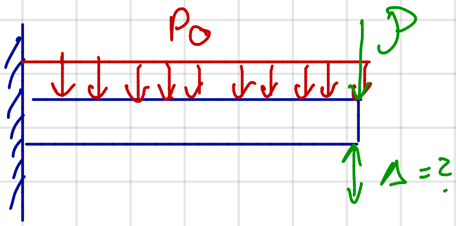
Dummy Load Method

* How can we get displacement at a point where no concentrated loads are applied

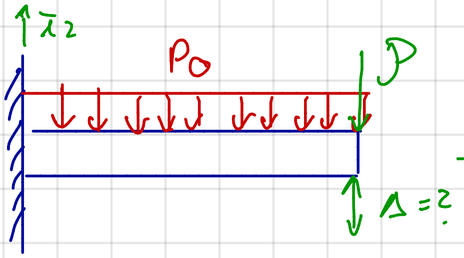


1) Apply a dummy load $\mathcal{P}$ and compute the corresponding displacement

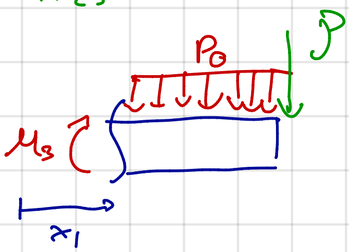
$$\hat{\Delta} = \frac{\partial A'}{\partial \mathcal{P}}$$

2) Remove the dummy load by taking the limit as it goes to zero and recover the real displacement

$$\Delta = \lim_{\mathcal{P} \to 0} \hat{\Delta} = \lim_{\mathcal{P} \to 0} \frac{\partial A'}{\partial \mathcal{P}}$$



$$H_{23}^c = 0$$



$$M_3 = -\frac{p_0 (L-x_1)^2}{2} - P(L-x_1)$$

$$A' = \int_0^L \frac{1}{2}\frac{M_3^2}{H_{33}^c}\,dx_1$$

$$\frac{\partial M_3}{\partial P} = -(L-x_1)$$

$$\hat{\Delta} = \frac{\partial A'}{\partial P} = \int_0^L \frac{M_3}{H_{33}^c}\cdot\frac{\partial M_3}{\partial P}\,dx_1$$

$$\hat{\Delta} = \int_0^L \frac{1}{H_{33}^c}\left(-\frac{p_0}{2}(L-x_1)^2 - P(L-x_1)\right)\cdot\left(-(L-x_1)\right)dx_1$$

$$\hat{\Delta} = \int_0^L \frac{1}{H_{33}^c}\left(\frac{p_0}{2}(L-x_1)^3 + P(L-x_1)^2\right)dx_1$$

$$\hat{\Delta} = \frac{1}{H_{33}^c}\left(\frac{p_0 L^4}{8} + \frac{P L^3}{3}\right)$$

$$\boxed{\Delta = \lim_{P\to 0}\hat{\Delta} = \frac{p_0 L^4}{8 H_{33}^c}}$$

Simplification: Take the limit before integration.

$$\Delta = \lim_{P\to 0}\hat{\Delta} = \lim_{P\to 0}\int_0^L \frac{1}{H_{33}^c}\left(\frac{p_0}{2}(L-x_1)^3 + \underbrace{\cancel{P(L-x_1)^2}}_{\to 0}\right)dx_1$$

$$= \int_0^L \frac{1}{H_{33}^c}\frac{p_0}{2}(L-x_1)^3\,dx_1 = \frac{p_0 L^4}{8 H_{33}^c} = \Delta$$