Particle Swarm Optimization

Introduction

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Presentation overview

- Origins
- The idea
- Continuous optimization
- The basic algorithm
- Main variants
- Parameter selection
- Research issues
- Our work at IRIDIA-CoDE

Particle swarm optimization: Origins

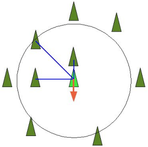
How can birds or fish exhibit such a coordinated collective behavior?

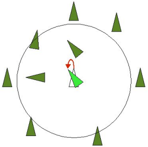
Reynolds [12] proposed a behavioral model in which each agent follows three rules:

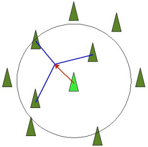
Separation. Each agent tries to move away from its neighbors if they are too close.

Alignment. Each agent steers towards the average heading of its neighbors.

Cohesion. Each agent tries to go towards the average position of its neighbors.



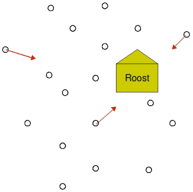


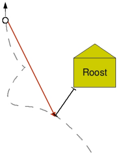


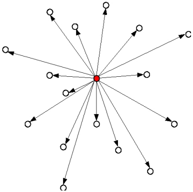
Particle swarm optimization: Origins

Kennedy and Eberhart [6] included a ‘roost’ in a simplified Reynolds-like simulation so that:

- Each agent was attracted towards the location of the roost.
- Each agent ‘remembered’ where it was closer to the roost.
- Each agent shared information with its neighbors (originally, all other agents) about its closest location to the roost.







Particle swarm optimization: The idea

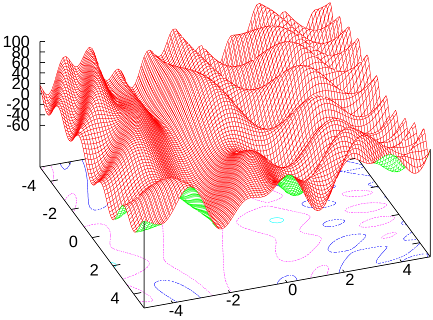
Eventually, all agents ‘landed’ on the roost.

What if the notion of distance to the roost is changed by an unknown function? Will the agents ‘land’ in the minimum?

J. Kennedy

R. Eberhart

The continuous optimization problem can be stated as follows:

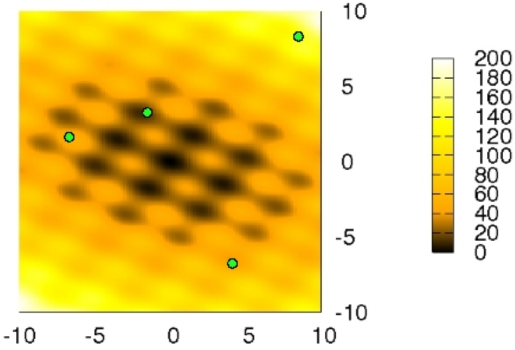


Find $\mathcal{X}^* \subseteq \mathcal{X} \subseteq \mathbb{R}^n$ such that

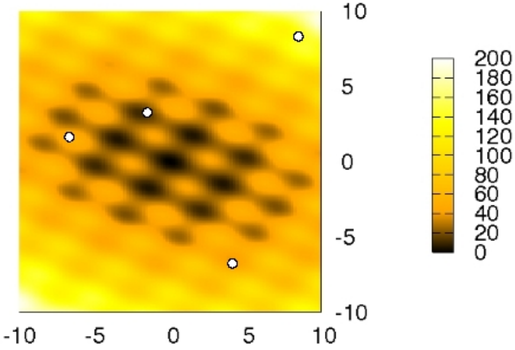
$$\mathcal{X}^* = \underset{\mathbf{x} \in \mathcal{X}}{\operatorname{argmin}} f(\mathbf{x}) = \{\mathbf{x}^* \in \mathcal{X} \,:\, f(\mathbf{x}^*) \leq f(\mathbf{x}) \ \forall \mathbf{x} \in \mathcal{X}\}$$

