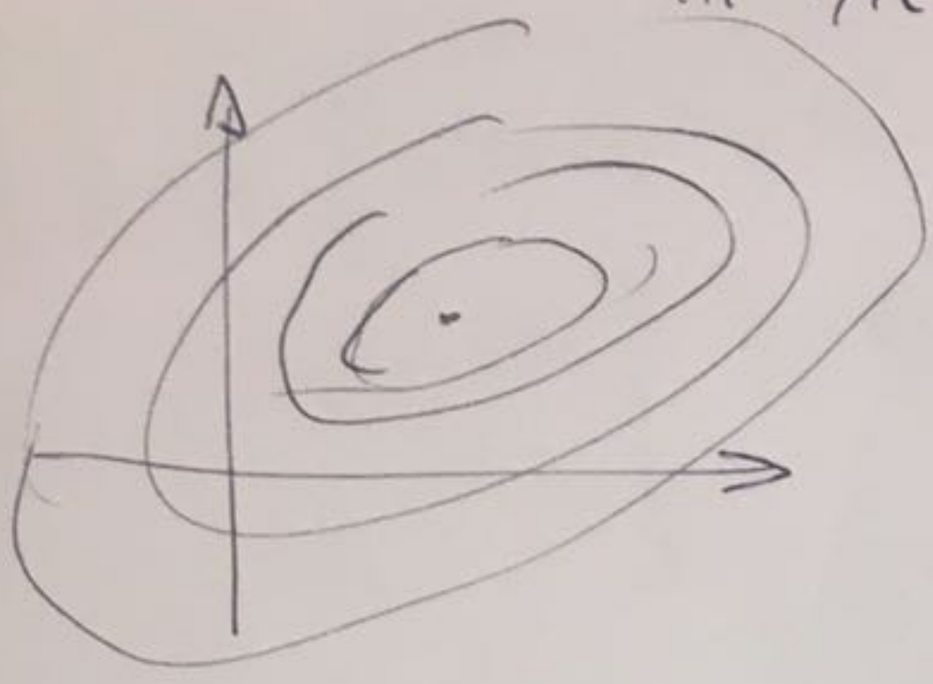
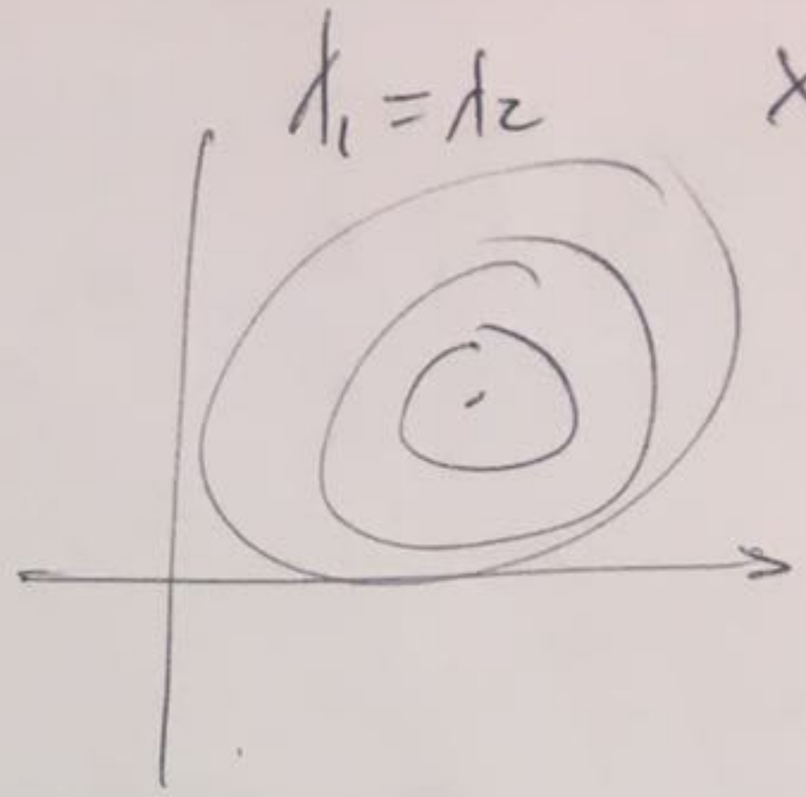
