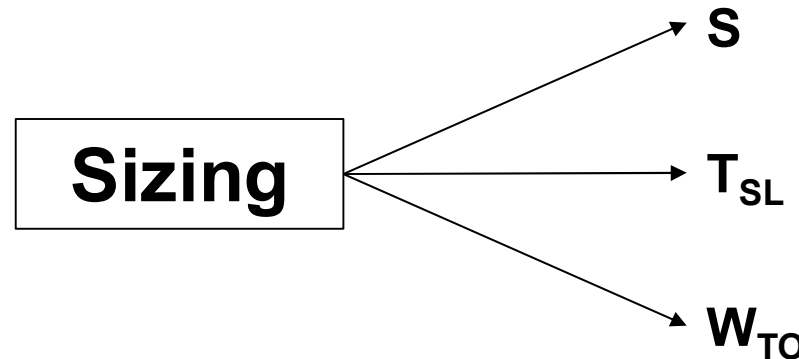

Sizing and Synthesis - A step-by-step walk-through

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Purpose of Mission Analysis

- Constraint analysis provides a solution mapped as a point in a thrust loading, T_{SL}/W_{TO} against wing loading, W_{TO}/S diagram. But we need the actual values of these parameters and not just their ratios
- A mission analysis will yield the additional equation needed
- A fuel required vs available iteration is performed to determine the actual size of the aircraft required to fly the mission



Disciplines Involved in Conceptual Design

- Propulsion
- Structures
- Aerodynamics

We need information from each of these disciplines before we proceed!

Discipline: Propulsion

- In constraint analysis, we used lapse rate and TSFC, but going forward, more information would be needed

$$F_N = \dot{m}_a (v_j - v_0) \quad v_j \text{ is a function of } \textbf{cycle analysis}$$

- For our purposes, we use an engine that is in the ball park and scale it up and down (rubberized engine) within reason to meet our requirement (fixed cycle analysis)
- How can engine be scaled?

$$F_N \propto \dot{m}_a$$
$$\dot{m}_a = \rho \pi \frac{d_f^2}{4} v_0$$

Bigger fan produces more thrust by increasing \dot{m}_a

Discipline: Propulsion

- The information about the performance of an engine on various conditions are contained in an engine deck. A typical engine deck looks like:

| h (altitude) | M | % $Thrust$ | $TSFC$ |
|----------------|-----|------------|--------|
| | | | |

- Because we are scaling the engine, instead of % $Thrust$, we use $\frac{\%Thrust}{\dot{m}_a}$, and instead of $TSFC$, we use \dot{m}_f

| h (altitude) | M | % T/\dot{m}_A | \dot{m}_f |
|----------------|-----|-----------------|-------------|
| | | | |

Discipline: Propulsion

- Importance of using $\frac{\%Thrust}{\dot{m}_a}$ over $\%Thrust$: scales the engine through \dot{m}_a , which can be used to calculate the diameter of the fan. This is the basis of a “rubberized” engine. Scaled up and down photographically through \dot{m}_a
- Note:
 - For cases where an existing engine is not suitable nor can easily be scaled, a new engine will need to be designed. That is accomplished through an engine cycle analysis (variation of FPR, CPR, efficiencies of stages, etc.)
 - This approach allows for a better integration of engine and airframe.

Discipline: Structures

- In the conceptual design phase, this discipline is represented by calculating the total weight and weight breakdown of the aircraft

$$W_{TO} = W_E + W_P + W_F$$

$$W_E = W_{wing} + W_{fuselage} + W_{empennage} + W_{prop} + W_{elec} + W_{hydraulic}$$

| |
|--|
| W_{TO} : Takeoff Weight W_E : Empty Weight W_P : Payload Weight W_F : Fuel Weight |
|--|

