2: Takeoff

- The airplane accelerated by thrust with no resisting forces in the ground roll
- Thrust is balanced by drag forces during the constant velocity rotation

Given these conditions, Takeoff equation becomes:

$$s_{TO} = \left\{ \frac{k_{TO}{}^{2} \beta^{2}}{\rho * g_{o} * C_{L\max} * a_{wet} \left(\frac{T_{SL}}{w_{TO}} \right)} \right\} \left(\frac{w_{TO}}{S} \right) + \{ t_{R} * k_{TO} \left(\frac{2\beta}{\rho C_{L\max}} \right)^{\frac{1}{2}} \sqrt{\frac{w_{TO}}{S}} \qquad (46)$$

Where: $$a(W_{TO} / S) + b\sqrt{W_{TO} / S} - c = 0$$

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Mission Phase 1-2: Takeoff

$$s_{TO} = \left\{\frac{k_{TO}{}^{2}\beta^{2}}{\rho * g_{o} * C_{L\max} * a_{wet}\left(\frac{T_{SL}}{w_{TO}}\right)}\right\}\left(\frac{w_{TO}}{S}\right) + \{t_{R} * k_{TO}\left(\frac{2\beta}{\rho C_{L\max}}\right)^{\frac{1}{2}}\sqrt{\frac{w_{TO}}{S}} \qquad (46)$$

Rewritten equation 46:

$$\left(\frac{w_{TO}}{S}\right) = \left\{\frac{-b + \sqrt{b^{2} + 4ac}}{2a}\right\}^{2} \qquad (47)$$

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Phase 1-2: Takeoff

We estimate α and β to be constant at appropriate mean values. A conservative estimate for ξ_{TO} for the AAF is obtained by assuming $(C_{DR} - \mu_{TO} C_L) = 0$ and evaluating C_D at $C_L = C_{Lmax}/k_{TO}^2$. From Table 2 and Fig 4, we get following data:

With:

• $\beta = 1.0$	• $K_1 = 0.18$
• $\rho = 0.002047$ slug/ft^3	• $\sigma = 0.8613$
• $C_{Lmax} = 2.0$	• $(\alpha_{wet})_{M=0.1} = 0.8775$
• $k_{TO} = 1.2$	• $\mu_{TO} = 0.05$
• $C_{Do} = 0.014$	• $t_R = 3.0$s
• $\xi_{TO} = 0.36$	• $s_{TO} = 1500$ ft..

Then:

$$a = -42.03 \ln\left\{1 - \frac{0.2601}{\left[0.8775\left(\frac{T_{SL}}{W_{TO}}\right) - 0.05\right]}\right\}$$

$$b = 79.57$$

$$c = 1500$$

Using values of a,b,c from above in eq. (47)

T_{SL}/W_{TO}	0.4	0.8	1.2	1.6	2.0
W_{TO}/S (lb./ft^2)	14.3	45.1	67.2	85.3	101

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Phase 6-7: Supersonic Penetration and Escape Dash

Assumptions:
1.5M/30k ft..
No afterburning

From (2.12):

$$\left(\frac{T_{SL}}{W_{TO}}\right)=\frac{\beta}{\alpha}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)+\frac{C_{Do}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

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Phase 6-7: Supersonic Penetration and Escape Dash

Assumptions (most from Case1):

dh/dt = 0	Constant Altitude
dV/dt =0	Constant Speed
n=1	Lift equals Weight
R=0	Not on the ground
h & v	Values are Given
K_2=0	Pure Parabolic Drag Polar

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + \cancel{K_2\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)} + C_{D_o} + \cancel{\frac{R}{qS}}\right] + \cancel{\frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)}\right\}$$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{D_o}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

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Phase 6-7: Supersonic Penetration and Escape Dash

With:

$\beta=0.78$ $\alpha_{dry}=0.3953$ $q=991.8$ psf

$\alpha=0.3747$ $K_1=0.28$ $C_{Do}=0.028$

$$\frac{T_{SL}}{W_{TO}} = 4.345*10^{-4}\left(\frac{W_{TO}}{S}\right) + \frac{70.25}{\left(\frac{W_{TO}}{S}\right)}$$

W_{TO}/S	20	40	60	80	100	120
T_{SL}/W_{TO}	2.35	1.77	1.2	0.913	0.746	0.638

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Phase 7-8: Combat Turn 1

Assumptions:	
dh/dt=0	Constant Altitude
dV/dt =0	Constant Speed
R=0	Not on the ground
h, n, & V	Values are Given
K2=0	Pure Parabolic Drag Polar

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + \cancel{K_2\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)} + C_{D_o} + \cancel{\frac{R}{qS}}\right] + \cancel{\frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)}\right\}$$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1 n^2 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{D_o}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

Phase 7-8: Combat Turn 1

With:

$\beta=0.78$ $\alpha_{wet}=0.7481$ $q=1128$ psf $n=5$

$\alpha=0.3747$ $K_1=0.30$ $C_{Do}=0.028$

$$\frac{T_{SL}}{W_{TO}} = 5.407 * 10^{-3}\left(\frac{W_{TO}}{S}\right) + \frac{42.22}{\left(\frac{W_{TO}}{S}\right)}$$

W_{TO}/S	20	40	60	80	100	120
T_{SL}/W_{TO}	2.22	1.27	1.03	0.96	0.963	1

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Phase 7-8: Combat Turn 2

--0.9 M/30K ft.., two 360 degree 5g sustained turns, with afterburning.

