An Energy Based Approach to Constraint Analysis – Part 2 Mission Analysis

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Preliminary Estimates For Constraint Analysis

- Preliminary Estimates for the aerodynamic characteristics of the airframe and the installed engine thrust lapse are needed for constraint analysis

- Aerodynamics
  - Maximum coefficient of lift (C_{Lmax}) is important in constraint analysis during takeoff and landing
  - For Fighter aircraft, a clean wing has C_{Lmax} between 1.0 and 1.2, and a wing with a leading edge slat has a C_{Lmax} between 1.2 and 1.6
  - Table 1 has typical C_{Lmax} values for cargo and passenger aircraft

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Preliminary Estimates For Constraint Analysis

Table 1 C_{Lmax} for High Lift Device

High Lift Device		Typical Flap Angle (deg)		$C_{Lmax}/\cos(\Lambda_{c/4})$	
Trailing	Leading Edge	Takeoff	Landing	Takeoff	Landing
Plain	--	20	60	1.4-1.6	1.7-2.0
Single Slot	--	20	40	1.5-1.7	1.8-2.2
Fowler	--	15	40	2.0-2.2	2.5-2.9
Double slotted	--	20	50	1.7-2.0	2.3-2.7
Double slotted	slat	20	50	2.3-2.6	2.8-3.2
Triple slotted	slat	20	40	2.4-2.7	3.2-3.5

$\Lambda_{c/4}$ is the sweep angle at the quarter chord

Lift-Drag Polar Estimation

$$K_1 = K' + K''$$

$$C_{Do} = C_{D\min} + K'' C_{L\min}^2$$

$$K_2 = -2K'' C_{L_{MIN}}$$

Lift-Drag Polar Estimation

- The lift-drag polar for most large cargo and passenger aircraft can be approximated through the use of Figure 1 and Equation (9) with

$$0.001 \le K'' \le 0.03$$

$$0.1 \le C_{L\min} \le 0.3$$

$$K' = \frac{1}{\pi AR e}$$

where wing planform efficiency factor (e) is between 0.75 and 0.85 and AR (wing aspect ratio) is between 7 and 10.

Lift-Drag Polar Estimation

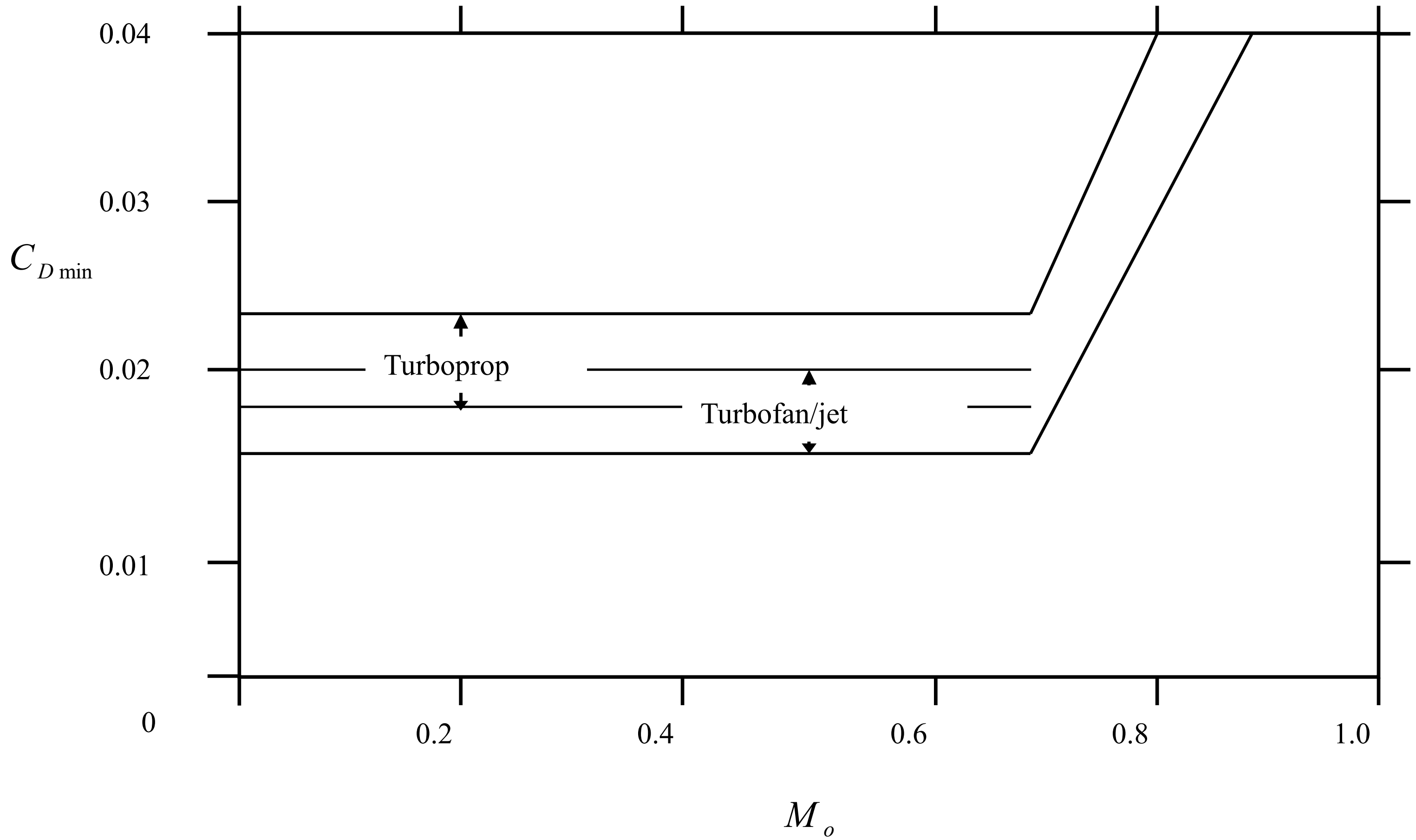


Figure 1: C_{Dmin} for Cargo and Passenger Aircraft

Lift-Drag Polar Estimation

The lift-drag polar for high-performance fighter type aircraft can be approximated through the use of Eq.(9), K_2=0, and Figures 1 and 2

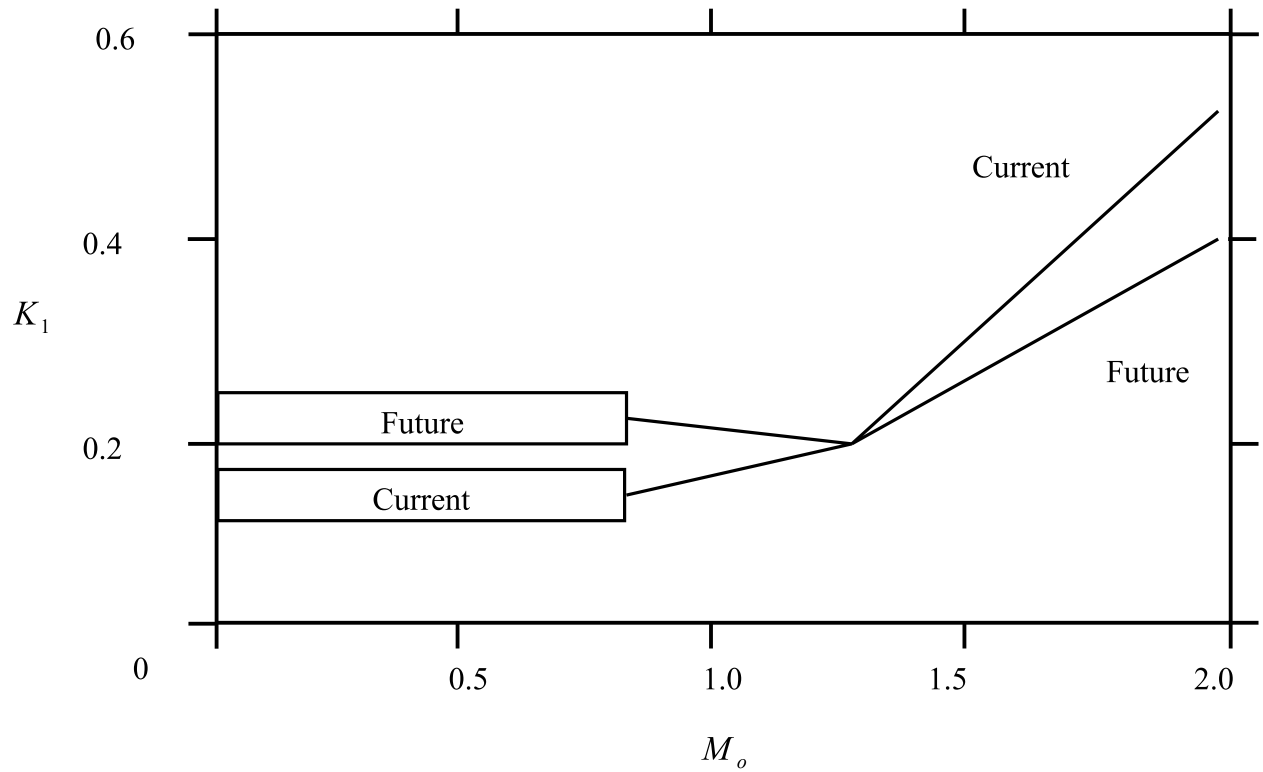


Figure 2: K_1 for Fighter Aircraft

Lift-Drag Polar Estimation

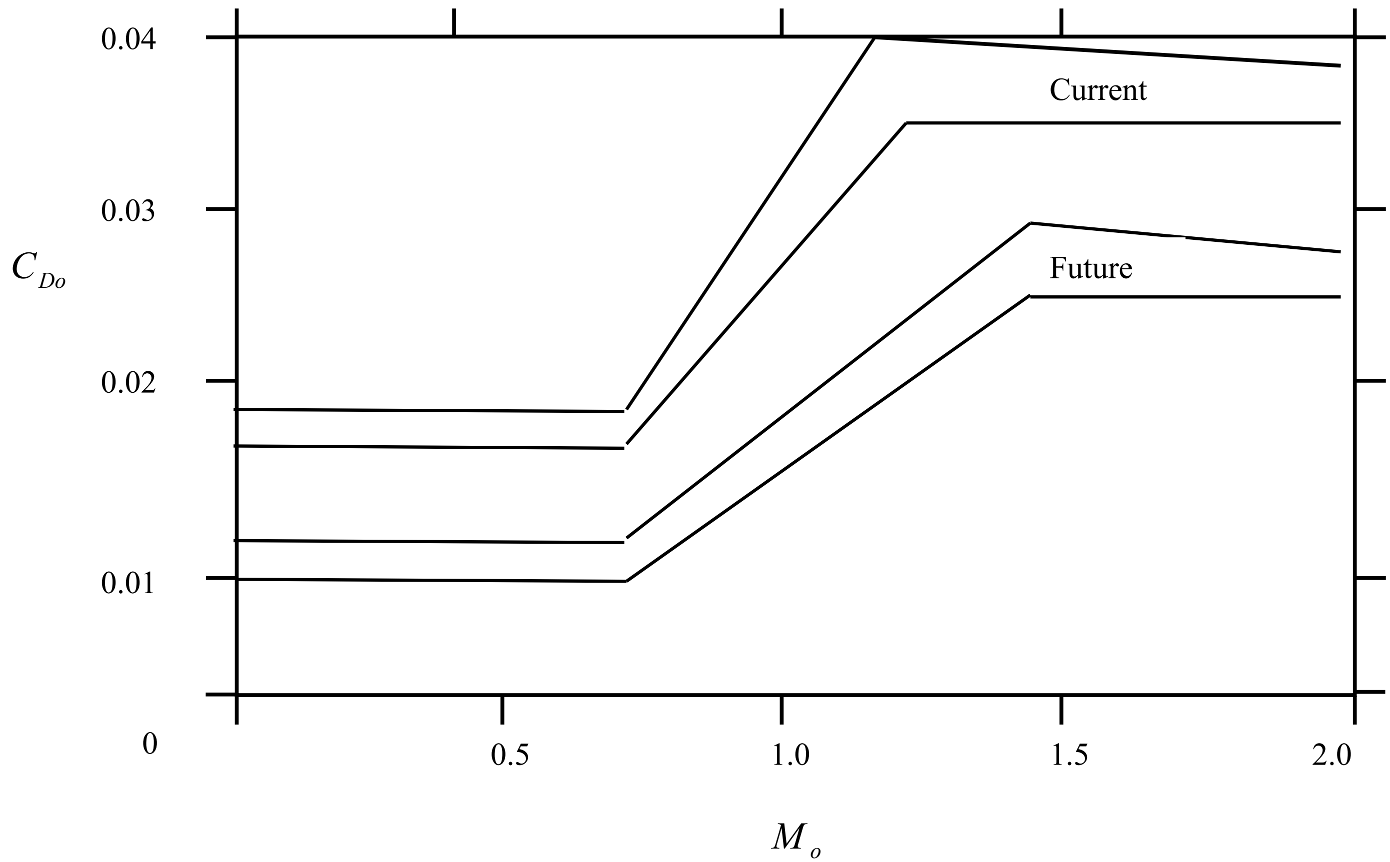


Figure 3: C_{Do} for Fighter Aircraft

Propulsion

- Thrust of installed engine varies with Mach number, altitude, and afterburner operation
- It can be described with a simple algebraic equation that has been fit to either existing data of company-published performance curves or predicted data based on the output of off-design cycle analysis with estimates made for installation losses
- The following algebraic equations for installed engine thrust lapse are based on expected performance of advanced engines in the 1990 era and beyond

Thrust Lapse

- For a High bypass ratio turbofan engine (M<0.9):

$$a = \left\{0.568 + 0.25\left(1.2 - M\right)^3\right\}\sigma^{0.6} \qquad (42)$$

- For a Low-bypass ratio mixed turbofan engine with afterburner:

$$a \cong a_{mil} = 0.72\left\{0.88 + 0.245\left(\left|M - 0.6\right|\right)^{1.4}\right\}\sigma^{0.7} \qquad (43a)$$

$$a_{wet} \cong a_{\max} = \left\{0.94 + 0.38\left(M - 0.4\right)^2\right\}\sigma^{0.7} \qquad (43b)$$

Thrust Lapse

- For an advanced turbojet with afterburning:

$$a_{dry} = a_{mil} = 0.76\left\{0.907 + 0.262\left(\left|M - 0.5\right|\right)^{1.5}\right\}\sigma^{0.7} \quad \text{(44a)}$$

$$a_{wet} = a_{\max} = \left\{0.952 + 0.3\left(M - 0.4\right)^{2}\right\}\sigma^{0.7} \quad \text{(44b)}$$

- For an advanced turboprop:

$$a = \sqrt{\sigma} \qquad \text{For } M \le 0.1 \quad \text{(45a)}$$

$$a = \frac{0.12}{M + 0.02}\sqrt{\sigma} \qquad \text{For } 0.1 < M < 0.8 \quad \text{(45b)}$$

