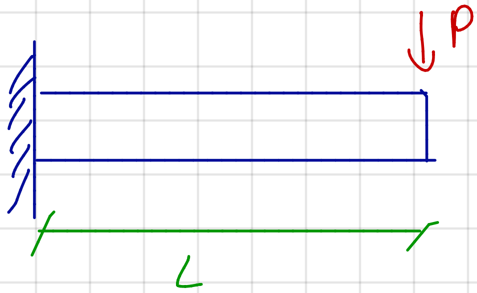
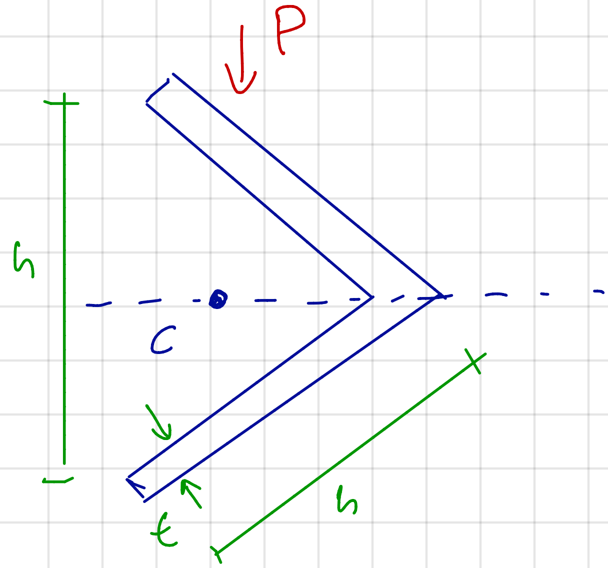
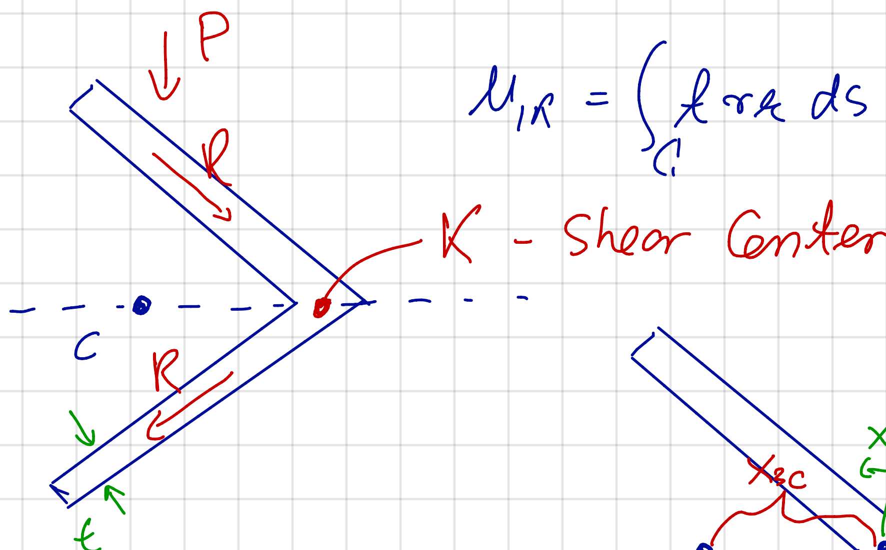




