Fundamentals of Aircraft Performance

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Aircraft Performance

- The previous discussion on aircraft performance concluded with a discussion on range and endurance

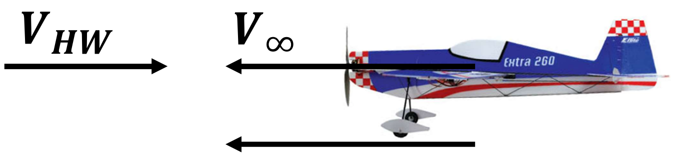
- In this lecture, we will analyze important aircraft performance characteristics for the following, classified as before, by the <u>nature</u> of flight:

  – **Steady flight:** No acceleration, rectilinear flight-path
    - Effect of wind on range, endurance, climb rate, and angle of climb
    - Payload-range characteristics

  – **Accelerated flight:** Non-zero acceleration, possibly curvilinear flight-path
    - Turning performance – turn rate, turn radius
    - Pull-up, pull-down, and push-over maneuvers
    - Accelerated rate of climb
    - Takeoff and landing performance

The Effect of Wind

- Range and Endurance calculations seen thus far assumed no wind. How are these affected by wind?
- Note that aerodynamic properties of the airplane depend its velocity w.r.t. to air or vice-versa, denoted as $\boldsymbol{V_\infty}$
- In the absence of wind, airplane velocity w.r.t. ground ($\boldsymbol{V_g}$) is equal to airplane velocity w.r.t. air ($\boldsymbol{V_\infty}$)
- However, in the presence of wind, the following scenarios may occur:



$$\boldsymbol{V_g = V_\infty - V_{HW}}$$

$$\boldsymbol{V_g = V_\infty + V_{TW}}$$

- We will look at the effect of wind on the following:
  - Range
  - Endurance
  - $\boldsymbol{V_\infty}$ for best rate of climb
  - $\boldsymbol{V_\infty}$ for best angle of climb

- Note that the airplane's airspeed ($\boldsymbol{V_\infty}$) is simply that for max. endurance. Ground speed is irrelevant to the consideration of endurance. Therefore, **wind *does not* affect endurance**.
- Similarly, since winds are defined as only affecting the horizontal component of $\boldsymbol{V_\infty}$, **they *do not* affect the rate of climb.**

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Effect of Wind on Range

- Since range is a function of the distance covered on the ground, V_g must be used in its calculation
- Recall, $V_g = \frac{ds}{dt}$ or, $ds = V_g dt$. This is the same relation we in terms of V_∞ in the absence of wind
- The Breguet range equation in the presence of wind is obtained by using V_g instead of V_∞
- V_∞ for max. range may be analytically derived. However, a graphical interpretation is more intuitive

Jet powered airplane

- Analytical expression,

$$\boldsymbol{R = \frac{V_g}{c_t}\frac{L}{D} ln \frac{W_{ini}}{W_{fin}}}$$

- Another way of expressing range,

$$\frac{lb\ of\ fuel\ consumed}{mi} = \frac{(TSFC)T_R}{V_g}$$

- Max. range is obtained at minimum $\frac{T_R}{V_g}$

Piston engine powered airplane

- Analytical expression,

$$\boldsymbol{R = \frac{\eta_{pr}}{c}\frac{V_g}{V_\infty}\frac{L}{D} ln \frac{W_{ini}}{W_{fin}}}$$

- Another way of expressing range,

$$\frac{lb\ of\ fuel\ consumed}{mi} = \frac{(SFC)P_R}{\eta_{pr} V_g}$$

- Max. range is obtained at minimum $\frac{P_R}{V_g}$

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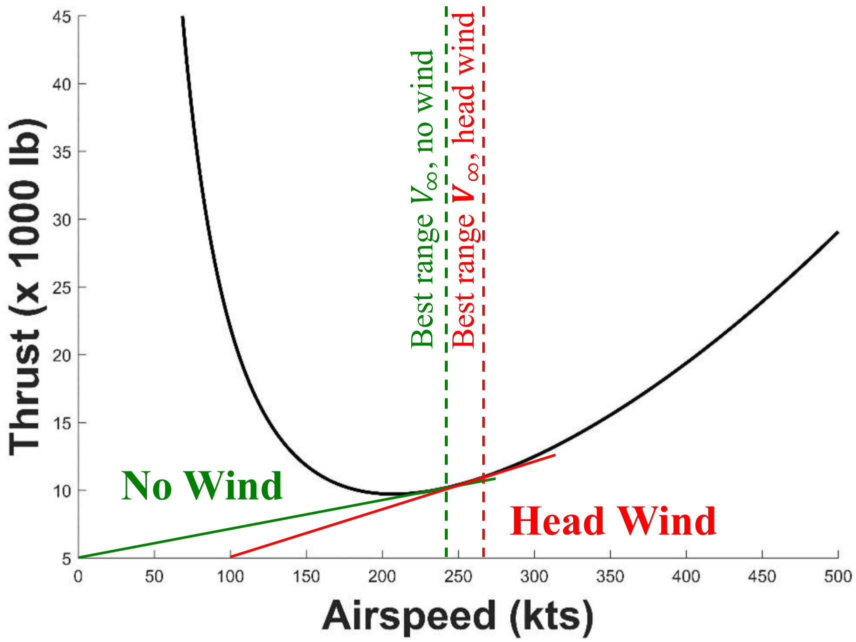
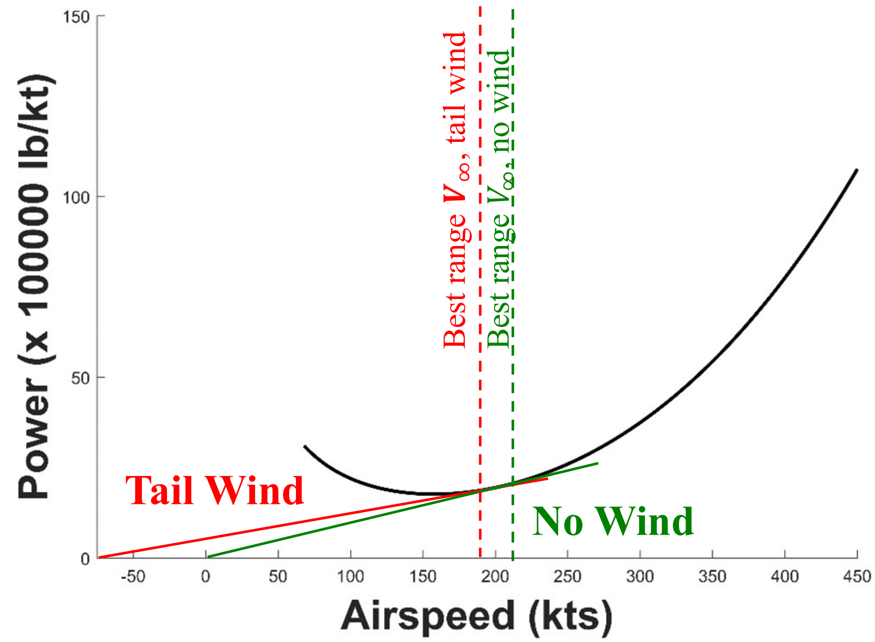
Effect of Wind on Range

Jet powered airplane



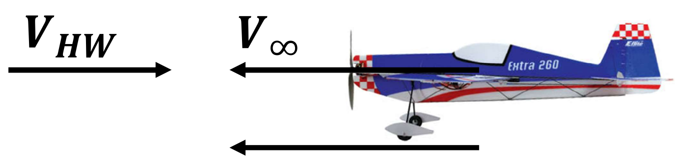
Piston engine powered airplane



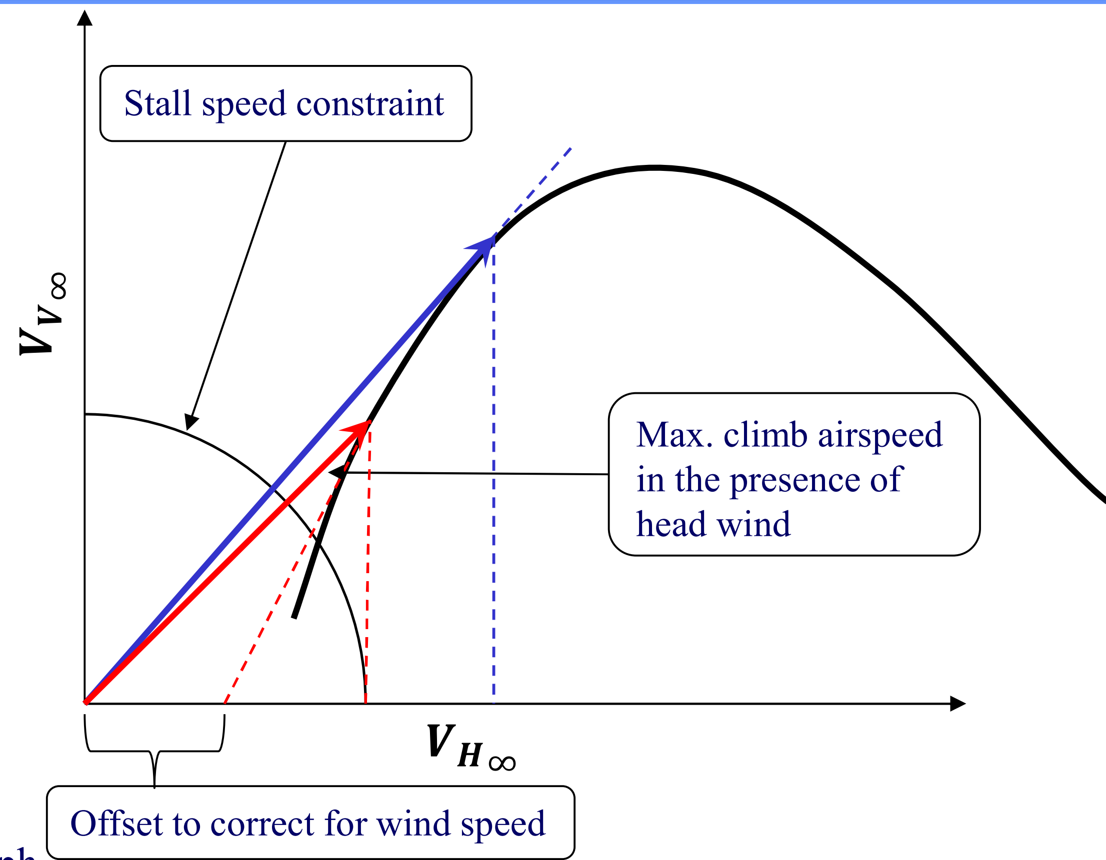
- The effect of wind can be analyzed by observing its effect on the resulting airspeed (V_∞), the x-axis
- The airspeed for maximum range is higher in the presence of a head wind and lower in the presence of a tail wind

Effect of Wind on Climb Angle

- Recall that climb angle γ may be viewed as the angle between the horizontal and airspeed (V_∞)
- Therefore, presence of wind manifests itself as an increase in the climb angle for a headwind, and a decrease in the climb angle for a tailwind for a fixed V_{H_∞} as observed from the ground
- Let's analyze wind's effect on max. climb angle
- Recall,



$$V_g = V_\infty - V_{HW}$$



- The graph shows a notional hodograph
- In the presence of a head wind, max. climb angle is achieved at a lower airspeed as compared to the airspeed with no wind, as shown in the figure

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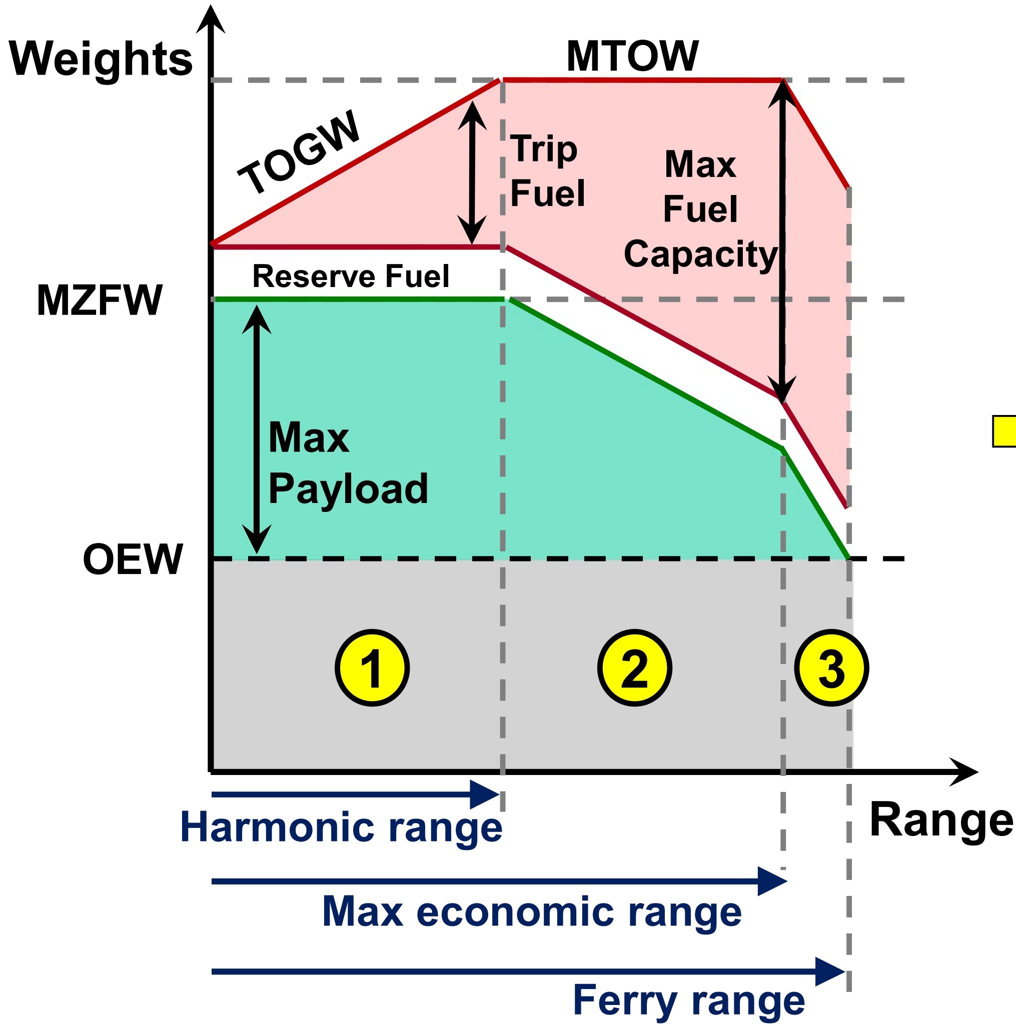
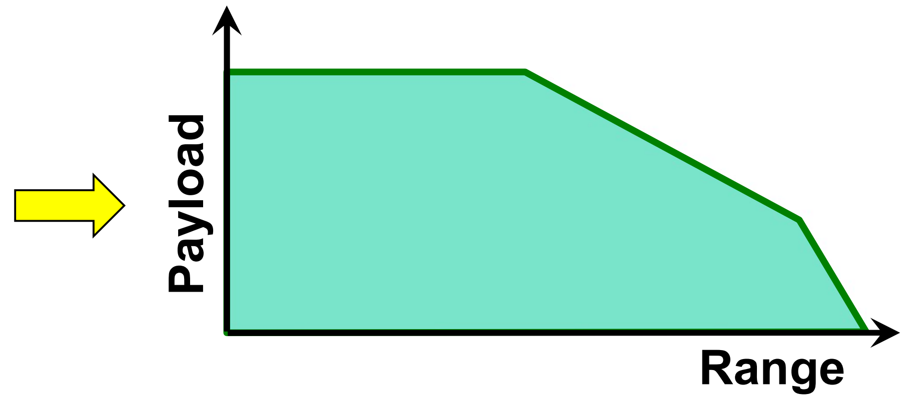
Definition of Some Aircraft Weights

- **Maximum Ramp Weight (MRW):** The max weight on the ramp prior to taxiing
- **Maximum Brake-Release Weight (MBRW):** Max weight at the start of the takeoff run
- **Maximum Takeoff Weight (MTOW):** Max weight at lift-off (wheels leave ground). Observe that MRW – MTOW = fuel burnt (by engines and APU) during taxi-out and takeoff run
- **Takeoff Gross Weight (TOGW):** Total weight at takeoff. TOGW $\leq$ MTOW
- **Maximum Landing Weight (MLW):** Max touchdown weight (as limited by landing gear or structural strength. Remember, landings are not always soft)
- **Maximum Payload Weight (PAY):** Max weight of passengers, baggage, cargo, munitions, equipment, etc. May be limited by weight (obviously), volume (e.g., for cargo pallets), or capacity (e.g., seating).
- **Manufacturer's Empty Weight (MEW):** Weight of aircraft including all integral parts (airframe, propulsion, systems, fixed equipment, ballast, closed system fluids)
- **Operating/Operational Empty Weight (OEW):** The MEW + additional removable items that are added due to operational reasons
- **Maximum Zero Fuel Weight (MZFW):** Max permissible weight on the ground excluding usable fuel Note: MZFW = OEW + PAY

Payload-Range Characteristics



- MTOW: Maximum Takeoff Weight
- TOGW: Takeoff Gross Weight
- MZFW: Maximum Zero Fuel Weight
- OEW: Operating Empty Weight

