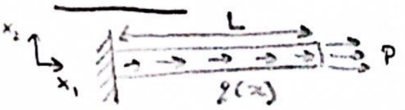
Problem 1



distributed
axial load per unit length

Young's modulus E

Cross-sectional area A

Find the corresponding displacement field by using the principle of minimum potential energy, that is, by finding the displacement field corresponding to $\delta\Pi = 0$.

1.) Assuming a uniaxial stress state ($\sigma_{11} \neq 0$, all others zero) and the full form of Hooke's law in 3D ($\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$), show that the stress reduces to $\sigma_{11} = E\varepsilon_{11}$.

Since uniaxial stress state ($\sigma_{11} \neq 0$) and assuming linear elastic isotropic solid, Hooke's Law becomes:

$$\varepsilon_{11} = \frac{1}{E}\left[\sigma_{11} - \nu(\underbrace{\sigma_{22}}_{=0} + \underbrace{\sigma_{33}}_{=0})\right]$$, $\sigma_{22} = 0$ and $\sigma_{33} = 0$ since uniaxial stress state ($\sigma_{11} \neq 0$)

$$\varepsilon_{11} = \frac{1}{E}\sigma_{11}$$

$$\sigma_{11} = E\varepsilon_{11}$$

2) Show the elastic potential for this problem:

Using Hooke's Law in 3D:

$$\sigma_{11} = E\varepsilon_{11}, \quad \varepsilon_{11} = \frac{\partial u_1}{\partial x_1}$$

$$W = \int_\Omega \frac{1}{2} E \varepsilon_{11}^2 \, d\Omega$$

$$= \int_0^L \int_A \frac{1}{2} E \left(\frac{\partial u_1}{\partial x_1}\right)^2 dA \, dx_1$$

$$= \frac{EA}{2} \int_0^L \left(\frac{\partial u_1}{\partial x_1}\right)^2 dx_1$$

$$E = \int_\Omega b_i u_i \, dV + \int_{\partial_2\Omega} t_i u_i \, dA$$

$$= A\int_0^L q(x_1) u_1 dx_1 + APu_1\big|_{x_1 = L}$$

$$\Pi = W - E, \quad u_1(x_1 = 0) = 0$$

$$= \frac{EA}{2}\int_0^L \left(\frac{\partial u_1}{\partial x_1}\right)^2 dx_1 - A\int_0^L q(x_1) u_1 dx_1 - APu_1\big|_{x_1 = L}$$

Problem 1 (continued)

3.) Using the properties of variations

$$\delta\Pi = \frac{AE}{2}\delta\left[\int_0^L \left(\frac{\partial u_1}{\partial x_1}\right)^2 dx_1\right] - A\delta\left[\int_0^L q(x_1)\, u_1 dx_1\right] - \delta\left[A P u_1 \big|_{x=L}\right]$$

Using: $\delta\left(\frac{\partial^n v}{\partial x^n}\right) = \frac{\partial^n \delta v}{\partial x^n}$

$\delta \int v\, dx = \int \delta v\, dx$

$\delta(v^n) = n v^{n-1}\delta v$

$$= 2 \times \frac{AE}{2}\int_0^L \frac{\partial u_1}{\partial x_1}\frac{\partial \delta u_1}{\partial x_1} dx_1 - A\int_0^L q(x_1)\delta u_1 dx_1 - AP\delta u_1\big|_{x_1=L}$$

$$\Rightarrow \delta\Pi = \underbrace{AE\int_0^L \frac{\partial u_1}{\partial x_1}\frac{\partial \delta u_1}{\partial x_1} dx_1}_{I} - A\int_0^L q(x_1)\delta u_1 dx_1 - AP\delta u_1\big|_{x_1=L}$$

Note: $u_1(x_1=0)=0$

4.) After integration by parts and considering that $\delta u_1 = 0$ at $x_1=0$, the first integral (I) becomes:

$$\int_0^L \frac{\partial u_1}{\partial x_1}\frac{\partial \delta u_1}{\partial x_1} dx_1 = \frac{\partial u_1}{\partial x_1}\delta u_1\Big|_{x_1=L} - \int_0^L \frac{\partial^2 u_1}{\partial x_1^2}\delta u_1 dx_1$$

$\delta\left(\frac{\partial u}{\partial x}\right) = \frac{\partial(\delta u)}{\partial x}$