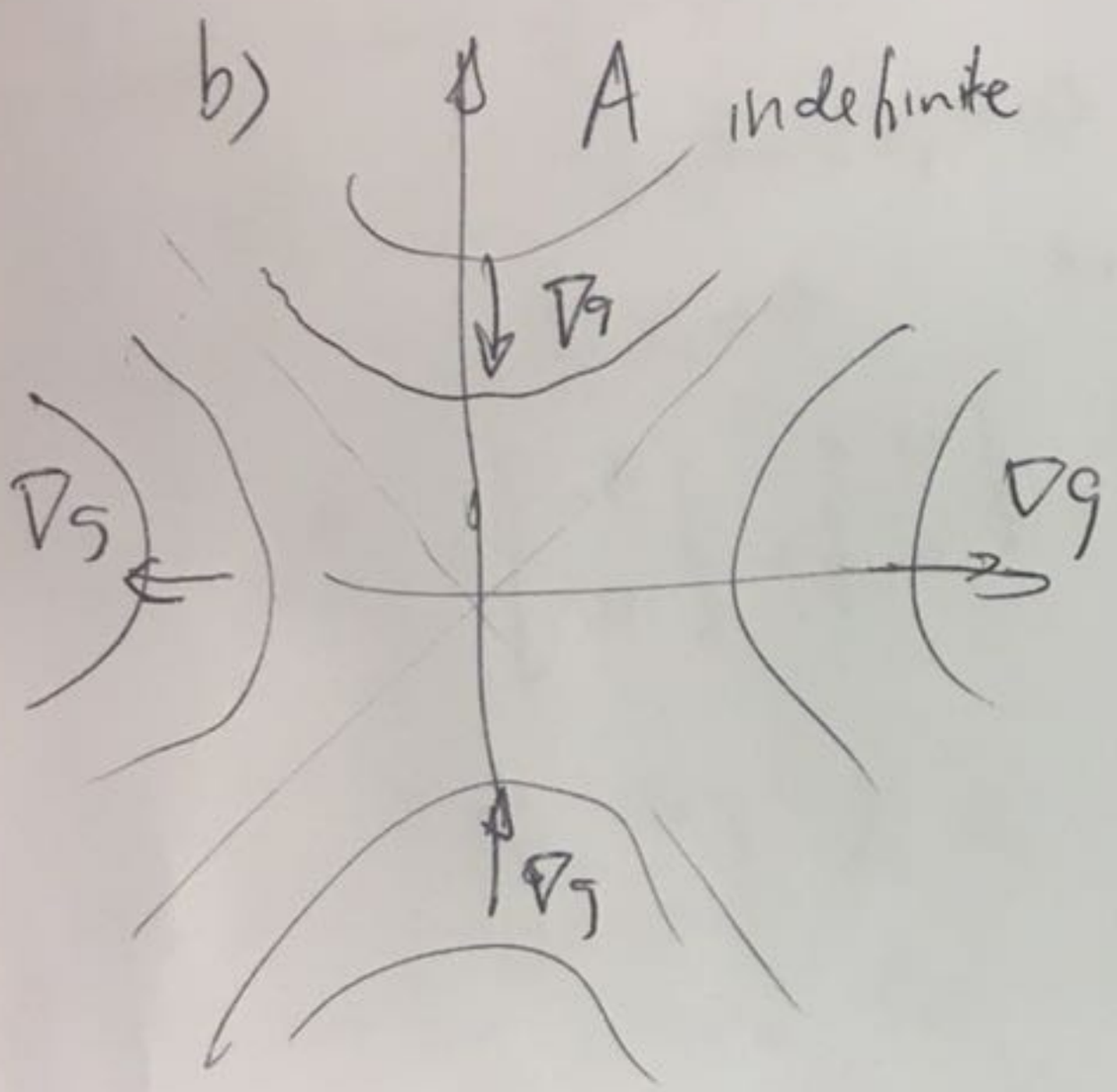
#1 a)

