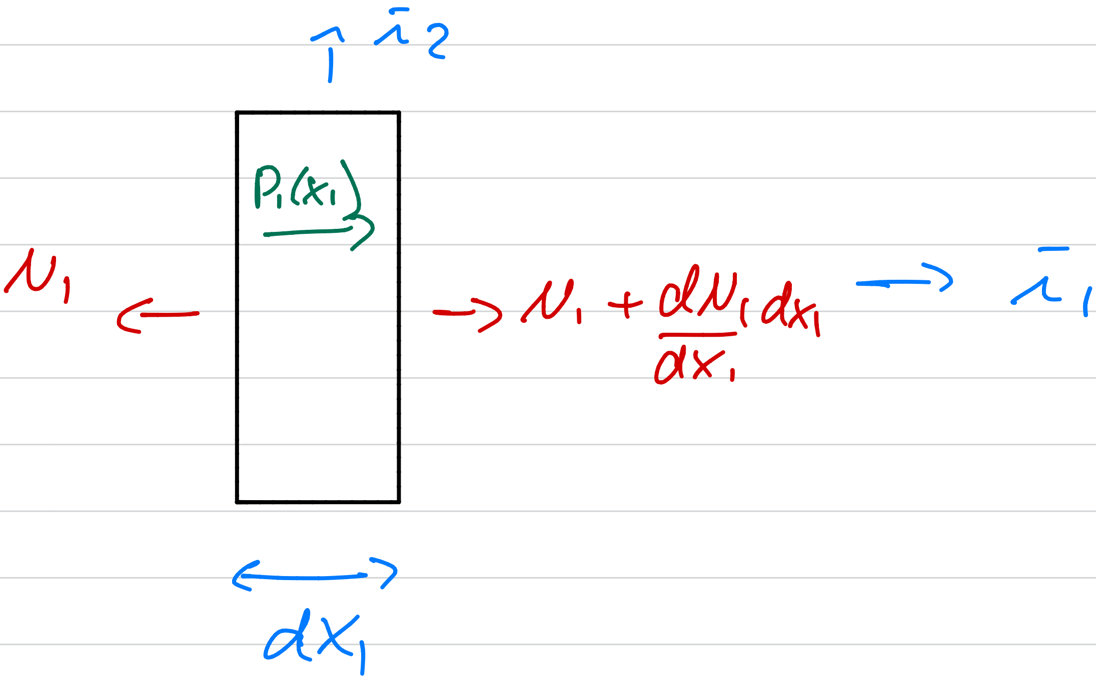
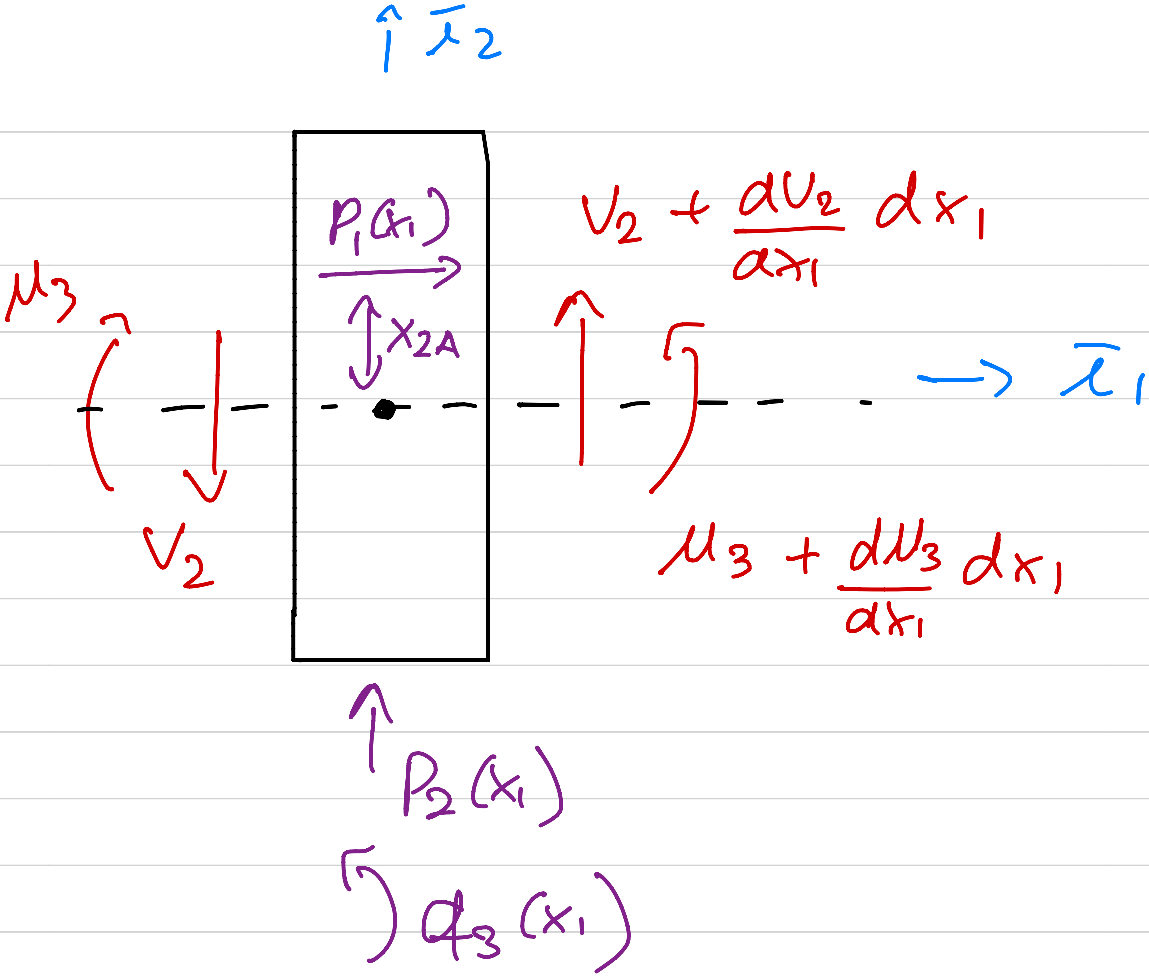
EQUILIBRIUM EQUATIONS.