$\int u\,dv = uv - \int v\,du$

$$\int_0^L \frac{\partial u_1}{\partial x_1}\frac{\partial \delta u_1}{\partial x_1} dx_1 = \int_0^L\left[\frac{\partial}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1}\delta u_1\right) - \frac{\partial^2 u_1}{\partial x_1^2}\delta u_1\right]dx_1$$

$$= \int_0^L \frac{\partial}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1}\delta u_1\right)dx_1 - \int_0^L \frac{\partial^2 u_1}{\partial x_1^2}\delta u_1 dx_1,$$

$x_1 = L,\ \delta u_1 = \delta u_1$
$x_1 = 0;\ \delta u_1 = 0$

$$= \frac{\partial u_1}{\partial x_1}\delta u_1\Big|_{x_1=L} - \int_0^L \frac{\partial^2 u_1}{\partial x_1^2}\delta u_1 dx_1$$

Problem 1 (continued)

5.) Using previous results, and considering that they must apply to all compatible variations δu_1 (in particular for which $\delta u_1|_{x_1=L} = 0$), show that $\delta\Pi = 0$ necessarily implies:

$$E\frac{\partial^2 u_1}{\partial x_1^2} + q(x_1) = 0 \quad \text{and} \quad E\frac{\partial u_1}{\partial x_1} = P \text{ at } x = L.$$

$$d\Pi = 0 = \underbrace{\cancel{A}E\int_0^L \frac{\partial u_1}{\partial x_1}\frac{\partial \delta u_1}{\partial x_1}dx_1}_{(I)} - \cancel{A}\int_0^L q(x_1)\,\delta u_1\,dx_1 - \cancel{A}P\delta u_1\Big|_{x_1=L}$$

| A cancels out. (common factor) → 0
| Sub in past result for (I)

$$0 = E\left(\frac{\partial u_1}{\partial x_1}\delta u_1\Big|_{x_1=L} - \int_0^L \frac{\partial^2 u_1}{\partial x_1^2}\delta u_1\,dx_1\right) - \int_0^L q(x_1)\,\delta u_1\,dx_1 - P\delta u_1\Big|_{x_1=L}$$

$$0 = E\frac{\partial u_1}{\partial x_1}\delta u_1\Big|_{x_1=L} - E\int_0^L \frac{\partial^2 u_1}{\partial x_1^2}\delta u_1\,dx_1 - \int_0^L q(x_1)\,\delta u_1\,dx_1 - P\delta u_1\Big|_{x_1=L}$$

| $\delta u_1|_{x_1=L} = 0$
| $\delta u_1|_{x_1=0} = 0$

$$0 = E\frac{\partial u_1}{\partial x_1}\delta u_1\Big|_{x_1=L} E\frac{\partial^2 u_1}{\partial x_1^2} - q(x_1) - P\delta u_1\Big|_{x_1=L}$$

$$P\delta u_1\Big|_{x_1=L} = E\frac{\partial u_1}{\partial x_1}\delta u_1\Big|_{x_1=L} - E\frac{\partial^2 u_1}{\partial x_1^2} - q(x_1)$$

Assuming δu_1 is arbitrary:

Eqn 1: $\quad P\,\delta u_1|_{x_1=L} = E\frac{\partial u_1}{\partial x_1}\delta u_1|_{x_1=L}$

$$\Rightarrow \quad \boxed{E\frac{\partial u_1}{\partial x_1} = P \text{ at } x = L}$$

Eqn 2: $\quad 0 = -E\frac{\partial^2 u_1}{\partial x_1^2} - q(x_1)$

$$\Rightarrow \quad \boxed{E\frac{\partial^2 u_1}{\partial x_1^2} + q(x_1) = 0}$$

Problem 1 (continued)

6.) What do these equations represent?

The first equation, $E\frac{\partial u_1}{\partial x_1} = P$ at $x_1 = L$, represents traction Boundary condition. The second equation, $E\frac{\partial^2 u_1}{\partial x_1} + q(x_1) = 0$, represents the conservation of linear momentum. They represent the equilibrium conditions at the boundary and inside the bar.

7) Solve for u_1 assuming $q(x) = \text{constant} = q$.