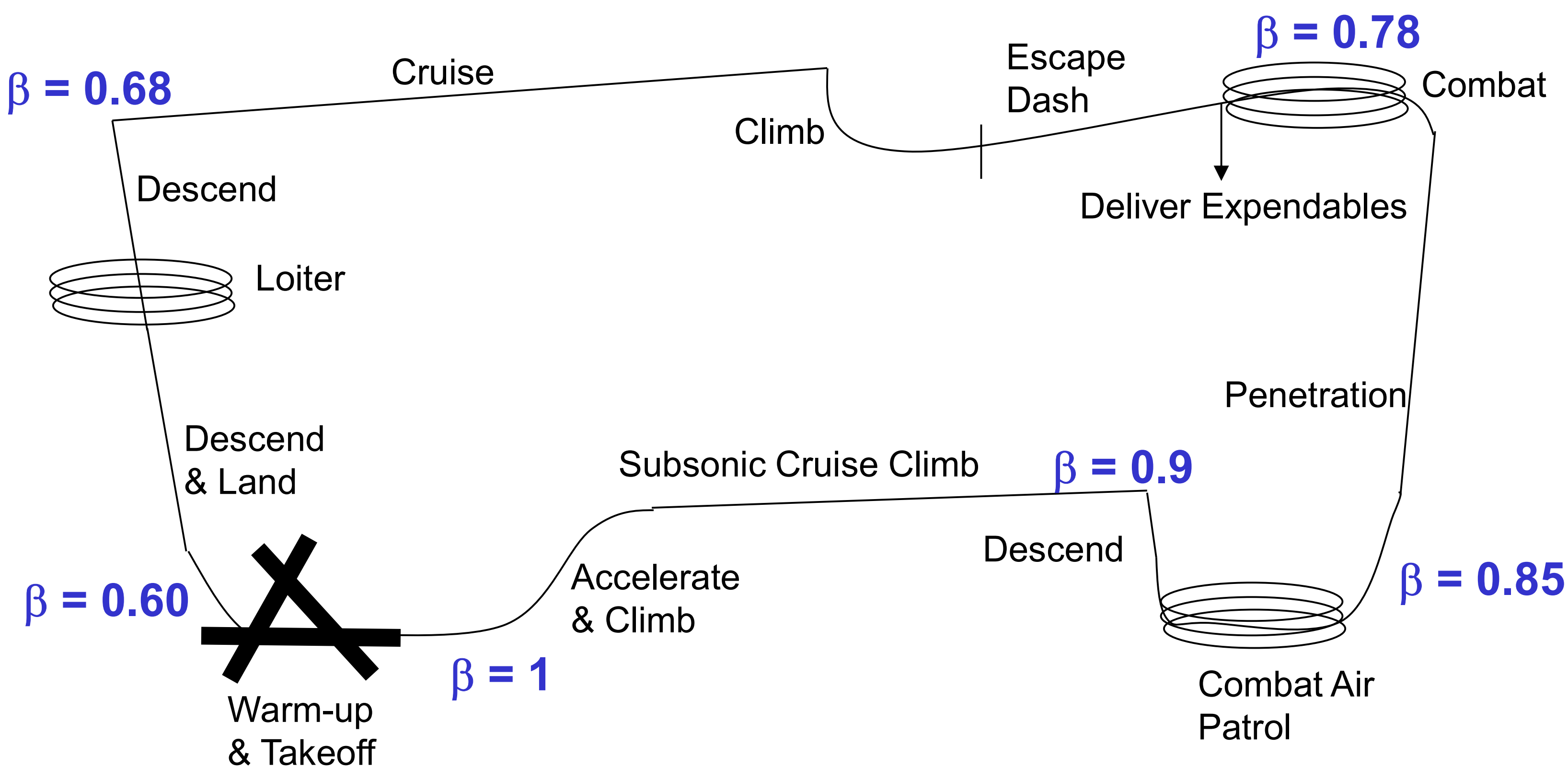
Weight Fraction

- Instantaneous weight fraction, β, is needed for computing the constraints
  - β is a fuel/payload correction
  - Initial value for β can be based on experience
  - Typical β values for different mission phases are shown on the mission profile figures on the following slides
    - Fig 4: β for Typical Fighter Aircraft
    - Fig 5: β for Typical Cargo and Passenger Aircraft

Weight Fraction (Fighter)

Instantaneous weight fraction, β, for a typical fighter aircraft

(Figure 4)



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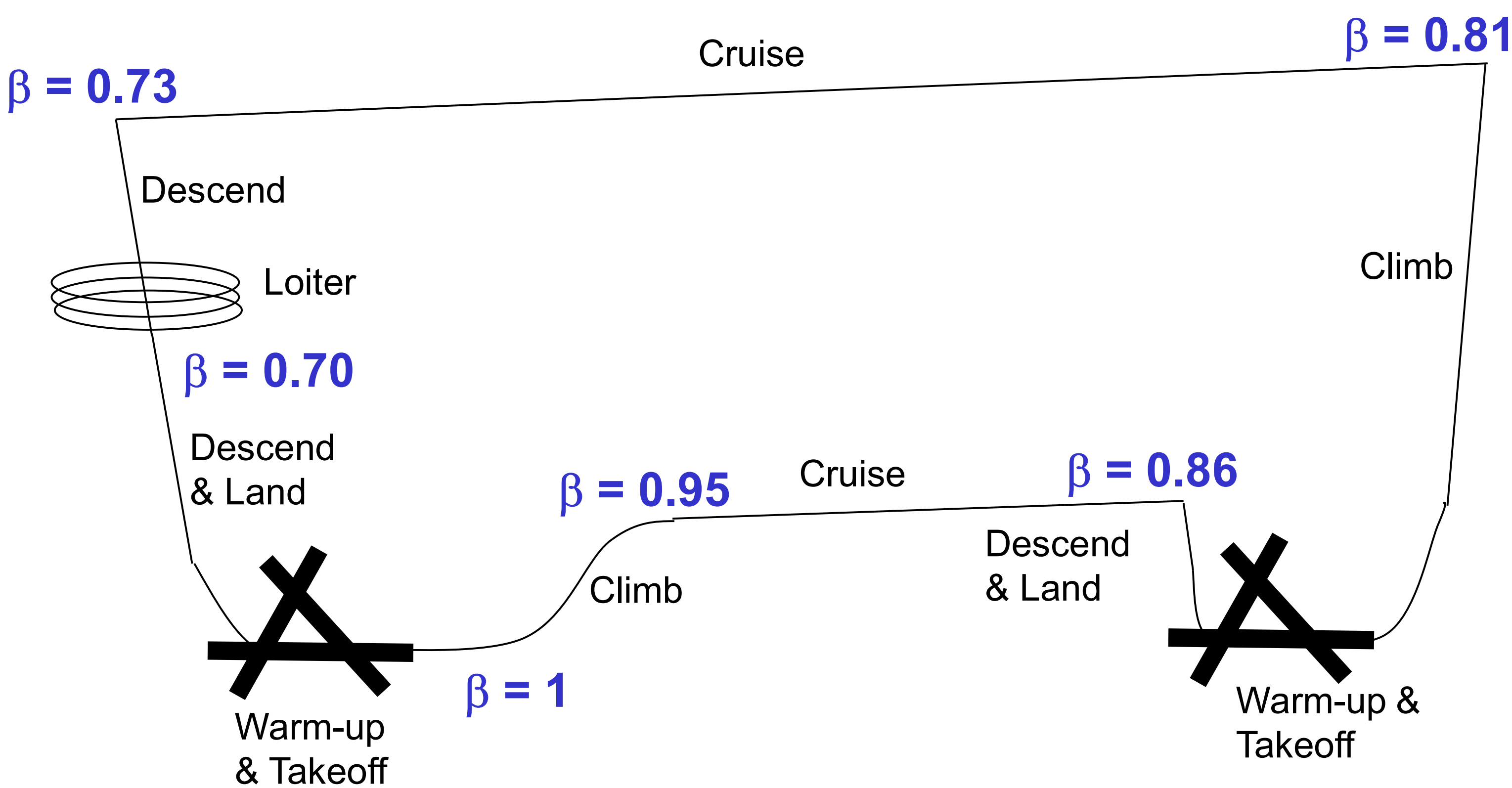
Weight Fraction (Passenger)

Instantaneous weight fraction, β, for a typical cargo and passenger aircraft

(Figure 5)



Example Constraint Analysis

- Creating constraint boundaries on a takeoff thrust loading (T_{SL}/W_{TO}) vs wing loading (W_{TO}/S) diagram
- An Air-to-Air Fighter (AAF) case study is provided here as a typical example
- Created by re-arranging terms so as to create an equation in the following form:

$$f\{(T_{SL}/W_{TO}),(W_{TO}/S)\}=0$$

Table 2 Selected AAF Specifications

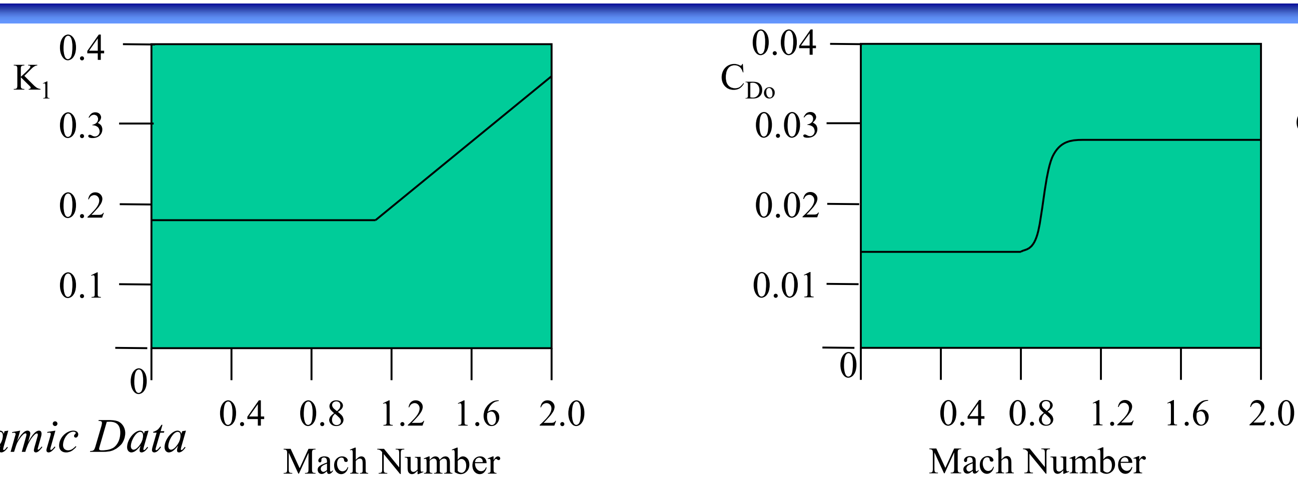
Mission Phases & Segments		Performance Requirements
1-2	Takeoff Acceleration Rotation	2000 ft.. PA, 100°F, $S_{TO} = S_G + S_R \leq 1500$ ft.. $k_{TO} = 1.2$, μ_{TO}=0.05, max power V_{TO}, $t_R = 3$ s, max power
6-7	Supersonic Penetration And Escape Dash	1.5 M/ 30 k ft.., no afterburning (if possible)
7-8	Combat Turn 1 Turn 2 Acceleration	30,000 ft.. 1.6M, one 360 deg 5g sustained turn, with afterburning 0.9M, two 360 deg 5g sustained turns, with afterburning 0.8 → 1.6M, $t \leq 50$ s, max power
13-14	Landing Free roll Braking	2000 ft.. PA, 100˚F, $s_L = s_{FR} + s_B \leq 1500$ ft.. $k_{TD} = 1.15$, $t_{FR} = 3$ s, $\mu_B = 0.18$ Drag chute diameter 15.6 ft.., deployment ≤ 2.5 s
Max Mach Number		2.0M/40k, max power

In order to proceed, you need:

- C_{Lmax}
- Lift drag polar
- Engine data, including lapse rates (see Fig. 6)

These are obtained from the figures and the equation to come…

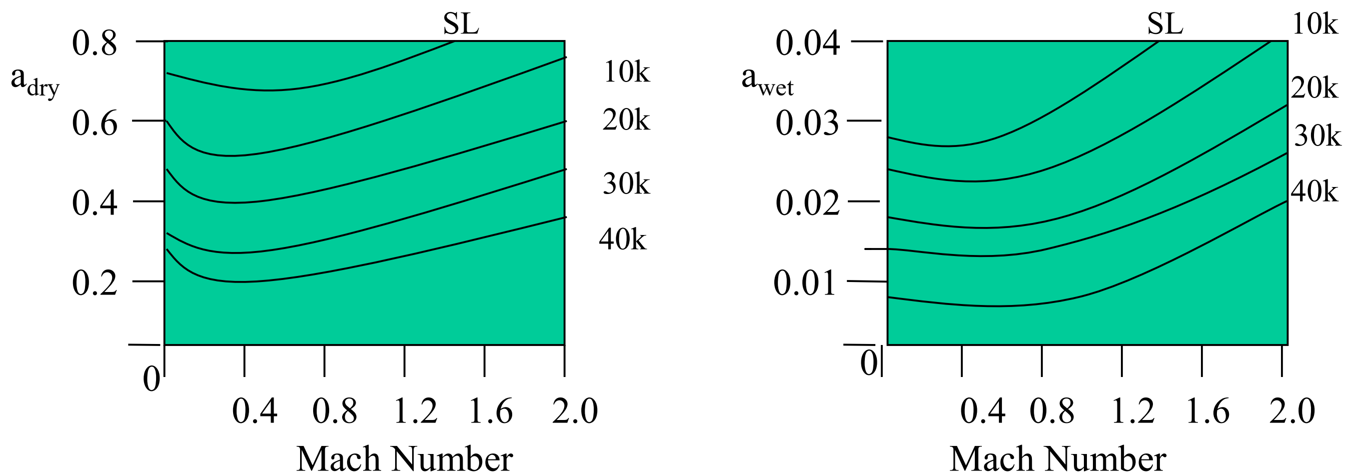
Figure 6: Preliminary AAF Data



(a) Aerodynamic Data

$C_D = K_1 C_L^2 + C_{Do}$

$C_{Lmax} = 2.0$



(b) Installed Thrust Lapse

$$\frac{T_{dry}}{T_{SL}} = a_{dry} = 0.72\{0.88 + 0.245(|M - 0.6|)^{1.4}\} * \sigma^{0.7}$$

NOTE: T_{SL} = Std.sea level max power static thrust

$$\frac{T_{wet}}{T_{SL}} = a_{wet} = \{0.94 + 0.38(M - 0.4)^2\} * \sigma^{0.7}$$

Procedure with Takeoff

- The airplane accelerated by thrust with no resisting forces in the ground roll
- Thrust is balanced by drag forces during the constant velocity rotation
- Given these conditions, takeoff equation becomes:

$$s_{TO} = \left\{ \frac{{k_{TO}}^2 \beta^2}{\rho * g_o * C_{L\max} * a_{wet} \left(\frac{T_{SL}}{w_{TO}} \right)} \right\} \left(\frac{w_{TO}}{S} \right) + \{ t_R * k_{TO} \left(\frac{2\beta}{\rho C_{L\max}} \right)^{\frac{1}{2}} \sqrt{\frac{w_{TO}}{S}} \qquad (46)$$

Where: $a(W_{TO} / S) + b\sqrt{W_{TO} / S} - c = 0$

From Table 2, Fig. 2 and Standard Atmosphere Tables

k_{TO}	=	1.2
β	=	1.0
ρ	=	0.002047 slugs/ft^3
g_o	=	32.17 ft/s^2
C_{Lmax}	=	2.0
σ	=	0.8613
$(a_{wet})_{M=1.0}$	=	0.8775
t_R	=	3.0 s
s_{TO}	=	1500 ft

$$s_{TO} = \left\{ \frac{k_{TO}{}^2 \beta^2}{\rho * g_o * C_{L\max} * a_{wet} \left(\frac{T_{SL}}{w_{TO}} \right)} \right\} \left(\frac{w_{TO}}{S} \right) + \{ t_R * k_{TO} \left(\frac{2\beta}{\rho C_{L\max}} \right)^{\frac{1}{2}} \sqrt{\frac{w_{TO}}{S}} \qquad (46)$$

Evaluate Coefficients a, b, and c from Equation 46

Rewritten Equation 46:

$$\left(\frac{w_{TO}}{S}\right) = \left\{\frac{-b + \sqrt{b^2 + 4ac}}{2a}\right\}^2 \qquad (47)$$

$$a = \frac{12.47}{t_{SL} / w_{TO}}$$

b = 79.57

c = 1500

To obtain the data:

T_{SL}/W_{TO}	0.4	0.8	1.2	1.6	2.0	2.4
W_{TO}/S (lb./ft^2)	33.4	57.5	77.1	93.7	108	121

Maximum Mach Number Cruise Constraint for Fighter Aircraft

Assumptions:	
$dh/dt = 0$	Constant Altitude
$dV/dt = 0$	Constant Speed
$n=1$	Lift equals Weight
$T=D$	Thrust equals Drag
$R=0$	Not on the ground
h & V	Values are Given
K_2	Small

Pertinent Data:
b= 0.78
$(a_{wet})_{M=2} = 0.7189$
$q = 1101$ lb./ft^2
$K_1 = 0.36$
$C_{Do} = 0.028$

Begin with Equation 2.12

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{D_o}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\} \qquad (2.12)$$

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Maximum Mach Number Cruise Constraint for Fighter Aircraft

Substitution of pertinent data into Equation 2.12 yields the following

$$\frac{T_{SL}}{W_{TO}} = 2.767 \times 10^{-4}\left(\frac{W_{TO}}{S}\right) + \frac{42.88}{\left(\frac{W_{TO}}{S}\right)} \qquad (48)$$

The maximum Mach number cruise constraint combined with the takeoff constraint enclose a solution space as illustrated in Figure 7.

