A Performance Comparison of Algebraic Multigrid Preconditioners on GPUs and MIC

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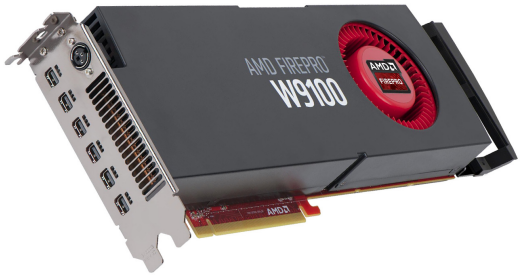
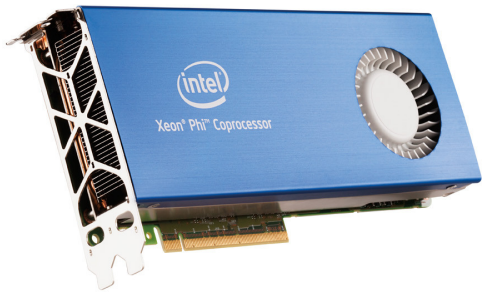
Introduction

Recent Many-Core Architectures

- High FLOP/Watt ratio
- High memory bandwidth
- Attached via PCI-Express



AMD FirePro W9100
320 GB/sec



INTEL Xeon Phi
320 (220?) GB/sec

NVIDIA Tesla K20
250 (208) GB/sec

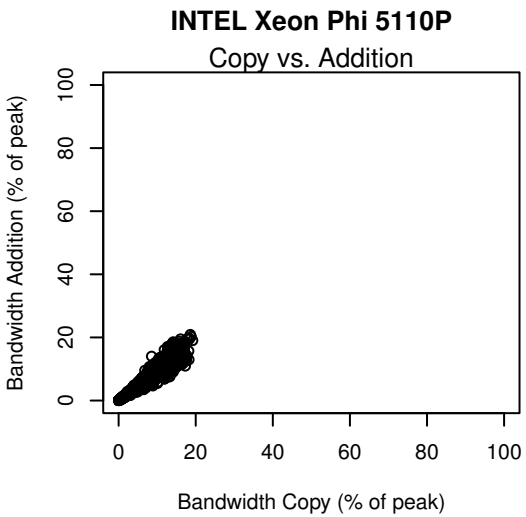
Programming Model

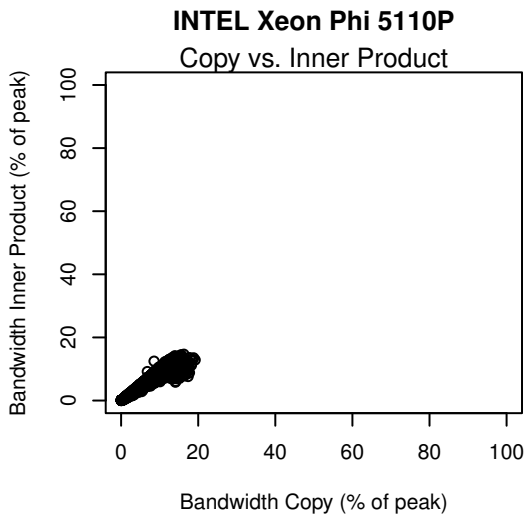
FirePro W9100: OpenCL

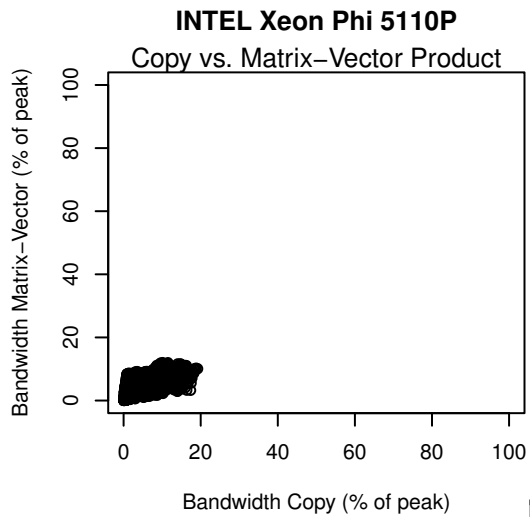
Tesla K20: CUDA, OpenCL

Xeon Phi: OpenCL, OpenMP

OpenCL for Everything?





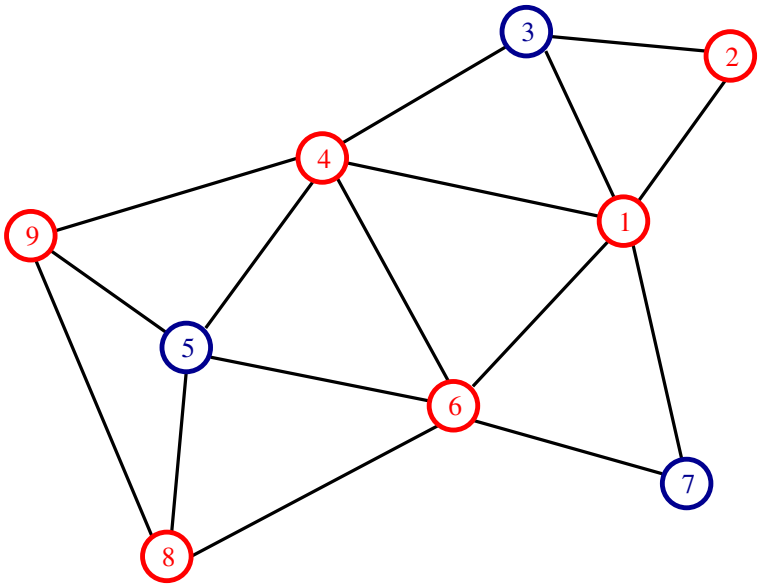


Ingredients of Algebraic Multigrid

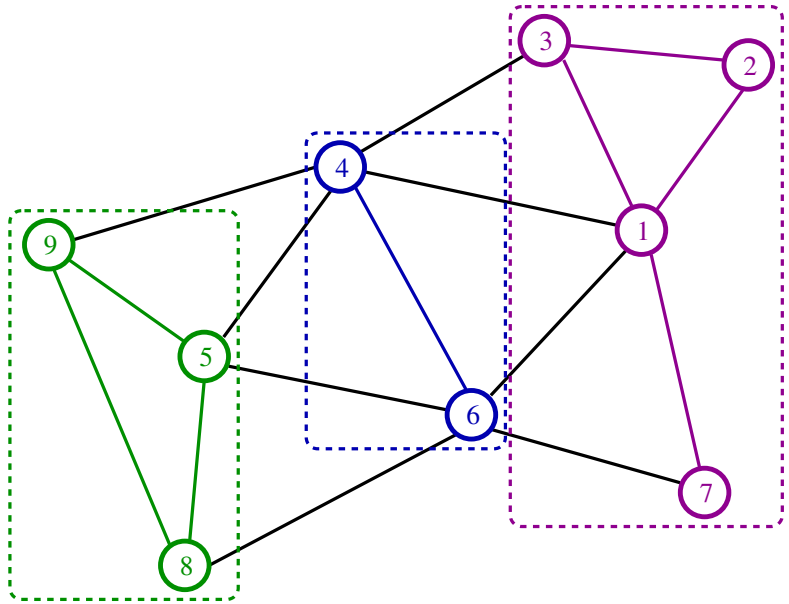
Smoother (Relaxation schemes, etc.)

Coarsening

Interpolation (Inter-grid transfer)



Classical coarsening



Aggregation coarsening

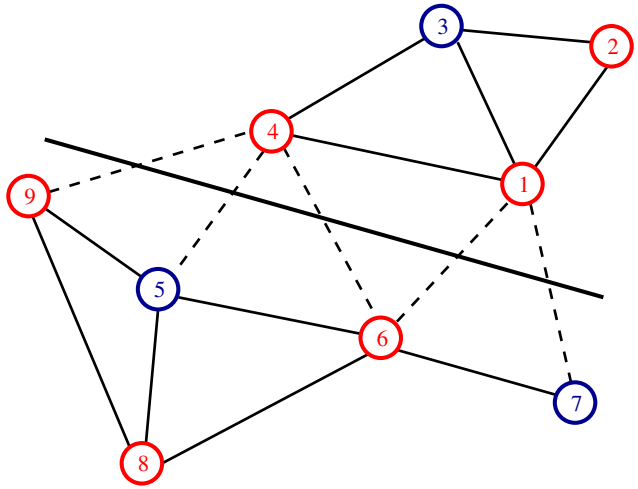
Multigrid Parallelization

Setup Phase

- Determination of coarse points in parallel by graph splitting
- Compute coarse operators $A^{k+1} = R^k A^k P^k$ (where $A^0 = A$)
- Datastructures: analyze and allocate
- Limited fine-grained parallelism

Cycle Phase

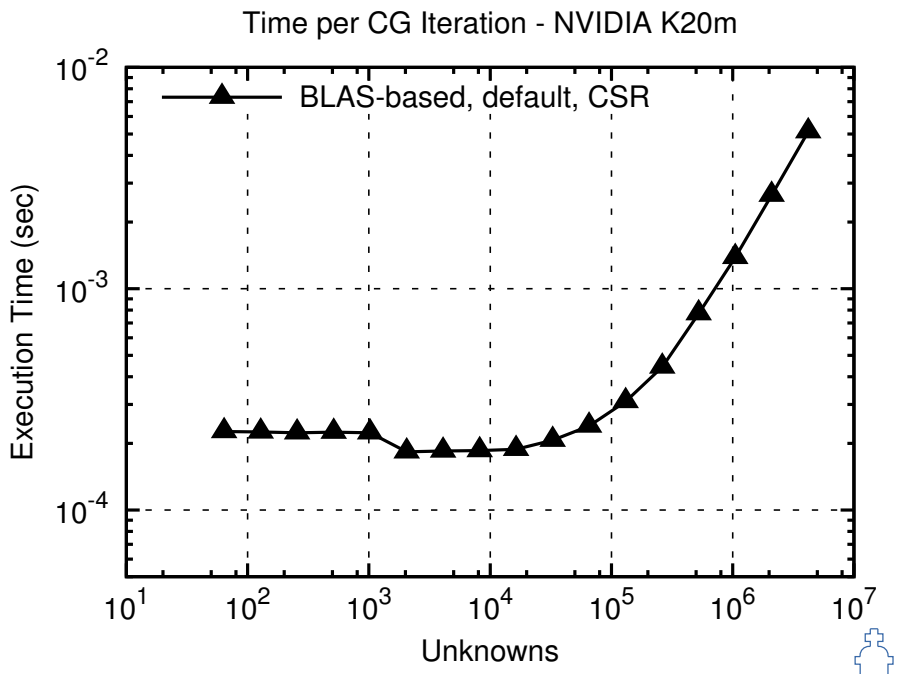
- Parallel Jacobi Smoother
- Restriction $R^k x^k$, prolongation $P^k x^{k+1}$
- Direct solution on coarsest level
- Static datastructures
- Enough fine-grained parallelism



Multigrid Parallelization

Why is AMG Hard?