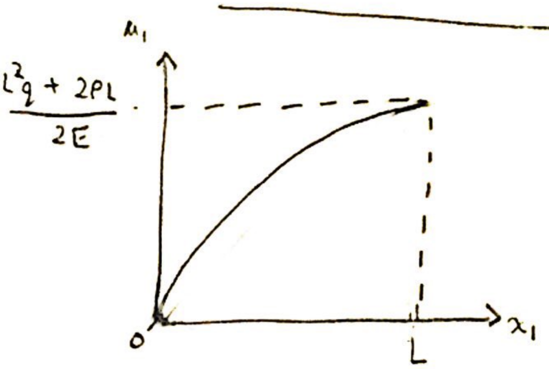
Plot.

$$E\frac{\partial^2 u_1}{\partial x_1^2} + q = 0 \Rightarrow -\frac{1}{2}\frac{q}{E}x_1^2 + C_1 x_1 + C_2$$

At $x_1 = 0$, $u_1 = 0 \Rightarrow C_2 = 0$

At $x = L$, $E\frac{\partial u_1}{\partial x_1} = P \Rightarrow C_1 = \frac{qL + P}{E}$

$$\therefore \quad u_1(x_1) = -\frac{1}{2}\frac{q x_1^2}{E} + \frac{(qL+P)}{E}x_1$$



@ $x_1 = L$:

$$u_1(x_1 = L) = -\frac{1}{2}\frac{q(L)^2}{E} + \frac{qL+P}{E}L$$

$$= -\frac{qL^2}{2E} + \frac{qL^2 + PL}{E}$$

$$= \frac{-qL^2 + 2L^2 q + 2PL}{2E}$$

$$= \frac{L^2 q + 2PL}{2E}$$

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Problem 1 (continued)

8.) Elastostatics Formulation - what would be different? Pros and cons?

The solution for the elastostatics problem is given by variation of $\Pi = 0$ ($\delta\Pi = 0$), instead of setting $d\Pi = 0$. The principle of minimum potential energy provides an alternative problem formulation to the boundary value problem in elastostatics. The principle of minimum potential energy is general and can be applicable to all problems in linear elasticity theory.

The principle of minimum potential energy is variational or weak formulation. The weak formulation contains only first derivatives of displacements, and it is the starting point (or one of them) for the finite element method. In comparison, the elastostatics formulation is a differential or strong formulation. The strong formulation contains derivatives of stress, which translate into second derivatives of displacements.

other pros and cons?

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Problem 2

Rayleigh-Ritz method used to solve Problem 1

Assume $q(x) = \text{constant} = q$

i.) First, consider an approximate solution $\hat{u} = a x_1 + b$, make sure it satisfies the displacement boundary conditions and minimize the corresponding functional. Plot this approximate solution and the exact one obtained for the previous problem. Comment on the results.

$\hat{u}_1$ is a linear function $\hat{u}_1(x_1) = a x_1 + b$

Displacement BC: $\hat{u}_1(0) = 0 \Rightarrow b = 0 \Rightarrow \hat{u}_1(x_1) = a x_1$

$$\frac{\partial \hat{u}_1}{\partial x_1} = a \quad (*)$$

Plug $(*)$ into potential:

$$\Pi = \frac{EA}{2}\int_0^L \left(\frac{\partial u_1}{\partial x_1}\right)^2 dx_1 - A\int_0^L q(x_1) u_1 dx_1 - APu_1\Big|_{x_1=L}, \quad q(x_1) = q \text{ and } (*)$$

Approximate;
$$\hat{\Pi} = \frac{EA}{2}\int_0^L a^2\, dx_1 - A\int_0^L q\, a x_1\, dx_1 - AP a x_1$$

$$\hat{\Pi} = \frac{1}{2} A E a^2 L - \frac{1}{2} a q A L^2 - APaL$$

$$\frac{\partial \hat{\Pi}}{\partial a} = 0 \Rightarrow AEaL - \frac{1}{2} q A L^2 - APL = 0$$

$$\Rightarrow a = \frac{qL + 2P}{2E} \quad (**)$$

$\therefore$ $(**)$ into $\hat{u}_1(x_1) = a x_1$.

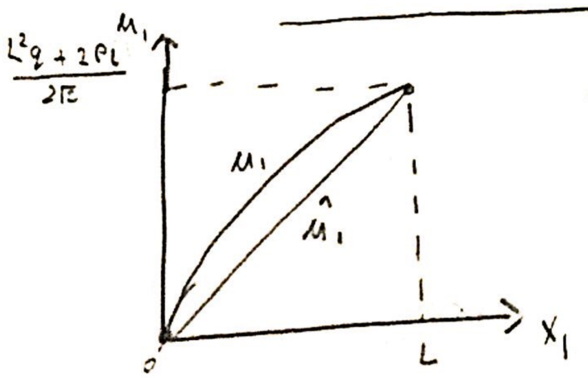
$$\Rightarrow \boxed{\hat{u}_1(x_1) = \left(\frac{qL + 2P}{2E}\right) x_1}$$

@ $x_1 = 0$

$\hat{u}_1(x_1 = 0) = 0$

@ $x_1 = L$

$$\hat{u}_1(x_1 = L) = \frac{L^2 q + 2PL}{2E}$$



$\hat{u}_1$ approx solution is constant while u_1 exact solution changes linearly.