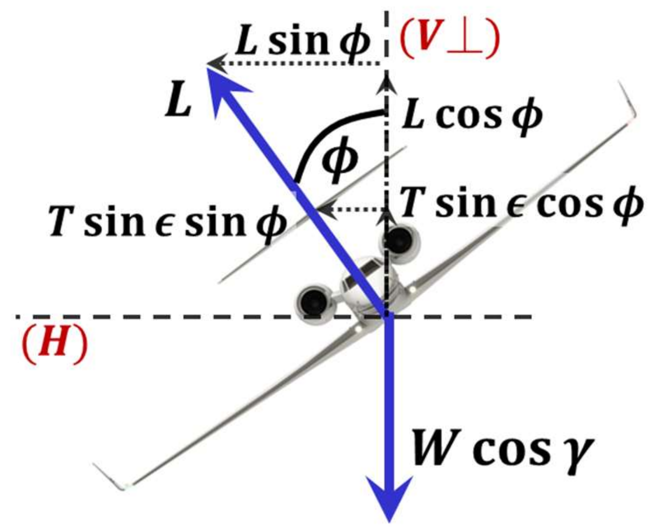


Range / Zone	Payload	Fuel	TOGW
Zone 1	= max	< full	< MTOW
Harmonic Range	= max	< full	= MTOW
Zone 2	< max	< full	= MTOW
Max econ range	< max	= full	= MTOW
Zone 3	< max	= full	< MTOW
Ferry range	0	= full	< MTOW

Level Turn – Equations of Motion

- Constant altitude: $\frac{dh}{dt} = V_\infty \sin\gamma = 0 \rightarrow \gamma = 0$
- Constant speed: $\frac{dV_\infty}{dt} = 0$
- Turn radius in horizontal plane: $r_H = R$
- No path curvature in vertical plane: $r_V = \infty$
- Assume thrust inclination angle $\epsilon = 0$

* http://www.gulfstream.com/assets/images/_550/img-slide03a-large.jpg



$(||)$ $$m\frac{dV_\infty}{dt} = T\cos\epsilon - D - W\sin\gamma \quad \Rightarrow \quad T = D$$

$(V\perp)$ $$m\frac{V_\infty^2}{r_V} = L\cos\phi + T\sin\epsilon\cos\phi - W\cos\gamma \quad \Rightarrow \quad W = L\cos\phi$$

(H) $$m\frac{(V_\infty\cos\gamma)^2}{r_H} = L\sin\phi + T\sin\epsilon\sin\phi \quad \Rightarrow \quad m\frac{V_\infty^2}{R} = L\sin\phi$$

Equations of motion for level turn

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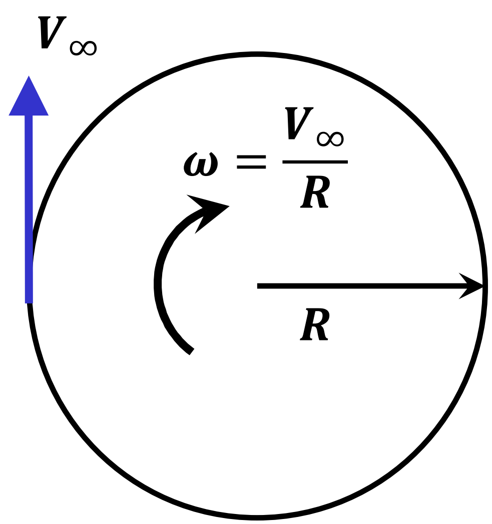
Level Turn – Parameters of Interest

$T = D$ — Engine thrust has to balance drag to allow a constant airspeed to be maintained, i.e., to allow the level turn to be sustained

$W = L\cos\phi$ — Only a component of the lift (cos) is available to balance weight. Therefore, magnitude of lift is greater than that in level flight

$m\frac{V_\infty^2}{R} = L\sin\phi$ — The remaining component of lift (sin) provides the centripetal force required to take the airplane around the turn



Some parameters of interest for turn performance

- **Turn radius, R:** how tight is the turn?
- **Turn rate, ω:** how quickly is the direction changing?
- **Velocity, V_∞:** how quickly is the aircraft moving?
- **Bank angle, ϕ:** how steeply is it banked over?
- **Load factor, n:** what loads are being generated?
- **Thrust, T:** what thrust is needed to sustain the turn?

- Relationships linking the above will now be developed

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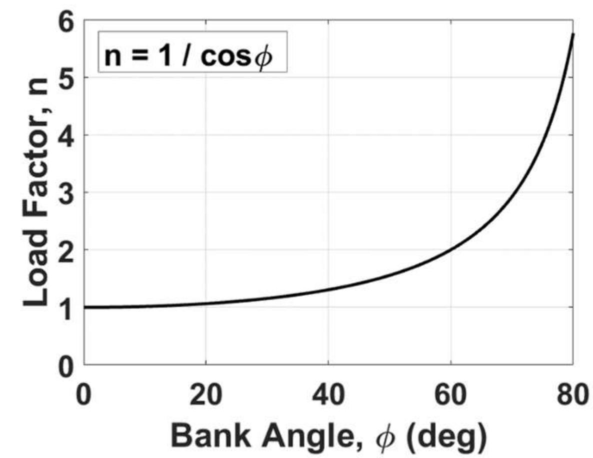
Relating the Turn Parameters

- Using the equation of motion $W = L\cos\phi$ and the definition of load factor $L = nW$, bank angle and turn load factor can be related as:

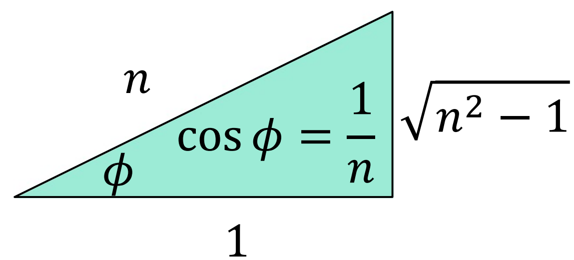
$$\boxed{n = \frac{1}{\cos\phi} \quad \rightarrow \quad \phi = \cos^{-1}\left(\frac{1}{n}\right)}$$



- Recall two of the equations of motion for a level turn: $W = L\cos\phi$ and $m\frac{V_\infty^2}{R} = L\sin\phi$
  - Dividing one by the other and re-arranging:

$$R = \frac{V_\infty^2}{g\tan\phi}$$

- Using basic trig to eliminate bank angle, turn radius and turn rate can be related to speed and load factor:



$$\boxed{R = \frac{V_\infty^2}{g\sqrt{n^2-1}}} \qquad \boxed{\omega = \frac{V_\infty}{R} = \frac{g\sqrt{n^2-1}}{V_\infty}}$$

$$\sin\phi = \frac{\sqrt{n^2-1}}{n} \qquad \tan\phi = \sqrt{n^2-1}$$

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Maximizing Turn Performance

- Maximizing turn performance is especially important for combat aircraft
  - Small turn radius: the ability to execute the turn within a small area
  - High turn rate: the ability to rapidly change the direction of flight

- From the relationships just derived, this means:
  - Flying at low velocity V_∞
  - Sustaining a high load factor n

$$\boxed{R = \frac{V_\infty^2}{g\sqrt{n^2-1}}} \quad \boxed{\omega = \frac{g\sqrt{n^2-1}}{V_\infty}}$$

- For a fixed V_∞, the maximum achievable load factor n_{max} is constrained by:
  - **Structural limit:** the maximum load factor that the airframe is designed for
  - **Stall limit:** where main lifting surfaces are at their critical angle of attack
  - **Available thrust limit:** where balancing the drag generated requires the maximum available thrust output of the engines

Constraints on Level Turn Performance

Structural Limit:

- The turn load factor must not exceed the limit load factor $n_{max,str}$ to which the airplane's structure is designed – otherwise, structural damage will result
  - This is a design characteristic that does not vary with flight condition

Structure limited level turn performance

$$\boxed{R_{min,str} = \frac{V_\infty^2}{g\sqrt{n_{max,str}^2 - 1}}}$$

$$\boxed{\omega_{max,str} = \frac{g\sqrt{n_{max,str}^2 - 1}}{V_\infty}}$$

Stall Limit:

- The maximum lift coefficient that the wing can generate also limits the load factor that can be generated without stalling the airplane
- Recall from definitions: $L = nW = \frac{1}{2}\rho V_\infty^2 S\, C_L$
- Re-arranging, the stall-limit max load factor is:

$$\boxed{n_{max,stall} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{L_{max}}}{\left(\frac{W}{S}\right)}}$$

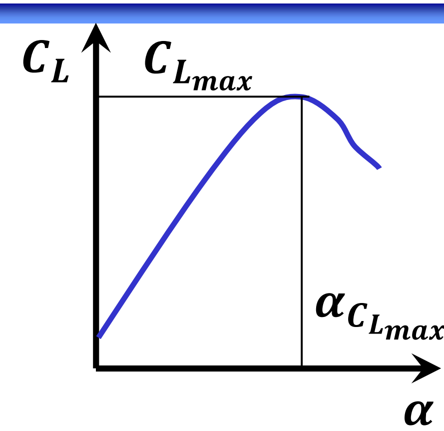
$$\boxed{R_{min,stall} = \frac{V_\infty^2}{g\sqrt{n_{max,stall}^2 - 1}}}$$

$$\boxed{\omega_{max,stall} = \frac{g\sqrt{n_{max,stall}^2 - 1}}{V_\infty}}$$

Stall limited level turn performance

Stall Speed vs. Load Factor – Life vs. Death

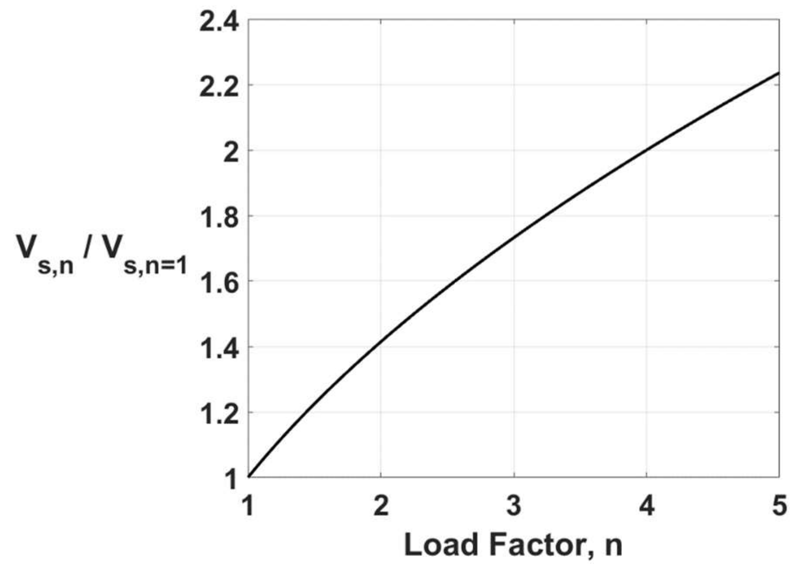
- Q: At what airspeed can an airplane stall?
- A: At <u>any</u> airspeed.
- Stall is an aerodynamic characteristic of a lifting surface that depends on angle of attack
  - Stall angle of attack is a constant, stall speed is not



- Recall from definitions alone: $L = nW = \frac{1}{2}\rho_\infty V_\infty^2 \; S \; C_L$
- Re-arranging this to solve for the stall speed at load factor n, or $V_{s,n}$:

$$V_{s,n} = \sqrt{\frac{2}{\rho_\infty}\left(\frac{W}{S}\right)\frac{n}{C_{L_{max}}}} = \sqrt{\frac{2}{\rho_\infty}\left(\frac{W}{S}\right)\frac{1}{C_{L_{max}}}} \;\; \sqrt{n}$$

$$\boxed{\boldsymbol{V_{s,n} = V_{s,n=1}\,\sqrt{n}}}$$



Stall speed increases with load factor: $\propto \sqrt{n}$

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Constraints on Level Turn Performance

Available Thrust Limit:

- Recall that the thrust must balance the drag in order to sustain the level turn

$$T = D = \frac{1}{2}\rho V_\infty^2 S\, C_D = \frac{1}{2}\rho V_\infty^2 S\left(C_{D_0} + K\, C_L^2\right)$$

- Recalling the definition of lift coefficient and load factor: $C_L = \dfrac{L}{\frac{1}{2}\rho V_\infty^2\, S} = \dfrac{nW}{\frac{1}{2}\rho V_\infty^2\, S}$
- Substituting this into the thrust equation and solving for load factor n yields:

$$n = \left[\frac{\frac{1}{2}\rho_\infty V_\infty^2}{K\left(\frac{W}{S}\right)}\left\{\left(\frac{T}{W}\right) - \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{D_0}}{\left(\frac{W}{S}\right)}\right\}\right]^{\frac{1}{2}}$$

- Maximizing this by applying maximum available thrust,

$$\boxed{n_{max,thr} = \left[\frac{\frac{1}{2}\rho_\infty V_\infty^2}{K\left(\frac{W}{S}\right)}\left\{\left(\frac{T_{A,max}}{W}\right) - \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{D_0}}{\left(\frac{W}{S}\right)}\right\}\right]^{\frac{1}{2}}}$$

Thrust limited level turn performance

$$\boxed{R_{min,thr} = \frac{V_\infty^2}{g\sqrt{n_{max,thr}^2 - 1}}}$$

$$\boxed{\omega_{max,thr} = \frac{g\sqrt{n_{max,thr}^2 - 1}}{V_\infty}}$$

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Constraints on Level Turn Performance - Recap

Structure limited level turn performance

$$R_{min,str} = \frac{V_\infty^2}{g\sqrt{n_{max,str}^2 - 1}}$$

$$\omega_{max,str} = \frac{g\sqrt{n_{max,str}^2 - 1}}{V_\infty}$$

- The level turn performance can be assessed by finding the load factor / velocity space where **all** the three constraints are satisfied

Stall limited level turn performance

$$R_{min,stall} = \frac{V_\infty^2}{g\sqrt{n_{max,stall}^2 - 1}}$$

$$n_{max,stall} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{L_{max}}}{\left(\frac{W}{S}\right)}$$

$$\omega_{max,stall} = \frac{g\sqrt{n_{max,stall}^2 - 1}}{V_\infty}$$

- Best explained using a numerical example

Thrust limited level turn performance

$$R_{min,thr} = \frac{V_\infty^2}{g\sqrt{n_{max,thr}^2 - 1}}$$

$$n_{max,thr} = \left[\frac{\frac{1}{2}\rho_\infty V_\infty^2}{K\left(\frac{W}{S}\right)}\left\{\left(\frac{T_{A,max}}{W}\right) - \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{D_0}}{\left(\frac{W}{S}\right)}\right\}\right]^{\frac{1}{2}}$$

$$\omega_{max,thr} = \frac{g\sqrt{n_{max,thr}^2 - 1}}{V_\infty}$$

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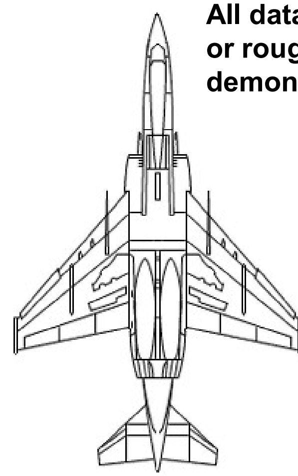
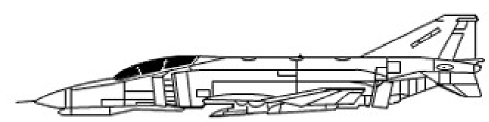
McDonnell Douglas F-4 Phantom II

- Gather the necessary data to feed into turn performance relationships
- Consider sea-level conditions

Geometry, Aero, Propulsion, and Structures	
Wing area, S	49.2 m^2 (530.0 ft^2)
Max lift coeff., $C_{L_{max}}$	1.2
Zero lift drag coeff., C_{D_0}	0.0291
Induced drag coeff. K	0.1166
Structural limit, $n_{max,str}$	7 g
Max thrust, $T_{A,max}$ (with afterburners)	2 x 79.4 kN (2 x 17,845 lbf)
Loading Condition 1: Max Takeoff Weight	
Gross weight, W	28,030 kg (61,795 lb)
Thrust-to-weight, T/W	0.58
Wing loading, W/S	569.7 kg/m^2 (116.6 lb/ft^2)
Loading Condition 2: Loaded (Intermediate) Weight	
Gross weight, W	18,825 kg (41,500 lb)
Thrust-to-weight, T/W	0.86
Wing loading, W/S	382.3 kg/m^2 (78.3 lb/ft^2)