$$N_1 + \frac{dN_1}{dx_1} dx_1 - N_1 + P_1(x_1) \cdot dx_1 = 0$$

$$\frac{dN_1}{dx_1} = -P_1(x_1)$$



$$\Sigma F_2 : \quad V_2 + \frac{dV_2}{dx_1} dx_1 - V_2 + p_2(x_1) \cdot dx_1 = 0$$

$$\longrightarrow \quad \frac{dV_2}{dx_1} = -p_2(x_1)$$

TRANSVERSE BALANCE

$$\Sigma M_3 : \quad \cancel{M_3} + \frac{dM_3}{dx_1} dx_1 - \cancel{M_3}$$

$$+ \frac{dx_1}{2}\left(V_2 + \overset{\approx 0}{\cancel{\frac{dV_2}{dx_1} dx_1}} + V_2 \right)$$

$$+ q_3(x_1) dx_1 - x_{2A}\, p_1(x_1)$$

$$\rightarrow \frac{dM_3}{dx_1} + V_2 = -q_3(x_1) + x_{2A}\, p_1(x_1)$$

BENDING Balance

COMBINEING

$$\begin{cases} \dfrac{d^2 M_3}{dx_1^2} = p_2(x_1) + \dfrac{d}{dx_1}\left[x_{2A}\, p_1(x_1) - q_3(x_1)\right] \\[2ex] \dfrac{d^2 M_2}{dx_1^2} = -p_3(x_1) - \dfrac{d}{dx_1}\left[x_{3A}\, p_1(x_1) + q_2(x_1)\right] \end{cases}$$

