Robust Design

AE 6310: Optimization for the Design of Engineered Systems

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Lecture Notes Developed By Dr. Brian German

Introduction to Robust Design

Robust design refers to a broad category of techniques that attempt to understand and mitigate the effects of *variability* of different factors on the performance of a design.

The factors that may affect this variability include the design variables themselves, e.g. unexpected variations in manufacturing, and *noise variables* that result from exogenous sources.

The concept of robust design can be thought of as a generalized optimization problem in which the objective function is formulated to incorporate the goal of minimizing variability.

Introduction to Robust Design

The field of robust design has roots in several areas including:

- **Taguchi methods**: Practical industrial approaches to design for variability developed by Genichi Taguchi in the 1960s. Popularized in the Japanese auto industry and corporate Design for Six Sigma initiatives.

- **Probabilistic robust design optimization methods**: Roots in stochastic programming from the operations research community and significant research in the engineering design community beginning in the 1990s.

Introduction to Robust Design

The field is very broad and we will only survey it in this course.

AE 6373: Advanced Design Methods I goes into considerable additional detail.

For a very readable review of the field that indicates how all of the topics fit together, the following paper is recommended:

Beyer, H-G. and Sendhoff, B., Robust Design Optimization – A Comprehensive Survey, 2007.

Robust Design via Probabilistic Approaches

The classical Taguchi approach treats robust design largely with deterministic methods, or with limited experiments in an inner/outer array to determine the effects of the noise variables.

In modern computational approaches to robust design, other approaches can be applied:

- Interval analysis
- Convex models and info-gap theory
- Fuzzy set theory
- Probabilistic approaches

We will discuss primarily probabilistic approaches.

Types of Uncertainty

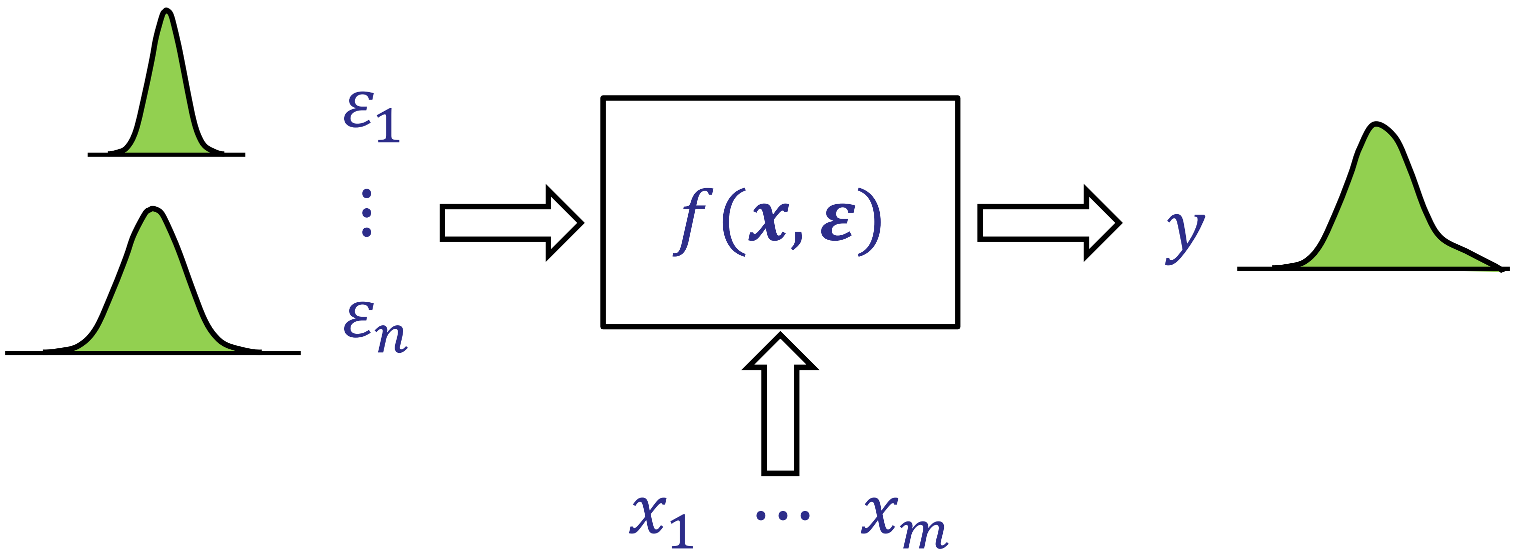
- **Aleatory (or irreducible) uncertainty**
  - Uncertainty due to our inability to exactly predict phenomena that affect a system or component, even with very high fidelity models
  - Examples:
    - Weather conditions on a particular day at a particular place
    - Fuel price on a particular day in the future
  - Alleatory uncertainties *cannot* be reduced by actions undertaken by design engineers
  - However, we may be able to characterize statistical information (mean, variance, etc.) about these uncertainties through observations or controlled experiments
  - Well-characterized by probabilistic approaches; “noise variables” in the previous slides are examples
  - This is the type of uncertainty typically addressed by robust design techniques