From Eq. (2.15)

$$\left(\frac{T_{SL}}{W_{TO}}\right) = \frac{\beta}{\alpha}\left\{K_1 n^2 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{Do}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

with $\beta = 0.78$ $\alpha_{wet} = 0.5206$ $n = 5$ $C_{Do} = 0.018$

$\sigma = 0.3747$ $K_1 = 0.18$ $q = 357.0$ lb./ft^2

then

$$\left(\frac{T_{SL}}{W_{TO}}\right) = 0.01473\left(\frac{W_{TO}}{S}\right) + \frac{12.34}{\left(\frac{W_{TO}}{S}\right)}$$

whence

W_{TO}/S (lb./ft^2)	20	40	60	80	100	120
T_{SL}/W_{TO}	0.910	0.900	1.09	1.33	1.60	1.87

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Phase 7-8: Horizontal Acceleration

--0.8 → 1.6 M/30K ft.., $\Delta t \leq 50$s, max power.

From Eq. (2.18)

$$\left(\frac{T_{SL}}{W_{TO}}\right) = \frac{\beta}{\alpha}\left\{K_1 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{Do}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)} + \frac{a\Delta M}{g_o \Delta t}\right\}$$

We obtain approximate constant values of ρ, K_1, C_{Do}, and α at a mean Mach number of 1.2.

with $\beta = 0.78$ $\quad K_1 = 0.23$ $\quad a = 994.8$ ft./s

$\sigma = 0.3747$ $\quad q = 634.7$ lb./ft^2 $\quad \Delta M = 0.8$

$\alpha_{\text{wet } M=1.2} = 0.5952$ $\quad C_{Do} = 0.025$ $\quad \Delta t = 50$ s

then

$$\left(\frac{T_{SL}}{W_{TO}}\right) = 3.704 * 10^{-4} \cdot \left(\frac{W_{TO}}{S}\right) + \frac{28.79}{\left(\frac{W_{TO}}{S}\right)} + 0.6484$$

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Phase 7-8: Horizontal Acceleration

whence,

$W_{TO}/S(lb/ft^2)$	20	40	60	80	100	120
T_{SL}/W_{TO}	2.10	1.38	1.15	1.04	0.973	0.933

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Phase 13-14: Landing

2000ft PA, 100F, $k_{TD} = 1.15$,
$t_{FR} = 3s$, $\mu_B = 0.18$,
$s_L = s_{FR} + s_B \leq 1500\,\text{ft}$
GFE drag chute, diameter 15.6 ft,
deployment $\leq 2.5s$

From Eqs. (2.33) and (2.37) with $s_L = s_{FR} + s_B$

$$\left(\frac{W_{TO}}{S}\right) = \left\{\frac{-b + \sqrt{b^2 + 4ac}}{2a}\right\}^2$$

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Phase 13-14: Landing(cont.)

Where,

$$a = \frac{\beta}{\rho g_0 \xi_L} \ln\left\{1 + \frac{\xi_L}{\left[\mu_B + \frac{(-a)}{\beta}\left(\frac{T_{ST}}{W_{TO}}\right)\right]\frac{C_{L_{\max}}}{k_{TD}^2}}\right\}$$

$$b = t_{FR} k_{TD} \sqrt{2\beta / \rho C_{L_{\max}}} \qquad c = s_L$$

Phase 13-14: Landing

Assumptions

$\text{Drag chute } C_D = 1.4$

$\text{RFP chute area} = 191\ \text{ft}^2$

$\text{Estimate airplane wing area} = 500\text{ft}^2$

$C_{DRc} = 0.5348$

A conservative estimate of ξ_L from C_D at 0.8 of touchdown lift coefficient $(C_{L_{max}} / k_{TD}{}^2)$ and by assuming $(C_{DR} - C_{DRc} - \mu_B C_L) = 0$

$$\beta = 0.56,\ k_{TD} = 1.15,\ \mathrm{C_D} = 0.2775,\ \mathrm{a} = 0$$

$$\rho = 0.002047\text{slug/ft}^3,\ \mathrm{C_L} = 1.210,\ \mathrm{C_{DRc}} = 0.5348,\ t_{FR} = 3s,$$

$$C_{L_{max}} = 2.0,\ \mathrm{C_{D_0}} = 0.014,\ \xi_{\mathrm{L}} = 0.8123,\ \mathrm{s_L} = 1500\,ft,$$

$$\mathrm{K_1} = 0.18,\ \mu_{\mathrm{B}} = 0.18$$

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Phase 13-14: Landing

then

$$a = 14.47 \qquad \mathrm{b} = 57.06 \qquad \mathrm{c} = 1500$$

whence

$$W_{TO} / S = 70.5 lb / ft^2$$

Maximum Mach Number Constraint

- Maximum Mach Number
  - M = 2 at 40,000 ft.., max power
  - From (Equation 2.12), we have:

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 + \frac{C_{D_o}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

With

$\beta = 0.78$	$a_{wet} = 0.7189$	$q = 1101$ lb./ft^2
$\sigma = 0.2471$	$K_1 = 0.36$	$C_{Do} = 0.028$

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Maximum Mach Number Constraint

- This gives

$$\frac{T_{SL}}{W_{TO}} = 2.767 * 10^{-4} \left(\frac{W_{TO}}{S} \right) + \frac{42.88}{\left(\frac{W_{TO}}{S} \right)}$$

The table below shows a few calculations

$W_{TO}/S\ (lb/ft^2)$	20	40	60	80	100	120
T_{SL}/W_{TO}	2.14	1.07	0.713	0.535	0.428	0.357