- Several thread launches with little work
- Sequential stages
- PCI-Express latency
- Unstructured data access



Scope of Comparison

Coarsening Strategies

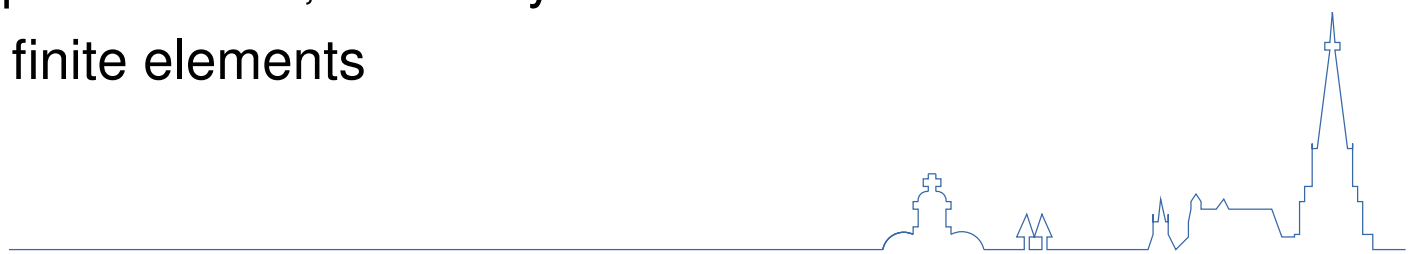
- Classical One-Pass Coarsening
- Aggregation-based Coarsening

Interpolation Strategies

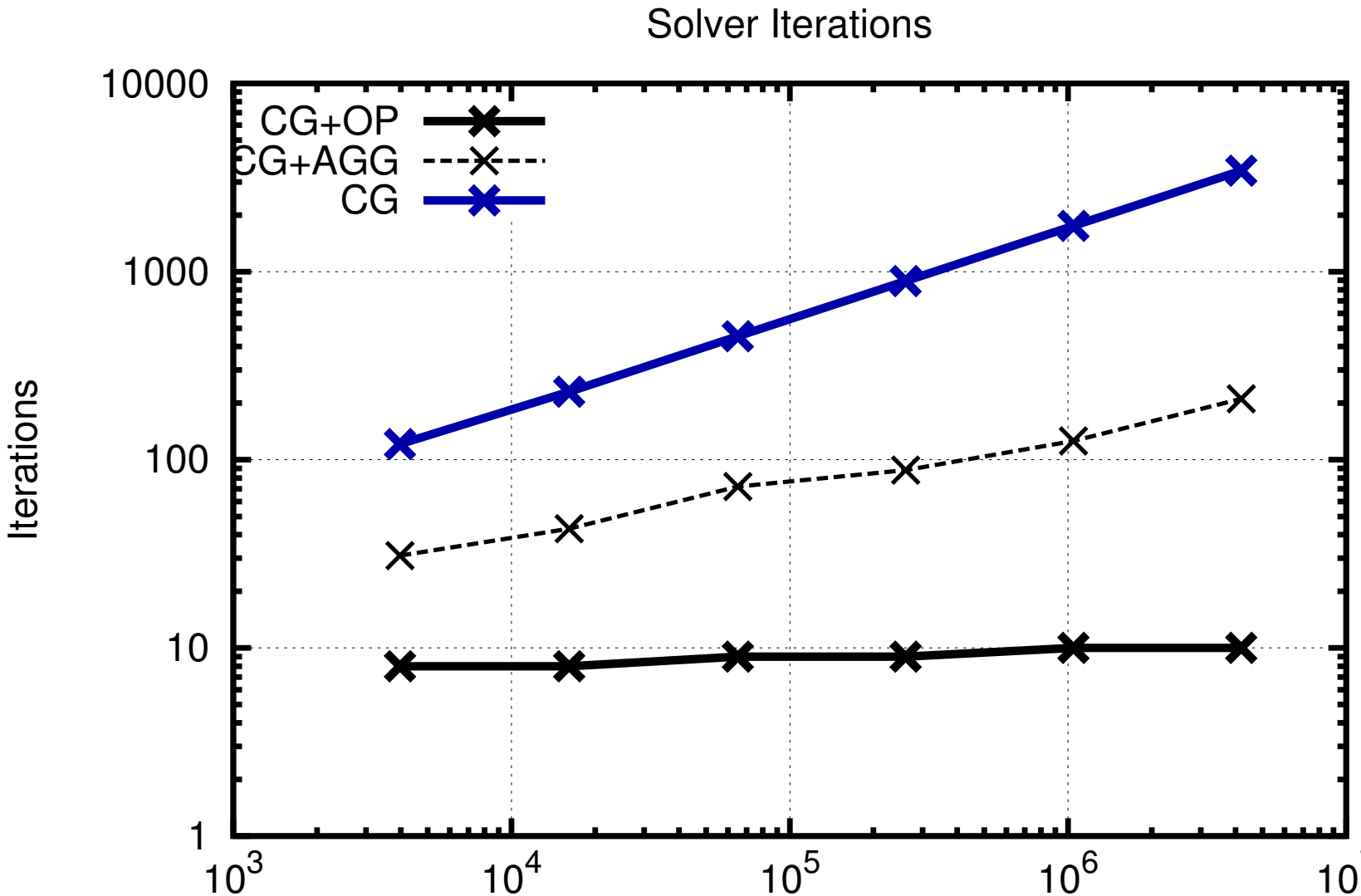
- Direct Interpolation
- Aggregation-based Interpolation

Systems

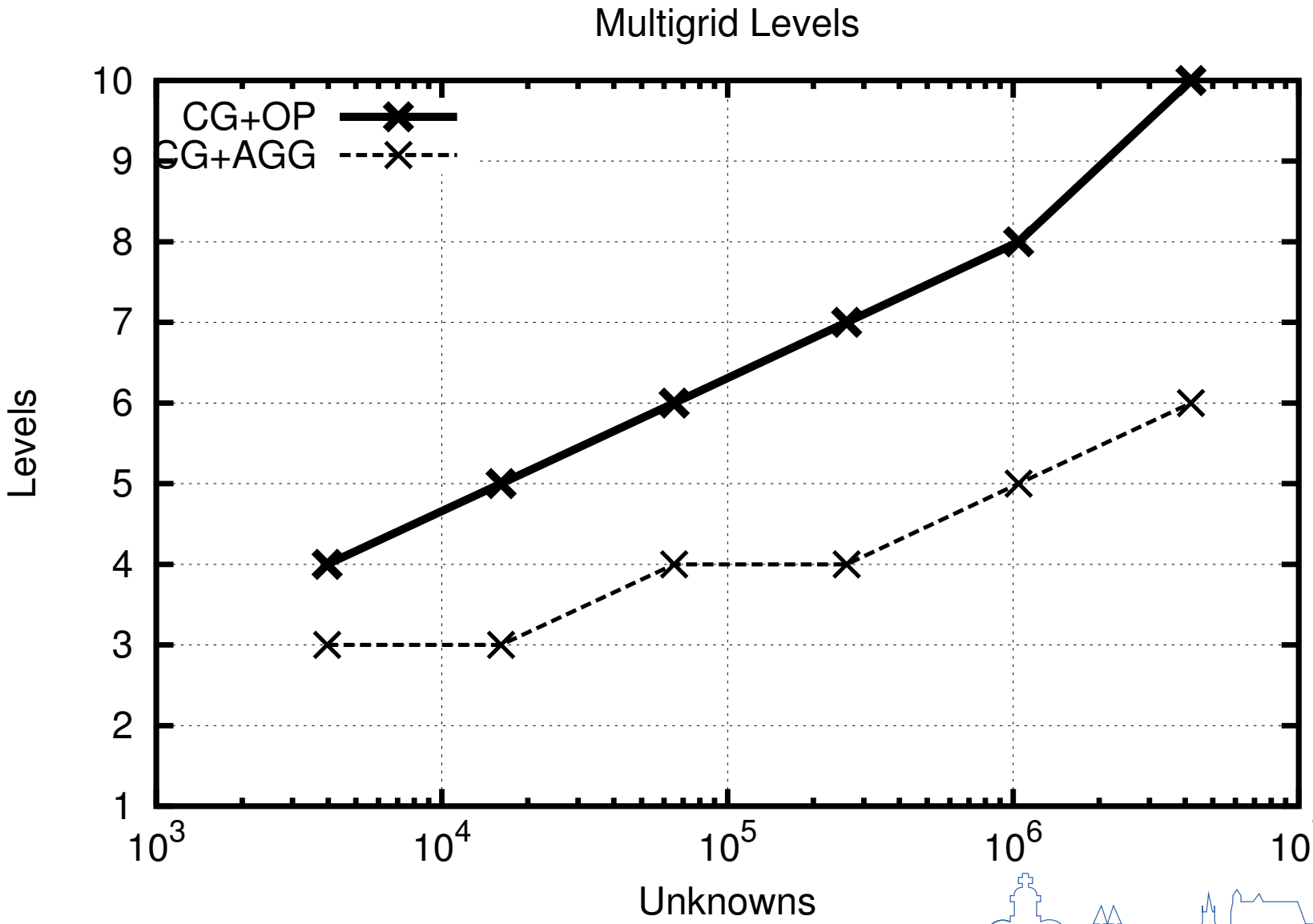
- Poisson equation in 2D, uniformly refined
- First-order finite elements



