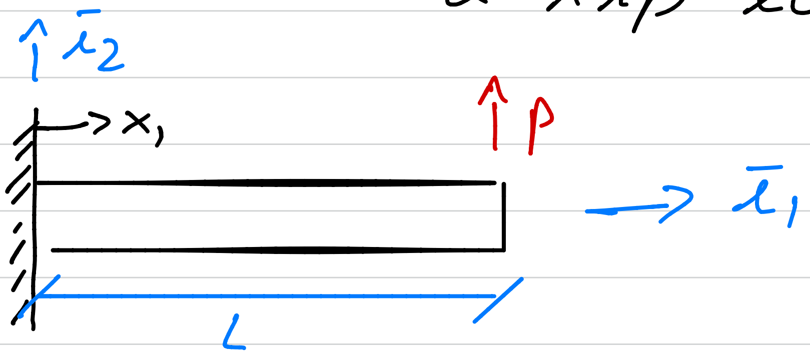
Example: Cantilevered beam w/ a tip load



$$\frac{d}{dx_1}\left(H_{33}^C \frac{d\Phi_3(x_1)}{dx_1}\right) + K_{22}\left(\frac{dU_2(x_1)}{dx_1} - \Phi_3(x_1)\right) = -\cancel{q_3(x_1)} \quad (1)$$

$$\frac{d}{dx_1}\left[K_{22}\left(\frac{dU_2(x_1)}{dx_1} - \Phi_3(x_1)\right)\right] = -\cancel{p_2(x_1)} \quad (2)$$

B.C.'s $\quad @\ x_1 = 0 \qquad U_2 = \Phi_3 = 0$

$\quad @\ x_1 = L \qquad M_3 = 0\ , \quad V_2 = P$

$$H_{33}^C \left.\frac{d\Phi_3}{dx_1}\right|_L = 0\ , \quad K_{22}\left.\left(\frac{dU_2}{dx_1} - \Phi_3\right)\right|_L = P$$

Integrating (2)

$$K_{22}\left(\frac{dU_2}{dx_1} - \Phi_3\right) = C_1 = P$$

Applying $V_2 = P$ @ $x_1 = L$

$$\Rightarrow \quad C_1 = P$$

Combining w/ (1)

$$H^c_{33}\frac{d^2\phi_3}{dx_1^2} + P = 0$$

$$H^c_{33}\frac{d\phi_3}{dx_1} = -Px_1 + C_2$$

Applying $M_3 = 0$ @ $x_1 = L$

$$0 = -PL + C_2 \implies C_2 = PL$$

$$H^c_{33}\frac{d\phi_3}{dx_1} = P(L - x_1)$$

$$\boxed{H^c_{33}\,\phi_3 = P\left(Lx_1 - \frac{x_1^2}{2}\right)} \cancel{+C_2}$$

Applying $\phi_3 = 0$ @ $x_1 = 0$

$$\rightarrow C_2 = 0$$

$$K_{22}\left(\frac{dU_2}{dx_1} - \frac{P}{H^c_{33}}\left(Lx_1 - \frac{x_1^2}{2}\right)\right) = P$$

$$\frac{du_2}{dx_1} = \frac{P}{K_{22}} + \frac{P}{H_{33}^C}\left(Lx_1 - \frac{x_1^2}{2}\right)$$

$$u_2 = \frac{P}{K_{22}}x_1 + \frac{P}{H_{33}^C}\left(\frac{Lx_1^2}{2} - \frac{x_1^3}{6}\right) + C_3$$

Applying $u_2 = 0$ @ $x_1 = 0$

$\rightarrow C_3 = 0$

$$\boxed{u_2 = \underbrace{\frac{P}{K_{22}}x_1}_{\text{Shear Contribution}} + \underbrace{\frac{P}{H_{33}^C}\left(\frac{Lx_1^2}{2} - \frac{x_1^3}{6}\right)}_{\text{EB Bending Solution.}}}$$

