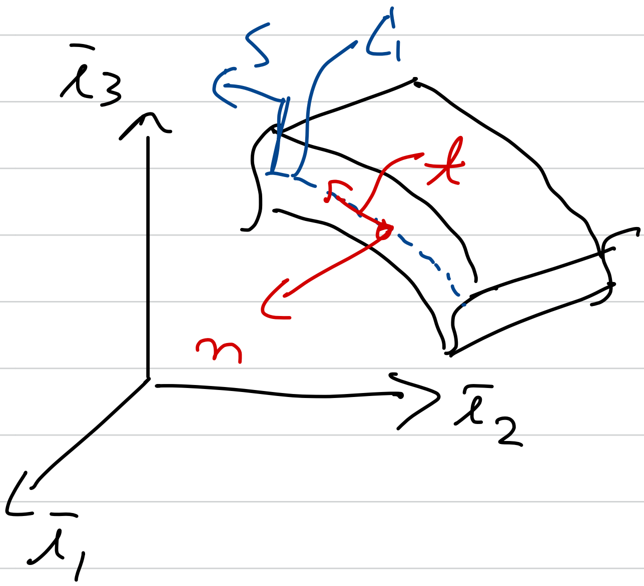
Thin-Walled Beams: Bending and Shear



Stress Resultants:

$$N_1 = \int_{C_1'} n \, ds$$

$$M_2 = \int_{C_1'} n \cdot x_3 \, ds$$

$$M_3 = \int_{C_1'} n \, x_2 \, ds$$

$$V_2 = \int_A \tau_{12} \, dA = \int_{C_1'} t \frac{dx_2}{ds} \cdot ds$$

$$V_3 = \int_{C_1'} t \frac{dx_3}{ds} \, ds$$

Equilibrium:

$$\frac{\partial n}{\partial x_1} + \frac{\partial t}{\partial s} = 0$$

Bending

* Euler - Bernoulli assumptions are equally applicable

$$\epsilon_1(x_1, x_2, x_3) = \bar{\epsilon}_1 + x_3 \kappa_2 - x_2 \kappa_3$$

$$\sigma_1 = E\,\epsilon_1$$

$$\begin{bmatrix} N_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} S & 0 & 0 \\ 0 & H_{22}^c & -H_{23}^c \\ 0 & -H_{23}^c & H_{33}^c \end{bmatrix} \begin{bmatrix} \bar{\epsilon}_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix}$$

$$\sigma_1 = E\left[\frac{N_1}{S} - \frac{x_2 H_{23}^c - x_3 H_{33}^c}{\Delta H} M_2 - \frac{x_2 H_{22}^c - x_3 H_{23}^c}{\Delta H} M_3 \right]$$

$$\Delta H = H_{22}^c H_{33}^c - H_{23}^c H_{23}^c$$

$$n(x_1, s) =$$

$$E(s)\,t(s)\left[\frac{N_1}{S} - \frac{x_2 H_{23}^c - x_3 H_{33}^c}{\Delta H} M_2 - \frac{x_2 H_{22}^c - x_3 H_{23}^c}{\Delta H} M_3 \right]$$

* You may use thin-walled assumption in computing

H_{22}^{c} , H_{33}^{c} , H_{23}^{c}

* Solve these problems in the same fashion that we solved solid section bending problems.

* Applies to both open and closed thin-walled sections.

Shearing

* Bending moments are usually accompanied by transverse shear forces.

$$\frac{\partial n}{\partial x_1} + \frac{\partial t}{\partial s} = 0 \rightarrow \frac{\partial t}{\partial s} = -\frac{\partial n}{\partial x_1}$$

$$\frac{\partial t}{\partial s} = -E(s)t(s)\left[\frac{1}{S}\frac{dN_1}{dx_1} - \frac{x_2 H_{23}^c - x_3 H_{33}^c}{\Delta H}\frac{dM_2}{dx_1} - \frac{x_2 H_{22}^c - x_3 H_{23}^c}{\Delta H}\frac{dM_3}{dx_1}\right]$$

