

For  $C_3$  and  $C_4$ , the carry generation, propagation and annihilation behavior of both cases are identical. Hence we only discuss  $C(\tau) = C_3$ . In this case, (??) is modified to give (1). We have  $Y_{[\tau+1]} = Y_{[\tau]}$  as the appending digit  $y_{\tau+\delta+1} = 0$ . Thus  $D$  only depends on the number of leading non-zero digits of  $Y_{[\tau]}$ .

$$P_{[\tau+1]} = r^{-\delta+1}(x_{\tau+\delta+1}Y_{[\tau+1]}) = r^{-\delta+1}(x_{\tau+\delta+1}Y_{[\tau]}) \quad (1)$$

For the first stage where  $\tau = -\delta$ , substituting this into (1) yields  $P_{[-\delta+1]} = r^{-\delta+1}(x_1y_1)$ . Therefore the value in (??) can only be obtained when  $C(-\delta) = C_2$ , otherwise  $d(-\delta) = 0$ . For the other stages where  $\tau > -\delta$ , there are 2 possible situations for  $\tau' = \tau - 1$  as stated below:

- If  $y_{\tau'+\delta+1} \neq 0$ , then  $D$  equals to the maximum word-length of  $Y_{[\tau'+1]}$ . Thus we have  $d(\tau) = d(\tau')_{max} - 1$ .
- If  $y_{\tau'+\delta+1} = 0$ , then  $Y_{[\tau']} = Y_{[\tau'-1]}$ . This is a recursive process and these two judgements will be performed again for  $\tau'' = \tau' - 1$ . It terminates when the first judgement is satisfied or .