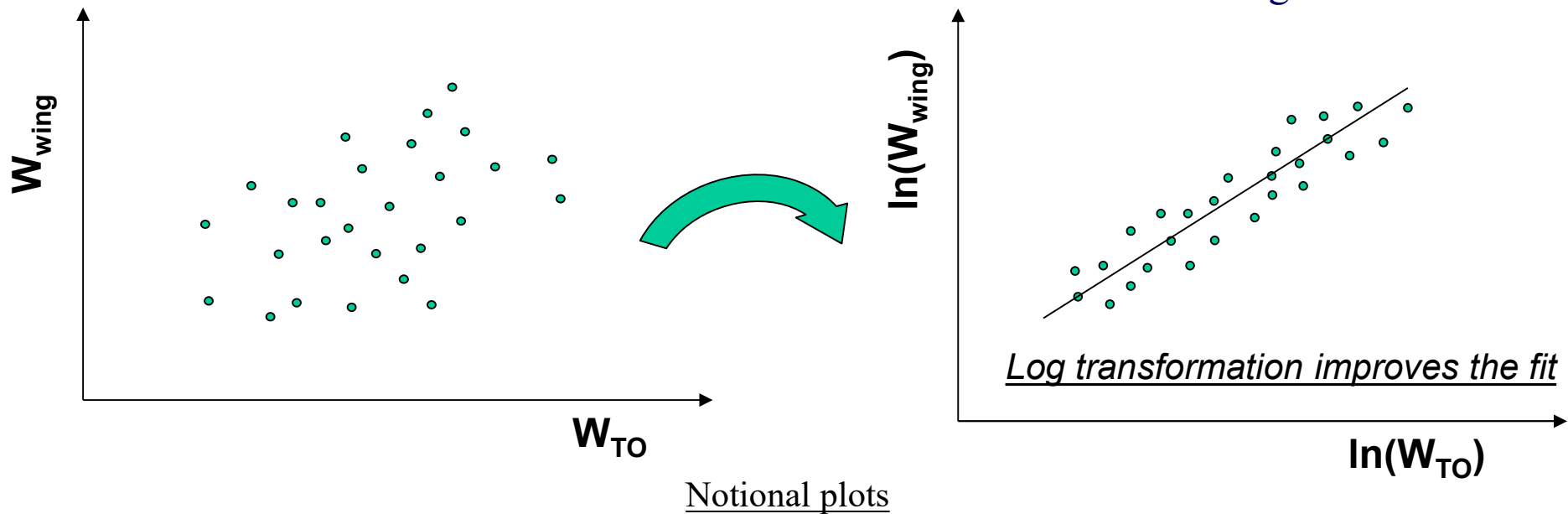
- Intuition tells us that W_E is somehow dependent on W_{TO}
 - A larger aircraft (large W_{TO}) will have a larger empty weight (W_E)
 - This implies W_{wing} , $W_{fuselage}$, $W_{empennage}$, etc. also depend on W_{TO}

Discipline: Structures

- For example, consider W_{wing} :

$$W_{\text{wing}} = f(\text{span}, \text{chord}, AR, \text{material}, \text{topology})$$

Since this information is not available in conceptual design, let's look at a historical correlation between W_{wing} and W_{TO}



Discipline: Structures

- Relational form for W_{wing} :

$$W_{wing} = \alpha' W_{TO}^{\beta'}$$

- Similar relations apply for the other component weights. The R^2 for such relation is very low on a component level. However, this is a useful approach when looking from an empty weight perspective. Although high, at component level, the error on the W_E using such form is very low for conceptual design level

$$W_E = W_{wing} + W_{fuselage} + \dots = \alpha W_{TO}^{\beta}$$

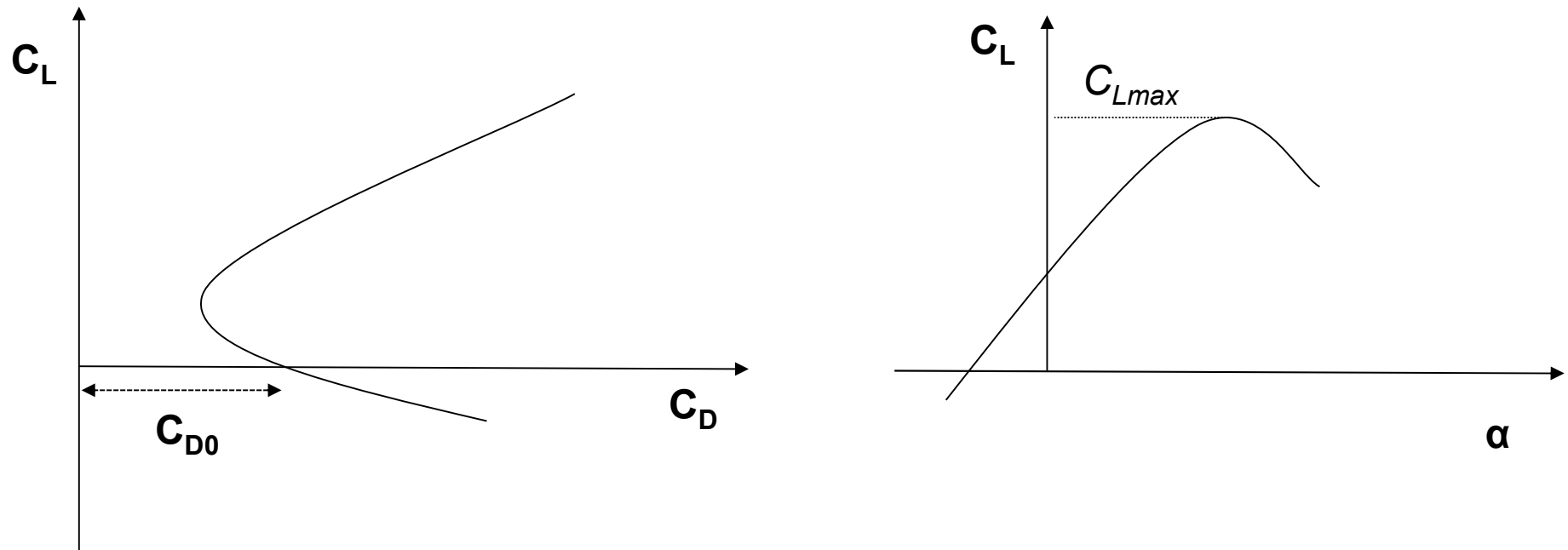
- α and β : found for a given aircraft using historical data

