Fundamentals of Aircraft Aerodynamics

Presented by:

Dr. Dimitri Mavris
Director of Aerospace Systems Design Laboratory
Boeing Professor for Advanced Aerospace Systems Analysis

The Standard Atmosphere

- *Why do we need to know about the atmosphere?*
  - The performance of aircraft, spacecraft, and their engines depend on the atmosphere in which they operate, primarily **density** and **viscosity**
  - Density and viscosity, in turn, are functions of **altitude**
    - Density (ρ) varies with pressure (p) and temperature (T)
    - Viscosity (μ) varies only with temperature (T)

- The "standard atmosphere" is defined from the equation of state of a perfect gas:

$$\boxed{p = \rho R T}$$ Perfect Gas Law

The Standard Atmosphere

- The standard atmosphere:
  - p = pressure in lb/ft^2 or N/m^2
  - ρ = density in slugs/ft^3 or kg/m^3
  - T = absolute temperature in Rankine (R) or Kelvin (K)
  - R = gas constant = 1718 ft-lb/slugsR or 287.05 N-m/kgK

- Remember:

$$R = {}^{\circ}F + 459.7$$
$$K = {}^{\circ}C + 273.15$$

- For our purposes, the atmosphere can be regarded as a **homogenous** gas of **uniform composition** that satisfies the **perfect gas law**

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Effect of Water Vapor on Atmosphere

- When there is a significant amount of water vapor in the air, the density is changed, but by a very small amount

ρ = 0.002243 slugs/ft^3 dry air

ρ = 0.002203 slugs/ft^3 100% humidity

- Although the effect of water vapor on air density is very small, water vapor does have a significant effect on **engine performance** and **supersonic aerodynamics**

International Standard Atmosphere

- To allow for comparison of the performance of airplanes, as well as calibration of altimeters, "standard" properties of the atmosphere have been established by the International Civil Aviation Organization (ICAO)

- The ICAO and the U.S. Standard Atmosphere are identical below 65,617 feet

- This standard atmosphere is representative of mid latitudes of the northern hemisphere

- "Standard" sea level properties are:

$$g_0 = 32.17 \text{ ft/s}^2 = 9.806 \text{ m/s}^2$$

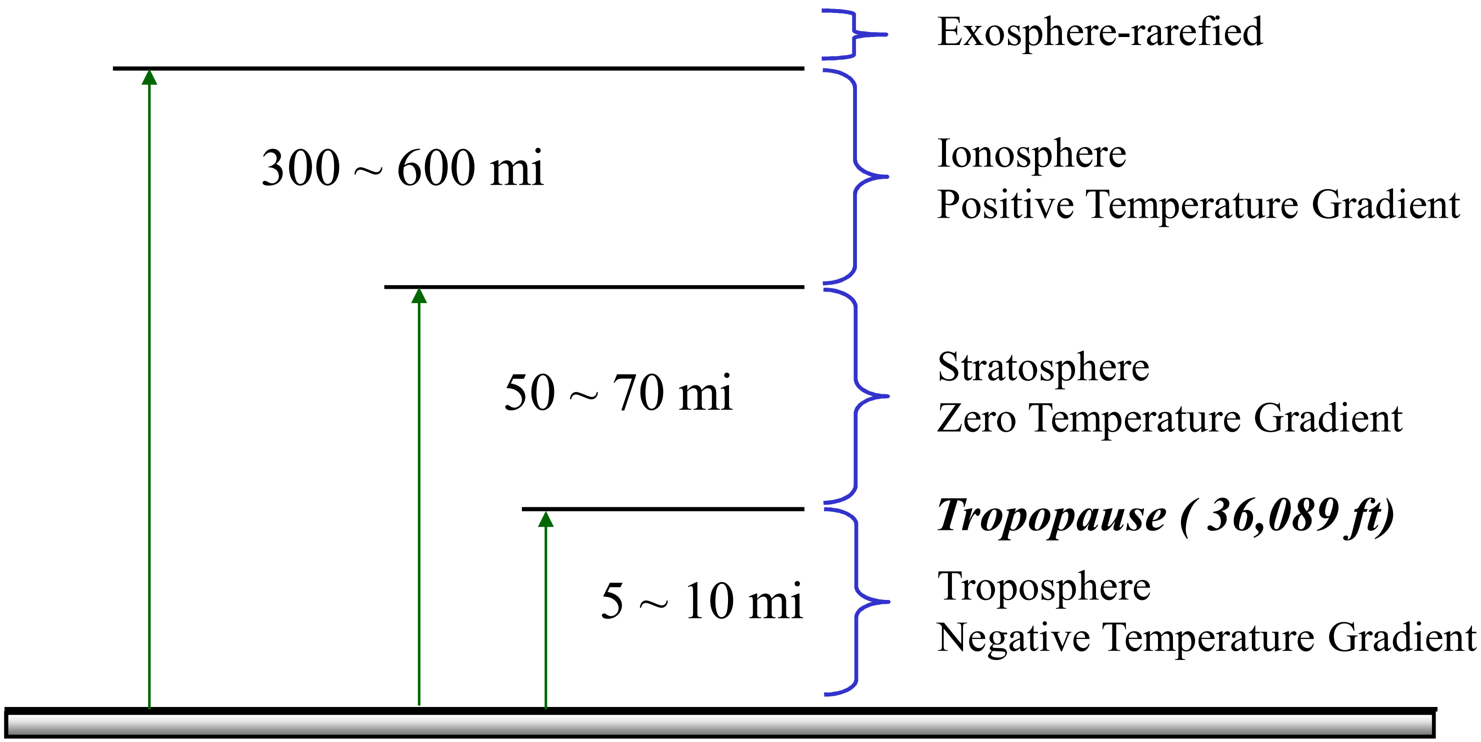
$$P_0 = 29.92 \text{ in Hg} = 2116.2 \text{ lb/ft}^2 = 1.013 \times 105 \text{ N/m}^2$$

$$T_0 = 59 \text{ F} = 518.7 \text{ R} = 15 \text{ C} = 288.2 \text{ K}$$

$$\rho_0 = 0.002377 \text{ slugs/ft}^3 = 1.225 \text{ Kg/m}^3$$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Regions of the Atmosphere



- In airplane aerodynamics, only the troposphere and stratosphere are important

Temperature Variation with Altitude

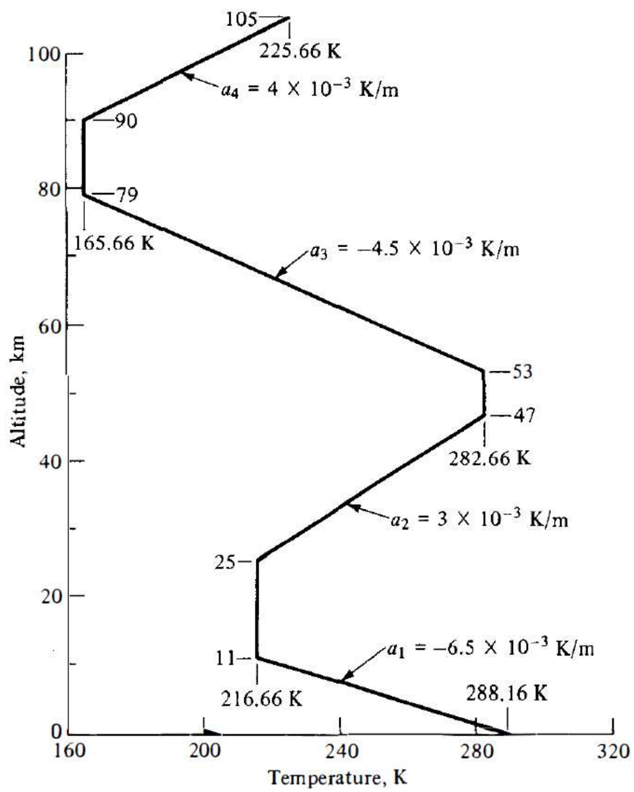
- Below 36,089 ft, we assume there is a constant drop of temperature from sea level to altitude

$$\boxed{T = T_1 + a(h - h_1)}$$

where:
  - a = "lapse rate" = -0.00356616 F/ft in the standard atmosphere

- T_1 and h_1 are reference temperatures
  - For sea level, $T_1 = T_0$ and $h_1 = 0$

- Above 36,089 ft in the stratosphere, the standard temperature is assumed constant and equal to -69.7 F



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Pressure/Density Variation with Altitude

- Below 36,089 ft (*relative to standard sea level values*):

$$\frac{T}{T_0} = \Theta = 1 + \frac{a}{T_0}h = 1 - 6.875 \times 10^{-6}h$$

$$\frac{p}{p_0} = \delta = \Theta^{5.2561}$$

$$\frac{\rho}{\rho_0} = \sigma = \Theta^{4.2561}$$

Pressure/Density Variation with Altitude

- Above 36,089 ft (*relative to standard sea level values*):

$$T = constant = -69.7\ F$$

$$\frac{p}{p_0} = 0.2234 \exp\left[-\frac{h - 36{,}089}{20806.7}\right]$$

$$\frac{\rho}{\rho_0} = 0.2971 \exp\left[-\frac{h - 36{,}089}{20806.7}\right]$$

Viscosity

- Viscosity varies primarily with temperature

- There is a strong relationship between air viscosity and boundary layer behavior
  - This will be discussed more when we review aerodynamics

$$\upsilon = \mu/\rho$$ Kinematic Viscosity

$$Re = \frac{Vl}{\upsilon}$$ Reynolds Number

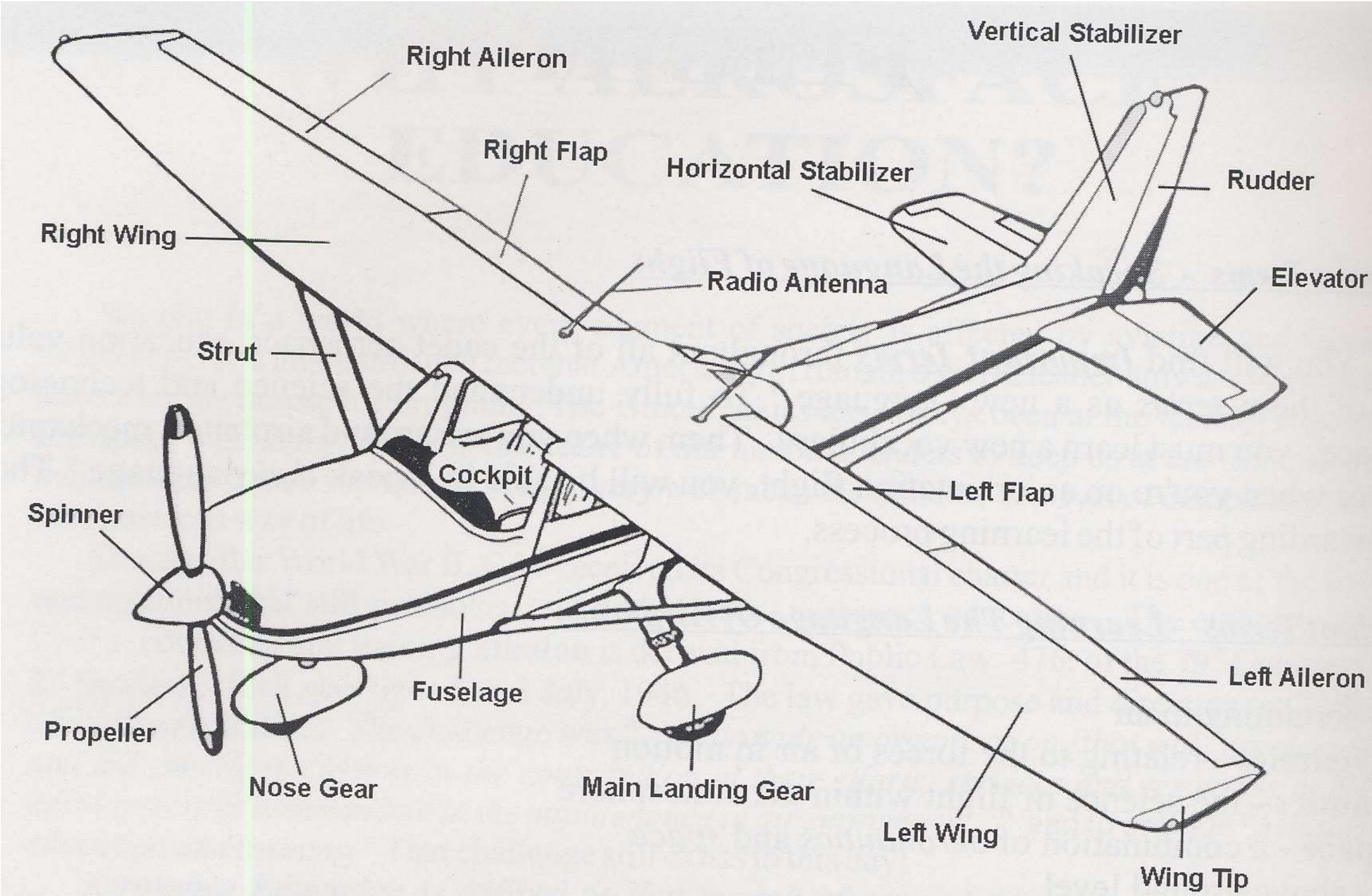
Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Types of Airspeeds

- **Indicated Airspeed (IAS)** - is the direct reading from the airspeed indicator
  - This represents the airplane's speed through the air, NOT necessarily its speed across the ground
- **Calibrated Airspeed (CAS)** - is the indicated airspeed corrected for instrument position and instrument error
  - This is a function of each unique aircraft and the position of its pitot tube
  - There is no direct reading of CAS in the cockpit!
  - The pilot must refer to the Pilot's Operating Handbook for a table corresponding to that particular aircraft
- **True Airspeed (TAS)** - because an airspeed indicator is calibrated for standard sea level conditions, when the airplane is flying at altitude, the airspeed is not correctly reflected
  - The amount of error is a function of temperature and altitude
  - TAS can be approximated by increasing the indicated airspeed by 2% per thousand feet of altitude

Fundamentals of Aircraft Aerodynamics

The Anatomy of the Airplane



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Slide 13

MDO2 Skipped this slide
Melek Dicle Ozcan, 10/23/2019

Introduction to Performance

Flight Mechanics is the study of the motions of bodies (aircraft and rockets), through a fluid

Stability and Control

The science of designing for steady and controllable flight characteristics

Aerodynamic Performance

Speed
Rate of climb
Range
Fuel consumption
Maneuverability
Runway length requirements

Aircraft Performance

- Aircraft performance is defined as how the aircraft responds (its motion) to the four forces of flight
- It is considered to be a branch of the Flight Mechanics discipline
- Performance is obtained once the aircraft weight and its aerodynamic and propulsion characteristics are defined. We use the following information in performance:

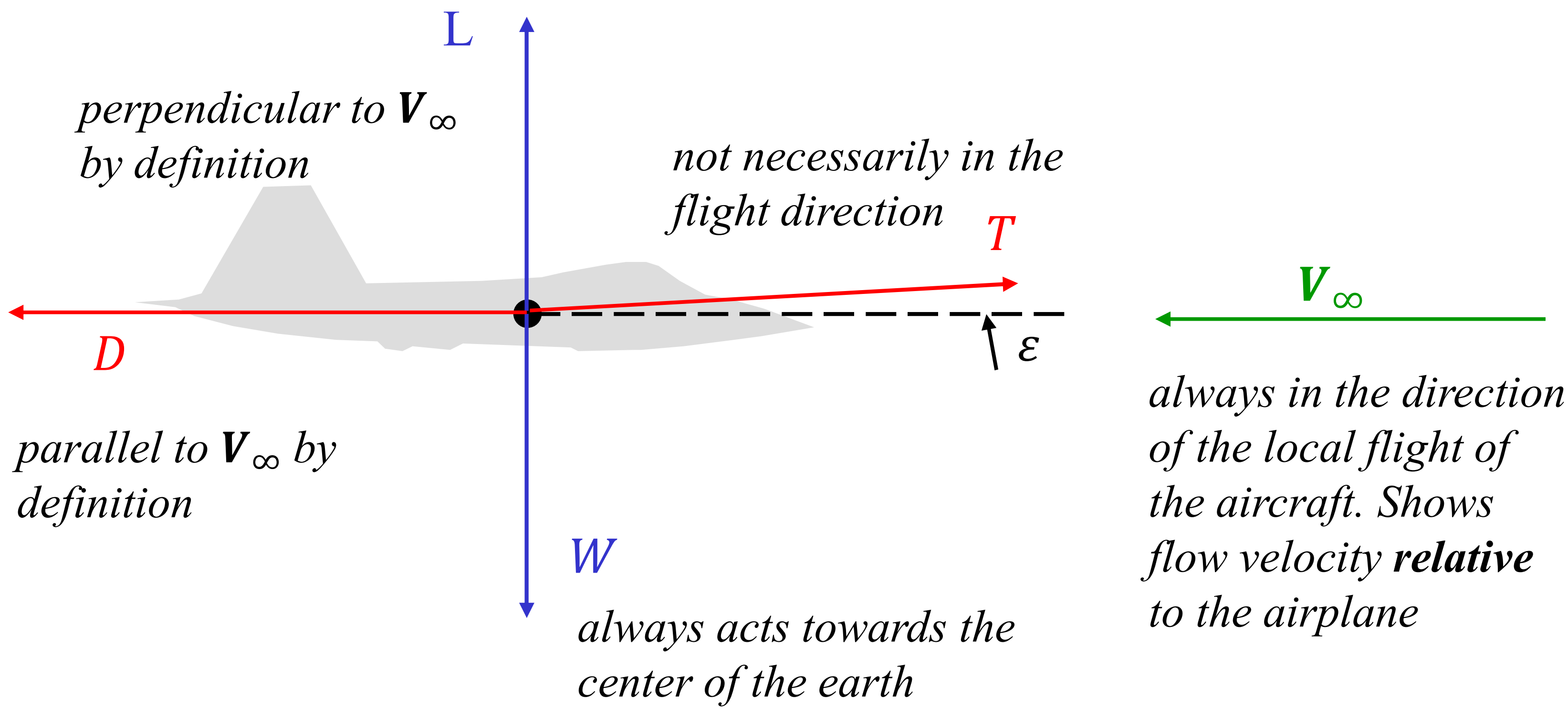
Aerodynamics $\longrightarrow$ Drag polar, lift coefficient

Propulsion $\longrightarrow$ Thrust or power, SFC

The Four Forces of Flight

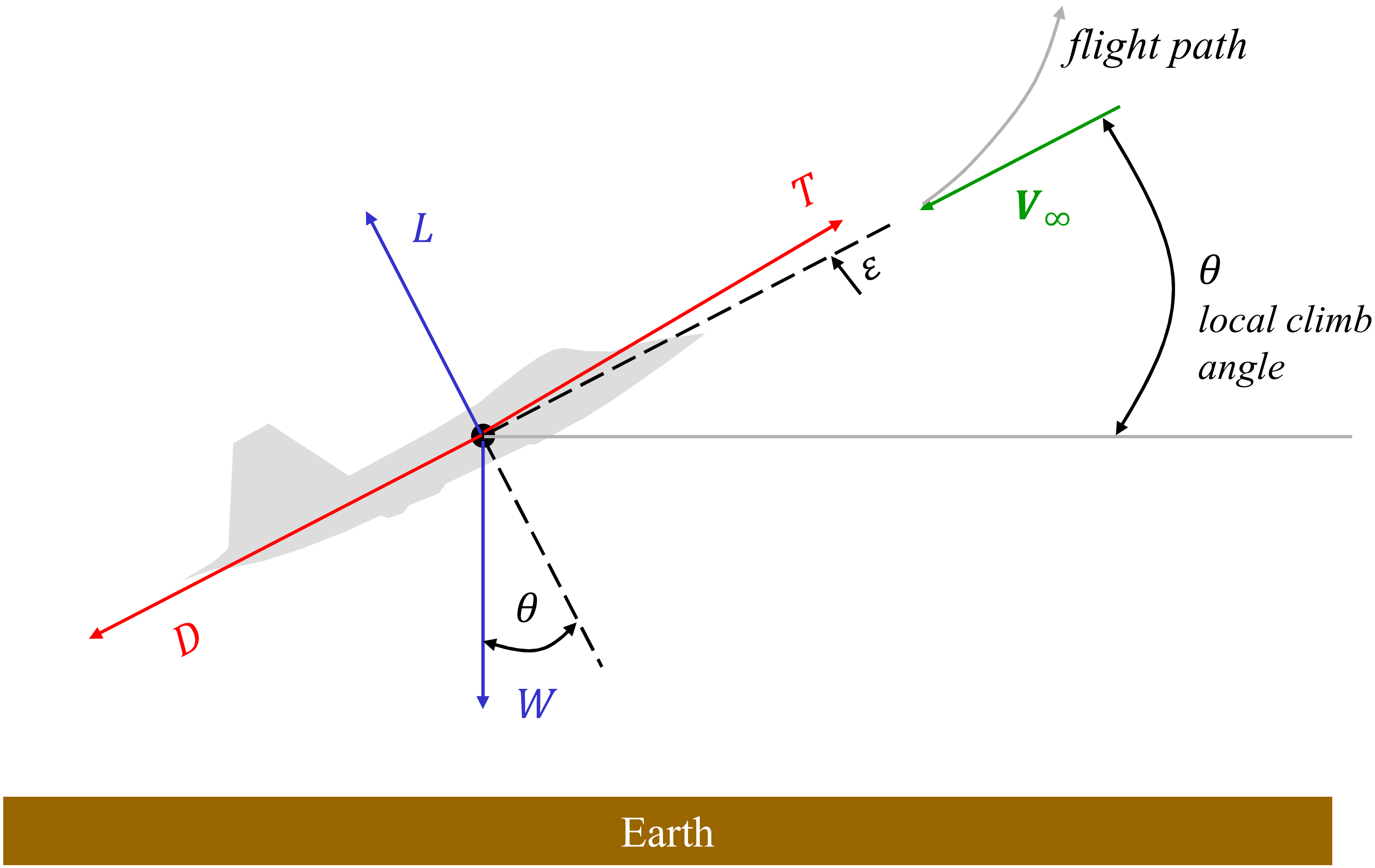
Lift, Drag, Weight, Thrust

Lift and Drag are for **complete** airplane



Steady, Level Flight

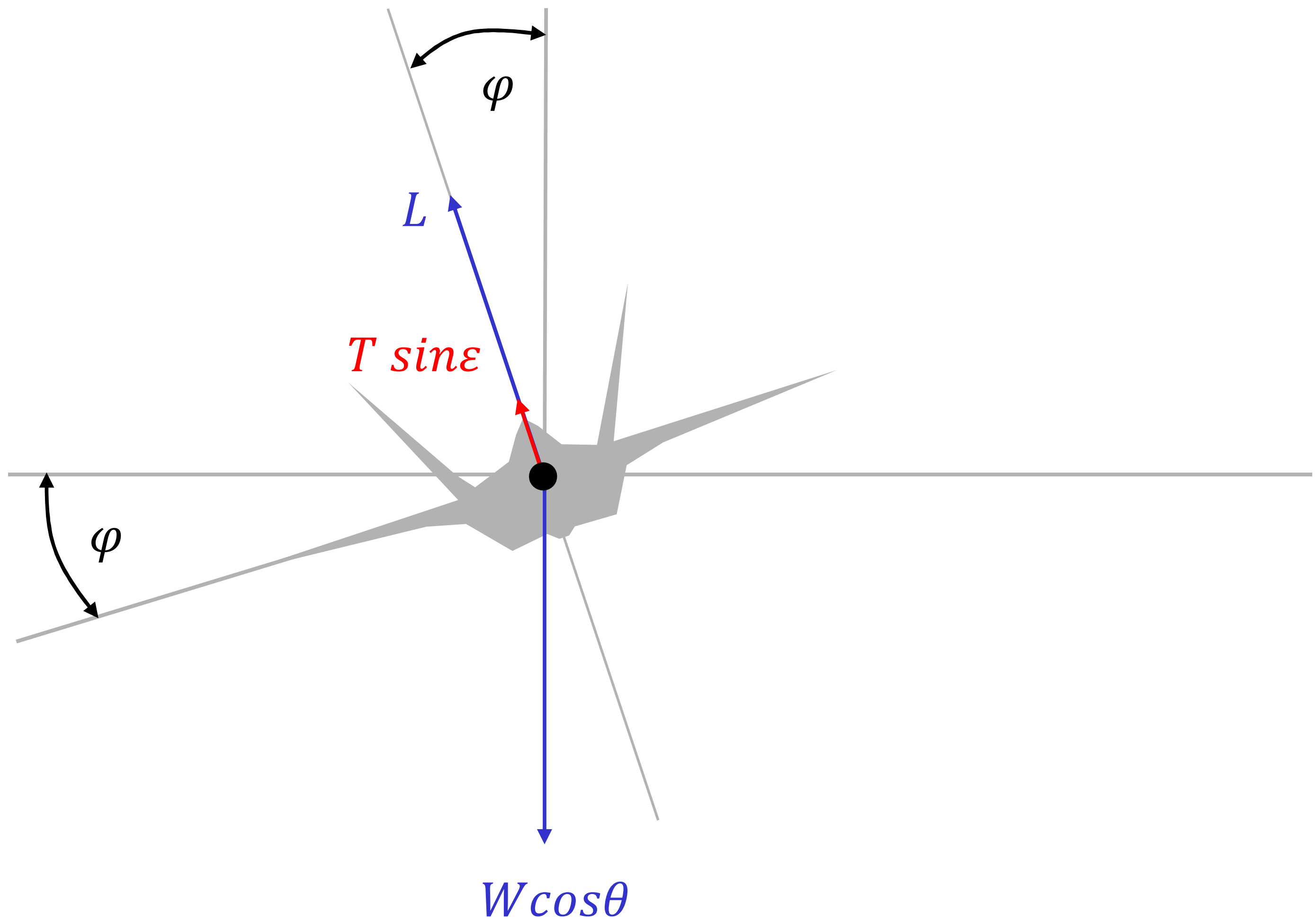
Four Forces in Climbing Flight



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Turning, Banking the Aircraft

φ – Bank (roll) angle



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

The Equations of Motion

- Based on Newton's Second Law:

$$\boldsymbol{F} = m\,\boldsymbol{a}$$

note this is in a vector form

- In scalar form, for arbitrary direction in space, *s*

$$F_s = m\,a_s$$

<u>General, Formal Derivation</u>

Rotating spherical earth

Acceleration of gravity with distance from center of the earth

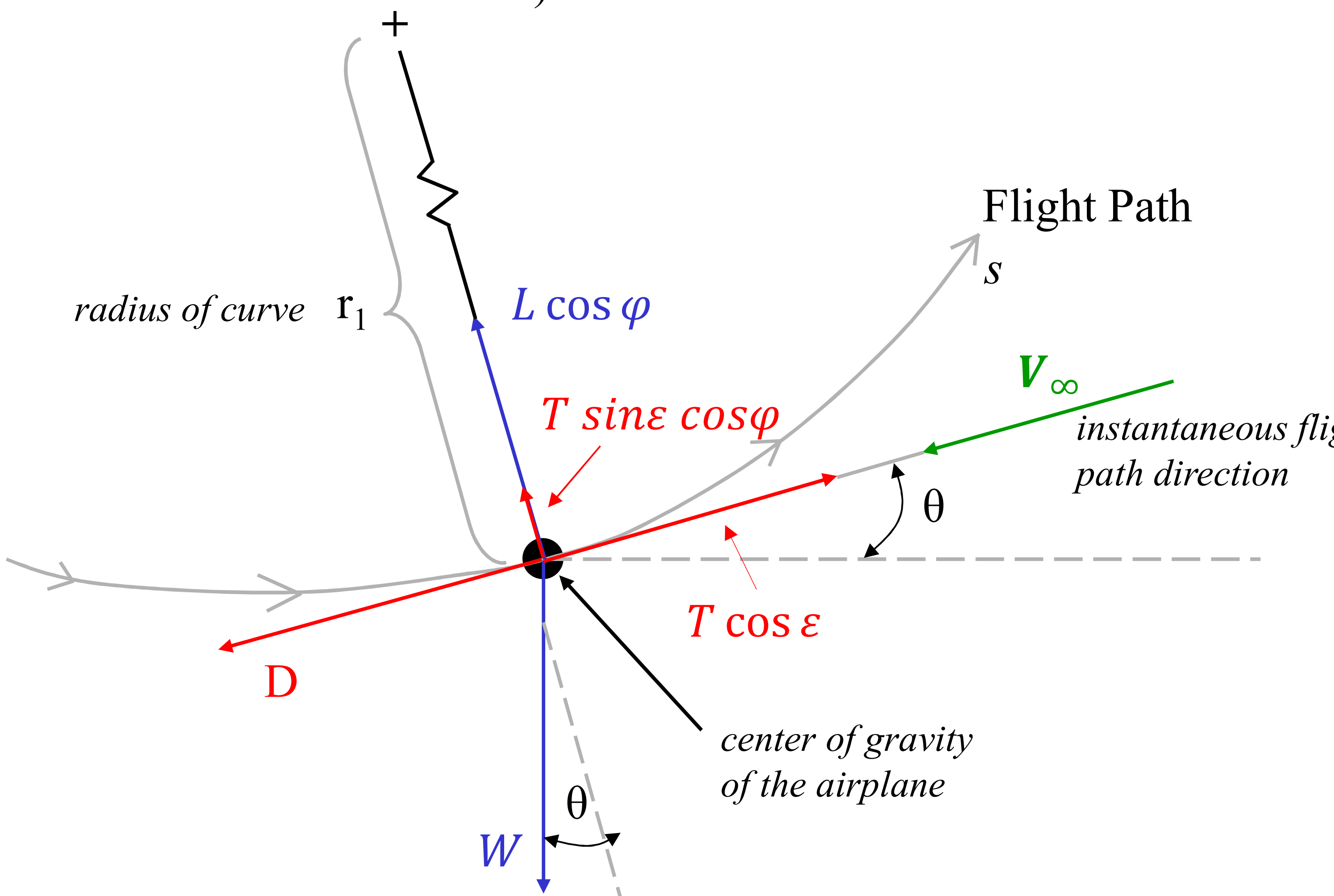
<u>Less Formal, more Physical Derivation</u>

★ Flat, stationary earth

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Climbing, Banking Flight

- Replace aircraft with point mass at its center of gravity (because we are only concerned with **translational** motion).



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

First Equation of Motion

Take components **parallel** to the flight path

- The force is $F_{\|} = T\cos\varepsilon - D - \mathrm{W}\sin\theta$
- The acceleration is $a_{||} = \dfrac{dV_\infty}{dt}$

- Therefore, Newton's Second Law parallel to the flight path is

First Equation of Motion

$$\boxed{m\frac{dV_\infty}{dt} = T\cos\varepsilon - D - \mathrm{W}\sin\theta}$$

$$ma = F$$

Second Equation of Motion

- Take components **perpendicular** to the flight path
  - The force is $F_{\perp} = L\cos\varphi + \mathrm{T}\sin\varepsilon\cos\varphi - \mathrm{W}\cos\theta$
  - The radial acceleration is $a_{\perp} = \dfrac{V_{\infty}^2}{r_1}$
- Therefore, Newton's Second Law perpendicular to the flight path is

Second Equation of Motion

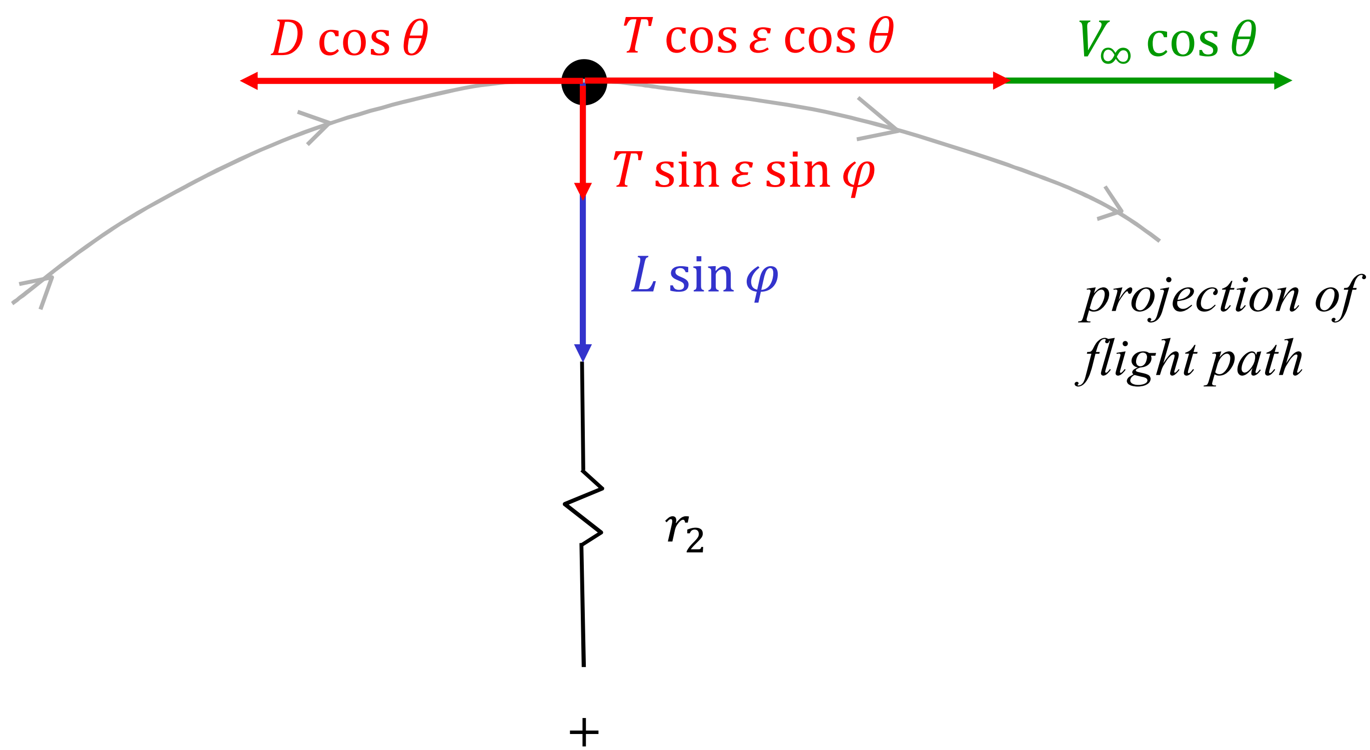
$$\boxed{m\frac{V_{\infty}^2}{r_1} = L\cos\varphi + \mathrm{T}\sin\varepsilon\cos\varphi - \mathrm{W}\cos\theta}$$

$$ma = F$$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Forces on Horizontal Plane

- Now look at flight path from a “top” view



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Third Equation of Motion

Take components **perpendicular** to the flight path in the horizontal plane (2)

- The force is $$F_2 = L \sin\varphi + \mathrm{T} \sin\varepsilon \sin\varphi$$
- The radial acceleration is $$a_2 = \frac{(V_\infty \cos\theta)^2}{r_2}$$
- Therefore, Newton's Second Law perpendicular to the horizontal flight path is

Third Equation of Motion

$$\boxed{m\frac{(V_\infty \cos\theta)^2}{r_2} = L \sin\varphi + \mathrm{T} \sin\varepsilon \sin\varphi}$$

$$ma = F$$

Summary

- The three Equations of Motion are simply statements of **Newton's Second Law**
- The three Equations of Motion describe the **translational** motion of an airplane through **three-dimensional** space over a **flat earth**
- There are three additional equations of motion that describe the **rotational** motion of the aircraft about its three axes
- Final note: The three equations of motion here **do not assume a yaw** component
  - The free stream velocity vector is assumed always parallel to the symmetry plane of the aircraft

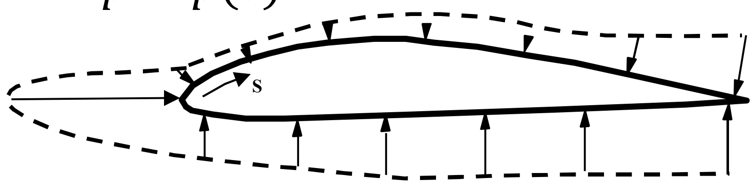
Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Sources of Aerodynamic Force

- A body immersed in an airflow will experience an **Aerodynamic Force** due to:

Pressure

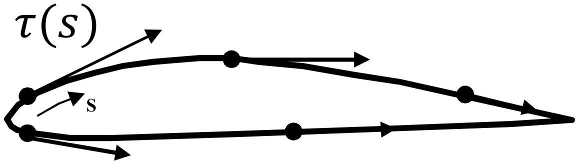
$p = p(s)$



acts perpendicular to the surface

Shear Stress

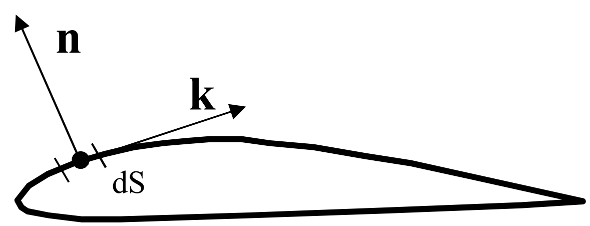
$\tau = \tau(s)$



acts parallel to the surface

- Integrate around the surface of the body to get the total force:

$$R = \iint_S pn\, dS + \iint_S \tau K\, dS$$



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

2-D Sources of Aerodynamic Forces

Only 2 sources of resultant aerodynamic force (R):

Integral of Pressure

Newton's 2nd Law:
Conservation of Momentum
gives the relationship between
pressure and velocity

$$p_0 = p_1 + \left(\frac{1}{2}\right)\rho V_1^2 = p_2 + \left(\frac{1}{2}\right)\rho V_2^2$$

Affected by:

Airfoil shape
Angle of attack
Shocks
Vortices

Integral of Shear Stress

Friction (Viscous Forces)
No slip condition at the surface
creates shear stress

Affected by:

Smoothness
Wetted area

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Derivation of Bernoulli's Equation

- Let's derive Bernoulli's equation from the x component of the momentum equation:

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \rho f_x + (F_x)_{viscous}$$

- For an inviscid flow with no body forces this equation becomes:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho \upsilon \frac{\partial u}{\partial y} + \rho \omega \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x}$$

- For steady flow, du/dt = 0 and the equation above is written as below:

$$u \frac{\partial u}{\partial x} + \upsilon \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

- Multiply by dx:

$$u \frac{\partial u}{\partial x} dx + \upsilon \frac{\partial u}{\partial y} dx + \omega \frac{\partial u}{\partial z} dx = -\frac{1}{\rho} \frac{\partial p}{\partial x} dx$$

References: Anderson; Fundamentals of Aerodynamics, 3rd Edition.