Problem 2 (continued)

2.) Now, consider an approximate solution of the form $\hat{u} = cx_1^2 + dx_1 + e$, and proceed as before. Plot this new solution versus the previous ones (exact and approx.) and comment about your results.

$\hat{u}_1(x_1) = cx_1^2 + dx_1 + e$

Displacement BC's: $\hat{u}_1(0) = e = 0 \Rightarrow \hat{u}_1(x_1) = cx_1^2 + dx_1 \quad (0)$

$$\frac{\partial \hat{u}_1}{\partial x_1} = 2cx_1 + d \quad (*)$$

Plug (*) into potential:

$$\hat{\pi} = \frac{AE}{2}\int_0^L \left(\frac{\partial \hat{u}_1}{\partial x_1}\right)^2 dx - A\int_0^L q\,\hat{u}_1\,dx$$

, with (*) and $q(x) = \text{constant} = q$:

$$= \frac{AE}{2}\int_0^L (2cx_1 + d)^2 dx - A\int_0^L q\,(cx_1^2 + dx_1)\,dx - AP(cx_1^2 + dx_1)\Big|_{x_1=L}$$

$$= \frac{AE}{2}\int_0^L (d^2 + 4c^2x_1^2 + 4cx_1 d)\,dx - A\int_0^L q(cx_1^2 + dx_1)\,dx - AP(cx_1^2 + dx_1)\Big|_{x_1=L}$$

$$= \frac{AE}{2}\left(d^2x_1 + \frac{4c^2x_1^3}{3} + \frac{4cx_1^2 d}{2}\right)\Big|_{x_1=L} - Aq\left(\frac{cx_1^3}{3} + \frac{dx_1^2}{2}\right)\Big|_{x_1=L} - AP(cx_1^2 + dx_1)\Big|_{x_1=L}$$

$$= AE\left(\frac{d^2L}{2} + \frac{4c^2L^3}{6} + \frac{4cL^2d}{4}\right) - A\left(\frac{cL^3}{3} + \frac{dL^2}{2}\right) - AP(cL^2 - dL)$$

$$= A\left(\frac{ELd^2}{2} + \frac{2EL^3c^2}{3} + EL^2cd - \frac{qL^3c}{3} - \frac{qL^2d}{2} - P(cL^2 - dL)\right)$$

$$= \frac{1}{6}AL(3Ed^2 + 4Ec^2L^2 + 6ELcd - 2qL^2c - 3qLd - 6PcL + 6Pd)$$

$$= \frac{1}{6}AL(4Ec^2L^2 + 6ELcd + 3Ed^2 - 2qL^2c - 3qLd - 6PcL + 6Pd)$$

To minimize, took derivative w.r.t. c and d.

$$\frac{\partial \hat{\pi}}{\partial c} = 0, \quad \frac{\partial \hat{\pi}}{\partial d} = 0 \quad \begin{cases} \frac{8Ec^3L^2}{3} + 6ELd - 2qL^2c - 6PL = 0 \\ 6ELc + 6Ed - 3qL + 6P = 0 \end{cases} \Rightarrow \quad c = -\frac{1}{2}\cdot\frac{q}{E} \quad (**), \qquad d = \frac{qL+P}{E} \quad (***)$$

Plug in (**) and (***) into (0):

$$\Rightarrow \hat{u}_1(x_1) = -\frac{1}{2}q\frac{x_1^2}{E} + \frac{qL+P}{E}x_1$$

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Problem 2 (continued)

With derived : $\hat{u}_1(x_1) = -\frac{1}{2} q \frac{x_1^2}{E} + \frac{qL+P}{E} x_1$ (quadratic)