All data are from public domain or rough estimates made for demonstration purposes



https://en.wikipedia.org/wiki/McDonnell_Douglas_F-4_Phantom_II

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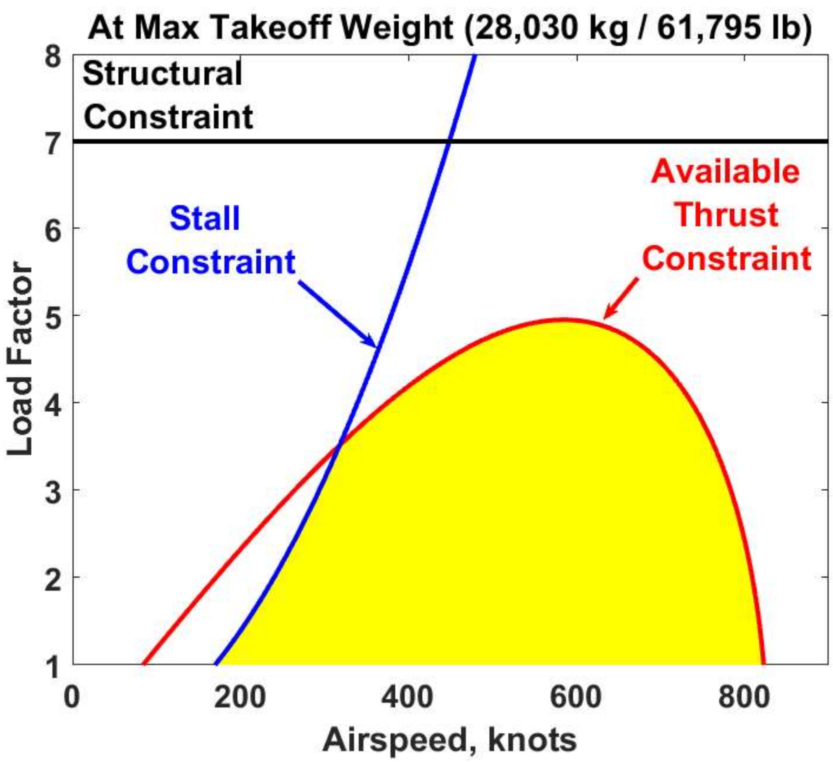
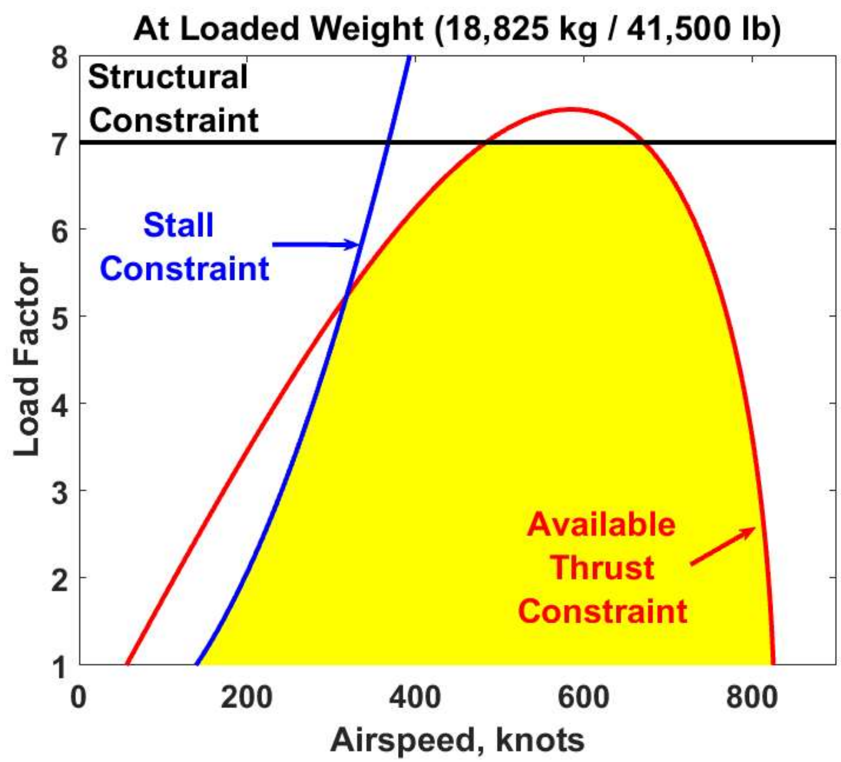
F-4 Phantom II – Turn Performance Assessment

- The shaded yellow area satisfies all three enforced constraints
- Note how change in aircraft weight affects the shape & size of the feasible area
  - At MTOW: Structural constraint never active. Max n lower, and limited by thrust
  - At lower weight: Maximum n is higher, and is limited by active structural constraint





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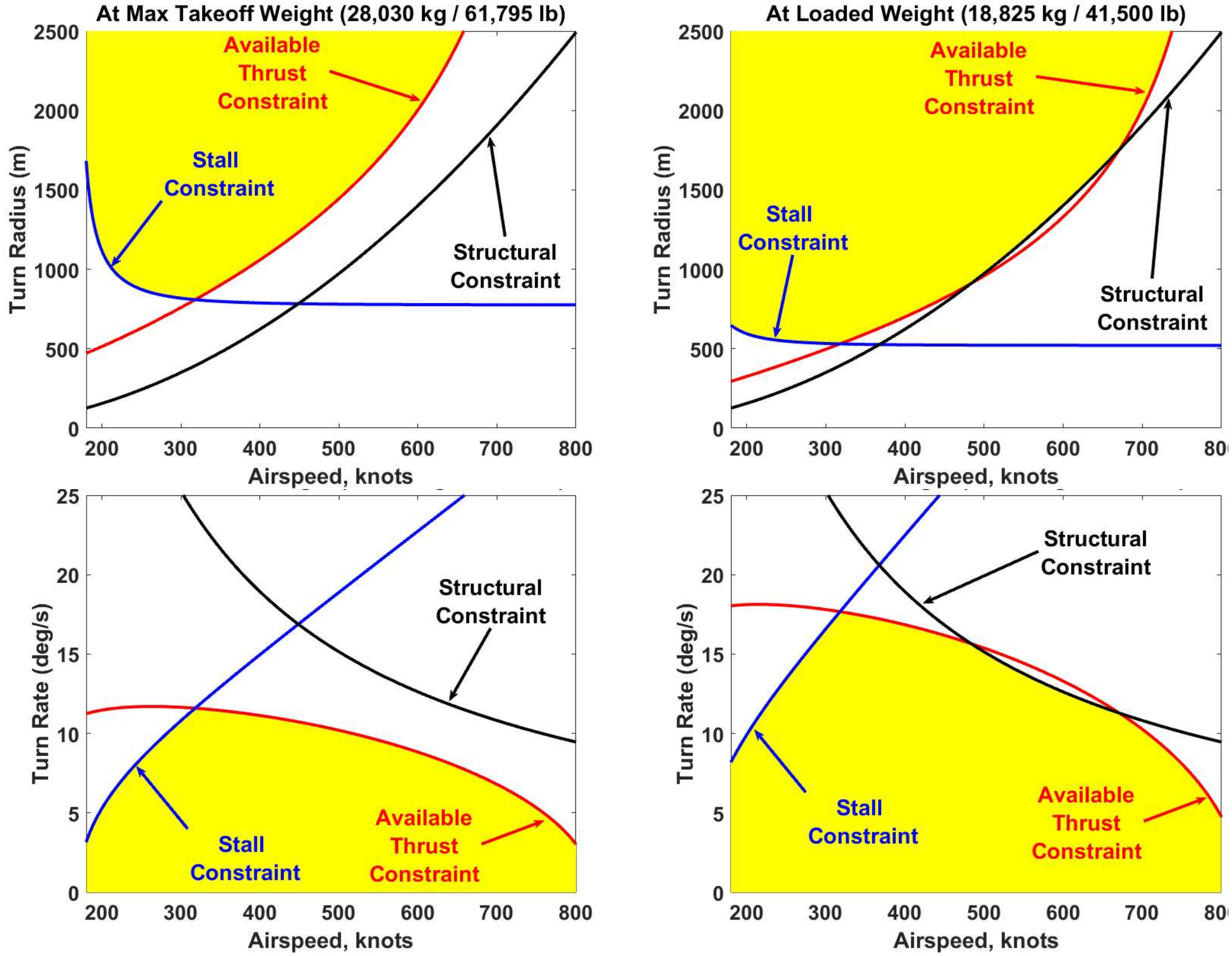
F-4 Phantom II – Turn Radius and Turn Rate



- At lower weight, the airplane can
  - Turn *tighter* (smaller *radius*)
  - Turn *faster* (higher *rate*)
- For this example, R_{min} and ω_{max} occur where stall and thrust constraint curves intersect
- Note the condition that the turn performance has to be <u>sustained</u>

What if a Turn Needn't be Sustained or Level?

- In other words, suppose the requirement is that an *instantaneous* load factor $n_{max,inst}$ be generated at a given flight condition (i.e., given $q = \frac{1}{2}\rho_\infty V_\infty^2$) and a stated weight fraction $\beta = W / W_{TO}$

- There is no requirement that (i) the load factor be sustained, (ii) that the turn be level, or (iii) that airspeed be maintained

- Recall from definitions: $L = nW = q\,S\,C_L$ $\qquad$ $n = q\,\dfrac{C_L}{\left(\frac{W}{S}\right)}$

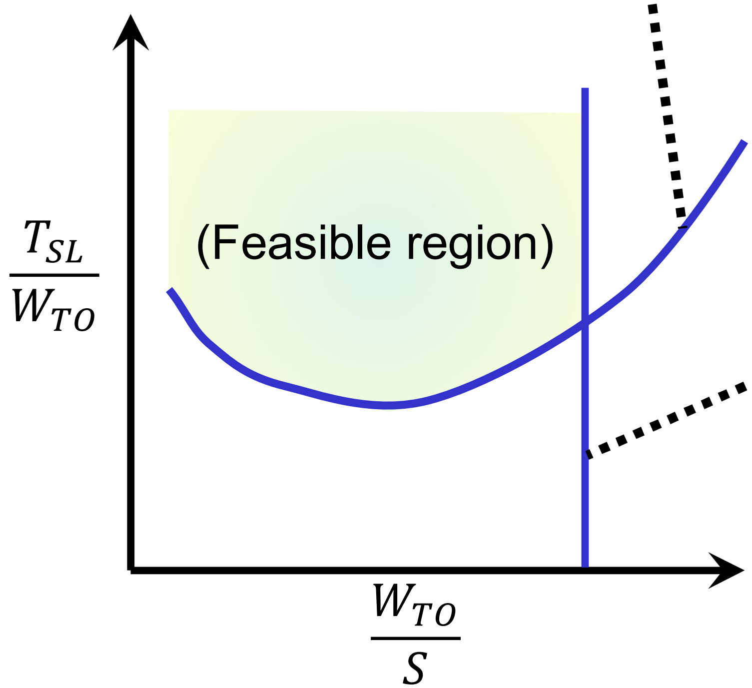
- The load factor is maximized when maximum lift coefficient is achieved $\qquad$ $n_{max,inst} = q\,\dfrac{C_{L_{max}}}{\beta\left(\frac{W_{TO}}{S}\right)}$

- Re-arranging, an *instantaneous load factor constraint* for wing loading is obtained: $\qquad$ $\boxed{\left(\dfrac{W_{TO}}{S}\right) \le q\,\dfrac{C_{L_{max}}}{\beta\, n_{max,inst}}}$

Sustained vs. Instantaneous Turn Constraints

<u>Sustained turn requirement:</u> Provides a <u>lower bound</u> (constraint) for thrust-to-weight ratio as a function of wing loading

$$\left(\frac{T_{SL}}{W_{TO}}\right) \geq \frac{\beta}{\alpha}\left\{\frac{q}{\beta\left(\frac{W_{TO}}{S}\right)}\left[K_1\left(\frac{\beta\, n_{max,sust}}{q}\left(\frac{W_{TO}}{S}\right)\right)^2 + K_2\left(\frac{\beta\, n_{max,sust}}{q}\left(\frac{W_{TO}}{S}\right)\right) + C_{D_0}\right]\right\}$$



<u>Instantaneous turn requirement:</u> Provides an <u>upper bound</u> (constraint) for wing loading

$$\left(\frac{W_{TO}}{S}\right) \leq \frac{C_{L_{max}}}{\beta\,\frac{n_{max,inst}}{q}}$$

- Instantaneous turn requirements impose no constraints on thrust-to-weight ratio
- This is because the requirements of constant altitude & airspeed (i.e., energy state) are removed

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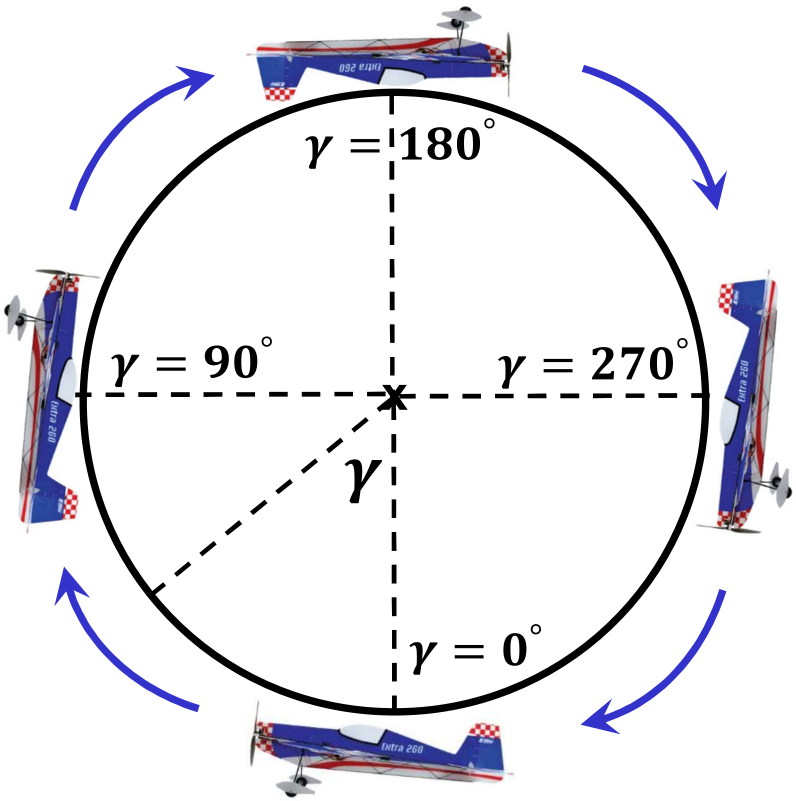
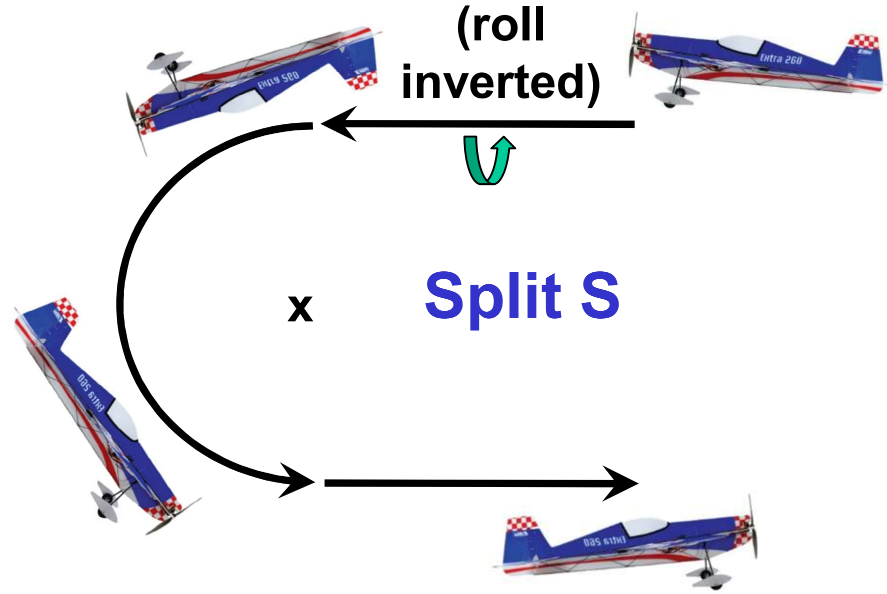
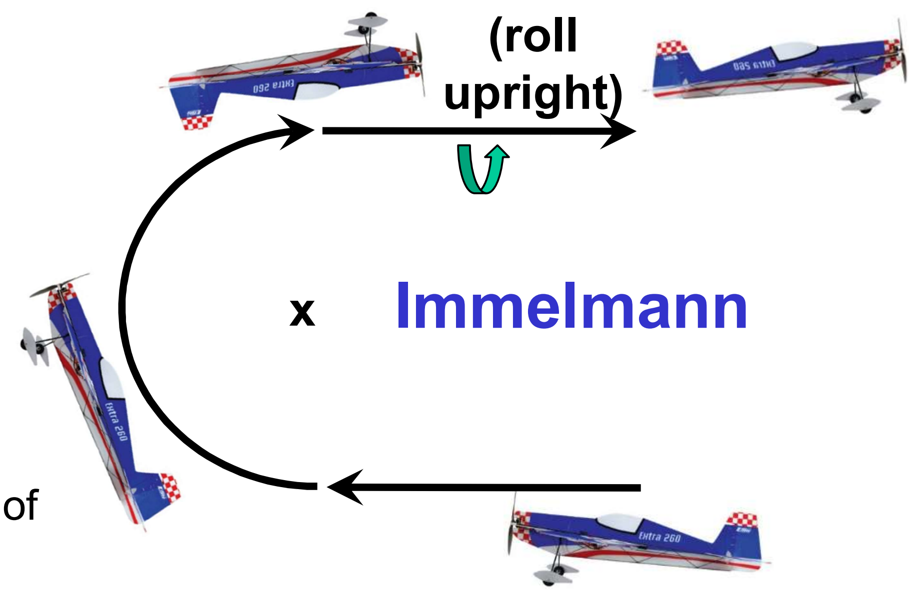
Accelerated Flight in the Vertical Plane



Inside Loop





<u>Note:</u> The relationships that follow will be easy to understand if you associate the aircraft's direction of flight with the angle convention shown above!