Derivation of Bernoulli's Equation

- Then substitute the Cartesian equations of a streamline shown below:

$$udz - \omega dx = 0$$

$$\upsilon dx - udy = 0$$

- And now the equation looks like this:

$$u\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

- Recall that given a function $u = u(x, y, z)$, the differential is:

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dy$$

- Substituting this you get:

$$udu = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

or: $$\frac{1}{2}d(u^2) = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

References: Anderson; Fundamentals of Aerodynamics, 3rd Edition.

Derivation of Bernoulli's Equation

- Similarly by applying these assumptions, inviscid, steady flow, to flow along a streamline, you can get the equations for the y and z components of the momentum equation:

$$\frac{1}{2}d(u^2) = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$

$$\frac{1}{2}d(\upsilon^2) = -\frac{1}{\rho}\frac{\partial p}{\partial y}dy$$

$$\frac{1}{2}d(\omega^2) = -\frac{1}{\rho}\frac{\partial p}{\partial z}dz$$

- Add these equations to get:

$$\frac{1}{2}d(u^2 + \upsilon^2 + \omega^2) = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right)$$

Or:

$$\frac{1}{2}d(V^2) = -\frac{dp}{\rho}$$

$$\boxed{dp = -\rho V dV}$$

Euler's Equation

References: Anderson; Fundamentals of Aerodynamics, 3rd Edition.

Derivation of Bernoulli's Equation

- Euler's Equation works for Incompressible and Compressible Flow under the previous assumptions:

$$\boxed{dp = -\rho V dV}$$

- However, with the incompressible flow assumption, where ρ = constant, the flow can easily be integrated between two points along a streamline to get Bernoulli's Equation:

$$\int_1^2 dp = -\rho \int_1^2 V dV$$

$$p_2 - p_1 = -\rho\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)$$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

Euler's vs Bernoulli's Equations

Euler's Equation

$$dp = -\rho V dV$$

- Assumptions:
  - Potential flow (inviscid, incompressible)
  - No body forces
  - Barotropic ($\rho = f(P)$)
  - Homogeneous fluid (uniform)
  - Continuous, steady flow along a streamline
  - Uniform velocity

Bernoulli's Equation

$$p_1 + \frac{V_1^2}{2} = p_2 + \frac{V_2^2}{2}$$

- Assumptions:
  - Potential flow (inviscid, incompressible)
  - No body forces
  - Neglect fluid friction
  - Idealized laminar flow
  - Barotropic ($\rho = f(P)$)
  - Continuous, steady flow along a streamline (points 1 and 2 are on a streamline)

Euler's vs Bernoulli's Equations

Euler's Equation

$$dp = -\rho V dV$$

- Uses:
  - Describe how velocity, pressure, and density are related
  - Aeroelastic problems
  - Useful for prediction of flow properties for a 2D thin airfoil at low angles of attack

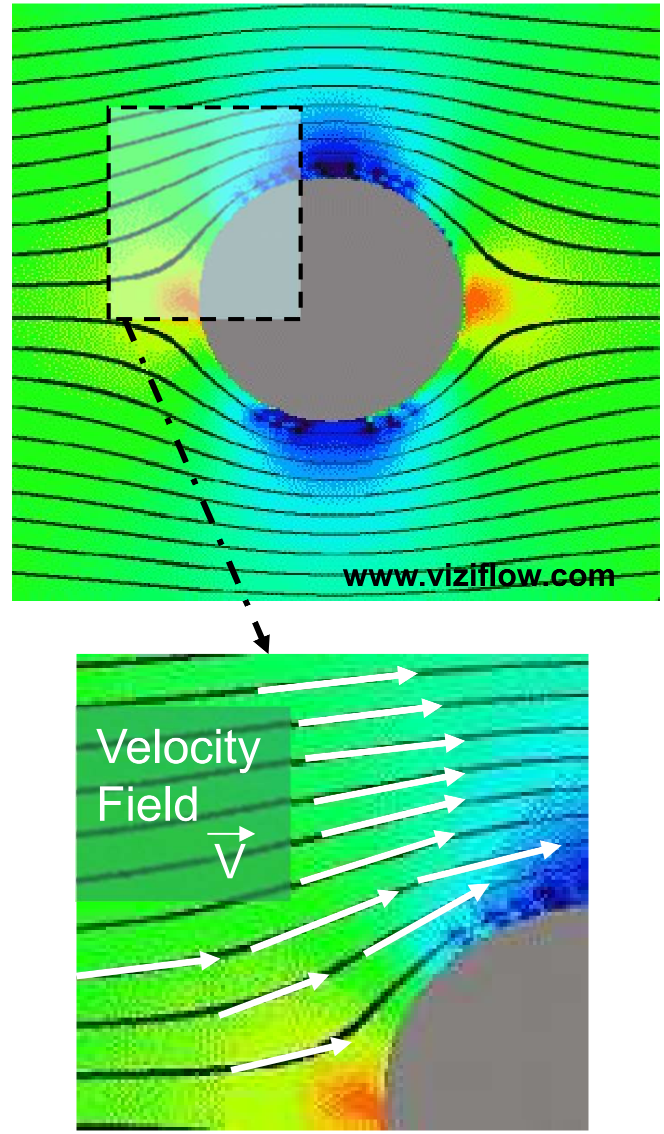
Bernoulli's Equation

$$p_1 + \frac{V_1^2}{2} = p_2 + \frac{V_2^2}{2}$$

- Uses:
  - Equivalent to the energy conservation principle for a fluid
  - Applicable only to steady flow
  - Useful for predicting low velocities over a displacement

Defining Streamlines

- **Purpose of a Streamline:**
  - **Streamlines** provide a means for **flow pattern visualization** based on the **Eulerian** Approach
- **Definition of a Streamline:**
  - A **streamline** is a line that is tangential to the **local velocity vector** of the flow field at a given instant
- **Properties of a Streamline:**
  - Describes the **velocity field** of the flow
  - At every point along the path of a streamline, the velocity is **tangent** to the path
  - The **density** of streamlines in the vicinity of a point is proportional to the **magnitude** of the velocity at that point
  - The **mass** contained between any two streamlines is **conserved** throughout the flow field (NO normal velocity components)
  - For a **rigid body** inside the flow field, a line on the surface of the body is a streamline

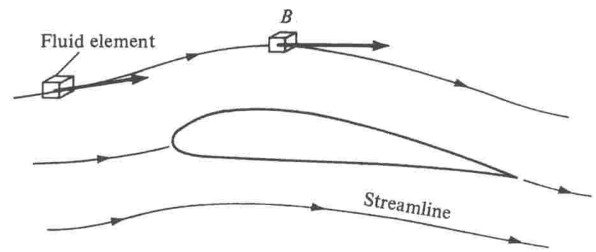


What are "streamlines"

Definition:

- A **streamline** is a curve whose tangent at any point is in the direction of velocity vector at that point.

$$(ds \times V = 0)$$

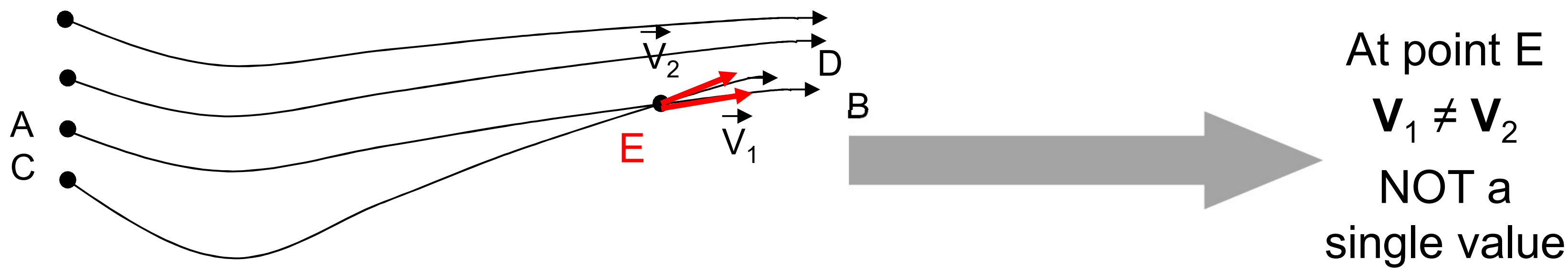


- In general, a set of streamlines is a snapshot of velocity vectors in the flow field at a given time
- For steady flow, streamline pattern is invariant at different time; while in unsteady flow, it changes with time

Can Streamlines intersect?

Important Remarks

- The velocity at **any** point in the flow has a **single** value
- The flow cannot go at more than **one direction** at the same instant



- Therefore one can conclude that by definition if two lines are streamlines, then they **CANNOT INTERSECT**!!
- If they could intersect, the velocity would NOT be **uniquely** determined at that point
- There is an exception concerning points where the velocity magnitude is **zero** (e.g. a stagnation point)

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

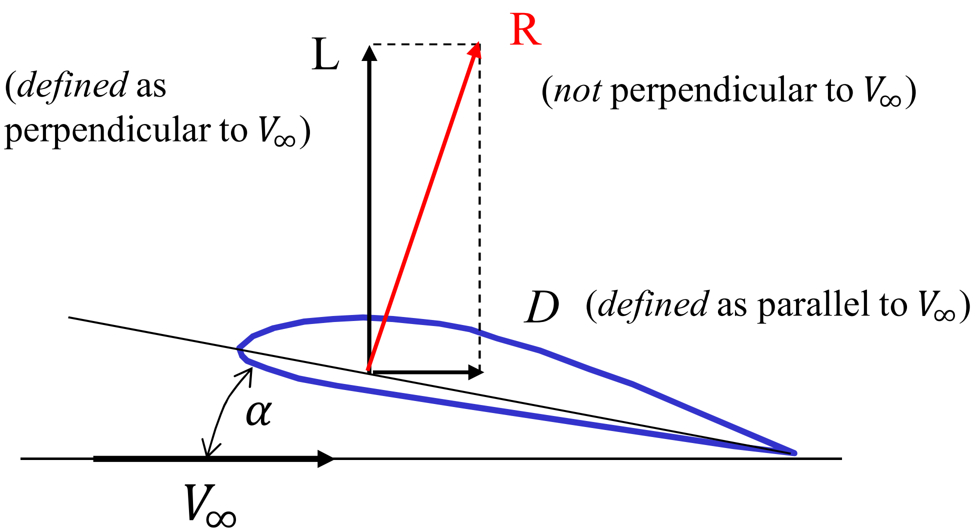
Is Bernoulli's Equation valid for Compressible Flow?

Answer: No Because it was derived with incompressible flow assumptions.

Reference: Anderson; Fundamentals of Aerodynamics, 3rd Edition.

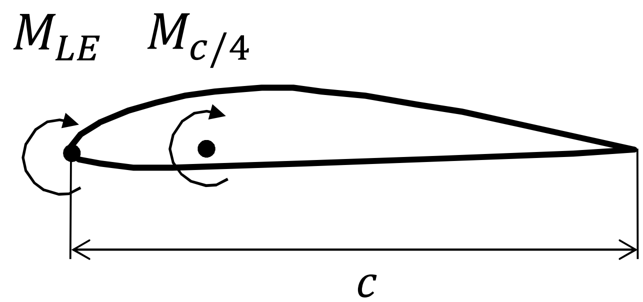
Aerodynamic Lift, Drag, and Moments

AERODYNAMIC FORCES



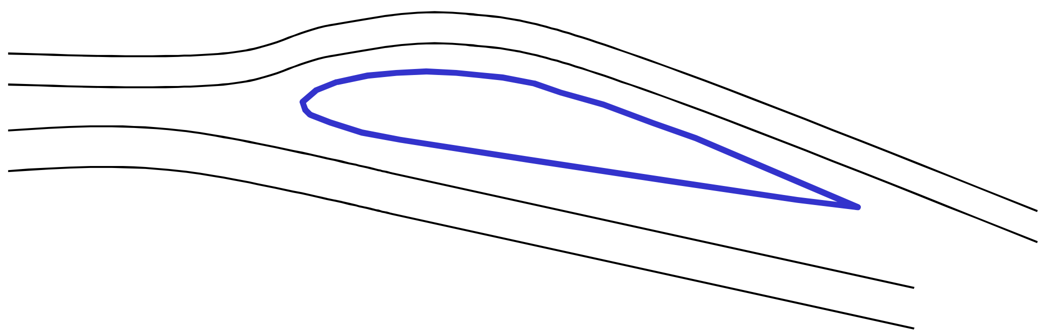
"free stream velocity" or "relative wind"

MOMENTS



- By convention, a moment which rotates a body causing an **increase** in angle of attack is **positive**

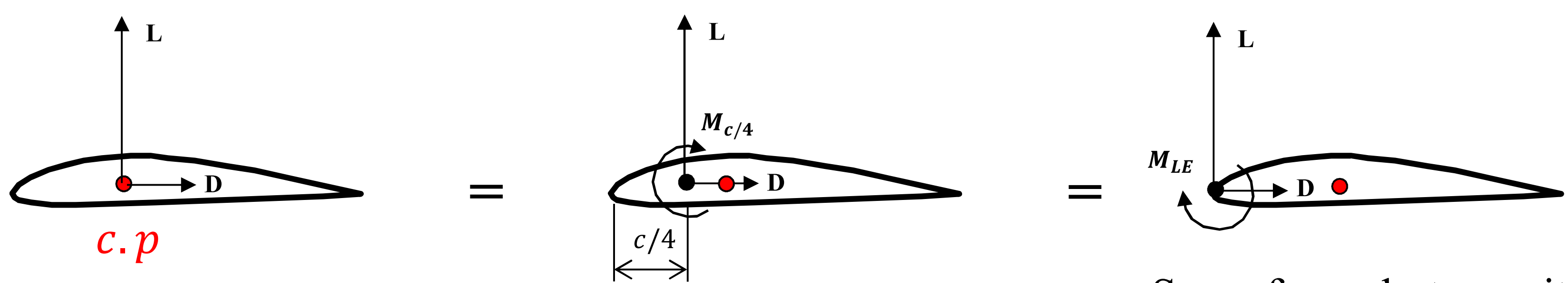
- What creates lift?
  - There is a net change of momentum in vertical plane between LE and TE of the airfoil
  - Airfoil pushes air downwards



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Center of Pressure vs Aerodynamic Center

- The center of pressure is the point where the resultant force due to the pressure field acts on the object
  - The magnitude of the resultant force is the value of the integral of the pressure field
- For an airfoil moving through a fluid, the center of pressure is the point where the pressure field can be reduced to a force with no associated moment



NO moment!

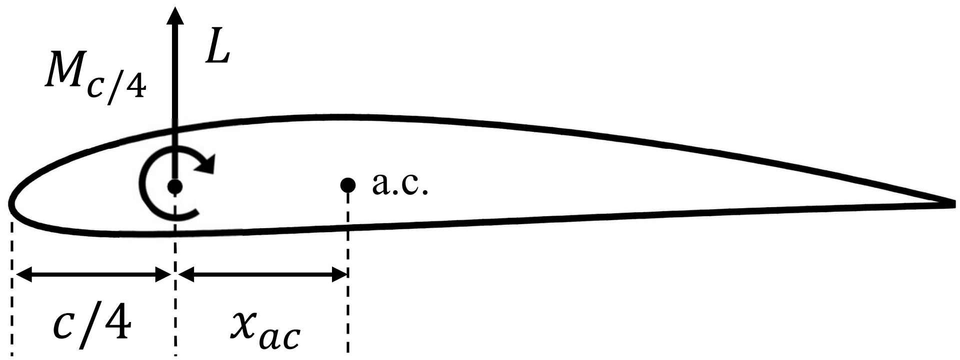
Same force, but move it to the quarter chord and add a moment

Same force, but now it's at the leading edge, along with a moment about the leading edge

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Center of Pressure vs Aerodynamic Center

- The aerodynamic center (x_{ac}) is the point on an airfoil about which the aerodynamic moment is independent of the lift coefficient or, equivalently, independent of angle of attack
  - Mathematically stated as $\left.\frac{dc_m}{dc_l}\right|_{x_{ac}} = 0$

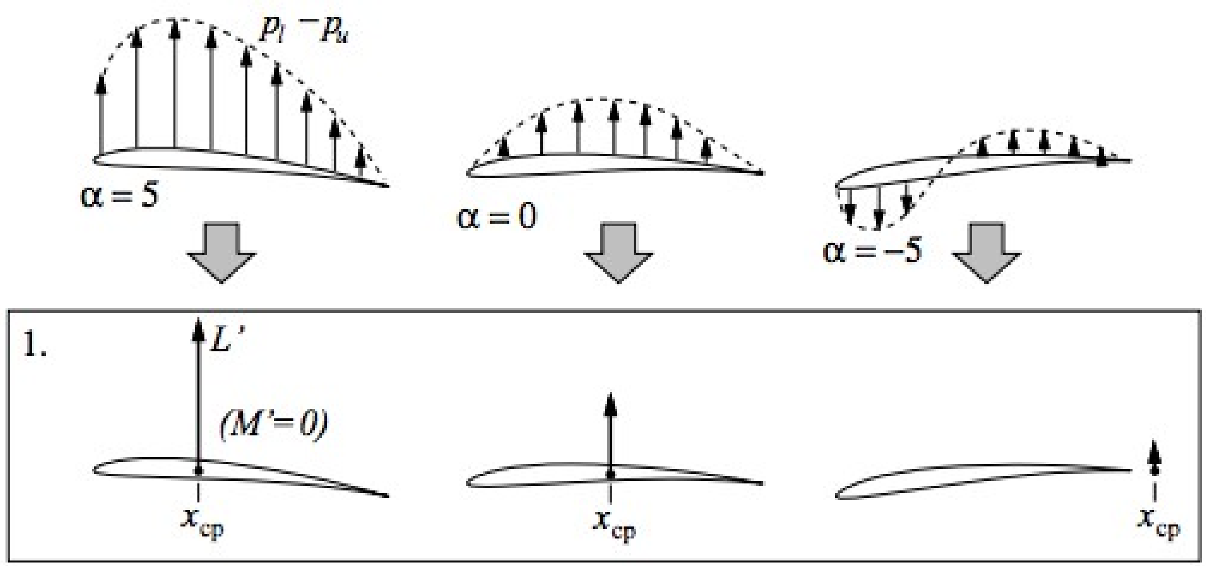


  - Important for stability calculations, since we need to know the point through which the lift is acting

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Center of Pressure vs Aerodynamic Center

- While the center of pressure allows for the flow field around the airfoil to be reduced to a point, it is not a fixed point
  - It changes with angle of attack, as pressure distribution changes shape



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Center of Pressure vs Aerodynamic Center

- The changing location of the center of pressure comes as a result of the asymmetric contour of the airfoil
  - Undesirable for most of our calculations since they would have to change with location → it is preferable to convert the lift acting at the c.p. to an equivalent force and moment combination about the a.c.
  - For the **symmetric airfoil**, both the c.p. and a.c. actually coincide at the quarter-chord point (thin airfoil theory!)
  - For the **cambered airfoil**, however, the location of the a.c. depends on the lift and moment curve slopes

Calculating the Aerodynamic Center

- Sum of moments about a.c:

$$M_{ac} = M_{c/4} + Lx_{ac}$$

- Divide by $q_\infty Sc$:

$$\frac{M_{a.c}}{q_\infty Sc} = \frac{M_{c/4}}{q_\infty Sc} + \frac{L}{q_\infty S} + \frac{x_{a.c}}{c} \qquad c_{ma.c} = c_{mc/4} + c_l \frac{x_{a.c}}{c}$$

- Then differentiate with respect to α:

$$\frac{dc_{ma.c}}{d\alpha} = \frac{dc_{mc/4}}{d\alpha} + \frac{dc_l}{d\alpha}\frac{x_{a.c}}{c}$$

- Using definition of a.c., then:

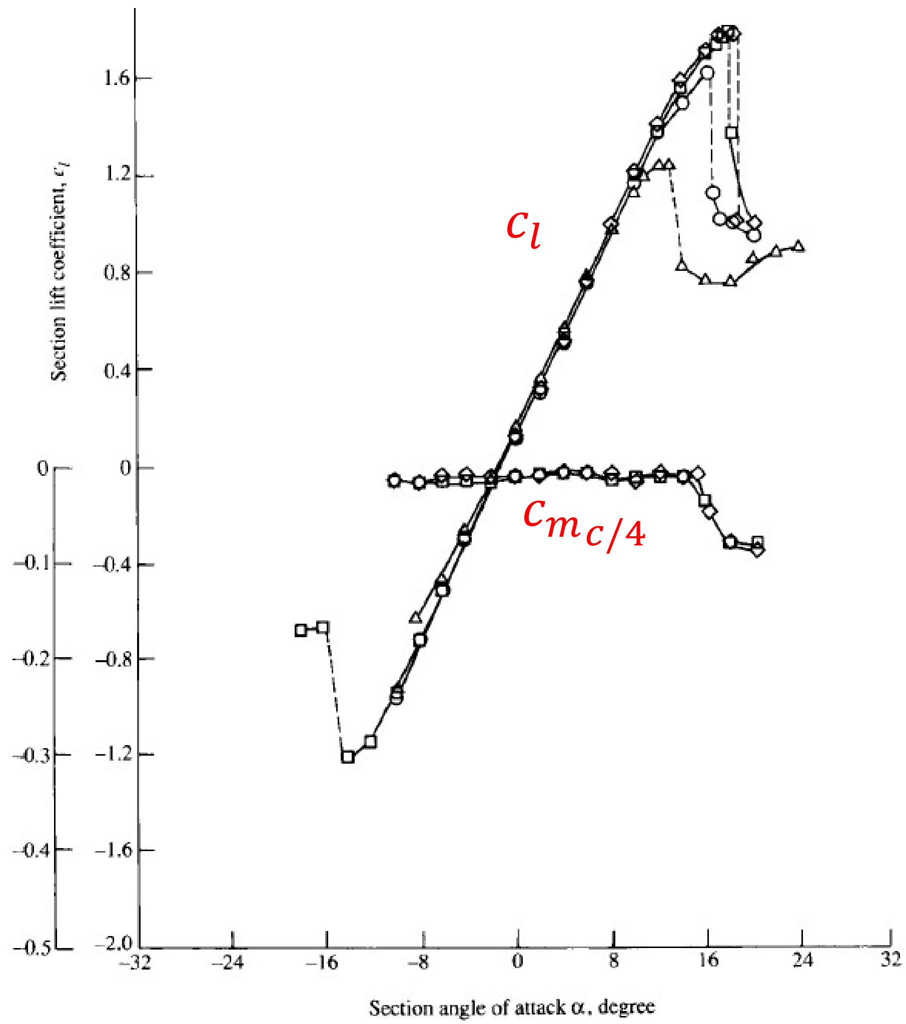
$$\frac{dc_{ma.c}}{d\alpha} = 0 \rightarrow \frac{dc_{mc/4}}{d\alpha} = -\frac{dc_l}{d\alpha}\frac{x_{a.c}}{c}$$

Calculating the Aerodynamic Center

- As seen, $\frac{dc_{m_{c/4}}}{d\alpha}$ and $\frac{dc_l}{d\alpha}$ are constant with respect to angle of attack over linear portions
- Hence, we can solve for x_{ac}/c (location of aerodynamic center with respect to airfoil chord)

$$\boxed{\frac{x_{ac}}{c} = \frac{-\frac{dc_{m_{c/4}}}{d\alpha}}{\frac{dc_l}{d\alpha}} = -\frac{m_0}{a_0}}$$

- This proves that, for a body with linear lift and moment curves, the aerodynamic center does exist as a **fixed point** on the airfoil



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Aerodynamic Coefficients

- From intuition and basic knowledge, we know:
  - Aerodynamic force = *f(velocity, density, size of body, angle of attack, viscosity, compressibility)*

$$L = L(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$$

$$D = D(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$$

$$M = M(\rho_\infty, V_\infty, S, \alpha, \mu_\infty, a_\infty)$$

- To find out how the lift on a given body varies with the parameters, we could run a series of wind tunnel tests in which the velocity, say, is varied and everything else stays the same
- From this we could extract the change in lift due to change in velocity
- If we did this for each parameter, and each force (moment), we would have to conduct experiments that resulted in 19 stacks of data (one for each variation plus a baseline)
  - This is bad: wind tunnel time is very expensive and the whole process is time consuming

Buckingham π Theorem

http://brainstorming.typepad.com/photos/london/

Edgar Buckingham

- 1867 ~ 1940
- Education
  - B.S. in Physics from Harvard
  - Grad school at University of Strasbourg
  - Ph.D. from University of Leipzig
- Known for
  - Buckingham π Theorem
  - Work on soil water (esp. unsaturated flow and capillary action in soils)

Buckingham π Theorem

- This is a key theorem in dimensional analysis
- A function with "n" variables can be described with "$n - k$" non-dimensional variables
- $(p = n - k)$
  - p is the # of dimensionless variables needed
  - n is the # of physical variables in physical relation
  - k is the # of fundamental dimensions required to describe the physical variables

Buckingham π Theorem

- Note
  - The dependent variable appears in only one of the π's
  - To calculate the π's, select $q_1, q_2, \ldots, q_k$ and one more q per π

$$f_1(q_1, q_2, \cdots; q_N) = 0$$

$$f_2(\pi_1, \pi_2, \cdots; \pi_{N-K}) = 0$$

$$\pi_1 = f_3(q_1, q_2, \cdots; q_k, q_{k+1})$$

$$\pi_2 = f_3(q_1, q_2, \cdots; q_k, q_{k+2})$$

$$\vdots$$

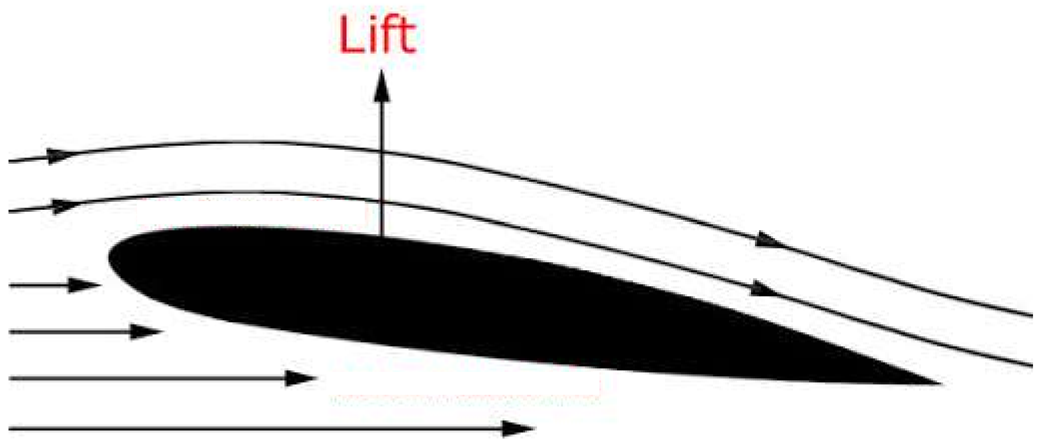
$$\pi_i = f_3(q_1, q_2, \cdots; q_k, q_N)$$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Lift Force on an Airfoil

$$L = f(\rho_\infty, V_\infty, c, \mu_\infty, a_\infty) \qquad 0 = g(L, \rho_\infty, V_\infty, c, \mu_\infty, a_\infty)$$

- Fundamental Dimensions
  - M = mass
  - L = length
  - T = time
- K = 3



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Lift Force on an Airfoil

- Physical variables
  - $[L] = MLT^{-2}$
  - $[\rho_\infty] = ML^{-3}$
  - $[V_\infty] = LT^{-1}$
  - $[c] = L$
  - $[\mu_\infty] = ML^{-1}T^{-1}$
  - $[a_\infty] = LT^{-1}$
- $N = 6$
- $N - K = 6 - 3 = 3$

$$f_2(\pi_1, \pi_2, \cdots; \pi_{N-K}) = 0$$

$$\pi_1 = f_3(q_1, q_2, \cdots; q_k, q_{k+1})$$

$$\pi_2 = f_3(q_1, q_2, \cdots; q_k, q_{k+2})$$

$$\vdots$$

$$\pi_i = f_3(q_1, q_2, \cdots; q_k, q_N)$$

Lift Force on an Airfoil

- Physical variables
  - $[L] = MLT^{-2}$
  - $[\rho_\infty] = ML^{-3}$
  - $[V_\infty] = LT^{-1}$
  - $[c] = L$
  - $[\mu_\infty] = ML^{-1}T^{-1}$
  - $[a_\infty] = LT^{-1}$
- $N = 6$
- $N - K = 6 - 3 = 3$

$$f(\pi_1,\pi_2,\pi_3)=0$$

$$\pi_1 = f_3(\rho_\infty, V_\infty, c, L)$$

$$\pi_2 = f_3(\rho_\infty, V_\infty, c, \mu_\infty)$$

$$\pi_3 = f_3(\rho_\infty, V_\infty, c, a_\infty)$$

Lift Force on an Airfoil

$$\pi_1 = \rho_\infty^b V_\infty^d c^e L$$

$$[\pi_1] = (ML^{-3})^b (LT^{-1})^d (L)^e (MLT^{-2})$$

- For each term:
  - M: $b + 1 = 1$
  - L: $-3b + d + e + 1 = 0$
  - T: $-d - 2 = 0$
- Solving the system of equations:
  - $b = -1$
  - $d = -2$
  - $e = -2$

Lift on an Airfoil

- Previously, we found that:

$$b = -1 \quad d = -2 \quad e = -2$$

- Hence, we create our first group:

$$\pi_1 = \rho_\infty^{-1} V_\infty^{-2} c^{-2} L$$

- Similarly:

$$\pi_1 = \frac{L}{\rho_\infty V_\infty^2 c^2} = \frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 S} = C_L$$

$$\pi_2 = \frac{\rho_\infty V_\infty c}{\mu_\infty} = Re \qquad \pi_3 = \frac{V_\infty}{a_\infty} = M_\infty$$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Aerodynamic Coefficients

- Let's define lift, drag, and moment coefficients for a given body:

$$C_L = \frac{L}{q_\infty S} \qquad C_D = \frac{D}{q_\infty S} \qquad C_M = \frac{M}{q_\infty S c}$$

  – q is defined as the *dynamic pressure*:

$$q = \frac{1}{2}\rho V_\infty^2$$

  – c is defined as a characteristic length of a body, usually the chord length

- Now define the following *similarity parameters*:

$$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty}$$

Reynolds Number (based on chord length)

$$M_\infty = \frac{V_\infty}{a_\infty}$$

Mach Number

Aerodynamic Coefficients

- Using dimensional analysis, we get the following results. For a *given body shape*:

$$C_L = f_1(\alpha, Re, M_\infty)$$

$$C_D = f_2(\alpha, Re, M_\infty)$$

$$C_M = f_3(\alpha, Re, M_\infty)$$

- If we conduct the same experiments, we can now get the equivalent data with 10 stacks of data

- But more fundamentally, dimensional analysis tells us that, *if* the Reynolds Number and the Mach Number are the *same* for two different flows (different density, velocity, viscosity, speed of sound), the lift coefficient **will be the same**, given two geometrically similar bodies at the same angle of attack.

- This is the driving principle behind wind tunnels

- But…be careful. In real life, it is very difficult to match both *Re* and *M*

Reference Area, S

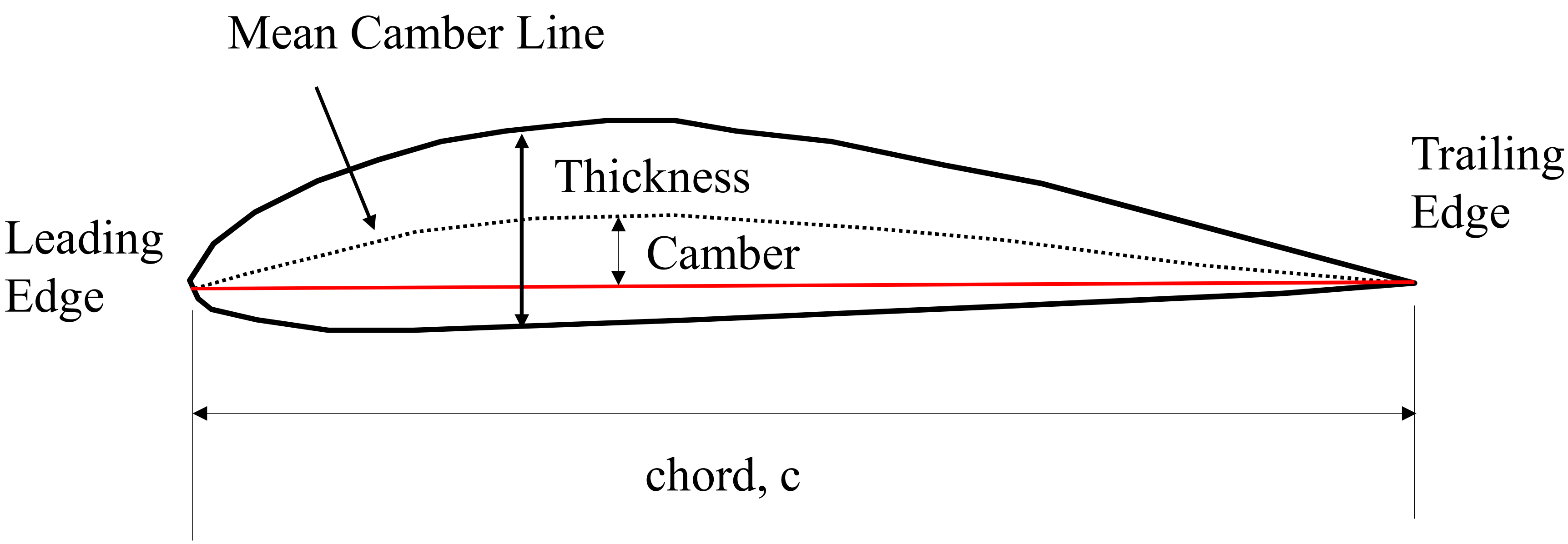
- *S* is some sort of reference area used to calculate the aerodynamic coefficients
  - *S as wetted area* - not common, but is the surface upon which the pressure and shear distributions act, so it is a meaningful geometric quantity when discussing aerodynamic force
  - *S as planform area* - the projected area we see when looking down at the wing or aircraft (the "shadow"). Most common definition of S used when calculating aerodynamic coefficients
  - *S as base area* - mostly used when analyzing slender bodies, such as missiles

The Point: it is crucial to know how S was defined when you look at or use technical data!

NACA Airfoil Nomenclature

- Today's aircraft airfoils are custom designed using CFD and Aero codes
  - Before all of this computing power, designers would use empirically designed airfoils
- NACA- National Advisory Committee for Aeronautics
  - 1920-1960 designed and tested airfoils
  - Still used today for those who don't have time or money to design own airfoil
- Until 1930, airfoil design had no rhyme or reason. NACA used a systematic approach
  - First, NACA defined airfoil nomenclature

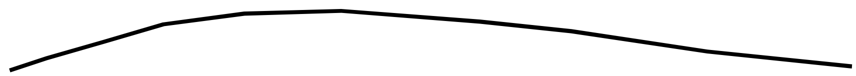
NACA Airfoil Nomenclature



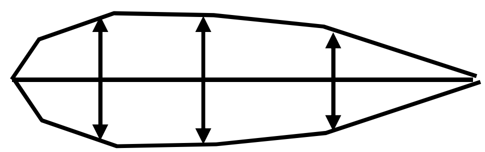
Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Designing an Airfoil

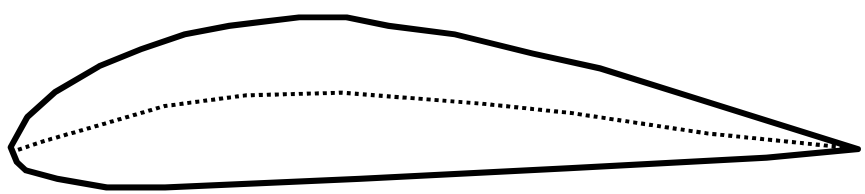
1. Pick a camber line



2. Choose a thickness distribution on a symmetric shape



3. Apply thickness distribution to camber line



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Four Digit Airfoils

#1 Digit	max camber in % of chord
#2 Digit	location of max camber in tenths of chord
#3,4 Digits	max thickness in % of chord

Ex: **NACA 2412**

Max camber of 2% of chord, located at 40% from leading edge. Max thickness 12%.

