

An Overview of GPU-accelerated Routines and Implementation Techniques in ViennaCL

Karl Rupp^{1,2}

`rupp@iue.tuwien.ac.at`

`https://karlrupp.net/`

`https://github.com/karlrupp/slides/`

 `@karlrupp`

with contributions from

Philippe Tillet¹, Florian Rudolf¹,

Josef Weinbub¹, Ansgar Jüngel², Tibor Grasser¹

(based on stimuli from PETSc+ViennaCL users)



¹ Institute for Microelectronics, TU Wien, Austria

² Institute for Analysis and Scientific Computing, TU Wien, Austria

Positions

PhD student at TU Wien (2009-2011)

Postdoc at ANL (09/2012-09/2013)

Postdoc at TU Wien (09/2013-current)

Research Interests

Semiconductor device simulation

Numerical solution of PDEs

Parallel computing

Software Development

PETSc

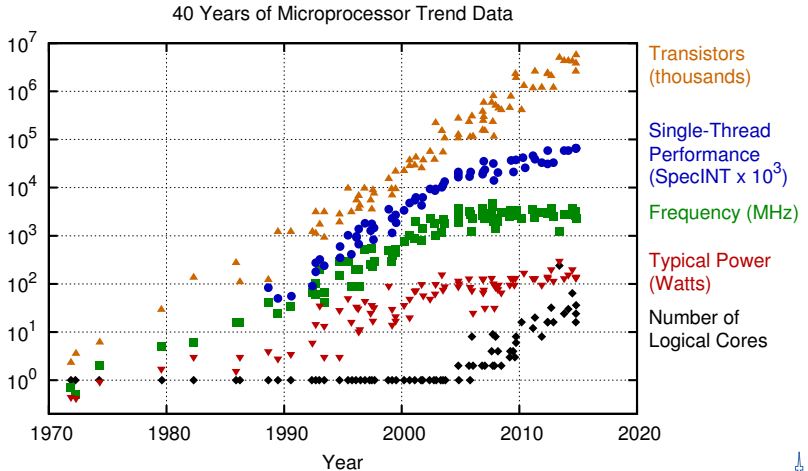
ViennaCL

ViennaSHE

...



Introduction

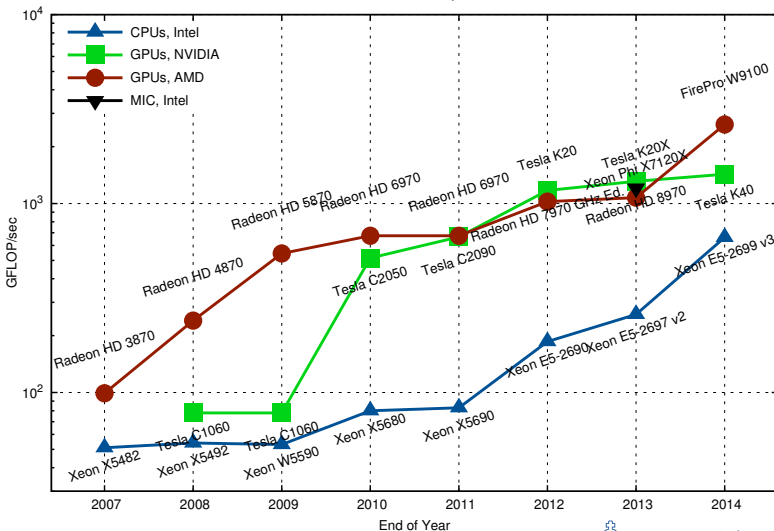


Original data up to the year 2010 collected and plotted by M. Horowitz, F. Labonte, O. Shacham, K. Olukotun, L. Hammond, and C. Batten
New plot and data collected for 2010-2015 by K. Rupp

Introduction

Theoretical Peak Performance

Theoretical Peak Performance, Double Precision

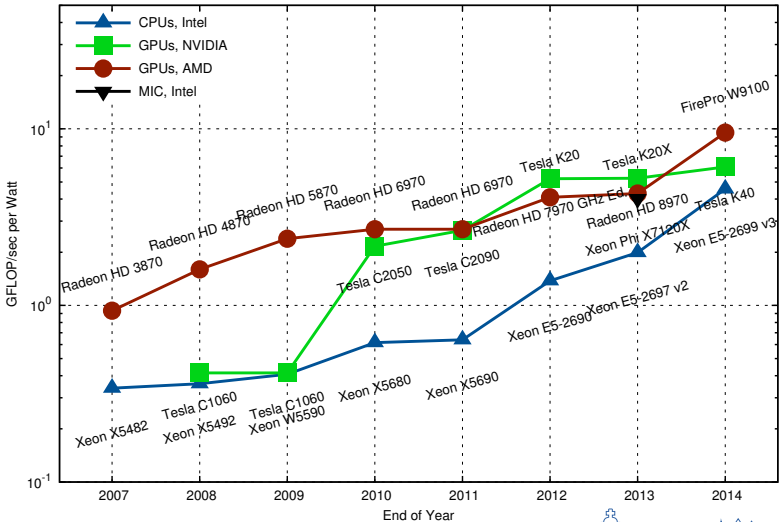


<https://www.karlsruhp.net/2013/06/cpu-gpu-and-mic-hardware-characteristics-over-time/>

Introduction

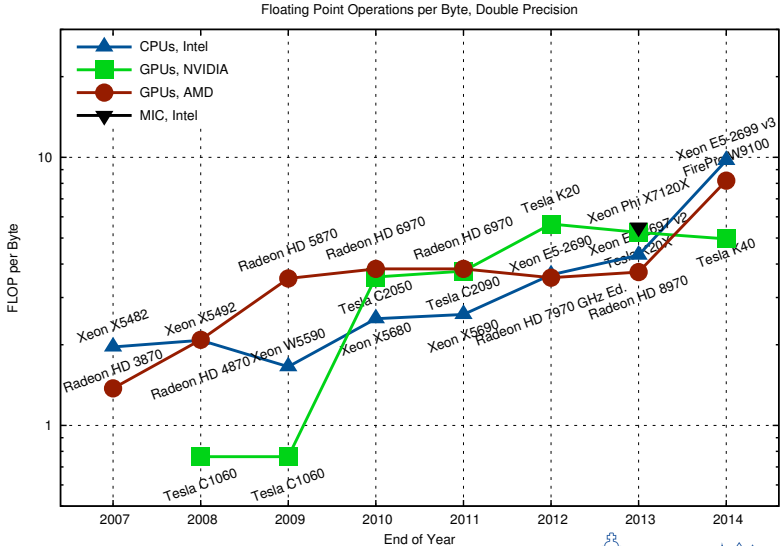
Theoretical Peak Performance per Watt

Peak Floating Point Operations per Watt, Double Precision



Introduction

Theoretical Peak Performance (FLOPs) per Byte of Memory Bandwidth



Consider Existing CPU Code (Boost.uBLAS)

```
using namespace boost::numeric::ublas;

matrix<double> A(1000, 1000);
vector<double> x(1000), y(1000);

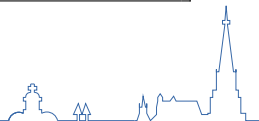
/* Fill A, x, y here */

double val = inner_prod(x, y);
y += 2.0 * x;
A += val * outer_prod(x, y);

x = solve(A, y, upper_tag()); // Upper tri. solver

std::cout << " 2-norm: " << norm_2(x) << std::endl;
std::cout << "sup-norm: " << norm_inf(x) << std::endl;
```