Focus on tip displacement $u_2(x_1 = L)$

$$u_2 = \frac{PL^3}{3H_{33}^C}\left(1 + \frac{3H_{33}^C}{K_{22}L^2}\right)$$

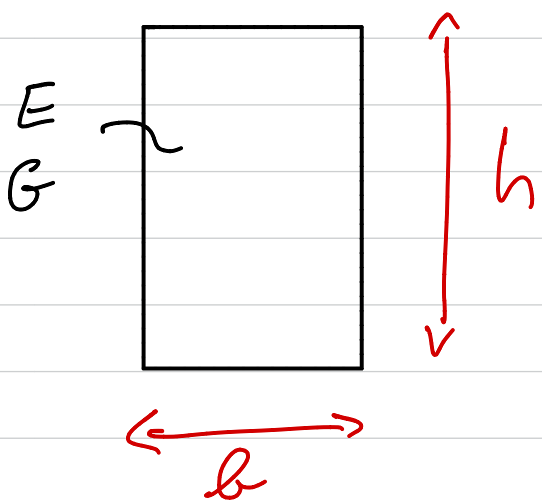
Tip deflection due to shear, δ_S

Tip deflection due to bending, δ_B

$$\frac{\delta_S}{\delta_B} = \frac{3H_{33}^C}{K_{22}L^2}$$

Now we need H_{33}^c & K_{22}

Solid Homogeneous Rectangular Bar



$$H_{33}^c = E\frac{bh^3}{12}$$

$$K_{22} = \frac{5}{6} G \cdot A$$ → From previous equilibrium analysis.

$$\frac{H_{33}^c}{K_{22}L^2} = \frac{E\cancel{b}h^3}{12}\,\frac{6}{5}\,\frac{1}{G\cancel{b}h}\,\frac{1}{L^2} = \frac{1}{10}\left(\frac{E}{G}\right)\left(\frac{h}{L}\right)^2$$

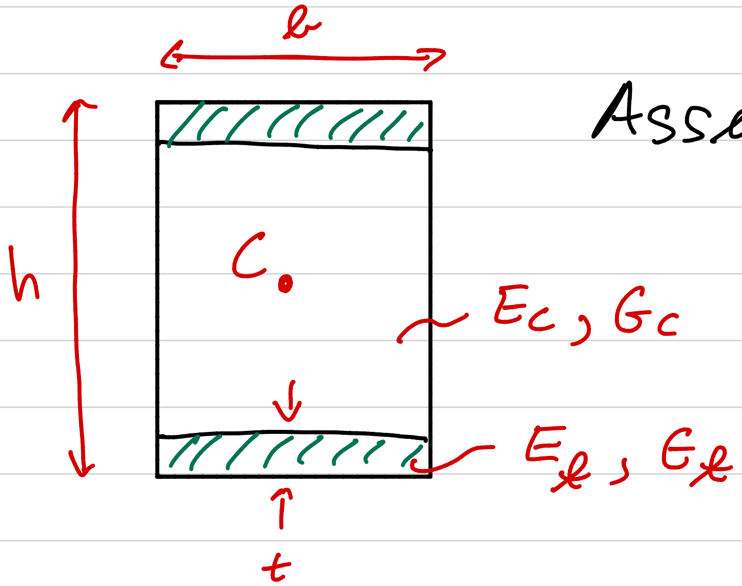
Rough Estimate for E/G?

$$\frac{E}{G} = 2(1+\nu) \qquad \nu \approx 0.3 \quad (Al, Ti)$$

$$\frac{H_{33}^c}{K_{22}L^2} = 0.26\left(\frac{h}{L}\right)^2$$

→ The importance of shear deformations decreases quadratically w/ beam slenderness.