Particle swarm optimization: The basic algorithm

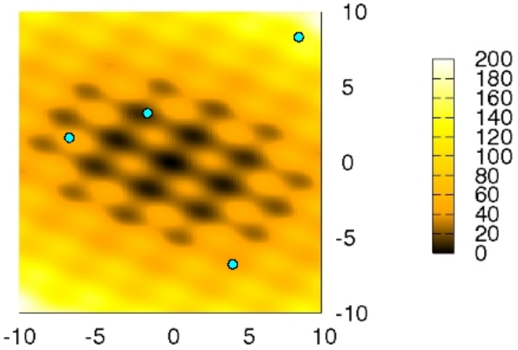
1. Create a 'population' of agents (called *particles*) uniformly distributed over $\mathcal{X}$.



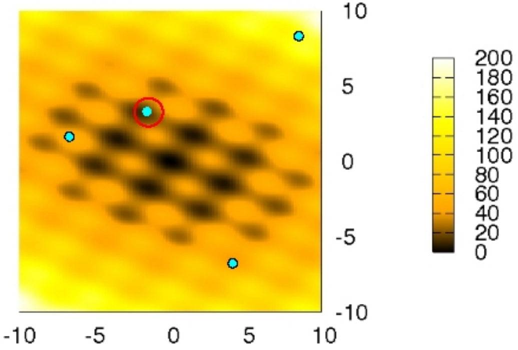
2. Evaluate each particle's position according to the objective function.



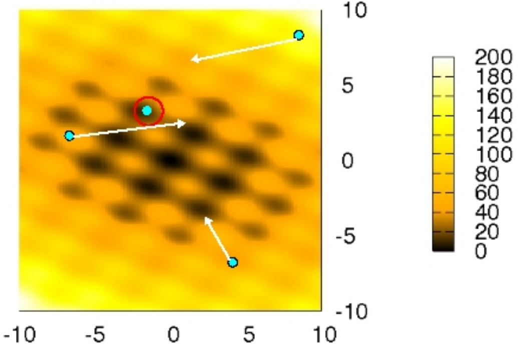
3. If a particle's current position is better than its previous best position, update it.



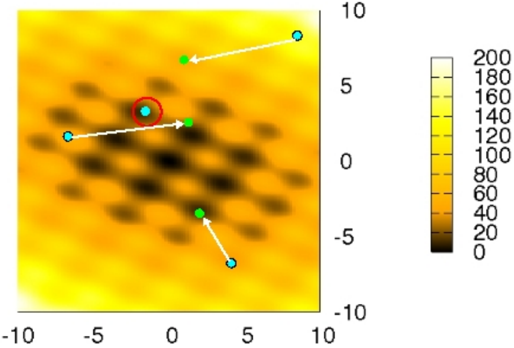
4. Determine the best particle (according to the particle's previous best positions).



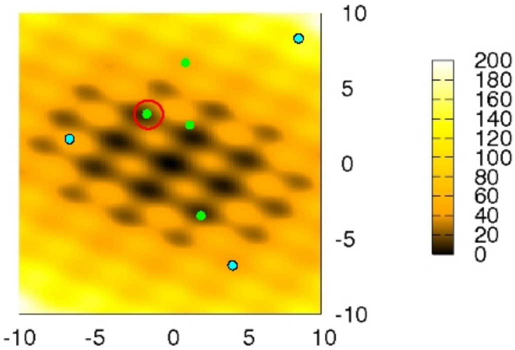
5. Update particles' velocities according to
$\mathbf{v}_i^{t+1} = \mathbf{v}_i^t + \varphi_1 \mathbf{U}_1^t(\mathbf{pb}_i^t - \mathbf{x}_i^t) + \varphi_2 \mathbf{U}_2^t(\mathbf{gb}^t - \mathbf{x}_i^t)$.