Benchmarks

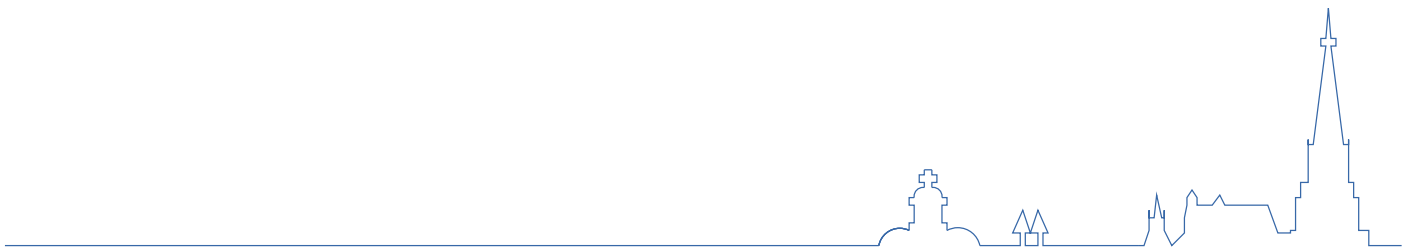

Solver Iterations
10000
1000
100
10
1
Iterations
CG+OP
CG+AGG
CG
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

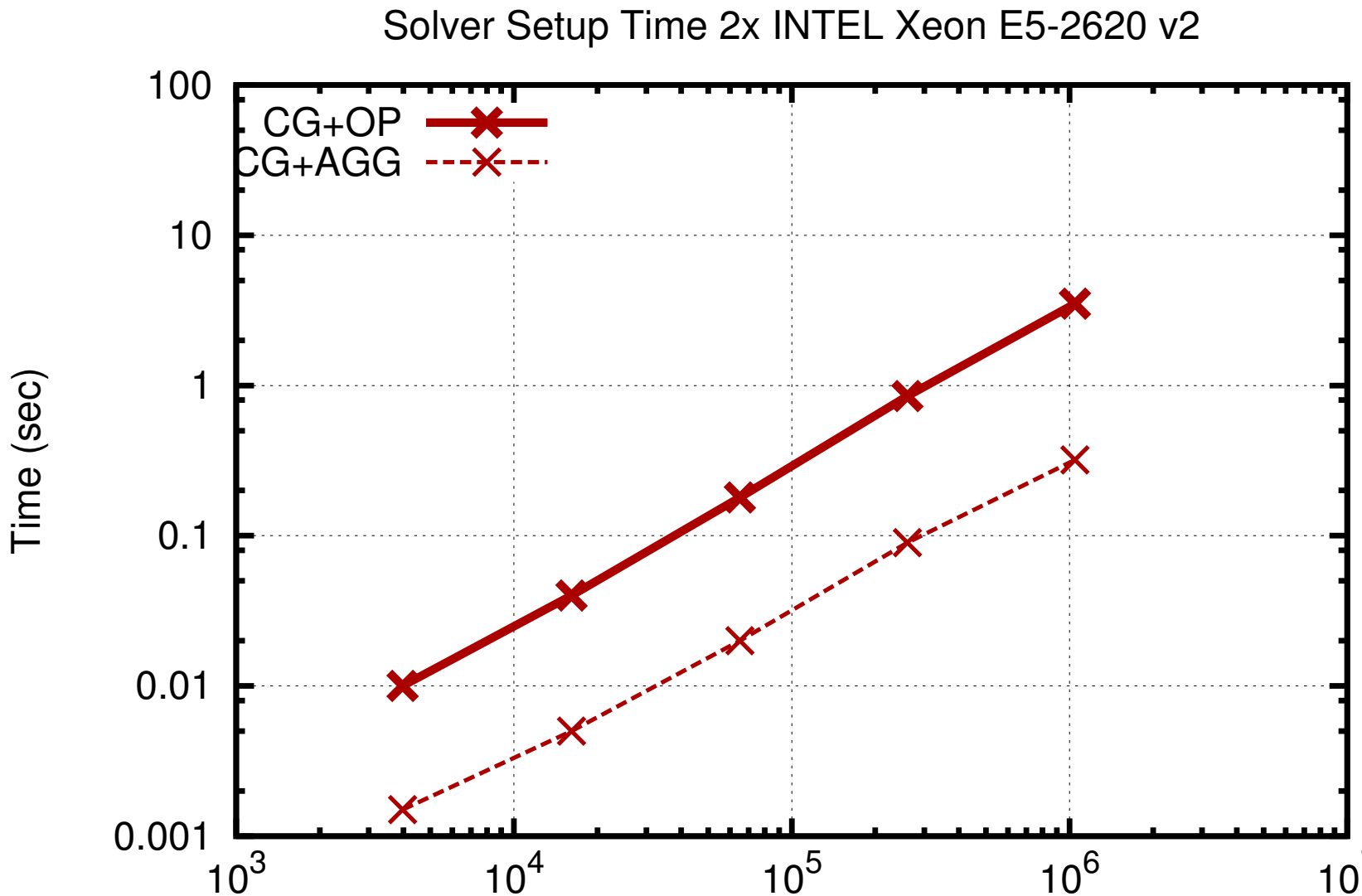

Multigrid Levels
CG+OP
CG+AGG
Levels
Unknowns
10
9
8
7
6
5
4
3
2
1
10^3
10^4
10^5
10^6
10^7