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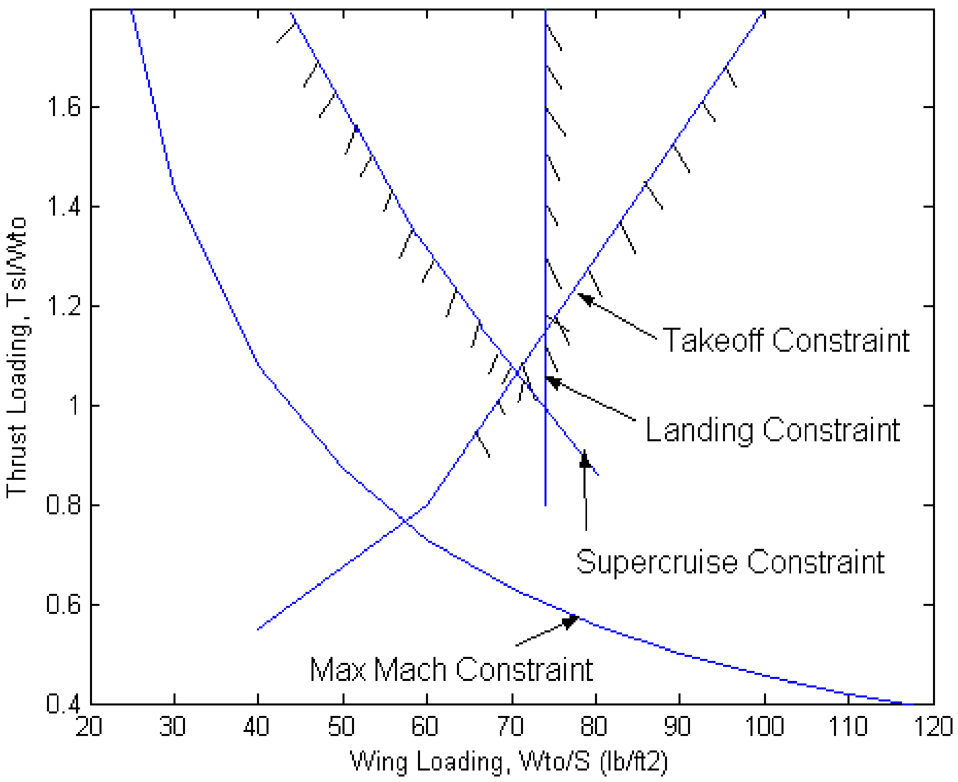
Selection of the Air-to-Air Fighter Design Point

The requirements defined for a new air-to-air fighter result in the constraint diagram shown in the figure.

Based on the problem assumptions, the constraints that dominate the design are the takeoff, landing, and supercruise constraints.

To keep cost and practicality of the AAF design under control, thrust loading is chosen near the region of previous experience.

Reasonable first choice:



AAF Design Constraints

$$\frac{T_{SL}}{W_{TO}} = 1.2 \qquad \frac{W_{TO}}{S} = 64 lb / ft^2$$

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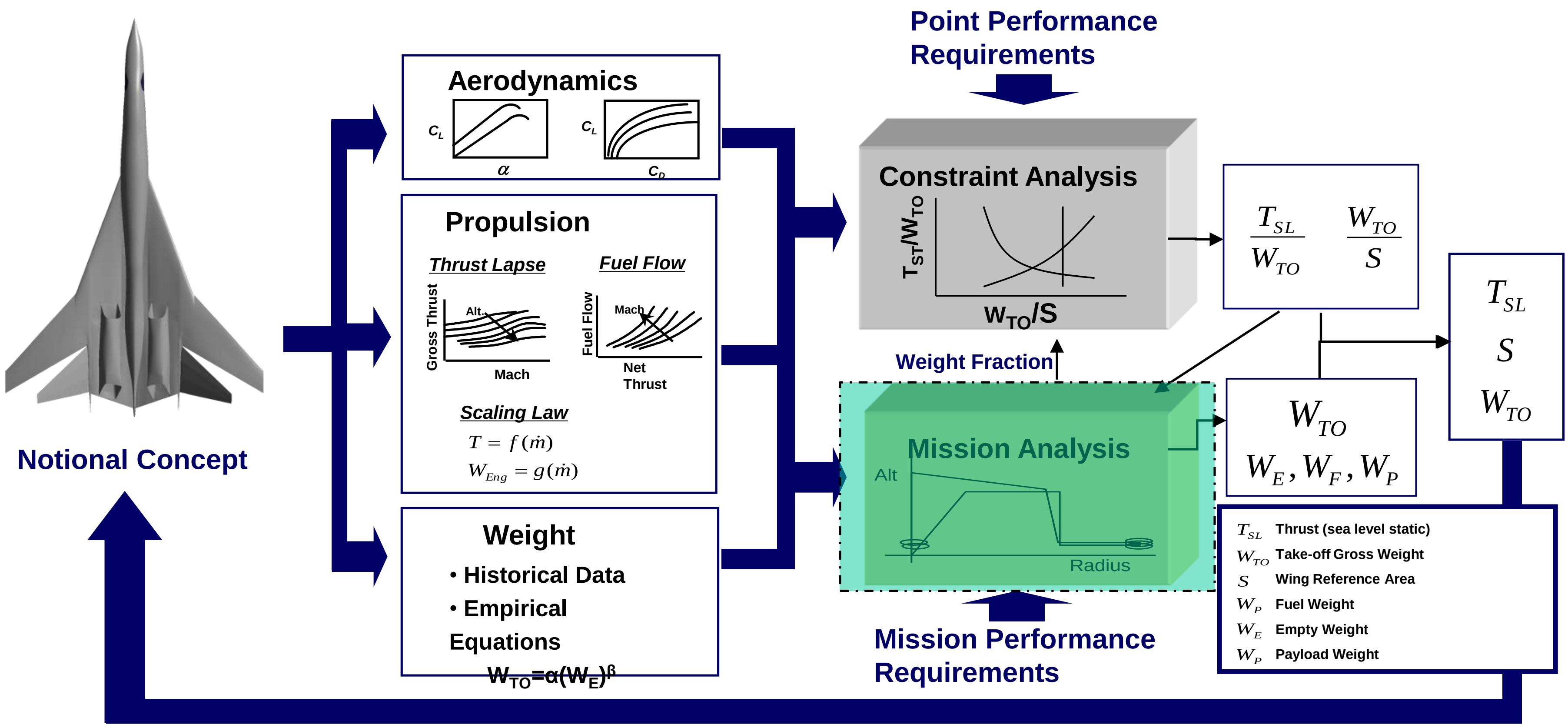
Selection of the Air-to-Air Fighter Design Point

- Capability requirements for short takeoff and landing and nonafterburning supercruise drive design
- Relaxing these constraints would not significantly change the design choice because other constraints are important just outside of the takeoff, landing, and supercruise constraints.
- For many flight segments, the aircraft will be operating at full thrust; thus, no one requirement will cause the size to become excessively large. This is a result of careful consideration and balancing of requirements.