6. Move particles to their new positions according to $\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1}$.



7. Go to step 2 until stopping criteria are satisfied.



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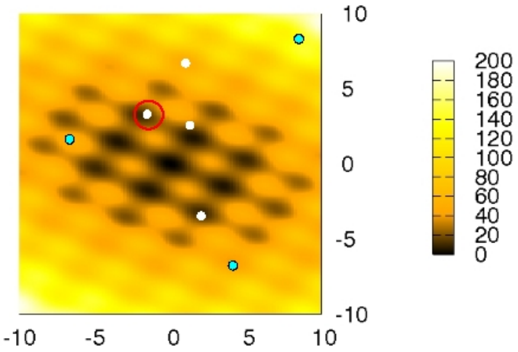

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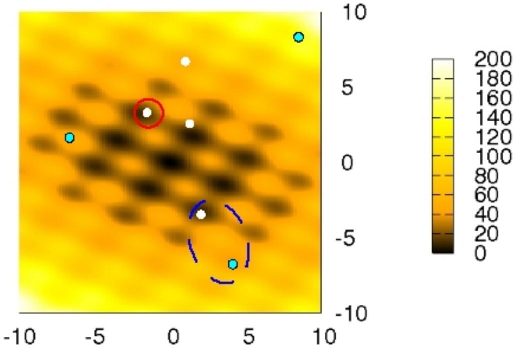

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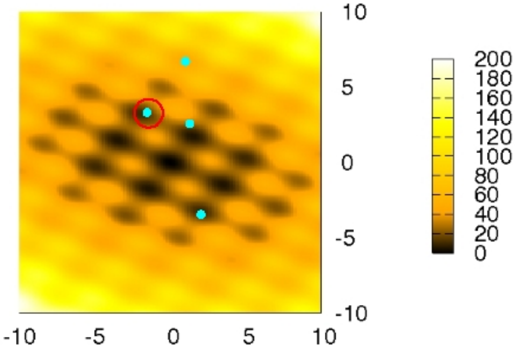

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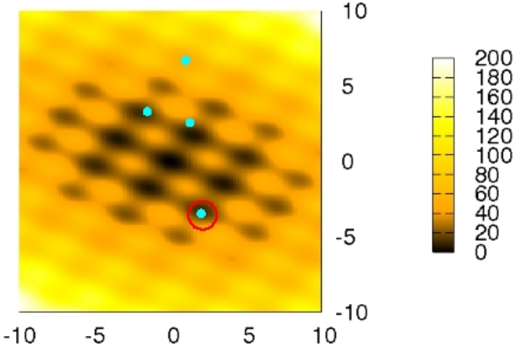

Particle swarm optimization: The basic algorithm

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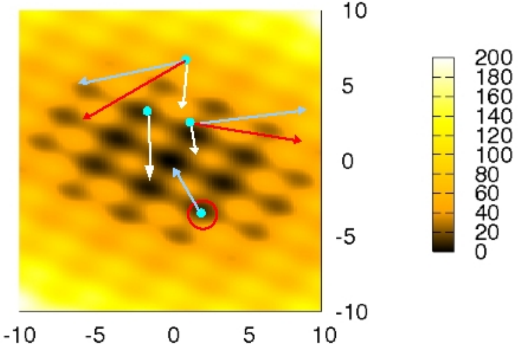
$\mathbf{v}_i^{t+1} = \mathbf{v}_i^{t} + \varphi_1 \mathbf{U}_1^t(\mathbf{pb}_i^t - \mathbf{x}_i^t) + \varphi_2 \mathbf{U}_2^t(\mathbf{gb}^t - \mathbf{x}_i^t)$.



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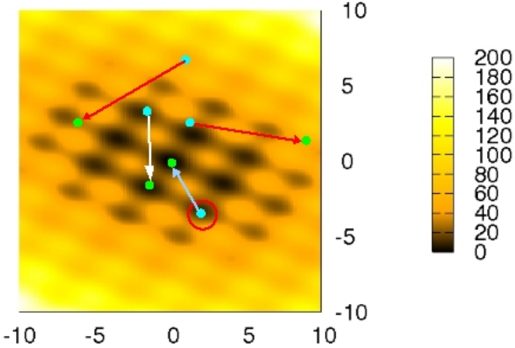

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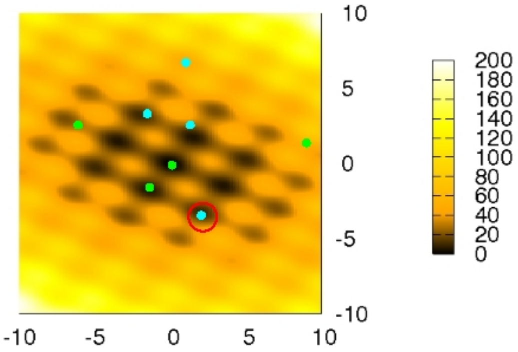


