AE6310: Optimization for the Design of Engineered Systems

Quiz 1
February 14th, 2019

Briefly answer the following questions on the paper provided. Organize your work and be careful to properly answer all parts of each question.
This quiz is closed book. The length of the quiz is 40 minutes.

1. (30 points) Find and characterize the critical points of the following function:

$$f(x_1, x_2) = x_1^2 + 4x_2^4 - 2x_2^2 + 1$$

2. (20 points) Sketch the contours of the following quadratic functions. Indicate any critical points and identify whether they are local minimum, maximum or saddle points.

   (a) A quadratic with a unique minimum

   (b) A quadratic function that is unbounded from below

   (c) A quadratic function with an indefinite Hessian matrix

3. (20 points) First state the strong Wolfe conditions and give representative values for the coefficients. Second, for each part below, sketch both the merit function $\phi(\alpha)$ and the derivative of the merit function $\phi'(\alpha)$ (immediately below the sketch of $\phi(\alpha)$).

   (a) Sketch a valid merit function and indicate all intervals which satisfy the sufficient decrease criterion, but *do not* satisfy the curvature condition from the strong Wolfe conditions.

   (b) Sketch a valid merit function and indicate all intervals which satisfy the curvature condition from the strong Wolfe conditions, but *do not* satisfy the sufficient decrease condition.

   Make sure that your merit function is valid and that the intervals shown are not empty.

4. (30 points) You are given a quadratic function with a Hessian matrix that has the following properties:

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad b = 0$$

   (a) Sketch the quadratic function

   (b) Re-drawn the sketch and illustrate the convergence behavior of the steepest descent method using exact line searches from the point $x_1 = 2$, $x_2 = 1$.

   (c) Re-drawn the sketch and illustrate the convergence behavior of the conjugate gradient method using exact line searches from the point $x_1 = 2$, $x_2 = 1$.

   (d) Describe why you expect to see a difference in performance between these two methods.

   (e) Describe any differences you would observe if you used inexact line searches.