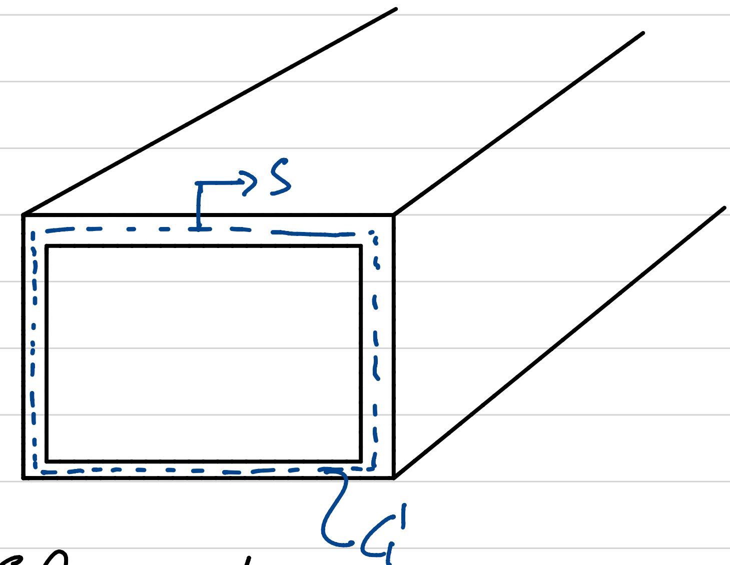
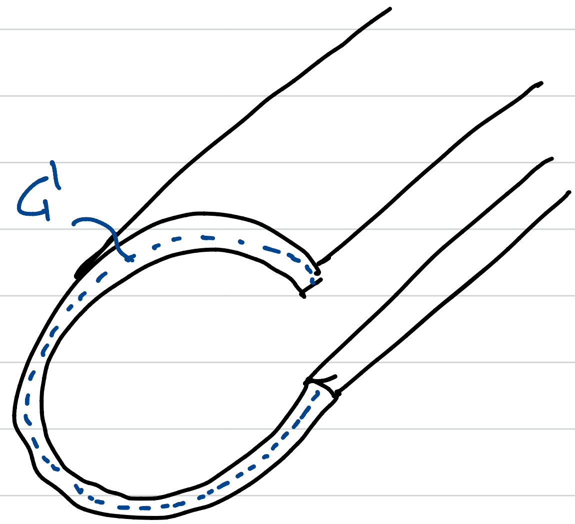
Thin-Walled Beams: Torsion

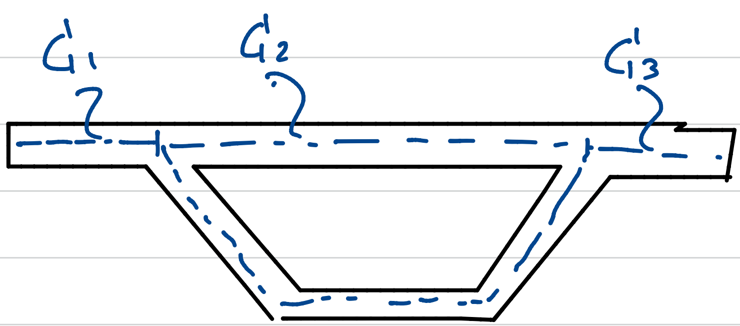
→ Closed or open cross-sections



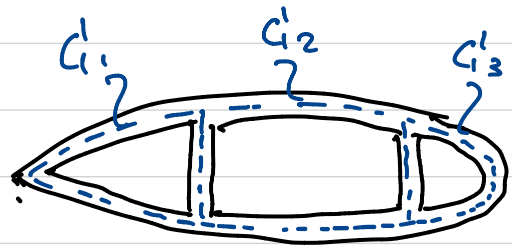
Closed



Open



Open and closed components



Multi-cellular Section

→ Geometry of the section is described by a curve C, drawn along the mid-thickness of the wall

→ The curvilinear variable s measures the length along this contour

→ Can have multiple curves C_i and multiple variables s.