Particle swarm optimization: Main variants

Almost all modifications vary in some way the velocity-update rule:

$$\mathbf{v}_i^{t+1} = \mathbf{v}_i^t + \varphi_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{x}_i^t) + \varphi_2 \mathbf{U}_2^t (\mathbf{gb}^{\,t} - \mathbf{x}_i^t)$$

Almost all modifications vary in some way the velocity-update rule:

$$\mathbf{v}_i^{t+1} = \underbrace{\mathbf{v}_i^{t}}_{inertia} + \varphi_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{x}_i^t) + \varphi_2 \mathbf{U}_2^t (\mathbf{gb}^t - \mathbf{x}_i^t)$$

Almost all modifications vary in some way the velocity-update rule:

$$\mathbf{v}_i^{t+1} = \mathbf{v}_i^t + \underbrace{\varphi_1 \mathbf{U}_1^t(\mathbf{pb}_i^t - \mathbf{x}_i^t)}_{\textit{personal influence}} + \varphi_2 \mathbf{U}_2^t(\mathbf{gb}^t - \mathbf{x}_i^t)$$

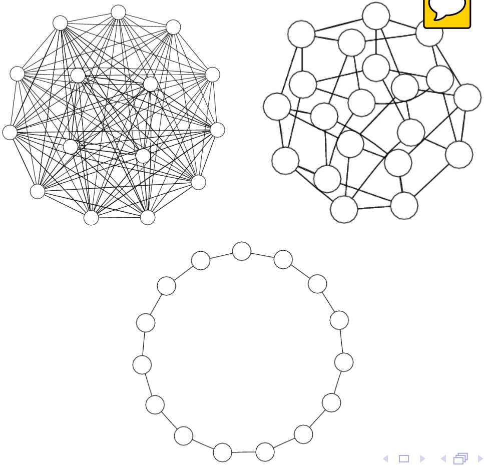
Almost all modifications vary in some way the velocity-update rule:

$$\mathbf{v}_i^{t+1} = \mathbf{v}_i^t + \varphi_1 \mathbf{U}_1^t(\mathbf{pb}_i^t - \mathbf{x}_i^t) + \underbrace{\varphi_2 \mathbf{U}_2^t(\mathbf{gb}^t - \mathbf{x}_i^t)}_{\textit{social influence}}$$

Particle swarm optimization: Different population topologies

Every particle i has a neighborhood N_i.

$$\mathbf{v}_i^{t+1} = \mathbf{v}_i^t + \varphi_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{x}_i^t) + \varphi_2 \mathbf{U}_2^t (\mathbf{lb}_i^t - \mathbf{x}_i^t)$$



Particle swarm optimization: Inertia weight

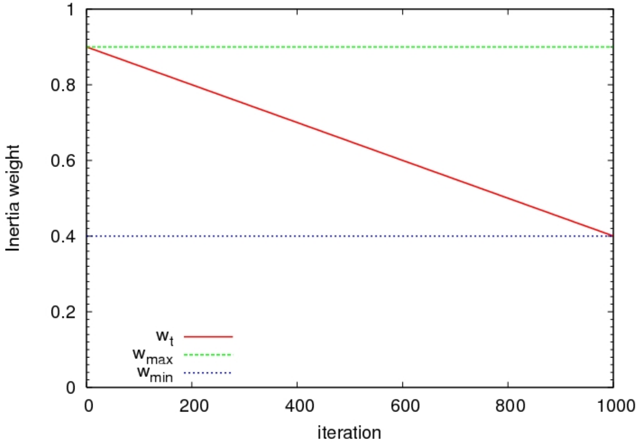
It adds a parameter called *inertia weight* so that the modified rule is:

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + \varphi_1 \mathbf{U}_1^t(\mathbf{pb}_i^t - \mathbf{x}_i^t) + \varphi_2 \mathbf{U}_2^t(\mathbf{lb}_i^t - \mathbf{x}_i^t)$$

It was proposed by Shi and Eberhart [14].