@ $x_1 = 0$

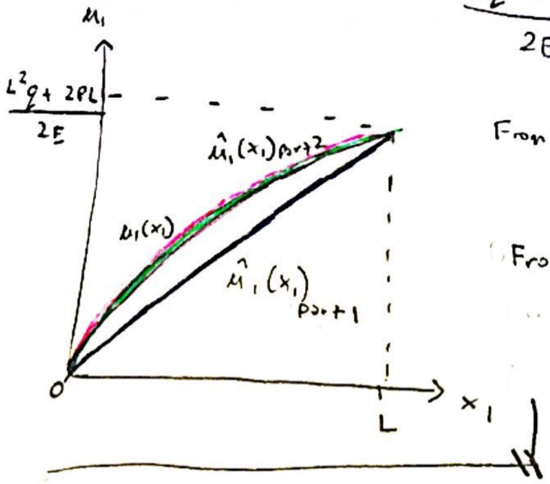
$$\hat{u}_1(x_1 = 0) = 0$$

@ $x_1 = L$

$$\hat{u}_1(x_1 = L) = \frac{-L^2 q}{2E} + \frac{qL^2 + PL}{E}$$

$$= \frac{-L^2 q + 2qL^2 + 2PL}{2E}$$

$$= \frac{L^2 q + 2PL}{2E}$$



From Problem 1:

- $u_1(x_1) = -\frac{1}{2} q \frac{x_1^2}{E} + \frac{qL+P}{E} x_1$ (exact)

From Problem 2:

- $\hat{u}_1(x_1)_{part1} = \left(\frac{qL + 2P}{2E}\right) x_1$ (Approx 1)
- $\hat{u}_1(x_1)_{part2} = -\frac{1}{2} q \frac{x_1^2}{E} + \frac{qL+P}{E} x_1$ (Approx 2)

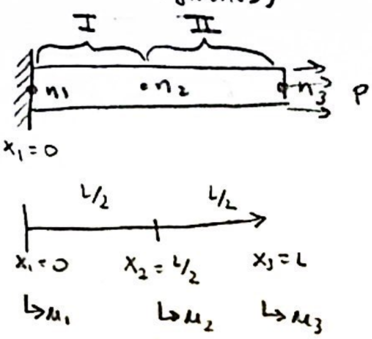
Approx 2 has the same curve as exact $u_1(x_1)$. Approx 2 and the exact $u_1(x_1)$ have quadratic solutions while Approx 1 has a constant solution. This shows that Approx 2 is sufficient to get a realistic solution.

Problem 3

Piece-wise linear approximation to solve Problem 1.
Divide the bar in n segments, assume the displacement field is linear with each segment, writing the linear function in terms of the (unknown) displacements at the end points of each segment. We call these points nodes, and the corresponding unknown values nodal displacements.

1.) Solve for in particular the cases for $n = 2, 4, 8$, and 16.

i.) $n=2$ (2 segments)



BC's: $\hat{u}(0) = 0 \Rightarrow u_1 = 0$

Assume linear displacement b/w n_1 and n_2, and n_2 and n_3.

$$\hat{u} = \begin{cases} \underbrace{\cancel{u_1}^{\,0} + \frac{(u_2 - \cancel{u_1}^{\,0})}{L/2}x}_{\hat{u}^{I}}, & 0 \le x < \frac{L}{2} \\ \underbrace{u_2 + \frac{(u_3 - u_2)}{L/2}\left(x - \frac{L}{2}\right)}_{\hat{u}^{II}}, & \frac{L}{2} \le x \le L \end{cases}$$

For $\Pi = \frac{EA}{2}\int_0^L \left(\frac{\partial \hat{u}}{\partial x}\right)^2 dx - A\int_0^L q\,\hat{u}\,dx - AP\hat{u}\big|_{x=L}$

$$u^{I} = \frac{2u_2}{L}x, \quad 0 \le x \le \frac{L}{2} \quad \Rightarrow \quad \frac{\partial \hat{u}^{I}}{\partial x} = \frac{2u_2}{L}$$

$$u^{II} = u_2 + \frac{(u_3 - u_2)}{L/2}\left(x - \frac{L}{2}\right), \quad \frac{L}{2} \le x \le L \quad \Rightarrow \quad \frac{\partial \hat{u}^{II}}{\partial x} = \frac{2(u_3 - u_2)}{L}$$

$$\Pi = \frac{EA}{2}\left[\int_0^{L/2}\left(\frac{2u_2}{L}\right)^2 dx + \int_{L/2}^{L}\left(\frac{2(u_3 - u_2)}{L}\right)^2 dx\right] - A\left[\int_0^{L/2} q\left(\frac{2u_2 x}{L}\right)dx + \int_{L/2}^{L} q\left(u_2 + \underbrace{\frac{(u_3 - u_2)}{L/2}\left(x - \frac{L}{2}\right)}_{\frac{2(u_3-u_2)}{L}x \,-\, \frac{2(u_3-u_2)}{L}\frac{L}{2}}\right)dx\right]$$

$$- AP\left(u_2 + \frac{(u_3 - u_2)}{L/2}\left(x - L/2\right)\right)\Big|_{x=L}$$