Recap of what has been done so far

- A performance requirements list of an AAF was given
- You want to come up with an values of T_{SL}, S, W_{TO}, W_P, W_E, and W_F
- Using an estimate of initial weight, weight fractions, aerodynamic and thrust parameters, constraint analysis was performed
- At the end of the constraint analysis, a design point was chosen. Two ratios – T_{SL}/W_{TO} and W_{TO}/S are chosen
- Now, we move on to mission analysis and will use the chosen values of T_{SL}/W_{TO} and W_{TO}/S wherever required
- At the end of mission analysis, you will get new weight fractions for each mission segment

Traditional Aircraft Sizing Process



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Installed thrust specific fuel consumption (TSFC)

The TSFC of an aircraft engine usually varies with Mach number, altitude, type of engine and throttle condition. Actual measurements of the airframe/engine system performed during flight tests can be obtained from the manufacturer's published data. For initial estimates though, the following relations provide a value for C depending on the altitude, for a given Mach number and engine type.

High by-pass turbofan (M < 0.9): $C = (0.45 + 0.54M_0)$

Low by-pass turbofan:

$C = (0.9 + 0.30M_0)$ mil power

$C = (1.6 + 0.27M_0)$ max power

Turbojet engine:

$C = (1.1 + 0.30M_0)$ mil power

$C = (1.5 + 0.23M_0)$ max power

Turboprop engine: $C = (0.18 + 0.8M_0)$

Phase 1–2 A: Warm-up

Assumptions: 2000 ft.. Pressure Altitude, 100 °F Temp, 60 second duration, min power

$$\Pi_{warm-up} = 1 - C\sqrt{\theta}\,\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)\Delta t$$

Δt = 60 sec, σ = 0.8613, β = 1.0, T_{SL}/W_{TO} = 1.2,

α_{dry} = 0.6484, θ = 1.08, C = 1.35 h^{-1}

$$\Pi_{warm-up} = 0.9818$$

$$\beta = 0.9818$$

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Phase 1–2 B: Takeoff Acceleration

Assumptions: 2000 ft.. Pressure Altitude, 100 °F Temp, k_{TO} = 1.2, μ_{TO} = 0.05, max power

$$\Pi_{Takeoff\ Accel} = \exp\left\{\left(\frac{-C\sqrt{\theta}}{1-u}\right)\frac{V_{TO}}{g_0}\right\}$$

$$u = \left\{\xi_{TO}\frac{q}{\beta}\left(\frac{S}{W_{TO}}\right)+\mu_{TO}\right\}\frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right)$$

Phase 1–2 B: Takeoff Acceleration

$\beta = 0.9818$, $a = 1160$ ft./s, $\mu_{TO} = 0.05$, $W_{TO}/S = 64$ lb./ft^2, $M_{TO} = 0.1812$,

$q = 11.31$ lb./ft^2, $\sigma = 0.8613$, $C = 2.0$ h^{-1}, $\xi_{TO} = 0.36$ (Estimated from 2.4.3.1),

$C_{L\,max} = 2.0$, $\theta = 1.08$, $u = 0.1067$, $k_{TO} = 1.2$, $T_{SL}/W_{TO} = 1.2$, $V_{TO} = 210.2$ ft./s,
$\alpha_{wet} = 0.8795$

$$\Pi_{takeoff\ accel} = 0.9958$$

$$\beta = 0.9777$$

Phase 1-2 C: Takeoff Rotation

M_{TO}/2000 ft.. PA, 100 degrees F, t_R = 3s, max power

$$\prod_C = 1 - C\sqrt{\theta}\,\frac{a}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)t_R$$

t_R = 3s, M_{TO}=.1812, Beta=.9777,
T_{SL}/W_{TO}=1.2a_{wet}=.8631,Theta=1.080,sigma=.8613, C=2.0 h^-1

$$\prod_C = .9982$$

The weight fraction for Mission phase 1-2 is the product of the weight fractions for Segments A,B, and C.

$$_1\prod_2 = \prod_A \prod_B \prod_C = 0.9759$$

$$\beta = 0.9759$$

Phase 2-3: Climb and Acceleration

For this Mach number change of 0.1812 to 0.70, a single interval calculation will suffice. The Climb/Acceleration Weight Fraction Calculation Method 3.4.1 is utilized with c' evaluated at the middle state point (M=.4406, 2000ft PA, 100 degrees F), and Beta = Beta sub i.

M_{CL}/2000 ft.. PA, 100 degrees F → BCM/BCA, mil power

The weight fraction of this mission segment is calculated along the minimum time-to-climb path depicted in Fig. 3.E.2. In our illustrative example of Sec. 3.4.1, we found the weight fraction of this segment, using a single gross interval, to be .9766. Here we shall use the three successive integration intervals given in Table 3.E4 for our calculations – applying Eq. 3.20 to each. The Climb/Acceleration Weight Fraction Calculation Method of Sec. 3.4.1 was used.

