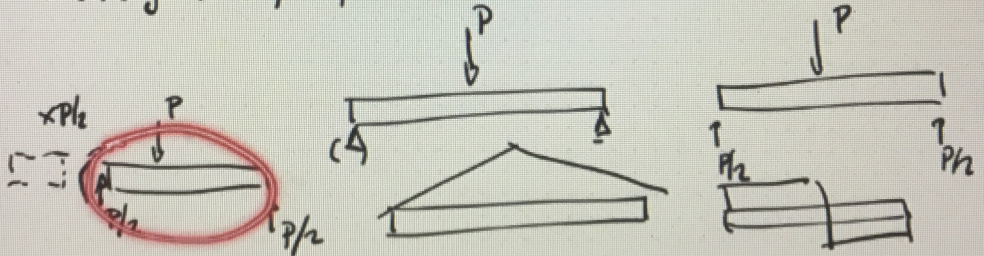
10/1

Recap:

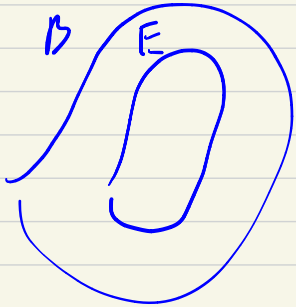
Balance of linear momentum

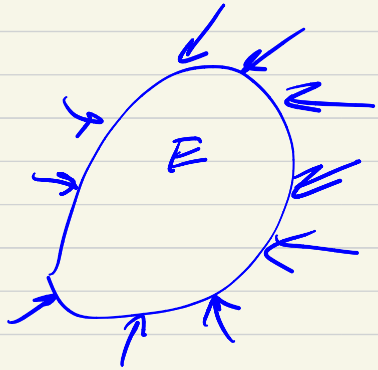
Recap:

* System of particles: $\frac{D}{Dt}(\underline{L}) = \underline{F}_{ext}$; $\underline{L} = \sum_{i=1}^{N} m^i \dot{\underline{r}}^i$; $\underline{F}^{ext} = \sum_{i=1}^{N} \underline{f}^i$
* Continuum system: $\frac{D}{Dt}\int_B \dot{\underline{x}}\, \rho \, dv = \int_B \underline{b}\, \rho \, dv + \int_{\partial B} \underline{t}\, dA$
* Using cons. of mass & kinematics: $\int_B \rho\, \ddot{\underline{x}}\, dv = \int_B \rho\, \underline{b}\, dv + \int_{\partial B} \underline{t}\, dA$
* Cauchy's stress principle



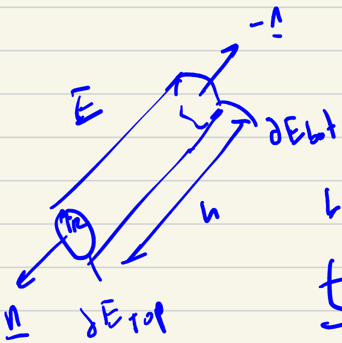
Cauchy stress principle.





$$\int_E \rho \, \ddot{\underline{x}} \, dv = \int_E \rho \, \underline{b} \, dv + \int_{\partial E} \underline{t} \, dA$$

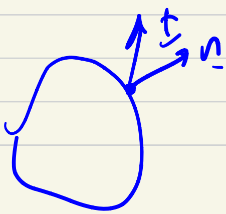
represent the effect of rest of the body on my subbody E



by limits,

$$\underline{t}(x)\Big|_{\partial E_{top}} = -\underline{t}(x)\Big|_{\partial E_{bot}}$$

(action = reaction) traction

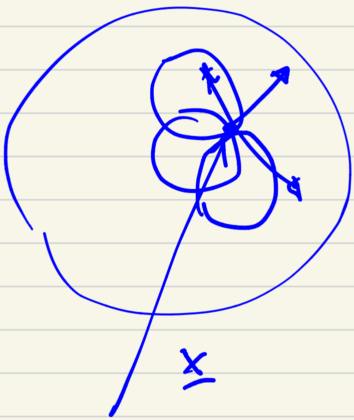


* As $R \to 0$, the only characteristic from the surface that remains is its normal.

Consequently, we expect $\underline{t}(\underline{x}, \underline{n})$.

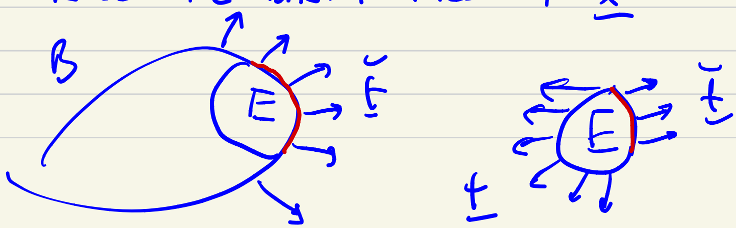
In this way, we can write:

$$\underline{t}(\underline{x}, \underline{n}) = -\underline{t}(\underline{x}, -\underline{n})$$



* This means there are an infinite number of tractions at each point!