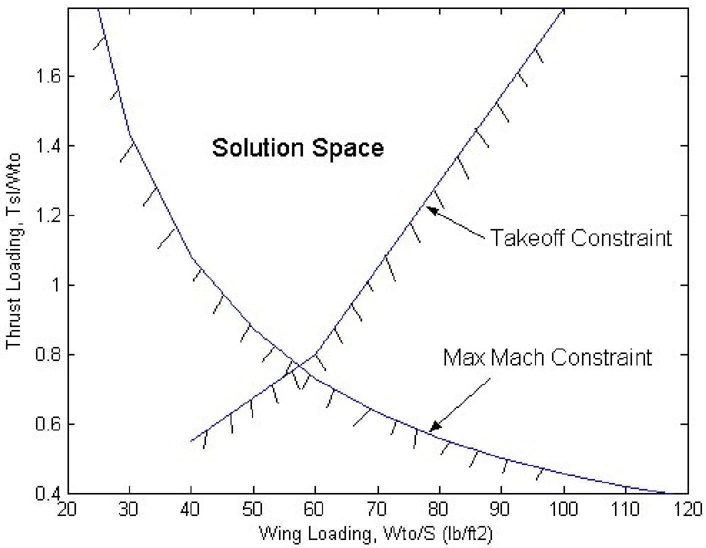


Figure 7 Constraints for Takeoff and Max Mach Number

Selection of the Air-to-Air Fighter Design Point

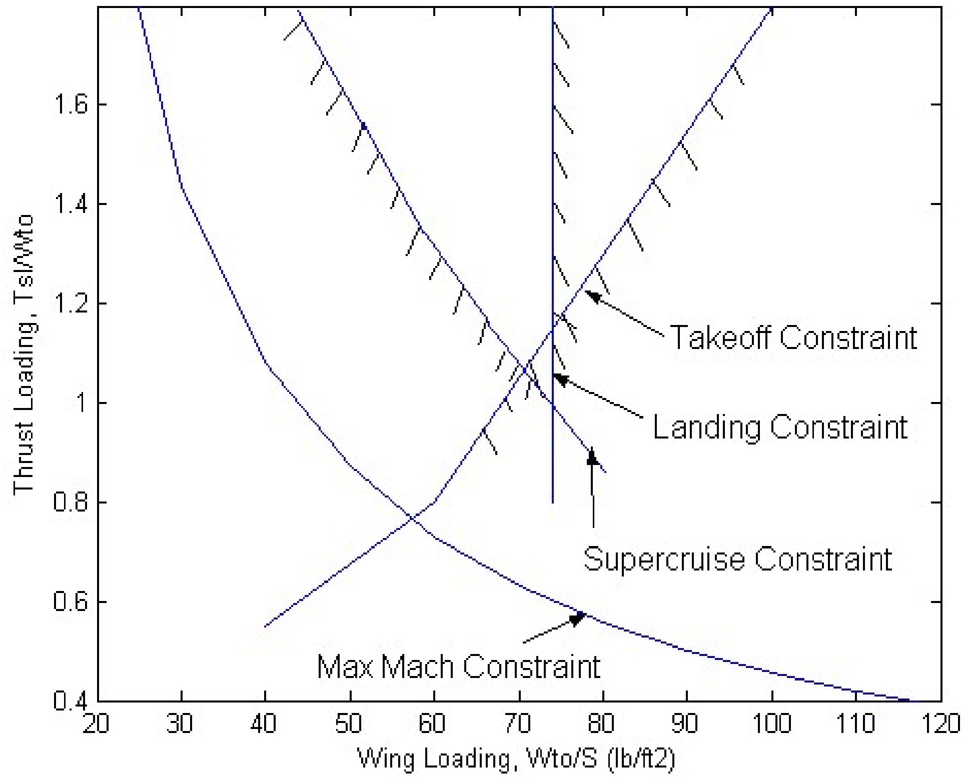
The requirements defined for a new air-to-air fighter result in the constraint diagram shown in the figure.

Based on the problem assumptions, the constraints that dominate the design are the takeoff, landing, and supercruise constraints.

To keep cost and practicality of the AAF design under control, thrust loading is chosen near the region of previous experience.

Reasonable first choice:

$$\frac{T_{SL}}{W_{TO}} = 1.2 \qquad \frac{W_{TO}}{S} = 64 lb / ft^2$$



AAF Design Constraints

Selection of the Air-to-Air Fighter Design Point

- Capability requirements for short takeoff and landing and nonafterburning supercruise drive design
- Relaxing these constraints would not significantly change the design choice because other constraints are important just outside of the takeoff, landing, and supercruise constraints.
- For many flight segments, the aircraft will be operating at full thrust; thus, no one requirement will cause the size to become excessively large. This is a result of careful consideration and balancing of requirements.

Selection of the Air-to-Air Fighter Design Point

If thrust loading and wing loading are selected, it is possible to determine weight specific excess power (P_s) for a given flight condition by combining Equations 47 and 2.11.

Taking R = 0, this combination yields the following relationship:

$$P_s = V\left\{\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right) - K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) - K_2 - \frac{C_{D_0}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\} \qquad (49)$$

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Selection of the Air-to-Air Fighter Design Point

This relationship is used to generate the contours of constant P_s in Figure 50 for specified loadings, b, and flight conditions.

This figure can be used to determine the minimum time-to-climb flight path; this flight path corresponds to the maximum P_s attainable for a given line of constant energy height, z_e.

This time-to-climb relationship is a modified version of Equation 2.2b:

$$\Delta t = \int_1^2 dt = \int_{z_{e1}}^{z_{e2}} \frac{dz_e}{P_s}$$

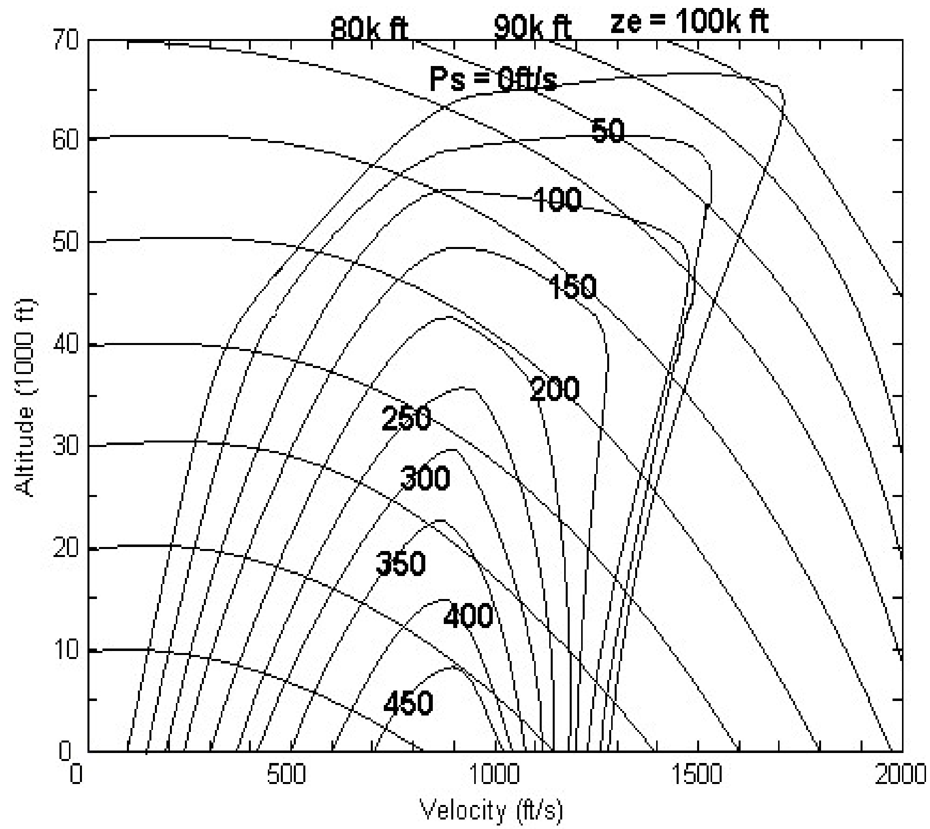

Figure 50 : AAF P_s Contours

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Mission Phase 1-2: Takeoff

Assumptions:
2000 ft. PA
100ºF
k_{TO}=1.2
µTO=0.05
tR=3 s
$s_{TO}=s_G+s_R \leq 1500$ ft.
Max Power

From (2.25) and (2.26), with $s_{TO}=s_G+s_R$:

$$\left(\frac{W_{TO}}{S}\right)=\left\{\frac{-b+\sqrt{b^2+4ac}}{2a}\right\}^2$$

where:

$$a=-\frac{\beta}{\rho g_o \xi_{TO}}\ln\left\{1-\frac{\xi_{TO}}{\left[\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)-\mu_{TO}\right]\frac{C_{L_{MAX}}}{k_{TO}^2}}\right\}$$

$$b=t_R k_{TO}\sqrt{\frac{2\beta}{\rho C_{L_{MAX}}}}$$

$$c=s_{TO}$$

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Mission Phase 1-2: Takeoff

We estimate α and β to be constant at appropriate mean values. A conservative estimate for ξ_{TO} for the AAF is obtained by assuming $(C_{DR} - \mu_{TO}C_L) = 0$ and evaluating C_D at $C_L = C_{Lmax}/k_{TO}^2$.

With:

• $\beta=1.0$	• $K_1=0.18$
• $\rho=0.002047$ slug/ft^3	• $\sigma=0.8613$
• $C_{Lmax}=2.0$	• $(\alpha_{wet})_{M=0.1}=0.8775$
• $k_{TO}=1.2$	• $\mu_{TO}=0.05$
• $C_{Do}=0.014$	• $t_R=3.0$s
• $\xi_{TO}=0.36$	• $s_{TO}=1500$ ft..

Then:

$$a = -42.03 \ln\left\{1 - \frac{0.2601}{\left[0.8775\left(\frac{T_{SL}}{W_{TO}}\right) - 0.05\right]}\right\}$$

$$b = 79.57$$

$$c = 1500$$

whence:

T_{SL}/W_{TO}	0.4	0.8	1.2	1.6	2.0
W_{TO}/S (lb./ft^2)	14.3	45.1	67.2	85.3	101

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Mission Phases 6-7 and 8-9: Supersonic Penetration and Escape Dash

Assumptions:
1.5M/30k ft.
No afterburning

From (2.12):

$$\left(\frac{T_{SL}}{W_{TO}}\right)=\frac{\beta}{\alpha}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)+\frac{C_{Do}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

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Mission Phase 6-7: Supersonic Penetration and Escape Dash

Assumptions (most from Case 1):	
dh/dt = 0	Constant Altitude
dV/dt =0	Constant Speed
n=1	Lift equals Weight
R=0	Not on the ground
h & v	Values are Given
K_2=0	Pure Parabolic Drag Polar

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + \cancel{K_2\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)} + C_{D_o} + \cancel{\frac{R}{qS}}\right] + \cancel{\frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)}\right\}$$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{D_o}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

Supersonic Penetration and Escape Dash Example

With:

$\beta=0.78$ $\alpha_{dry}=0.3953$ $q=991.8$ psf

$\alpha=0.3747$ $K_1=0.28$ $C_{Do}=0.028$

$$\frac{T_{SL}}{W_{TO}} = 4.345*10^{-4}\left(\frac{W_{TO}}{S}\right) + \frac{70.25}{\left(\frac{W_{TO}}{S}\right)}$$

W_{TO}/S	20	40	60	80	100	120
T_{SL}/W_{TO}	2.35	1.77	1.2	0.913	0.746	0.638

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Mission Phase 7-8: Combat Turn 1

Assumptions:	
dh/dt=0	Constant Altitude
dV/dt =0	Constant Speed
R=0	Not on the ground
h, n, & V	Values are Given
K_2=0	Pure Parabolic Drag Polar

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{\frac{qS}{W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + \cancel{K_2\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)} + C_{D_o} + \cancel{\frac{R}{qS}}\right] + \cancel{\frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)}\right\}$$

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1 n^2 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{D_o}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

Combat Turn 1 Example

With:

β=0.78 α_{wet}=0.7481 q=1128 psf n=5

α=0.3747 K_1=0.30 C_{Do}=0.028

$$\frac{T_{SL}}{W_{TO}} = 5.407 * 10^{-3}\left(\frac{W_{TO}}{S}\right) + \frac{42.22}{\left(\frac{W_{TO}}{S}\right)}$$

W_{TO}/S	20	40	60	80	100	120
T_{SL}/W_{TO}	2.22	1.27	1.03	0.96	0.963	1

Mission Phase 7-8: Combat Turn 2

--0.9 M/30K ft., two 360 degree 5g sustained turns, with afterburning. From Eq. (2.15)

$$\left(\frac{T_{SL}}{W_{TO}}\right)=\frac{\beta}{\alpha}\left\{K_1 n^2\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)+\frac{C_{Do}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$$

with $\beta = 0.78$ $\alpha_{wet} = 0.5206$ $n = 5$ $C_{Do} = 0.018$

$\sigma = 0.3747$ $K_1 = 0.18$ $q = 357.0$ lb./ft^2

then

$$\left(\frac{T_{SL}}{W_{TO}}\right)=0.01473\left(\frac{W_{TO}}{S}\right)+\frac{12.34}{\left(\frac{W_{TO}}{S}\right)}$$

whence

W_{TO}/S (lb./ft^2)	20	40	60	80	100	120
T_{SL}/W_{TO}	0.910	0.900	1.09	1.33	1.60	1.87

Mission Phase 7-8: Horizontal Acceleration

--0.8 → 1.6 M/30K ft., $\Delta t \leq 50s$, max power.

From Eq. (2.18)

$$\left(\frac{T_{SL}}{W_{TO}}\right) = \frac{\beta}{\alpha}\left\{K_1 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + \frac{C_{Do}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)} + \frac{a\Delta M}{g_o \Delta t}\right\}$$

We obtain approximate constant values of ρ, K_1, C_{Do}, and α at a mean Mach number of 1.2.

with $\beta = 0.78$ | $K_1 = 0.23$ | $a = 994.8$ ft./s

$\sigma = 0.3747$ | $q = 634.7$ lb./ft^2 | $\Delta M = 0.8$

$\alpha_{\text{wet } M=1.2} = 0.5952$ | $C_{Do} = 0.025$ | $\Delta t = 50$ s

then

$$\left(\frac{T_{SL}}{W_{TO}}\right) = 3.704 * 10^{-4} \cdot \left(\frac{W_{TO}}{S}\right) + \frac{28.79}{\left(\frac{W_{TO}}{S}\right)} + 0.6484$$

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Mission Phase 7-8: Horizontal Acceleration

whence,

$W_{TO}/S(lb/ft^2)$	20	40	60	80	100	120
T_{SL}/W_{TO}	2.10	1.38	1.15	1.04	0.973	0.933

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Mission Phase 13-14: Landing

Assumptions:
2000 ft PA, 100F, k_{TD}=1.15,
tFR=3s, μB=0.18,
sL=sFR+sB≤1500 ft
GFE drag chute, diameter 15.5 ft,
deployment≤2.5s

From Eqs. (2.33) and (2.37) with $s_L = s_{FR} + s_B$

$$\left(\frac{W_{TO}}{S}\right) = \left\{\frac{-b + \sqrt{b^2 + 4ac}}{2a}\right\}^2$$

Mission Phase 13-14: Landing (cont.)

Where,

$$a = \frac{\beta}{\rho g_0 \xi_L} \ln \left\{ 1 + \frac{\xi_L}{\left[\mu_B + \frac{(-a)}{\beta} \left(\frac{T_{ST}}{W_{TO}} \right) \right] \frac{C_{L_{\max}}}{k_{TD}^2}} \right\}$$

$$b = t_{FR} k_{TD} \sqrt{2\beta / \rho C_{L_{\max}}} \qquad c = s_L$$

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Mission Phase 13-14: Landing(cont.)

