AE 6114

8/18

2 parts

1) continuum mechanics (focused on solids)
2) Elasticity /BC problem / Energy / FEM

Tests: closed books but open notes

- notes, HW, equation sheet (1 pg, both sides)

Sept 24th, Oct 29th

→ Cumulative

* HW format

Math Preliminaries — Tensor Math / Continuum Mechanics

Tensors

- Abstract entities that behave following certain transformation rules
- They can be defined in terms of those rules
  - ↳ kind of like generalization of vector into higher dimensions

For more details:
- Aris book (fluid mechanics)
- Sokolnikoff book

Tensor notation

Rank 1: column vector
Rank 2: Matrix

- Direct:

Rank 2

$$\underline{t} = \underline{\underline{\sigma}} \cdot \underline{n}$$

Rank 1

- Matrix: $[\underline{t}] = [\underline{\underline{\sigma}}][\underline{n}]$ → lose directional information → need to define coordinate sys

- Indicial notation: $t_i = \sigma_{ij} n_j$

Note: In direct & matrix notation, the rank of the tensor is denoted by the # of underscores.
In indicial notation, denoted by the # of indices.

The summation convention

Let's consider the following sum:

$$s = a_1 x_1 + a_2 x_2 + \dots + a_n x_n$$ (dot product)

(a, x vectors)

we can write this as a sum

$$s = \sum_{i=1}^{N} a_i x_i = \sum_{j=1}^{N} a_j x_j = \sum_{k=1}^{N} a_k x_k$$

we can see that the choice of letter for the indices is irrelevant

↳ Indices w/ this property are called "dummy" indices

we can write the same using "summation convention"

$$s = a_i x_i = a_j x_j = a_k x_k$$

(this is missing N, but usually known by context)

Examples : $a_i x_i = a_1 x_1 + a_2 x_2 + a_3 x_3$

$(n = 3)$

$$a_i a_i = a_1 a_1 + a_2 a_2 + a_3 a_3 + a_4 a_4$$

$$= a_1^2 + a_2^2 + a_3^2 + a_4^2$$

Note : $a_i a_i \neq (a_i)^2$

$\uparrow$ No sum.

Note: a product having more than 2 of the same dummy indices is meaning less

(Indicial notation assumes summation when there are 2 repeated indices)

$\hookrightarrow$ "dummy indices" = 2 indices sum assumed unless noted otherwise.

ex: $t_i = \delta_{ij} n_j$ $\rightarrow$ dummy indices on j.

$\rightarrow$ sum on j

(i is free index)

An index that appears only once in each product term in an equation is referred to as a "free index"

$$\overbrace{A_{\underset{\uparrow}{i}\underset{\uparrow}{j}}\; x_{\underset{\uparrow}{j}}}^{\text{product term}} = \overbrace{b_{\underset{\uparrow}{i}}}^{\text{product term}}$$

free dummy free

Expanding:

Assume $n = 3$

$$\sum_{j=1}^{3} A_{ij}\, x_j = b_i \Rightarrow A_{i1}x_1 + A_{i2}x_2 + A_{i3}x_3 = b_i$$

$$A_{ij}x_j = b_j \Rightarrow \begin{cases} A_{11}x_1 + A_{12}x_2 + A_{13}x_3 = b_1 \\ A_{21}x_1 + A_{22}x_2 + A_{23}x_3 = b_2 \\ A_{31}x_1 + A_{32}x_2 + A_{33}x_3 = b_3 \end{cases}$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Indicial notation is compact!

Note: All terms in an expression must have the same free indices

↳ they have to appear on every term

For example:

$A_{ij} x_j = b_k$ is meaningless