Benchmarks

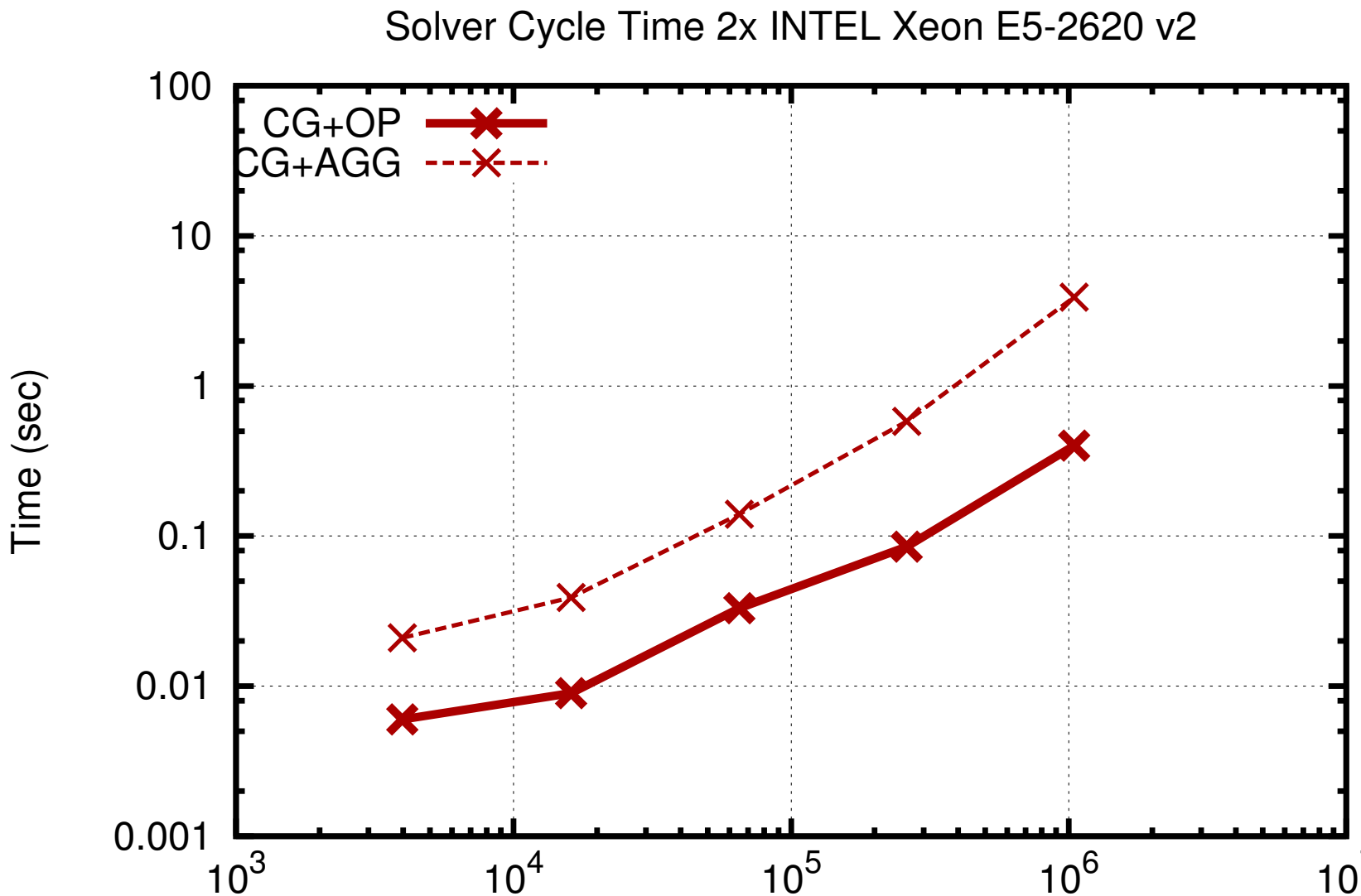
2x INTEL Xeon E5-2620 v2



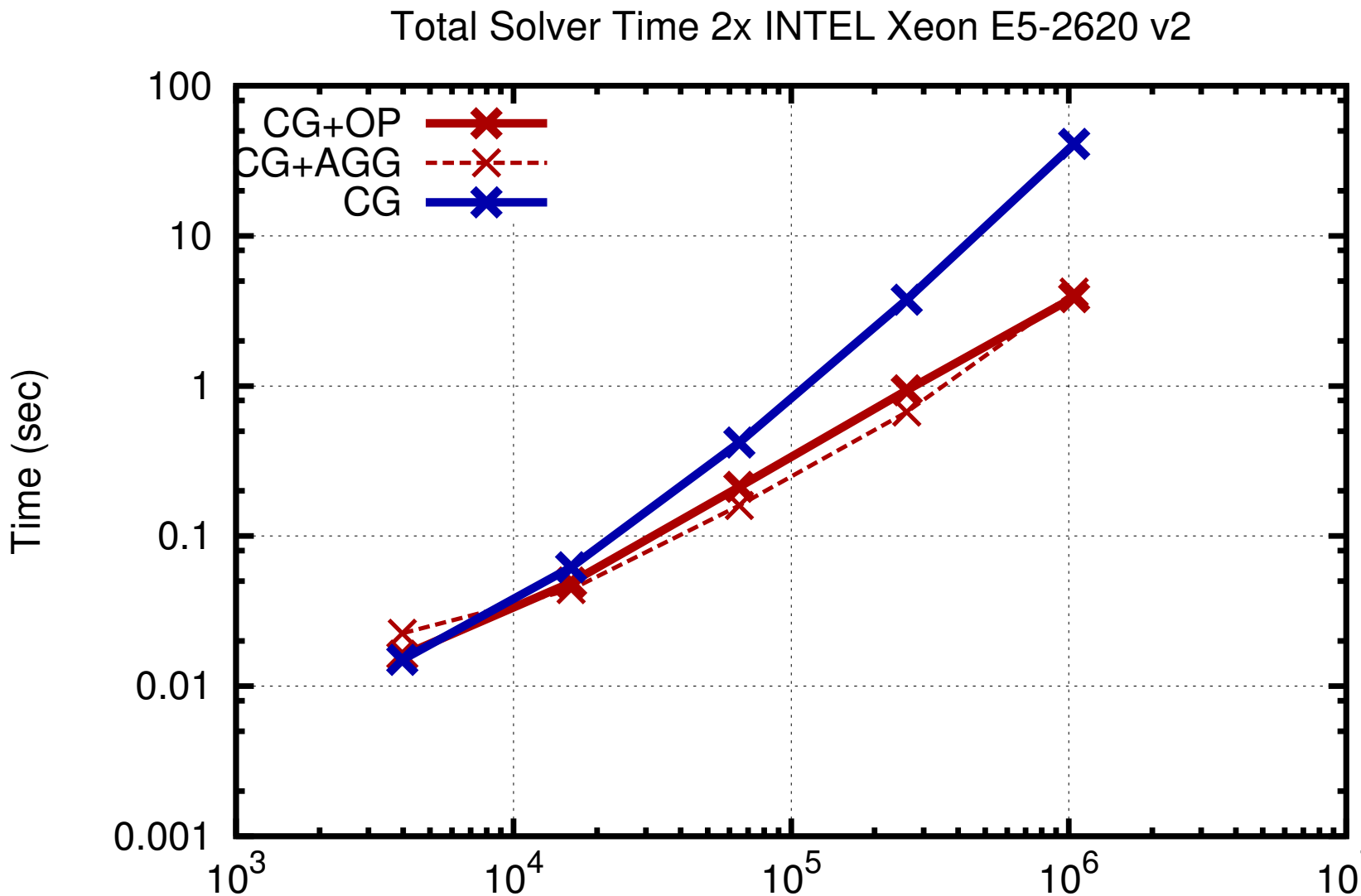
Benchmarks


Solver Setup Time 2x INTEL Xeon E5-2620 v2
CG+OP
CG+AGG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

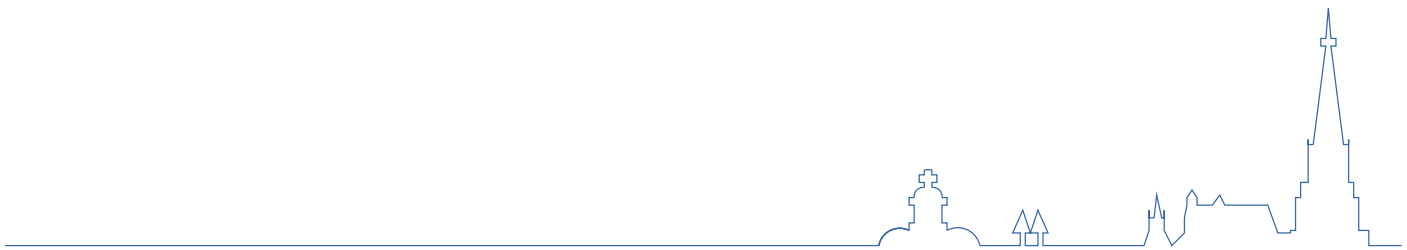

Solver Cycle Time 2x INTEL Xeon E5-2620 v2
CG+OP
CG+AGG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

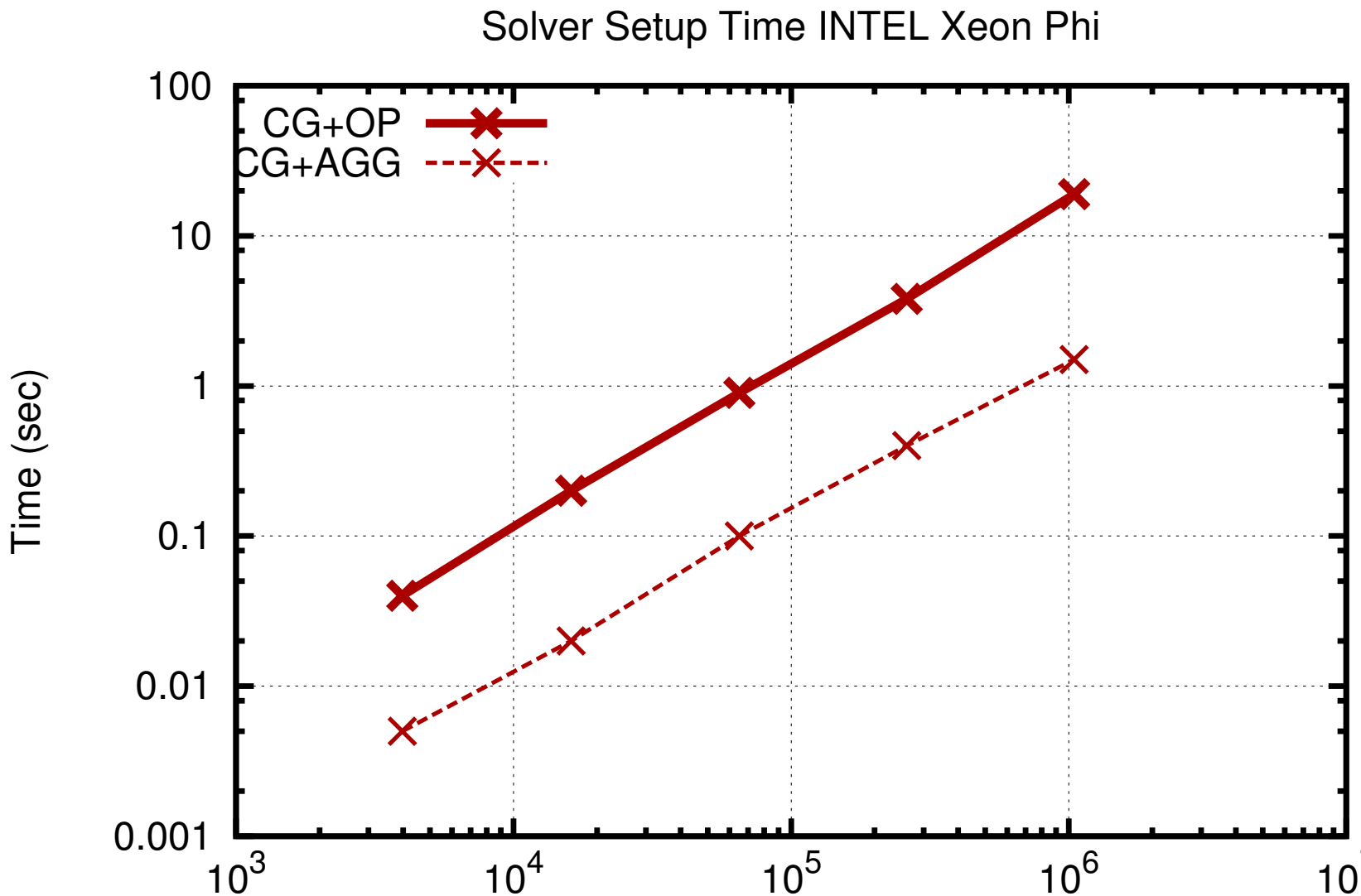

Total Solver Time 2x INTEL Xeon E5-2620 v2
CG+OP
CG+AGG
CG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