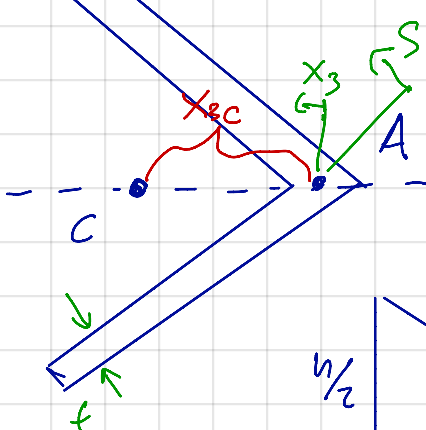
1) Find τ_s & $\tau_{s,max}$

2) Find σ_1 & $\sigma_{1,max}$

3) Compare these

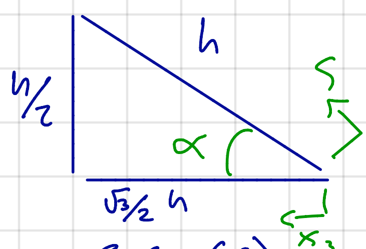


$$M_{1,K} = \int_C f\, r_K \, ds = 0$$



$\sin(\alpha) = \frac{1}{2}$

$\cos(\alpha) = \frac{\sqrt{3}}{2}$



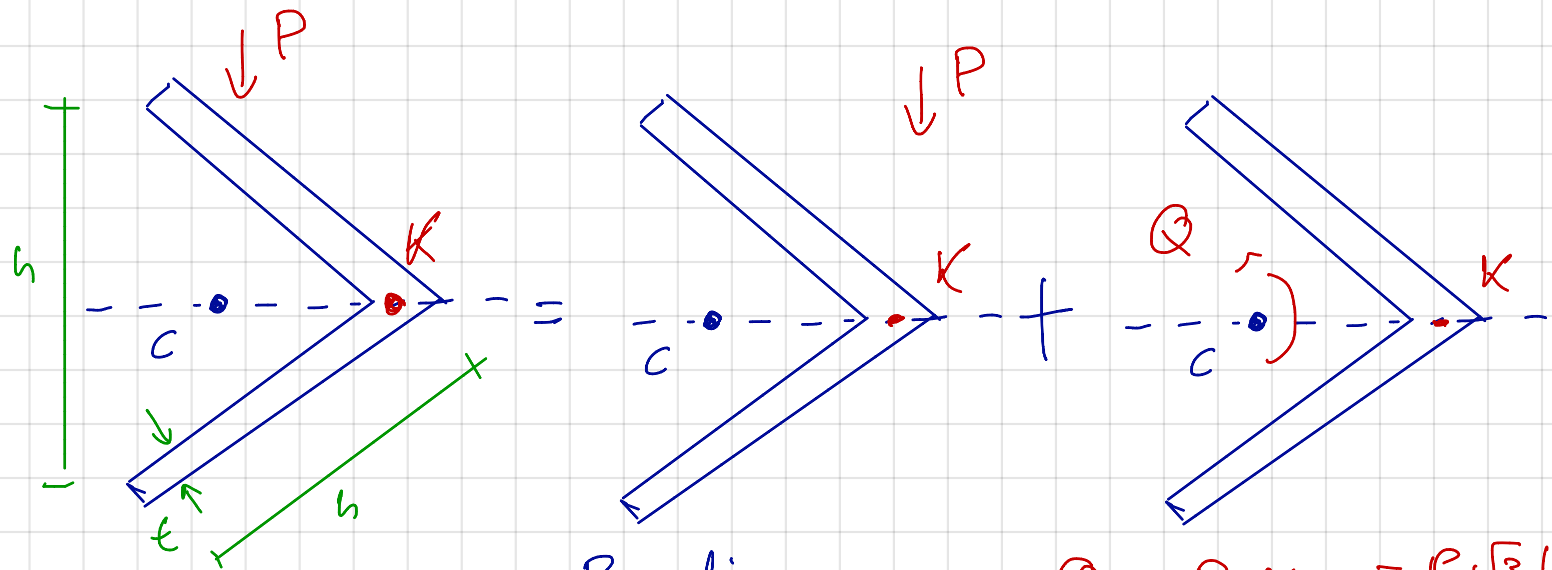
$$x_{3c} = S_3 / S$$

$$S = 2 \cdot E \cdot t \cdot h$$

$$S_3 = \int_A E x_3 \, dA = 2 \cdot \int_0^h E\, s \cos(\alpha)\, t\, ds \qquad x_3 = s \cdot \cos(\alpha)$$