Assumptions

Drag chute $C_D = 1.4$

RFP chute area $= 191\ \text{ft}^2$

Estimate airplane wing area $= 500\text{ft}^2$

$C_{DRc} = 0.5348$

A conservative estimate of ξ_L from C_D at 0.8 of touchdown lift coefficient $(C_{L_{\max}}/k_{TD}^{\,2})$ and by assuming $(C_{DR} - C_{DRc} - \mu_B C_L) = 0$

$\beta = 0.56,\ k_{TD} = 1.15,\ \mathrm{C_D} = 0.2775,\ \mathrm{a} = 0$

$\rho = 0.002047\ \text{slug/ft}^3,\ \mathrm{C_L} = 1.210,\ \mathrm{C_{DRc}} = 0.5348,\ t_{FR} = 3s,$

$C_{L_{\max}} = 2.0,\ \mathrm{C_{D_0}} = 0.014,\ \xi_{\mathrm{L}} = 0.8123,\ \mathrm{s_L} = 1500\ ft,$

$\mathrm{K_1} = 0.18,\ \mu_{\mathrm{B}} = 0.18$

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Mission Phase 13-14: Landing(cont.)

then

$$a = 14.47 \quad \mathrm{b} = 57.06 \quad \mathrm{c} = 1500$$

whence

$$W_{TO} / S = 70.5 lb / ft^2$$

Maximum Mach Number

- Maximum Mach Number
  - M = 2 at 40,000 ft., max power
  - From (Equation 2.12), we have:

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{ K_1 \frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 + \frac{C_{D_o}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)} \right\}$$

With

$\beta = 0.78$	$a_{wet} = 0.7189$	$q = 1101$ lb./ft^2
$\sigma = 0.2471$	$K_1 = 0.36$	$C_{Do} = 0.028$

Maximum Mach Number

This gives

$$\frac{T_{SL}}{W_{TO}} = 2.767 * 10^{-4}\left(\frac{W_{TO}}{S}\right) + \frac{42.88}{\left(\frac{W_{TO}}{S}\right)}$$

The table below shows a few calculations

W_{TO}/S (lb/ft^2)	20	40	60	80	100	120
T_{SL}/W_{TO}	2.14	1.07	0.713	0.535	0.428	0.357

Mission Analysis

- With the values of thrust loading (T_{SL}/W_{TO}) and wing loading (W_{TO}/S) determined, now the focus shifts to the estimation of the gross takeoff weight, which is the sum of the component weights:

$$W_{TO} = W_P + W_E + W_F \tag{3.1}$$

- W_p is specified in the RFP and is separated into payload weight (W_{PE}) that consists of the cargo being delivered during a mission (i.e. ammunition) and the permanent payload weight (W_{PP}), which consists of the crew, passengers, their equipment, and anything else that is carried the entire mission

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Mission Analysis

- W_E consists of the basic aircraft structure and any permanently attached equipment
- W_F represents the required fuel weight to fly the mission
- W_F depends on the rate of fuel consumption, which depends on the thrust specific fuel consumption (TSFC)
- Thrust can be found from (Equation 2.1) while the TSFC depends on the engine cycle, flight conditions (altitude, Mach #, etc.)

Mission Analysis

- Fuel consumption analysis results in multiple benefits:
  - Little information needed for calculation
  - Reveals optimum solution (minimum fuel) for flying certain segments of the mission
  - Shows the fuel consumed during each mission segment as a fraction of the aircraft weight at the beginning of the mission segment, which makes W_F a calculable fraction of W_{TO}

Mission Analysis

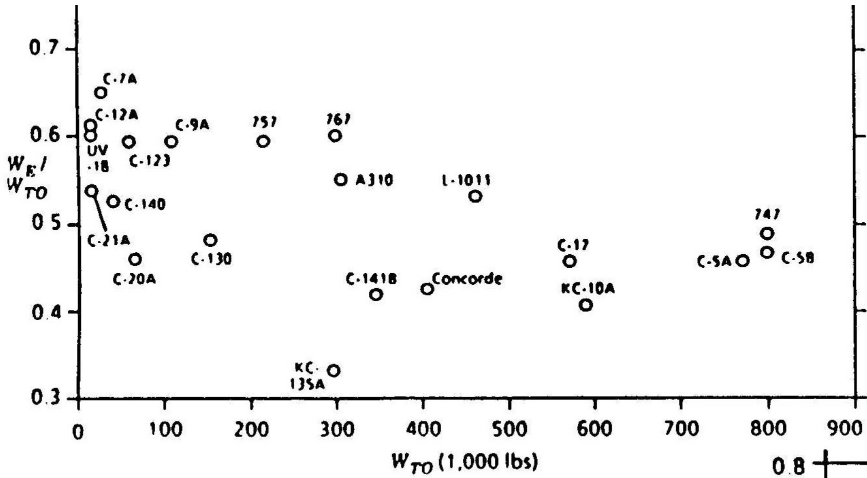

Figure 3.1 Weight Fractions of Cargo and Passenger Aircrafts

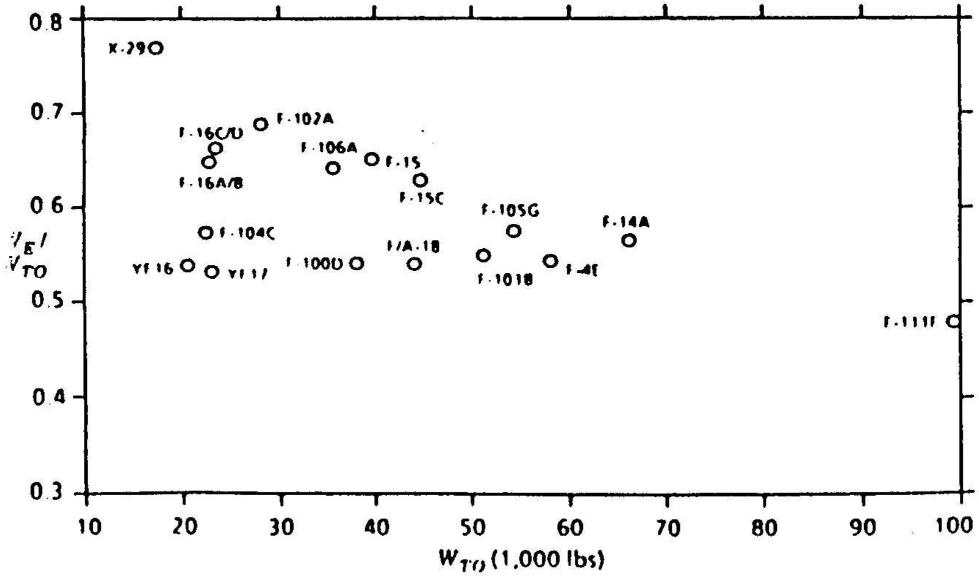

Figure 3.2 Weight Fractions of Fighter Aircraft

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Mission Analysis

- This section deals with the thermodynamics of the flight and the way the thrust work of the engine is used by the aircraft
- Begin by considered the change in weight of aircraft from fuel burn

$$\frac{dW}{dt} = \frac{-dW_F}{dt} = -TSFC * T \qquad (3.2)$$

Mission Analysis

- Equation 3.2 can be rewritten as:

$$\frac{dW}{W} = -TSFC\left(\frac{T}{W}\right)dt \qquad (3.3)$$

- Note that T is the installed thrust and TSFC is the installed thrust specific fuel consumption
- Also, since:

$$\frac{T}{W}dt = \frac{T}{W}\frac{dt}{ds}ds = \frac{Tds}{WV}$$

Mission Analysis

- The above equation shows the incremental weight-velocity specific engine thrust work done as an amount of fuel is consumed (dW_F)
- This work is partly used in the mechanical energy (PE + KE) of the airplane mass and partly dissipated into the atmosphere (wing tip vortices, heating, etc…)

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Weight Fractions

- The goal of the subsequent slides is to integrate Eq.(3.3) to get "weight fractions"

$$\frac{W_{final}}{W_{initial}} \doteq \frac{W_f}{W_i}$$

- Requires knowledge of TSFC behavior and "instantaneous thrust loading" as a function of time along flight path. $\left\{ \frac{T}{W} = \left(\frac{\alpha}{\beta}\right)\left(\frac{T_{SL}}{W_{TO}}\right) \right\}$
- Two distinct classes of integration
  - $P_s > 0$ (Type A)
  - $P_s = 0$ (Type B)

Instantaneous Thrust Loading Behavior: Type A

- $P_s > 0$; examples of Type A found in the following cases:
  1. *Constant speed climb*
  2. *Horizontal acceleration*
  3. *Climb and acceleration*
  4. *Takeoff acceleration*
- Using Eq.(2.2a) to move towards the desired solution:

$$\frac{T}{W}V dt = \frac{T}{W} ds = \frac{d\left(h + V^2/2g_0\right)}{1-u} = \frac{dz_e}{1-u} \quad where \quad u = \frac{D+R}{T} \qquad (3.4), (3.5)$$

- Combining Eqs.(3.3) and (3.4)

$$\frac{dW}{W} = -\frac{TSFC}{V(1-u)} d\left(h + \frac{V^2}{2g_0}\right) \quad or \quad \frac{dW}{W} = -\frac{TSFC}{V(1-u)} dz_e \qquad (3.6a), (3.6b)$$

- u = how total engine thrust work is distributed between mechanical energy and dissipation
- Equation 3.5 shows u is fraction of thrust work dissipated

Instantaneous Thrust Loading Behavior: Type A (continued)

- Integration of Equation (3.6a) which largely depends on variation of $\{TSFC/V(1\text{-}u)\}$ with altitude and velocity. When that is relatively constant:

$$\frac{W_f}{W_i} = \exp\left\{-\frac{TSFC}{V(1-u)}\Delta\left(h+\frac{V^2}{2g_0}\right)\right\} \qquad (3.7a)$$

$$\text{or} \quad \frac{W_f}{W_i} = \exp\left\{-\frac{TSFC}{V(1-u)}\Delta z_e\right\} \qquad (3.7b)$$

- Δz_e = total change in energy height
- Equations (3.6) and (3.7) show how potential and kinetic energy can be “traded” within certain limits
- Example of when bad: No drag ($u = 0$) and constant z_e
  - Unpowered dive or zoom climb
  - No fuel consumed and $dW = 0$ or $W_f = W_i$

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(from Type A, cont.) Note: Minimum Fuel Path

- Fuel consumption analysis also shows the supposed "best" way to fly certain legs for minimum fuel used.
- For Type A legs, Equations (3.2) (2.2b) and (2.3) produce

$$dW_F = T \times TSFC \times dt = T_{SL} \frac{aTSFC}{P_s} dz_e$$

Type A (Cont.)

$$P_s = \frac{T-(D+R)}{W}V = \frac{T}{W}(1-u)V$$

Defining the fuel consumed specific work (f_s) as:

$$f_s = \frac{P_s}{a\,TSFC} \tag{3.8}$$

Then

$$W_{F_{1-2}} = \int_1^2 dW_F = T_{SL}\int_{z_{e1}}^{z_{e2}} \frac{dz_e}{f_s} \tag{3.9}$$

Eq. 3.9 shows that the minimum fuel-to-climb flight from $z_{e1}(h_1,V_1)$ to $z_{e2}(h_2,V_2)$ corresponds to a flight path that produces the max. thrust work per unit weight of fuel at each energy height level in the climb (i.e. max value of f_s at each z_e)

Type A (Cont.).

Graphical method for finding max. f_s at any z_e (thus min. fuel-to-climb path from z_{e1} to z_{e2})

- Constant f_s and z_e contours in the altitude-velocity plane (Fig. 3.E5)
- Analogous to using the P_s and z_e contours (Fig. 50) to find min time-to-climb path (Fig. 3.E2) in conjunction with the time-to-climb equation from Eq. 2.2b.

$$\Delta t = \int_1^2 dt = \int_{z_{e_1}}^{z_{e_2}} \frac{dz_e}{P_s}$$

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Instantaneous Thrust Loading Behavior (Type B)

When $P_s=0$, it is generally true that the speed and altitude are essentially constant and specific information is given regarding the total amount of time (Δt or $\Delta s/V$) which elapses. Examples of Type B cases are:

- Constant speed cruise
- Constant speed turn
- Best cruise Mach number and altitude
- Loiter
- Warm-up
- Take off rotation
- Constant energy height maneuver

For this type of flight, the thrust work is completely dissipated and the required thrust may not be known in advance. The thrust is modulated or throttled so that $T=(D+R)$ or $u=1$ by Equation 3.5 and $dz_e=0$ by Equation 3.4. Using Equation 2.2 it follows that:

$$\frac{dW}{W} = -TSFC\left(\frac{D+R}{W}\right)dt \qquad (3.10)$$

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Instantaneous Thrust Loading Behavior (Type B)

Often TSFC (D+R)/W remains relatively constant over the flight leg. So, Equation 3.10 can be integrated:

$$\frac{W_f}{W_i} = \exp\left(-TSFC\left(\frac{D+R}{W}\right)\Delta t\right) \tag{3.11}$$

$$\Delta t = total\ flight\ time$$

If desired, the required thrust may be computed after the fact.

TSFC Behavior (Thrust Specific Fuel Consumption)

- Complex function:
  - Altitude
  - Speed
  - Throttle setting (especially if has afterburner)

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Instantaneous Thrust Loading Behavior (Type B)

Good starting point at this stage is the assumption:

$$TSFC = C\sqrt{\theta} \tag{3.12}$$

C = constant

θ = represents the usual thermodynamic cycle improvement due to a lower ambient temperature at higher altitude

Basis of this assumption is the observation that:

TSFC is not a strong function of speed & throttle setting

C could be selected differently for different legs of the mission

NOTE: Assumptions for TSFC other than Equation 3.12 can be made and the weight fraction analysis continued to completion but this model is a good for turbojet and turbofan. Different assumptions would be required for turboprop engines which TSFC is primarily proportional to flight Mach # (Equation 1.25). The same process could be applied.

Summary of Weight Fraction Equations

- The equations previously mentioned for dW/W and W_f/W_i (Equations 3.6, 3.7, 3.10, 3.11) are combined with the equation for TSFC (Equation 3.12) in order to get the equations for the Type A and Type B situations.

Summary of Weight Fraction Equations (cont.)

- TYPE A ($P_s > 0$, $T = \alpha T_{SL}$ given)

$$\frac{dW}{W} = -\frac{C\sqrt{\theta}}{V(1-u)} d\left(h + \frac{V^2}{2g_o}\right)$$

Instantaneously exact (3.13)

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V(1-u)} \Delta\left(h + \frac{V^2}{2g_o}\right)\right\}$$

Exact when sqrt(θ)/ V(1-u) constant (3.14)

Summary of Weight Fraction Equations (cont.)

- TYPE B (P_s=0, T=D+R given)

$$\frac{dW}{W} = -C\sqrt{\theta}\left(\frac{D+R}{W}\right)dt$$

Instantaneously exact (3.15)

$$\frac{W_f}{W_i} = \exp\left\{-C\sqrt{\theta}\left(\frac{D+R}{W}\right)\Delta t\right\}$$

Exact when sqrt(θ)(D+R)/ W constant (3.16)

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Special Cases

- The following slides will discuss various special cases to give both specific design tools and more insight into weight fraction analysis.

- Assumptions
  - α, β, K_1, K_2, C_{Do} are known
  - $n=1$ and $L=W$
    - Except for Cases 4,8 ($n>1$), and 10
  - $R=0$
    - Except where the aircraft is on the ground (Cases 4 and 10)

CASE 1: Constant Speed Climb (P_s=dh/dt)

- Given:
  - dV/dt = 0
  - n=1(L=W)
  - R=0
  - $T = \alpha T_{SL}$ (Full Thrust)
  - $h_{initial}$
  - h_{final}
  - V

- Conditions

$$u = \frac{C_D}{C_L}\frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right)$$

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Case 1: Constant Speed Climb

Equation (3.14) becomes Equation (3.17)

$$\frac{W_f}{W_i} = \exp\left\{ -\frac{C\sqrt{\theta}}{V} \left[\frac{\Delta h}{1 - (C_D / C_L)(\beta / \alpha)(W_{TO} / T_{SL})} \right] \right\}$$

where the level flight relationships (3.18)

$$\frac{D}{W} = \frac{D}{L} = \frac{qC_D S}{qC_L S} = \frac{C_D}{C_L}$$

have been used, and the change in altitude is

$$\Delta h = h_{final} - h_{initial}$$

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Case 1: Constant Speed Climb

Notes on using Equation 3.17

- Typically, variables values are averaged for the altitude interval used in the analysis although the variance in the quantity

$$\frac{\sqrt{\theta}}{1-u}$$

  should be checked to verify the accuracy of this averaging approach
  - If the accuracy is poor, the climb should be broken into intervals and Eq. (3.17) should be applied to each interval
  - The overall W_f/W_i the then the product of the interval results

- Note that the fraction of engine thrust dissipated is $\left(C_D/C_L\right)\left(\beta/\alpha\right)\left(W_{TO}/T_{SL}\right)$ while $\left\{1-\left(C_D/C_L\right)\left(\beta/\alpha\right)\left(W_{TO}/T_{SL}\right)\right\}$ is the fraction of the engine thrust invested in potential energy

Case 2: Horizontal Acceleration

Horizontal Acceleration $$P_s = \frac{VdV}{g_o dt}$$

Given: $dh/dt = 0$, $n = 1$ ($L = W$), $R = 0$, full thrust ($T = a_{TSL}$) and the values of h, $V_{initial}$, and V_{final}.

