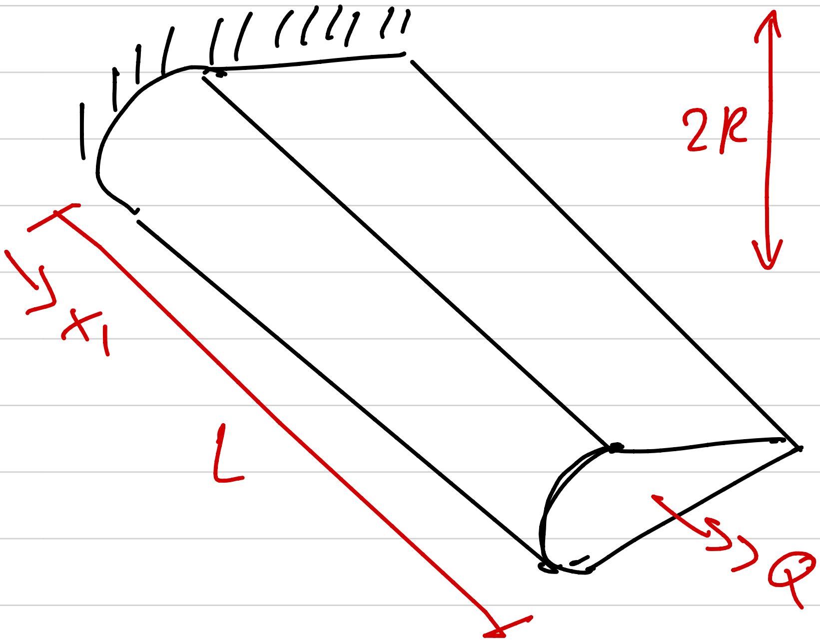
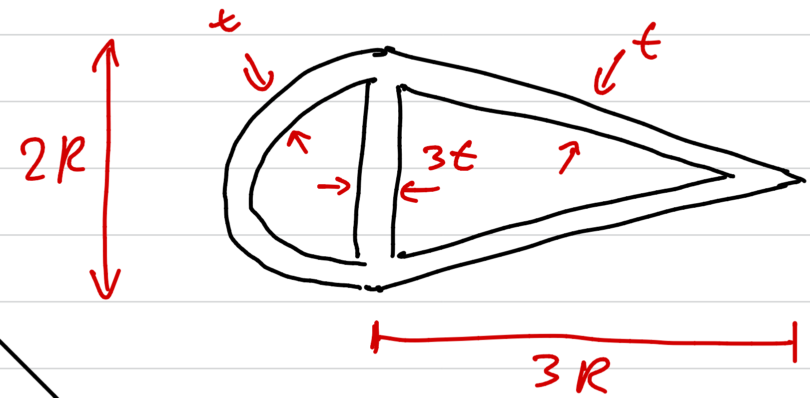
Example: Multi-Cellular Thin-Walled Beam in Torsion