$$= 2 E t \frac{h^2}{2} \cos(\alpha) = \frac{\sqrt{3}}{2} E t h^2$$

$$\boxed{x_{3c} = \frac{\sqrt{3}}{4} h}$$



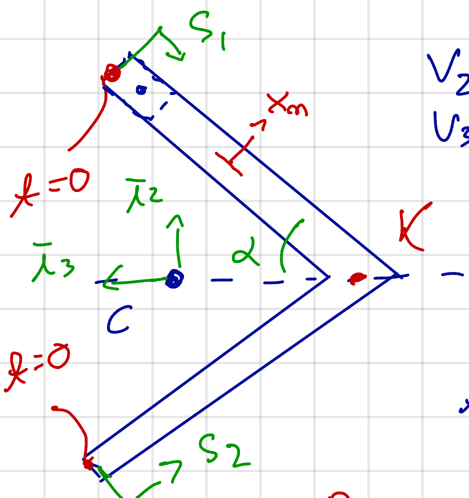
Bending

$$Q = P \cdot x_{sc} = P\,\frac{\sqrt{3}\,h}{4}$$

Torsion

$$\tau_s = \tau_s^{BEND} + \tau_s^{TORSION}$$

a) Find $\tau_s^{Bending}$ $\quad H_{23}^c = 0$



$V_2(x_1) = -P$

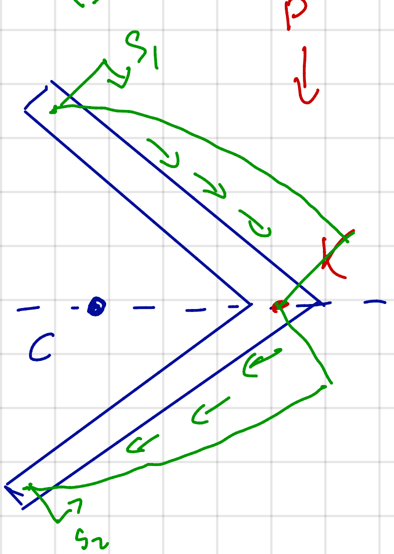
$V_3 = 0$

$$f(s) = c - \frac{Q_3(s)}{H_{33}^c} V_2$$

$$f(s_1) = 0 - E(ts_1)\left(h\sin(\alpha) - \frac{s_1}{2}\sin(\alpha)\right)\frac{V_2}{H_{33}^c}$$

$$f(s_1) = E(ts_1)\left(\frac{h}{2} - \frac{s_1}{4}\right)\frac{(P)}{H_{33}^c}$$

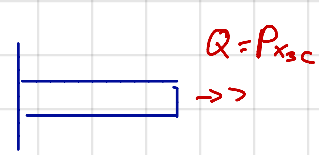
$$f(s_1) = -f(s_2) = -E(ts_2)\left(\frac{h}{2} - \frac{s_2}{4}\right)\frac{P}{H_{33}^c}$$



b) Find $\tau_s^{TORSION}$

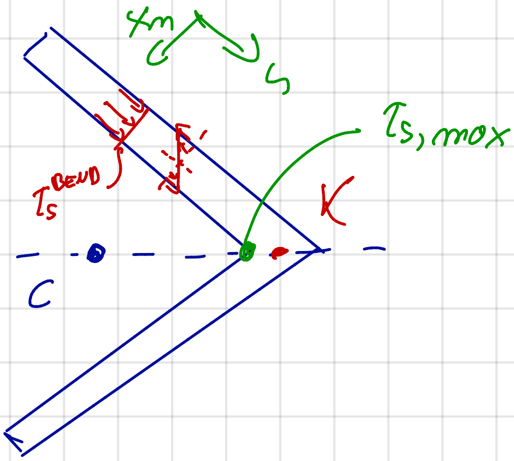
$$\tau_s^{TOR} = 2 \cdot G \cdot \kappa_1 \cdot x_n$$