Types of Uncertainty

- **Epistemic (or reducible) uncertainty**
  - Uncertainty due to abstractions or simplifications in our physical/economic models
  - Examples:
    - Uncertainty associated with the Euler equations because the viscous terms were omitted from the Navier-Stokes equations
    - Geometry representation uncertainty—running a Navier-Stokes code on an abstract aircraft geometry available in conceptual design may produce meaningless results because of discontinuities in surface slopes
  - Epistemic uncertainties *can* be reduced by actions undertaken by design engineers by improving design analysis codes, geometry representation, etc.
  - Not well characterized by probabilistic approaches; typically manifested as deterministic bias errors across the design space
  - Robust design typically does not address this type of uncertainty

Robust Design via Probabilistic Approaches

The general idea of probabilistic approaches for robust design is to **propagate uncertainty** by treating the noise variables, $\boldsymbol{\varepsilon}$, as random variables:



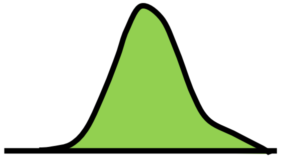
We then base our decision of the design variables, $\boldsymbol{x}$, in part on the corresponding variability of the response y.

Propagating Uncertainty

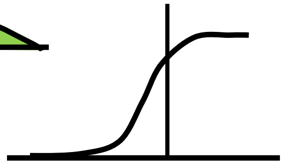
There are several ways to propagate uncertainty.

Our choice of method depends in part on what we need to know about the response:

- ❖ Probability density function (PDF)



- ❖ Cumulative distribution function (CDF)



- ❖ Moments (mean, variance, skew, kurtosis) $\mu, \sigma, \ldots$

Ways to Propagate Uncertainty

Monte Carlo Simulation (MCS)

- Sample points from the PDFs of the noise variables with a random number generator
- Propagate through the analysis code or a surrogate model
- Bin the results to estimate the PDF of the response

Pros: Very accurate. Entire PDF of response is obtained.

Cons: Computationally expensive. MANY samples from the noise variables must be propagated through the analysis function.

Ways to Propagate Uncertainty

First-Order and Second-Order Reliability Methods (FORM and SORM)

- Approximate the cumulative probability that the response is less than some value
- Based on asymptotic expansions of the CDF integral
- Estimates the Most Probable Point (MPP) of failure by solving a constrained optimization problem

Pros: Lower computational cost than MCS.

Cons: Only the cumulative probability of the response is returned.

Ways to Propagate Uncertainty

Other techniques for uncertainty propagation:

- **Bayesian Monte Carlo technique**
  - Uses Gaussian process (GP) surrogate models. GP models have a built-in characterization of uncertainty.

- **Polynomial chaos (PC) expansion**
  - Uses multidimensional Hermite polynomials to represent uncertainty distributions as stochastic processes.

- **“Possibilistic” interval methods**
  - Methods that are intended to be robust to the shapes of the noise variable uncertainty distributions. Determine “belief” and “plausibilty” bounds of the response.

Types of Robust Design Problem Formulations

Reliability-Based Design Optimization

- Focus is on "guaranteeing", with a specified cumulative probability, that a critical constraint is not violated:

$$P[g_i(\boldsymbol{x},\boldsymbol{\varepsilon}) \leq 0] \geq R_{g_i}$$

- This is in the spirit of reliability engineering and connotes analogies with factors of safety
- The objective function could be chosen as a deterministic response or to move a mean to target and minimize variance

Types of Robust Design Problem Formulations

Traditional robust design (inspired by Taguchi)