$x^2/2$

$$= \frac{EA}{2}\left[\frac{4u_2^2}{L^2}\left(\frac{L}{2} - 0\right) + \frac{4(u_3 - u_2)^2}{L^2}\left(L - \frac{L}{2}\right)\right] - Aq\left[\frac{u_2}{L}\left(\frac{L}{2}\right)^2 + \frac{u_2 L}{2} + \frac{2(u_3 - u_2)}{L}\cdot\frac{L^2}{8} - \frac{u_3 L}{2} - \frac{u_2 L}{2}\right]$$

$$- AP(u_3)$$

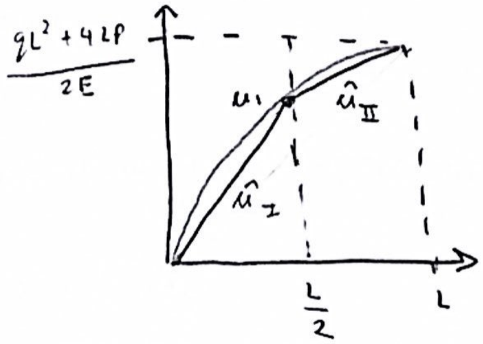
$$\Rightarrow \Pi = \frac{A}{4L}\left(8Eu_2^2 + 4Eu_3^2 - 8Eu_2u_3 - 2qL^2u_2 - qL^2u_3\right) - APu_3$$

$$\frac{\partial \Pi}{\partial u_2} = \frac{A}{2L}\left(8Eu_2 - 4Eu_3 - qL^2\right) = 0 \Rightarrow u_2 = \frac{3}{8}\frac{qL^2}{E}$$

$$\frac{\partial \Pi}{\partial u_3} = \frac{A}{4L}\left(8Eu_3 - 8Eu_2 - qL^2 - 4LP\right) = 0 \Rightarrow u_3 = \frac{qL^2 + 4LP}{2E}$$

Problem 3 (continued)

1.) $n = 2$



For the remaining plots, as n increases, the closer the piece-wise function is to the exact u_1. Therefore, $n = 16$ has a more accurate approximation than $n = 2$.

plots for $n = 4, 8, 16$?

comment of Π vs. n | plot for Π vs. n

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Problem 3 Results

N = 2

Potential =

```
(A*E*((2*(u2 - u3)^2)/L + (2*u2^2)/L))/2 - A*P*u3 - A*q*((L*u2)/4 + (L*(u2 + u3))/4)
```

potential_dU2 =

```
(A*E*((2*(2*u2 - 2*u3))/L + (4*u2)/L))/2 - (A*L*q)/2
```

potential_dU3 =

```
- A*P - (A*L*q)/4 - (A*E*(2*u2 - 2*u3))/L
```

N = 4

Potential =

```
(A*E*((16*(u2 - u3)^2)/L + (16*(u3 - u4)^2)/L + (16*(u4 - u5)^2)/L + (16*u2^2)/L))/2 - A*P*(u4 + (8*(u4 - u5)*((3*L)/4 - x))/L) - A*q*((L*u2)/4 + (L*u3)/4 + (L*u4)/4 + (L*u5)/4)
```

potential_dU2 =

```
(A*E*((16*(2*u2 - 2*u3))/L + (32*u2)/L))/2 - (A*L*q)/4
```

potential_dU3 =

```
- (A*L*q)/4 - (A*E*((16*(2*u2 - 2*u3))/L - (16*(2*u3 - 2*u4))/L))/2
```

potential_dU4 =

```
- A*P*((8*((3*L)/4 - x))/L + 1) - (A*L*q)/4 - (A*E*((16*(2*u3 - 2*u4))/L - (16*(2*u4 - 2*u5))/L))/2
```