Particle swarm optimization: Time-decreasing inertia weight

The value of the inertia weight is decreased during a run



It was proposed by Shi and Eberhart [15].

Particle swarm optimization: Canonical PSO

It is a special case of the inertia weight variant derived from:

$$\mathbf{v}_i^{t+1} = \chi \left[\mathbf{v}_i^t + \varphi_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{x}_i^t) + \varphi_2 \mathbf{U}_2^t (\mathbf{lb}_i^t - \mathbf{x}_i^t)\right] ,$$

where χ is called a "constriction factor" and is fixed.

It has been very influential after its proposal by Clerc and Kennedy [3].

Particle swarm optimization: Fully Informed PSO

In the Fully Informed PSO, a particle is attracted by every other particle in its neighborhood:

$$\mathbf{v}_i^{t+1} = \chi \left[\mathbf{v}_i^t + \sum_{p_k \in \mathcal{N}_i} \varphi_k \mathbf{U}_k^t (\mathbf{pb}_k^t - \mathbf{x}_i^t) \right] .$$

It was proposed by Mendes et al. [9].

Particle swarm optimization: Other variants

There are many other variants reported in the literature. Among others:

- with dynamic neighborhood topologies (e.g., [16], [10])
- with enhanced diversity (e.g., [2], [13])
- with different velocity update rules (e.g., [11], [8])
- with components from other approches (e.g., [1], [5])
- for discrete optimization problems (e.g., [7], [18])
- ...

Consider a one-particle one-dimensional particle swarm. This particle's velocity-update rule is

$$v^{t+1} = av^{t} + b_1 U_1^t(pb^{t} - x^{t}) + b_2 U_2^t(gb^{t} - x^{t})$$

Additionally, if we make

$$E[U_*^t(0,1)] = \frac{1}{2}\,,$$

$$b = \frac{b_1 + b_2}{2}\,,$$

$$pb^{\,t+1} = pb^{\,t+1}\,, gb^{\,t+1} = gb^{\,t}\,,$$

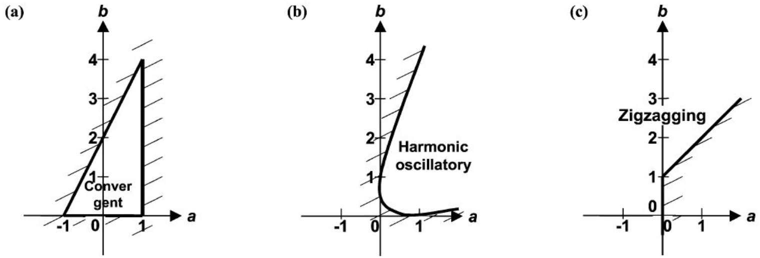
and

$$r = \frac{b_1}{b_1 + b_2} pb^{\,t} + \frac{b_2}{b_1 + b_2} gb^{\,t}\,.$$

Then, we can say that

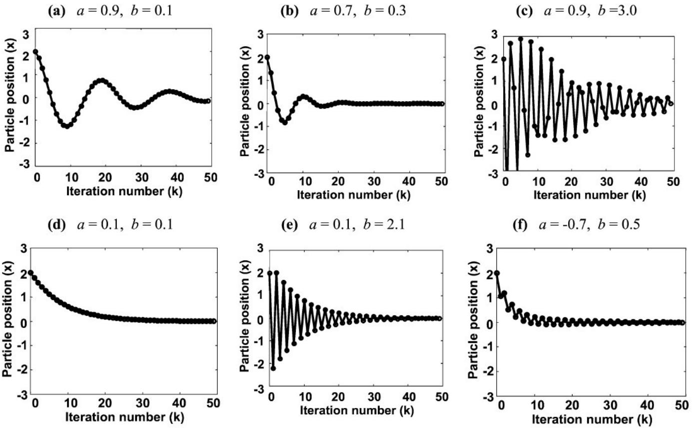
$$v^{t+1} = av^{t} + b(r - x^{t}) \,.$$

It can be shown that this system will behave in different ways depending on the value of a, b.



Graph taken from Trelea [17].

Some examples



Graph taken from Trelea [17].