High-level code with syntactic sugar



Previous Code Snippet Rewritten with ViennaCL

```
using namespace viennacl;
using namespace viennacl::linalg;

matrix<double> A(1000, 1000);
vector<double> x(1000), y(1000);

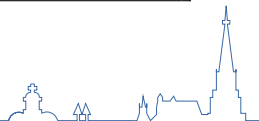
/* Fill A, x, y here */

double val = inner_prod(x, y);
y += 2.0 * x;
A += val * outer_prod(x, y);

x = solve(A, y, upper_tag()); // Upper tri. solver

std::cout << " 2-norm: " << norm_2(x) << std::endl;
std::cout << "sup-norm: " << norm_inf(x) << std::endl;
```

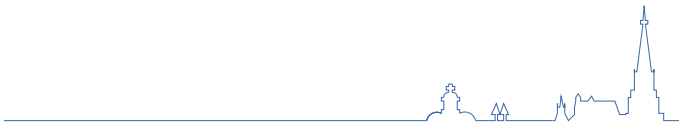
High-level code with syntactic sugar



ViennaCL in Addition Provides Iterative Solvers

```
using namespace viennacl;  
using namespace viennacl::linalg;  
  
compressed_matrix<double> A(1000, 1000);  
vector<double> x(1000), y(1000);  
  
/* Fill A, x, y here */  
  
x = solve(A, y, cg_tag());           // Conjugate Gradients  
x = solve(A, y, bicgstab_tag());      // BiCGStab solver  
x = solve(A, y, gmres_tag());         // GMRES solver
```

No Iterative Solvers Available in Boost.uBLAS...



Thanks to Interface Compatibility

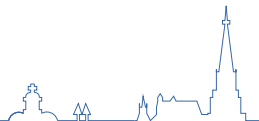
```
using namespace boost::numeric::ublas;  
using namespace viennacl::linalg;  
  
compressed_matrix<double> A(1000, 1000);  
vector<double> x(1000), y(1000);  
  
/* Fill A, x, y here */  
  
x = solve(A, y, cg_tag());           // Conjugate Gradients  
x = solve(A, y, bicgstab_tag());      // BiCGStab solver  
x = solve(A, y, gmres_tag());         // GMRES solver
```

Code Reuse Beyond GPU Borders

Armadillo <http://arma.sourceforge.net/>

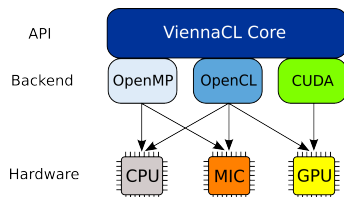
Eigen <http://eigen.tuxfamily.org/>

MTL 4 <http://www.mtl4.org/>



About

High-level linear algebra C++ library
OpenMP, OpenCL, and CUDA backends
Header-only
Multi-platform



Dissemination

Free Open-Source MIT (X11) License
<http://viennacl.sourceforge.net/>
50-100 downloads per week

Design Rules

Reasonable default values
Compatible to Boost.uBLAS whenever possible
In doubt: clean design over performance

Basic Types

scalar

vector

matrix

compressed_matrix, coordinate_matrix, (sliced_)ell_matrix, hyb_matrix

Data Initialization

Using `viennacl::copy()`

```
std::vector<double>      std_x(100);
ublas::vector<double>    ublas_x(100);
viennacl::vector<double> vcl_x(100);

for (size_t i=0; i<100; ++i){
    std_x[i] = rand();
    ublas_x[i] = rand();
    vcl_x[i] = rand(); //possible, inefficient
}
```

Basic Types

scalar

vector

matrix

compressed_matrix, coordinate_matrix, (sliced_)ell_matrix, hyb_matrix

Data Initialization

Using `viennacl::copy()`

```
std::vector<double>      std_x(100);
ublas::vector<double>    ublas_x(100);
viennacl::vector<double> vcl_x(100);

/* setup of std_x and ublas_x omitted */

viennacl::copy(std_x.begin(), std_x.end(),
               vcl_x.begin()); //to GPU
viennacl::copy(vcl_x.begin(), vcl_x.end(),
               ublas_x.begin()); //to CPU
```

Basic Types

scalar

vector

matrix

compressed_matrix, coordinate_matrix, (sliced_)ell_matrix, hyb_matrix

Data Initialization

Using `viennacl::copy()`

```
std::vector<std::vector<double> >    std_A;  
ublas::matrix<double>                ublas_A;  
viennacl::matrix<double>              vcl_A;  
  
/* setup of std_A and ublas_A omitted */  
  
viennacl::copy(std_A,  
               vcl_A);    // CPU to GPU  
viennacl::copy(vcl_A,  
               ublas_A);  // GPU to CPU
```

Vector Addition

```
x = y + z;
```

Naive Operator Overloading

```
vector<T> operator+(vector<T> & v, vector<T> & w);
```

$t \leftarrow y + z, x \leftarrow t$

Temporaries are extremely expensive!

Expression Templates

```
vector_expr<vector<T>, op_plus, vector<T> >  
operator+(vector<T> & v, vector<T> & w) { ... }  
  
vector::operator=(vector_expr<...> const & e) {  
    viennacl::linalg::avbv(*this, 1,e.lhs(), 1,e.rhs());  
}
```



Vector Addition

```
// x = y + z
void avbv(...) {
    switch (active_handle_id(x))
    {
        case MAIN_MEMORY:
            host_based::avbv(...);
            break;
        case OPENCL_MEMORY:
            opencl::avbv(...);
            break;
        case CUDA_MEMORY:
            cuda::avbv(...);
            break;
        default:
            raise_error();
    }
}
```

Memory buffers can switch memory domain at runtime

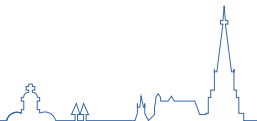


Generalizing Compute Kernels

```
// x = y + z
__kernel void avbv(
    double * x,

    double * y,

    double * z, uint size)
{
    i = get_global_id(0);
    for (size_t i=0; i<size; i += get_global_size(0))
        x[i] = y[i] + z[i];
}
```



Generalizing Compute Kernels

```
// x = a * y + b * z
__kernel void avbv(
    double * x,
    double a,
    double * y,
    double b,
    double * z, uint size)
{
    i = get_global_id(0);
    for (size_t i=0; i<size; i += get_global_size(0))
        x[i] = a * y[i] + b * z[i];
}
```



Generalizing Compute Kernels

```
// x[4:8] = a * y[2:6] + b * z[3:7]
__kernel void avbv(
    double * x, uint off_x,
    double a,
    double * y, uint off_y,
    double b,
    double * z, uint off_z, uint size)
{
    i = get_global_id(0);
    for (size_t i=0; i<size; i += get_global_size(0))
        x[off_x + i] = a * y[off_y + i] + b * z[off_z + i];
}
```