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Equations of Motion in Vertical Plane

- The relevant equations of motion are
  (i) parallel to flight-path and
  (ii) perpendicular to flight-path in vertical plane

$$m\frac{dV_\infty}{dt} = T - D - W\sin\gamma$$

$$m\frac{V_\infty^2}{R} = L - W\cos\gamma$$

- The following assumptions were enforced:
  – Thrust inclination angle $\epsilon = 0$
  – Bank angle $\phi = 0$

- Recall from definition that $L = nW = n\,mg$. Substituting into the second equation of motion gives a relationship for the maneuver load factor

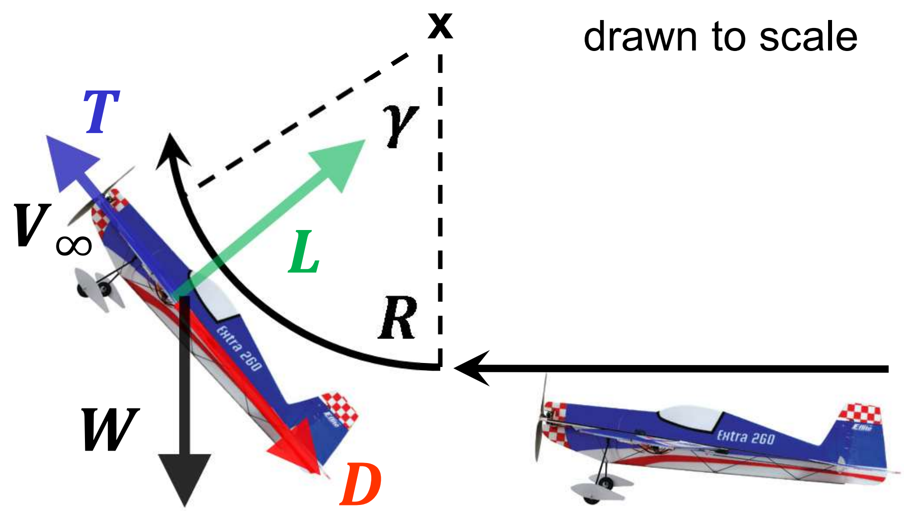
$$m\frac{V_\infty^2}{R} = n\,mg - mg\cos\gamma \quad \Rightarrow \quad \boxed{n = \cos\gamma + \frac{V_\infty^2}{Rg}}$$

Load factor depends on
(i) Flight-path angle, γ
(ii) Speed, V_∞
(iii) Path radius of curvature, R

The Pull-up Maneuver

- The pull-up is initiated by pulling back (aft) on the stick/yoke
- This moves the longitudinal control (e.g., elevator), creating a nose-up pitching moment that results in an increase in the angle of attack
- The increased lift thus generated exceeds weight, resulting in a net centripetal force (normal to flight path), causing the trajectory to arc upwards
- Instantaneous (rather than sustained) pull-up is of greater interest

Force vectors not drawn to scale



$$m\frac{dV_\infty}{dt} = T - D - W\sin\gamma$$

$$m\frac{V_\infty^2}{R} = L - W\cos\gamma$$

$$n = \cos\gamma + \frac{V_\infty^2}{Rg}$$

The Pull-up Maneuver

- At the start of the pull-up, $\gamma = 0$
- Thus, the starting load factor is

$$n = 1 + \frac{V_\infty^2}{Rg}$$

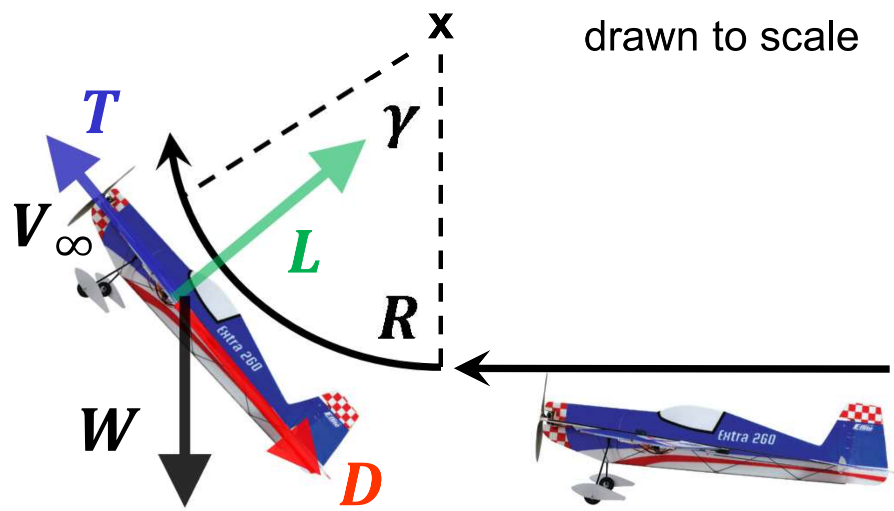
- Solving this for the radius R yields

$$R = \frac{V_\infty^2}{g\,(n-1)}$$

- The instantaneous pitch rate is given by

$$\omega = \frac{d\gamma}{dt} = \frac{V_\infty}{R} = \frac{g(n-1)}{V_\infty}$$

Force vectors not drawn to scale



$$m\frac{dV_\infty}{dt} = T - D - W\ \sin\gamma$$

$$m\frac{V_\infty^2}{R} = L - W\cos\gamma$$

$$n = \cos\gamma + \frac{V_\infty^2}{Rg}$$

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The Immelmann Maneuver (KE → PE, 180° reversal)

Finish pull-up (inverted):

$\gamma = 180^\circ, R \rightarrow \infty, n = -1$

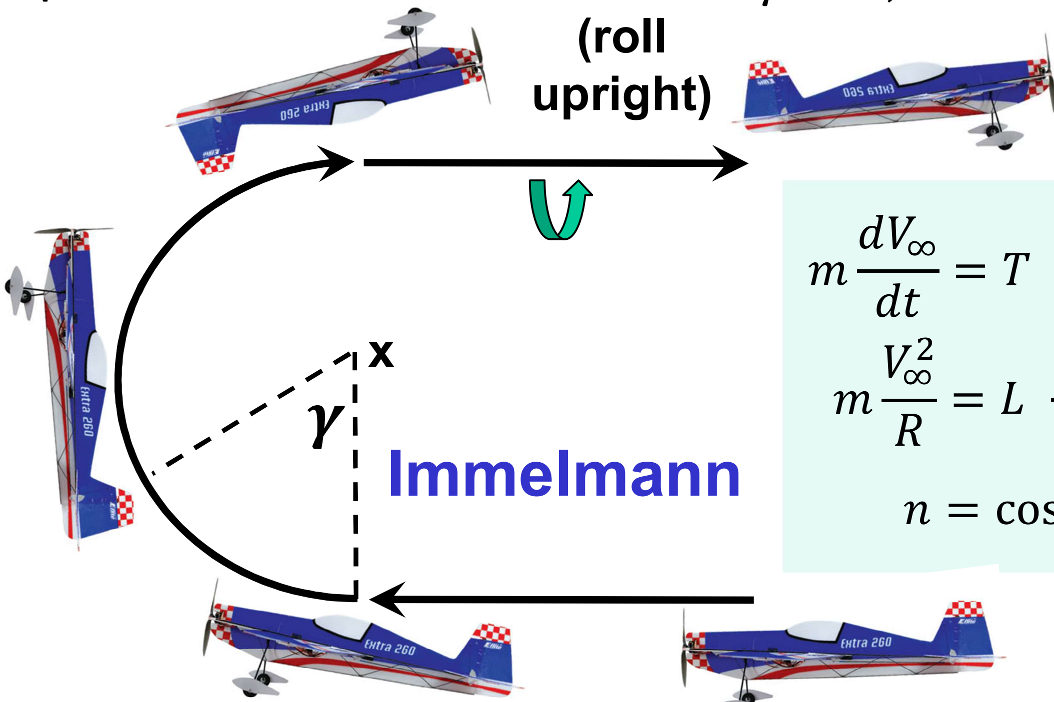
Straight-and-level:

$\gamma = 0, R \rightarrow \infty, n = 1$

During pull-up:

$0^\circ < \gamma < 180^\circ$

$$n = \cos\gamma + \frac{V_\infty^2}{Rg}$$



$$m\frac{dV_\infty}{dt} = T - D - W\sin\gamma$$

$$m\frac{V_\infty^2}{R} = L - W\cos\gamma$$

$$n = \cos\gamma + \frac{V_\infty^2}{Rg}$$

Begin pull-up:

$$\gamma = 0, n = 1 + \frac{V_\infty^2}{Rg}$$

Straight-and-level:

$\gamma = 0, R \rightarrow \infty, n = 1$

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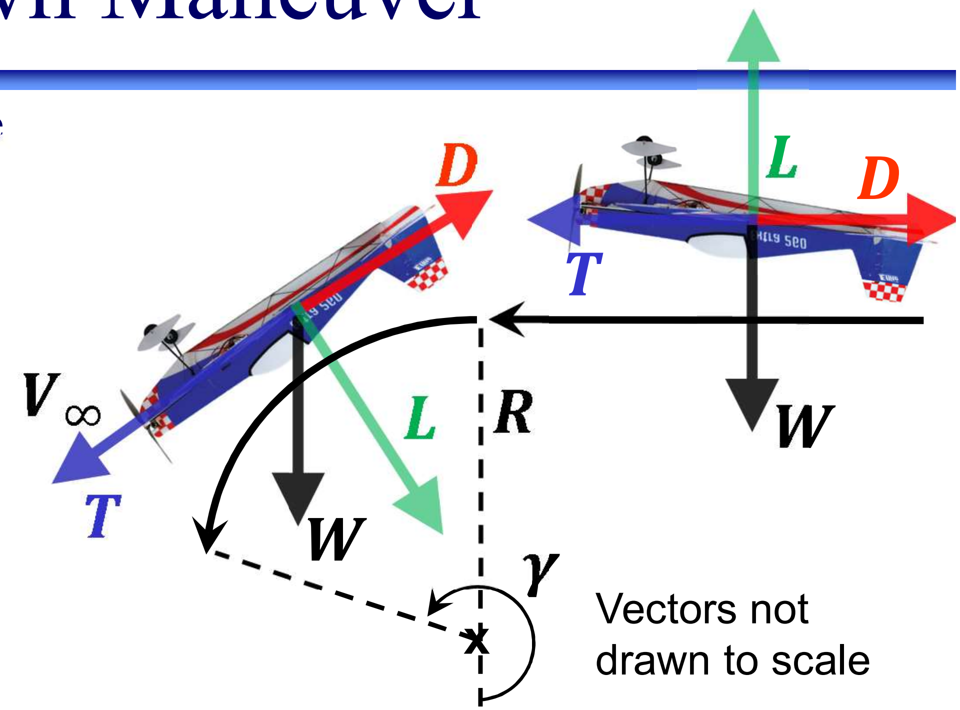
The Pull-down Maneuver

- The pull-down maneuver starts with the airplane in an inverted position
- The maneuver is then initiated by pulling back (aft) on the stick/yoke
- The change in lift creates a net centripetal force directed downward, causing the trajectory to arc downward
- Instantaneous (rather than sustained) pull-down is of greater interest
- Observe the direction of the lift vector for the two positions shown



$$m\frac{dV_\infty}{dt} = T - D - W\sin\gamma$$

$$m\frac{V_\infty^2}{R} = L - W\cos\gamma$$

$$n = \cos\gamma + \frac{V_\infty^2}{Rg}$$

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The Pull-down Maneuver

- At the start of the pull-up, $\gamma = 180°$
- Thus, the starting load factor is

$$n = -1 + \frac{V_\infty^2}{Rg}$$

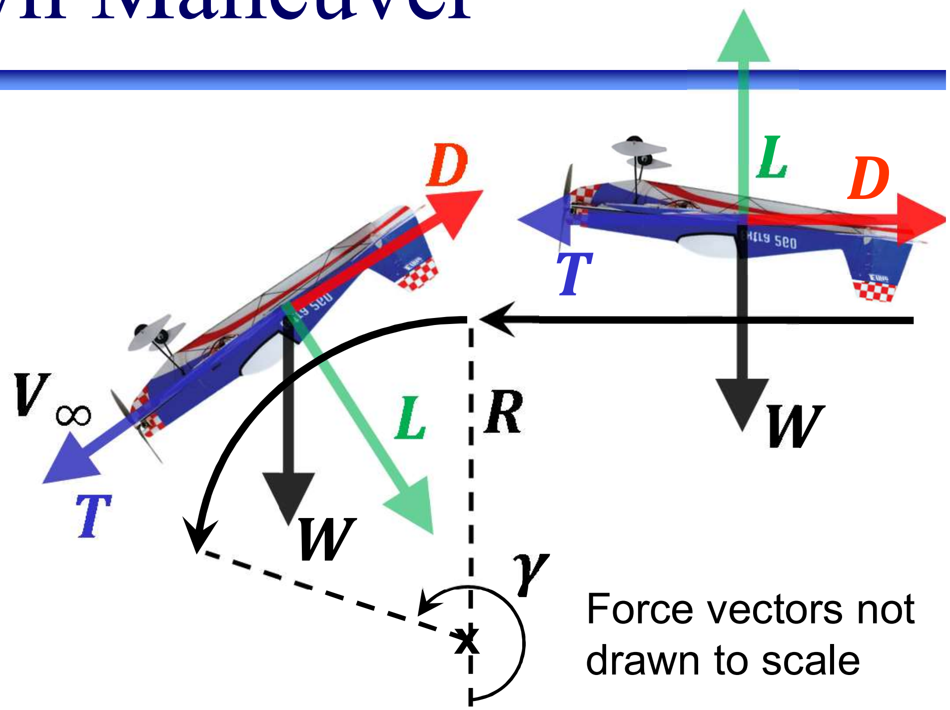
  - What is the sign of the sum?

- Solving this for the radius R yields

$$R = \frac{V_\infty^2}{g\,(n+1)}$$

- The instantaneous pitch rate is given by

$$\omega = \frac{d\gamma}{dt} = \frac{V_\infty}{R} = \frac{g(n+1)}{V_\infty}$$



Force vectors not drawn to scale

$$m\frac{dV_\infty}{dt} = T - D - W\,\sin\gamma$$

$$m\frac{V_\infty^2}{R} = L - W\cos\gamma$$

$$n = \cos\gamma + \frac{V_\infty^2}{Rg}$$

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The Split S Maneuver (PE → KE, 180° reversal)

Begin pull-down:

$$\gamma = 180^\circ, n = -1 + \frac{V_\infty^2}{Rg}$$

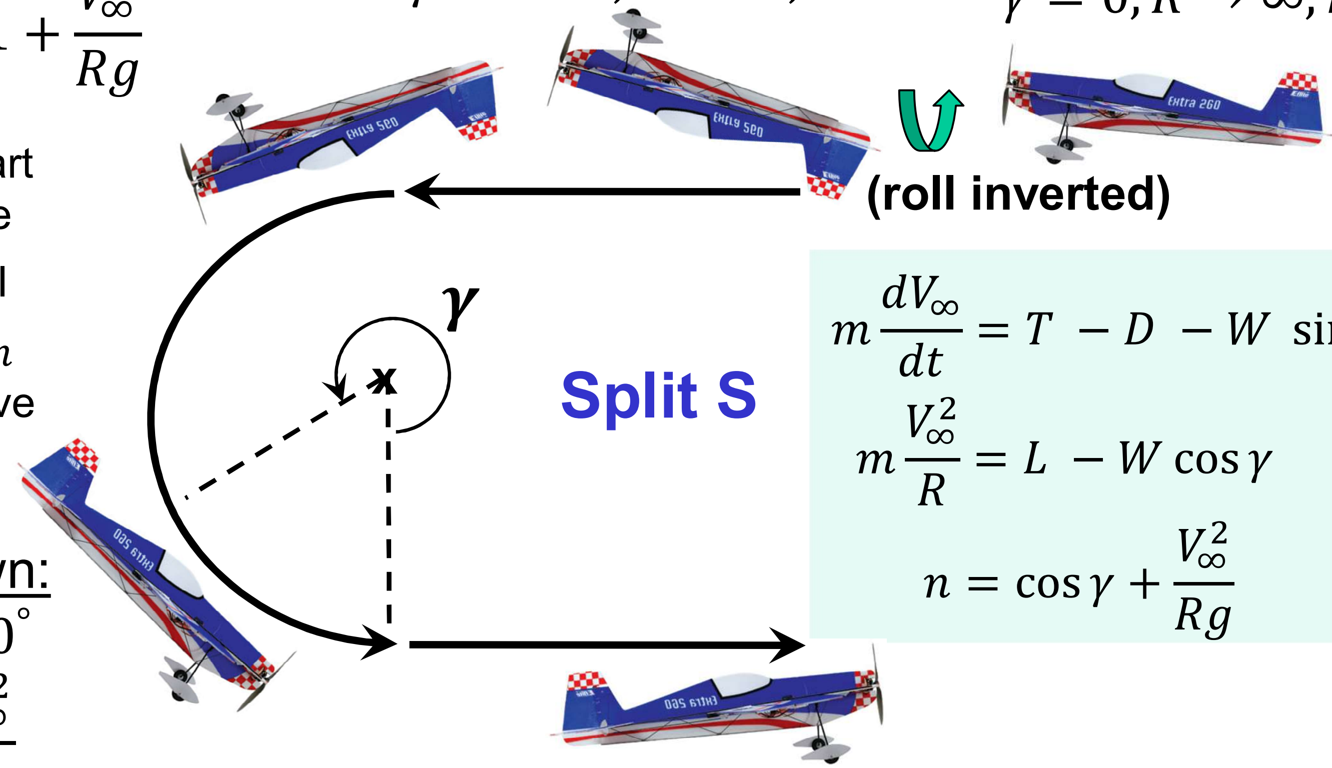
Roll inverted:

$$\gamma = 180^\circ, R \to \infty, n = -1$$

Straight-and-level:

$$\gamma = 0, R \to \infty, n = 1$$

Note: In the early part of the maneuver, the magnitude of $\frac{V_\infty^2}{Rg}$ will determine whether n Is positive or negative



$$m\frac{dV_\infty}{dt} = T - D - W \sin\gamma$$

$$m\frac{V_\infty^2}{R} = L - W\cos\gamma$$

$$n = \cos\gamma + \frac{V_\infty^2}{Rg}$$

During pull-down:

$$180^\circ < \gamma < 360^\circ$$

$$n = \cos\gamma + \frac{V_\infty^2}{Rg}$$

Straight-and-level:

$$\gamma = 0, R \to \infty, n = 1$$

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Limiting Case for Large Load Factor