Recall: Balance equations

$$\frac{dN_1}{dx_1} = -p_1(x_1)$$

$$\frac{dM_3}{dx_1} + V_2 = -\cancel{q_3(x_1)} + \cancel{x_{2A}\, p_1(x_1)}$$

$$\frac{dM_2}{dx_1} - V_3 = -\cancel{q_2(x_1)} - \cancel{x_{3A}\, p_1(x_1)}$$

* Assume: $q_2 = q_3 = 0$, $x_{2A} = x_{3A} = 0$

$$\frac{\partial t}{\partial s} = -E(s)t(s)\left[-\frac{x_2 H_{23}^c - x_3 H_{33}^c}{\Delta H} \underline{V_3} + \frac{x_2 H_{22}^c - x_3 H_{23}^c}{\Delta H} \underline{V_2} \right]$$

$$t(s) = C - \int_0^s \frac{\partial t}{\partial s}\, ds$$

↳ Value at shear flow t at $s=0$.

* $V_2, V_3, H_{22}^c, H_{23}^c, H_{33}^c$ are not functions at s

$$t(s) = C + \frac{H_{23}^c}{\Delta H} V_3 \overbrace{\int_0^s E t x_2\, ds}^{Q_3(s)}$$

$$- \frac{H_{33}^c}{\Delta H} V_3 \overbrace{\int_0^s E t x_3\, ds}^{Q_2(s)}$$

$$- \frac{H_{22}^c}{\Delta H} V_2 \int_0^s E t x_2\, dS$$

$$+ \frac{H_{23}^c}{\Delta H} V_2 \int_0^s E t x_3\, dS$$

Defining:

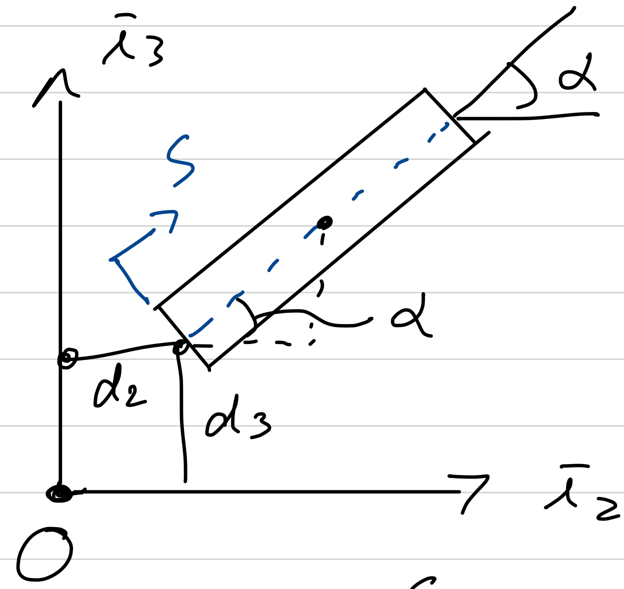
$$\left.\begin{aligned} Q_2(s) &= \int_0^s E\,t\,x_3\,ds \\ Q_3(s) &= \int_0^s E\,t\,x_2\,ds \end{aligned}\right\} \text{Stiffness Static Moments}$$

→ Functions of s! NOT constants.

$$\boxed{f(s) = C + \frac{Q_3 H_{23}^c - Q_2 H_{33}^c}{\Delta H} V_3 - \frac{Q_3 H_{22}^c - Q_2 H_{23}^c}{\Delta H} V_2}$$

* Given V_2 & V_3, and the geometry and material properties → we can solve for f!

