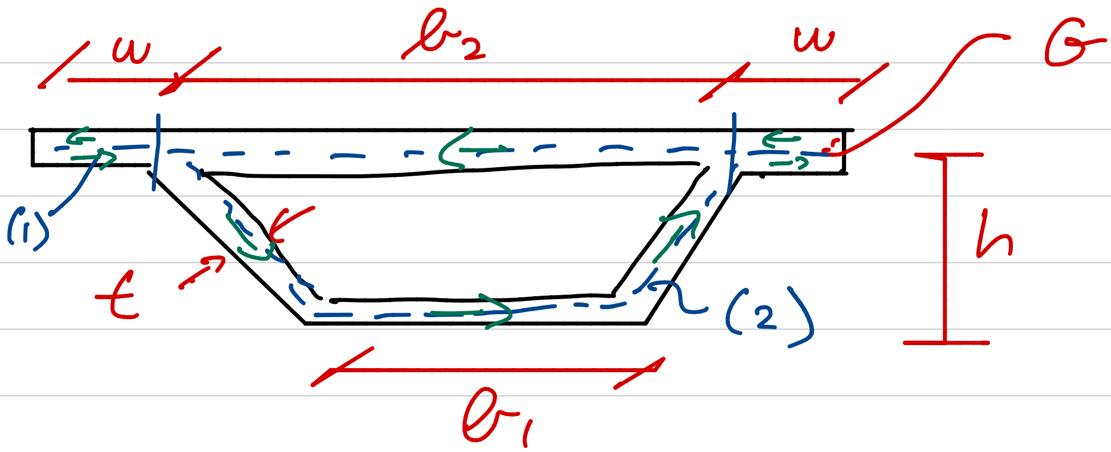
Example: Multi-Component Thin-Walled beam in Torsion



1) Find H_{11}

2) Find τ_s as a function of M_1

$\rightarrow$ $M_1 = 2 M_1^{(1)} + M_1^{(2)}$

$\rightarrow$ $K_1 = K_1^{(1)} = K_1^{(2)}$

$M_1^{(1)} = H_{11}^{(1)} K_1$, $M_1^{(2)} = H_{11}^{(2)} K_1$

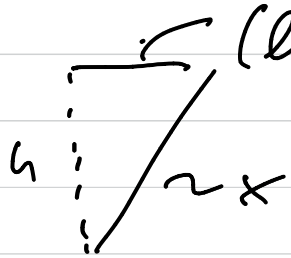
$$M_1 = \underbrace{\left(2 H_{11}^{(1)} + H_{11}^{(2)}\right)}_{H_{11}} K_1$$

$\rightarrow$ $H_{11} = 2 H_{11}^{(1)} + H_{11}^{(2)}$

$$H_{11}^{(1)} = \frac{1}{3} G \cdot \ell \cdot t^3 = \frac{1}{3} G w t^3$$

$$H_{11}^{(2)} = \frac{4 \cdot A_c^2}{\int_{C_1'} \frac{ds}{Gt}} = \frac{4 A_c^2 G \cdot t}{\ell}$$

$$A_c^2 = b_1 h + 2 \cdot \frac{1}{2} \left(\frac{b_2 - b_1}{2} \right) h = \frac{h}{2} (b_2 + b_1)$$



$$x = \left(\left(\frac{b_2 - b_1}{2} \right)^2 + h^2 \right)^{1/2}$$

$$\ell = b_1 + b_2 + 2x$$

$$H_{11}^{(2)} = 4 \left(\frac{h}{2} (b_2 + b_1) \right)^2 \cdot \frac{G \cdot t}{\ell}$$

$$H_{11}^{(2)} = \frac{h^2 (b_2 + b_1)^2 \cdot G \cdot t}{\ell}$$

$$H_{11} = 2 H_{11}^{(1)} + H_{11}^{(2)}$$

$$H_{11} = H_{11}^{(2)} \left(1 + 2 \cdot \frac{H_{11}^{(1)}}{H_{11}^{(2)}} \right)$$

$$H_{11} = H_{11}^{(2)}\left(1 + 2\,\frac{1}{3}\,\cancel{G}\,w\,t^{\cancel{3}\,2}\cdot\frac{l}{h^2(b_2+b_1)^2\,\cancel{Gt}}\right)$$

$$H_{11} = H_{11}^{(2)}\left(1 + \frac{2}{3}\,\frac{wl}{(b_2+b_1)^2}\left(\frac{t}{h}\right)^2\right)$$

Very Small!

$\rightarrow$ $H_{11} \approx H_{11}^{(2)}$

$\rightarrow$ The stiffness is nearly equal to that at the closed trapezoidal section!

$$\tau_{MAX}^{(1)} = G\,t\,K_1 \qquad K_1 = \frac{M_1}{H_{11}} \approx \frac{M_1}{H_{11}^{(2)}}$$

$$= G\,t\,\frac{M_1}{H_{11}^{(2)}}$$

$$\tau_{MAX}^{(2)} = \frac{\cancel{t}}{t} \qquad M_1 = 2\,A_c \cdot \cancel{t}$$

$$\tau_{MAX}^{(2)} = \frac{M_1}{2 A_c t}$$

$$\frac{\tau_{MAX}^{(1)}}{\tau_{MAX}^{(2)}} = \cancel{G}\cancel{t}\cancel{M_1}\,\frac{\ell}{h^2(\ell_2+\ell_1)^2\cancel{G}\cancel{t}} \cdot \cancel{2}\,\frac{\cancel{h}}{\cancel{2}}\,\frac{\cancel{(\ell_2+\ell_1)}\,t}{\cancel{M_1}}$$

$$\frac{\tau_{MAX}^{(1)}}{\tau_{MAX}^{(2)}} = \frac{\ell}{\ell_2+\ell_1}\left(\frac{t}{h}\right)$$

← Small stresses in the open strips!

Very Small!