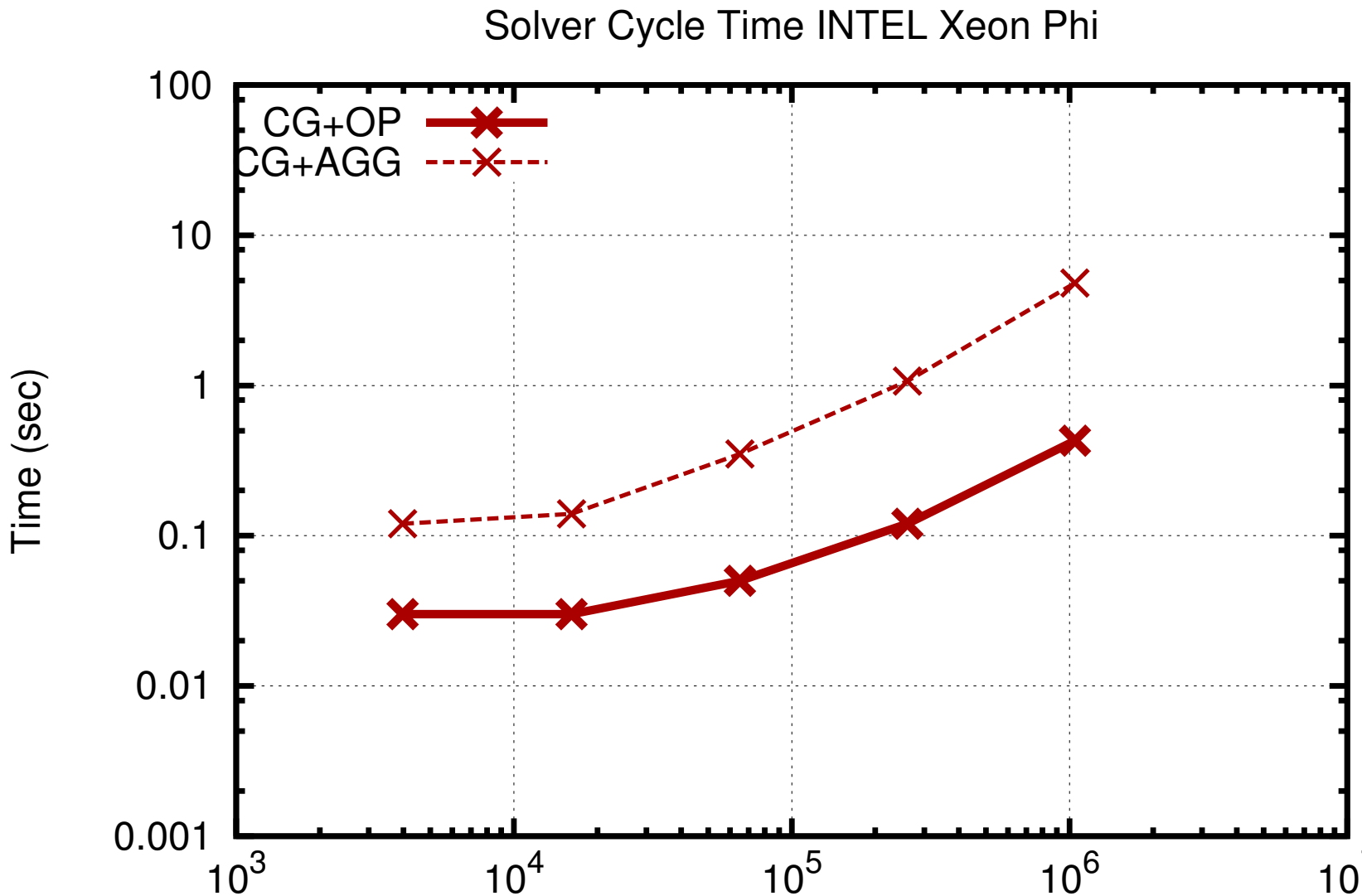
INTEL Xeon Phi



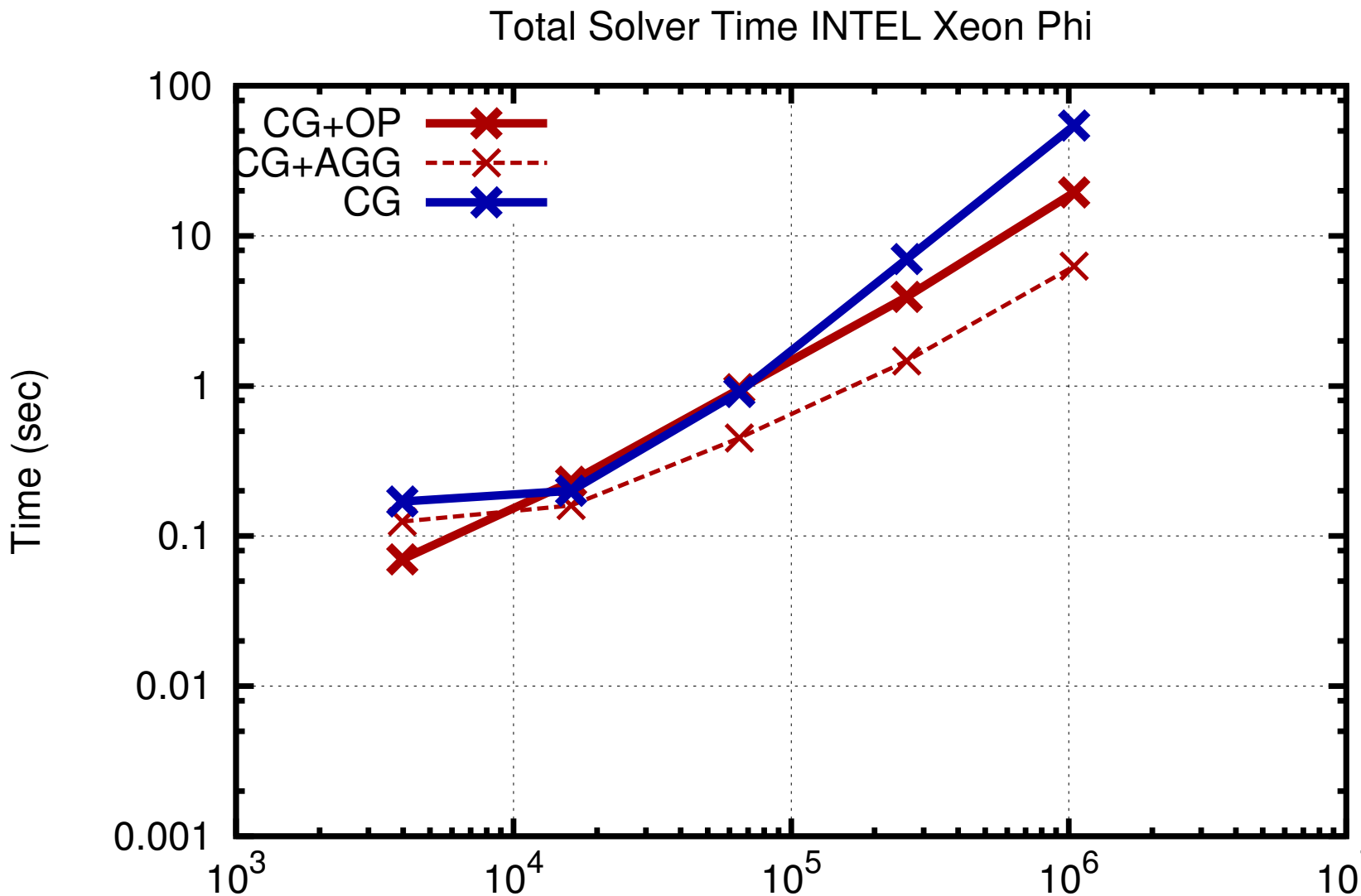
Benchmarks


Solver Setup Time INTEL Xeon Phi
CG+OP
CG+AGG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

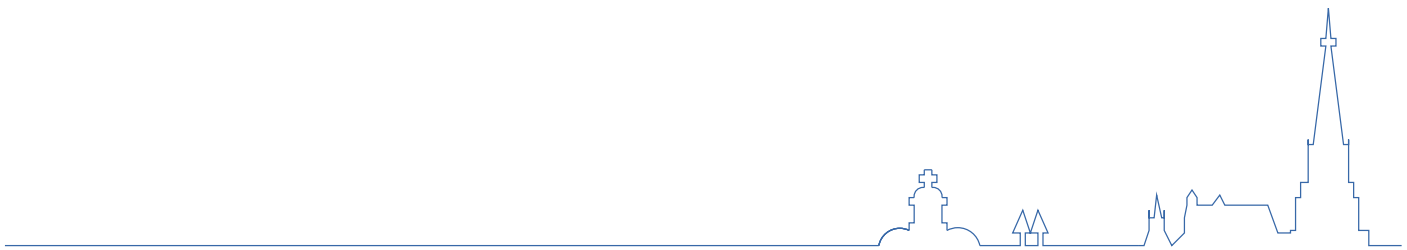

Solver Cycle Time INTEL Xeon Phi
CG+OP
CG+AGG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