Example



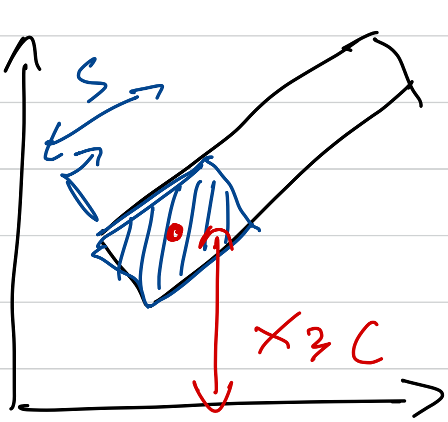
$$x_2 = d_2 + s\cos(\alpha)$$

$$x_3 = d_3 + s\,\text{sen}(\alpha)$$

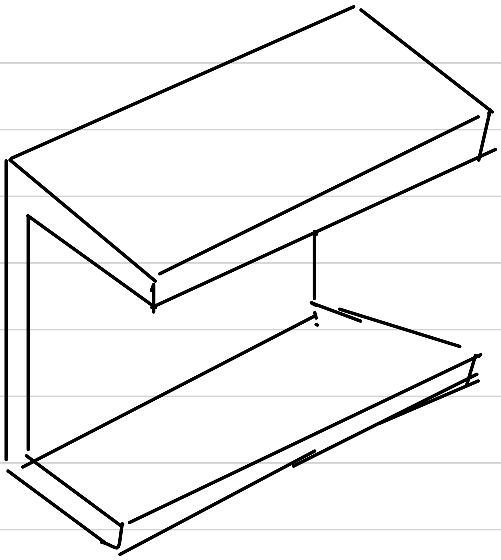
$$Q_2 = \int_0^s E t\, x_3\, ds = Et\int_0^s (d_3 + s\,\text{Sen}(\alpha))\,ds$$

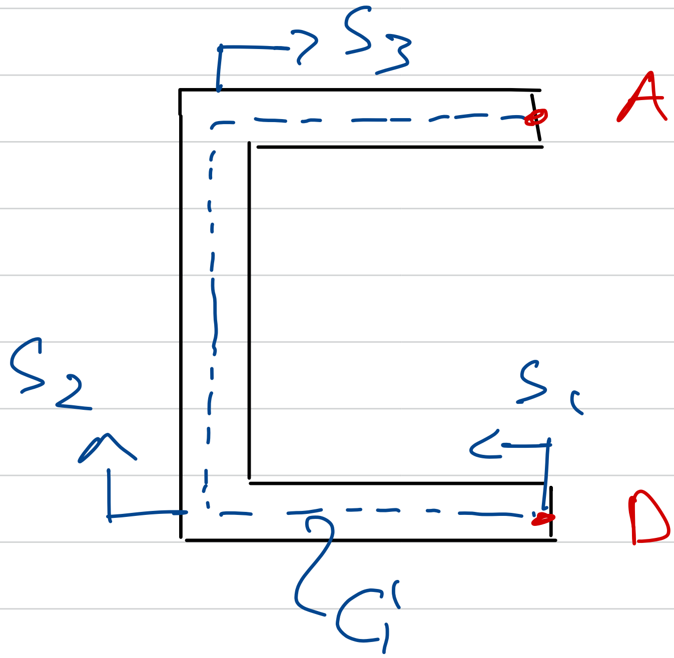
$$Q_2 = Et\left[d_3 s + \frac{s^2}{2}\,\text{Sen}(\alpha)\right]$$

$$Q_2 = E\underbrace{(ts)}_{\text{Area at a given } s}\underbrace{\left[d_3 + \frac{s}{2}\,\text{Sen}(\alpha)\right]}_{\text{Distance to the centroid at the area at a given } s \text{ in the } \bar{\imath}_3 \text{ direction, } x_{3C}}$$



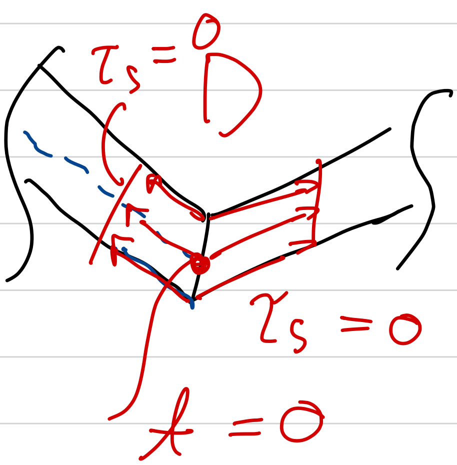
Shear flow in Open Sections





* For open sections, the shear stress vanishes at the end points at the curve C_1'

→ Points A and D



* If we choose the origin at s at such an endpoint

→ $c = 0$!

Given a shear force V_2, V_3 compute t

Procedure:

1) Find centroid and compute H_{22}^c, H_{33}^c, H_{23}^c
2) Select a coordinate s. Several coordinates may be required.
3) Evaluate $Q_2(s)$ and $Q_3(s)$
4) Compute t.