RECALL

$$\begin{bmatrix} N_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} S & 0 & 0 \\ 0 & H_{22}^c & -H_{23}^c \\ 0 & -H_{23}^c & H_{33}^c \end{bmatrix} \begin{bmatrix} \bar{\epsilon}_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix}$$

$$\bar{\epsilon}_1 = \frac{d\bar{u}_1}{dx_1}, \quad \kappa_2 = -\frac{d^2 u_3}{dx_1^2}, \quad \kappa_3 = \frac{d^2 u_2}{dx_1^2}$$

$$N_1 = S \frac{d\bar{u}_1}{dx_1}$$

$$M_2 = -H_{22}^c \frac{d^2 u_3}{dx_1^2} - H_{23}^c \frac{d^2 u_2}{dx_1^2}$$

$$M_3 = H_{23}^c \frac{d^2 u_3}{dx_1^2} + H_{33}^c \frac{d^2 u_2}{dx_1^2}$$

$$\frac{d}{dx_1}\left(S\frac{d\bar{u}_1}{dx_1}\right) = -p_1(x_1)$$

$$\frac{d^2}{dx_1^2}\left[H_{23}^c\frac{d^2u_3}{dx_1^2} + H_{33}^c\frac{d^2u_2}{dx_1^2}\right]$$

$$= p_2(x_1) + \frac{d}{dx_1}\left[x_{2A}\,p_1(x_1) - q_3(x_1)\right]$$

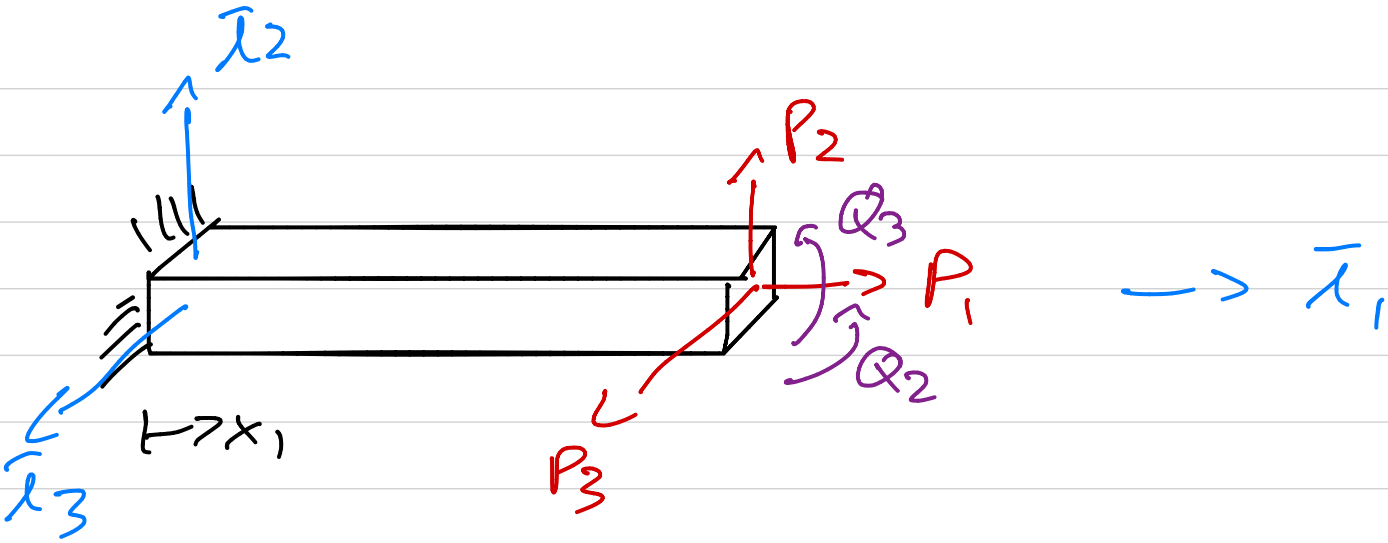
$$\frac{d^2}{dx_1^2}\left[H_{22}^c\frac{d^2u_3}{dx_1^2} + H_{23}^c\frac{d^2u_2}{dx_1^2}\right]$$

$$= p_3(x_1) + \frac{d}{dx_1}\left[x_{3A}\,p_1(x_1) + q_2(x_1)\right]$$

→ 3 COUPLED DIFFERENTIAL EQUATIONS FOR $\bar{u}_1$, u_2, u_3

→ 2ND ORDER IN $\bar{u}_1$ AND 4TH ORDER IN u_2 & u_3.