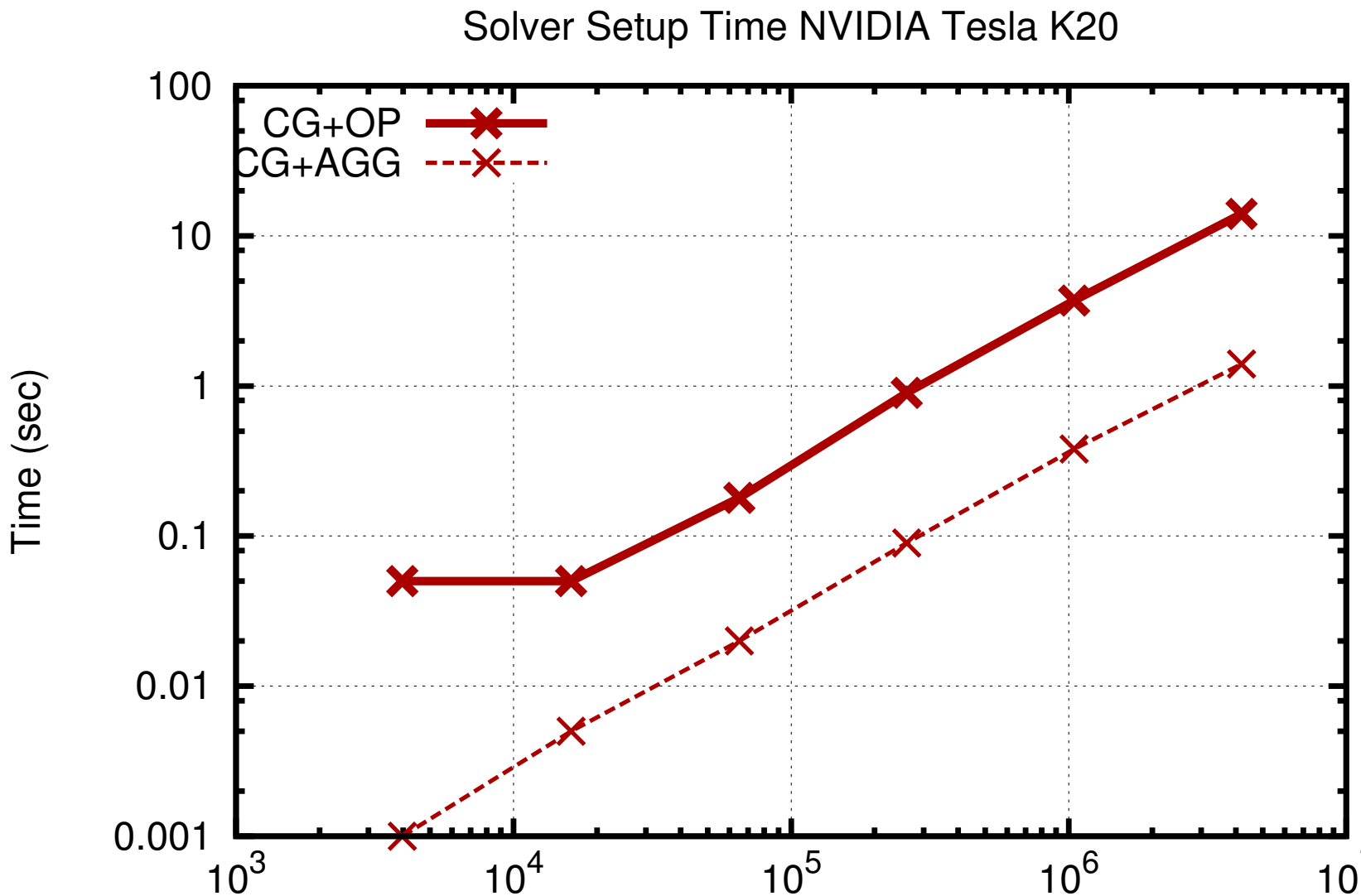

Total Solver Time INTEL Xeon Phi
CG+OP
CG+AGG
CG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

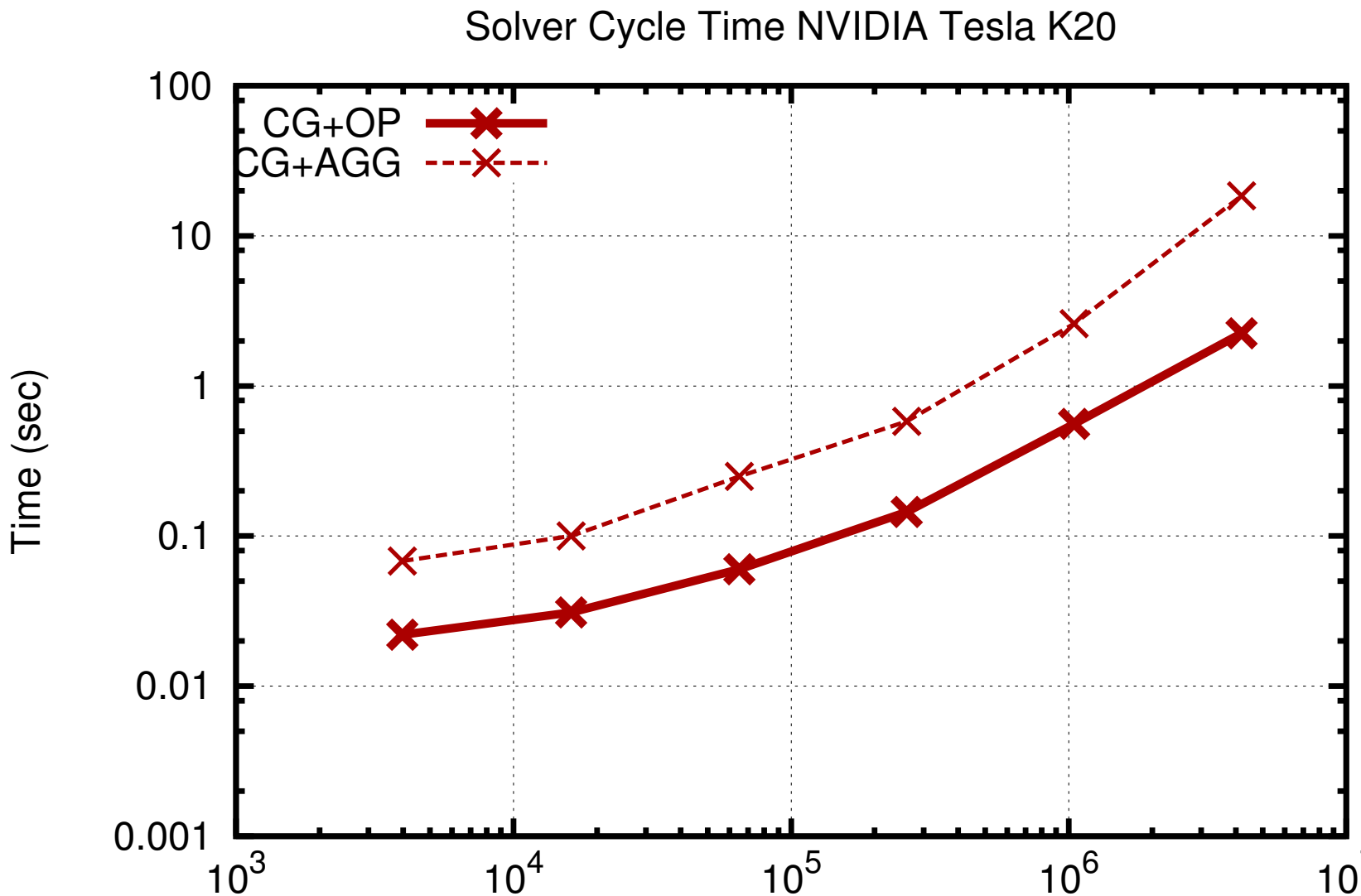
NVIDIA Tesla K20



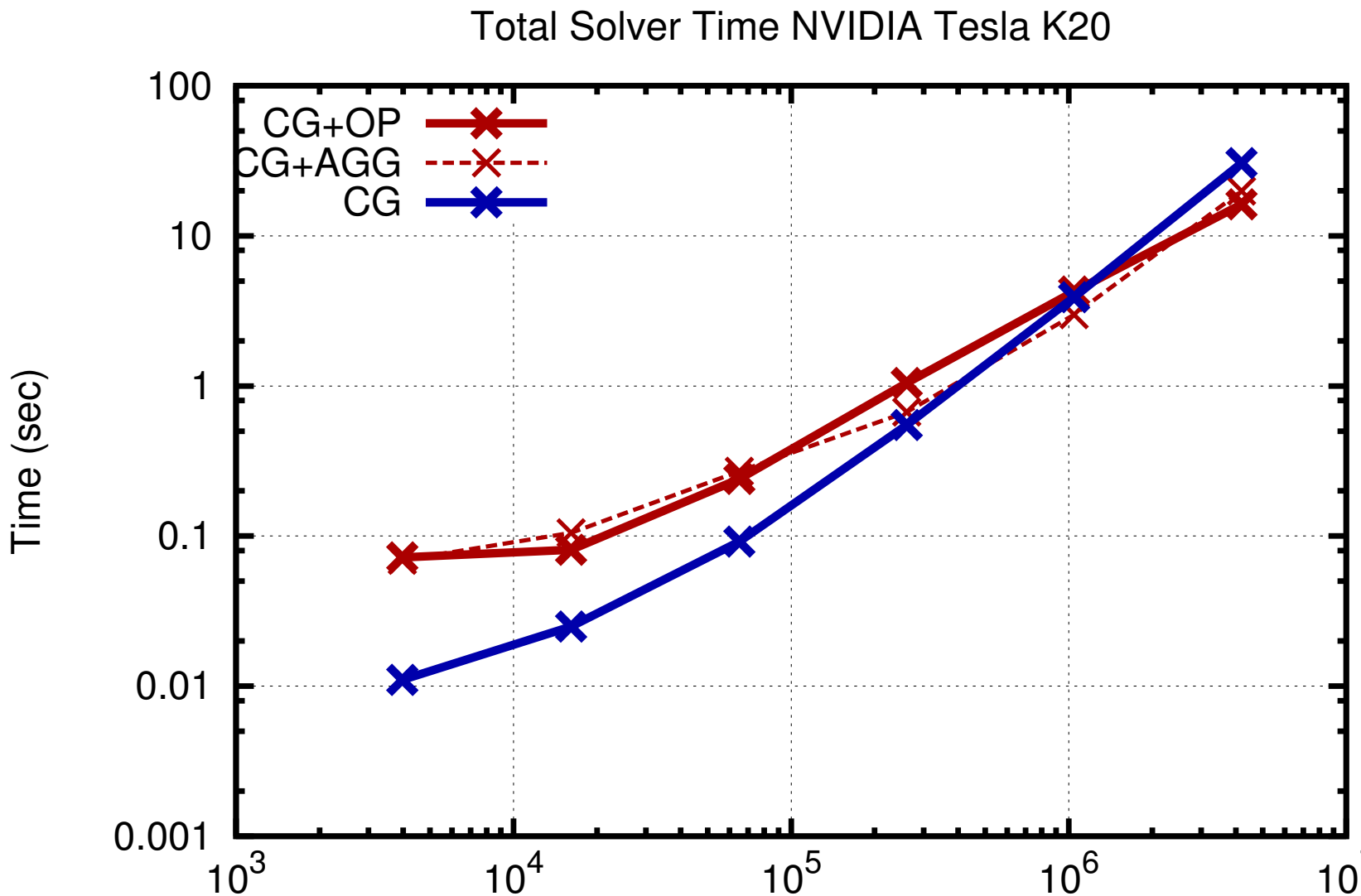
Benchmarks


Solver Setup Time NVIDIA Tesla K20
CG+OP
CG+AGG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