Discipline: Aerodynamics

- During analysis of takeoff, landing, cruise, or any other segment, the aerodynamic characteristics of the airframe are needed
 - C_{Lmax}
 - *Lift-drag polar*
 - $C_{L\text{ cruise}}$
- Maximum coefficient of lift (C_{Lmax}) is important in analysis during takeoff and landing

Discipline: Aerodynamics

- Lift-drag polar estimation
 - Lift-drag polar can be estimated for the notional design or approximated using experience or historical trends



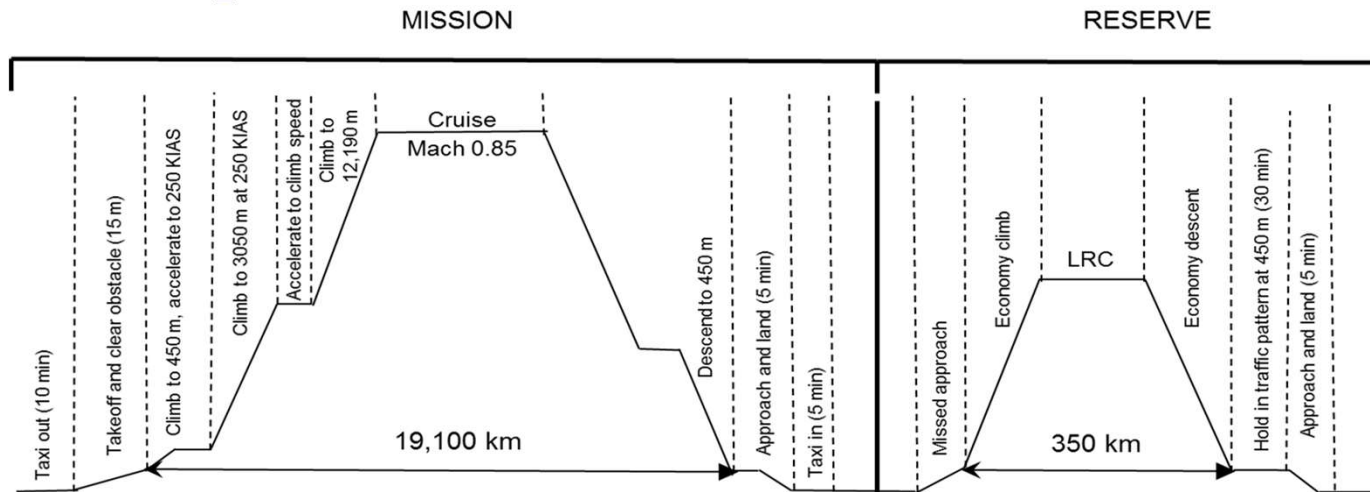
Notional plots

Discipline: Aerodynamics

- For sizing of horizontal tail:
 - A three Degrees of Freedom of model is needed
 - Lift, drag and pitching moment considered
 - CG calculation required
- To size the vertical tail, rudder: lateral analysis needs to be considered:
 - Need a six Degrees of Freedom depiction
 - In conceptual design, it is common to size the empennage through the use of tail volume coefficients

Sizing and Synthesis

- A mission profile is needed before we size the aircraft



Example mission profile

- Usually, we want to test our aircraft under critical conditions to ensure that it can withstand stringent conditions like one engine failure, a lifting surface not employed, etc.