Generalizing Compute Kernels

```
// x[4:2:8] = a * y[2:2:6] + b * z[3:2:7]
__kernel void avbv(
    double * x, uint off_x, uint inc_x,
    double a,
    double * y, uint off_y, uint inc_y,
    double b,
    double * z, uint off_z, uint inc_z, uint size)
{
    i = get_global_id(0);
    for (size_t i=0; i<size; i += get_global_size(0))
        x[off_x + i * inc_x] = a * y[off_y + i * inc_y]
                               + b * z[off_z + i * inc_z];
}
```

No penalty on GPUs because FLOPs are for free



Standard Functionality

BLAS levels 1-3

Sparse matrix times {vector, dense matrix, sparse matrix}

Triangular solvers (dense and sparse)

Iterative Solvers

Krylov solvers: CG, BiCGStab, GMRES

Preconditioners: Jacobi, serial + parallel ILU0, ILUT, AMG, SPAI

Eigen Solvers

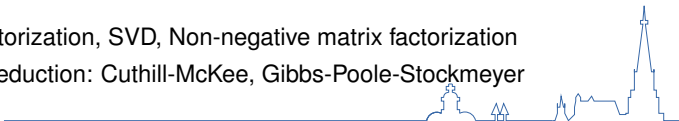
Lanczos, power iteration

QR method (experimental), bisection

Miscellaneous

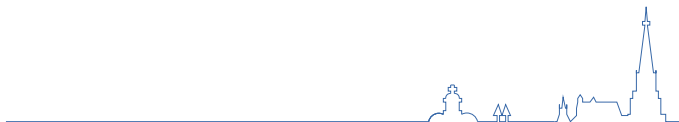
FFT, QR factorization, SVD, Non-negative matrix factorization

Bandwidth reduction: Cuthill-McKee, Gibbs-Poole-Stockmeyer



Case Study: Optimizing Iterative Solvers

A Story in Three Parts



Pseudocode

Choose x_0

$$p_0 = r_0 = b - Ax_0$$

For $i = 0$ until convergence

1. Compute and store Ap_i
2. Compute $\langle p_i, Ap_i \rangle$
3. $\alpha_i = \langle r_i, r_i \rangle / \langle p_i, Ap_i \rangle$
4. $x_{i+1} = x_i + \alpha_i p_i$
5. $r_{i+1} = r_i - \alpha_i Ap_i$
6. Compute $\langle r_{i+1}, r_{i+1} \rangle$
7. $\beta_i = \langle r_{i+1}, r_{i+1} \rangle / \langle r_i, r_i \rangle$
8. $p_{i+1} = r_{i+1} + \beta_i p_i$

EndFor

BLAS-based Implementation

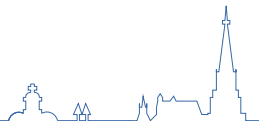
-

SpMV, AXPY

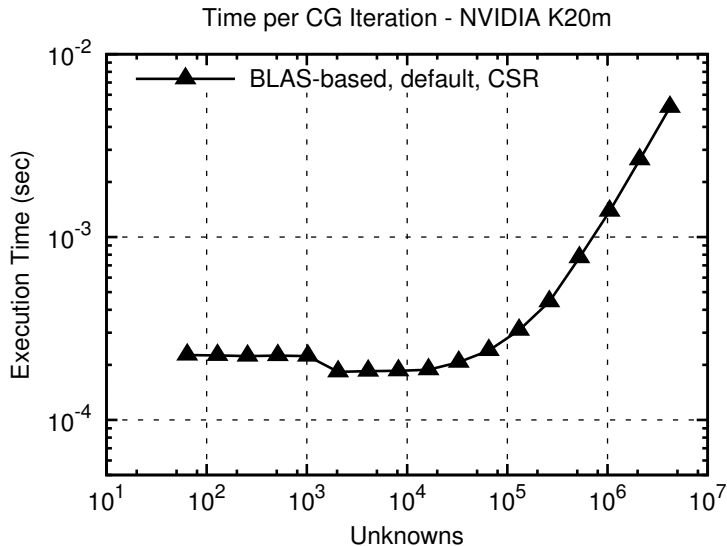
For $i = 0$ until convergence

1. SpMV \leftarrow No caching of Ap_i
2. DOT \leftarrow Global sync!
3. -
4. AXPY
5. AXPY \leftarrow No caching of r_{i+1}
6. DOT \leftarrow Global sync!
7. -
8. AXPY

EndFor



Conjugate Gradients



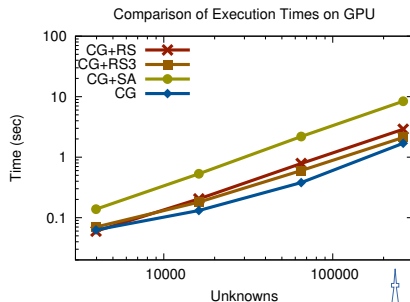
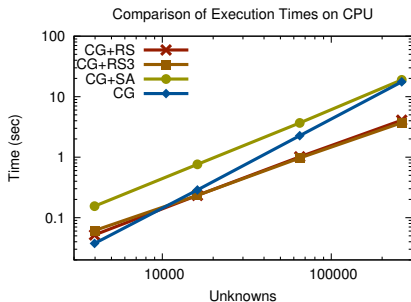
(2D Finite Difference Discretization)

Conjugate Gradient

Implications

Kernel launches expensive

Delicate balance for preconditioners

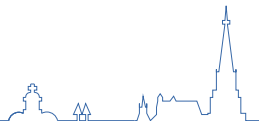


Optimization 1

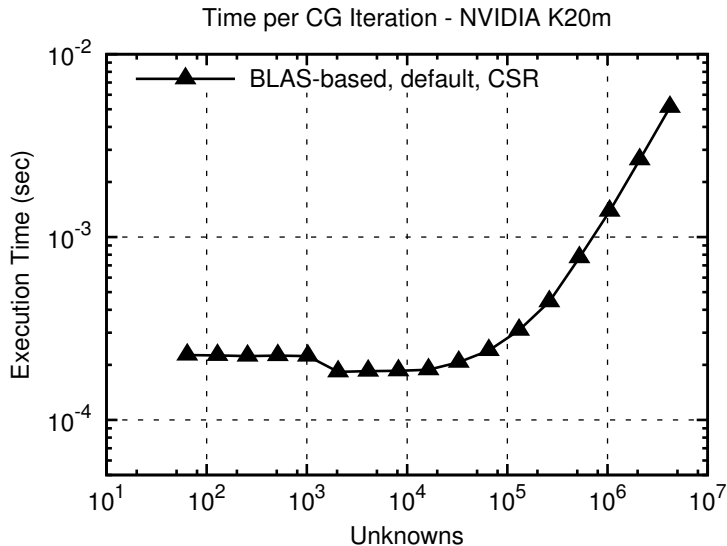
Get best performance out of SpMV

Compare different sparse matrix types

Cf.: N. Bell: Implementing sparse matrix-vector multiplication on throughput-oriented processors. *Proc. SC '09*

