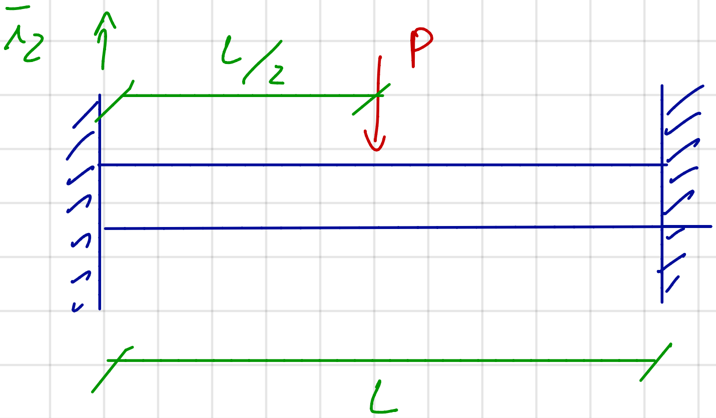
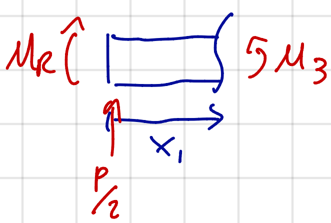
$H_{23}^{c} = 0$



1) Find $M_3(x_1)$?

$$\theta(0) = \frac{\partial A'}{\partial M_R} = 0 \quad \rightarrow \text{ Find } M_R$$



$$M_3 - M_R - \frac{P}{2} x_1 = 0$$

$$M_3 = M_R + \frac{P}{2} x_1 \qquad 0 < x_1 < L/2$$

$$A' = 2 \cdot \int_0^{L/2} \frac{1}{2} \frac{M_3^2}{H_{33}^c} \, dx_1$$

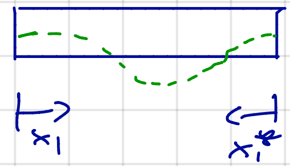
$$\theta(0) = 0 = \frac{\partial A'}{\partial M_R} = 2 \cdot \int_0^{L/2} \frac{M_3}{H_{33}^c} \frac{\partial M_3}{\partial M_R} \, dx_1$$

$$0 = 2 \cdot \int_0^{L/2} \frac{1}{H_{33}^c} \left(M_R + \frac{P}{2} x_1 \right) (1) \, dx_1$$

$$= \frac{2}{H_{33}^c} \cdot \left(M_R \frac{L}{2} + \frac{P}{4} \frac{L^2}{4} \right) = 0 \qquad \boxed{M_R = -\frac{PL}{8}}$$

$$M_3 = -\frac{PL}{8} + \frac{P}{2} x_1 = P \left(\frac{x_1}{2} - \frac{L}{8} \right)$$

$$\mathcal{M}_3 = -\frac{PL}{8} + \frac{P}{2}x_1 = P\left(\frac{x_1}{2} - \frac{L}{8}\right) \qquad x_1 < L/2$$



$$\mathcal{M}_3 = P\left(\frac{(L-x_1)}{2} - \frac{L}{8}\right) = P\left(-\frac{x_1}{2} + \frac{3L}{8}\right)$$

$$x_1^* = L - x_1$$

$$* \quad \mathcal{M}_3 = \begin{cases} P\left(\frac{x_1}{2} - \frac{L}{8}\right) & x_1 < L/2 \\ P\left(-\frac{x_1}{2} + \frac{3L}{8}\right) & x_1 > L/2 \end{cases}$$

Part 2) Find $u_2(x_1)$?

Unit Load System

$$\hat{\mathcal{M}}_3 = \begin{cases} \delta P(d - x_1) & x_1 < d \\ 0 & x_1 > d \end{cases}$$

$$\Delta(d) = \int_0^L \frac{\mathcal{M}_3 \hat{\mathcal{M}}_3}{H_{33}^c}$$

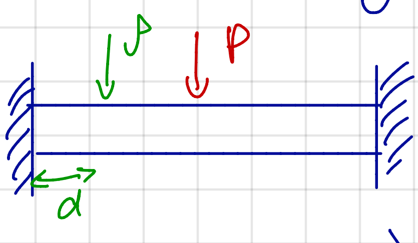
$$\Delta = \int_0^d \frac{P}{H_{33}^c}\left(\frac{x_1}{2} - \frac{L}{8}\right)(d - x_1)\,dx_1$$

$$\Delta = \int_0^d \frac{P}{H_{33}^c}\left(\frac{x_1 d}{2} - \frac{L}{8}d - \frac{x_1^2}{2} + \frac{x_1 L}{8}\right)dx_1$$

$$\Delta = \frac{P}{H_{33}^c}\left(\frac{d^3}{4} - \frac{L}{8}d^2 - \frac{d^3}{6} + \frac{d^2 L}{16}\right)$$

$$\Delta = \frac{P}{H_{33}^c}\left(\frac{d^3}{12} - \frac{L d^2}{16}\right) \rightarrow u_2(x_1) = \frac{P}{H_{33}^c}\left(\frac{x_1^3}{12} - \frac{L x_1^2}{16}\right) \qquad x_1 < L/2$$

Aside: Dummy Load



Find $\mathcal{M}_3(P, D)$

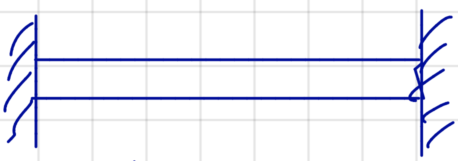
$\rightarrow$ Difficult!