Factors to consider when choosing a particular variant and/or a parameter set:

- The characteristics of the problem ("modality", search ranges, dimension, etc)
- Available search time (wall clock or function evaluations)
- The solution quality threshold for defining a satisfactory solution

Particle swarm optimization: Research issues

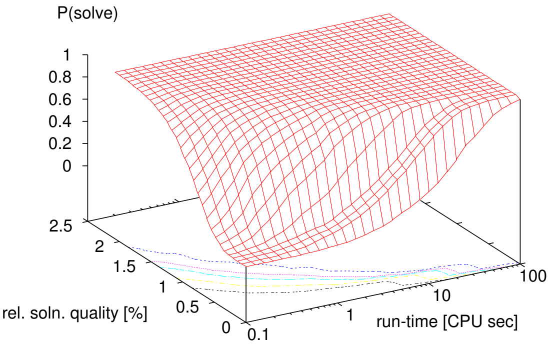
A number of research directions are currently pursued:

- Matching algorithms (or algorithmic components) to problems
- Application to different kind of problems (dynamic, stochastic, combinatorial)
- Parameter selection. (How many particles, which topology?)
- Identification of "state-of-the-art" PSO algorithms (comparisons)
- New variants (modifications, hybridizations)
- Theoretical aspects (particles behavior, stagnation)

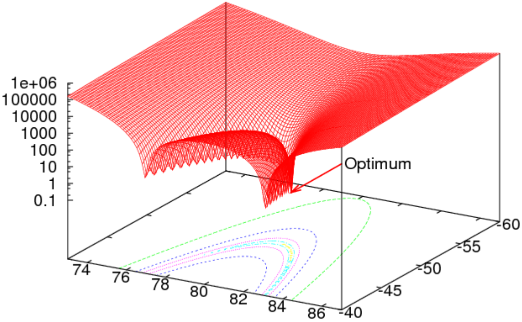
We have been working on three of the previously mentioned directions:

- Identification of "state-of-the-art" PSO algorithms (comparisons)
- Matching algorithms (or algorithmic components) to problems
- New variants (modifications, hybridizations)

We used *run-time and solution-quality distributions* [4] to compare several PSO variants.

