AE6114 - Fundamentals of Solid Mechanics

Fall 2020

Homework 5: Linear Elasticity - Part II

Due at the indicated time on Canvas, on Tuesday, November 10^{th} 2020

Problem 1

Consider two bricks of linear elastic materials of different properties glued together at the interface $x_2 = 0$ as shown in the figure below. The body is subject to uniaxial tension in the direction of x_2 through a uniform traction s, and no body forces are present. Formulate the corresponding elastostatics boundary value problem.

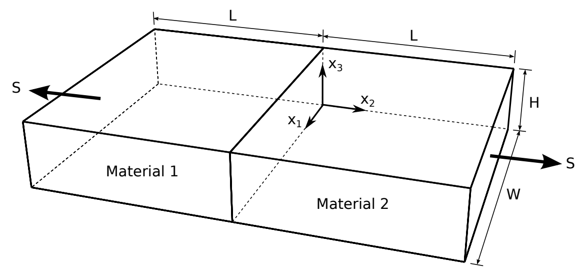

Figure 1: Schematics for problem 3

Problem 2

Consider a 2-dimensional problem in the $x_1 - x_2$ plane consisting of a rectangular body with a crack at the middle of one of its edges. A constant shear load is applied to two of its edges as depicted in the figure below. Formulate the corresponding elastostatics boundary value problem.

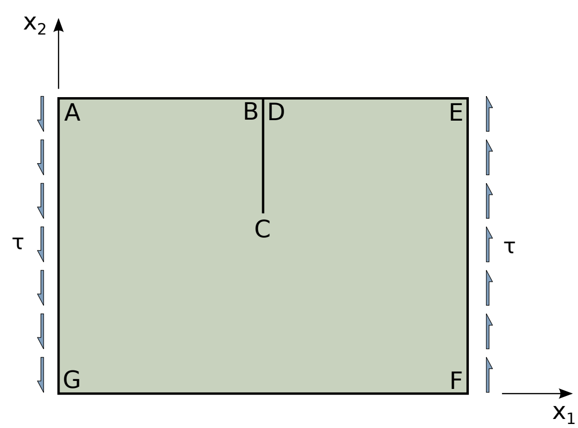

Figure 2: Schematics for problem 4

Problem 3

Consider a bar of density ρ with a circular cross section of unit area. The top of the bar is glued on the flat bottom surface of a rigid horizontal beam. On the other end of the bar, a traction $\mathbf{p}$ pointing downwards is uniformly applied across the surface. The bar is deformed under its own weight and the applied traction $\mathbf{p}$.

1. Specify the body force acting on the bar.
2. What are the traction boundary conditions for the bar?
3. Assume that the problem can be treated as one-dimensional for stresses and strains, i.e., stresses and strains are only functions of x_1 and independent of x_2 and x_3. Using the traction boundary conditions and equilibrium equations, show that all components of the stress tensor are zero except σ_{11}, and that $\sigma_{11} = P + \rho g(L - x_1)$.
4. Find the infinitesimal strains ϵ_{ij}, assuming that the bar is made of a linear elastic material.
5. Find the corresponding displacement field u_i. Explain why the displacements at the top surface of the bar should be identically zero. Discuss why you cannot fit this displacement boundary condition with your solution.

Problem 4

1. Consider the following 2D stress distribution on a body:

$$\sigma_{11} = 12x_1^4 x_2^2, \quad \sigma_{22} = 12x_1^2 x_2^4, \quad \sigma_{12} = -16x_1^3 x_2^3$$

   (a) Show that this stress distribution satisfies the equation of equilibrium when no body forces are present.
   (b) Assuming isotropic linear elastic material, find the corresponding 2D strains $\epsilon_{11}, \epsilon_{12}$, and ϵ_{22}.
   (c) Show that these strains do not satisfy the compatibility equation

$$\frac{\partial^2 \epsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \epsilon_{22}}{\partial x_1^2} = 2\frac{\partial^2 \epsilon_{12}}{\partial x_1 x_2}$$

   (d) Integrate the ϵ_{11} strains to obtain an expression for the displacement component u_1 and the ϵ_{22} strains to obtain an expression for the displacement component u_2. Show that it is impossible to obtain the prescribed strain ϵ_{12} from the obtained expressions for u_1 and u_2.

2. Consider the following 2D stress distribution on a body:

$$\sigma_{11} = -e^{x_1} \sin x_2, \quad \sigma_{22} = e^{x_1} \sin x_2, \quad \sigma_{12} = -e^{x_1} \cos x_2$$

   (a) Show that this stress distribution satisfies the equation of equilibrium when no body forces are present.
   (b) Assuming isotropic linear elastic material, find the corresponding 2D strains $\epsilon_{11}, \epsilon_{12}$, and ϵ_{22}.
   (c) Show that these strains *do satisfy* the compatibility equation

$$\frac{\partial^2 \epsilon_{11}}{\partial x_2^2} + \frac{\partial^2 \epsilon_{22}}{\partial x_1^2} = 2\frac{\partial^2 \epsilon_{12}}{\partial x_1 x_2}$$

   (d) Integrate the ϵ_{11} strains to obtain an expression for the displacement component u_1 and the ϵ_{22} strains to obtain an expression for the displacement component u_2. Show that it is possible to obtain the prescribed strain ϵ_{12} from the obtained expressions for u_1 and u_2.

3. Comment on the differences between cases 1 and 2. Does a stress field that satisfies conservation of linear and angular momentum necessarily make physical sense? Please elaborate.