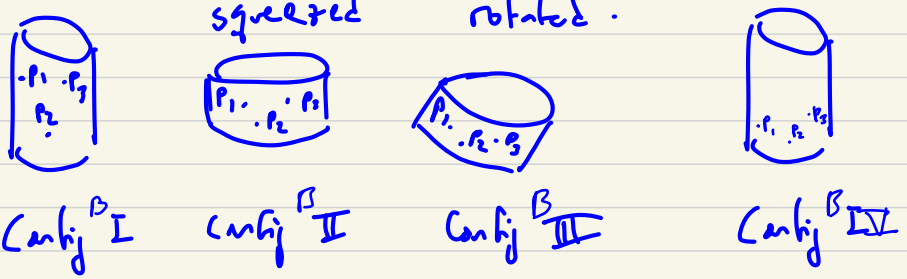
8/27
AE 6114

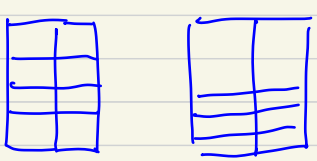
Kinematics

Body & its configurations

- A body is a set of material particles that occupy a region in space
- A particular correspondence between the particles in the body and points in space is called a configuration of the body



But can't just look at the outside to tell config.



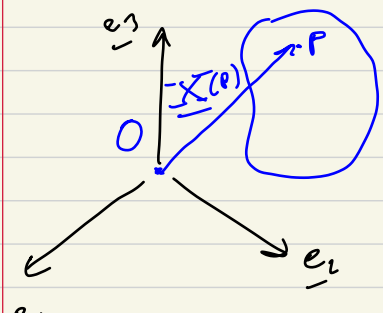
← structured internally different config!

How to describe a body?

- select a convenient configuration as a reference configuration B_r
- Label each material particle P of the body w/ its position vector $\underline{X}(P)$ in the configuration B_r relative to a convenient origin O.

(convenient, for example, is no stress)
↳ make math simpler

The collection of $\underline{X}$ obtained describes the body. The position can be expressed w.r.t. a chosen basis



(Reference configuration)

(For this class, assume orthonormal basis)

Then each position vector can be expressed as:

$$\underline{X} = X_1 \underline{e}_1 + X_2 \underline{e}_2 + X_3 \underline{e}_3 \quad \text{(properties)}$$

Deformation Mapping

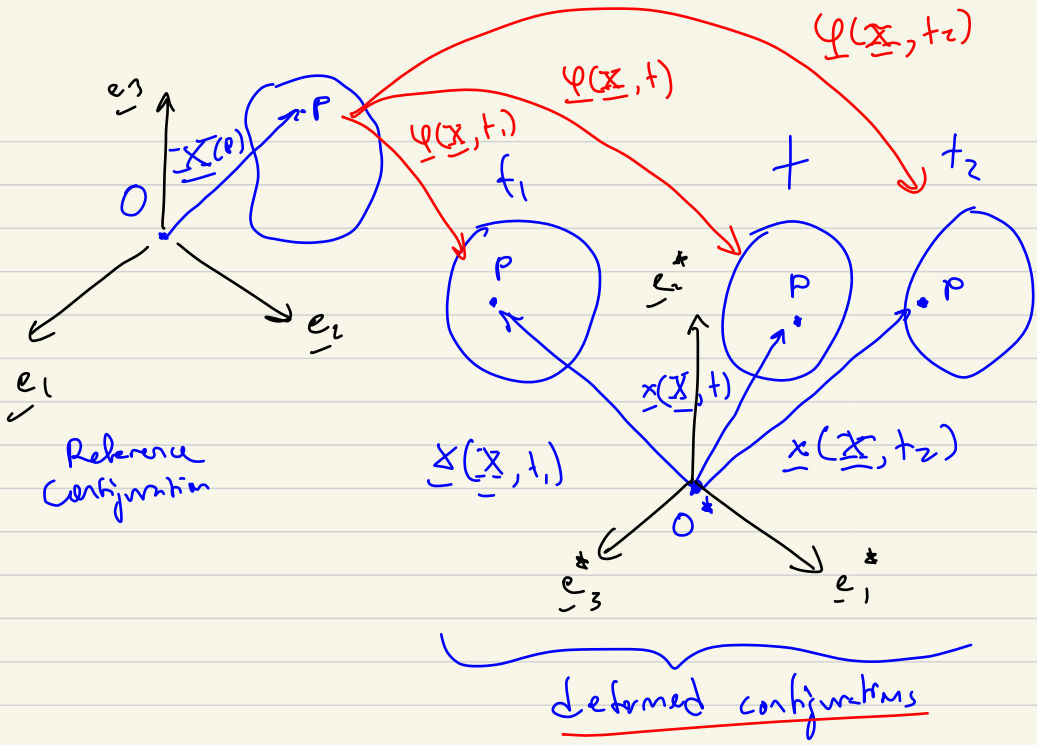
- Motion: change of the body conf. w/ time