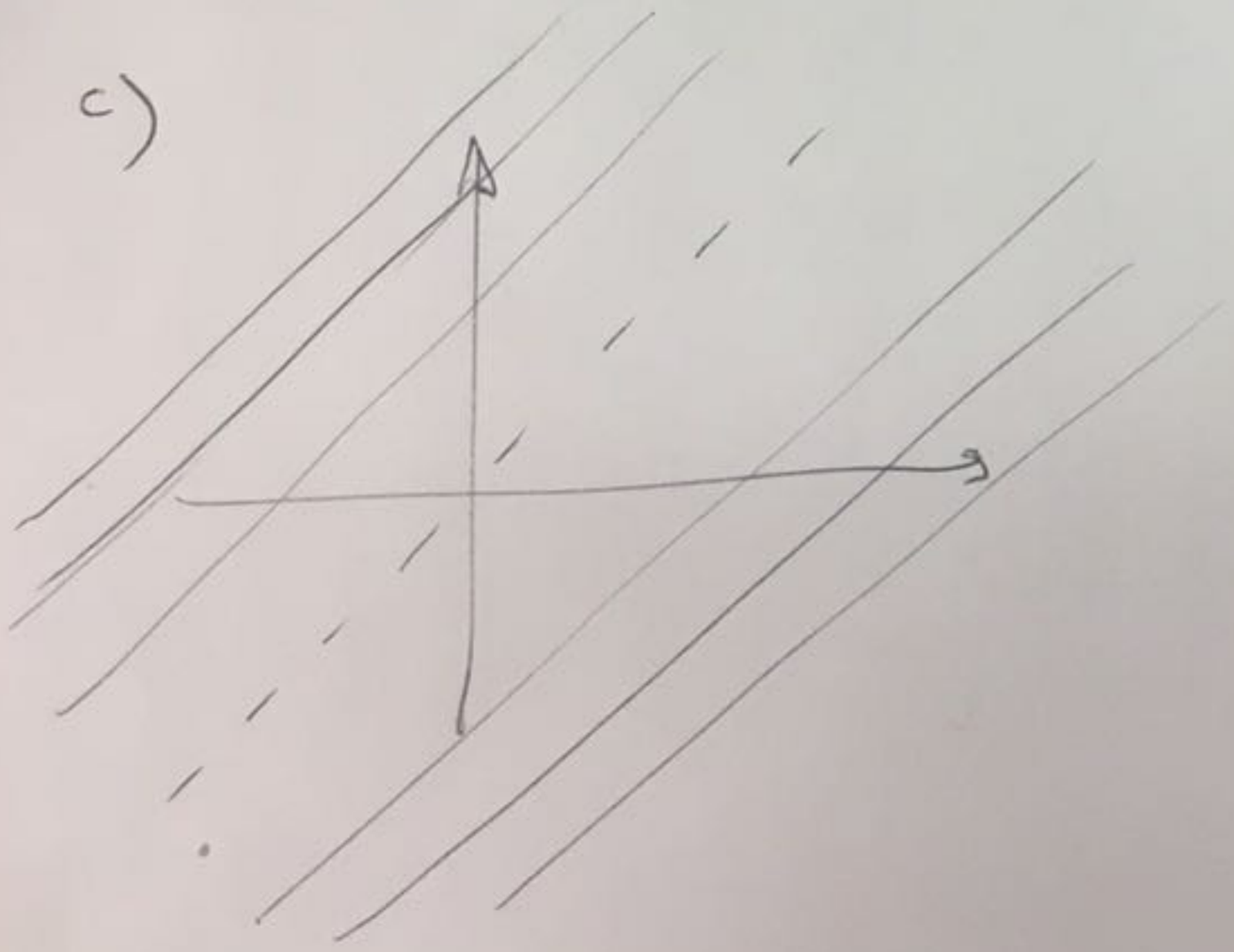

$\lambda_1 \neq \lambda_2$


$\lambda_1 = \lambda_2$
X

b)


A indefinite
∇g
∇g
∇g
∇g

c)



#2) $f(x_1, x_2) = \frac{1}{2}x_1^4 - x_1^2 + x_2^2 - 3$

$$Df = \begin{bmatrix} 2x_1^3 - 2x_1 \\ 2x_2 \end{bmatrix} \qquad H = \begin{bmatrix} 6x_1^2 - 2 & 0 \\ 0 & 2 \end{bmatrix}$$ (5)

Find critical points:

$Df = 0 \quad 2x_1^3 - 2x_1 = 0 \rightarrow 2x_1(x_1^2 - 1) = 0$

$x_1 = 0 \qquad x_1 = \pm 1 \qquad x_2 = 0$

$$x_{cr}^* = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ (10)

$x_c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$H = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \quad \lambda_1, \lambda_2 = 4, 2$

Pos. definite. (5)

$\Rightarrow x_{cr}^* = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ is a local min

$H = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$

H is indefinite. (5)

saddle point.

not a min.