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Five Digit Airfoils

2 3 0 1 2

x 3/2

design c_l in tenths

x 1/2

location of max camber
in tenths of chord

Thickness in % of chord

So, 23012 = c_l of 0.3 at 15% chord with 12% thickness

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

6 Series Airfoils

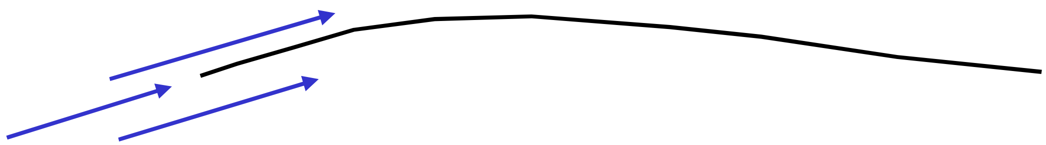
Designed for laminar flow, creating a reduction in skin friction drag

64-212

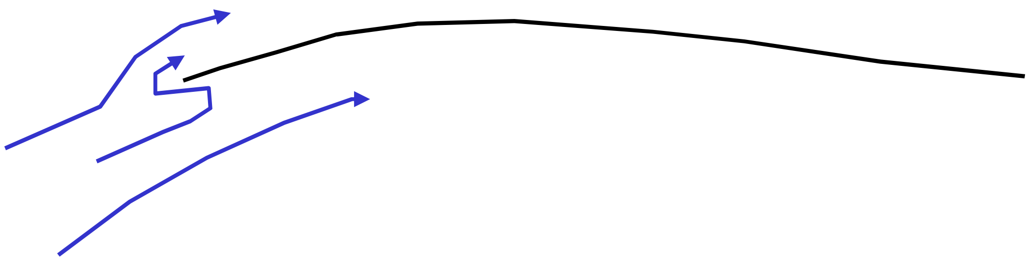
6	is just series designation
4	location of minimum pressure in tenths of chord (based on symmetric design section at $\alpha = 0$)
2	design lift coefficient in tenths
12	% max thickness

Design Lift Coefficient

- Replace airfoil with its camber line

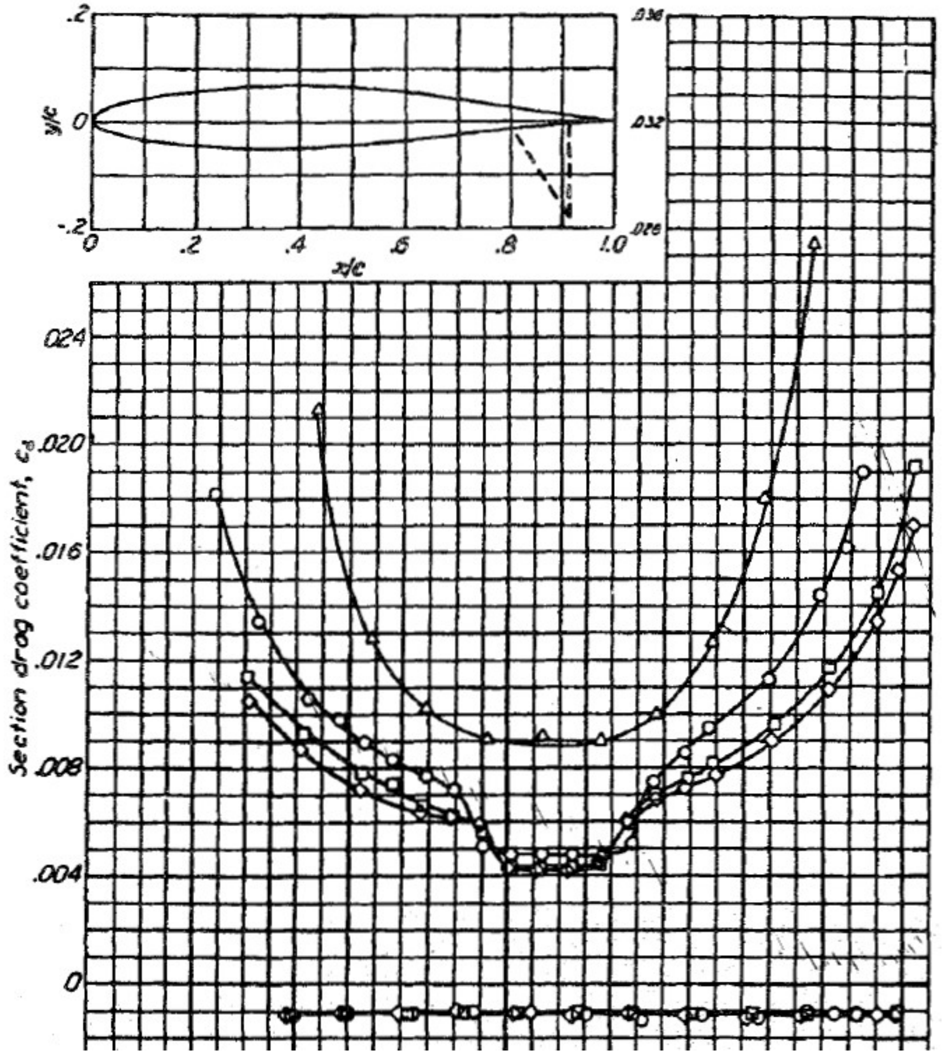


- There is only one angle of attack where flow is tangent to leading edge. Else,



- The C_l at this angle of attack is called the design lift coefficient

Drag Bucket-6 Series Laminar Airfoil



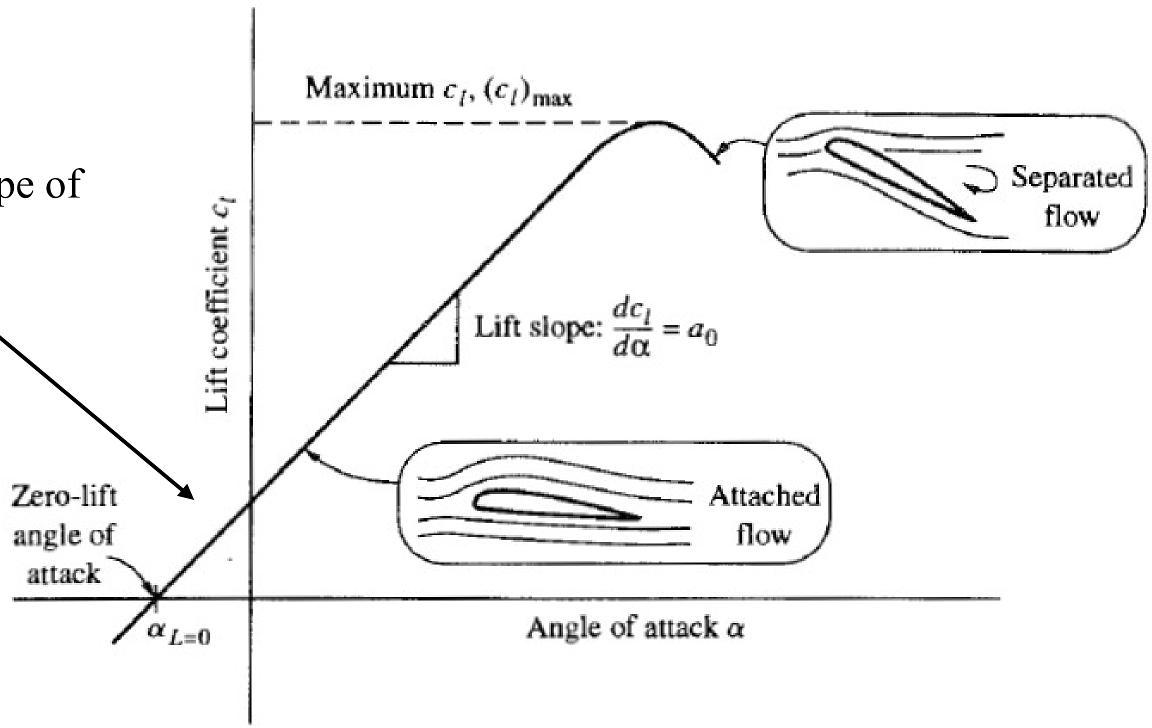
P-51 Mustang

Used 6-Series NACA Airfoil

Variation of Coefficients with Parameters

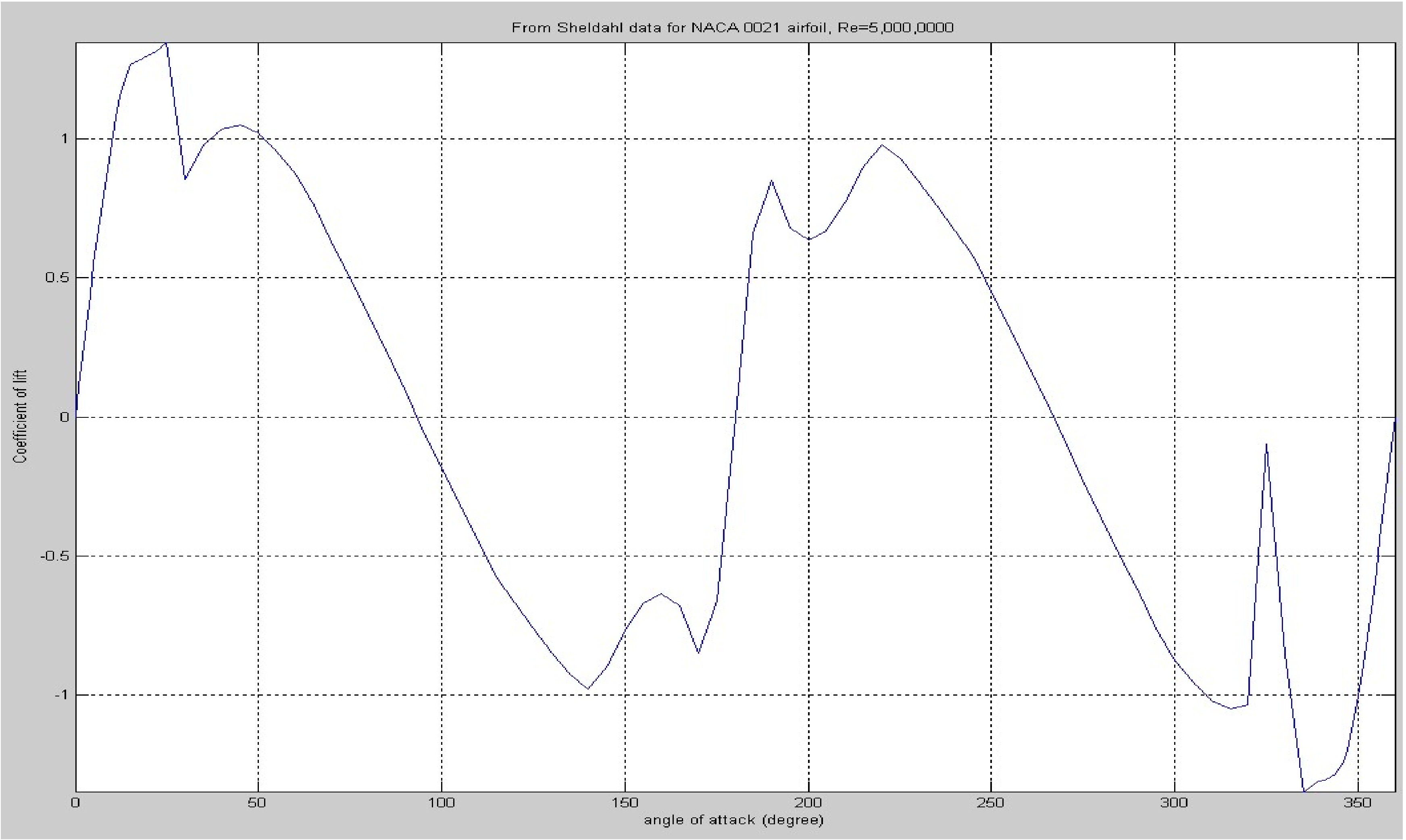
- How do the coefficients vary with α, Re, and M?
  - Answer: It depends on the flow regime and the shape of the body. Primarily, the effect of the three parameters is that they change the pressure distribution, and thus R

Note *linear* shape of slope



Generic Lift Curve Slope

Lift Coefficient dependency of angle of attack



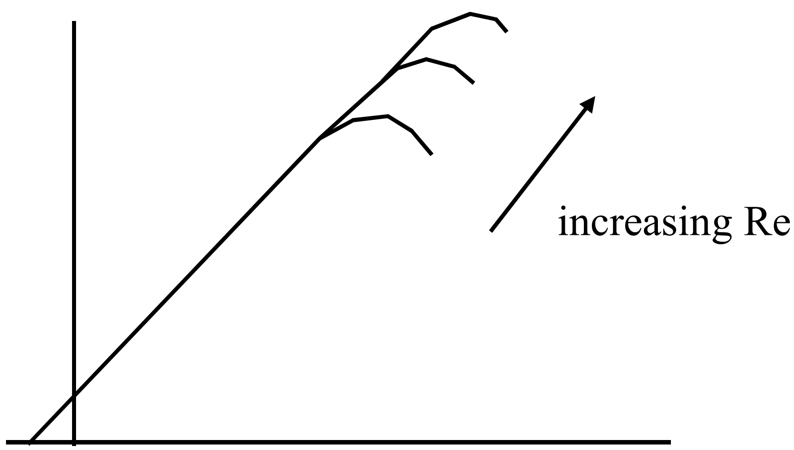
Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Variation of c_l vs. α

- **First question:** What is difference between C_L and c_l?
  - C_L is for the whole (3-D) aircraft. c_l is for a 2-D shape, usually just an airfoil
- **Features of Typical Airfoils and Lift Curve Slope:**
  - Slope is mostly linear over practical range of alpha
  - For thin airfoils, theoretical maximum of lift curve slope is 2π per radian
  - For most conventional airfoils, experimentally measured lift slopes are very close to theoretical values
  - All positively cambered airfoils have negative zero-lift angles of attack
    - A symmetric airfoil has a zero-lift angle of attack equal to zero ($\alpha_{ZL=0} = 0\ deg$)
  - At high angles of attack, slope becomes non-linear and airfoil exhibits stall due to separated flow

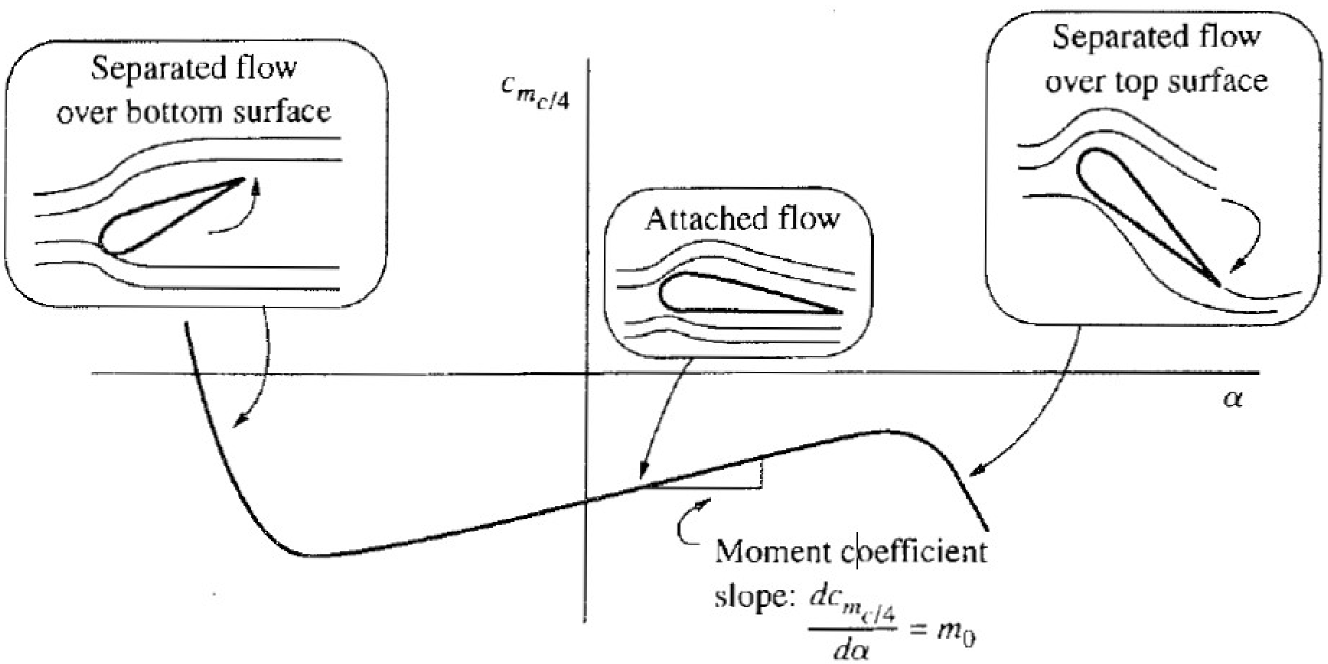
Variation of c_l vs. *Re*

- Virtually no effect on lift curve slope in linear region (a_0 is essentially insensitive to Reynolds number)
- However, at low Reynolds Numbers ($Re < 100{,}000$), there is a substantial *Re* effect
  - Small model airplanes
  - Small UAV's
- Important Re effect on $(c_l)_{max}$ due to viscous effects.



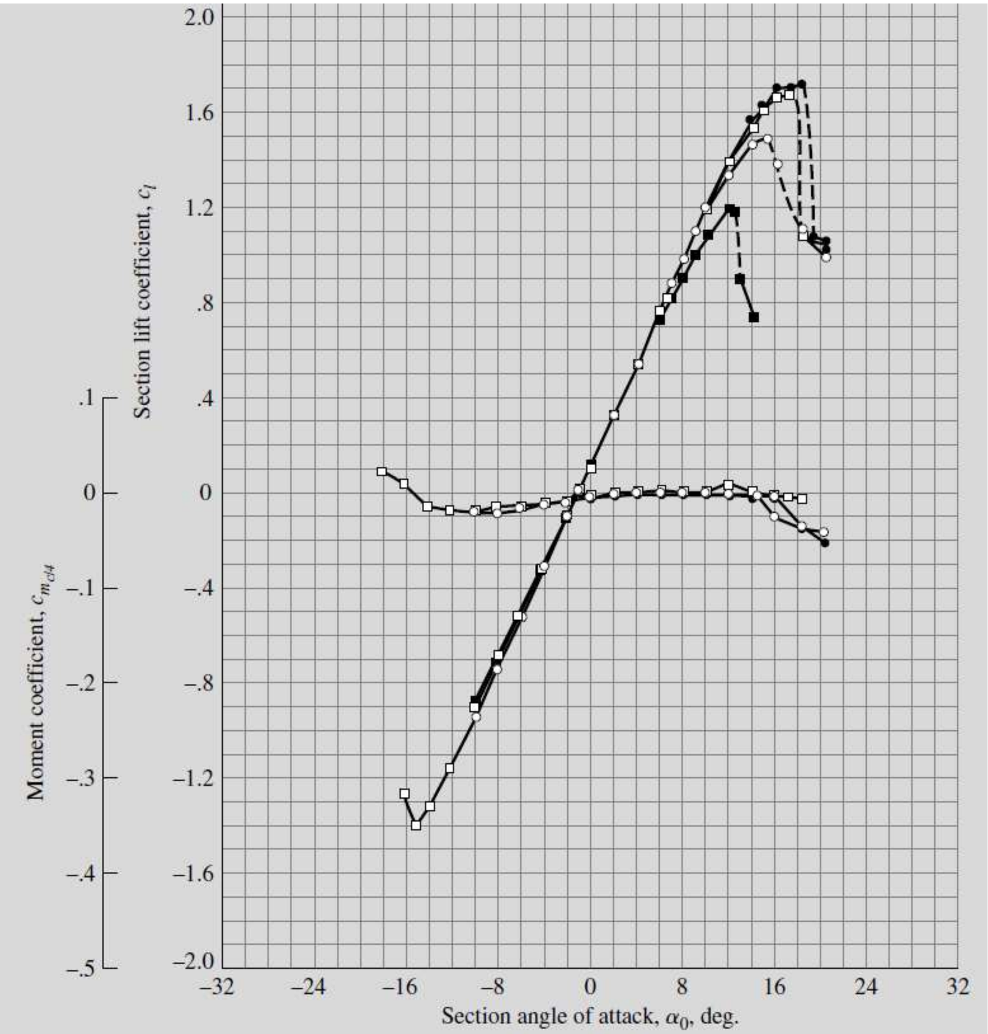
Variation of $c_{mc/4}$ vs. α and Re

- Essentially linear over practical range of angle of attack
- Slope is positive for some airfoils, negative for others
- Variation becomes non-linear at high angles of attack, when flow separates
- Linear portion of curve is essentially independent of *Re*



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Airfoil Data-NACA 23015



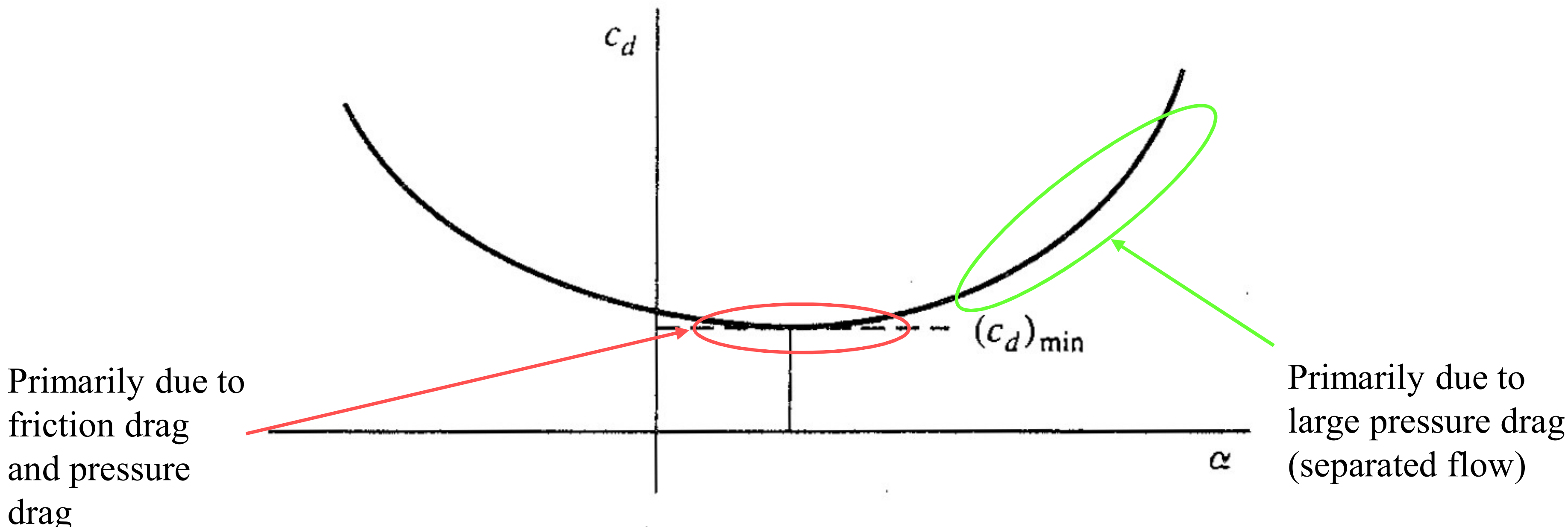
NACA 23015 Wing Section

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Drag Polar: c_d vs. c_l

- Remembering that the lift coefficient is a linear function of the angle of attack, c_l could be effectively replaced by α for trend

- For a cambered airfoil, the minimum drag value does not necessarily occur at zero angle of attack, but rather at some finite but small α



Generic Drag Curve

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Georgia Institute of Technology
Aerospace Systems Design Laboratory

c_d vs. Re and $c_{mc/4}$

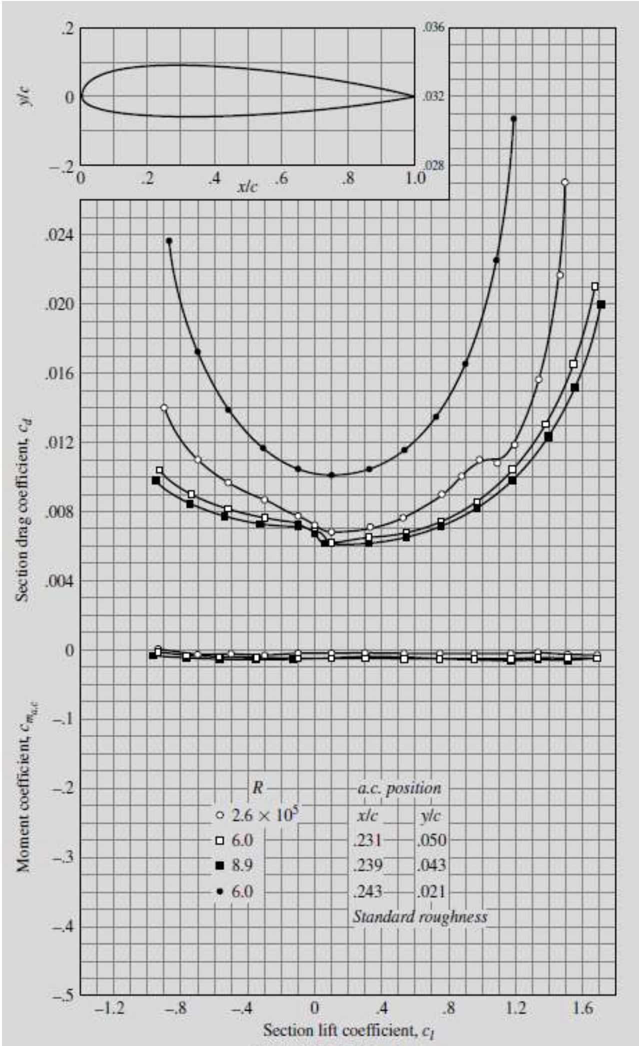
- Viscous flow theory states that the local skin friction coefficient, c_f, varies as

$$c_f \propto \frac{1}{\sqrt{Re}} \qquad \text{for laminar flow}$$

$$c_f \propto \frac{1}{(Re)^{0.2}} \qquad \text{for turbulent flow}$$

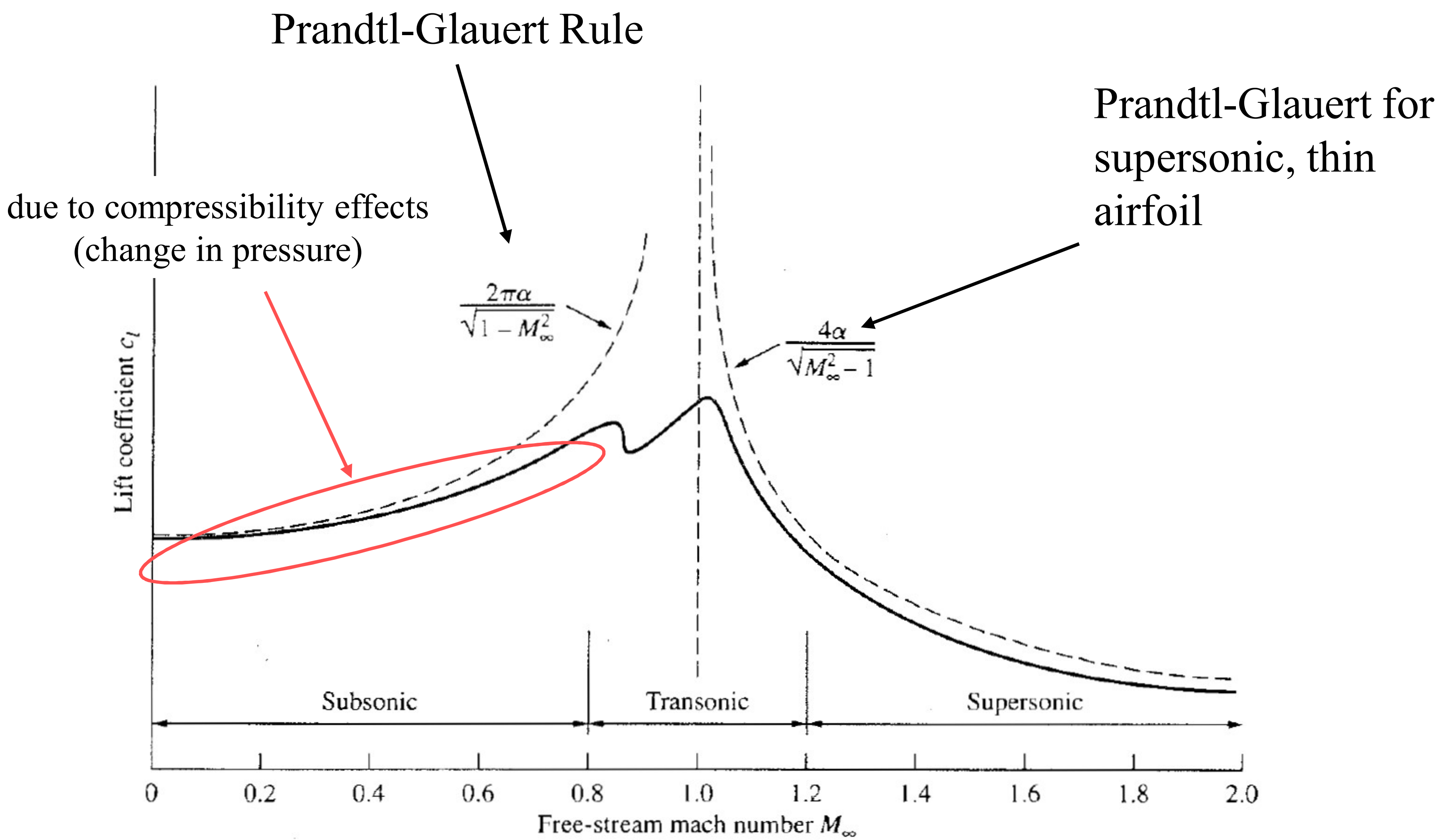
- Therefore, conclude that c_d IS sensitive to *Re*, and is larger at lower *Re*
- $c_{mc/4}$ is essentially constant over the range of angle of attack

Airfoil Data-NACA 23015



NACA 23015 Wing Section

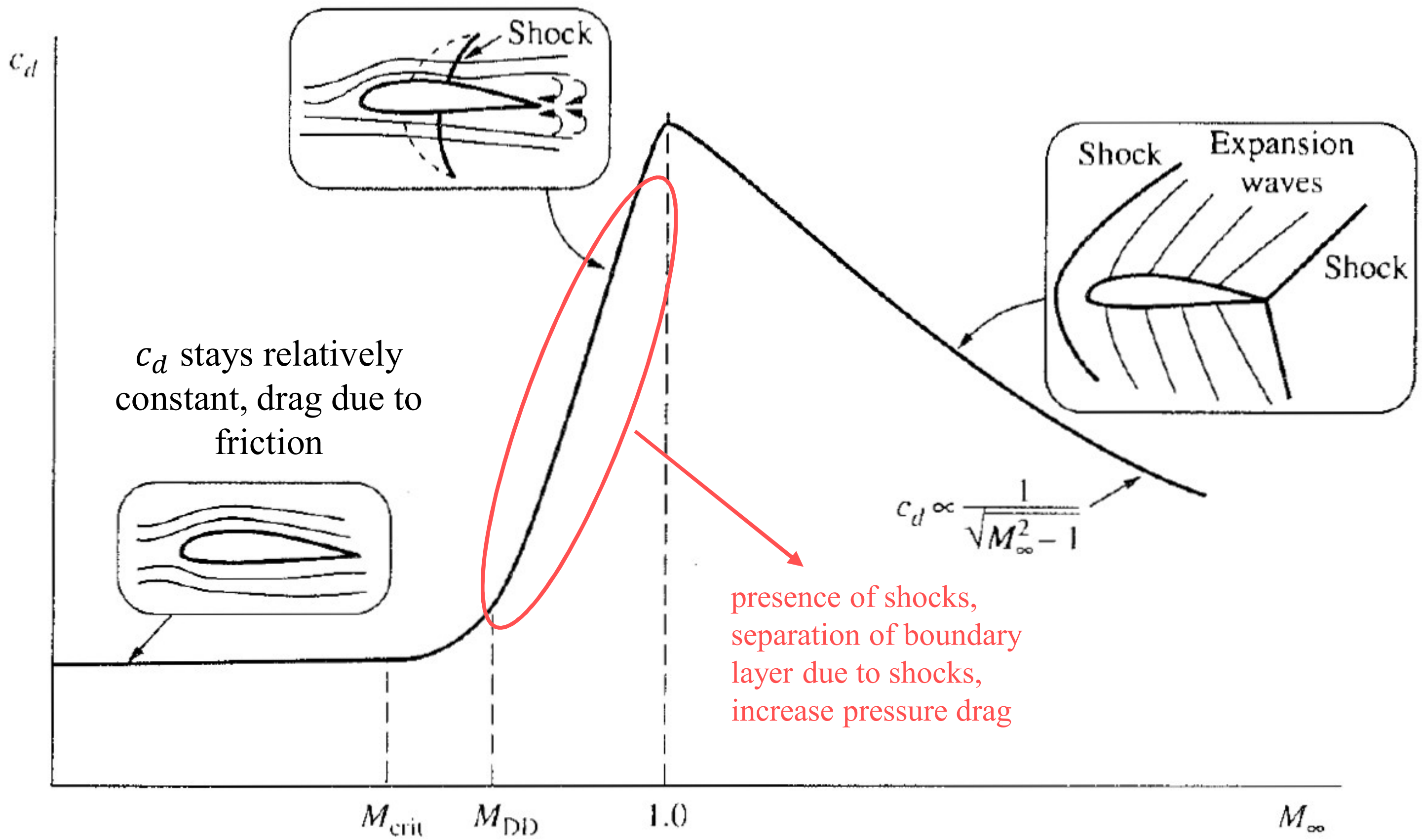
c_l vs. Mach variation



Since the moment coefficient is mainly due to pressure distribution, the variation of c_m with Mach will *qualitatively* resemble the c_l vs. Mach curve.