- Optimization approach to balance desire to match a target and minimize noise

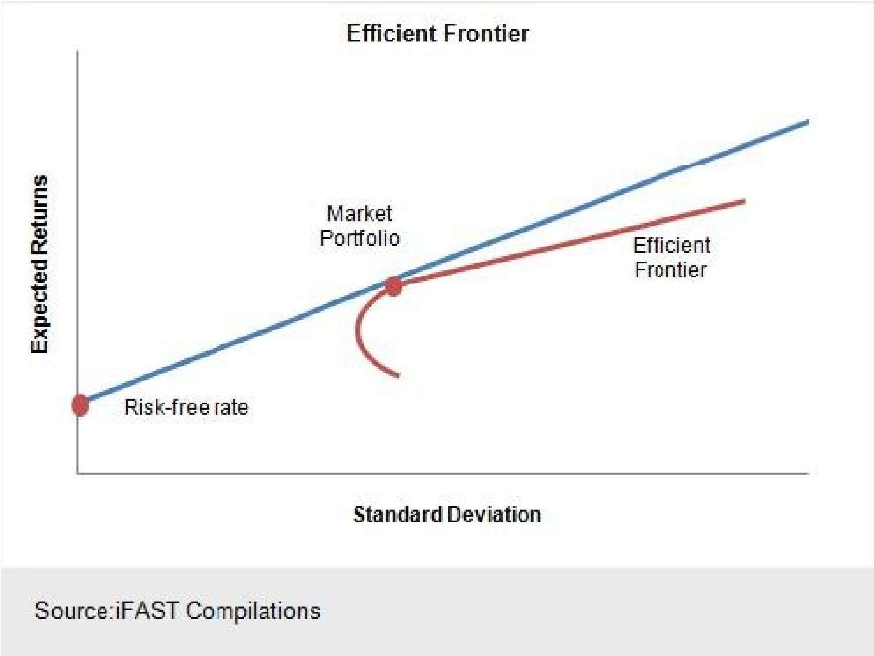
$$\min \{(\mu_y - y_T)^2 + \sigma_y^2\}$$

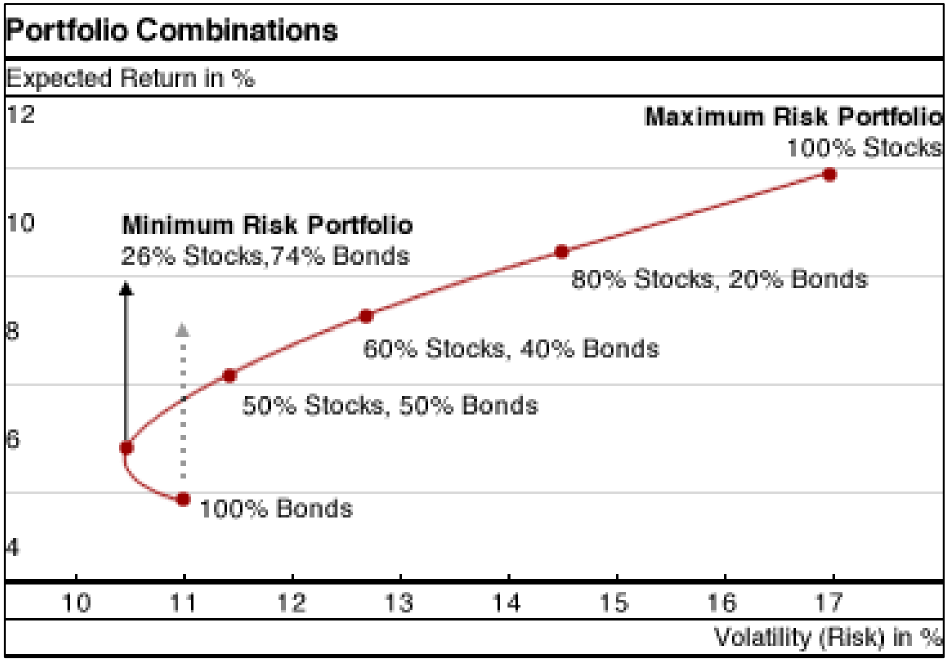
- Minimization or maximization of y also possible
- Constraints added as needed
- It is also possible to formulate the problem to move multiple responses to target simultaneously via techniques such as Joint Probabilistic Decision Making (JPDM)

Types of Robust Design Problem Formulations

Robust design Pareto frontiers

- Inspired by analogy with Modern Portfolio Theory
- Pareto frontiers sometimes called “efficient frontiers”





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Types of Robust Design Problem Formulations

Robust design Pareto frontiers

- Weighted sum formulation

$$\min\{\mu_y + \lambda \sigma_y\}$$

Vary λ to draw the frontier.

- General formulation

$$\min\{\mu_y, \sigma_y\}$$

The general formulation can capture non-convex efficient frontiers. Different metrics to capture variability in a reliability sense can be used in place of σ_y, e.g. Value at Risk (VAR).