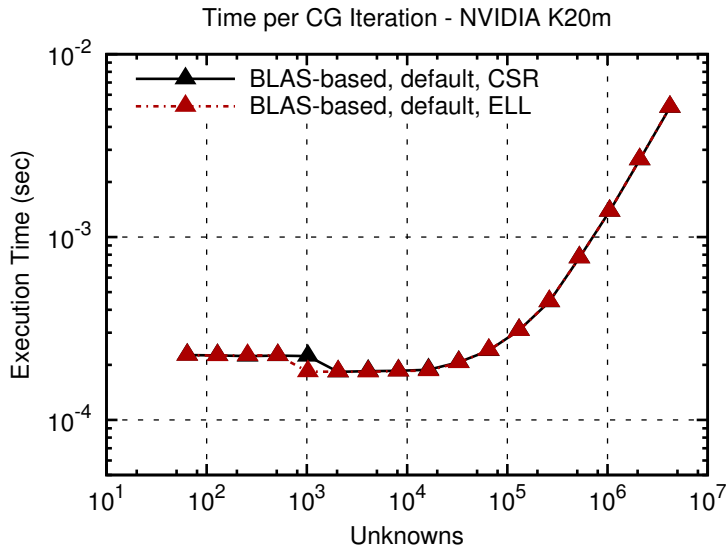


Conjugate Gradients



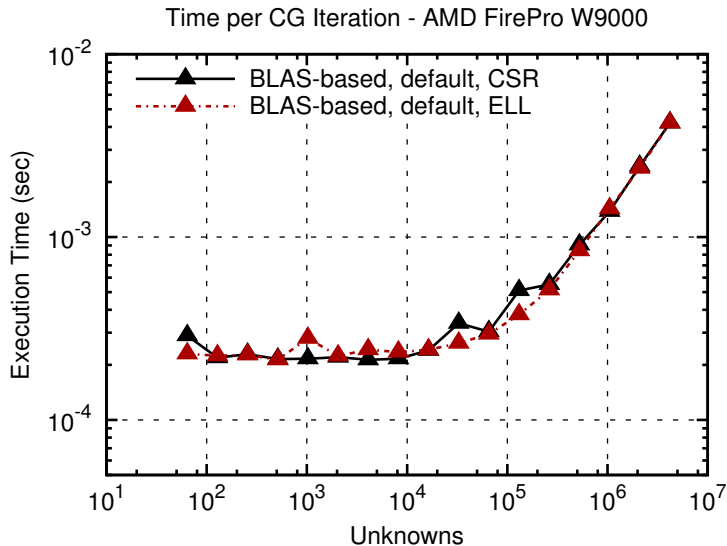
(2D Finite Difference Discretization)

Conjugate Gradients



(2D Finite Difference Discretization)

Conjugate Gradients



(2D Finite Difference Discretization)

Optimization 2

Optimize kernel parameters for each operation



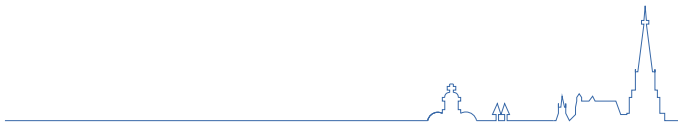
Scope for Portability Study

Vector and matrix-vector operations (BLAS levels 1 and 2)

Limited by memory bandwidth

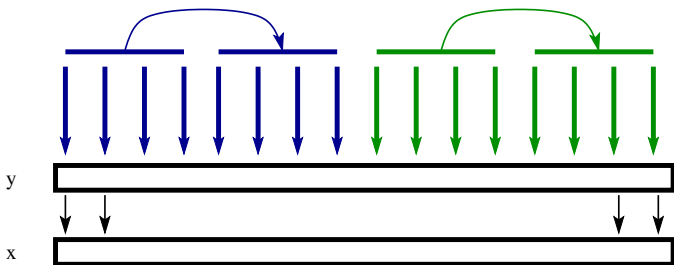
Key Question (Memory-Bandwidth-Limited Kernels)

Good performance of complicated kernels
by optimizing the simplest kernel?



Vector Assignment (Copy) Kernel

$x \leftarrow y$ for (large) vectors x, y



Parameters (1900 variations)

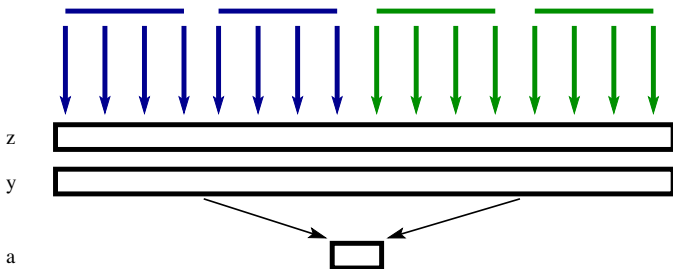
```
for (size_t i = group_start + get_local_id(0);  
     i < group_end; i+= get_local_size(0))  
    x[i] = y[i];
```