Climb Schedule		Integration Intervals (state points)		
Altitude (ft..)	Mach #	a	b	c
2000 (100 deg)	0.7	initial		
9000	0.83	middle		
16000	0.85	final	initial	
23000	0.88		middle	
30000	0.9		final	initial
36000	0.9			middle
43000	0.9			final

Phase 2-3: Climb and Acceleration

<u>Segment E – Climb/Acceleration Results</u>

	Δ h (ft..)	$\Delta(V^2/2g_o)$ (ft..)	Δ z_e (ft..)	TΔs/W (ft..)	Δt (min)	Δs (nmi)	c $(ft./s)^{-1}$	Π
a	14,000	2208	16,210	23,250	0.6452	5.683	0.001557	0.9906
b	14,000	3.105	14,000	20,420	0.7589	6.749	0.001485	0.9922
c	13,000	-661.6	12,340	18,520	0.9810	8.437	0.001494	0.9931
$\sum$	41,000	1550	42,550	62,190	2.385	20.87	n/a	n/a

$$\Pi_E = \Pi_a \Pi_b \Pi_c = 0.9761$$

The results do NOT differ insignificantly, therefore the phase should be broken down into finer intervals. In summary, Mission Phase 2-3:

$$\Delta s_{23} = \Delta s_D + \Delta s_E = 23.43 nmi$$

$$\Delta t_{23} = \Delta t_D + \Delta t_E = 2.892 \min$$

$$\Pi_{23} = \Pi_D \Pi_E = 0.9678$$

$$\beta = 0.9445$$

Phase 2-3: Climb and Acceleration

Mission Phase 3-4: Subsonic Cruise Climb

BCM/BCA, Δs23+Δs34 = 150 nmi

Assume: (Ps = 0 , T = D+R , K2 = 0)

$$\Pi_{34} = \exp\left\{\left(\frac{\sqrt{4C_{DO}K_1}}{M_{CRIT}}\right)\left(\frac{C\Delta s_{34}}{a_{SL}}\right)\right\}$$

Where: Δs_{34} = 126.6 nmi K1 = 0.18

C = 1.35 h-1 C_{do} = 0.018

a_{SL} = 1116 ft./s M_{crit} = 0.9

Then:

$$\Pi_{34} = 0.9678$$
$$\beta = 0.9141$$

Mission Phase 4-5: Descend

BCM/BCA -> Mcap/30k ft..

$$\Pi_{45} = 1.0$$
$$\beta = 0.9141$$

Mission Phase 5-6: Combat Air Patrol

30k ft.. , 20 min.

Assume: (K2 = 0)

Where: Δt = 1200 sec θ = 0.7940

K1 = 0.18 C = 1.35 h-1

Cdo = 0.014

Then:

$$\Pi_{56} = \exp\left\{-C\sqrt{\vartheta}\sqrt{4C_{DO}K_1}\Delta t\right\}$$

$$\Pi_{56} = 0.9605$$
$$\beta = 0.8780$$

Phase 6-7: Supersonic Penetration

This phase consists of segments

- F (Horizontal Acceleration) and
- G (Supersonic Penetration)

Phase 6-7 F: Horizontal Acceleration
– MCAP 1.5M/30k ft.., max power

- Horizontal acceleration calculation is divided into three intervals as shown in Table 1
- The initial, final and average Mach number of each interval and *h*=30,000 ft.. are used in the Climb/Acceleration Weight Fraction Calculation method of Sec. 3.4.1
- The calculated results are given in Table 1.

Table 1: Segment F-Horizontal Acceleration

	M_i	M_f	M_{avg}	Δz_e (ft)	$T\Delta s/W$ (ft)	Δt (min)	Δs (n mi)	c' (ft/s)	Π
a	0.6762	0.95	0.8131	6844	8324	0.2484	1.982	664.8	0.9949
b	0.95	1.23	1.09	9382	13,780	0.2735	2.926	738.1	0.9937
c	1.23	1.5	1.365	11,330	21,770	0.297	3.979	706.6	0.9921
			Σ	27,560	43,870	0.8189	8.887		

Phase 6-7 F: Horizontal Acceleration – MCAP 1.5M/30k ft.., max power

then

$$\prod_F = \prod_a \prod_b \prod_c = 0.9808$$

$$\beta = 0.8611$$

- Note: the total mission weight specific work is 43,870 ft.. with 62.82% utilized to increase the mechanical kinetic energy of the airplane
- The remaining 37.18% is dissipated into non mechanical energy of the airplane/atmosphere system
- A single gross interval calculation for this segment gives
  - higher value of the weight fraction by 0.18%
  - lower value of the total time by 6.34%
  - lower value of the ground distance by 7.68% and
  - lower value of the specific thrust work by 12.38%
- Also note that $\Delta_{sF} = 8.887$ n mi.

Phase 6-7 G: Supersonic Penetration

– 1.5M/30k ft.., ΔsF + ΔsG = 100 n mi., no afterburning

From Eq. (3.23)

with

Δ_{sG} = 91.11 n mi.
$\beta = 0.8611$
$W_{TO}/S = 64\ lb/ft^2$
$\delta = 0.2975$
$M = 1.5$

$C_L = 0.5558$
$C_{Do} = 0.028$
$K_1 = 0.28$
$C_D/C_L = 0.5193$
$\theta = 0.794$

$C = 1.5\ h^{-1}$
$(a/a_{SL}) = 0.8911$
$V = 1492$ ft/s

then

$$\Pi_G = 0.9331$$

Segments F and G together yield

$$\Pi_{67} = \Pi_F \, \Pi_G = 0.9152$$

$$\beta = 0.8035$$

Mission Phase 7-8: Combat

This phase consists of segments

- H (Combat Turn 1)
- I (Combat Turn 2) and
- J (Horizontal Acceleration)

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Phase 7-8 H: Combat Turn 1

— 1.6M/30k ft.., one 360 deg 5*g* sustained turn with afterburning

From Eq. (3.25)

$$\prod_H = \exp\left\{-C\sqrt{\theta}\left(\frac{nC}{C_L}\right)\frac{2nNV}{g\sqrt{n-1}}\right\}$$

$n = 5$		
$N = 1$	$C_L = 0.2279$	$C = 2.0\ h^{-1}$
$\beta = 0.8035$	$C_{Do} = 0.028$	$(a/a_{SL}) = 0.8911$
$W_{TO}/S = 64\ lb/ft^2$	$K_1 = 0.298$	$V = 1591\ ft/s$
$\delta = 0.2975$	$C_D/C_L = 0.5193$	
$M = 1.6$	$\theta = 0.794$	

