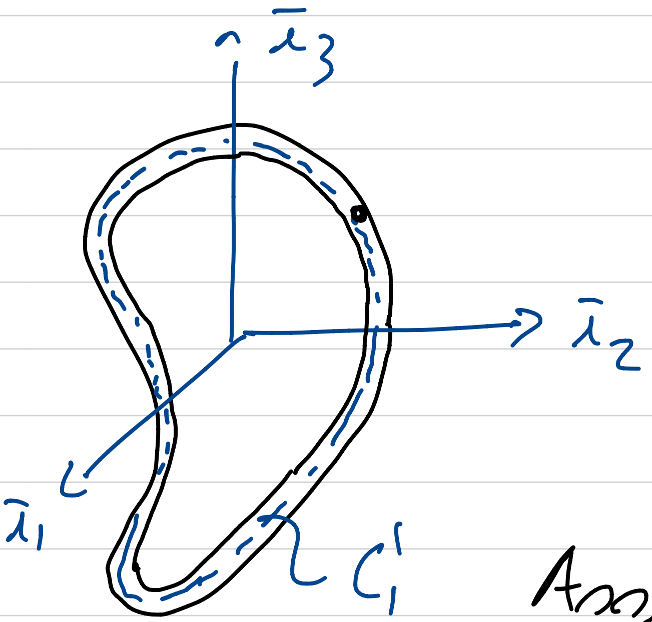
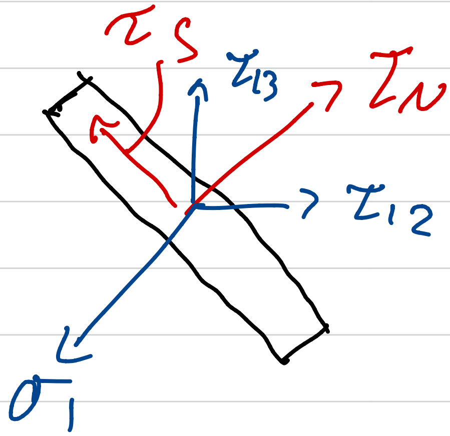
Stress in a thin-walled closed section





Assume that

→ $\sigma_1 \gg \sigma_2$, $\sigma_1 \gg \sigma_3$

$\sigma_2 \approx 0$ & $\sigma_3 \approx 0$

→ $\tau_{12}, \tau_{13} \gg \tau_{23}$

$\tau_{23} \approx 0$

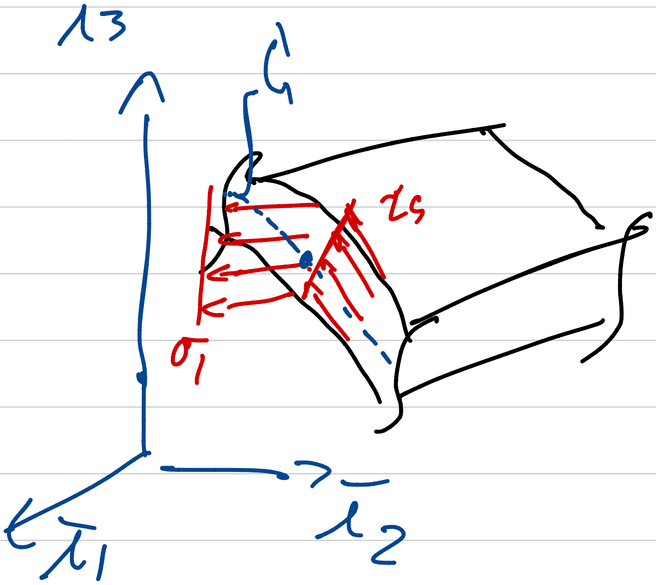
→ At the edges $\tau_N = 0$. Since the section is thin we will assume