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

c_d vs. Mach variation



M_{crit} - Mach number at which sonic flow is first encountered at some location on the airfoil

M_{DD} - free stream Mach number at which drag rapidly diverges

High-Lift Devices

- As speeds and wing loadings of airplanes increased, artificial means became necessary for to increase $(C_L)_{max}$ for landing and takeoff. This increase is provided by mechanical high-lift devices. The most common of these are shown schematically in the Figure below:

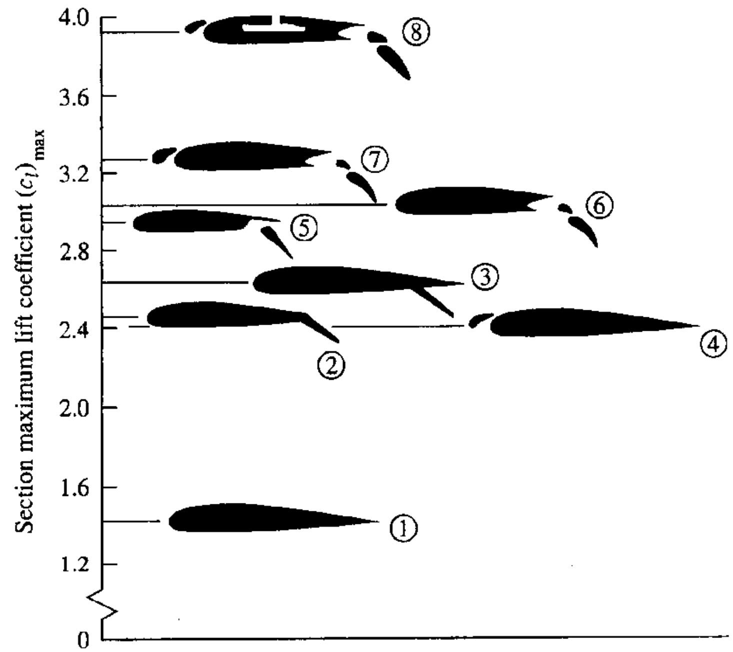
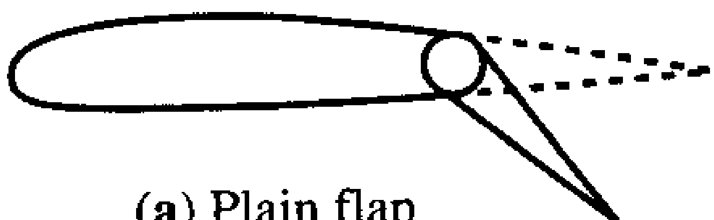


Figure 5.28 Typical values of airfoil maximum lift coefficient for various types of high-lift devices: (1) airfoil only, (2) plain flap, (3) split flap, (4) leading-edge slat, (5) single-slotted flap, (6) double-slotted flap, (7) double-slotted flap in combination with a leading-edge slat, (8) addition of boundary-layer suction at the top of the airfoil. (*From Loftin, Ref. 13.*)

1. The plain airfoil
2. The plain flap
3. The split flap
4. The leading-edge slat
5. The single-slotted flap
6. The double-slotted flap
7. The double-slotted flap in combination with a leading-edge slat
8. Addition of boundary layer suction

Plain flap

- Rear section of the airfoil hinged so that it can be rotated downward
- Creates more lift by mechanically increasing the effective camber
- $(c_l)_{max}$ can be almost doubled
- Also increases the drag and pitching moment

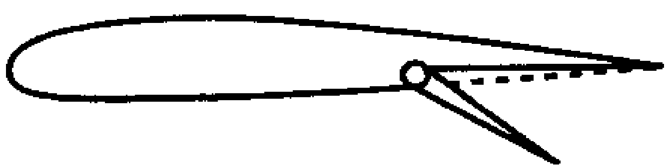


(a) Plain flap

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Split flap

- Only the bottom surface of the airfoil is hinged
- Creates more lift by mechanically increasing the effective camber
- Slightly higher $(c_l)_{max}$ than a plain flap
- Split flap produces more drag and less change in pitching moment compared to a plain flap
- Because of higher drag, rarely used on modern airplanes

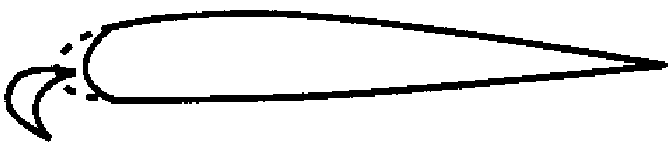


(b) Split flap

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Leading-edge slat

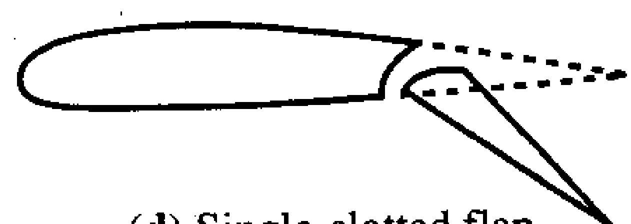
- Small highly camber airfoil located slightly forward of the leading edge
- There is a gap between the flap and the leading edge
- Modifies the pressure distribution over the top surface of the airfoil mitigating the adverse pressure gradient on the main airfoil section and delaying flow separation
- $(c_l)_{max}$ is increased with no significant increase in drag
- Produces about the same increase in $(c_l)_{max}$ as the plain flap



(c) Leading-edge slat

Single-slotted flap

- Unlike the plain flap, the single-slotted flap allows a gap between the top and bottom surfaces
- This allows higher-pressure air on the bottom surface to flow through the gap, modifying and stabilizing the boundary layer on the top surface of the airfoil
- Creates a low pressure on the leading edge of the flap, essentially forming a new boundary layer over the flap allowing the flow to remain attached to very high flap deflections
- Generates a higher $(c_l)_{max}$ than a plain flap
- In common use on light, general aviation

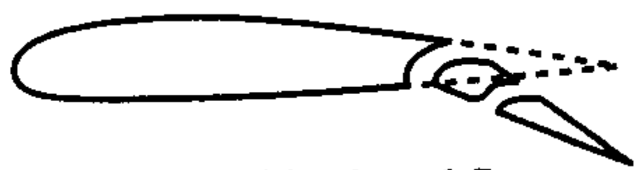


(d) Single-slotted flap

High-Lift Devices

Double-slotted flap

- The flap is divided into two segments, each with a slot
- If one slot is good, two are better, giving a slight increase in $(c_l)_{ma}$
- The benefit is achieved at the cost of increased mechanical complexity

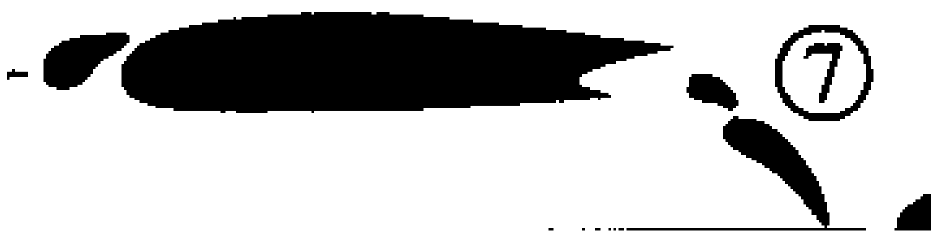


(**e**) Double-slotted flap

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Double-slotted flap with a leading-edge slat

- The mutual benefits obtained by employing the combination on the same airfoil give an increase in $(c_l)_{max}$



Addition of boundary layer suction

- By mechanically sucking away a portion of the boundary layer through small holes or slots in the top surface of the airfoil, flow separation can be delayed.
- Can lead to a substantial increase in $(c_l)_{max}$
- The increased mechanical complexity and cost of this device, along with the power requirements of the pumps, diminish its attractiveness as a design option
- Active boundary layer suction has not been used on any standard production airplanes

8

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

High-Lift Devices

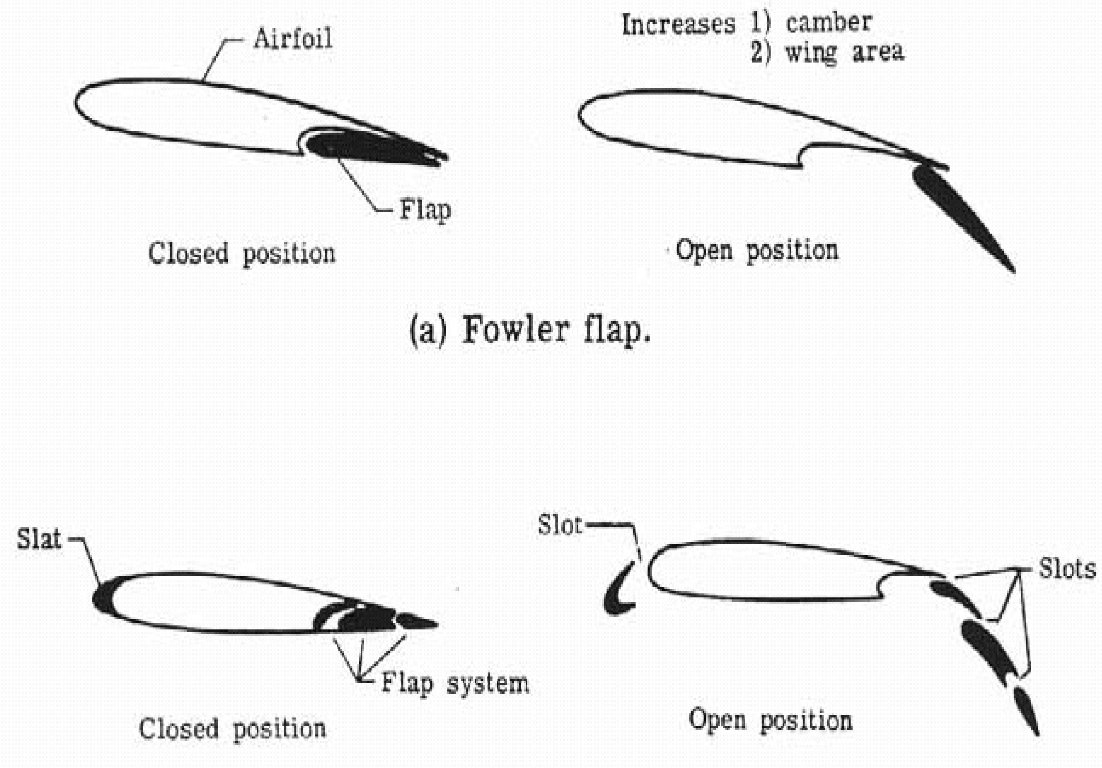
Triple-slotted flap

- Used on several commercial transports with high wing loadings (Boeing 747)
- An airfoil equipped with leading edge devices and a triple-slotted flap generates about the ultimate in high $(c_l)_{max}$ associated with purely mechanical high-lift systems
- Because of design complexity, recent airplanes have returned to simpler mechanisms

(f) Triple-slotted flap

Fowler flap

- When deployed, not only deflects downward increasing effective camber, but also translates or tracks to the trailing edge of the airfoil; hence, increasing exposed wing area and further increasing lift
- Today the concept of the Fowler flap is combined with the double-slotted and triple-slotted flaps

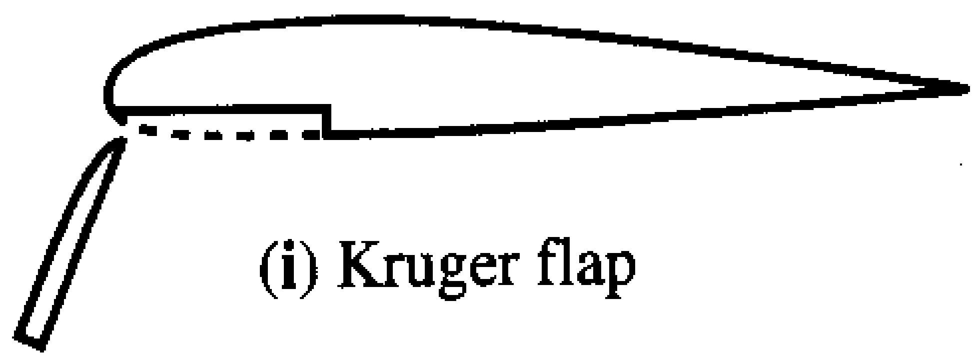


(a) Fowler flap.

(b) Complex slotted flap of Boeing 737.

Kruger flap

- Essentially a leading edge slat that is thinner and lies flush with the bottom surface of the airfoil when not deployed
- Suitable for thinner airfoils

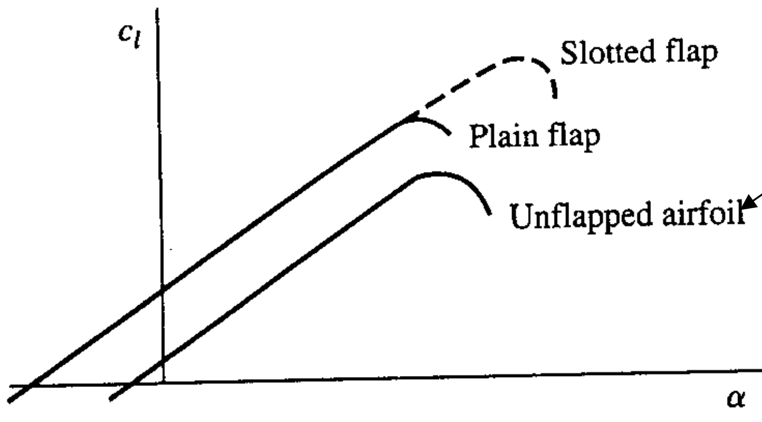


(i) Kruger flap

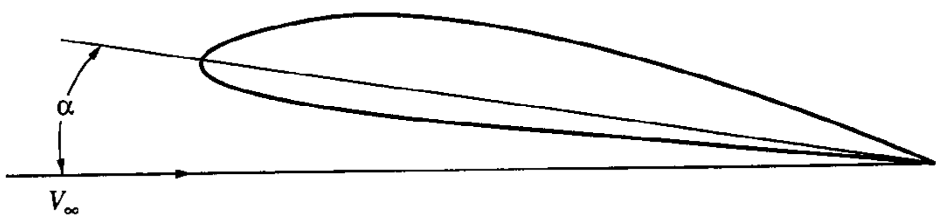
High-Lift Devices

- The effect of slats and flaps on the lift curve is shown below

Unflapped airfoil—no flap deflection on either the leading or the trailing edge



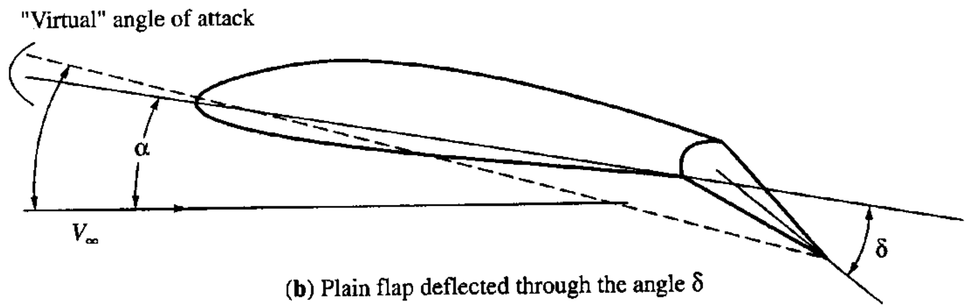
(a)



(a) Basic airfoil

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

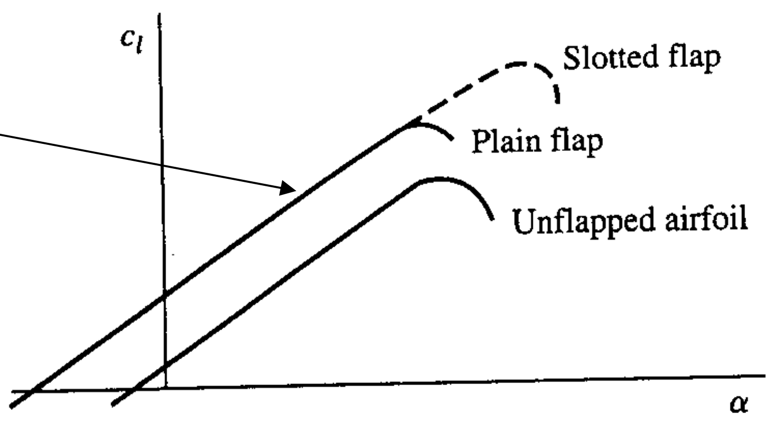
High-Lift Devices



(b) Plain flap deflected through the angle δ

- Airfoil with a plain flap deflected through angle δ
- Flap deflection is fixed and the airfoil is pitched through a range of angle of attack α
- α is the angle between the original chord line and the free-stream velocity direction

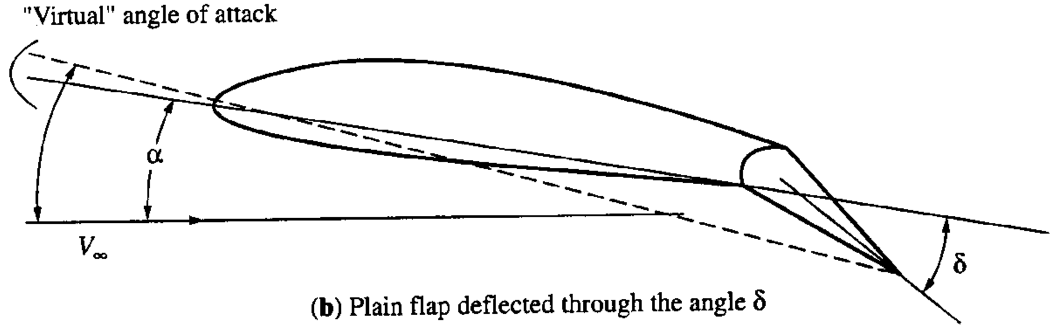
- Note the effect of flap deflection is to shift the lift curve to the left
- Lift slope for the flapped airfoil is essentially the same as for the basic airfoil
- Zero-lift angle of attack is shifted to a lower value



(a)

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

High-Lift Devices



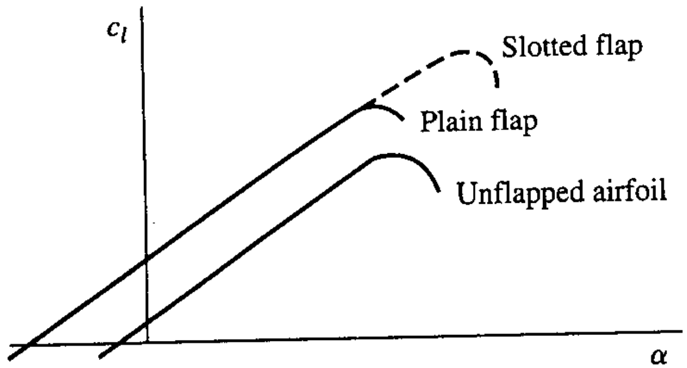
(b) Plain flap deflected through the angle δ

Reasons for the left shift of the lift curve:

1. When the flap is deflected downward, the effective camber of the airfoil is increased
   - A more cambered airfoil yields a more negative zero-lift angle of attack
2. A line drawn from the airfoil trailing edge to the leading edge of the airfoil is treated as a "virtual" chord line
   - Thus the flapped airfoil is at a "virtual" angle of attack which is larger than α
   - The flapped airfoil appears to the free-stream to have a slightly higher angle of attack

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

High-Lift Devices

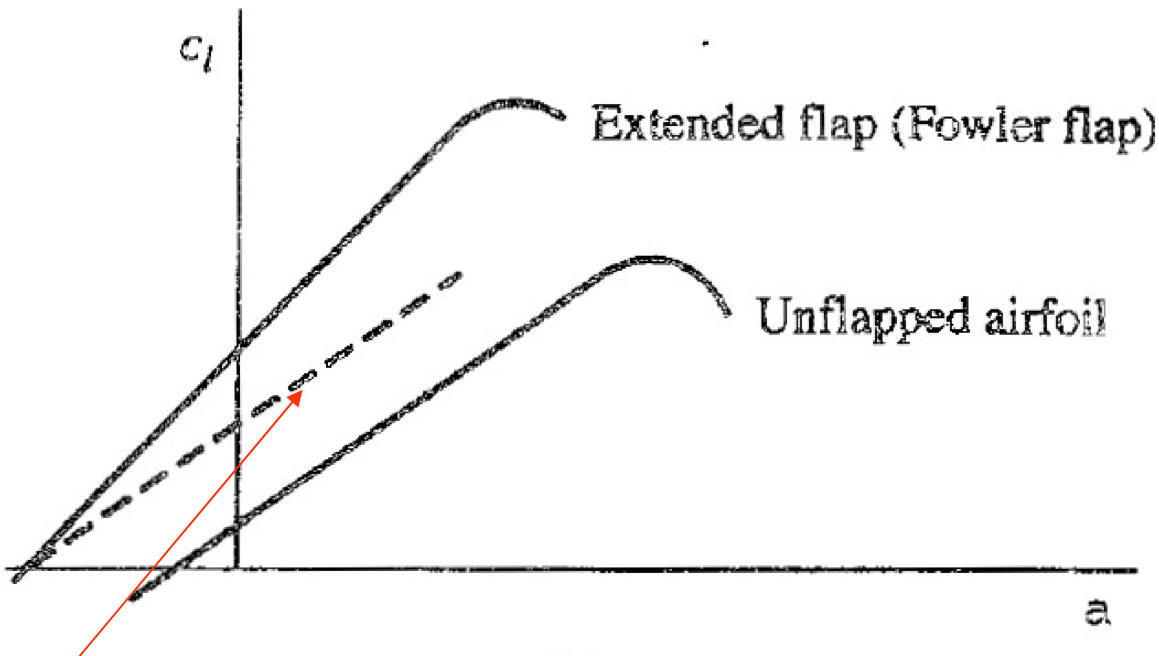


(a)

- Also note that the plain flap results in a larger $(c_l)_{max}$
- This maximum lift coefficient generally occurs at a smaller angle of attack than the unflapped airfoil
- Note also the difference between the plain flap and the slotted flap
  - The lift curve is extended as shown by the dashed line in the figure
  - The high-energy flow through the slot delays flow separation
  - The airfoil can be pitched to a higher angle of attack for incurring stall
  - Does not materially effect the lift curve slope or the zero-lift angle of attack

Effect of the Fowler flap:

1. Increases effective camber, which shifts zero-lift angle of attack to the left
2. Increases effective planform area, which increases the ***slope*** of the lift curve



(b)

Dashed line would hold for flap deflection with no area extension.

High-Lift Devices

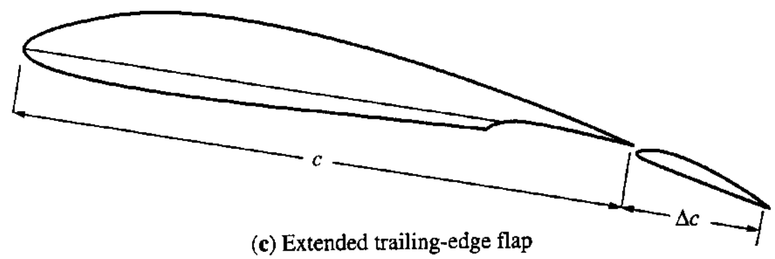
Why does the extended flap increase the lift slope?

- For simplicity, look at the case of extension only, with no flap deflection
  - For the basic airfoil with no extension, the lift per unit span is:

$$L = q_\infty c c_l$$

where c is the chord of the basic airfoil

  - Now L^* is the lift per unit span with the flap extended by the distance Δc



(c) Extended trailing-edge flap

$$L^* = q_\infty (c + \Delta c) c_l$$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

High-Lift Devices

- Dividing by cq_∞:

$$\frac{L^*}{q_\infty c} = \left(1 + \frac{\Delta c}{c}\right) c_l \qquad \text{(A)}$$

- Now we want to base the lift coefficient for the airfoil with the extended flap on the chord of the basic airfoil with no extension. Denoting this lift coefficient by c_l^*

$$c_l^* = \frac{L^*}{q_\infty C}$$

- From Equation (A)

$$c_l^* = \left(1 + \frac{\Delta c}{c}\right) c_l$$

- Differentiating with respect to α, we have

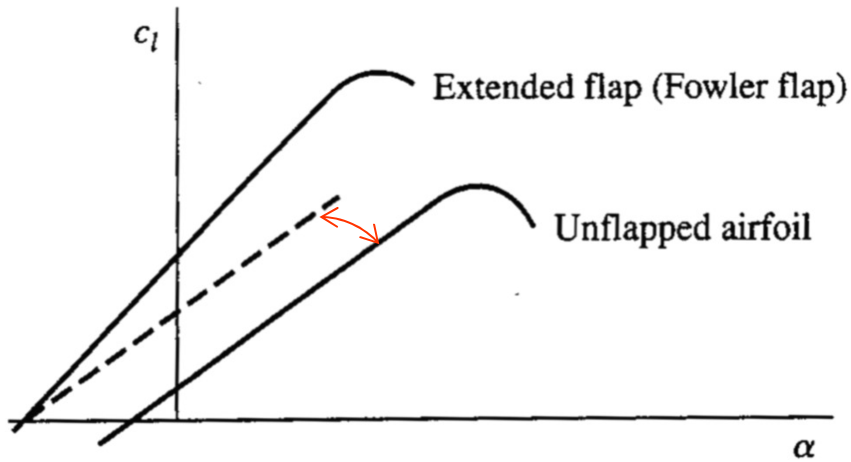
$$\frac{dc_l^*}{d\alpha} = \left(1 + \frac{\Delta c}{c}\right) \frac{dc_l}{d\alpha} \qquad \text{(B)}$$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

High-Lift Devices

$$\frac{dc_l^*}{d\alpha} = \left(1 + \frac{\Delta c}{c}\right)\frac{dc_l}{d\alpha} \qquad \textbf{(B)}$$

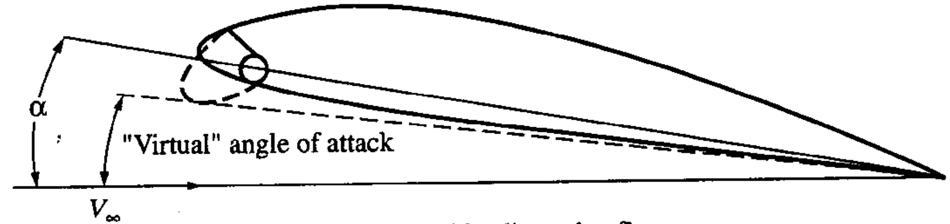
- In Equation (B), $dc_l/d\alpha$ is the lift slope for an airfoil with no flap extension.
- Note that the lift curve is increased by the amount $\Delta c/c$ with the flap extension.
- This is the increase in lift slope sketched in the Figure below.



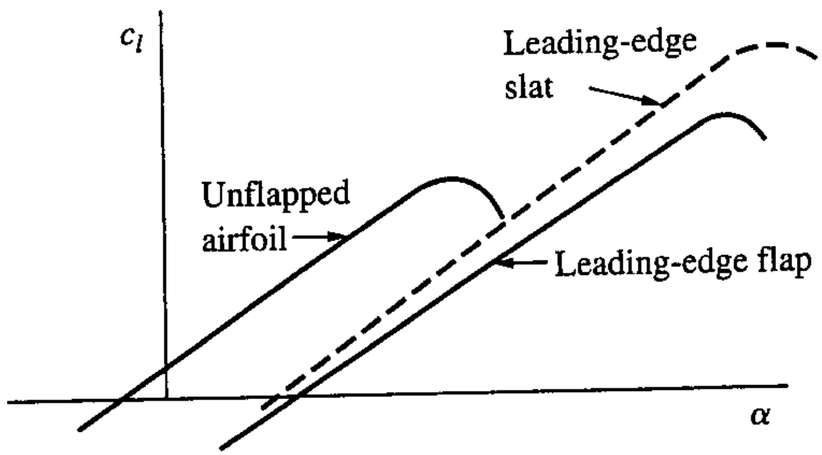
(b)

High-Lift Devices

- Consider now a leading-edge flap



(d) Deflected leading-edge flap



(c)

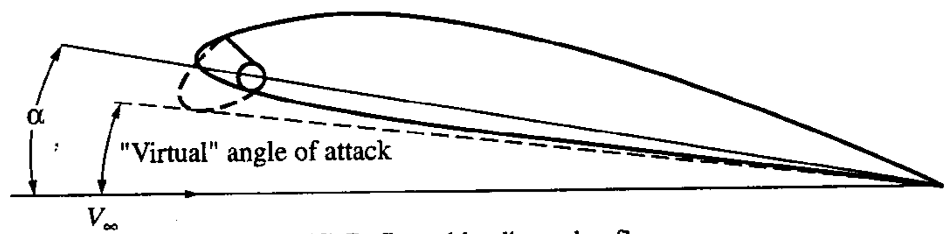
- Note the lift curve slope is shifted to the right, with virtually no change in lift slope

<u>Why is there a shift to the right?</u>

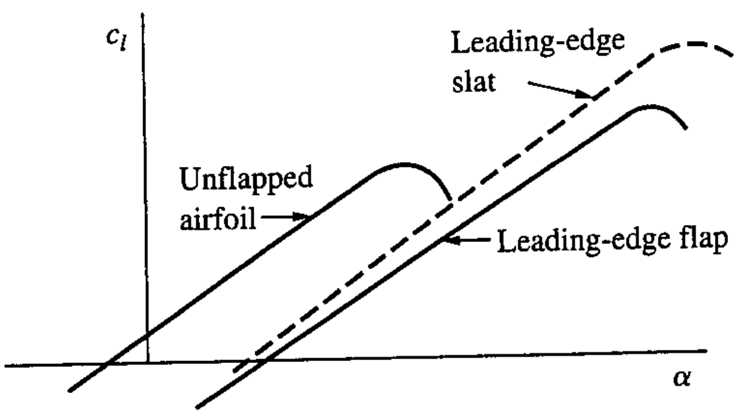
High-Lift Devices

Leading-edge flap:

- Note when the flap is extended the effective camber is increased, which shifts the curve to the left. However, this is more than compensated by the influence of the "virtual" angle of attack



(d) Deflected leading-edge flap



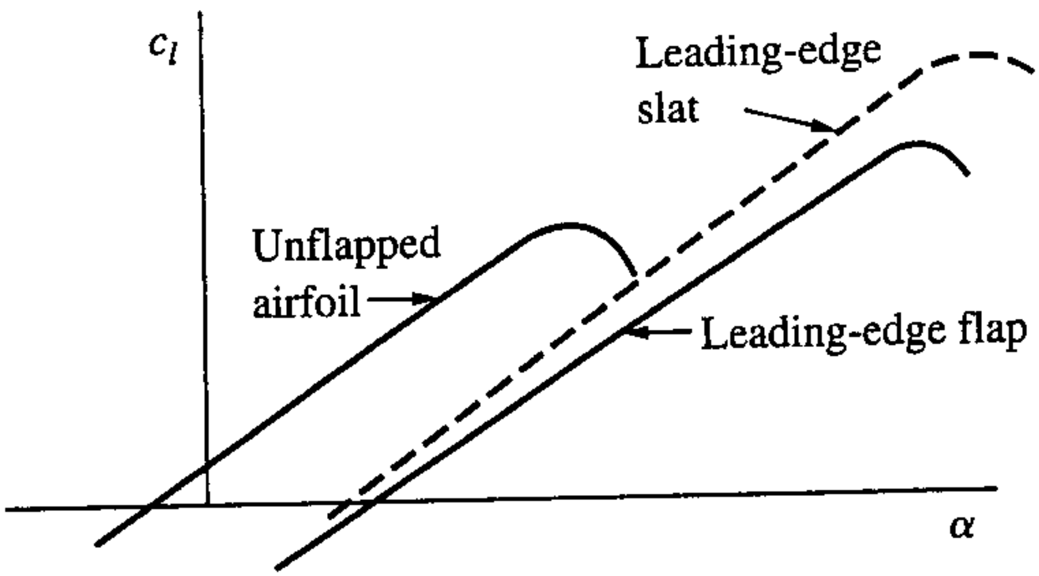
(c)

- The "virtual" angle of attack is smaller than α
- This is usually the case
- Leading-edge flap results in a higher $(c_l)_{max}$

High-Lift Devices

Leading-edge slat:

- If the leading-edge flap is replaced by a leading-edge slat a small effective increase in the planform area results
- This results in a small increase in the lift curve slope
- The favorable influence of the flow through the slot between the slat and the airfoil results in an a higher $(c_l)_{ma}$ for leading-edge slat when compared to the leading-edge flap



(c)

The Pressure Coefficient

- Dimensionless parameters like M, Re, Lift and Drag coefficients are useful for comparing flows around physically similar objects
  - *"It makes sense; therefore, that a dimensionless pressure would also find use in aerodynamics"*
- The pressure coefficient is just that, a defined quantity:

$$C_p = \frac{p - p_\infty}{q_\infty}$$

where

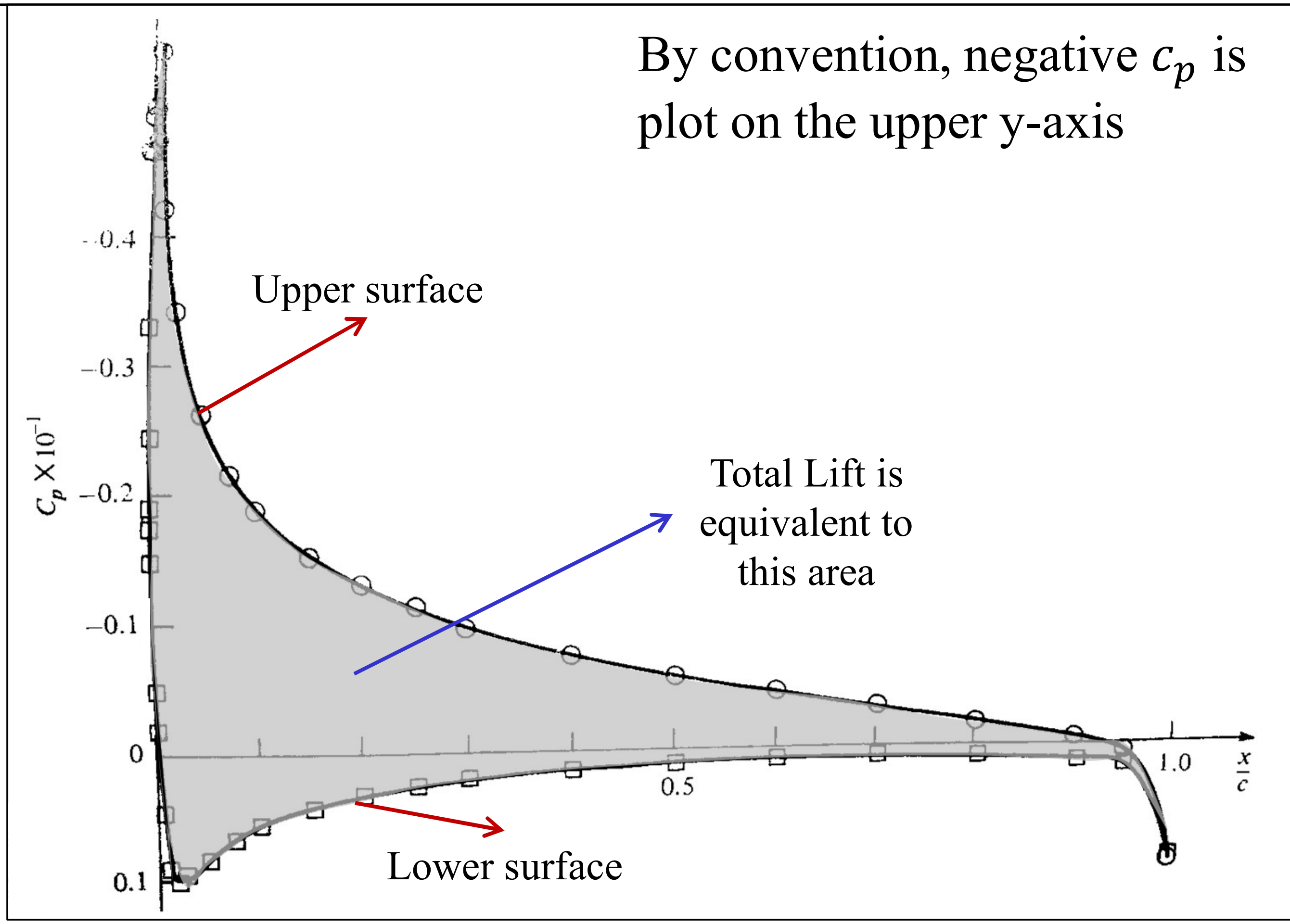
$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$$

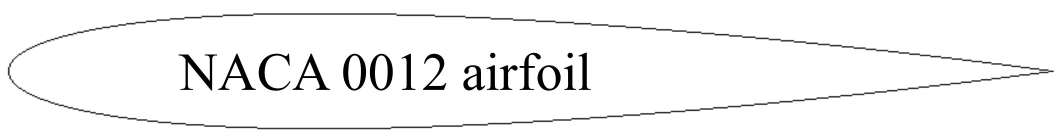
$$p = \text{static pressure at point of interest}$$

$$p_\infty = \text{free stream static pressure}$$

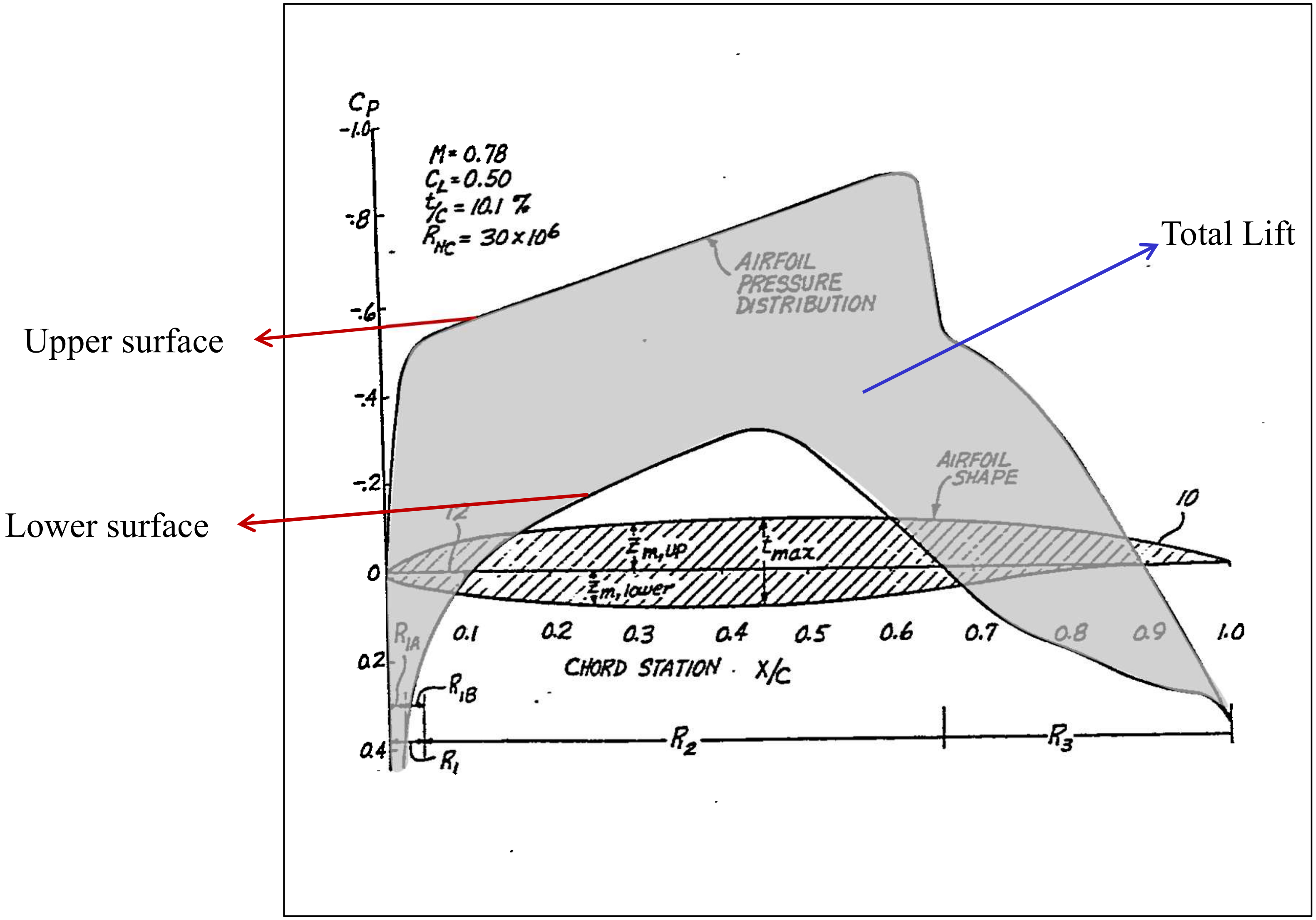
Anderson, John D. Jr., *Fundamentals of Aerodynamics,* 3rd Edition, McGraw-Hill, 2001