→ THERE ARE 10 BCS.



@ $x_1 = 0$ $\quad u_1 = u_2 = u_3 = 0$

$$\frac{du_2}{dx_1} = \frac{du_3}{dx_1} = 0$$

@ $x_1 = L$ $\quad N_1 = P_1 \,,\; V_2 = P_2$

$$V_3 = P_3$$

$$M_2 = Q_2 + x_{3A} P_1$$

$$M_3 = Q_3 - x_{2A} P_1$$

$\rightarrow$ $$S \frac{d\bar{u}_1}{dx_1} = P_1 \qquad \frac{d\bar{u}_1}{dx_1} = \frac{P_1}{S}$$

$\rightarrow$ $$- H_{22}^{c} \frac{d^2 u_3}{dx_1^2} - H_{23}^{c} \frac{d^2 u_2}{dx_1^2} = Q_2 + x_{3A} P_1$$

$$H_{23}^{C}\frac{d^2u_3}{dx_1^2} + H_{33}^{C}\frac{d^2u_2}{dx_1^2} = Q_3 - x_{2A}P_1$$

Recall:

$$\frac{du_3}{dx_1} + V_2 = -\cancel{q_3(x_1)} + \cancel{x_{2A}P_1(x_1)}$$

$$\frac{d}{dx_1}\left(H_{23}^{C}\frac{d^2u_3}{dx_1^2} + H_{33}^{C}\frac{d^2u_2}{dx_1^2}\right) = -P_2$$

$$\frac{d}{dx_1}\left(-H_{22}^{C}\frac{d^2u_3}{dx_1^2} - H_{23}^{C}\frac{d^2u_2}{dx_1^2}\right) = P_3$$

THE NEUTRAL AXIS

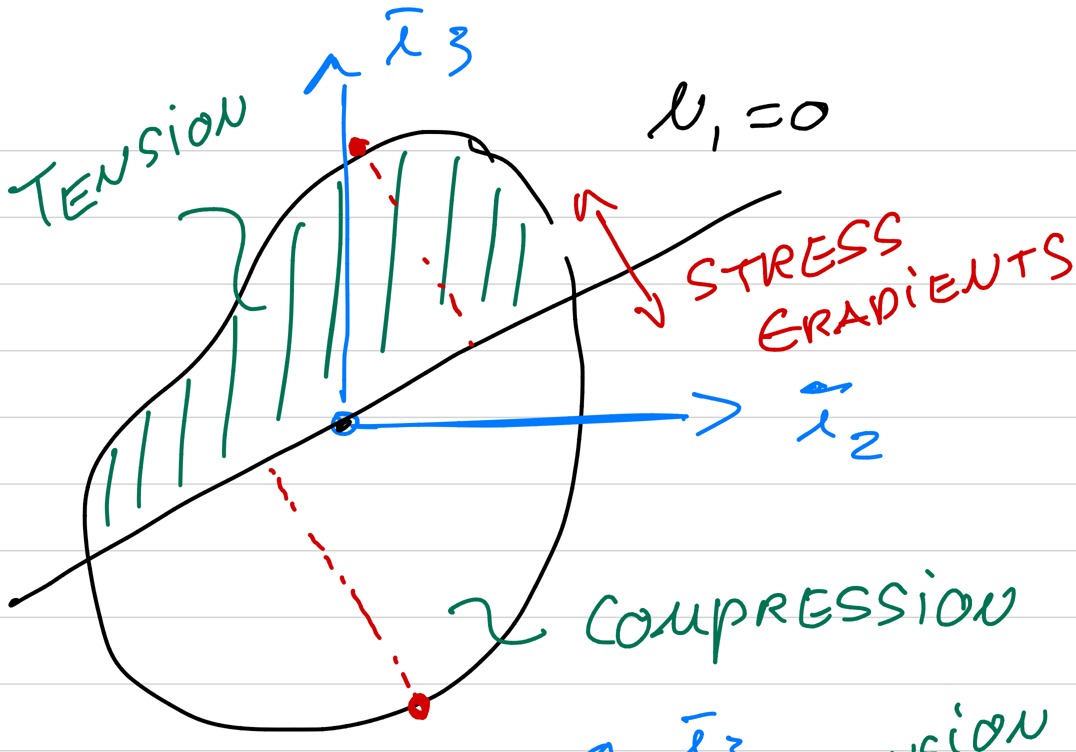
* LINE DEFINED BY THE VANISHING OF THE AXIAL STRESS