Benchmark Setting

Operations

Vector copy, vector addition, inner product

Matrix-vector product

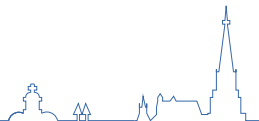


Devices

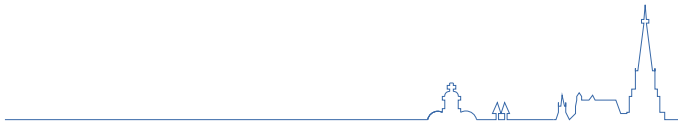
AMD: A10-5800 APU, HD 5850 GPU

INTEL: Dual Socket Xeon E5-2670, Xeon Phi

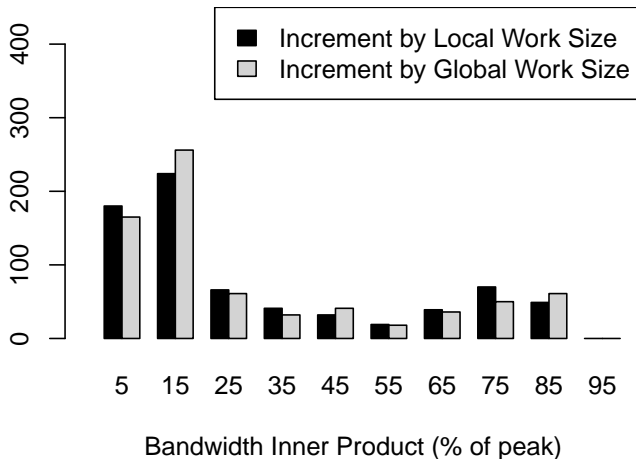
NVIDIA: GTX 285, Tesla K20m



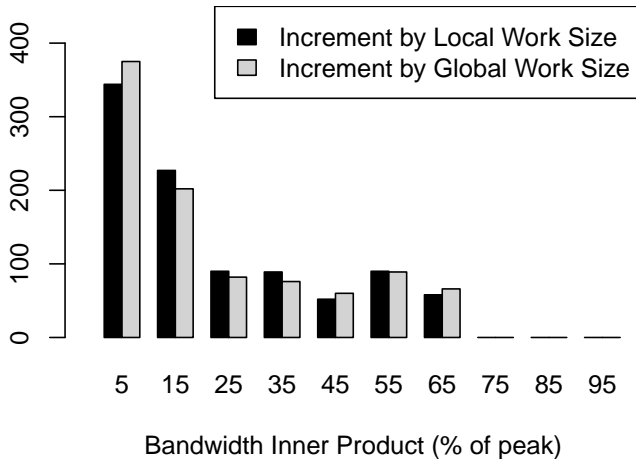
Histograms



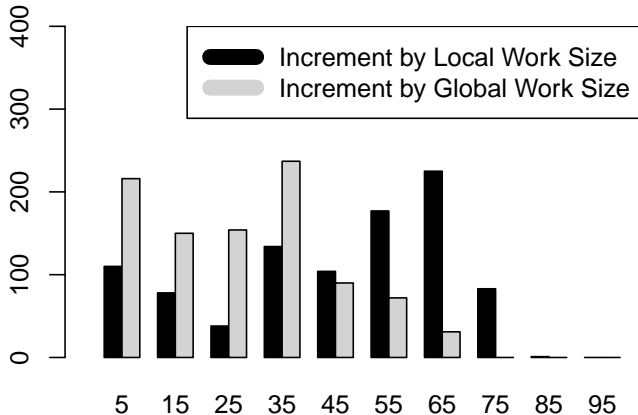
AMD Radeon HD 5850



NVIDIA Tesla K20m

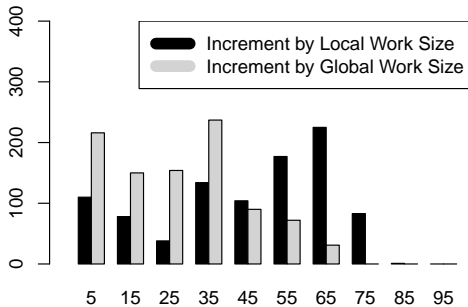


Intel Xeon E5-2670

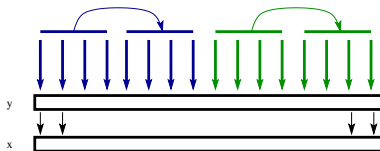


Bandwidth Inner Product (% of theoretical peak)

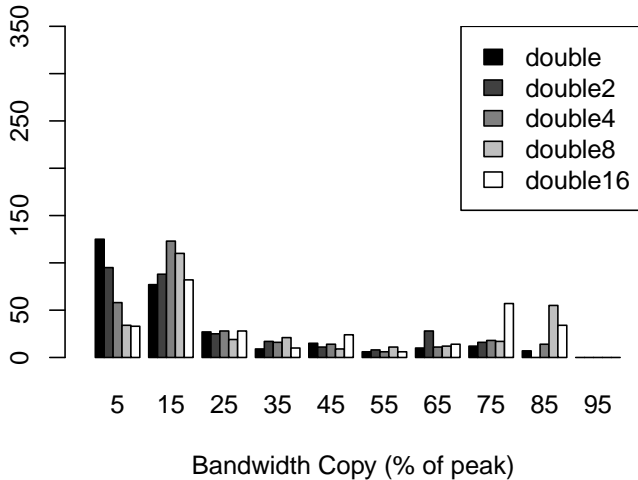
Intel Xeon E5-2670



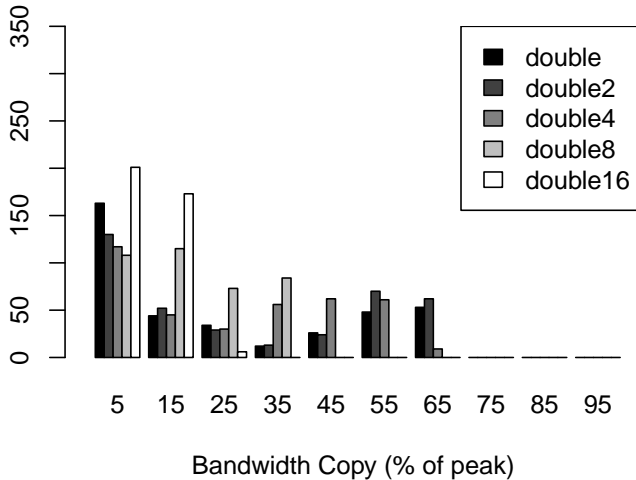
Bandwidth Inner Product (% of theoretical peak)