$x_{cr} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ positive definite.

$H = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ is a local min (5)

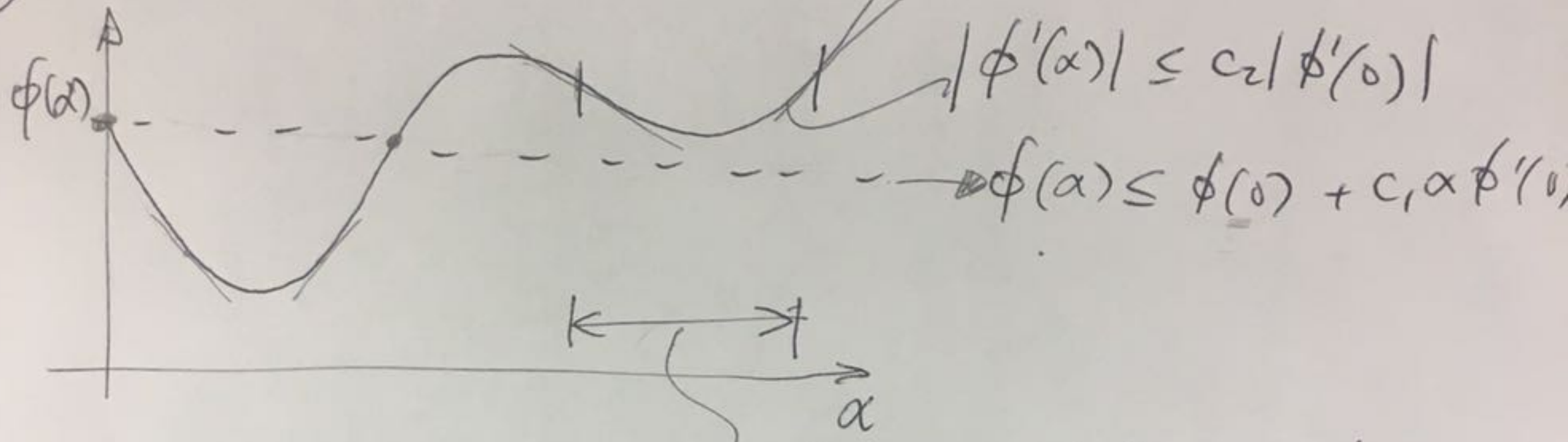
$\phi(\alpha)$

* Strong Wolfe conditions:

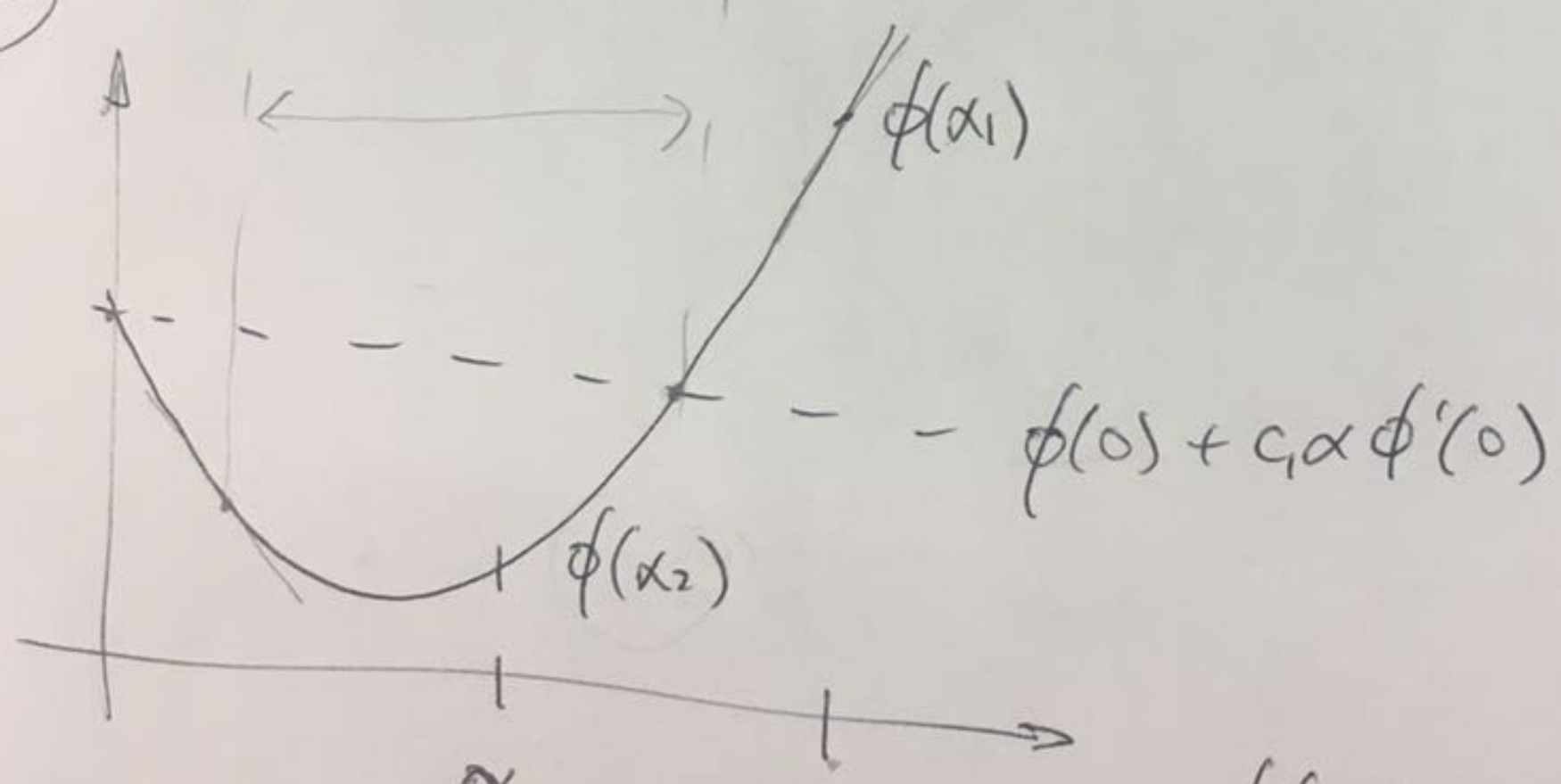
① $\phi(\alpha) \leq \phi(0) + c_1 \alpha \phi'(0)$. (5)

② $|\phi'(\alpha)| \leq c_2 |\phi'(0)| \left(= -c_2 \phi'(0)\right)$

(10)



(c) (10)



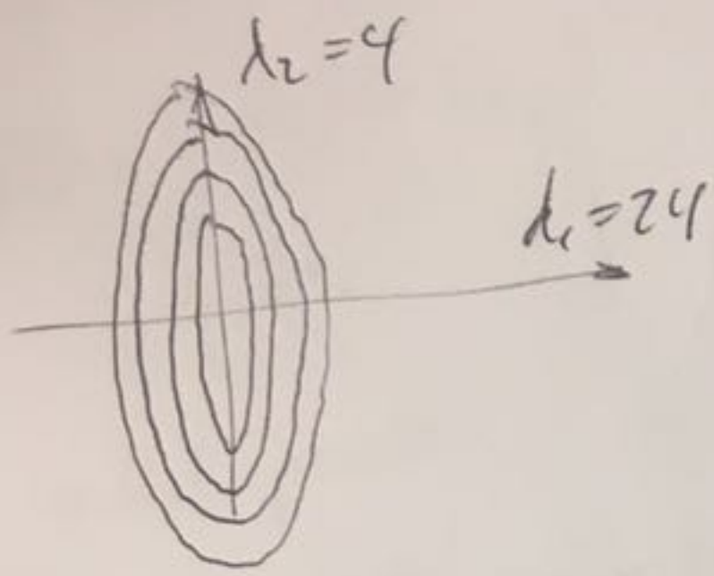
$\phi(\alpha_2) \leq \phi(0) + c_1 \alpha \phi'(0)$