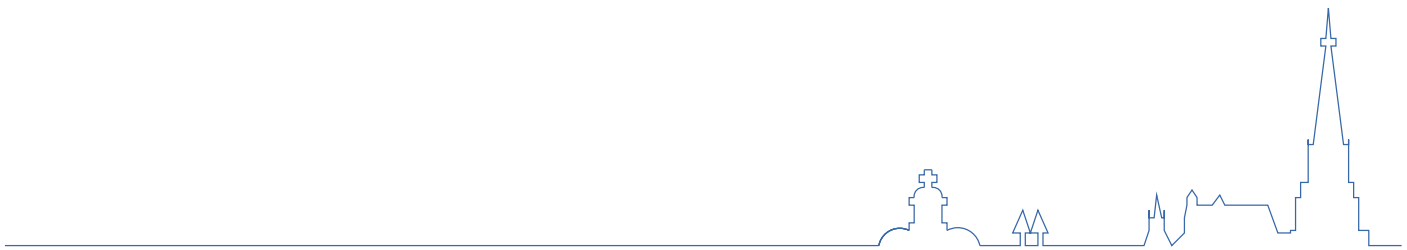

Solver Cycle Time NVIDIA Tesla K20
CG+OP
CG+AGG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

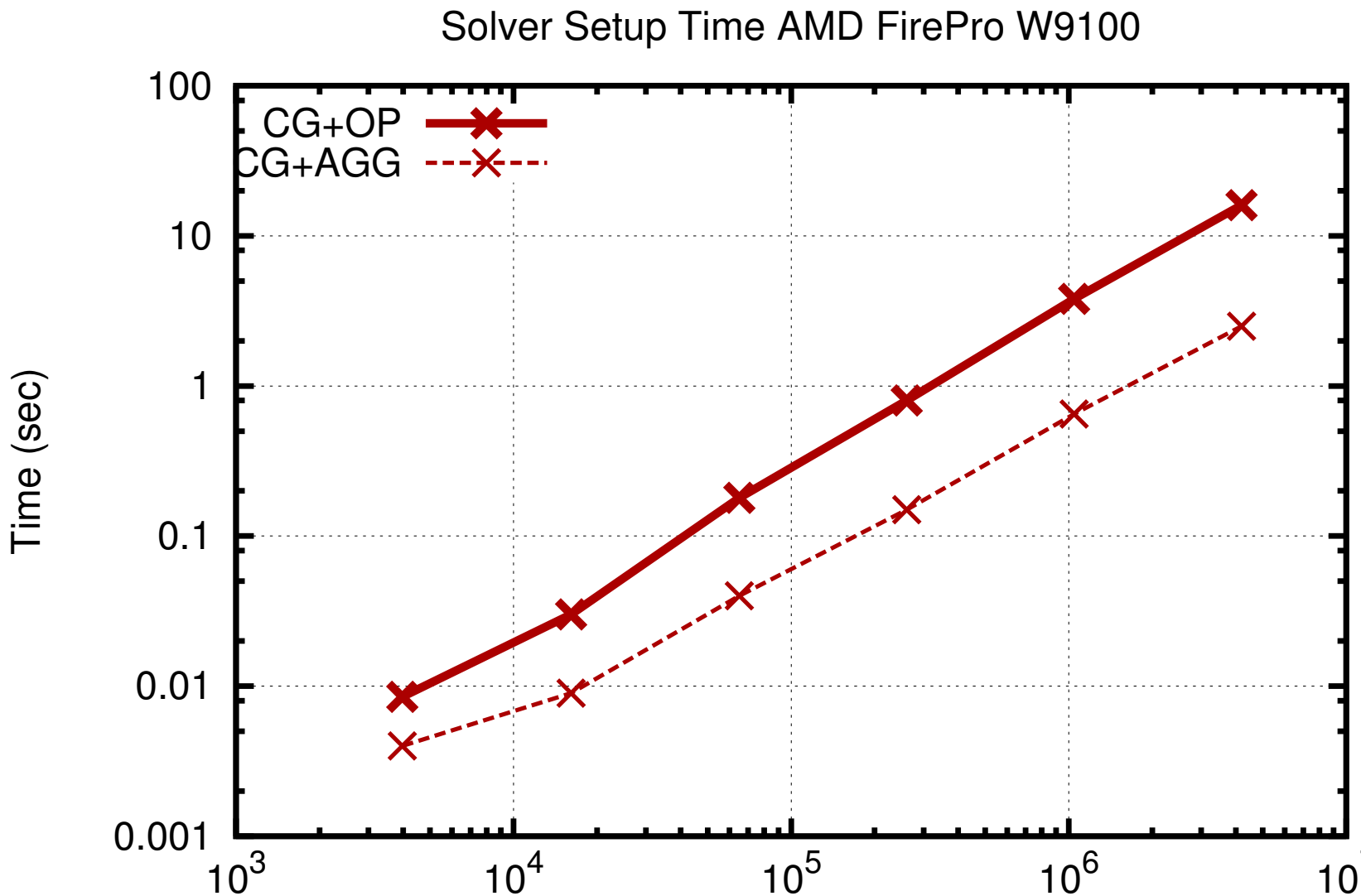

Total Solver Time NVIDIA Tesla K20
CG+OP
CG+AGG
CG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