Then,

$$\prod_H = 0.9705$$

$$\beta = 0.7798$$

Phase 7-8 J: Horizontal Acceleration – 0.8 1.6M/30k ft.., max power

- Horizontal acceleration calculation is divided into the three intervals shown in Table 2
- The initial, final, and average Mach number of each interval and h = 30,000 ft.. are used in the Climb/Acceleration Weight Fraction Calculation Method of Sec. 3.4.1.
- The calculated results are given in Table 2

	M_i	M_f	M_{avg}	Δz_e (ft)	$T\Delta s/W$ (ft)	Δt (min)	Δs (n mi)	c' (ft/s)	Π
a	0.8	1.06	0.93	7433	9444	0.2405	1.866	728	0.995
b	1.06	1.33	1.2	9919	15,980	0.2361	2.781	740.5	0.9934
c	1.33	1.6	1.47	12,160	25,040	0.2581	3.724	709.8	0.9916
			Σ	29,510	50,460	0.6987	8.371		

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Phase 7-8 J: Horizontal Acceleration – 0.8 1.6M/30k ft.., max power

$$\prod_J = \prod_a \prod_b \prod_c = 0.9801$$

- The weight specific thrust work of Segment J is 50,460 ft.. with 58.48% going to an increase of the airplane's mechanical kinetic energy and the rest is dissipated into nonmechanical energy by the drag forces
- A single interval calculation for this segment gives essentially
  - the same weight fraction value
  - lower value of time by 5.15%
  - lower value of distance by 6.77%
  - lower values of specific thrust work by 11.53%

For mission Phase 7-8 we have

$$\prod = \prod_H \prod_I \prod_J = 0.9261$$

$$\beta = 0.7441$$

Phase: Deliver Expendables – 1.6M/30k ft.., 1309 lb.

From Eq. (3.44), since Wj = W8 here,

$$\frac{W_8 - W_{PE}}{W_8} = 1 - \frac{W_{PE}}{W_{TO} \prod_{1\ 8}}$$

with WPE = 1309 lb.

$$\prod_{1\ 8} = 0.7441$$

W_{TO} = 25,000 lb. (assumed) Source: Fig 3.2

then $$\frac{W_8 - W_{PE}}{W_8} = 0.9296$$

β = 0.6917

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Phase 8-9: Escape Dash

-1.5M/30k ft.., $\Delta s89 = 35$ n mi., no afterburning.

- The computational procedure is identical with that of Segment G
- Here the distance is smaller (25n. mi. vs 91.11 n. mi.) and the fraction ……………………………………… ……………..

$$\prod_{8\ 9} = 0.9769$$

$$\beta = 0.6757$$

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Phase 9-10: Minimum Time Climb

BCM/BCA = 1.5M/30kft

Airplane climbs from 30,00 ft.. to 50,000 ft.. and reduces speed from 1.5 to 0.9 Ma

From Eq 3.42:

$$\prod_{9\ 10} = \exp\left\{-c\sqrt{\theta}\left(\frac{C_D}{C_L}\right)\Delta t\right\}$$

With (assuming vertical speed = 0.7 V_{avg})

$$\Delta t = \frac{\Delta h}{0.7V_{avg}} \qquad \frac{V_{mid}^2}{2g_o} = z_{e_i} - h_{mid}$$

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Phase 9-10: Minimum Time Climb

Given information:

h_i = 30,00 ft..
M_i = 1.5
$(a/a_{SL})_i$= 0.8911
V_i = 1492 ft./s
Z_{ei} = 64,600 ft..
H_f = 50,000 ft..
Δ_h = 20,000 ft..
M_f = 0.9

$(a/a_{SL})_f$= 0.8671
V_f = 870.9 ft./s
V_{avg} = 1181 ft./s
Δt = 24.19 s
H_{mid} = 40,000 ft..
V_{mid} = 1258 ft./s
$(a/aSL)_{mid}$= 0.8671
M_{mid} = 1.3
W_{TO}/S = 64 lb./ft^2

β= 0.6757
δ= 0.1858
C_L = 0.093
K_1 = 0.23
K_2 = 0
C_{D0} = 0.023
C_D/C_L = 0.2687
θ= 0.7519
C = 1.35 h^{-1}

Calculation from given information

$$\prod_{9}\prod_{10} = 0.9979$$

$$\beta = 0.6743$$

Phase 10-11: Subsonic Cruise Climb

BCM/BCA, $\Delta s_{10\text{-}11}$ = 150 nmi.

$$\prod_{10\ \ 11} = .9620$$

$$\beta = 0.6487$$

Mission Phase 11-12: Descend

BCM/BCA = M_{loiter}/10kft

$$\prod_{11\ \ 12} = 1$$

$$\beta = 0.6487$$

Phase 12-13: Loiter

BCM/BCA = M_{loiter}/10kft, 20 min

$$\prod_{12}^{13} = .9573$$

$$\beta = 0.6210$$

Phase 13-14: Descend and Land

M_{loiter}/10kft --> 2000 ft.. PA, 100^{o}F

$$\prod_{13}^{}\prod_{14}^{} = 1$$

$$\beta = 0.6210$$

Table 3.E3 Summary of Results – Mission Analysis

Mission Phases & Segments		**$\beta = W/W_{TO}$ End of Leg**	**$W_f/W_i=\prod_i^f$**
1-2	Warm-up and Takeoff	0.9759	0.9759
2-3	Accelerate and Climb	0.9445	0.9678
3-4	Subsonic Cruise Climb	0.9141	0.9678
4-5	Descend	0.9141	1.0000
5-6	Combat Air Patrol	0.8780	0.9605
6-7	Supersonic Penetration	0.8035	0.9152
7-8	Combat	0.7441	0.9261
	Deliver Expendables	0.6917	-------
8-9	Escape Dash	0.6757	0.9769
9-10	Minimum Time Climb	0.6743	0.9979
10-11	Subsonic Cruise Climb	0.6487	0.9620
11-12	Descend	0.6487	1.0000
12-13	Loiter	0.6210	0.9573
13-14	Descend and Land	0.6210	1.0000

