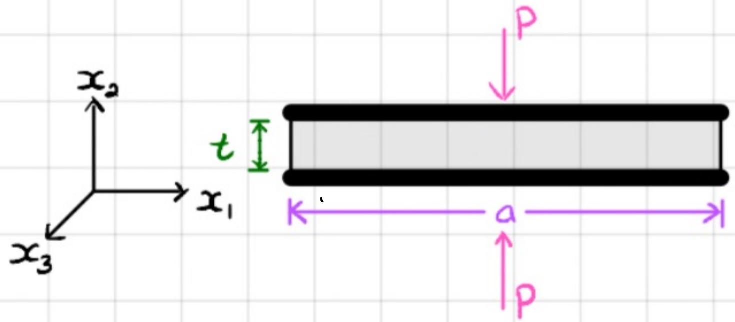
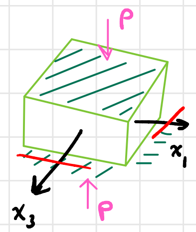
Final Exam - Problem 2 (second part)

Stress-strain relations: Test 2



Known: $\begin{cases} \sigma_{22} = \dfrac{P}{A} & \text{b/c applied load} \\ \varepsilon_{11} = \varepsilon_{33} = 0 & \text{b/c laterally confined} \end{cases}$



Using: $\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}$

$$\sigma_{11} = \lambda(\cancel{\varepsilon_{11}} + \varepsilon_{22} + \cancel{\varepsilon_{33}}) + 2\mu \cancel{\varepsilon_{11}}$$
$$\sigma_{22} = \lambda(\cancel{\varepsilon_{11}} + \varepsilon_{22} + \cancel{\varepsilon_{33}}) + 2\mu \varepsilon_{22}$$
$$\sigma_{33} = \lambda(\cancel{\varepsilon_{11}} + \varepsilon_{22} + \cancel{\varepsilon_{33}}) + 2\mu \cancel{\varepsilon_{33}}$$

Therefore:

$$\sigma_{11} = \lambda \varepsilon_{22}$$
$$\sigma_{22} = \lambda \varepsilon_{22} + 2\mu \varepsilon_{22} = \varepsilon_{22}(\lambda + 2\mu)$$
$$\sigma_{33} = \lambda \varepsilon_{22}$$

$$\therefore \; \sigma_{11} = \sigma_{33} = \lambda \varepsilon_{22}$$
$$\sigma_{22} = \underline{\varepsilon_{22}}(\lambda + 2\mu)$$

Stress-strain response for Test 2

But! Can re-write in terms of ν and E using ε_{ij}.

To get ε_{22}:

Using: $$\varepsilon_{ij} = \frac{-\nu}{E}\sigma_{kk}\delta_{ij} + \frac{1+\nu}{E}\sigma_{ij}$$

$$\varepsilon_{11} = 0 = \frac{-\nu}{E}(\sigma_{11}+\sigma_{22}+\sigma_{33}) + \frac{1+\nu}{E}\sigma_{11} \quad [1]$$

$$\varepsilon_{22} = \frac{-\nu}{E}(\sigma_{11}+\sigma_{22}+\sigma_{33}) + \frac{1+\nu}{E}\sigma_{22} \quad [2]$$

$$\varepsilon_{33} = 0 = \frac{-\nu}{E}(\sigma_{11}+\sigma_{22}+\sigma_{33}) + \frac{1+\nu}{E}\sigma_{33} \quad [3]$$

[3] - [1]: $$0 = \frac{1+\nu}{E}(\sigma_{33}-\sigma_{11})$$

$$\Rightarrow \sigma_{33} = \sigma_{11}$$

Need to solve for σ_{22} $\therefore$ using [1] and $\sigma_{33} = \sigma_{11}$:

$$0 = \frac{-\nu}{E}(\sigma_{22} + 2\sigma_{11}) + \frac{1+\nu}{E}\sigma_{11}$$

$$0 = \frac{-\nu}{E}\sigma_{22} - \frac{\nu}{E}\cdot 2\sigma_{11} + \frac{1+\nu}{E}\sigma_{11}$$

$$\frac{\nu}{E}\sigma_{22} = \sigma_{11}\left(\frac{1+\nu}{E} - \frac{2\nu}{E}\right)$$

$$\sigma_{22} = \frac{\sigma_{11}}{\nu}(1+\nu-2\nu)$$

$$\sigma_{22} = \frac{1-\nu}{\nu}\sigma_{11}$$

$$\Rightarrow \sigma_{11} = \frac{\nu}{1-\nu}\sigma_{22}$$

Plugging into ε_{22}:

$$\varepsilon_{22} = \frac{-\nu}{E}\left(\frac{\nu}{1-\nu}\sigma_{22} + \sigma_{22} + \frac{\nu}{1-\nu}\sigma_{22}\right) + \frac{1+\nu}{E}\sigma_{22}$$

$$= \frac{-\nu^2}{E}\cdot\frac{\sigma_{22}}{(1-\nu)} - \frac{\nu}{E}\sigma_{22} - \frac{\nu^2}{E}\cdot\frac{\sigma_{22}}{(1-\nu)} + \frac{1+\nu}{E}\sigma_{22}$$

$$= \frac{\sigma_{22}}{E}\left(\frac{-\nu^2}{(1-\nu)} - \nu - \frac{\nu^2}{(1-\nu)} + 1+\nu\right)$$

$$= \frac{\sigma_{22}}{E}\left(\frac{-\nu^2}{(1-\nu)} - \frac{(1-\nu)\nu}{(1-\nu)} - \frac{\nu^2}{(1-\nu)} + \frac{(1-\nu)(1+\nu)}{(1-\nu)}\right)$$

$$\begin{array}{c|cc} & 1 & -\nu \\ \hline 1 & 1 & -\nu \\ +\nu & \nu & -\nu^2 \end{array}$$

$$= \frac{\sigma_{22}}{E(1-\nu)}\left[-\nu^2 - \nu + \cancel{\nu^2} \cancel{-\nu^2} + 1 - \nu^2\right]$$

$$= \sigma_{22}\left[\frac{-2\nu^2 - \nu + 1}{E(1-\nu)}\right]$$

$$\begin{array}{ccc} & -2 & \\ -2 & \times & 1 \\ & -1 & \end{array}$$

$$\Rightarrow \varepsilon_{22} = \frac{(1-2\nu)(1+\nu)}{E(1-\nu)}\sigma_{22}$$

Therefore:

$$\boxed{\begin{aligned} \sigma_{22} &= \frac{E(1-\nu)}{(1-2\nu)(1+\nu)}\varepsilon_{22} \\ \sigma_{11} &= \sigma_{33} = \lambda\varepsilon_{22} \end{aligned}}$$

Stress - strain response for test 2.