- For a large load factor, the expressions for turn radius and rate for level turn, pull-up, and pull-down maneuvers approximate to the same expressions:

Level turn	Pull-up	Pull-down		Large load factor
$R = \dfrac{V_\infty^2}{g\sqrt{n^2-1}}$	$R = \dfrac{V_\infty^2}{g(n-1)}$	$R = \dfrac{V_\infty^2}{g(n+1)}$	$\Rightarrow$	$R \approx \dfrac{V_\infty^2}{g\,n}$
$\omega = \dfrac{g\sqrt{n^2-1}}{V_\infty}$	$\omega = \dfrac{g(n-1)}{V_\infty}$	$\omega = \dfrac{g(n+1)}{V_\infty}$	$\Rightarrow$	$\omega \approx \dfrac{g\,n}{V_\infty}$

- Removing V_∞ using the expression $L = nW = \frac{1}{2}\rho_\infty V_\infty^2\, S\, C_L$ yields

$$R = \frac{2}{\rho_\infty g}\left(\frac{W}{S}\right)\frac{1}{C_L} \quad \Rightarrow \quad \boxed{R_{min} = \frac{2}{\rho_\infty g}\left(\frac{W}{S}\right)\frac{1}{C_{L_{max}}}}$$

$$\omega = g\sqrt{\frac{\rho_\infty}{2}\frac{C_L\, n}{\left(\frac{W}{S}\right)}} \quad \Rightarrow \quad \boxed{\omega_{max} = g\sqrt{\frac{\rho_\infty}{2}\frac{C_{L_{max}} n_{max}}{\left(\frac{W}{S}\right)}}}$$

Note the prominent role of $\left(\frac{W}{S}\right)$, $C_{L_{max}}$, and n_{max} on maneuvering performance

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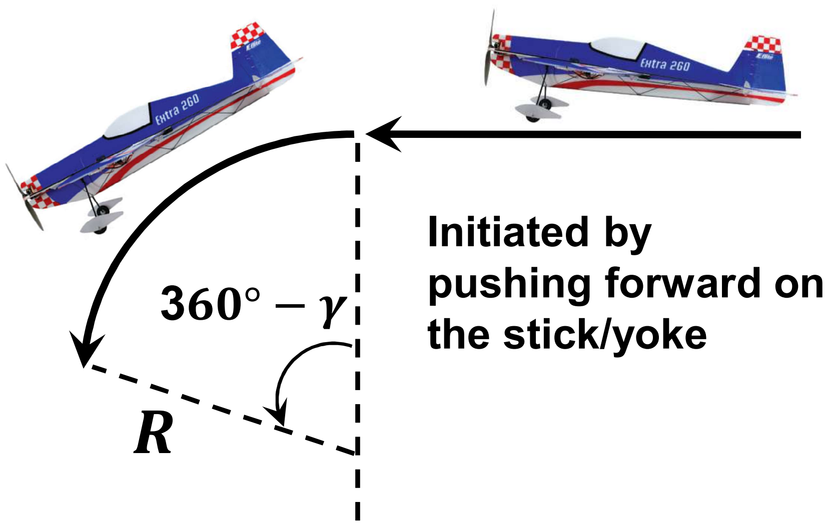
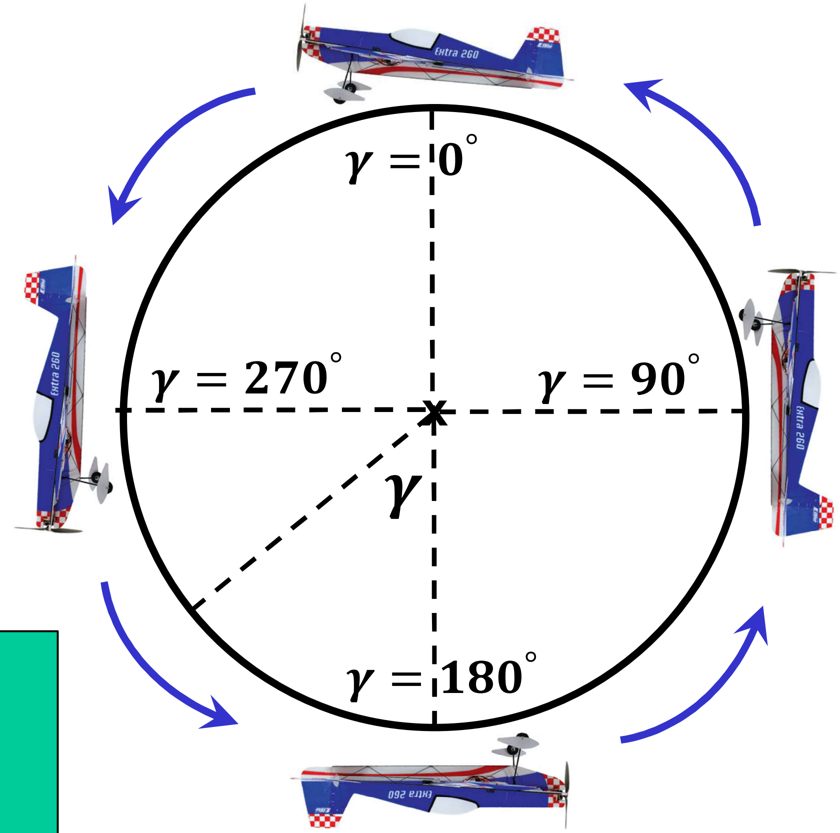
The Push-over and the Outside Loop





$$m\frac{dV_\infty}{dt} =$$

$$-m\frac{V_\infty^2}{R} =$$

$$n =$$

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A Discussion on Airspeeds

Indicated Airspeed (IAS)

Correct for instrument & position error

Calibrated Airspeed (CAS)

Correct for compressibility effects

Equivalent Airspeed (EAS)

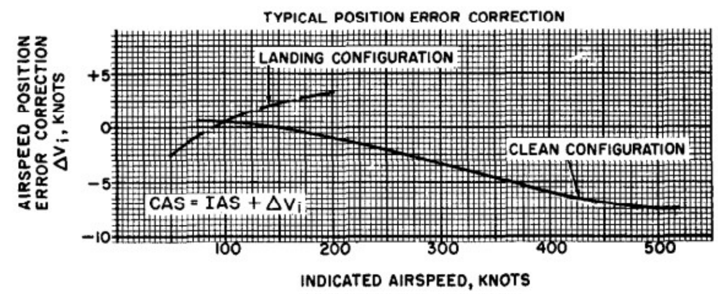
Correct for non-standard density

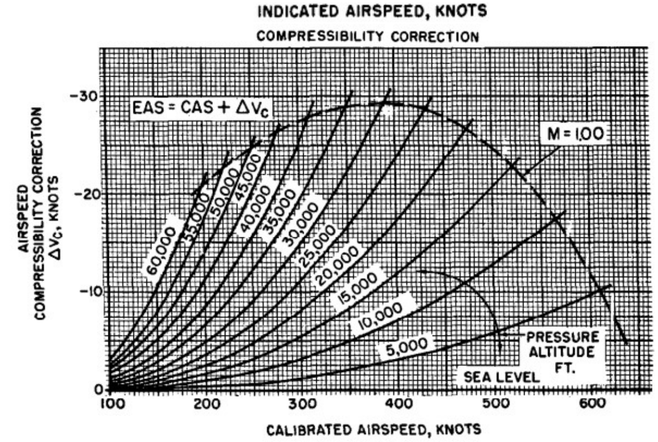
True Airspeed (TAS)

- This is the reading of the cockpit Airspeed Indicator (ASI)
- Variations between the IAS and actual flight speed occur due to (i) instrument & position error, (ii) compressibility effects, and (iii) altitudes other than sea-level
- Each effect must be corrected for, resulting in multiple airspeeds

- **Instrument error:** Should be small for well-designed ASI
- **Position error:** Due to pressure distribution variations over AOA range at location of instrument
- **Compressibility correction:** At higher Mach numbers, compressibility results in a magnified stagnation pressure reading
- **Density correction:** EAS at sea-level gives same dynamic pressure as TAS at flight altitude $\frac{1}{2}\rho_\infty V_{TAS}^2 = \frac{1}{2}\rho_{SL} V_{EAS}^2$

$$\boldsymbol{V_{TAS} = V_{EAS}\sqrt{\rho_{SL}/\rho}}$$





Airspeeds are often stated in knots. Thus, KTAS stand for "Knots, True Airspeed". Similarly, KEAS, KCAS, KIAS.

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Charts: H.H. Hurt, "Aerodynamics for Naval Aviators", 1965
Image: https://en.wikipedia.org/wiki/Airspeed_indicator

The V-n Diagram – Limit & Ultimate Load Factors

- Limit load factor (n^+, n^-): Structure must withstand limit loads without permanent deformation. Any temporary deformation must not interfere with safe operation
- Ultimate load factor (n^+_{ULF}, n^-_{ULF}): Limit load factor multiplied by factor of safety (typically 1.5). Structure must withstand ultimate loads for at least three seconds without failure
- $n^- \leq n \leq n^+$ - permissible (safe) operation
- $n^+ < n \leq n^+_{ULF}$ or $n^-_{ULF} \leq n < n^-$
  - structural damage/deformation, which requires grounding, inspection, and repair
- $n > n^+_{ULF}$ or $n < n^-_{ULF}$
  - structural failure will occur

(Addl. info, see: FAR 25.303, 25.305, 25.337)

Aircraft Type	n^+	n^-
Normal	2.5 to 3.8	-1 to -1.5
Utility	4.4	-1.8
Aerobatic	6	-3
Extreme aerobatic	10 to 12	-10 to -12
Homebuilt	5	-2
Transport	3 to 4	-1 to -2
Fighter	7.5	-3
Bomber	2 to 4	-1 to -2

The V-n Diagram – Recall a Few Concepts

- Design Dive Speed (V_D): This speed exceeds that attainable in level flight. Requires a dive
  - Airplane must be free of flutter, control reversal, and divergence up to 1.2 V_D

- In developing the V-n diagram, equivalent airspeed (EAS) is used
  - This removes the altitude dependence, and the dynamic pressure varies to the square of EAS ($q_\infty \propto V_{EAS}^2$)
  - **Caution:** The EAS **cannot** be used to compute accelerations!

$$\boxed{\boldsymbol{V_{TAS} = V_{EAS}\sqrt{\rho_{SL}/\rho}}}$$

$$\cancel{\boldsymbol{a_r = \frac{V_{EAS}^2}{R}}}$$

- Recall from prior discussions the expression for the maximum load factor as limited by the aerodynamic (stall) constraint, this time written in terms of EAS ($V_{\infty,EAS} = V_E$):

$$n_{max} = \frac{1}{2}\rho_\infty V_{\infty,TAS}^2 \frac{C_{L_{max}}}{\left(\frac{W}{S}\right)} = \frac{1}{2}\rho_{SL} V_{\infty,EA}^2 \frac{C_{L_{max}}}{\left(\frac{W}{S}\right)} = \frac{1}{2}\rho_{SL} V_E^2 \frac{C_{L_{max}}}{\left(\frac{W}{S}\right)}$$

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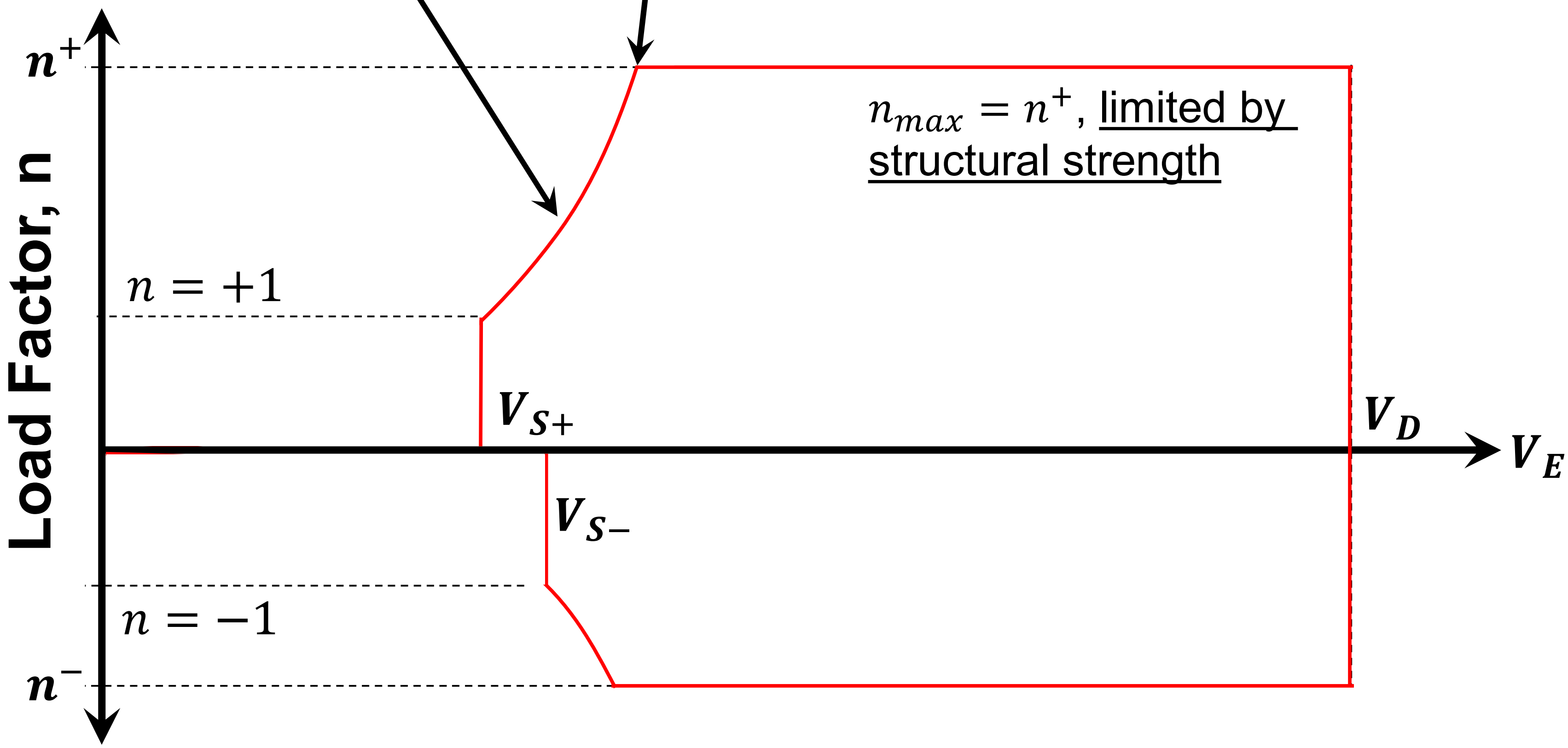
The V-n Diagram

n_{max} limited by stall

$$n_{max} = \frac{1}{2}\rho_{SL}V_E^2\frac{C_{L_{max}}}{\left(\frac{W}{S}\right)}$$

Maneuver (corner) speed

$$V_A = \sqrt{\frac{2}{\rho_{SL}}\left(\frac{W}{S}\right)\frac{n^+}{C_{L_{max}}}}$$



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The V-n Diagram – Significance of V_A

- It is clear from the expression that if the aircraft weight is lower, so is the maneuver speed V_A

$$V_A = \sqrt{\frac{2}{\rho_{SL}}\left(\frac{W}{S}\right)\frac{n^+}{C_{L_{max}}}}$$

- How does the TAS corresponding to V_A change with increase in altitude?

$$V_{A,TAS} = V_{A,EAS}\sqrt{\rho_{SL}/\rho}$$

- V_A is very significant for instantaneous maneuvering performance. Corresponds to:
  - Minimum instantaneous radius, R_{min}
  - Maximum instantaneous turn rate, ω_{max}

$$R_{min} = \frac{2}{\rho_\infty g}\left(\frac{W}{S}\right)\frac{1}{C_{L_{max}}}$$

$$\omega_{max} = g\sqrt{\frac{\rho_\infty}{2}\frac{C_{L_{max}} n_{max}}{\left(\frac{W}{S}\right)}}$$

(For large load factors)

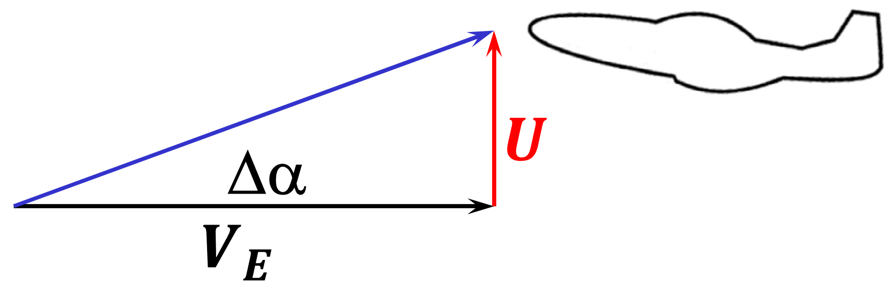
- V_A is also significant for structural considerations
  - Below V_A, the airplane will stall before it reaches limit load factor
  - Thus, air loads (e.g., from abrupt maneuvers) cannot cause structural damage

The V-n Diagram – The Impact of Gusts

- Consider an aircraft in steady, level flight at airspeed V_E. From force balance:

$$L_0 = W = \frac{1}{2}\rho_{SL} V_E^2 S \ \ C_{L,0} \qquad \textbf{(1)}$$

- The airplane then encounters a vertical gust of speed U. This changes the angle of attack by an amount $\Delta\alpha$, and the lift coefficient by ΔC_L