Weight Fractions (β)

- After the β values have been estimated or calculated, they must be reapplied to the constraint equations to confirm that the design point thrust and wing loading is still acceptable for all constraints and mission segments

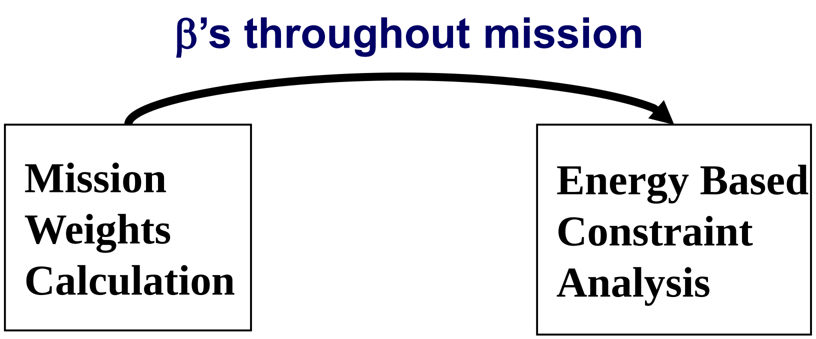

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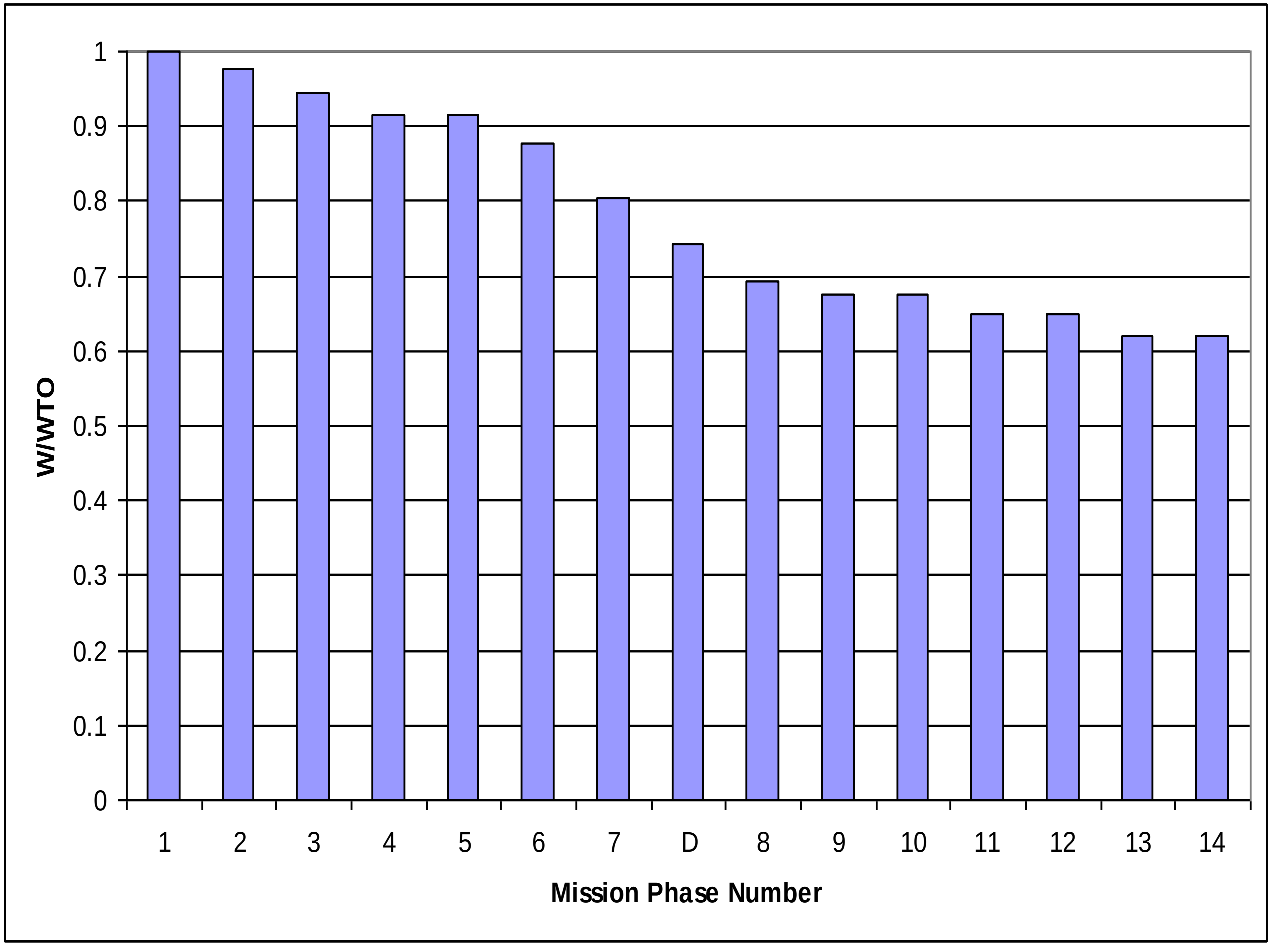
Figure 3.E3 Fraction of Takeoff Weight vs Mission Phase



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3.4.2 Determination of W_{TO}, T_{SL}, and S

- Now compute the takeoff weight for the AAF (Air-to-Air Fighter) using Equation 3.46 shown below.

$$W_{TO} = \frac{W_{PP} + W_{PE}\Pi_8^{14}}{\Pi_1^{14} - \Gamma} \qquad (3.46)$$

Where,

W_{PP}= 200 lb.	Pilot plus equipment	
270	Cannon	
405	Ammunition feed system	
275	Returning ammunition	
198	Casings weight	
1,348 lb.		

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Continued

W_{PE} =	382 lb.	Sidewinder missiles
	652	AMRAAMs
	<u>275</u>	Spent Ammunition
	1,309 lb.	