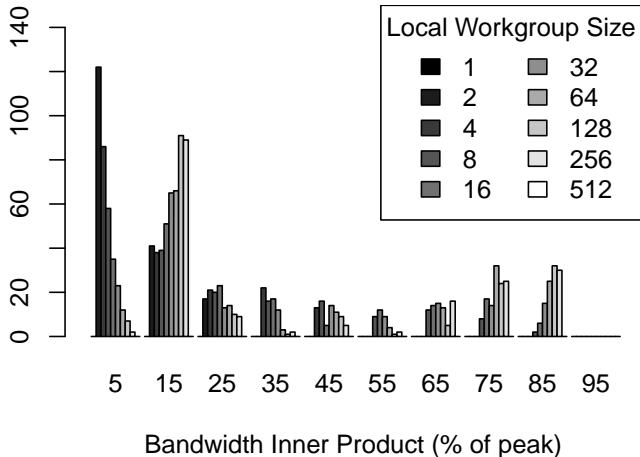
AMD Radeon HD 5850

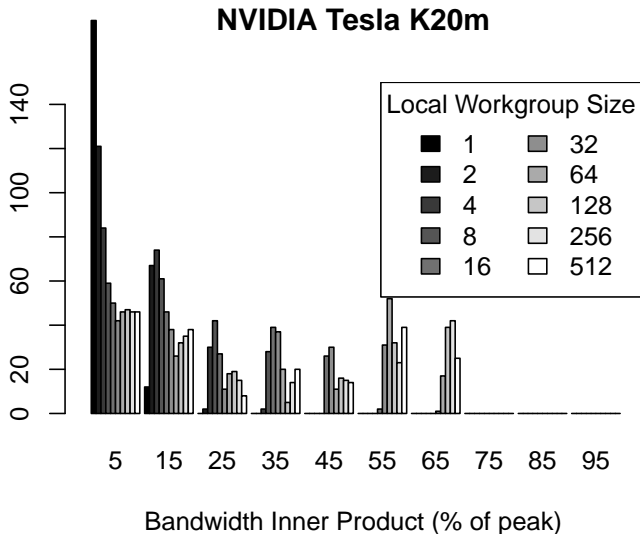


NVIDIA Tesla K20m

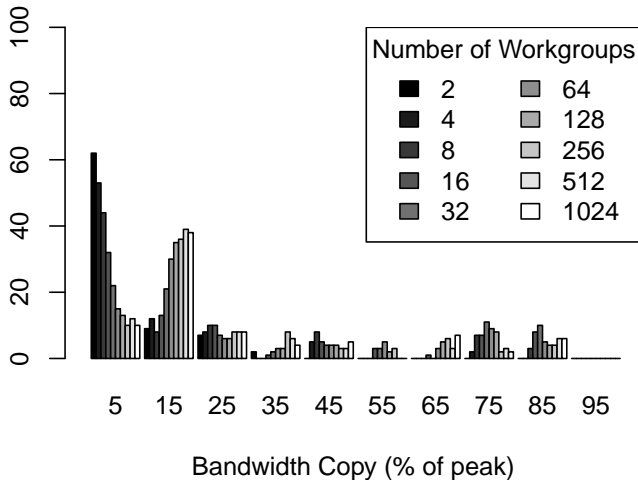


AMD Radeon HD 5850

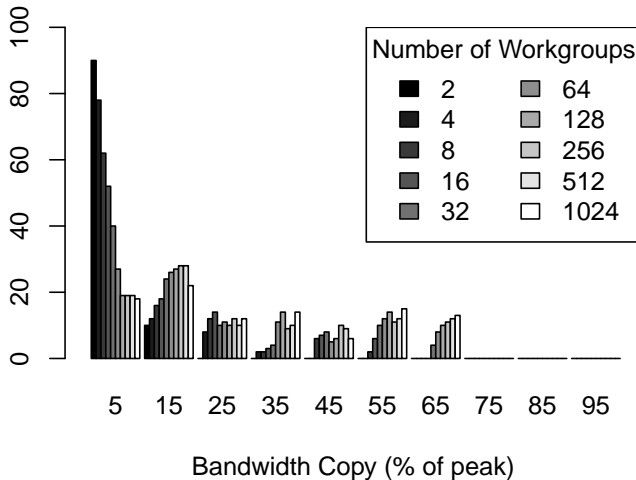




AMD Radeon HD 5850

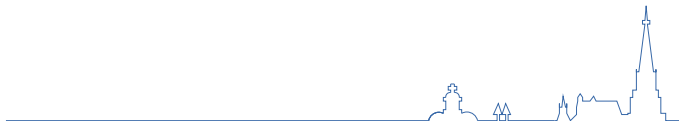


NVIDIA Tesla K20m

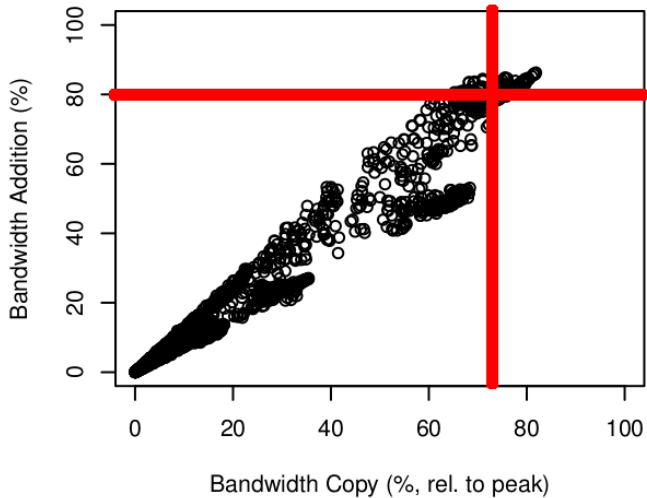


[Addition|Inner Product|Matrix-Vector] vs. Copy Kernel

Same Device

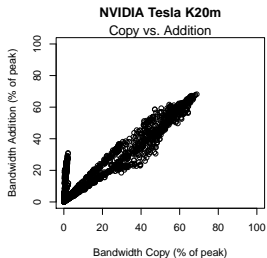


NVIDIA GeForce GTX 285

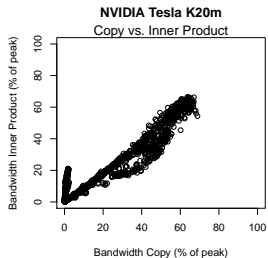


NVIDIA Tesla K20m

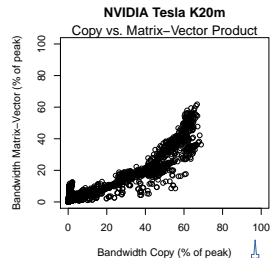
Addition



Inner Product

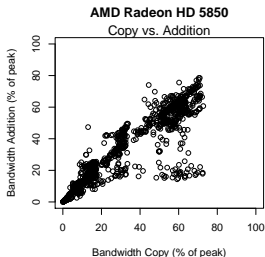


Mat-Vec Product

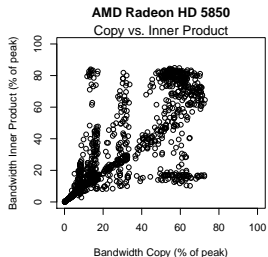


AMD Radeon HD 5850

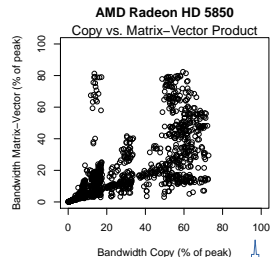
Addition



Inner Product

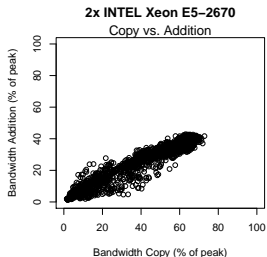


Mat-Vec Product

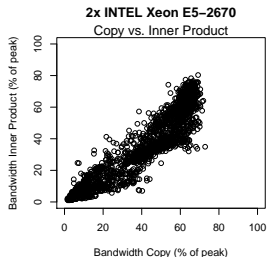


INTEL Dual Xeon E5-2670

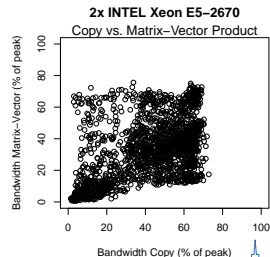
Addition



Inner Product

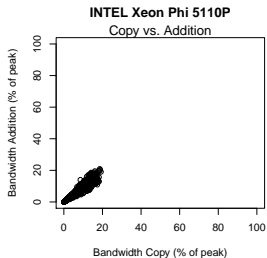


Mat-Vec Product

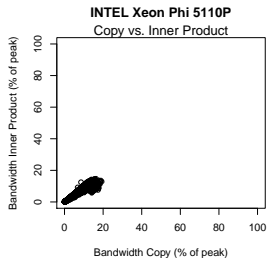


INTEL Xeon Phi

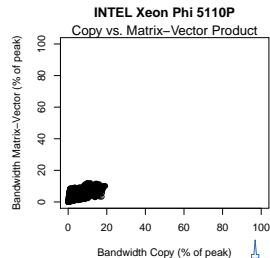
Addition



Inner Product



Mat-Vec Product



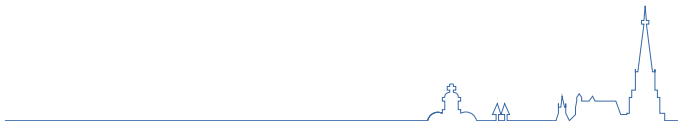
Conclusio:

Focus on fastest configurations for copy-kernel sufficient

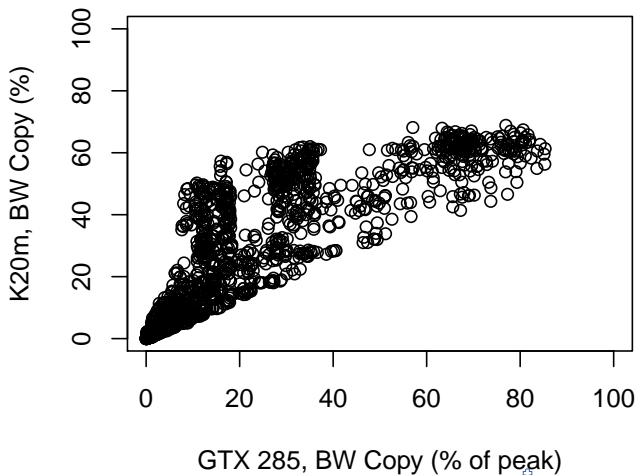


[Copy|Addition|Inner Product|Matrix-Vector] vs. Copy Kernel

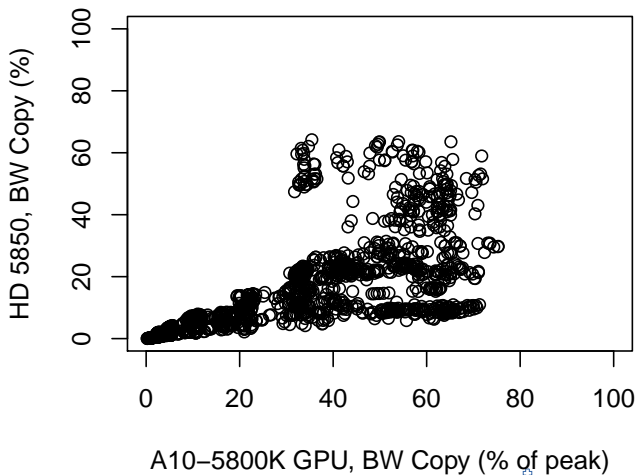
Different Device, Same Vendor



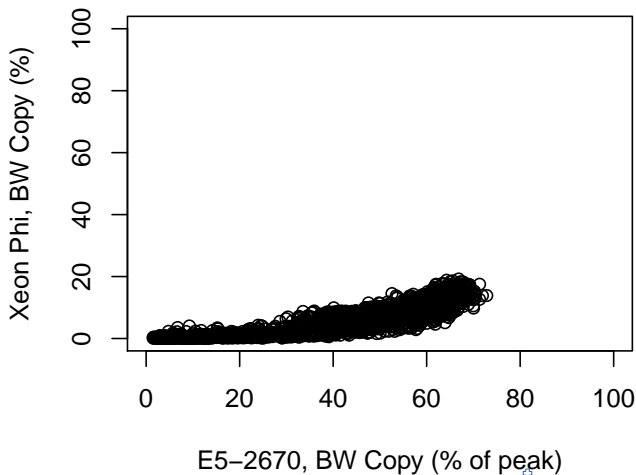
NVIDIA Hardware (x: GTX 285, y: K20m)



AMD Hardware (x: A10-5800K GPU, y: HD 5850)

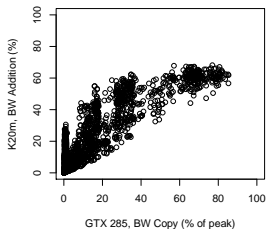


INTEL Hardware (x: Xeon Phi, E5-2670)

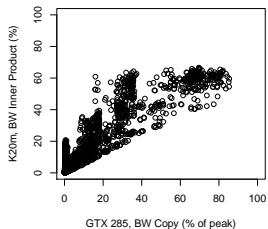


NVIDIA Hardware (x: GTX 285, y: K20m)

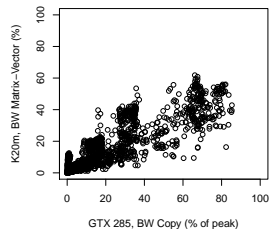
Addition



Inner Product

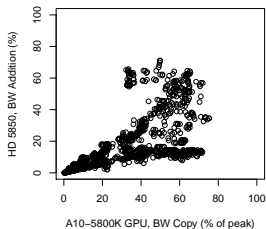


Mat-Vec Product

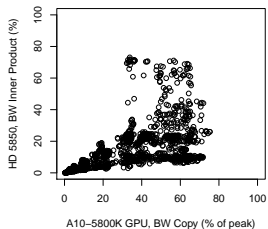


AMD Hardware (x: A10-5800K GPU, y: HD 5850)

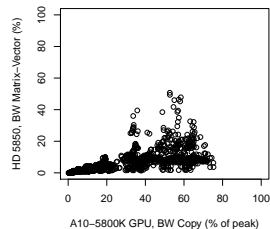
Addition



Inner Product



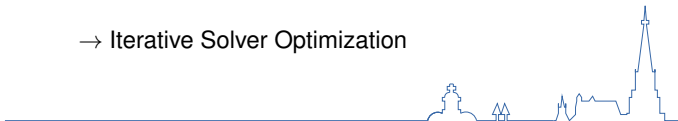
Mat-Vec Product



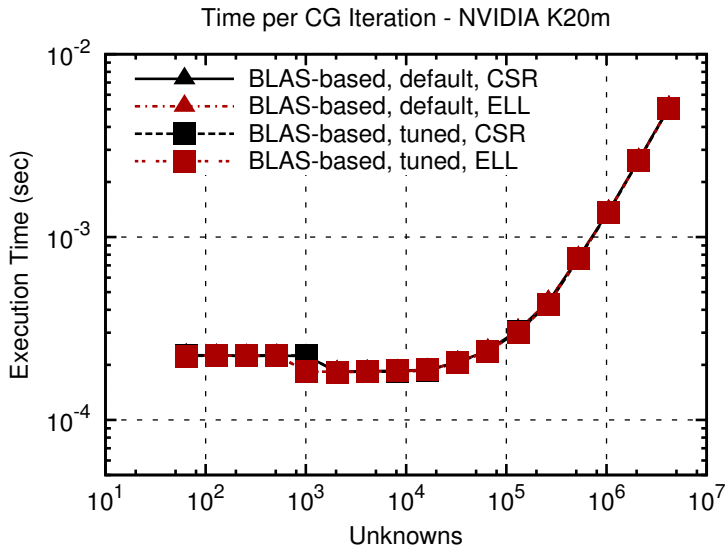
Conclusio:

Certain Performance Portability per Vendor

→ Iterative Solver Optimization

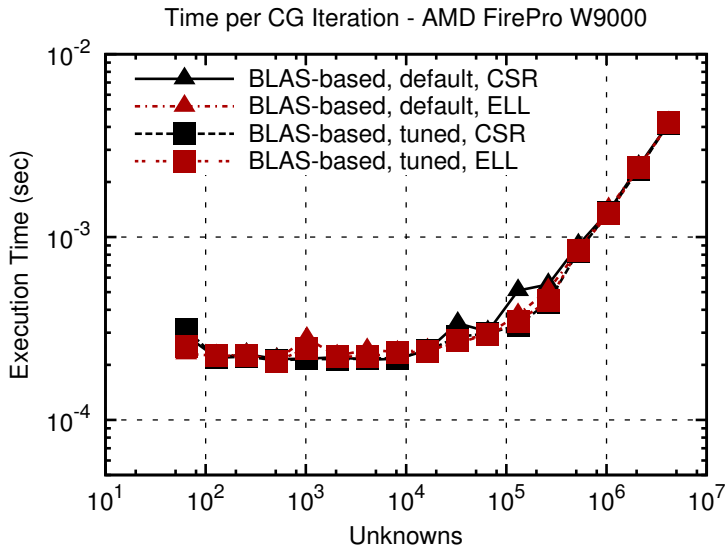


Conjugate Gradients



(2D Finite Difference Discretization)