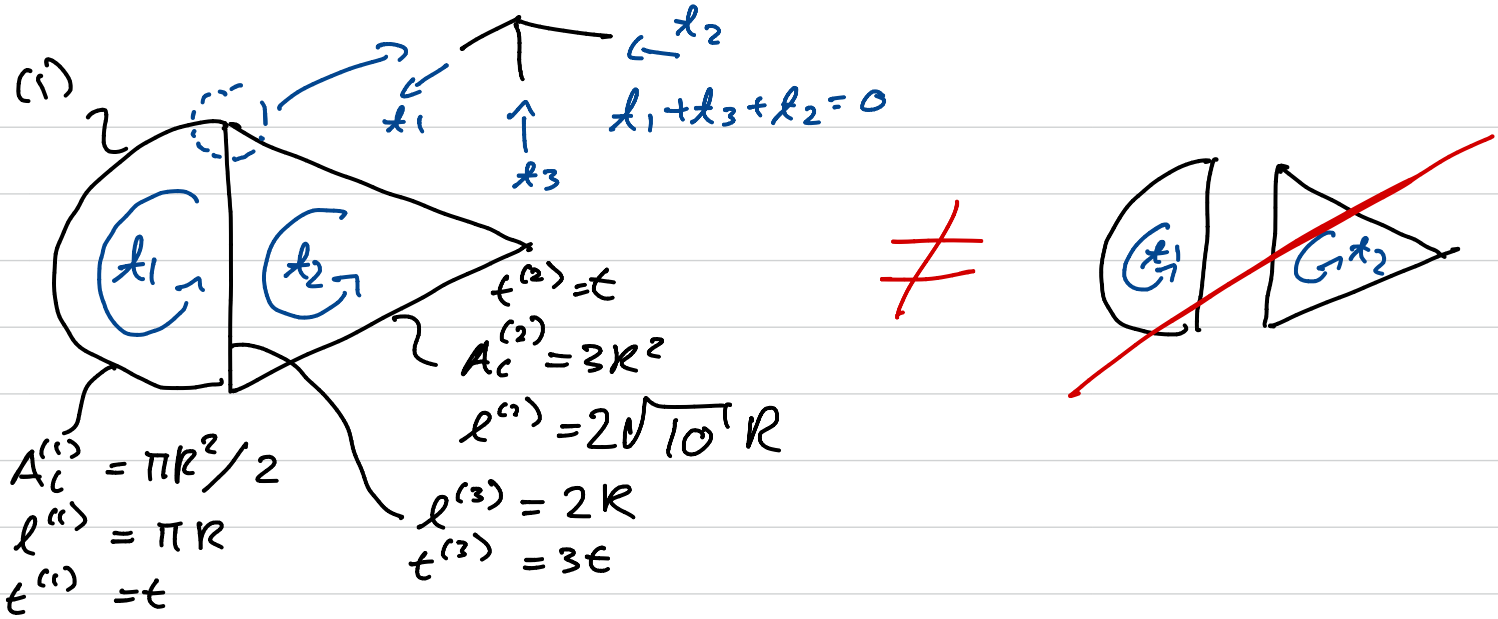


a) Find H_{11}

b) Find $\tau_{s,max}$

c) Analyze the importance of the vertical spar

d) Find the tip rotation $\Phi_1(x_1 = L)$.



$$\left.\begin{aligned} M_1 &= M_1^{(1)} + M_1^{(2)} \\ K_1 &= K_1^{(1)} = K_1^{(2)} \end{aligned}\right\} \text{Solve for } t_1 \text{ and } t_2$$

$$M_1^{(i)} = 2A_c^{(i)}\,t^{(i)}$$

$$K_1^{(i)} = \frac{M_1^{(i)}}{4A_c^2}\oint_{C_i}\frac{ds}{Gt} = \frac{t^{(i)}}{2A_c^{(i)}}\oint_{C_i}\frac{ds}{Gt}$$

$$K_1^{(1)} = \frac{1}{G\,2\,\frac{\pi R^2}{2}}\left[\frac{t_1}{t}(\pi R) + \frac{t_1 - t_2}{3t}(2R)\right]$$

$$K_1^{(1)} = \frac{1}{\pi G R t}\left[\pi t_1 + \frac{2}{3}(t_1 - t_2)\right]$$

$$K_1^{(2)} = \frac{1}{G\,2\cdot 3R^2}\left[\frac{t_2}{t}\left(2\sqrt{10}\,R\right) + \frac{t_2 - t_1}{3t}\left(2R\right)\right]$$

$$K_1^{(2)} = \frac{1}{\pi G R t}\left[t_2\,\frac{\pi}{3}\sqrt{10} + \left(t_2 - t_1\right)\frac{\pi}{9}\right]$$