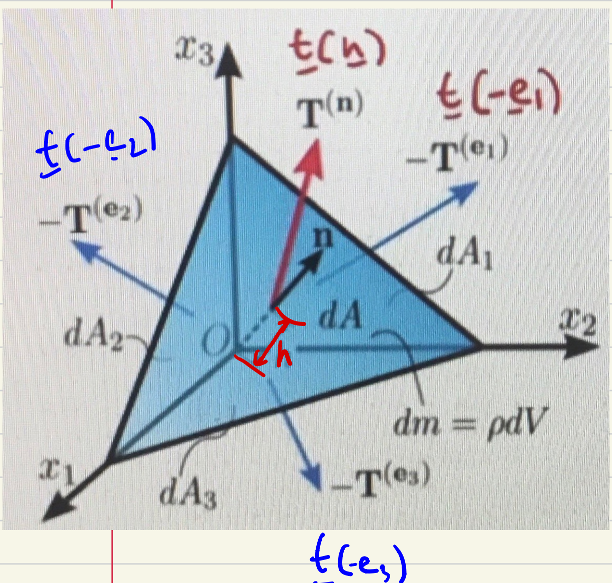
* The totality of these, called the stress state, characterizes the internal forces at $\underline{x}$



* If one side of our subbody coincides w/ an external surface, we can relate the boundary conditions to the stress state:

$$\boxed{\underline{t}(\underline{x},\underline{n}) = \check{\underline{t}}(\underline{x}) \text{ on } \partial B}$$

(not ∂E)



Cauchy stress tensor

By simple geometric projection:

$$\Delta A_1 = \Delta A_n (\underline{n}\cdot\underline{e}_1) = \Delta A_n n_1$$

$$\Delta A_2 = \Delta A_n (\underline{n}\cdot\underline{e}_2) = \Delta A_n n_2$$

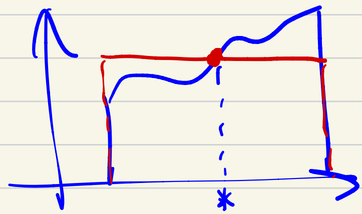
$$\Delta A_3 = \Delta A_n (\underline{n}\cdot\underline{e}_3) = \Delta A_n n_3$$

Balance of linear momentum for the tetrahedron is:

$$\int_T \rho(\ddot{\underline{x}} - \underline{b})\, dv$$

$$= \int_{\partial T} \underline{t}\, dA = \int_{\partial T_1} \underline{t}\, dA + \int_{\partial T_2} \underline{t}\, dA + \int_{\partial T_3} \underline{t}\, dA + \int_{\partial T_n} \underline{t}\, dA$$

By mean value theorem, there's a point that the function gives the value of the integral



Applying the mean value theorem:

$$\underbrace{\rho^*(\underline{x}^* - \underline{b}^*)\left(\frac{1}{3} h \, \Delta A_n\right)}_{\Delta V}$$

$$= \underline{t}^*(-\underline{e}_1)\Delta A_1 + \underline{t}^*(-\underline{e}_2)\Delta A_2$$
$$+ \underline{t}^*(-\underline{e}_3)\Delta A_3 + \underline{t}^*(\underline{n})\Delta A_n$$

substituting all ΔA_i and dividing by ΔA_n we get:

$$\frac{1}{3} h \rho^*(\underline{\bar{x}}^* - \underline{b}^*) = \underline{t}^*(-\underline{e}_1)n_1 + \underline{t}^*(-\underline{e}_2)n_2$$
$$+ \underline{t}^*(-\underline{e}_3)n_3 + \underline{t}^*(\underline{n})$$

Substituting $\underline{t}^*(-e_i) = -\underline{t}^*(e_i)$

& taking $\lim h \to 0$:

$$\boxed{\underline{t}(\underline{n}) = \underline{t}(\underline{e}_1)n_1 + \underline{t}(\underline{e}_2)n_2 + \underline{t}(\underline{e}_3)n_3}$$

(points collapsed to same point)

Note: $\underline{t}(e_1) = \begin{Bmatrix} t_1(e_1) \\ t_2(e_1) \\ t_3(e_1) \end{Bmatrix}$

- full vector $\underline{t}$, function of direction looking at
- not $\underline{t}$ in direction of $\underline{e}_1$
- This is $\underline{t}$ in the face of $\underline{e}_1$
  - ↳ doesn't have to be perpendicular to face.

full vector!

Let us define the vectors $\underline{t}_1 \equiv \underline{t}(\underline{e}_1)$

$$\underline{t}_2 \equiv \underline{t}(\underline{e}_2)$$

$$\underline{t}_3 \equiv \underline{t}(\underline{e}_3)$$

The previous expression becomes:

$$\underline{t}(\underline{n}) = \underline{t}_1 n_1 + \underline{t}_2 n_2 + \underline{t}_3 n_3$$

we can see that $\underline{t}(\underline{n})$ is a vector w/ components

$$t_i(\underline{n}) = (\underline{t}_1)_i n_1 + (\underline{t}_2)_i n_2 + (\underline{t}_3)_i n_3$$

$$= \underbrace{(\underline{e}_i \cdot \underline{t}_1)}_{\sigma_{i1}} n_1 + \underbrace{(\underline{e}_i \cdot \underline{t}_2)}_{\sigma_{i2}} n_2 + \underbrace{(\underline{e}_i \cdot \underline{t}_3)}_{\sigma_{i3}} n_3$$

$$\boxed{t_i(\underline{n}) = \sigma_{ij} n_j}$$

(Cauchy's relation)

9 values.

we claim that σ_{ij} are the components of a rank 2 tensor $\underline{\underline{\sigma}}$ called Cauchy stress Tensor: $\boxed{\underline{t}(\underline{n}) = \underline{\underline{\sigma}} \cdot \underline{n}}$