- Under the assumption that $U \ll V_E$

$$\Delta\alpha = \tan^{-1}\left(\frac{U}{V_E}\right) \approx \frac{U}{V_E} \qquad \textbf{(2)}$$

- The net lift L_g generated during the gust encounter is now given by

$$L_g = \frac{1}{2}\rho_{SL} V_E^2 \ S \ \left(C_{L,0} + \Delta \mathrm{C_L}\right) = \frac{1}{2}\rho_{SL} V_E^2 \ S \ C_{L,0} + \frac{1}{2}\rho_{SL} V_E^2 \ S \ \Delta \mathrm{C_L}$$

$$L_g = W + \frac{1}{2}\rho_{SL} V_E^2 \ S \ \Delta \mathrm{C_L} \qquad \textbf{(3)}$$

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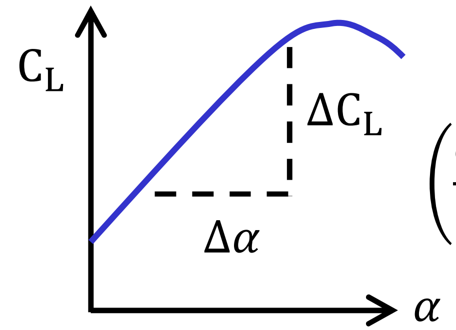
The V-n Diagram – The Impact of Gusts

- Assuming operation within the linear region of the lift-curve, the change in lift coefficient ΔC_L is related to the change in AOA $\Delta\alpha$ as follows:

$$\Delta C_L = \left(\frac{dC_L}{d\alpha}\right)\Delta\alpha = \left(\frac{dC_L}{d\alpha}\right)\frac{U}{V_E} \quad \textbf{(4)}$$



$$\left(\frac{dC_L}{d\alpha}\right) = \frac{\Delta C_L}{\Delta\alpha}$$

- Using (4) in (3) yields

$$L_g = W + \frac{1}{2}\rho_{SL}\,S\left(\frac{dC_L}{d\alpha}\right)U\,V_E \quad \textbf{(5)}$$

- Dividing both sides of (5) by weight W yields the gust load factor $n_g = \frac{L_g}{W}$:

$$\boxed{n_g = 1 + \left[\frac{\rho_{SL}}{2}\,\frac{1}{\left(\frac{W}{S}\right)}\left(\frac{dC_L}{d\alpha}\right)U\right]\,V_E} \quad \textbf{(6)}$$

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The V-n Diagram – The Impact of Gusts

$$n_g = 1 + \left[\frac{\rho_{SL}}{2} \frac{1}{\left(\frac{W}{S}\right)} \left(\frac{dC_L}{d\alpha}\right) U\right] V_E \quad \textbf{(6)}$$

- The incremental load factor due to gusts decreases with increase in wing loading
  - All else being equal, airplanes with higher wing loading are less sensitive to gusts/turbulence (better ride quality)

- The incremental load factor depends on gust velocity U (magnitude & direction)
  - These are specified in FAR 25.341 for both upward & downward gusts
  - A higher value is specified for design cruise speed V_C and a lower value for design dive speed V_D

- The incremental load factor varies linearly with airspeed V_E
  - Large load factors may occur if flying fast in gusty / turbulent conditions

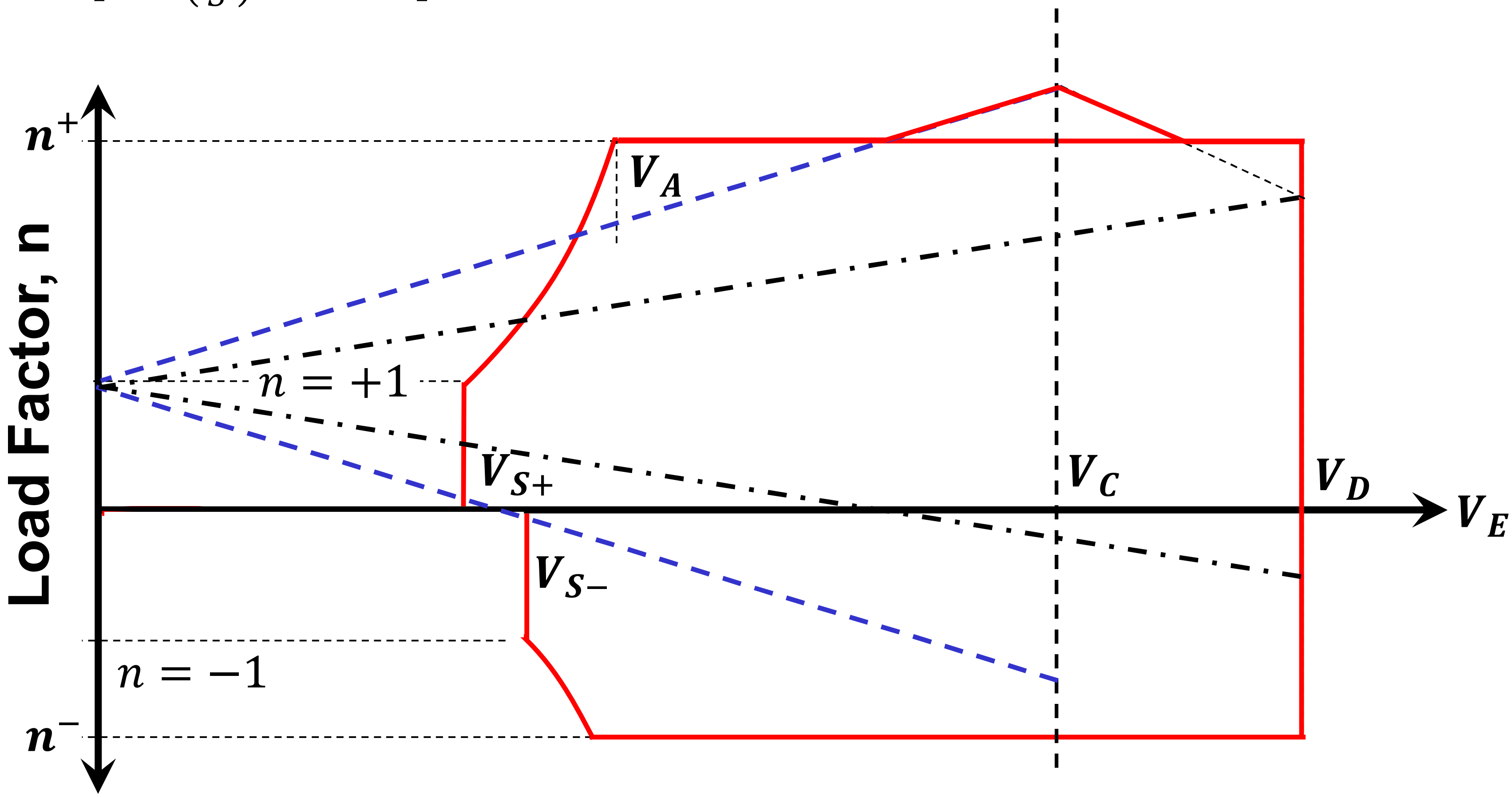
- The "gust lines" given by (6) can now be factored into the V-n diagram

The V-n Diagram – Showing Gust Lines

$$n_g = 1 \pm \left[\frac{\rho_{SL}}{2}\ \frac{1}{\left(\frac{W}{S}\right)}\left(\frac{dC_L}{d\alpha}\right) U\right]\ V_E$$

U as specified for V_C (FAR 25.341)

U as specified for V_D (FAR 25.341)



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Energy Concepts - Introduction

- The discussion of rate of climb in Performance Lecture #1 was limited to the unaccelerated case, here we will look into accelerated climb
- Up to this point, we have primarily used Newton's first two laws to obtain performance equations
- In this section, we will focus on using the principle of conservation of energy to understand accelerated climb as it offers a different and interesting perspective

Energy Concepts - Introduction

<u>Some historical perspective:</u>

- Rutowski, in 1954, published a graphical method for an optimal flight profile to attain a particular speed and altitude
- Boyd (A Korean War era fighter pilot) and Christie (around 1970s) extended this to the Energy-Maneuverability (E-M) theory to rank relative performance of fighter aircraft
  - F-14, F-15, F-16, and F-18 designs incorporated objectives of bettering Soviet designs in terms of E-M
  - Generation of "sky-maps" – plotted as altitude versus velocity for various performance parameters such as climb rate, acceleration, maximum instantaneous and sustained turn rates
- More recently, Takahashi[1] has utilized E-M presentation to visualize various aircraft conceptual design parameters

[1] Takahashi, T., Aircraft Concept Design Performance Visualization Using an Energy-Maneuverability Presentation," AIAA Aviation, Sept. 2012

Review

- **Specific Excess Power** is given by:

$$P_s = \frac{(T-D)V_\infty}{W}$$

In the case of unaccelerated climb, this is also equal to maximum rate of climb

- **Energy height** is defined as the total mechanical energy per unit weight:

$$H_e = \left(\frac{V_\infty^2}{2g} + h\right)$$

Recall from earlier lecture on energy-based constraint analysis:

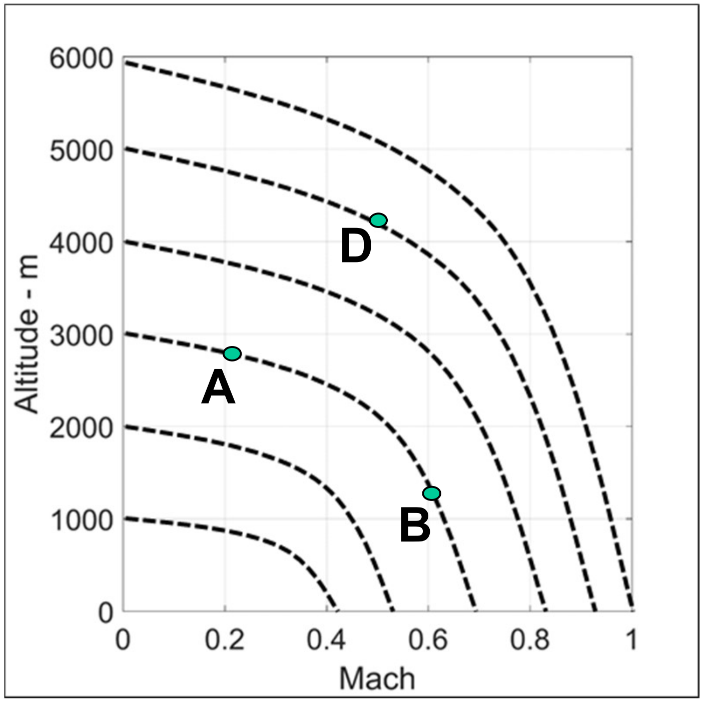
$$\frac{(T-D)V_\infty}{W} = \frac{d}{dt}\left(\frac{V_\infty^2}{2g} + h\right)$$

$$\Rightarrow \boxed{P_s = \frac{dH_e}{dt}}$$

Therefore, to increase the energy height of the aircraft, we need to have positive specific excess power

Sky-Maps: Energy Height

Sky-map is a plot with x-axis as Mach number and y-axis as the altitude in which various aircraft point performance characteristics can be plotted and visualized

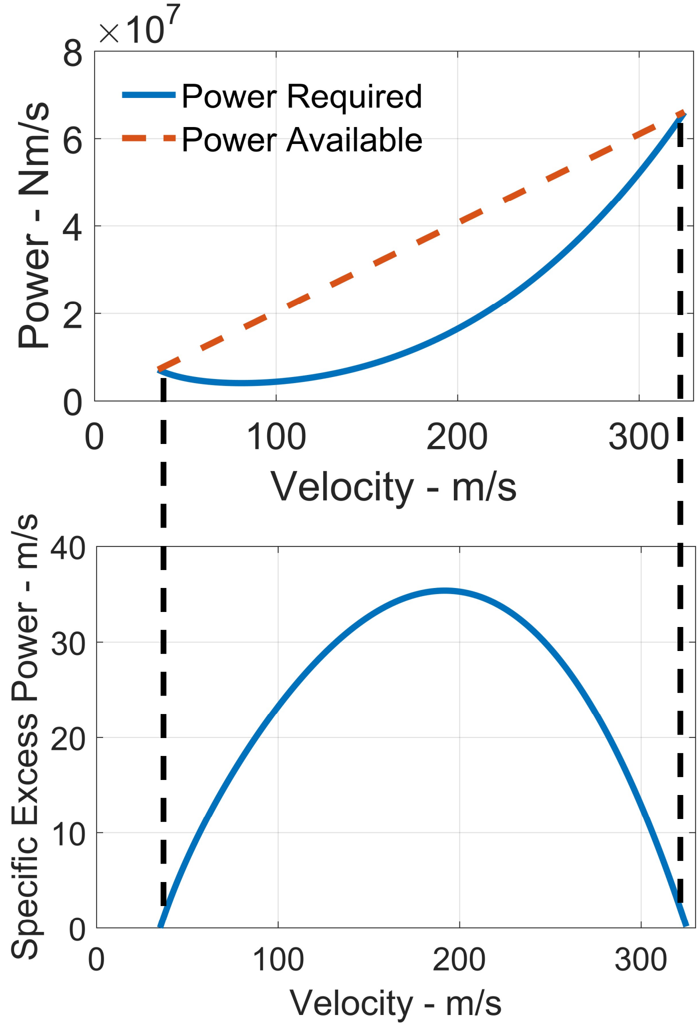


- In this figure, dashed lines are of constant energy height
- Along each line, airplane can trade between PE and KE, but energy height remains constant
- Question: Does energy height depend on aircraft?
  - No! These are universal curves
- Airplanes A,B flying at same energy height, airplane D at a higher energy height

In combat, other things held constant, you would want to be at a higher energy height than your enemy

Sky-Maps: Specific Excess Power

- To obtain sky-map for specific excess power, let us proceed in a step-wise manner
- Recall how power required and power available vary with velocity (and by extension, Mach number)
- If specific excess power is now plotted versus velocity (or Mach number) – it will look like the figure



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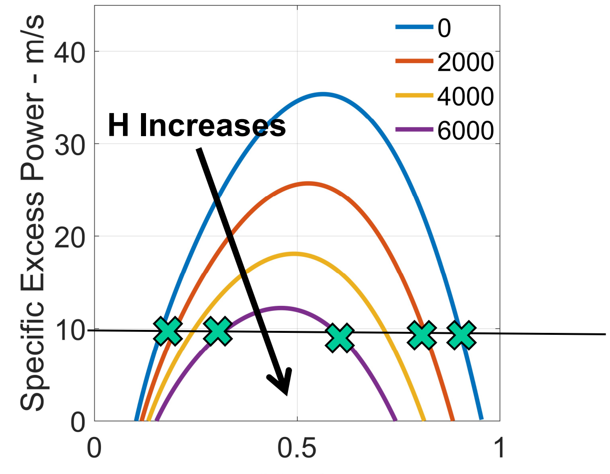
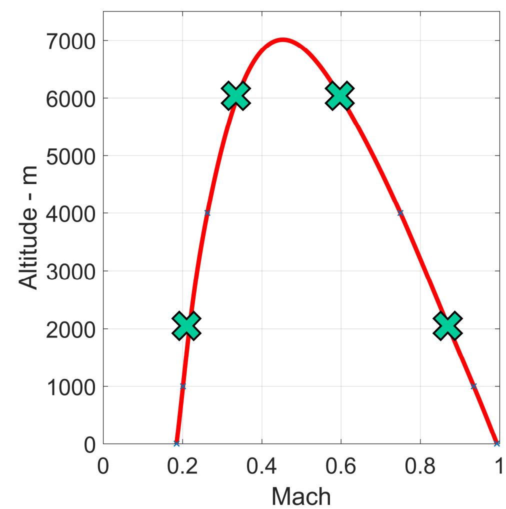
Sky-Maps: Specific Excess Power

- Cross plotting the earlier plot by changing the y-axis to altitude will yield the sky-map representation

- The curve shown here is a typical specific excess power plot for a subsonic airplane