Under these conditions $$u = \frac{C_D}{C_L}\frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right)$$

so that Equation (3.14) becomes Equation (3.19)

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V}\left[\frac{\Delta\left(V^2/2g_0\right)}{1-\left(C_D/C_L\right)\left(\beta/\alpha\right)\left(W_{TO}/T_{SL}\right)}\right]\right\}$$

where Equation (3.18) has been used and $\Delta V^2 = V_{final}^2 - V_{initial}^2$

Similar to the previous analysis, average values for the speed intervals should be used unless the quantity $V(1\text{-}u)$ varies too much.

Case 3: Climb and Acceleration ($P_S = dh/dt + VdV/g_o dt$)

Given: n=1 (L=W), R=0, full thrust ($T=aT_{SL}$) and values of $h_{initial}$, h_{final}, $V_{initial}$, and V_{final}

This solution combines Case 1 and 2.

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{V}\left[\frac{\Delta\left(h+V^2/2g_o\right)}{1-\left(C_D/C_L\right)\left(\beta/a\right)\left(W_{TO}/T_{SL}\right)}\right]\right\}$$

In this equation, h and V are independent of each other until the flight trajectory is chosen.

$\left(C_D / C_L\right)\left(\beta / a\right)\left(W_{TO} / T_{SL}\right)$ Represents the fraction of the engine thrust work that is dissipated

$1-\left(C_D / C_L\right)\left(\beta / a\right)\left(W_{TO} / T_{SL}\right)$ Fraction that is invested in mechanical energy or energy height

Case 4: Takeoff Acceleration ($P_S=VDV/g_odt$)

Given: dh/dt=0, n=1, full thrust ($T=aT_{SL}$) and values of ρ, D, C_{Lmax}, k_{TO}, and R

$$\frac{W_f}{W_i} = \exp\left\{-\frac{C\sqrt{\theta}}{g_o}\left[\frac{V_{TO}}{1-u}\right]\right\}$$

Where u can be obtained from Eq. (2.21) as

$$u = \left[\xi_{TO}\left(\frac{q}{\beta}\right)\left(\frac{S}{W_{TO}}\right) + \mu_{TO}\right]\frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right) \qquad (22)$$

and V_{TO} is as given by Eq. (2.20).

Case 5: Constant Altitude/Speed Cruise (P_s=0)

Given: dh/dt=0, dV/dt=0, n=1 (L=W), R=0, and values of h, V and cruise range Δs

Under these conditions, Equations (3.16) and (3.18) become

$$\frac{W_f}{W_i} = \exp\left[-\frac{C\sqrt{\theta}}{V}\left(\frac{C_D}{C_L}\right)\Delta s \right] \qquad (3.23)$$

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Case 5: Constant Altitude/Speed Cruise (P_s=0)

Where C_L is obtained by setting lift equal to weight and C_D is computed from Eq. (2.9). Since C_D/C_L will vary as fuel is consumed and the weight of the aircraft decreases, this result is not, strictly speaking, exact. Hence, it might become advisable to break the cruise into several intervals and apply Eq. (3.23) to each.

The Type B cases 6-10 that follow are, however, exact integrals.

Case 6: Constant Altitude/Speed Turn (P_s=0)

Given: dh/dt=0, dV/dt=0, R=0, and values of h, V, n>1, and number of turns N

This situation can be treated much the same as Case 5, except that L=nW. The duration of the turning can be shown, with the help of Eq. (2.17), to equal:

$$\Delta t = \frac{2nR_c N}{V} = \frac{2nNV}{g_0\sqrt{n^2-1}} \qquad (24)$$

Under these conditions, Eqs. (3,16) and (3.24) become

$$\frac{W_f}{W_i} = \exp\left[-C\sqrt{\theta}\left(\frac{nC_D}{C_L}\right)\frac{2nNV}{g_0\sqrt{n^2-1}}\right] \qquad (25)$$

Case 7: Best Cruise Mach Number and Altitude (BCM/BCA) (P_s=0)

Assumptions:	
dh/dt = 0	Constant Altitude
dV/dt = 0	Constant Speed
n=1	L=W
R=0	Not on the Ground
And value of cruise range is Δs	

$$C_L = \frac{n\beta}{q}\left(\frac{W_{TO}}{S}\right)$$

Replacing dt with ds/V, Eq. (3.15) becomes:

$$\frac{dW}{W} = -\left(\frac{C_d}{C_L}\frac{1}{M}\right)\frac{Cds}{a_{SL}} \qquad (3.26)$$

Case 7: BCM/BCA

From Equation (3.26) weight reduction can be minimized by operating at lowest value of $(1/M)(C_D/C_L)$.

This is known as the best cruise condition and the parameters associated with it contain a * superscript. The below equation shows that drag and lift do not depend on M below critical drag rise Mach number:

$$\frac{C_D}{C_L} = \frac{K_1 C_L^{\,2} + K_2 C_L + C_{Do}}{C_L}$$

From the above equation, a minimum C_D/C_L can be found by differentiating the equation with respect to C_L and setting the result equal to zero. This leads to:

$$\left(\frac{C_D}{C_L}\right)^{*} = \sqrt{4 C_{Do} K_1} + K_2 \qquad (3.27)$$

Mission Analysis

This may be substituted into Equation (3.27) to yield

$$C_D{*} = 2C_{D0} + K_2\sqrt{\frac{C_{D0}}{K_1}} \qquad (3.29)$$

$$\left(\frac{C_D}{C_L}\frac{1}{M}\right)^{*} = \frac{\sqrt{4C_{D0}K_1} + K_2}{M_{CRIT}} \qquad (3.30)$$

Mission Analysis

which may be substituted into Eq. (3.26) to yield

$$\frac{dW}{W} = -\left\{ \frac{\sqrt{4C_{D0}K_1} + K_2}{M_{CRIT}} \frac{C}{a_{SL}} \right\} ds \qquad (31)$$

This may be directly integrated to yield the equivalent of Eq. (3.16),

$$\frac{W_f}{W_i} = \exp\left\{ -\frac{\sqrt{4C_{D0}K_1} + K2}{M_{CRIT}} \frac{C}{a_{SL}} \Delta s \right\} \qquad (32)$$

Finally, since the aircraft must sustain its weight under this condition,

$$qC_L S = \frac{\gamma P M^2}{2} C_L S = W$$

or

$$\delta = \frac{P}{P_{SL}} = \frac{2\beta}{\gamma P_{SL} M^2{}_{CRIT}} \frac{1}{\sqrt{C_{D0}/K_1}} \left(\frac{W_{TO}}{S} \right) \qquad (33)$$

Mission Analysis

- Which may be used to find the altitude of “best” cruise or BCA
  - Since β must gradually diminish as the mission progresses, this last result shows that the altitude must gradually increase
- Since the flight Mach number is fixed at M_{CRIT}, it also follows that the speed, which is proportional to $\sqrt{\theta}$, must gradually decrease
- In most cases, these changes occur so slowly that the assumption of $P_S = 0$ remains true

Case 8: Loiter (P_s=0)

Assumptions:	
dh/dt=0	Constant Altitude
dV/dt =0	Constant Speed
R=0	Not on the ground
n=1	(L=W)
Δt	Flight Duration

Putting these assumptions into Equation 3.15, we get:

$$\frac{dW}{W} = -C\sqrt{\theta}\left(\frac{C_D}{C_L}\right)dt \qquad (34)$$

From inspection in case 7, we know that the optimum C_D/C_L for minimum fuel consumption is equal to: $C_D/C_L = \sqrt{4C_{Do}K_1} + K_2$

Thus,

$$\frac{dW}{W} = -C\sqrt{\theta}\left(\sqrt{4C_{Do}K_1} + K_2\right)dt \qquad (35)$$

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Case 8: Loiter (P_s=0)

Integrating Equation 35 for weight from W_i to W_f and for the time interval Δt, we get:

$$\frac{W_f}{W_i} = \exp\left\{-C\sqrt{\theta}\left(\sqrt{4C_{Do}K_1} + K_2\right)\Delta t\right\} \qquad (36)$$

Variation in pressure altitude (δ) can be found as a function of Mach number for loiter by equating Lift and Weight.

$$\delta = \frac{2\beta}{\gamma P_{SL} M^2} \frac{1}{\sqrt{C_{Do}/K_1}} \left(\frac{W_{TO}}{S}\right) \qquad (37)$$

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Case 9: Warm-Up (P_s=0)

Assumptions:	
dh/dt=0	Constant Altitude
dV/dt =0	Constant Speed
R=T=αTSL	
V=0	Standing Still
D=0	Not Moving
Given:	
h	Altitude
Δt	Warm-up time

Equation (15) under these assumptions becomes:

$$dW = -C\sqrt{\theta}\left(\alpha T_{SL}\right)dt$$

Mission Analysis

$$\frac{W_f}{W_i} = 1 - C\sqrt{\theta}\,\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)\Delta t \qquad (39)$$

Weight Fraction

Where β is evaluated at the beginning of warm-up

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CASE 10: Takeoff Rotation ($P_S = 0$)

Given: $dh/dt = 0$, $dV/dt = 0$, $(D+R) = T = \alpha T_{SL}$ and values of h, V, and rotation time t_R

Under these conditions, Eq. 3.15 becomes

$$dW = -C\sqrt{\theta}\left(\alpha T_{SL}\right)dt \tag{40}$$

$$\frac{W_f}{W_i} = 1 - C\sqrt{\theta}\,\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)t_R \tag{41}$$

where β is evaluated at the beginning of rotation

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CASE 11: Constant Energy Height Maneuver ($P_S = 0$)

Given: $\Delta z_e = 0$, $n = 1$ ($L=W$), $R = 0$ and the values of $h_{initial}$, h_{final}, $V_{initial}$, and V_{final}

In this case potential energy is exchanged for kinetic energy and any loss to aerodynamic drag is balanced by the engine thrust.

Under these conditions, Equations 16 and 18 become

$$\frac{W_f}{W_i} = \exp\left\{-C\sqrt{\theta}\left(\frac{C_D}{C_L}\right)\Delta t\right\} \tag{42}$$

where C_L is obtained by setting lift equal to weight and C_D is computed from Eq. 9.

Calculation of Takeoff Weight

- Weight fraction for each mission segment is the ratio of final weight to the initial weight for that segment

$$\frac{W_f}{W_i} = \prod_{i \to f} \leq 1 \qquad (43)$$

- For the weight ratio of mission legs where <u>only</u> fuel is consumed

Calculation of Takeoff Weight (cont.).

- For example, for several consecutive mission segments:

$$\frac{W_5}{W_2} = \frac{W_3}{W_2}\frac{W_4}{W_3}\frac{W_5}{W_4} = \prod_{2\to 3}\prod_{3\to 4}\prod_{4\to 5} = \prod_{2\to 5}$$

Expendable Payload Case

- For the special case where the expendable payload is delivered at some point ***j*** in the mission, we write instead:

$$\frac{W_j - W_{PE}}{W_j} = 1 - \frac{W_{PE}}{W_j} \tag{44}$$

Takeoff Weight

- Recall that takeoff weight is defined as

$$W_{TO} = W_F + W_E + W_{PE} + W_{PP} \qquad (3.45a)$$

Fuel Weight

- For a mission with *n* junction and payload delivery at junction *j* (*j*<*n*), W_F is expressed in terms of W_{TO} by the sum of the aircraft weight decrements between takeoff and junction *j* and between junction *j* (after delivery of expendable payload) and landing in the following manner:

$$W_F = \left\{W_{TO} - W_{TO} \prod_{1\to j}\right\} + \left\{\left(W_{TO} \prod_{1\to j} - W_{PE}\right) - \left(W_{TO} \prod_{1\to j} - W_{PE}\right) \prod_{j\to n}\right\}$$

which reduces to

$$W_F = W_{TO}\left(1 - \prod_{1\to n}\right) - W_{PE}\left(1 - \prod_{j\to n}\right) \qquad (3.45b)$$

Historical Correlations

- Figures 3.1 and 3.2 relate empty weight to takeoff weight via

$$\Gamma \doteq \frac{W_E}{W_{TO}} = f(W_{TO}) \qquad (45c)$$

- Combining Equations (45a) – (45c) and solving for $f(W_{TO})$ yields

where W_{TO} is given by Equations (47)-(50).

$$W_{TO} = \frac{W_{PP} + W_{PE} \prod_{j \to n}}{\prod_{1 \to n} - \Gamma} \qquad (46)$$

Historical Correlations

Figures 3.1 and 3.2 relate empty weight to takeoff weight via

$$\Gamma \doteq \frac{W_E}{W_{TO}} = f(W_{TO}) \tag{45c}$$

Combining Equations (45a) – (45c) and solving for $f(W_{TO})$ yields W_{TO} where is given by Equations (47)-(50).

$$W_{TO} = \frac{W_{PP} + W_{PE} \prod\limits_{j \to n}}{\prod\limits_{1 \to n} - \Gamma} \tag{46}$$

Summary

- Takeoff weight is proportional to W_{PP} and W_{PE} (where W_{PE} matters more because it is carried the whole way)
- Improved structural materials, which reduce W_E (and therefore Γ), can reduce W_{TO} dramatically
- An initial estimate of W_{TO} must be made in order to place values upon the W_{PE}/W_{TO} of the payload deliveries and Γ of the structural weight
- If the initial estimate of takeoff weight is far from the value given by the Equation (46), an iterative solution of Equation (46) is will be necessary

Empty Weight Fraction (Γ)

Based on the historical correlations of various authors, the empty weight fraction for a particular "class" of aircraft versus take-off weight can be obtained for initial sizing estimates.

Cargo aircraft: $$\Gamma = 1.26 W_{TO}^{-0.08} \quad (47)$$

Passenger aircraft: $$\Gamma = 1.02 W_{TO}^{-0.06} \quad (48)$$

Fighter aircraft: $$\Gamma = 2.34 W_{TO}^{-0.13} \quad (49)$$

Twin turboprop aircraft: $$\Gamma = 0.96 W_{TO}^{-0.05} \quad (50)$$

Installed Thrust Specific Fuel Consumption (TSFC)

The TSFC of an aircraft engine usually varies with Mach number, altitude, type of engine and throttle condition. Actual measurements of the airframe/engine system performed during flight tests can be obtained from the manufacturer's published data. For initial estimates though, the following relations provide a value for C depending on the altitude, for a given Mach number and engine type.