Source: AAF RFP Sec. D

$$\Pi_8^{14} = \Pi_8^9 \cdots \Pi_{13}^{14}$$

$$\Pi_1^{14} = \Pi_1^2 \cdots \Pi_{13}^{14}$$

Source: Table 3.E3

$$\Gamma = \frac{W_E}{W_{TO}} = 0.6273$$

Source: Eq. (3.49)
Assumed W_{TO} = 25,000 lb.

Thus,

$$W_{TO} = \frac{1348 + (1309)(0.8978)}{0.6680 - 0.6273} = 62,000 lb$$

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Example Mission Analysis (cont.)

Reducing the payload provides little relief because the large W_{TO} is the result of the very small denominator.

Another assumption can be the reducing the fuel usage, for example by shortening the range or duration of mission. Yet, another assumption would be reducing the empty weight, for example by using advanced lightweight construction materials as composites.

With the latter assumption, the aircraft's Γ will be reduced by approximately 10%. Therefore, with this reduction, W_{TO} is recalculated for the assumed W_{TO} of 25,000 lbs.,

$$W_{TO} = \frac{1348 + (1309)(0.8978)}{0.6680 - 0.6273(0.90)} = 24{,}400 lbs$$

Example Mission Analysis (cont.)

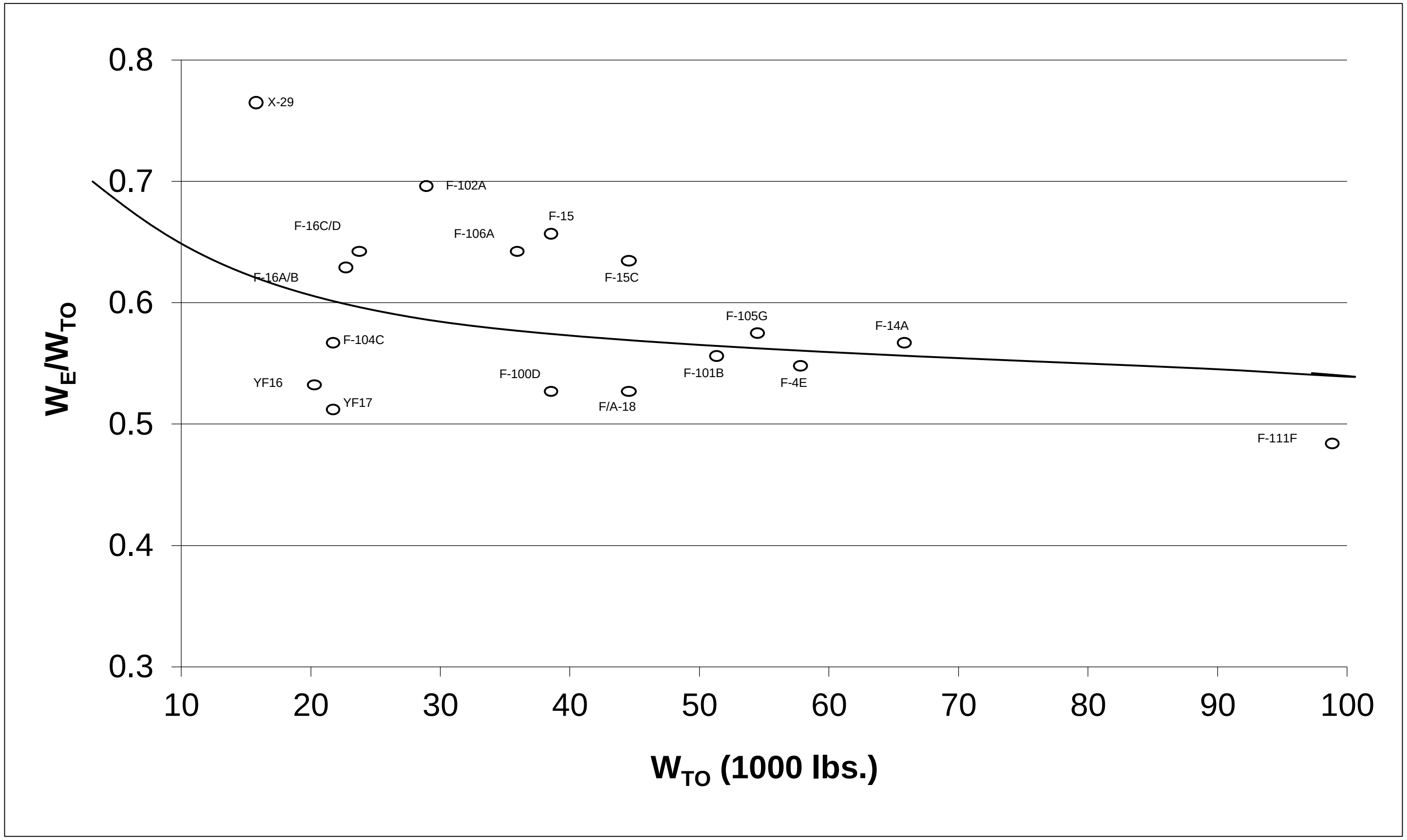

Fig. 3.E4 Weight Fractions of Fighter Aircraft

Notice (Figure 3.E4) that this conventional metal version of the AAF would be considerably heavier than existing lightweight fighters (e.g., F-16).

Example Mission Analysis (cont.)

The conclusion is that AAF can have a practical size. This is true only if reliable non metallics with competitive strength, durability, and reparability become available according to schedule. With the choices of T_{SL}/W_{TO} = 1.20 and W_{TO}/S = 64.0 made, the description of the AAF at this stage of design, using W_F/W_{TO} = 0.3265, is:

W_{TO} = 24,400 lb.

T_{SL} = 29,300 lb.

S = 381 ft^2

W_P = 2,660 lb.

W_E= 13,800 lb.

W_F = 7,970 lb.

This information will give perspective and insight regarding the nature and shape of the corresponding aircraft.