* $u_2(x_1) = \frac{P}{H^C_{33}}\left(\frac{x_1^3}{12} - \frac{L x_1^2}{16}\right) \qquad x_1 < L/2$

For $x_1 > L/2$ substitute $x_1 = L - x_1$

* $u_2(x_1) = \frac{P}{H^C_{33}}\left(\frac{(L-x_1)^3}{12} - L\frac{(L-x_1)^2}{16}\right)$

3a)



$$\hat{u}_2 = \cancel{q_0} + \cancel{q_1 x_1} + q_2 x_1^2 + q_3 x_1^3 + q_4 x_1^4$$

$$\underline{\hat{u}_2 = q\, x_1^2 (L-x_1)^2}$$

$$\Pi = \int_0^L \frac{1}{2} H^C_{33} \left(\frac{d^2\hat{u}_2}{dx_1^2}\right)^2 dx_1 + P \cdot u_2(L/2)$$

$\hat{u} = q\, x_1^{\alpha} (L - x_1)^{\beta}$

$\left.\begin{array}{l} \alpha = 0 \rightarrow \text{free} \\ \alpha = 1 \rightarrow \text{pin} \\ \alpha = 2 \rightarrow \text{cantilever} \end{array}\right\} x_1 = 0$

$$\hat{u}_2 = q\left(L^2 x_1^2 - 2L x_1^3 + x_1^4\right) \qquad u_2(L/2) = q\frac{L^4}{16}$$

$$\hat{u}_2' = q\left(2L^2 x_1 - 6L x_1^2 + 4x_1^3\right)$$

$$u_2'' = q\left(2L^2 - 12L x_1 + 12 x_1^2\right)$$

$$\Pi = \int_0^L \frac{1}{2} H^C_{33} \left(q\left(2L^2 - 12L x_1 + 12x_1^2\right)\right)^2 dx_1 + q\frac{L^4}{16}$$

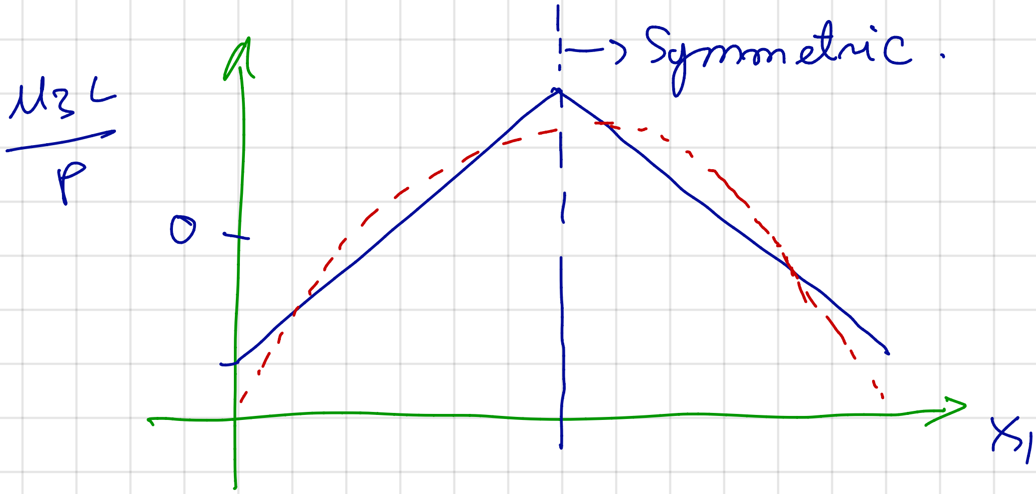
$$\Pi = H^C_{33}\, q^2 \frac{4}{10} L^5 + q\frac{L^4}{16} \qquad \frac{\partial \Pi}{\partial q} = 2 H^C_{33}\, q \frac{4}{10} L^5 + q\frac{L^4}{16}$$

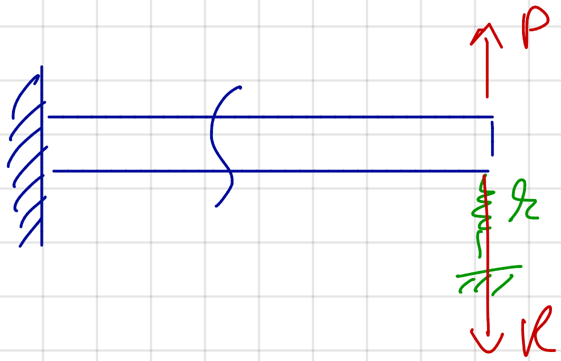
$$\boxed{q = -\frac{1}{64}\frac{P}{H^C_{33} L}}$$

$$* \quad \hat{u}_2 = -\frac{1}{64}\frac{P}{H_{33}^c L} x_1^2 (L - x_1)^2$$

b) Find $\hat{u}_3$?

$$\hat{u}_3 = H_{33}^c \frac{d^2\hat{u}_2}{dx_1^2} = -\frac{5}{128}\left(2L^2 - 12Lx_1 + 12x_1^2\right)\frac{P}{L}$$





Find M_3

$$M_3 = (P-R)(L-x_1) \qquad P_{TOT} = P-R$$

$$A' = \int_0^L \frac{1}{2}\frac{1}{H_{33}^c}\Big((P-R)(L-x_1)\Big)^2 dx_1$$

$$\Delta = \frac{\partial A'}{\partial P_{TOT}} = \int_0^L \frac{1}{H_{33}^c}(P-R)(L-x_1)(L-x_1)\,dx_1 = \frac{R}{k}$$

$$\Delta = \frac{1}{H_{33}^c}(P-R)\frac{L^3}{3} = \frac{R}{k}$$

$$\boxed{R = \frac{P\,k L^3}{3H_{33}^c + kL^3}}$$

$$* \quad M_3 = P\left(1 - \frac{kL^3}{3H_{33}^c + kL^3}\right)(L-x_1)$$