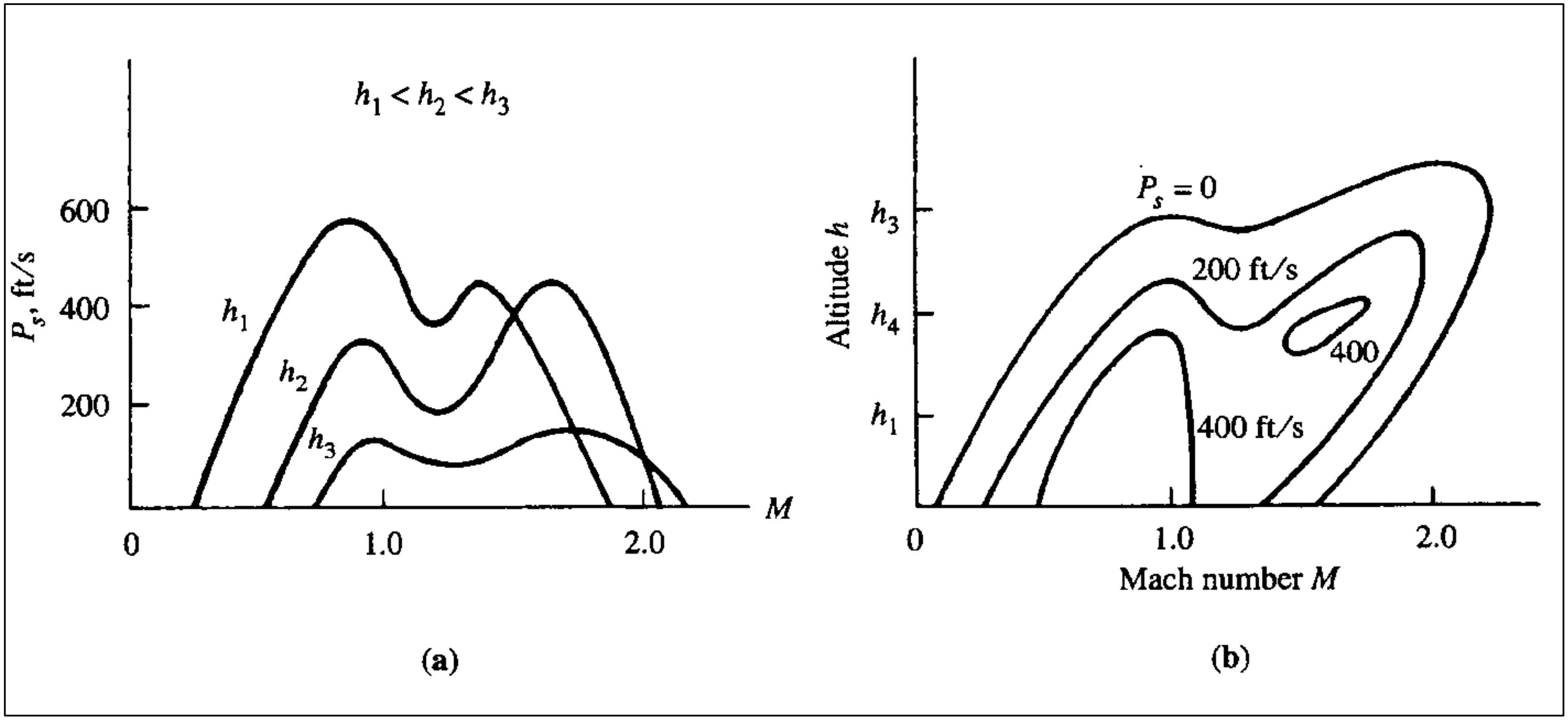
- Outermost contour ($P_s = 0$) corresponds to the aircraft's sustained envelope





Specific Excess Power – Supersonic Aircraft

Curves for supersonic aircraft look different due to drag divergence effects



The "dent" in the curves around Mach 1 is due to large drag in the transonic regime

Figure source: Anderson: Aircraft Performance and Design

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Energy Concepts: Accelerated Climb

For accelerated rate of climb we have:

$$\boldsymbol{P_s = \left(\frac{V_\infty}{2g} \times \frac{dV_\infty}{dt}\right) + \left(\frac{dh}{dt}\right)}$$

Therefore, if the acceleration is A

$$\boldsymbol{\frac{dh}{dt} = P_s - A \quad \Rightarrow \quad \int \frac{dh}{P_s - A} = \int dt}$$

This expression can be integrated to obtain the total time required to go from one altitude to another

However, it requires you to know the acceleration at each point in time and is therefore not very useful if we want to calculate say, the minimum time to climb

Energy Concepts: Minimum time to climb

For accelerated rate of climb we have:

$$P_s = \frac{dH_e}{dt} \Rightarrow dt = \frac{dH_e}{P_s}$$

Now, let us assume that the aircraft can rapidly exchange kinetic and potential energy while maintaining the same energy height (called a "zoom maneuver" in literature)

In that case, we can integrate the above expression to get

$$t_2 - t_1 = \int_{H_{e,1}}^{H_{e,2}} \frac{dH_e}{P_s}$$

Therefore, if we look at small step increases in energy height ΔH_e, then the time required to achieve that change $\boldsymbol{\Delta t}$ is minimized when $\boldsymbol{P_s}$ is maximized. This directly yields a strategy to go from one energy height to another!

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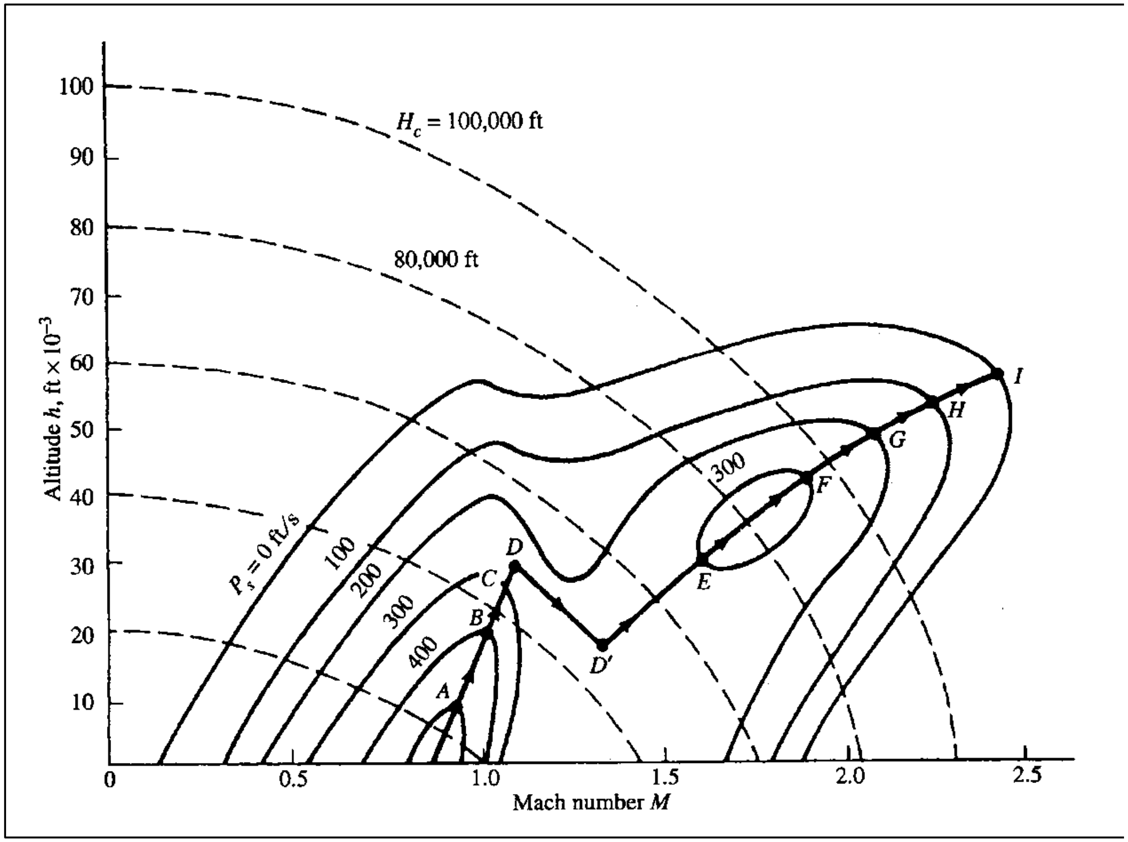
Energy Concepts: Minimum time to climb

- In accelerated climb, the time to climb between two energy heights is not a unique value, it is path dependent
- But the minimum time to climb is a unique value
- As noted earlier, time to climb is minimum along path where P_s is maximum
- This means points where the P_s curve is tangent to H_e curves
- Move along H_e curve till you reach tangent to P_s curve and then increase your H_e



In transonic regime, sometimes you have to dive first to climb faster!

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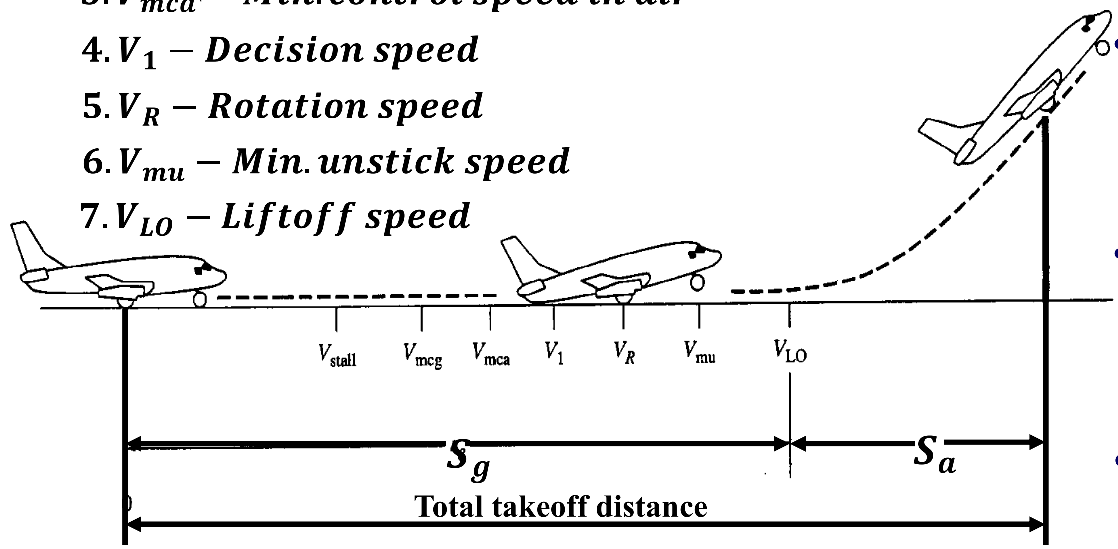
Takeoff Performance

1. V_{stall} – Stall speed
2. V_{mcg} – Min. control speed on ground
3. V_{mca} – Min. control speed in air
4. V_1 – Decision speed
5. V_R – Rotation speed
6. V_{mu} – Min. unstick speed
7. V_{LO} – Liftoff speed



- We will now analyze takeoff performance of an airplane
- Takeoff is segmented into a ground roll(S_g), and distance to clear an obstacle(S_a)
- Obstacle height:
  - 50 ft for military aircraft
  - 35 ft for commercial aircraft
- $\text{Total takeoff distance} = S_g + S_a$

- Note that all the relative velocities are sandwiched between the value of V_{stall} and V_{LO}. Usually $V_{LO} \approx 1.1\ V_{stall}$

- A *balanced field length* is defined as follows. Let A be the distance traveled by the airplane along the ground from the original starting point(point 0) to the point where V_1 is reached, and let B be the additional distance traveled with an engine failure. This is the distance to clear an obstacle at the end of takeoff or to brake to a stop. The balanced field length is by definition A + B.

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Calculation of Ground Roll

- The forces acting on an airplane during takeoff can be used to calculate ground roll as follows:

$$\frac{\boldsymbol{W}}{\boldsymbol{g}}\frac{\boldsymbol{dV_\infty}}{\boldsymbol{dt}} = \boldsymbol{T} - \boldsymbol{D} - \boldsymbol{R}$$

- Thrust as a function of velocity:
- Turbojet - $\mathrm{T} = constant$
- Turbofan - $T = k_1 - k_2 V_\infty + \mathrm{k}_3 \mathrm{V}_\infty^2$
- Piston Powered - $T = \frac{k}{V_\infty}$

- These equations are used to obtain thrust at any point during the takeoff ground roll or to obtain a closed form analytical solution for distance

- Rolling friction is defined as $\mu * normal\ force$ Therefore,

$$R = \mu_r(W - L)$$

- We will now develop an expression for drag

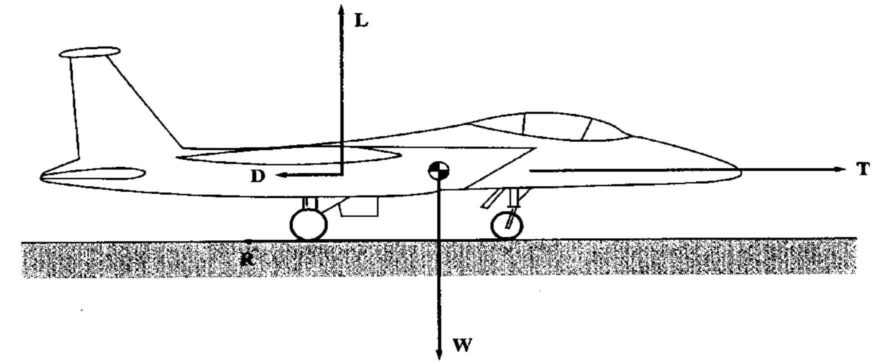

Figure 6.14 Forces acting on an airplane during takeoff and landing.

Surface	μ_r (Typical Values) Brakes off	Brakes on
Dry concrete/asphalt	0.03–0.05	0.3–0.5
Wet concrete/asphalt	0.05	0.15–0.3
Icy concrete/asphalt	0.02	0.06–0.10
Hard turf	0.05	0.4
Firm dirt	0.04	0.3
Soft turf	0.07	0.2
Wet grass	0.08	0.2

Drag During Ground Roll

- Recall the conventional drag polar $\boldsymbol{C_D = C_{D0} + KC_L^2}$

- During ground roll, there are additional effects that contribute to the drag:
  1. Additional $\boldsymbol{C_{D0}}$ due to a fully extended landing gear

$$\Delta C_{D0} = \frac{W}{S} K_{uc} m^{-0.215}$$

  - W/S - wing loading
  - m - maximum mass of the airplane
  - K_{uc} – accounts for amount of flap deflection

  2. Reduction in induced drag due to ground effect

$$\frac{C_D(in-ground\,effect)}{C_{D_i}(out-of-ground\,effect)} \equiv G = \frac{\left(16\frac{h}{b}\right)^2}{1+\left(16\frac{h}{b}\right)^2}$$

  - h – height of wing above ground
  - b – wing span

- The effective drag polar during takeoff turns out to be, $\boldsymbol{C_D = C_{D0} + \Delta C_{D0} + (k_1 + Gk_3)C_L^2}$
- Takeoff ground roll distance is computed using numerical integration techniques by discretizing the takeoff segment into small parts
- Thrust, drag, and lift values are computed using equations describing their functional dependence on velocity, as derived above

Graphical Analysis of Forces in Ground Roll

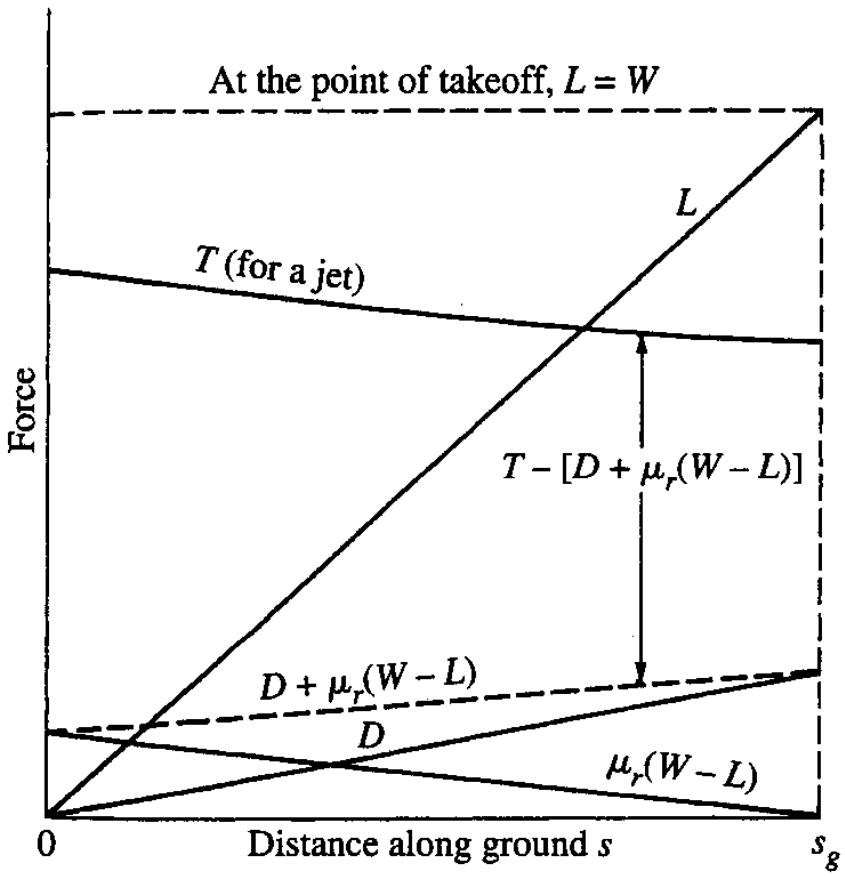

Figure 6.15 Schematic of a typical variation of forces acting on an airplane during takeoff.

- Thrust for a turbojet is fairly constant with increasing velocity
- Lift, and drag increase linearly with distance along the ground since $L, D \propto V^2, and\ V^2 = 2as$
- $\mu_r(W - L)$ decreases linearly with distance along ground as lift increases linearly
- The difference between thrust, and the sum of drag and friction results in the airplane's acceleration leading to takeoff when lift equals weight

- We will now look at an analytical closed form solution that enables identification of trends, and dependence of takeoff on important aircraft design parameters

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Closed Form Expression for Takeoff Ground Roll Distance

- Under major simplifying assumptions, and some trivial and rather involved algebra, we obtain an expression for the takeoff ground roll distance,

$$s_g = \frac{1.21\left(\frac{W}{S}\right)}{g\rho_\infty C_{Lmax}\left(\frac{T}{W} - \frac{D}{W} - \mu_r\left(1 - \frac{L}{W}\right)\right)_{0.7V_{LO}}} + 1.1N\sqrt{\left(\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{C_{Lmax}}\right)}$$

- Note that this expression is rarely used to perform any useful calculation. It is exclusively intended to reveal the concrete dependence of s_g on important aircraft design parameters

- Important observations:

1. **s_g increases with an increase in W/S.**
2. **s_g decreases with and increase in $(C_L)_{max}$.**
3. **s_g decreases with an increase in T/W.**

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Calculation of Distance While Airborne to Clear an Obstacle(S_a)

Recall that the total takeoff distance is equal to the ground roll s_g and the extra distance required to clear an obstacle after becoming airborne S_a. Now let's consider the calculation of S_a.