C_p Distribution Incompressible Flow



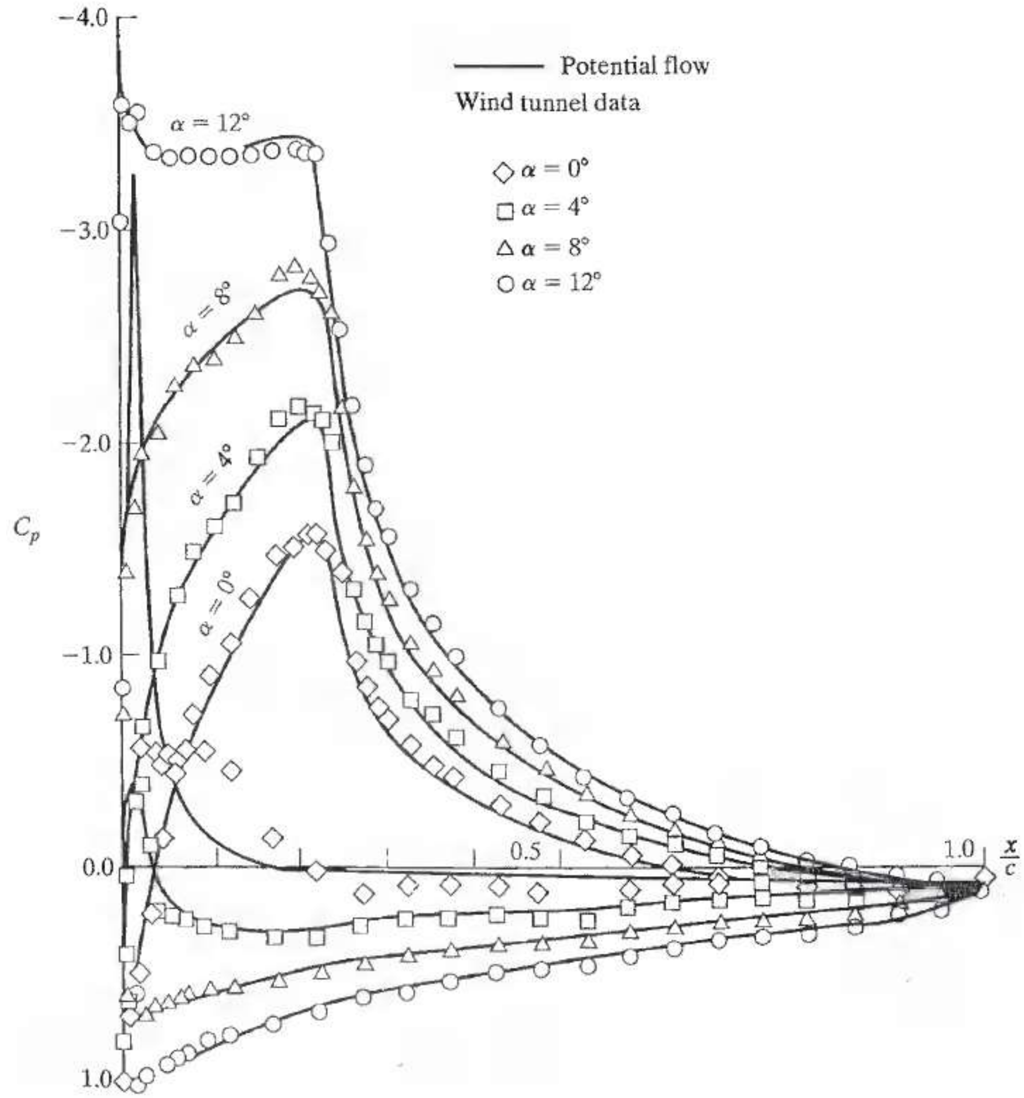


C_p distribution compressible flow

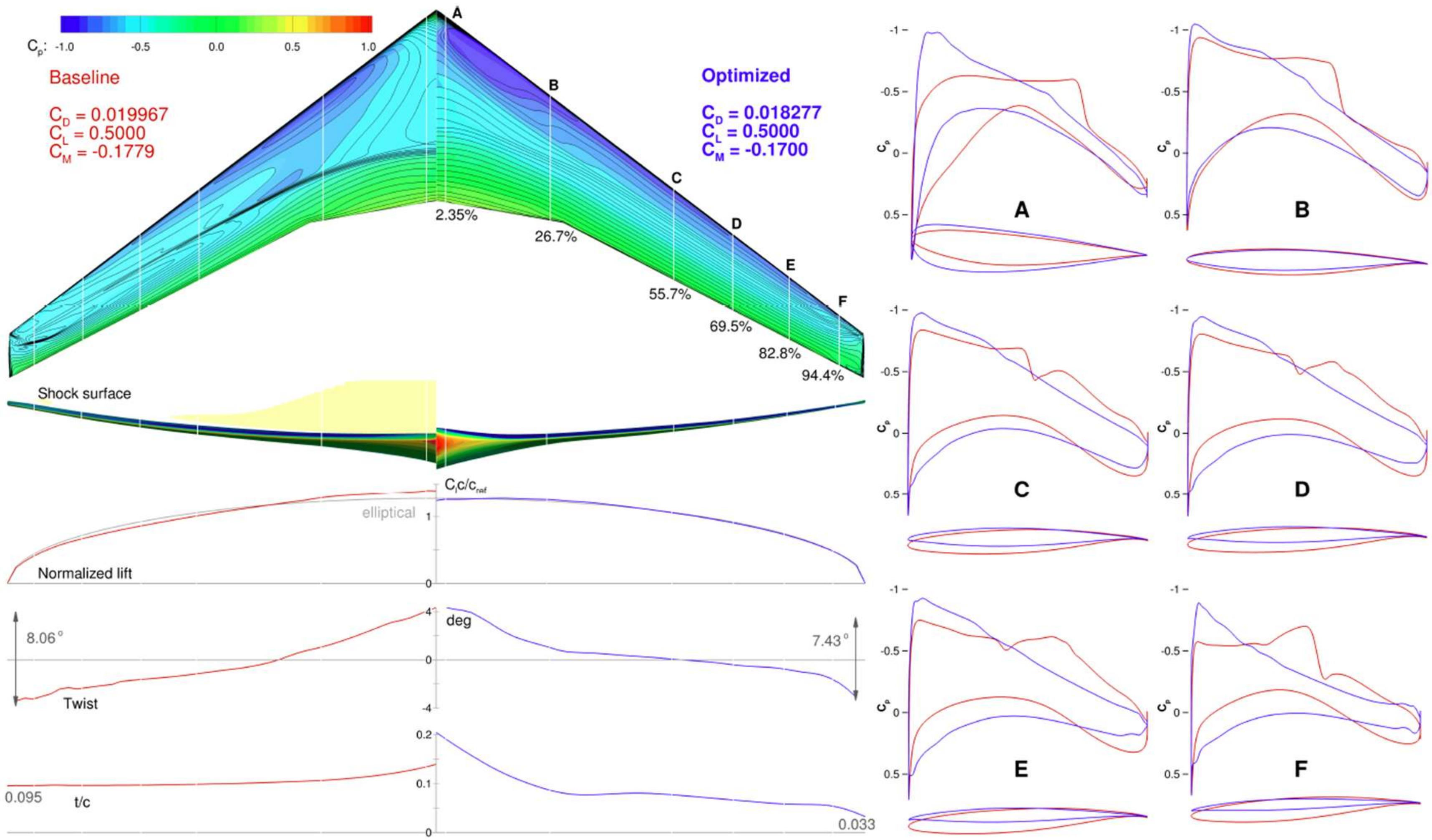


Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

C_p vs. Angle of Attack



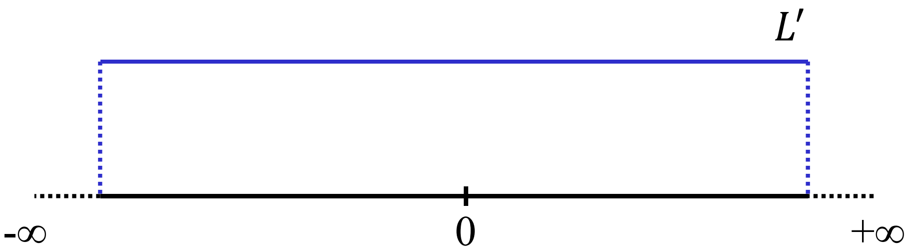
C_p Span Wise



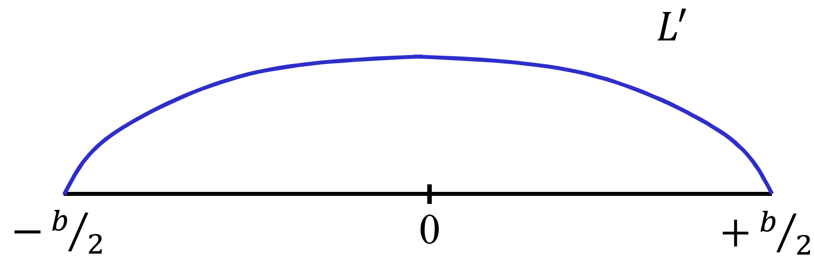
Cp: -1.0 -0.5 0.0 0.5 1.0
Baseline
CD = 0.019967
CL = 0.5000
CM = -0.1779
Optimized
CD = 0.018277
CL = 0.5000
CM = -0.1700
A
B
C
D
E
F
2.35%
26.7%
55.7%
69.5%
82.8%
94.4%
Shock surface
Clc/cref
elliptical
Normalized lift
8.06°
7.43°
deg
Twist
0.095
t/c
0.033

Lift and Drag Buildup

- So far we have only considered airfoil
  - Wing with infinite span
  - Every section sees exactly the same lift
- Now we look at finite wing
  - Lift is directly linked with the lift capability of a given airfoil
  - Lift goes to zero at the wing tips



Infinite Wing

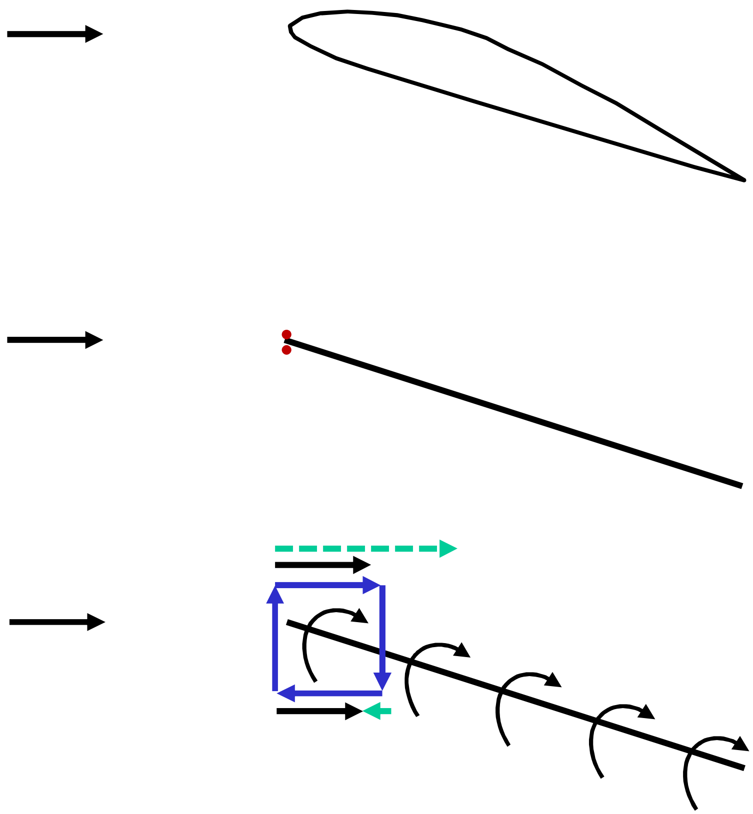


Finite Wing

L': Lift per unit wing span, also called lift distribution

Lift and Drag Buildup

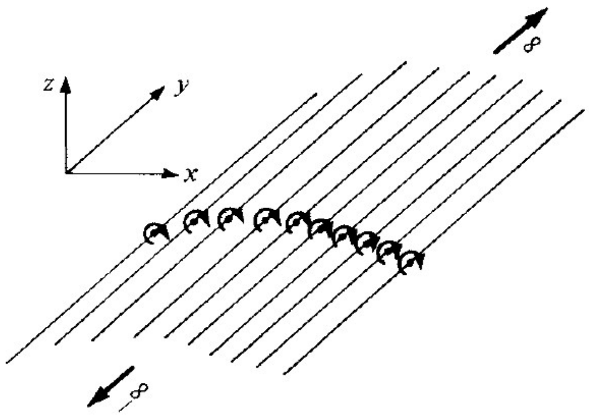
<u>Question:</u> How could you approximate an airfoil's lift capability?



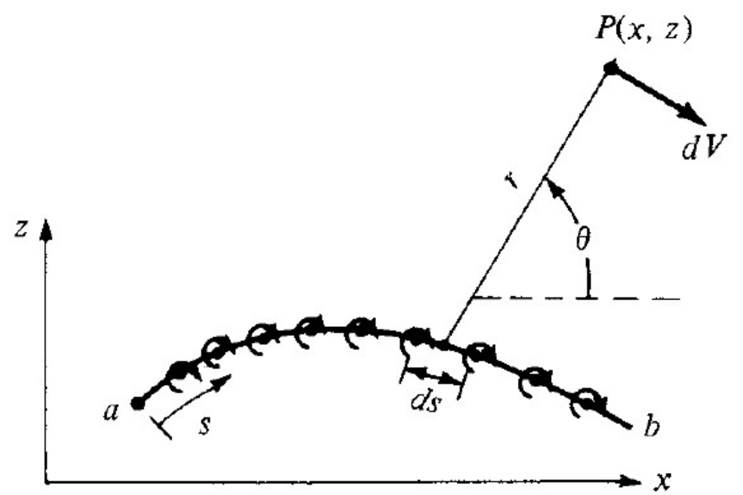
- Simplify your model
  - Infinitely thin, flat plate at an angle of attack to an oncoming uniform stream
- There is a velocity differential above and below
  - Two points at the edge that are infinitesimally apart have different velocities
  - How can you represent this discontinuity in velocity?
    - Vortex → mathematical model
- How can you model this flat plate mathematically?
  - Vortex Sheet

Vortex Sheet

- Vortex sheet is formed by side by side vortex filaments
- Define $\gamma(s)$ as the strength of the vortex sheet per unit length along s



Vortex sheet in perspective



Edge view of sheet

References: "Fundamentals of Aerodynamics", J.D. Anderson, page #255-257.

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

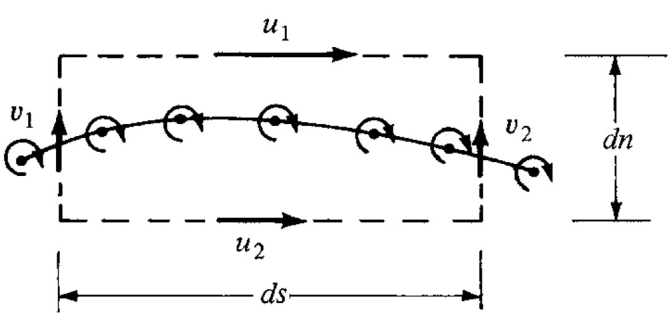
Vortex Sheet

- Consider rectangular dashed path enclosing a section of the sheet of length ds
- From definition of circulation, calculate circulation around the dashed path:

$$\Gamma = -\oint \vec{V}\,\overrightarrow{dl} = u_1 ds - v_2 dn - u_2 ds + v_1 dn$$

$$\gamma ds = (u_1 - u_2) ds + (v_1 - v_2) dn$$

$$dn \rightarrow 0 \quad \Rightarrow \quad \boxed{\gamma = (u_1 - u_2)}$$



The *local jump in tangential velocity* across a vortex sheet is equal to the *local sheet strength!*

References: "Fundamentals of Aerodynamics", J.D. Anderson, page #255-257.

Lift and Drag Buildup

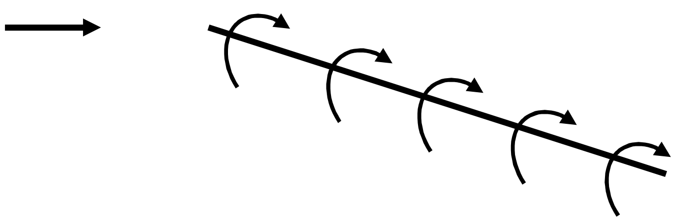
- Integral of γ from leading edge to trailing edge gives the circulation

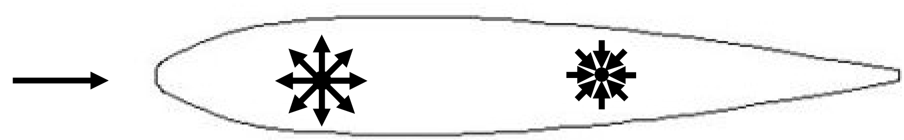
$$\Gamma = \int_0^c \gamma_{(s)} ds$$

- Circulation, Γ, is linked to L' through Kutta-Joukowski theorem

$$L' = -\rho V_\infty \Gamma$$

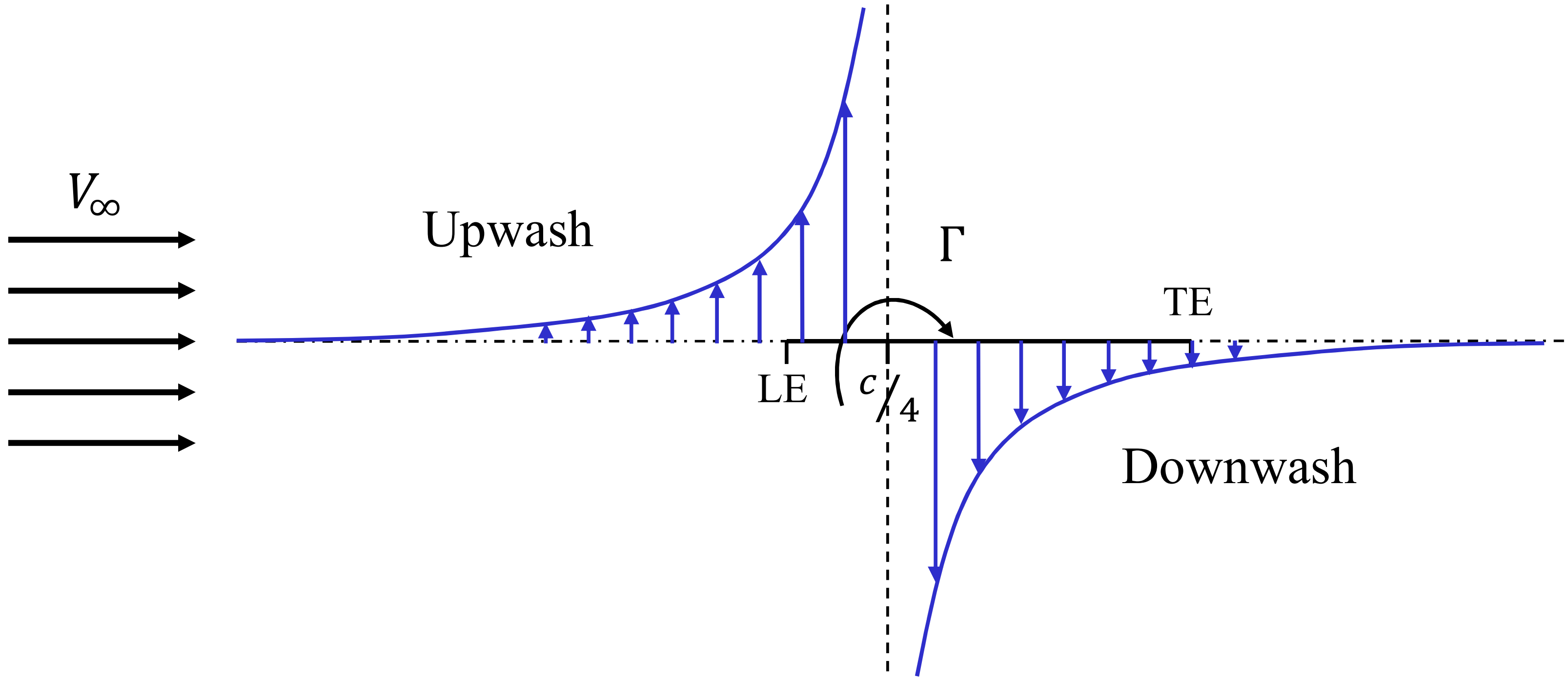
- This airfoil model have no thickness
  - Add other singularities





Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Induced Flow Field

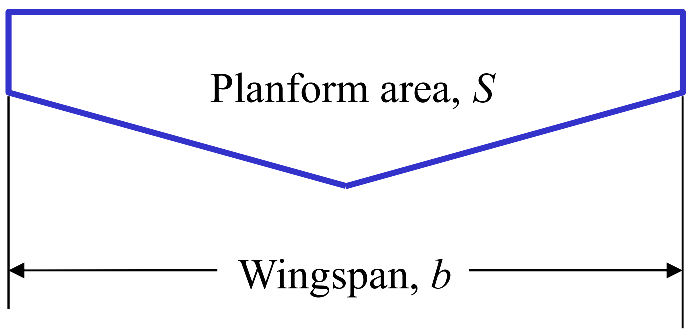


- In 2D, all the sections are the same
  - Symmetrical → effects cancel out
- How would this induced flow field affect the lift distribution in a 3D case?
  - Downwash → leads to drag due to lift

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

$$\boxed{C_{D,i} = \frac{C_L^2}{\pi ARe}}$$

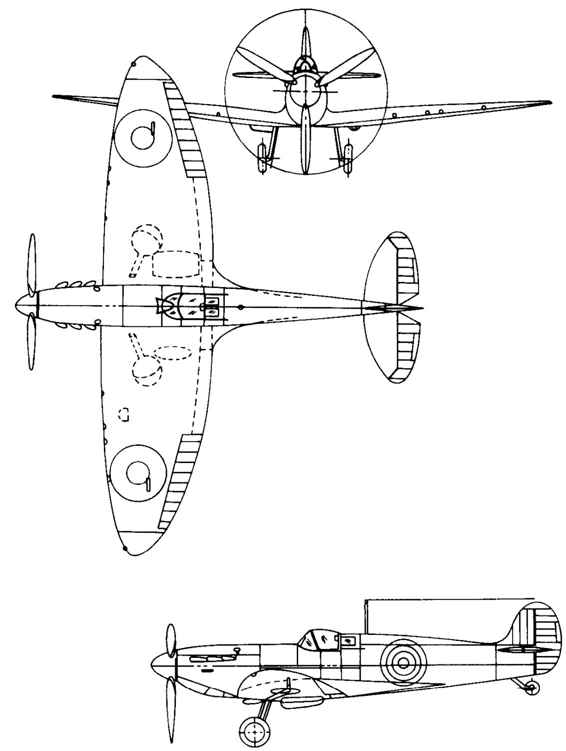
- Aspect Ratio, $AR = {}^{b^2}/_{S}$
  - Shows how finite the wing is
  - $AR \rightarrow \infty \Rightarrow C_{D,i} \rightarrow 0$
    - No induced drag
  - Technology indicator



- Oswald factor, e
  - Correction factor
  - $e = 1 \Rightarrow$ lift distribution is perfect ellipse
    - Minimum induced drag

Elliptical Lift Distribution

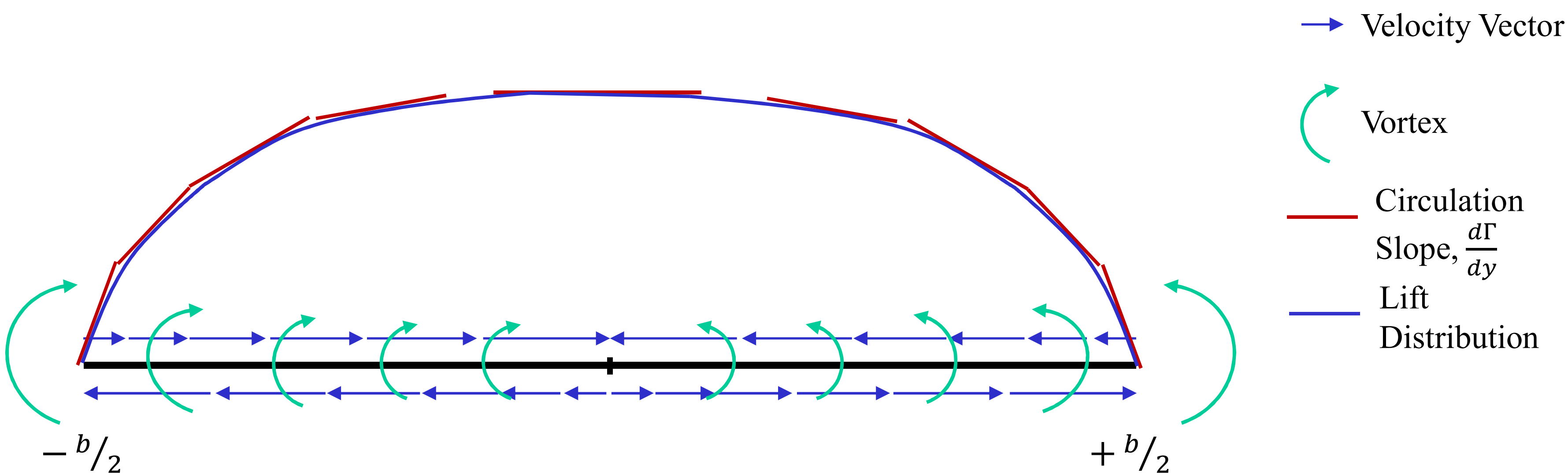
- **<u>Goal</u>:** Minimize the deviation from elliptical lift distribution
- How can one create an elliptical load distribution?
  - Twist
    - Effective angle of attack
  - Taper ratio
    - Takes weight out
  - Elliptical planform
    - Supermarine Spitfire
    - Stability issues!
  - Sweep
    - Compressibility effects



- Designer decides how to achieve highest *e* possible
- Depending on tool sophistication, one can choose to use Euler's equation, Navier-Stokes equation, etc.

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Lift Distribution

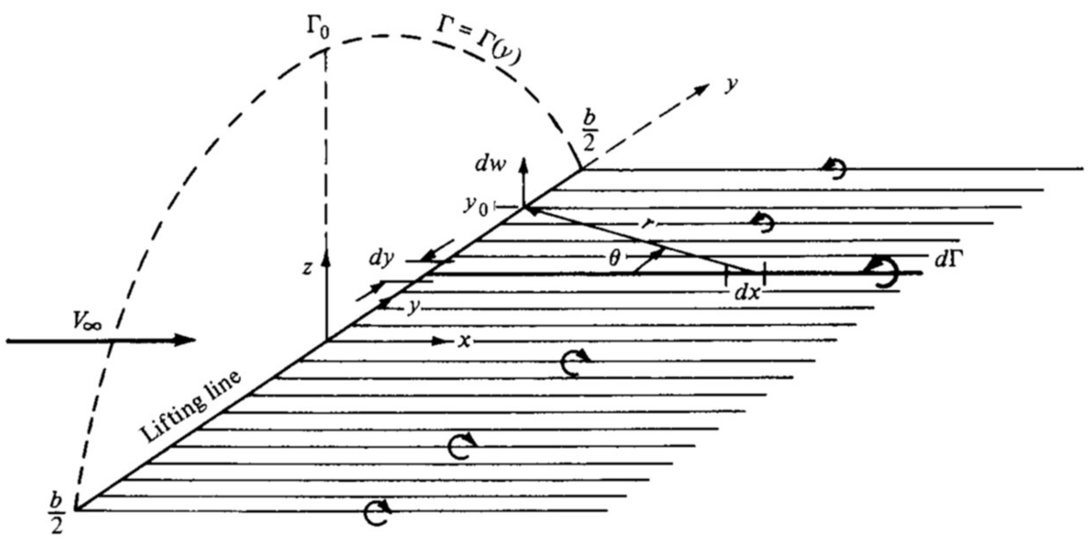


- These vortices are perpendicular to the ones before
- Vortices are stronger towards the tip
- Vortices are directly linked to the shape of the lift distribution
  - Slope, $\frac{d\Gamma}{dy}$
    - Higher slope, stronger vortex

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Lifting Line Theory

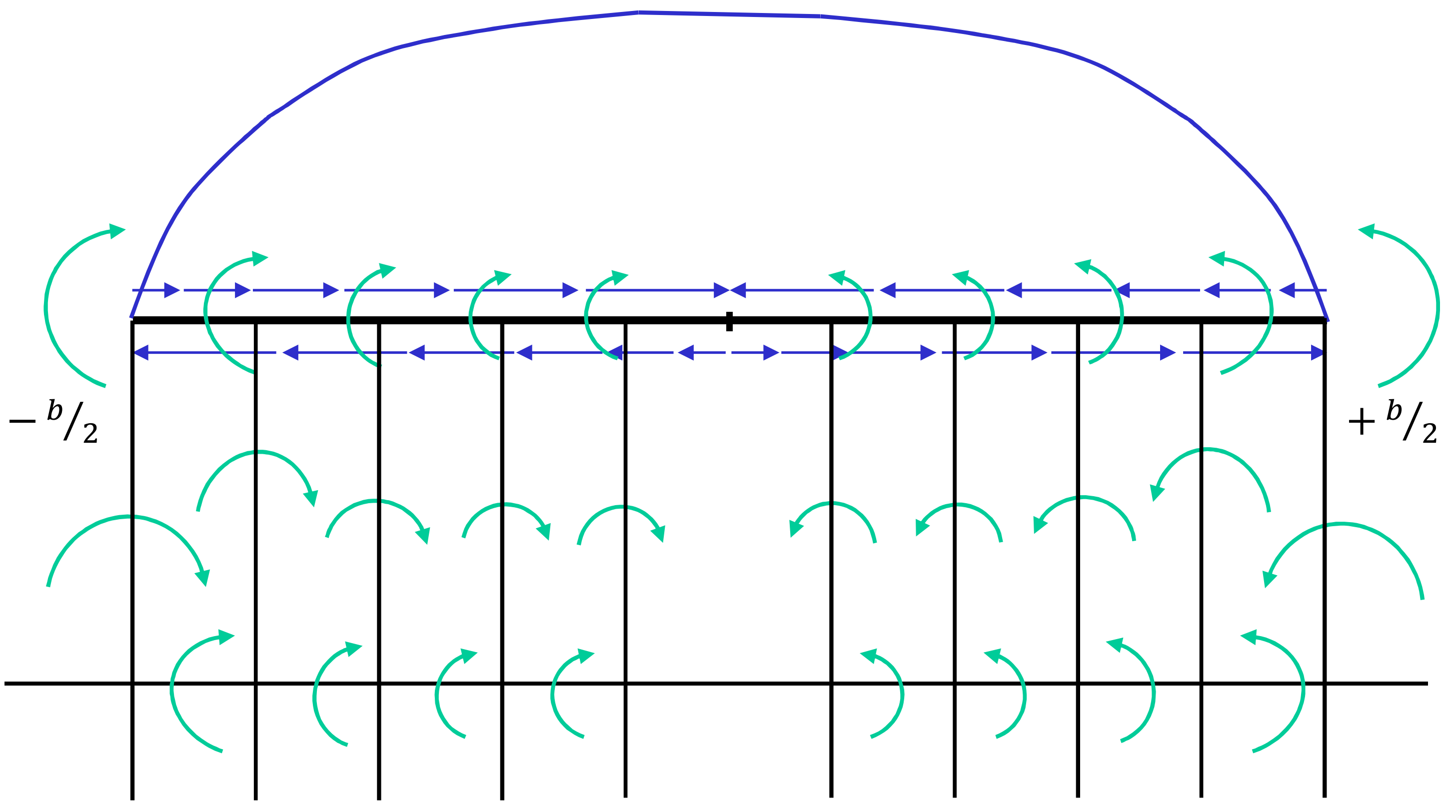
- The wing is represented by a large number of horseshoe vortices, each with a different length of the bound vortex, but with all the bound vortices coincident along a single line, called the ***lifting line***
- This vortex sheet is parallel to the direction of V_∞
- The total strength of the sheet integrated across the span of the wing is zero
  - It consists of pairs of trailing vortices of equal strength but in opposite directions



References: "Fundamentals of Aerodynamics", J.D. Anderson.

Lifting Line Theory

- All vortices induce flow field around them
  - Wake droop down
  - The filaments are not straight lines behind the wing, they are at an angle
- Vortices induce a downward velocity on the lifting line
- Apply Biot-Savart law to find downwash velocities



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

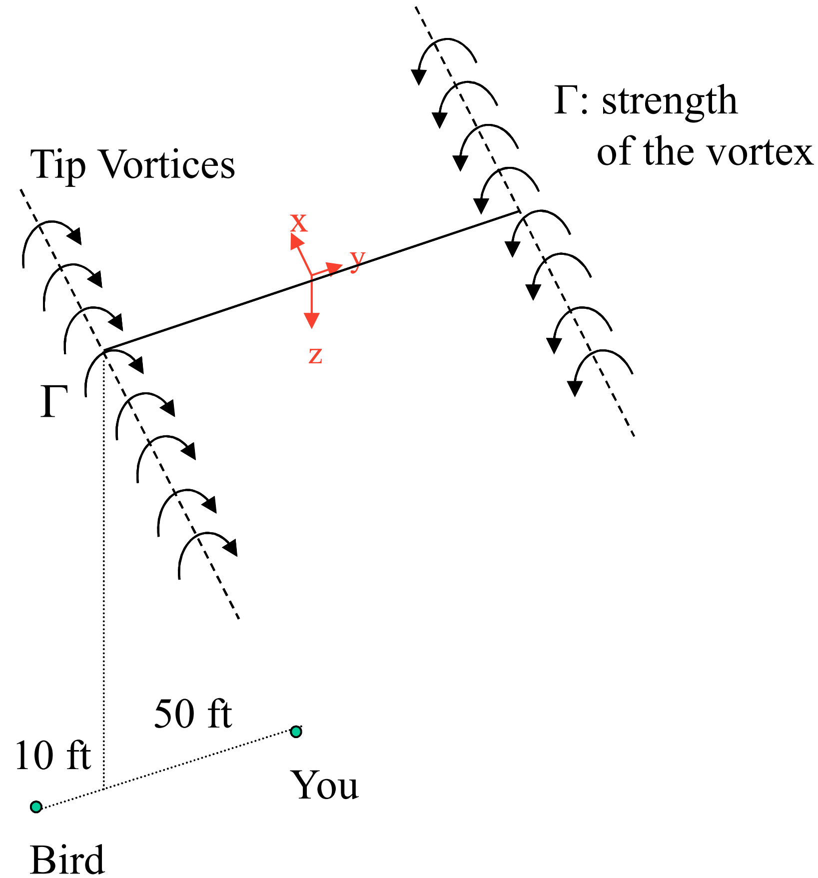
Lifting Line Theory

- Approximate velocity directions and magnitudes applying **Biot-Savart Law**

$$d\vec{V} = \frac{\Gamma}{4\pi}\frac{d\vec{l}\,x\vec{r}}{|\vec{r}|^3}$$

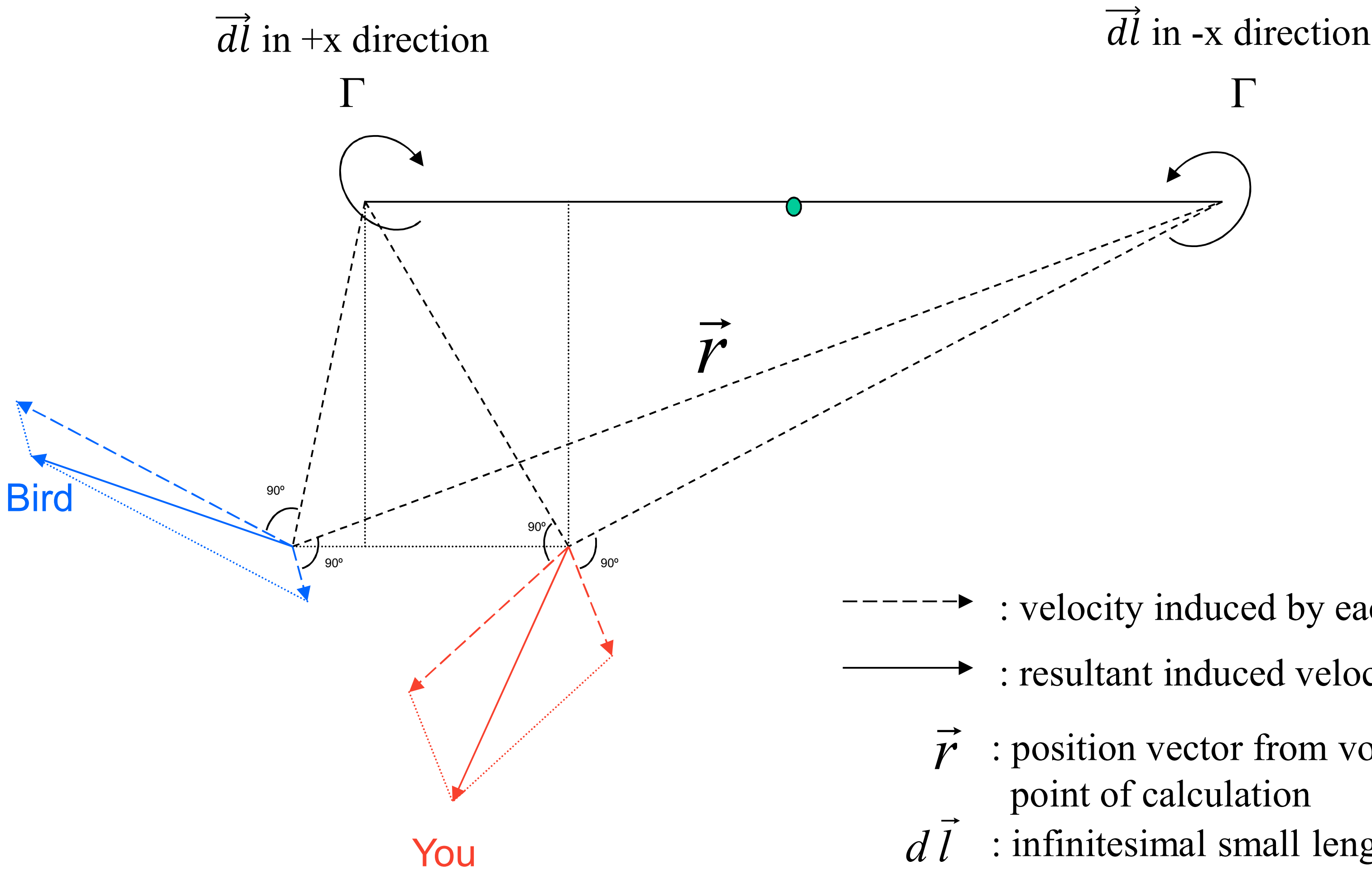
$d\vec{l}\,x\vec{r} \longrightarrow$ Velocity directions

$|d\vec{V}|\ \alpha\ \frac{1}{|\vec{r}|^2} \longrightarrow$ Velocity magnitudes



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Y-Z Plane Qualitative Picture:



- - - - → : velocity induced by each tip vortex

——→ : resultant induced velocity

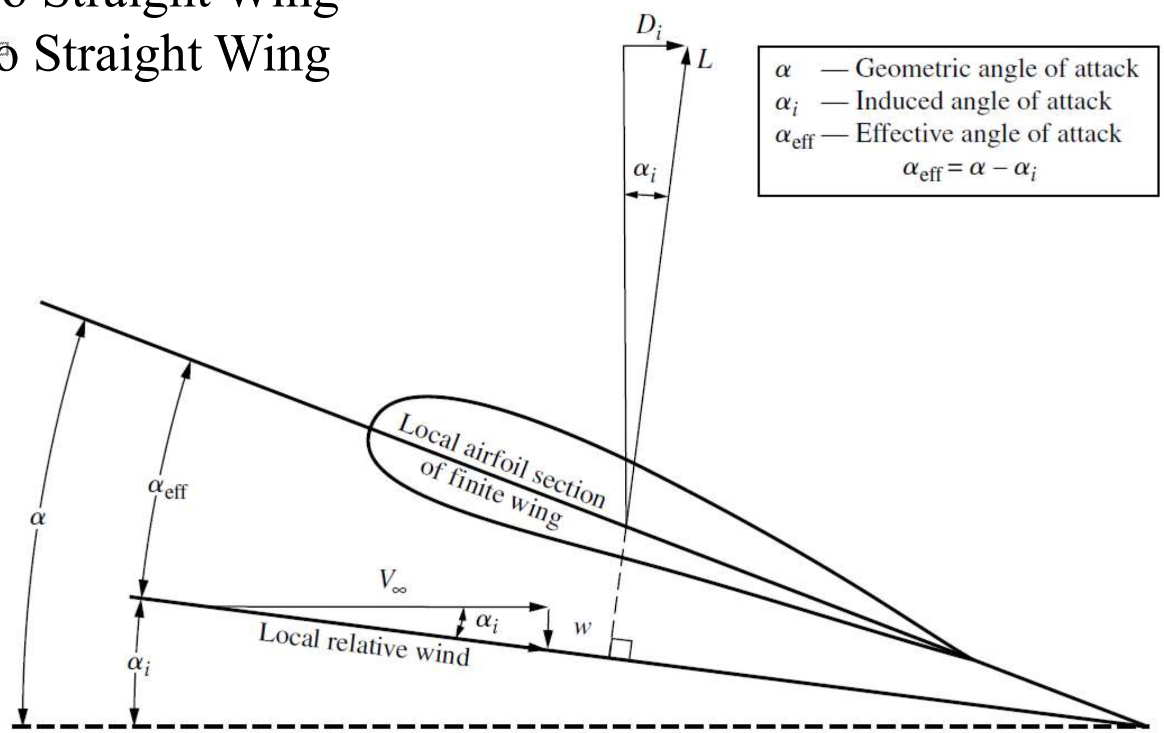
$\vec{r}$: position vector from vortex to the point of calculation