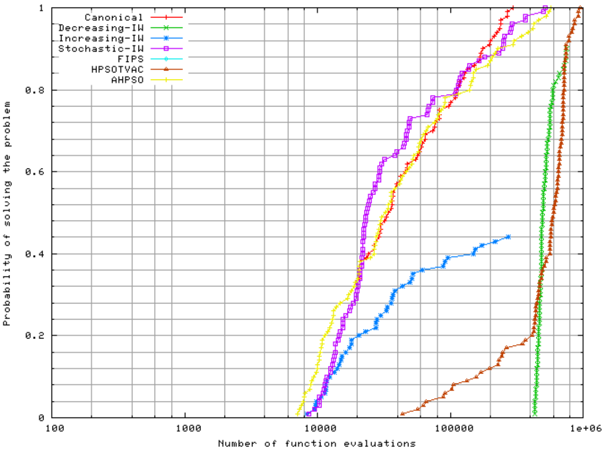


Rosenbrock function



Particle swarm optimization: Comparisons: Different Topologies

(20 particles, Rosenbrock) : Fully connected topology



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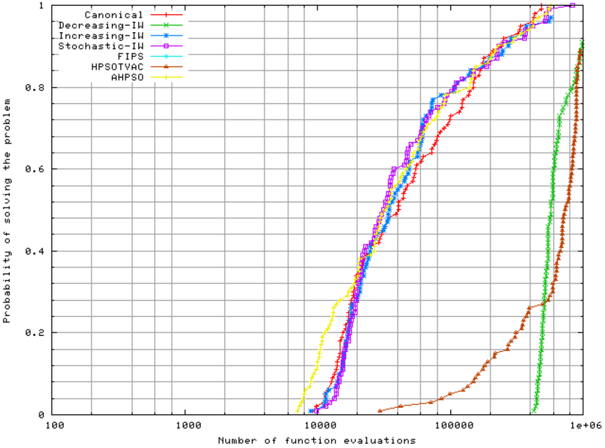
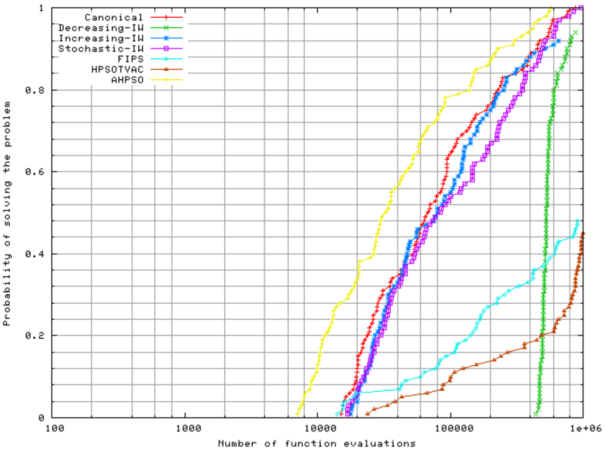
(20 particles, Rosenbrock) : Square topology



Particle swarm optimization: Comparisons: Different Topologies

(20 particles, Rosenbrock) : Ring topology



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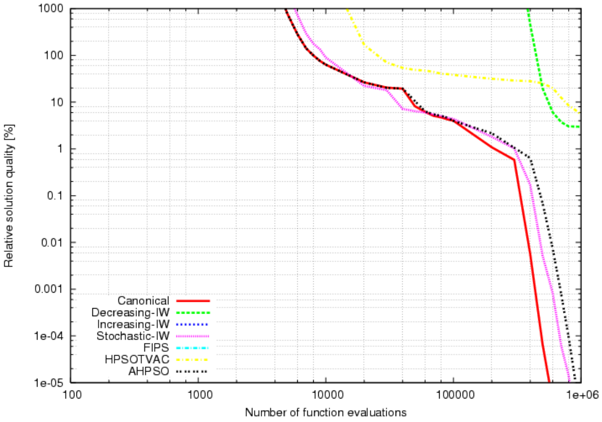

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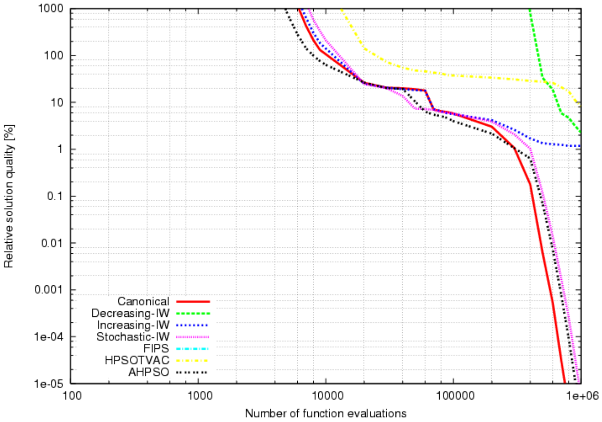

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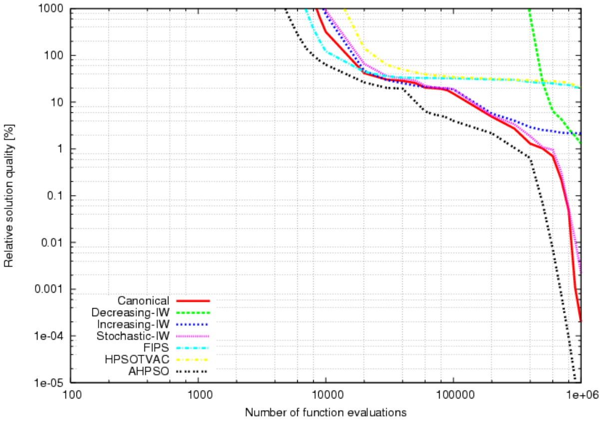
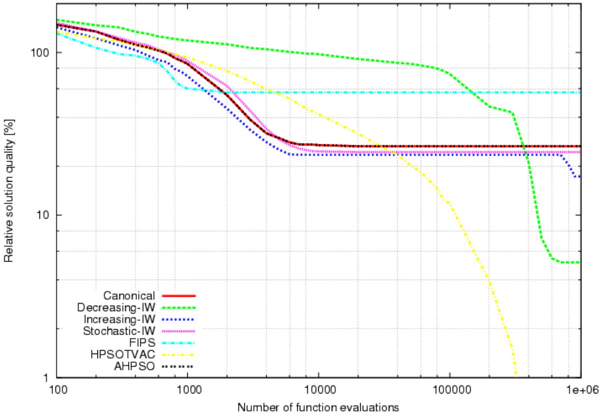