$$* \quad \tau_s^{TOR} = 2G\,\frac{P x_{3c}}{H_{11}} \cdot x_n$$



$$M_1 = H_{11}\kappa_1 = Q$$

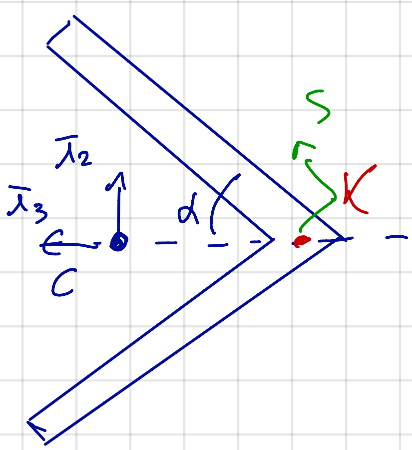
$$\begin{cases} \tau_s(s_1, x_n) = E\, s_1 \left(\dfrac{h}{2} - \dfrac{s_1}{4} \right) \dfrac{P}{H_{33}^{c}} - 2G \dfrac{P x_{3c}}{H_{11}} x_n \\ \text{for} \quad 0 \geq s_1 \geq h \quad \& \quad -t/2 \geq x_n \geq t/2 \\ \tau(s_2, x_n) = -E\, s_2 \left(\dfrac{h}{2} - \dfrac{s_2}{4} \right) \dfrac{P}{H_{33}^{c}} - 2G \dfrac{P x_{3c}}{H_{11}} x_n \\ \text{for} \quad 0 \geq s_2 \geq h \quad \& \quad -t/2 \geq x_n \geq t/2 \end{cases}$$



$$\tau_{s,max} = E\, h \left(\frac{h}{2} - \frac{h}{4} \right) \frac{P}{H_{33}^{c}} + \frac{G P x_{3c} t}{H_{11}}$$

$$\tau_{s,max} = E \frac{h^2}{4} \frac{P}{H_{33}^{c}} + G P \cdot \frac{\sqrt{3}}{4} \frac{h t}{H_{11}}$$

$$H_{33}^{c} = \int_A E x_2^2 \, dA = 2 \int_0^h E\, s^2 \sin^2(\alpha)\, t\, ds$$



$$* \; H_{33}^{c} = 2 \cdot E \frac{h^3}{3} \cdot \frac{1}{4} t = \frac{E h^3 t}{6}$$

$$* \; H_{11} = \frac{1}{3} G \ell t^3 = \frac{1}{3} G (2h) t^3 = \frac{2}{3} G h t^3$$

Find σ_1 & $\sigma_{1,max}$ $\qquad H_{23}^c = 0$, $M_2 = 0$

$$\sigma_1 = -E \frac{M_3 x_2}{H_{33}^c} \qquad M_3 = -P(L - x_1)$$

$$* \quad \sigma_1 = \frac{E}{H_{33}^c} P(L - x_1) x_2$$

$$\sigma_{1,max} = \frac{E}{H_{33}^c} PL \frac{h}{2} \qquad @ \quad x_1 = 0, \; x_2 = h/2$$

c) Scale $\tau_{s,max}$ & $\sigma_{1,max}$

$$* \quad \tau_{s,max} = E \frac{h^2}{4} \frac{P}{H_{33}^c} + GP \cdot \frac{\sqrt{3}}{4} \frac{ht}{H_{11}} = \frac{3}{2} \frac{P}{ht} + \frac{3\sqrt{3}}{8} \frac{P}{t^2} \sim \frac{P}{t^2}$$

$$* \quad \sigma_{1,max} = \frac{E}{H_{33}^c} PL \frac{h}{2} = 3 \frac{PL}{h^2 t}$$

$t \ll h$ - Thin wall

$h \ll L$ - Slender

$$\frac{\sigma_{1,max}}{\tau_{s,max}} \sim \frac{PL}{h^2 t} \frac{t^2}{P} = \underbrace{\frac{L}{h}}_{\gg 1} \cdot \underbrace{\frac{t}{h}}_{\ll 1}$$

* It is possible for both stresses to be at the same order!

→ Neither Dominates!

$$\tau_{s,max} \sim \frac{P}{ht} + \frac{P}{ht} \sim \frac{P}{ht} \qquad \tau_{s,torsion} \sim \frac{M_1}{A_c \cdot t} \sim \frac{Ph}{h^2 t} = \frac{P}{ht}$$

$$\sigma_{1,max} \sim \frac{PL}{h^2 t}$$

$$\frac{\sigma_{1,max}}{\tau_{s,max}} \approx \frac{PL}{h^2 t} \cdot \frac{ht}{P} = \frac{L}{h} >> 1$$

Here the axial stress always dominates.