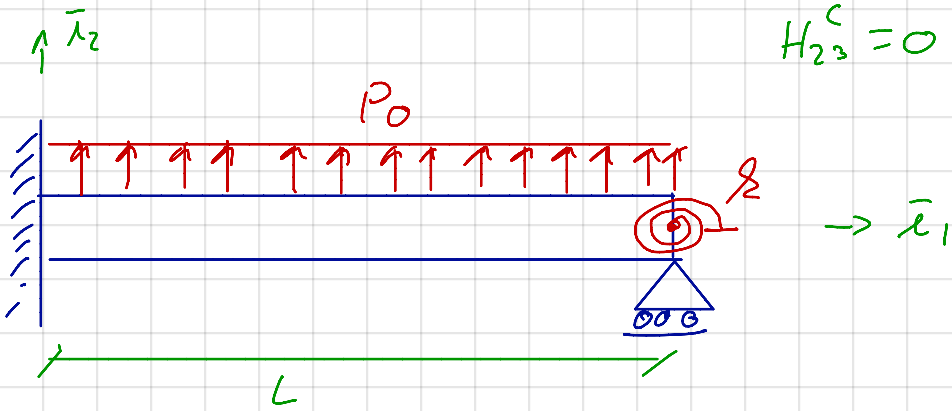
$H_{23}^{c} = 0$



1) Find an approximate solution using Rayleigh Ritz.

$$u_2(x_1) = q_0 + q_1 x_1 + q_2 x_1^2 + q_3 x_1^3$$

Apply B.C.s

$$u_2(x_1=0) = 0 = q_0 \rightarrow q_0 = 0$$

$$du_2/dx_1(x_1=0) = 0 = q_1 \rightarrow q_1 = 0$$

$$u_2(x_1=L) = 0 = q_2 L^2 + q_3 L^3 \rightarrow q_2 = -q_3 L$$

$$u_2(x_1) = q_3(-L x_1^2 + x_1^3)$$

$$* \quad u_2(x_1) = q_3 x_1^2 (L - x_1) = q_3(L x_1^2 - x_1^3)$$

$$* \quad \frac{du_2}{dx_1} = q_3(2L x_1 - 3x_1^2)$$

$$* \quad \frac{d^2u_2}{dx_1^2} = q_3(2L - 6x_1)$$

$$\Pi = \frac{1}{2}\int_0^L H_{33}^{c}\left(q_3(2L - 6x_1)\right)^2 dx_1 + \overbrace{\frac{1}{2}k\left(q_3(2L^2 - 3L^2)\right)^2}^{\text{Strain Energy at Spring } \frac{1}{2}k\theta^2}$$

$$- \int_0^L P_0\, q_3(L x_1^2 - x_1^3)\, dx_1$$

$$\Pi = \frac{1}{2}\int_0^L H_{33}^c \left(q_3(2L-6x_1)\right)^2 dx_1 + \frac{1}{2}k\left(q_3(2L^2-3L^2)\right)^2$$

$$- \int_0^L P_0\, q_3\left(Lx_1^2 - x_1^3\right)dx_1$$

$$\Pi = \frac{1}{2}\int_0^L H_{33}^c\, q_3^2\left(4L^2 - 24Lx_1 + 36x_1^2\right)dx_1 + \frac{1}{2}k\, q_3^2 L^4$$

$$- P_0\, q_3\left(L\frac{L^3}{3} - \frac{L^4}{4}\right)$$

$$\Pi = \frac{1}{2}H_{33}^c\, q_3^2\left(4L^2\cdot L - 24L\cdot\frac{L^2}{2} + 36\frac{L^3}{3}\right) + \frac{1}{2}k\, q_3^2 L^4$$

$$- \frac{P_0\, q_3 L^4}{12}$$

$$\Pi = \frac{1}{2}H_{33}^c\, q_3^2\, 4L^3 + \frac{1}{2}k\, q_3^2 L^4 - \frac{P_0\, q_3 L^4}{12}$$

$$\frac{\partial \Pi}{\partial q_3} = 0 \qquad \frac{\partial \Pi}{\partial q_3} = 4H_{33}^c\, q_3 L^3 + k\, q_3 L^4 - \frac{P_0 L^4}{12} = 0$$

$$q_3\left(4H_{33}^c L^3 + kL^4\right) = \frac{P_0 L^4}{12}$$

$$* \quad q_3 = \frac{P_0 L^4}{12\cdot(4H_{33}^c L^3 + kL^4)} = \frac{P_0 L}{12(4H_{33}^c + kL)}$$

$$\boxed{u_2(x_1) = \frac{P_0 L}{12(4H_{33}^c + kL)}\, x_1^2(L - x_1)}$$