(20 particles, Rastrigin) : Fully connected topology



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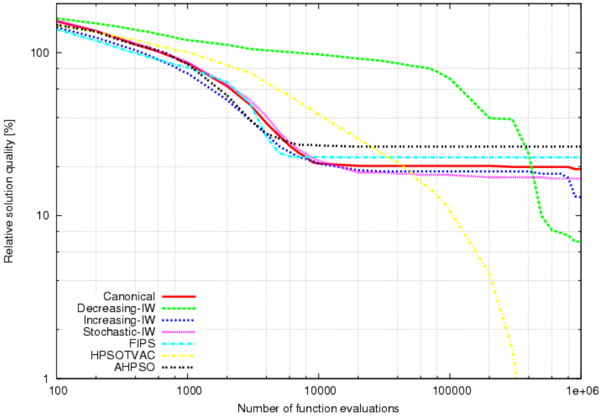

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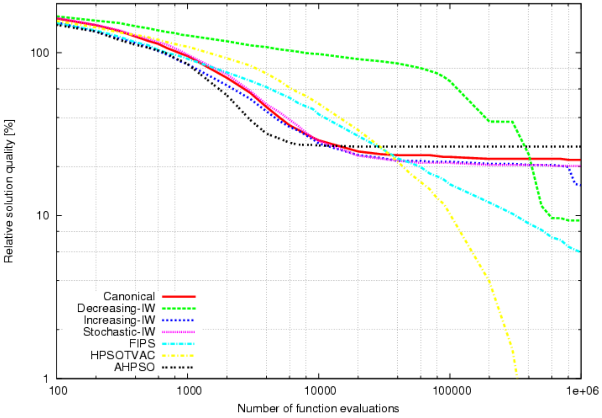
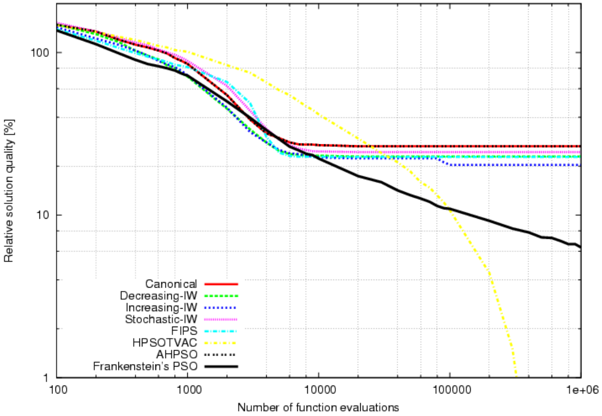


Particle swarm optimization: New variants (Frankenstein's PSO)

Rastrigin (best configurations for speed)



Particle swarm optimization: New variants (Frankenstein's PSO)

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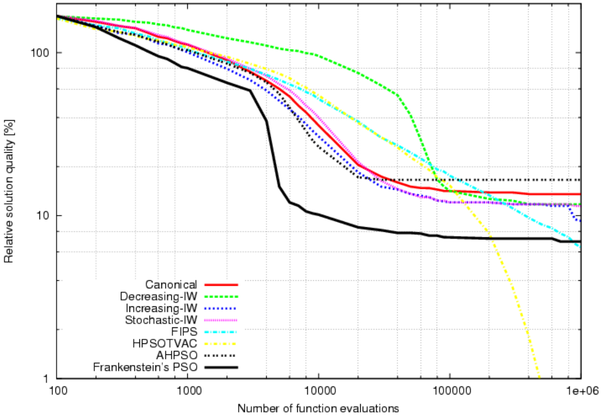

Thank you

(More) Questions?

http://iridia.ulb.ac.be/~mmontes/slidesCIL/slides.pdf

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