$K_1 = K_1^{(1)} = K_1^{(2)}$ $\rightarrow$ Find t_1 as a function of t_2

$$t_2\left(\frac{\pi}{3}\sqrt{10} + \frac{\pi}{9}\right) - t_1\,\frac{\pi}{9}$$

$$= -t_2\,\frac{2}{3} + t_1\left(\pi + \frac{2}{3}\right)$$

$$t_1\left(\frac{\pi}{9} + \pi + \frac{2}{3}\right) = t_2\left(\frac{\pi}{3}\sqrt{10} + \frac{\pi}{9} + \frac{2}{3}\right)$$

$$\boxed{t_1 = 1.04\,t_2}$$

$$K_1 = K_1^{(1)} = \frac{1}{\pi G R t}\left[\pi\left(1.04\right) + \frac{2}{3}\left(1.04 - 1\right)\right]t_2$$

$$H_{11} = \frac{M_1}{K_1} = \frac{M_1}{K_1^{(1)}}$$

$$M_1 = 2A_c^{(1)} t_1 + 2A_c^{(2)} t_2$$

$$= \left[2\,\frac{\pi R^2}{2}(1.04) + 2(3R^2)\right] t_2$$

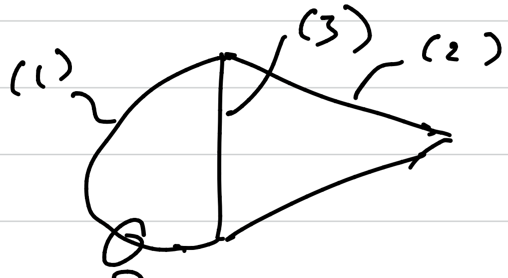
$$M_1 = (\pi(1.04) + 6) R^2 t_2$$

$$H_{11} = \frac{(\pi(1.04) + 6) R^2 \cancel{t_2}}{\left(\pi(1.04) + \frac{2}{3}(1.04 - 1)\right) \cancel{t_2}} \pi G R t$$

$$H_{11} = \left[\frac{\pi(\pi(1.04) + 6)}{\pi(1.04) + \frac{2}{3}(1.04 - 1)}\right] G R^3 t$$

$$H_{11} = 2.81\pi\, G R^3 t$$

Shear Stress



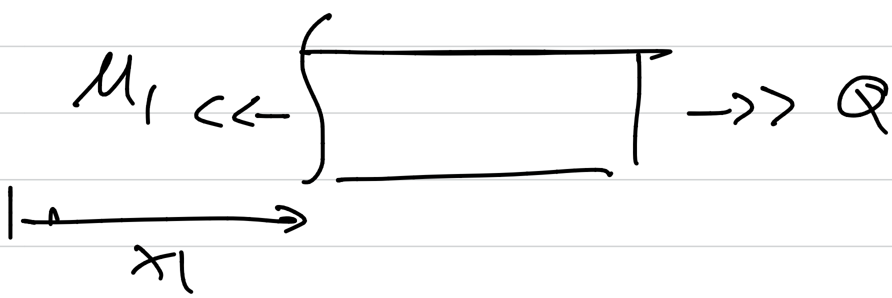
$$\tau = \frac{f^{(i)}}{t^{(i)}}$$

$$\tau^{(1)} = \frac{f_1}{t} = \frac{1.04}{t} f_2 = \tau_{s,max}$$

$$\tau^{(3)} = \frac{f_1 - f_2}{t} = \frac{0.04}{t} f_2 \approx 0 \,!$$

$$\tau^{(2)} = \frac{f_2}{t}$$

$$\boxed{M_1 = Q}$$



$$\boxed{\tau_{s,max} = \frac{1.04}{t} \frac{Q}{(\pi(1.04) + 6)R^2}}$$

d) Find ϕ_1

$$M_1 = H_{11} K_1 = H_{11} \frac{d\phi_1}{dx_1} = Q$$

$$\frac{d\phi_1}{dx_1} = \frac{Q}{H_{11}} \qquad B.C. \quad \phi_1(x_1 = 0) = 0$$

$$\phi_1 = \frac{Q}{H_{11}} x_1$$

At the tip $\phi_1(x_1 = L) = \frac{Q}{H_{11}} L$

$$\boxed{\phi_1(x_1 = L) = \frac{Q L}{(2.81\pi) G R^3 t}}$$