$d\vec{l}$: infinitesimal small length vector along each vortex

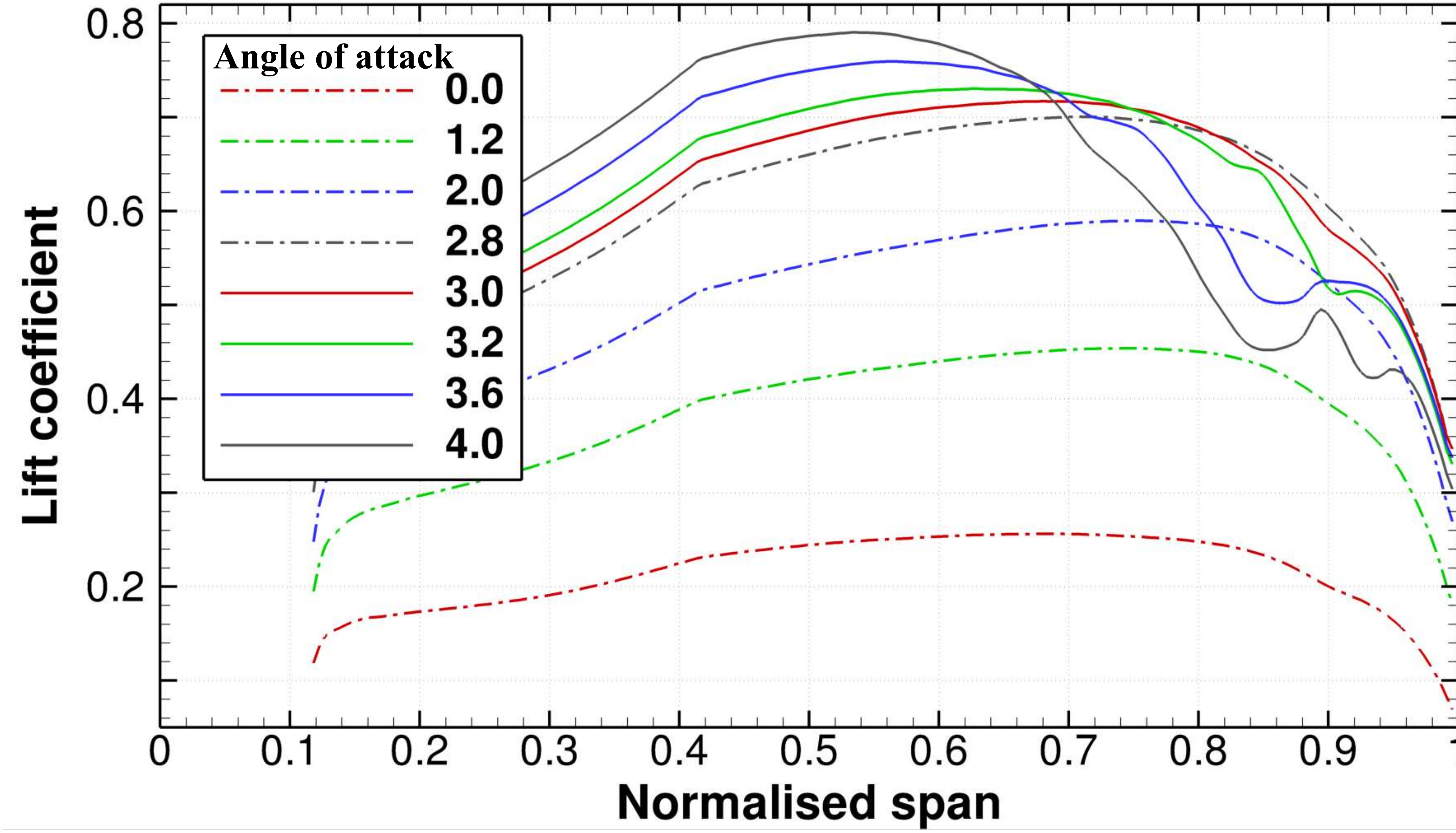
Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Lifting Line Theory

- **Downwash** or **induced velocity**
- The result is a **smaller effective angle of attack**, and thus a smaller lift component
  - How small? It depends on the wing geometry:
    - High Aspect Ratio Straight Wing
    - Low Aspect Ratio Straight Wing
    - Swept Wing
    - Delta Wing



C_L vs. Angle of Attack – Span Wise



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

High Aspect Ratio Straight Wing

Used primarily for relatively low speed subsonic airplanes

- Classic theory for such wings, most straightforward engineering approach to estimate aerodynamic coefficients is Prandtl's Lifting Line Theory

- To estimate the lift curve slope of a finite wing:

$$\boxed{a = \frac{a_0}{1 + a_0/(\pi e AR)}}$$

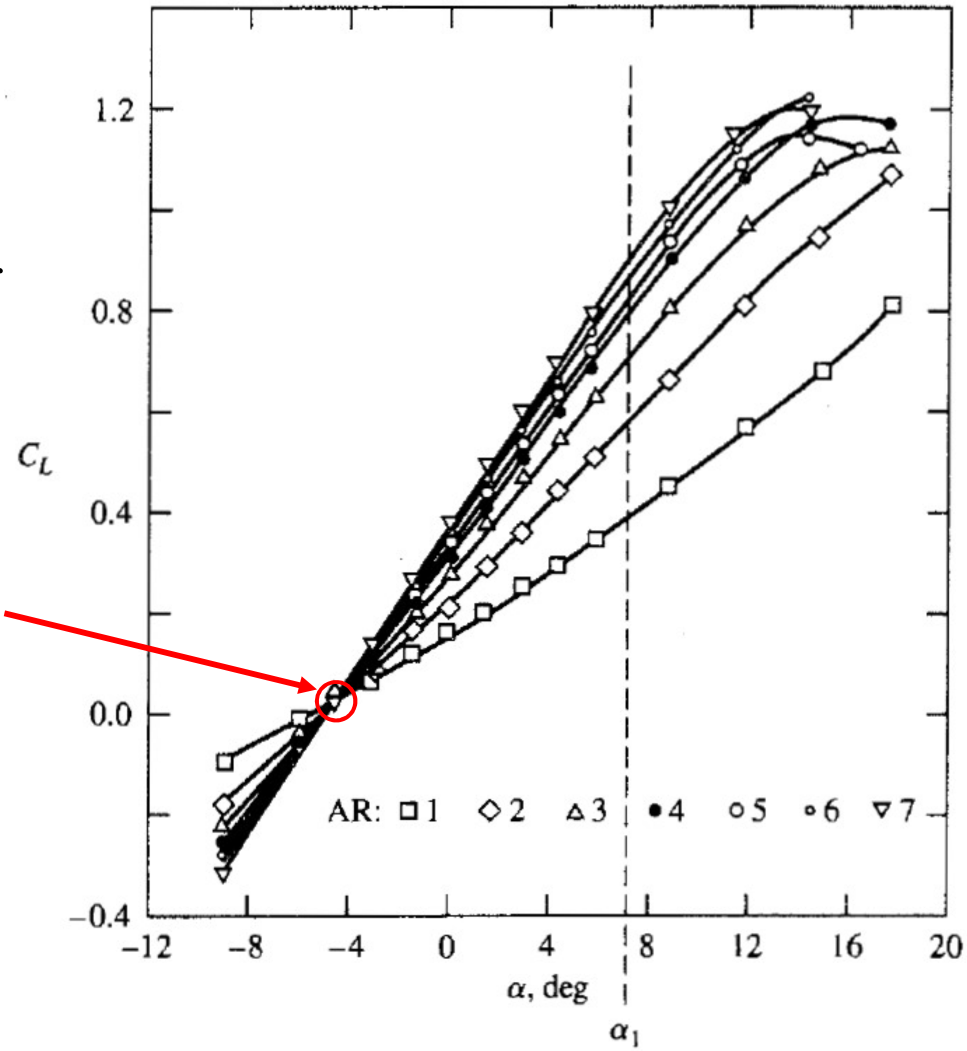
a_0 is section lift curve slope (per radian)

e is wing efficiency factor (usually about 0.95)

- Equation good for <u>high</u> aspect ratio, straight wings in incompressible flow
  - Good for wings with aspect ratio 4 or higher

Effect of Aspect Ratio on Lift Curve

- Lift curve slope decreases as aspect ratio decreases
  - As aspect ratio decreases, induced effects from wing tip vortices become stronger and lift is decreased for a given angle of attack
- Note that that zero lift angle of attack is same for all wings, but lift curve slope varies with aspect ratio



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Compressibility Correction

- Prandtl's Lifting Line Theory may be corrected for compressibility effects at higher speeds.

Prandtl-Glauert Rule

$$a_{0,comp} = \frac{a_0}{\sqrt{1 - M_\infty^2}}$$

Now replace a_0 in Prandtl's Lifting Line Theory with $a_{0,comp}$ to get corrected lift curve slope.

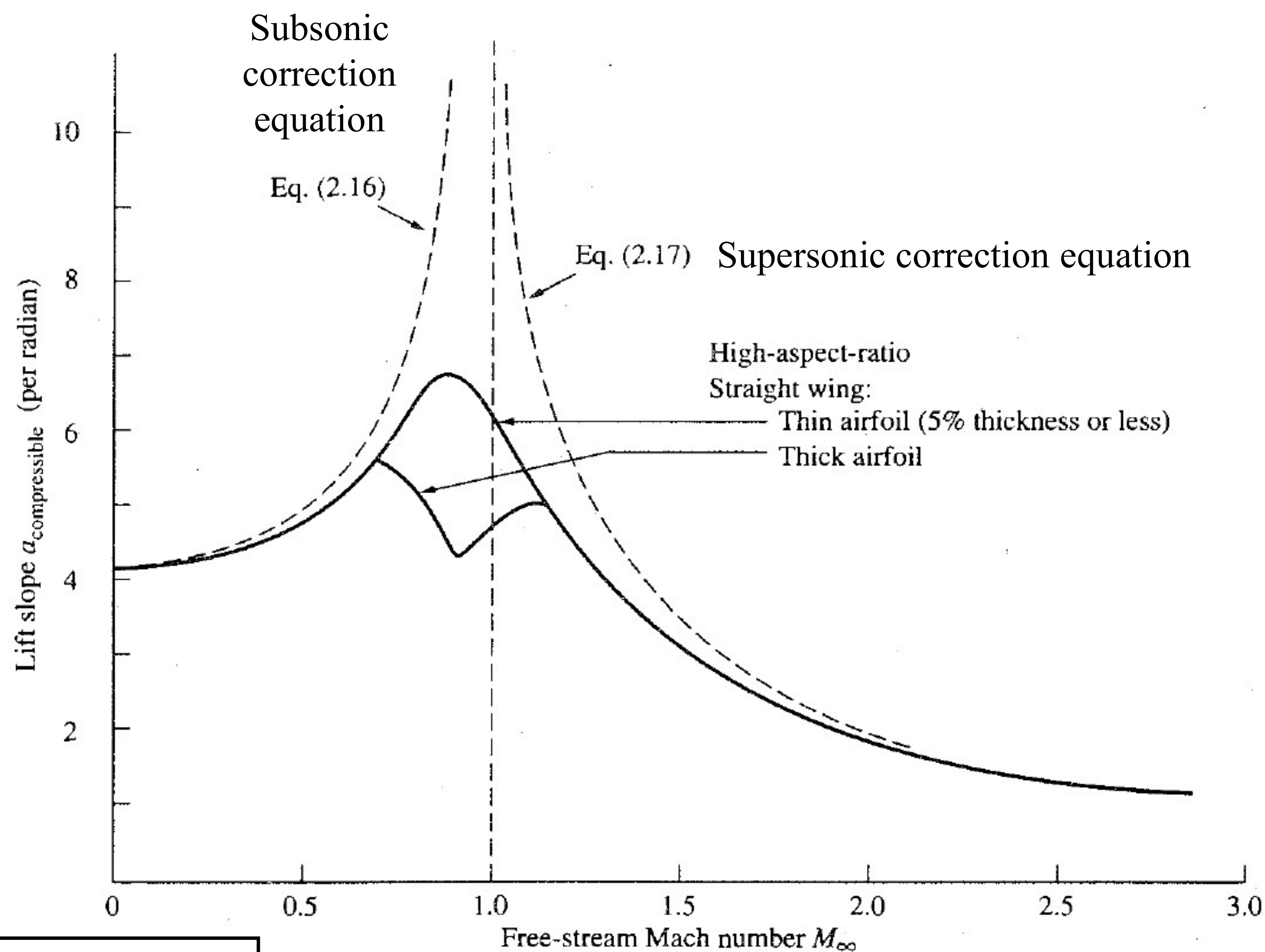
$$\boxed{a_{comp} = \frac{a_0}{\sqrt{1 - M_\infty^2} + a_0/(\pi e AR)}}$$

good for Mach numbers up to about 0.7

For Supersonic High Aspect Ratio Wings, use this equation derived from supersonic linear theory:

$$\boxed{a_{comp} = \frac{4}{\sqrt{M_\infty^2 - 1}}}$$

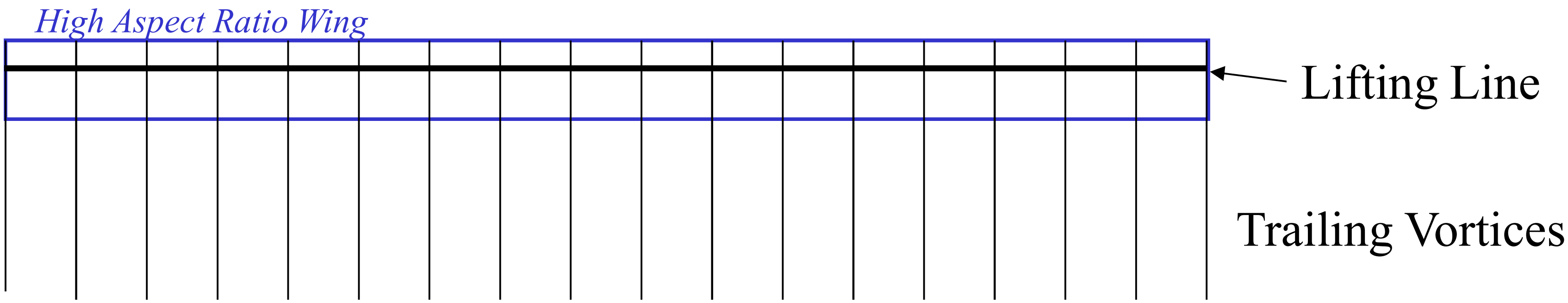
Effect of Mach Number on Lift Slope

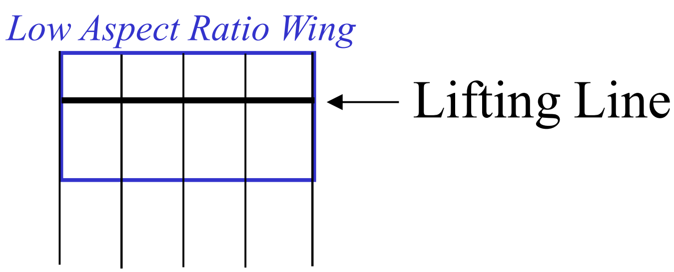


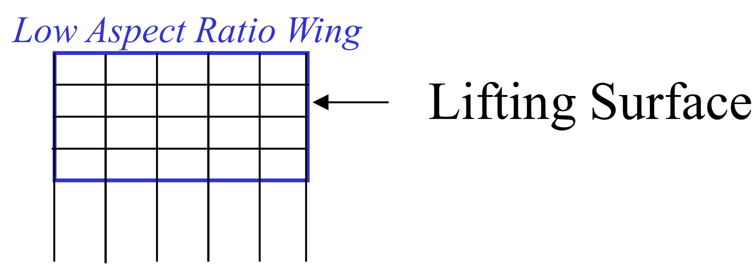
For transonic region, use CFD

Low Aspect Ratio Straight Wings

- Cannot use Prandtl's Lifting Line Theory for low aspect ratio wings (AR < 4)
  - It is based on lifting line theory, which models high aspect ratio wings well, but models low aspect ratio wings poorly
  - It is more appropriate to use lifting surface







Lifting line produces bad model for low AR wings

Lifting surface provides better model for low AR wings

Low AR Wing Lift Approximations

Helmbold's Equation

$$a = \frac{a_0}{\sqrt{1 + [a_0/(\pi AR)]^2} + a_0/(\pi AR)}$$

For incompressible flow

based on lifting surface solution for elliptical wings, but used for straight wings

$$a = \frac{a_0}{\sqrt{1 - M_\infty^2 + [a_0/(\pi AR)]^2} + a_0/(\pi AR)}$$

For subsonic, compressible flow

modified from above equation

4

$$a_{comp} = \frac{4}{\sqrt{M_\infty^2 - 1}}\left[1 - \frac{1}{2AR\sqrt{M_\infty^2 - 1}}\right]$$

For supersonic flow, low AR, straight wing

Valid as long as Mach cones from the two wing tips do not overlap

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

When do we want a low AR wing?

- Most aircraft do not use low AR wings
- At subsonic speeds, low AR wings have high induced drag
- At supersonic speeds, low AR wings have low supersonic wave drag

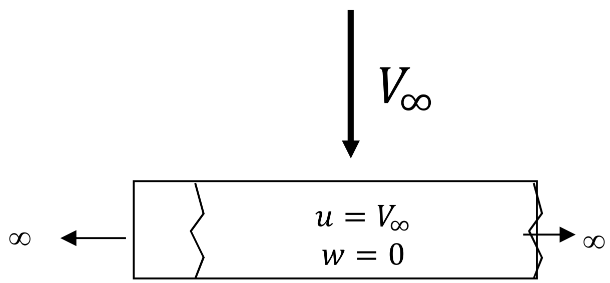
NASA Dryden Flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC90-224 Date: 1990

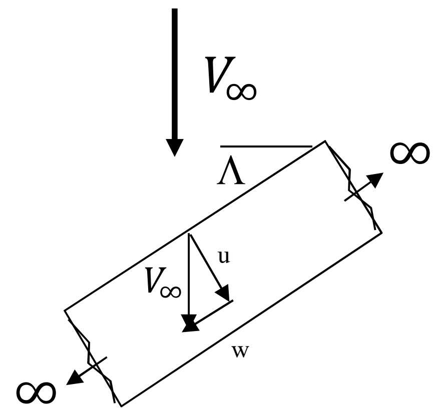
F-104

Lockheed F-104, AR = 2.97

Swept Wings

- Purpose of using a swept wing is to reduce wave drag at transonic and supersonic speeds





Here, $u = V \cos \Lambda$

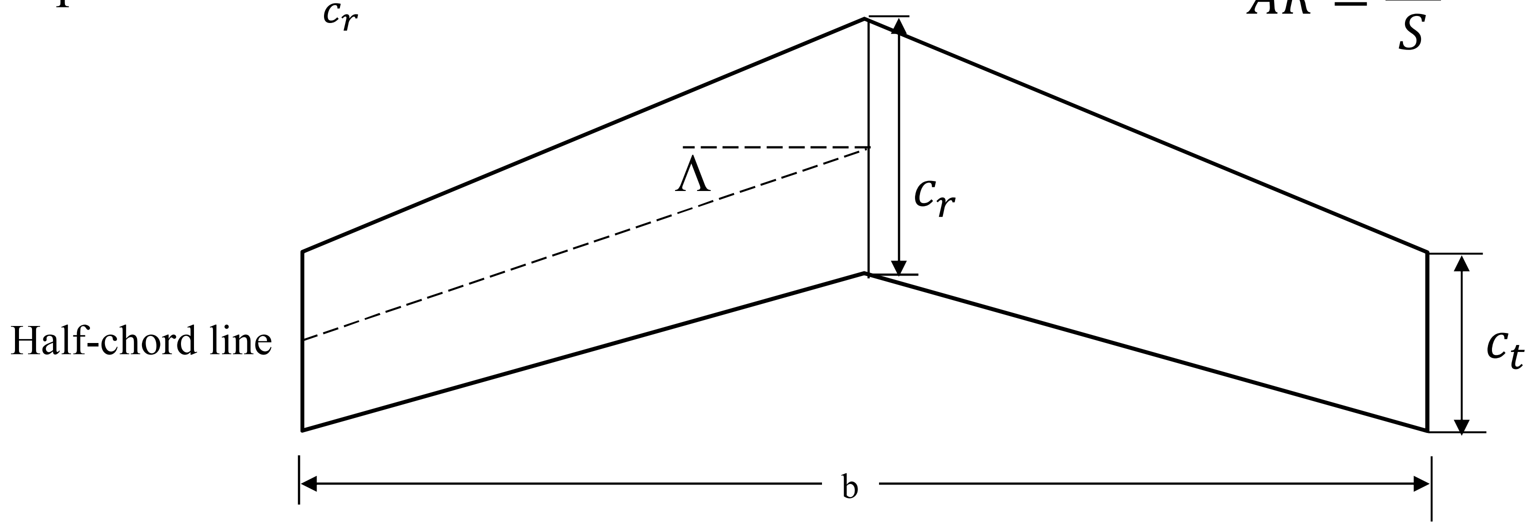
- Since u for swept wing is less than u for straight wing, the difference in pressure between top and bottom surfaces of the swept wing will be less than the difference in pressures on the straight wing **Result: swept wing has less lift**

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Swept Wing Geometry and Approximations

Taper Ratio $= \frac{c_t}{c_r}$

$AR = \frac{b^2}{S}$



$$a = \frac{a_0 \cos\Lambda}{\sqrt{1 + [a_0 \cos\Lambda/(\pi AR)]^2} + a_0 \cos\Lambda/(\pi AR)}$$

Lift curve slope approximation based on lifting surface theory

incompressible flow

Note: this equation is same as for low aspect ratio wings, with "new" a_0

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Swept Wing, Compressibility Corrections

Let $M_{\infty,n}$ be the component of the free stream Mach number perpendicular to the half chord line (equivalent to $M_{\infty,n} = M_\infty \cos\Lambda$).
The lift curve slope becomes:

$$\frac{a_0}{\sqrt{1 - M_{\infty,n}}}$$

Let

$$\beta = \sqrt{1 - M_\infty^2 \cos^2\Lambda}$$

Replace a_0 with a_0/β in incompressible swept wing equation to get:

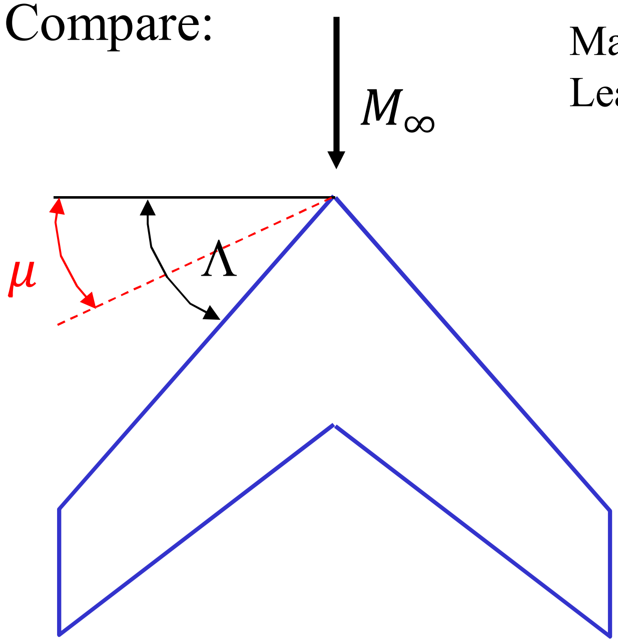
$$\boxed{a_{comp} = \frac{a_0 \cos\Lambda}{\sqrt{1 - M_\infty^2 \cos^2\Lambda + [a_0 \cos\Lambda/(\pi AR)]^2} + a_0 \cos\Lambda/(\pi AR)}}$$

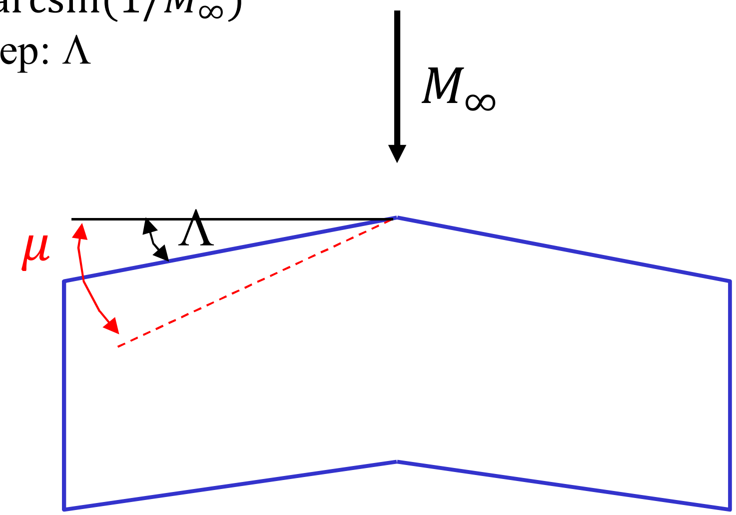
for subsonic, compressible flow over a swept wing

Supersonic Swept Wings

Compare:

Mach Angle: $\mu = \arcsin(1/M_\infty)$
Leading Edge Sweep: Λ





Wing is inside Mach cone. Component of M_∞ perpendicular to leading edge is subsonic
subsonic leading edge

Weak shock at apex, NO shock on leading edge

Behaves as subsonic wing even though $M_\infty > 1$

Wing leading edge is outside of Mach cone
supersonic leading edge

Shock wave attached along entire leading edge

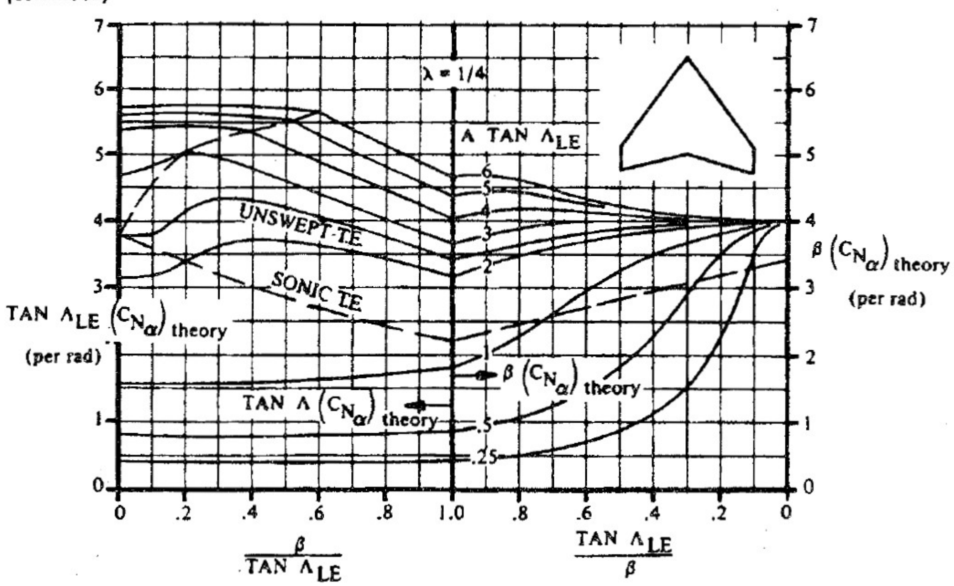
Behaves as supersonic flat plate at angle of attack

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

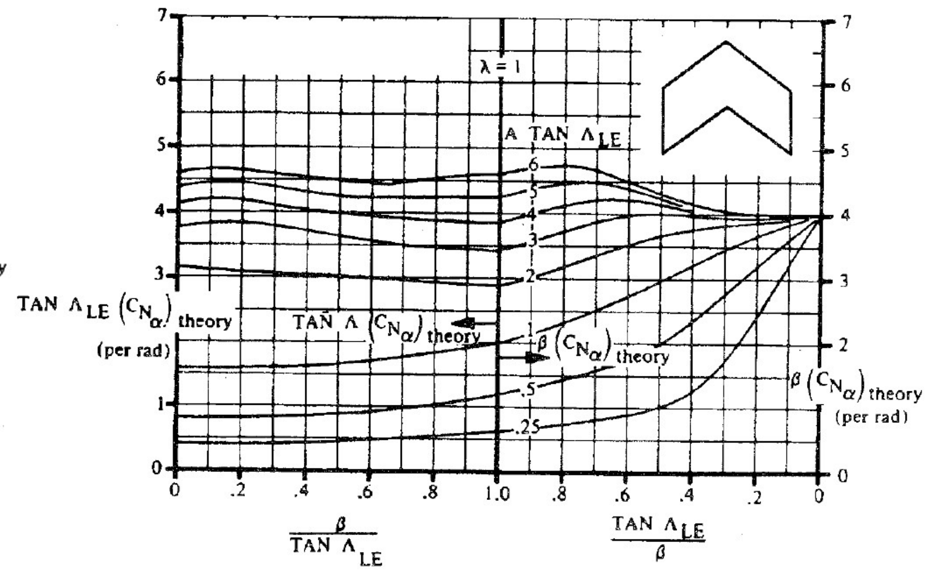
Estimating Lifting Properties of Supersonic Wings

- No quick way to estimate lifting properties. Normally use CFD techniques to get pressure distribution and then integrate. Or use charts...

(continued)



(c)



(f)

$C_{N,\alpha}$, the **normal lift coefficient**, can be made analogous to C_L

$$C_L = C_{N,\alpha}\alpha$$

Swept Wings Overview

- Used to reduce transonic and supersonic wave drag (used on high speed airplanes)

- However, **<u>wing sweep is usually a detriment at low speeds</u>**:

  low speed lift coefficient is reduced by sweeping the wings

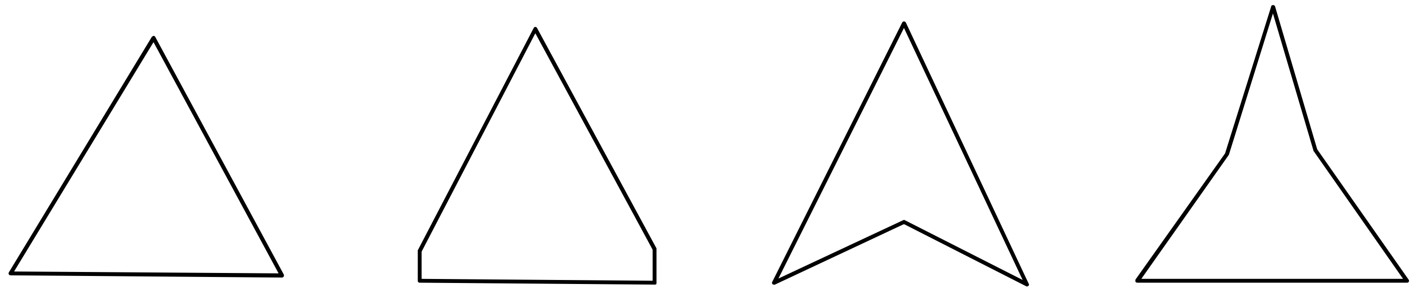
  ↓

  degraded takeoff and landing performance

- Therefore, swept wings must often be designed with elaborate high lift devices
  - Expensive
  - Complicated

Delta Wings

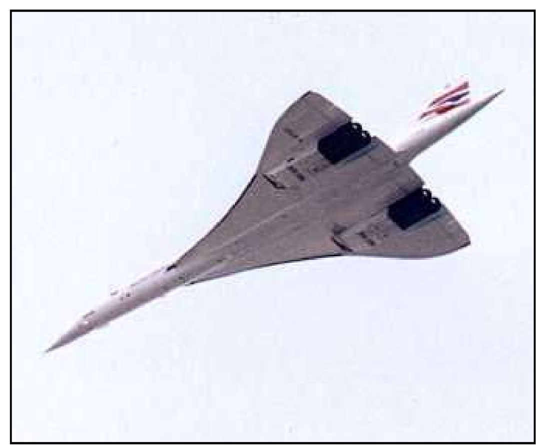
- Delta Wings- triangular shaped, highly swept wings



Simple Delta **Cropped Delta** **Notched Delta** **Double Delta**

- First explored in 1930s in Germany by Lisspisch
- Used primarily for supersonic flight

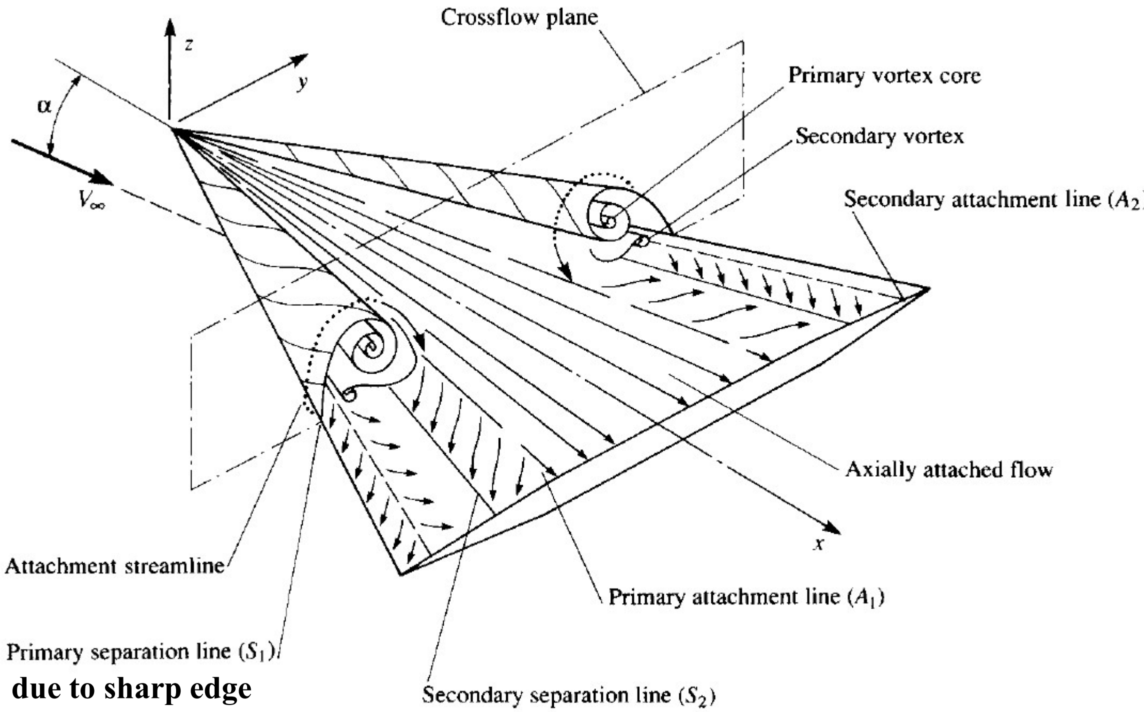
F-102



Concorde

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

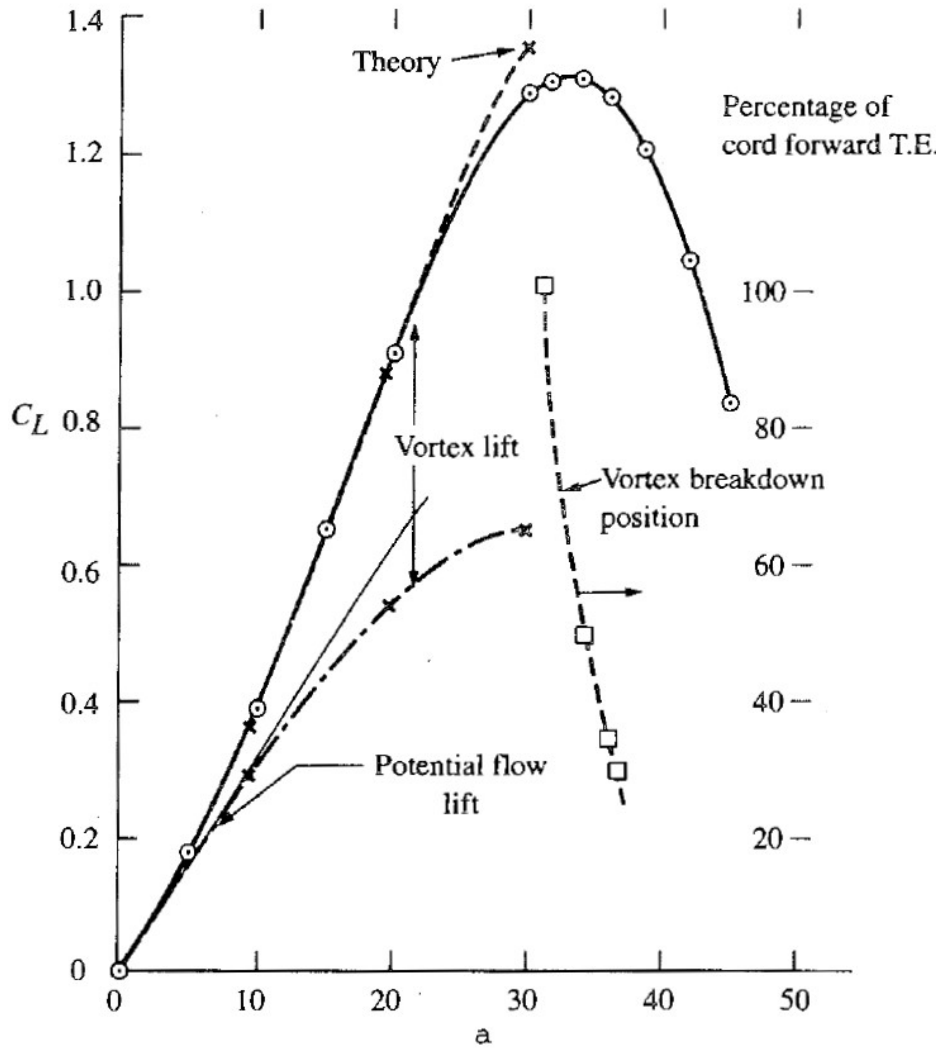
Delta Wings-Subsonic Flow



- Dominant flow is the two vortices that develop along the highly swept leading edges:
  - The pressure on the upper surface is lower than the pressure on the bottom. This induces a flow that curls around the leading edge. If this edge is sharp, the flow separates and curls into a primary vortex. A secondary vortex forms beneath the primary one
- This vortex flow is "good" stable vortices, high energy – Vortex Lift