N = 8

Potential =

(A*E*((8*(u2 - u3)^2)/L + (8*(u3 - u4)^2)/L + (8*(u4 - u5)^2)/L + (8*(u6 - u7)^2)/L + (8*(u7 - u8)^2)/L + (8*(u8 - u9)^2)/L + (L*(9*u5 - 7*u6))/16 + (8*u2^2)/L))/2 - A*P*(u8 + (8*(u8 - u9)*((7*L)/8 - x))/L) - A*q*((L*u2)/16 - L*(3*u7 - 4*u8) + (3*L*(3*u8 - 4*u9))/4 + (L*(u2 + u3))/16 + (L*(u3 + u4))/16 + (L*(u4 + u5))/16 + (L*(u5 + u6))/16 + (L*(u6 + u7))/16)

potential_dU2 =

(A*E*((8*(2*u2 - 2*u3))/L + (16*u2)/L))/2 - (A*L*q)/8

potential_dU3 =

- (A*L*q)/8 - (A*E*((8*(2*u2 - 2*u3))/L - (8*(2*u3 - 2*u4))/L))/2

potential_dU4 =

- (A*L*q)/8 - (A*E*((8*(2*u3 - 2*u4))/L - (8*(2*u4 - 2*u5))/L))/2

potential_dU5 =

(A*E*((9*L)/16 - (8*(2*u4 - 2*u5))/L))/2 - (A*L*q)/8

potential_dU6 =

- (A*E*((7*L)/16 - (8*(2*u6 - 2*u7))/L))/2 - (A*L*q)/8

potential_dU7 =

(47*A*L*q)/16 - (A*E*((8*(2*u6 - 2*u7))/L - (8*(2*u7 - 2*u8))/L))/2

potential_dU8 =

- A*P*((8*((7*L)/8 - x))/L + 1) - (25*A*L*q)/4 - (A*E*((8*(2*u7 - 2*u8))/L - (8*(2*u8 - 2*u9))/L))/2

```
%HMW 7 - AE 6114
clear;
clc;

n = 8; %input n = 2, 4, 8
u1 = 0;
syms u2 u3 u4 u5 u6 u7 u8 u9 u10 u11 u12 u13 u14 u15 u16 u17
syms L x A E q int1 fun1 P

if n == 2 % 2 segments
    function1 = u1 + ((u2 - u1)/(L/2))*x
    function2 = u2 + ((u3 - u2)/(L/2))*(x - (L/2))
    limit1 = 0
    limit2 = L/2
    limit3 = L/2
    limit4 = L
    %Part 1
    Du_dx_1 = diff(function1, x)
    Du_dx_2 = diff(function2, x)
    function1_1 = (Du_dx_1)^2;
    integrate1_1 = int(function1_1, x,limit1,limit2)
    function1_2 = (Du_dx_2)^2;
    integrate1_2 = int(function1_2, x, limit3, limit4)
    %Part 2
    function2_1 = function1
    intgrate2_1 = int(function2_1, x, limit1, limit2)
    function2_2 = function2
    integrate2_2 = int(function2_2, x, limit3, limit4)

    Potential = (A*E/2)*(integrate1_1 + integrate1_2) - A*q*(intgrate2_1 +
integrate2_2) - A*P*(u2 + ((u3 - u2)/(L/2))*(L - (L/2)))
    potential_dU2 = diff(Potential, u2)
    potential_dU3 = diff(Potential, u3)

elseif n == 4 % 4 segments
    function1 = u1 + ((u2 - u1)/(L/8))*x
    function2 = u2 + ((u3 - u2)/(L/8))*(x - (L/4))
    function3 = u3 + ((u4 - u3)/(L/8))*(x - (L/2))
    function4 = u4 + ((u5 - u4)/(L/8))*(x - (3*L/4))
    limit1 = 0
    limit2 = L/4
    limit3 = L/4
    limit4 = L/2
    limit5 = L/2
    limit6 = (3*L)/4
    limit7 = (3*L)/4
    limit8 = L
    %Part 1
    Du_dx_1 = diff(function1, x)
    Du_dx_2 = diff(function2, x)
    Du_dx_3 = diff(function3, x)
    Du_dx_4 = diff(function4, x)
    function1_1 = (Du_dx_1)^2;
    integrate1_1 = int(function1_1, x,limit1,limit2)
    function1_2 = (Du_dx_2)^2;
```

```
        integrate1_2 = int(function1_2, x, limit3, limit4)
        function1_3 = (Du_dx_3)^2;
        integrate1_3 = int(function1_3, x, limit5, limit6)
        function1_4 = (Du_dx_4)^2;
        integrate1_4 = int(function1_4, x, limit7, limit8)
        %Part 2
        function2_1 = function1
        intgrate2_1 = int(function2_1, x, limit1, limit2)
        function2_2 = function2
        integrate2_2 = int(function2_2, x, limit3, limit4)
        function2_3 = function3
        integrate2_3 = int(function2_3, x, limit5, limit6)
        function2_4 = function4
        integrate2_4 = int(function2_4, x, limit7, limit8)

    Potential = (A*E/2)*(integrate1_1 + integrate1_2 + integrate1_3 +