High by-pass turbofan ($M < 0.9$): $C = 1.0\ h^{-1}$

Low by-pass turbofan: $C = 1.35\ h^{-1}$ ($M<1$) mil power
$C = 1.45\ h^{-1}$ (M1) mil power
$C = 2\ h^{-1}$ max power

Turbojet engine: $C = 1.45\ h^{-1}$ ($M<1$) mil power
$C = 1.65\ h^{-1}$ (M1) mil power
$C = 2\ h^{-1}$ max power

Turboprop engine: $C = 0.6\ h^{-1}$

Aircraft Sizing & Mission Analysis

A Simplified Approach

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Initial Weight Estimation

Requirements

- Often put forth in a document called an RFP (Request for Proposal)
- Outlines basic design requirements and goals
- Basic mission or mission profile
- Examples
  - Payload and type of payload
  - Range/loiter requirements
  - Cruise speed and altitude
  - Field length for takeoff/landing
  - Fuel reserves
  - Climb requirements
  - Maneuvering requirements
  - Certification base (experimental, FAR 23, FAR 25, military)

FAR 23- normal, utility, aerobatic, commuter

FAR 25- transports

- Can be specific or vague

Initial Weight Estimation

$$W_{TO} = W_E + W_f + W_C + W_{PL}$$

W_{TO}	Takeoff Gross Weight
W_{OE}	Empty Weight
W_f	Mission Fuel Weight
W_C	Crew Weight
W_{PL}	Payload Weight

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Estimation Strategy

Use weight break-down for crew, payload fuel and empty components. Express Fuel and Empty weights as weight fractions

$$W_0 = W_c + W_p + \frac{W_f}{W_0} W_0 + \frac{W_e}{W_0} W_0$$

Solve for the take-off gross weight in terms of the crew and payload weights (constants), the fuel weight ratio (function of mission, aerodynamics and fuel consumption) and empty weight fraction (function of take-off gross weight from empirical data)

$$W_0 = \frac{W_c + W_p}{1 - {W_f}/{W_0} - {W_e}/{W_0}}$$

Crew and Payload Weight

For initial weight estimation, we want to estimate W_{TO}, W_E and W_f, and it is assumed that the crew and payload weight as well as the mission are known.

Determine mission crew payload weight

- Passengers and baggage
- Cargo
- Military loads (bombs, ammunition, etc.)

Rules of Thumb for Passenger Aircraft:

175 lbs. per person

30 lbs. luggage per person, short to medium flights

40 lbs. luggage per person, long flights

Mission Fuel Weight Fraction

To find W_f, use fuel fraction method.

Break down mission into a number of mission phases and calculate fuel used in each phase based on simple calculations or experience.

Fuel Fraction: for each phase is defined as the ratio of end weight to begin weight

Phase 1: Engine Start and Warm Up

W_1/W_{TO}

Use Existing Data for Comparable Aircraft

Phase 2: Taxi

W_2/W_1

Use Existing Data for Comparable Aircraft

Phase 3: Takeoff

W_3/W_2

Use Existing Data for Comparable Aircraft

Mission Fuel Weight Fraction

Phase 4: Climb to Cruise and Accelerate to Cruise Speed

W_4/W_3

Use Existing Data for Comparable Aircraft and

Use Breguet's Endurance Equation

(you will need to assume appropriate values for L/D_{climb}, specific fuel consumption, time to climb [or rate of climb])

$$E = \frac{550\eta_{pr}}{cV}\frac{L}{D}\ln\frac{W_0}{W_1} \quad \text{(prop)}$$

$$E = \frac{1}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1} \quad \text{(jet)}$$

Be careful using these equations. You must use consistent units!

Phase 5: Cruise

W_5/W_4

Use Breguet's Range Equation

$$R = \frac{550\eta_{pr}}{c}\frac{L}{D}\ln\frac{W_0}{W_1} \quad \text{(prop)}$$

$$R = \frac{V}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1} \quad \text{(jet)}$$

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Mission Fuel Weight Fraction

Phase 6: Loiter

W_6/W_5

Breguet's Endurance Equation

Phase 7: Descent

W_7/W_6

Use Existing Data for Comparable Aircraft

Phase 8: Landing, Taxi, and Shutdown

W_7/W_6

Use Existing Data for Comparable Aircraft

Mission Fuel Weight Fraction

Now calculate the mission fuel fraction:

Final to Initial Weight Ratio: $$\frac{W_1}{W_0}\cdot\frac{W_2}{W_1}\cdot\frac{W_3}{W_2}\cdots\frac{W_n}{W_{n-1}}=\frac{W_n}{W_0}$$

Mission Fuel Weight Fraction: $$\frac{W_f}{W_0}=1-\frac{W_n}{W_0}$$

Trapped fuel considerations (e.g. 6%) : $$\frac{W_f}{W_0}=1.06\left(1-\frac{W_n}{W_0}\right)$$

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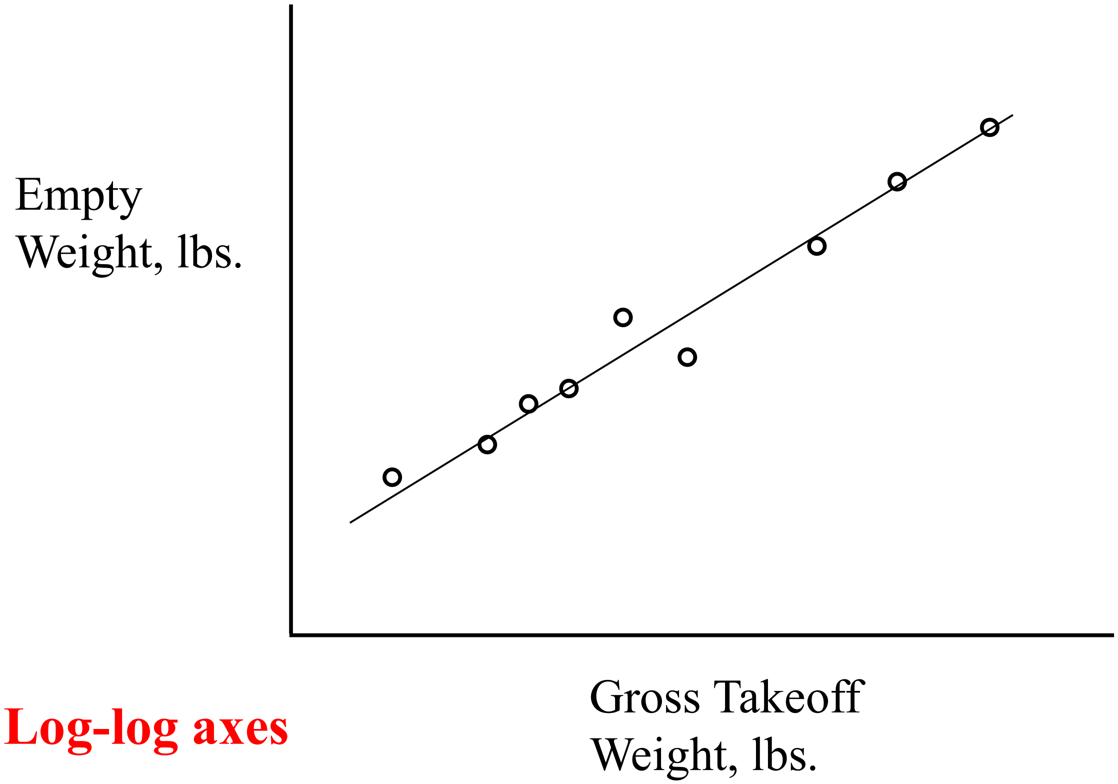
Empty Weight Estimation

Empty weight is a function of W_0 based on empirical data. This relationship looks like this:



Log-log axes

Regression Curve: Assumes W_E was designed to be lowest possible for best cost/performance, so each point represents "state of the art".

Regression Equation:

$$\text{inv } \log_{10} \{[\log_{10} W_{TO} - A]/B\}$$

A and B are the regression constants (slope and intercept) of the line. Use Historical data tables for values and types of aircraft, or plot your own regression curve and estimate your own A and B.

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Initial Weight Estimation

With these relationships and known values of payload and crew weight the takeoff gross weight can be guessed. This guess is used to find the mission fuel weight fraction and empty weight fraction.

$$\frac{W_c + W_p}{1 - \underbrace{W_f / W_0 - W_e / W_0}_{\text{Function of guessed } W_0}}$$

Resulting value of W_0

Use Resulting value of W_0 as new guessed value. Estimate new values for weight fractions. Iterate until convergence of W_0 value.

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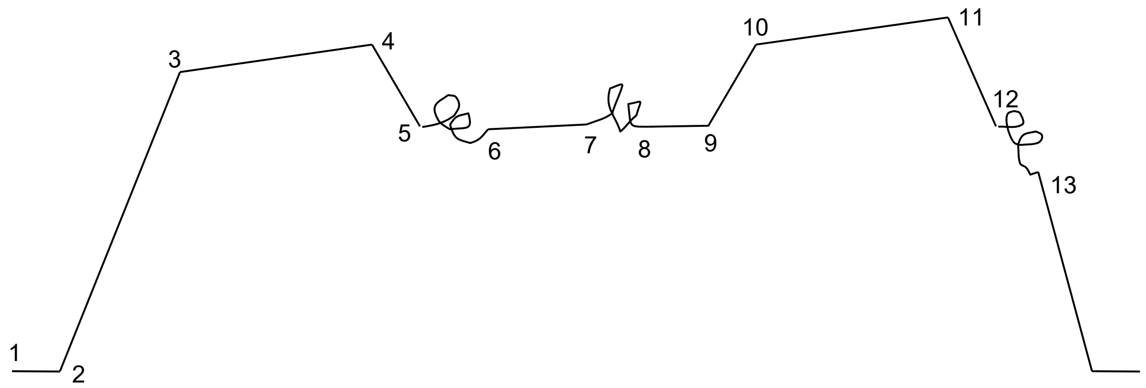
Sample Mission Analysis

Mattingly's Method

Example Mission Analysis



- The purpose of this section is to get a first estimate for the wing area for a fighter mission, whose mission profile is depicted above
- To do this, the fuel fraction for each mission segment is calculated, and hence obtaining a total take-off weight
- From this, the thrust-weight ratio can be selected (as previously outlined) and thus an appropriate wing-loading and wing area can be found
- Given next is a table indicating the flight conditions for each of the above mission segments

AAF Mission

Mission Phases & Segments		Conditions
1-2	Warm-up & take-off A – Warm-up B – Acceleration C - Rotation	2000 ftPA, 100^{o}F 60s mil power k_{TO}=1.2, μ_{TO}=0.05 max power $M_{TO}.t_R$ = 3s, max power
2-3	Accelerate & climb D – Acceleration E – Climb/acceleration	Min. time-to-climb path M_{TO} =>M_{CL}/2000ft PA, 100^{o}F, mil power M_{CL}/2000ft PA, 100^{o}F => BCM/BCA, mil power
3-4	Subsonic cruise climb	BCM/BCA, $\Delta s_{23} + \Delta s_{34}$ = 150 nmi.
4-5	Descend	BCM/BCA => M_{CAP}/30k ft..
5-6	Combat air patrol	30k ft.., 20 min
6-7	Supersonic penetration F – Acceleration G – Penetration	30,000 ft.. M_{CAP} => 1.5M/30 kft, max power 1.5 M, $\Delta s_F + \Delta s_G$ = 100 n. mi., no after-burn (AB)
7-8	Combat H – Turn 1 I – Turn 2 J – Acceleration	30,000 ft.. 1.6 M, one 360^o 5g sustained turn, with AB 0.9 M, two 360^o 5g sustained turn, with AB 0.8 => 1.6 M, Δt <50s, max power
	Deliver expendables	1.6 M /30k ft.., 1309 lb.
8-9	Escape dash	1.5 M/30k ft.., Δs_{89} = 25 n. mi., no AB
9-10	Min. time climb	1.5 M/30k ft.. => BCM/BCA
10-11	Subsonic cruise climb	BCM/BCA, Δs_{89} = 150 n. mi.
11-12	Descend	BCM/BCA => M_{loiter}/10k ft..
12-13	Loiter	M_{loiter}/10k ft.., 20 min
13-14	Descend & land	M_{loiter}/10k ft.. => 2000 ft.. PA, 100^{o}F

Weight Fraction Analysis

Now, using other preliminary data provided in figures 2.E1 & 2.E5, table 3.E2 (below), section 3.3 and the standard atmosphere tables, the weight fraction calculations can be completed. Figure 2.E5 was used to determine minimum time-to-climb path and the result is presented below in figure 3.E2.

Table 3.E2 Values of C

$TSFC = C\sqrt{\theta}$	Military Power		Maximum Power
	Subsonic	Supersonic	
$C\,(h^{-1})$	1.35	1.45	2.00

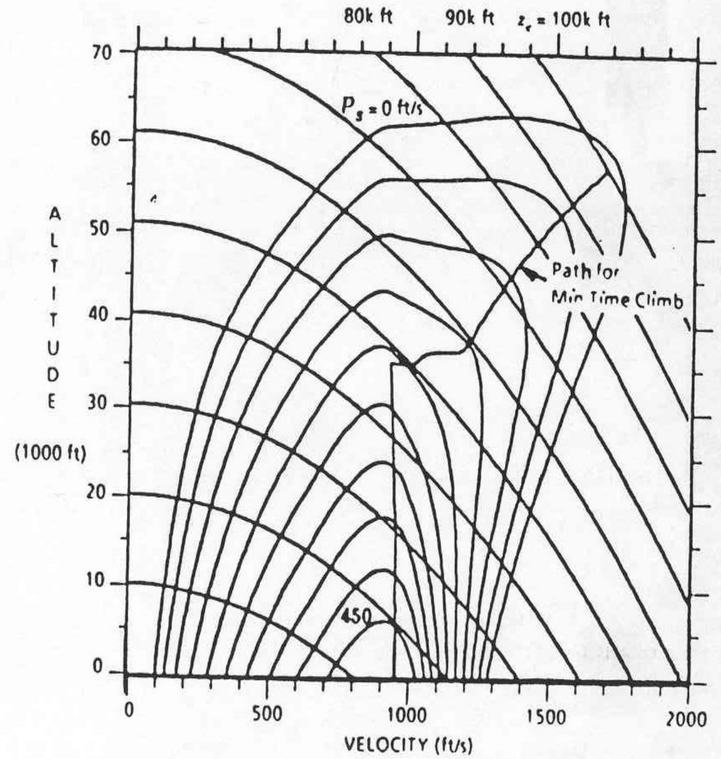


Fig. 3.E2 Minimum Time-to-Cimb Path on P_s Chart

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Engine Design

Example mission analysis

Altitude	Mach	Computational interval
2000	0.7	initial state point
9000	0.83	
16000	0.85	
23000	0.88	middle state point
30000	0.9	
36000	0.9	
43000	0.9	final state point

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Equation 33 yields BCA:

$$\delta_{BCA} = \frac{2\beta\left(\frac{w_{TO}}{S}\right)}{\gamma P_{SL} M_{CRIT}^{2}\sqrt{\frac{C_{D_0}}{K_1}}} = 0.1632, h = 43000\,ft$$

$$Where$$

$$\frac{w_{TO}}{S} = 64 lb / ft^2$$

$$M_{CRIT} = 0.9$$

$$C_{D_0} = 0.018$$

$$K_1 = 0.18$$

$$\beta = 0.9676$$

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Weight Fraction Calculation

$$\Pi_E = \left(\frac{w_f}{w_i}\right)_E = \exp\left(-Cc\Delta z_e\right)$$

$$where$$

$$c = \frac{\sqrt{\theta}}{V(1-u)}$$

$$u = \frac{C_D}{C_L}\frac{\beta}{a}\left(\frac{W_{TO}}{T_{ST}}\right)$$

Given

- Initial state point quantities: $h[i]$, $M[i]$, $(a/a[SL])[i]$, $\beta[i]$
- Final state point quantities: $h[f]$, $M[f]$, $(a/a[SL])[f]$, $\beta[f]$
- Initial state point quantities: h, M, $a/a[SL]$, θ, δ, σ, $(a/a[SL])$, $K1$, $K2$, $C[D0]$
- Other quantities: $a[SL]$, $g[0]$, $W[TO]/S$, γ, $P[SL]$, $T[SL]/W[TO]$, C