  ie, a family of configurations $\{\mathcal{B}_t, t \in [t_1, t_2]\}$
- To describe a motion, select a convenient origin O^* and provide the position occupied at time t by the particle w/ position $\underline{X}$ in the reference configuration

(Need to map particles position from original config to new config)

$$\underline{x} = \underline{x}(\underline{X}, t) = \underline{\varphi}(\underline{X}, t)$$

- $\underline{x}$: position in space
- $\underline{X}$: label of the particle
- t: time
- $\underline{\varphi}$: deformation mapping



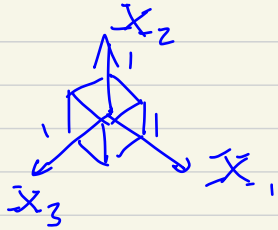
Note: In general $\{O, \underline{e}_i\}$ and $\{O^*, \underline{e}_i^*\}$ (coordinate sys) can be two different coordinate systems.

(For this course, we will always pick them to be the same)

Example

Body: $\underline{X}; \in [0,1] \longrightarrow$ a cube.

Ref config.



Example

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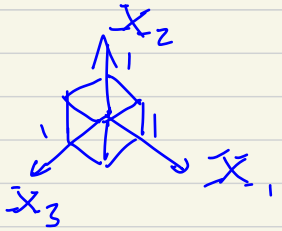
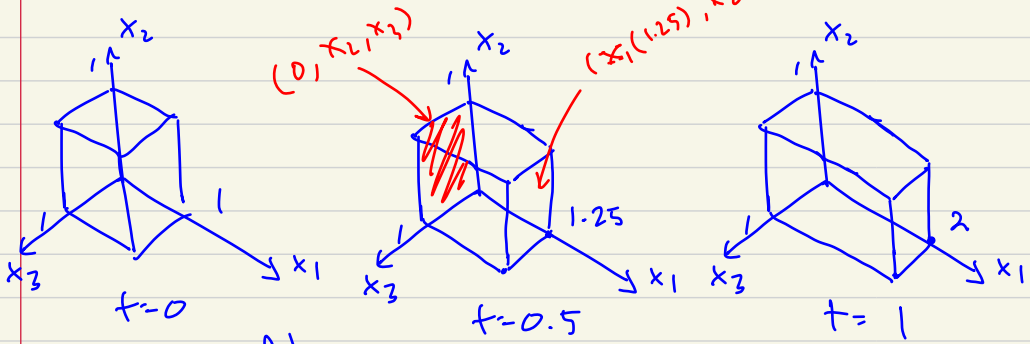
Ref config.



Def mapping: ϕ

$$\begin{cases} x_1 = X_1(1+t^2) \\ x_2 = X_2 \\ x_3 = X_3 \end{cases}$$



generally, we also want to track properties as well ... density, etc.

- In cont. mechanics problems we are usually interested in tracking the evolution of different quantities: density, stress, position, velocity, strains, internal variables, etc.

- we could express them in terms of:

a) $(\underline{X}, t)$:
- how the quantity changes following each material particle in time.
- called material or lagrangian description
- most convenient in solid mech.

b) $(\underline{x}, t)$:
- how the quantity changes in a particular point in space
- called spatial or Eulerian description
- Most convenient in fluid mech.