Conjugate Gradients



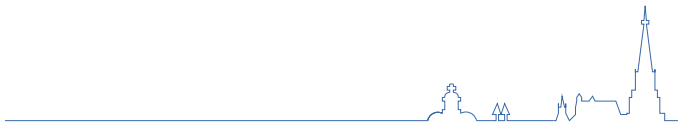
(2D Finite Difference Discretization)

Optimization 3: Rearrange the algorithm

- Remove unnecessary reads

- Remove unnecessary synchronizations

- Use custom kernels instead of standard BLAS



Standard CG

Choose x_0

$$p_0 = r_0 = b - Ax_0$$

For $i = 0$ until convergence

1. Compute and store Ap_i
2. Compute $\langle p_i, Ap_i \rangle$
3. $\alpha_i = \langle r_i, r_i \rangle / \langle p_i, Ap_i \rangle$
4. $x_{i+1} = x_i + \alpha_i p_i$
5. $r_{i+1} = r_i - \alpha_i Ap_i$
6. Compute $\langle r_{i+1}, r_{i+1} \rangle$
7. $\beta_i = \langle r_{i+1}, r_{i+1} \rangle / \langle r_i, r_i \rangle$
8. $p_{i+1} = r_{i+1} + \beta_i p_i$

EndFor

Pipelined CG

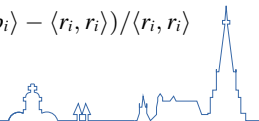
Choose x_0

$$p_0 = r_0 = b - Ax_0$$

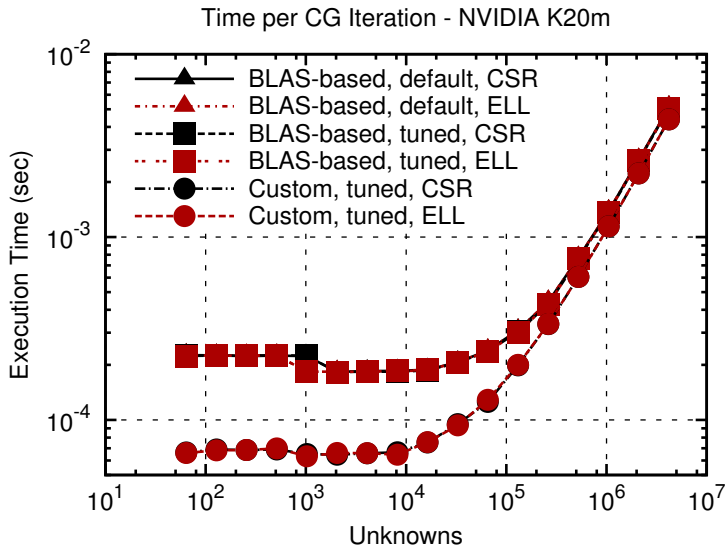
For $i = 1$ until convergence

1. $i = 1$: Compute α_0, β_0, Ap_0
2. $x_i = x_{i-1} + \alpha_{i-1} p_{i-1}$
3. $r_i = r_{i-1} - \alpha_{i-1} Ap_{i-1}$
4. $p_i = r_i + \beta_{i-1} p_{i-1}$
5. Compute and store Ap_i
6. Compute $\langle Ap_i, Ap_i \rangle, \langle p_i, Ap_i \rangle, \langle r_i, r_i \rangle$
7. $\alpha_i = \langle r_i, r_i \rangle / \langle p_i, Ap_i \rangle$
8. $\beta_i = (\alpha_i^2 \langle Ap_i, Ap_i \rangle - \langle r_i, r_i \rangle) / \langle r_i, r_i \rangle$

EndFor

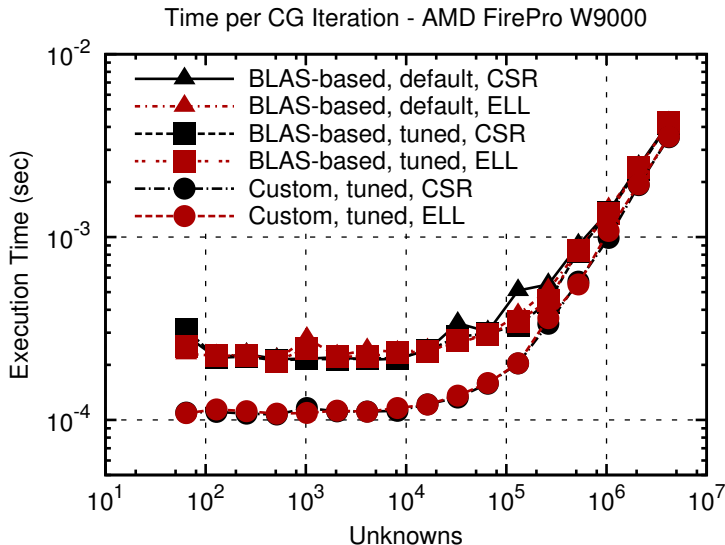


Conjugate Gradients



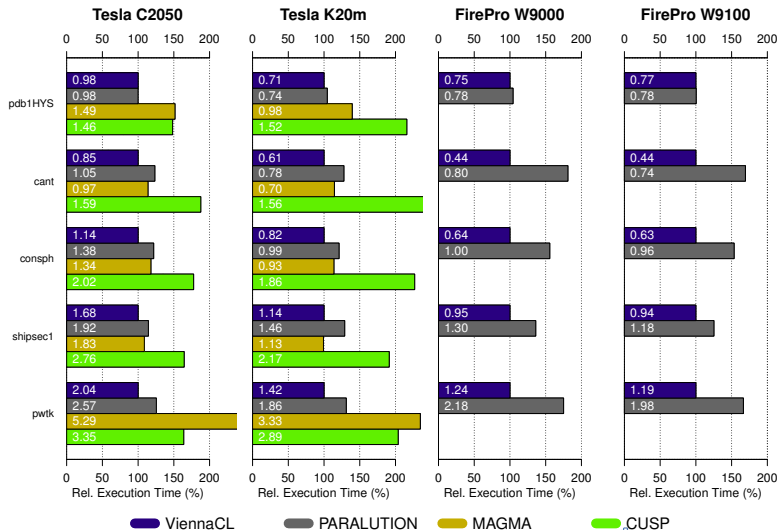
(2D Finite Difference Discretization)

Conjugate Gradients



(2D Finite Difference Discretization)

Benefits of Pipelining also for Large Matrices



Pick Proper Algorithms

- FLOPs are (almost always) for free
- Avoid unnecessary PCI-Express communication
- Expose fine-grained parallelism
- Pipelining and overlapping computations

Fuse Lightweight Kernels

- Reduced number of kernel launches
- Less PCI-Express traffic
- Case study: faster than BLAS-based implementations

Parameterize Kernels for Performance-Portability

- 128 work items, 128 work groups is a good starting point
- Vector datatypes (float4, etc.) often not necessary
- Let each workgroup operate on a contiguous piece of memory