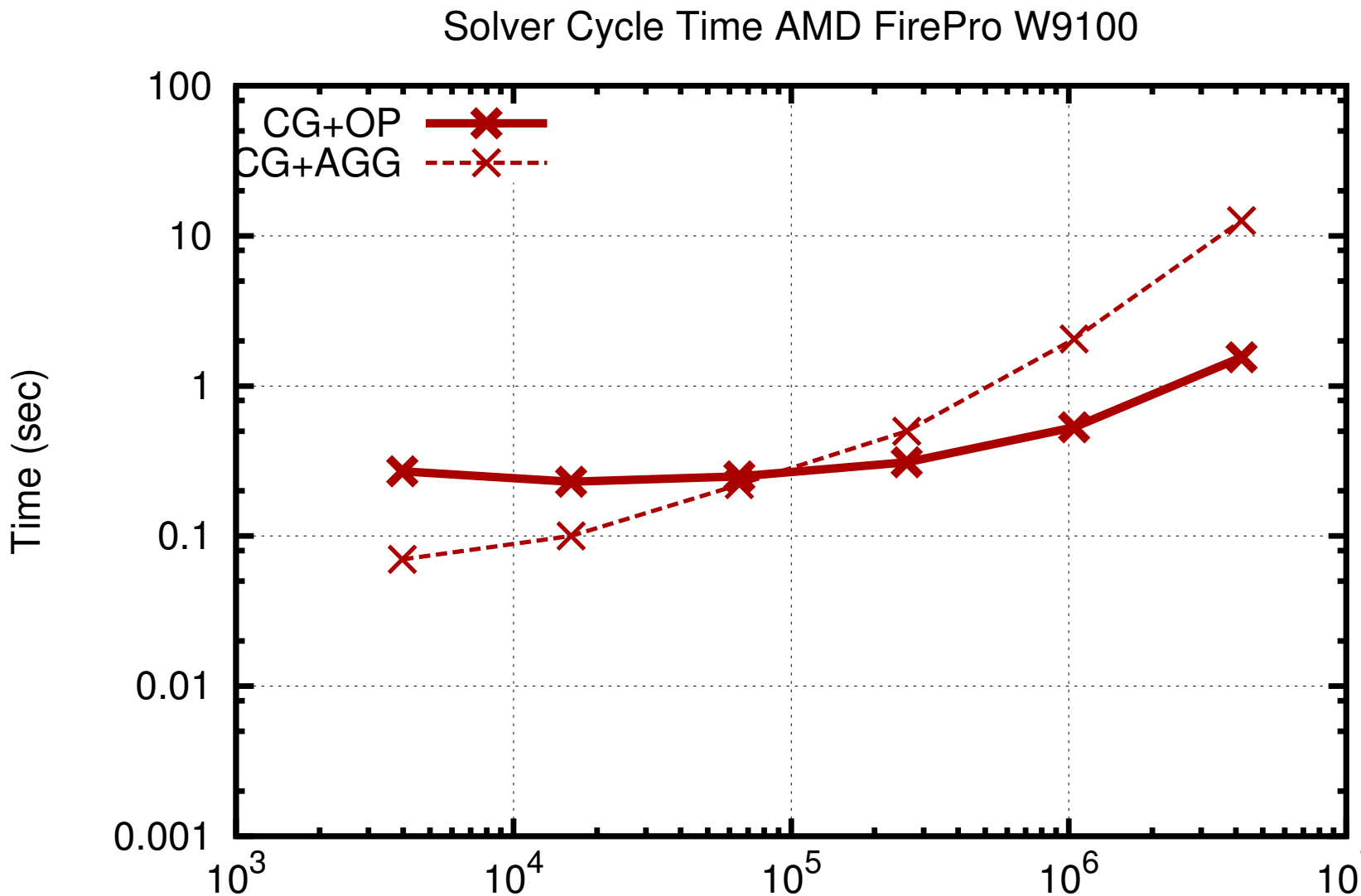
AMD FirePro W9100



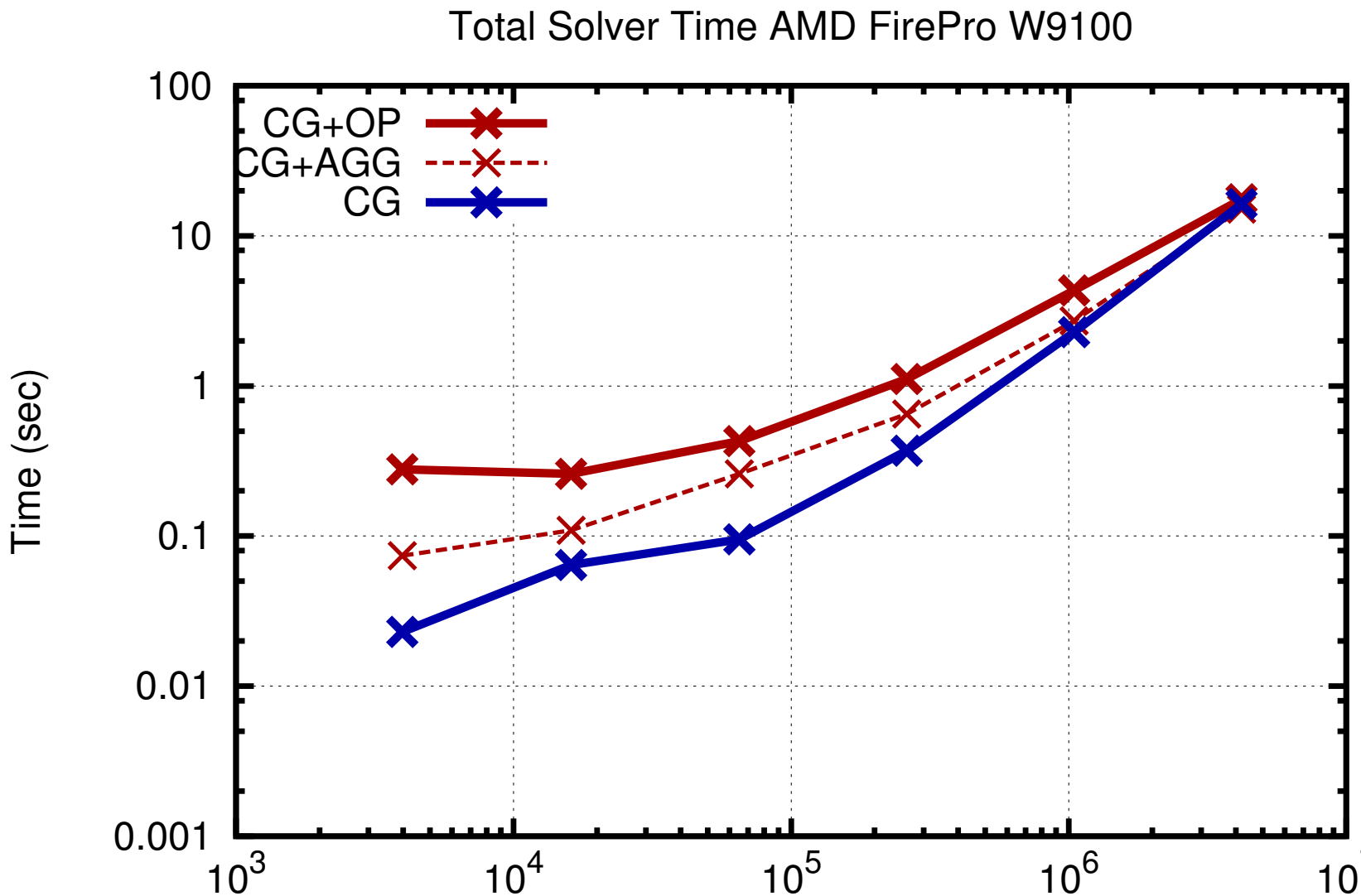
Benchmarks


Solver Setup Time AMD FirePro W9100
CG+OP
CG+AGG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

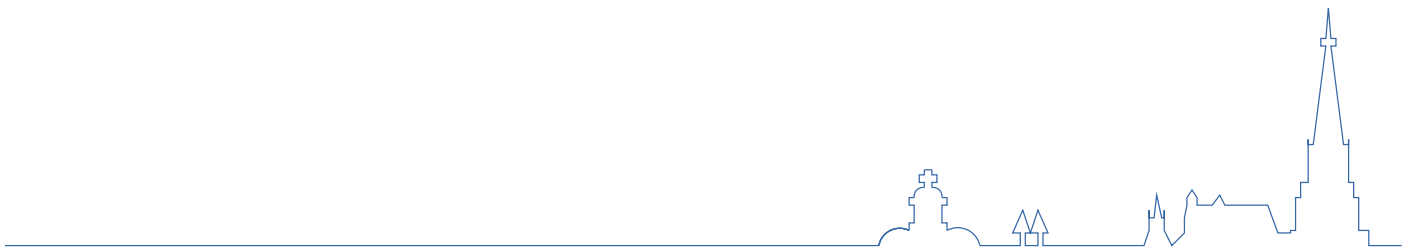

Solver Cycle Time AMD FirePro W9100
CG+OP
CG+AGG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

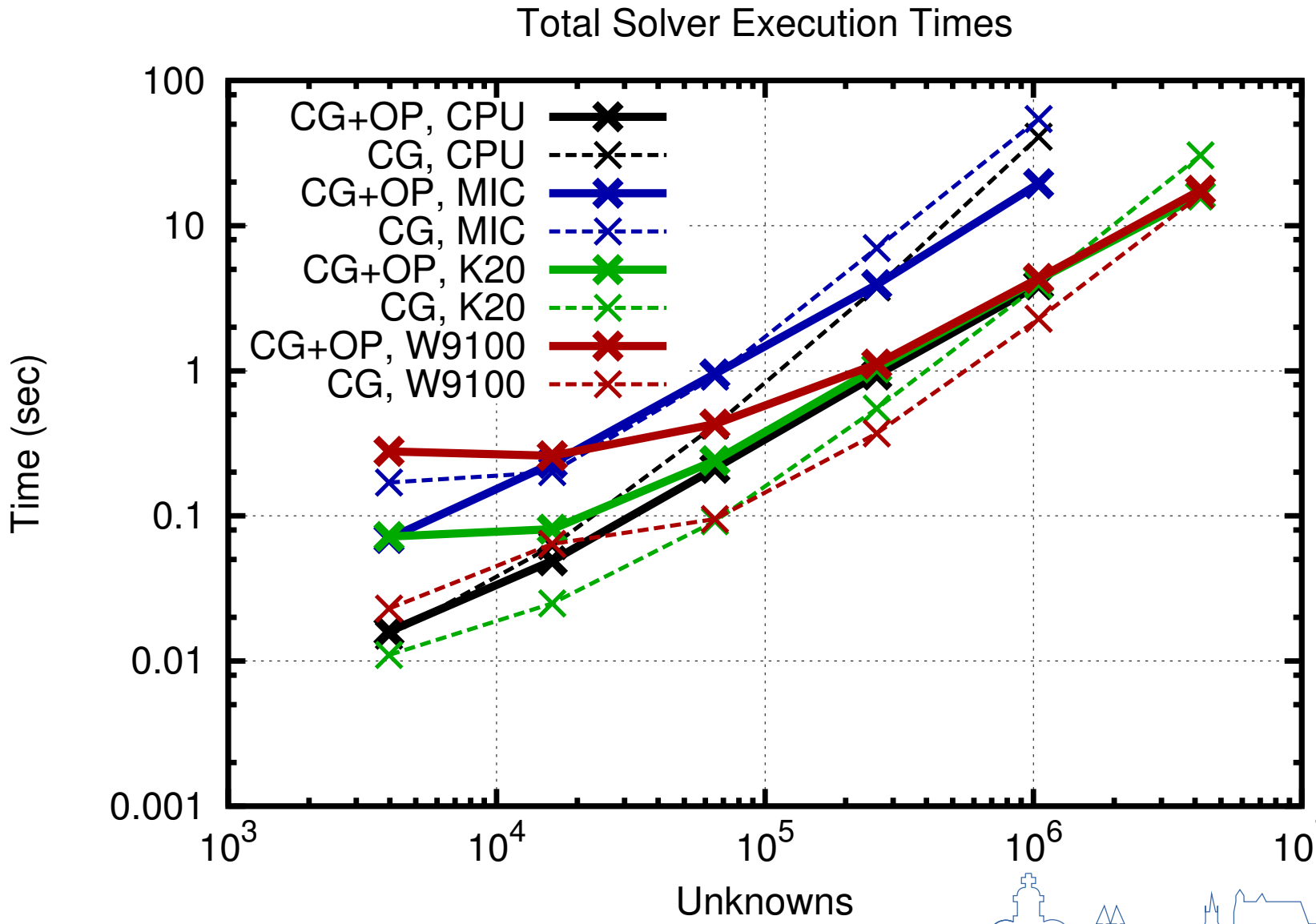

Total Solver Time AMD FirePro W9100
CG+OP
CG+AGG
CG
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns

Benchmarks

Putting all together...



Benchmarks


Total Solver Execution Times
Time (sec)
100
10
1
0.1
0.01
0.001
10^3
10^4
10^5
10^6
10^7
Unknowns
CG+OP, CPU
CG, CPU
CG+OP, MIC
CG, MIC
CG+OP, K20
CG, K20
CG+OP, W9100
CG, W9100

Summary and Conclusion

Parallel AMG

- Setup on CPU, Cycles on GPUs
- Sweet spot for GPUs above 1 million unknowns, below 10 million
- Sweet spot for MIC still to be found

Parallel Setup

- PCI-Express and sequential stages a bottleneck
- Matrix transposition hard on MIC, easier on GPU
- Galerkin-products fastest on CPU

Availability

`http://viennacl.sourceforge.net/`