Vortex Lift

- The vortices on a delta wing in subsonic flow create a lower pressure region on the top of the wing that would not normally exist. This creates more lift, called **Vortex Lift**
- In Figure: data for an AR=1.46 wing
- Potential Flow lift is the theoretical calculation of lift without the leading edge vortices.
- Actual lift was obtained experimentally



Features of Vortex Lift

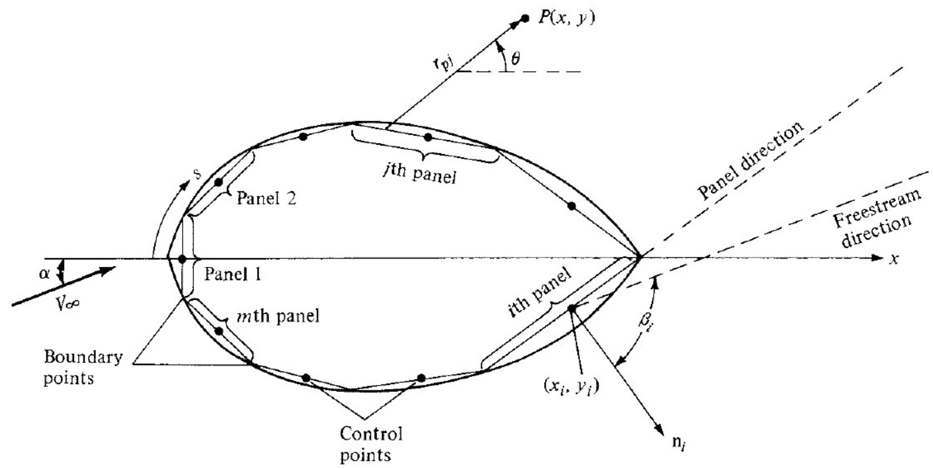
- The lift slope is small, on the order of 0.05 per degree
- The lift continues to increase over a large range of angle of attack. A reasonable $C_{L_{max}}$ would be on the order of 1.35 with a stalling angle of 35 deg
- The lift curve is *non-linear*, due to the effects of vortex lift
- Note: large angles of attack are used for takeoff and landing for vehicles with delta wings (Space Shuttle, Concorde). Realize that this high angle of attack makes visibility for the pilot very difficult, leading to such solutions as the droop nose on the Concorde

Other Methods to Calculate Lift Distribution

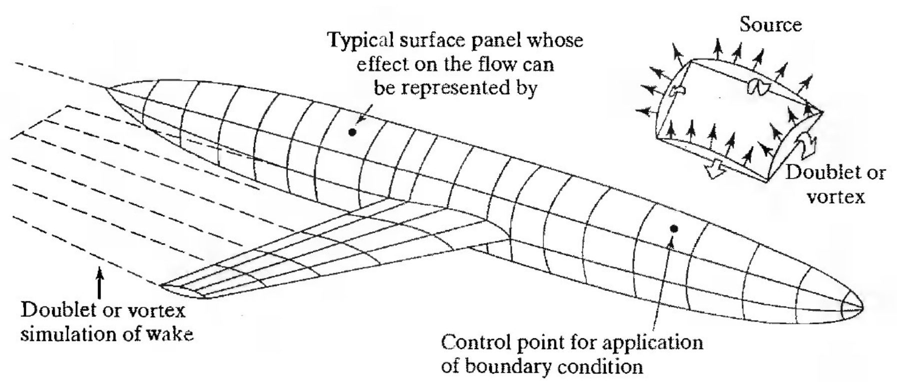
- So far, we have talked about lifting line theory. However, we recognize the limitations of this theory
- In practice, we use more advanced methods for the characterization of the flowfield around an aerodynamic body
  - Panel methods
  - Computational Fluid Dynamics
    - Euler
    - Navier Stokes
    - Reynold’s Average Navier-Stokes

Panel Methods

- Panel methods:
  - Used to represent the flowfield for more complex geometries where conventional lifting line theory is no longer valid
  - In general, you need three elements for the application of a panel method:
    - A geometry, which is to be discretized into a series of panels and control points
    - A set of boundary conditions
    - A singularity to be applied at each panel



Application of panel method to a 2-D body



3-D geometry discretized into panels

Panel Methods

- Panel methods:
  - You set up the problem numerically to find the strength of your vortices, sources, etc. such that the surface becomes a streamline of the flow
- Source panel method:
  - Panels are given a uniform source sheet which "induces" a velocity potential at a given point
  - Used to calculate the flow around **non-lifting surfaces** → it has zero circulation
  - It can be used to calculate velocity and pressure distributions
- Vortex panel method:
  - Now we wrap around a **vortex sheet** over the complete surface of the body
  - Lift can be found from total circulation, pressure from Bernoulli's

Incompressible Flow about Wings of Finite Span Chap. 7

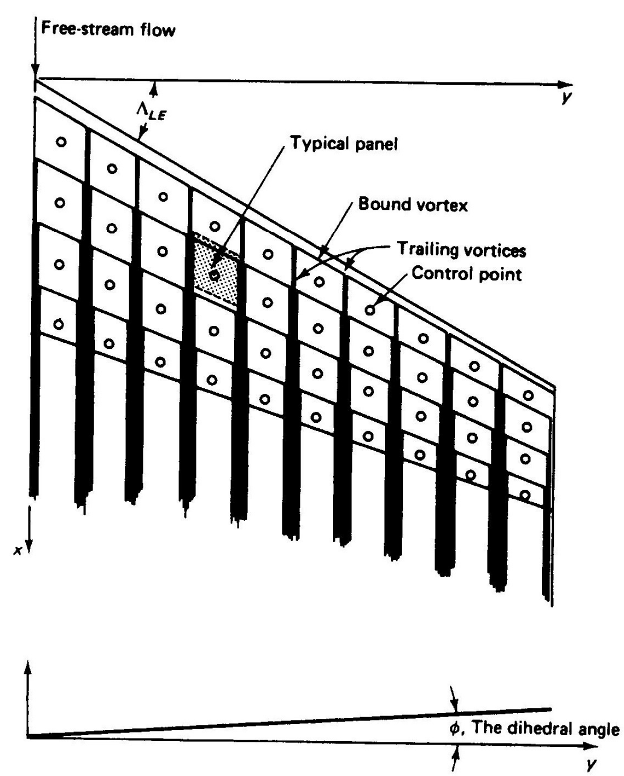


Figure 7.21 Coordinate system, elemental panels, and horseshoe vortices for a typical wing planform in the vortex lattice method.

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Georgia Institute of Technology
Aerospace Systems Design Laboratory

Vortex Lattice Method

- Consider the lifting line:

FUNDAMENTALS OF AERODYNAMICS

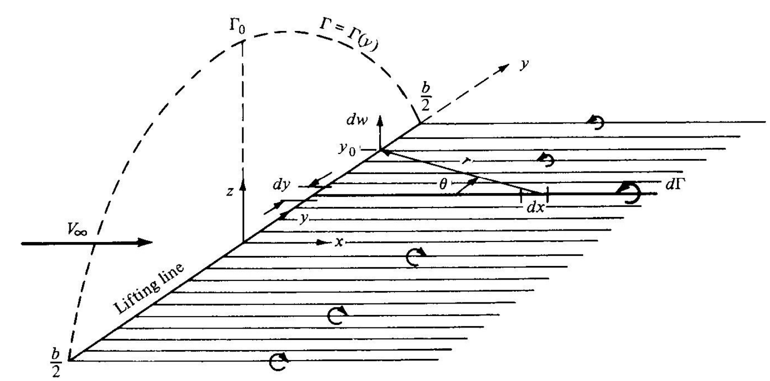


Figure 5.13 Superposition of an infinite number of horseshoe vortices along the lifting line.

- Now imagine creating a grid or lattice of "horseshoe vortices":

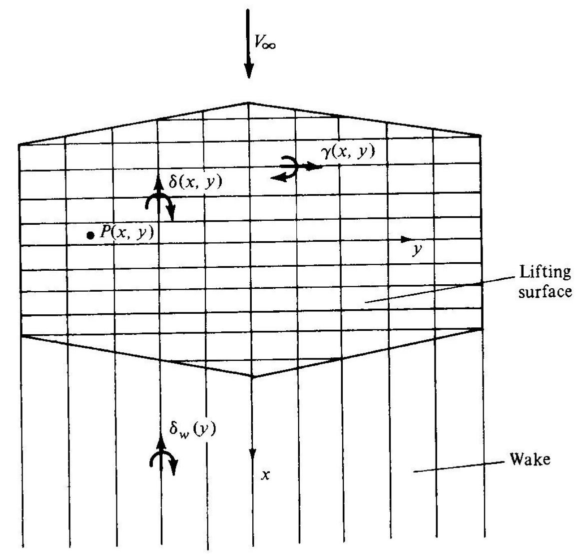


Figure 5.31 Schematic of a lifting surface.

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Vortex Lattice Method

- Every point P on the wing experiences flow normal to the surface due to vortices from every other panel dξ x dη
  - Velocities are evaluated at each control point using Biot-Savart
- We need to find equations γ(x,y) and δ(x,y)
  - Setup as a simultaneous system of algebraic equations

Circulation vector

$$\sum A_{i,j}\Gamma_i = -V_\infty \frac{ds}{dx}\Big|_I$$

Boundary condition

Geometry of the panels

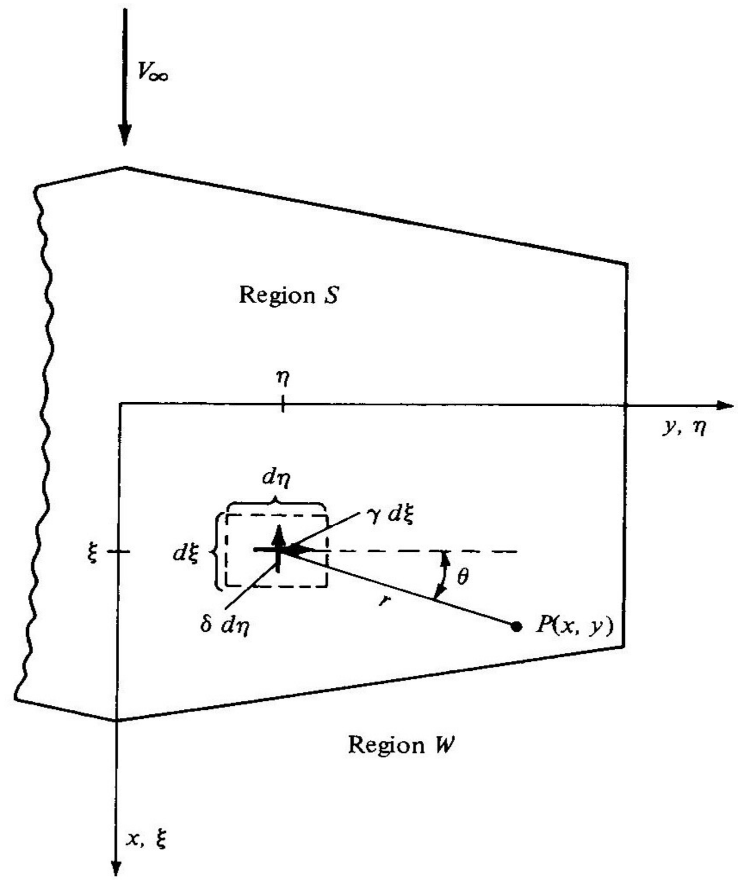


Figure 5.32 Velocity induced at point P by an infinitesimal segment of the lifting surface. The velocity is perpendicular to the plane of the paper.

What is Potential Flow?

- A potential flow is an irrotational flow
- An Irrotational flow has the following characteristic:
  - The vorticity is zero at every point in the flow:

$$\xi = \nabla \times \overline{V} = 0$$

- Note that a potential flow must, by definition, be inviscid, as all viscous flows are rotational

What is the Advantage of Potential Flow?

- A potential flow is easier to analyze
- For a potential flow there exists a scalar function whose gradient is equal to the velocity vector at all points
  - i.e. $\overline{V} = \nabla \phi$
- This function is called the velocity potential, and allows the velocity components to be described with a single equation, greatly simplifying irrotational flow analyses

What good is it?

- In incompressible flow that is also irrotational, Bernoulli's equation holds between any two points in the flow, not necessarily just points located along the same streamline
- Irrotational flows are much easier to analyze than rotational flows
  - For example, subsonic flows over airfoils, supersonic flow over slender bodies at small angle of attack, etc.
- Allows the study of the region outside the boundary layer of viscous flow adjacent to the surface of an airfoil in a simplified manner
  - Often used instead of using Boundary Layer Theory which is more complicated

Potential Theory and Drag on an Airfoil

- In potential flow there is assumed to be zero rotation
  - No rotation means no viscosity
  - No viscosity means no Boundary Layer!
- Types of drag on a wing: form, skin friction, induced, and wave
  - Form drag results from separation of flow over a body
  - Skin friction drag results from the friction in the boundary layer
  - Induced drag is a 3-D effect not pertinent to the 2-D airfoil
  - Wave drag is related to pressure drag at supersonic velocity

So this means?

- Potential theory cannot account for skin friction drag because it does not model boundary layers

- Skin friction drag is an important component of overall drag

- THEREFORE, ***<u>no</u>***, potential flow cannot be used to predict *all* drag contributions on an airfoil or a wing

Computational Fluid Dynamics

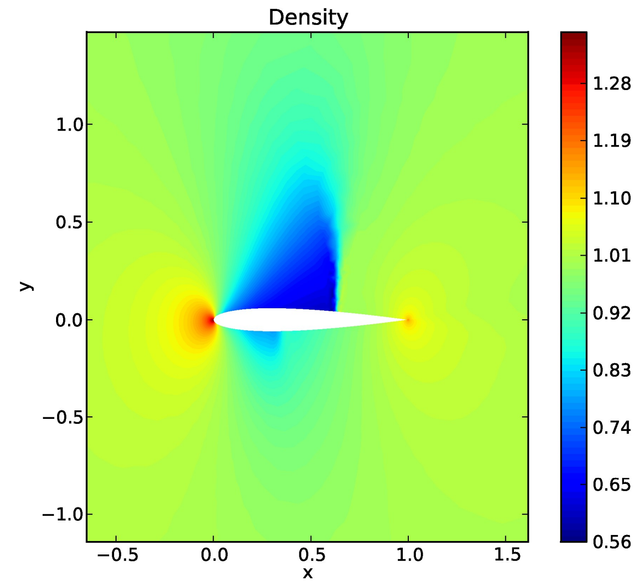
- While at first aerodynamic design was based on experience and experimentation, advancements in computer power and development of numerical methods have enabled the use of CFD
  - However, there are still numerous challenges to be faced when using CFD for the calculation of the flowfield and aerodynamic coefficients
    - Inviscid simulations can be performed relatively efficiently and have some practical applications → use of **Euler methods**
    - The estimation of lift-independent drag due to skin friction and flow separation requires the use of viscous models with high resolution → **Navier Stokes**
    - Turbulent flow calculations still rely on empirical methods and introduce uncertainty into the process → require Reynolds Averaged Navier-Stokes equations **(RANS)**, Large Eddy Simulation **(LES)**, or Direct Numerical Simulation **(DNS)**

Computational Fluid Dynamics

- Recall Euler's equation:

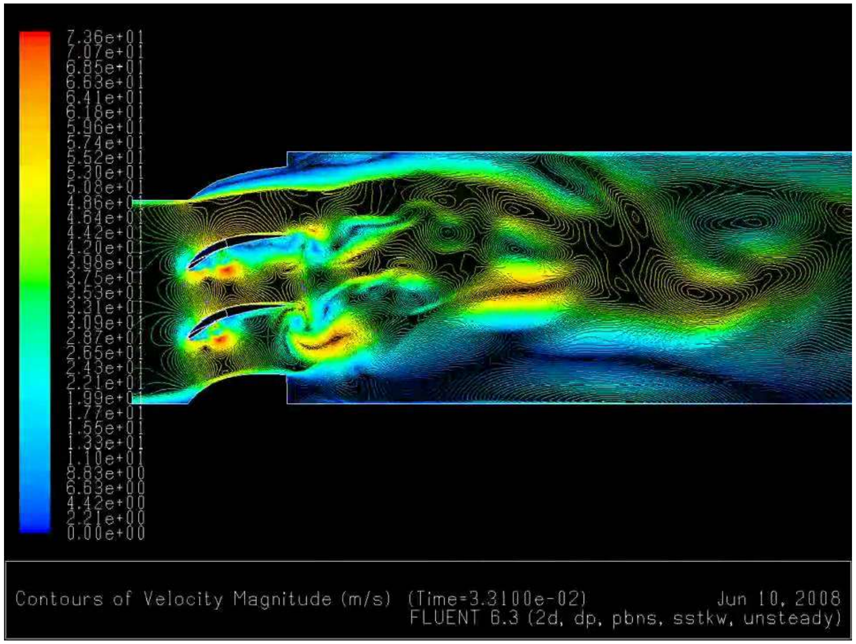
$$dp = -\rho V dV$$

  - Only valid for steady, inviscid flow, with no other body forces
  - Can be used to calculate lift and the flowfield around an aerodynamic body
  - Not suitable for drag calculations, as it cannot calculate the boundary layer, and other viscous flow phenomena



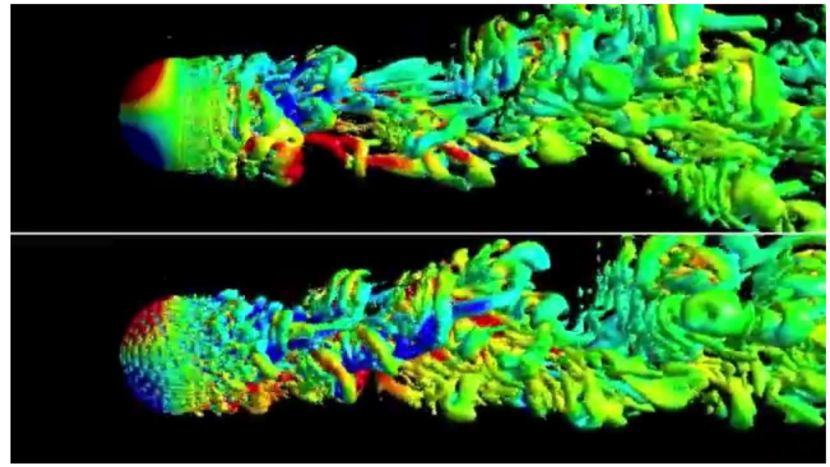
Computational Fluid Dynamics

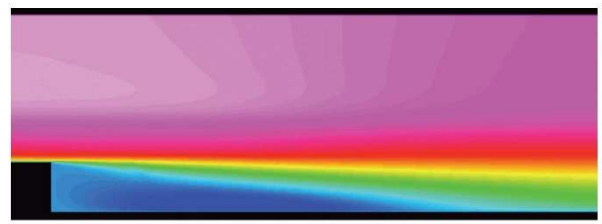
- The Navier Stokes equations can be used to model more complex aerodynamic effects
  – Set of continuity and momentum equations that include the effects of viscosity and flow unsteadiness
  – Require proper mesh refinement to capture these effects
  – Computational cost often makes it prohibited to perform design space exploration studies



Computational Fluid Dynamics

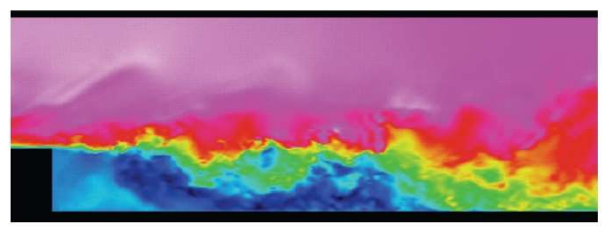
- What about turbulence?
  - **Laminar flow**: "layered flow", with little exchange of fluid particles perpendicular to flow direction
  - **Turbulent flow**: irregular transverse motions of fluid particles and fluctuations; must consider unsteady motion of particles
- Computational modeling of turbulence requires the use of advanced techniques such as:
  - Reynolds Averaged N-S (RANS)
  - Large Eddy Simulations (LES)
  - Direct Numerical Simulation (DNS)





Source: Rémy Fransen, 3rd INCA colloquium, ONERA, Toulouse (2011)

RANS



Source: Rémy Fransen, 3rd INCA colloquium, ONERA, Toulouse (2011)

LES

Source: https://earthscience.stackexchange.com/questions/2932/what-are-the-differences-between-an-les-sgs-model-and-a-rans-based-turbulence-mo

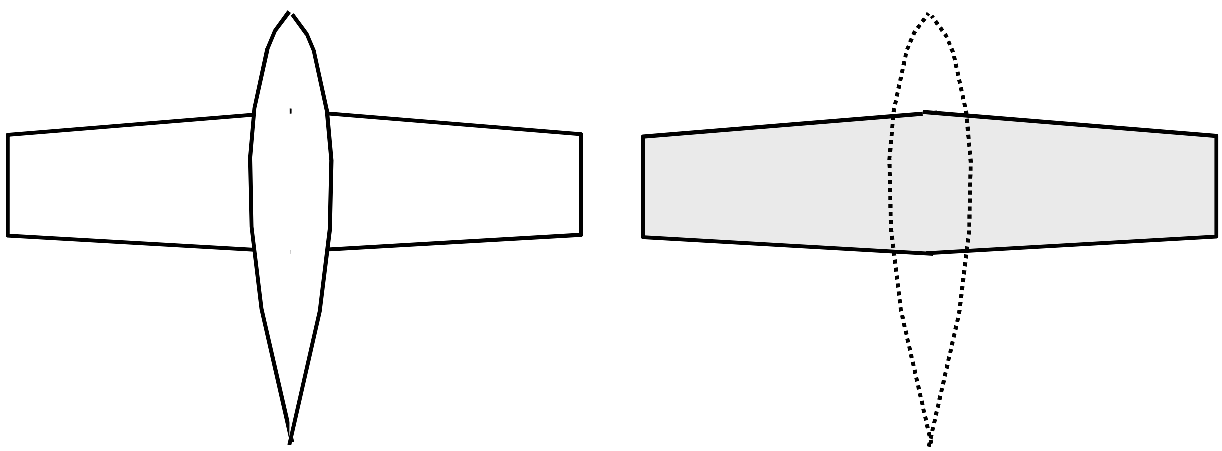
Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Wing Body Combinations

$$\text{Lift}_{\text{wing}} + \text{Lift}_{\text{body}} \neq \text{Lift}_{\text{wing/body combination}}$$

- No accurate analytical way to predict lift of wing-body interaction
  - Wind tunnel tests
  - CFD analysis
  - Can’t even say if it will be more or less

- However, work by Hoerner and Borst shows that the lift of the wing-body combination can be treated as simply the lift on the complete wing by itself, including that portion which is covered by the fuselage.



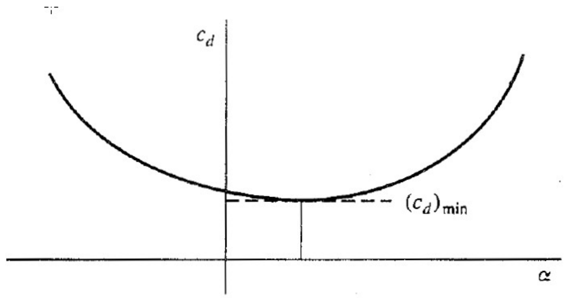
reasonable approximation for preliminary design and performance at subsonic speeds

Aerodynamic Drag Contributions

- Performance goal → Design for minimum drag (or maximum L/D)
- Remember, there are 2 sources of aerodynamic force:
  - Pressure → pressure drag
  - Shear stress → skin friction drag
- Just like the two aerodynamic forces, there are two corresponding types of drag. All drag types can be classified under one of these two headings
- In general, drag is difficult to predict analytically. Often must rely on empirical relationships

Subsonic Drag-Airfoils

- Section drag, also called profile drag, is what you see in typical airfoil c_l vs c_d data, like the NACA airfoil data



$$c_d \quad = \quad c_f \quad + \quad c_{d,p}$$

profile drag = skin-friction drag + pressure drag due to separation

<u>Skin friction drag</u> due to frictional shear stress acting on the surface of the airfoil

- For thin airfoils and wings, c_f can be estimated by using formulas for a flat plate

$$\boxed{c_f = \frac{1.328}{\sqrt{Re}}}$$ laminar

Exact theoretical for laminar incompressible flow over a flat plate, but we use it as an *approximation* of an airfoil

Skin Friction Drag

$$c_d = \frac{D_f}{q_\infty S}$$

$$Re = \frac{\rho_\infty V_\infty c}{\mu_\infty}$$

D_f friction on one side of plate

c length of plate in flow direction

S planform area of plate

- For turbulent flow, must use approximations

$$\boxed{(c_f)^{-1/2} = 4.13 \log(Re\ c_f)}$$

turbulent (solve implicitly) Karman-Schoenherr

or

$$\boxed{c_f = \frac{0.42}{\ln^2(0.056\ Re)}}$$

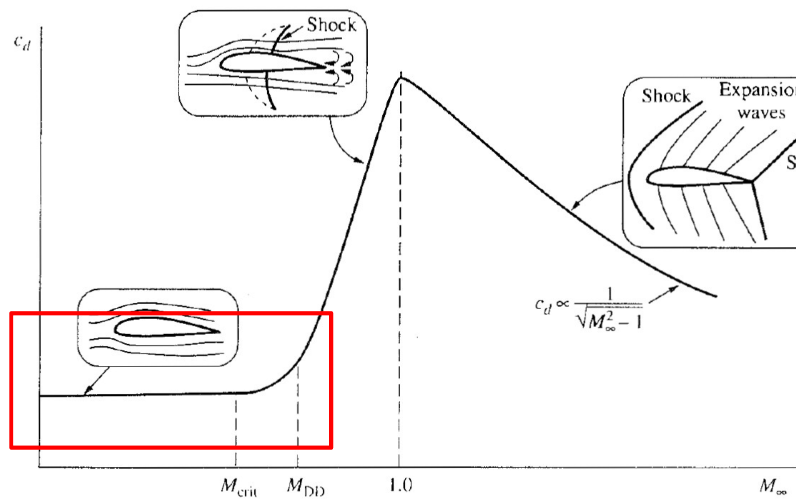
accurate within +/- 4% for $10^5 < Re < 10^9$

- Now, when do you apply these equations?
- Assumption: for high Re normally encountered in flight, laminar flow region is very small, so assume entire surface is turbulent

Pressure Drag (Form Drag)

- Pressure drag due to flow separation (form drag)
  - Caused by the imbalance of the pressure distribution in the drag direction when the boundary layer separates from the surface
  - $C_{d,p}$ is usually found experimentally

- In general, at subsonic speeds below M_{DD}, the variation of c_d with Mach number is small, so we can assume c_d is relatively constant across subsonic Mach number range



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Subsonic Drag-Finite Wings

- Now we need to add in induced drag, which is a form of pressure drag

- For a high aspect ratio straight wing, use Prandtl's Lifting Line Theory to get:

$$\boxed{C_{D_i} = \frac{C_L^2}{\pi e AR}} \qquad C_{D_i} = \frac{D_i}{q_\infty S}$$

e is efficiency factor

$0 < e < 1$ function of aspect ratio and taper

- Realize that induced drag and lift are caused by the same mechanism: change in pressure distribution between top and bottom surfaces.

- So, it makes sense that C_{D_i} and C_L are strongly coupled.

- Induced drag is the "cost" of lift.

Subsonic Drag-Finite Wings

- To reduce drag:

$$C_{D_i} = \frac{C_L^2}{\pi e AR}$$

- Want e to be as close to unity as possible. $e = 1$ is a wing with an elliptical spanwise lift distribution. But for modern aircraft, e ~ 0.95 - 1.0, so it's not as critical to have an elliptical wing

- Aspect ratio has a very strong effect: doubling AR reduces induced drag by a factor of 2

$$AR = \frac{b^2}{S}$$

Increasing AR moves the wingtip vortices further apart, which reduces their effect

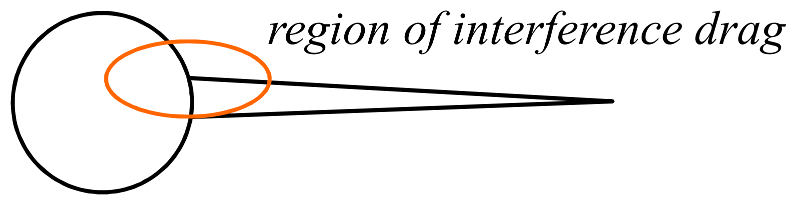
Subsonic Drag-Finite Wings

- Although high aspect ratios are aerodynamically best, they are structurally expensive.
  - Most subsonic aircraft today: $6 < AR < 9$
  - Modern sailplanes: $10 < AR < 30$

Subsonic Drag- Fuselages

Fuselages experience substantial drag:

- skin-friction - function of wetted area
- pressure drag due to flow separation
- Interference drag - interaction that occurs at the junction of the wing and fuselage



No analytical way to predict interference drag. Use experimental data.

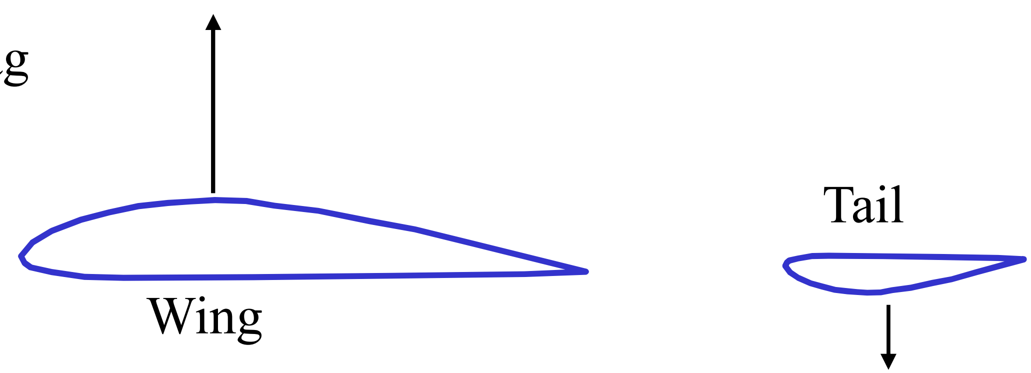
Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Summary of Subsonic Drag

- Skin friction drag - due to frictional shear stress over the surface
- Pressure drag due to flow separation (form drag) - due to pressure imbalance caused by flow separation
- Profile drag (section drag) - sum of skin friction drag and form drag
- Interference drag - additional pressure drag that is caused when two surfaces (components) meet
- Parasite drag - term used for the profile drag of the *complete aircraft*, including interference drag
- Induced drag - pressure drag caused by the creation of wing tip vortices (induced lift) of finite wings
- Zero-lift drag - parasite drag of complete aircraft that exists at its zero-lift angle of attack
- Drag due to lift - total aircraft drag minus zero lift drag. It measures the change in parasite drag as α changes from $\alpha_{L=0}$

Other Contributions to Drag

- The previous drags were the main categories of drag. Sometimes they are broken into more detailed categories. For example:
  - External Store Drag
  - Landing Gear Drag
  - Protuberance Drag
  - Leakage Drag
  - Engine Cooling Drag (reciprocating engines)
  - Flap Drag
  - Trim Drag



Transonic Drag

- The distinguishing feature between the subsonic region and the transonic region is shock waves
  - In transonic, $M_\infty < 1$ but…
  - Local pockets of supersonic flow on the aircraft are usually terminated by shock waves
- Transonic drag is exclusively a pressure drag effect
  - Strong adverse pressure gradient across shock causes separation. Therefore, it is a pressure drag due to flow separation
  - Total loss of pressure across shock wave
- No closed form analytical formulas to predict transonic drag rise
  - CFD often misses the calculation of the shock induced separated flow
  - Empirical data best bet

Reduction of Transonic Drag

- Two ways of reducing transonic drag rise: i) Area ruling, ii) supercritical airfoils
- Area rule:
  - Kinks in cross sectional area distribution cause large transonic drag rise, so design fuselage/wing to smooth out distribution
  - Richard Whitcomb, using empirical information and intuition, developed idea of area rule in the 1950's

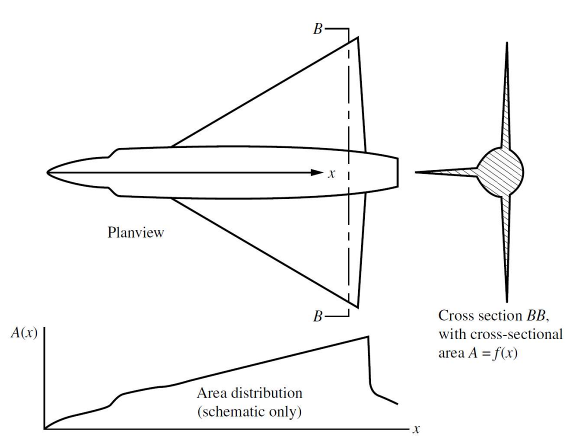


Figure: Without the Area Rule

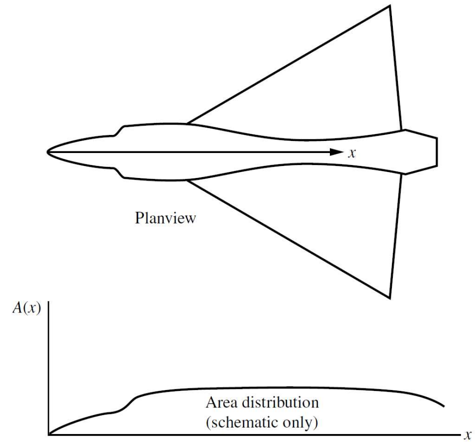
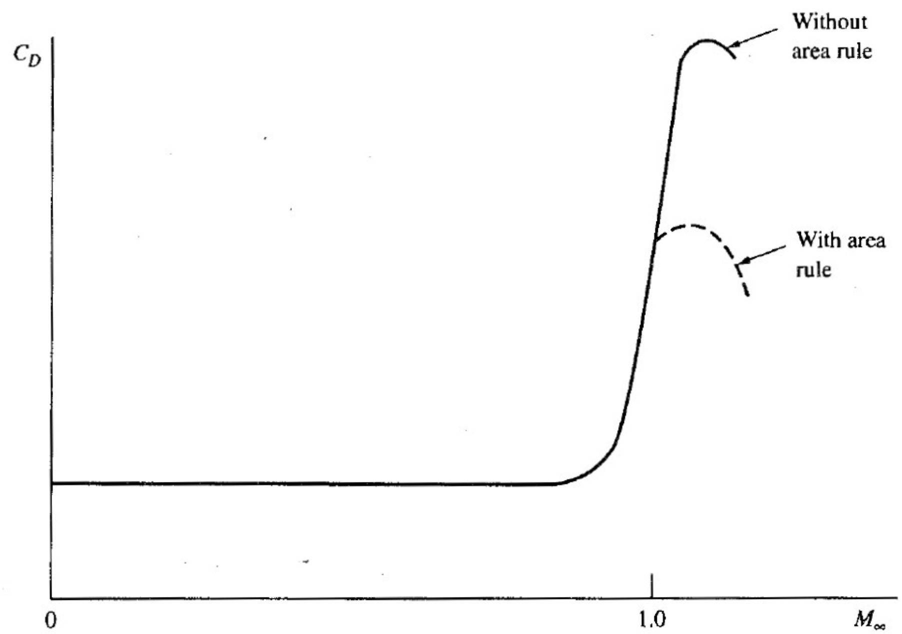
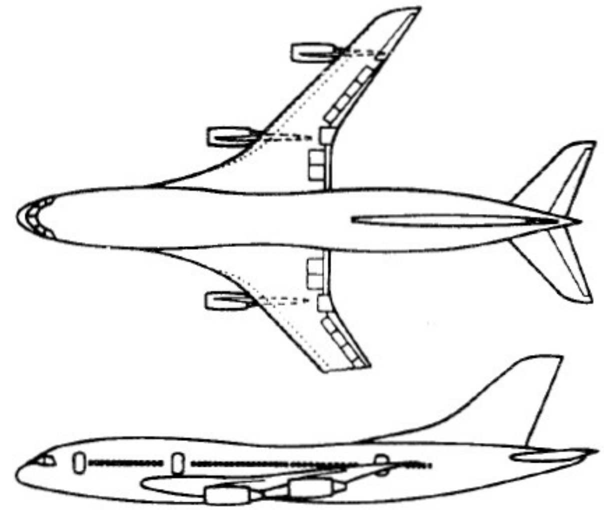
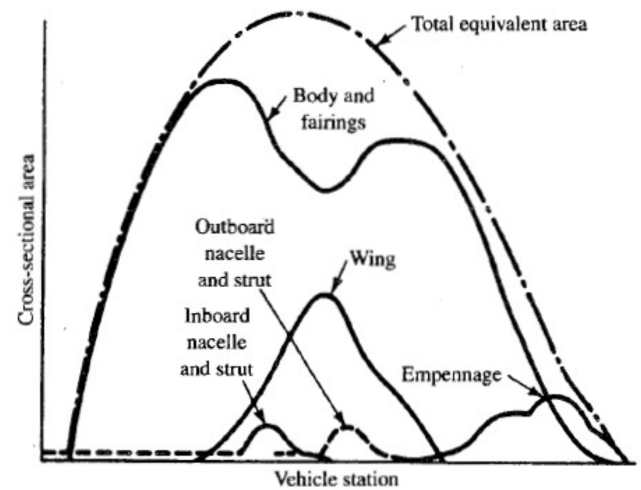


Figure: With the Area Rule

Aerodynamic Area Ruling for Wave Drag

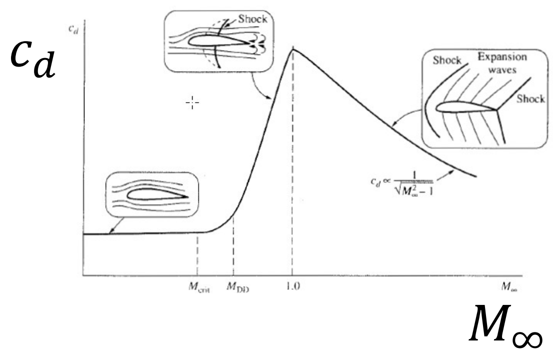






Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Supercritical Airfoil



- After WWII, it was thought the only way to increase M_{DD} was to increase M_{crit}
- Although the NACA 6-series airfoils were designed for laminar flow, they had a higher M_{crit}, so they were used on higher Mach aircraft

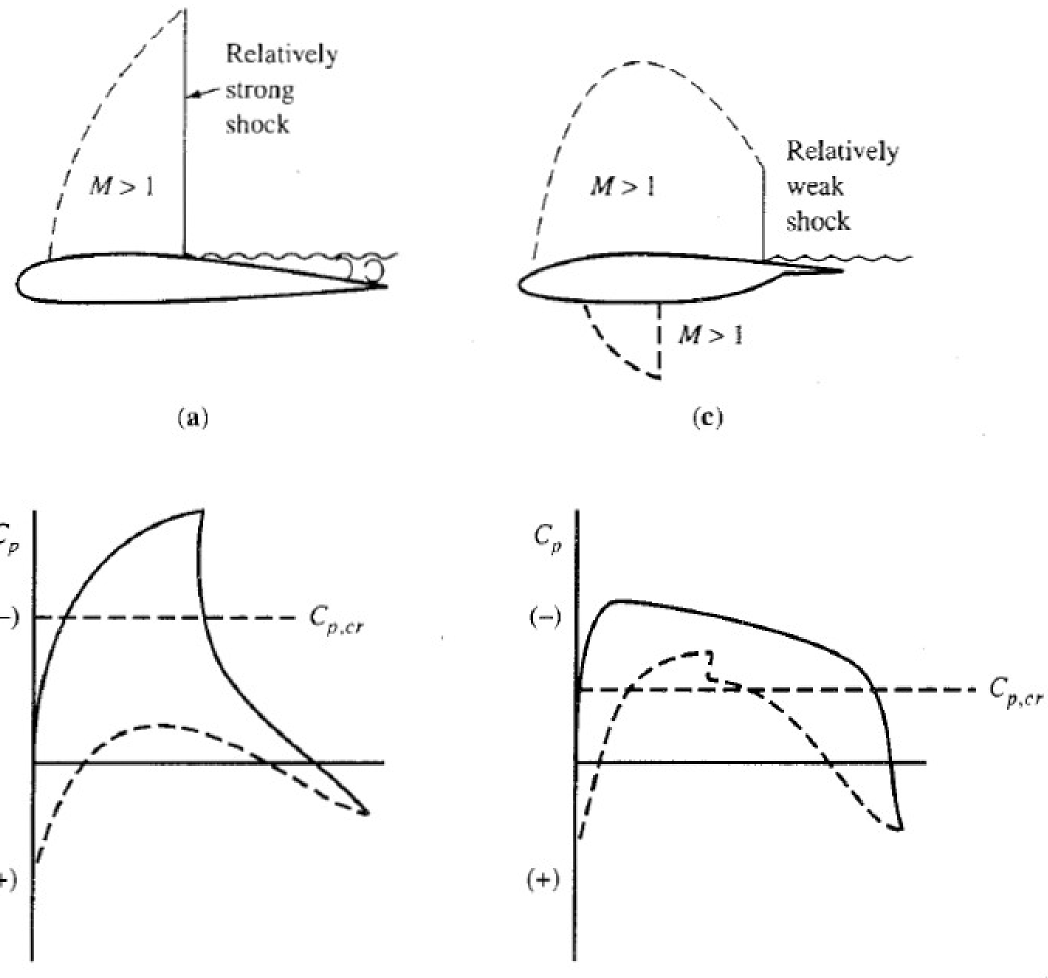
- Whitcomb, in 1965, took a different approach. He wanted to increase the distance between M_{crit} and M_{DD}
- The supercritical airfoil design came out of this pursuit

Higher M_{DD} {

- flat top encourages a region of supersonic flow with lower local values of M than the 6-series
- terminating shock is weaker, causing less drag
- extent of supersonic flow is closer to the airfoil

Supercritical Airfoil

- Because top of airfoil is flat, it actually has a negative camber in the first 60% of the airfoil.
- To compensate for this loss of lift, the aft 30% of the airfoil has extreme positive camber, giving the airfoil its distinctive look.