$$\tau_N \approx 0$$

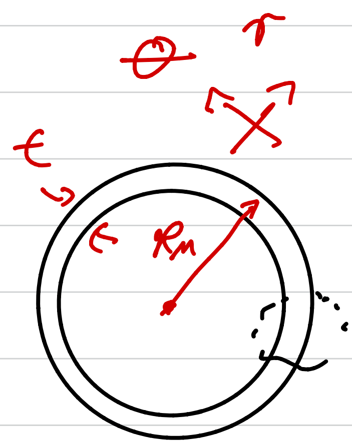
→ Because the section is thin, we will assume that

$$\tau_s \text{ and } \sigma_1$$

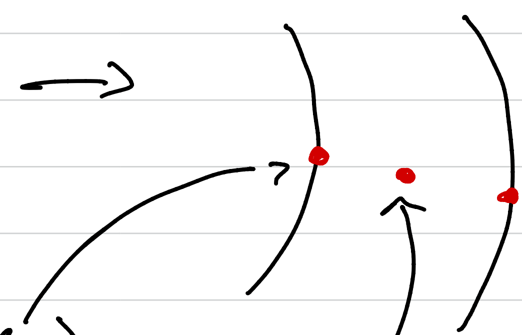
are <u>uniformly distributed</u> across the thickness



<u>Aside</u>: Thin-walled circular cylinder



$$\tau_s = G K_1 r$$



$$\tau_s = G K_1 \left(R_M - \frac{t}{2}\right)$$

$$\tau_s = G K_1 R_M \underbrace{\left(1 - \frac{1}{2}\frac{t}{R_M}\right)}_{\approx 1}$$

$$\tau_s = G K_1 R_M$$

Stress Flow

Define

$$n(x_1, s) = \sigma_1(x_1, s) \cdot t(s)$$

$$f(x_1, s) = \tau_s(x_1, s) \cdot t(s)$$

n - Axial Stress Flow

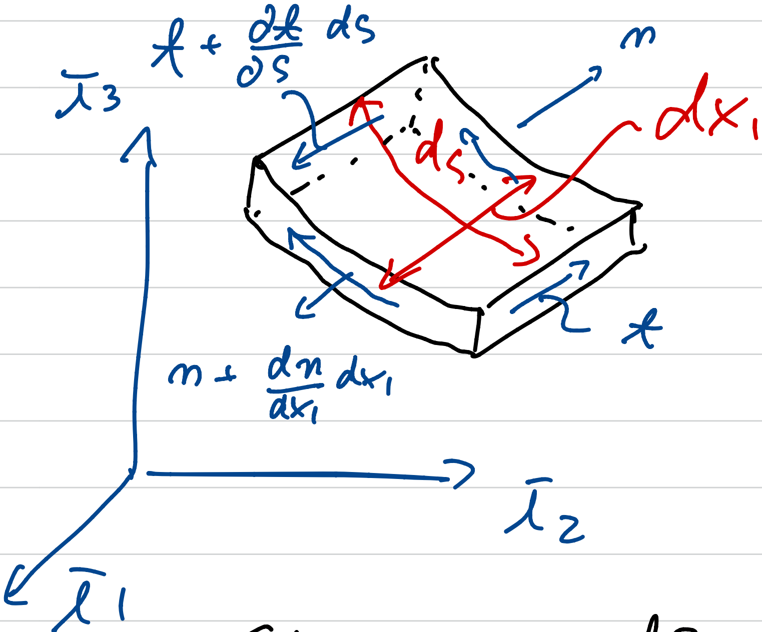
f - Shear Stress Flow

$$\underline{M}_0 = \int_{C_i'} \underline{r}_p \times \underline{f} \, ds$$

$$\rightarrow M_{10} = \int_{C_i'} r_0 f \, ds$$

$$r_0 = \left(x_2 \frac{dx_3}{ds} - x_3 \frac{dx_2}{ds} \right)$$

Local Equilibrium



$$\Sigma F_1: \quad -n\, ds + \left(n + \frac{\partial n}{\partial x_1} dx_1\right) ds$$