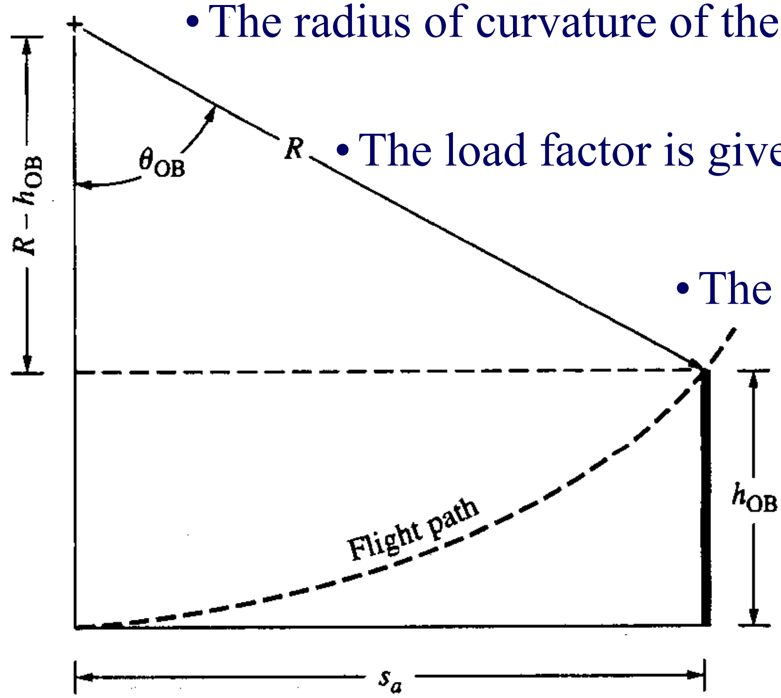


Figure 6.16 Sketch for the calculation of distance while airborne.

- The radius of curvature of the pull-up arc, $\boldsymbol{R = \frac{V_\infty^2}{g\,(n-1)}}$

FAR requirement

Safety factor

- The load factor is given by, $\boldsymbol{n = \frac{L}{W} = \frac{0.5\rho_\infty(1.15V_{stall})^2(0.9)SC_{Lmax}}{0.5\rho_\infty(V_{stall})^2 SC_{Lmax}} = 1.19}$

- The radius then becomes, $\boldsymbol{R = \frac{(1.15V_{stall})^2}{g\,(1.19-1)} = \frac{6.96(V_{stall})^2}{g}}$

- The angle $\boldsymbol{\theta_{ob} = \cos^{-1}(1 - \frac{h_{ob}}{R})}$

- Finally, the distance to clear an obstacle is calculated as,

$$\boldsymbol{S_a = R\sin\theta_{ob}}$$

Landing Performance

- The analysis of landing performance is analogous to takeoff, only in reverse. Consider an airplane on a landing approach

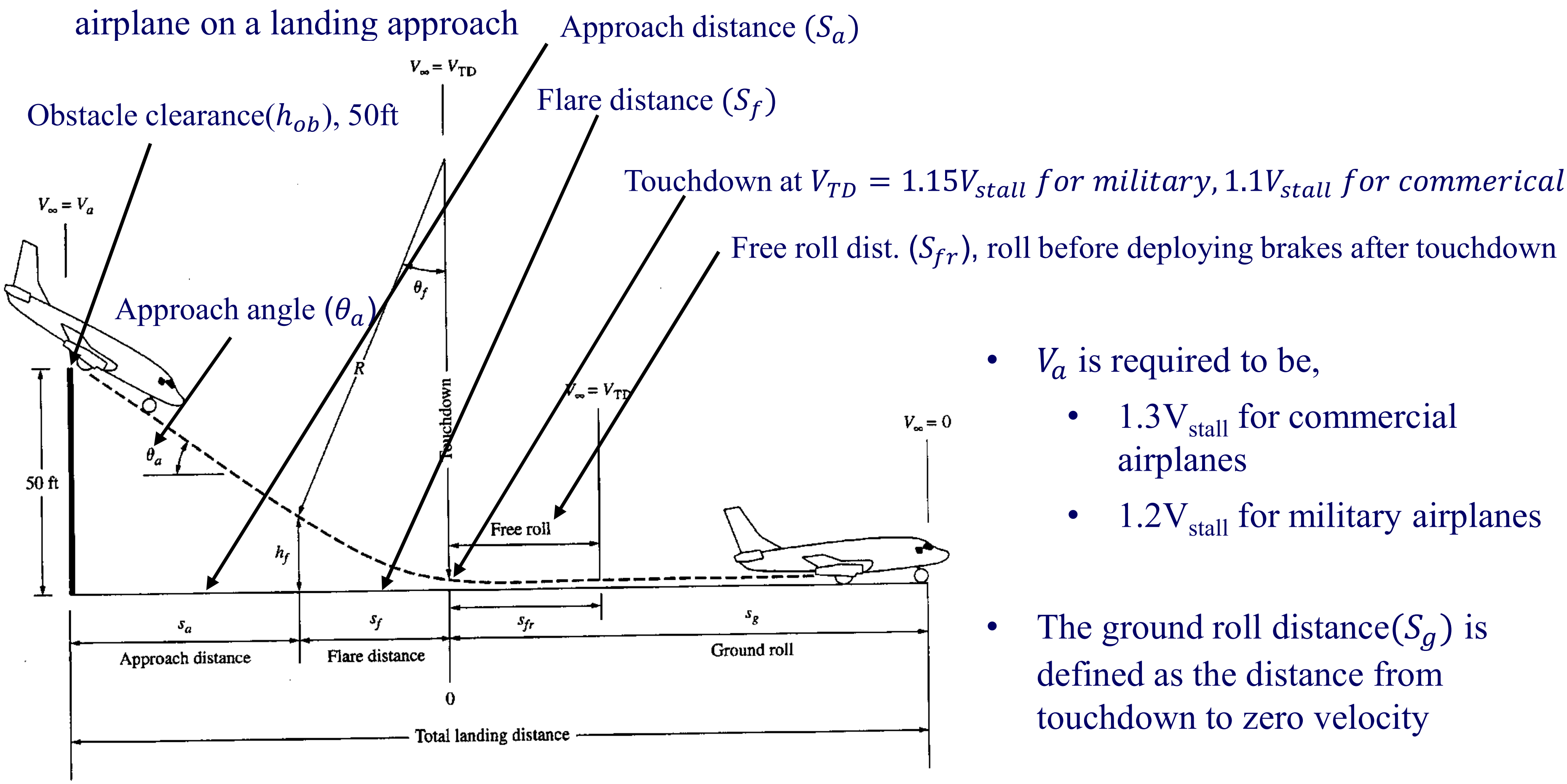


Figure 6.17 The landing path and landing distance.

- V_a is required to be,
  - $1.3V_{stall}$ for commercial airplanes
  - $1.2V_{stall}$ for military airplanes
- The ground roll distance(S_g) is defined as the distance from touchdown to zero velocity

Calculation of Approach (S_a) and Flare (S_f) Distances

- The approach distance (S_a) depends on the approach angle (θ_a) and the flare height (h_f).
- In turn θ_a depends on T/W and L/D. This can be seen in figure below, which shows the force diagram for an aircraft on the approach flight path, assuming equilibrium flight conditions

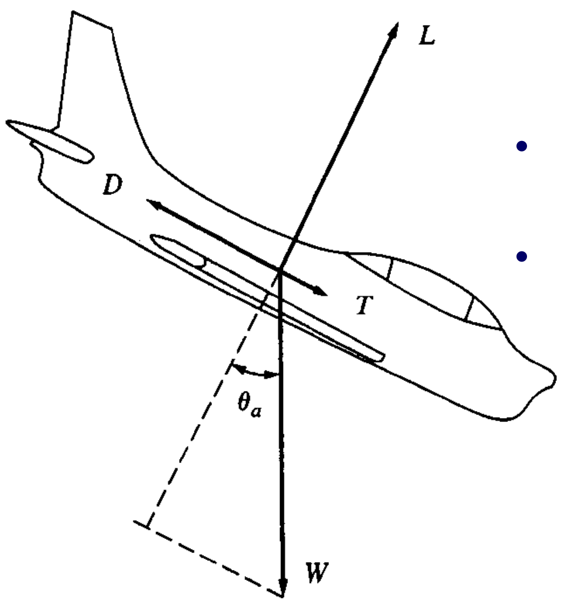
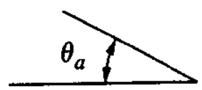


Figure 6.18 Force diagram for an airplane on the landing approach flight path.

- The approach angle is usually small for most cases. Raymer states that for transport aircraft $\theta_a \leq 3°$. Hence, $\cos\theta_a \approx 1$ and $L \approx W$, and

$$\boldsymbol{\sin\theta_a = \frac{1}{L/D} - \frac{T}{W}}$$

- The circular path is tangent to both the approach path and the ground, therefore, $\boldsymbol{\theta_f = \theta_a}$
- Also, $\boldsymbol{h_f = R(1 - \cos\theta_f) = R(1 - \cos\theta_a)}$



- V_∞: $1.3V_{stall} - 1.15V_{stall}$, *commerical*
- V_∞: $1.2V_{stall} - 1.1V_{stall}$, *military*
- $V_f = 1.23V_{stall}$, *commercial*
- $V_f = 1.23V_{stall}$, *military*
- The radius is obtained using the pull-up equation at a stipulated load factor of 1.2,

$$\boldsymbol{R = \frac{V_f^2}{0.2g}}$$

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Calculation of Approach (S_a) and Flare (S_f) Distances

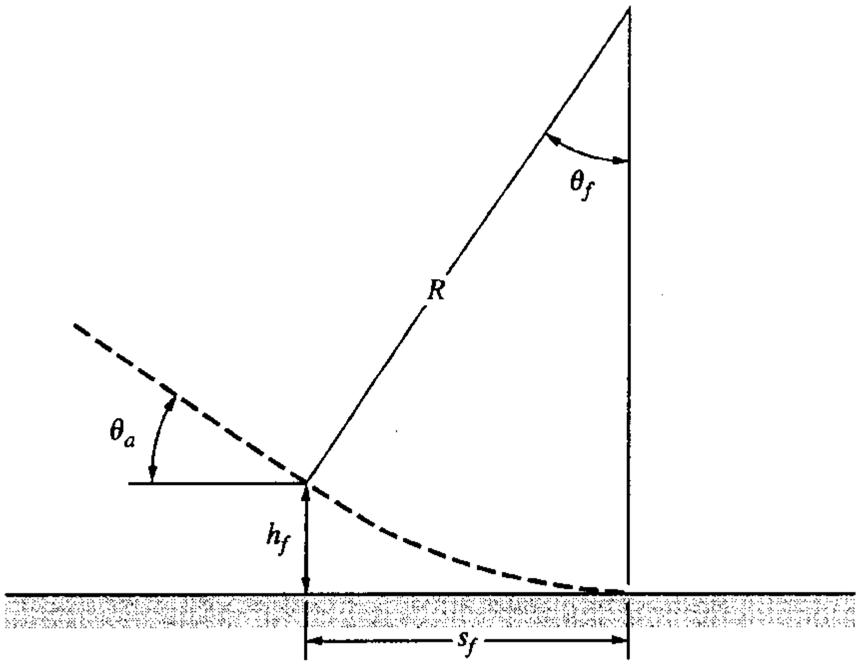

Figure 6.19 Geometry of the landing flare.

- The approach distance can be calculated as,

$$s_a = \frac{50 - h_f}{\tan\theta_a}$$

- The flare distance calculation, analogous to the takeoff case is,

$$s_f = R\sin\theta_f$$

- Since, $\theta_f = \theta_a$ as discussed earlier,

$$s_f = R\sin\theta_a$$

- We will now develop and analyze methods to calculate ground roll as we did for takeoff

Calculation of Ground Roll(S_g)

- Recall the forces acting on an airplane during ground roll,

$$\frac{W}{g}\frac{dV_\infty}{dt} = T - D - R$$

- Normal landing practice assumes that upon touchdown the engine thrust is reduced to idle (essentially zero). In this case T = 0,

$$\frac{W}{g}\frac{dV_\infty}{dt} = -D - \mu_r(W - L)$$

- If the airplane is equipped with thrust reversers which typically produce ~50% reverse thrust,

$$\frac{W}{g}\frac{dV_\infty}{dt} = -T_{rev} - D - \mu_r(W - L)$$

- Note that the drag term can be used to account for spoilers, speed brakes etc.

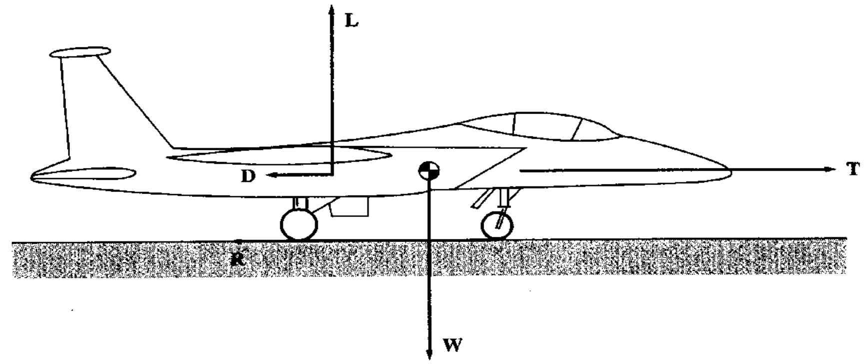

Figure 6.14 Forces acting on an airplane during takeoff and landing.

	μ_r(Typical Values)	
Surface	**Brakes off**	**Brakes on**
Dry concrete/asphalt	0.03–0.05	0.3–0.5
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Graphical Analysis of Forces in Landing Ground Roll

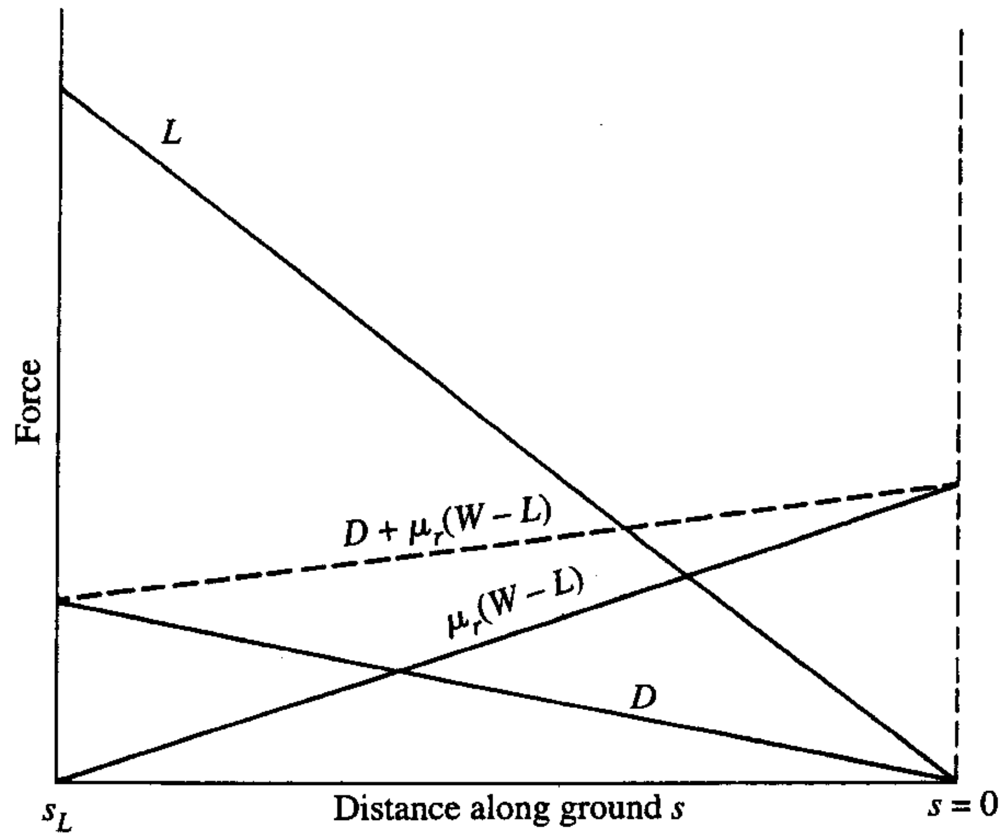

Figure 6.20 Schematic of a typical variation of forces acting on an airplane during landing.

- Lift, and drag decrease linearly with distance along the ground since $L, D \propto V^2, and\ V^2 = 2as$
- $\mu_r(W - L)$ increases linearly with distance along ground as lift decreases linearly
- With no brakes applied $D + \mu_r(W - L)$, the net retarding force increases linearly with distance on the ground
- We will now look at an analytical closed form solution that enables identification of trends, and dependence of takeoff on important aircraft design parameters

Closed Form Expression for Landing Ground Roll Distance

- Under major simplifying assumptions, and some trivial and rather involved algebra, we obtain an expression for the landing ground roll distance,

$$s_g = jN\sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{C_{Lmax}}} + \frac{j^2(\frac{W}{S})}{g\rho_\infty C_{Lmax}\left(\frac{T_{rev}}{W} + \frac{D}{W} + \mu_r\left(1 - \frac{L}{W}\right)\right)_{0.7V_{LO}}}$$

- Important observations:

1. s_g increases with an increase in W/S.
2. s_g decreases with and increase in $(C_L)_{max}$.
3. s_g decreases with an increase in T_{rev}/W.
4. s_g increases with an decrease in ρ_∞.

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