$$\sigma_1 = 0$$

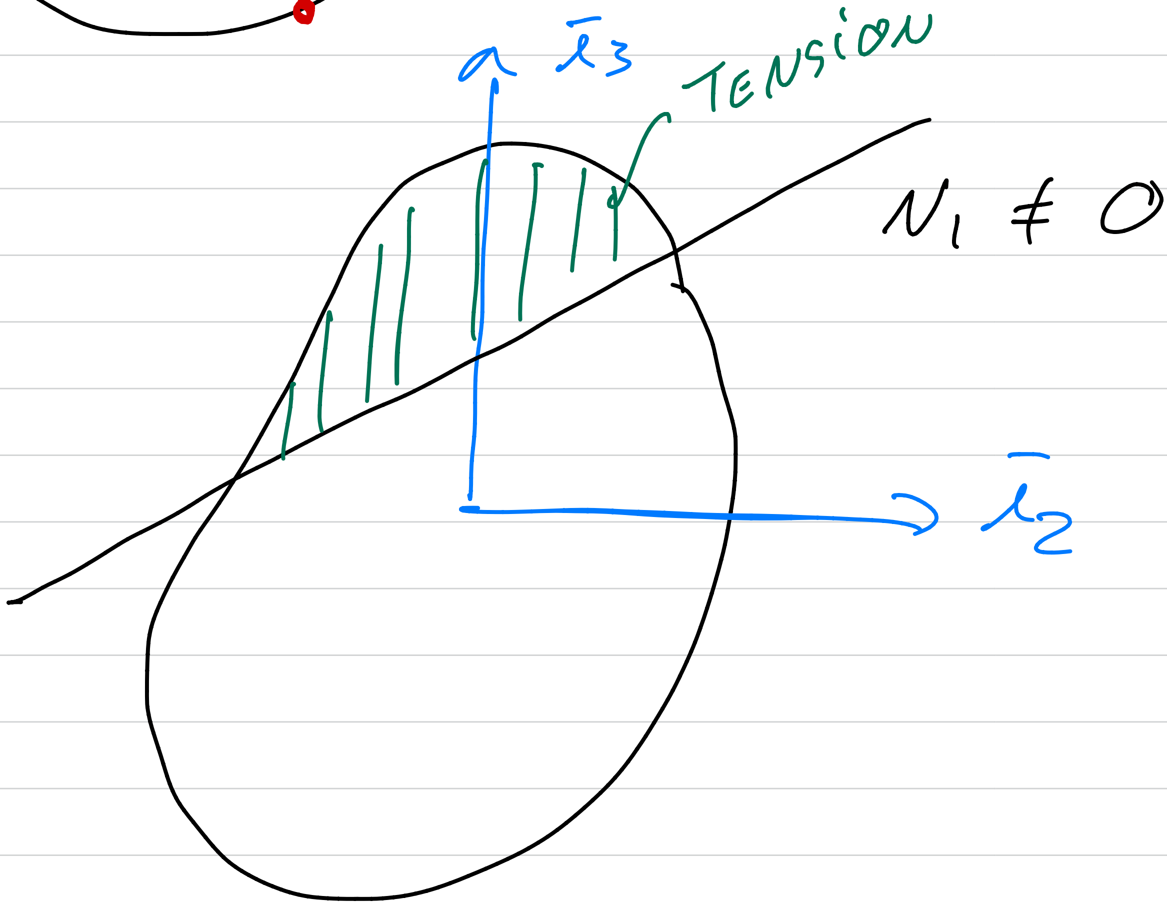
$$\sigma_1 = E\left(\frac{N_1}{S} + x_3 \frac{H_{33}^c M_2 + H_{23}^c M_3}{\Delta H} - x_2 \frac{H_{23}^c M_2 + H_{22}^c M_3}{\Delta H} \right) = 0$$

$$\frac{N_1}{S} + x_3 \frac{H_{33}^c M_2 + H_{23}^c M_3}{\Delta H} - x_2 \frac{H_{23}^c M_2 + H_{22}^c M_3}{\Delta H}) = 0$$