$\tau \in (0,1)$ $\boxed{\alpha_2 = \tau \alpha_1 = \frac{1}{2}\alpha_1}$

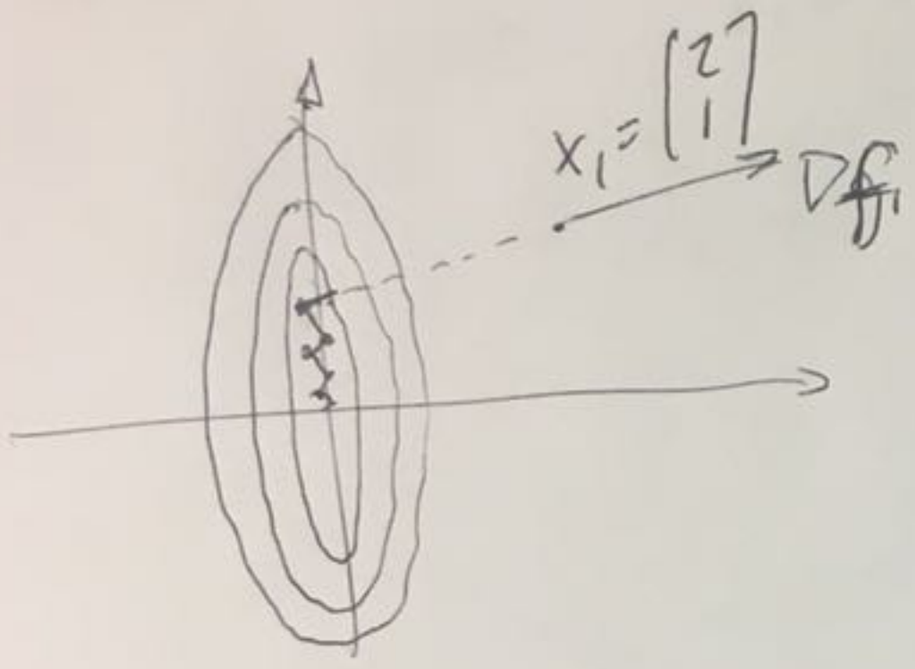
• The backtracking method works by decreasing the step size at each iteration.

#4 $A = \begin{bmatrix} 24 & 0 \\ 0 & 4 \end{bmatrix}$ $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $b = 0$

a)

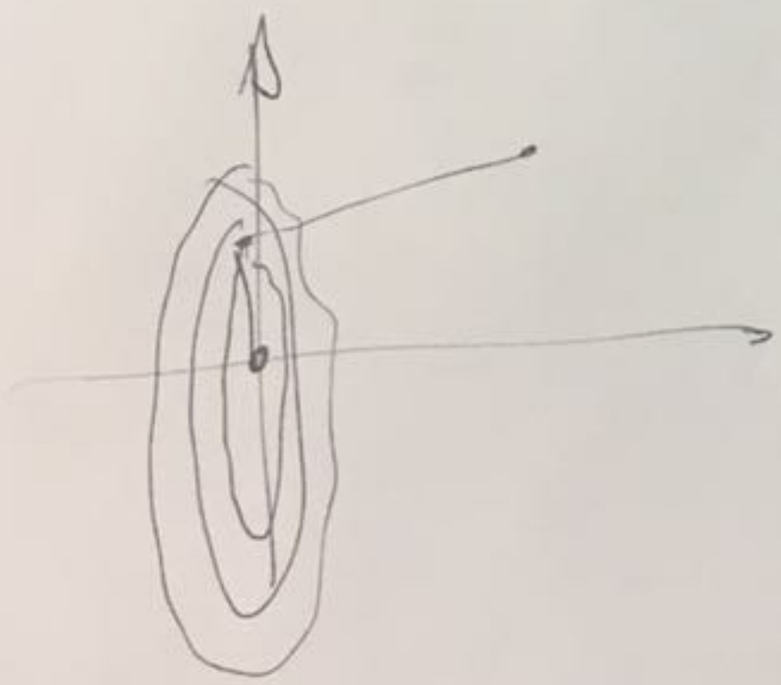


b)



- Key points:
  1) Subsequent line search directions should be perpendicular.
  2) It should take more than 2 iterations.

c)



Key pruk:
1) Subsequent directions are conjugate
2) It should take two iterations.

d) In the steepest descent, the search direction is $\perp$ and in C.G, the search direction is conjugate, finish in n steps (in the problem 2)

e)
1. More than 2 steps for C.G not conjugate
2. steepest descent no longer $\perp$