ENERGY PRINCIPLES

* Closely related to work is the concept of energy

* In each type of system, forces are present which may be associated a "capacity" to displace and thereby perform work

* Energy can be thought of as a capacity to do work.

* Work is done when a form of energy is changed

  → Since only changes matter, the reference w.r.t. which we measure energy is arbitrary

* Energy is conserved

Example:

$$\underset{m}{\bigcirc} \to F = m \cdot \frac{dV}{dt}$$

Work done $\quad W = \int_1^2 m \frac{dV}{dt} dx$

Aside

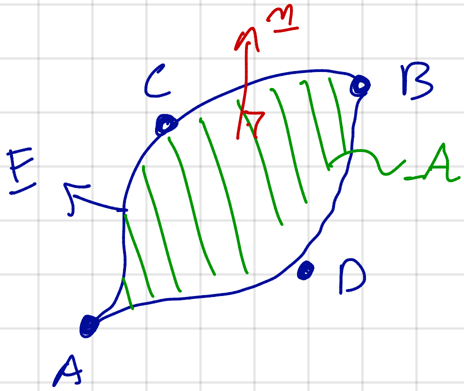
$v = \frac{dx}{dt}$

$dx = v\,dt$

$$W = m \int_1^2 v\,dv$$

$$\underset{\text{WORK DON}}{W} = \underbrace{m\frac{v_2^2}{2} - m\frac{v_1^2}{2}}_{\text{CHANGE IN KINETIC ENERGY}}$$

CONSERVATIVE FORCES



* Force is conservative if the work done is path independent

$$W = \int_{ACB} \underline{F} \cdot \underline{dr} = \int_{ADB} \underline{F} \cdot \underline{dr}$$

* Work vanishes when performed over an arbitrary closed path

$$\int_{ACB} \underline{F} \cdot \underline{dr} = W \quad , \quad \int_{BDA} \underline{F} \cdot \underline{dr} = -W$$

$$W = \oint_{C_1} \underline{F} \cdot \underline{dr} = 0$$

Stokes Theorem

$$\oint_{C_1} \underline{F} \cdot \underline{dr} = \int_A \underline{n} \cdot (\nabla \times \underline{F}) \, dA = 0$$

We need $\nabla \times \underline{F} = \underline{0}$ for $\underline{F}$ to be conservative

-> THIS is always true if

$\underline{F} = -\nabla \phi$ $\quad$ ϕ is a scalar

PROOF : $\nabla \times \underline{F} = 0$ If $\underline{F} = \nabla \phi$

$$(\nabla \times \underline{F}) = \left(\frac{\partial F_3}{\partial x_2} - \frac{\partial F_2}{\partial x_3}\right)\underline{e}_1 + \left(\frac{\partial F_1}{\partial x_3} - \frac{\partial F_3}{\partial x_1}\right)\underline{e}_2 + \left(\frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2}\right)\underline{e}_3$$

$$\underline{F} = \nabla \phi = \underbrace{\frac{\partial \phi}{\partial x_1}}_{F_1}\underline{e}_1 + \underbrace{\frac{\partial \phi}{\partial x_2}}_{F_2}\underline{e}_2 + \underbrace{\frac{\partial \phi}{\partial x_3}}_{F_3}\underline{e}_3$$

$$(\nabla \times (\nabla \phi)) = \left(\frac{\partial \phi}{\partial x_2 \partial x_3} - \frac{\partial \phi}{\partial x_3 \partial x_2}\right)\underline{e}_1 \to 0$$

$$+ \left(\frac{\partial \phi}{\partial x_3 \partial x_1} - \frac{\partial \phi}{\partial x_1 \partial x_3}\right)\underline{e}_2 \to 0$$

$$+ \left(\frac{\partial \phi}{\partial x_2 \partial x_1} - \frac{\partial \phi}{\partial x_1 \partial x_2}\right)\underline{e}_3$$

$$= 0$$

If $\underline{F} = -\nabla \phi$ the force is conservative

ϕ - Potential (Potential Function)

$$W = \int_{\underline{r}_1}^{\underline{r}_2} \underline{F} \cdot d\underline{r} = -\int_{\underline{r}_1}^{\underline{r}_2} \nabla\phi \cdot d\underline{r}$$

$$= -\int_{\underline{r}_1}^{\underline{r}_2} \left(\frac{\partial \phi}{\partial x_1} \cdot dx_1 + \frac{\partial \phi}{\partial x_2} dx_2 + \frac{\partial \phi}{\partial x_3} dx_3 \right)$$

$$W = -\int_{r_1}^{r_2} d\phi = \phi(\underline{r}_1) - \phi(\underline{r}_2)$$

$$W = \phi(\underline{r}_1) - \phi(\underline{r}_2) = -\Delta\phi$$