$$- t \cdot dx_1 + \left(t + \frac{\partial t}{\partial s} ds\right) dx_1 = 0$$

Divide through by $ds\, dx_1$

$$\boxed{\frac{\partial n}{\partial x_1} + \frac{\partial t}{\partial s} = 0}$$

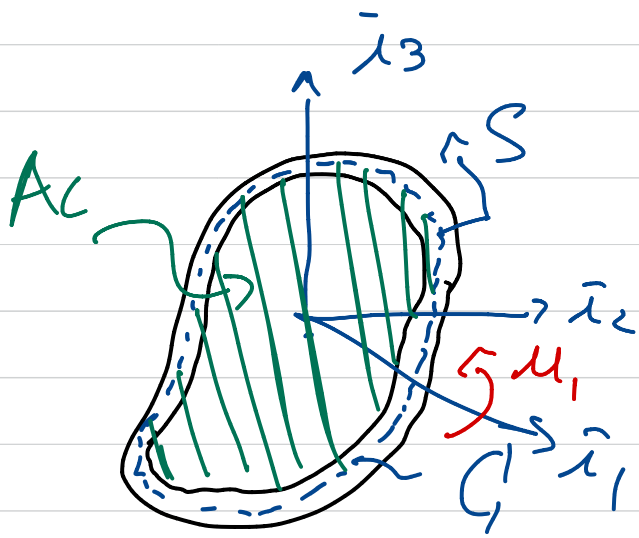
Torsion of a closed thin-walled section

Axial strain/stress vanishes

$$n(s) = 0$$

Equilibrium

$$\frac{\partial f}{\partial s} = 0 \rightarrow f = \text{constant}.$$



$$M_1 = \int_{C_1'} f\, r_0\, ds$$

$$M_1 = f \underbrace{\int_{C_1'} r_0\, ds}_{\text{Purely Geometric}}$$

$= 2 \cdot A_C$ $\quad$ A_C – Area enclosed by C_1'

$$M_1 = 2 A_C f$$

→ Bredt-Batho Formula

$$\tau_s(s) = \frac{M_1}{2 A_C \cdot t(s)}$$

Need to relate K_1 and M_1 to find H_{11}

Energy Argument

$$\underbrace{M_1 \cdot K_1}_{\text{Work Done on System}} = \underbrace{\int_{C_1'} \tau_s \cdot \gamma_s \cdot t \, ds}_{\text{Strain Energy}}$$

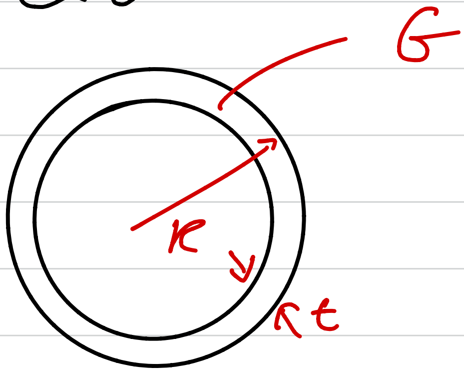
$$K_1 = \frac{M_1}{4 A_c^2} \int_{C_1'} \frac{ds}{G(s)\, t(s)}$$

$$\boxed{H_{11} = \frac{4 A_c^2}{\displaystyle\int_{C_1'} \frac{ds}{G \cdot t}}}$$