integrate1_4) - A*q*(intgrate2_1 + integrate2_2 + integrate2_3 +
integrate2_4) - A*P*(u4 + ((u5 - u4)/(L/8))*(x - (3*L/4)))
    potential_dU2 = diff(Potential, u2)
    potential_dU3 = diff(Potential, u3)
    potential_dU4 = diff(Potential, u4)
elseif n == 8 % 8 segments
    function1 = u1 + ((u2 - u1)/(L/8))*x
    function2 = u2 + ((u3 - u2)/(L/8))*(x - (L/8))
    function3 = u3 + ((u4 - u3)/(L/8))*(x - (L/4))
    function4 = u4 + ((u5 - u4)/(L/8))*(x - (3*L/8))
    function5 = u5 + ((u6 - u5)/(L/8))*(x - (L/2))
    function6 = u6 + ((u7 - u6)/(L/8))*(x - (5*L/8))
    function7 = u7 + ((u8 - u7)/(L/8))*(x - (3*L/4))
    function8 = u8 + ((u9 - u8)/(L/8))*(x - (7*L/8))
    limit1 = 0
    limit2 = L/8
    limit3 = L/8
    limit4 = L/4
    limit5 = L/4
    limit6 = (3*L)/8
    limit7 = (3*L)/8
    limit8 = L/2
    limit9 = L/2
    limit10 = (5*L)/8
    limit11 = (5*L)/8
    limit12 = (3*L)/4
    limit13 = (3*L)/4
    limit14 = (7*L)/4
    limit15 = (7*L)/4
    limit16 = L
    %Part 1
    Du_dx_1 = diff(function1, x)
    Du_dx_2 = diff(function2, x)
    Du_dx_3 = diff(function3, x)
    Du_dx_4 = diff(function4, x)
    Du_dx_5 = diff(function5, x)
    Du_dx_6 = diff(function6, x)
    Du_dx_7 = diff(function7, x)
    Du_dx_8 = diff(function8, x)
    function1_1 = (Du_dx_1)^2;
    integrate1_1 = int(function1_1, x,limit1,limit2)
```

```
    function1_2 = (Du_dx_2)^2;
    integrate1_2 = int(function1_2, x, limit3, limit4)
    function1_3 = (Du_dx_3)^2;
    integrate1_3 = int(function1_3, x, limit5, limit6)
    function1_4 = (Du_dx_4)^2;
    integrate1_4 = int(function1_4, x, limit7, limit8)
    function1_5 = (Du_dx_5)^2;
    integrate1_5 = int(function5, x,limit1,limit2)
    function1_6 = (Du_dx_6)^2;
    integrate1_6 = int(function1_6, x, limit3, limit4)
    function1_7 = (Du_dx_7)^2;
    integrate1_7 = int(function1_7, x, limit5, limit6)
    fununction1_8 = (Du_dx_8)^2;
    integrate1_8 = int(fununction1_8, x, limit7, limit8)
    %Part 2
    function2_1 = function1
    intgrate2_1 = int(function2_1, x, limit1, limit2)
    function2_2 = function2
    integrate2_2 = int(function2_2, x, limit3, limit4)
    function2_3 = function3
    integrate2_3 = int(function2_3, x, limit5, limit6)
    function2_4 = function4
    integrate2_4 = int(function2_4, x, limit7, limit8)
    function2_5 = function5
    integrate2_5 = int(function2_5, x, limit9, limit10)
    function2_6 = function6
    integrate2_6 = int(function2_6, x, limit11, limit12)
    function2_7 = function7
    integrate2_7 = int(function2_7, x, limit13, limit14)
    function2_8 = function8
    integrate2_8 = int(function2_8, x, limit15, limit16)

    Potential = (A*E/2)*(integrate1_1 + integrate1_2 + integrate1_3 +
integrate1_4 + integrate1_5 + integrate1_6 + integrate1_7 + integrate1_8) -
A*q*(intgrate2_1 + integrate2_2 + integrate2_3 + integrate2_4 + integrate2_5
+ integrate2_6 + integrate2_7 + integrate2_8) - A*P*(u8 + ((u9 -
u8)/(L/8))*(x - (7*L/8)))
    potential_dU2 = diff(Potential, u2)
    potential_dU3 = diff(Potential, u3)
    potential_dU4 = diff(Potential, u4)
    potential_dU5 = diff(Potential, u5)
    potential_dU6 = diff(Potential, u6)
    potential_dU7 = diff(Potential, u7)
    potential_dU8 = diff(Potential, u8)
end
```