Sizing and Synthesis

- Step 1:
 - It is assumed that W_P is known (from RFP)
 - Payload weight may be comprised of passengers, crew, baggage, military loads, etc.

$$W_{TO} = W_E + \mathbf{W}_P + W_F$$

- Step 2:
 - Guess a value for W_{TO}

$$\mathbf{W}_{TO} = W_E + \mathbf{W}_P + W_F$$

- Step 3:
 - Calculate S and T_{SL} using values of W_{TO} (from Step 2), T_{SL}/W_{TO} and W_{TO}/S (from constraint analysis)

Sizing and Synthesis

- Step 4:

- Using empty weight regressions, find W_E/W_{TO} for guessed W_{TO}

$$\frac{W_E}{W_{TO}} = f(W_{TO})$$

$$W_{TO} = W_E + W_P + W_F$$

- Step 5:

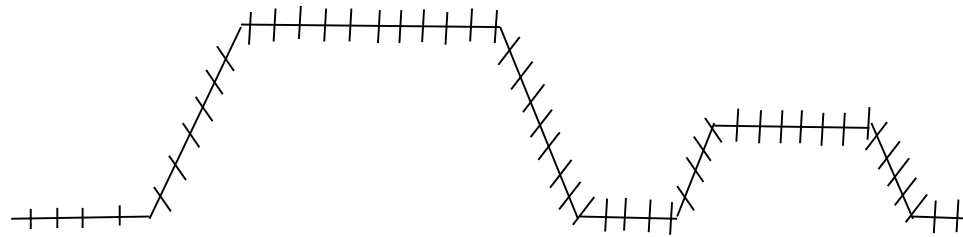
- Calculate W_F available to fly the mission

$$W_{F,avail} = W_{TO} - W_E - W_P$$

- If density of fuel (ρ_F) is known, then volume available (V_F) is also known. For our purpose, V_F is not needed (for military designs, V_F is needed as such designs are limited by volume)

Sizing and Synthesis

- Step 6: Fuel required calculation
 - Split the mission into smaller segments and march through all segments of the mission
 - For any point in mission, h and M are known
 - Vehicle balance is done to calculate Lift (L) and Drag (D)



Mission split into smaller segments

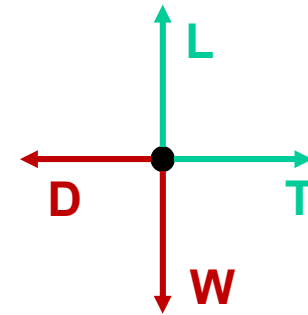
Sizing and Synthesis

- Step 6...

$$L = \frac{1}{2} \rho V^2 C_L S$$

Altitude → ρ Aerodynamics → C_L
Mach → V

From FBD, L balances instantaneous weight



Free Body Diagram (FBD)

- S falls out from above. Now, Drag(D) can be calculated

$$D = \frac{1}{2} \rho V^2 C_D S$$

- Once drag at a point is known, part thrust can be calculated from engine deck, and if it's linked to \dot{m}_a , required mass flow rate of air can be obtained

Sizing and Synthesis

- Step 6...

- The required fuel flow rate ($\Delta \dot{W}_f$) for this segment can also be obtained from the table. Multiply this by the time of segment 'i' to get fuel burn for that segment (ΔW_{Fi})

$$\text{Total fuel required} = W_{F,req} = \sum_i \Delta W_{Fi}$$

- Step 7:

- Check difference between fuel available ($W_{F,avail}$) and fuel required ($W_{F,req}$)

$$\Delta F = W_{F,avail} - W_{F,req}$$

- If the difference is within tolerance, you have a converged solution (Now you have a sized vehicle!)

Sizing and Synthesis

- Step 7...

- If not, update W_{TO} in Step 2 as:

$$W_{TO} = W_{TO} - \Delta F$$

(or the weight difference can be split between $W_{F,avail}$ and W_{TO})

- In the calculations, it might turn out that there was not enough area
 - You might want to update S as the largest area required or change the aerodynamics that can provide enough C_L
 - Update on the thrust would be \dot{m}_a multiplier
 - Now, you have new values for T_{SL} , S , and W_{TO} .
 - Repeat Step 4 – Step 7 until $|\Delta F|$ converges to 0 or a specified tolerance

Sizing and Synthesis

Notes:

- Volume analysis can be done by converting weight of fuel required to volume required
 - The volume available calculation is not straightforward for vehicles that are volume limited
- To include horizontal and vertical tail analysis, CG needs to be determined
 - The knowledge of topology of components is needed
 - For a quick estimation, you can look at other similar aircraft and see how the components can be placed.