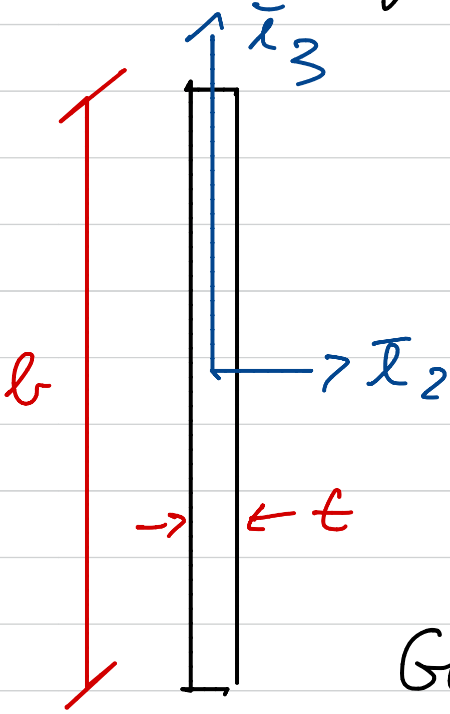
Thin - Walled

$$t(s)/b \ll 1, \quad t(s)/h \ll 1 \Big\} \text{ Thin Wall}$$

or $t(s)/\sqrt{b^2+h^2} \ll 1$

and $\sqrt{b^2+h^2}/L \ll 1 \Big\}$ Beam Theory.

Torsion at a thin-walled rectangular cross-section



Since $t \ll b$ we assume that the stress function Φ is constant along $\bar{\imath}_3$-direction.

$$\frac{\partial \Phi}{\partial x_3} \approx 0$$

Governing Eq. for Φ

$$\frac{\partial^2 \Phi}{\partial x_2^2} + \overset{\approx 0}{\cancel{\frac{\partial^2 \Phi}{\partial x_3^2}}} = -2GK_1$$

$$\Phi = -GK_1 x_2^2 + C_1 x_2 + C_2$$

Boundary Condition

$$\Phi = 0 \quad @ \quad x_2 = \pm t/2$$

$$\boxed{\Phi = -GK_1\left(x_2^2 - \frac{t^2}{4}\right)}$$

$$H_{11} = \frac{M_1}{K_1} \quad , \quad M_1 = 2\int_A \Phi \, dA$$

$$M_1 = -2GK_1 \int_{-t/2}^{t/2} \left(x_2^2 - \frac{t^2}{4} \right) dx_2 \cdot b$$

$$= -2GK_1 \left[\frac{x_2^3}{3} - \frac{t^2 x_2}{4} \right]_{-t/2}^{t/2} \cdot b$$

$$M_1 = \frac{1}{3} G K_1 b \cdot t^3$$

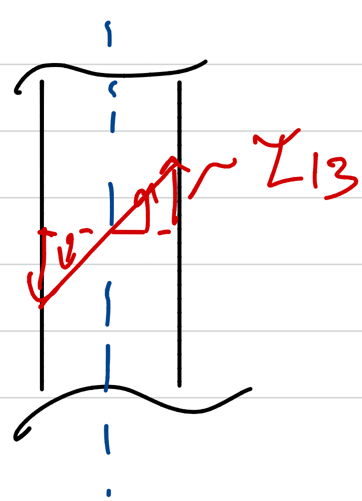
$$\boxed{H_{11} = \frac{1}{3} G b t^3}$$

$$\tau_{12} = \frac{\partial \Phi}{\partial x_3} = 0 \quad , \quad \tau_{13} = -\frac{\partial \Phi}{\partial x_2} = 2GK_1 x_2$$

$$\tau_{13} = 6 \frac{M_1}{b t^3} x_2$$

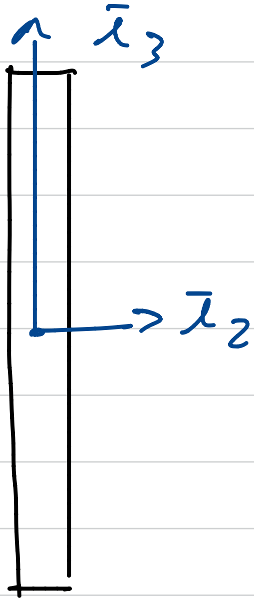
$$|\tau_{MAX}| = |\tau_{13}(x_2 = \pm t/2)|$$