Equations

$$V_i = (a/a_{SL})_f a_{SL} M_i$$

$$V_f = (a/a_{SL})_f a_{SL} M_f$$

$$\Delta\left(\frac{V^2}{2g_0}\right) = \frac{V_f^2 - V_i^2}{2g_0}$$

$$\Delta h = \Delta h_f - h_i$$

$$\Delta z_e = \Delta h + \left(\frac{V^2}{2g_0}\right)$$

$$C_L = \frac{2\beta\left(\frac{W_{TO}}{S}\right)}{\gamma P_{SL}\delta M^2}$$

$$\frac{C_D}{C_L} = \frac{K_1 C_L^2 + K_2 C_L + C_{D_0}}{C_L}$$

$$a = a_{wet}(\sigma, M)$$

$$a = a_{dry}(\sigma, M)$$

$$u = \frac{C_D}{C_L}\frac{\beta}{a}\left(\frac{W_{TO}}{T_{ST}}\right)$$

$$\frac{T\Delta_s}{W} = \frac{\Delta z_e}{1-u}$$

$$V = \left(\frac{a}{a_{SL}}\right) a_{SL} M$$

$$\Delta t = \frac{T\Delta_s}{W}\frac{1}{V}\frac{\beta}{a}\left(\frac{W_{TO}}{T_{ST}}\right)$$

$$\Delta s = V\Delta t$$

$$c' = V(1-u)$$

$$c = \frac{\sqrt{\theta}}{c'}$$

$$\Pi_E = \left(\frac{w_f}{w_i}\right)_E = \exp(-Cc\Delta z_e)$$

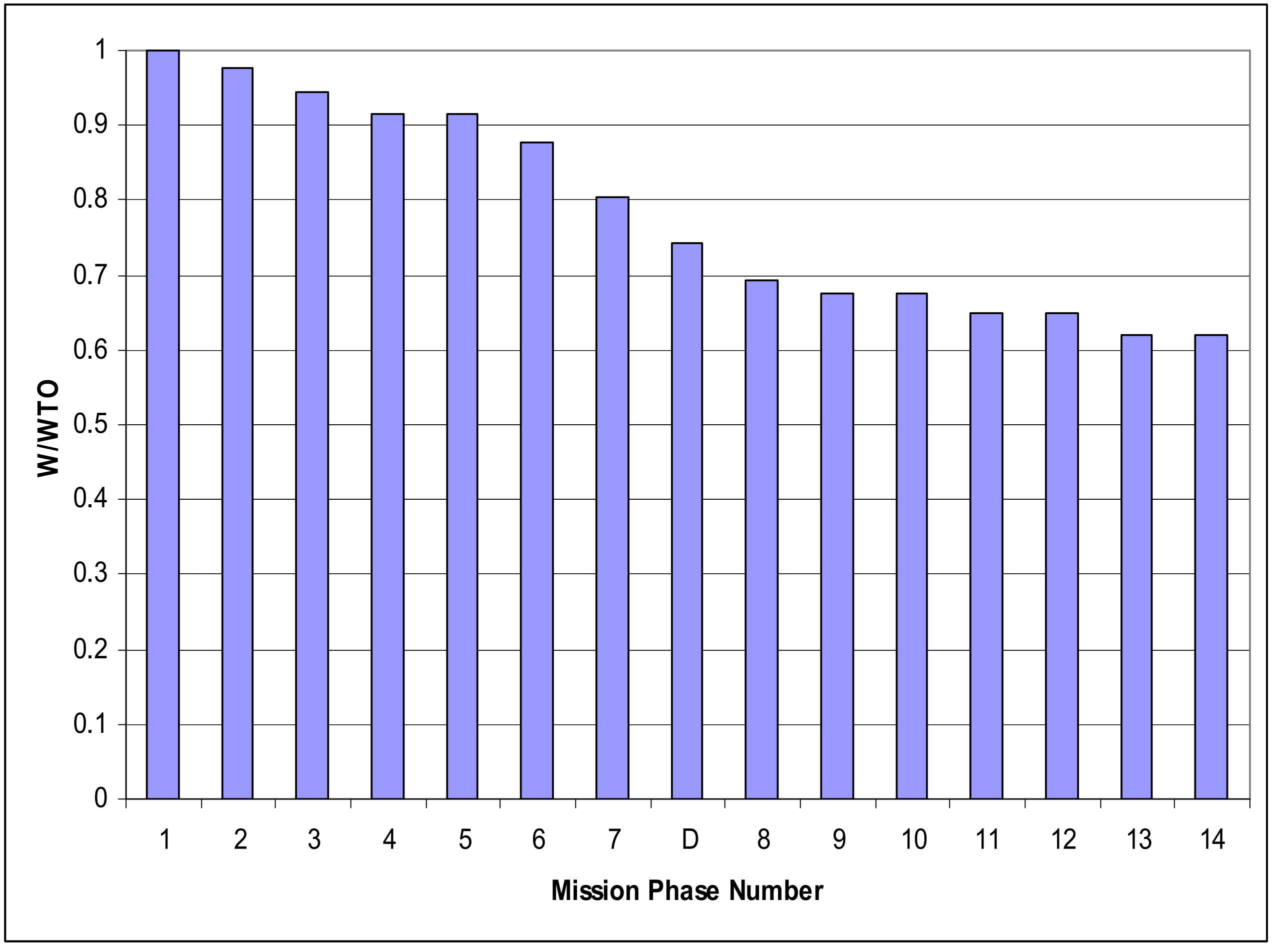
$$V_i = 881.7\,ft/s$$
$$V_f = 870.9\,ft/s$$
$$\left(\frac{a}{a_{SL}}\right)_i = 1.039$$
$$\left(\frac{a}{a_{SL}}\right)_f = 0.8671$$
$$\left(\frac{a}{a_{SL}}\right) = 0.9176$$
$$\Delta\left(\frac{V^2}{2g_0}\right) = 1549\,ft$$
$$\Delta h = 41000\,ft$$
$$\Delta z_e = 42550\,ft$$
$$K_1 = 0.18$$
$$K_2 = 0.0$$
$$C_{D_0} = 0.0175$$
$$C_L = 0.1333$$
$$\frac{C_D}{C_L} = 0.1553$$
$$a_{dry} = 0.3974$$

$$M = 0.88$$
$$\sigma = 0.4811$$
$$\theta = 0.8420$$
$$\frac{T\Delta_s}{W} = 62120\,ft$$
$$\beta = 0.9676$$
$$V = 901.2\,ft/s$$
$$\Delta t = 2.331\min$$
$$\Delta s = 20.73\,NM$$
$$c = 0.001487(ft/s)^{-1}$$
$$C = 1.35h^{-1}$$
$$\frac{W_{TO}}{S} = 64\,lb/ft^2$$
$$\frac{T_{SL}}{W_{TO}} = 1.2$$
$$\delta = 0.4051$$
$$u = 0.3151$$
$$\Pi_E = 0.9766$$

Table 3.E3 Summary of Results – Mission Analysis

Mission Phases & Segments		$\beta = W/W_{TO}$ End of Leg	$W_f/W_i = \prod_i^f$
1-2	Warm-up and Takeoff	0.9759	0.9759
2-3	Accelerate and Climb	0.9445	0.9678
3-4	Subsonic Cruise Climb	0.9141	0.9678
4-5	Descend	0.9141	1.0000
5-6	Combat Air Patrol	0.8780	0.9605
6-7	Supersonic Penetration	0.8035	0.9152
7-8	Combat	0.7441	0.9261
	Deliver Expendables	0.6917	-------
8-9	Escape Dash	0.6757	0.9769
9-10	Minimum Time Climb	0.6743	0.9979
10-11	Subsonic Cruise Climb	0.6487	0.9620
11-12	Descend	0.6487	1.0000
12-13	Loiter	0.6210	0.9573
13-14	Descend and Land	0.6210	1.0000

Figure 3.E3 Fraction of Takeoff Weight vs Mission Phase



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3.4.2 Determination of W_{TO}, T_{SL}, and S

Now compute the takeoff weight for the AAF (Air-to-Air Fighter) using Equation 3.46 shown below.

$$W_{TO} = \frac{W_{PP} + W_{PE}\Pi_8^{14}}{\Pi_1^{14} - \Gamma} \qquad (3.46)$$

where

W_{PP}=	200 lb.	Pilot plus equipment
	270	Cannon
	405	Ammunition feed system
	275	Returning ammunition
	<u>198</u>	Casings weight
	1,348 lb.	

Continued

W_{PE} = 382 lb. Sidewinder missiles
652 AMRAAMs
275 Spent Ammunition
1,309 lb.

Source: AAF RFP Sec. D

$$\Pi_8^{14} = \Pi_8^9 \cdots \Pi_{13}^{14}$$

$$\Pi_1^{14} = \Pi_1^2 \cdots \Pi_{13}^{14}$$

Source: Table 3.E3

$$\Gamma = \frac{W_E}{W_{TO}} = 0.6273$$

Source: Eq. (3.49)
Assumed W_{TO} = 25,000 lb.

Thus,

$$W_{TO} = \frac{1348 + (1309)(0.8978)}{0.6680 - 0.6273} = 62,000 lb$$

Example Mission Analysis (cont.)

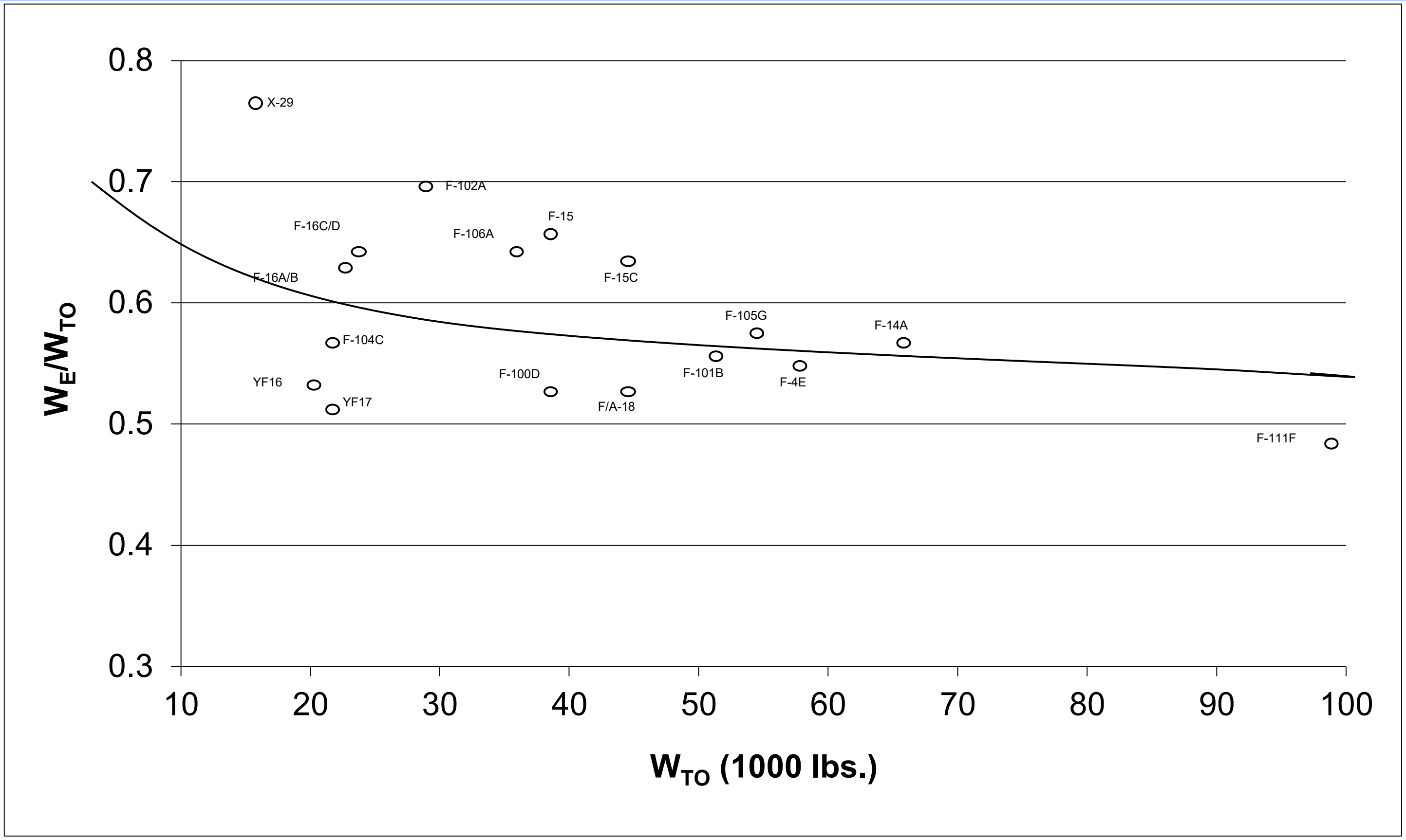

Fig. 3.E4 Weight Fractions of Fighter Aircraft

Notice (Figure 3.E4) that this conventional metal version of the AAF would be considerably heavier than existing lightweight fighters (e.g., F-16).

Example Mission Analysis (cont.)

Reducing the payload provides little relief because the large W_{TO} is the result of the very small denominator.

Another assumption can be the reducing the fuel usage, for example by shortening the range or duration of mission. Yet, another assumption would be reducing the empty weight, for example by using advanced lightweight construction materials as composites.

With the latter assumption, the aircraft's Γ will be reduced by approximately 10%. Therefore, with this reduction, W_{TO} is recalculated for the assumed W_{TO} of 25,000 lbs.,

$$W_{TO} = \frac{1348 + (1309)(0.8978)}{0.6680 - 0.6273(0.90)} = 24{,}400 lbs$$

Example Mission Analysis (cont.)

The conclusion is that AAF can have a practical size. This is true only if reliable non metallics with competitive strength, durability, and reparability become available according to schedule. With the choices of $T_{SL}/W_{TO} = 1.20$ and $W_{TO}/S = 64.0$ made, the description of the AAF at this stage of design, using $W_F/W_{TO} = 0.3265$, is:

W_{TO} = 24,400 lb.

T_{SL} = 29,300 lb.

S = 381 ft^2

W_P = 2,660 lb.

W_E = 13,800 lb.

W_F = 7,970 lb.

This information will give perspective and insight regarding the nature and shape of the corresponding aircraft.

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3.4.3 First Reprise

Here, the key assumptions made so far will be evaluated. Hopefully, the assumptions made are accurate and the results acceptable. If not, the whole process will have to be reiterate with new assumptions.

Earlier, the wing area (S) was assumed to have a value of 500 ft^2, whereas the recently derived wing area is 381 ft^2. This in turn will increase the solution space in Fig. 2.E3, but it is important to note that it wont provide any design points superior to the one already selected.

Warm-up

Assumptions: 2000 ft.. Pressure Altitude, 100 °F Temp, 60 second duration, min power

$$\Pi_{warm-up} = 1 - C\sqrt{\theta}\,\frac{\alpha}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)\Delta t$$

$\Delta t = 60$ sec, $\sigma = 0.8613$, $\beta = 1.0$, $T_{SL}/W_{TO} = 1.2$,

$\alpha_{dry} = 0.6484$, $\theta = 1.08$, $C = 1.35\ h^{-1}$

$$\Pi_{warm-up} = 0.9818$$

$$\beta = 0.9818$$

Takeoff Acceleration

Assumptions: 2000 ft.. Pressure Altitude, 100 °F Temp, $k_{TO} = 1.2$, $\mu_{TO} = 0.05$, max power

$$\Pi_{Takeoff\,Accel} = \exp\left\{\left(\frac{-C\sqrt{\theta}}{1-u}\right)\frac{V_{TO}}{g_0}\right\}$$

$$u = \left\{\xi_{TO}\frac{q}{\beta}\left(\frac{S}{W_{TO}}\right) + \mu_{TO}\right\}\frac{\beta}{\alpha}\left(\frac{W_{TO}}{T_{SL}}\right)$$

Takeoff Acceleration

$\beta = 0.9818$, $a = 1160$ ft./s, $\mu_{TO} = 0.05$, $W_{TO}/S = 64$ lb./ft^2, $M_{TO} = 0.1812$,

$q = 11.31$ lb./ft^2, $\sigma = 0.8613$, $C = 2.0$ h^{-1}, $\xi_{TO} = 0.36$ (Estimated from 2.4.3.1),

$C_{L\,max} = 2.0$, $\theta = 1.08$, $u = 0.1067$, $k_{TO} = 1.2$, $T_{SL}/W_{TO} = 1.2$, $V_{TO} = 210.2$ ft./s, $\alpha_{wet} = 0.8795$

$$\Pi_{takeoff\ accel} = 0.9958$$

$$\beta = 0.9777$$

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Takeoff Rotation

M_{TO}/2000 ft.. PA, 100 degrees F, t_R = 3s, max power

$$\prod_C = 1 - C\sqrt{\theta}\,\frac{a}{\beta}\left(\frac{T_{SL}}{W_{TO}}\right)t_R$$

t_R = 3s, M_{TO}=.1812, Beta=.9777, T_{SL}/W_{TO}=1.,2
a_{wet}=.8631,θ=1.080, σ=.8613, C=2.0 h^{-1}

$$\prod_C = .9982$$

The weight fraction for Mission phase 1-2 is the product of the weight fractions for Segments A,B, and C.

$$_1\prod_2 = \prod_A \prod_B \prod_C = 0.9759$$

$$\beta = 0.9759$$

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Climb and Acceleration

For this Mach number change of 0.1812 to 0.70, a single interval calculation will suffice. The Climb/Acceleration Weight Fraction Calculation Method 3.4.1 is utilized with c' evaluated at the middle state point (M=.4406, 2000ft PA, 100 degrees F), and $\beta = \beta_i$.