Slope $\tan(\beta) = \dfrac{H_{23}^c M_2 + H_{22}^c M_3}{H_{33}^c M_2 + H_{23}^c M_3}$



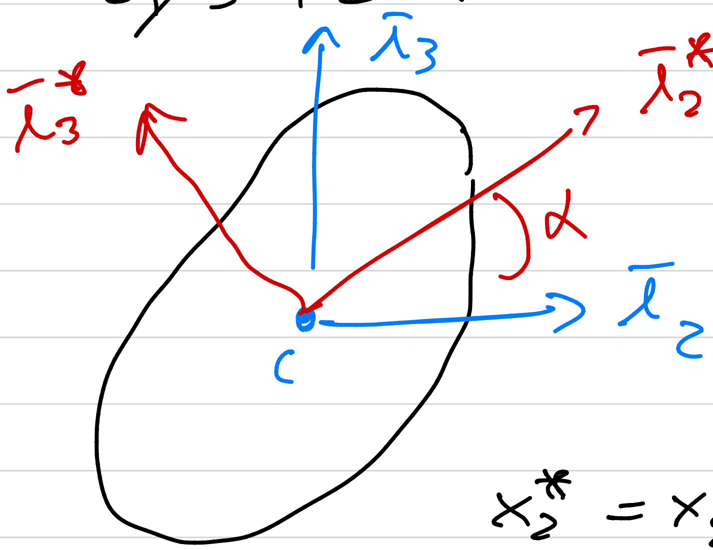
$\bar{x}_3$
$N_1 = 0$
TENSION
STRESS GRADIENTS
$\bar{x}_2$
COMPRESSION



$\bar{x}_3$
TENSION
$N_1 \neq 0$
$\bar{x}_2$

PRINCIPAL AXIS OF BENDING

* CONSIDER THE ORIENTATION OF THE CENTROIDAL COORDINATE SYSTEM



FIND α SUCH THAT

$$H_{23}^{c*} = 0 \, ?$$

$$x_2^* = x_2 \cos(\alpha) + x_3 \operatorname{Sen}(\alpha)$$

$$x_3^* = -x_2 \operatorname{Sen}(\alpha) + x_3 \cos(\alpha)$$

$$H_{23}^{c*} = \int_A E \, x_2^* x_3^* \, dA$$

$$= \int_A E \left[x_2 \cos(\alpha) + x_3 \operatorname{Sen}(\alpha) \right] \cdot \left[-x_2 \operatorname{Sen}(\alpha) + x_3 \cos(\alpha) \right] dA$$

$$H_{23}^{C*} = \int_A dA\, E\Big(-x_2^2 \cos(\alpha)\sin(\alpha) + x_2 x_3 \cos^2(\alpha) - x_2 x_3 \sin^2(\alpha) + x_3^2 \sin(\alpha)\cos(\alpha)\Big)$$

$$H_{23}^{C*} = \cos(\alpha)\sin(\alpha)\left(H_{22}^{C} - H_{33}^{C}\right) + H_{23}^{C}\left(\cos^2(\alpha) - \sin^2(\alpha)\right) = 0$$

$$H_{23}^{C*} = \frac{H_{22}^{C} - H_{33}^{C}}{2}\sin(2\alpha) + H_{23}^{C}\cos(2\alpha) = 0$$

$$\boxed{\tan(2\alpha) = -\frac{2H_{23}^{C}}{H_{22}^{C} - H_{33}^{C}}}$$

$$\begin{bmatrix} N_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} S & 0 & 0 \\ 0 & H_{22}^{C*} & 0 \\ 0 & 0 & H_{33}^{C*} \end{bmatrix} \begin{bmatrix} \bar{\varepsilon}_1 \\ K_2 \\ K_3 \end{bmatrix}$$