Sandwich Beam



Assume → $t \ll h$
→ $E_C \ll E_f$
$E_c \approx 0 \quad (G_c \neq 0)$

$$H_{33}^{C} = 2 \cdot E_f \left[\overset{\approx 0}{\cancel{\frac{bt^3}{12}}} + (bt)\left(\frac{h}{2}\right)^2 \right]$$

$$H_{33}^{C} = \frac{1}{2} E_f \, b t \underline{h^2}$$

→ Efficient since $H_{33}^{C} \sim h^2$ & core increases h.

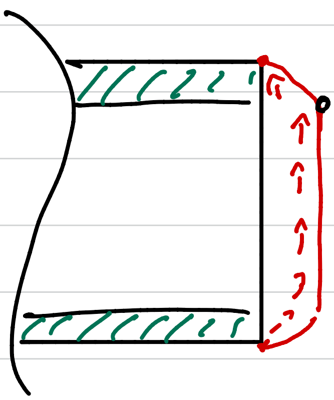
Recall from equilibrium analysis

$$\tau_{12} = \frac{E}{2}\left(x_2^2 - \left(\frac{h}{2}\right)^2\right)\frac{d^2\phi_3}{dx_1^2}$$

$$\tau_{12}^{*} = \frac{E_*}{2}\left(x_2^2 - \left(\frac{h+t}{2}\right)^2\right)\frac{d^2\phi_3}{dx_1^2}$$

Since the core has no axial stiffness $(E_c \approx 0) \rightarrow \sigma_1 = 0$

$$\cancel{\frac{d\sigma_1}{dx_1}} + \frac{d\tau_{12}}{dx_2} = 0 \rightarrow \underline{\tau_{12}^{c} = \text{constant}}$$



$$\tau_{12}^{c} = \tau_{12}^{*}\left(x_2 = \frac{h-t}{2}\right)$$

$$\tau_{12}^{c} = \frac{E_*}{2}\left(\left(\frac{h-t}{2}\right)^2 - \left(\frac{h+t}{2}\right)^2\right)\frac{d^2\phi_3}{dx_1^2}$$

$$\tau_{12}^{c} = -\frac{1}{2}E_* h t \frac{d^2\phi_3}{dx_1^2}$$

$$V_2 = \int_{-h/2}^{h/2} \tau_{12}^{c}\, dA = -\frac{1}{2}E_* h^2 t b \frac{d^2\phi_3}{dx_1^2}$$

$$\tau_{12}^{c} = \frac{V_2}{hb}$$ → CORE CARRIES ALL SHEAR!

$$dE = \frac{1}{2}\int \frac{\tau_{12}^2}{G_C}\, dA\, dx_1 = \frac{1}{2}\frac{V_2^2}{(hb)^2}\frac{(hb)}{G_C}dx_1$$

$$dE = \frac{1}{2}\frac{V_2^2}{G_C b h} \rightarrow \boxed{K_{22} = G_C\, b \cdot h}$$

FOR A SANDWICH BEAM

$$\frac{\delta_S}{\delta_B} = \frac{H_{33}^{c}}{K_{22}L^2} = \frac{1}{2}E_t\, b t h^2 \cdot \frac{1}{G_C b h}\cdot\frac{1}{L^2}$$

$$\frac{\delta_S}{\delta_B} = \frac{1}{2}\left(\frac{E_t}{G_C}\right)\left(\frac{t}{h}\right)\left(\frac{h}{L}\right)^2$$

No intrinsic relationship between these!

Ex: Aluminium faces, $E_f = 73\,GPa$

Aluminum Honeycomb Core, $G_c = 1\,GPa$

$E_f/G_c = 73$, compared to $E/G = 2.6$ for pure aluminium.

Comparison: Solid: $E/G = 2.6$

Sandwich: $E_f/G_c = 73$, $t/h = 1/10$

	SOLID		SANDWICH	
h/L	1/10	1/5	1/10	1/5
σ_S/σ_B	0.78%	3.1%	10.5%	42%