M_{CL}/2000 ft.. PA, 100 degrees F → BCM/BCA, mil power

The weight fraction of this mission segment is calculated along the minimum time-to-climb path depicted in Fig. 3.E.2. In our illustrative example of Sec. 3.4.1, we found the weight fraction of this segment, using a single gross interval, to be .9766. Here we shall use the three successive integration intervals given in Table 3.E4 for our calculations – applying Eq. 3.20 to each. The Climb/Acceleration Weight Fraction Calculation Method of Sec. 3.4.1 was used.

Climb Schedule		Integration Intervals (state points)		
Altitude (ft..)	Mach #	a	b	c
2000 (100 deg)	0.7	initial		
9000	0.83	middle		
16000	0.85	final	initial	
23000	0.88		middle	
30000	0.9		final	initial
36000	0.9			middle
43000	0.9			final

Example Mission Analysis

Segment E – Climb/Acceleration Results

	Δ h (ft..)	$\Delta(V^2/2g_o)$ (ft..)	Δz_e (ft..)	$T\Delta s/W$ (ft..)	Δt (min)	Δs (nmi)	c $(ft./s)^{-1}$	Π
a	14,000	2208	16,210	23,250	0.6452	5.683	0.001557	0.9906
b	14,000	3.105	14,000	20,420	0.7589	6.749	0.001485	0.9922
c	13,000	-661.6	12,340	18,520	0.9810	8.437	0.001494	0.9931
$\sum$	41,000	1550	42,550	62,190	2.385	20.87	n/a	n/a

$$\Pi_E = \Pi_a \Pi_b \Pi_c = 0.9761$$

The results do NOT differ insignificantly, therefore the phase should be broken down into finer intervals.

In summary, Mission Phase 2-3:

$$\Delta s_{23} = \Delta s_D + \Delta s_E = 23.43 nmi$$

$$\Delta t_{23} = \Delta t_D + \Delta t_E = 2.892 \min$$

$$\Pi_{23} = \Pi_D \Pi_E = 0.9678$$

$$\beta = 0.9445$$

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Example Mission Analysis

Mission Phase 3-4: Subsonic Cruise Climb

BCM/BCA, Δs23+Δs34 = 150 nmi

Assume: (Ps = 0 , T = D+R , K2 = 0)

$$\Pi_{34} = \exp\left\{\left(\frac{\sqrt{4C_{DO}K_1}}{M_{CRIT}}\right)\left(\frac{C\Delta s_{34}}{a_{SL}}\right)\right\}$$

Where: Δs_{34} = 126.6 nmi   K1 = 0.18

C = 1.35 h-1 C_{do} = 0.018

a_{SL} = 1116 ft./s M_{crit} = 0.9

Then:

$$\Pi_{34} = 0.9678$$
$$\beta = 0.9141$$

Mission Phase 4-5: Descend

BCM/BCA -> Mcap/30k ft..

$$\Pi_{45} = 1.0$$
$$\beta = 0.9141$$

Mission Phase 5-6: Combat Air Patrol

30k ft.. , 20 min.

Assume: (K2 = 0)

Where: Δt = 1200 sec   θ = 0.7940

K1 = 0.18   C = 1.35 h-1

Cdo = 0.014

Then:

$$\Pi_{56} = \exp\left\{-C\sqrt{\vartheta}\sqrt{4C_{DO}K_1}\Delta t\right\}$$

$$\Pi_{56} = 0.9605$$
$$\beta = 0.8780$$

Case 3.4.4.6 Mission Phase 6-7: Supersonic Penetration

This phase consists of segments

- F (Horizontal Acceleration) and
- G (Supersonic Penetration)

F. Horizontal Acceleration
– MCAP 1.5M/30k ft.., max power

- Horizontal acceleration calculation is divided into three intervals as shown in Table 1
- The initial, final and average Mach number of each interval and *h*=30,000 ft.. are used in the Climb/Acceleration Weight Fraction Calculation method of Sec. 3.4.1
- The calculated results are given in Table 1.

Table 1: Segment F-Horizontal Acceleration

	M_i	M_f	M_{avg}	Δz_e (ft)	$T\Delta s/W$ (ft)	Δt (min)	Δs (n mi)	c' (ft/s)	Π
a	0.6762	0.95	0.8131	6844	8324	0.2484	1.982	664.8	0.9949
b	0.95	1.23	1.09	9382	13,780	0.2735	2.926	738.1	0.9937
c	1.23	1.5	1.365	11,330	21,770	0.297	3.979	706.6	0.9921
			Σ	27,560	43,870	0.8189	8.887		

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F. Horizontal Acceleration
– MCAP 1.5M/30k ft.., max power

then

$$\prod_F = \prod_a \prod_b \prod_c = 0.9808$$

$$\beta = 0.8611$$

- Note: the total mission weight specific work is 43,870 ft.. with 62.82% utilized to increase the mechanical kinetic energy of the airplane
- The remaining 37.18% is dissipated into non mechanical energy of the airplane/atmosphere system
- A single gross interval calculation for this segment gives
  - higher value of the weight fraction by 0.18%
  - lower value of the total time by 6.34%
  - lower value of the ground distance by 7.68% and
  - lower value of the specific thrust work by 12.38%
- Also note that Δ_{sF} = 8.887 n mi.

G. Supersonic Penetration

– 1.5M/30k ft.., $\Delta sF + \Delta sG$ = 100 n mi., no afterburning

From Eq. (3.23)

with

Δ_{sG} = 91.11 n mi.
β = 0.8611
W_{TO}/S = 64 lb/ft^2
δ = 0.2975
M = 1.5

C_L= 0.5558
C_{Do} = 0.028
K_1 = 0.28
C_D/C_L =0.5193
θ = 0.794

C = 1.5 h^{-1}
(a/a_{SL}) = 0.8911
V = 1492 ft/s

then

$$\prod_G = 0.9331$$

Segments F and G together yield

$$\prod_{6\,7} = \prod_F \prod_G = 0.9152$$

$$\beta = 0.8035$$

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Mission Phase 7-8: Combat

This phase consists of segments

- H (Combat Turn 1)
- I (Combat Turn 2) and
- J (Horizontal Acceleration)

H. Combat Turn 1

— 1.6M/30k ft.., one 360 deg 5g sustained turn with afterburning

From Eq. (3.25)

$$\prod_H = \exp\left\{-C\sqrt{\theta}\right\}\left(\frac{nC}{C_L}\right)\frac{2nNV}{g\sqrt{n-1}}$$

$n = 5$		
$N = 1$	$C_L = 0.2279$	$C = 2.0\ h^{-1}$
$\beta = 0.8035$	$C_{Do} = 0.028$	$(a/a_{SL}) = 0.8911$
$W_{TO}/S = 64\ lb/ft^2$	$K_1 = 0.298$	$V = 1591\ ft/s$
$\delta = 0.2975$	$C_D/C_L = 0.5193$	
$M = 1.6$	$\theta = 0.794$	

Then,

$$\prod_H = 0.9705$$

$$\beta = 0.7798$$

Horizontal Acceleration – 0.8 1.6M/30k ft.., max power

- Horizontal acceleration calculation is divided into the three intervals shown in Table 2
- The initial, final, and average Mach number of each interval and h = 30,000 ft.. are used in the Climb/Acceleration Weight Fraction Calculation Method of Sec. 3.4.1.
- The calculated results are given in Table 2

	M_i	M_f	M_{avg}	Δz_e (ft)	$T\Delta s/W$ (ft)	Δt (min)	Δs (n mi)	c' (ft/s)	Π
a	0.8	1.06	0.93	7433	9444	0.2405	1.866	728	0.995
b	1.06	1.33	1.2	9919	15,980	0.2361	2.781	740.5	0.9934
c	1.33	1.6	1.47	12,160	25,040	0.2581	3.724	709.8	0.9916
			Σ	29,510	50,460	0.6987	8.371		

J. Horizontal Acceleration

– 0.8 1.6M/30k ft.., max power

$$\prod_J = \prod_a \prod_b \prod_c = 0.9801$$

- The weight specific thrust work of Segment J is 50,460 ft.. with 58.48% going to an increase of the airplane's mechanical kinetic energy and the rest is dissipated into nonmechanical energy by the drag forces
- A single interval calculation for this segment gives essentially
  - the same weight fraction value
  - lower value of time by 5.15%
  - lower value of distance by 6.77%
  - lower values of specific thrust work by 11.53%

For mission Phase 7-8 we have

$$\prod = \prod_H \prod_I \prod_J = 0.9261$$

$$\beta = 0.7441$$

Deliver Expendables – 1.6M/30k ft.., 1309 lb.

From Eq. (3.44), since Wj = W8 here,

$$\frac{W_8 - W_{PE}}{W_8} = 1 - \frac{W_{PE}}{W_{TO}\prod_{1}^{8}}$$

with WPE = 1309 lb.

$$\prod_{1}^{8} = 0.7441$$

W_{TO} = 25,000 lb. (assumed) Source: Fig 3.2

then

$$\frac{W_8 - W_{PE}}{W_8} = 0.9296$$

β = 0.6917

3.4.4.9 Mission Phase 8-9: Escape Dash

-1.5M/30k ft.., Δs89 = 35 n mi., no afterburning.

- The computational procedure is identical with that of Segment G
- Here the distance is smaller (25n. mi. vs 91.11 n. mi.) and the fraction ………………………………………… …………..

Mission Phase 8-9 Continued

$$\prod_{8\ 9} = 0.9769$$

$$\beta = 0.6757$$

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Mission Phase 9-10: Minimum Time Climb

BCM/BCA = 1.5M/30kft

Airplane climbs from 30,00 ft.. to 50,000 ft.. and reduces speed from 1.5 to 0.9 Ma

From Eq 3.42:

$$\prod_{9\ \ 10} = \exp\left\{-c\sqrt{\theta}\left(\frac{C_D}{C_L}\right)\Delta t\right\}$$

With (assuming vertical speed = 0.7 $V_{avg)}$

$$\Delta t = \frac{\Delta h}{0.7V_{avg}} \qquad \frac{V_{mid}^2}{2g_o} = z_{e_i} - h_{mid}$$

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Mission Phase 9-10: Minimum Time Climb

Given information:

h_i = 30,00 ft..
M_i = 1.5
$(a/a_{SL})_i$= 0.8911
V_i = 1492 ft./s
Z_{ei} = 64,600 ft..
H_f = 50,000 ft..
Δ_h = 20,000 ft..
M_f = 0.9

$(a/a_{SL})_f$= 0.8671
V_f = 870.9 ft./s
V_{avg} = 1181 ft./s
Δt = 24.19 s
H_{mid} = 40,000 ft..
V_{mid} = 1258 ft./s
$(a/aSL)_{mid}$= 0.8671
M_{mid} = 1.3
W_{TO}/S = 64 lb./ft^2

β= 0.6757
δ= 0.1858
C_L = 0.093
K_1 = 0.23
K_2 = 0
C_{D0} = 0.023
C_D/C_L = 0.2687
θ= 0.7519
C = 1.35 h^{-1}

Calculation from given information

$$\prod_{9\quad 10} = 0.9979$$

$$\beta = 0.6743$$

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Mission Phase 10-11: Subsonic Cruise Climb

BCM/BCA, Δs_{10-11} = 150 nmi.

$$\prod_{10}^{11} = .9620$$

$$\beta = 0.6487$$

Mission Phase 11-12: Descend

BCM/BCA = M_{loiter}/10kft

$$\prod_{11}^{12} = 1$$

$$\beta = 0.6487$$

Mission Phase 12-13: Loiter

BCM/BCA = M_{loiter}/10kft, 20 min

$$\prod_{12}^{13} = .9573$$

$$\beta = 0.6210$$

Mission Phase 13-14: Descend and Land

M_{loiter}/10kft --> 2000 ft.. PA, 100°F

$$\prod_{13}^{14} = 1$$

$$\beta = 0.6210$$

Case X: Minimum Fuel-to-Climb Path

NOTE: AAF is not required to fly a climb path for minimum fuel burn.

Fuel Consumed Specific Work (f_s) is calculated with Eq. (3.8) at military power using:

$$T_{SL}/W_{TO} = 1.2$$

$$W_{TO}/S = 64 lb/ft^2$$

$$\beta = 0.97$$

$$C = 1.35h^{-1} \quad for \quad M_0 < 1$$

$$C = 1.45h^{-1} \quad for \quad M_0 \geq 1$$

Case X: Minimum Fuel-to-Climb Path

Contours of constant f_s and constant energy heights (z_e) are shown in the velocity-altitude space (figure 3.E5).

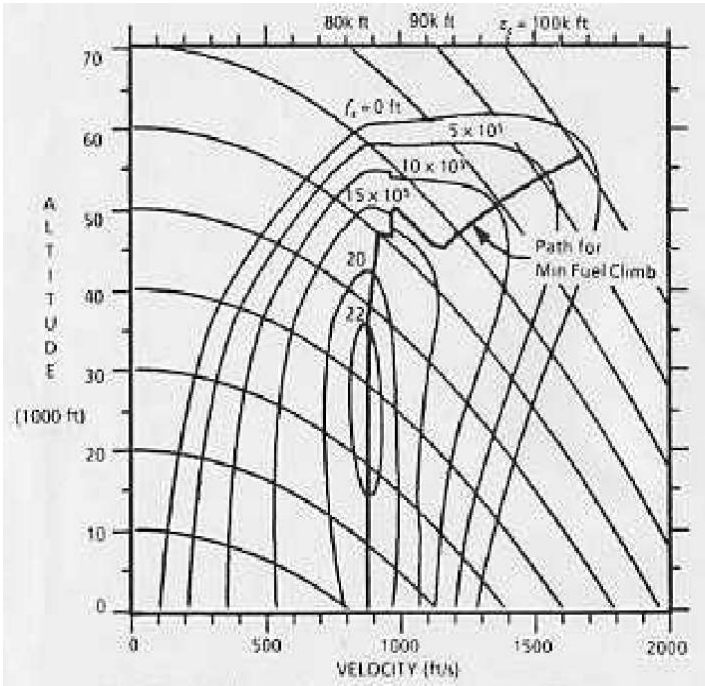

Fig. 3.E5 Minimum Fuel-to-Climb Path on f_s Chart

From energy height $z_{e1}(h_1,V_1)$ to $z_{e2}(h_2,V_2)$, the min fuel-to-climb path has a maximum value of f_s at each z_e.

z_e = 12 kft @ SL to z_e = 100 kft @ 57 kft is also shown in figure 3.E5

Case X: Minimum Fuel-to-Climb Path

By comparing figures 3.E2 and 3.E5:

- $P_s = 0$ and $f_s = 0$ are the same [from Eq (3.8)]

- Due to the step change in C across M = 1, each f_s contour has a discontinuity
  - Not including $f_s = 0$

- Minimum time-to-climb path occurs at about 50 fps greater than subsonic minimum fuel-to-climb path

- Minimum time-to-climb paths reach transonic and then supersonic speeds at a lower altitude than minimum fuel-to-climb paths

Case X: Minimum Fuel-to-Climb Path

By dividing Eq. (3.9) by $W_1 = \beta W_{TO}$ and given $W_{F1\text{-}2} = W_1 - W_2$ & ${}_1\Pi_2 = W_2/W_1$

$$\frac{W_{F_{1-2}}}{W_1} = 1 - \prod_{1\ \ 2} = \frac{T_{SL}}{\beta W_{TO}} \int_{z_{e_1}}^{z_{e_2}} \frac{dz_e}{f_s}$$

The aircraft weight fraction (${}_1\Pi_2$) results in:

$$\prod_{1\ \ 2} = 1 - \frac{T_{SL}}{\beta W_{TO}} \int_{z_{e_1}}^{z_{e_2}} \frac{dz_e}{f_s}$$

In figure 3.E5 (z_{e1} = 12 kft, z_{e2} = 64 kft):

$$\prod_{1\ \ 2} = 0.9685$$

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