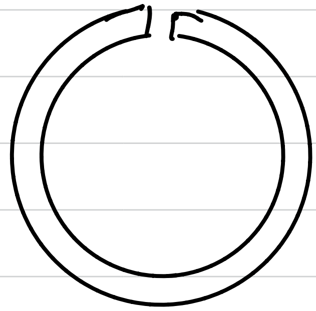
If G = constant and t = constant

$$H_{11} = \frac{4 G t A_c^2}{\ell}$$

Closed



Open



$$H_{11}^{CL} = \frac{4 G t A_c^2}{\ell} = \frac{4 G t (\pi R^2)^2}{2\pi R}$$

$$H_{11}^{OP} = \frac{1}{3} G \ell t^3 = \frac{1}{3} G (2\pi R) t^3$$

$$H_{11}^{CL} = 2\pi G t R^3$$

$$H_{11}^{OP} = \frac{2}{3} \pi G t^3 R$$

$$\frac{H_{11}^{CL}}{H_{11}^{OP}} = 3\left(\frac{R}{t}\right)^2 \approx 1200$$

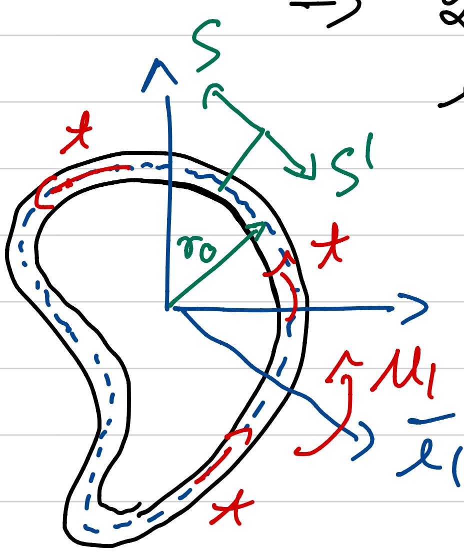
for $\frac{R}{t} = 20$

Notes

→ $l = \int_{C_i'} ds$ is the perimeter at the curve C_i'.

→ The cross-section at max torsional stiffness is the thin-walled circular tube

→ Largest $A_C = \pi R^2$ for a given $l = 2\pi R$.



→ r_0 is an algebraic quantity whose sign depends on the direction at s

→ A_C is an algebraic area

→ $A_C > 0$ when the curvilinear variable describes C_i' with A_C to the left.

s → $M_1 = 2 A_C t$

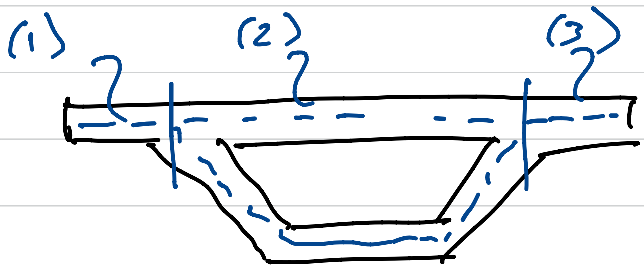
$t = \frac{M_1}{2A_C}$

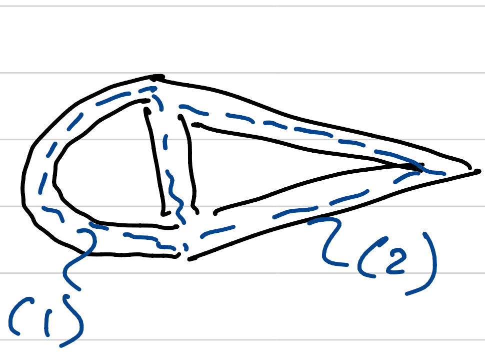
$A_C > 0 \rightarrow t > 0$

s' → $A_C < 0$

$\rightarrow t < 0$

Torsion of Multi-Component & Multi-Cellular Structures





→ The total torque M_1 carried by the cross-section is the sum of torques carried by each segment

$$M_1 = \sum_i M_1^{(i)}$$

→ The twist rate of all segments is equal

$$\kappa_1 = \kappa_1^{(1)} = \dots = \kappa_1^{(i)}$$

For a given applied torque Q we can solve for $M_1^{(i)}$ and κ_1

→ Solve for shear stress in each segment

→ Solve for twist angle.