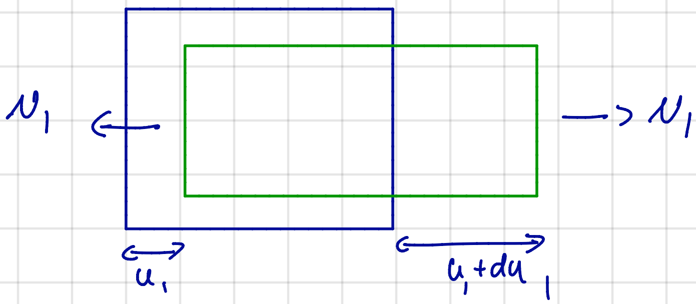
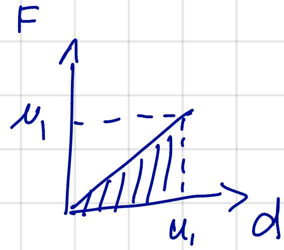
Strain Energy

Uniaxial Loading





$$dW_I = \left[-\frac{1}{2} N_1 u_1 + \frac{1}{2} N_1 \left(u_1 + \frac{du_1}{dx_1} dx_1\right)\right]$$

$$dW_I = \frac{1}{2} N_1 \underbrace{\frac{du_1}{dx_1}}_{\bar{\epsilon}_1} dx_1 = \frac{1}{2} N_1 \cdot \bar{\epsilon}_1 dx_1$$

$$N_1 = S \bar{\epsilon}_1$$

$$a(\bar{\epsilon}_1) = \frac{1}{2} S \bar{\epsilon}_1^2$$ - Strain energy density functional

$$a'(N_1) = \frac{1}{2} \frac{N_1^2}{S}$$ - Stress energy density functional

Total Energies

$$A(\bar{\epsilon}_1) = \int_0^L a(\bar{\epsilon}_1) dx_1 = \int_0^L \frac{1}{2} S \bar{\epsilon}_1^2 dx_1$$

$$A'(N_1) = \int_0^L a'(N_1) dx_1 = \int_0^L \frac{1}{2} \frac{N_1^2}{S} dx_1$$

For linear elastic materials $A(\bar{\epsilon}_1) = A'(N_1)$

Aside: Virtual

$$\delta W_I = \int_0^{\bar{\epsilon}_1} N_1 \cdot \delta \bar{\epsilon}_1 = N_1 \int_0^{\bar{\epsilon}_1} \delta \bar{\epsilon}_1 = N_1 \bar{\epsilon}_1$$

Real

$$dW_I = \int_0^{\bar{\epsilon}_1} N_1 \cdot d\bar{\epsilon}_1 = \int_0^{\bar{\epsilon}_1} S \bar{\epsilon}_1 \cdot d\bar{\epsilon}_1 = \frac{S}{2} \bar{\epsilon}_1^2$$

Varies w/ $\bar{\epsilon}_1$

$$= \frac{1}{2} N_1 \bar{\epsilon}_1$$

Beams in Torsion

$$a(\kappa_1) = \frac{1}{2} H_{11} \kappa_1^2 \qquad A(\kappa_1) = \frac{1}{2} \int_0^L H_{11} \kappa_1^2 \, dx_1$$

$$a'(M_1) = \frac{1}{2} \frac{M_1^2}{H_{11}} \qquad A'(M_1) = \frac{1}{2} \int_0^L \frac{M_1^2}{H_{11}} \, dx_1$$

3D Beam

$$W_I = \frac{1}{2} \int_0^L \left(N_1 \bar{\epsilon}_1 + M_2 \kappa_2 + M_3 \kappa_3 \right) dx_1$$

$$A = \frac{1}{2} \int_0^L \left(S \bar{\epsilon}_1^2 + H_{22}^c \kappa_2^2 + H_{33}^c \kappa_3^2 - 2 H_{23}^c \kappa_2 \kappa_3 \right) dx_1$$

$$A' = \frac{1}{2} \int_0^L \left(\frac{N_1^2}{S} + \frac{H_{33}^c}{\Delta H} M_2^2 + \frac{H_{22}^c}{\Delta H} M_3^2 + 2 \frac{H_{23}^c}{\Delta H} M_2 M_3 \right) dx_1$$

General Solid

$$W = \frac{1}{2}\int_V \underline{\sigma}^T \underline{\epsilon}\, dV$$

$$a(\underline{\epsilon}) = \frac{1}{2}\underline{\epsilon}^T \underline{\underline{C}}\,\underline{\epsilon}$$

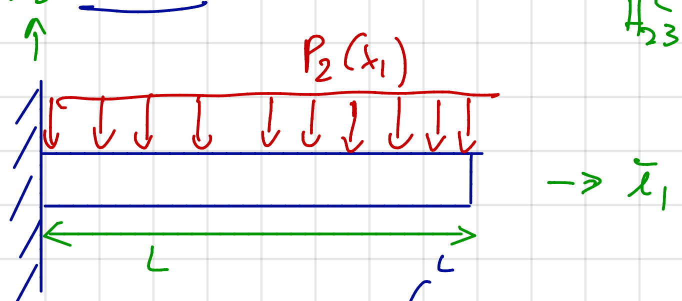
Stiffness Matrix

$$a'(\underline{\sigma}) = \frac{1}{2}\underline{\sigma}^T \underline{\underline{S}}\,\underline{\sigma}$$

Compliance Matrix

$\bar{\imath}_2$ Beam

$H_{23}^C = 0$



$$\underline{\Phi} = -\int_0^L P_2(x_1) \cdot u_2(x_1)\, dx_1$$

$$A = \frac{1}{2}\int_0^L H_{33}^C \left(\frac{d^2 u_2(x_1)}{dx_1^2}\right)^2 dx_1$$

$$\Pi = A + \underline{\Phi} = \frac{1}{2}\int_0^L H_{33}^C \left(\frac{d^2 u_2}{dx_1^2}\right)^2 dx_1 - \int_0^L P_2(x_1)\, u_2(x_1)\, dx_1$$

$\Pi = \hat{\Pi}(u_2(x_1))$ → Is a functional. "Function of a function".

* Infinite # of DOF!

* Need to know or approximate $u_2(x_1)$!