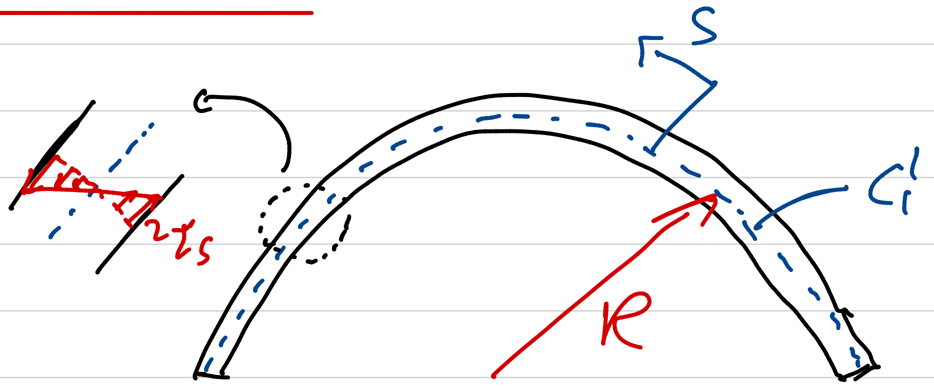
$$= GK_1 t = \frac{3 M_1}{b t^2}$$



Torsion at thin-walled

open Sections





Solution based on

$$\frac{\partial \Phi}{\partial x_3} \approx 0$$

More Generally

$$\frac{\partial \Phi}{\partial s} \approx 0$$

For a thin-walled open section

$$H_{11} = \frac{1}{3} G \, \ell \, t^3$$

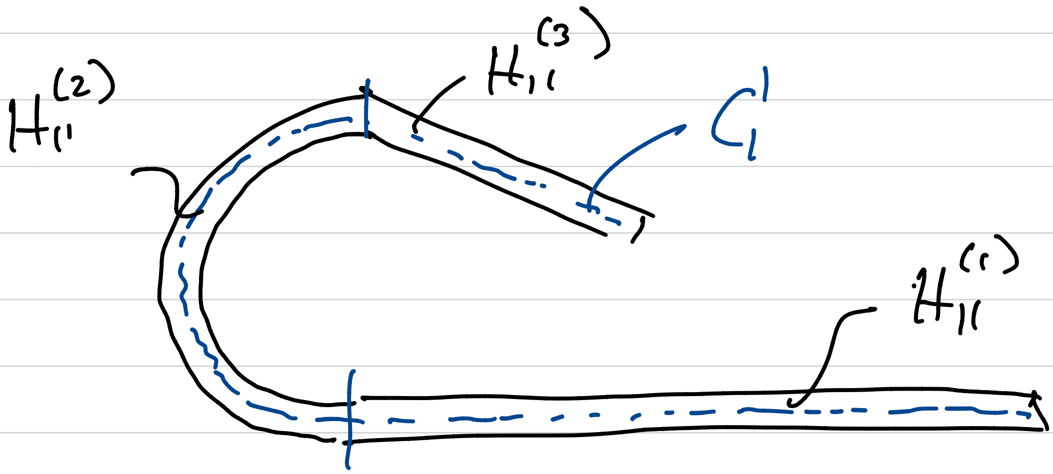
ℓ - length at the curve C_1'

Ex : Semi-circle

$$H_{11} = \frac{1}{3} G (\pi R) t^3$$

$$\tau_s^{MAX} = 2 G K_1 \frac{t}{2} = G K_1 t$$

$$\tau_s^{MAX} = \frac{3 \mathcal{M}_1}{\ell t^2}$$



$$H_{11} = \sum_i H_{11}^{(i)} = \sum_i \frac{1}{3} G_i \ell_i t_i^3$$

At each segment

$$\tau_s^{MAX, i} = G_i t_i K_1$$

$$= G_i t_i \frac{\mathcal{M}_1}{H_{11}}$$

Assume G = constant

$$\tau_s^{MAX} = G t_{MAX} \frac{\mathcal{M}_1}{H_{11}}$$

Summary

→ For a thin-walled open section, shear stresses are linearly distributed through the thickness

→ Torsional stiffness H_{11} is proportional to the cube of the wall thickness

→ Hence thin-walled open sections have very limited torque carrying capacity!