Rayleigh-Ritz Method

* Achieve an approximate solution by reducing a continuous system to one with a finite number of degrees of freedom

Write the solution as a linear combination of functions

$$\bar{u}_1 = h_1(x_1, x_2, x_3) \cdot q_1 + h_2(x_1, x_2, x_3) q_2$$

$$+ \dots + h_i(x_1, x_2, x_3) q_i$$

$$\bar{u}_1 = \sum_{i=1}^{N} \underbrace{h_i(x_1, x_2, x_3)}_{\text{Shape functions (known)}} \cdot q_i \leftarrow \text{Degrees of Freedom}$$

* Chose shape function such that kinematic boundary conditions are satisfied.

  1) Compute $\Pi = \hat{\Pi}(q_i)$

  2) Solve for q_i such that

$$\frac{\partial \Pi}{\partial q_i} = 0$$

Notes:

* Usually, the accuracy of the assumed displacement increases with the number of degrees of freedom

  * Although displacements can be fairly accurate, the stresses can vary significantly since they depend on derivatives of $\underline{u}$.

* Equilibrium is satisfied in an average sense through minimization of the potential, but might not be satisfied at every point.

* The approximate solution is less flexible since it has less degrees of freedom. Usually overestimates stiffn. Buckling loads are greater or equal.

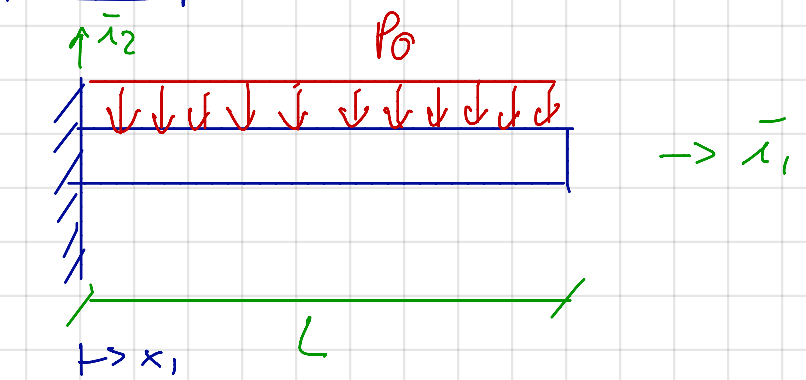
Example: $H^c_{23} = 0$

@ $x_1 = 0$

$u_2 = 0$

$\dfrac{du_2}{dx_1} = 0$

Polynomial



$$u_2 = q_0 + q_1 x_1 + q_2 x_1^2$$

Apply B.C.

$$u_2(x_1 = 0) = q_0 \rightarrow q_0 = 0$$

$$du_2/dx_1\,(x_1 = 0) = q_1 \rightarrow q_1 = 0$$

$$u_2 = q_2\, \underbrace{x_1^2}_{x_1^2}$$

DOF

$$\Pi = \int_0^L \frac{1}{2} H^c_{33} \cdot (2q_2)^2\, dx_1 - \int_0^L P_0 \cdot (q_2 x_1^2)\, dx_1$$

$$\Pi = 2 H^c_{33} q_2^2 L - P_0 q_2 \frac{L^3}{3} = \Pi(q_2)$$

$$\frac{\partial \Pi}{\partial q_2} = 0 \qquad \frac{\partial \Pi}{\partial q_2} = 4 H^c_{33} q_2 L - P_0 \frac{L^3}{3} = 0$$

$$q_2 = \frac{P_0 L^2}{12 H^c_{33}}$$

$$\boxed{u_2 = \frac{P_0 L^2}{12 H^c_{33}} \cdot x_1^2}$$

$$u_2 = q_2 x_1^2 + q_3 x_1^3$$

$$\pi = \int_0^L \frac{1}{2} H_{33}^c \cdot (2q_2 + 6q_3 x_1)^2 dx_1 - \int_0^L p_0 \cdot (q_2 x_1^2 + q_3 x_1^3) dx_1$$

$$\pi = \frac{1}{2} \frac{H_{33}^c}{L^3} \left(4q_2^2 + \frac{36}{3} q_3^2 + \frac{24}{2} q_2 q_3 \right) - p_0 L \left(\frac{q_2}{3} + \frac{q_3}{4} \right)$$

$$\pi = \pi(q_2, q_3)$$

$$\frac{\partial \pi}{\partial q_2} = 0 = \frac{H_{33}^c}{L^3} \left(4q_2 + 6q_3 \right) - \frac{p_0 L}{3}$$

$$\frac{\partial \pi}{\partial q_3} = 0 = \frac{H_{33}^c}{L^3} \left(12q_3 + 6q_2 \right) - \frac{p_0 L}{4}$$

$$\begin{bmatrix} 4 & 6 \\ 6 & 12 \end{bmatrix} \begin{bmatrix} q_2 \\ q_3 \end{bmatrix} = \frac{p_0 L^4}{H_{33}^c} \begin{bmatrix} 1/3 \\ 1/4 \end{bmatrix}$$

$$\boxed{u_2 = \frac{p_0 L^2}{H_{33}^c} \frac{1}{24} \left(5x_1^2 - 2x_1^3 \right)}$$

$$H_{33}^c \frac{d^4 u_2}{dx_1^4} = p_0$$

$$u_2 = q_2 x_1^2 + q_3 x_1^3 + q_4 x_1^4$$

$$* \quad u_2 = \frac{p_0 L^2}{H_{33}^c} \frac{1}{24} \left(6x_1^2 - 4x_1^3 + x_1^4 \right) \rightarrow$$

Exact Solution!