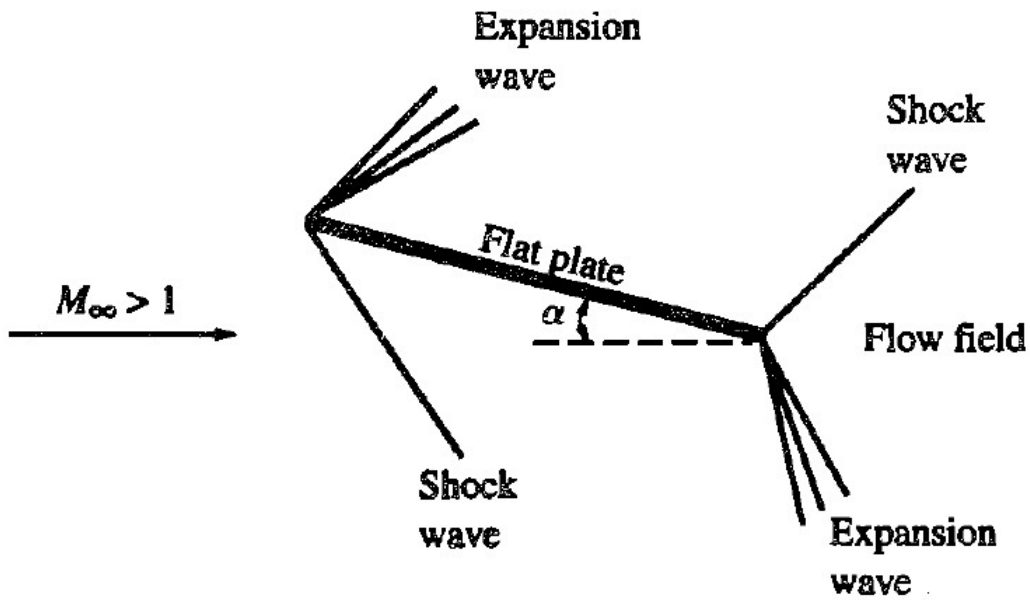


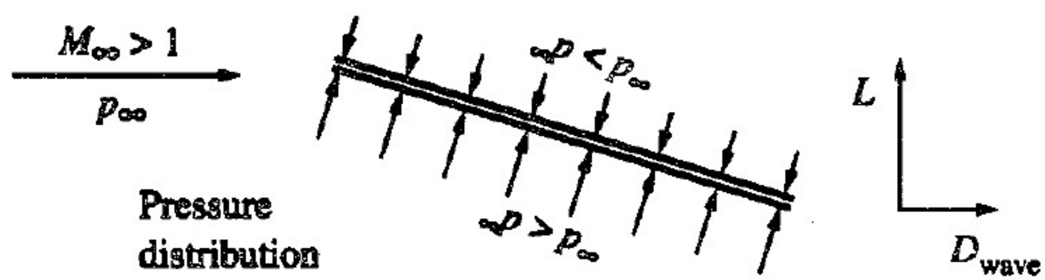
(b) NACA 64_2-A215 airfoil
$M_\infty = 0.69$

(d) Supercritical airfoil (13.5% thick)
$M_\infty = 0.79$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

- Shock waves are the dominant feature of the flow field around an aircraft at supersonic speeds
- Wave drag caused by pressure pattern around aircraft, so it is a pressure drag





Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Supersonic Drag

Recall, in slightly different format,

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

supersonic, small α's
high AR
straight wing

corresponding for drag:

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$$

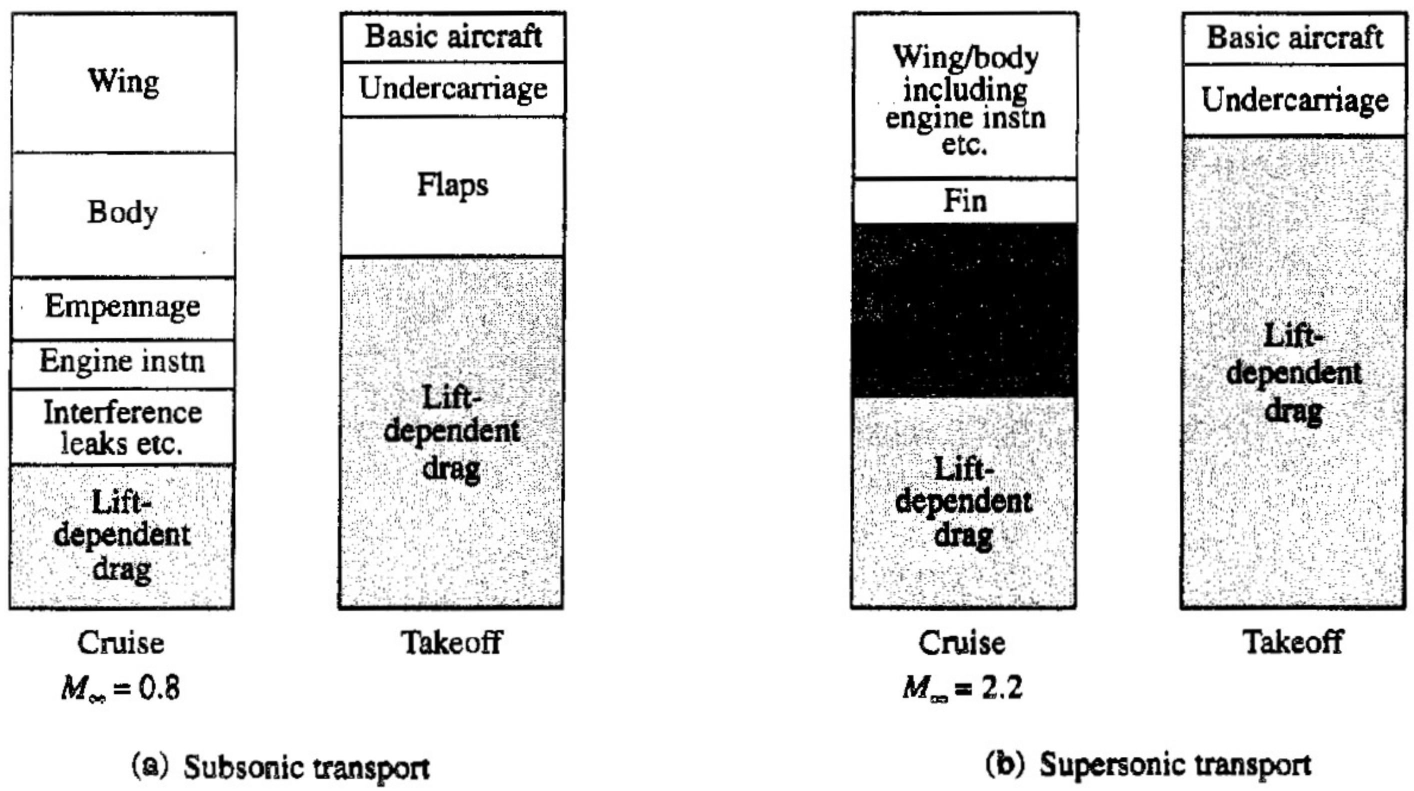
also called wave drag due to lift
Note: $C_{d,w} = 0$ at $\alpha = 0$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Drag build-up

We now focus on the drag of the complete aircraft, which is presented in the form of a drag polar

Drag Breakdown



Drag Breakdown

- For subsonic:
  - Most drag at cruise is parasite drag
  - Most drag at takeoff is lift-dependent drag
- For supersonic:
  - Most drag at cruise is wave drag (both kinds)
  - Most drag at takeoff is lift-dependent drag

- About 2/3 of total parasite drag in cruise is due to skin friction, the rest is interference and form drag

- Recall friction drag is a function of total *wetted* surface area, so to estimate friction drag, we should get an estimate of wetted area

- Wetted surface area, S_{wet}, is usually 2 to 8 times the reference planform area of the wing, S

Wetted Area Estimation

- The zero-lift parasite drag, D_0, can be written

$$D_0 = q_\infty S_{wet} C_{fe}$$

- The zero-lift drag coefficient, C_{D_0}, is defined as

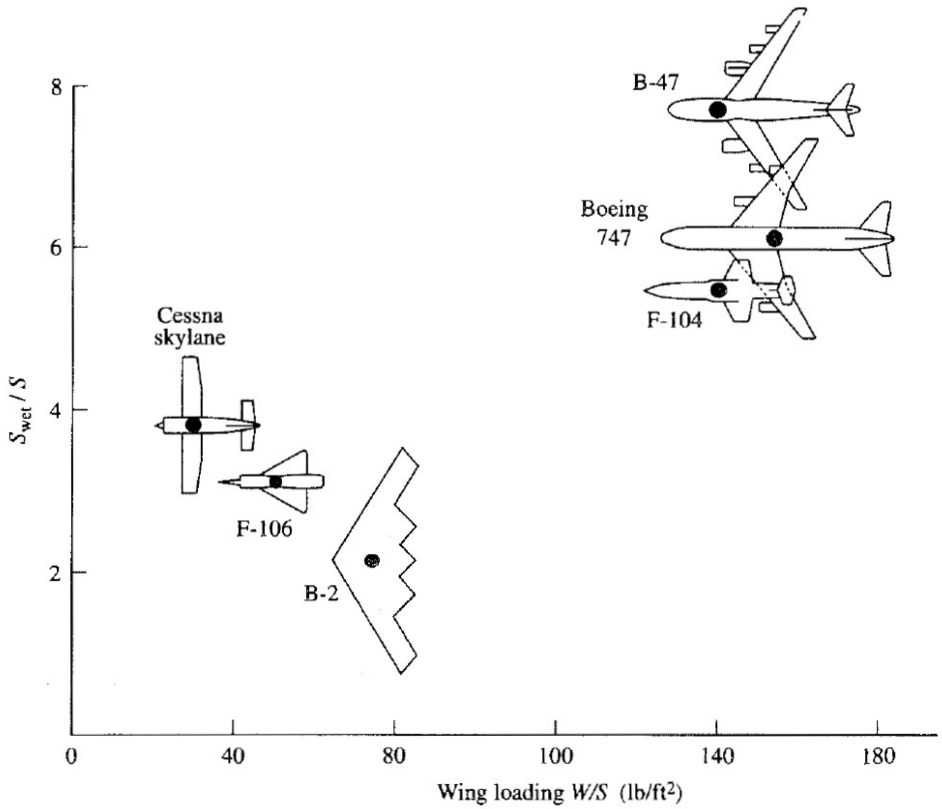
$$C_{D_0} = \frac{D_0}{q_\infty S}$$

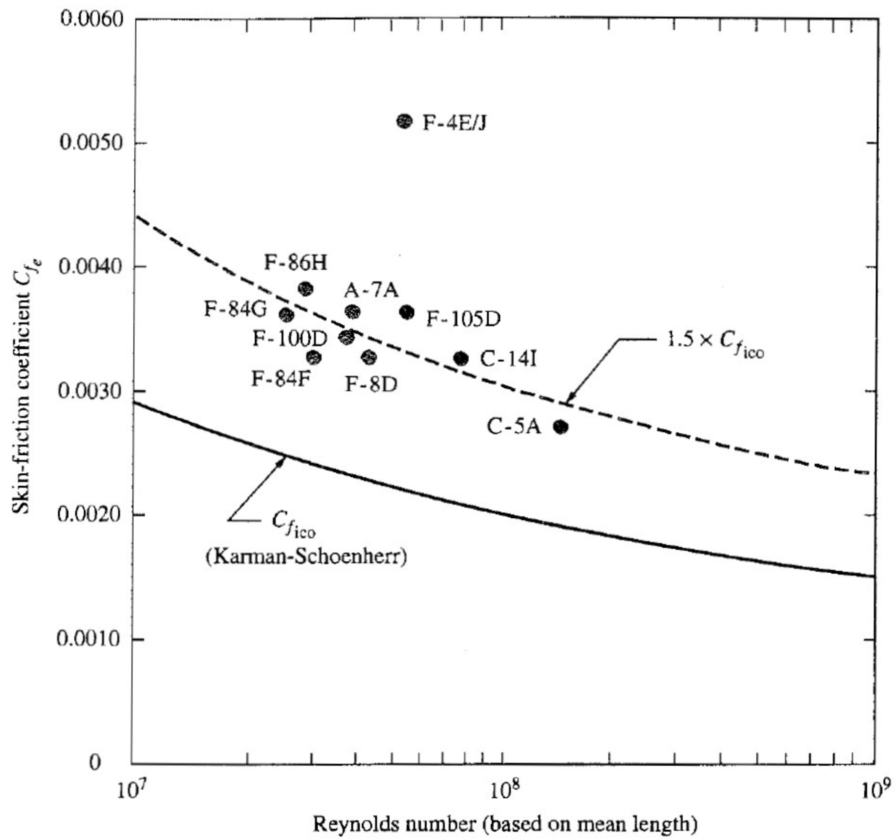
- Substituting, we get

$$\boxed{C_{D_0} = \frac{q_\infty S_{wet} C_{fe}}{q_\infty S} = \frac{S_{wet}}{S} C_{fe}}$$

Wetted Area Estimation

Now use the figures below to estimate zero lift drag





Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

The Drag Polar

For every aerodynamic body, there exists a relationship between C_L and C_D. This relationship can be expressed as either an equation or a graph. *Both* are called "drag polar".

Virtually all information necessary for a performance analysis is contained in the drag polar.

Recall

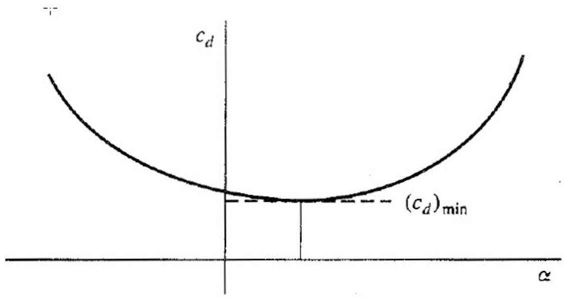
Total Drag = parasite drag + wave drag + induced drag

$$C_D = C_{D,e} + C_{D,w} + \frac{C_L^2}{\pi e AR}$$

Parasite Drag

- First, look at $C_{D,e}$

$$\underbrace{C_{D,e}}_{} = \underbrace{C_{D,e,0}}_{\substack{\text{parasite drag}\\ \text{at zero lift}}} + \underbrace{\Delta C_{D,e}}_{\substack{\text{increment in}\\ \text{parasite drag}\\ \text{due to lift}}}$$



- Now realize

  $\Delta C_{D,e}$ is a function of α and

  c_d varies as c_l^2

- This implies that $\Delta C_{D,e}$ varies w/ C_L^2. So:

$$\boxed{C_{D,e} = C_{D,e,0} + \Delta C_{D,e} = C_{D,e,0} + k_1 C_L^2}$$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Wave Drag

- Similar arguments can be made for wave drag, $C_{D,w}$

$$C_{D,w} = C_{D,w,0} + \Delta C_{D,w}$$

- From our supersonic discussion, we can combine equations to get

$$c_{d,w} = \frac{c_l^2 \sqrt{M_\infty^2 - 1}}{4}$$

so $C_{D,w}$ does vary with C_L^2

- So,

$$\boxed{C_{D,w} = C_{D,w,0} + \Delta C_{D,w} = C_{D,w,0} + k_2 C_L^2}$$

Drag Polar Calculations

- Putting it all together,

$$C_D = C_{D,e} + C_{D,w} + \frac{C_L^2}{\pi e AR}$$

$$C_D = C_{D,e,0} + C_{D,w,0} + k_1 C_L^2 + k_2 C_L^2 + \frac{C_L^2}{\pi e AR}$$

- Define $\mathrm{k}_3 = \frac{1}{\pi e AR}$, then:

$$C_D = \underbrace{C_{D,e,0} + C_{D,w,0}}_{\substack{\text{defines } C_{D_0} \\ \text{Zero lift drag coefficient}}} + \underbrace{(k_1 + k_2 + k_3)}_{\text{defined as K}} C_L^2$$

- So, complete drag polar can be written as $\boxed{C_D = C_{D,0} + K C_L^2}$

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Drag Polar Calculations

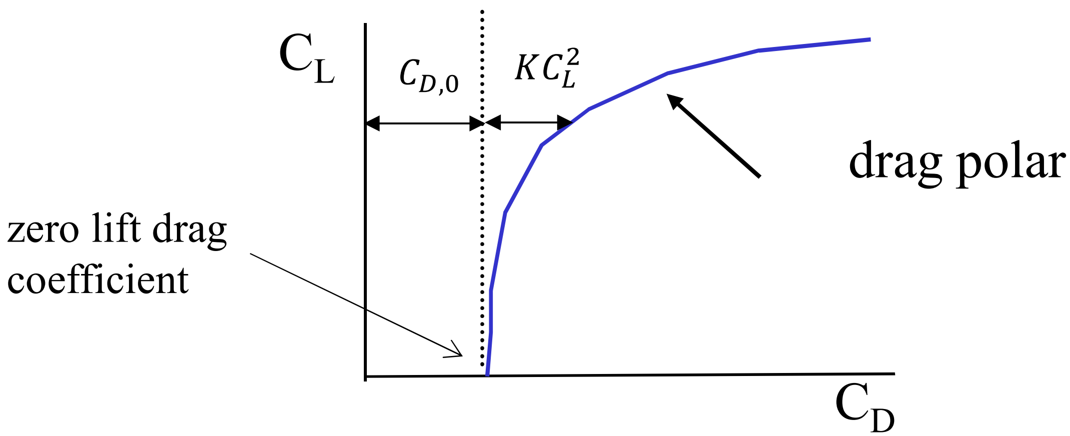
$$C_D = C_{D,0} + KC_L^2$$

C_D total drag coefficient

$C_{D,0}$ zero lift parasite drag coefficient
or "zero lift drag coefficient"

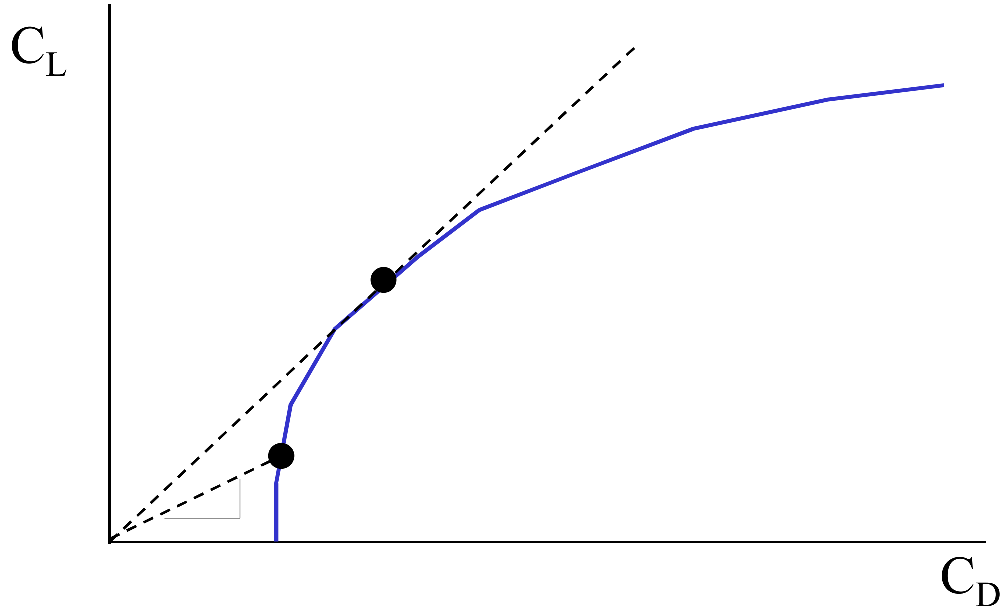
KC_L^2 drag due to lift

- Equation is valid for both subsonic and supersonic
- At supersonic, $C_{D,0}$ contains wave drag at zero lift, friction drag, form drag. The value for wave drag due to lift is part of K



Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

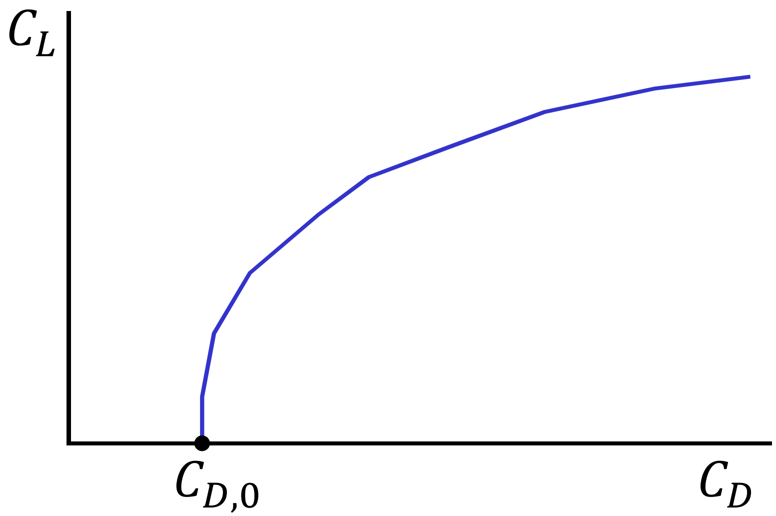
Graphic Drag Polar

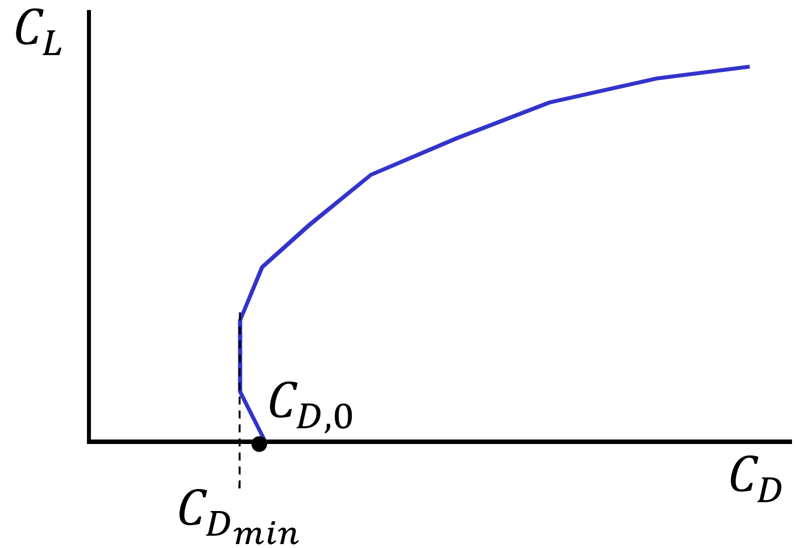


- The slope of the line from the origin to any point on the drag polar is the L/D at that point. It will have a corresponding α
- A line drawn from the origin tangent to the drag polar identifies the (L/D)max of the aircraft
  - Sometimes called the "design point:
  - Corresponding C_L is called the "design lift coefficient"
- Note: $(L/D)_{max}$ does not occur at a point of minimum drag

Graphic Drag Polar

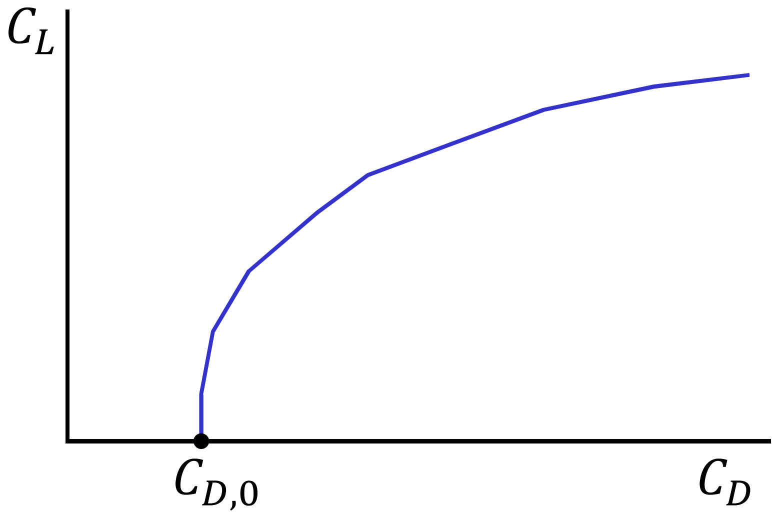
- Note: for most real aircraft, minimum drag point is NOT the same as zero lift point, although we have been drawing it that way

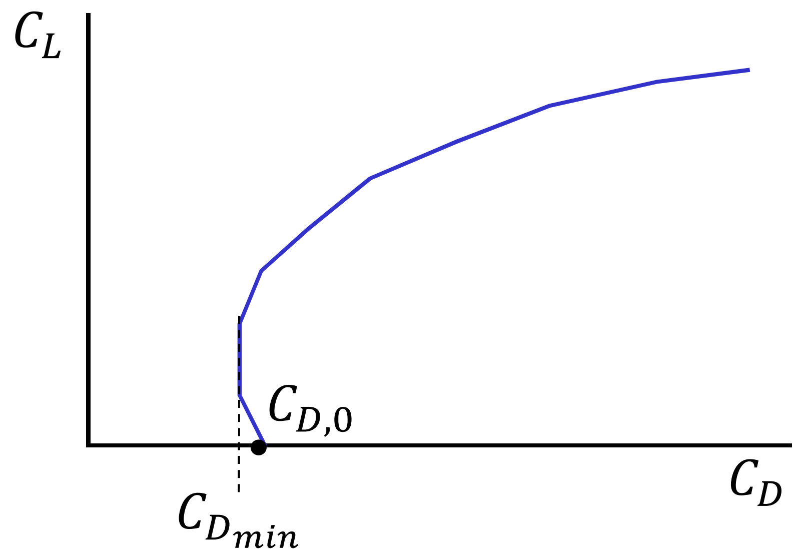




- But for airplanes with wings of moderate camber, the difference between $C_{D,0}$ and $C_{D_{min}}$ are small and can be ignored

Information from Polar

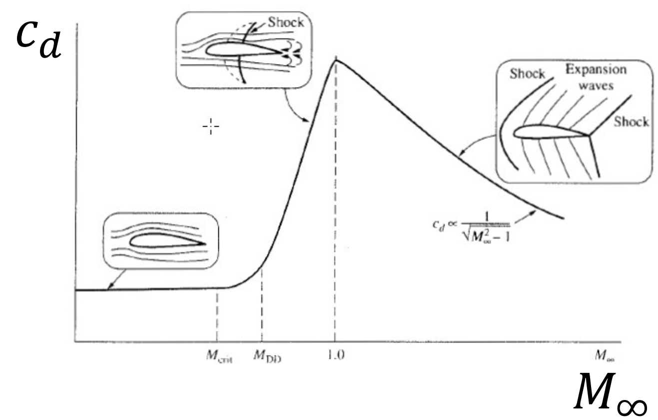


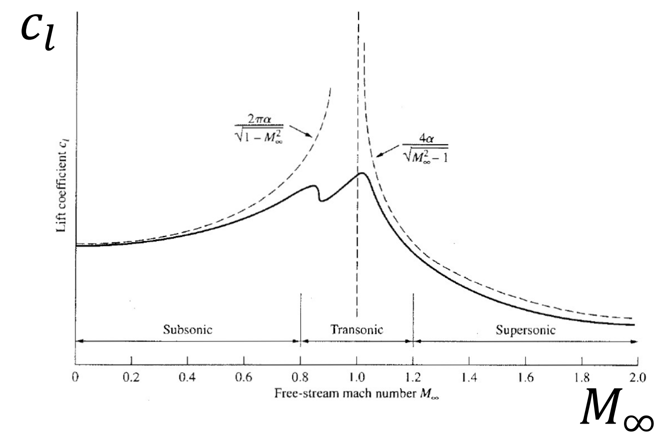


- Min drag coefficient at zero lift
  - Symmetric fuselage
  - Wing with symmetric airfoil
  - Zero incidence angle of attack
- Zero lift drag coefficient not same as minimum lift
  - Some effective camber
  - Zero lift drag coefficient obtained at some α not equal to zero

General Drag Polar Notes

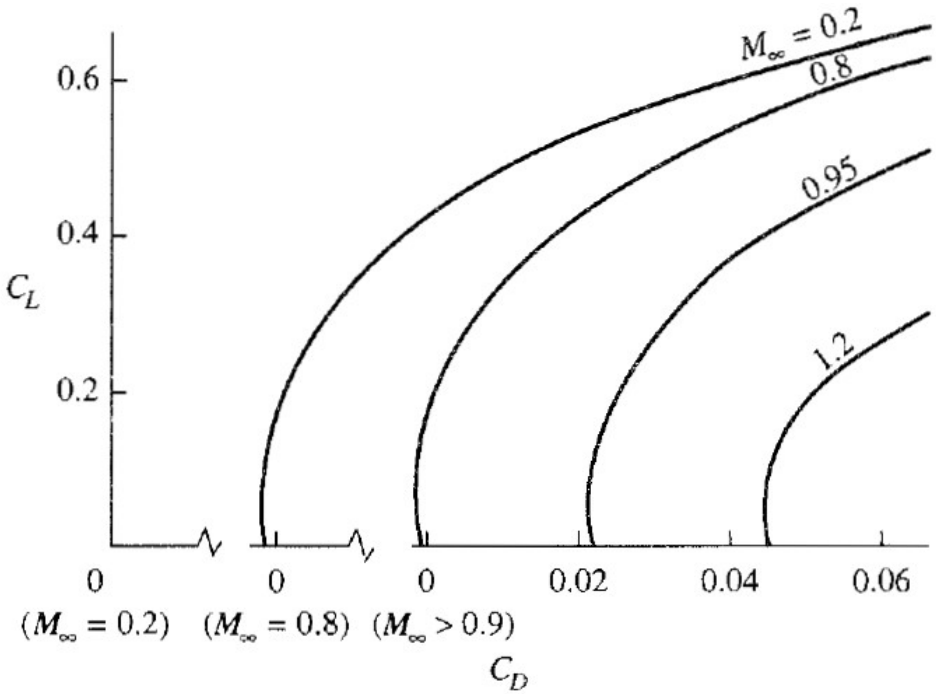
- The same aircraft will have different drag polars for different Mach numbers
  - At low M, this can be effectively ignored
  - At high M, differences are significant
- Subsonic:
  - As M_∞ increases, curve shifts to the right
  - $C_{D_{min}}$ increases due to drag divergence effects
- Supersonic:
  - As M_∞ increases, curve shifts to the left and "squashes"
  - $C_{D_{min}}$ decreases
  - C_L decreases



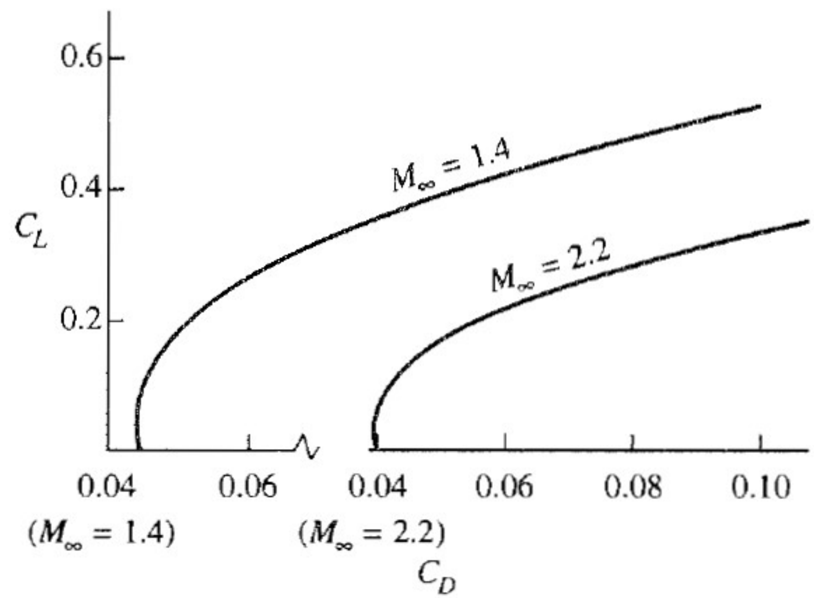


Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

General Drag Polar Notes



(a) Subsonic and Transonic



(b) Supersonic

Dr. Dimitri N. Mavris, Director ASDL
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
dimitri.mavris@ae.gatech.edu

Some Key References

S.F. Hoerner, *Fluid Dynamic Drag*, Hoerner Fluid Dynamics, Brick Town, NJ 1965

S.F. Hoerner and H.V. Borst, *Fluid Dynamic Lift*, Hoerner Fluid Dynamics, Brick Town, NJ 1975

Ira H. Abbott and Albert E. Von Doenhoff, *Theory of Wing Sections*, McGraw-Hill, New York, 1991.

John D. Anderson, Jr. *Introduction to Flight*, 3rd Edition, McGraw-Hill, New York, 1989

John D. Anderson, Jr. *Fundamentals of Aerodynamics*, 2nd Edition, McGraw-Hill, New York, 1991

Joseph Katz and Allen Plotkin, *Low-Speed Aerodynamics*, McGraw-Hill, New York, 1991

Deitrich Kuchemann, *The Aerodynamic Design of Aircraft*, Pergamon Press, Oxford, 1978

Daniel P. Raymer, *Aircraft Design: A Conceptual Approach*, 2nd Edition, AIAA Education Series, American Institute of Aeronautics and Astronautics, Washinton, 1992.